AMPT at the High Baryon Density Region

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Outline

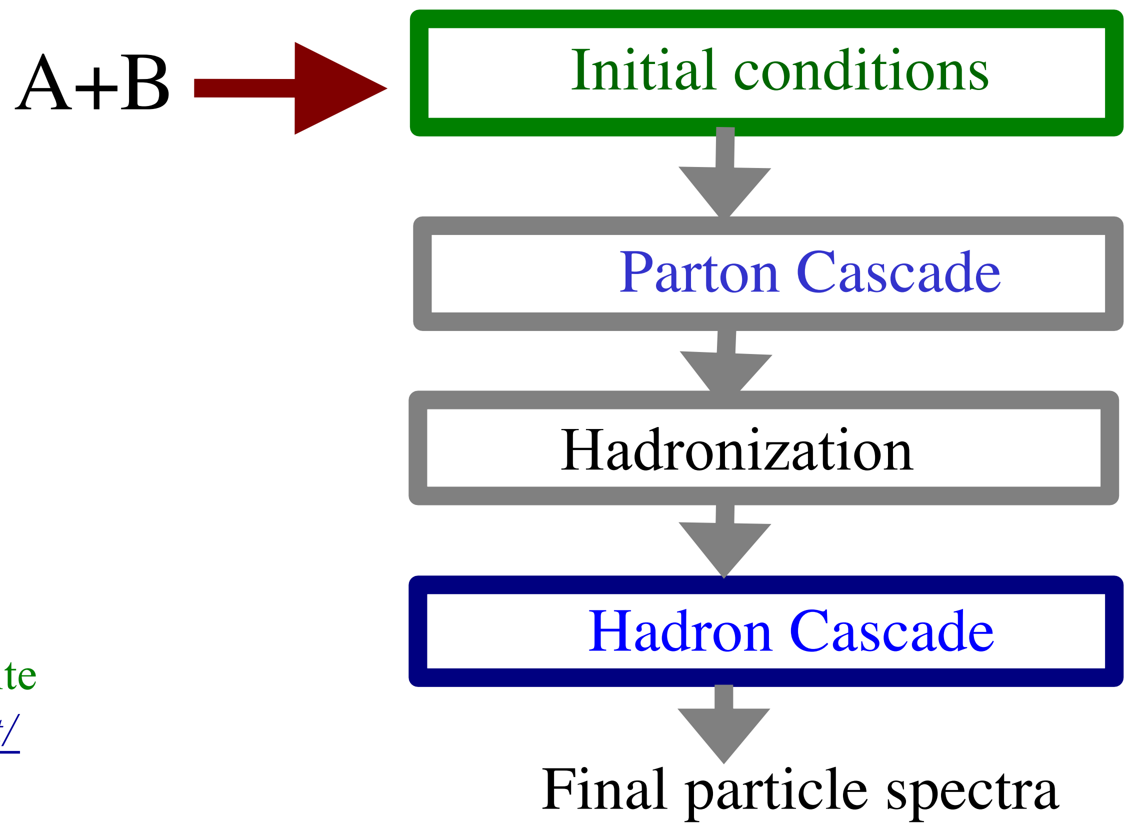
- Overview of the AMPT model
- Importance of nuclear thickness at lower energies
- Improved parton cascade and extraction of parton η/s
- Future developments for high baryon density physics
- Summary

Based on arXiv:1704.08418, 2001.10140, 2012.13825, 2102.06937, 2103.10815
(in collaboration with Todd Mendenhall, Xinli Zhao, Guo-Liang Ma, Yu-Gang Ma, Han-Sheng Wang, Wei-jie Fu, Chao Zhang, Liang Zheng, Shusu Shi)

A Multi-Phase Transport (AMPT)

was constructed as a self-contained kinetic description of heavy ion collisions:

- evolves the system from initial condition to final observables;
- particle productions of different flavours at different P_T & y;
- non-equilibrium initial condition & dynamics (e.g. fluctuations & correlations).



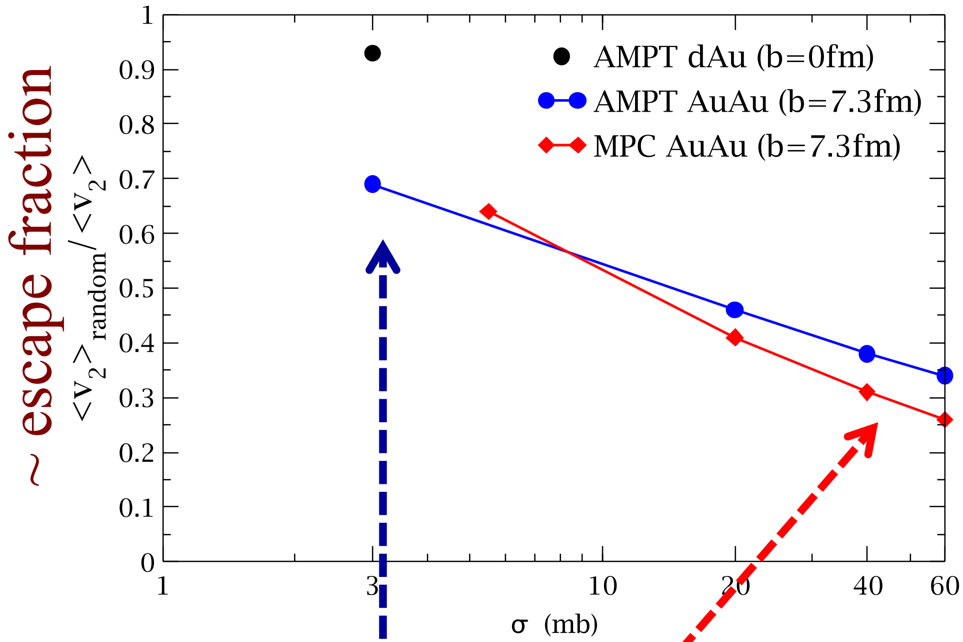
Source codes at the ECU website
http://myweb.ecu.edu/linz/ampt/

ZWL, Ko, Li, Zhang & Pal, PRC (2005);
ZWL & Zheng, NST (in press, 2021)

Transport versus hydrodynamics for finite systems

The escape mechanism:
interaction-induced response to anisotropic geometry from kinetic theory.

L He et al. PLB (2016),
ZWL et al. NPA (2016)



- Escape mechanism dominates v_2 for small systems & even semi-central AuAu@RHIC.
- At very large opacity (large system/energy/σ), hydrodynamic collective flow will dominate v_2

Heiselberg and Levy, PRC (1999), Borghini, Feld and Kersting, EPJC (2018),
Kurkela, Wiedemann and Wu, PLB (2018) & EPJC (2019)

- It is important to develop transport model/kinetic theory & compare with hydrodynamics to understand physics/collectivity of finite size systems.

- Transport model (non-equilibrium, microscopic picture) & hydrodynamics (EoS, transport coefficients) nicely complement each other.

String Melting version of AMPT (AMPT-SM)

From the Bjorken formula for initial energy density in central AA collisions:

$$\epsilon(\tau) = \frac{1}{\tau A_T} \frac{dE_T(\tau)}{dy}$$

~ 5	12	40	GeV/fm^3
SPS	**RHIC**	**LHC**	

τ: Proper formation time, taken as 0.5 *fm/c*

A_T: Transverse area

>> critical energy density for QCD phase transition:
$\varepsilon_c \sim O(1/2)$ GeV/fm^3

➔ At high-enough energies,

hadronic matter such as strings cannot exist at early times,

the initial matter should be represented by a high density partonic matter

➔ the string melting version of AMPT ZWL & Ko, PRC (2002)

AMPT codes are available online since 2004

myweb.ecu.edu/linz/ampt/

AMPT source codes

(updated December 25, 2018):

A Multi-Phase Transport (AMPT) model is a Monte Carlo transport model for nuclear collisions at relativistic energies.
Each of the following versions contains:
the source codes, an example input file, a Makefile, a readme, a required subdirectory for storing output files, and a script to run the code.

1. ampt-v1.11-v2.11.tgz (11/2004)
2. ampt-v1.21-v2.21.tgz (10/2008)
3. *Other older versions inbetween*
4. ampt-v1.26t5-v2.26t5.zip (4/2015)
5. ampt-v1.26t7-v2.26t7.zip (10/2016)
6. ampt-v1.26t7b-v2.26t7b.zip (5/2018)
7. ampt-v1.26t9-v2.26t9.zip (9/2018)
8. ampt-v1.26t9b-v2.26t9b.zip (12/2018)

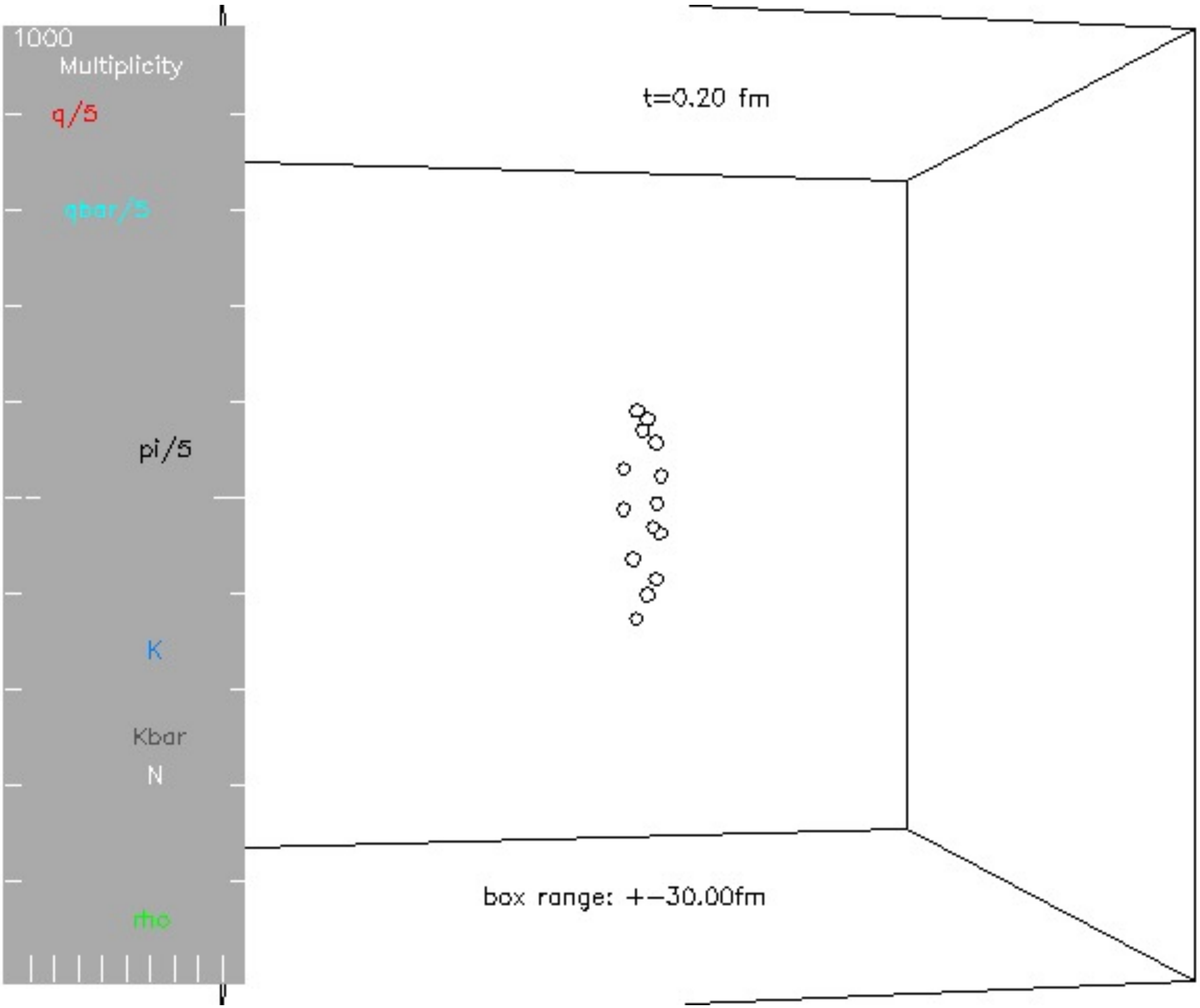
String Melting AMPT since 4/2015 can reasonably describe the bulk matter at high energies at RHIC and LHC.

This readme file lists the main changes up to version v1.26t9b-v2.26t9b ("t" means a version under test):

```
AMPT Users' Guide

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
12/2018 test version v1.26t9b/v2.26t9b:
     * Fixed bugs that can cause segmentation fault (especially for default
       AMPT at high energies):
       exclude endpoints of 0. and 1. in random values from RANART()
         in amptsub.f in order to avoid crash in case of 0 branching ratio,
       resample random value from RLU(0) when TEVB in PYSSPA does not change
         (this may happen when RLU(0)>~0.9999 and thus
         ex==|LOG(RLU(0))*B0/(5.*WTSUM)|<~3e-8 and thus EXP(ex)=1.00000000,
         which may cause IS(2) to be undefined and be >10^8 (out of bound)).
   The above modification can be found by searching "clin-12/2018".
```

A central Au+Au event at 200AGeV from the AMPT-SM



60fm-long box

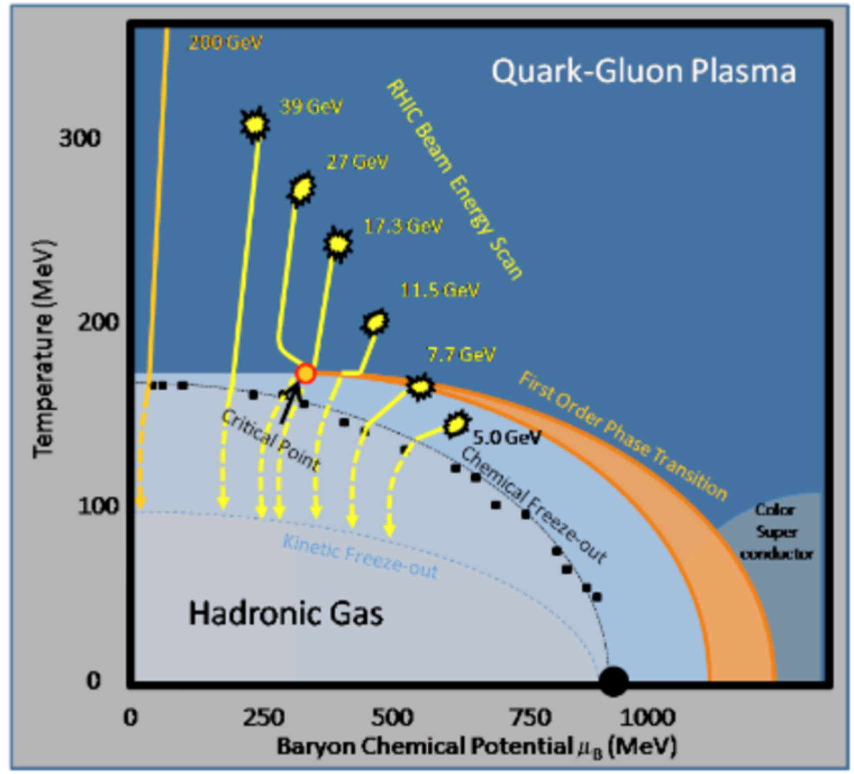
Side view:

Beam axes

Higher baryon densities / lower energies

- For lower energies such as BES, particular interests are in high baryon density physics including the QCD critical end point (CEP).

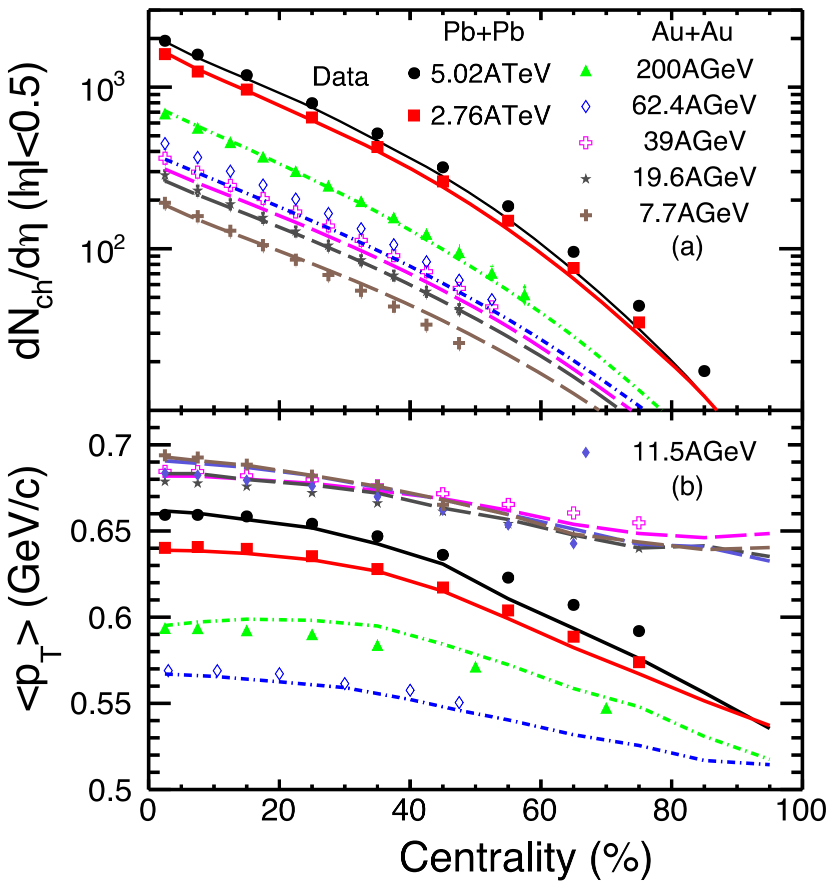
- Before addressing possible effects of the critical point,
  one needs to know trajectory
  of nuclear collisions on
  the QCD phase diagram,
  including time evolutions
  of energy density ε *&*
  net-baryon density n_B
  (or T *&* μ_B*)*

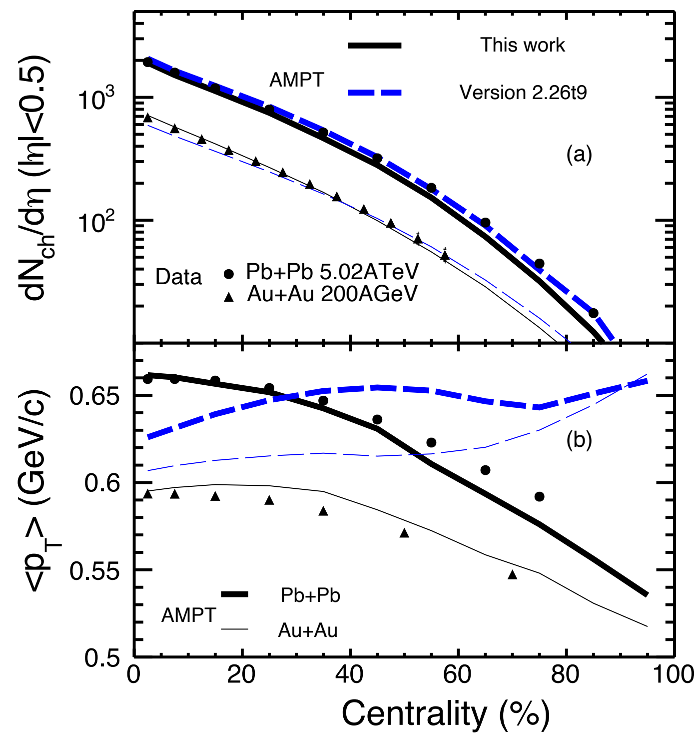


from STAR arXiv:1007.2613

Results from AMPT-SM at different energies

String melting AMPT can now reasonably describe both large and small systems, including their centrality dependence after we apply local nuclear scaling on 2 key input parameters (*Lund b parameter & minijet pT cutoff p_0*):
Chao Zhang et al. 2103.10815





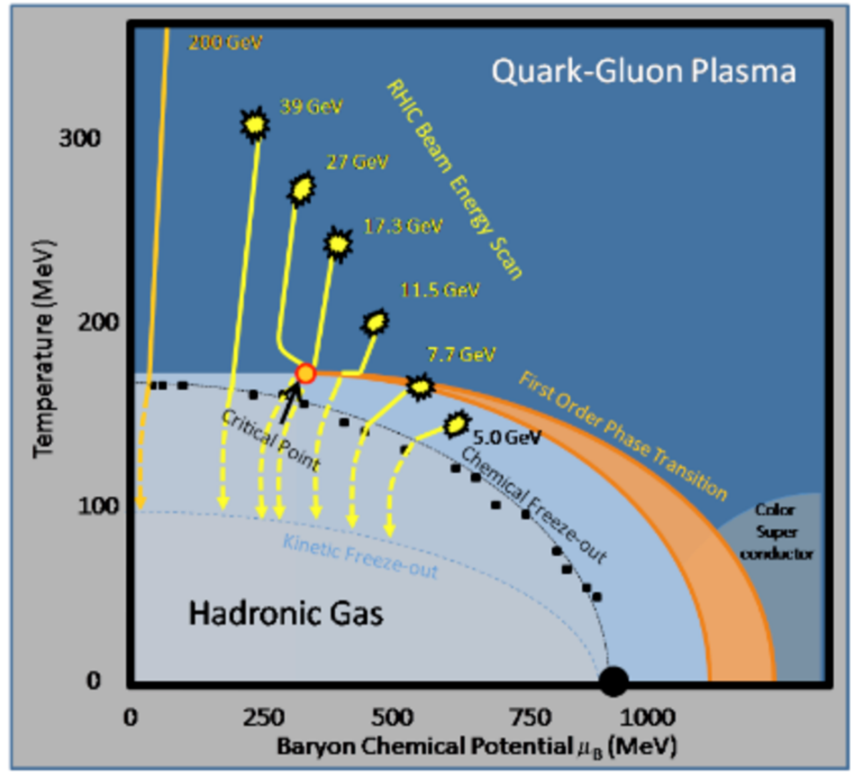
Centrality dependence of <pT> is now much better than public AMPT

However, finite nuclear thickness is neglected in AMPT-SM as it was constructed for high energy nuclear collisions.

Importance of nuclear thickness at lower energies

- Effects of nuclear thickness at low energies are big on *energy density* ε *& net-baryon density* n_B *(consequently on* T *&* μ_B*)*

  ZWL, 1704.08418,
  Mendenhall & ZWL, 2012.13825,
  H.S. Wang et al. 2102.06937



from STAR arXiv:1007.2613

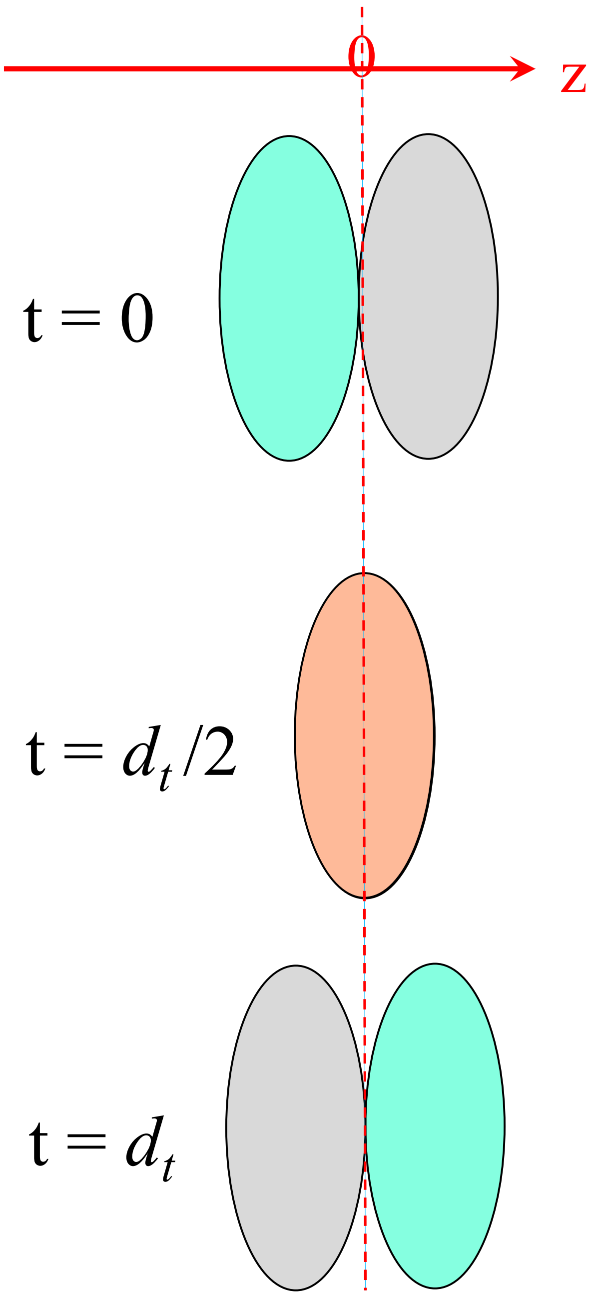
- For hydrodynamics models: dynamical initialization scheme is needed instead of initial condition at a fixed proper time.

  Okai et al. 1702.07541,
  Shen et al. 1704.04109

Effect of nuclear thickness on initial energy density

For central A+A collisions in CM frame
with the hard sphere model for nucleus:
crossing time is

$$d_t = \frac{2R_A}{\sinh y_{CM}} = \frac{2R_A}{\gamma\,\beta}$$



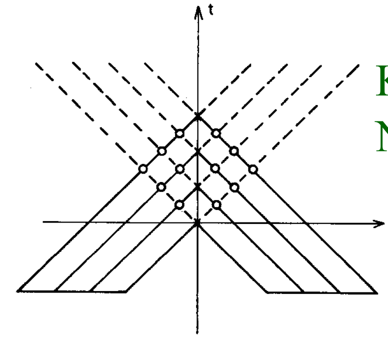
For central Au+Au collisions:

$\sqrt{s_{NN}}$ (GeV)	3	5	11.5	27	50	200
$d_t\ (fm/c)$	10.5	5.3	2.2	0.91	0.49	0.12

→ the Bjorken formula $\epsilon(\tau) = \frac{1}{\tau\,A_T}\frac{dE_T(\tau)}{dy}$

is only valid when $d_t \ll \tau_F$
or $\sqrt{s_{NN}} > \sim 50$ GeV *for* $\tau_F = 0.5\ fm/c$

Effect of nuclear thickness on initial energy density



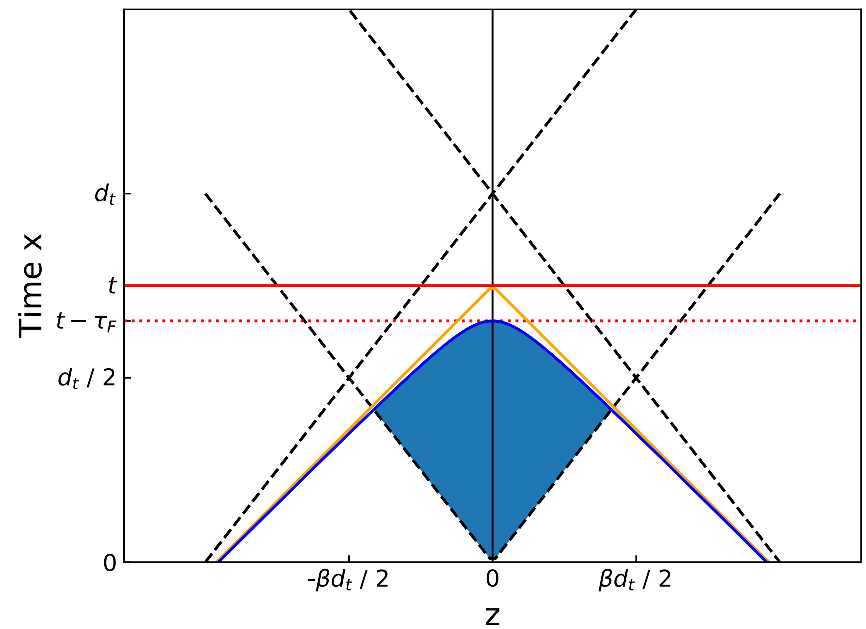
Kajantie et al.
NPB (1983)

Fig. 5. An alternative description of the A + A collision. In addition to the pairwise N + N collisions on the time axis (crosses), the secondaries may further interact with the incoming nucleons (circles). This would enhance the energy density in the central region.

A schematic picture:

The shaded area is the primary collision region that can contribute to ***ε(t)***, after considering formation time $t_F = \tau_F \cosh(y)$.

At late t ($> d_t + \tau_F$), ***ε(t)*** comes from the full primary collision region (the big diamond area).

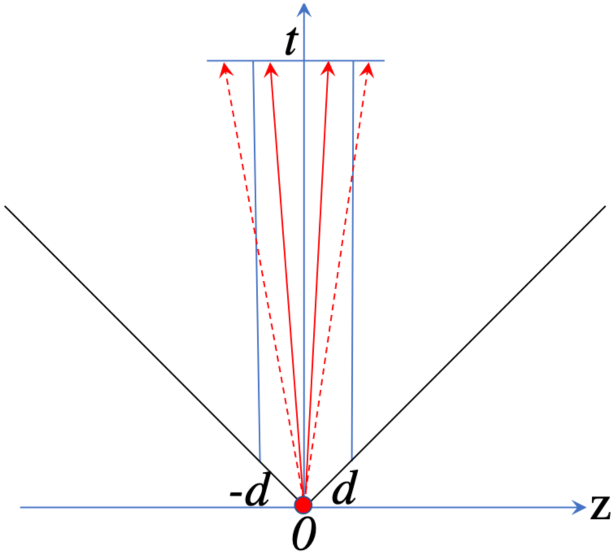


Mendenhall & ZWL 2012.13825

x: *production time*, $\in [0, d_t]$

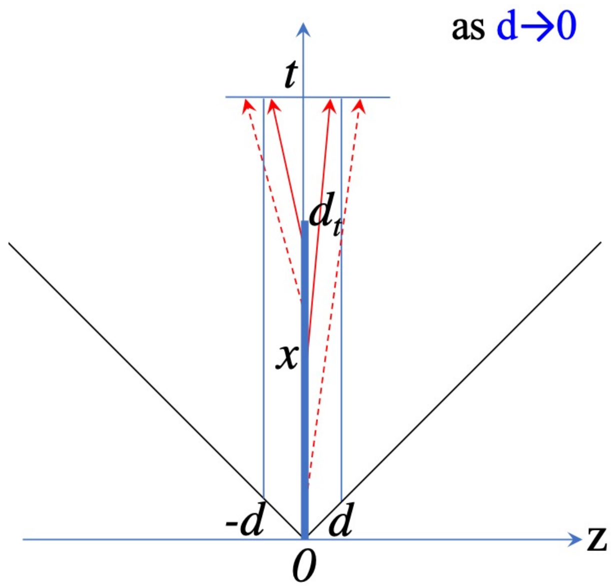
In these semi-analytical studies,
we only consider the central region ($\eta_s \sim 0$)
& neglect secondary scatterings or transverse expansion.

Extension of Bjorken ε formula with nuclear thickness: 2)



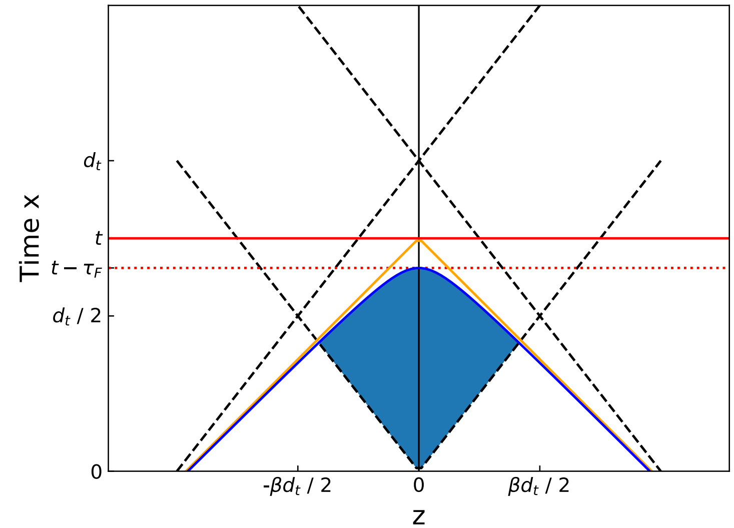
1) Without finite t or z: the Bjorken ε formula

Bjorken, PRD (1983)



2) With finite t *(but not finite z-width)*

ZWL 1704.08418



3) With both finite t & z

Mendenhall & ZWL 2012.13825

We first use this simpler method to illustrate the qualitative effect of nuclear thickness on initial ***ε(t)***

Extension of the Bjorken ε formula: the uniform time profile

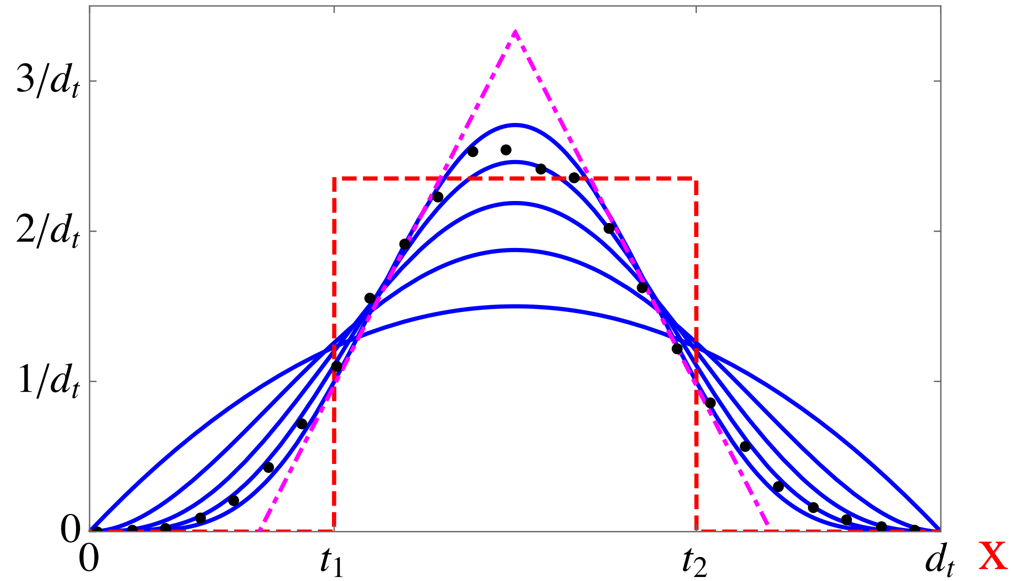
$$\varepsilon(t) = \frac{1}{A_T}\int_0^{t-\tau_F} \frac{dx}{(t-x)} \frac{d^2E_T}{dy_0 dx}$$

ZWL 1704.08418

For the simplest uniform profile, initial energy (at $\eta_s \sim y_0 \sim 0$) is produced uniformly from time $\boldsymbol{t_1}$ to $\boldsymbol{t_2}$:

$$\frac{d^2E_T}{dy_0 dx} = \frac{1}{t_{21}} \frac{dE_T}{dy_0}$$

for $x \in [t_1, t_2]$,
with $t_{21} \equiv t_2 - t_1$

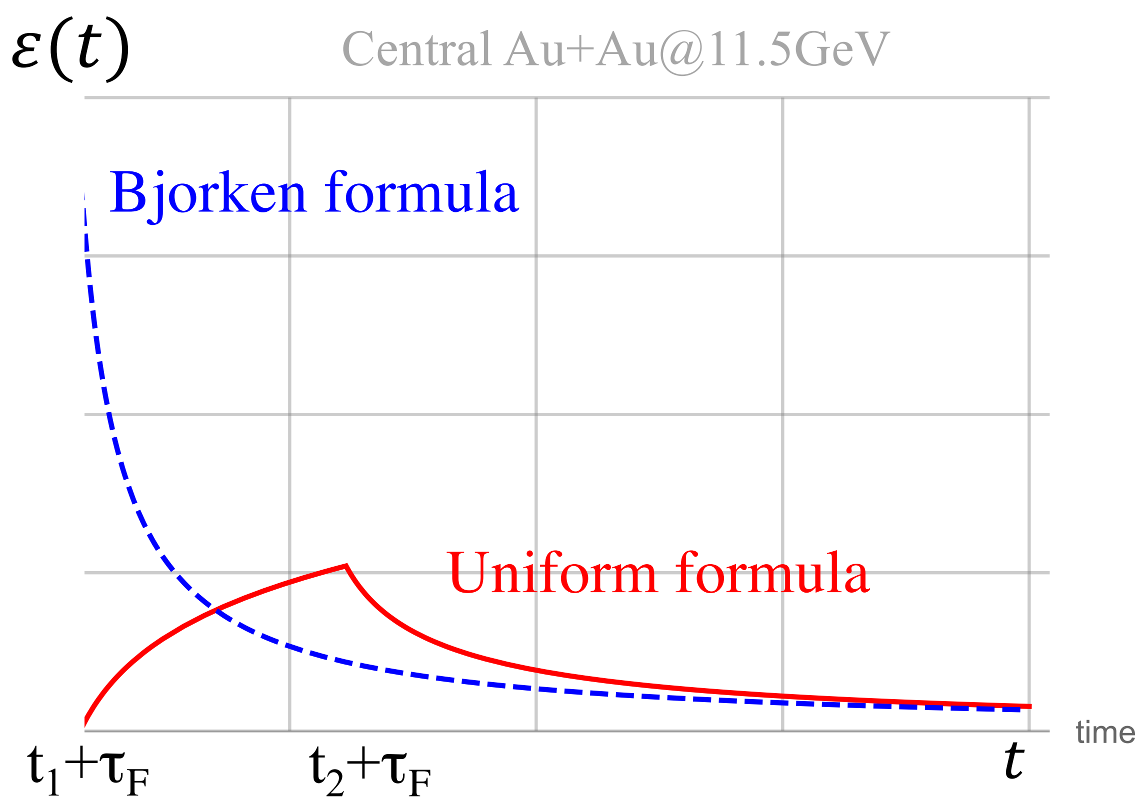


Circles: AMPT results

$dE_T/dy(y=0)$ parameterization from PHENIX PRC (2005)

Extension of the Bjorken ε formula: the uniform time profile

→ solution:
$$\epsilon_{\rm uni}(t) = \frac{1}{A_{\rm T} t_{21}} \frac{dE_{\rm T}}{dy} \ln\left(\frac{t-t_1}{\tau_{\rm F}}\right), \text{if } t \in [t_1+\tau_{\rm F}, t_2+\tau_{\rm F}];$$
$$= \frac{1}{A_{\rm T} t_{21}} \frac{dE_{\rm T}}{dy} \ln\left(\frac{t-t_1}{t-t_2}\right), \text{if } t \geq t_2+\tau_{\rm F}.$$



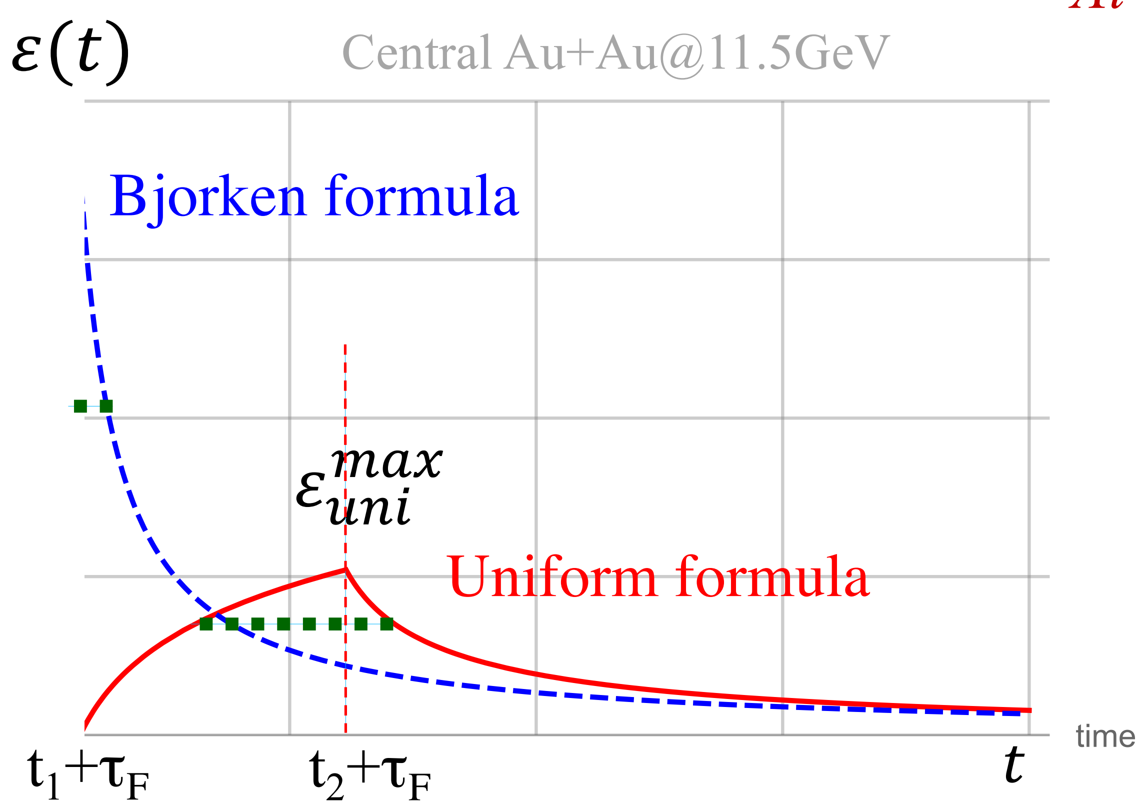
- *At high energies* (thin nuclei, or $t_{21}/\tau_F \to 0$): $\varepsilon_{uni}(t) \to \varepsilon_{Bj}(t)$ *analytically.*

- *At lower energies:* very different from Bjorken.

Extension of the Bjorken ε formula: the uniform time profile

Peak energy density $\epsilon_{\rm uni}^{max} = \epsilon_{\rm uni}(t_2 + \tau_{\rm F}) = \frac{1}{A_{\rm T} t_{21}} \frac{dE_{\rm T}}{dy} \ln\left(1 + \frac{t_{21}}{\tau_{\rm F}}\right)$

→ ratio over Bjorken: $\frac{\epsilon_{\rm uni}^{max}}{\epsilon_{\rm Bj}(\tau_{\rm F})} = \frac{\tau_{\rm F}}{t_{21}} \ln\left(1 + \frac{t_{21}}{\tau_{\rm F}}\right)$. $\lesssim 1$ always.



At very low energies (t_{21}/τ_F >>1):
ratio over Bjorken → 0;

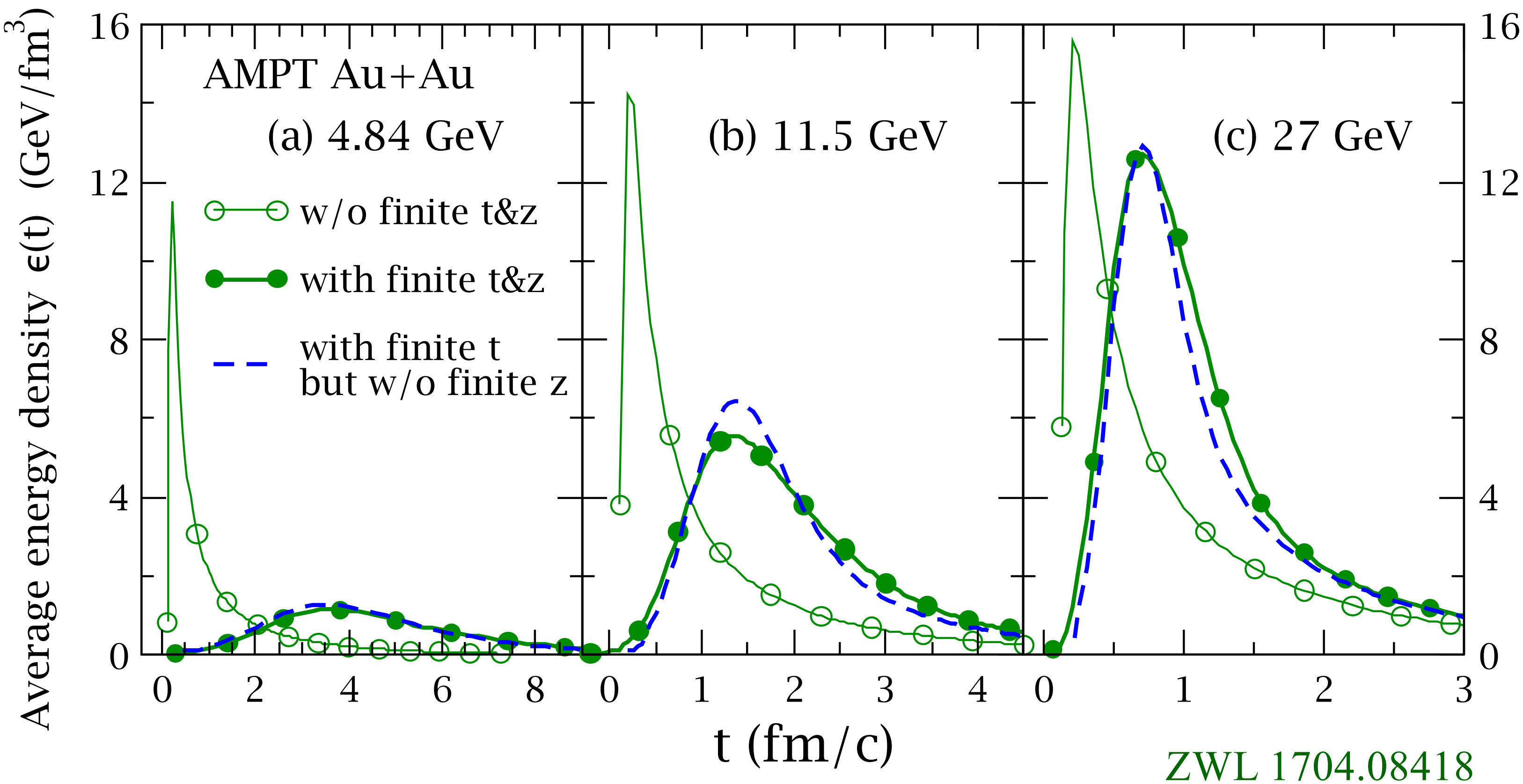
$\varepsilon_{uni}^{max} \propto \ln\left(\frac{1}{\tau_F}\right)$, not $\frac{1}{\tau_F}$.

So the peak energy density

- << Bjorken value
- much less sensitive to τ_F

- FWHM width in t >> Bjorken

Nuclear thickness effects from AMPT-SM



ZWL 1704.08418

We incorporated nuclear thickness in AMPT initial condition in a test version; the results show:

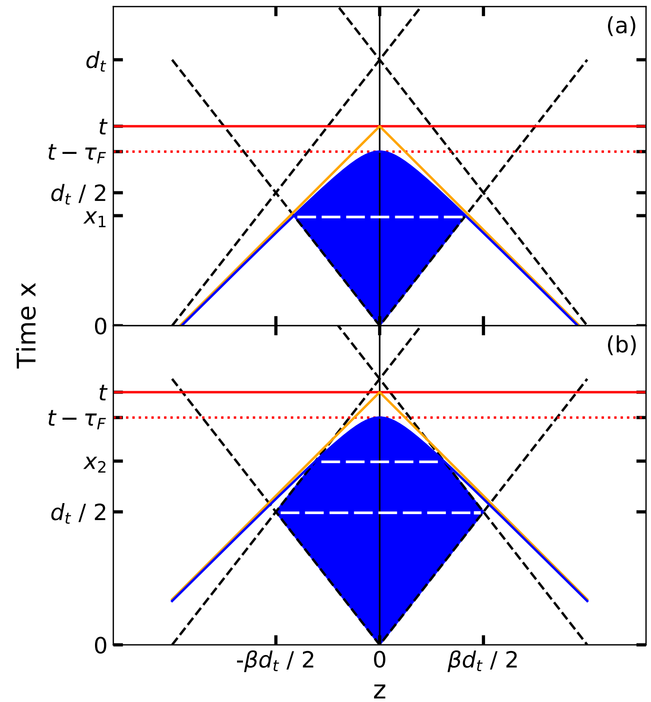
- Same qualitative features as our semi-analytical studies.
- Effect of nuclear thickness could be very important at low/BES energies.
- Peak energy density ε^{max} increases with $\sqrt{s_{NN}}$ much faster than Bjorken.

Extension of Bjorken ε formula with nuclear thickness: 3)

3) With both finite t & z

Mendenhall & ZWL 2012.13825

$$\varepsilon(t) = \frac{1}{A_T} \int\int_S \frac{dxdz}{(t-x)} \frac{d^3 m_T}{dy_0\, dxdz} ch^3 y_0$$



S: *integration area (shaded), has 2 or 3 pieces depending on **t**:*

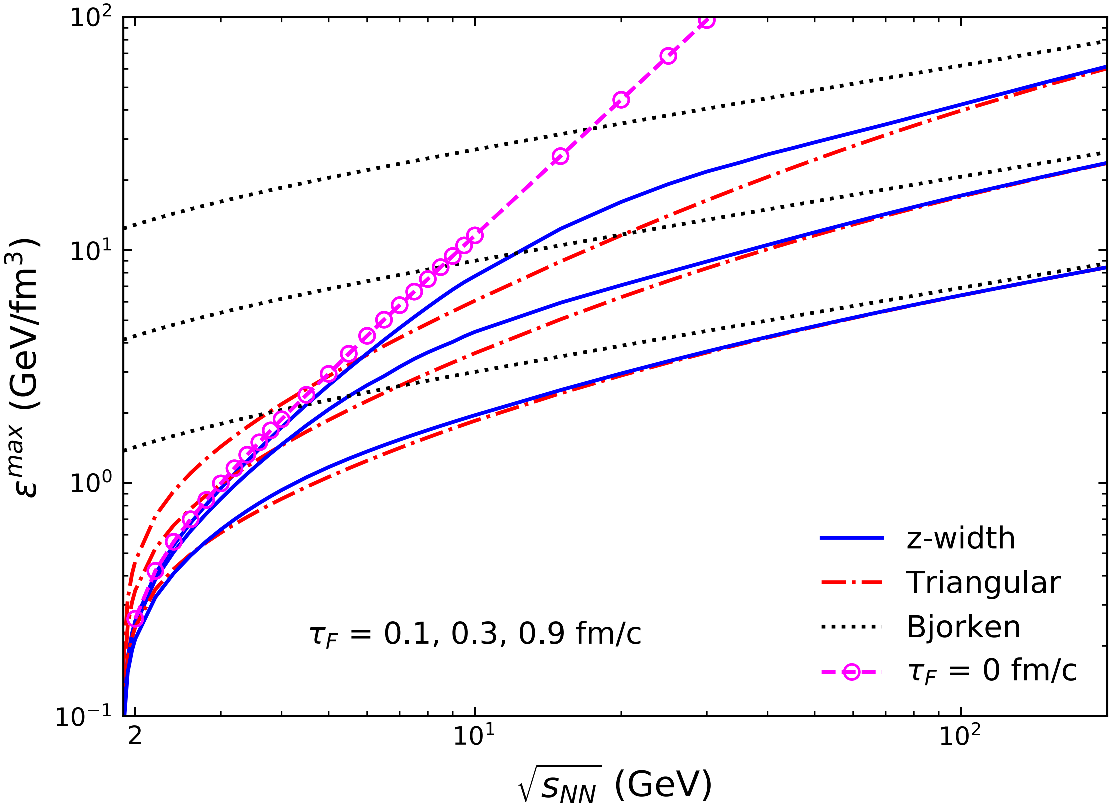
Depends on $\frac{d^3 m_T}{dy_0\, dxdz}$: initial dm_T/dy production density in x-z plane:
assumed to be uniform in x-z plane
(similar to triangular time profile of earlier study ZWL 1704.08418*).*

dm_T/dy is based on improving $dE_T/dy(y=0)$ parameterization from PHENIX PRC (2005)
by including net-baryon contribution (*important at low energies*).

Extension of Bjorken ε formula with nuclear thickness: 3)

3) With both finite t & z

Mendenhall & ZWL 2012.13825



- Qualitatively similar to earlier study ZWL 1704.08418

  ε^{max} << Bjorken value at low energies, ≈Bjorken value at high energies;

  ε^{max} & ***ε(t)*** depend on τ_F more weakly than Bjorken at lower energies.

- **Surprise finding:** ε^{max} is **finite** at $\tau_F = 0$ at any colliding energy.

Extension of Bjorken ε formula with nuclear thickness: 3)

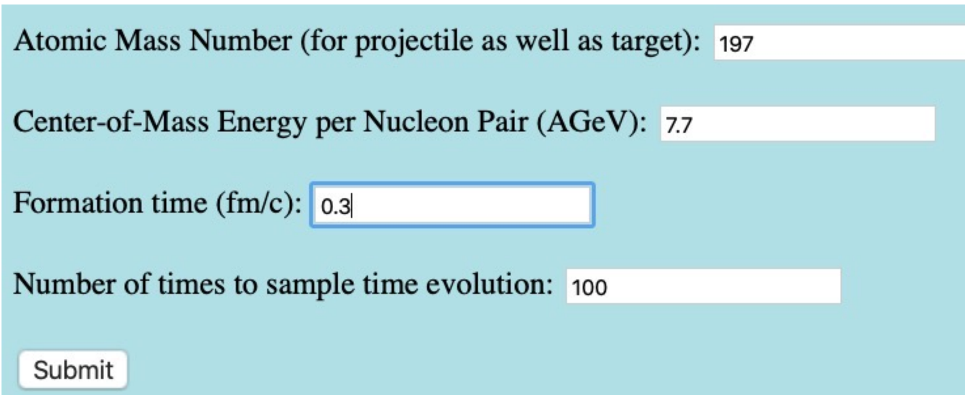
3) With both finite t & z

Mendenhall & ZWL 2012.13825

Todd Mendenhall has written a web interface
to calculate *the initial energy density* ***ε(t)***

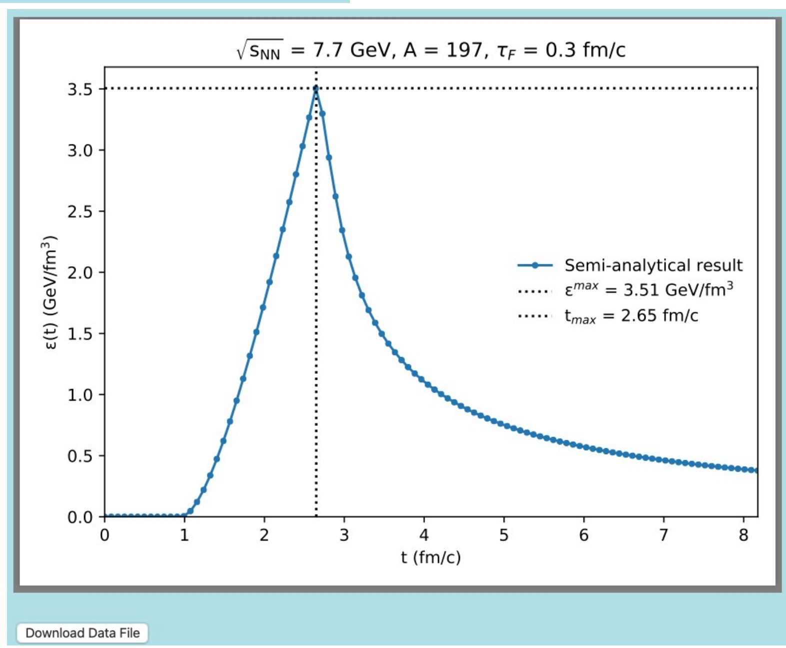
- Can be accessed at bottom of the AMPT webpage

- Takes input from user:

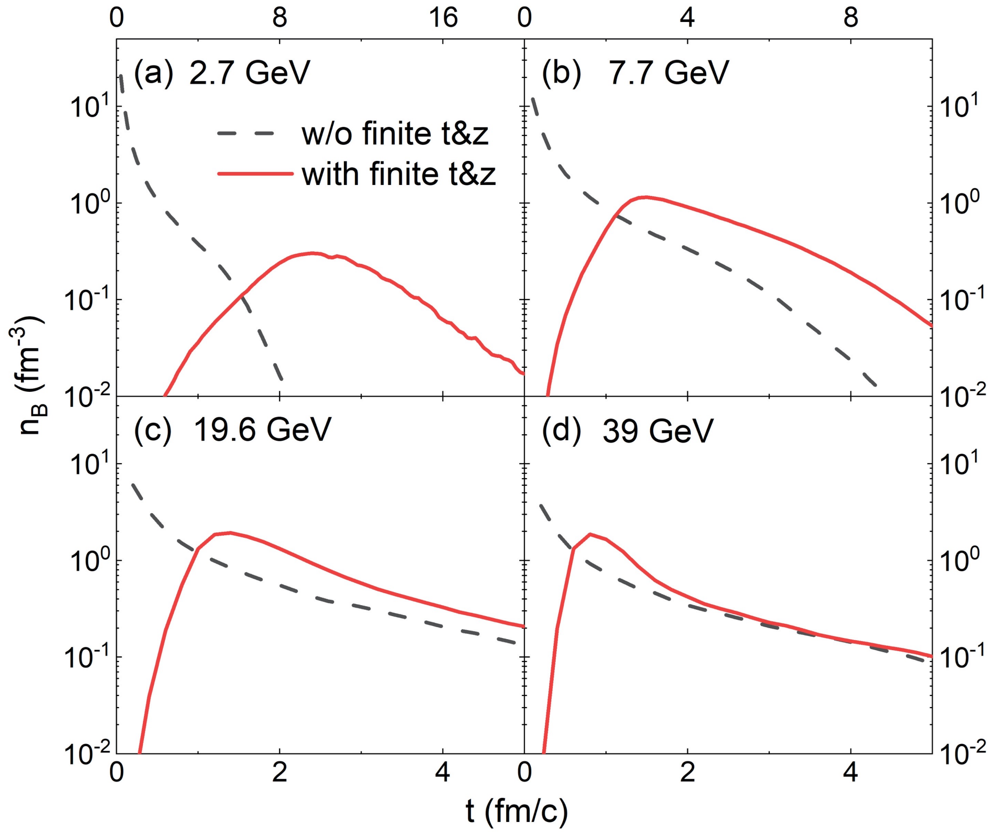


- Outputs ***ε(t)*** plot,

user can download data file:



Nuclear thickness effects from AMPT-SM on μ_B

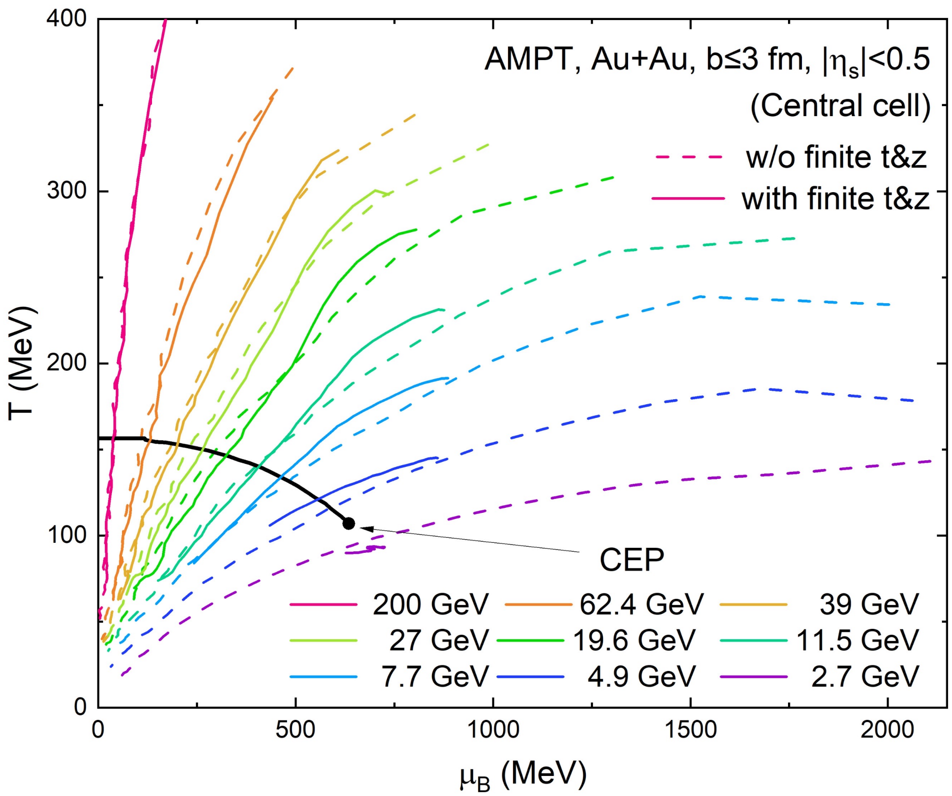


Han-Sheng Wang et al.
2102.06937,
preliminary

- Qualitatively similar to ***ε(t)*** results ZWL 1704.08418, Mendenhall & ZWL 2012.13825:

  $n_B{}^{max}$ decreases drastically at very low energies (after including thickness); smaller change at higher energies.

Nuclear thickness effects from AMPT-SM on T-μ_B



Han-Sheng Wang et al.
2102.06937,
preliminary

- Trajectory is plotted from t^{max} (time of reaching ε^{max}) to t=10 fm/c.
- T & μ_B both decrease significantly at very low energies (after including thickness); little change at high energies.

Improved parton cascade and extraction of parton η/s

Flows like v_2 & v_3 of large systems
mostly come from parton cascade in AMPT.

But ZPC/MPC cascade solution of the Boltzmann equation
is well known to suffer from causality violation.

Parton subdivision can resolve this problem:
but is very CPU-consuming
& alters e-by-e fluctuation/correlation.

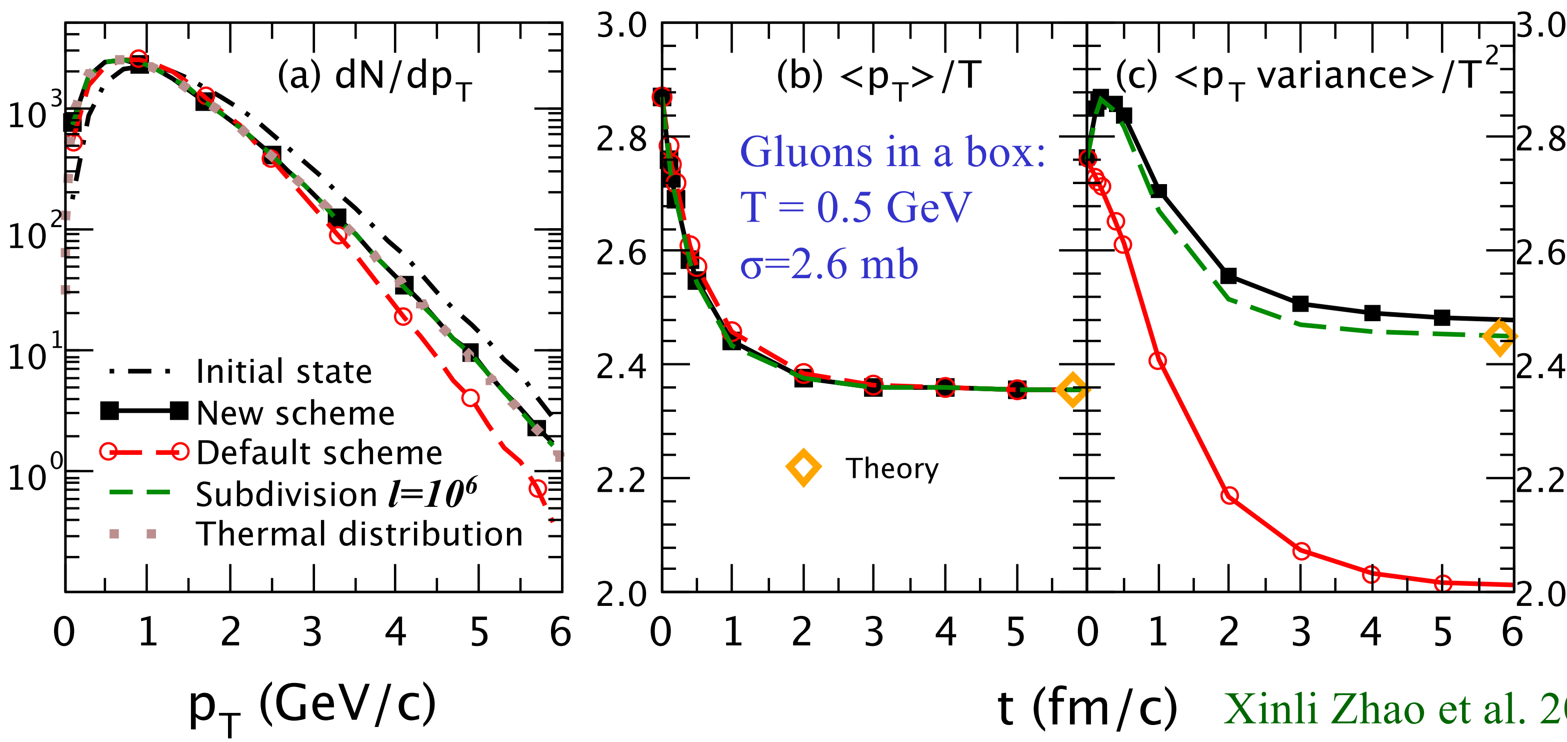
We study
→ how accurate ZPC is under expected densities/opacities
→ how to accurately solve Boltzmann equation w/o subdivision
→ extract parton η *or* η/s

Xinli Zhao et al. 2001.10140, Mendenhall & ZWL (ongoing)

Improved parton cascade and extraction of parton η/s

ZPC in AMPT numerically solves the Boltzmann equation for 2-body collisions:

$$p^{\mu}\partial_{\mu}f(\boldsymbol{r},\boldsymbol{p},t)=\mathcal{C}[|\mathcal{M}|^2 f_1(\boldsymbol{r}_1,\boldsymbol{p}_1,t)f_2(\boldsymbol{r}_2,\boldsymbol{p}_2,t)]$$



Xinli Zhao et al. 2001.10140

We have found a new scheme to perform parton collisions:

it almost eliminates causality violation

& is more accurate than default ZPC collision scheme

Improved parton cascade and extraction of parton η/s

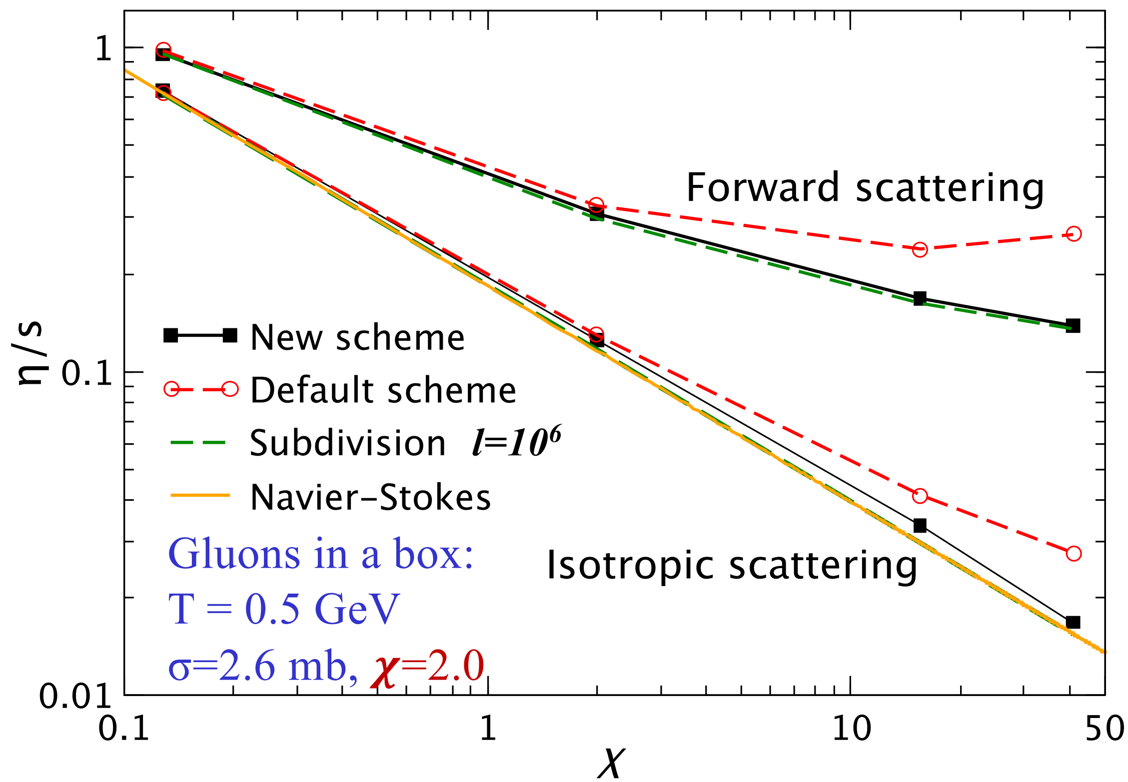
We calculate η with the Green-Kubo relation:

$$\eta = \frac{V}{T}\int_0^\infty dt\, \langle \bar{\pi}^{xy}(t+t')\,\bar{\pi}^{xy}(t')\rangle$$

Xinli Zhao et al. 2001.10140

New scheme
≈ Subdivision results
= Navier-Stokes values
(for isotropic scatterings):

$$\left(\frac{\eta}{s}\right)^{NS} \approx \frac{0.1839}{\chi^{2/3}}$$



Opacity parameter $\chi = \sqrt{\frac{\sigma}{\pi}}/\lambda_{mfp} = n\sqrt{\frac{\sigma^3}{\pi}}$

Bin Zhang, Gyulassy and Pang, PRC (1998)

Next step: extract parton η/s of a 3-d expanding system

Future developments for high baryon density physics

Above developments of the AMPT model
lay a good foundation for studies of nuclear collisions
at low energies /high baryon densities.

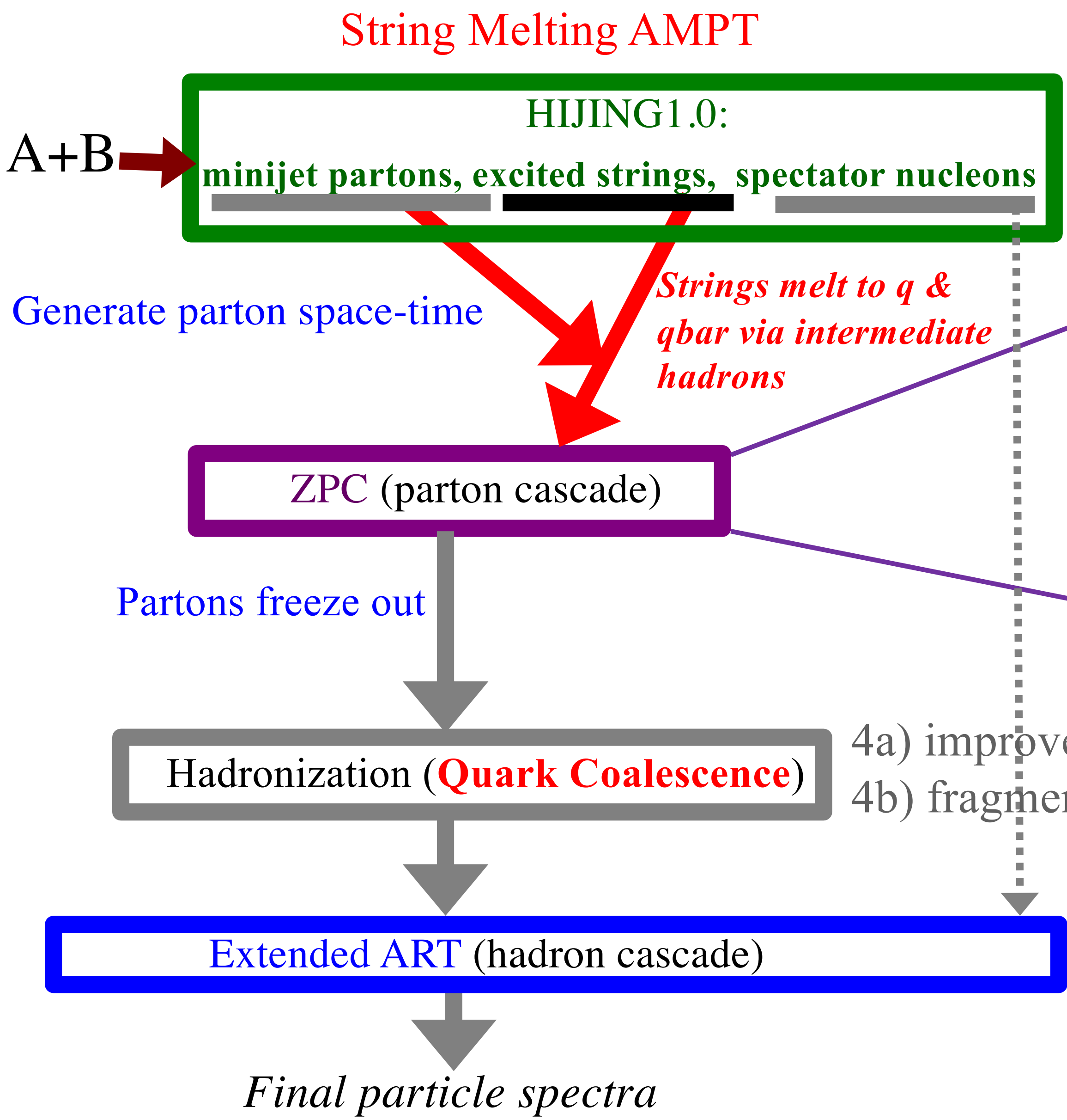
Still, much important work needs to be done:

1) realistic Equation of State of the dense matter (*including QCD critical end point*)
2) inelastic parton reactions of different flavors (*QGP chemical composition*)
3) link parton cross sections with η/s and other transport coefficients
4) hadronization (*parton recombination/quark coalescence/fragmentation)*
5) potentials (*partonic and hadronic*)

+more

better hadron cascade
kinetics of spin polarizations
transport under electromagnetic fields
better model maintenance & integration to experiments

Future developments for high baryon density physics



Important work to be done:

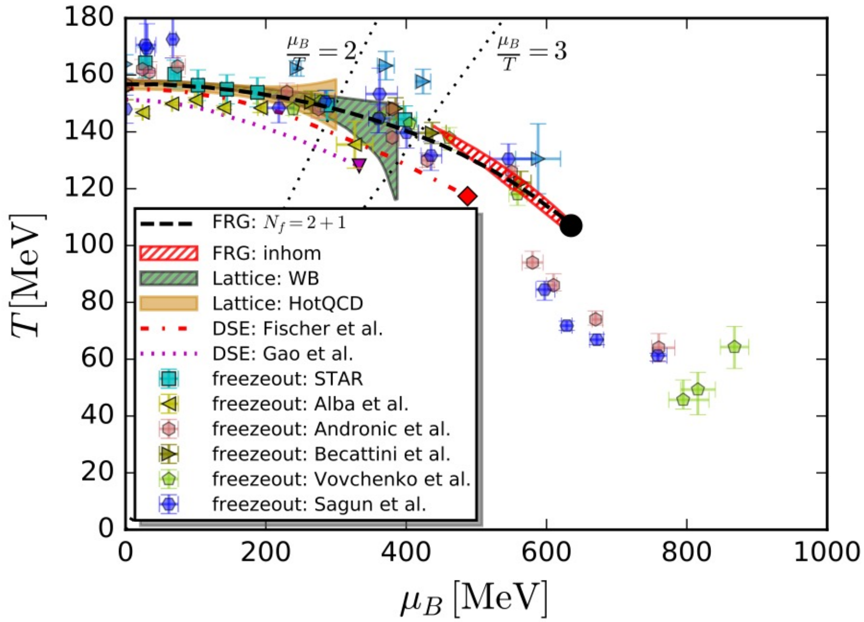
1) QCD equation of state & CEP
(FRG / dynamical parton mass / NJL)
2) inelastic parton reactions
with flavor-dependent cross sections
3) link $\boldsymbol{\sigma}$ with η/s
5a) parton potentials (*FRG / NJL)*

4a) improve coalescence with energy conservation
4b) fragmentation of partons w/o coalescing partners

5b) hadron potentials

Future developments for high baryon density physics

From Wei-Jie Fu



FRG is applicable at high μ_B
and well suited for search of CEP.

Fu, Pawlowski and Rennecke, PRD (2020)

One direction is to couple AMPT with FRG since they complement each other:

AMPT: a **dynamical** non-equilibrium model, can be directly compared with experimental data.

FRG (functional renormalization group):
A non-perturbative but **static** QCD approach, consistent with lattice QCD.

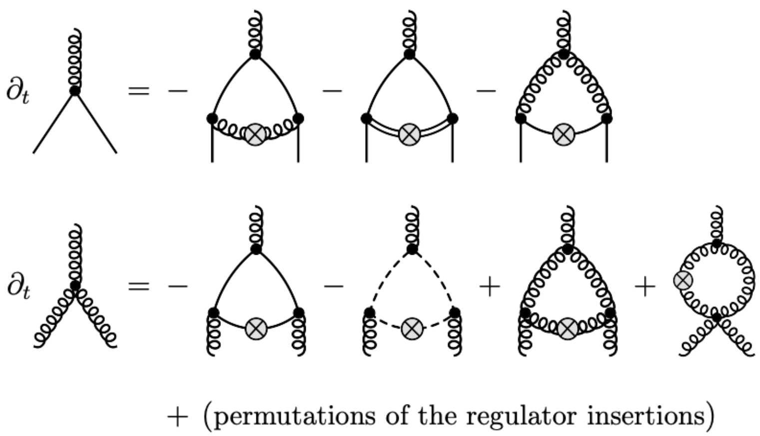
With the coupling of AMPT with FRG:

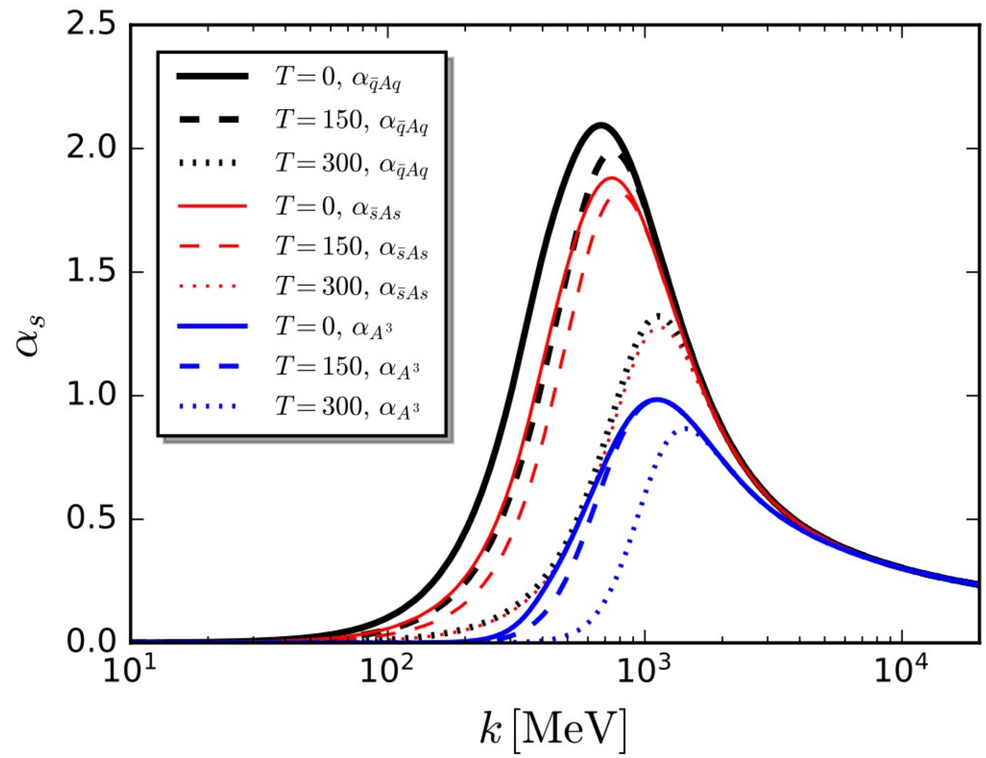
- first-principle QCD EoS from FRG provides information to improve EoS in AMPT.
- it enables calculation of dynamical evolution of fluctuations and possible effects of CEP.

Future developments for high baryon density physics

From Wei-Jie Fu

QCD strong couplings among quarks and gluons from FRG:





FRG could provide proper treatment of QCD interactions in AMPT

Fu, Pawlowski and Rennecke, PRD (2020)

Summary

- The string melting AMPT can now reasonably describe bulk observables of both large and small systems, including the centrality dependence

- At low energies like the BES, finite nuclear thickness has big effects on *energy density* ε *& net-baryon density* n_B , and consequently on event trajectories and relation to CEP

- We have incorporated finite nuclear thickness into AMPT to provide a better foundation for further studies at high baryon densities

- We have found a more accurate new collision scheme that almost eliminates causality violation in parton cascade and gives correct parton η values

- Much important work needs to be done, especially on realistic Equation of State of the dense matter including the CEP