Critical Fluctuation Dynamics

M. Stephanov

Outline

1. Introduction: critical point and heavy-ion collisions

2. Equilibrium fluctuations near QCD critical point
   - Critical fluctuations
   - Intriguing data from RHIC BES I

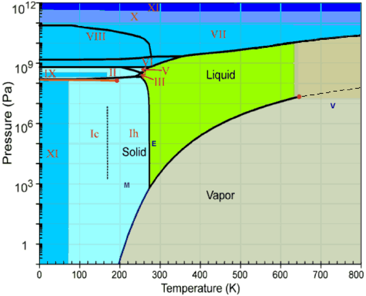
3. Non-equilibrium dynamics of fluctuations (work in progress)
   - Hydrodynamics and fluctuations
   - Hydro+
   - General formalism

4. Summary and Outlook

Substance[13][14]	Critical temperature	Critical pressure (absolute)
Argon	−122.4 °C (150.8 K)	48.1 atm (4,870 kPa)
Ammonia[15]	132.4 °C (405.5 K)	111.3 atm (11,280 kPa)
Bromine	310.8 °C (584.0 K)	102 atm (10,300 kPa)
Caesium	1,664.85 °C (1,938.00 K)	94 atm (9,500 kPa)
Chlorine	143.8 °C (416.9 K)	76.0 atm (7,700 kPa)
Ethanol	241 °C (514 K)	62.18 atm (6,300 kPa)
Fluorine	−128.85 °C (144.30 K)	51.5 atm (5,220 kPa)
Helium	−267.96 °C (5.19 K)	2.24 atm (227 kPa)
Hydrogen	−239.95 °C (33.20 K)	12.8 atm (1,300 kPa)
Krypton	−63.8 °C (209.3 K)	54.3 atm (5,500 kPa)
CH_4 (methane)	−82.3 °C (190.8 K)	45.79 atm (4,640 kPa)
Neon	−228.75 °C (44.40 K)	27.2 atm (2,760 kPa)
Nitrogen	−146.9 °C (126.2 K)	33.5 atm (3,390 kPa)
Oxygen	−118.6 °C (154.6 K)	49.8 atm (5,050 kPa)
CO_2	31.04 °C (304.19 K)	72.8 atm (7,380 kPa)
N_2O	36.4 °C (309.5 K)	71.5 atm (7,240 kPa)
H_2SO_4	654 °C (927 K)	45.4 atm (4,600 kPa)
Xenon	16.6 °C (289.8 K)	57.6 atm (5,840 kPa)
Lithium	2,950 °C (3,220 K)	652 atm (66,100 kPa)
Mercury	1,476.9 °C (1,750.1 K)	1,720 atm (174,000 kPa)
Sulfur	1,040.85 °C (1,314.00 K)	207 atm (21,000 kPa)
Iron	8,227 °C (8,500 K)	
Gold	6,977 °C (7,250 K)	5,000 atm (510,000 kPa)
Water[2][16]	373.946 °C (647.096 K)	217.7 atm (22.06 MPa)

Critical point
– end of phase coexistence –
is a ubiquitous phenomenon

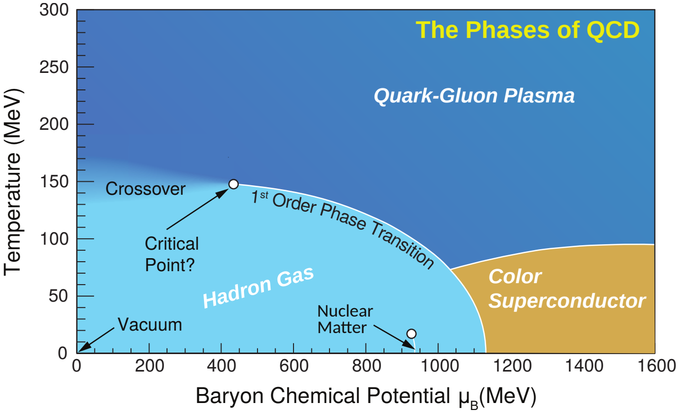
Water:



Is there one in QCD?

Is there a CP between QGP and hadron gas phases?

- Lattice QCD at $\mu_B = 0$ – a crossover.



- Unfortunately, lattice QCD cannot reach beyond $\mu_B \sim 2T$.
- But 1st order transition (and thus C.P.) is ubiquitous in models of QCD: NJL, RM, Holography, Strong coupl. Lattice QCD, …

How can one discover the QCD critical point?

Essentially two approaches to discovering the QCD critical point.
Each with its own challenges.

- Lattice simulations.

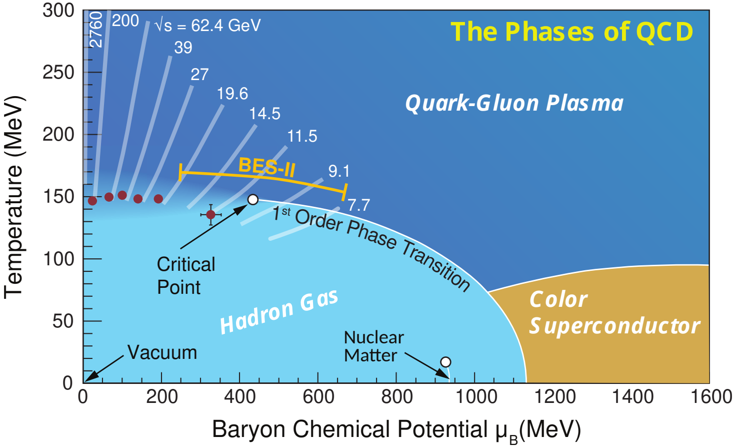
  The *sign problem* restricts reliable lattice calculations to $\mu_B = 0$.

  Under different assumptions one can estimate the position of the critical point, assuming it exists, by extrapolation from $\mu_B = 0$.

- Heavy-ion collisions. *Non-equilibrium.*

Beam Energy Scan.

- Expansion accompanied by cooling, followed by freezeout. Frezeout at a point tunable via $\sqrt{s}$.



Assumption for the next part of this talk

H.I.C. are sufficiently close to equilibrium that we can study thermodynamics at freezeout T and μ_B — as a first approximation.

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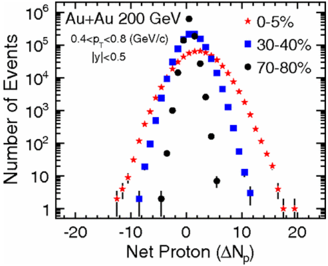
- NB: Event-by-event fluctuations:

  Heavy-ion collisions create systems which are large enough (for thermodynamics), but not too large ($N \sim 10^2 - 10^4$ particles)

  EBE fluctuations are small ($1/\sqrt{N}$), but measurable.



Outline

What are the signatures of the critical point?

EBE fluctuations vs $\sqrt{s}$ [PRL81(1998)4816]

- Equilibrium = maximum entropy.

$$P(\sigma) \sim e^{S(\sigma)}$$ *(Einstein 1910)*

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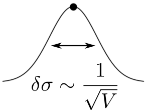

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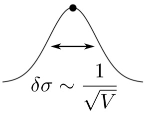
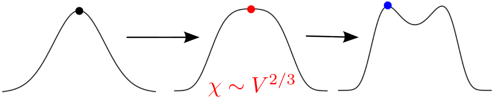


- At the critical point $S(\sigma)$ “flattens”. And $\chi \equiv \langle \delta\sigma^2 \rangle V \to \infty$.



CLT?

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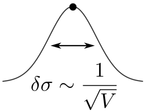

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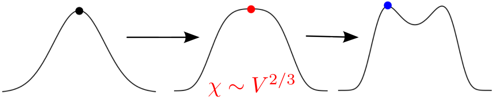
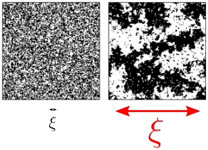




CLT?

$\delta\sigma$ is not an average of ∞ many *uncorrelated* contributions: $\xi \to \infty$

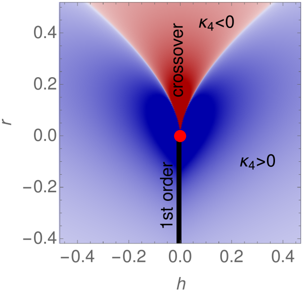
In fact, $\langle \delta\sigma^2 \rangle \sim \xi^2 / V$.

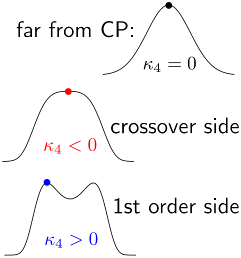
Higher order cumulants

- $n > 2$ cumulants (shape of $P(\sigma)$) depend stronger on ξ.

  E.g., $\langle \sigma^2 \rangle \sim \xi^2$ while $\kappa_4 = \langle \sigma^4 \rangle_c \sim \xi^7$ *[PRL102(2009)032301]*
- For $n > 2$, sign depends on which side of the CP we are.

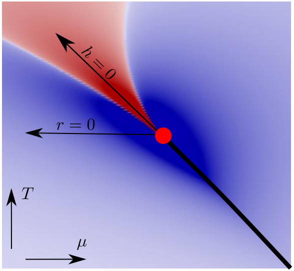
  This dependence is also universal. *[PRL107(2011)052301]*
- Using Ising model variables:





Mapping Ising to QCD and observables near CP

κ_4 vs μ_B and T:



- In QCD $(r, h) \to (\mu - \mu_{\rm CP}, T - T_{\rm CP})$

Parotto *et al*, 1805.05249

Pradeep-MS 1905.13247

Mrozcek *et al*, 2008.04022

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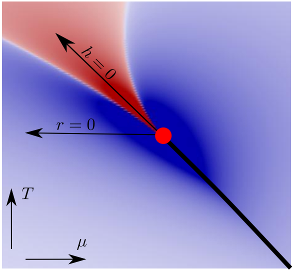

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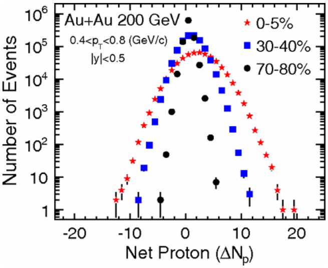
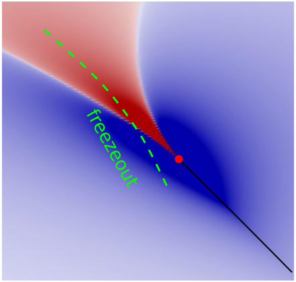
- $\kappa_n(N) = N + \mathcal{O}(\kappa_n(\sigma))$

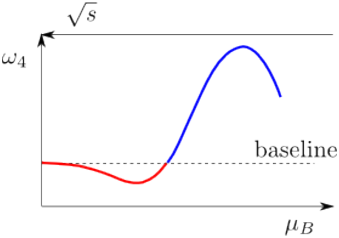
  1104.1627



Beam Energy Scan I: intriguing hints

Equilibrium κ_4 vs μ_B and T:





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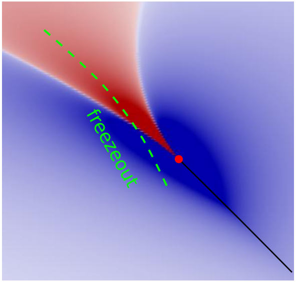

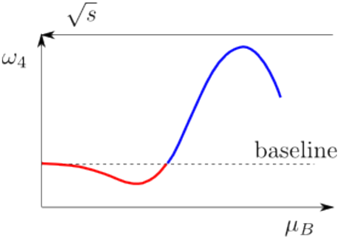

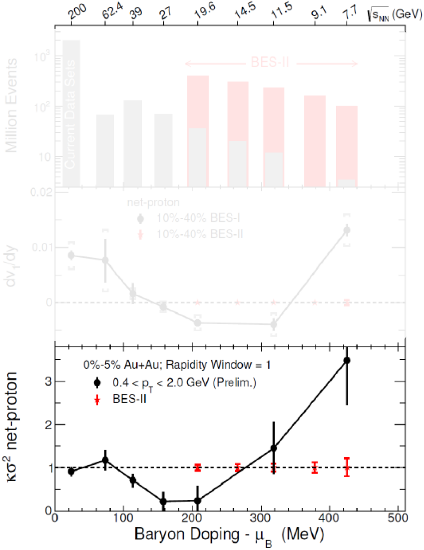

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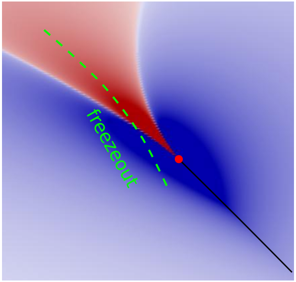

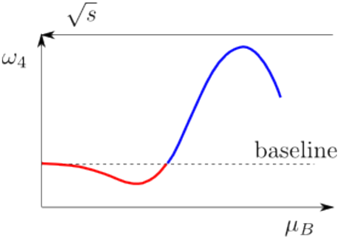

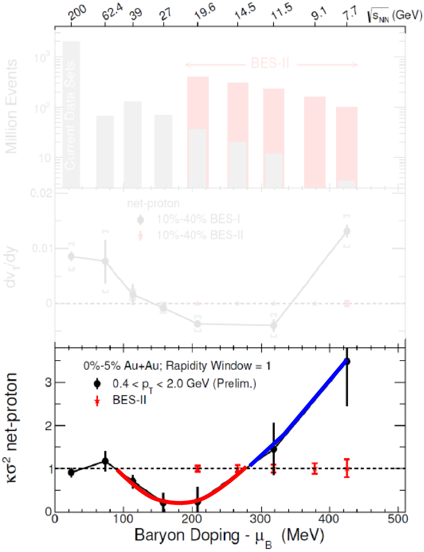

"intriguing hint" (2015 LRPNS)

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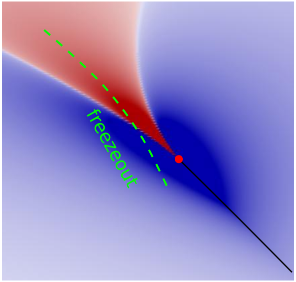

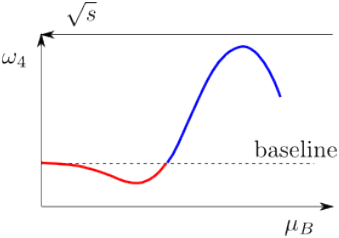

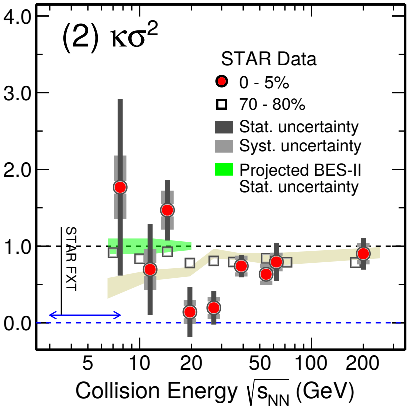

STAR 2001.02852

Outline

Non-equilibrium physics is essential near the critical point.

The challenge taken on by BEST COLLABORATION

- Goal: build a *quantitative* theoretical framework describing critical point signatures for comparison with experiment.
- Strategy:
  - Parameterize QCD EOS with yet unknown $T_{\rm CP}$ and $\mu_{\rm CP}$ as variable parameters (e.g., Parotto *et al*, 1805.05249) .
  - Use the EOS in a hydrodynamic simulation and compare with experiment to determine or constrain $T_{\rm CP}$ and $\mu_{\rm CP}$.

Stochastic hydrodynamics

- Hydrodynamic eqs. are conservation equations ($\partial_\mu T^{\mu\nu} = 0$):

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- Linearized version has been considered and applied to heavy-ion collisions (Kapusta-Muller-MS, Kapusta-Torres-Rincon, . . .)

- Non-linearities + point-like noise $\Rightarrow$ UV divergences. In numerical simulations – cutoff dependence.

Deterministic approach

- Variables are one- and two-point functions:
  $\psi = \langle\breve{\psi}\rangle$ and $G = \langle\breve{\psi}\breve{\psi}\rangle - \langle\breve{\psi}\rangle\langle\breve{\psi}\rangle$ – equal-time correlator

  Nonlinearities lead to dependence of flux on G.

$$\partial_t\psi = -\nabla\cdot\mathsf{Flux}[\psi, G]; \qquad \text{(conservation)}$$

$$\partial_t G = \mathsf{L}[G;\psi]. \qquad \text{(relaxation)}$$

- In Bjorken flow by Akamatsu *et al*, Martinez-Schaefer.
  For arbitrary relativistic flow – by An *et al* (this talk).
  Earlier, in *nonrelativistic* context, – by Andreev in 1970s.

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  Earlier, in *nonrelativistic* context, – by Andreev in 1970s.
- Advantage: deterministic equations.

  "Infinite noise" causes UV renormalization of EOS and transport coefficients – can be taken care of *analytically* *(1902.09517)*

Fluctuation dynamics near CP: Hydro+

Yin, MS, 1712.10305
Rajagopal et al, 1908.08539
Du et al, 2004.02719

- Fluctuation dynamics near CP requires two main ingredients:
  - Critical fluctuations ($\xi \to \infty$)
  - Slow relaxation mode with $\tau_{\mathrm{relax}} \sim \xi^3$ (leading to $\zeta \to \infty$)

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  - Critical fluctuations ($\xi \to \infty$)
  - Slow relaxation mode with $\tau_{\text{relax}} \sim \xi^3$ (leading to $\zeta \to \infty$)
- Both described by the same object: the two-point function of the slowest hydrodynamic mode $m \equiv (s/n)$, i.e., $\langle\, \delta m(x_1)\, \delta m(x_2)\, \rangle$.
- Without this mode, hydrodynamics would break down near CP when $\tau_{\text{expansion}} \sim \tau_{\text{relax}} \sim \xi^3$.

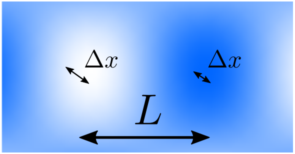
Additional variables in Hydro+

- At the CP the *slowest* new variable is the 2-pt function $\langle \delta m \delta m \rangle$ of the slowest hydro variable:

$$\phi_{\boldsymbol{Q}}(\boldsymbol{x}) = \int_{\Delta \boldsymbol{x}} \langle \delta m\left(\boldsymbol{x}_{+}\right) \, \delta m\left(\boldsymbol{x}_{-}\right) \rangle \; e^{i \boldsymbol{Q} \cdot \Delta \boldsymbol{x}}$$

  where $\boldsymbol{x} = (\boldsymbol{x}_{+} + \boldsymbol{x}_{-})/2$ and $\Delta \boldsymbol{x} = \boldsymbol{x}_{+} - \boldsymbol{x}_{-}$.

- Wigner transformed b/c dependence on $\boldsymbol{x}$ ($\sim L$) is slow and relevant $\Delta x \ll L$. Scale separation similar to kinetic theory.



Relaxation of fluctuations towards equilibrium

- As usual, equilibration maximizes entropy $S = \sum_i p_i \log(1/p_i)$:

$$s_{(+)}(\epsilon, n, \phi_{\boldsymbol{Q}}) = s(\epsilon, n) + \frac{1}{2}\int_{\boldsymbol{Q}} \left(\log \frac{\phi_{\boldsymbol{Q}}}{\bar{\phi}_{\boldsymbol{Q}}} - \frac{\phi_{\boldsymbol{Q}}}{\bar{\phi}_{\boldsymbol{Q}}} + 1 \right)$$

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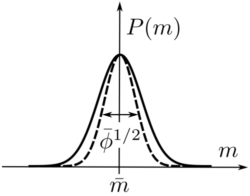
- Entropy = log # of states, which depends on the width of $P(m_{\boldsymbol{Q}})$, i.e., $\phi_{\boldsymbol{Q}}$:
  - Wider distribution – more microstates – more entropy: $\log(\phi/\bar{\phi})^{1/2}$;

    vs
  - Penalty for larger deviations from peak entropy (at $\delta m = 0$): $-(1/2)\phi/\bar{\phi}$.

Maximum of $s_{(+)}$ is achieved at $\phi = \bar{\phi}$.



--- equilibrium (variance $\bar{\phi}$)

— actual (variance ϕ)

Hydro+ mode kinetics

- The equation for $\phi_{\boldsymbol{Q}}$ is a relaxation equation:

$$(u\cdot\partial)\phi_{\boldsymbol{Q}} = -\gamma_\pi(\boldsymbol{Q})\pi_{\boldsymbol{Q}}, \quad \pi_{\boldsymbol{Q}} = -\left(\frac{\partial s_{(+)}}{\partial \phi_{\boldsymbol{Q}}}\right)_{\epsilon,n}$$

  $\gamma_\pi(\boldsymbol{Q})$ is known from mode-coupling calculation in 'model H'.

  It is universal (Kawasaki function).

  $\gamma_\pi(\boldsymbol{Q}) \approx 2DQ^2$ for $Q \ll \xi^{-1}$. ($D \sim 1/\xi$).

- Characteristic rate: $\Gamma(Q) \sim \gamma_\pi(Q) \sim \xi^{-3}$ at $Q \sim \xi^{-1}$.

- Slowness of this relaxation process is behind the divergence of $\zeta \sim 1/\Gamma \sim \xi^3$ and the breakdown of *ordinary* hydro near CP (frequency depedence of ζ at $\omega \sim \xi^{-3}$).

Towards a general deterministic formalism

An, Basar, Yee, MS, 1902.09517,1912.13456

- To embed Hydro+ into a unified theory for critical as well as non-critical fluctuations we develop a general *deterministic* (*hydro-kinetic*) formalism.
  - We expand hydrodynamic eqs. in $\{\delta m, \delta p, \delta u^\mu\} \sim \phi$ and then average, using equal-time correlator $G(x, y) \equiv \langle\, \phi(x + y/2)\, \phi(x - y/2)\, \rangle$.
  - What is “equal-time” in *relativistic* hydro?
  - Renormalization.

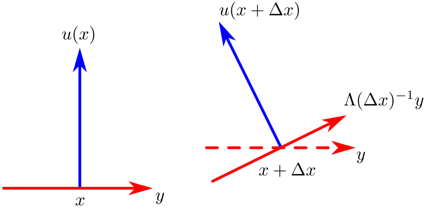
Equal time

We use equal-time correlator $G = \langle \phi(t, \boldsymbol{x}_+) \phi(t, \boldsymbol{x}_-) \rangle$.

But what does "equal time" mean? Needs a frame choice.

The most natural choice is local $u(x)$ ($x = (x_+ + x_-)/2$).

Derivatives wrt x at "y-fixed" should take this into account:



using $\Lambda(\Delta x) u(x + \Delta x) = u(x)$:

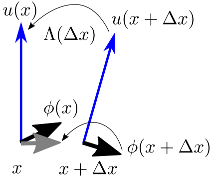
$$\Delta x \cdot \bar{\nabla} G(x, y) \equiv$$
$$G(x + \Delta x, \Lambda(\Delta x)^{-1} y) - G(x, y)\,.$$

not $G(x + \Delta x, y) - G(x, y)\,.$

Confluent derivative, connection and correlator

Take out dependence of *components* of ϕ due to change of $u(x)$:

$$\Delta x \cdot \bar{\nabla}\phi = \Lambda(\Delta x)\phi(x+\Delta x) - \phi(x)$$



Confluent two-point correlator:

$$\bar{G}(x,y) = \Lambda(y/2)\,\langle\phi(x+y/2)\,\phi(x-y/2)\rangle\,\Lambda(-y/2)^T$$

(boost to $u(x)$ – rest frame at midpoint)

$$\bar{\nabla}_\mu \bar{G}_{AB} = \partial_\mu \bar{G}_{AB} - \bar{\omega}_{\mu A}^{C}\bar{G}_{CB} - \bar{\omega}_{\mu B}^{C}\bar{G}_{AC} - \mathring{\omega}_{\mu a}^{b}\, y^a \frac{\partial}{\partial y^b}\bar{G}_{AB}\,.$$

Connection $\bar{\omega}$ corresponds to the boost Λ.

Connection $\mathring{\omega}$ makes sure derivative is independent of the choice of local space triad e_a needed to express $\boldsymbol{y} \equiv \boldsymbol{x}_+ - \boldsymbol{x}_-$.

We then define the Wigner transform $W_{AB}(x,q)$ of $\bar{G}_{AB}(x,y)$.

Sound-sound correlation and phonon kinetic equation

- Upon lots of algebra with many *miraculous* cancellations we arrive at "hydro-kinetic" equations for components of W.

  The longitudinal components, corresponding to p and u^μ fluctuations at $\delta(s/n) = 0$, obey the following eq. ($N_L \equiv W_L/(wc_s|q|)$)

$$\underbrace{\left[(u+v)\cdot\bar{\nabla} + f\cdot\frac{\partial}{\partial q}\right] N_L}_{\mathcal{L}[N_L]\text{ – Liouville op.}} = -\gamma_L q^2 \left(N_L - \underbrace{\frac{T}{c_s|q|}}_{N_L^{(0)}} \right)$$

- Kinetic eq. for phonons with $E = c_s|q|$, $v = c_s q/|q|$ ($q\cdot u = 0$)

$$f_\mu = \underbrace{-E(a_\mu + 2v^\nu\omega_{\nu\mu})}_{\text{inertial + Coriolis}} \underbrace{-q^\nu\partial_{\perp\mu}u_\nu}_{\text{"Hubble"}} - \bar{\nabla}_{\perp\mu}E\,.$$

- $N_L^{(0)}$ is equilibrium Bose-distribution.

Diffusive mode fluctuations

Fluctuations of $m \equiv s/n$ and transverse components of u^μ obey

(entropy-entropy) $$\mathcal{L}[N_{mm}] = -2\Gamma_\lambda \left(N_{mm} - \frac{c_p}{n} \right) + \dots$$

(entropy-velosity) $$\mathcal{L}[N_{mi}] = -2(\Gamma_\eta + \Gamma_\lambda) N_{mi} + \dots$$

(velocity-velocity) $$\mathcal{L}[N_{ij}] = -2\Gamma_\eta \left(N_{ij} - \frac{Tw}{n} \right) + \dots$$

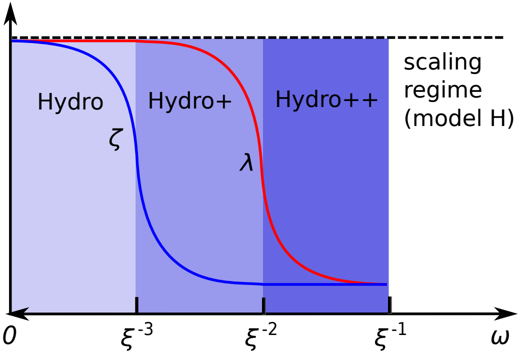
- $\mathcal{L}$ is Liouville operator with $v = f = 0$, i.e., no propagation, but diffusion: $\Gamma_X = \gamma_X q^2$, where $\gamma_\lambda = \lambda/c_p$ and $\gamma_\eta = \eta/w$.
- "$\dots$" are terms $\sim$ background grads, mixing $N_{mm} \leftrightarrow N_{mi} \leftrightarrow N_{ij}$.
- Near critical point Γ_λ is smallest, $\gamma_\lambda = \lambda/c_p \sim 1/\xi \to 0$.

  N_{mm} equation decouples and matches Hydro+ ($\phi_{\boldsymbol{Q}} = nN_{mm}$). Very nontrivially!

Beyond Hydro+

- Hydro+ breaks down when hydro frequency/rate is of order ξ^{-2} due to next-to-slowest modes (N_{mi} and N_{ij}).
  λ is frequency dependent.
- The formalism extends Hydro+ to higher frequencies, i.e., shorter hydrodynamic scales (all the way to ξ.)

  Fluctuations (N_{mi}) enhance conductivity λ for small ω.



Renormalization

Expansion of $\langle T^{\mu\nu} \rangle$ contains $\langle \phi(x)\phi(x) \rangle = G(x, 0) = \int \frac{d^3q}{(2\pi)^3}\, W(x, q)$.

This integral is divergent (equilibrium $G^{(0)}(x, y) \sim \delta^3(y)$).

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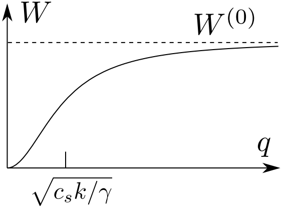

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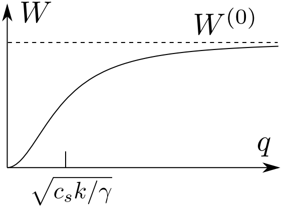

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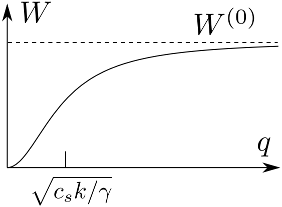

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$$\begin{aligned}\langle T^{\mu\nu}(x)\rangle &= \epsilon u^\mu u^\nu + p(\epsilon,n)\Delta^{\mu\nu} + \Pi^{\mu\nu} + \Big\{G(x,0)\Big\} \\ &= \epsilon_R u_R^\mu u^\nu + p_R(\epsilon_R,n_R)\Delta_R^{\mu\nu} + \Pi_R^{\mu\nu} + \Big\{\tilde{G}(x,0)\Big\}\ .\end{aligned}$$

Work in progress and outlook

- Add higher-order correlators for *non-gaussian* fluctuations.
- Connect *fluctuating* hydro with freezeout kinetics and implement in full hydrodynamic code and event generator.

  Compare with experiment.
- First-order transition in fluctuating hydrodynamics?
- Connection to action principle (SK) formulation.

Summary

- A fundamental question about QCD phase diagram:

  Is there a critical point on the QGP-HG boundary?

- Intriguing results from experiments (BES-I).

  More to come from BES-II (also FAIR/CBM, NICA, J-PARC).

  Quantitative theoretical framework is needed $\Rightarrow$ BEST COLLABORATION .

- In H.I.C., the magnitude of the fluctuation signatures of CP is controlled by dynamical *non-equilibrium effects*.

  In turn, critical fluctuations affect hydrodynamics.

  The interplay of critical and dynamical phenomena: Hydro+.