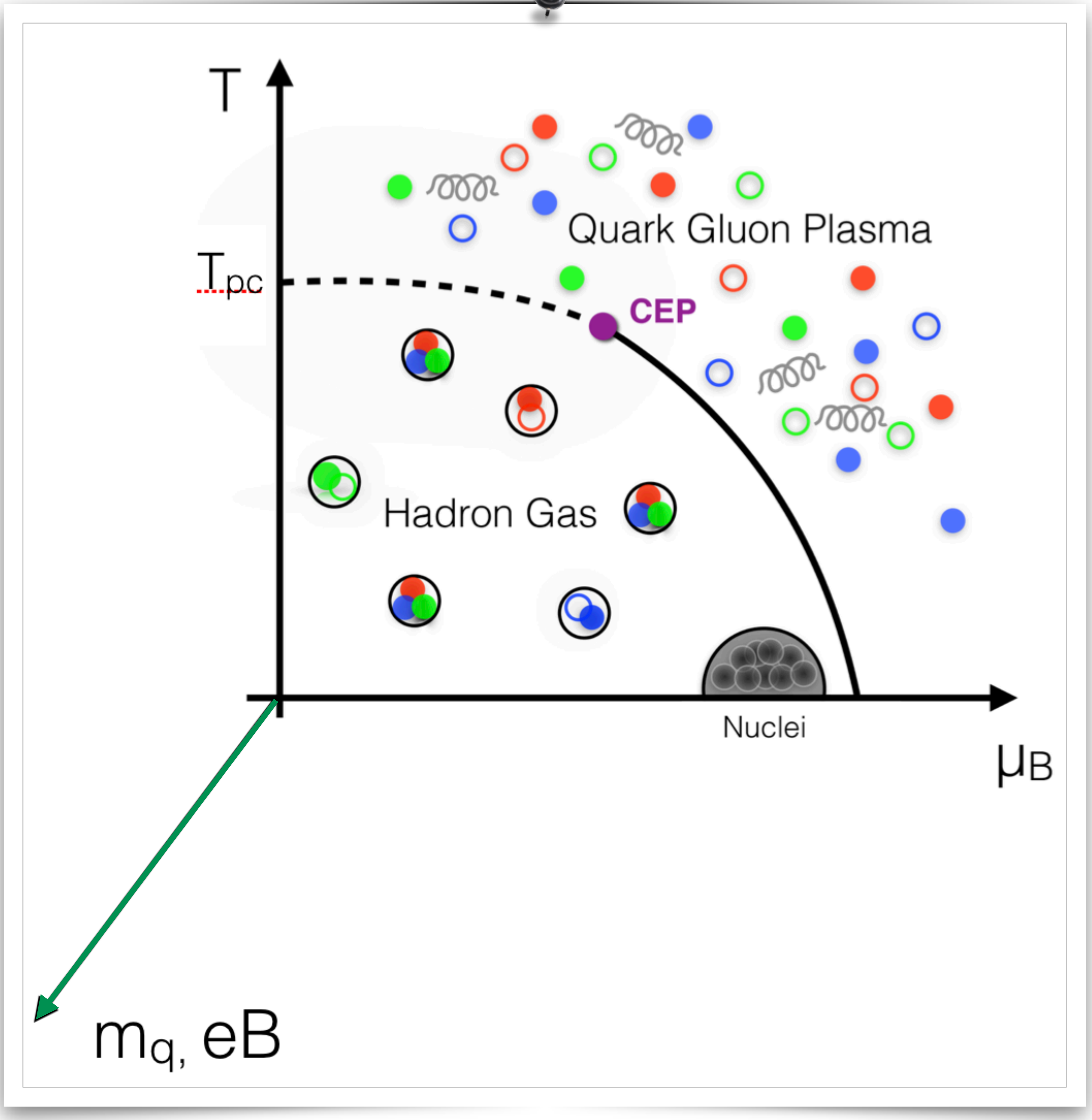
Nuclear Science Computing Center at CCNU

QCD phase structure from lattice QCD

Heng-Tong Ding (丁亨通)
Central China Normal University (华中师范大学)

The 7th RHIC-BES Theory and Experiment on-line seminar
Sep. 15, 2020

Outline: QCD phase structure



HTD, F. Karsch, S. Mukherjee, arXiv: 1504.05274
Int.J.Mod.Phys. E24 (2015) no.10, 1530007

- QCD transition in external magnetic field

HTD, S.-T. Li, A. Tomiya, X.-D. Wang & Y. Zhang,
arXiv:2008.00493 & work in progress

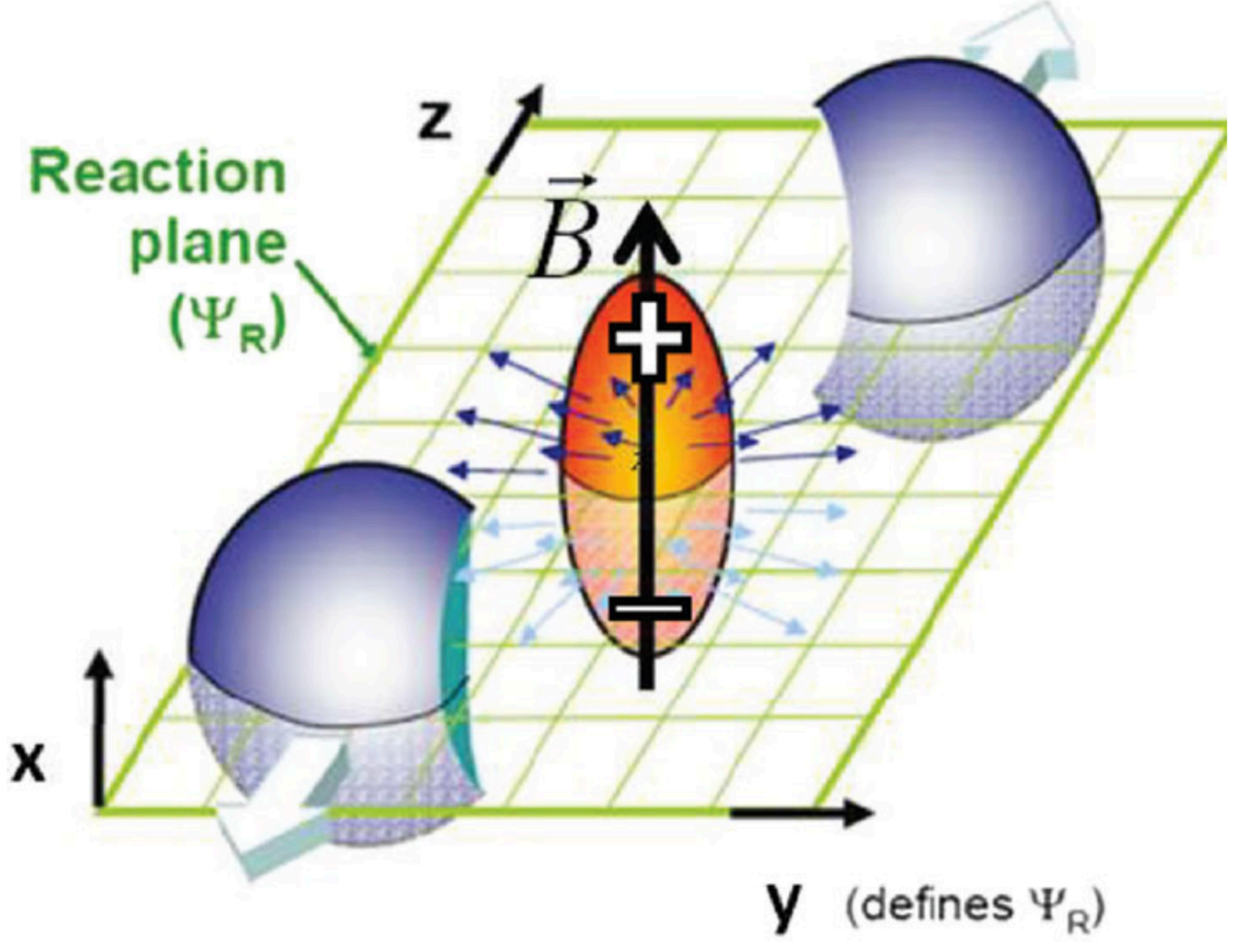
- Chiral crossover and chiral phase transition T

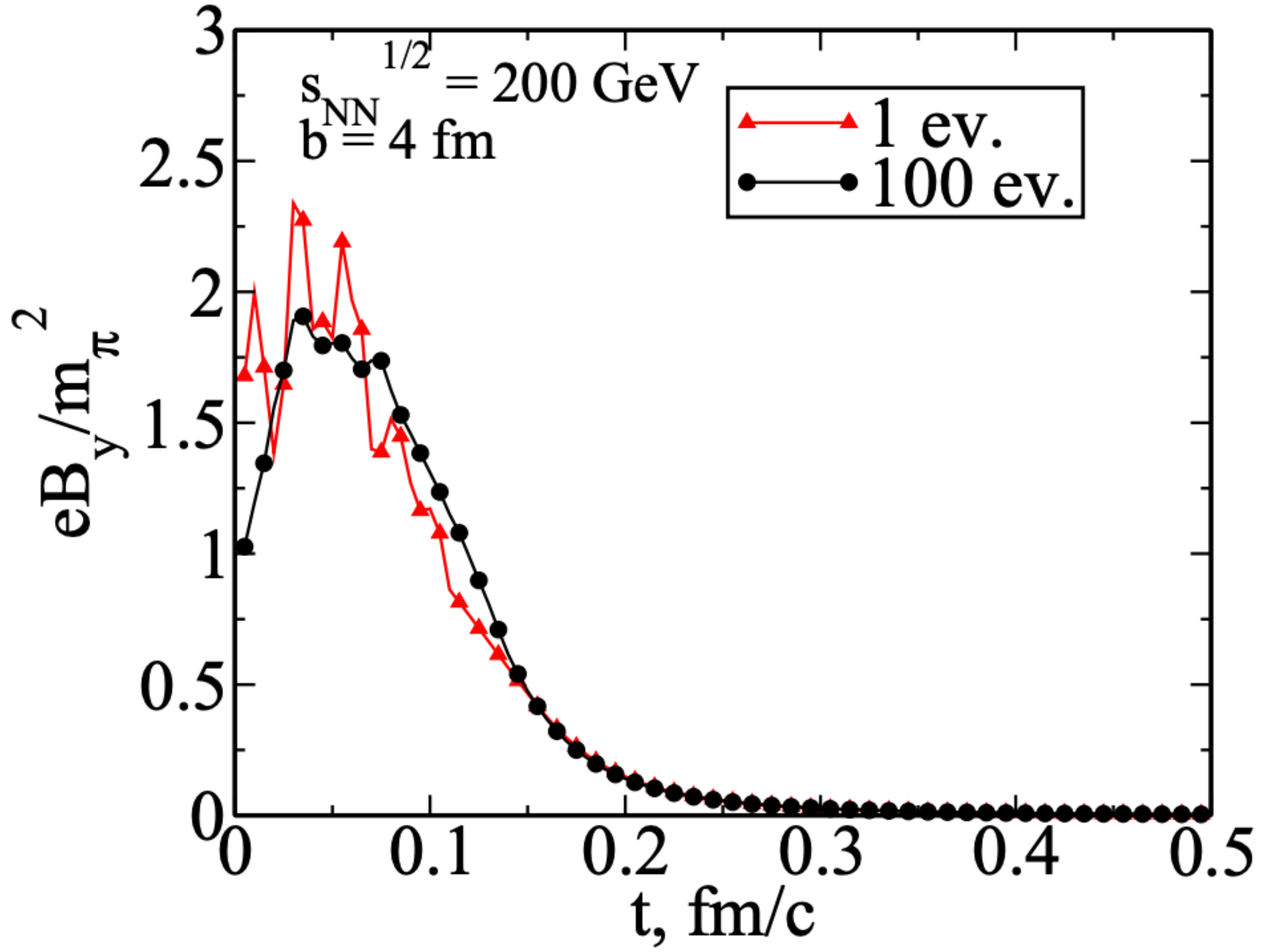
HTD, P. Hegde, O. Kaczmarek et al.,[HotQCD], PRL 123 (2019) 062002
A. Bazavov, HTD, P. Hegde et al.,[HotQCD], PLB795 (2019) 15

- LQCD meet experiments—Fluctuations of conserved charges

A. Bazavov, D. Bollweg, HTD et al., [HotQCD],
PRD101 (2020) 074502, PRD96 (2017) 074510

Magnetic fields created in HIC



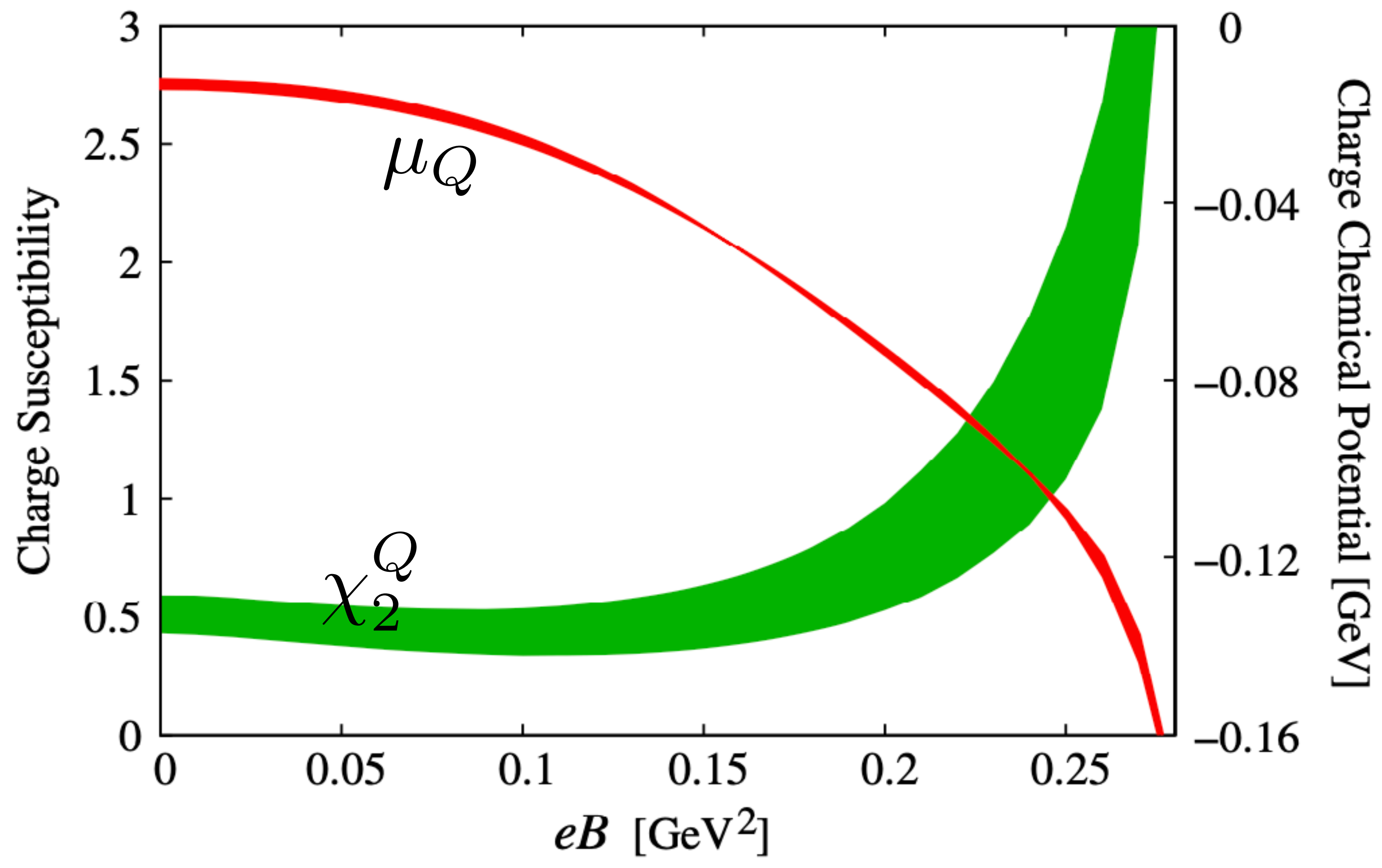


Skokov, Illarionov and V.Toneev, IJMPA 24 (2009) 5925

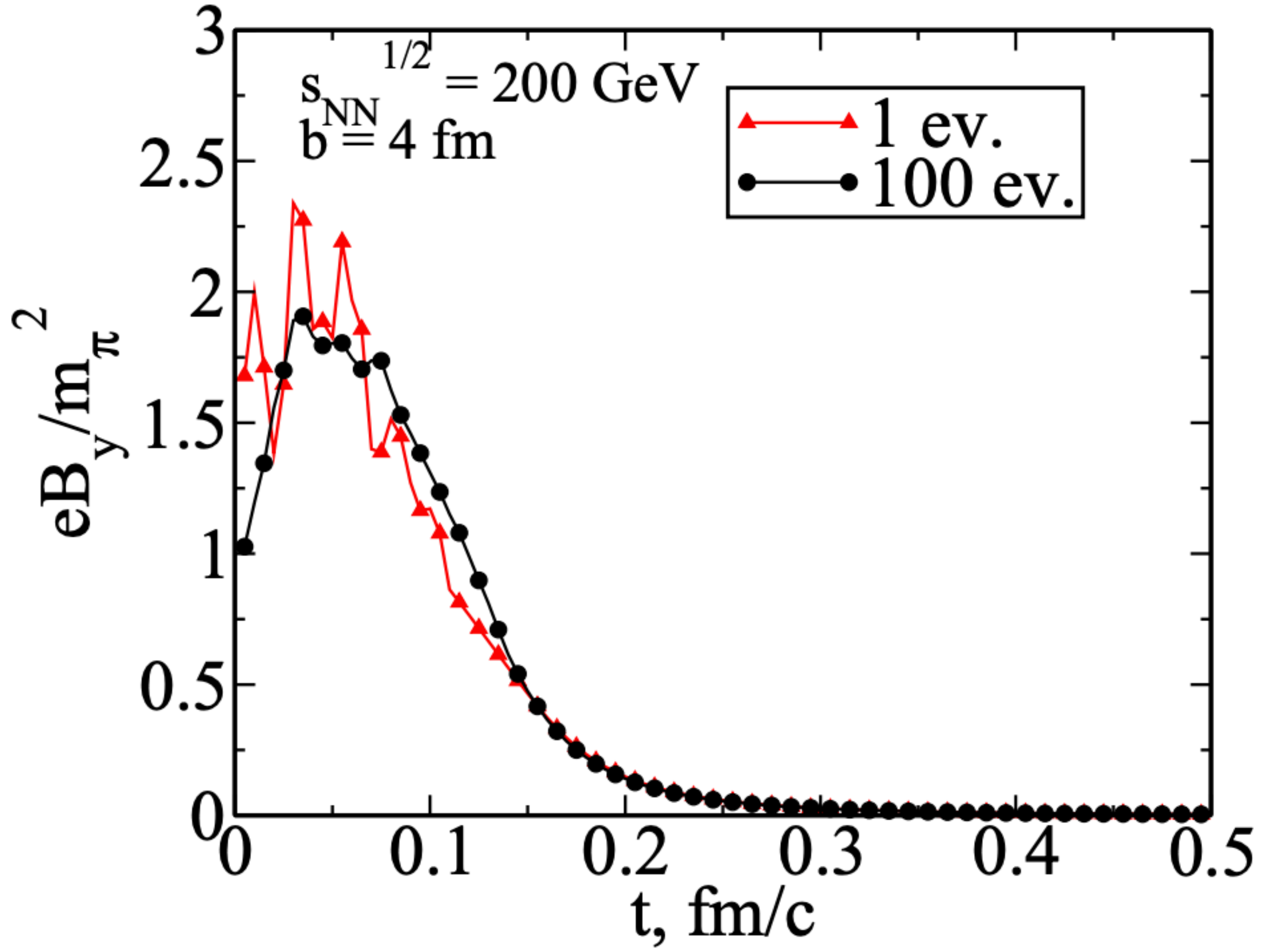
t=0: RHIC: $eB \sim m_\pi^2$

LHC: $eB \sim 15m_\pi^2$

Magnetic fields created in HIC



Fukushima & Hidaka, PRL 117, 102301 (2016)



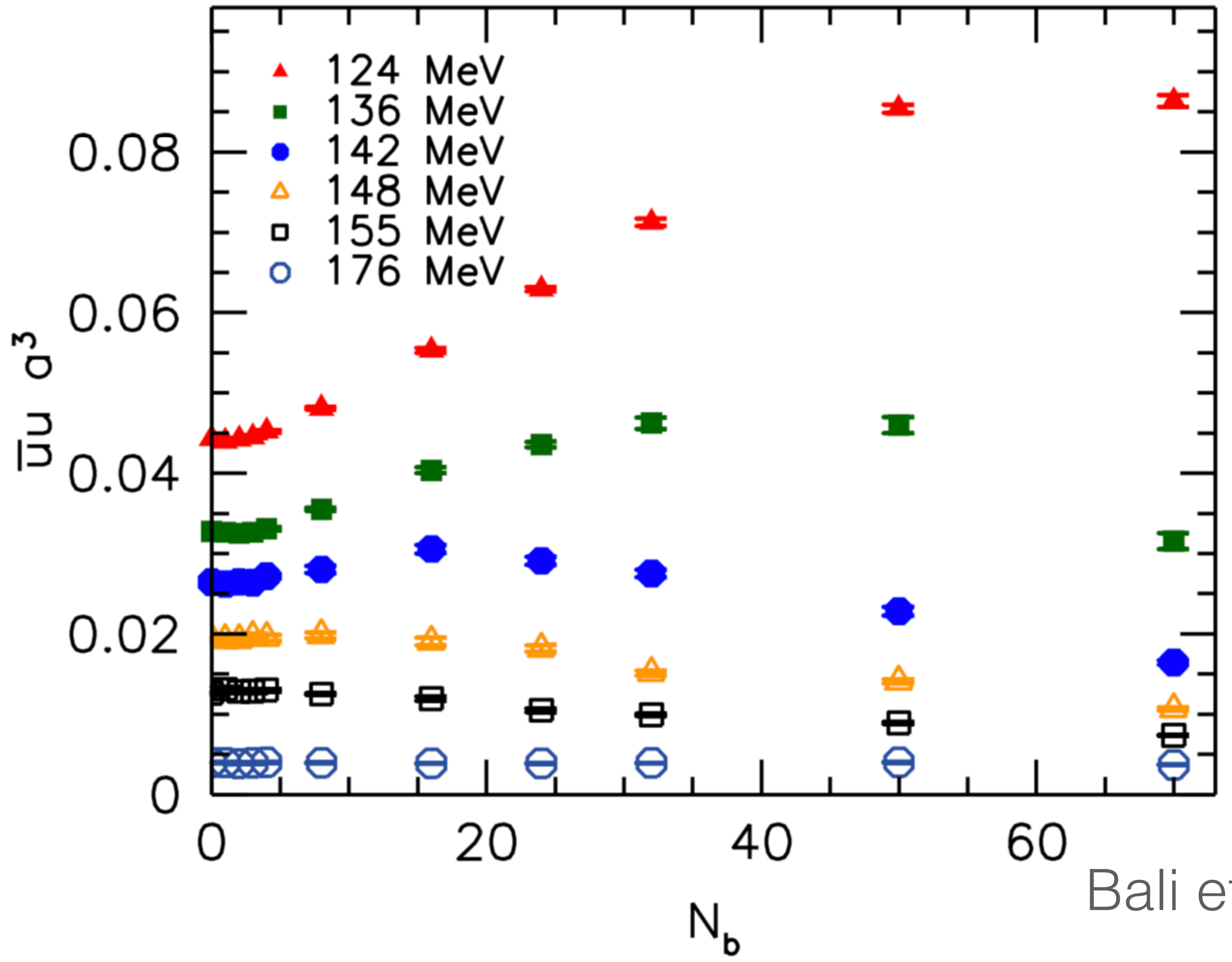
Skokov, Illarionov and V.Toneev, IJMPA 24 (2009) 5925

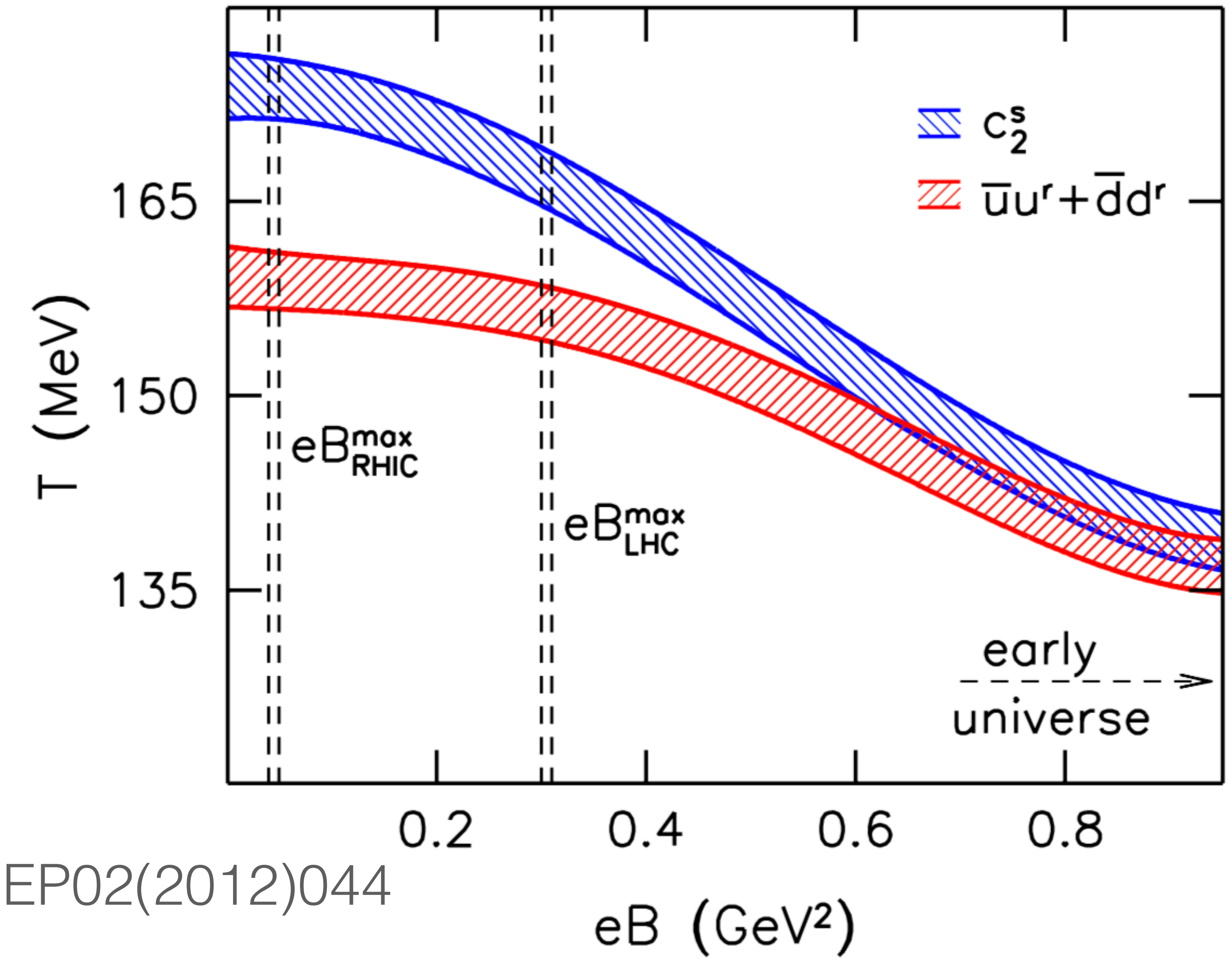
Based on HRG with μ_B=0.6 GeV, detect eB by comparing 2nd order electrical charge fluctuation in peripheral to central collisions

t=0: RHIC: $eB \sim m_\pi^2$

LHC: $eB \sim 15 m_\pi^2$

(Inverse) magnetic catalyses v.s. reduction of T_{pc} in a background magnetic field





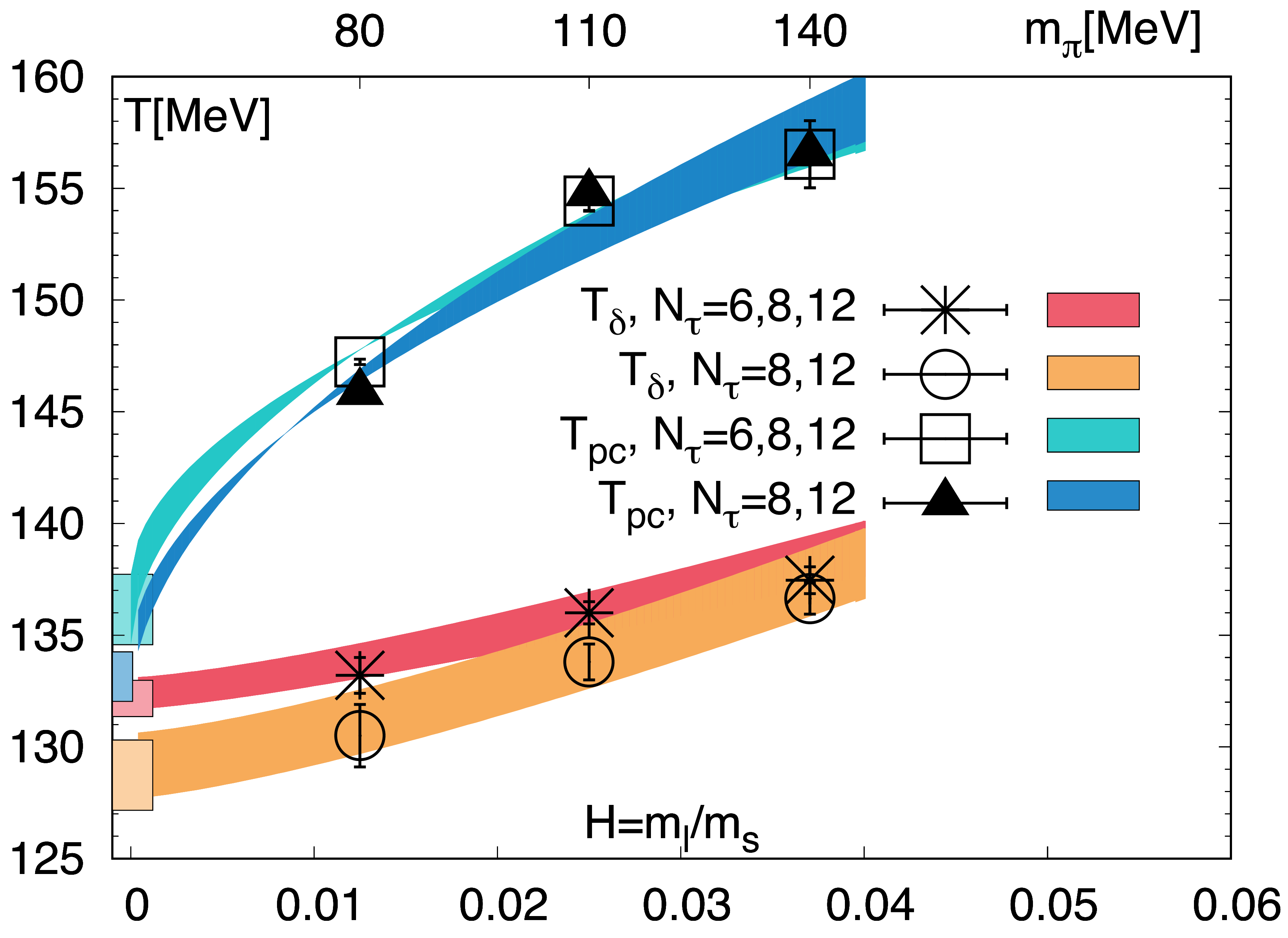
Bali et al., JHEP02(2012)044

Chiral condensate always increases as eB at $T \ll T_{pc}$

Its connection to the reduction of T_{pc} is highly non-trivial

Reduction of T_{pc} v.s. lighter pion

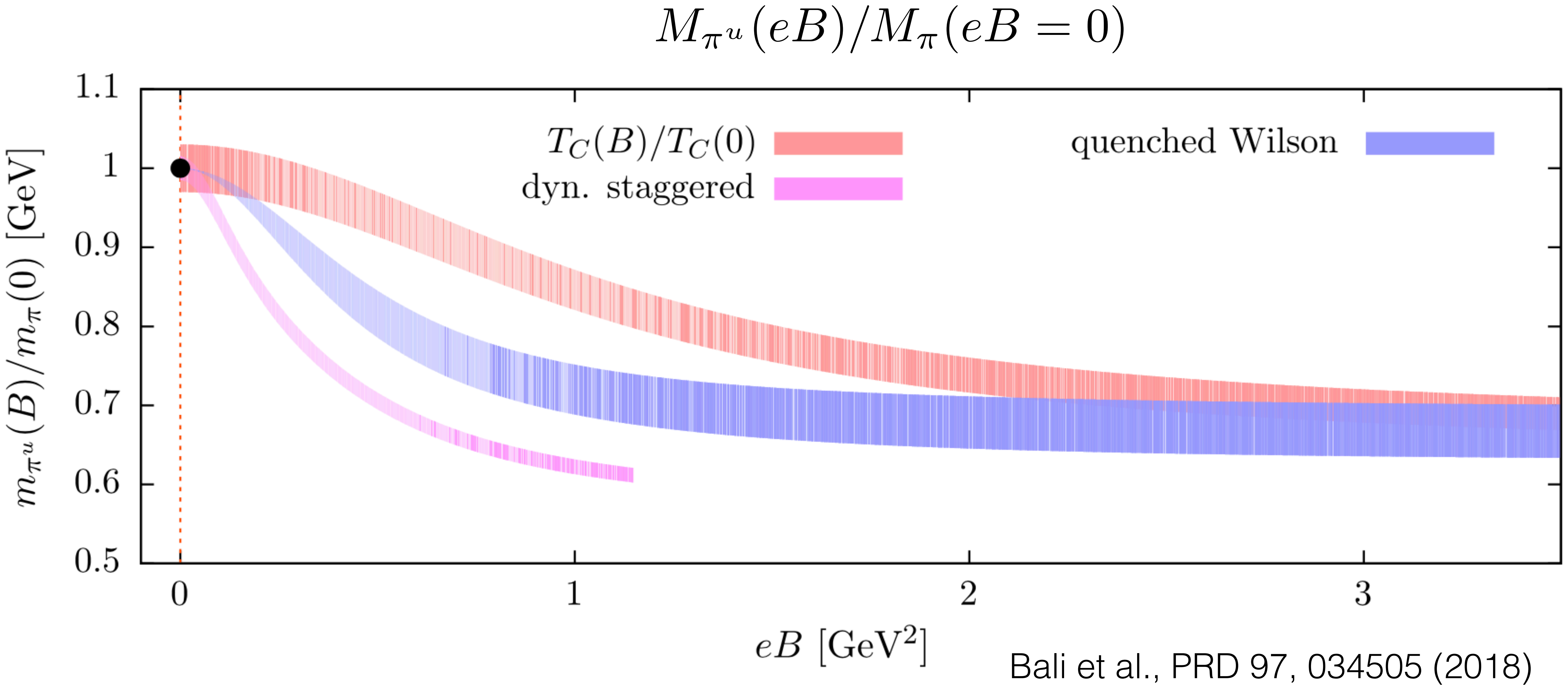
eB=0



HTD, P. Hegde, O. Kaczmarek et al.[HotQCD], Phys. Rev. Lett. 123 (2019) 062002

HTD, arXiv:2002.11957

eB=/=0



Bali et al., PRD 97, 034505 (2018)

Is (neutral) pion still a Goldstone boson at eB=/=0 ?

Gell-Mann-Oakes-Renner relation

M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968), J. Gasser and H. Leutwyler, Nucl. Phys. B 250, 465 (1985)

$$(m_u + m_d)\ \left(\langle\bar{\psi}\psi\rangle_u + \langle\bar{\psi}\psi\rangle_d\right) = 2f_\pi^2 M_\pi^2$$

- At T=0, in the weak magnetic field the 2-flavor GMOR relation holds true for chiral (& point-like) pions from LO ChPT

Shushpanov and Smilga, PLB402(1997)351

- At eB=/=0, additional pion decay constants appear due to a nonzero pion-to-vacuum transition via the vector electroweak current

Fayazbakhsh & Sadooghi, PRD 88(2013)065030

Bali et al., PRD121(2018)072001

Coppola et al., PRD.99 (2019)0540312

Gell-Mann-Oakes-Renner relation

M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968), J. Gasser and H. Leutwyler, Nucl. Phys. B 250, 465 (1985)

Explicit chiral symmetry breaking

$$(m_u + m_d)\left(\langle\bar{\psi}\psi\rangle_u + \langle\bar{\psi}\psi\rangle_d\right) = 2f_\pi^2 M_\pi^2$$

- At T=0, in the weak magnetic field the 2-flavor GMOR relation holds true for chiral (& point-like) pions from LO ChPT

Shushpanov and Smilga, PLB402(1997)351

- At eB=/=0, additional pion decay constants appear due to a nonzero pion-to-vacuum transition via the vector electroweak current

Fayazbakhsh & Sadooghi, PRD 88(2013)065030

Bali et al., PRD121(2018)072001

Coppola et al., PRD.99 (2019)0540312

Gell-Mann-Oakes-Renner relation

M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968), J. Gasser and H. Leutwyler, Nucl. Phys. B 250, 465 (1985)

Explicit chiral symmetry breaking

$$(m_u + m_d)\left(\langle\bar{\psi}\psi\rangle_u + \langle\bar{\psi}\psi\rangle_d\right) = 2 f_\pi^2 M_\pi^2$$

Spontaneous chiral symmetry breaking

- At T=0, in the weak magnetic field the 2-flavor GMOR relation holds true for chiral (& point-like) pions from LO ChPT

Shushpanov and Smilga, PLB402(1997)351

- At eB=/=0, additional pion decay constants appear due to a nonzero pion-to-vacuum transition via the vector electroweak current

Fayazbakhsh & Sadooghi, PRD 88(2013)065030

Bali et al., PRD121(2018)072001

Coppola et al., PRD.99 (2019)0540312

Gell-Mann-Oakes-Renner relation

M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968), J. Gasser and H. Leutwyler, Nucl. Phys. B 250, 465 (1985)

Explicit chiral symmetry breaking

$$(m_u + m_d) \left(\langle\bar{\psi}\psi\rangle_u + \langle\bar{\psi}\psi\rangle_d\right) = 2f_\pi^2 M_\pi^2 \,(1-\delta_\pi)$$

Spontaneous chiral symmetry breaking

At physical pion mass $\delta_\pi \sim 6\%$

- At T=0, in the weak magnetic field the 2-flavor GMOR relation holds true for chiral (& point-like) pions from LO ChPT

Shushpanov and Smilga, PLB402(1997)351

- At eB=/=0, additional pion decay constants appear due to a nonzero pion-to-vacuum transition via the vector electroweak current

Fayazbakhsh & Sadooghi, PRD 88(2013)065030

Bali et al., PRD121(2018)072001

Coppola et al., PRD.99 (2019)0540312

Lattice setup

arXiv:2008.00493

- Symanzik-improved gauge action with HISQ fermions

- 32^3x96 lattices, with a=0.117 fm (a^{-1}=0.17 GeV), m_l/m_s =1/10 (M_π =220 MeV)

- In our setup f_π= 96.93(2) MeV, f_K=112.50(2) MeV, f_K/f_π=1.1606(3)

FLAG 2019: At physical mass point f_π= 92.1(6) MeV, f_K=110.1(5) MeV, f_K/f_π=1.1917(37)

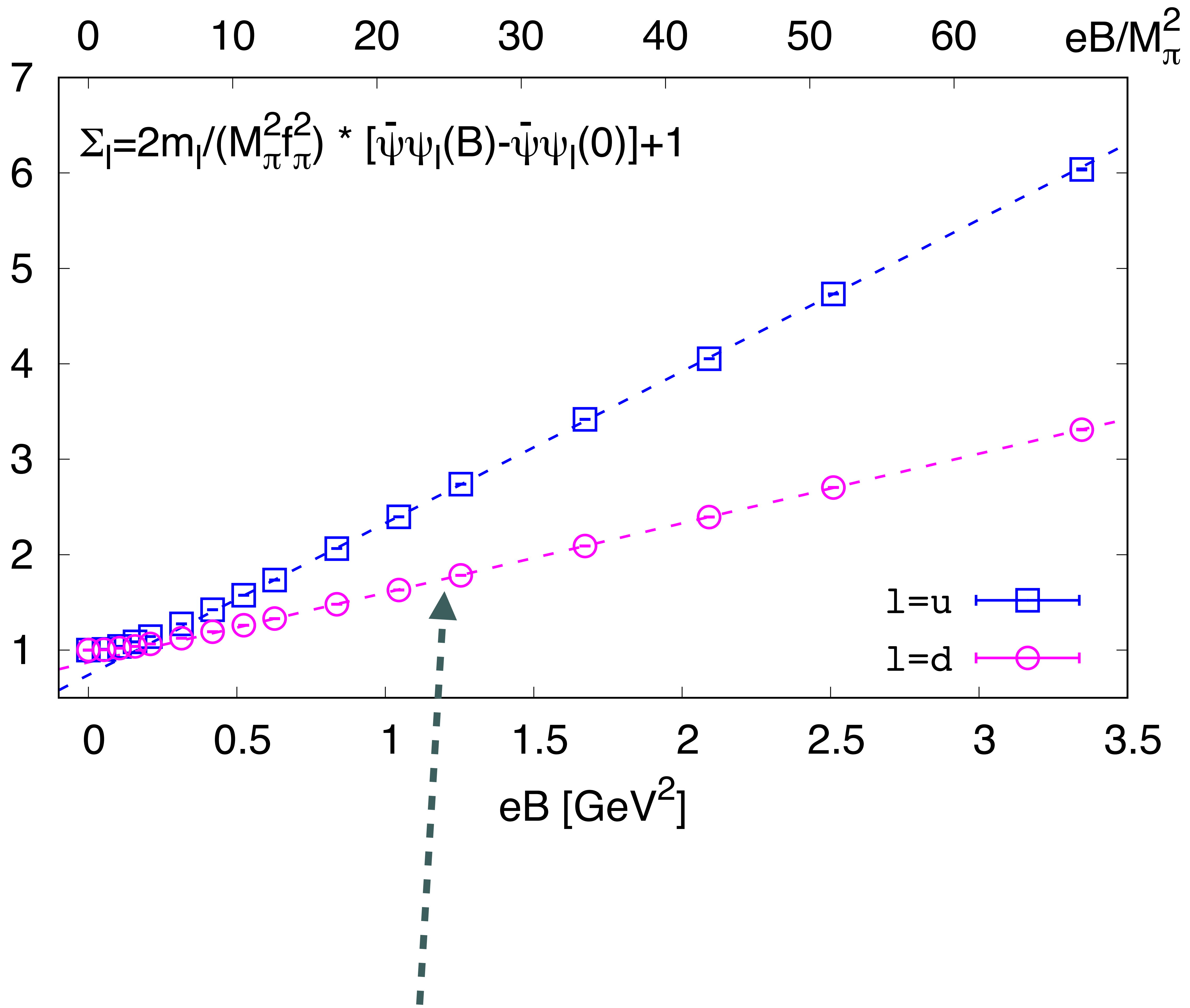
✦ Magnetic field is quantized as $eB = \frac{6\pi N_b}{N_x N_y} a^{-2}$,

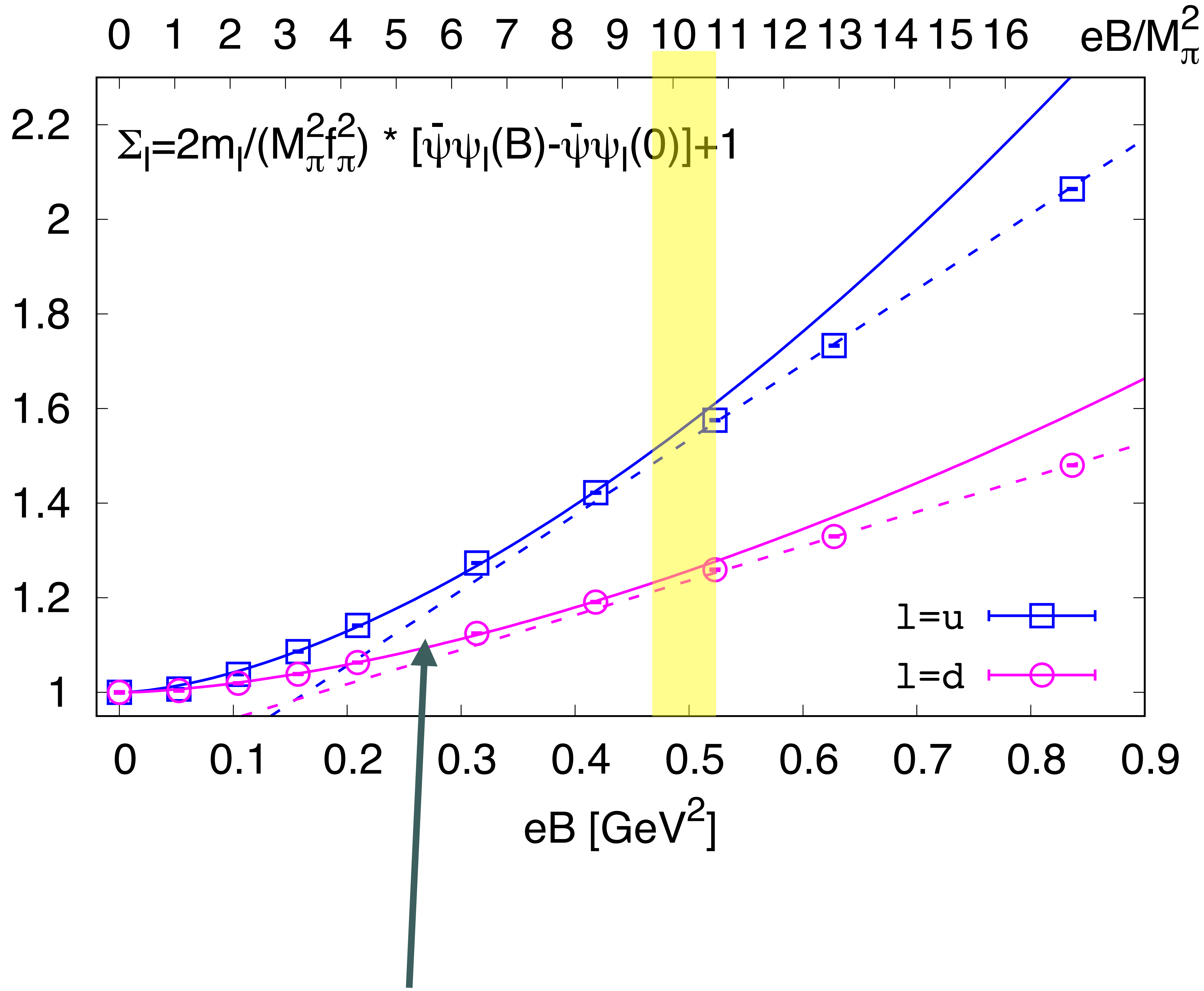
✦ Magnetic flux: N_b=0,1,2,3,4,6,8,10,12,16,20,24,32,48 & 64

✦ 0 ≤ eB ≤ 3.35 GeV^2 (~70 M_π^2)

✦ Fixed scale approach to nonzero T

Magnetic catalysis at T=0

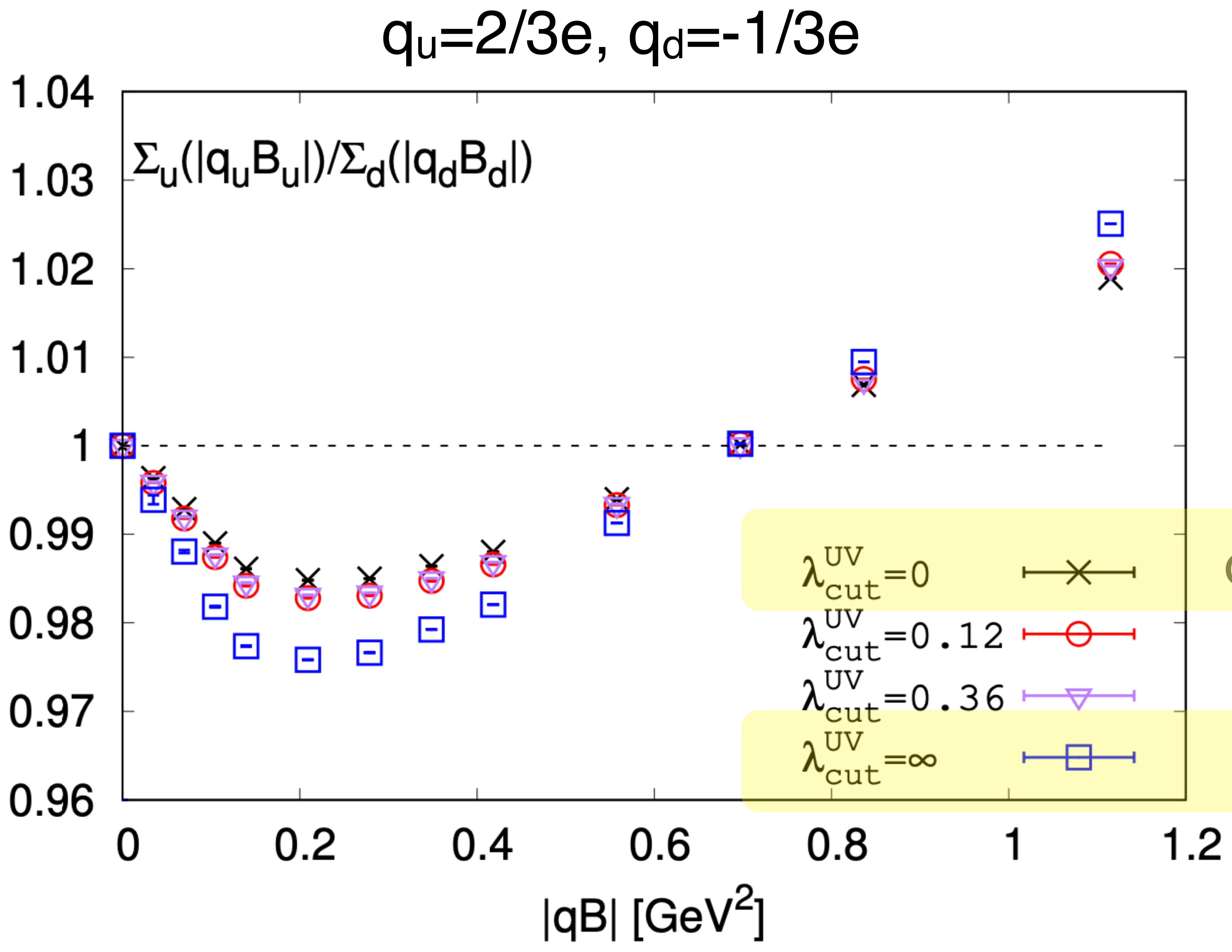




Linear in eB at large eB ≳ 0.5 GeV^2

Power law in eB at small eB ≲ 0.5 GeV^2

qB scaling of light quark chiral condensate at T=0

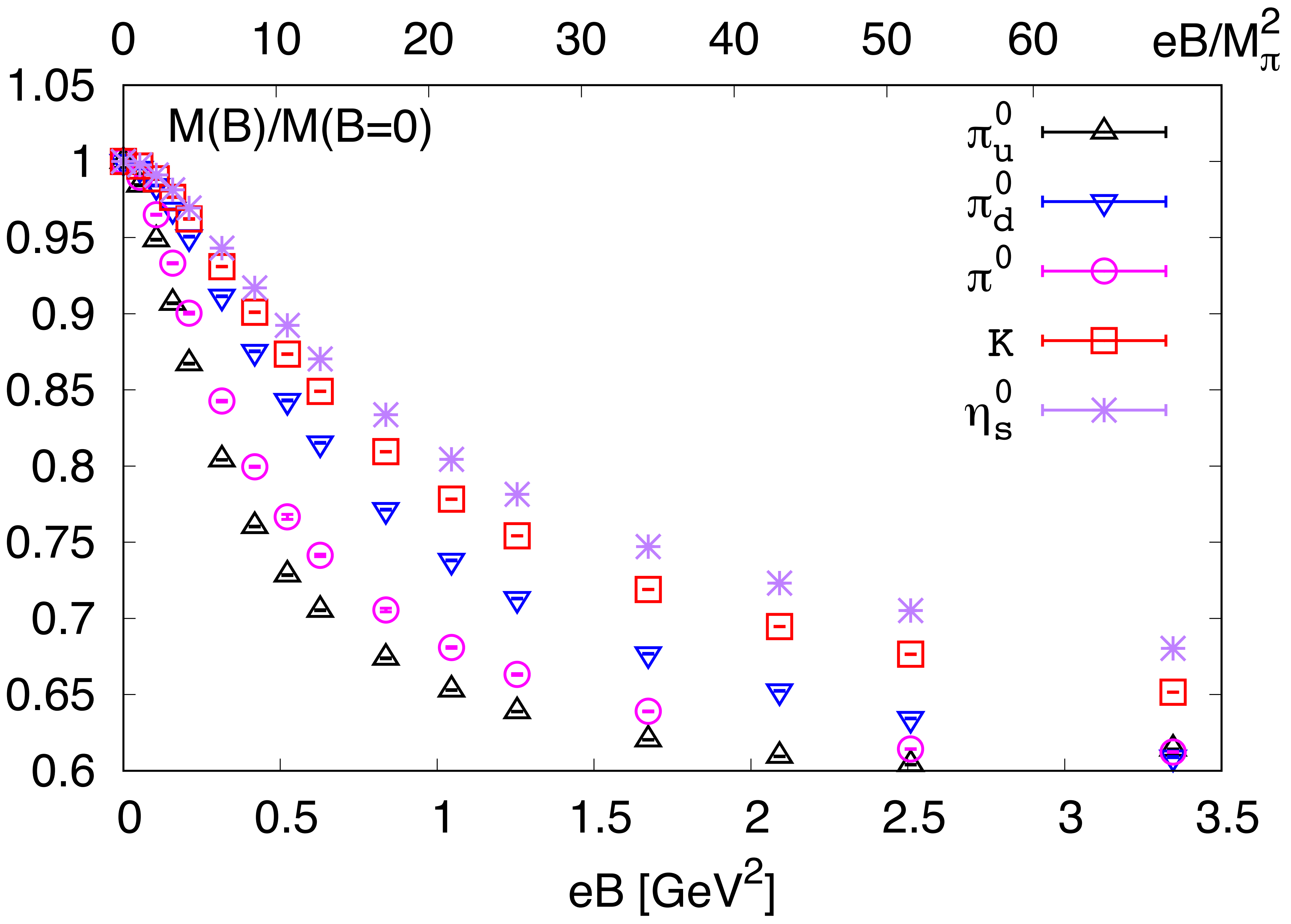


$\lambda_{\text{cut}}^{\text{UV}}$: different estimates of UV divergence in the chiral condensate are removed

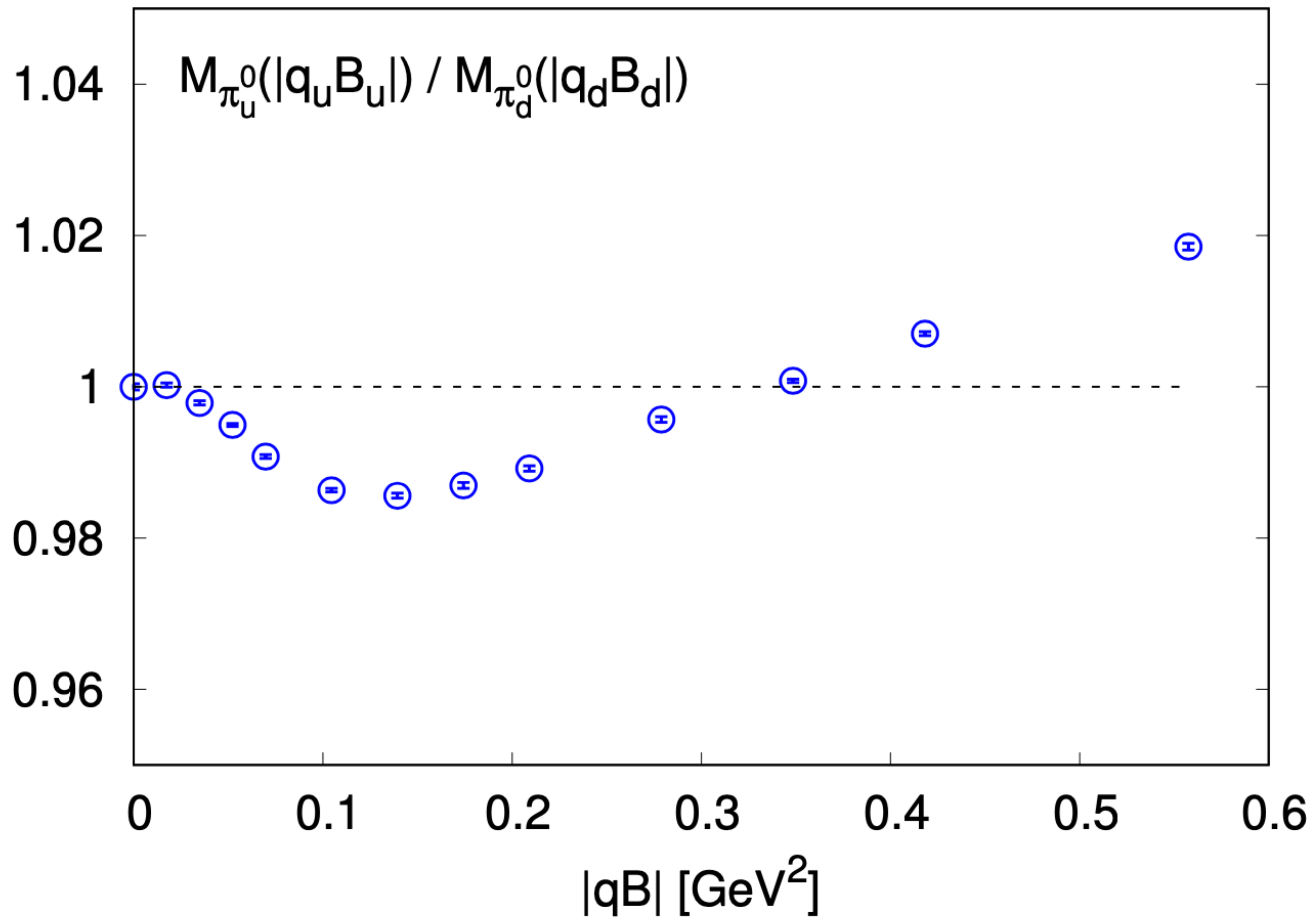
Chiral condensate at T=0&B=0 is subtracted

No subtraction

Masses of neutral and charged pseudo scalar mesons



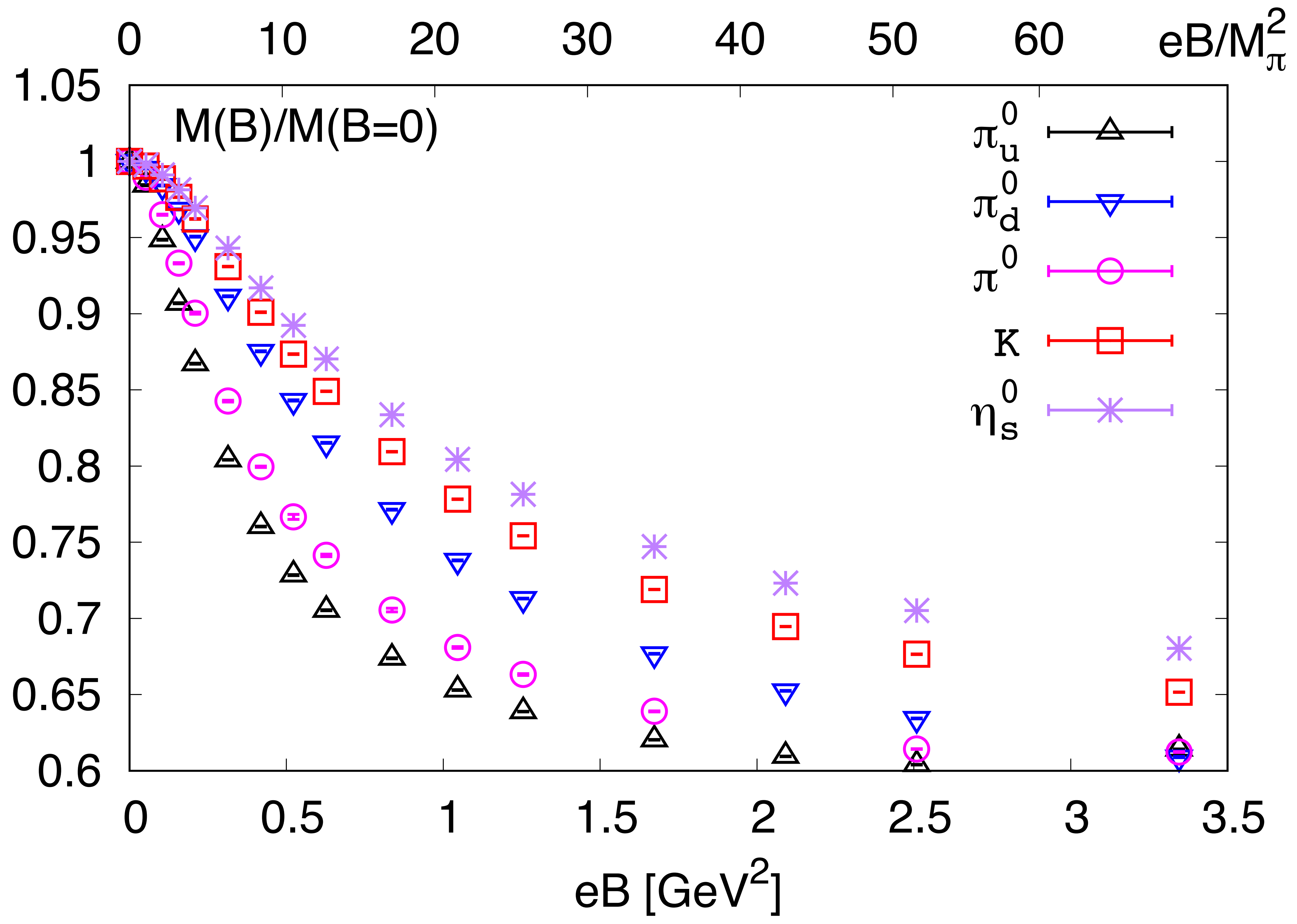
Mass of neutral pseudo scalar meson decreases with eB

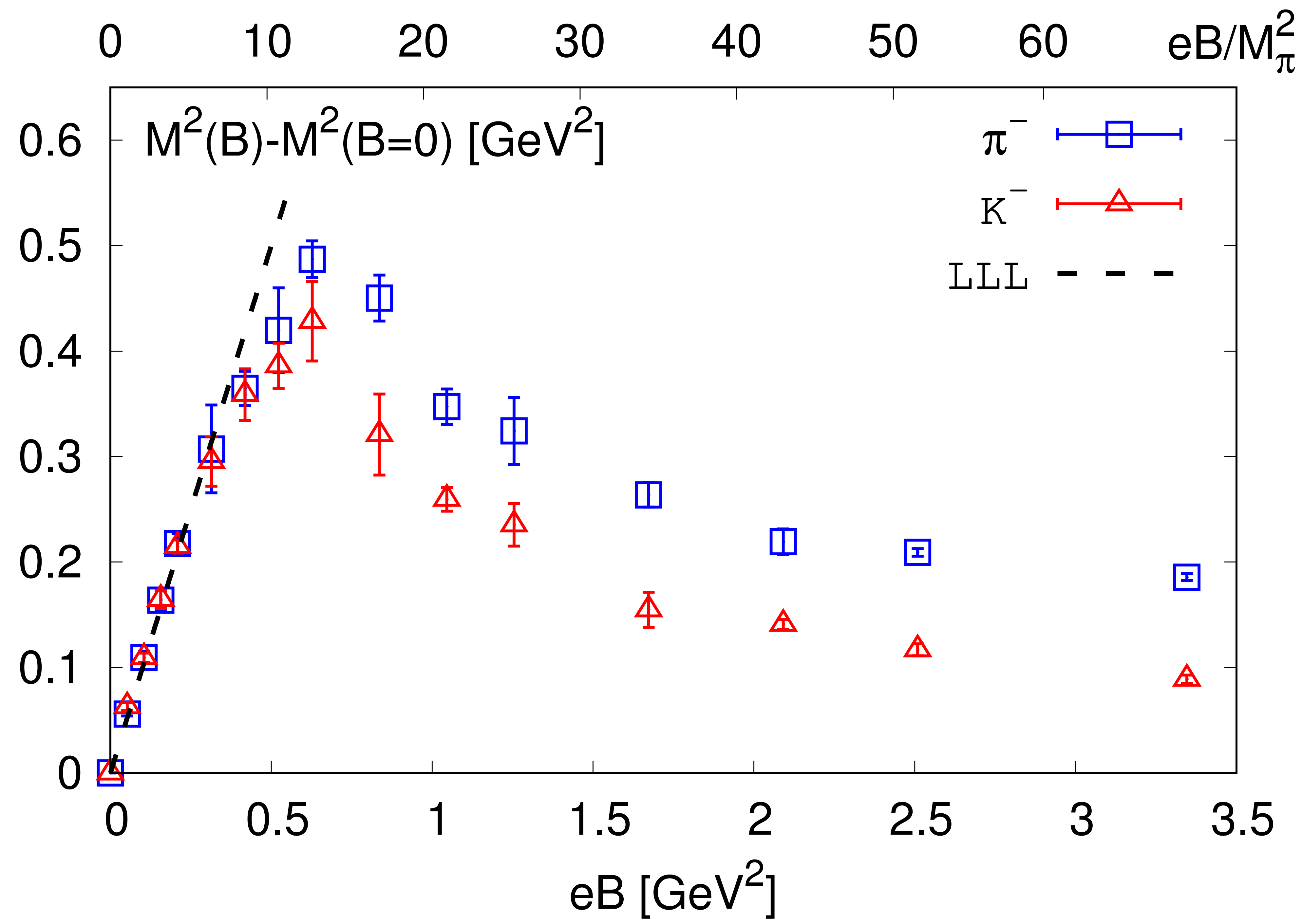


$$|\pi^0\rangle = \alpha|u\gamma_5\bar{u}\rangle - \beta|d\gamma_5\bar{d}\rangle$$

qB scaling observed in the up and down quark flavor components of neutral pion mass

Masses of neutral and charged pseudo scalar mesons

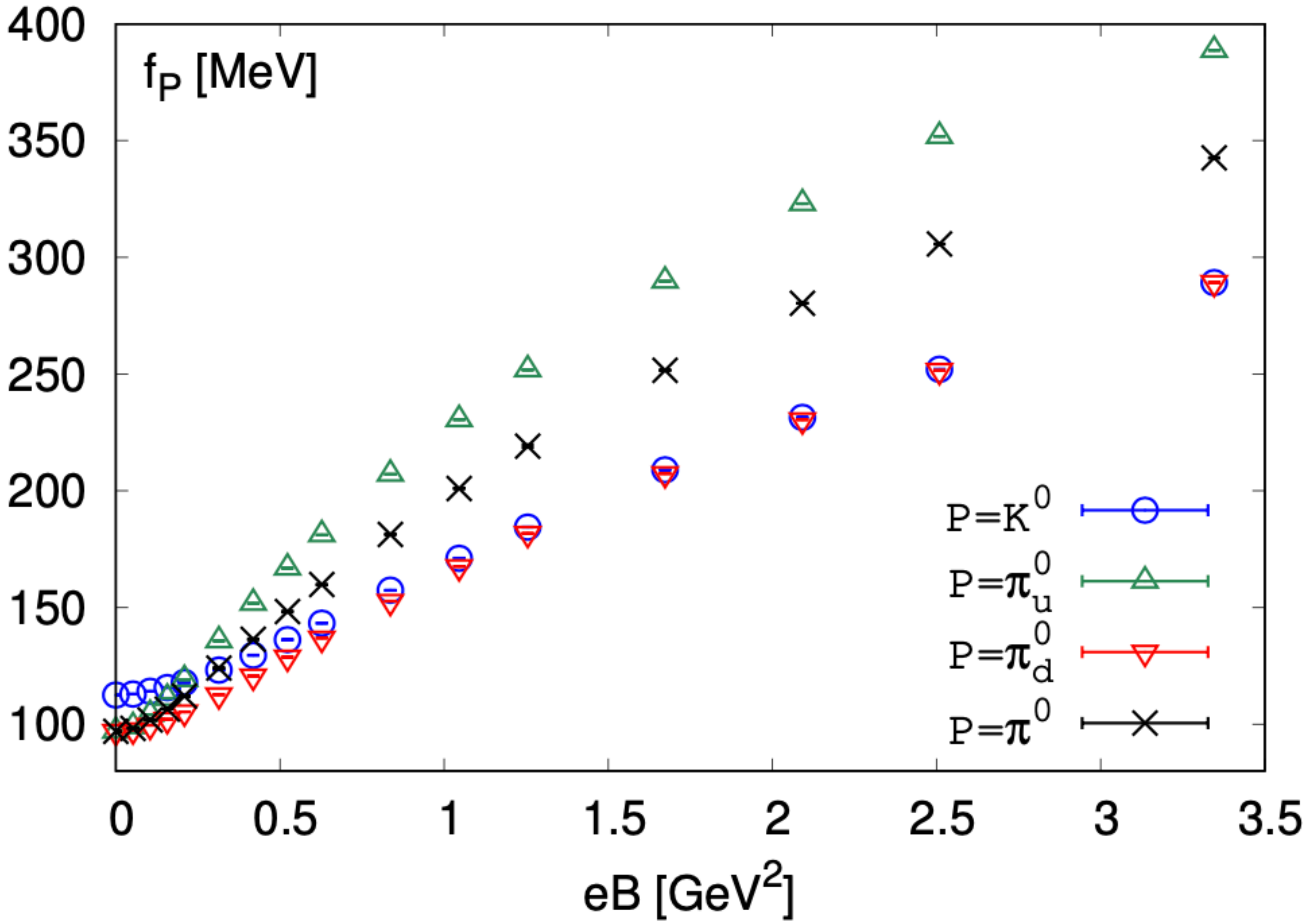


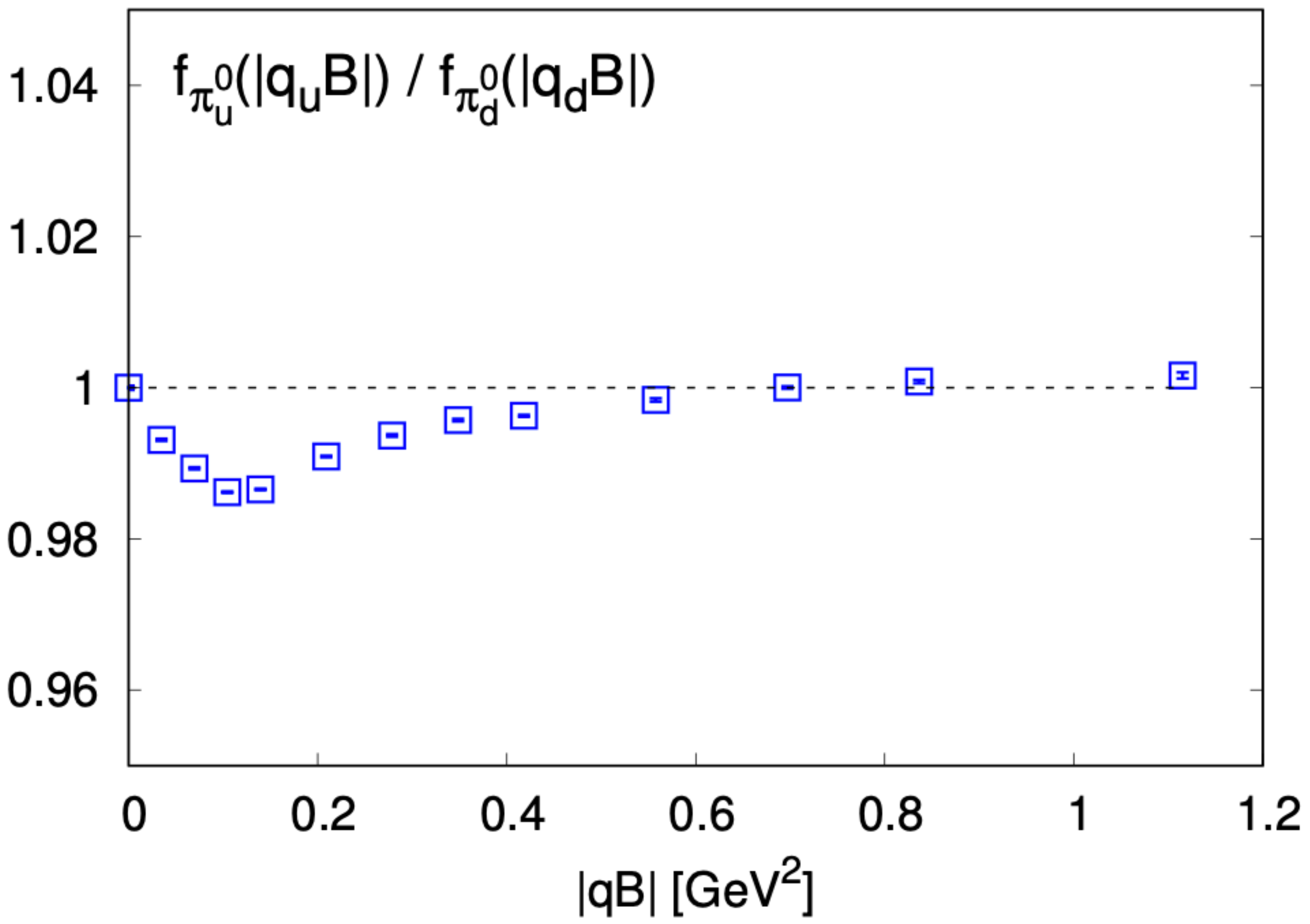


Mass of neutral pseudo scalar meson decreases with eB

Mass of charged pseudo scalar mesons firstly increases and then decreases with eB

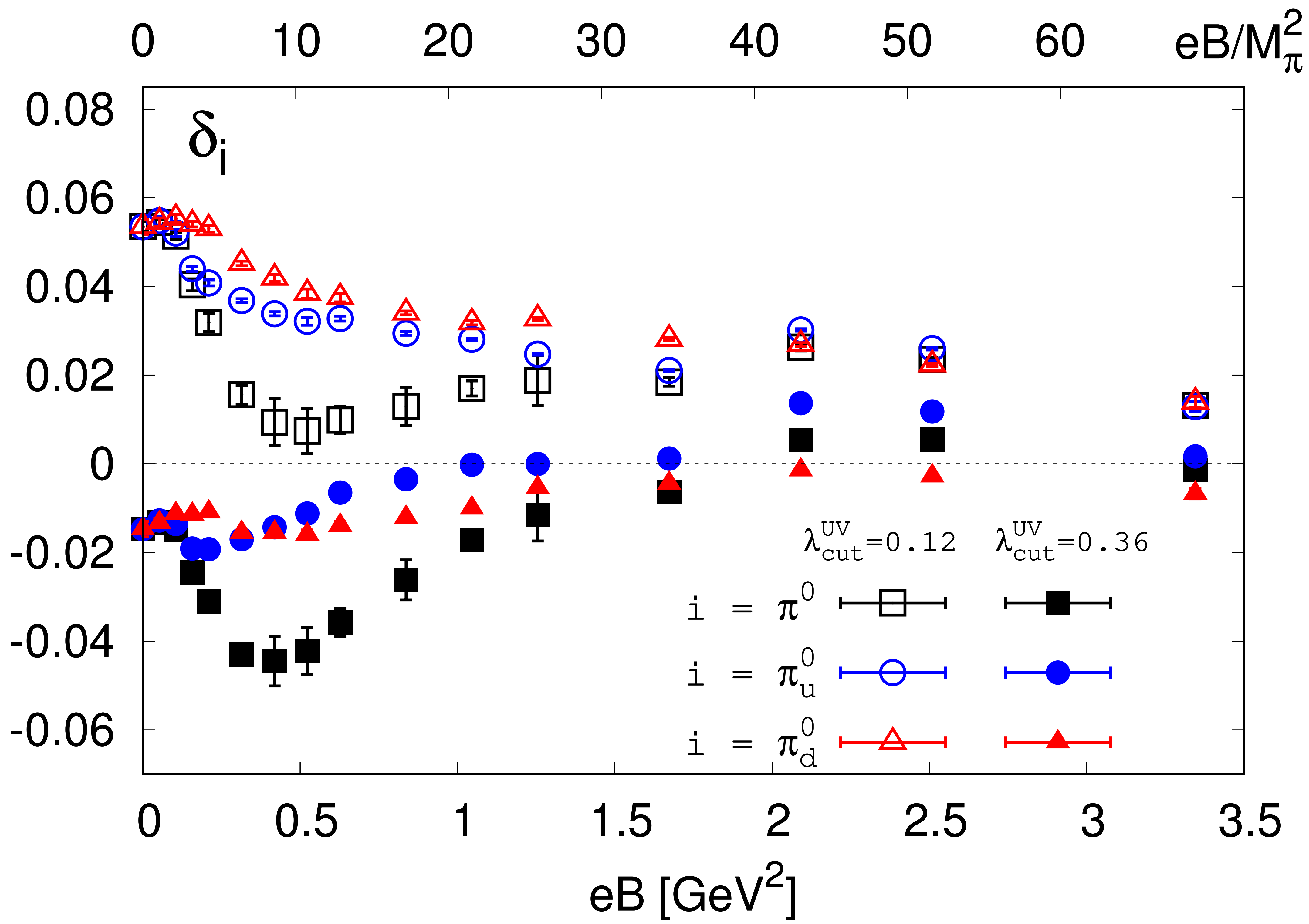
Decay constants of neutral pion and kaon





- All the decay constants increase with eB
- qB scaling observed in u and d quark flavor components of f_π

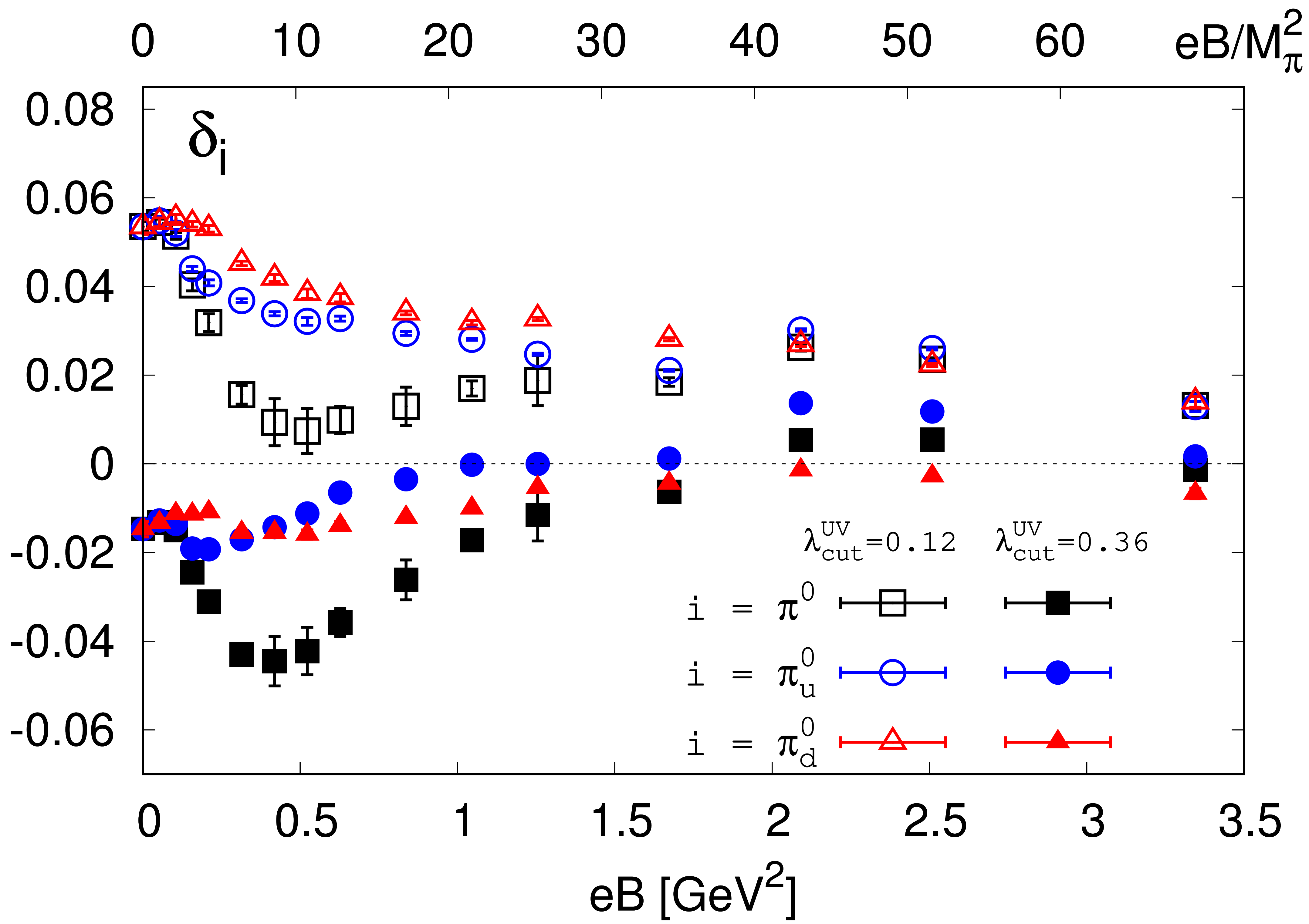
Gell-Mann-Oakes-Renner relation

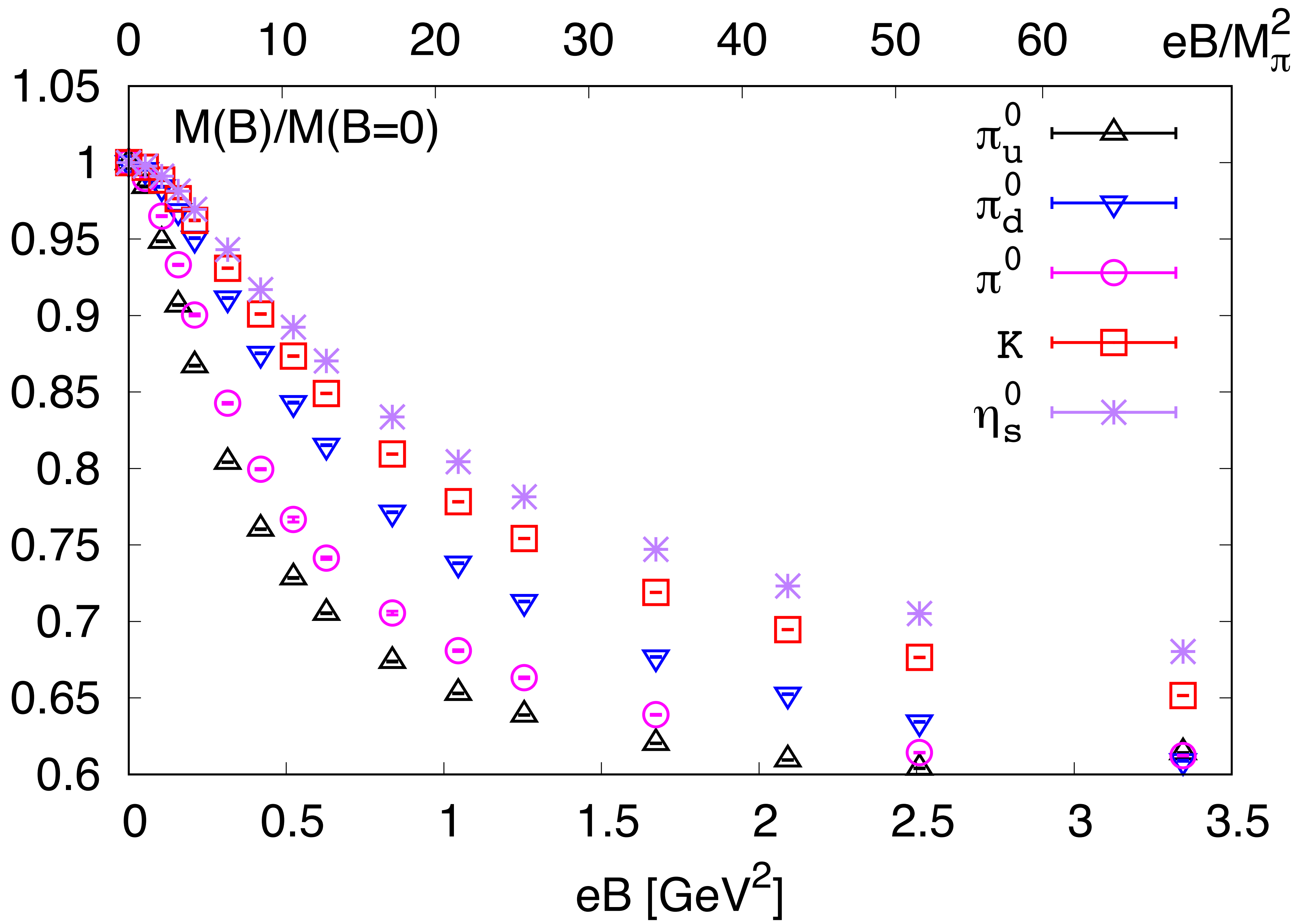


$$4m_u \,\langle\bar\psi\psi\rangle_u = 2f^2_{\pi^0_u} M^2_{\pi^0_u}\,(1-\delta_{\pi^0_u})$$
$$4m_d \,\langle\bar\psi\psi\rangle_d = 2f^2_{\pi^0_d} M^2_{\pi^0_d}\,(1-\delta_{\pi^0_d})\,.$$
$$(m_u+m_d)\,\left(\langle\bar\psi\psi\rangle_u+\langle\bar\psi\psi\rangle_d\right) = 2f^2_\pi M^2_\pi\,(1-\delta_\pi)$$

neutral pion remains as a Goldstone boson with eB up to ~3.5 GeV^2

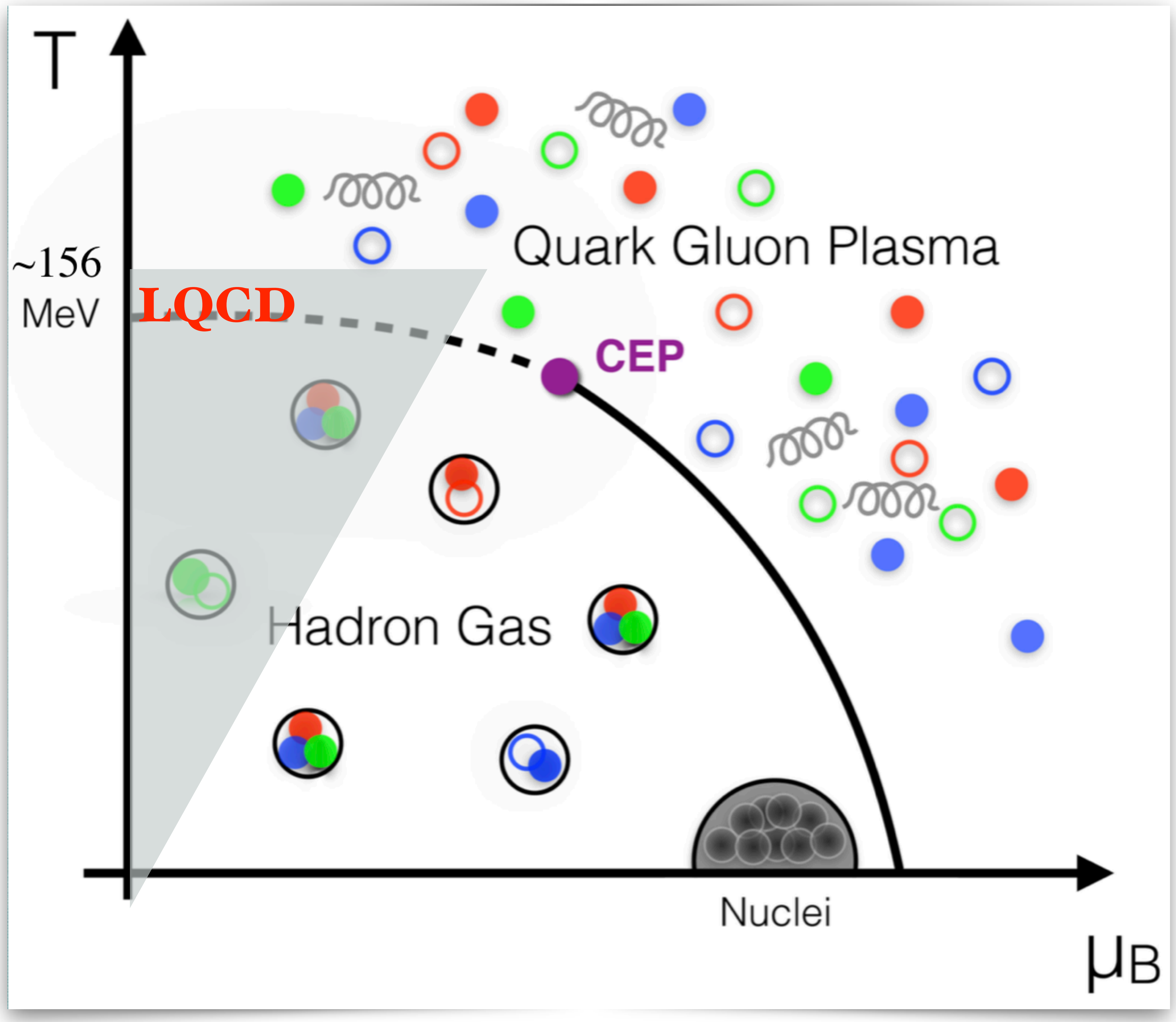
Gell-Mann-Oakes-Renner relation





T_{pc} decreases with eB regardless of (inverse) magnetic catalysis

Fluctuations of conserved charges



HTD, F. Karsch, S. Mukherjee, arXiv:1504.05274

Taylor expansion of the QCD pressure:

Allton et al., Phys.Rev. D66 (2002) 074507
Gavai & Gupta et al., Phys.Rev. D68 (2003) 034506

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln \mathcal{Z}(T, V, \hat{\mu}_u, \hat{\mu}_d, \hat{\mu}_s) = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{BQS}}{i!j!k!} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

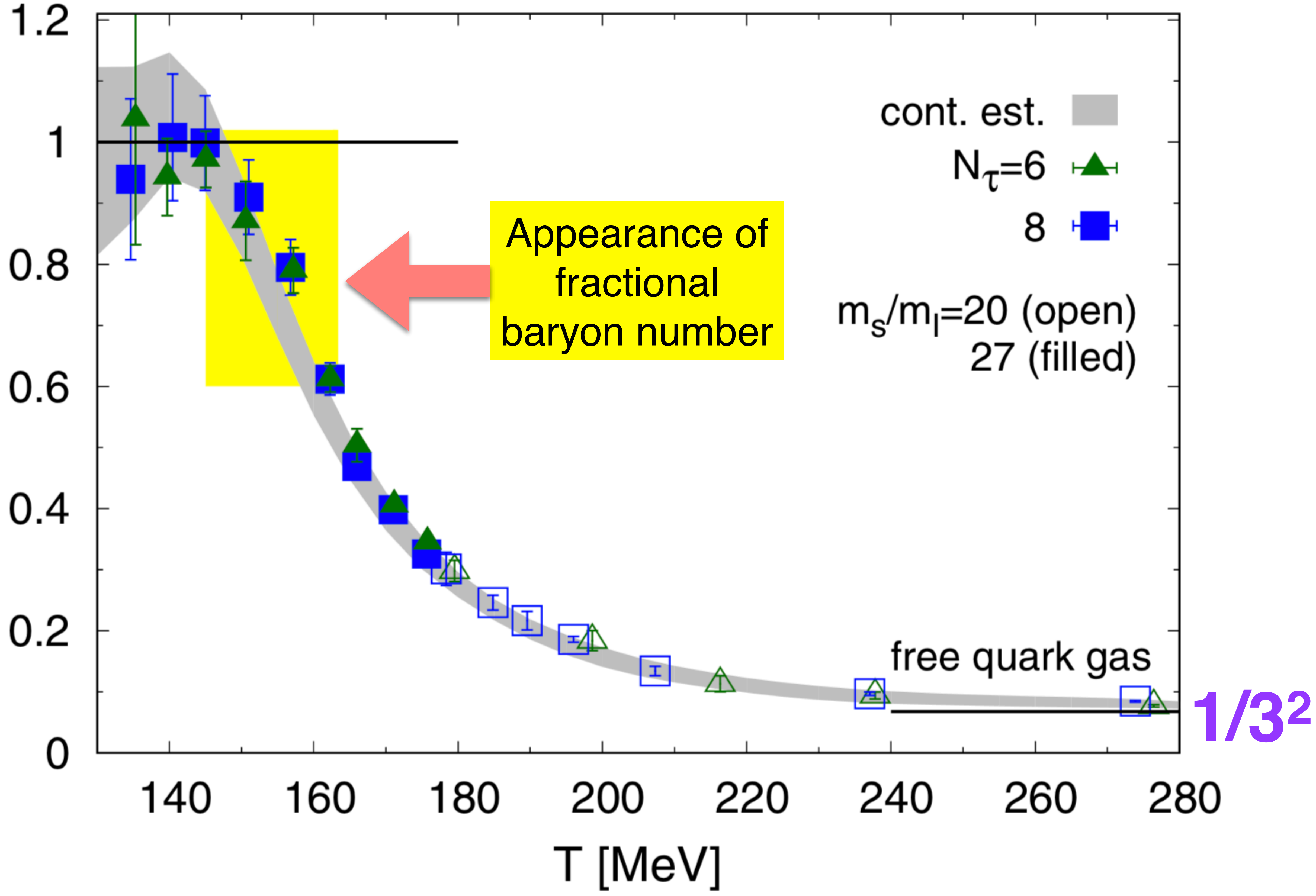
- Taylor expansion coefficients at μ=0 are computable in LQCD

fluctuations of conserved charges:

$$\chi_{ijk}^{BQS} \equiv \chi_{ijk}^{BQS}(T) = \frac{1}{VT^3} \frac{\partial^{i+j+k} P(T,\mu)/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k}\Big|_{\hat{\mu}_{B,Q,S}=0}$$

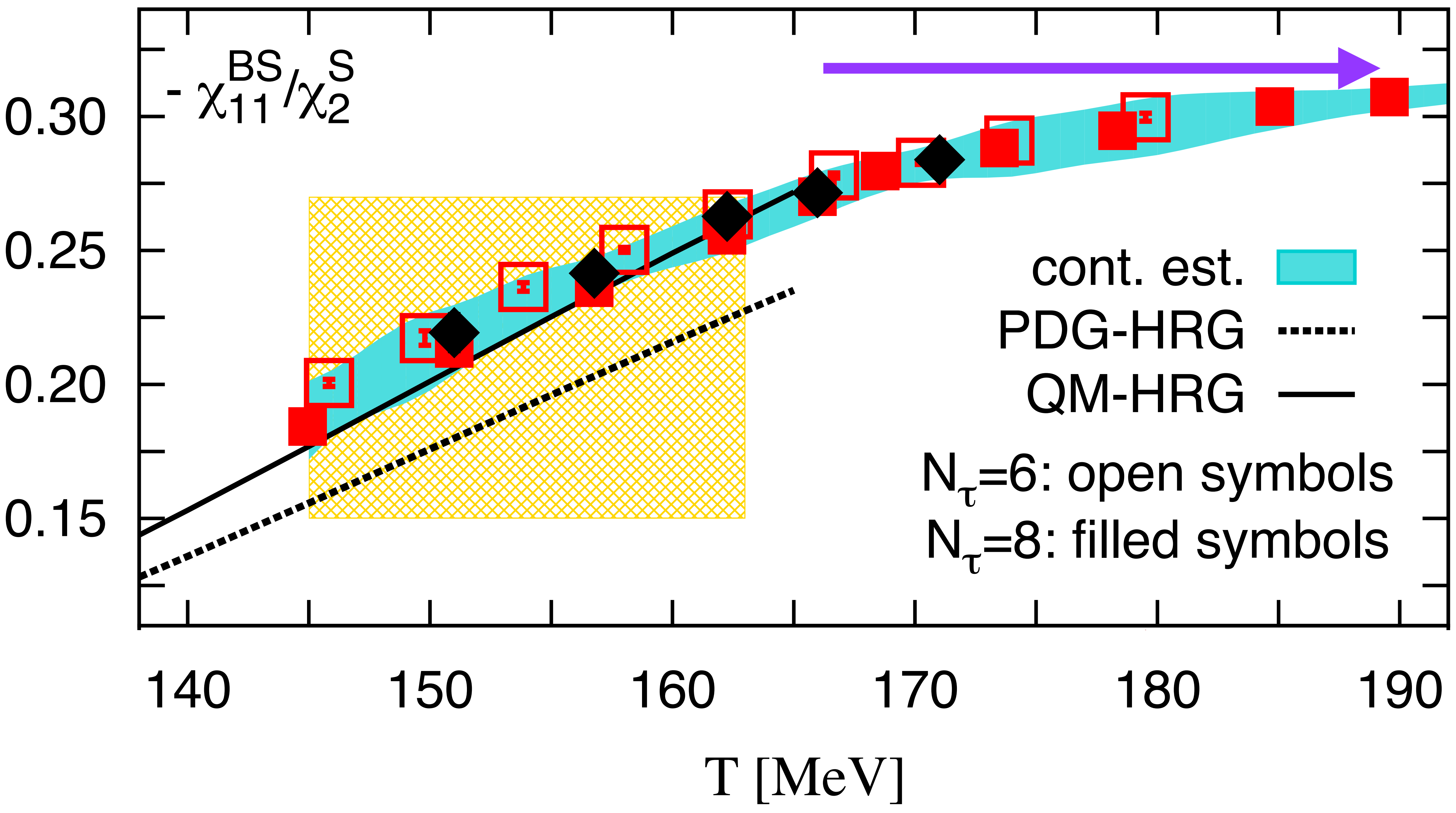
Change of degree of freedom in thermal QCD

χ_4^B/χ_2^B measure of fluctuations of Baryon number



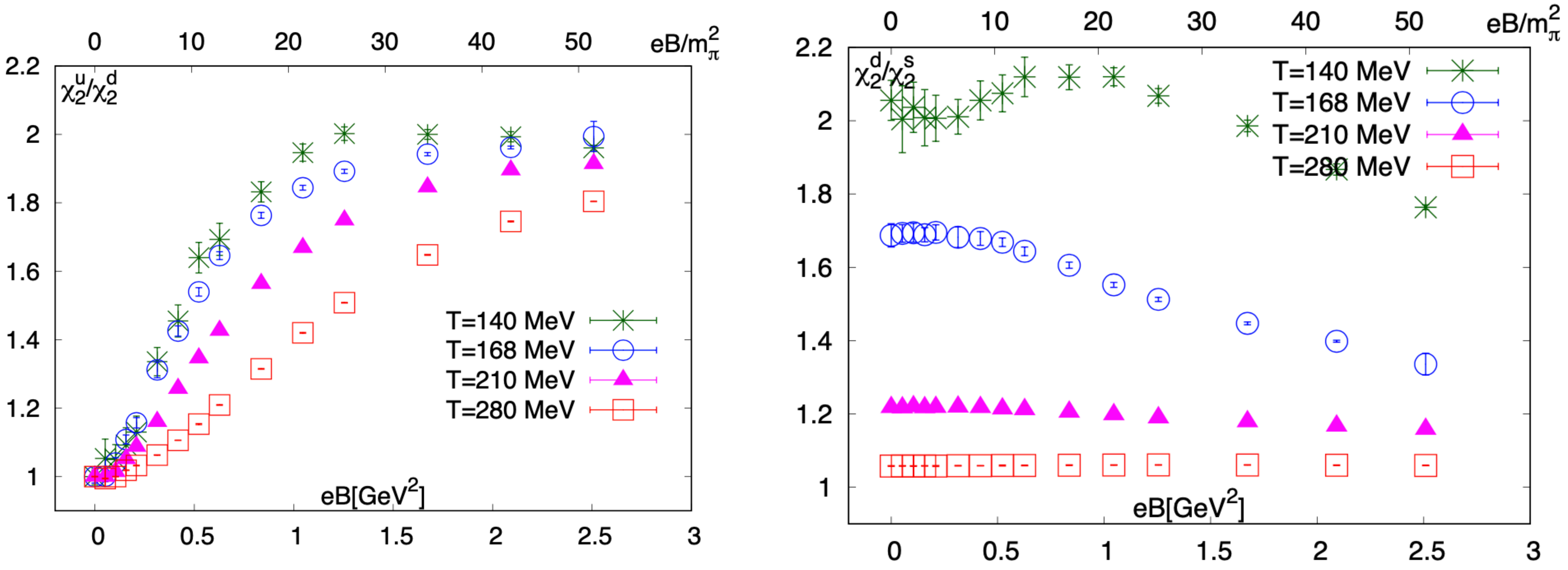
HotQCD: PRL 111(2013) 082301,
HTD, F. Karsch, S. Mukherjee, arXiv: 1504.05274

1/3, free gas limit



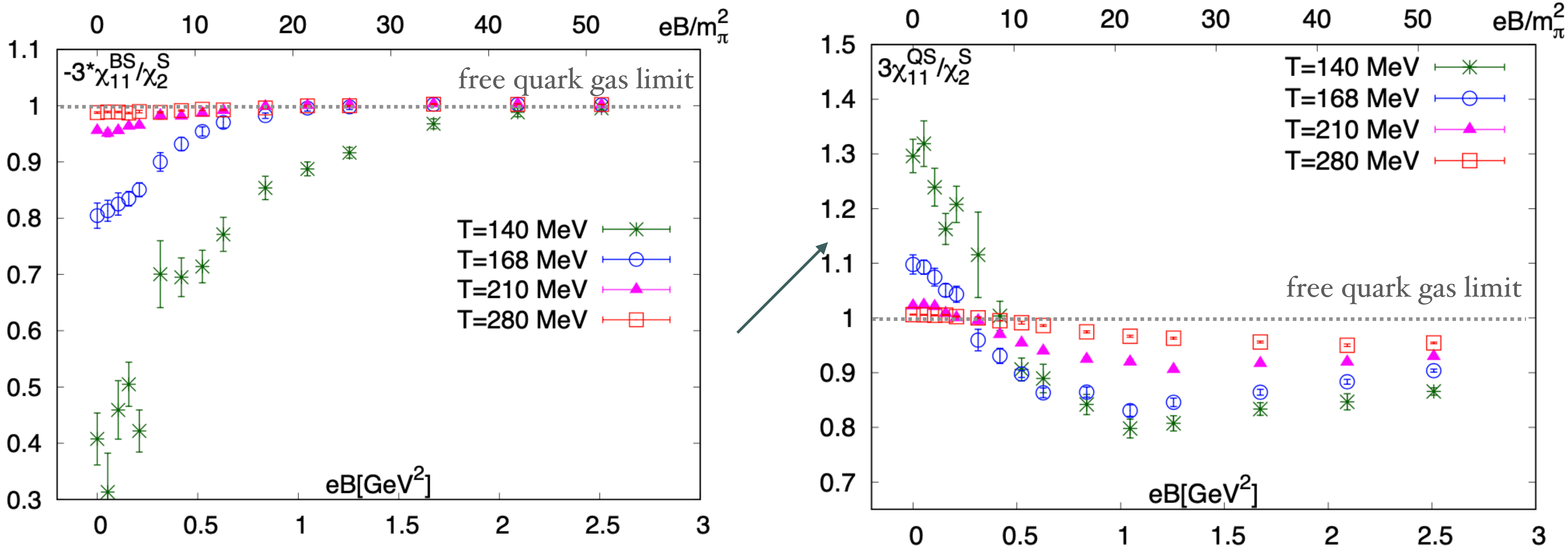
Bielefeld-BNL-CCNU, PRL 113 (2014) 072001
V. Koch, A. Majumder, and J. Randrup , PRL95 (2005) 182301

Quark number susceptibilities in nonzero magnetic field



- Up and down quark sus. are degenerate at eB=0 and start to deviate at eB=/=0
- Ratio for down to strange quark sus. is independent on eB at high T, while decreases at two low temperatures

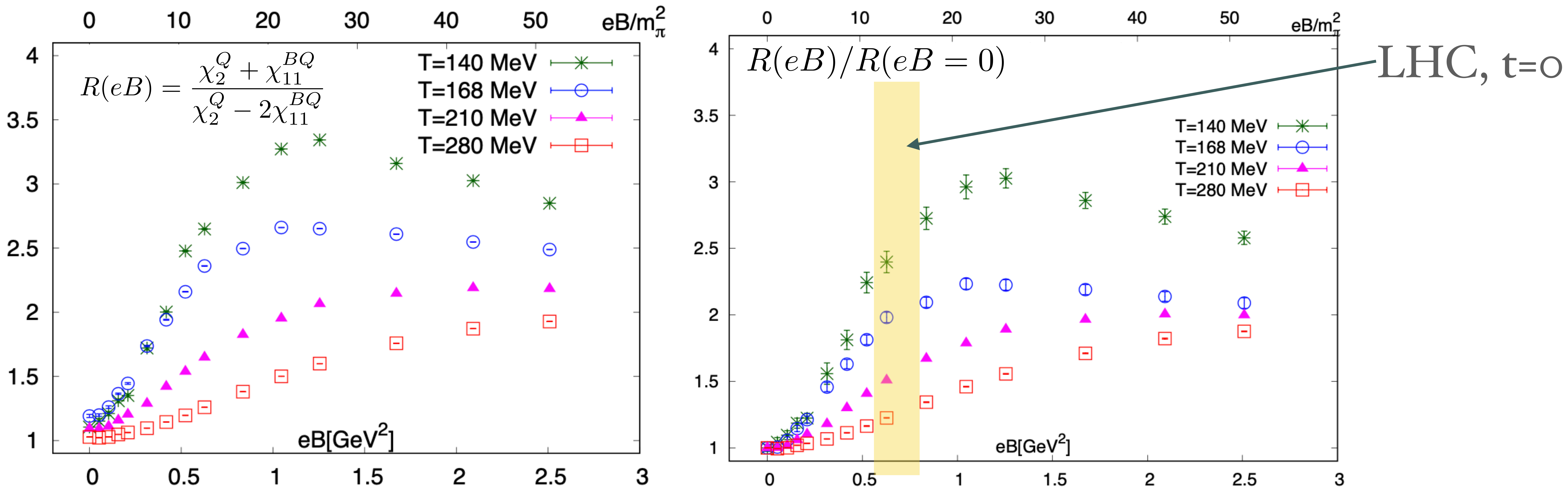
Fluctuations of conserved charges in strong magnetic field



Magnetic field accelerates the transition from hadronic matter to QGP

Fluctuations of conserved charges

Quantities that probes χ_2^u/χ_2^d at high T at eB=/=0

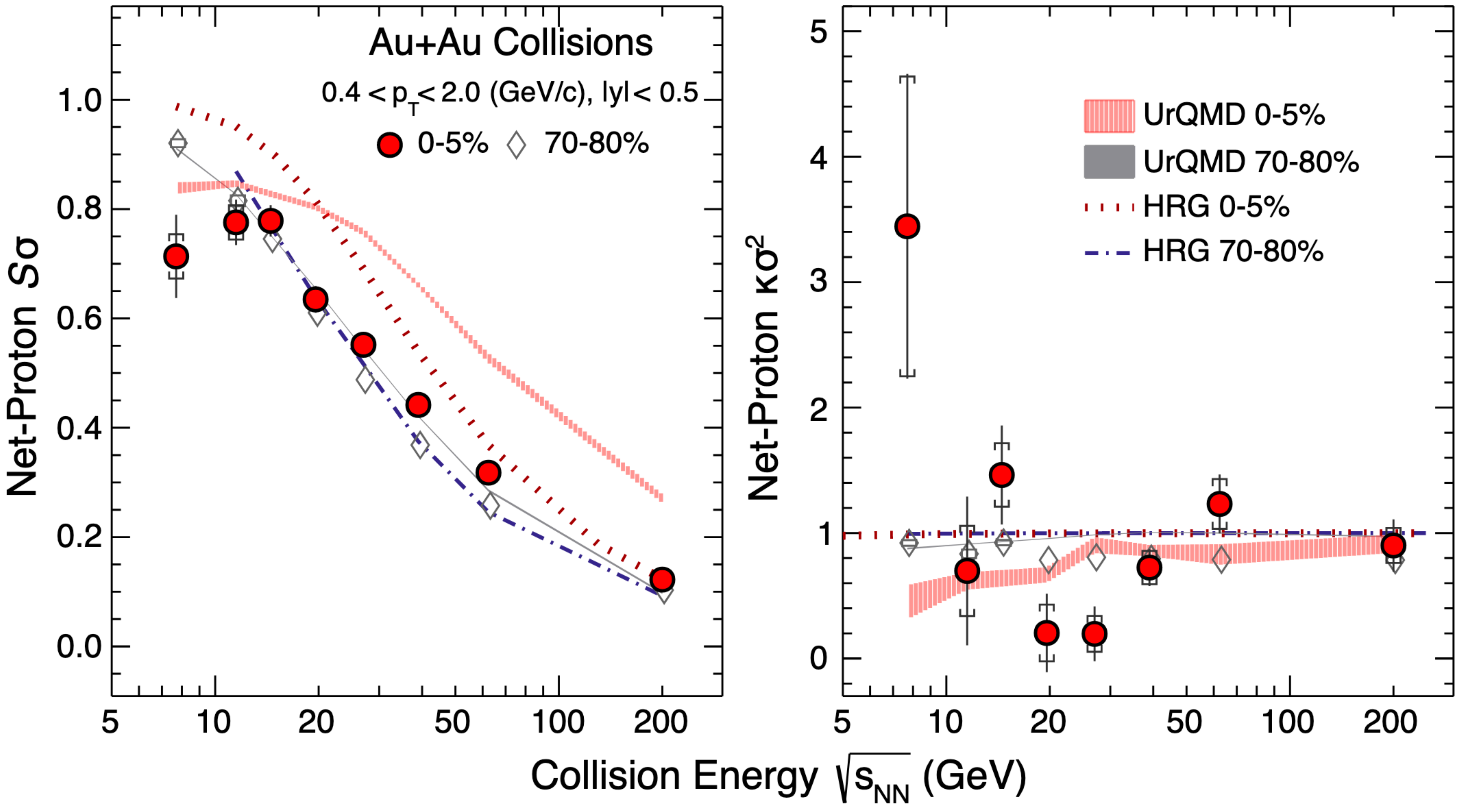


- Comparison between fluctuations in peripheral and central collisions to check the impact of the magnetic field ?
- …

Search for Criticality

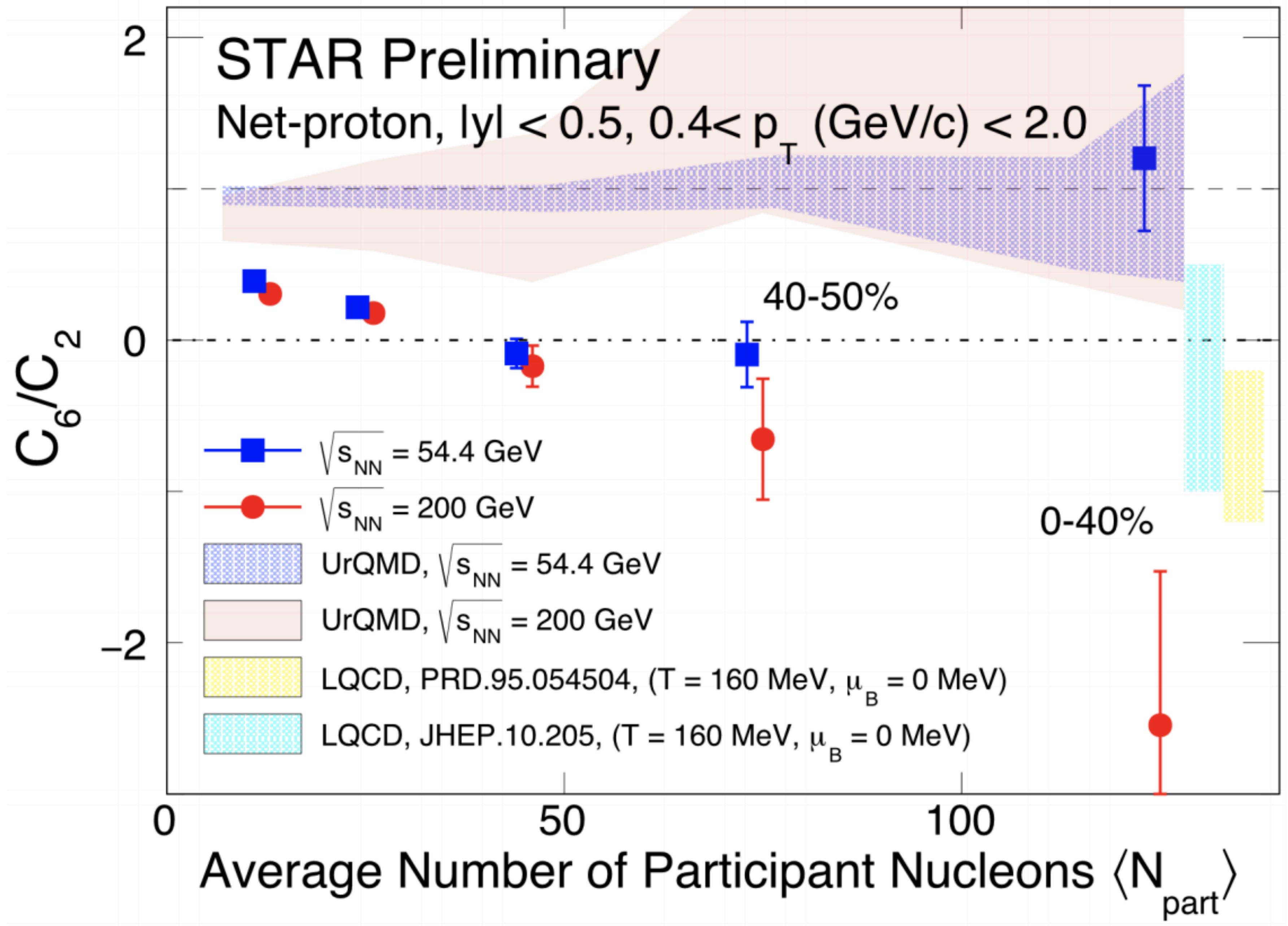
3rd to 1st order cumulant ratio

4th to 2nd order cumulant ratio



STAR, arXiv: 2001.02852

6th to 2nd order cumulant ratio



Toshihiro Nonaka, arXiv:2002.12505
Ashish Pandav, arXiv:2005.13398

Ginzburg-Landau-Wilson approach

Partition function: $$Z = \int [\mathrm{d}\sigma] \exp\left(- \int \mathrm{dx}\, \mathcal{L}_{eff}\,(\sigma(\mathrm{x}); K) \right)$$

Landau function: $$\mathcal{L}_{eff} = \frac{1}{2}(\nabla\sigma)^2 + \sum_n a_n(K)\sigma^n$$

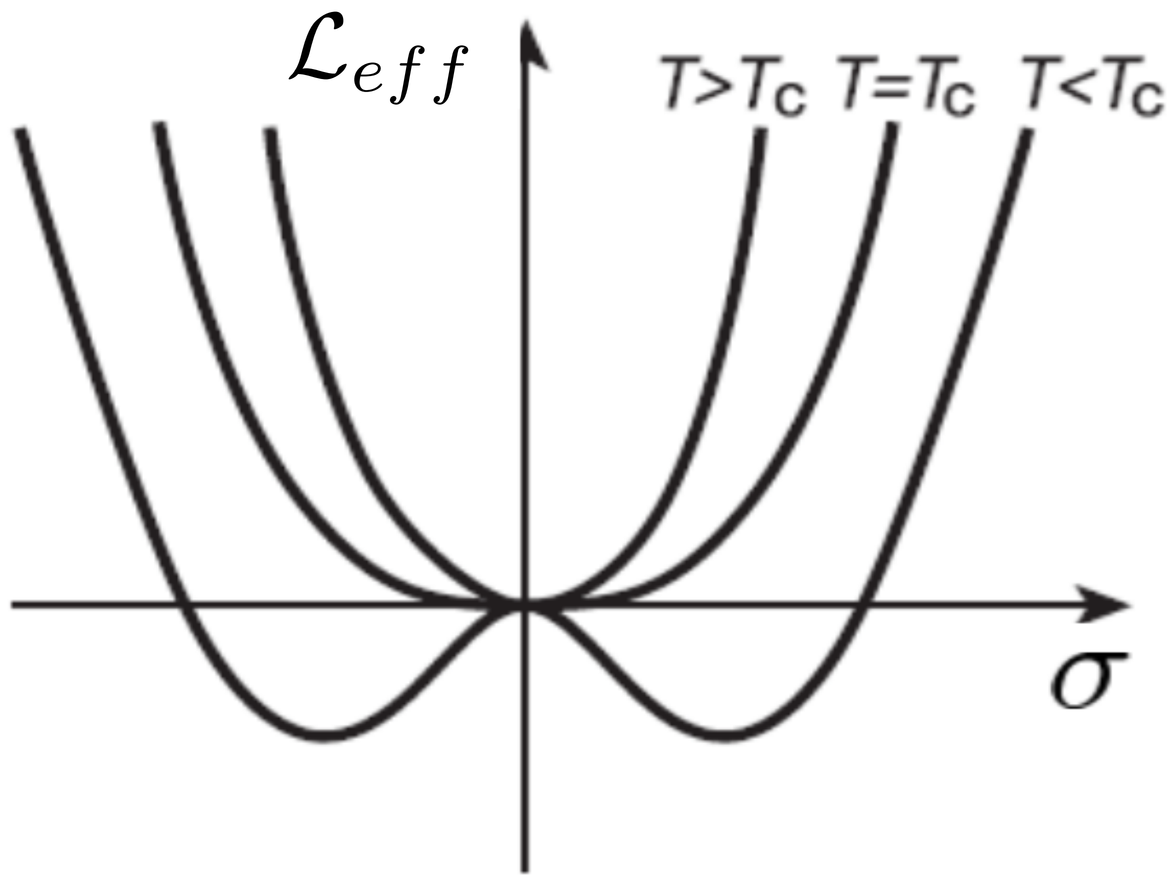
Same symmetry with the underlying theory

σ(x): order parameter field;
K={m,μ}: external parameters

2nd order phase transition

Z(2) Ising model, Nf=2 QCD

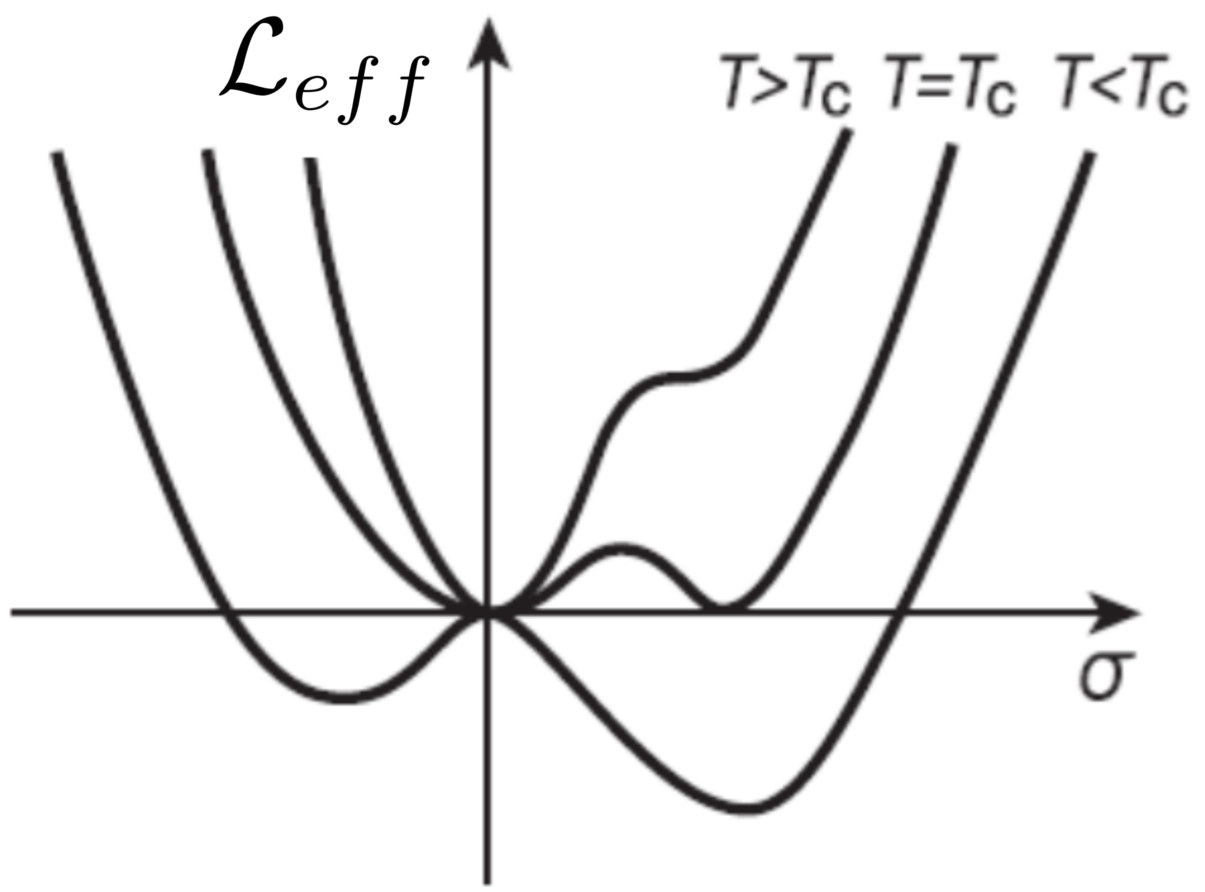
$$\mathcal{L}_{eff} = \frac{1}{2}a\sigma^2 + \frac{1}{4}b\sigma^4$$



1st order phase transition

Z(3) Potts model, Nf=3 QCD

$$\mathcal{L}_{eff} = \frac{1}{2}a\sigma^2 - \frac{1}{3}c\sigma^3 + \frac{1}{4}b\sigma^4$$



Ginzburg-Landau-Wilson approach

Partition function: $$Z = \int [\mathrm{d}\sigma] \exp\left(- \int \mathrm{dx}\, \mathcal{L}_{eff}\, (\sigma(\mathrm{x}); K) \right)$$

Landau function: $$\mathcal{L}_{eff} = \frac{1}{2} (\nabla\sigma)^2 + \sum_n a_n(K)\sigma^n$$

Same symmetry with the underlying theory

σ(x): order parameter field;
K={m,μ}: external parameters

2nd order phase transition

order parameter M:
continuous in T

fluctuations of M:

$$\chi(T) = \frac{T}{V}(\langle M^2 \rangle - \langle M \rangle^2)$$

$$\chi(T_c) \sim V^{(2-\eta)/3}$$

1st order phase transition

M:
discontinuous in T

fluctuations of M:

$$\chi(T_c) \sim V$$

Landau functional of QCD

Pisarski & Wilczek, PRD 84'

Symmetry: $SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A$

Chiral field: $\Phi_{ij} \sim \frac{1}{2}\bar{q}^j(1-\gamma_5)q^i = \bar{q}^j_R q^i_L$

Chiral transformation: $\Phi \to e^{-2i\alpha_A} V_L \, \Phi \, V_R^\dagger$

$$\mathcal{L}_{eff} = \frac{1}{2}\operatorname{tr}\partial\Phi^\dagger\partial\Phi + \frac{a}{2}\operatorname{tr}\Phi^\dagger\Phi + \frac{b_1}{4!}\left(\operatorname{tr}\Phi^\dagger\Phi\right)^2 + \frac{b_2}{4!}\operatorname{tr}\left(\Phi^\dagger\Phi\right)^2$$

→ SU(N$_f$)$_L$xSU(N$_f$)$_R$xU(1)$_A$

$$- \frac{c}{2}\left(\det\Phi + \det\Phi^\dagger\right)$$

→ SU(N$_f$)$_L$xSU(N$_f$)$_R$

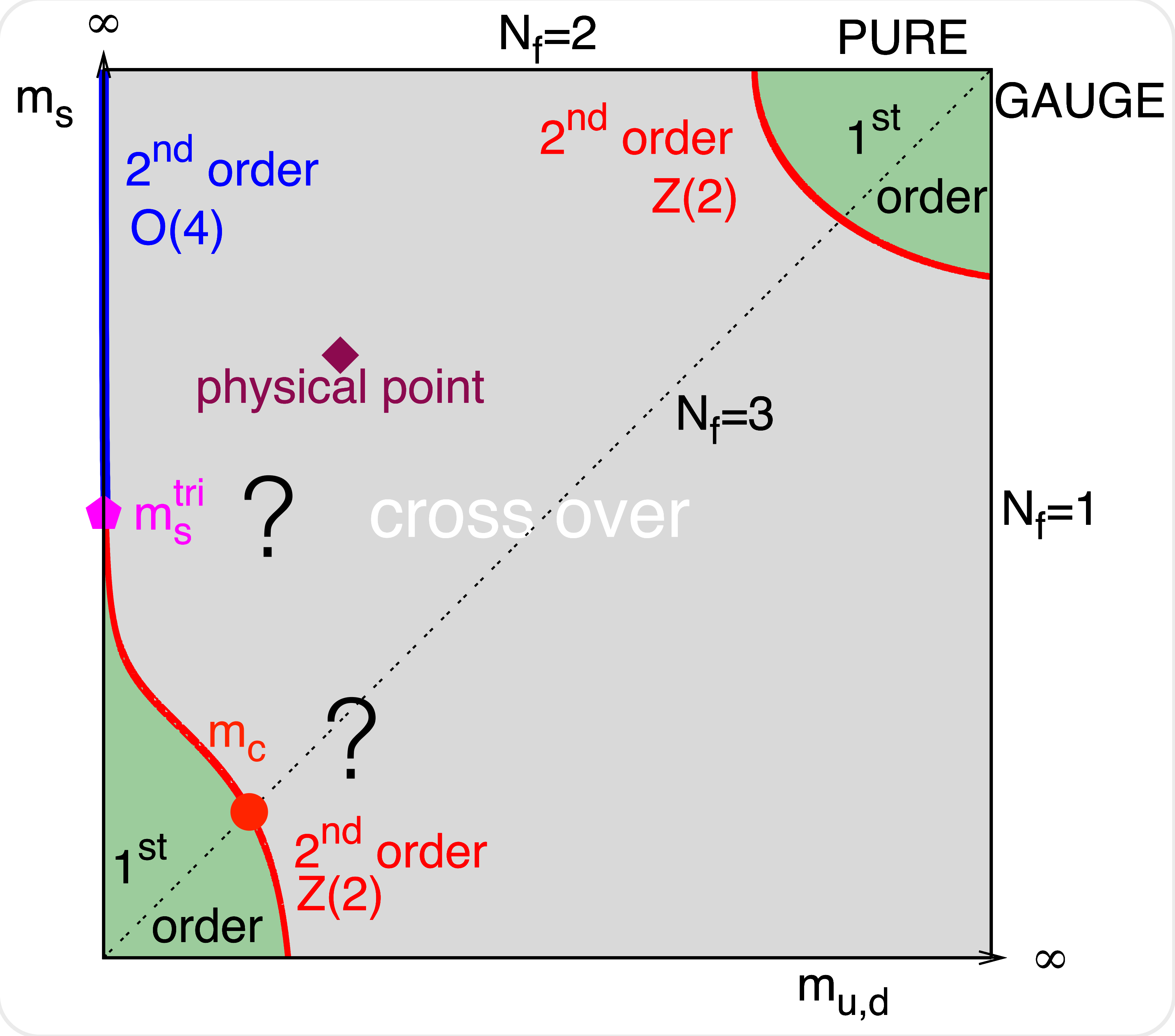
$$- \frac{d}{2}\operatorname{tr} h\left(\Phi + \Phi^\dagger\right).$$

→ Quark mass term

Results on phase transitions should be eventually checked by Lattice QCD

QCD criticality relevant to the real world

Columbia plot:
QCD phase diagram in quark mass plane



- At physical point: crossover type transition

[HotQCD] PRL,113(2014)082001, PRD 85(2012)054503
[Wuppertal-Budapest], JHEP 1009 (2010) 073

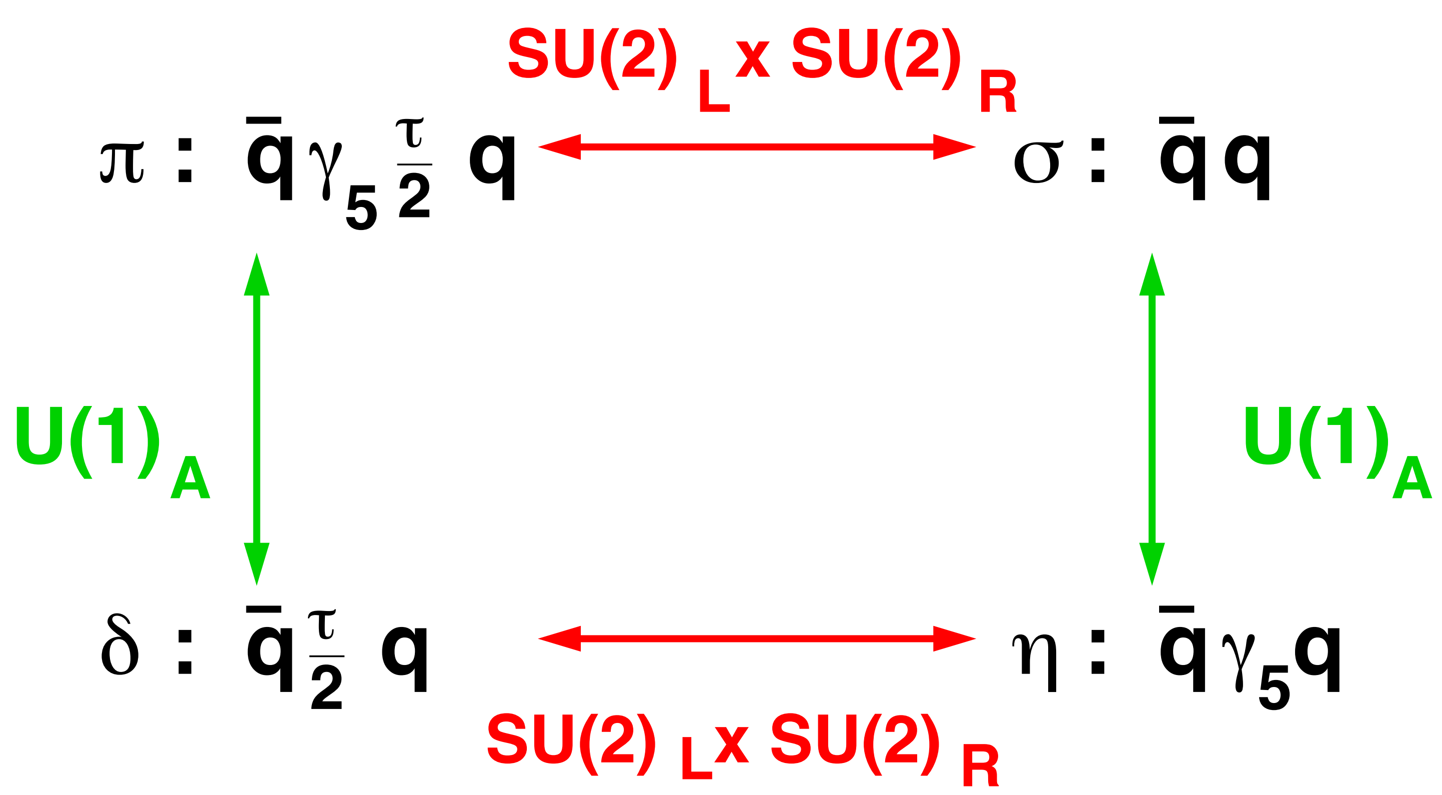
- Critical lines of second order transition

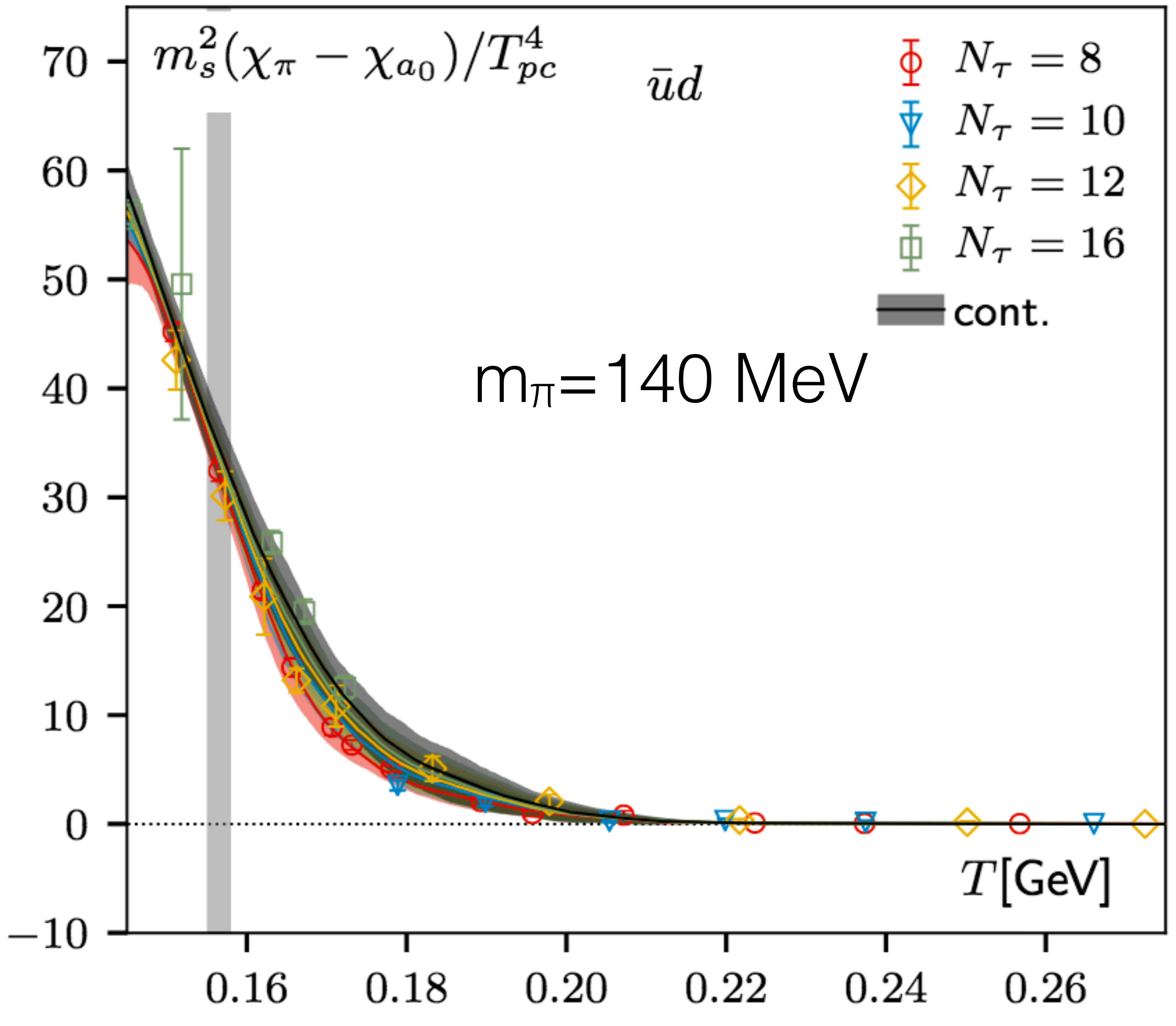
N_f=2: O(4) universality class Pisarski & Wilczek PRD '84,Kogut & Sinclair, PRD '06 …

N_f=3: Ising universality class Karsch, Laermann, Schmidt PLB '04,...

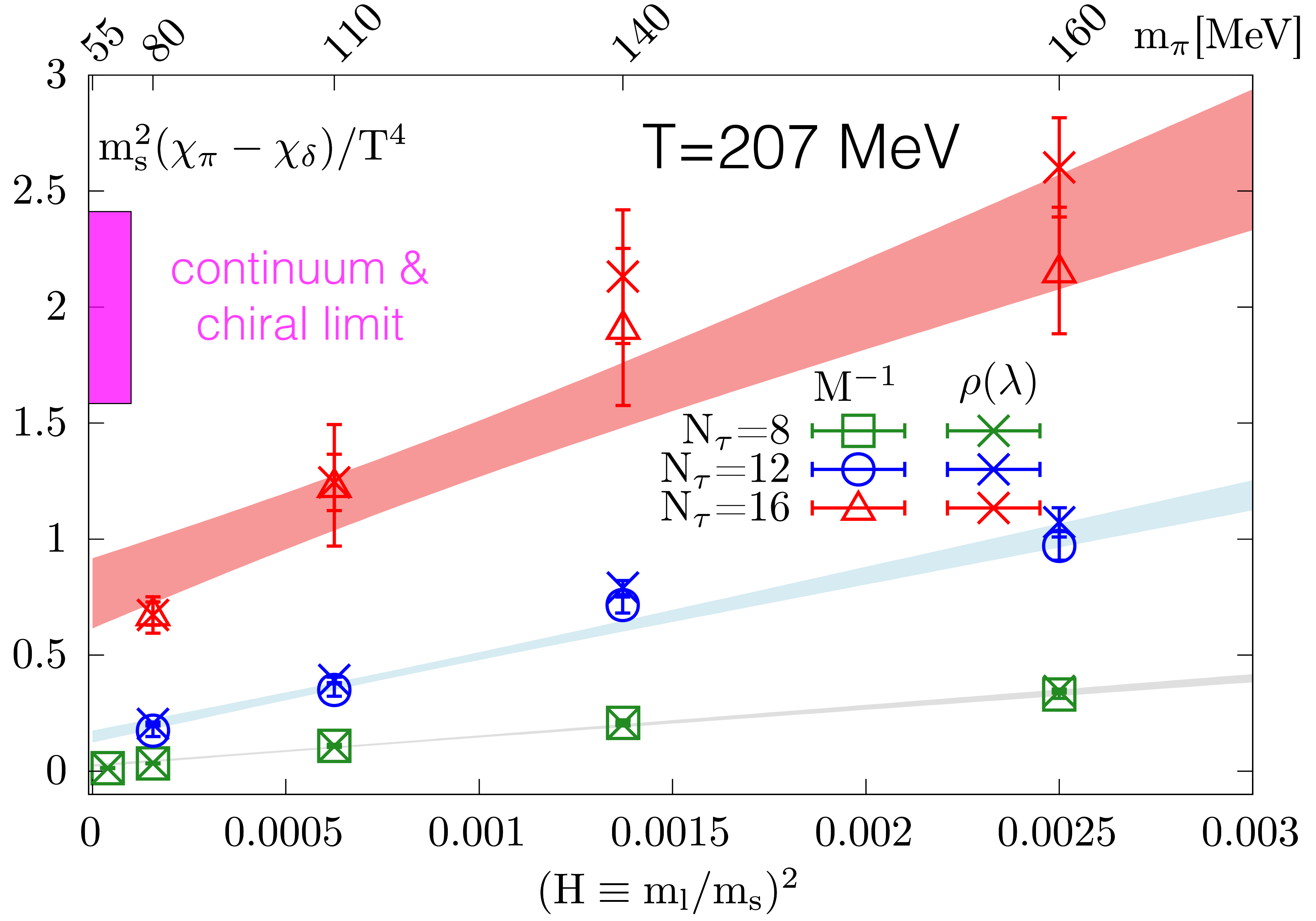
- N_f=2+1 QCD: fate of axial U(1) symmetry ? O(4) or 1st order ?
- Criticalities that are relevant to QCD thermodynamics at the physical point ?
- Fundamental scale of QCD: chiral T_c^0 ?

Fate of axial U(1) symmetry at T=/=0





Bazavov et al.[HotQCD], PRD 19', 1908.09552



HTD, S.-T. Li, A. Tomiya, S. Mukherjee, X.-D. Wang, Y. Zhang, work in progress

$$\chi_\pi - \chi_\delta = 0$$ ➡ Effective restoration of $U_A(1)$ symmetry

- Axial U(1) symmetry remains broken at $T_{\chi SB}$ see more in H. Fukaya EPJ Web Conf. 175 (2018) 01012,
- In the chiral limit of Nf=2 QCD the transition is most likely 2nd order belonging to O(4) universality class

Critical behavior in QCD

- Close to the critical window QCD transition is governed by the universal singular behavior

singular regular

$$\frac{p}{T^4} = \frac{1}{VT^3}\ln Z(V,T,\vec{\mu}) = -h^{(2-\alpha)/\beta\delta} f_s(t/h^{1/\beta\delta}) - f_r(V,T,\vec{\mu})$$

"Magnetic coupling": $h \sim m_q$ **"Thermal coupling":** $t \sim \frac{T-T_c}{T_c} + \kappa_2 \left(\frac{\mu_B}{T}\right)^2$

Critical behavior in QCD

- Close to the critical window QCD transition is governed by the universal singular behavior

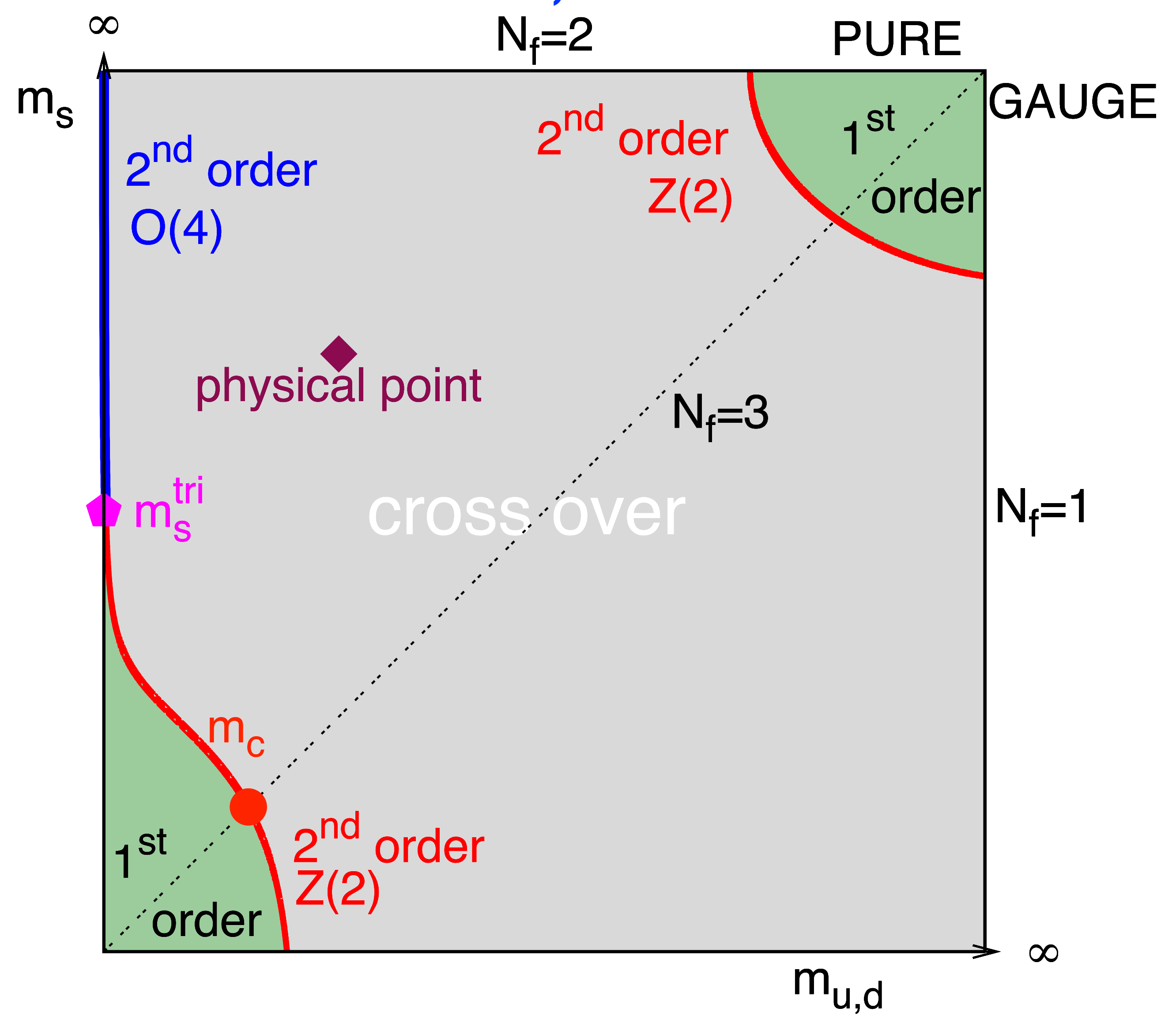
singular / regular

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\vec{\mu}) = -h^{(2-\alpha)/\beta\delta} f_s(t/h^{1/\beta\delta}) - f_r(V,T,\vec{\mu})$$

"Magnetic coupling": $h \sim m_q$ **"Thermal coupling":** $t \sim \frac{T-T_c}{T_c} + \kappa_2 \left(\frac{\mu_B}{T}\right)^2$

critical lines:

h=0,t=0



Critical behavior in QCD

- Close to the critical window QCD transition is governed by the universal singular behavior

singular — regular

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\vec{\mu}) = -h^{(2-\alpha)/\beta\delta} f_s(t/h^{1/\beta\delta}) - f_r(V,T,\vec{\mu})$$

"Magnetic coupling": $h \sim m_q$ **"Thermal coupling":** $t \sim \frac{T-T_c}{T_c} + \kappa_2 \left(\frac{\mu_B}{T}\right)^2$

2nd order susceptibilities

	Magnetic	Mixed	Thermal
	$\frac{\partial^2 \ln Z}{\partial h^2}$	$\frac{\partial^2 \ln Z}{\partial h \partial t}$	$\frac{\partial^2 \ln Z}{\partial t^2}$
	$\sim h^{1/\delta-1}$	$\sim h^{(\beta-1)/\beta\delta}$	$\sim h^{-\alpha/\beta\delta}$
O(4)	-0.793	-0.34	0.11
Z(2)	-0.792	-0.43	-0.07

critical lines:

h=0,t=0

Divergences

Magnetic sus.: strong

Mixed sus.: mild

Thermal sus.: only at higher orders for O(4)

Critical behavior in QCD

- Close to the critical window QCD transition is governed by the universal singular behavior

singular regular

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\vec{\mu}) = -h^{(2-\alpha)/\beta\delta} f_s(t/h^{1/\beta\delta}) - f_r(V,T,\vec{\mu})$$

"Magnetic coupling": $h \sim m_q$ **"Thermal coupling":** $t \sim \frac{T-T_c}{T_c} + \kappa_2 \left(\frac{\mu_B}{T}\right)^2$

2nd order susceptibilities

	Magnetic	Mixed	Thermal
	$\frac{\partial^2 \ln Z}{\partial h^2}$	$\frac{\partial^2 \ln Z}{\partial h \partial t}$	$\frac{\partial^2 \ln Z}{\partial t^2}$
	$\sim h^{1/\delta-1}$	$\sim h^{(\beta-1)/\beta\delta}$	$\sim h^{-\alpha/\beta\delta}$
O(4)	-0.793	-0.34	0.11
Z(2)	-0.792	-0.43	-0.07

critical lines:

h=0,t=0

Divergences

Magnetic sus.: strong
Mixed sus.: mild

Chiral phase (crossover) transition

Thermal sus.: only at higher orders for O(4)

Critical behavior in QCD

- Close to the critical window QCD transition is governed by the universal singular behavior

singular regular

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\vec{\mu}) = -h^{(2-\alpha)/\beta\delta} f_s(t/h^{1/\beta\delta}) - f_r(V,T,\vec{\mu})$$

"Magnetic coupling": $h \sim m_q$ **"Thermal coupling":** $t \sim \frac{T-T_c}{T_c} + \kappa_2 \left(\frac{\mu_B}{T}\right)^2$

2nd order susceptibilities

	Magnetic	Mixed	Thermal
	$\frac{\partial^2 \ln Z}{\partial h^2}$	$\frac{\partial^2 \ln Z}{\partial h \partial t}$	$\frac{\partial^2 \ln Z}{\partial t^2}$
	$\sim h^{1/\delta-1}$	$\sim h^{(\beta-1)/\beta\delta}$	$\sim h^{-\alpha/\beta\delta}$
O(4)	-0.793	-0.34	0.11
Z(2)	-0.792	-0.43	-0.07

critical lines:

h=0,t=0

Divergences

Magnetic sus.: strong

Mixed sus.: mild

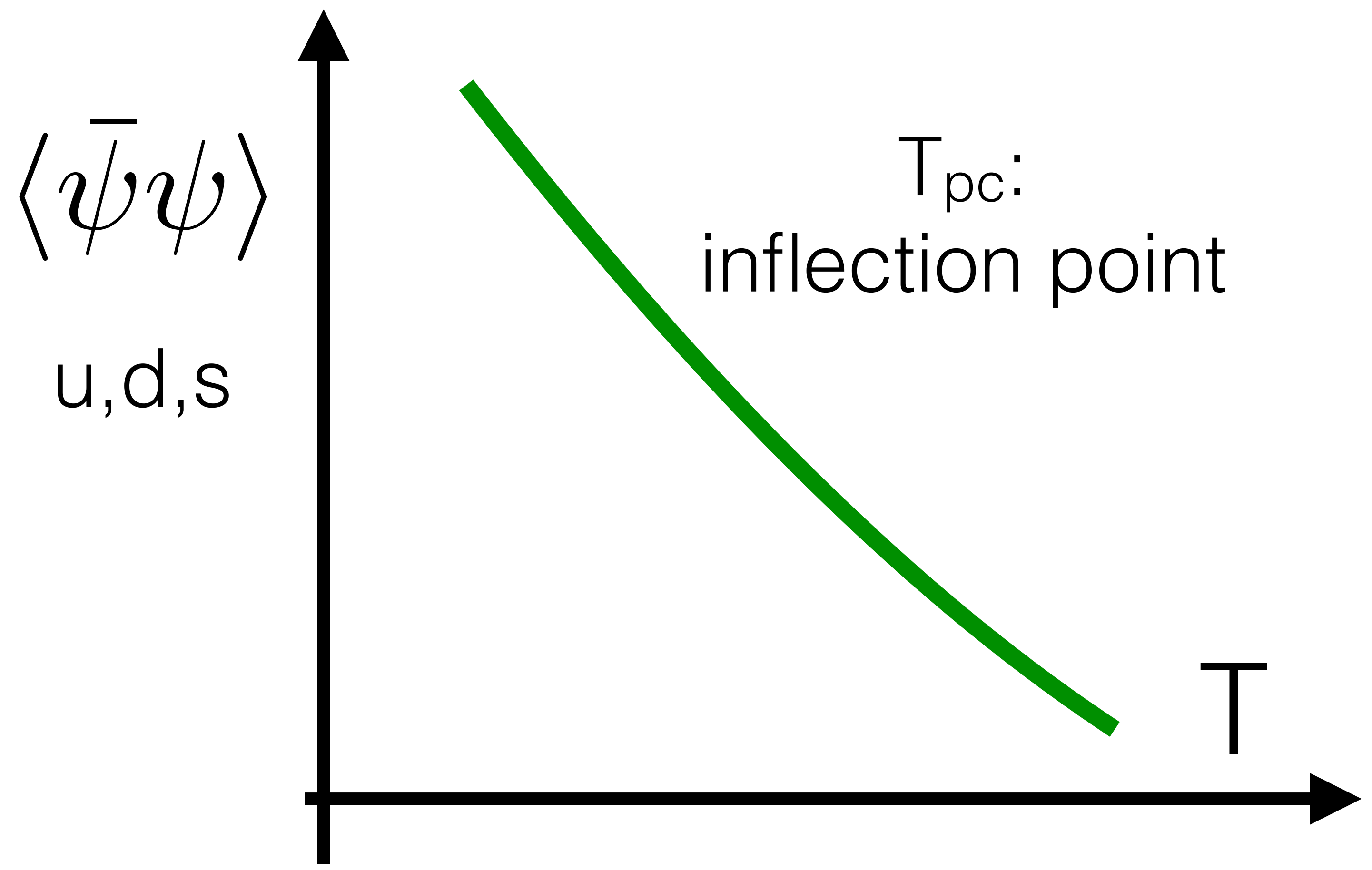
Chiral phase (crossover) transition

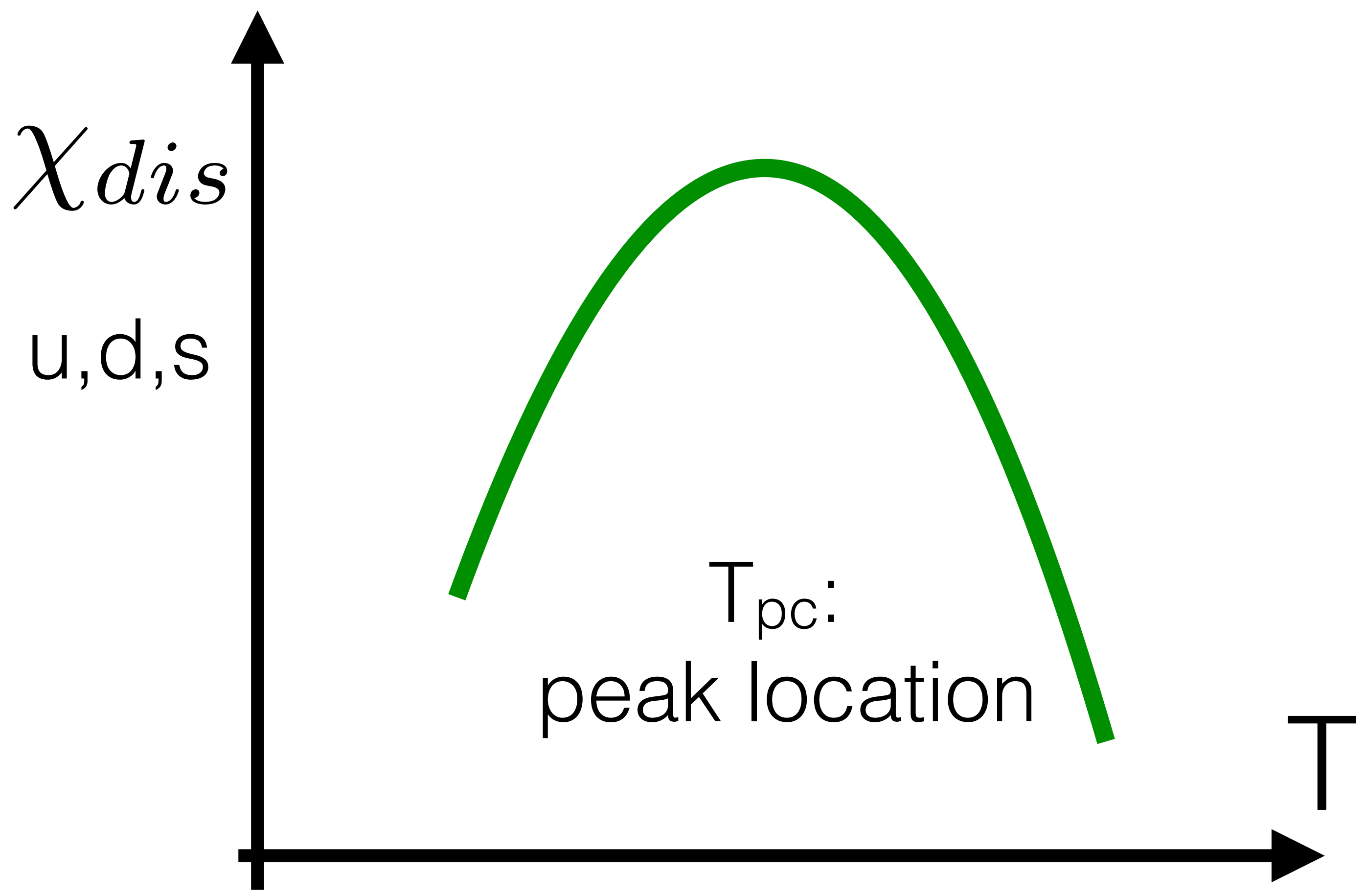
Thermal sus.: only at higher orders for O(4)

Fluctuations

Crossover transition temperature T_{pc} in the real world

- Crossover nature of the transition





- Chiral phase transition: most likely 2nd order, 3d O(4)

Ejiri et al., PRD 80(2009)094505,
HTD et al. [HotQCD], arXiv:1903.04801…

- A well-defined **chiral crossover transition temperature**: based on scaling properties of QCD

HotQCD, Phys. Lett. B795 (2019) 15

Scaling behavior of chiral observables

chiral condensate:

$$\Sigma(T,\mu_B) \sim m^{1/\delta} f_G$$

chiral susceptibility:

$$\chi^{\Sigma}(T,\mu_B) \sim m^{1/\delta-1} f_\chi$$

Taylor expansions:

$$\Sigma(T,\mu_X) = \sum_{n=0}^{\infty} \frac{C_{2n}^{\Sigma}(T)}{(2n)!}\left(\frac{\mu_X}{T}\right)^{2n}$$

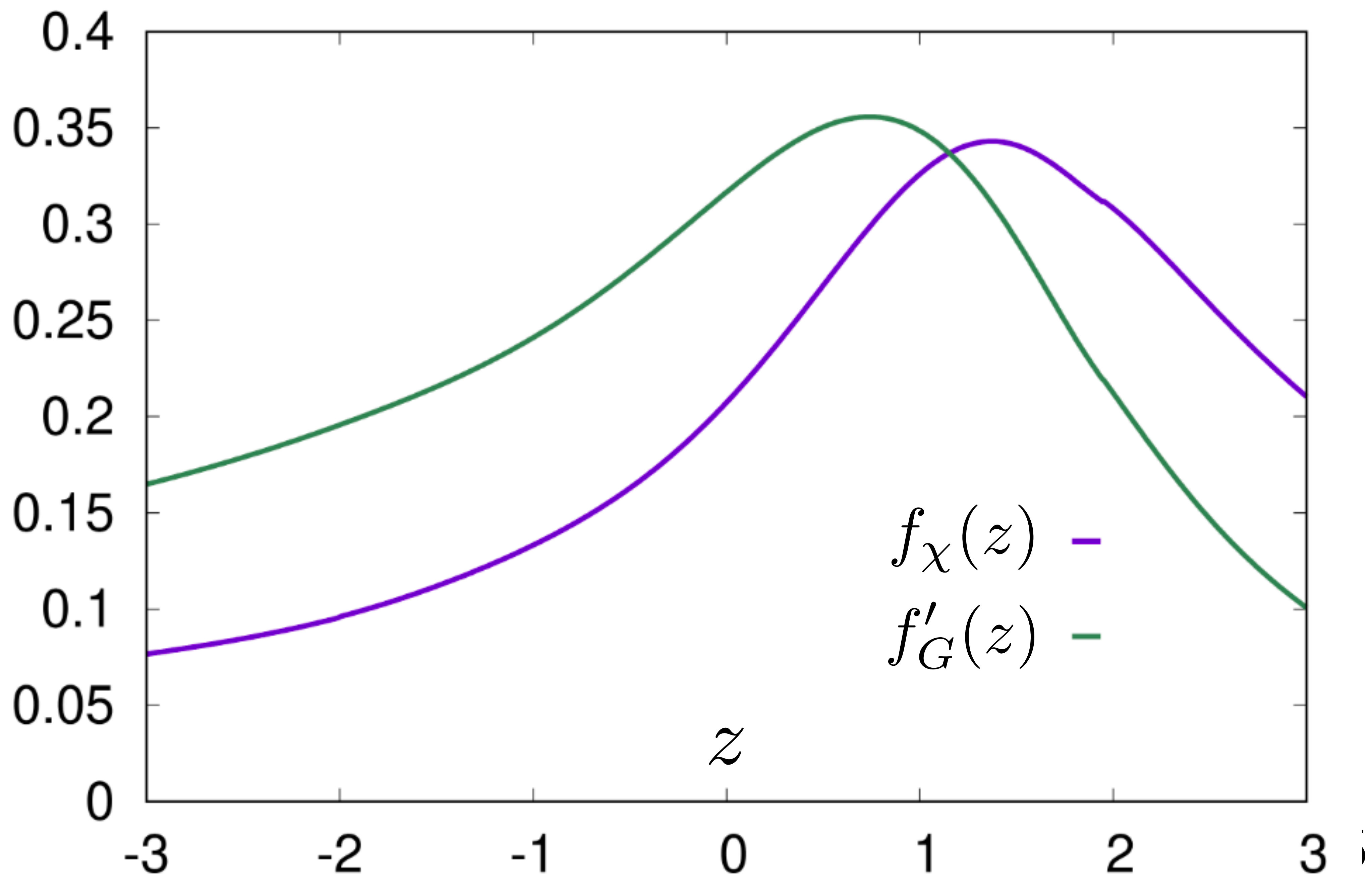
$$\frac{\partial f_\chi}{\partial T} = \frac{\partial f_\chi}{\partial z}\frac{\partial z}{\partial T} \equiv f_\chi' m^{-1/\beta\delta}$$

$$z \sim \left((T-T_c^0)/T_c^0 + K(\mu_B/T)^2\right)/m^{\frac{1}{\beta\delta}}$$

$$\frac{\partial}{\partial T} \simeq \frac{\partial^2}{\partial \mu_B^2}$$

$$\left.\begin{array}{l}\partial_T \chi^{\Sigma}(T)\\ \partial_T C_0^{\chi}(T)\\ C_2^{\chi}(T)\end{array}\right\} \sim m^{1/\delta-1-1/\beta\delta} f_\chi'(z)$$

$$\left.\begin{array}{l}\partial_T^2 C_0^{\Sigma}(T)\\ \partial_T C_2^{\Sigma}(T)\end{array}\right\} \sim m^{1/\delta-1/\beta\delta} f_G''(z)$$



Well-defined notation of chiral crossover transition T

- 5 conditions to extract T_c: maxima of f_χ and f'_G

$\partial_T \chi^\Sigma(T) = 0$ $\partial_T C_0^\chi(T) = 0$ $C_2^\chi(T) = 0$ $\partial_T^2 C_0^\Sigma(T) = 0$ $\partial_T C_2^\Sigma(T) = 0$

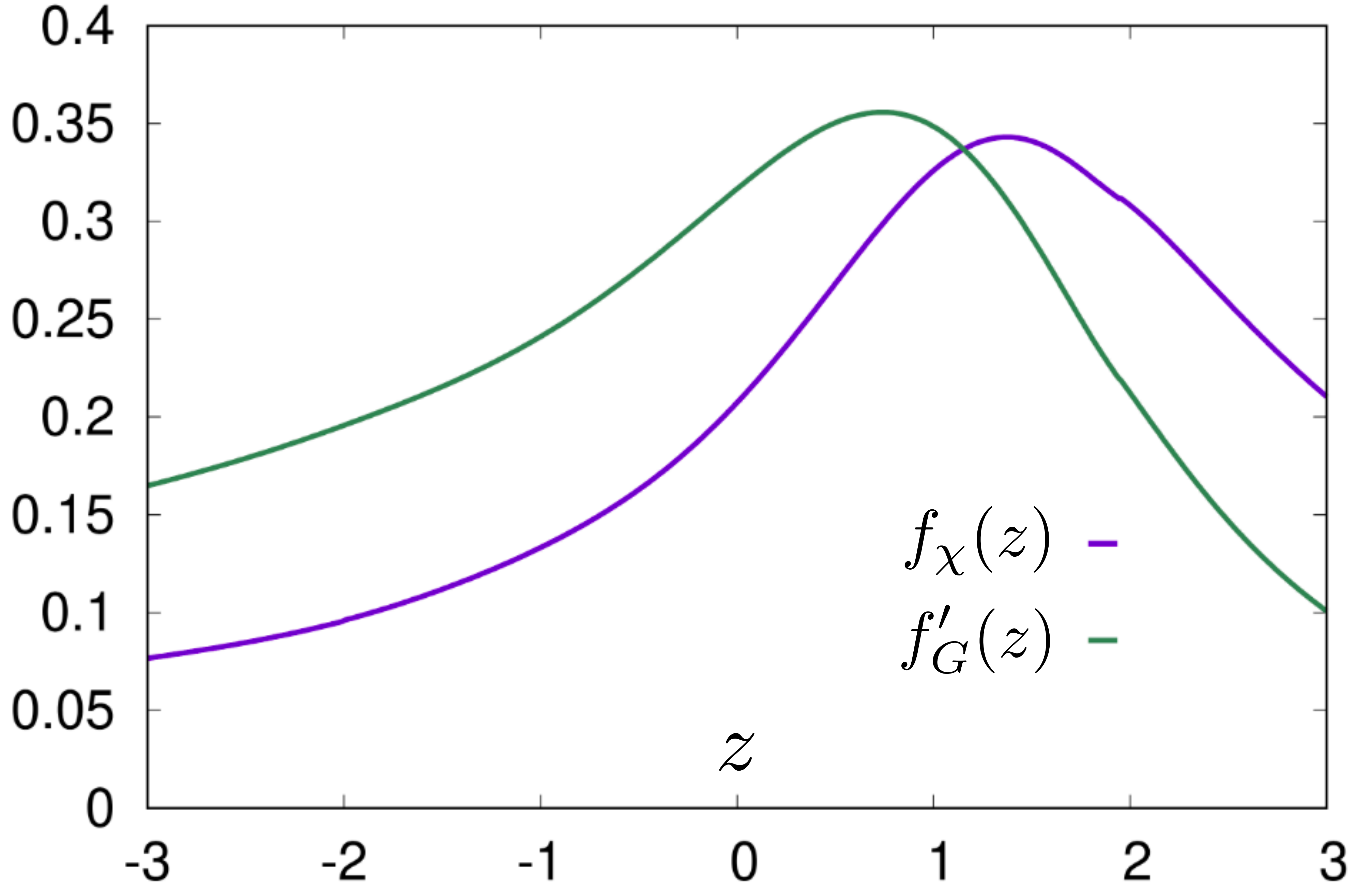
- m=0: all these susceptibilities diverge at a unique T

- m=/=0: non-unique temperatures, crossover

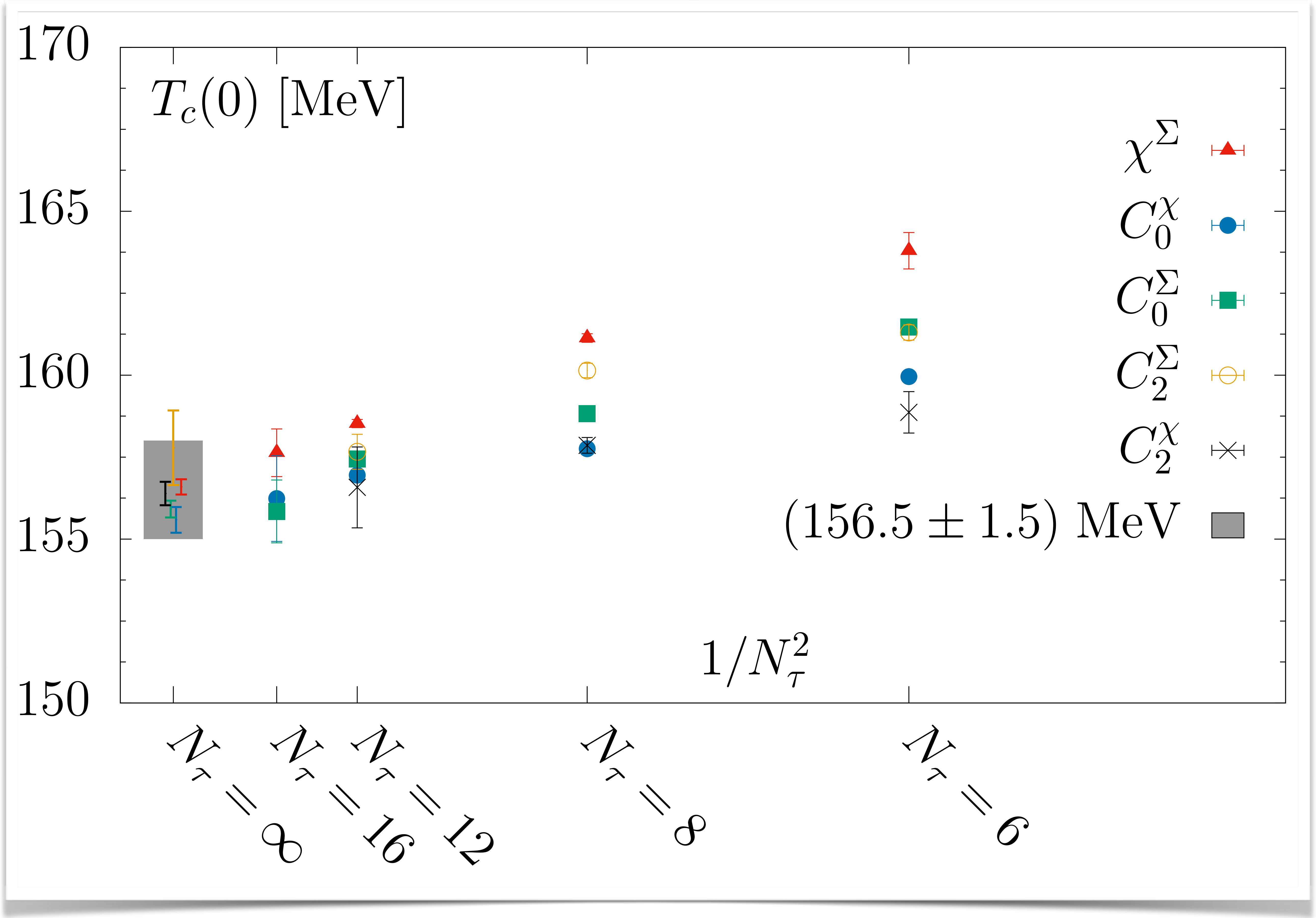
$$\partial_T \chi^\Sigma(T),\ \partial_T C_0^\chi(T),\ C_2^\chi(T) \sim m^{1/\delta-1-1/\beta\delta} f'_\chi(z)$$

$$\partial_T^2 C_0^\Sigma(T),\ \partial_T C_2^\Sigma(T) \sim m^{1/\delta-1/\beta\delta} f''_G(z)$$

$$z \sim \left((T-T_c^0)/T_c^0 + K(\mu_B/T)^2\right)/m^{\frac{1}{\beta\delta}}$$



QCD chiral crossover transition temperature with m_π =140 MeV



A. Bazavov, HTD, P. Hegde et al. [HotQCD], Phys. Lett. B795 (2019) 15

Chiral crossover temperature in the continuum limit:

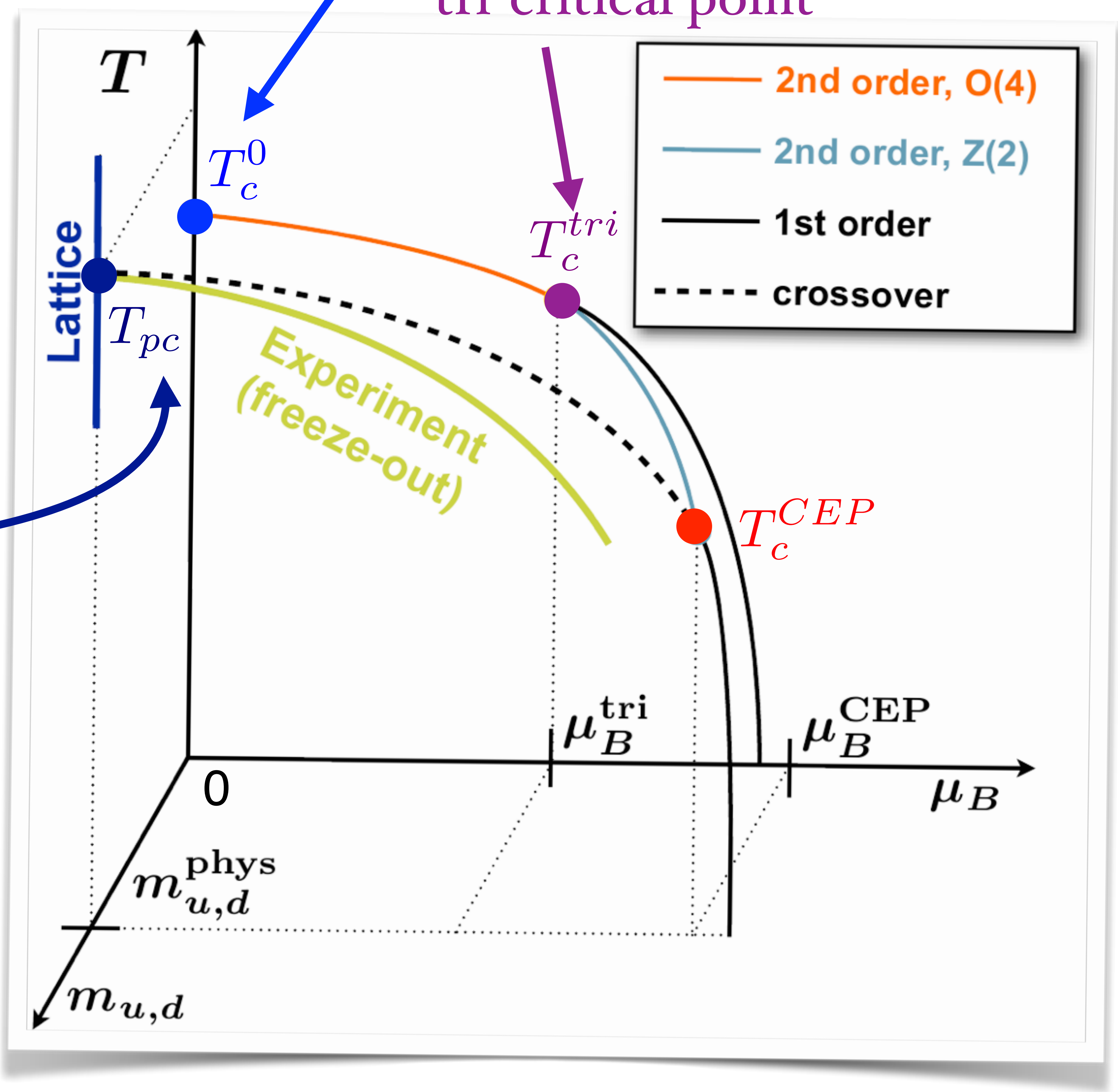
T_{pc} =156.5(1.5)MeV

Consistent results from Wuppertal-Budapest arXiv:2002.02821

QCD phase diagram in 3D: quark mass, μ_B, T

$M_\pi \to 0$, O(4)
Chiral phase transition T at μ_B=0

tri-critical point



Chiral crossover transition T at μ_B=0 and M_π=140 MeV

$T_{pc} = 156.5 \pm 1.5$ MeV

- $T_c^0(\mu_B)$ decreases as μ_B up to NLO from LQCD

P. Hegde & HTD, PoS LATTICE2015 (2016) 141
O. Kaczmarek et al., PRD83 (2011) 014504

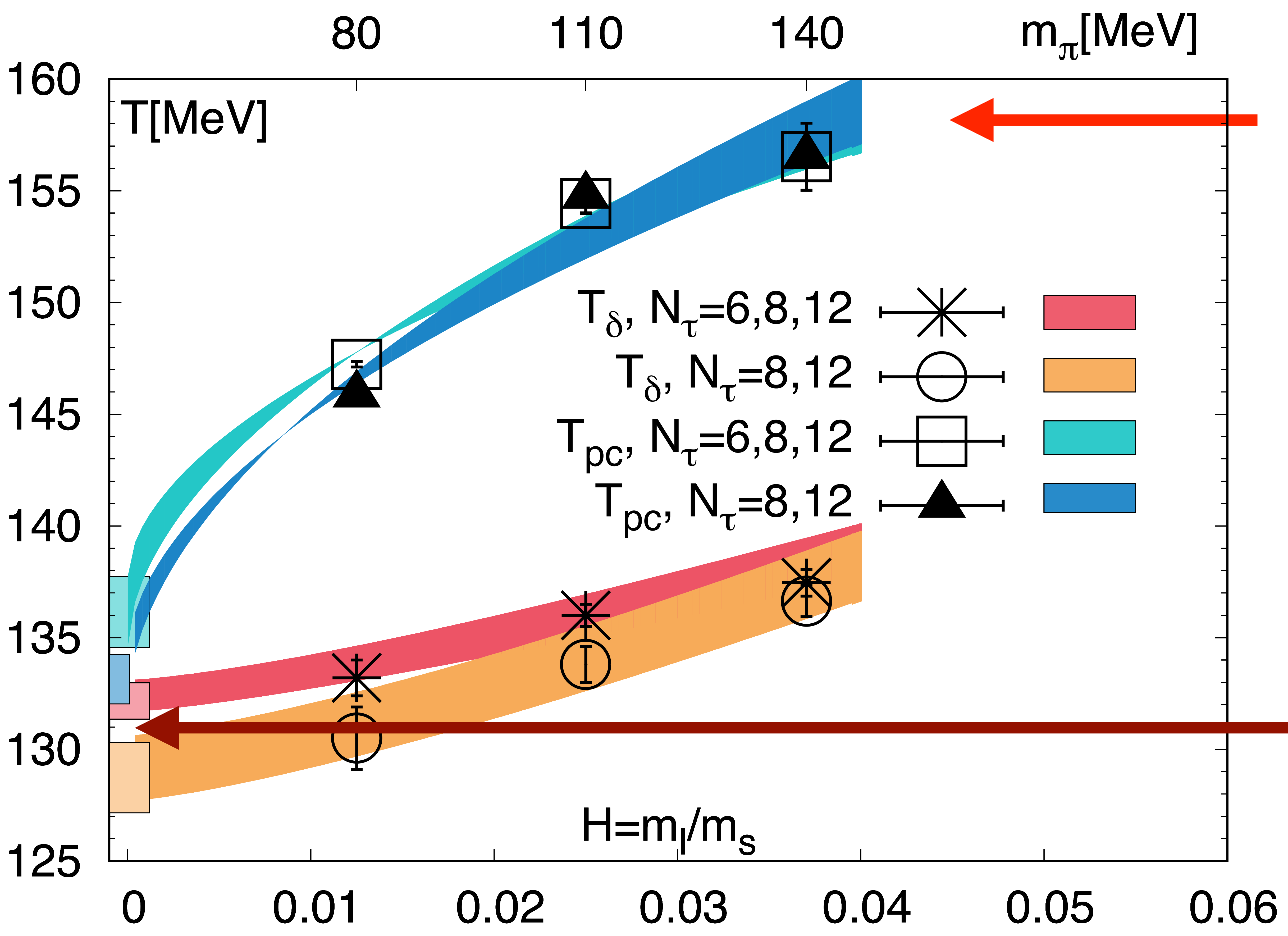
- Random Matrix Model & NJL suggests:

M. A. Halasz et al, PRD 58 (1998) 096007
M. Buballa, S. Carignano, PLB791(2019)361
Y. Hatta & T. Ikeda, PRD67 (2003) 014028

$$T_c^{tri} - T_c^{CEP}(m_q) \propto m_q^{2/5}$$

Indications:

$$T_{pc}^{phys} > T_c^0 > T_c^{tri} > T_c^{CEP}$$



HTD, P. Hegde, O. Kaczmarek et al.[HotQCD], PRL 123 (2019) 062002

HTD, arXiv:2002.11957

$$T_{pc}^{phys} = 156.5(1.5)\text{MeV}$$

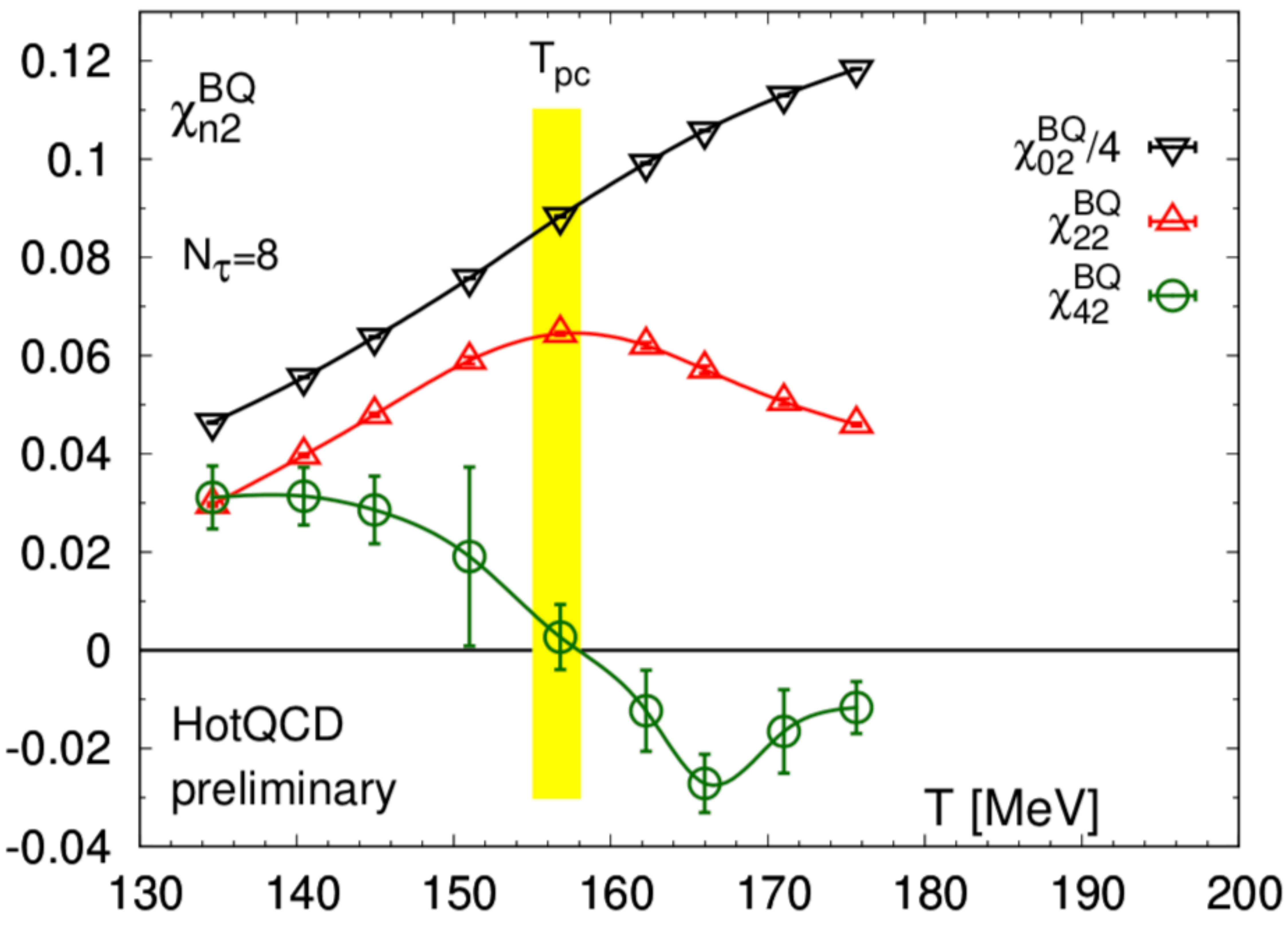
$$T_{pc}(H) = T_c^0 \left(1 + \frac{z_p}{z_0} \; H^{\frac{1}{\beta\delta}}\right)$$

$$T_c^0 = 132^{+3}_{-6} \text{ MeV}$$

See also in QCD-inspired model calculations:
e.g. J. Berges, D. U. Jungnickel and C. Wetterich, Phys. Rev.D59, 034010 (1999)
J. Braun, B. Klein, H.-J. Pirner and A. H. Rezaeian,Phys. Rev. D73, 074010 (2006)

Indication of $T_{CEP} \lesssim 132$ MeV

Critical behavior and higher order cumulants



$$\chi_{ijk}^{BQS}(T) = \frac{\partial^{i+j+k} P(T,\hat{\mu})/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k}\Big|_{\hat{\mu}=0}$$

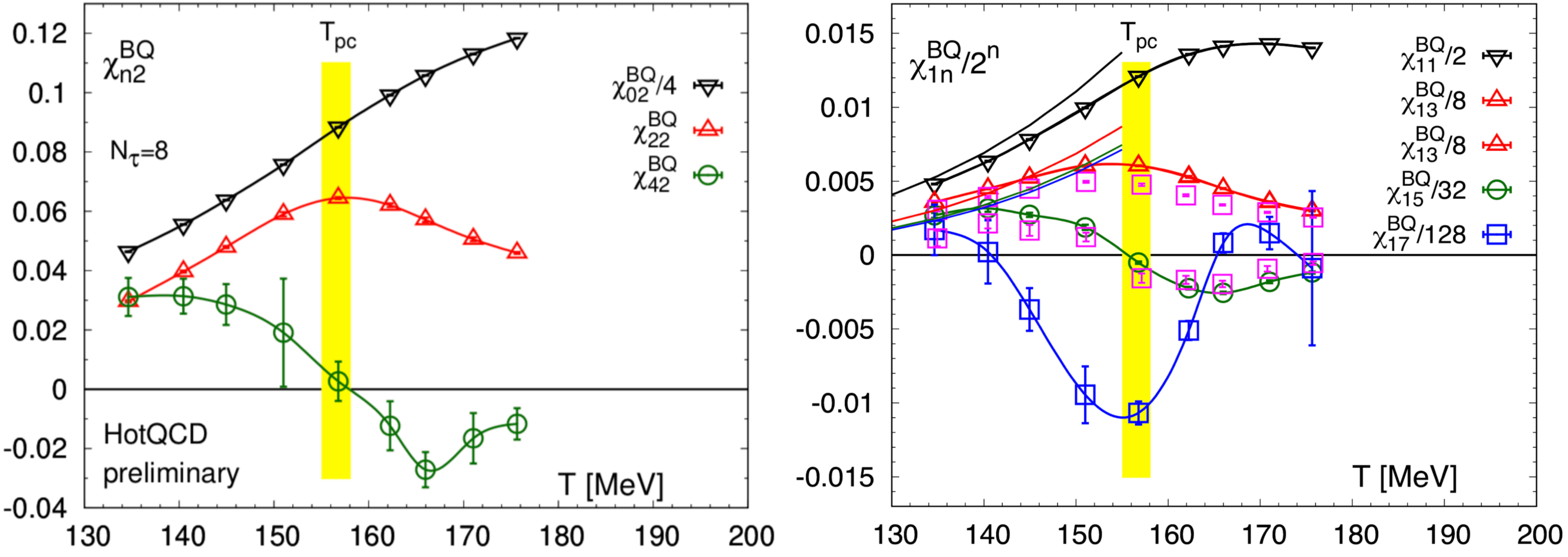
$$t \sim \frac{T - T_c^0}{T_c^0} + \kappa_2^{B,0} \left(\frac{\mu_B}{T}\right)^2$$

In the scaling regime: $\frac{\partial}{\partial T} \simeq \frac{\partial^2}{\partial \mu_B^2}$

Irregular sign change seen at $T>T_{pc}$ in χ_{42}^{BQ}

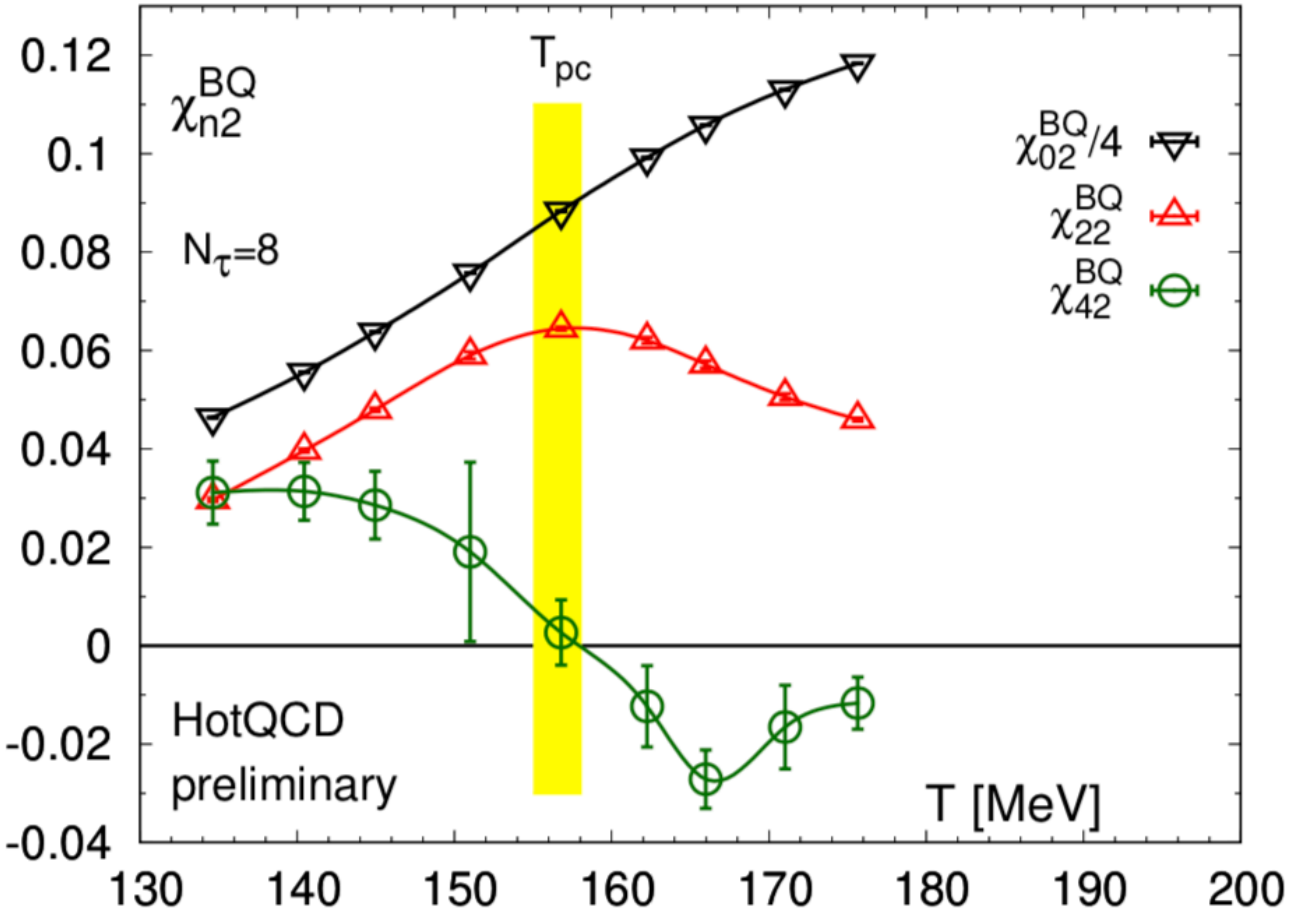
Irregular sign change expected at $T \gtrsim 135$ MeV in χ_{62}^{BQ}

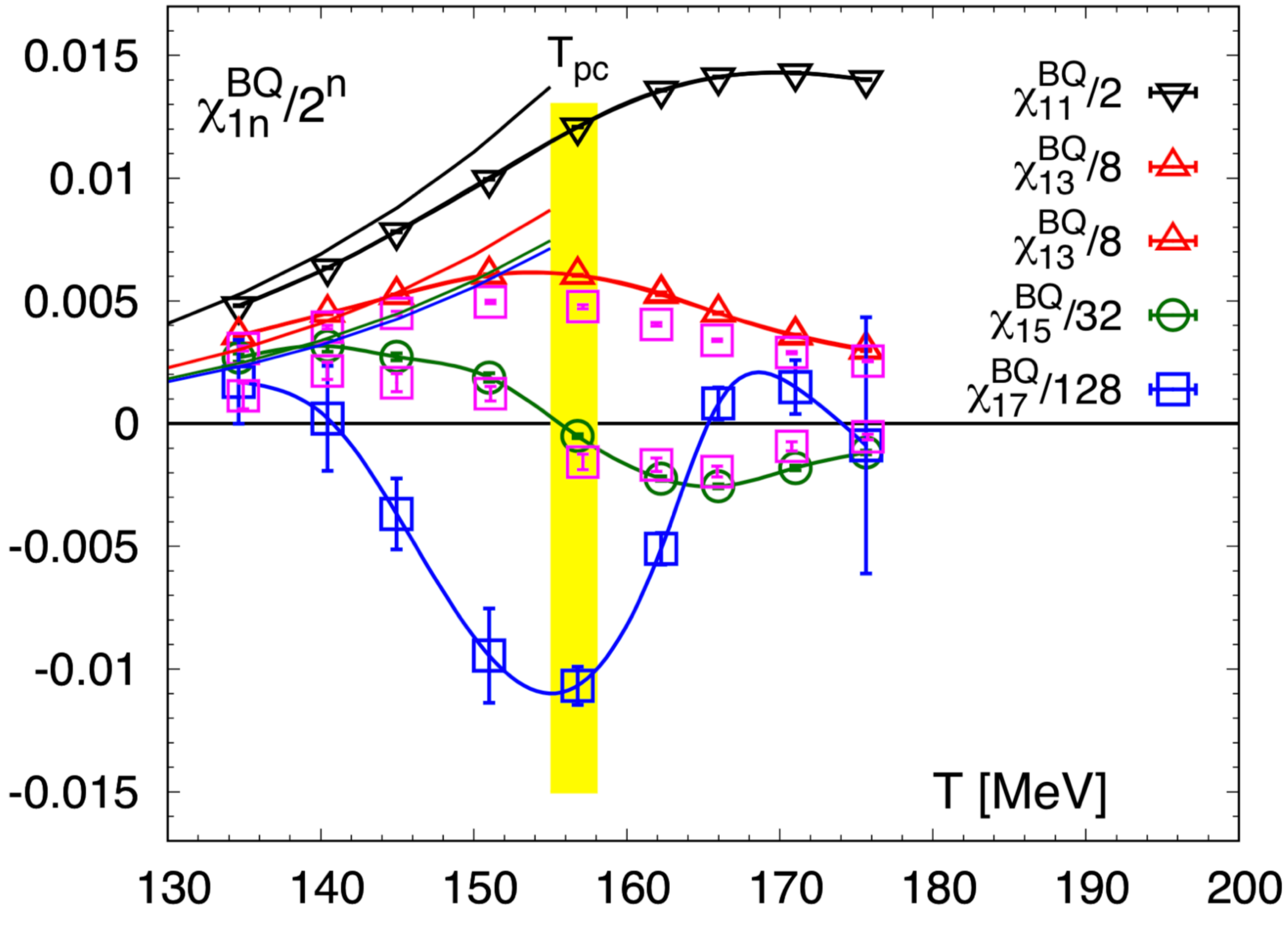
Critical behavior and higher order cumulants



Many 8th order fluctuations turn to be negative at T ≽ 135-140 MeV

Critical behavior and higher order cumulants





Many 8th order fluctuations turn to be negative at T ⩾ 135-140 MeV

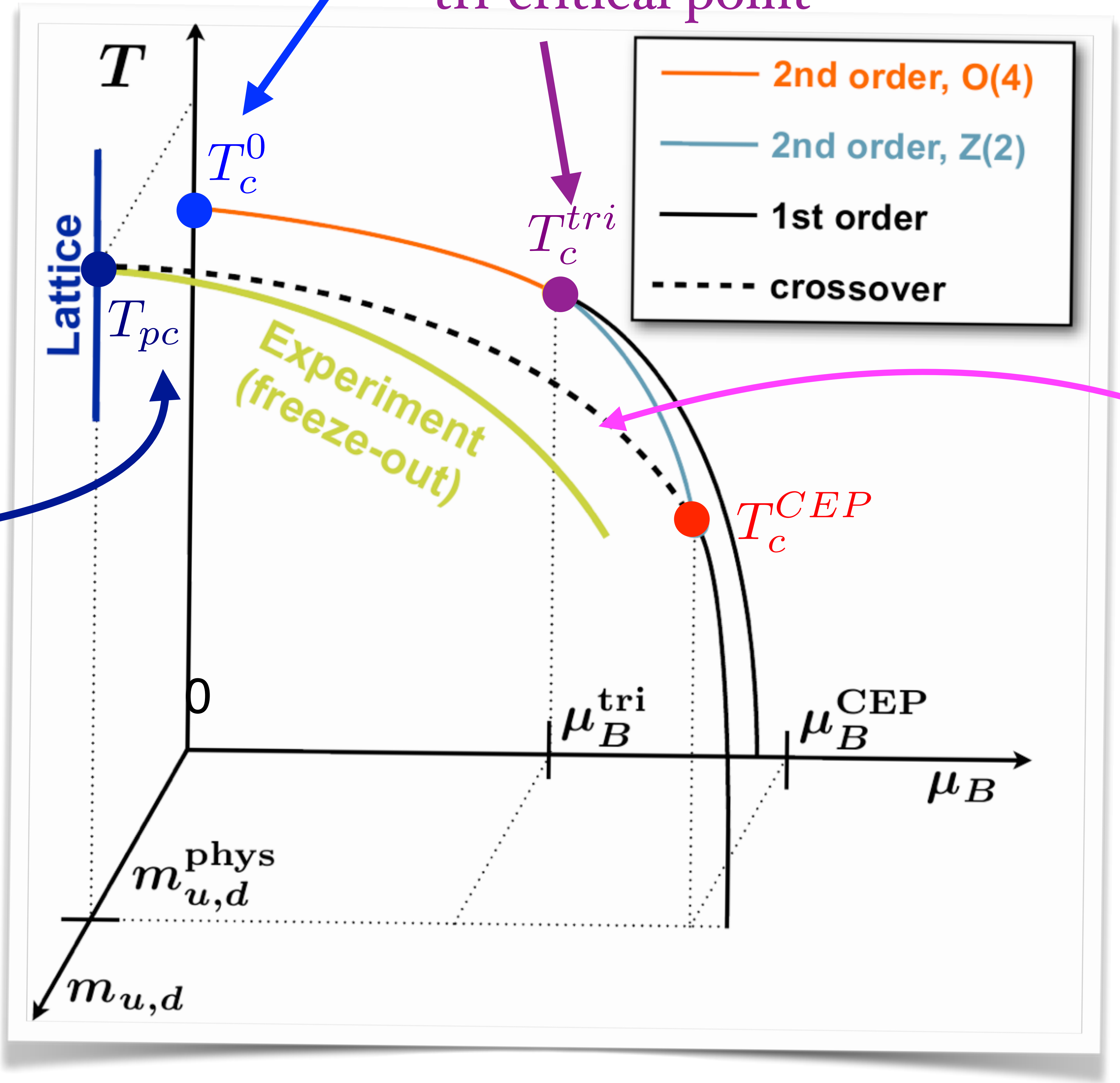
Suggests singularity in the complex plane at T>135-140 MeV

More support for $T_{CEP} < T_c^0$

QCD phase diagram in 3D: quark mass, μ_B, T

$M_\pi \to 0$, O(4)
Chiral phase transition T at μ_B=0

tri-critical point



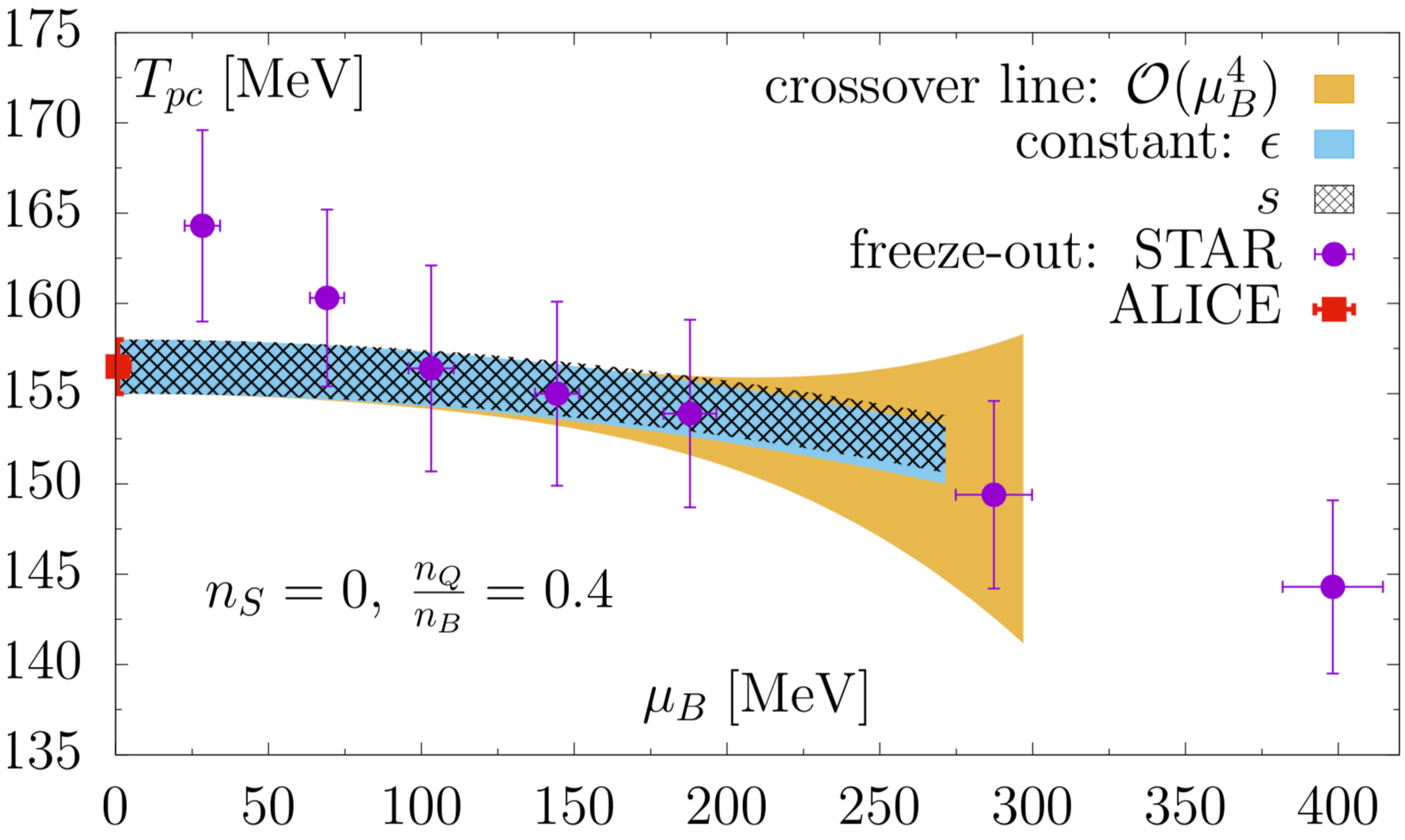
Chiral crossover transition T at μ_B=0 and M_π=140 MeV

$T_{pc} = 156.5 \pm 1.5$ MeV

$T_{CEP} < T_c^0 \sim 132$ MeV

How about the strength of transition along the crossover transition line?

Chiral crossover line: $T_{pc}(\mu_B) = T_{pc}(0)\left(1 - \kappa_2\left(\frac{\mu_B}{T}\right)^2 - \kappa_4\left(\frac{\mu_B}{T}\right)^4\right)$



Bazavov, HTD, P. Hegde et al. [HotQCD], Phys. Lett. B795 (2019) 15

ALICE data point:

$T_f = 156.5(1.5)$ MeV

Andronic et al, Nature 561 (7723) (2018) 321

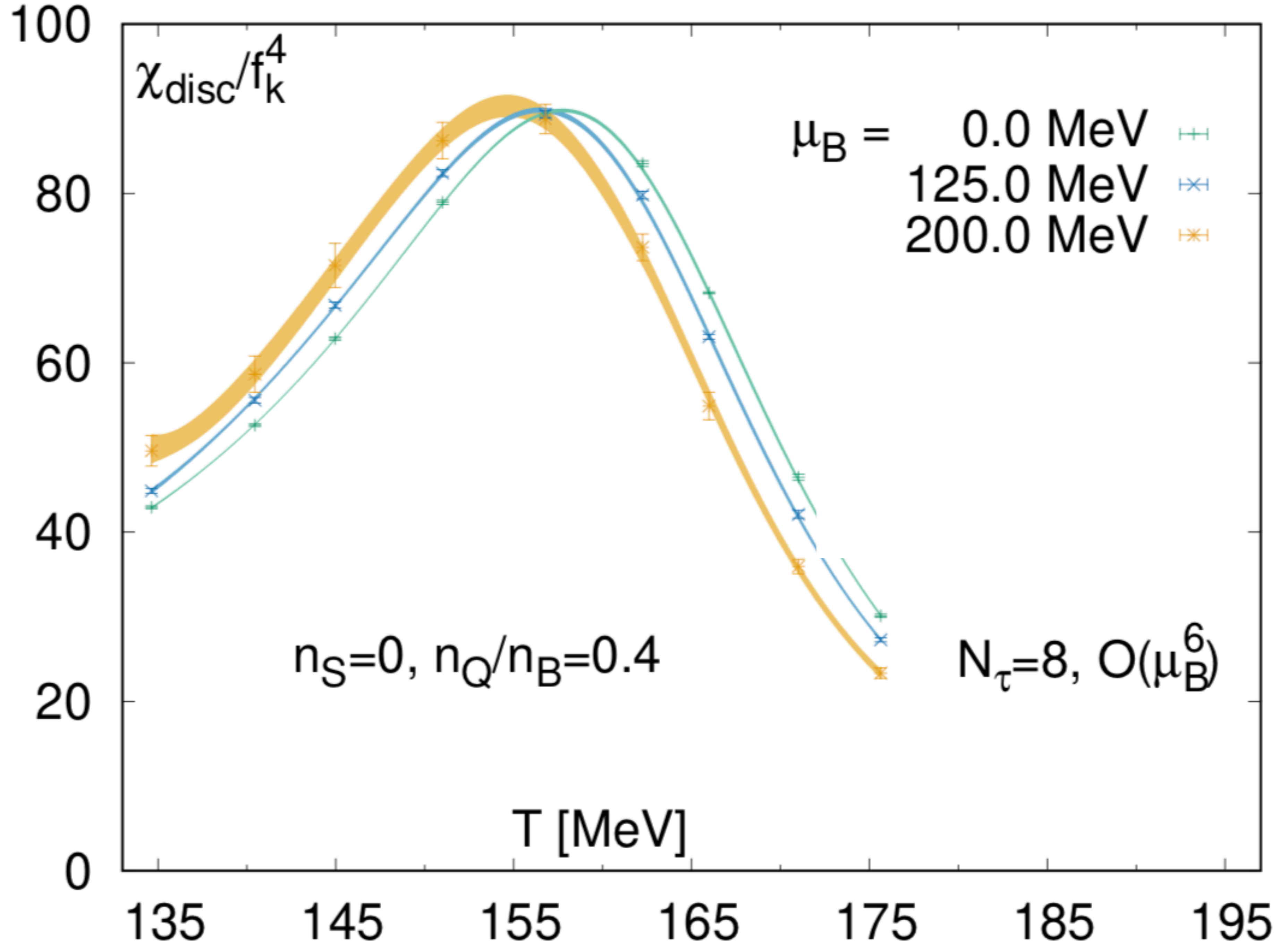
STAR data points:

Adamczyk et al., Phys. Rev. C 96 (4) (2017) 044904

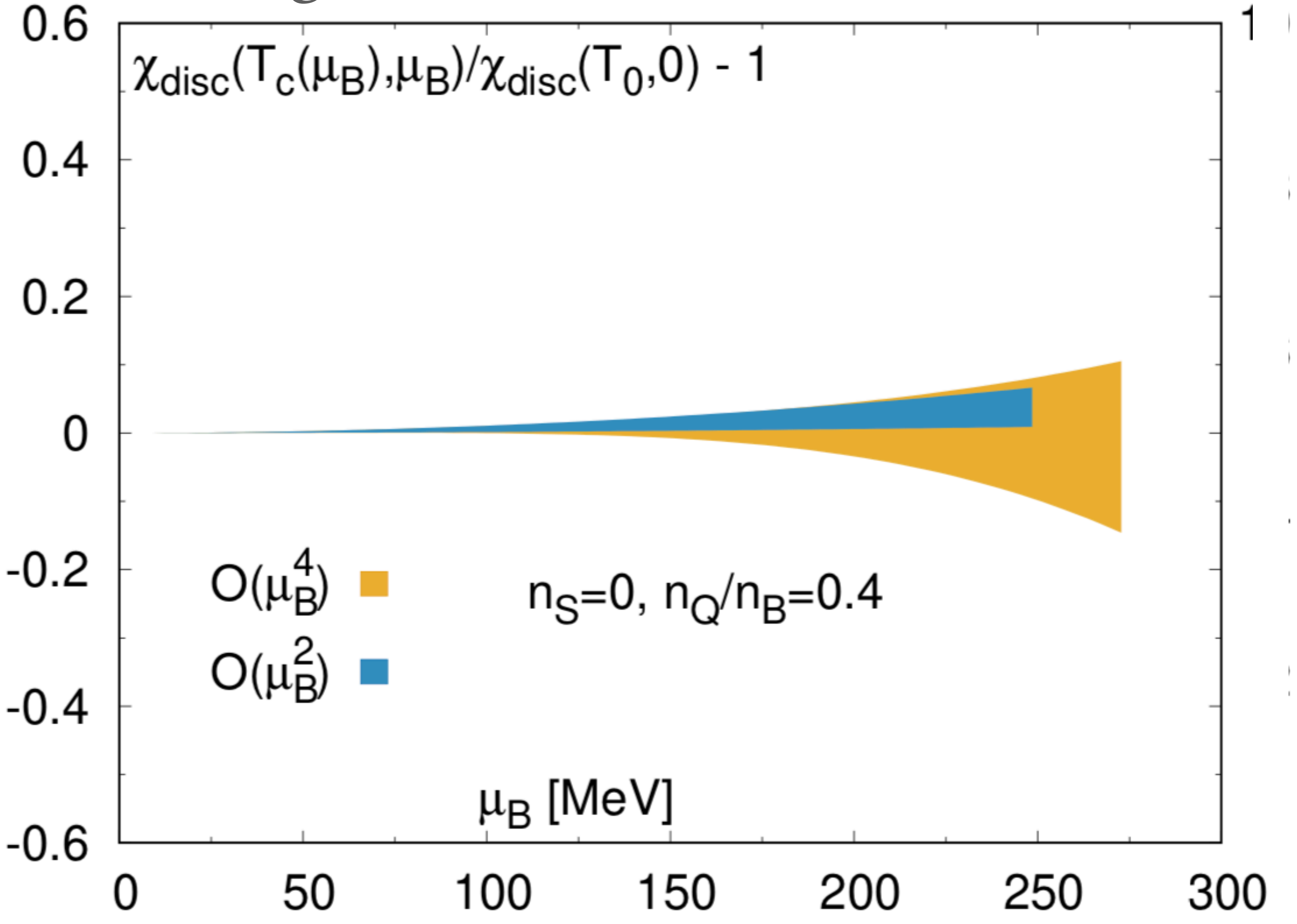
Consistent results on the chiral crossover line from Wuppertal-Budapest, arXiv:2002.02821

Strength of QCD transition at $\mu_B \lesssim 300$ MeV

disconnected susceptibility



Along the crossover transition line

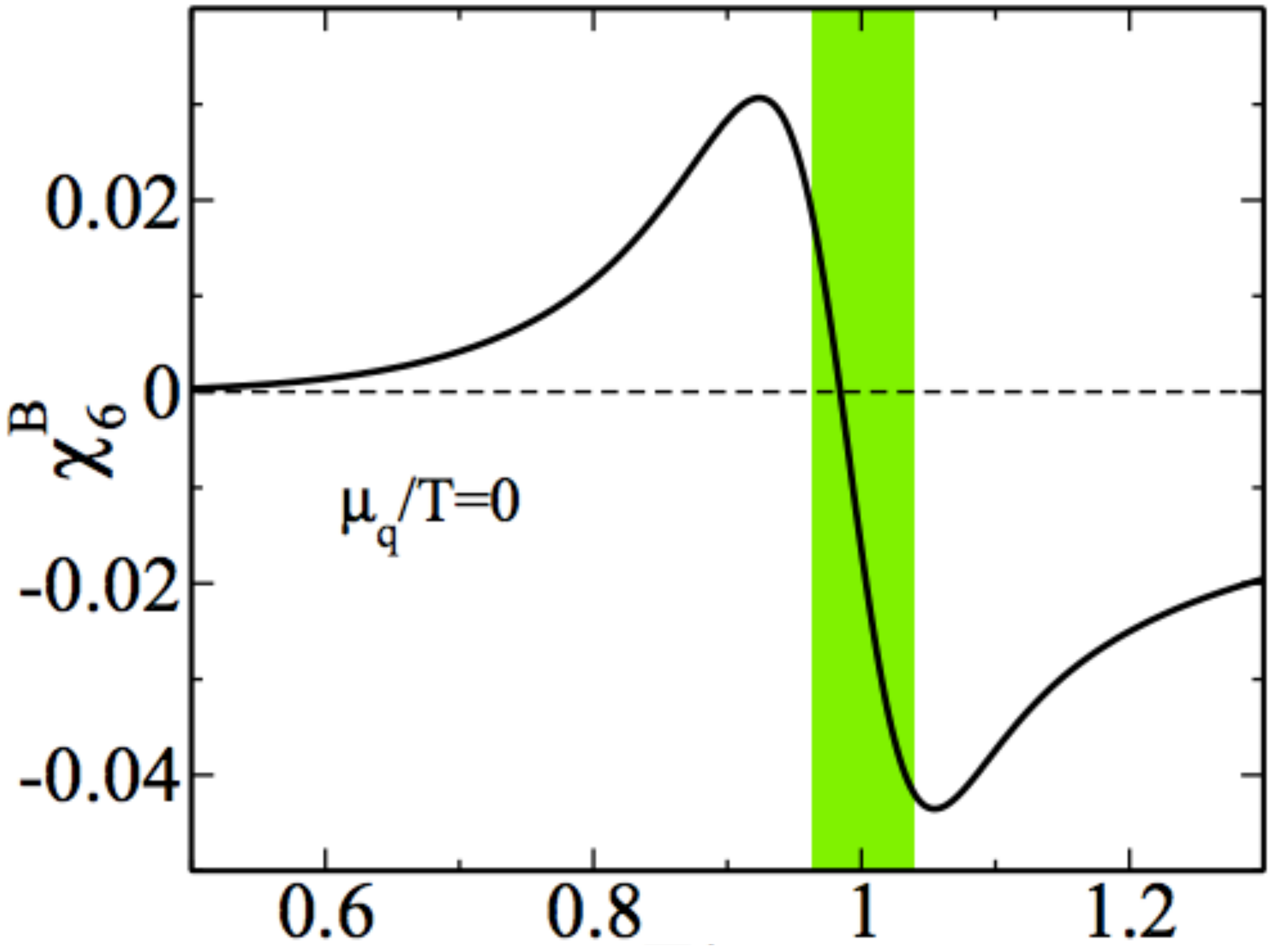


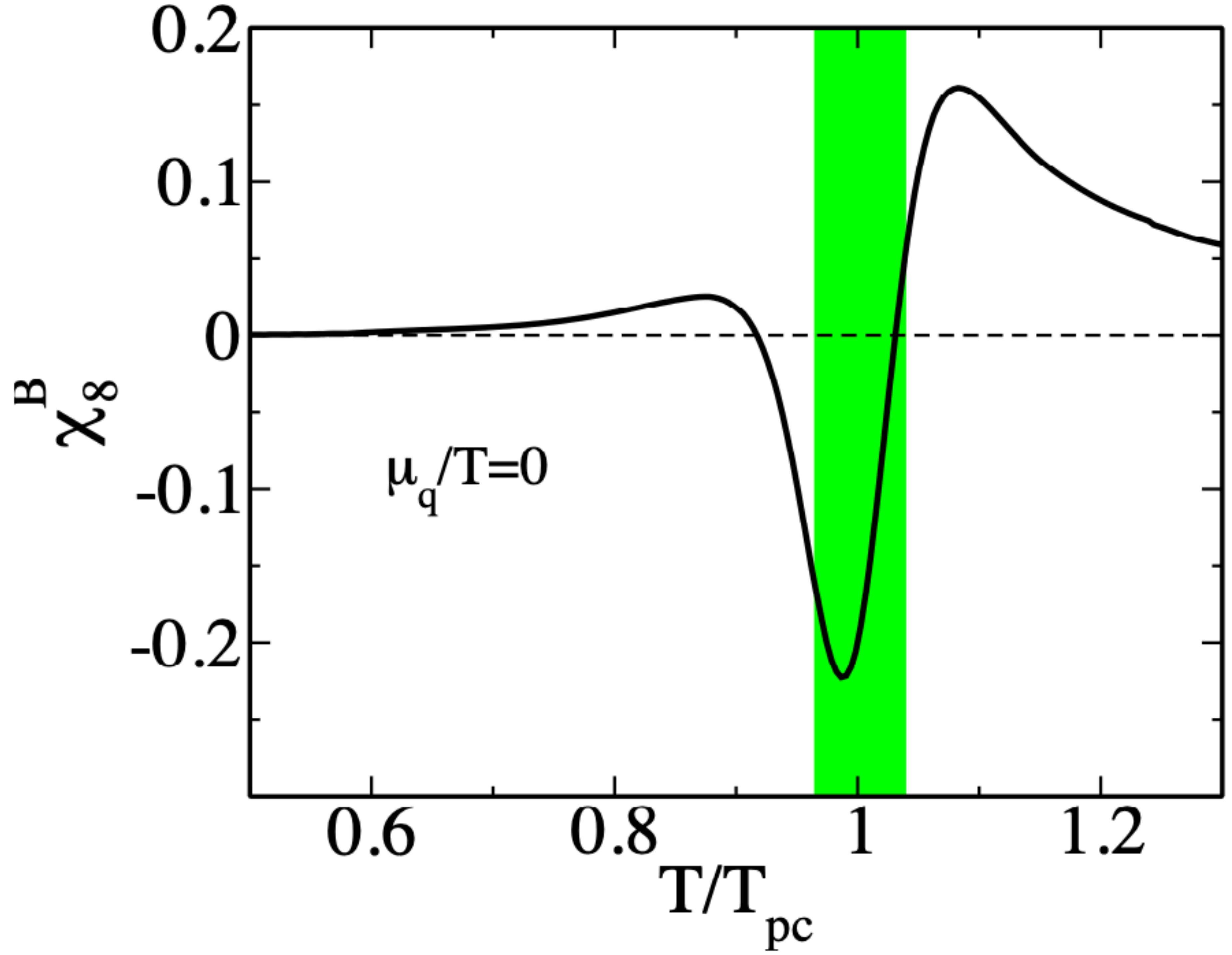
Lattice data suggests

$$T_c^{CEP} < 135 - 140\text{MeV}, \;\; \mu_B^{CEP} > 300\text{MeV}$$

Baryon number fluctuations according to 3-d O(4) universality class

B. Friman et al., Eur.Phys.J. C71 (2011) 1694



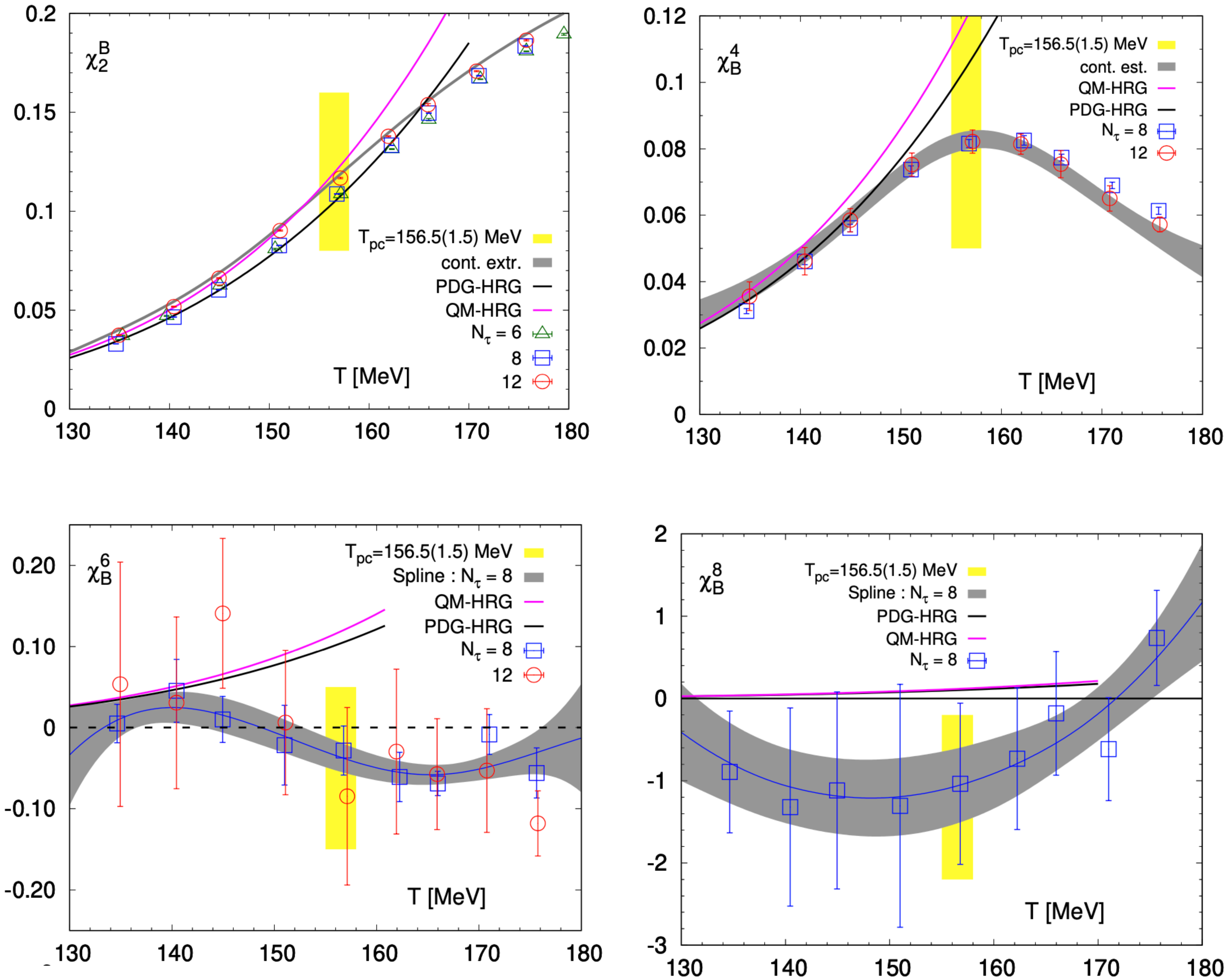


$$\chi_n^B \sim \begin{cases} -(2\kappa_q)^{n/2}|t|^{2-\alpha-n/2} f_\pm^{(n/2)} & , \text{ for } \mu_q/T = 0, \text{ and } n \text{ even} \\ -(2\kappa_q)^n \left(\frac{\mu_q}{T}\right)^n |t|^{2-\alpha-n} f_\pm^{(n)} & , \text{ for } \mu_q/T > 0 \, , \end{cases}$$

3-d O(4) : $\boldsymbol{\alpha}$=-0.21,

The exponent of t: $2 - \alpha - n/2$

becomes negative only with n>=4



Taylor expansion coefficients at μ_B=0

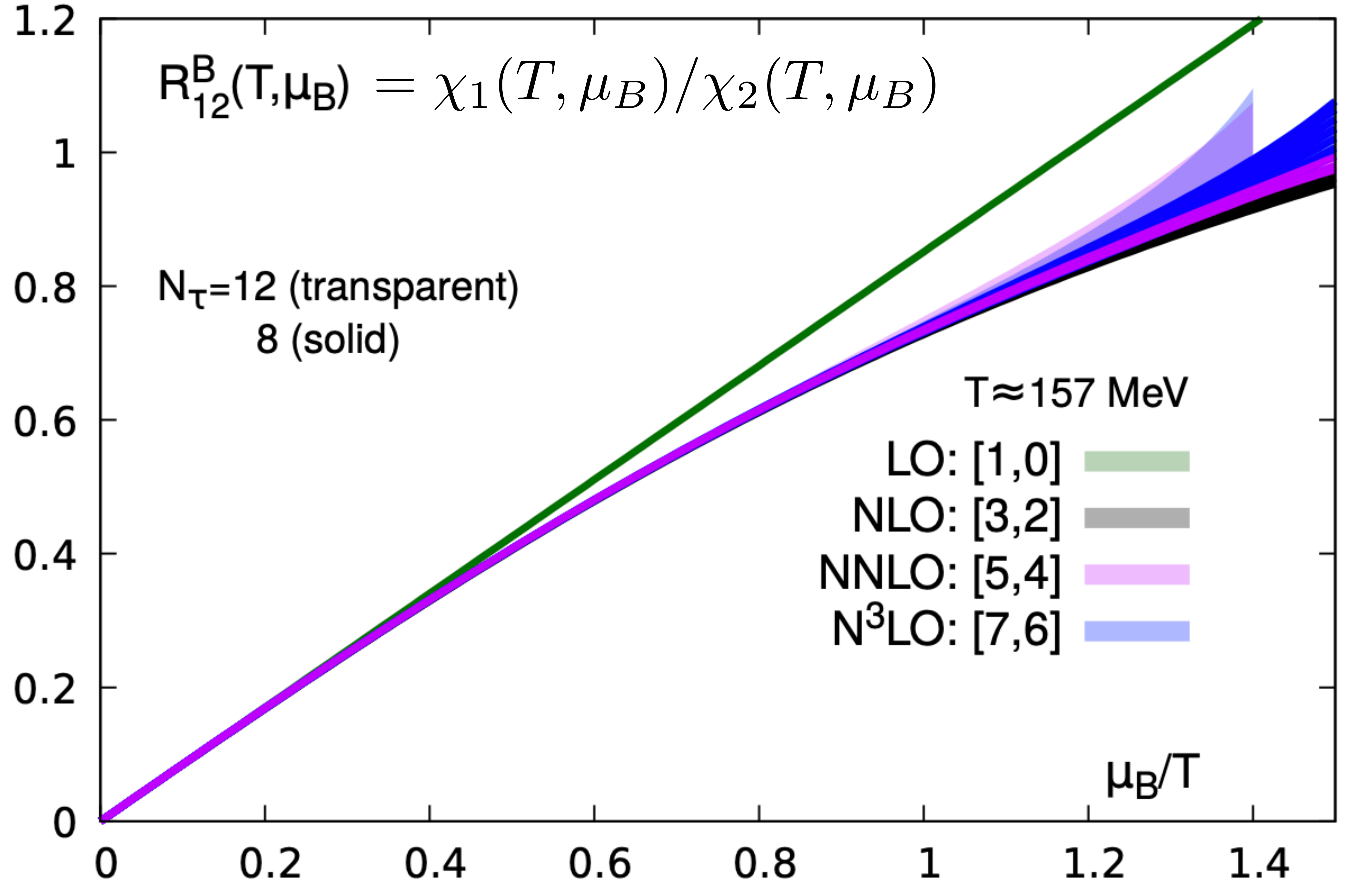
$$\chi_n^B(T) = \frac{\partial^n P(T,\mu_B)/T^4}{\partial \hat{\mu}_B^n}\Big|_{\hat{\mu}_B=0}$$

HotQCD, 2001.08530

Fluctuations of baryon numbers at μB=/=0

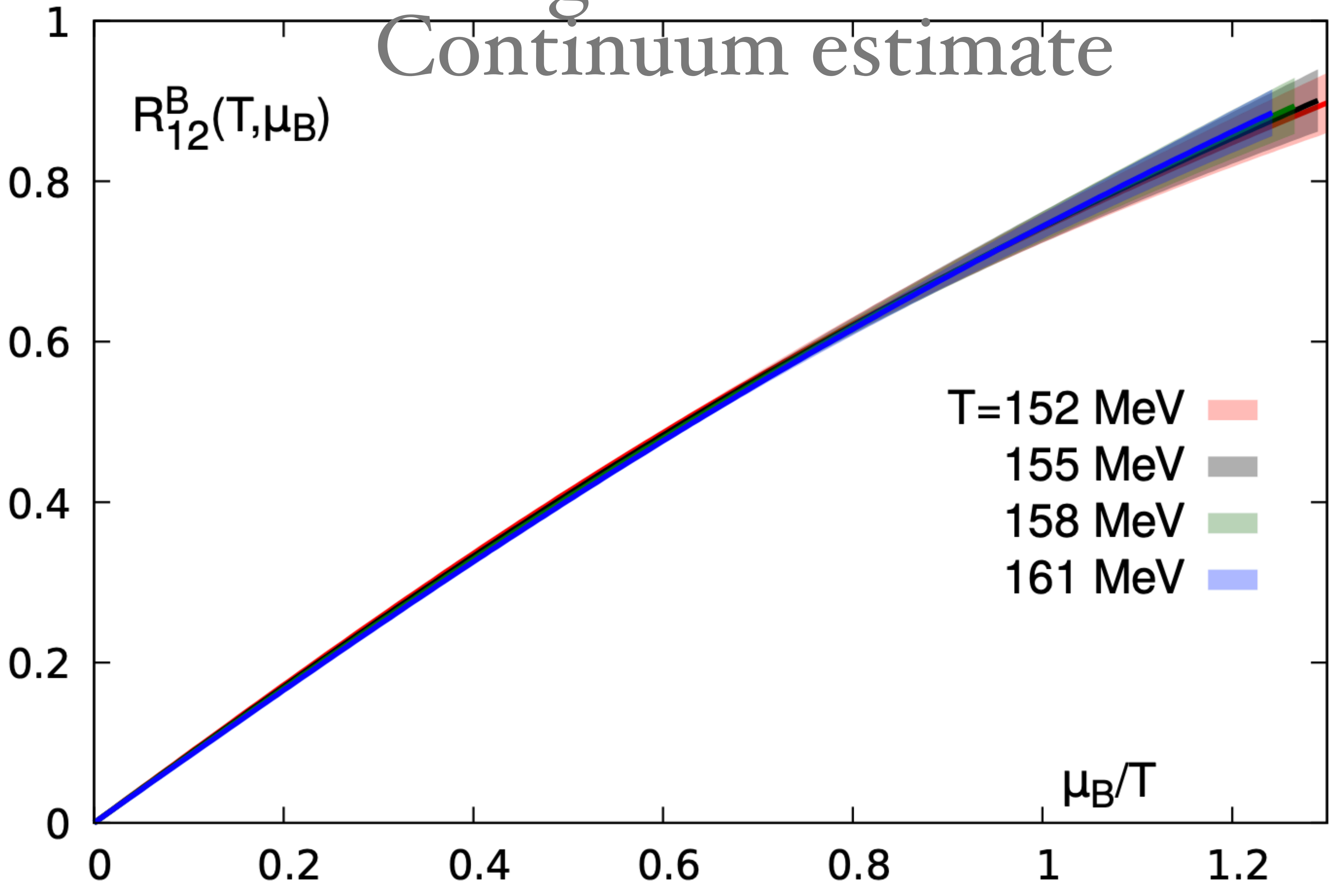
$$\chi_n^B(T,\mu_B) = \sum_{k=0}^{k_{max}} \tilde{\chi}_n^{B,k}(T)\hat{\mu}_B^k$$

$$R_{nm}^B = \frac{\chi_n^B(T,\mu_B)}{\chi_m^B(T,\mu_B)} = \frac{\sum_{k=1}^{k_{max}} \tilde{\chi}_n^{B,k}(T)\hat{\mu}_B^k}{\sum_{l=1}^{l_{max}} \tilde{\chi}_m^{B,l}(T)\hat{\mu}_B^l}$$



in strangeness neutral case

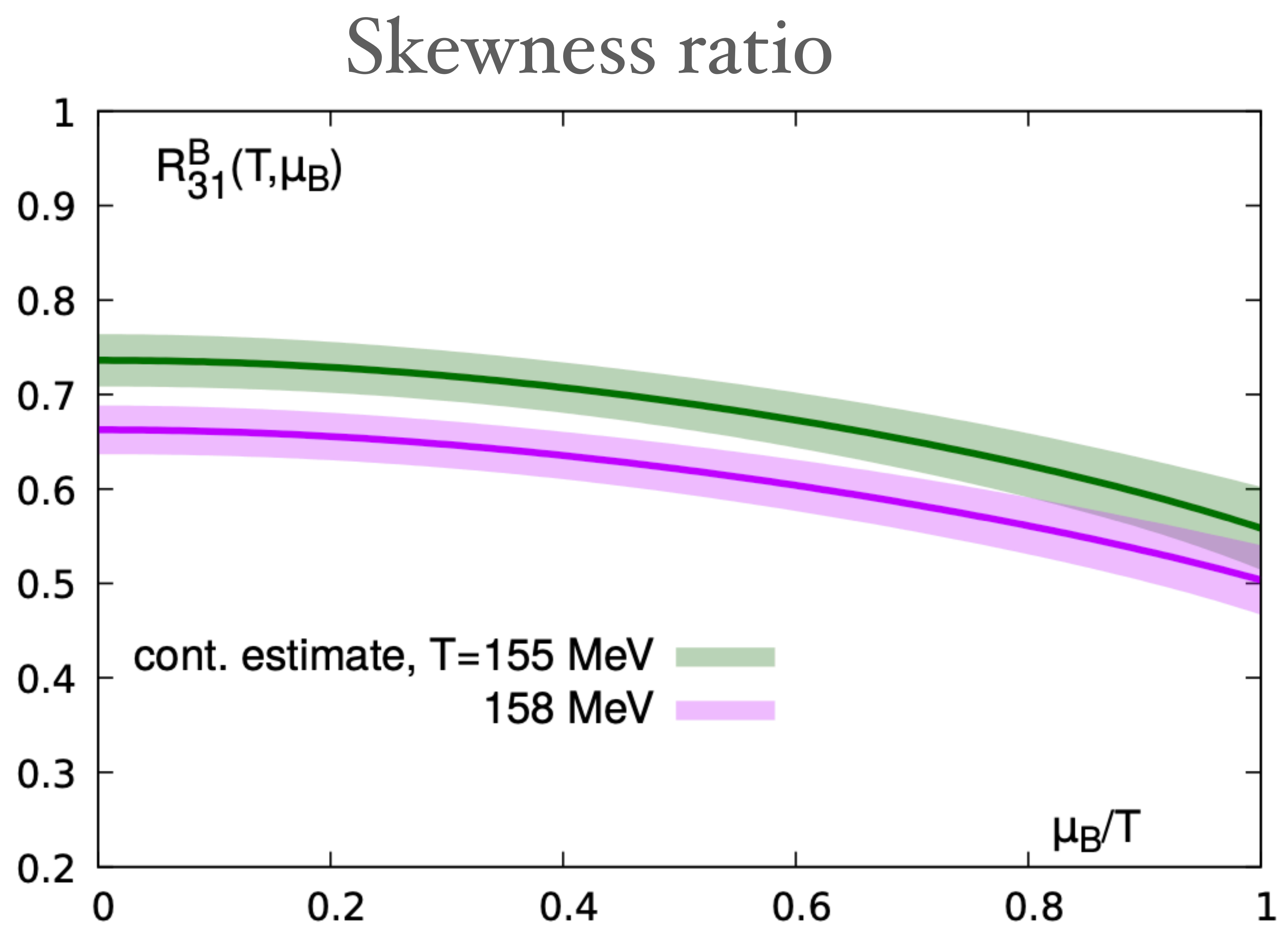
Continuum estimate

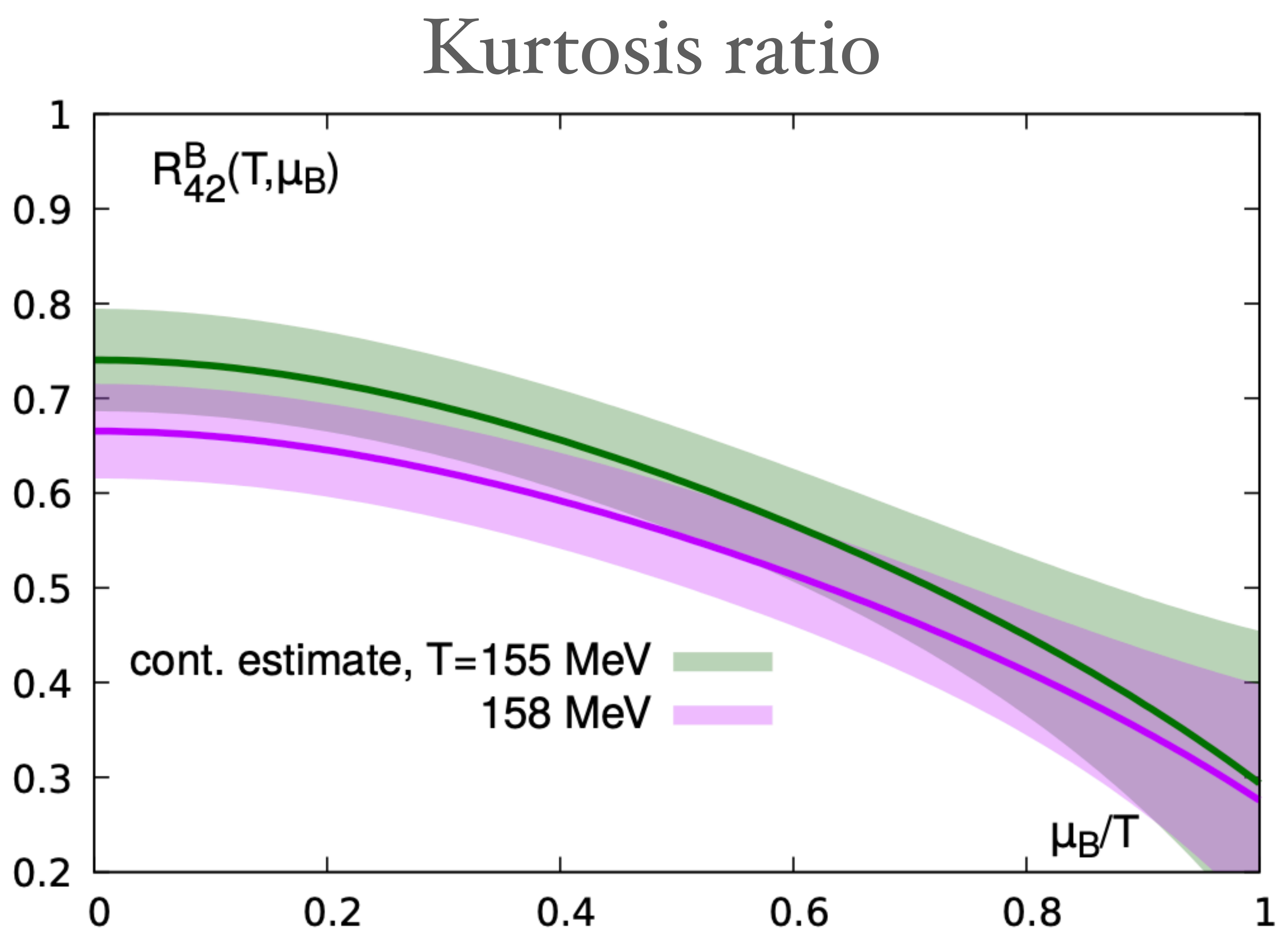


$R_{12}(\mu_B/T)$ is almost independent on T and will be used to convert μ_B to $\sqrt{s_{NN}}$

HotQCD, 2001.08530

Continuum estimates for skewness and kurtosis ratios in NNLO

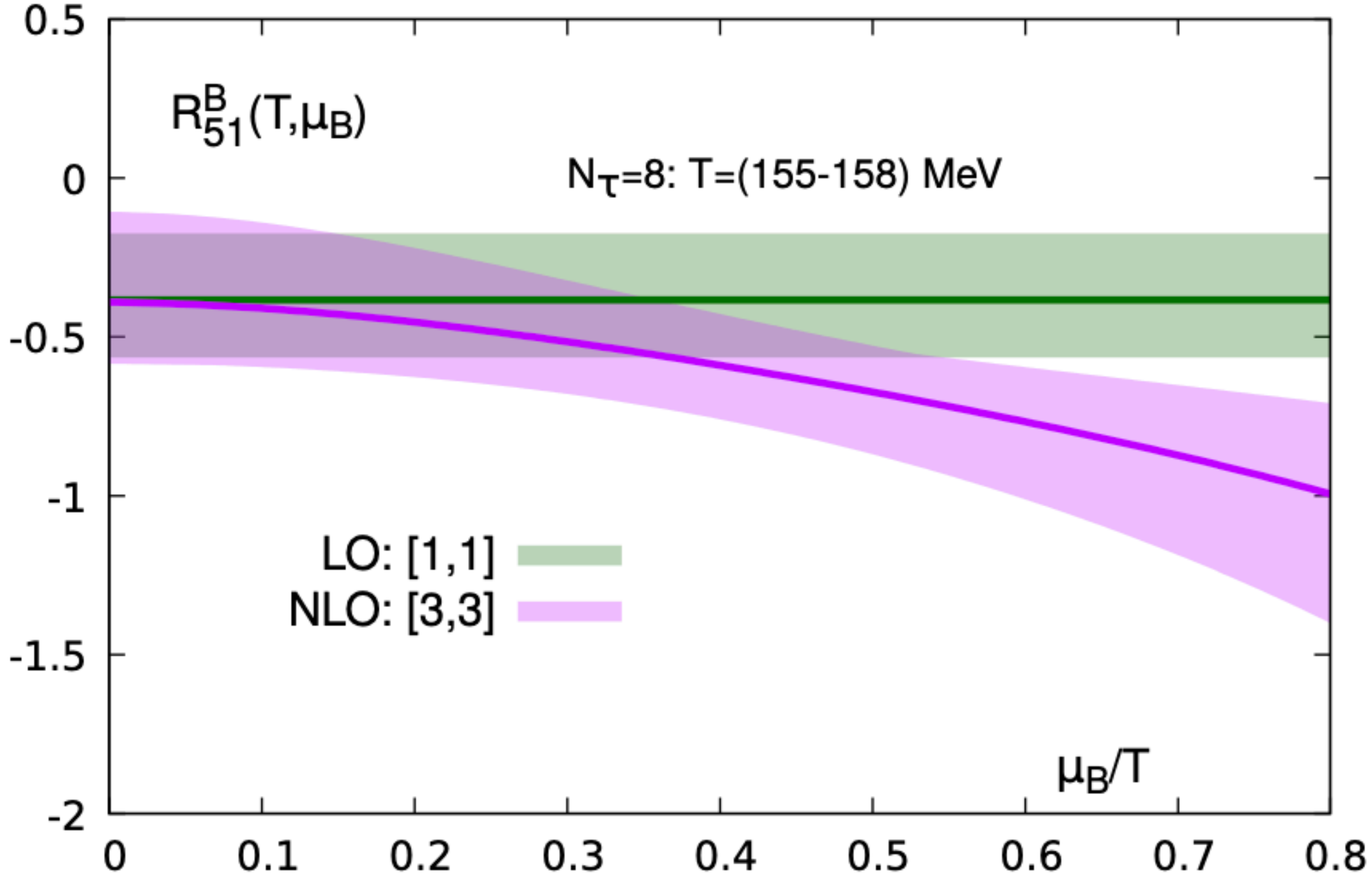


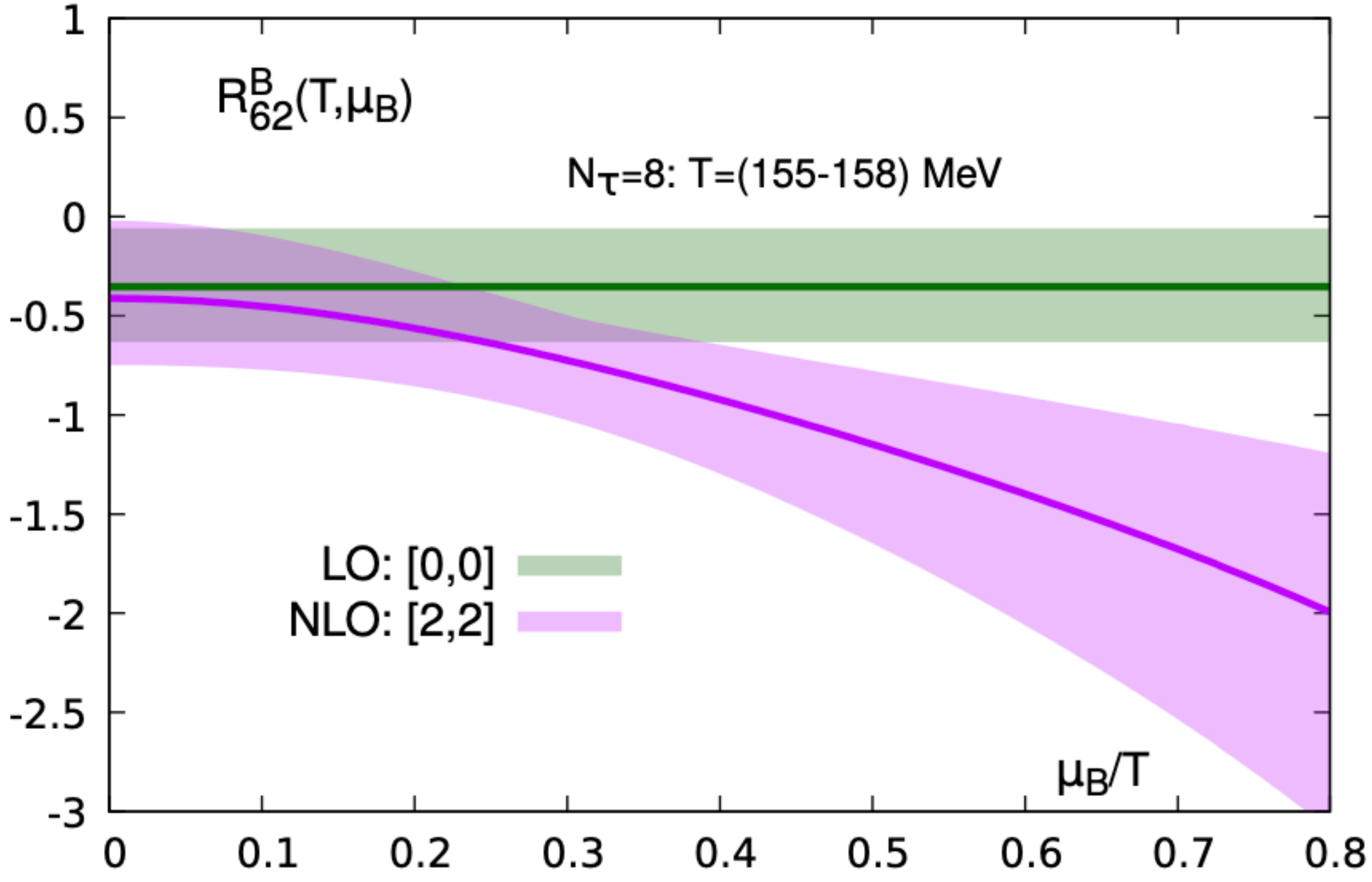


Both ratios decrease with μ_B/T and becomes larger at smaller T

HotQCD, 2001.08530

hyper-skewness and hyper-kurtosis ratios





Both C5/C1 & C6/C2 are negative in NLO!

HotQCD, 2001.08530

Explore the QCD phase diagram through fluctuations of conserved charges x=B,Q,S

HIC

mean: M_x

variance: σ_x^2

skewness: S_x

kurtosis: κ_x

hyper-skewness: S_x^h

hyper-kurtosis: κ_x^h

Proxies:
proton, charge particles, kaons

$$\frac{M_x(\sqrt{s})}{\sigma_x^2(\sqrt{s})} = \frac{\langle N_x \rangle}{\langle (\delta N_x)^2 \rangle} = \frac{\chi_1^x(T,\mu_B)}{\chi_2^x(T,\mu_B)} = R_{12}^x(T,\mu_B)$$

$$\frac{S_x(\sqrt{s})\,\sigma_x^3(\sqrt{s})}{M_x(\sqrt{s})} = \frac{\langle (\delta N_x)^3 \rangle}{\langle N_x \rangle} = \frac{\chi_3^x(T,\mu_B)}{\chi_1^x(T,\mu_B)} = R_{31}^x(T,\mu_B)$$

$$\kappa_x(\sqrt{s})\,\sigma_x^2(\sqrt{s}) = \frac{\langle (\delta N_x)^4 \rangle}{\langle (\delta N_x)^2 \rangle} = \frac{\chi_4^x(T,\mu_B)}{\chi_2^x(T,\mu_B)} = R_{42}^x(T,\mu_B)$$

$$\frac{S_x^h(\sqrt{s})\,\sigma_x^5(\sqrt{s})}{M_x(\sqrt{s})} = \frac{\langle (\delta N_x)^5 \rangle}{\langle N_x \rangle} = \frac{\chi_5^x(T,\mu_B)}{\chi_1^x(T,\mu_B)} = R_{51}^x(T,\mu_B)$$

$$\kappa_x^h(\sqrt{s})\,\sigma_x^4(\sqrt{s}) = \frac{\langle (\delta N_x)^6 \rangle}{\langle (\delta N_x)^2 \rangle} = \frac{\chi_6^x(T,\mu_B)}{\chi_2^x(T,\mu_B)} = R_{62}^x(T,\mu_B)$$

LQCD

generalized susceptibilities

$$\chi_n^x(T,\mu_B) = \frac{1}{VT^3}\frac{\partial^n \ln Z(T,\vec{\mu})}{\partial(\mu_x/T)^n}$$

Many caveats

Non-equilibrium effects

S. Mukherjee, R. Venugopalan, Yi Yin PRL(2016). ...

Proton v.s. Baryon

M. Kitazawa and M. Asakawa, PRC(2012)...

Detector effects: cuts in acceptance & kinematics...

V. Koch, S. Jeon, PRL (2000)
A.Bzdak,V.Koch, PRC (2012)
V. Skokov et al., PRC (2013)...

Final-state interactions in the hadronic phase J.Steinheimer et al., PRL (2013)...

... ...

Many caveats

Non-equilibrium effects

S. Mukherjee, R. Venugopalan, Yi Yin PRL(2016). ...

Proton v.s. Baryon

M. Kitazawa and M. Asakawa, PRC(2012)...

Detector effects: cuts in acceptance & kinematics...

V. Koch, S. Jeon, PRL (2000)
A.Bzdak,V.Koch, PRC (2012)
V. Skokov et al., PRC (2013)...

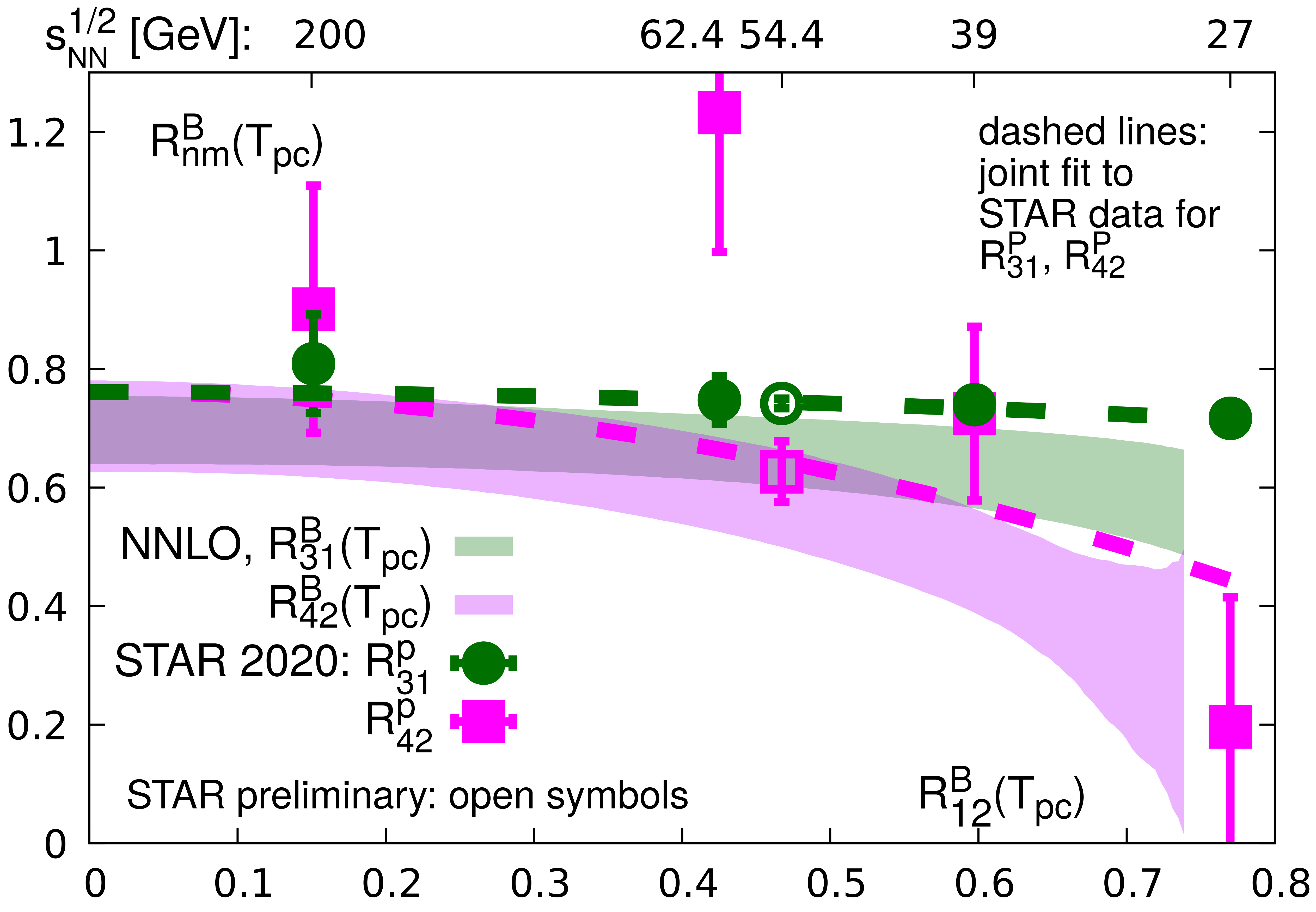
Final-state interactions in the hadronic phase J.Steinheimer et al., PRL (2013)...

... ...

Baseline from thermal equilibrated QCD

LQCD meet experiment

LQCD data are obtained at $T_{pc}(\mu_B)$ in NNLO



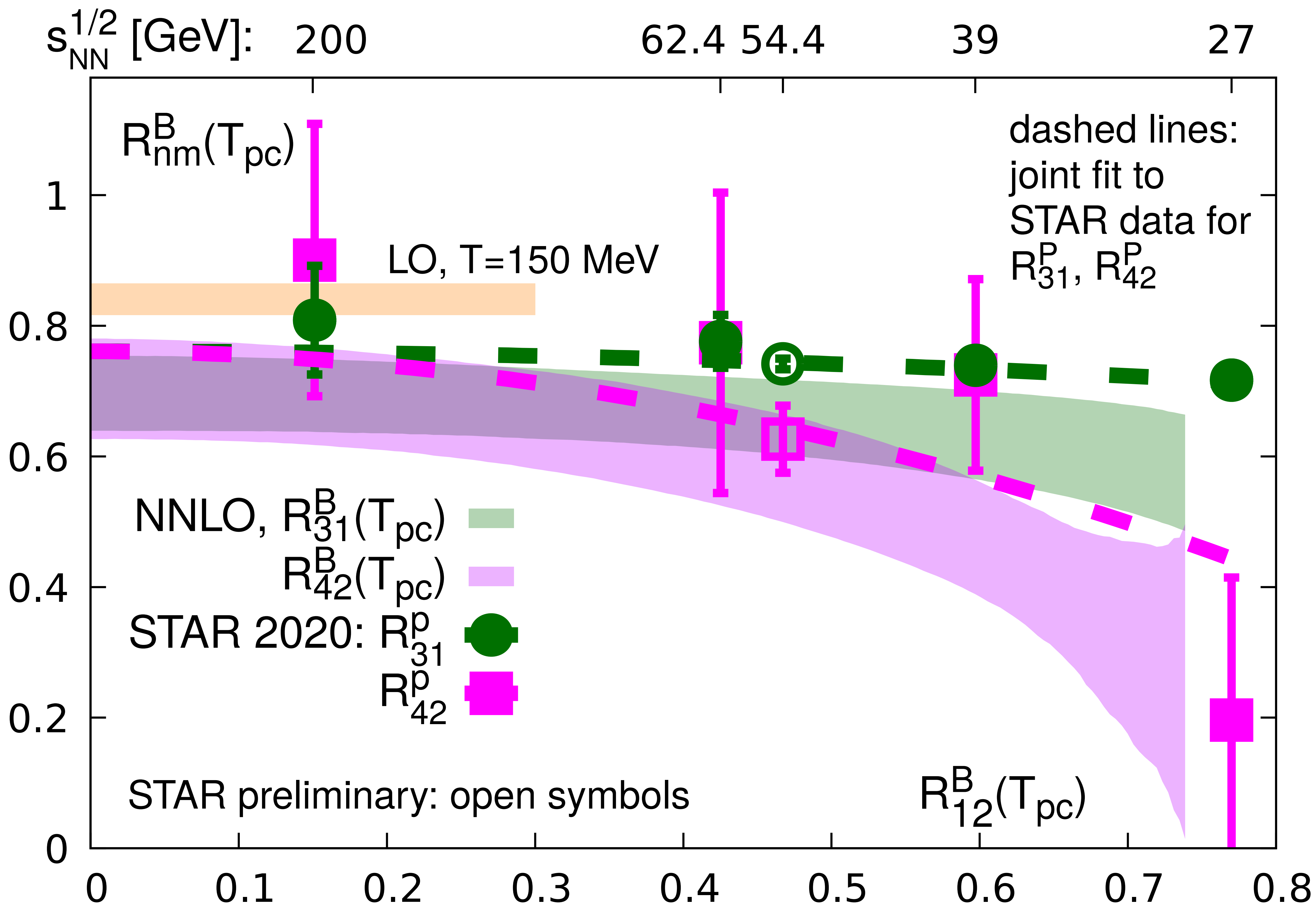
General trend of kurtosis & skewness ratios are consistent

High statistics data at 54.4 GeV are in good agreement

62.4 GeV kurtosis data is off the trend

LQCD meet experiment

LQCD data are obtained at $T_{pc}(\mu_B)$ in NNLO

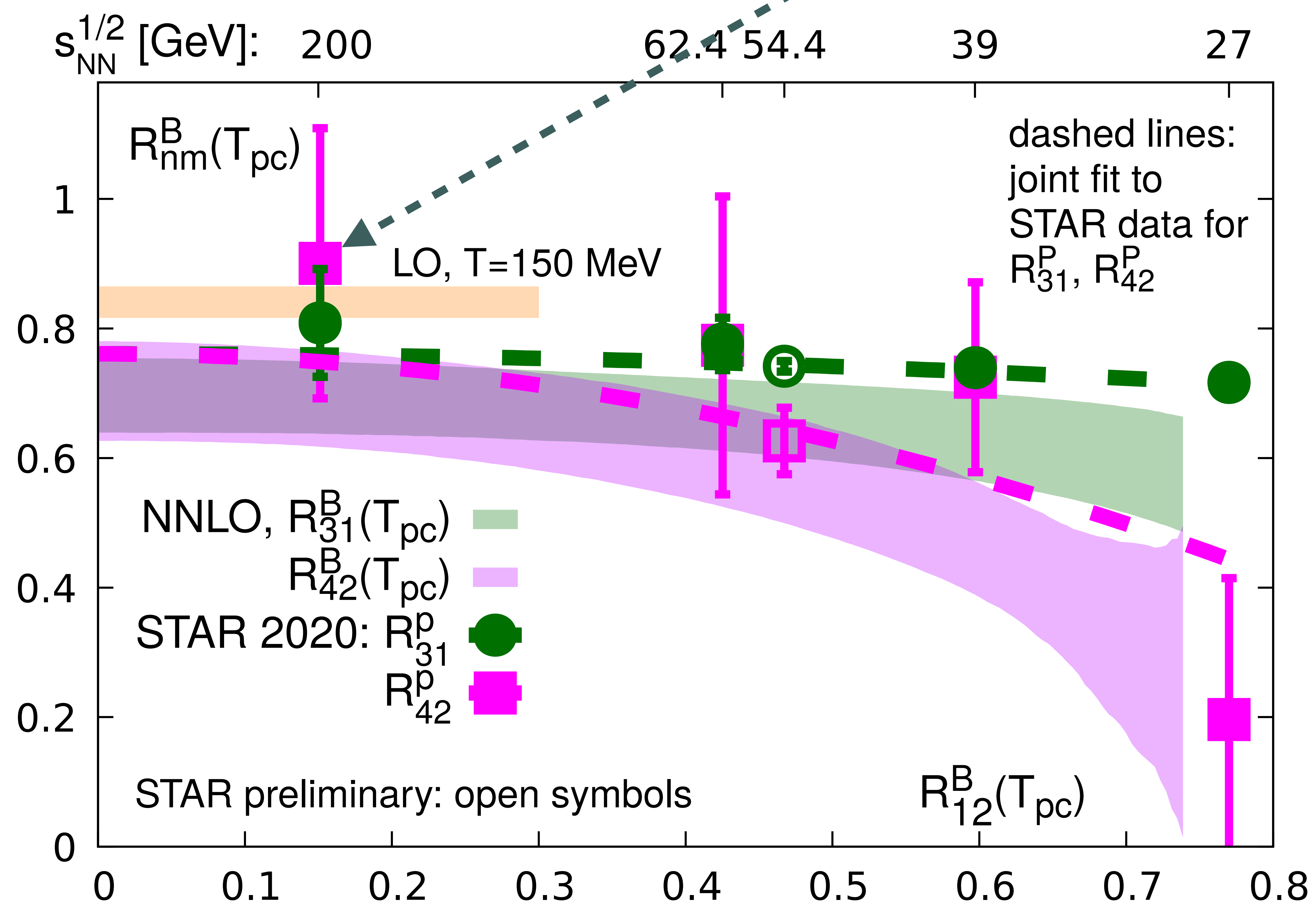


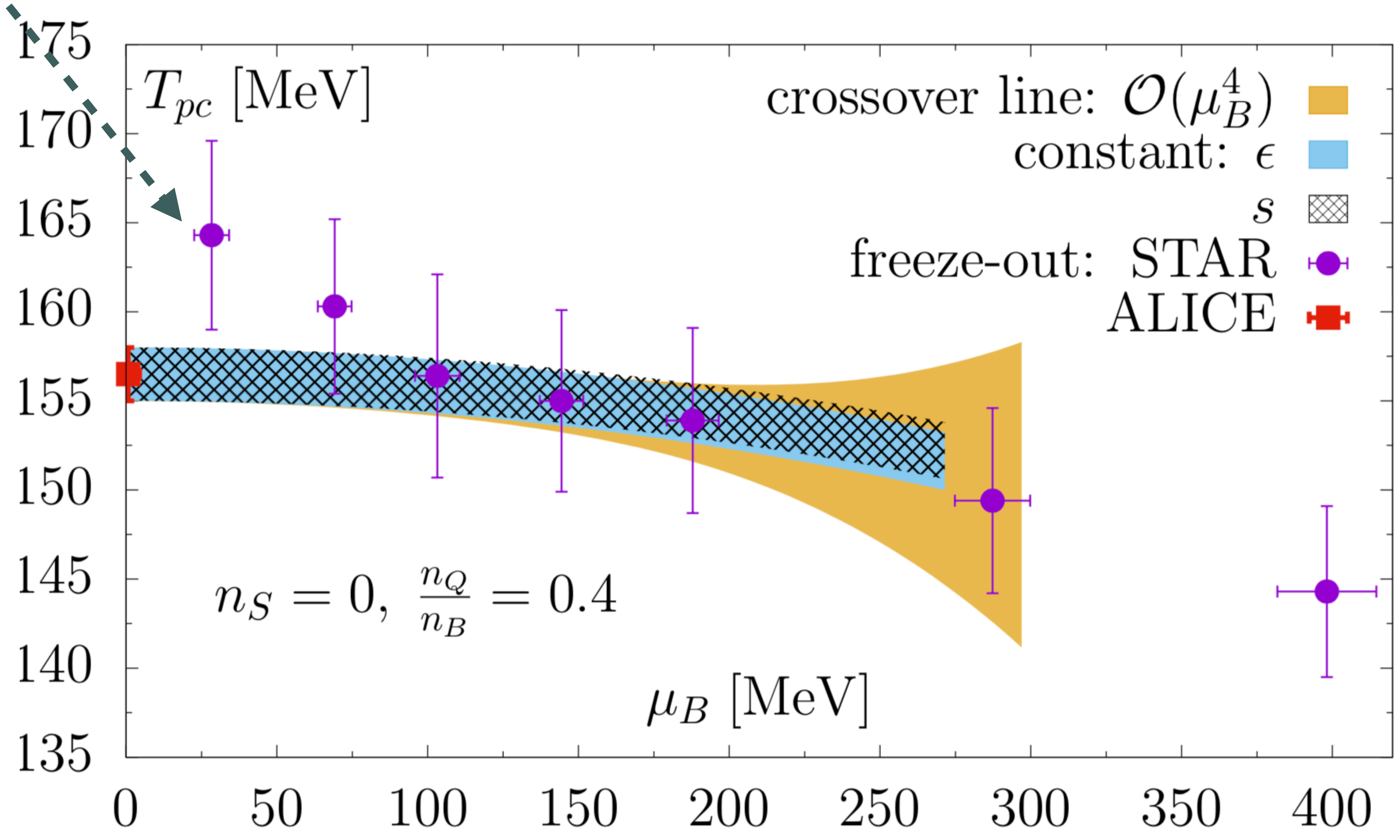
General trend of kurtosis & skewness ratios are consistent

High statistics data at 54.4 GeV are in good agreement

62.4 GeV kurtosis data are now consistent with updated STAR data

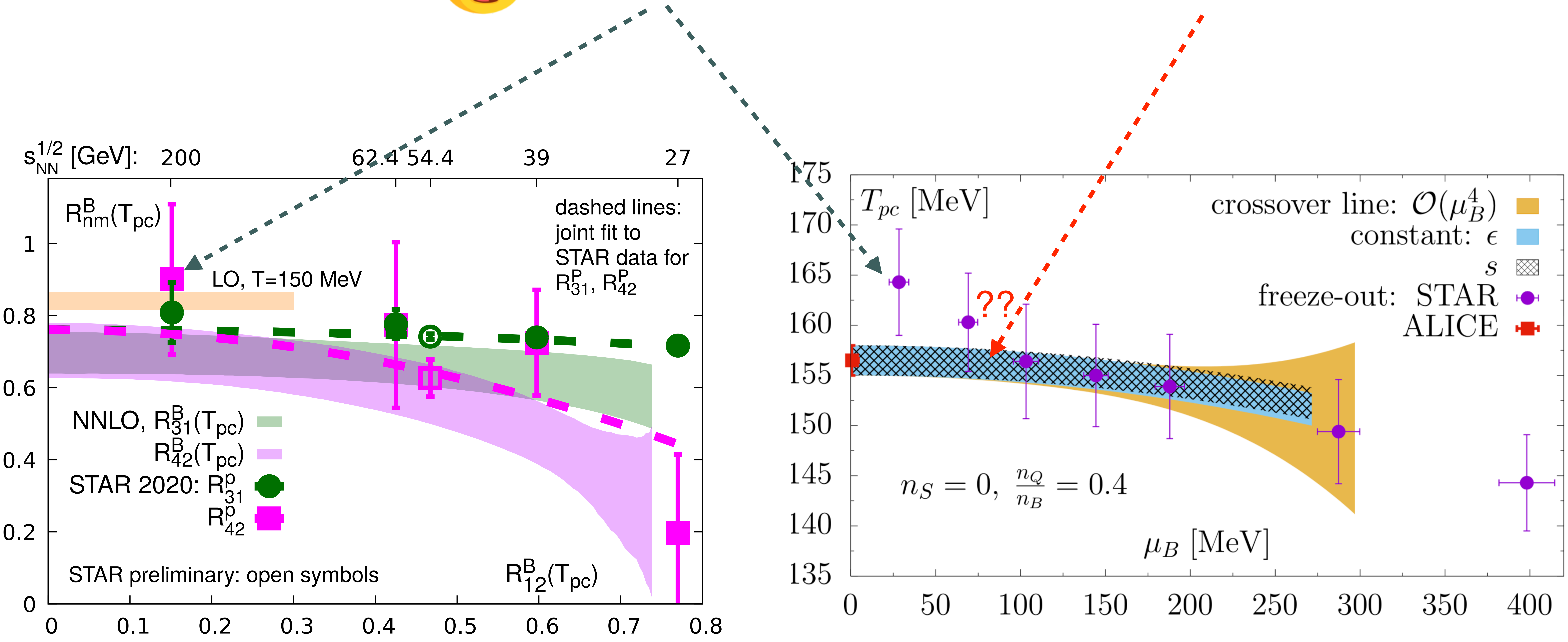
$\sqrt{s_{NN}}$ =200 GeV

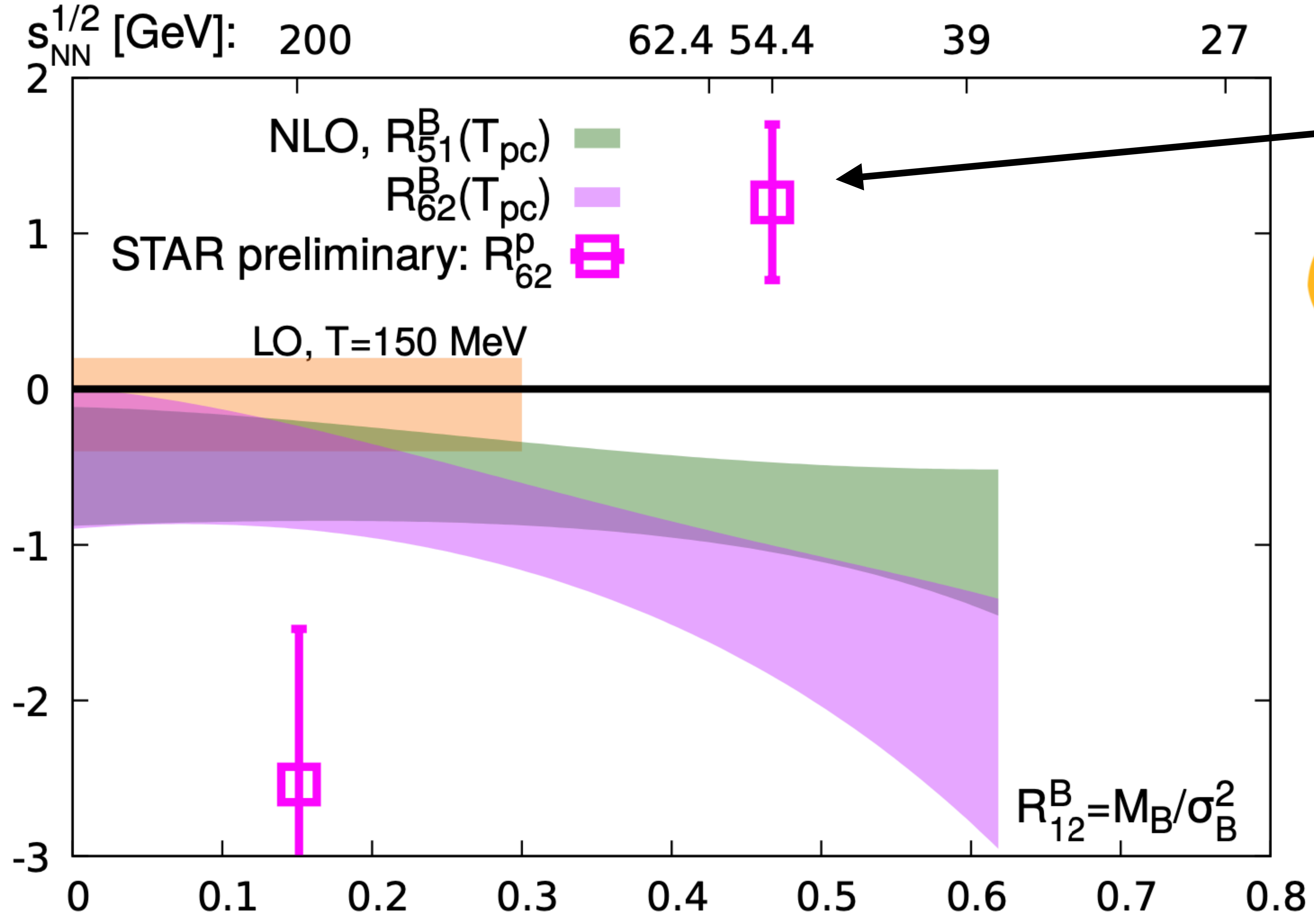




$\sqrt{s_{NN}}$ =200 GeV

$\sqrt{s_{NN}}$ =54.4 GeV



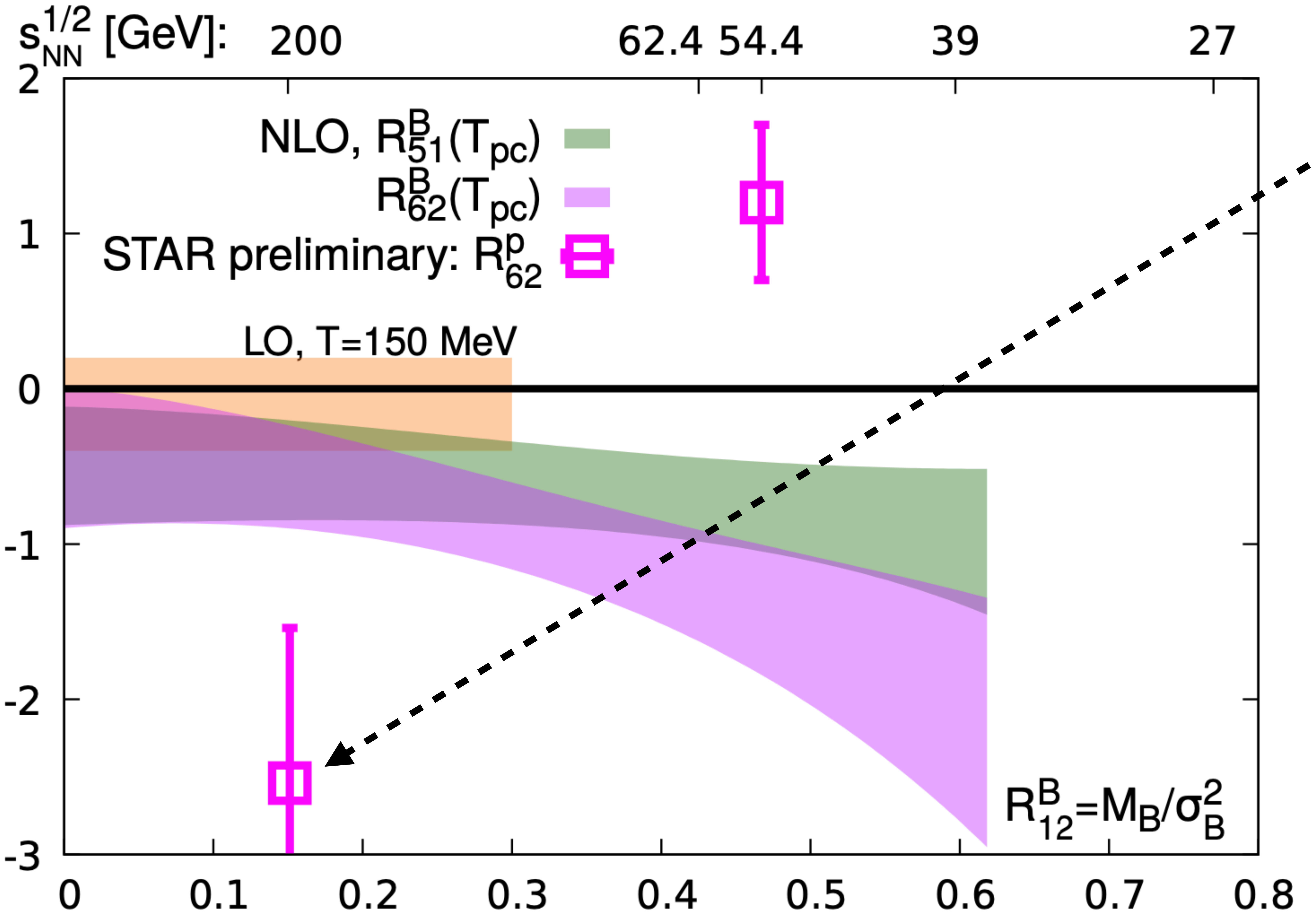


Qualitatively different sign!!!

1st, 2nd, 3rd & 4th order cumulant (ratios) at 54.4 GeV are consistent while C6/C2 is not

Statistics?
NNLO contributions in LQCD computations? Continuum extrapolation?
Off equilibrium effects?

A large higher order (10th) Taylor expansion coefficient is needed to turn C6/C2 positive, however, such a large coefficient is not expected since 54.4 GeV is not expected be close or in the critical region



LQCD and STAR data:
same sign

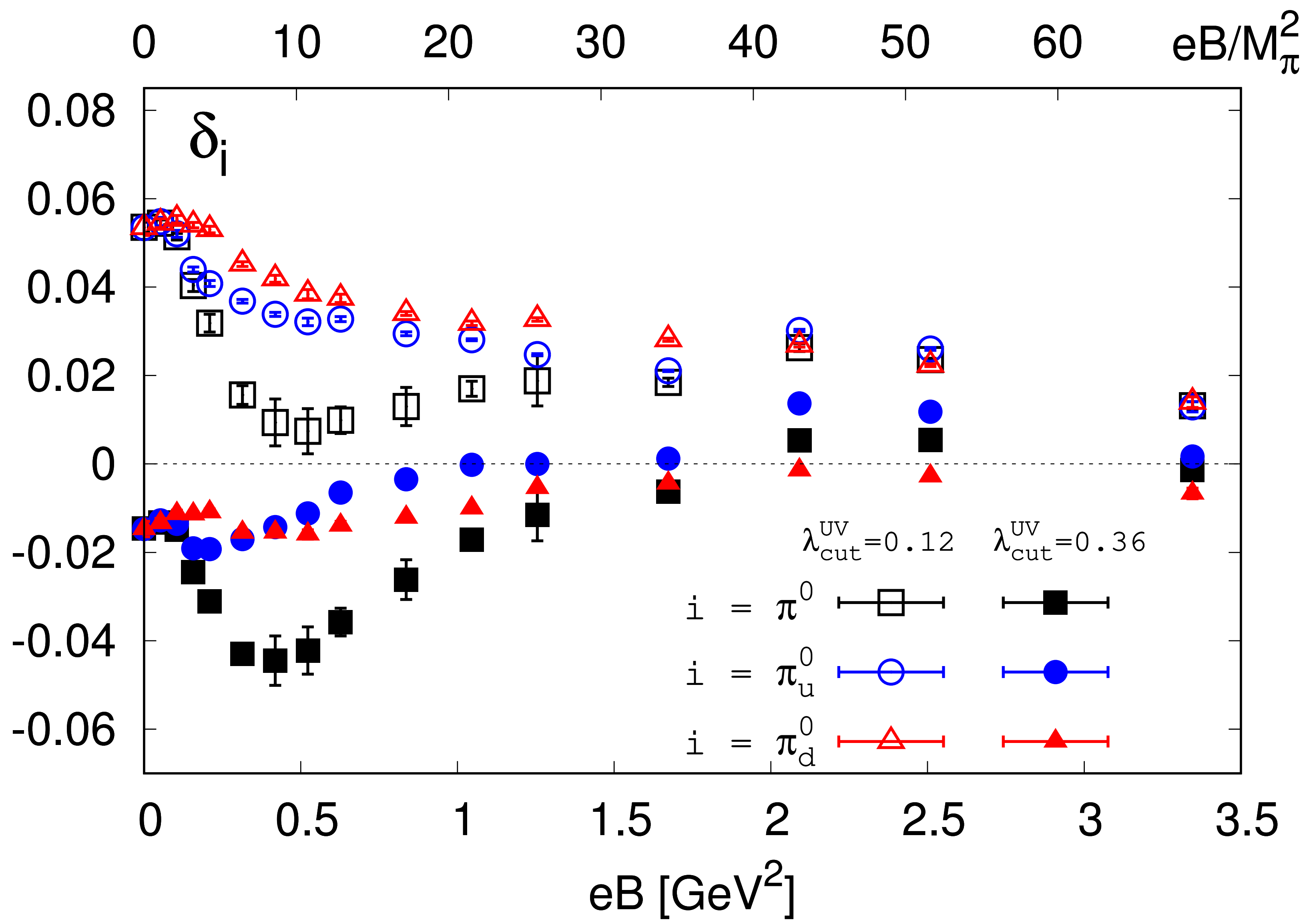
The LQCD results at 200 GeV is more reliable than that at 54.4 GeV, as the contribution from the higher expansion coefficients is more under control in the former case

Due to the agreement in C3/C1 & C4/C2 at 54.4 GeV, more statistics seems to be needed at 200 GeV to reconcile the conflict

STAR: C5/C1 ?

Summary

- Chiral and deconfinment properties in strong magnetic field: GMOR relation, 2nd order fluctuation and correlations of conserved charges



- Negative 6th and 8th order cumulants as well as $T_c^0 \approx 132$ MeV suggests a possible critical end point can located only at

$$T_c^{CEP} < 135 - 140 \text{MeV} \qquad \mu_B^{CEP} > 300 \text{MeV}$$

- Up to 6th order cumulants of net baryon number along the crossover line