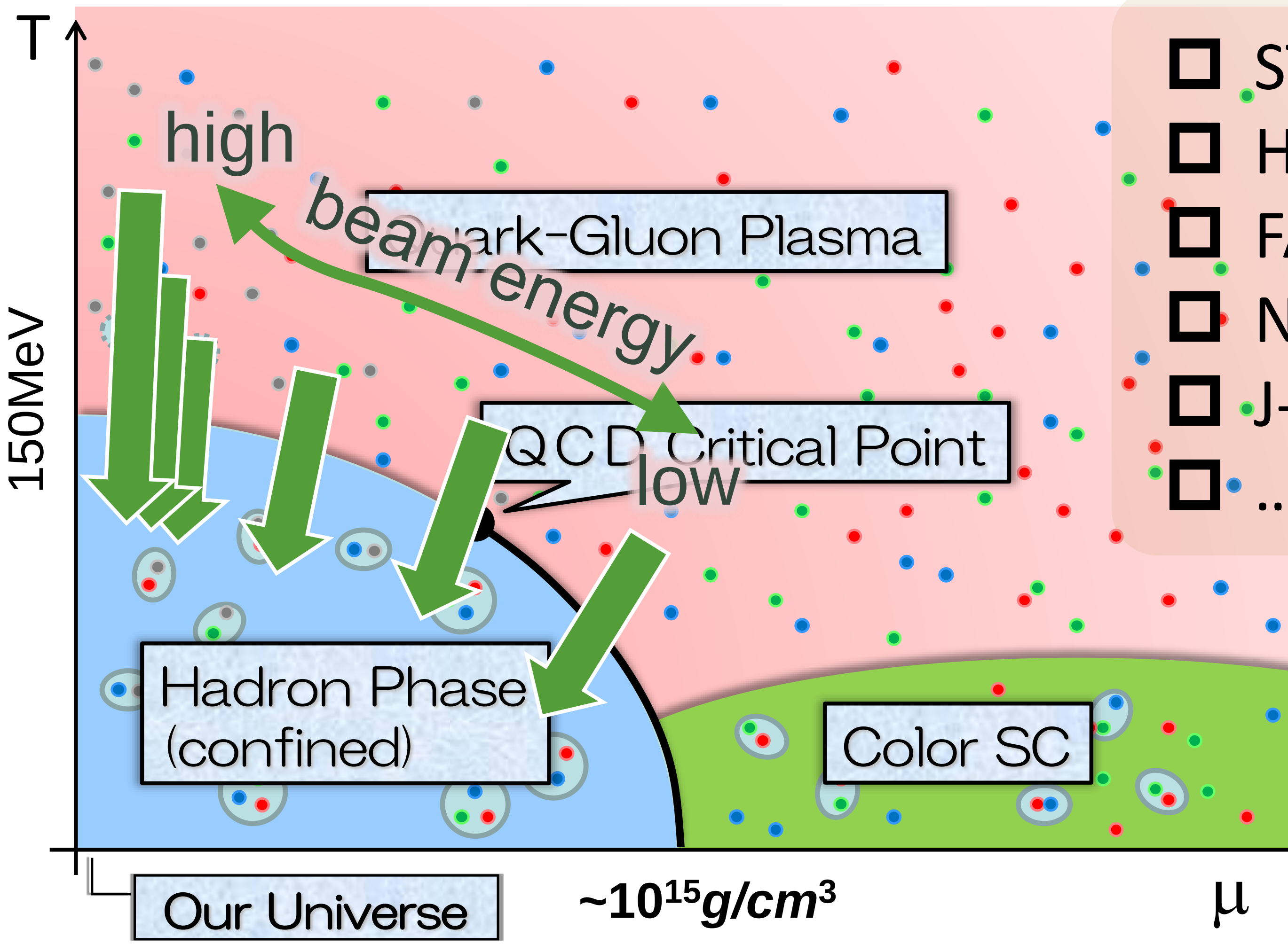
Diffusion Dynamics of Fluctuations of Conserved Charges

Masakiyo Kitazawa

(Osaka U.)

RHIC-BES Theory and Experiment on-line seminar
on Sep. 8, 2020, online

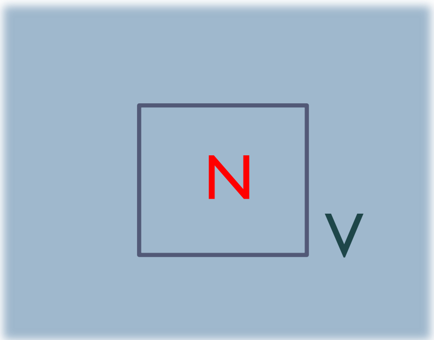
Beam-Energy Scan Program in Heavy-Ion Collisions

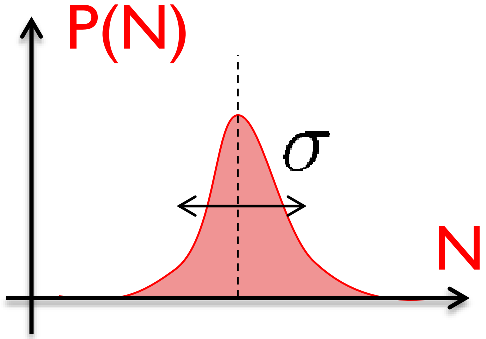


- ☐ STAR-BES
- ☐ HADES
- ☐ FAIR
- ☐ NICA
- ☐ J-PARC-HI
- ☐ …

Thermal Fluctuations

Observables are fluctuating even in an equilibrated medium.



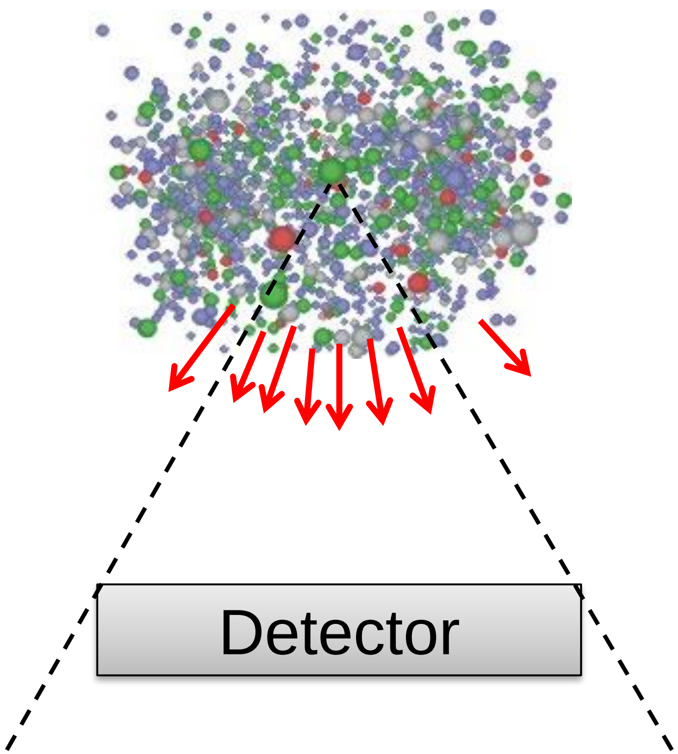


The noise is the signal.

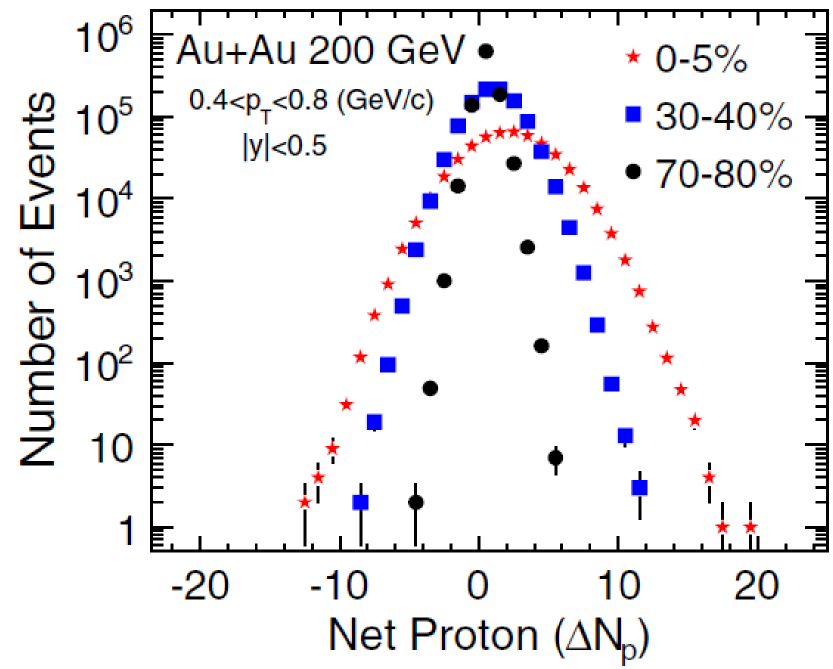
R. Landauer

1998

Event-by-Event Fluctuations



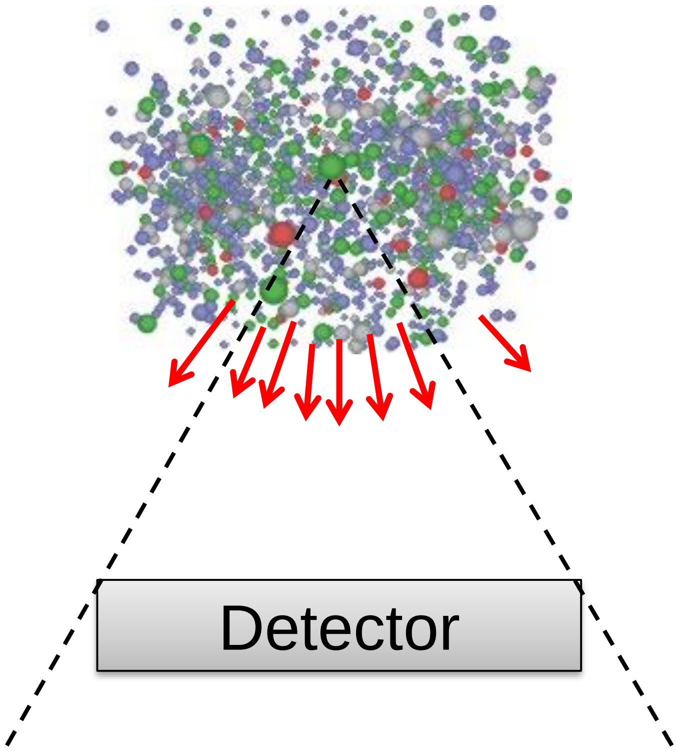
STAR, PRL**105** (2010)

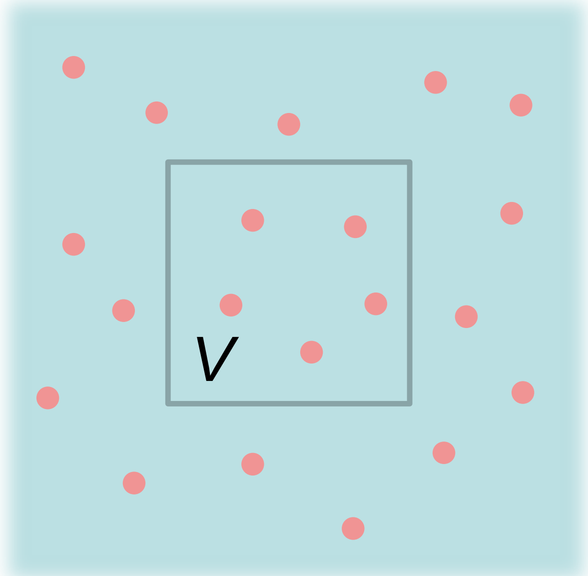


Form of distribution reflects microscopic properties

Cumulants: $\langle \delta N_p^2 \rangle, \langle \delta N_p^3 \rangle, \langle \delta N_p^4 \rangle_c$

Event-by-Event Fluctuations





Form of distribution reflects microscopic properties

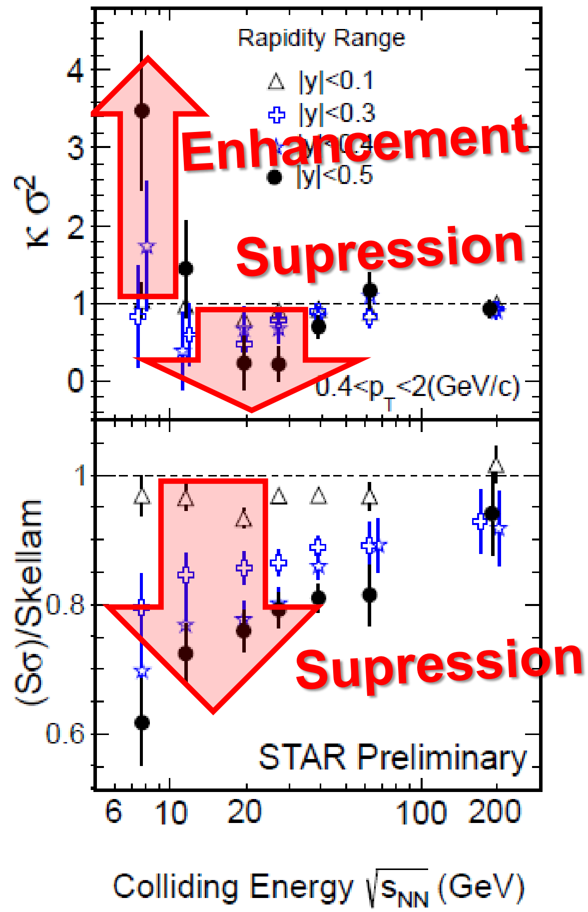
Cumulants: $\langle \delta N_p^2 \rangle, \langle \delta N_p^3 \rangle, \langle \delta N_p^4 \rangle_c$

Higher Order Cumulants

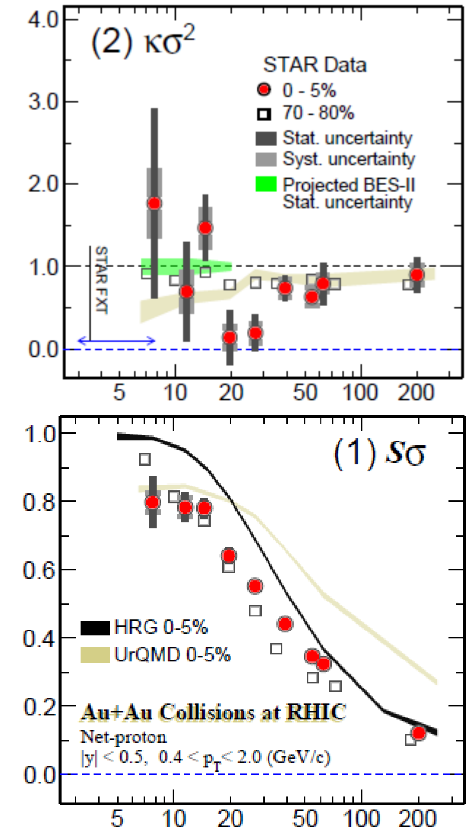
STAR, 2015

$$\frac{\langle N_p^4 \rangle_c}{\langle N_p^2 \rangle_c}$$

$$\langle N_p^3 \rangle_c$$



STAR, 2020 (2001.02852)



- Higher order cumulans have non-trivial enhancement and suppressions ➔ signal of the QCD critical point?

Why Fluctuations?

A Coin Game

① Bet 25 Euro
② You get head coins of

A. 50 x 1 Euro

B. 25 x 2 Euro

Same expectation value.

A Coin Game

① Bet 25 Euro

② You get head coins of

A. 50 x 1 Euro

B. 25 x 2 Euro

C. 1 x 50 Euro

Same expectation value.

But, different fluctuation.

Fluctuations in HIC: 2^{nd} Order

Search for QCD CP

Fluctuation
increases

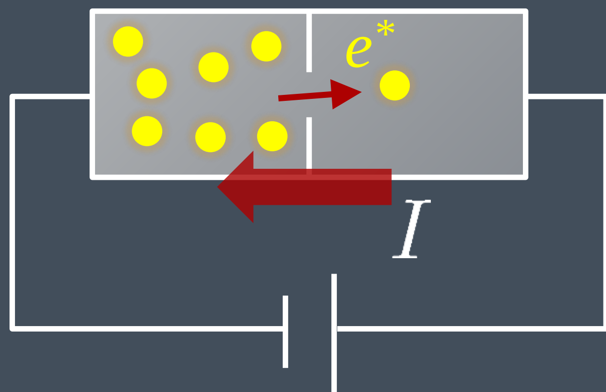
Stephanov, Rajagopal, Shuryak, 1998; 1999

Onset of QGP

Fluctuation
decreases

Asakawa, Heinz, Muller, 2000;
Jeon, Koch, 2000

Shot Noise



$$S_{\rm shot} \sim \langle \delta I^2 \rangle$$

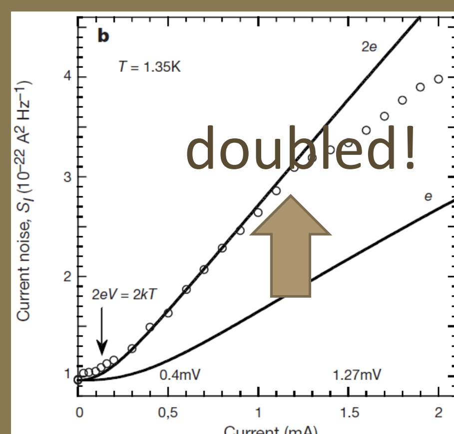
$$S_{\rm shot} = 2e^* \langle I \rangle$$

charge of quasi-particles

Superconductors with Cooper Pairs

$$e^* = 2e$$

Jehl+, Nature **405**,50 (2000)



Fractional Quantum Hall Systems

$$e^* = \frac{q}{p} e$$

Saminadayar+, PRL**79**,2526 (1997)

Higher orders: 3rd: ex. Beenakker+, PRL90,176802(2003)
up to 5th: Gustavsson+, Surf.Sci.Rep.**64**,191(2009)

Higher-order Cumulants

A. 50 x 1 Euro

B. 25 x 2 Euro

$2\langle \delta \text{€}^2 \rangle$  $=$ $\langle \delta \text{€}^2 \rangle$ 

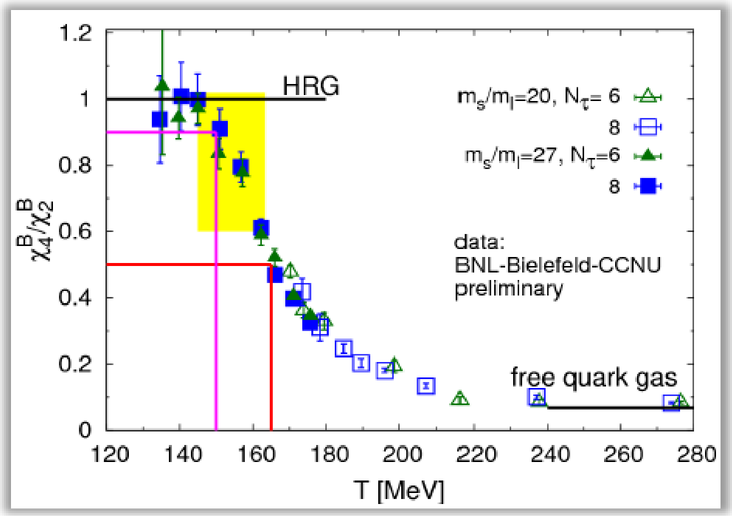
$4\langle \delta \text{€}^3 \rangle$  $=$ $\langle \delta \text{€}^3 \rangle$ 

$8\langle \text{€}^4 \rangle_c$  $=$ $\langle \text{€}^4 \rangle_c$ 

Asakawa, MK,
PPNP 90, 299
(2016)

Non-Gaussian Fluctuations

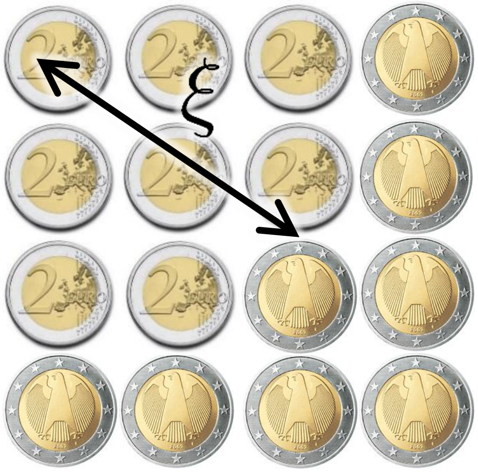
Onset of QGP



Fluctuation
decreases

Ejiri, Karsch, Redlich, 2006

Search for QCD CP



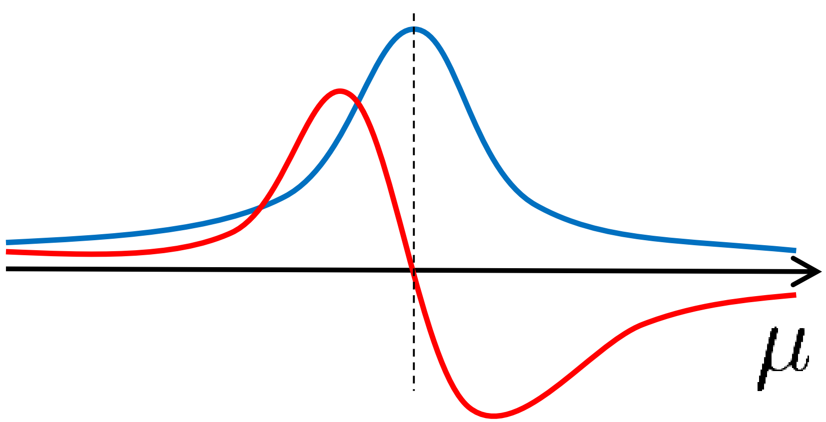
Fluctuation
increases

Stephanov, 2009

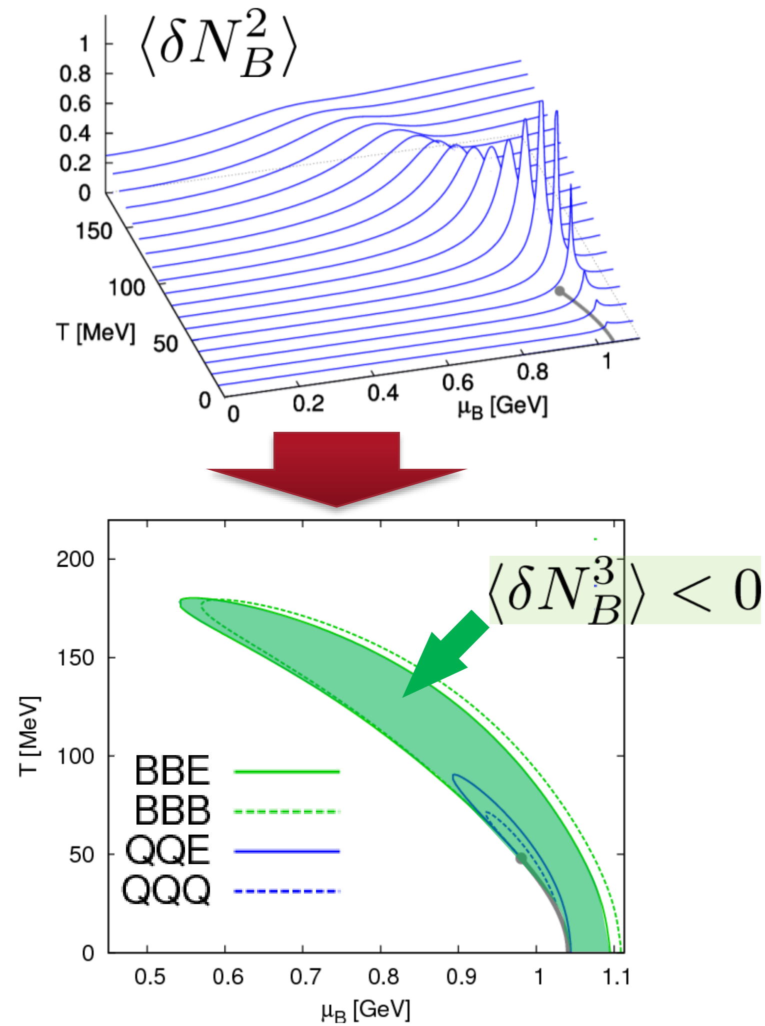
Sign of Higher-order Cumulants

Higher order cumulants can change sign near CP.

$$\langle \delta N^m \rangle = T \frac{\partial \langle \delta N^{m-1} \rangle}{\partial \mu}$$



Asakawa, Ejiri, MK, 2009



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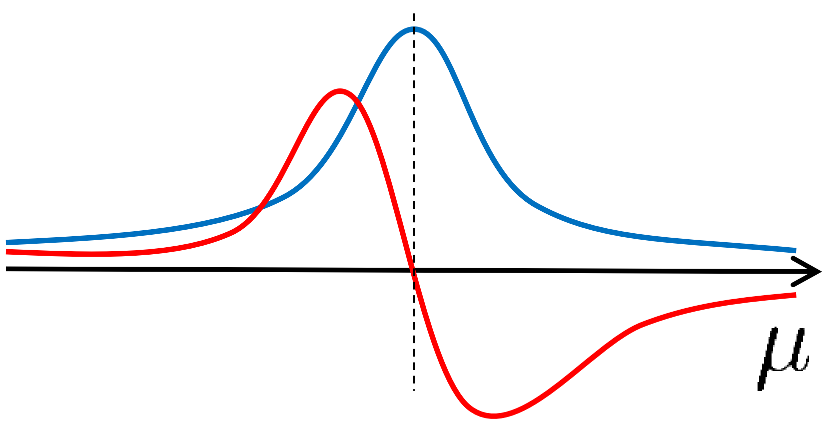
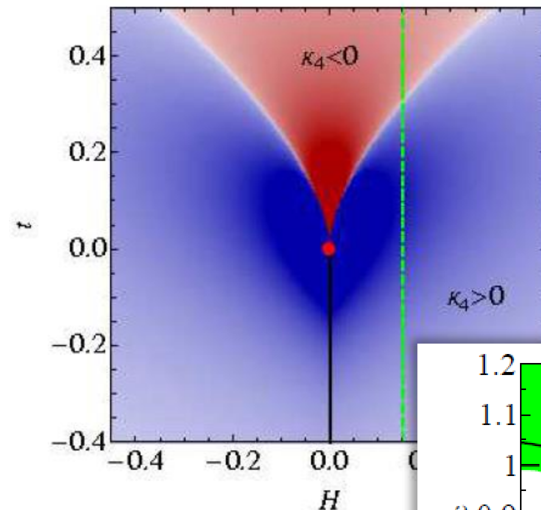
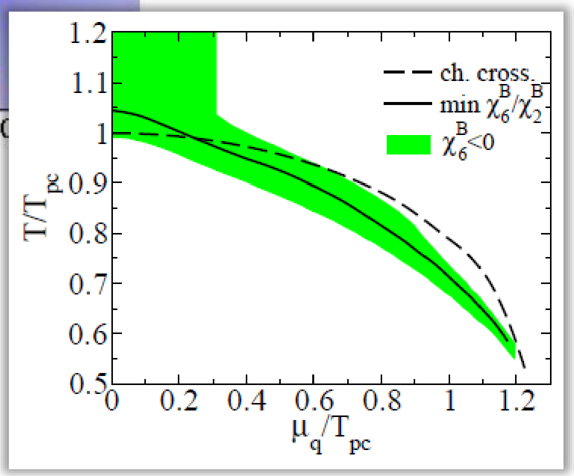






Asakawa, Ejiri, MK, 2009

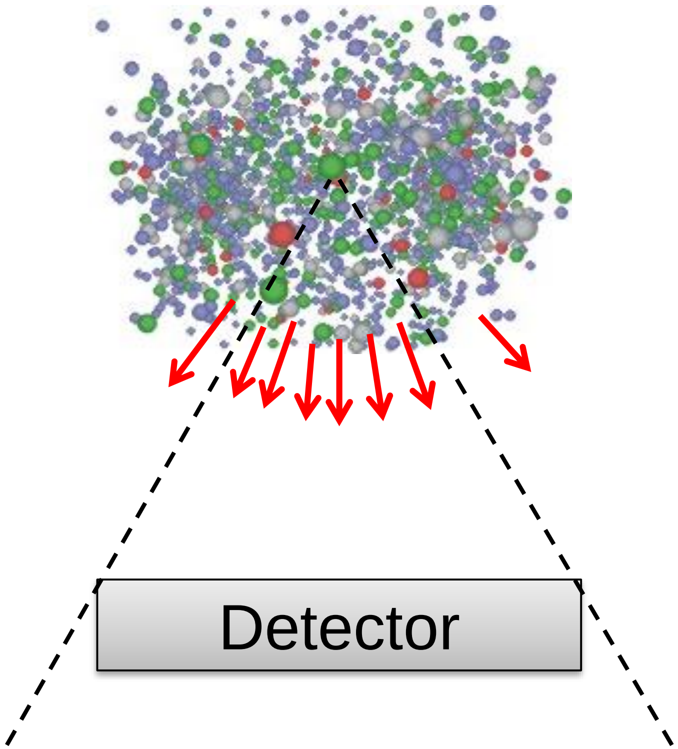
Stephanov, 2011;
Friman, Karsch, Redlich, Skokov, 2011; …

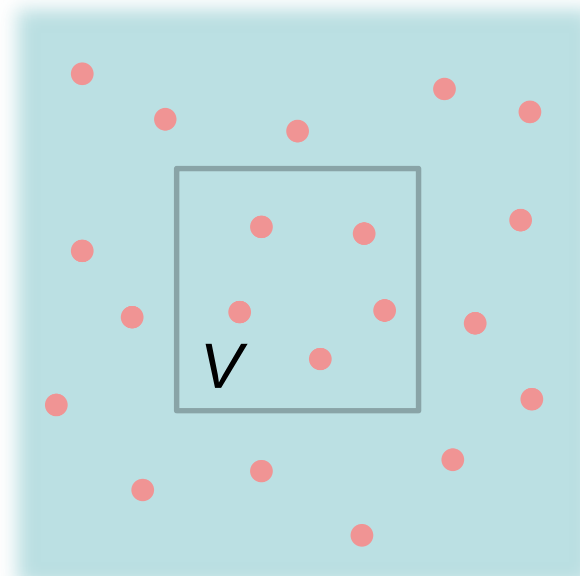
Various Issues

to compare exp. results with thermodynamics

- ☐ Volume (initial) fluctuations
- ☐ Dynamical evolution
- ☐ Resonance decays
- ☐ Measurement in momentum space
- ☐ Difference b/w proton & baryon number flucts.
- ☐ Efficiency/acceptance correction
- ☐ …

Event-by-Event Fluctuations



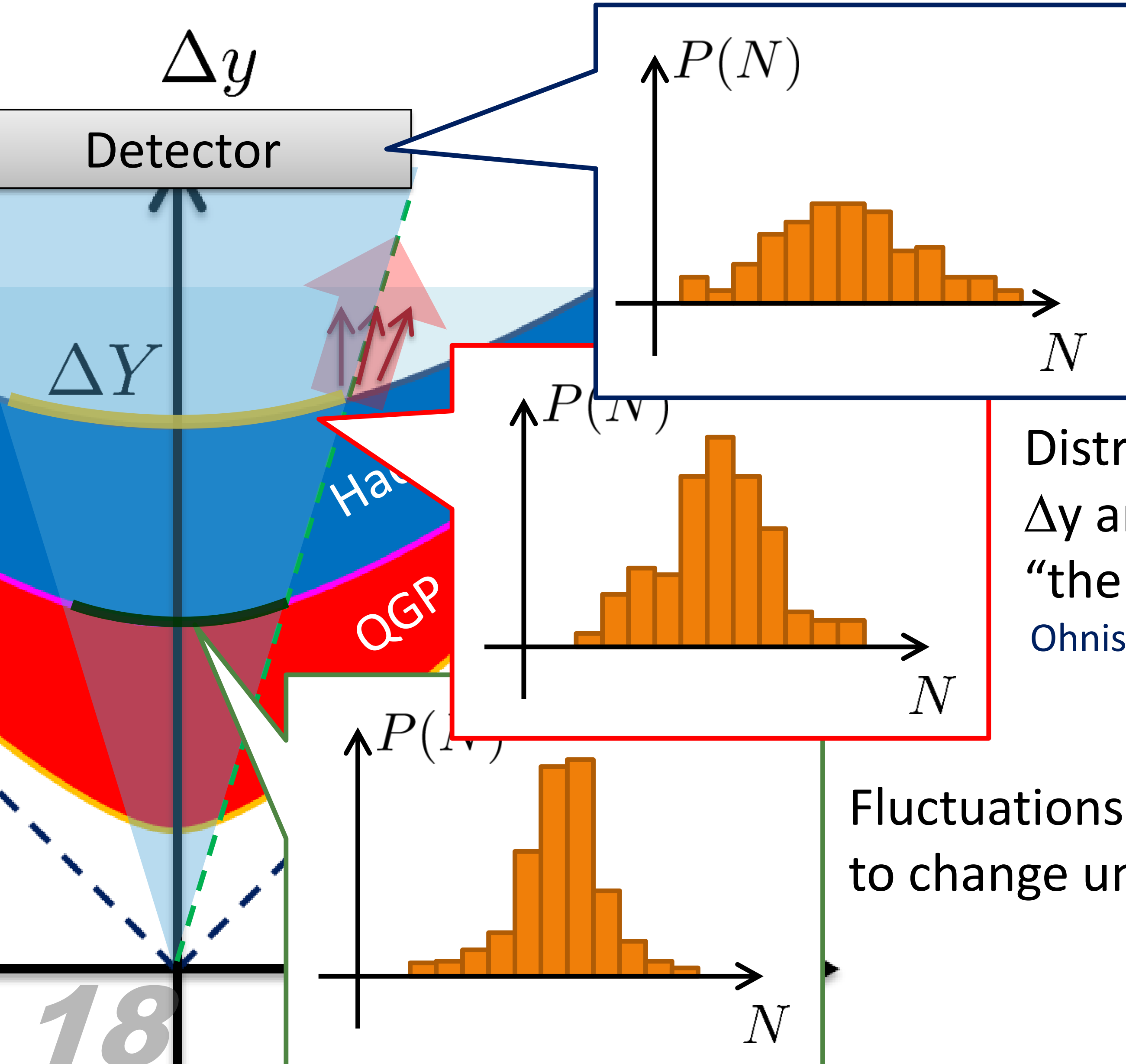


When are observed fluctuations generated?

Observed fluc. ≠ fluc. at chemical freeze out

Asakawa, MK, Muller, PRC (2020)

Time Evolution of Fluctuations

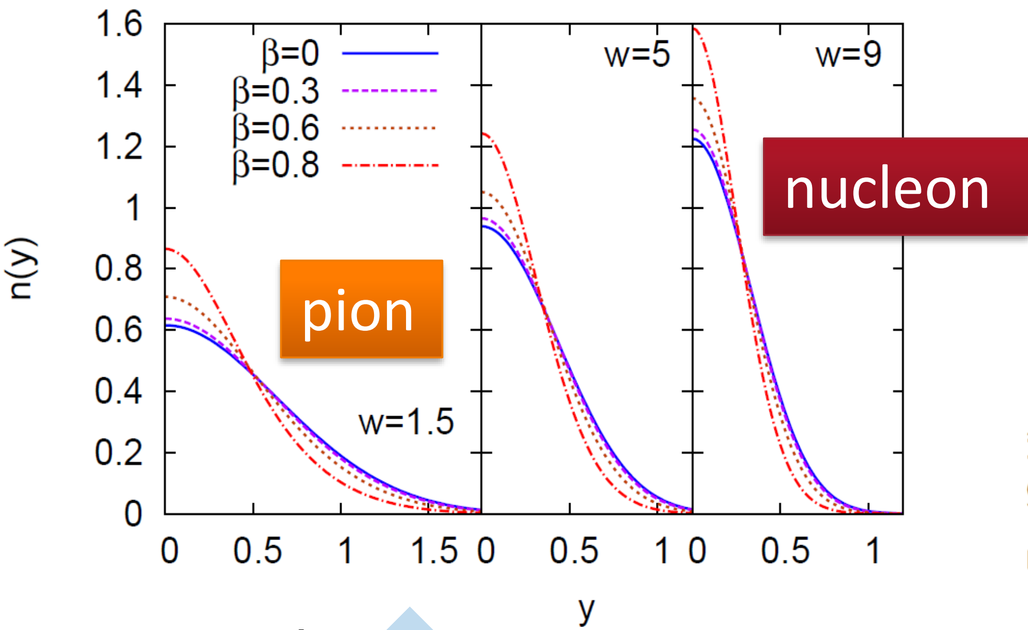


Distributions in ΔY and Δy are different due to "thermal blurring".

Ohnishi, MK, Asakawa, PRC(2016)

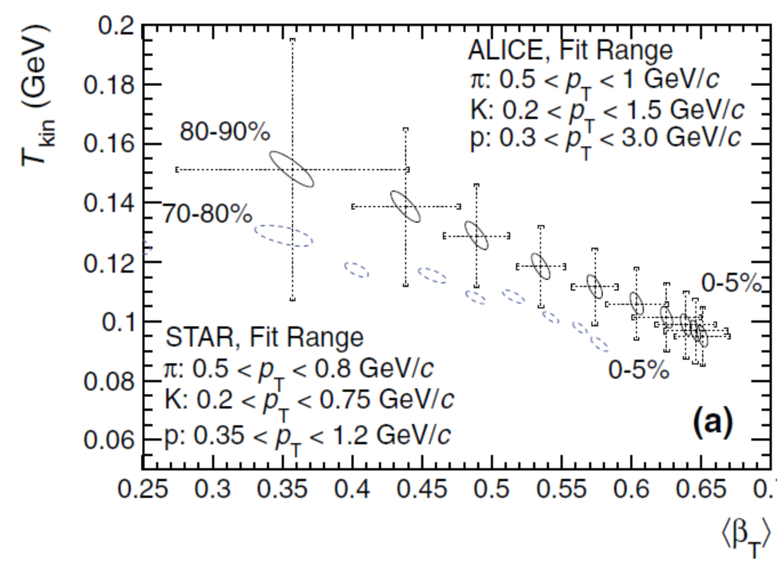
Fluctuations in ΔY continue to change until kinetic f.o.

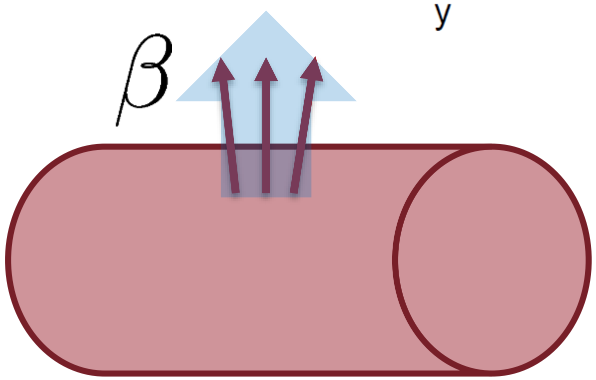
Thermal Distribution in y Space



$$w = \frac{m}{T}$$

- pions $w \simeq 1.5$
- nucleons $w \simeq 9$





Blast wave squeezes the distribution in rapidity space

- assume Bjoroken picture
- blast wave
- flat freezeout surface

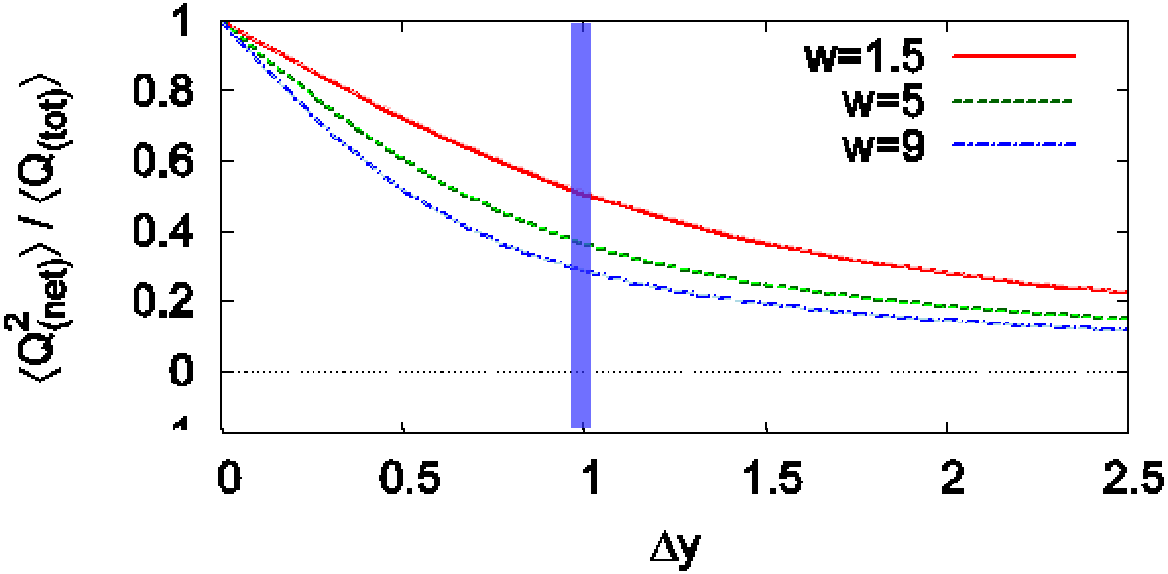
Δy Dependence

Ohnishi, MK, Asakawa, PRC (‘16)

Initial condition
(before blurring)
no e-v-e fluctuations

Cumulants *after* blurring
can take nonzero values

Cumulants after blurring

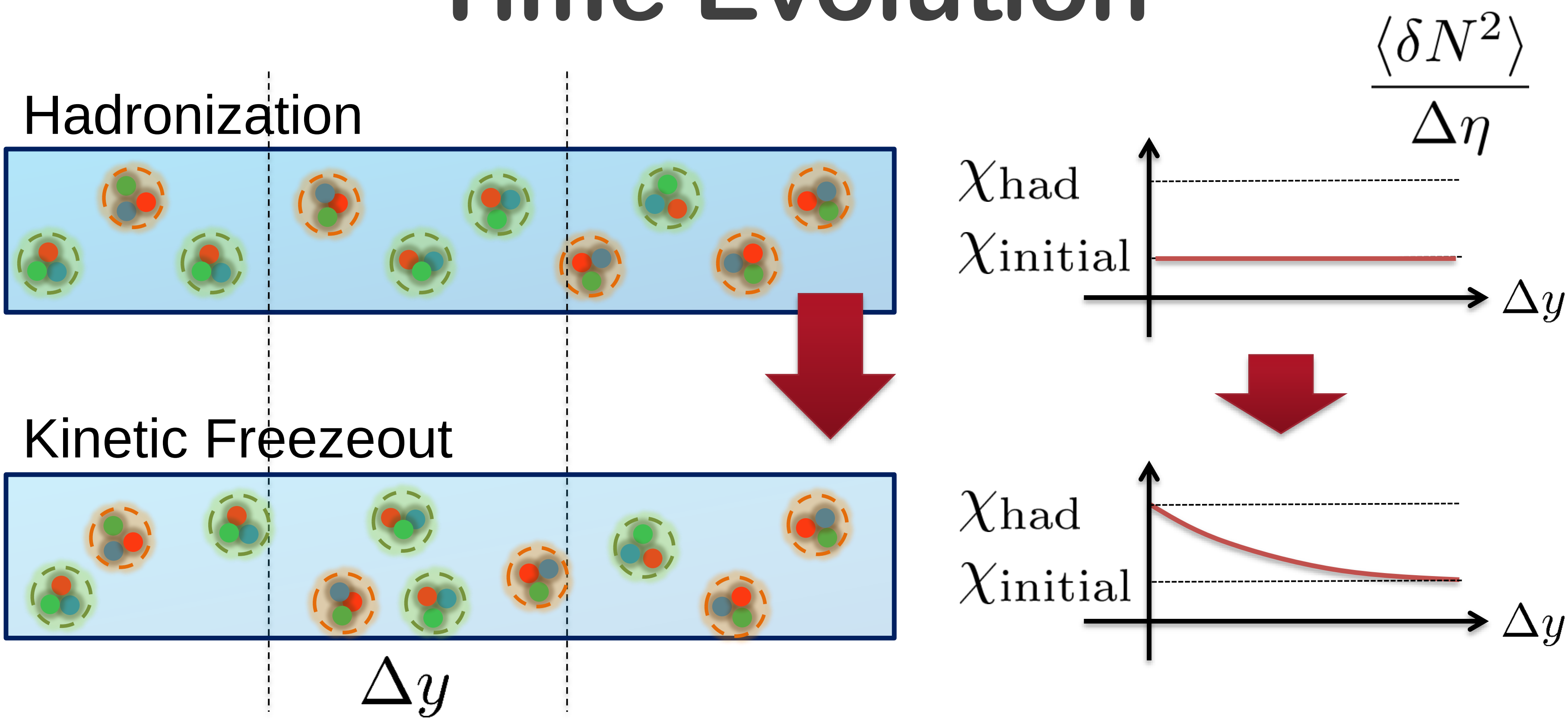


At Δy=1, the effect is
not well suppressed

$$w = \frac{m}{T}$$

- pions $w \simeq 1.5$
- nucleons $w \simeq 9$

Time Evolution



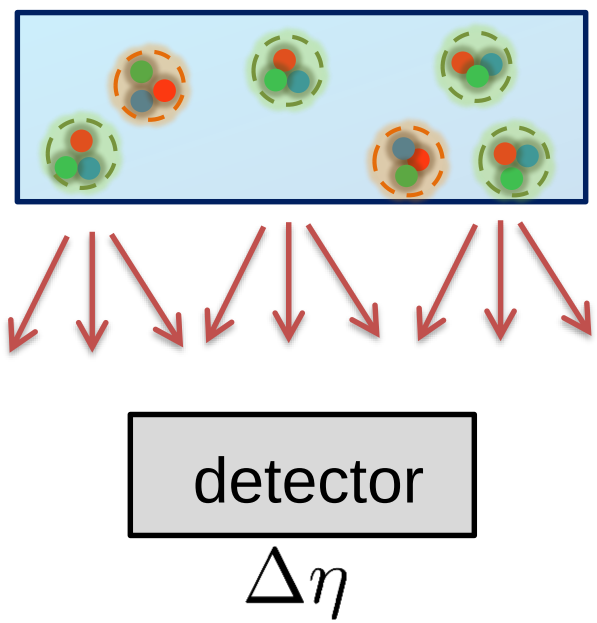
Variation of a conserved charge is achieved only through diffusion. → The larger $\Delta\eta$, the slower diffusion

Same physics as balance func. / 2-particle correlation

Pratt, Steinhorst, 2008.08623

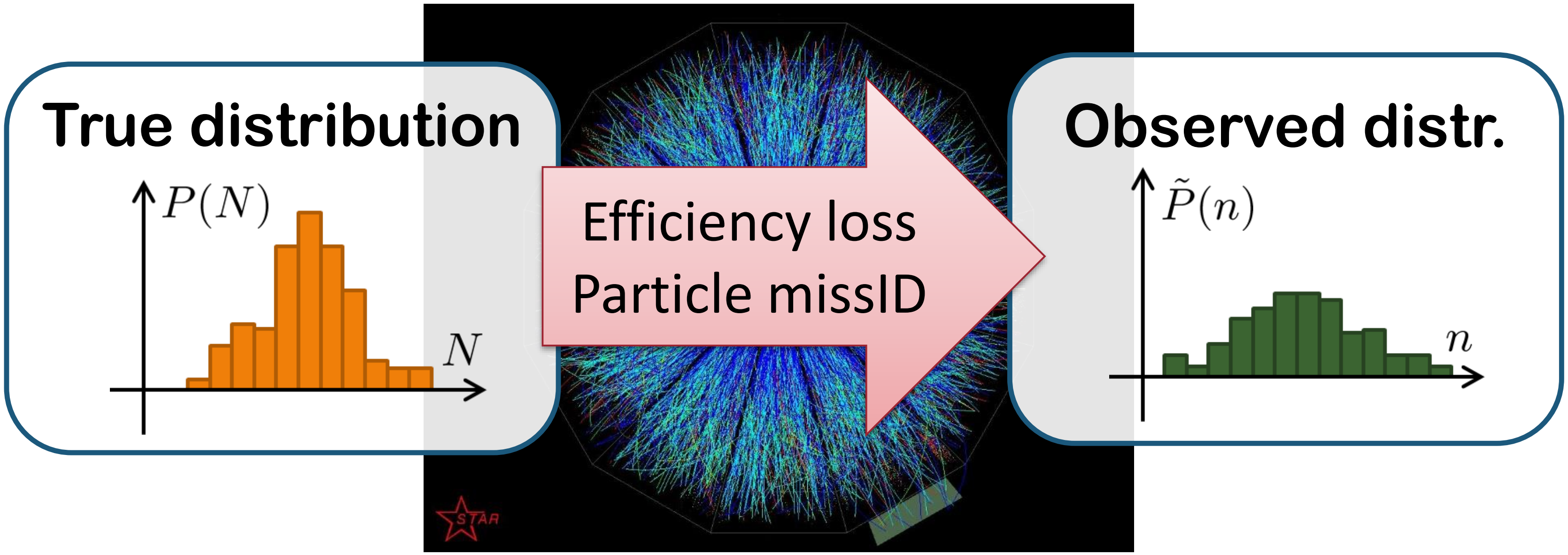
Low Energy Collisions

- Large contribution of global charge conservation
- Violation of Bjorken scaling



Careful treatment is required to interpret fluctuations at low beam energies!

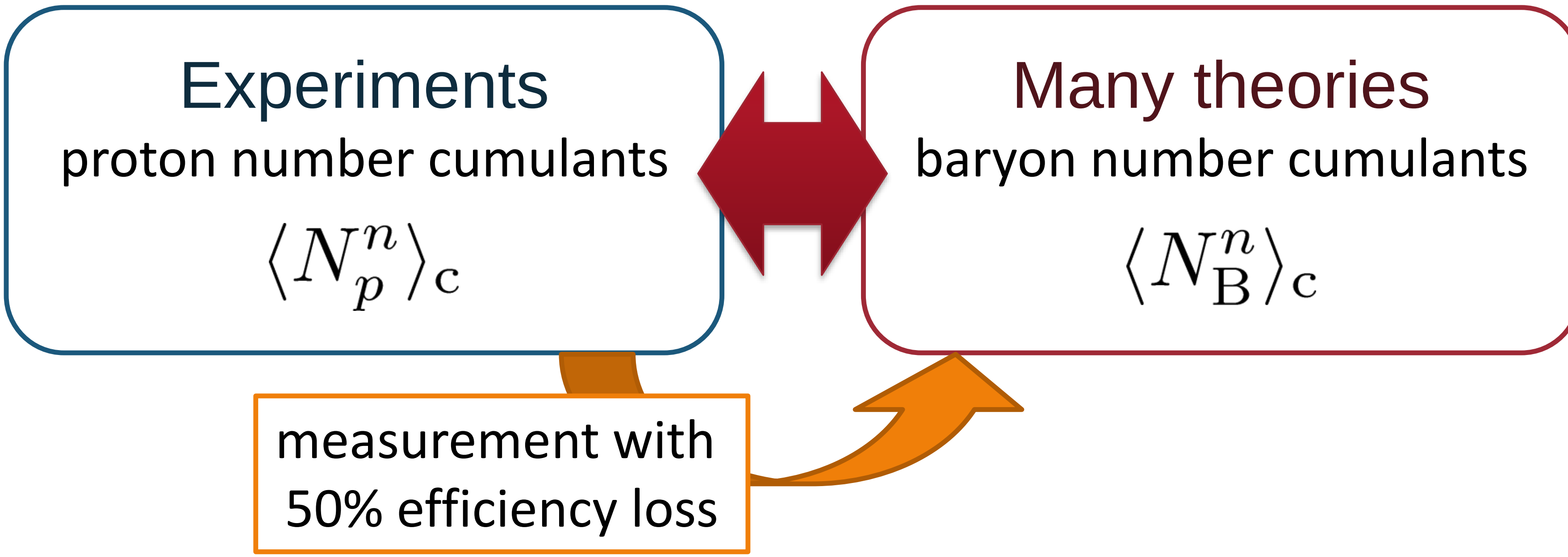
Detector-Response Correction



- **Correction assuming a binomial response**
  Bialas, Peschanski (1986);
  MK, Asakawa (2012); Bzdak, Koch (2012); ....
- **Correction for general case**
  Nonaka, MK, Esumi (2018)

Proton vs Baryon Cumulants

MK, Asakawa, 2012; 2012



- Clear difference b/w proton and baryon # cumulants.
- **Isospin randomization** justifies the reconstruction of $\langle N_B^n \rangle_c$ via the binomial model.
- Similar problem on the **momentum cut**…

Fragile Higher Orders

Ex.: Relation b/w baryon & proton # cumulants

(with approximations)

MK, Asakawa, 2012

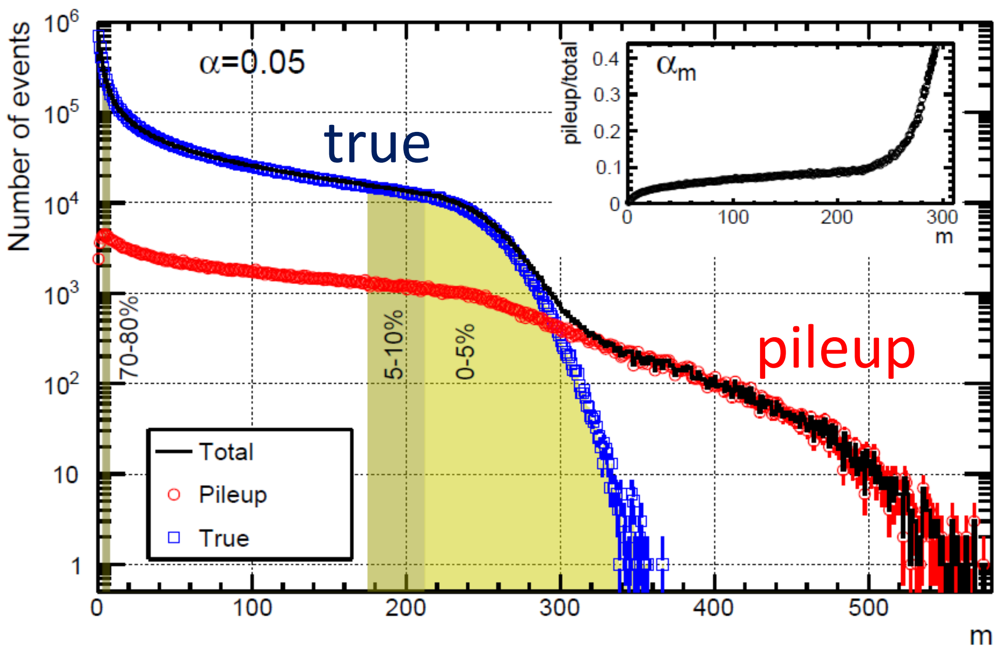
$$\left\{\begin{aligned} 2\langle(\delta N_p^{(\mathrm{net})})^2\rangle &= \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle + \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle_{\mathrm{free}} \\ 2\langle(\delta N_p^{(\mathrm{net})})^3\rangle &= \frac{1}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle + \frac{3}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle_{\mathrm{free}} \\ 2\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c &= \frac{1}{8}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^4\rangle_c + \cdots \end{aligned}\right.$$

genuine info. | Poisson noise

- Higher orders are more seriously affected by efficiency loss.
- Reconstruction of true cumulants is more desirable.
- Reliable analysis is more difficult for higher orders.

Pileup Correction

Nonaka, MK, Esumi, NIM, in press, 2006.15809

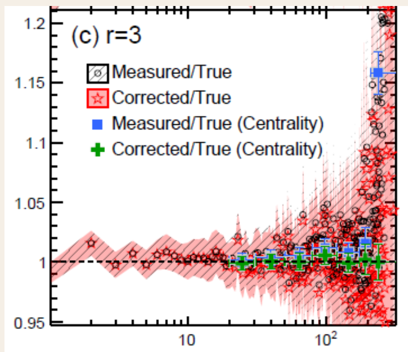


“Pileup events”
Collisions with small space-time interval identified as a single event.

Fluctuations are distorted by the pileup events.

Sonbun+ ('18); Garg, Mishra ('17)

- This effect can be corrected by a systematic procedure.
- Strong effects at the most central bin.

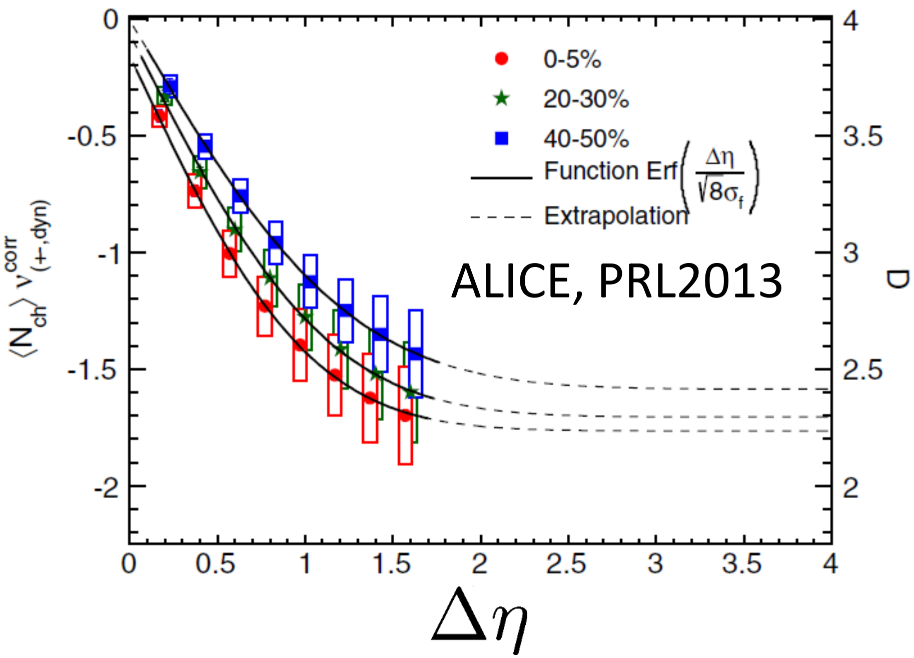


Fluctuations in Dynamically Expanding Systems

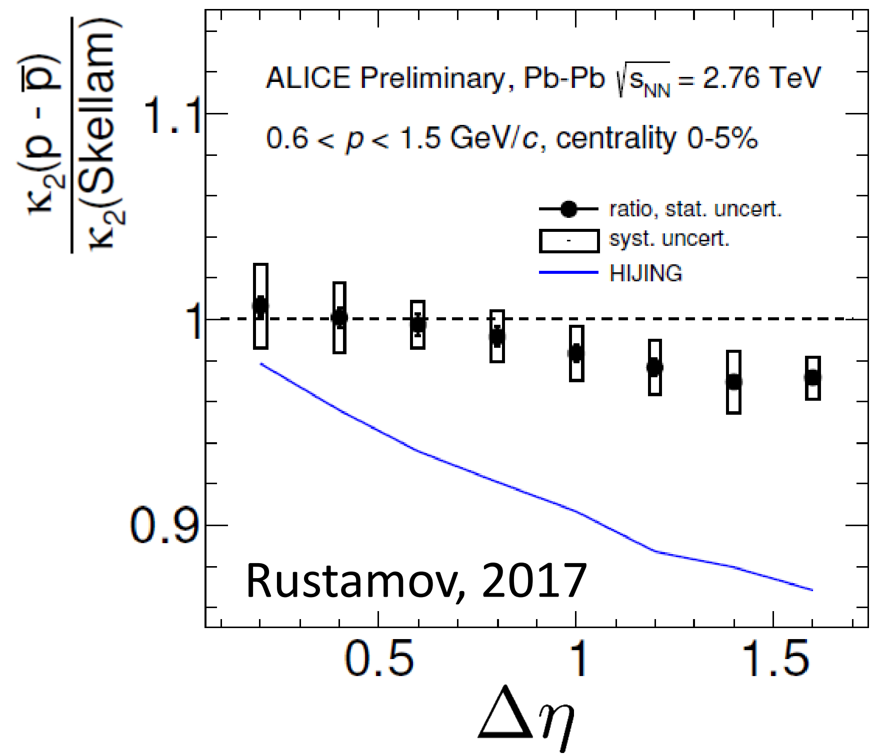
Thanks to:
A. Chatterjee, M. Bluhm, S. Esumi,
M. Nahrgang, T. Nonaka

2^{nd} Order @ ALICE

Net charge fluctuation



Net proton fluctuation



- Net-charge fluctuation has a suppression,
- but net-proton fluctuation does not. Why??
- Earlier fluctuations or resonance decays?

What is the VOLUME?

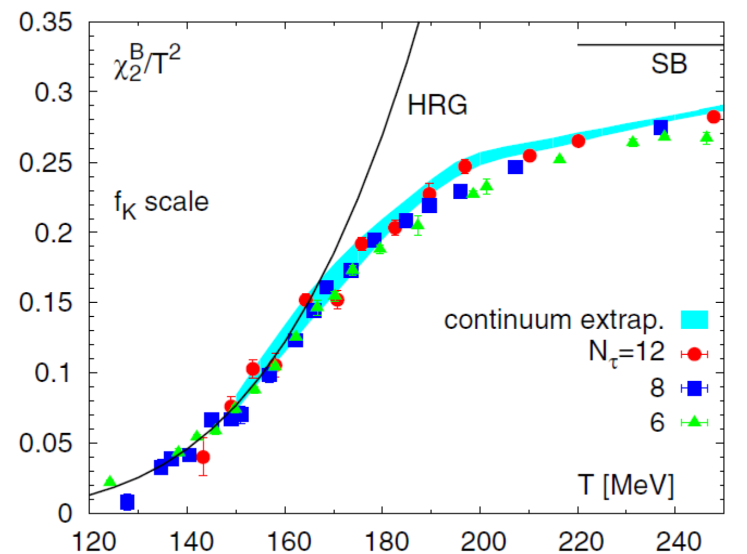
$$\langle \delta N^2 \rangle = \chi_2 V$$

fluctuation in volume V

susceptibility (lattice observable)

- $\chi_2 \to T^3$ in the high T limit
- In HIC, $V \to 0$ as $\tau \to$ small

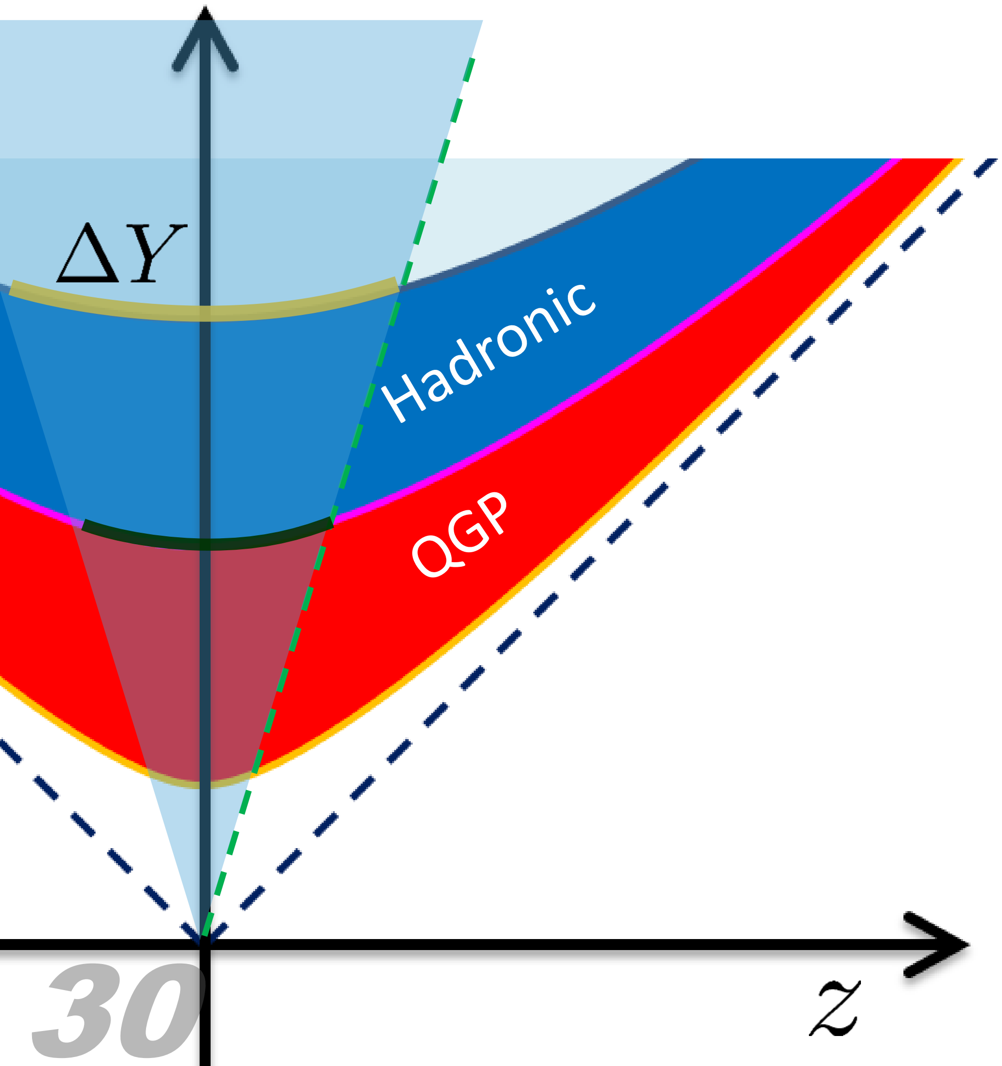
How do these effects compete in real evolution?



HotQCD, PRD86 ('12)

Revisiting AHM-JK's Argument

Asakawa-Heinz-Müller/Jeon-Koch ('00)



System in a space-time rapidity *ΔY*

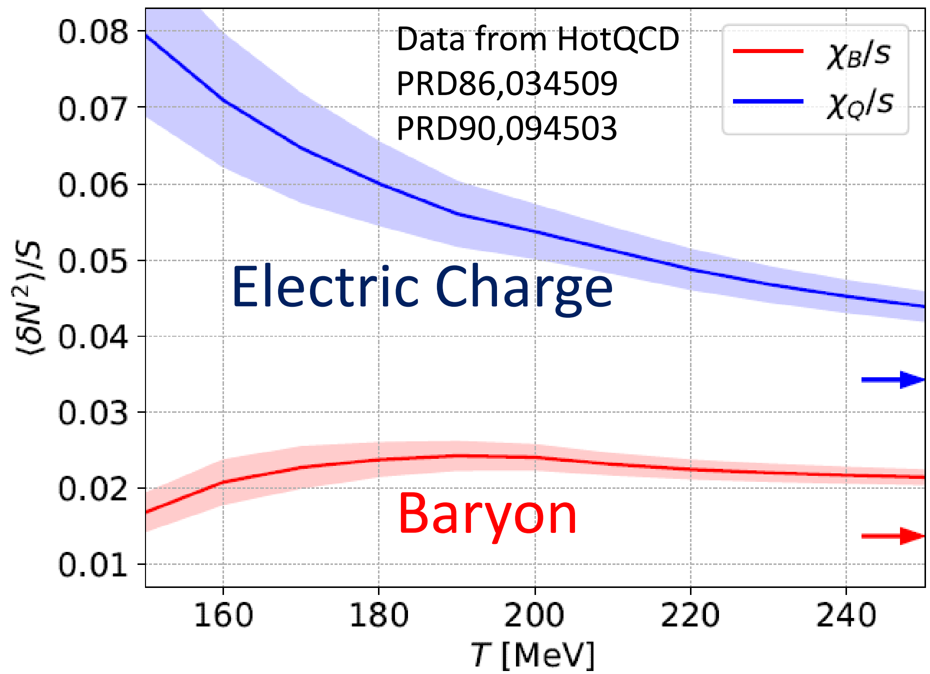
Entropy $S_{\Delta Y}$ is conserved in perfect hydro. expansion

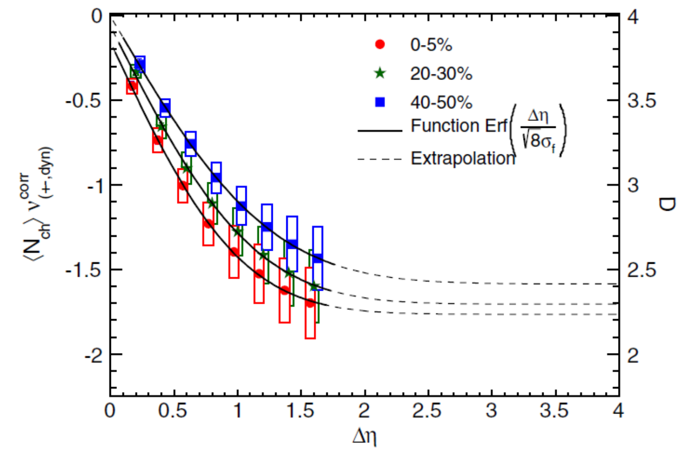
$$S_{\Delta Y} = sV_{\Delta Y}$$

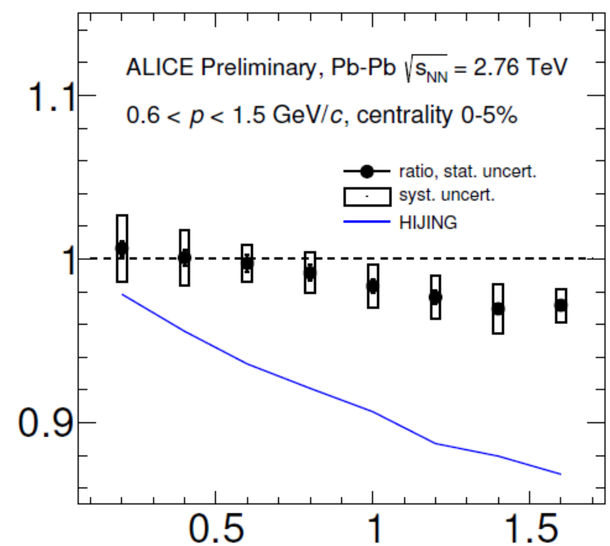
$$\frac{\langle \delta N^2 \rangle_{\Delta Y}}{S_{\Delta Y}} = \frac{\chi_2}{s}$$

Described only by thermodynamic quantities

χ_2/s







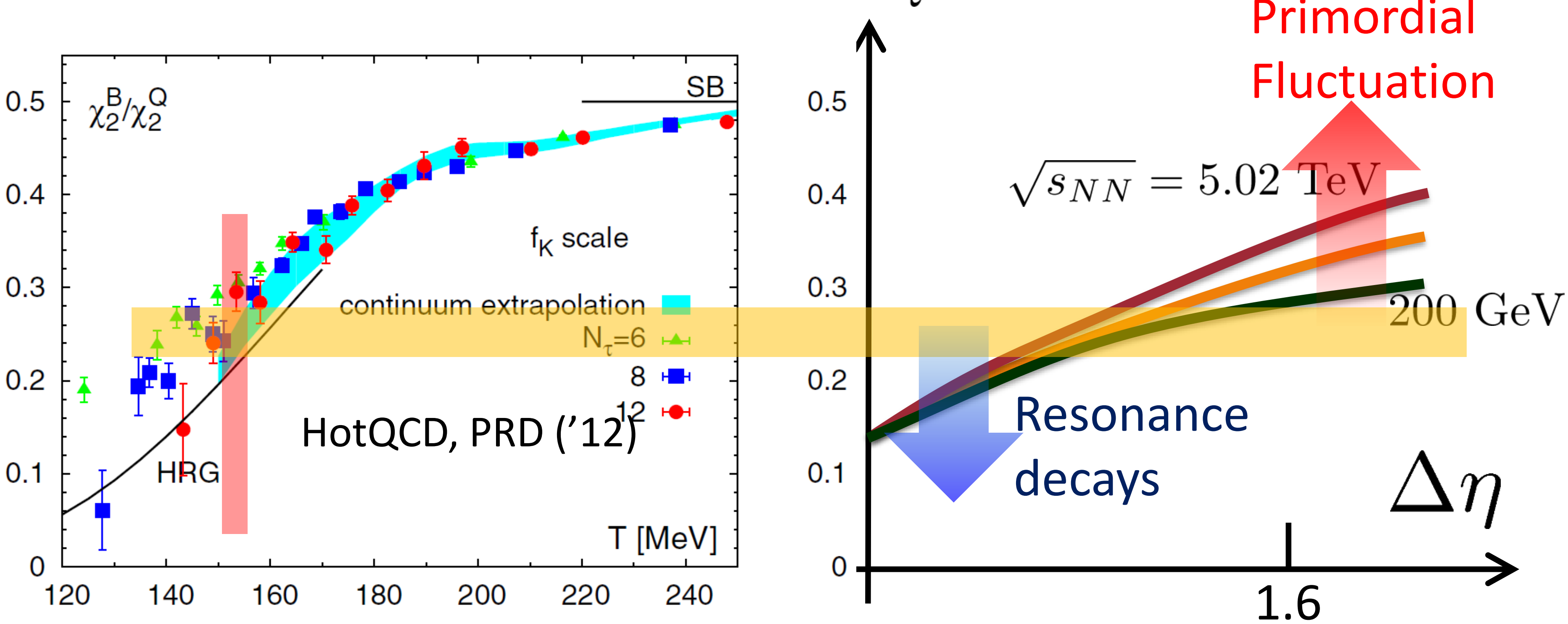
- ☐ Baryon: Almost T independent
- ☐ Electric charge: suppression

➡ Consistent with ALICE results

Presentation by MK
RRTF, GSI, Apr. 2019

B & Q Cumulatns

LATTICE | $\frac{\langle \delta N_B^2 \rangle}{\langle \delta N_Q^2 \rangle}$ | **Experiment**



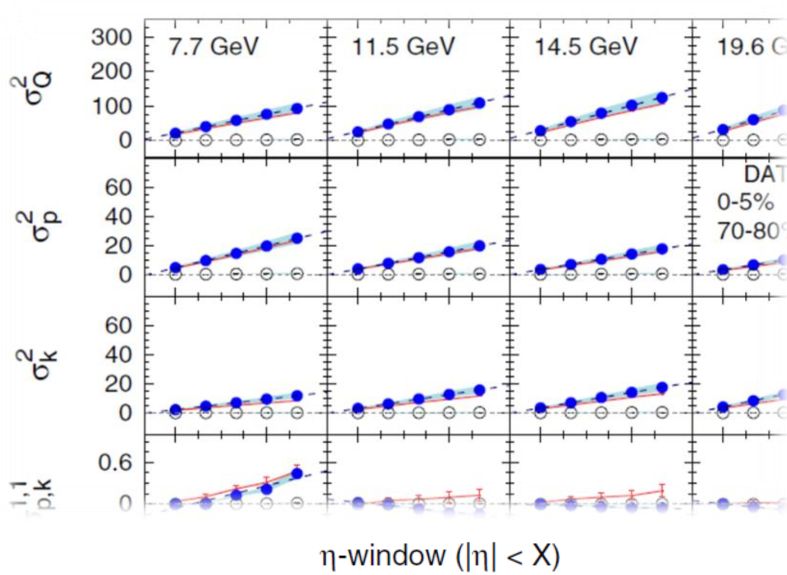
Δη dependence for tracing back the history!

Exp. Result @ STAR

PHYSICAL REVIEW C 100, 014902 (2019)

Collision-energy dependence of second-order off-diagonal and diagonal cumulants of net-charge, net-proton, and net-kaon multiplicity distributions in Au + Au collisions

STAR, PRC**100** ('19)



$\langle \delta N_B^2 \rangle / \langle \delta N_Q^2 \rangle$

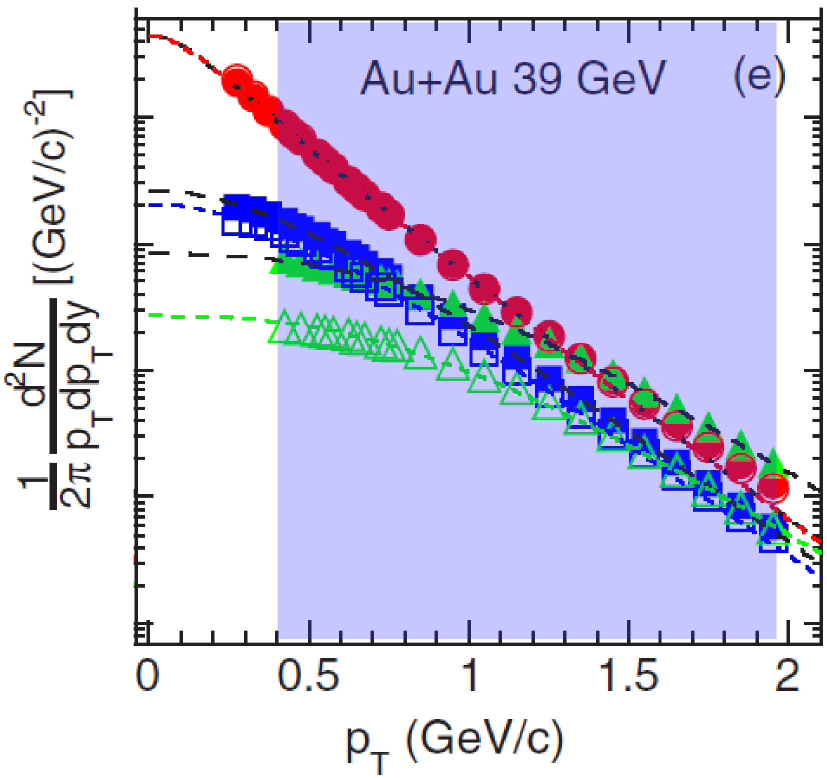
is directly calculable!

total particle numbers:
private comm. with A. Chatterjee

Remarks:

- ☐ Reconstructed baryon # cumulants are used.
- ☐ a non-trivial factor to convert η to y
- ☐ p_T cut correction is imposed

p_T-Cut Correction



Experimental measurement:

$0.4 < p_T < 2.0$ [GeV/c]

In the blast wave model,

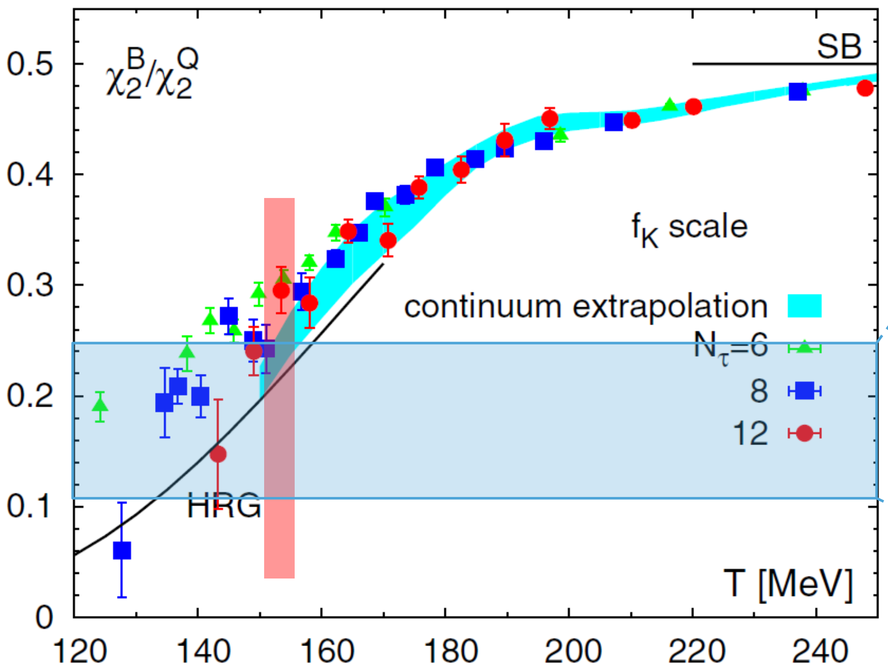
- 54% of charged pions
- 81% of protons

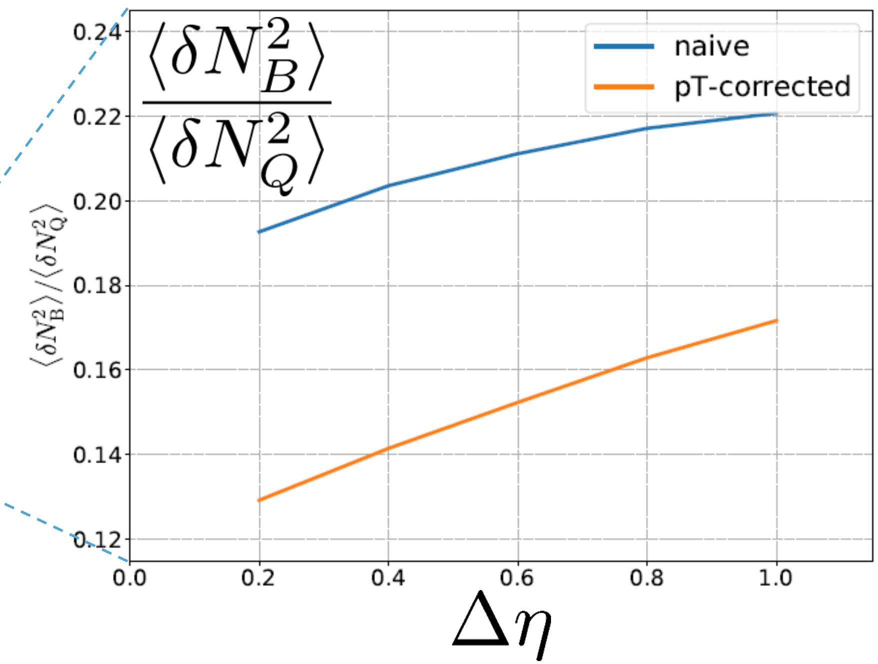
are observed @ $\sqrt{s_{NN}}$=200 GeV

We try acceptance correction via binomial response.

B/Q: Lattice vs Exp.

LATTICE **STAR, 200GeV**



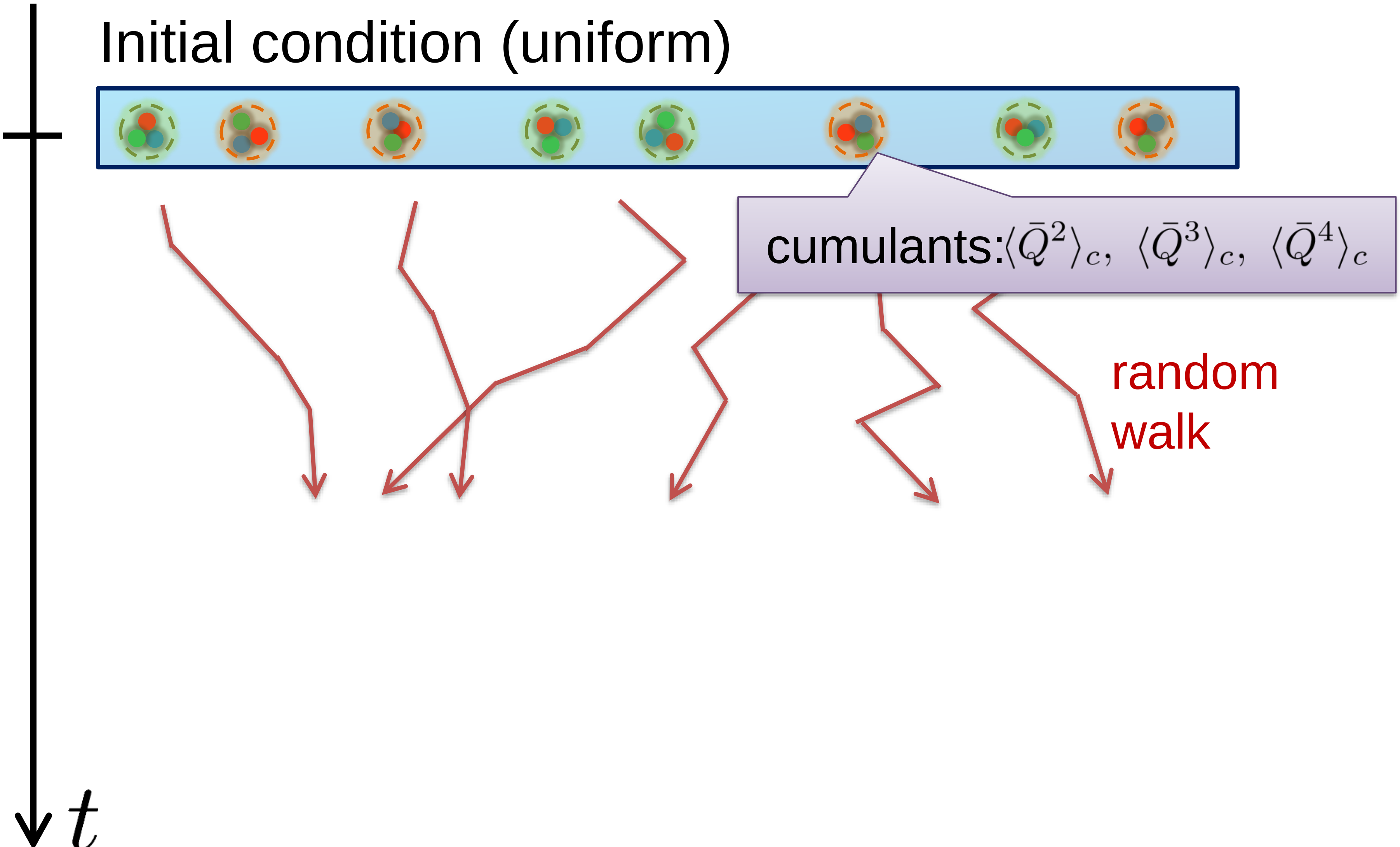


- Extracted "temperature" doesn't agree with thermal model.
- Observed charge fluctuation would be dominated by the effects of resonance decays.
- No direct evidence of primordial fluctuations at $\Delta\eta$ =1.0.

Diffusion of Higher-order Cumulants

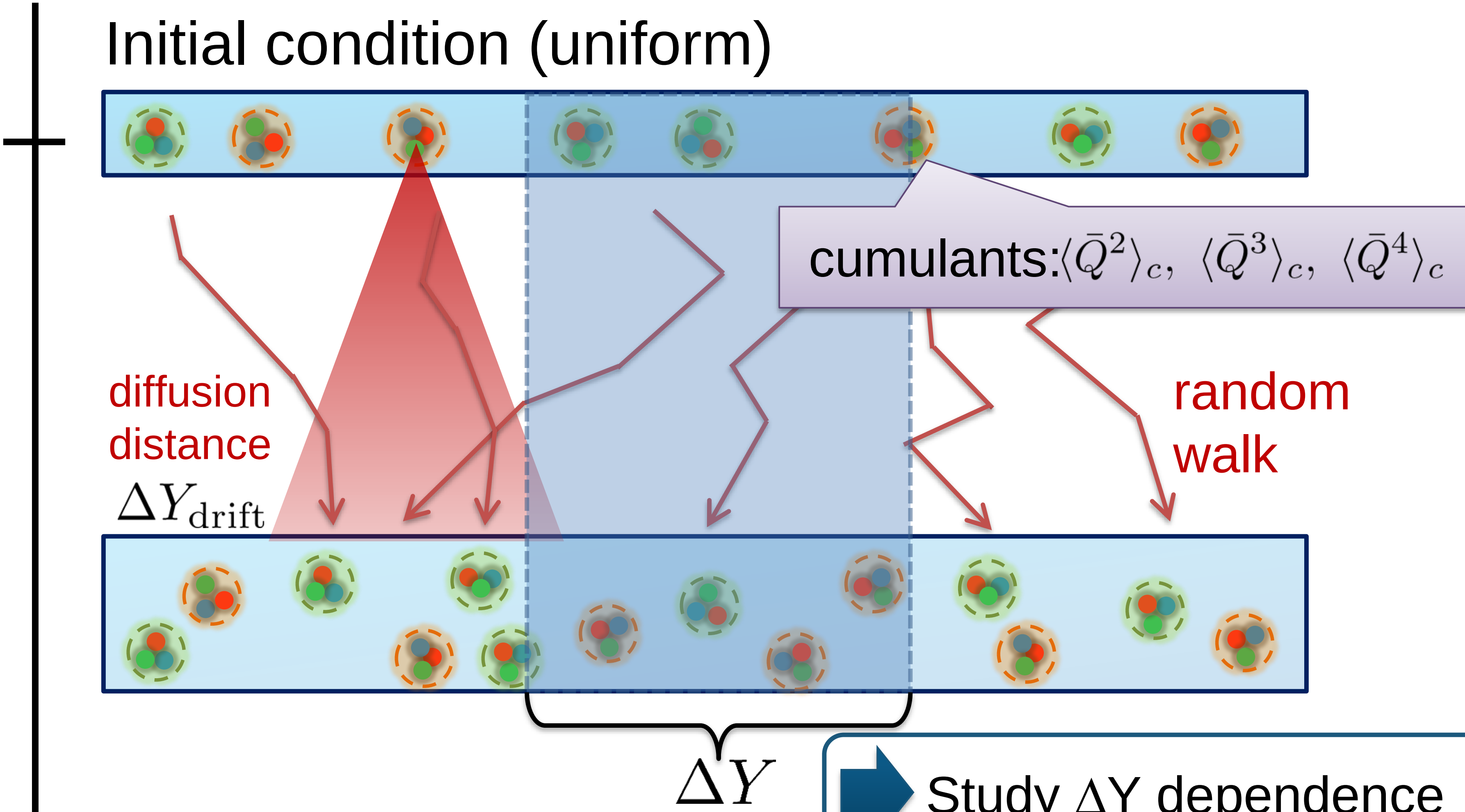
in a simple diffusion model

Diffusion in Hadronic Stage



diffusion master equation: MK+, PLB(2014)
probabilistic argument: Ohnishi+, PRC(2016)

Diffusion in Hadronic Stage

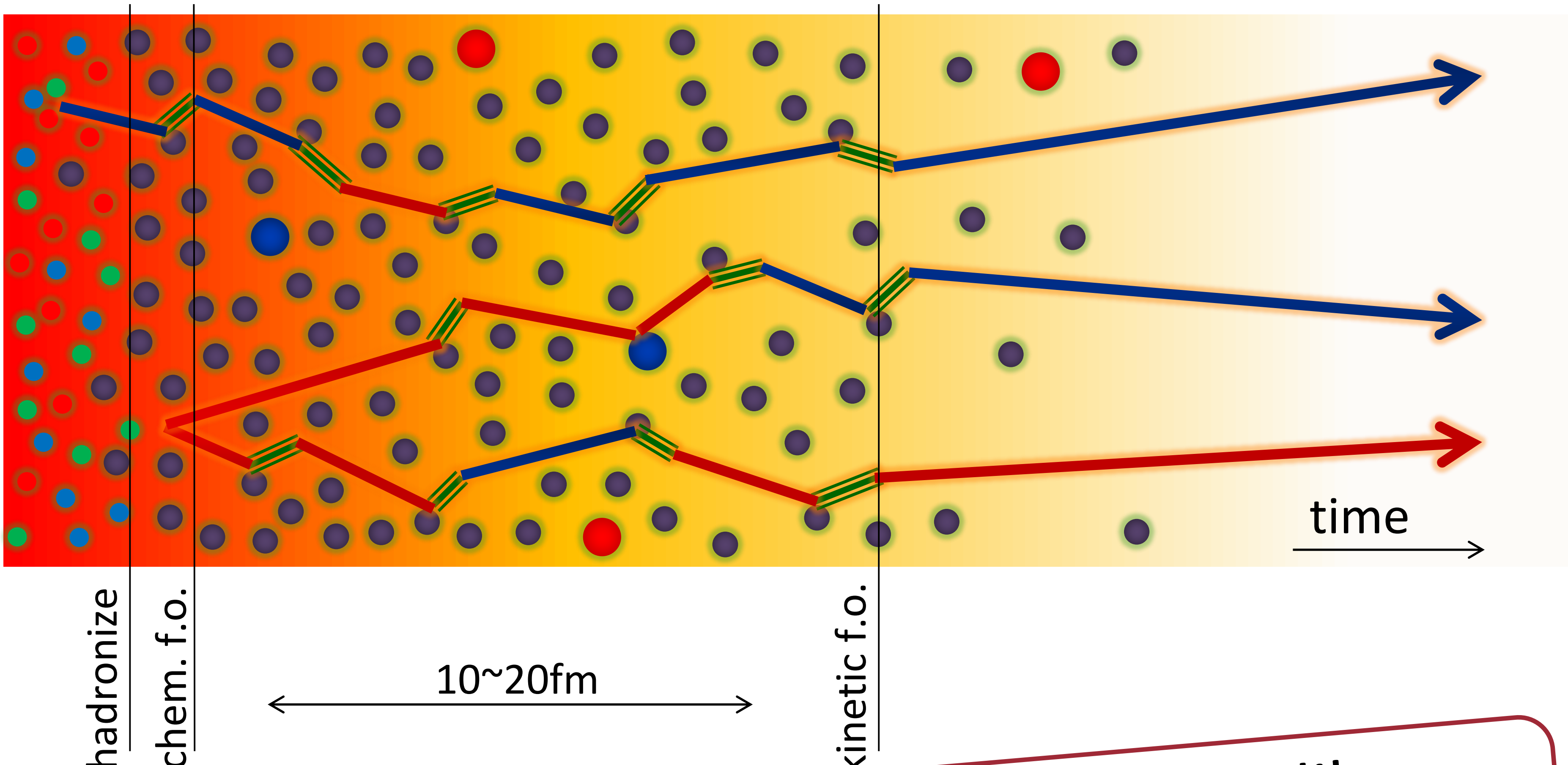


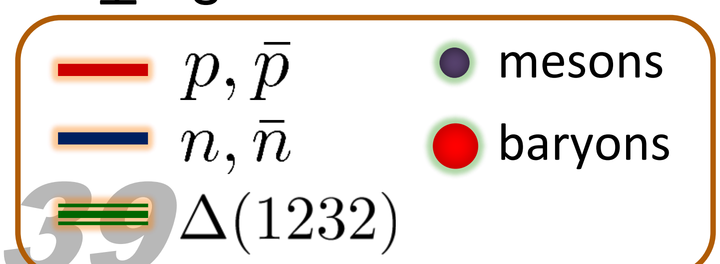
$t \to \infty$
Poisson distribution

➡ Study ΔY dependence

diffusion master equation: MK+, PLB(2014)
probabilistic argument: Ohnishi+, PRC(2016)

Baryons in Hadronic Phase



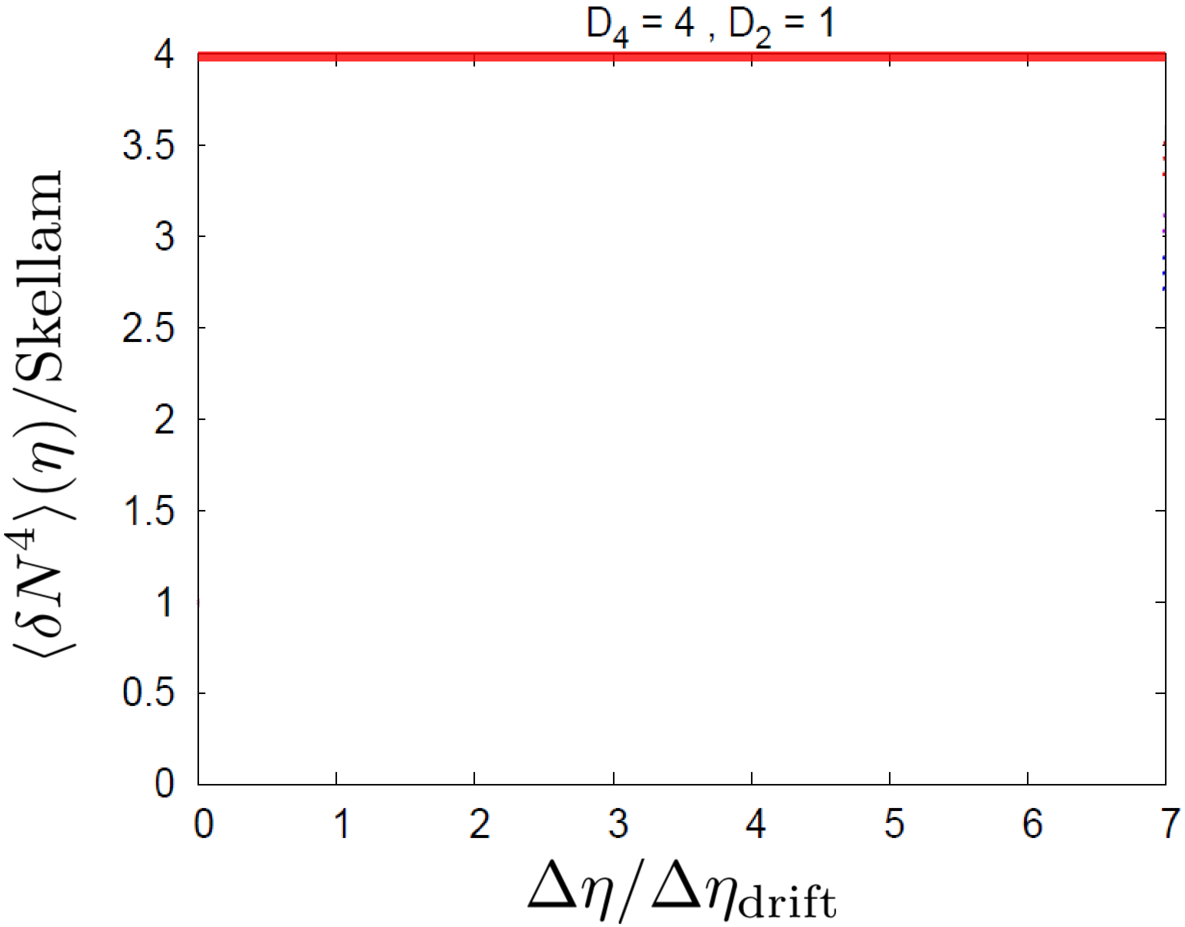


Baryons behave like
Brownian pollens in water

4^{th} Order Cumulant

MK+ (2014)
MK (2015)

Before the diffusion



Initial Condition

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 4$$

$$b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

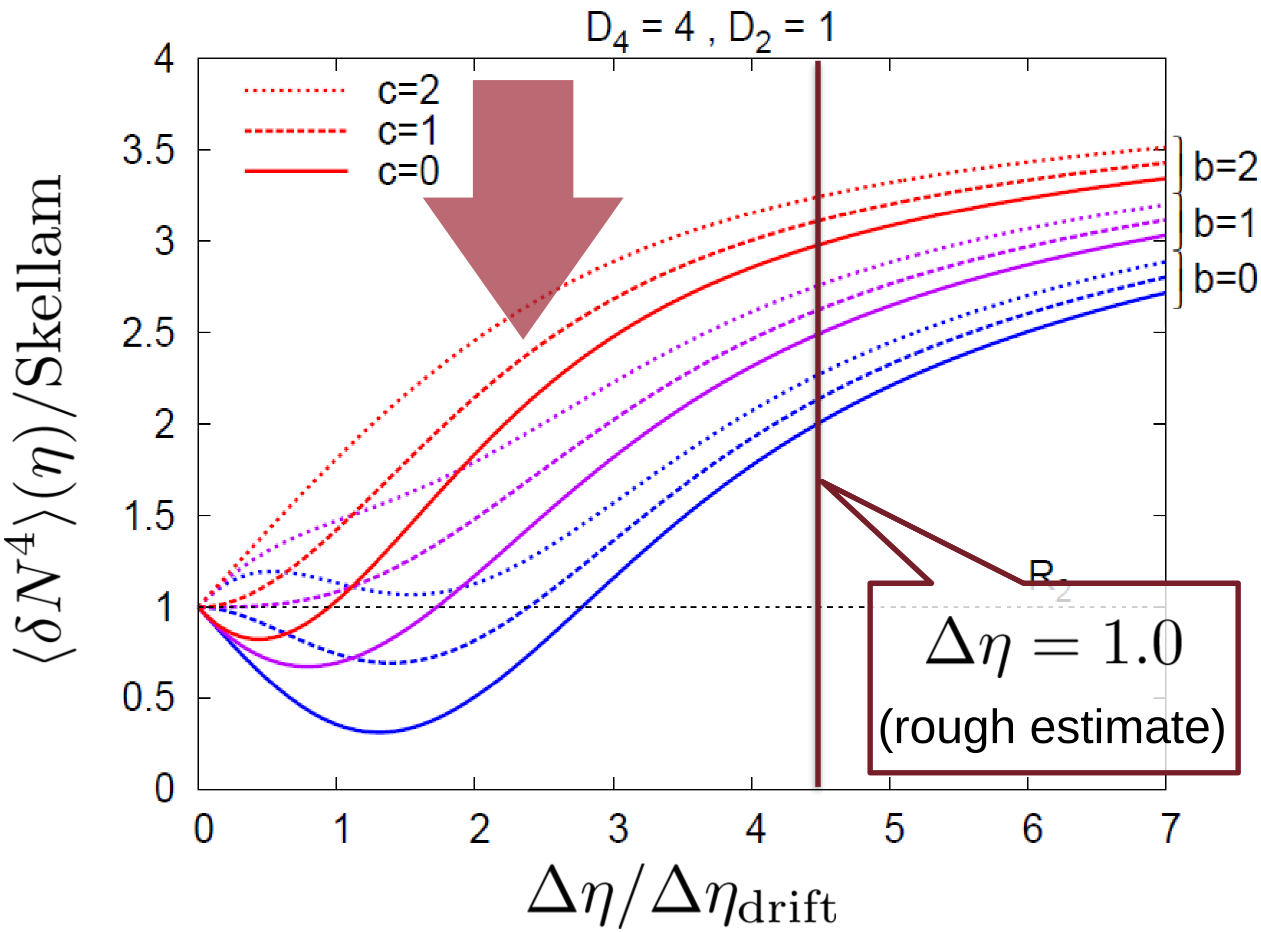
$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 1$$

4^{th} Order Cumulant

MK+ (2014)
MK (2015)

After the diffusion



Initial Condition

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 4$$

$$b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 1$$

- Cumulant at small $\Delta\eta$ is modified toward a Poisson value.
- Non-monotonic behavior can appear.

Rapidity Window Dep.

4th-order cumulant

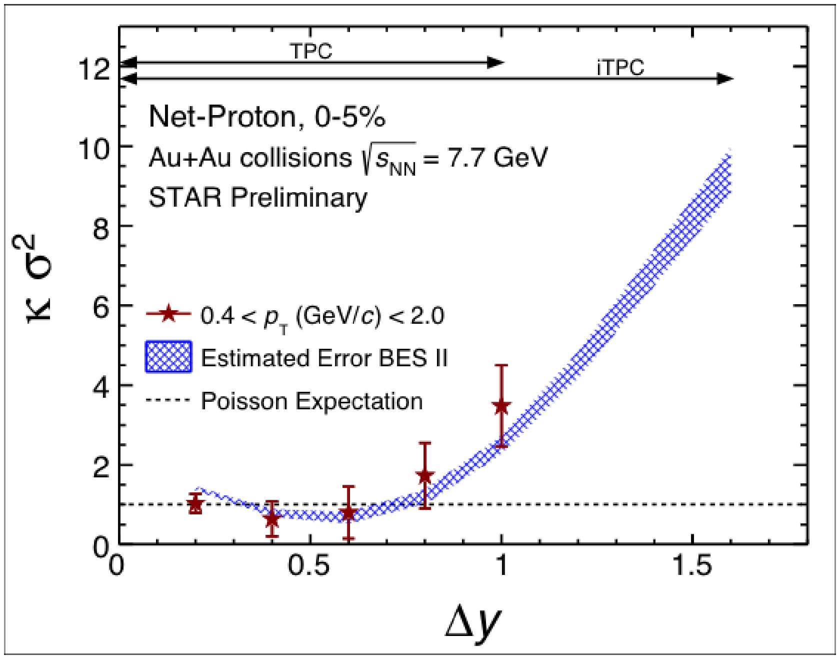
MK+, 2014
MK, 2015

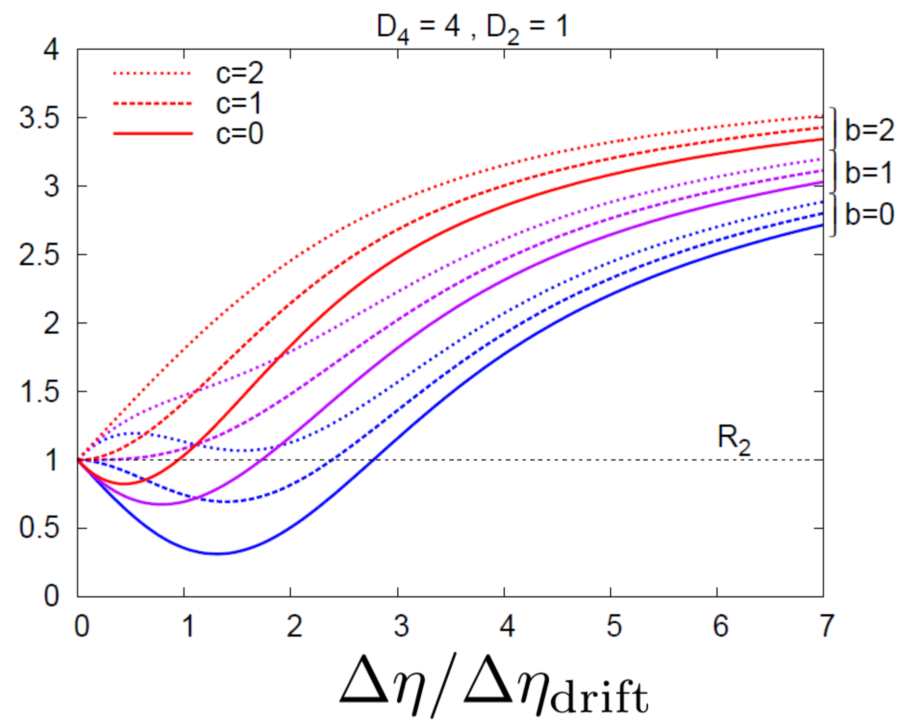
Initial Conditions

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle} \qquad b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})}\rangle_c}{\langle Q_{(\mathrm{net})}\rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle} \qquad c = \frac{\langle Q^2_{(\mathrm{tot})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle}$$

STAR Collab. (X. Luo, CPOD2014)





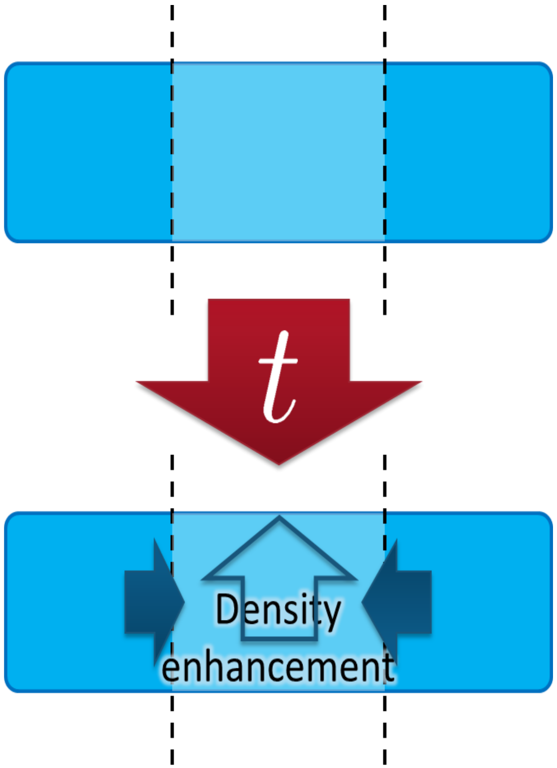
- Is non-monotonic Δη dependence already observed?
- Factorial cumulants will give us better understanding.

Ling, Stephanov, PRC ('16); MK, Luo, PRC('17)

Finite volume effects: Sakaida+, PRC90 (2015)

Evolution of Cumulant near QCD-CP

Evolution of Conserved-Charge Fluctuations



Equations describing transport of *n*:

- Diffusion Equation

$$\frac{\partial n}{\partial t} = D\nabla^2 n$$

- Stochastic Diffusion Equation (SDE)

$$\frac{\partial n}{\partial t} = D\nabla^2 n + \nabla\xi(x,t)$$

- SDE with non-linear terms

$$\frac{\partial n}{\partial t} = \kappa\nabla^2 \frac{\delta\mathcal{F}}{\delta n} + \frac{\partial}{\partial x}\xi(x,t)$$

$$\langle\xi(1)\xi(2)\rangle = 2D\chi_2\delta(1-2)$$

$$\mathcal{F} = \int dx\left(a\Delta n^2 + c(\nabla n)^2 + \lambda_3\Delta n^3 + \cdots\right)$$

Analysis of 2nd-order Cumulant

Sakaida+ (2017)

Evolution of baryon number density

Stochastic Diffusion Equation

$$\partial_t n = D(t)\partial_x^2 n + \partial_x \xi$$

$$\langle \xi(x_1,t_1)\xi(x_2,t_2)\rangle = 2D\chi_2 \delta^{(2)}(1-2)$$

$D(t),\ \chi_2(t)$:parameters characterizing criticality

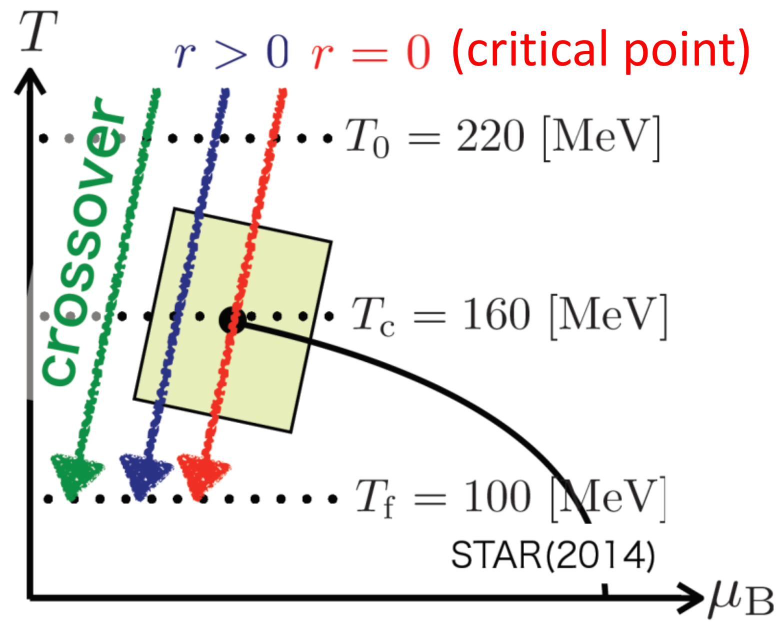
- Analytic solution is obtained.
- Study 2nd order cumulant & correlation function.

Parametrizing D(τ) and χ(τ)

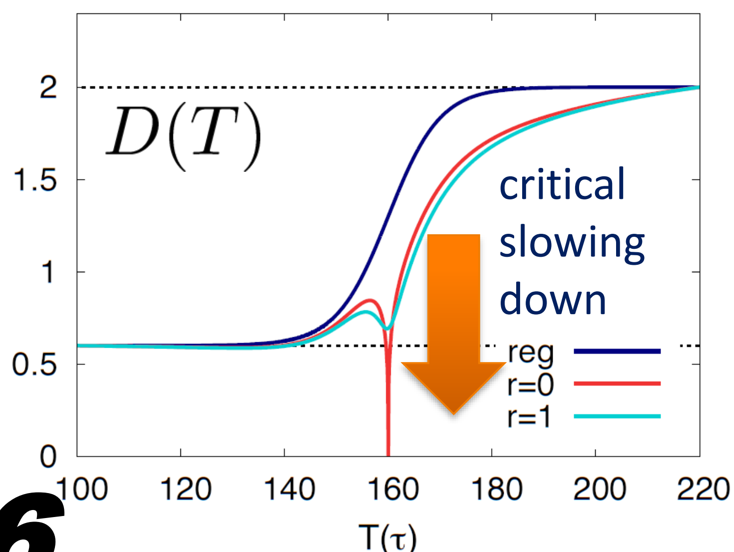
☐Critical behavior

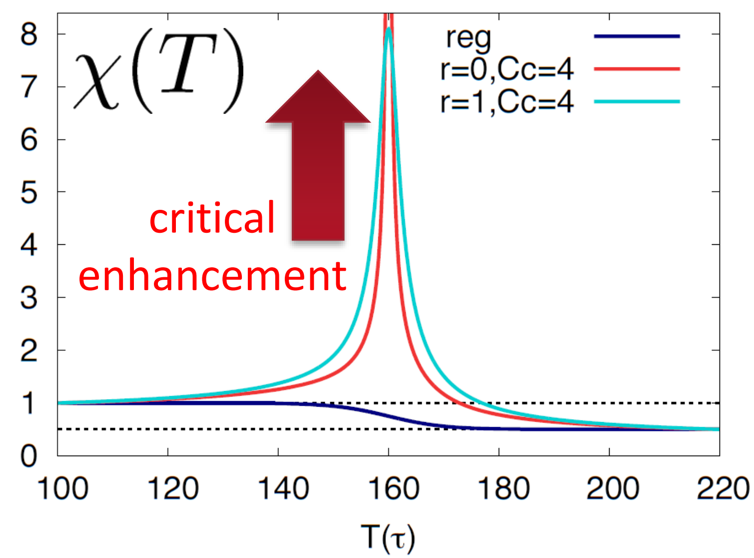
- 3D Ising (r,H)
- model H

Berdnikov, Rajagopal (2000)
Stephanov (2011); Mukherjee+(2015)



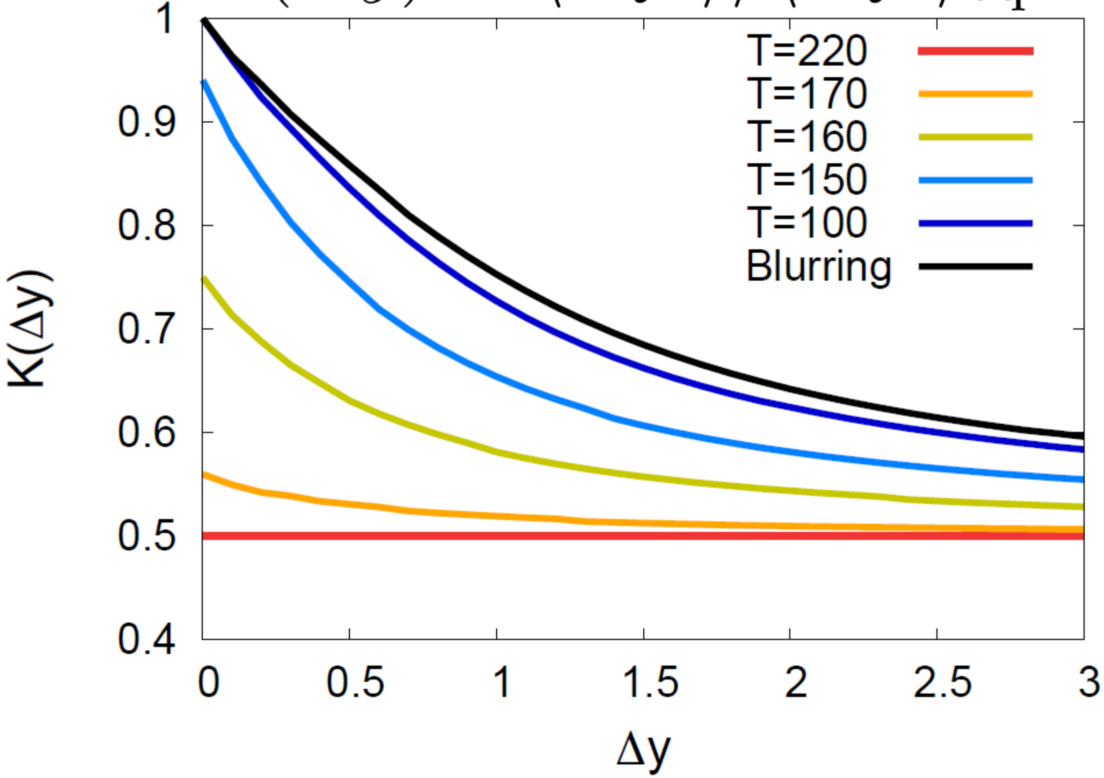
☐Temperature dep.

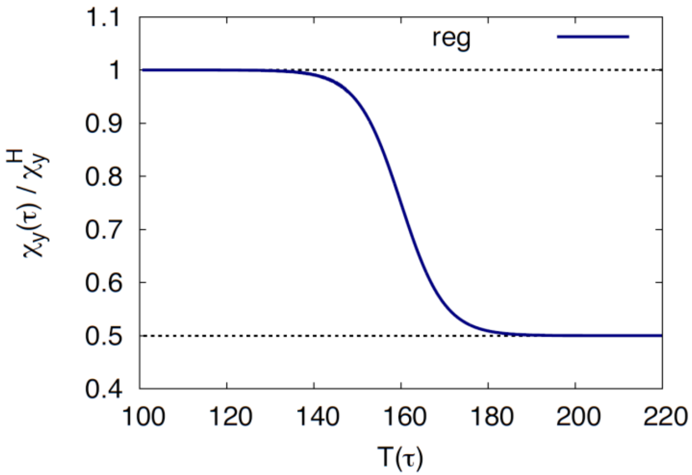


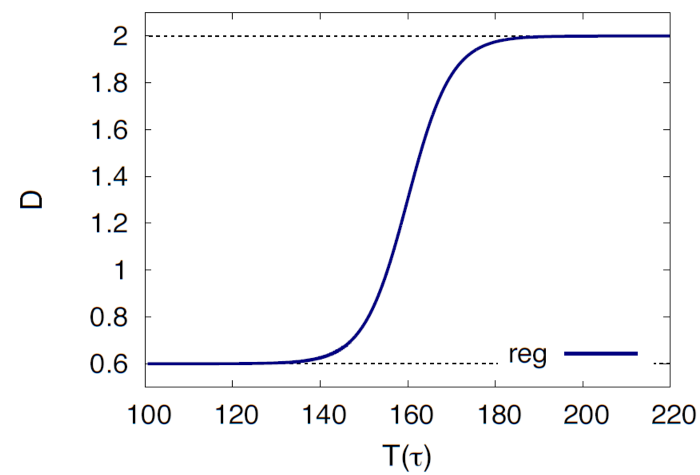


Crossover / Cumulant

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\text{eq.}}$$







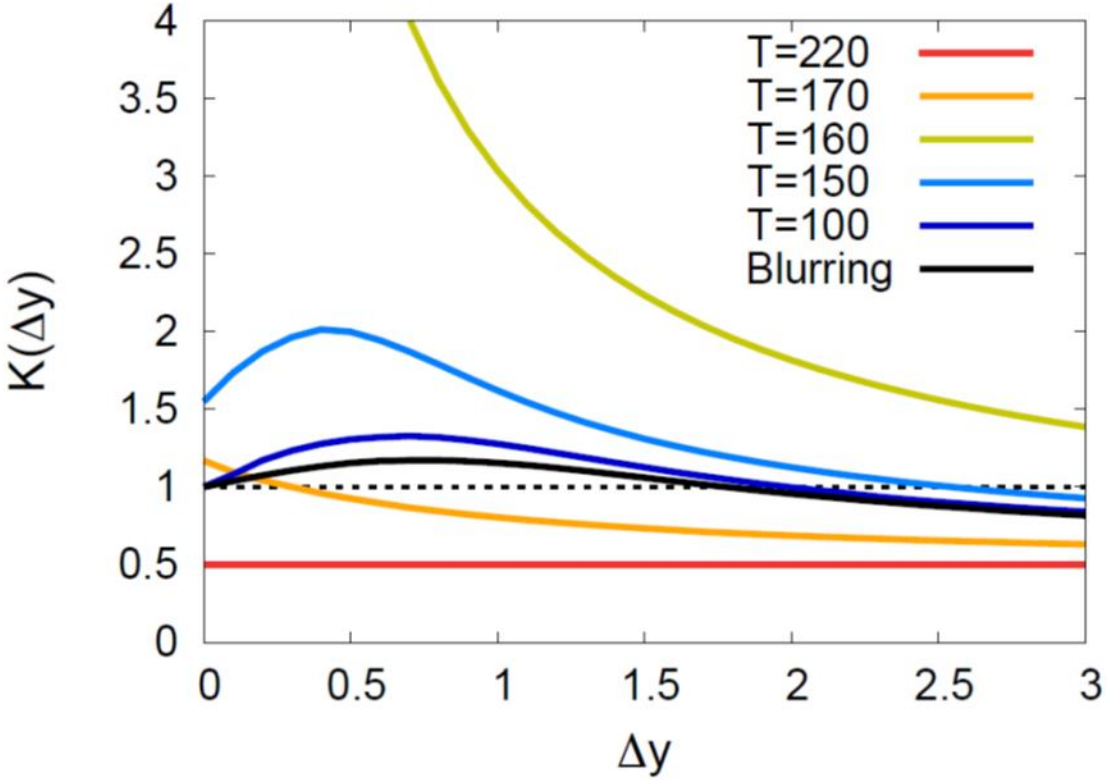
- monotonically decresing

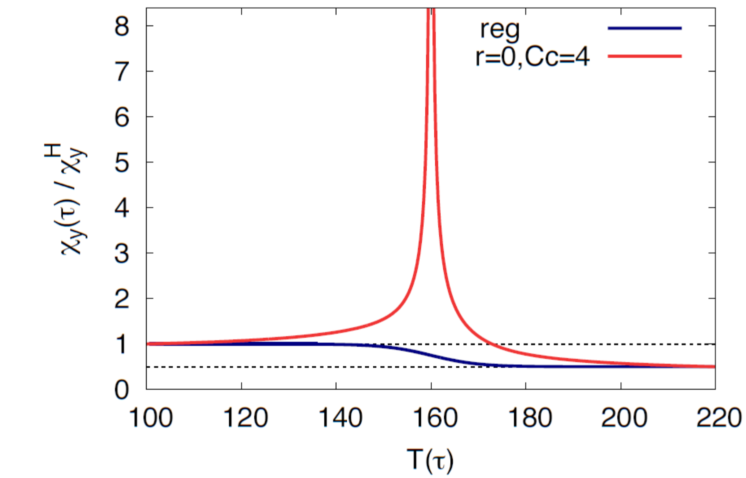
Analytic result

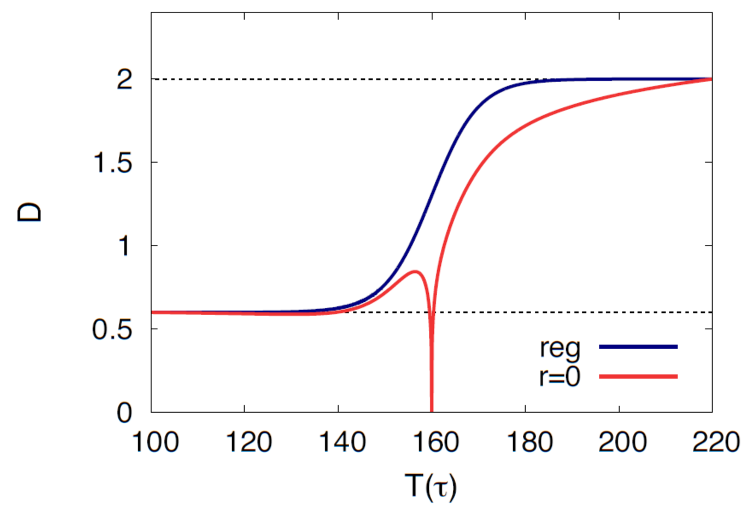
$\chi(\tau)$ monotonically increasing → $K(\Delta y)$ monotonically decreasing

Critical Point / Cumulant

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\mathrm{eq.}}$$







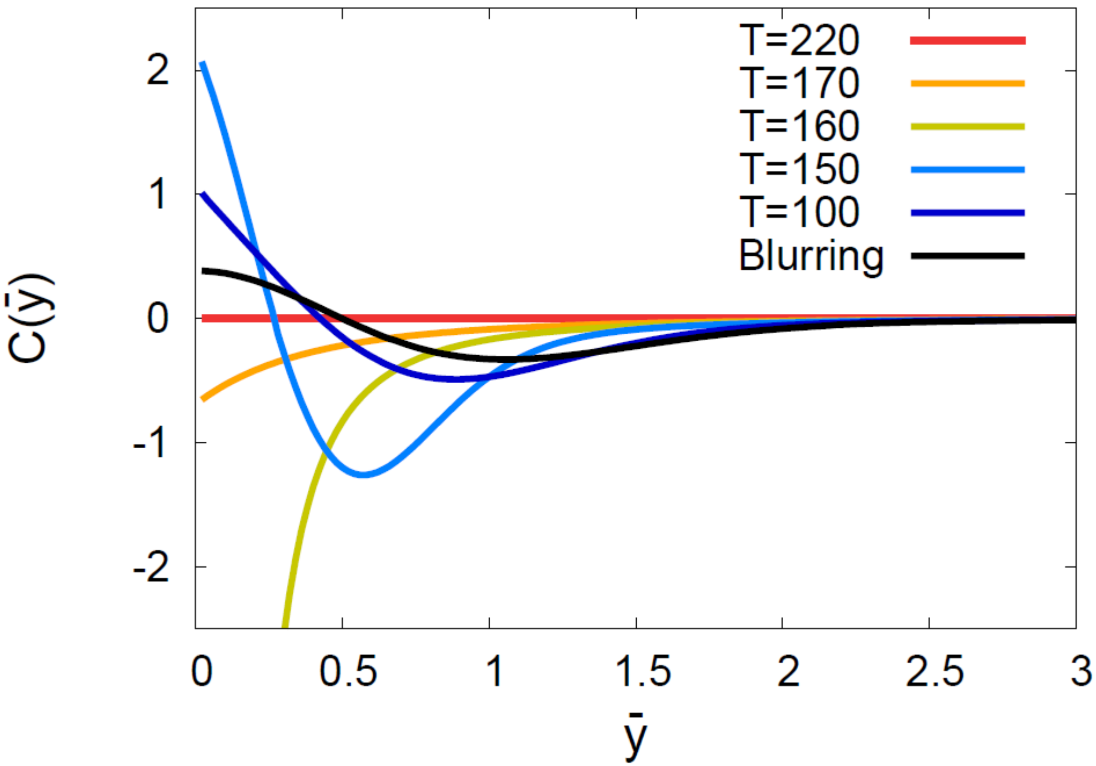
- non-monotonic Δy dep.

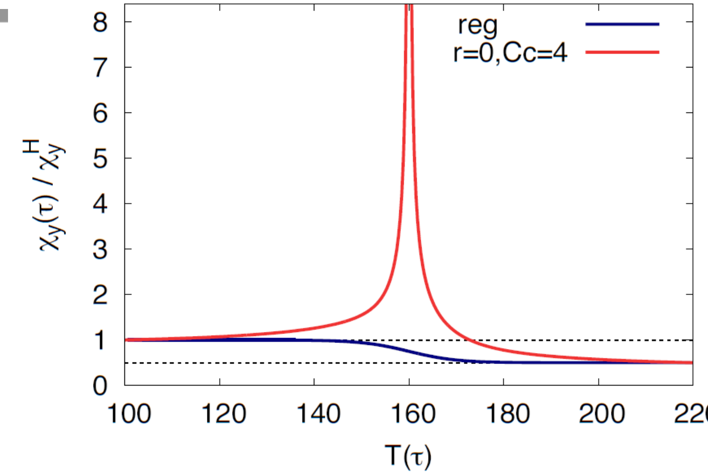
Analytic result: $K(\Delta y)$ non-monotonic ⇒ $\chi(\tau)$ non-monotonic

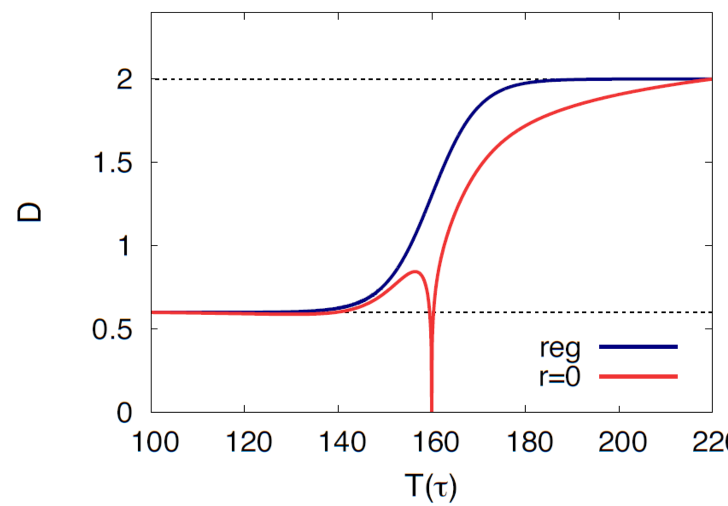
See also,
Wu, Song
Chin. Phys. C ('19)

Criticap Point / Correlation Func.

$$C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\text{hadron}}$$







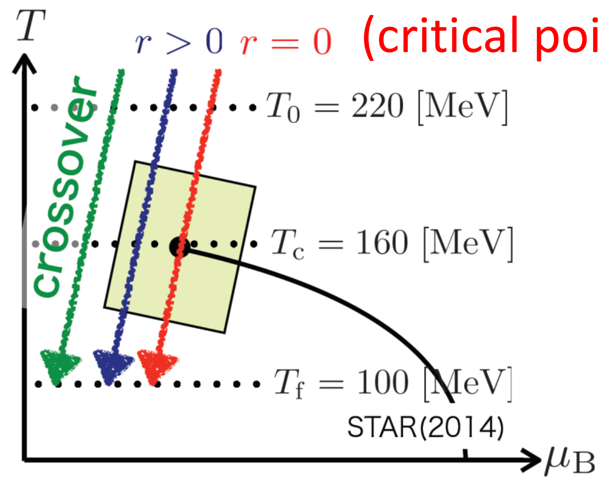
- non-monotonic Δy dep.

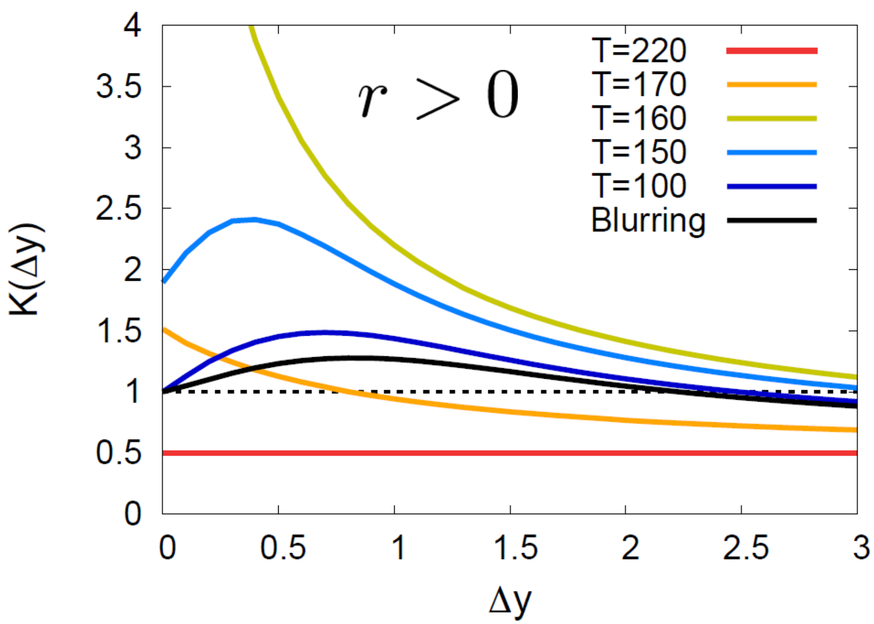
Analytic result: $C(\Delta y)$ non-monotonic ⇒ $\chi(\tau)$ non-monotonic

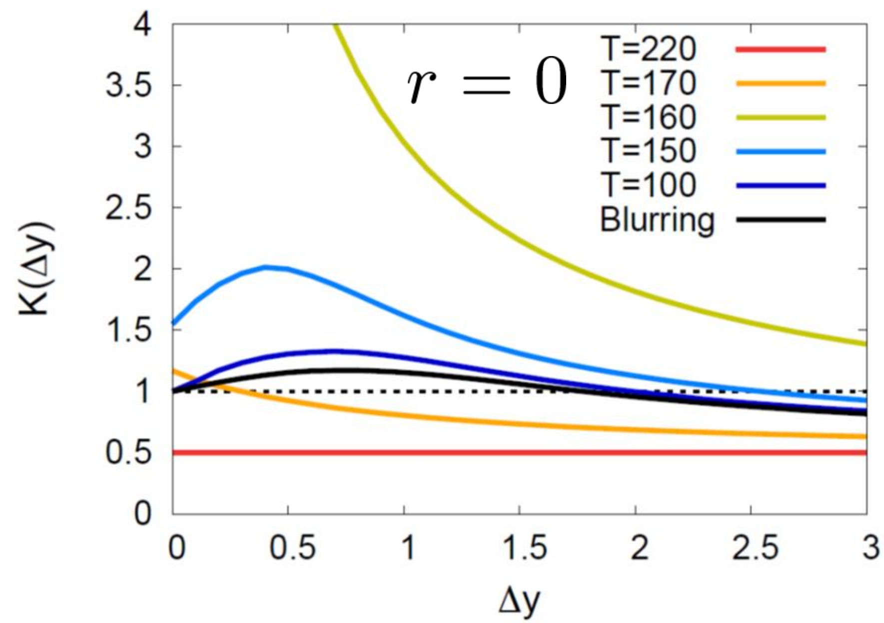
See also,
Wu, Song
Chin. Phys. C ('19)

Away from the CP

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\text{eq.}}$$







- Signal of the critical enhancement can be clearer on a path away from the CP.

Away from the CP → Weaker critical slowing down

SDE with Non-Linear Terms

$$\frac{\partial n}{\partial t} = \kappa \nabla^2 \frac{\delta \mathcal{F}}{\delta n} + \frac{\partial}{\partial x} \xi(x,t)$$

$$\mathcal{F} = \int dx \left(a \Delta n^2 + c(\nabla n)^2 + \lambda_3 \Delta n^3 + \cdots \right)$$

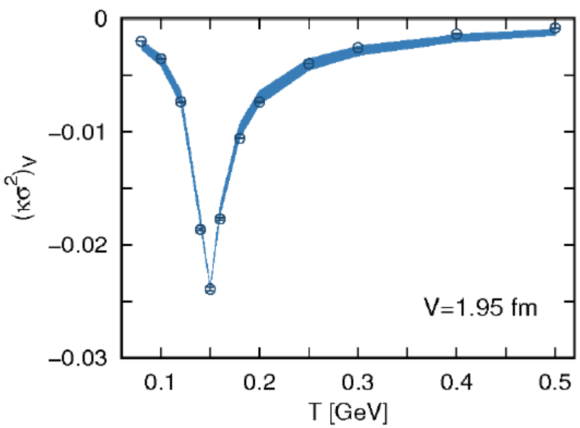
- Non-linear terms are necessary to intoruduce non-Gaussian fluctuations in equilibrium.
- No analytic solution
- Careful numerical implementation is needed.

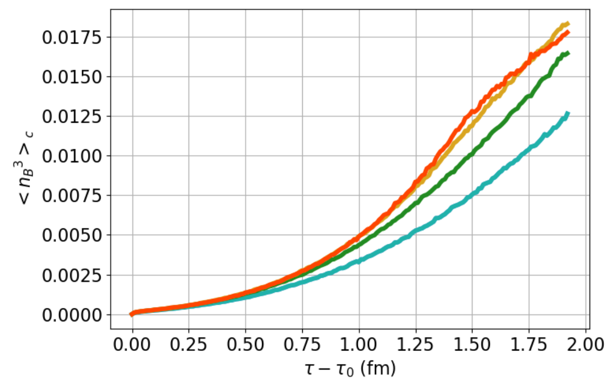
Nahrgang, Bluhm, Schaefr, Bass, PRD99 ('19)
Nahrgang, Bluhm, 2007.10371

SDE with Non-Linear Terms

☐ Higher order cumulants

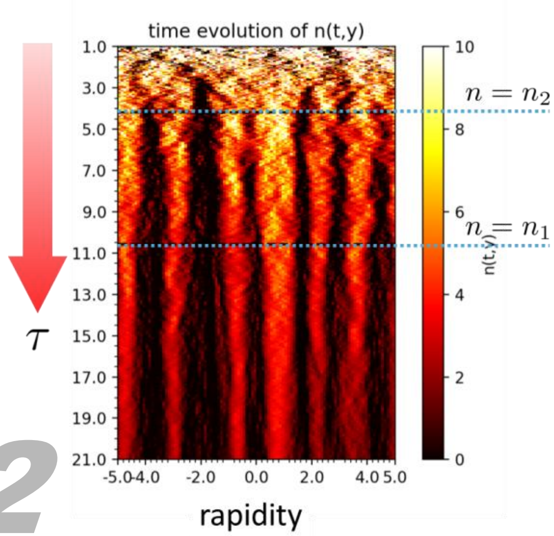
Pihan, Touroux, Nahrgang, Bluhm, Sami, MK, in prep.

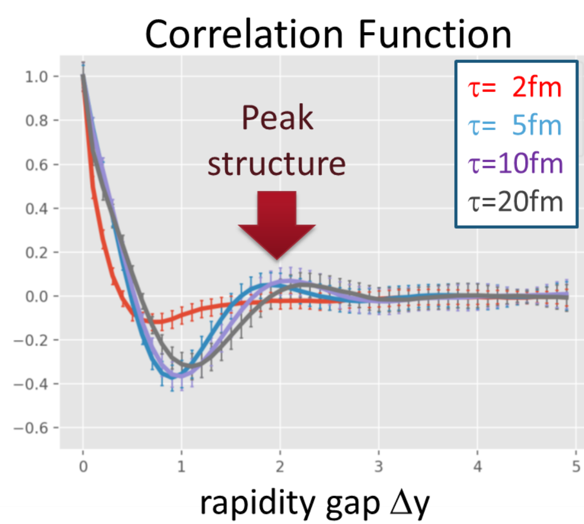




Time evolution of 4th cumulant can be described.

☐ 1st order transition





Domain formation and peak structure in the correlation function are found.

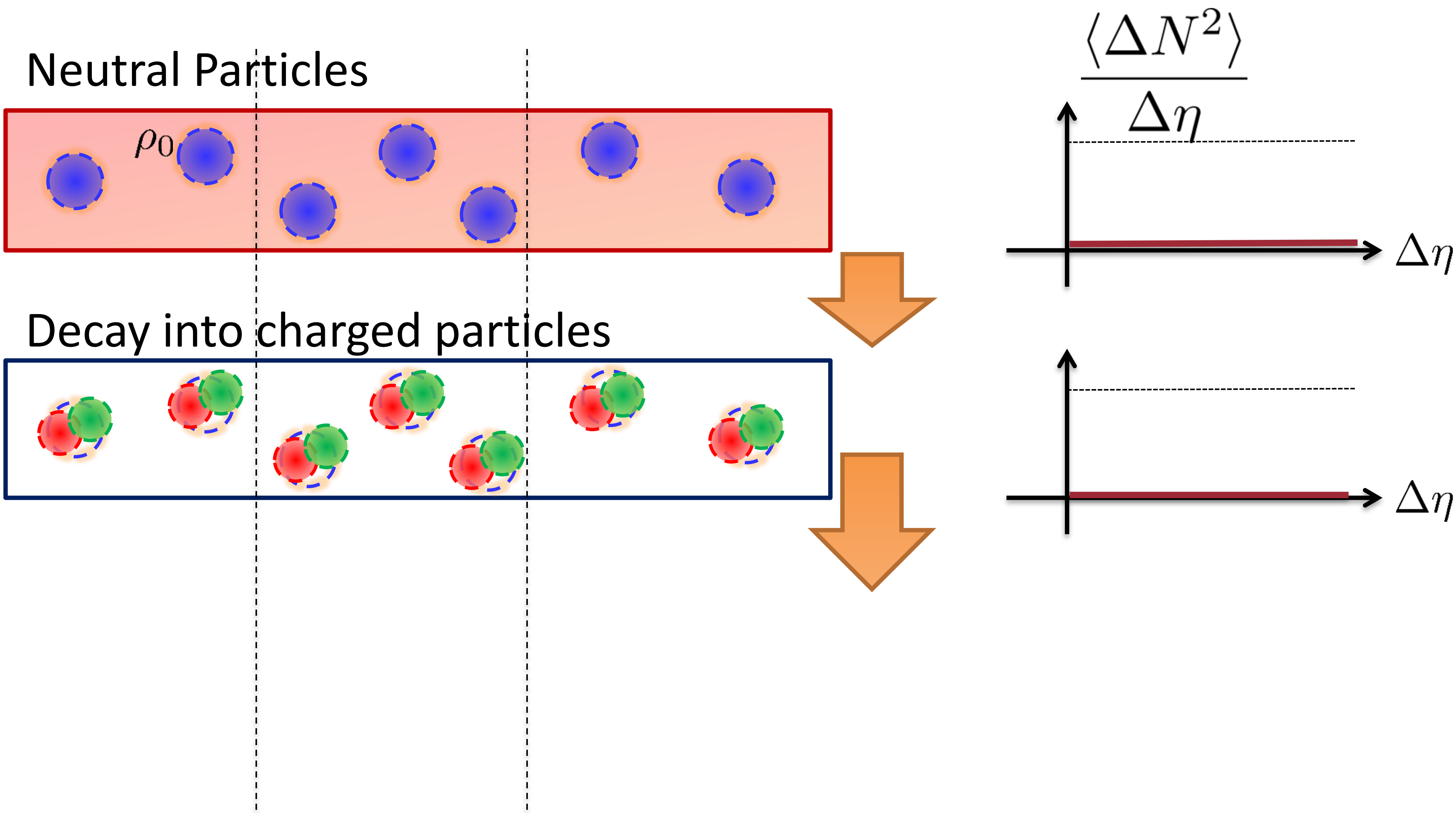
Summary

- Comprehensive understanding of various order cumulants are needed to understand .
- 2nd-order cumulants already contain important physics.

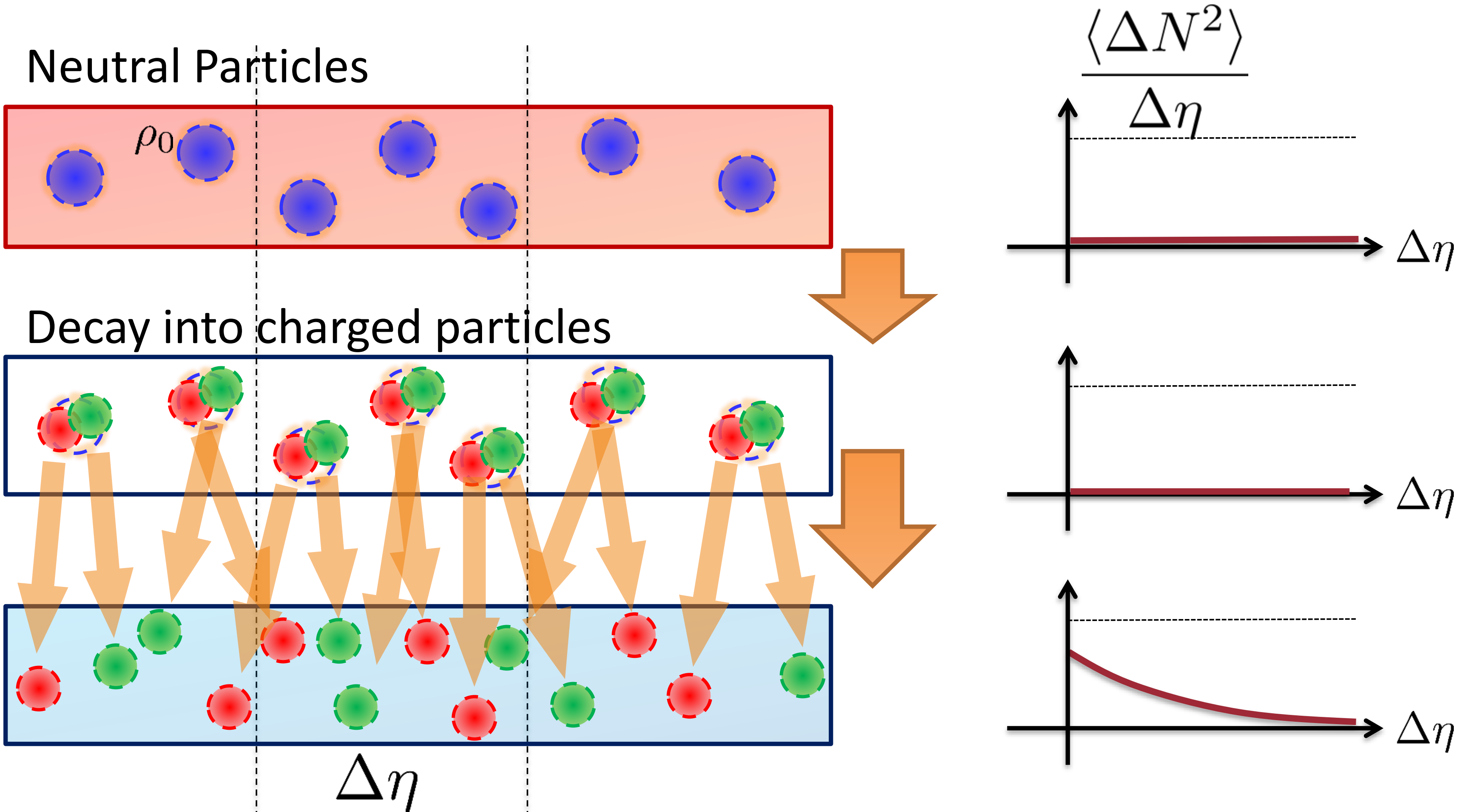
- Fluctuations observed in HIC are not in equilibrium.
- Plenty of information encoded in rapidity window dependences.

- Future
  - Wider rapidity coverage
  - Evolution of fluctuations in realistic dynamical models

Resonance Decay

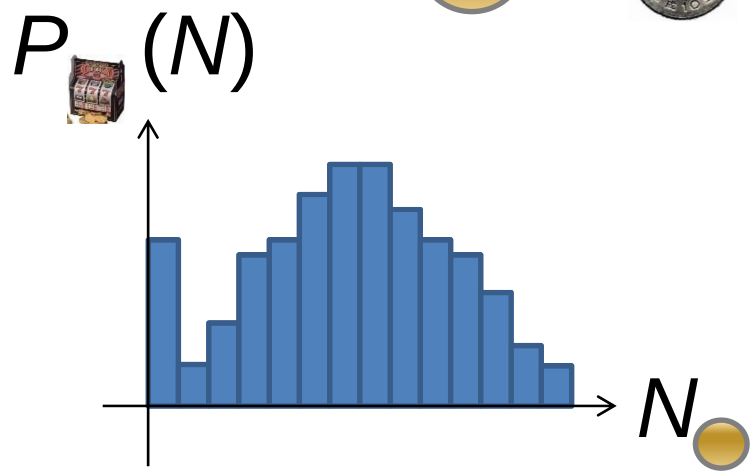


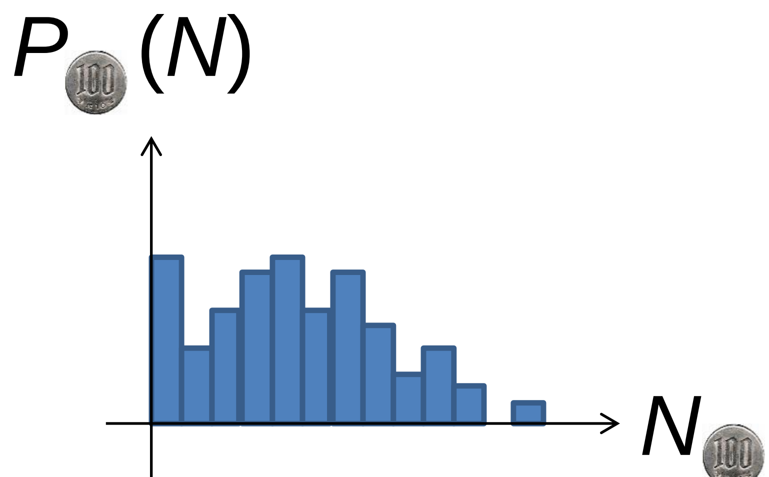
Resonance Decay



The larger Δη, the slower diffusion.

Slot Machine Analogy

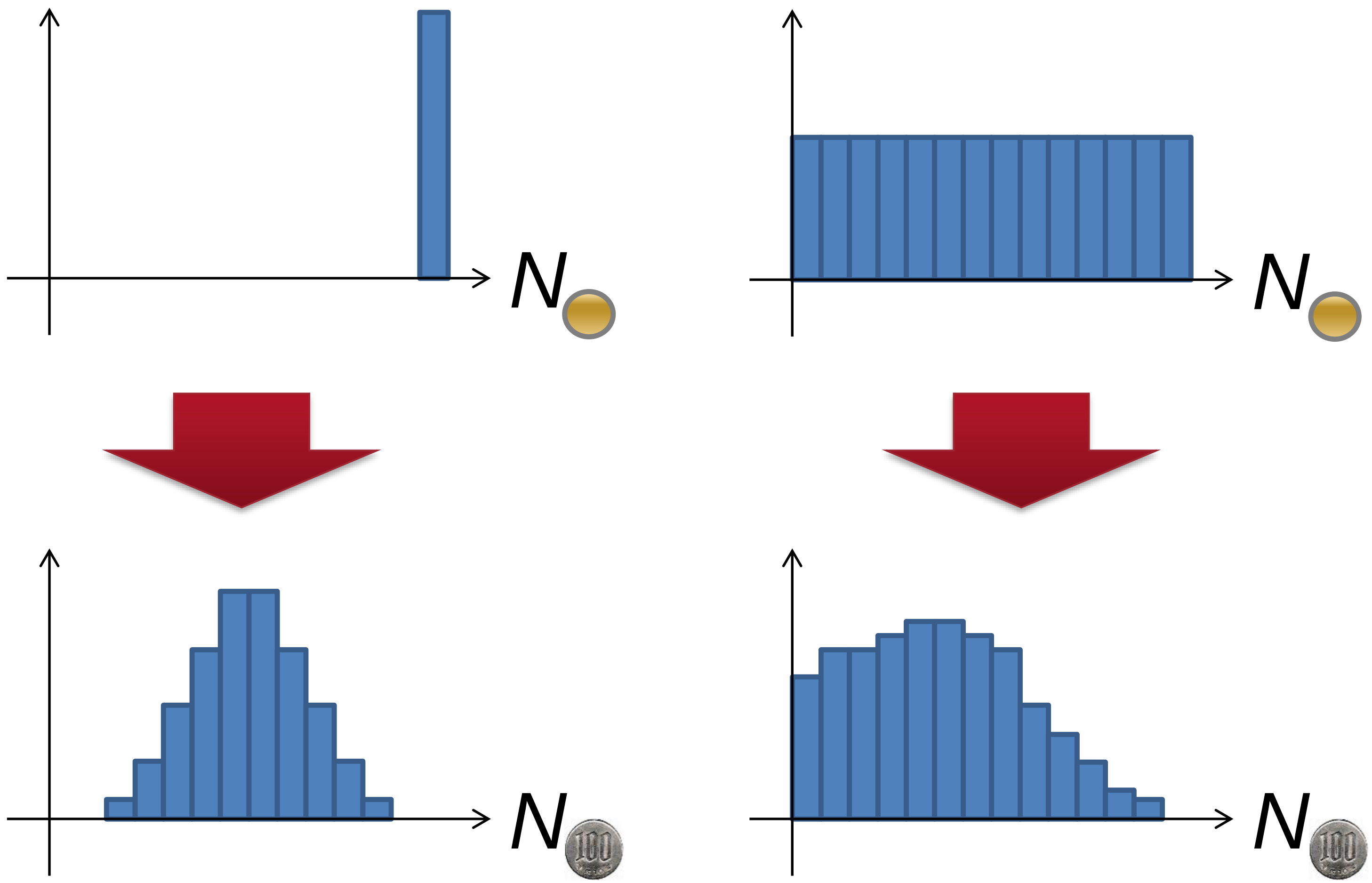




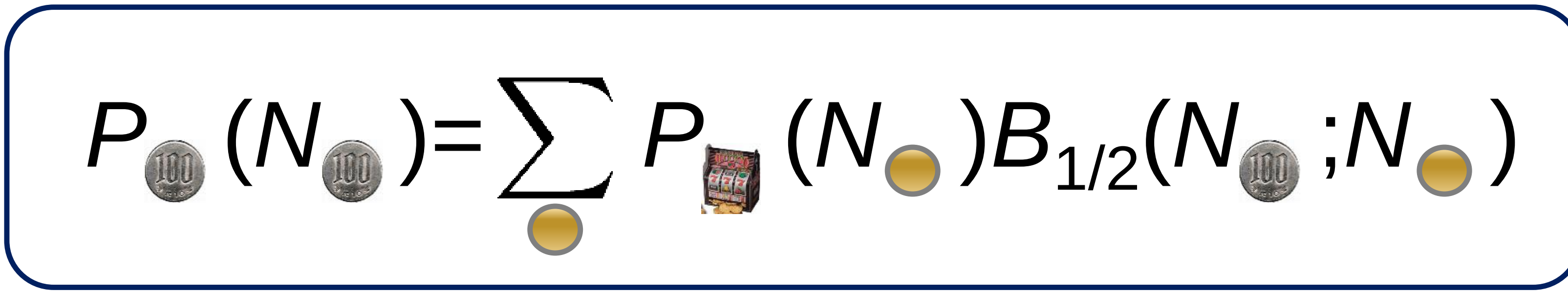
Extreme Examples

Fixed # of coins

Constant probabilities



Reconstructing Total Number



$$B_p(k;N) = p^k(1-p)^{N-k}\ {}_kC_N$$:binomial distr. func.

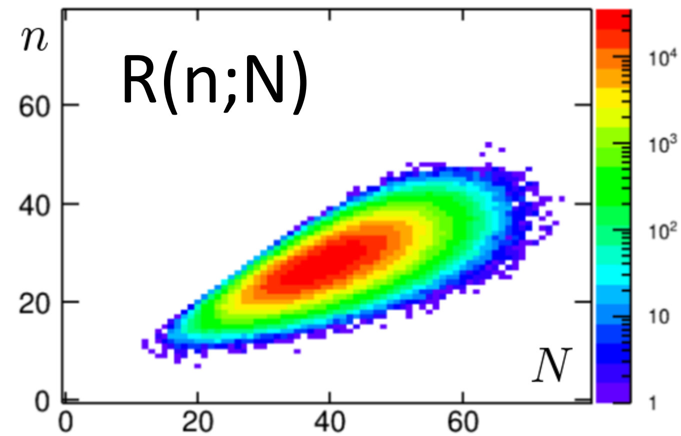
Non-Binomial Correction

- Response matrix

$$\tilde{P}(n) = \sum_{N} \mathcal{R}(n;N) P(N)$$

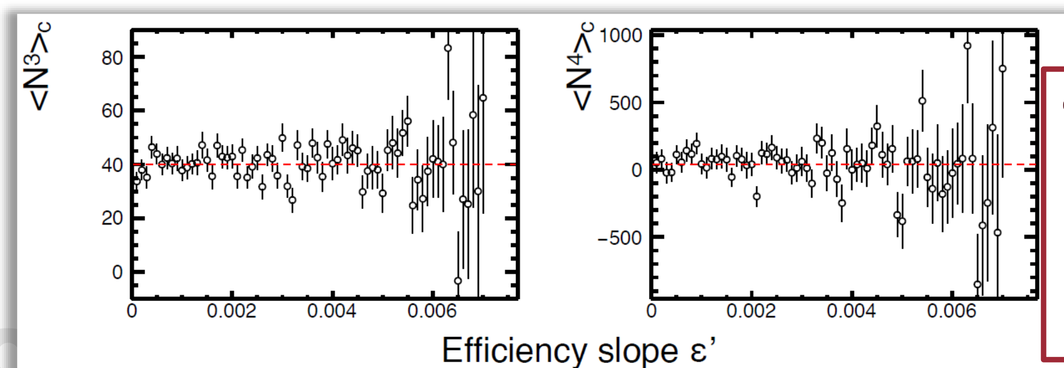
Reconstruction for any R(n;N)

Nonaka, MK, Esumi (2018)



Example

multiplicity-dependent efficiency $\epsilon(N) = \epsilon_0 + (N - N_{\rm ave})\epsilon'$



True cumulants are reproduced within statistics!

want	quickly	then	not	achieve
欲	速	則	不	達
		则		达

孔子

Confucius

Haste makes waste.