

From the stable to the exotic: clustering in light nuclei \*

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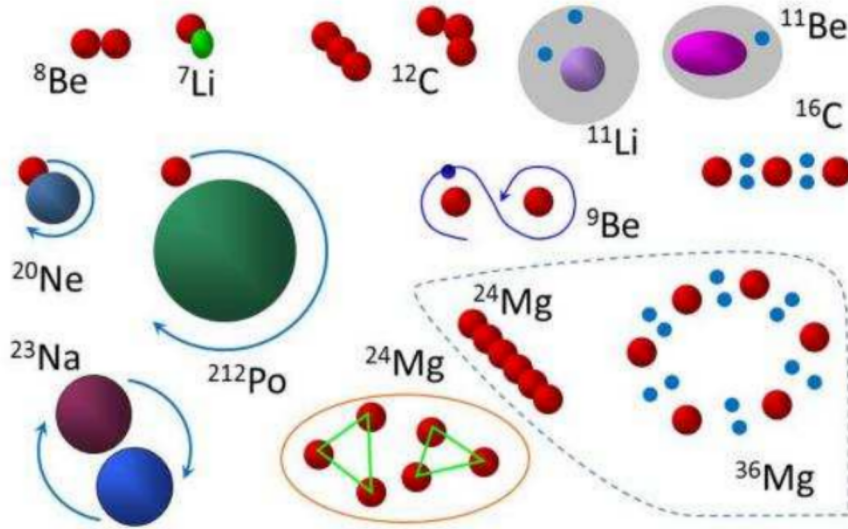


Fig. 1. Different types of clustering in nuclei that have been discussed the last two decades [2].

A great deal of research work has been undertaken in  $\alpha$ -clustering study since the pioneering discovery of  $^{12}\text{C}+^{12}\text{C}$  molecular resonances half a century ago. Our knowledge on physics of nuclear molecules has increased considerably and nuclear clustering remains one of the most fruitful domains of nuclear physics, facing some of the greatest challenges and opportunities in the years ahead. The occurrence of “exotic” shapes in light  $N=Z$   $\alpha$ -like nuclei is investigated. Various approaches of the superdeformed and hyperdeformed bands associated with quasimolecular resonant structures are presented. Evolution of clustering from stability to the drip-lines is examined: clustering aspects are, in particular, discussed for light exotic nuclei with large neutron excess such as neutron-rich Oxygen isotopes with their complete spectroscopy.

## 1. Introduction

One of the greatest challenges in nuclear science is the understanding of the structure of light nuclei from both the experimental and theoretical perspectives [1]. Figure 1 summarizes the different types of clustering discussed during the last two decades [2]. Most of these structures were investigated in an experimental context by using either some new approaches or developments of older methods [3]. Starting in the 1960s the search for resonant structures in the excitation functions for various combinations of light  $\alpha$ -cluster ( $N=Z$ ) nuclei in the energy regime from the Coulomb

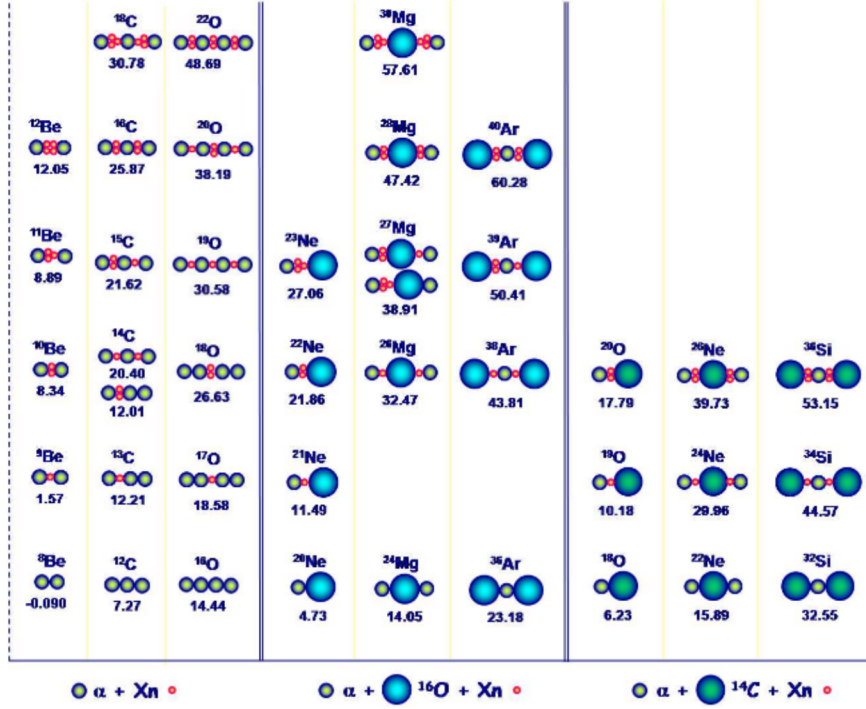


Fig. 2. Schematic illustration of the structures of molecular shape isomers in light neutron-rich isotopes of nuclei consisting of  $\alpha$ -particles,  $^{16}\text{O}$ - and  $^{14}\text{C}$ -clusters plus some covalently bound neutrons ( $Xn$  means  $X$  neutrons) [11]. The so called "Extended Ikeda-Diagram" [10] with  $\alpha$ -particles (left panel) and  $^{16}\text{O}$ -cores (middle panel) can be generalized to  $^{14}\text{C}$ -cluster cores (right panel). The lowest line of each configuration corresponds to parts of the original Ikeda diagram [9]. However, because of its deformation, the  $^{12}\text{C}$  nucleus is not included, as it was earlier [9]. Threshold energies are given in MeV.

barrier up to regions with excitation energies of  $E_x=20-50$  MeV remains a subject of contemporary debate [1, 4]. These resonances [4] have been interpreted in terms of nuclear molecules [1]. The question of how quasimolecular resonances may reflect continuous transitions from scattering states in the ion-ion potential to true cluster states in the compound systems was still unresolved in the 1990s [1]. In many cases, these resonant structures have been associated with strongly-deformed shapes and with  $\alpha$ -clustering phenomena [5, 6], predicted from the Nilsson-Strutinsky approach, the cranked  $\alpha$ -cluster model [5], or other mean-field calculations [6, 7]. In light  $\alpha$ -like nuclei clustering is observed as a general phenomenon at high excitation energy close to the  $\alpha$ -decay thresholds [5, 8]. This exotic behavior has been

perfectly illustrated by the famous "Ikeda-diagram" for  $N=Z$  nuclei in 1968 [9], which has been recently modified and extended by von Oertzen [10] for neutron-rich nuclei, as shown in the left panel of Fig.2. Clustering is a general feature [11] not only observed in light neutron-rich nuclei [12], but also in halo nuclei such as  ${}^{11}\text{Li}$  [13] or  ${}^{14}\text{Be}$ , for instance [14]. The problem of cluster formation has also been treated extensively for very heavy systems by R.G. Gupta [7], by Zagrebaev and W. Greiner [15] and by C. Simenel [16] where giant molecules and collinear ternary fission may co-exist [17]. Finally, signatures of  $\alpha$  clustering have also been discovered in light nuclei surviving from ultrarelativistic nuclear collisions [18, 19].

In this talk, I will limit myself first to the light  ${}^{12}\text{C}$ ,  ${}^{16}\text{O}$  and  ${}^{20}\text{Ne}$   $\alpha$ -like nuclei in Section 2, then to  $\alpha$  clustering, nuclear molecules and large deformations for heavier light nuclei in Section 3. The search for electromagnetic transitions and  $\alpha$  condensates in heavier  $\alpha$ -like nuclei will be discussed in Sections 4 and 5, respectively, and, finally, clustering effects in light neutron-rich nuclei (oxygen isotopes) will be presented in Section 6 before conclusions of Section 7.

## 2. Renewed interest in the spectroscopy of ${}^{12}\text{C}$ , ${}^{16}\text{O}$ and ${}^{20}\text{Ne}$ $\alpha$ -like nuclei

The renewed interest in  ${}^{12}\text{C}$  was mainly focused to a better understanding of the nature of the so called "Hoyle" state [20, 21] that can be described in terms of a bosonic condensate, a cluster state and/or a  $\alpha$ -particle gas [22, 23, 24]. Much experimental progress has been achieved recently as far as the spectroscopy of  ${}^{12}\text{C}$  near and above the  $\alpha$ -decay threshold is concerned [25, 26, 27, 28, 29]. More particularly, the  $2_2^+$  "Hoyle" rotational excitation in  ${}^{12}\text{C}$  has been observed by several experimental groups [25, 27]. The most convincing experimental result comes from measurements of the  ${}^{12}\text{C}(\gamma, \alpha){}^8\text{Be}$  reaction performed at the HIGS facility [27]. The measured angular distributions of the alpha particles are consistent with an  $L=2$  pattern, including a dominant  $2^+$  component. This  $2_2^+$  state that appears at around 10 MeV is considered to be the  $2^+$  excitation of the "Hoyle" state (in agreement with the previous experimental investigation of Itoh et al. [25]) according to the  $\alpha$  cluster [30] and  $\alpha$  condensation models [22]. On the other hand, the experiment  ${}^{12}\text{C}(\alpha, \alpha){}^{12}\text{C}^*$  carried out at the Birmingham cyclotron [29], UK, populates a new state compatible with an equilateral triangle configuration of three  $\alpha$  particles. Still, the structure of the "Hoyle" state remains controversial as experimental results of its direct decay into three  $\alpha$  particles are found to be in disagreement [31, 32, 33, 34, 35, 36].

In the study of Bose-Einstein Condensation (BEC), the  $\alpha$ -particle states in light  $N=Z$  nuclei [22, 23, 24], are of great importance. At present, the

search for an experimental signature of BEC in  $^{16}\text{O}$  is of highest priority. A state with the structure of the "Hoyle" state [20] in  $^{12}\text{C}$  coupled to an  $\alpha$  particle is predicted in  $^{16}\text{O}$  at about 15.1 MeV (the  $0_6^+$  state), the energy of which is  $\approx 700$  keV above the  $4\alpha$ -particle breakup threshold [37]. However, any state in  $^{16}\text{O}$  equivalent to the "Hoyle" state [20] in  $^{12}\text{C}$  is most certainly going to decay exclusively by particle emission with very small  $\gamma$ -decay branches, thus, very efficient particle- $\gamma$  coincidence techniques will have to be used in the near future to search for them. BEC states are expected to decay by alpha emission to the "Hoyle" state and could be found among the resonances in  $\alpha$ -particle inelastic scattering on  $^{12}\text{C}$  decaying to that state. In 1967 Chevallier et al. [38] could excite these states in an  $\alpha$ -particle transfer channel leading to the  $^8\text{Be}$ - $^8\text{Be}$  final state and proposed that a structure corresponding to a rigidly rotating linear arrangement of four alpha particles may exist in  $^{16}\text{O}$ . Very recently, a more sophisticated experimental setup was used at Notre Dame [39]: although the excitation function is generally in good agreement with the previous results [38] a phase shift analysis of the angular distributions does not provide evidence to support the reported hypothesis of a  $4\alpha$ -chain state configuration. Experimental investigations are still underway to understand the nuclear structure of high spin states of both  $^{16}\text{O}$  and  $^{20}\text{Ne}$  nuclei for instance at Notre Dame and/or iThemba Labs [40] facilities. Another possibility might be to perform Coulomb excitation measurements with intense  $^{16}\text{O}$  and  $^{20}\text{Ne}$  beams at intermediate energies.

### 3. Alpha clustering, nuclear molecules and large deformations

The real link between superdeformation (SD), nuclear molecules and  $\alpha$  clustering [6, 41] is of particular interest, since nuclear shapes with major-to-minor axis ratios of 2:1 have the typical ellipsoidal elongation for light nuclei i.e. with quadrupole deformation parameter  $\beta_2 \approx 0.6$ . Furthermore, the structure of possible octupole-unstable 3:1 nuclear shapes - hyperdeformation (HD) with  $\beta_2 \approx 1.0$  - has also been discussed for actinide nuclei in terms of clustering phenomena. Typical examples for possible relationship between quasimolecular bands and extremely deformed (SD/HD) shapes have been widely discussed in the literature for  $A = 20 - 60$   $\alpha$ -conjugate  $N=Z$  nuclei, such as  $^{28}\text{Si}$  [42],  $^{32}\text{S}$  [6],  $^{36}\text{Ar}$  [43],  $^{40}\text{Ca}$  [44],  $^{44}\text{Ti}$  [6],  $^{48}\text{Cr}$  [45] and  $^{56}\text{Ni}$  [46, 47].

In fact, highly deformed shapes and SD rotational bands have been discovered in several light  $\alpha$ -conjugate nuclei, such as  $^{36}\text{Ar}$  and  $^{40}\text{Ca}$  by using  $\gamma$ -ray spectroscopy techniques [43]. In particular, the extremely deformed rotational bands in  $^{36}\text{Ar}$  (shown as crosses in Fig. 3) might be comparable in shape to the quasimolecular bands observed in both  $^{12}\text{C}+^{24}\text{Mg}$  (shown as open triangles) and  $^{16}\text{O}+^{20}\text{Ne}$  (shown as full rectangles) reactions. These

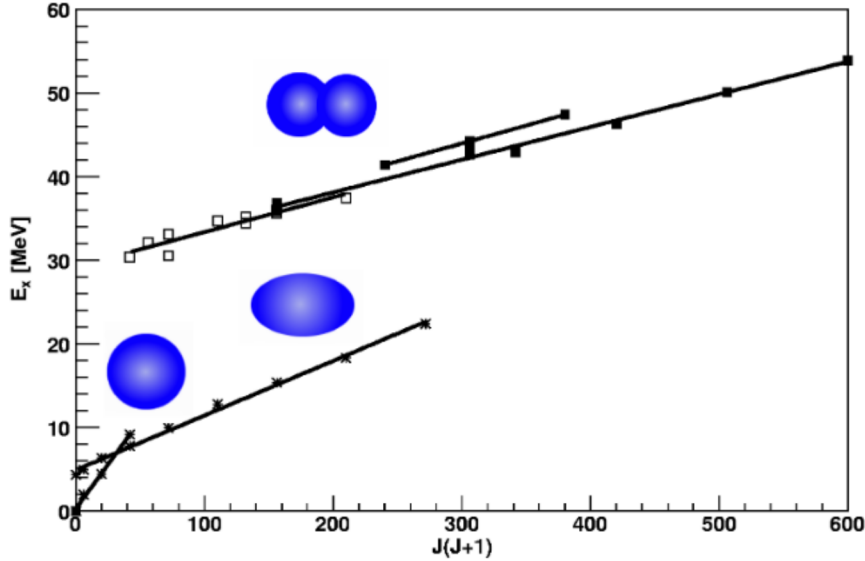


Fig. 3. Rotational bands and deformed shapes in  $^{36}\text{Ar}$ . Excitation energies of the ground state (spherical shape) and SD (ellipsoidal shape) bands, respectively, and the energies of HD (dinuclear shape) band from the quasimolecular resonances observed in the  $^{12}\text{C}+^{24}\text{Mg}$  (open rectangles) and  $^{16}\text{O}+^{20}\text{Ne}$  (full rectangles) reactions are plotted as a function of  $J(J+1)$ . This figure has been adapted from Refs. [43].

resonances belong to a rotational band, with a moment of inertia close to that of a HD band provided by both the cranked  $\alpha$ -cluster model [5] and the Nilsson-Strutinsky calculations. The fact that similar quasi-molecular states observed in the two reactions fall on the same rotational band gives further support to our interpretation of the  $^{36}\text{Ar}$  composite system resonances. An identical conclusion was reached for the  $^{40}\text{Ca}$  composite system where SD bands have been discovered [43]. Therefore, similar investigations are underway for heavier  $\alpha$ -like composite systems such as  $^{44}\text{Ti}$  [6],  $^{48}\text{Cr}$  [45] and  $^{56}\text{Ni}$  [46, 47].

Ternary clusterizations in light  $\alpha$ -like composite systems are also predicted theoretically, but were not found experimentally in  $^{36}\text{Ar}$  so far [43]. On the other hand, ternary fission of  $^{56}\text{Ni}$  – related to its HD shapes – was identified from out-of-plane angular correlations measured in the  $^{32}\text{S}+^{24}\text{Mg}$  reaction with the Binary Reaction Spectrometer (BRS) at the VIVITRON Tandem facility of the IPHC, Strasbourg [48]. This finding [48] is not limited to light  $N=Z$  compound nuclei, true ternary fission [15, 17, 49] can also occur for very heavy [17, 49] and superheavy [50] nuclei.

#### 4. Electromagnetic transitions as a probe of quasimolecular states and clustering in light nuclei

Clustering in light nuclei is traditionally explored through reaction studies, but observation of electromagnetic transitions can be of high value in establishing, for example, that highly-excited states with candidate cluster structure do indeed form rotational sequences.

There is a renewed interest in the spectroscopy of the  $^{16}\text{O}$  nucleus at high excitation energy [43]. Exclusive data were collected on  $^{16}\text{O}$  in the inverse kinematics reaction  $^{24}\text{Mg}+^{12}\text{C}$  studied at  $E_{lab}(^{24}\text{Mg}) = 130$  MeV with the BRS in coincidence with the EUROBALL IV installed at the VIVITRON facility [43]. From the  $\alpha$ -transfer reactions (both direct transfer and deep-inelastic orbiting collisions [51]), new information has been deduced on branching ratios of the decay of the  $3^+$  state of  $^{16}\text{O}$  at  $11.085$  MeV  $\pm 3$  keV. The high-energy level scheme of  $^{16}\text{O}$  shown in Ref. [43] indicated that this state does not  $\alpha$ -decay because of its non-natural parity (in contrast to the two neighbouring  $4^+$  states at 10.36 MeV and 11.10 MeV), but it  $\gamma$  decays to the  $2^+$  state at 6.92 MeV ( $54.6 \pm 2\%$ ) and to the  $3^-$  state at 6.13 MeV (45.4%). By considering all the four possible transition types of the decay of the  $3^+$  state (i.e. E1 and M2 for the  $3^+ \rightarrow 3^-$  transition and, M1 and E2 for the  $3^+ \rightarrow 2^+$  transition), our calculations yield the conclusion that  $\Gamma_{3^+} < 0.23$  eV, a value fifty times lower than known previously, which is an important result for the well studied  $^{16}\text{O}$  nucleus [43]. Clustering effects in the light neutron-rich oxygen isotopes  $^{17,18,19,20}\text{O}$  will also be discussed in Section 5.

$\alpha$  clustering plays an important role in the description of the ground state and excited states of light nuclei in the  $p$  shell. For heavier nuclei, in the  $sd$ -shell, cluster configurations may be based on heavier substructures like  $^{12}\text{C}$ ,  $^{14}\text{C}$  and  $^{16}\text{O}$  as shown by the "Extended Ikeda-diagram" proposed in Fig. 1. This was already well discussed to appear in  $^{24}\text{Mg}(^{12}\text{C}-^{12}\text{C})$  and  $^{28}\text{Si}(^{12}\text{C}-^{16}\text{O})$  both theoretically and experimentally. The case of the mid- $sd$ -shell nucleus  $^{28}\text{Si}$  is of particular interest as it shows the coexistence of deformed and cluster states at rather low energies [42]. Its ground state is oblate, with a partial  $\alpha$ - $^{24}\text{Mg}$  structure, two prolate normal deformed bands are found, one built on the  $0_2^+$  state at 4.98 MeV and on the  $0_3^+$  state at 6.69 MeV. The SD band candidate with a pronounced  $\alpha$ - $^{24}\text{Mg}$  structure is suggested [42]. In this band, the  $2^+$  (9.8 MeV),  $4^+$  and  $6^+$  members are well identified.

In the following we will briefly discuss a resonant cluster band which is predicted to start close to the Coulomb barrier of the  $^{12}\text{C}+^{16}\text{O}$  collision, i.e. around 25 MeV excitation energy in  $^{28}\text{Si}$ . We have studied the  $^{12}\text{C}(^{16}\text{O},\gamma)^{28}\text{Si}$  radiative capture reaction at five resonant energies around

the Coulomb barrier by using the zero degree DRAGON spectrometer installed at Triumf, Vancouver [52, 53]. Details about the setup, that has been optimized for the  $^{12}\text{C}(^{12}\text{C},\gamma)^{24}\text{Mg}$  radiative capture reaction in our of previous DRAGON experiments, can be found in Ref. [54]. The  $^{12}\text{C}(^{16}\text{O},\gamma)^{28}\text{Si}$  data clearly show [52, 53] the direct feeding of the prolate  $4_3^+$  state at 9.16 MeV and the octupole deformed  $3^-$  at 6.88 MeV. This state is the band head of an octupole band which mainly decays to the  $^{28}\text{Si}$  oblate ground state with a strong  $E_3$  transition. Our results are very similar to what has been measured for the  $^{12}\text{C}+^{12}\text{C}$  radiative capture reaction above the Coulomb barrier in the first DRAGON experiment [54] where the enhanced feeding of the  $^{24}\text{Mg}$  prolate band has been measured for a  $4^+-2^+$  resonance at  $E_{c.m.} = 8.0$  MeV near the Coulomb barrier. At the lowest energy of  $^{12}\text{C}+^{16}\text{O}$  radiative capture reaction, an enhanced feeding from the resonance  $J^\pi = 2^+$  and  $1^+$  T=1 states around 11 MeV is observed in  $^{28}\text{Si}$ . Again this is consistent with  $^{12}\text{C}+^{12}\text{O}$  radiative capture reaction data where  $J^\pi = 2^+$  has been assigned to the entrance resonance and an enhanced decay has been measured via intermediate  $1^+$  T=1 states around 11 MeV in  $^{24}\text{Mg}$ . A definitive scenario for the decay of the resonances at these low bombarding energies in both systems will come from the measurement of the  $\gamma$  decay spectra with a  $\gamma$ -spectrometer with better resolution than BGO but still rather good efficiency such as  $\text{LaBr}_3$  crystals.

## 5. Condensation of $\alpha$ clusters in light nuclei

In principle the nucleus is a quasi-homogeneous collection of protons and neutrons, which adopts a spherical configuration i.e. a spherical droplet of nuclear matter. For light nuclei the nucleons are capable to arrange themselves into clusters of a bosonic character. The very stable  $\alpha$ -particle is the most favorable light nucleus for quarteting -  $\alpha$  clustering - to occur in dense nuclear matter. These cluster structures have indeed a crucial role in the synthesis of elements in stars. The so called "Hoyle" state [20, 21], the main portal through which  $^{12}\text{C}$  is created in nucleosynthesis with a pronounced three- $\alpha$ -cluster structure, is the best exemple of  $\alpha$  clustering in light nuclei. In  $\alpha$  clustering a geometric picture can be proposed in the framework of point group symmetries [19]. For instance, in  $^8\text{Be}$  the two  $\alpha$  clusters are separated by as much as  $\approx 2\text{fm}$ ,  $^{12}\text{C}$  exhibits a triangle arrangement of the three  $\alpha$  particles  $\approx 3\text{fm}$  apart,  $^{16}\text{O}$  forms a tetrahedron, etc. Evidence for tetrahedral symmetries in  $^{16}\text{O}$  was given by the algebraic cluster model [55]. A density plot for  $^{20}\text{Ne}$  nucleus calculated as an arrangement of two  $\alpha$  particles with a  $^{12}\text{C}$  core is displayed in Fig. 4 to illustrate the enhancement of the symmetries of the  $\alpha$  clustering.

In the study of the Bose-Einstein Condensation (BEC) the  $\alpha$ -particle



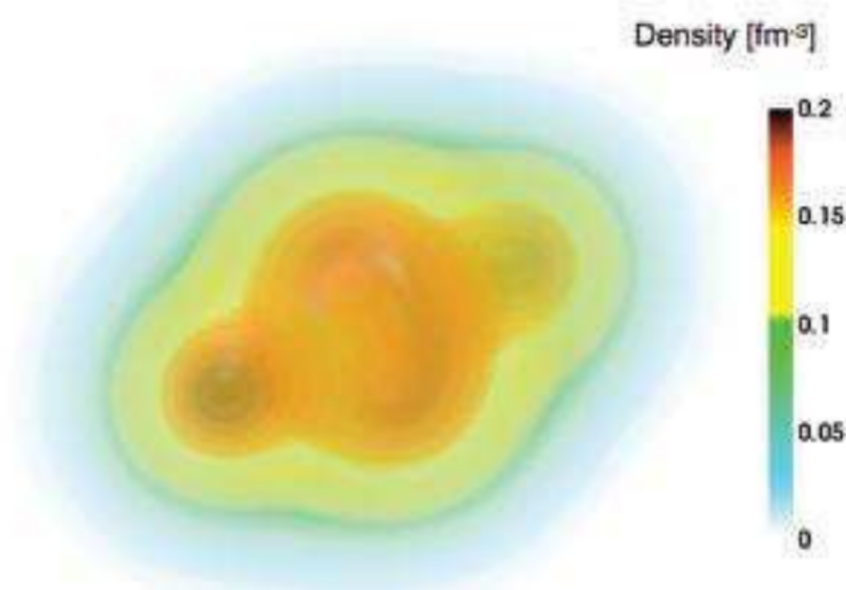


Fig. 4. Self-consistent ground-state densities of  $^{20}\text{Ne}$  as calculated with EDF. Densities (in units of  $\text{fm}^{-3}$ ) are plotted in the intrinsic frame of reference that coincides with the principal axes of the nucleus. This figure has been adapted from Refs. [57].

states were first described for  $^{12}\text{C}$  and  $^{16}\text{O}$  [22, 56] and later on generalized to heavier light  $N=Z$  nuclei [23, 24, 57, 58]. The structure of the “Hoyle” state and the properties of its assumed rotational band have been studied very carefully from measurements of the  $^{12}\text{C}(\gamma,3\alpha)$  reaction performed at the HIGS facility, TUNL [27]. At present, the search for an experimental signature of BEC in  $^{16}\text{O}$  is of highest priority. A state with the structure of the “Hoyle” state in  $^{12}\text{C}$  coupled to an  $\alpha$  particle is predicted in  $^{16}\text{O}$  at about 15.1 MeV (the  $0_6^+$  state), the energy of which is  $\approx 700$  keV above the  $4\alpha$ -particle breakup threshold [37, 59, 60]: in other words, this  $0_6^+$  state might be a good candidate for the dilute  $4\alpha$  gas state. However, any state in  $^{16}\text{O}$  equivalent to the “Hoyle” state in  $^{12}\text{C}$  is most certainly going to decay by particle emission with very small, probably un-measurable,  $\gamma$ -decay branches, thus, very efficient particle-detection techniques will have to be used in the near future to search for them.

BEC states are expected to decay by  $\alpha$  emission to the “Hoyle” state and could be found among the resonances in  $\alpha$ -particle inelastic scattering on  $^{12}\text{C}$  decaying to that state or could be observed in an  $\alpha$ -particle transfer

channel leading to the  ${}^8\text{Be}$ - ${}^8\text{Be}$  final state. The attempts to excite these states by  $\alpha$  inelastic scattering [25] was confirmed recently [61]. Another possibility, that has not been yet explored, might be to perform Coulomb excitation measurements with intense  ${}^{16}\text{O}$  beams at intermediate energies.

Clustering of  ${}^{20}\text{Ne}$  has also been described within the density functional theory [57] (EDF) as illustrated by Fig. 4 that displays axially and reflection symmetric self-consistent equilibrium nucleon density distributions. We note the well known quasimolecular  $\alpha$ - ${}^{12}\text{C}$ - $\alpha$  structure although clustering effects are less pronounced than the ones predicted by Nilsson-Strutinsky calculations and even by mean-field calculations (including Hartree-Fock and/or Hartree-Fock-Bogoliubov calculations) [5, 6, 7, 58].

The most recent work of Girod and Schuck [58] validates several possible scenarios for the influence of clustering effects as a function of the neutron richness that will trigger more experimental works. We describe in the following Section recent experimental investigations on the Oxygen isotopes chain.

## 6. Clustering in light neutron-rich nuclei

As discussed previously, clustering is a general phenomenon observed also in nuclei with extra neutrons as it is presented in an "Extended Ikeda-diagram" [9] proposed by von Oertzen [10] (see the left panel of Fig. 2). With additional neutrons, specific molecular structures appear with binding effects based on covalent molecular neutron orbitals. In these diagrams  $\alpha$ -clusters and  ${}^{16}\text{O}$ -clusters (as shown by the middle panel of the diagram of Fig. 2) are the main ingredients. Actually, the  ${}^{14}\text{C}$  nucleus may play similar role in clusterization as the  ${}^{16}\text{O}$  one since it has similar properties as a cluster: i) it has closed neutron p-shells, ii) first excited states are well above  $E^* = 6$  MeV, and iii) it has high binding energies for  $\alpha$ -particles.

A general picture of clustering and molecular configurations in light nuclei can be drawn from a detailed investigation of the light oxygen isotopes with  $A \geq 17$ . Here we will only present recent results on the even-even oxygen isotopes:  ${}^{18}\text{O}$  [62] and  ${}^{20}\text{O}$  [63]. But very striking cluster states have also been found in odd-even oxygen isotopes such as:  ${}^{17}\text{O}$  [64] and  ${}^{19}\text{O}$  [65].

Fig.5 gives an overview of all bands in  ${}^{20}\text{O}$  as a plot of excitation energies as a function of  $J(J+1)$  together with their respective moments of inertia. In the assignment of the bands both the dependence of excitation energies on  $J(J+1)$  and the dependence of measured cross sections on  $2J+1$  [63] were considered. Slope parameters obtained in a linear fit to the excitation energies [63] indicate the moment of inertia of the rotational bands given in Fig. 5. The intrinsic structure of the cluster bands is reflection asymmetric,

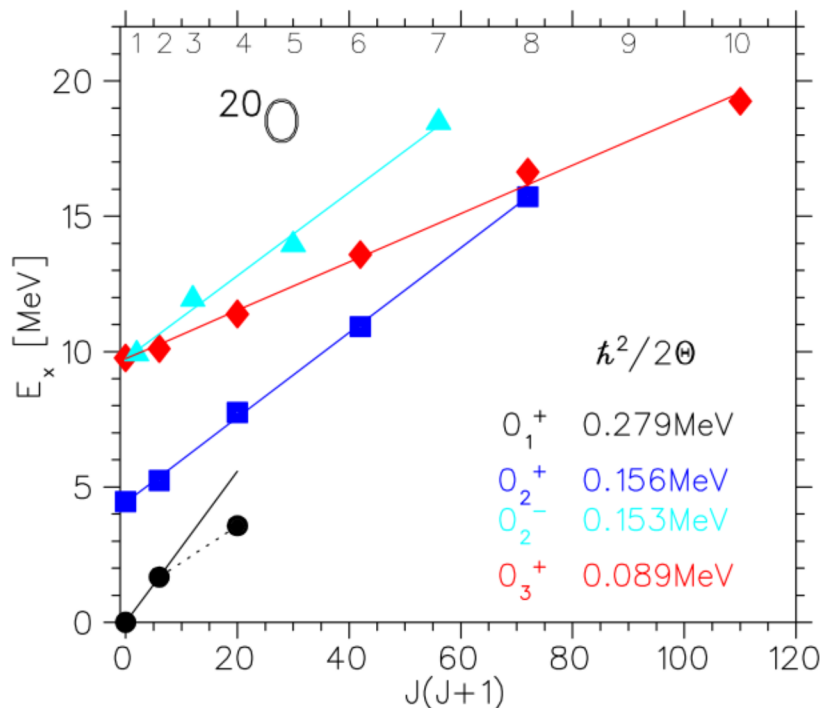


Fig. 5. Overview of 4 rotational band structures observed in  $^{20}\text{O}$ . This figure is adapted from [63]

the parity projection gives an energy splitting between the partner bands. The assignments of the experimental molecular bands are supported by both the Generator-Coordinate-Method [66] and the Antisymmetrized Molecular Dynamics (AMD) calculations [67].

We can compare the bands of  $^{20}\text{O}$  [63] shown in Fig. 5. The first doublet ( $K=0_2^\pm$ ) has a slightly larger moment of inertia (smaller slope parameter) in  $^{20}\text{O}$ , which is consistent with its interpretation as  $^{14}\text{C}-^6\text{He}$  or  $^{16}\text{C}-^4\text{He}$  molecular structures (they start well below the thresholds of 16.8 MeV and 12.32 MeV, respectively). The second band, for which the negative parity partner is yet to be determined, has a slope parameter slightly smaller than in  $^{18}\text{O}$ . This is consistent with the study of the bands in  $^{20}\text{O}$  by Furutachi et al. [67], which clearly establishes parity inversion doublets predicted by AMD calculations for the  $^{14}\text{C}-^6\text{He}$  cluster and  $^{14}\text{C}-2\text{n}-\alpha$  molecular structures. The corresponding moments of inertia given in Fig. 5 are strongly suggesting large deformations for the cluster structures. We may conclude that the reduction of the moments of inertia of the lowest bands of  $^{20}\text{O}$  is consistent with the assumption that the strongly bound  $^{14}\text{C}$  nucleus hav-

ing equivalent properties to  $^{16}\text{O}$ , has a similar role as  $^{16}\text{O}$  in relevant, less neutron rich nuclei. Therefore, the Ikeda-diagram [9] and the "extended Ikeda-diagram" consisting of  $^{16}\text{O}$  cluster cores with covalently bound neutrons [10] must be further extended to include also the  $^{14}\text{C}$  cluster cores as illustrated in Fig. 2.

### 7. Summary, conclusions and outlook

The link of  $\alpha$ -clustering, quasimolecular resonances, orbiting phenomena and extreme deformations (SD, HD, ...) has been discussed in this talk. In particular, the BEC picture of light (and medium-light)  $\alpha$ -like nuclei appears to be a good way of understanding most of properties of nuclear clusters. New results regarding cluster and molecular states in neutron-rich oxygen isotopes in agreement with AMD predictions are presented. Consequently, the "Extended Ikeda-diagram" has been further modified for light neutron-rich nuclei by inclusion of the  $^{14}\text{C}$  cluster, similarly to the  $^{16}\text{O}$  one. Of particular interest is the quest for the  $4\alpha$  states of  $^{16}\text{O}$  near the  $^8\text{Be}+^8\text{Be}$  and  $^{12}\text{C}+\alpha$  decay thresholds, which correspond to the so-called "Hoyle" state. The search for extremely elongated configurations (HD) in rapidly rotating medium-mass nuclei, which has been pursued by  $\gamma$ -ray spectroscopy measurements, will have to be performed in conjunction with charged-particle techniques in the near future since such states are most certainly going to decay by particle emission (see [3, 48]). Marked progress has been made in many traditional and novel subjects of nuclear cluster physics. The developments in these subjects show the importance of clustering among the basic modes of motion of nuclear many-body systems. All these open questions will require precise coincidence measurements [3] coupled with state-of-the-art theory.

### 8. Acknowledgments

This talk is dedicated to the memory of my friends Alex Szanto de Toledo and Valery Zagrebaev who passed away beginning of last year. I would like to acknowledge Christian Caron (Springer) for initiating in 2008 the series of the three volumes of **Lecture Notes in Physics** entitled "Clusters in Nuclei" and edited between 2010 and 2014.

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