From non-abelian superfluids to soft pion production

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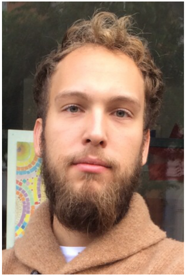
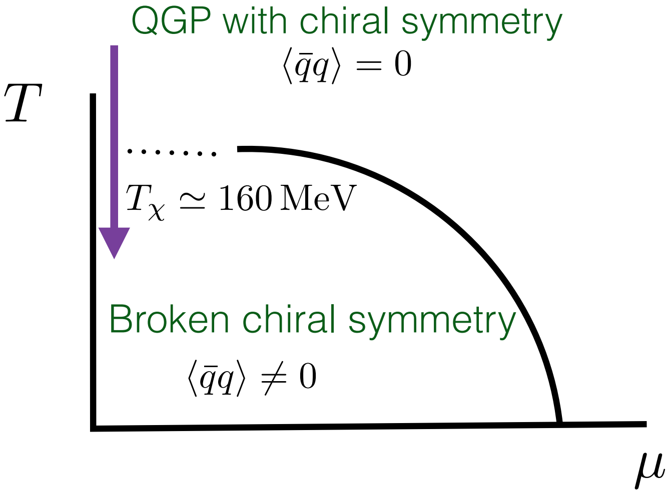
- Eduardo Grossi, Alex Soloviev, DT, Fanglida Yan: PRD, arxiv:arXiv:2005.02885
- Eduardo Grossi, Alex Soloviev, DT, Fanglida Yan: in progress

Trajectory of a heavy ion collision in the phase diagram

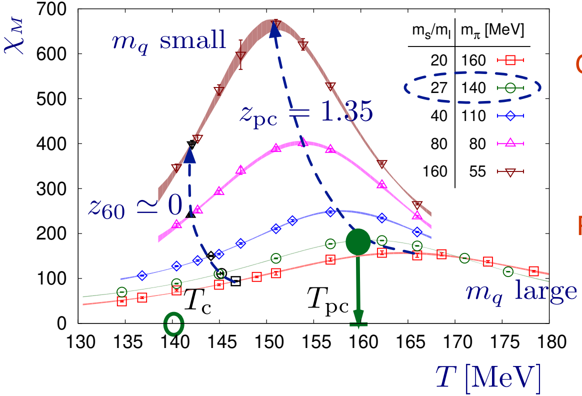


Chiral symmetry plays no role in the "Standard Model" of heavy ions ...

Chiral symmetry and $O(4)$ scaling in lattice QCD:

Hot QCD, PRL 2019

$$\chi_M \propto \frac{\partial \langle \bar{q} q \rangle}{\partial m_q} = \frac{\partial^2 \log Z}{\partial m_q^2}$$



O(4) Scaling predictions

$$\chi_M = m_q^{1/\delta - 1} f_\chi(z)$$

$$z \equiv t m_q^{-1/\beta\delta}$$

Pseudo critical point:

$$T_{\rm pc} \simeq 158\,\mathrm{MeV}$$

$$z_{\rm pc} = 1.35$$

The QCD lattice knows about the $O(4) = SU_L(2) \times SU_R(2)$ critical point!

QCD and the Chiral limit and Broken Symmetry:

Son hep-ph/9912267; Son and Stephanov hep-ph/020422

1. The approximately conserved quantities $\hat{J}^{\mu}_{a} = \bar{\psi}\gamma^{5}\gamma^{\mu}\tau^{a}\psi$

$$\underbrace{T^{\mu\nu}}_{\text{stress}} \qquad \underbrace{J^{\mu}_{B}}_{\text{Baryon number}} \qquad \underbrace{J^{\mu}_{a}}_{\text{isovector}} \quad \textit{and} \quad \underbrace{\hat{J}^{\mu}_{a}}_{\text{iso-axial vector}}$$

2. There is the phase of the chiral condensate and pion field: $\varphi^{a} = \pi^{a}/F$

$$\Sigma = \sigma \cdot U = \sigma \cdot \text{Phase of } \langle \bar{q}q \rangle \equiv \sigma \cdot e^{i\tau^{a}\varphi^{a}}$$

3. The pion φ^{a} is like T, $\vec{u}$, μ_{I}, *and* $\hat{\mu}$ in the constitutive relations
4. Include a mass term so the Goldstone fields decay at large distances

Need to write down a theory of superfluid hydro for φ (Son '99)

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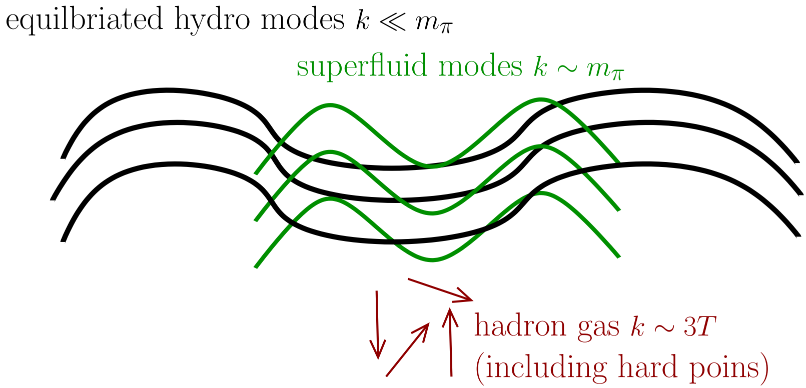
Near the critical point the $\sigma(t, \boldsymbol{x})$ should be included too!

Picture for $T \lesssim T_c$

- Work in the regime

$$k \ll m_\pi \ll \pi T \sim \pi \Lambda_{QCD}$$



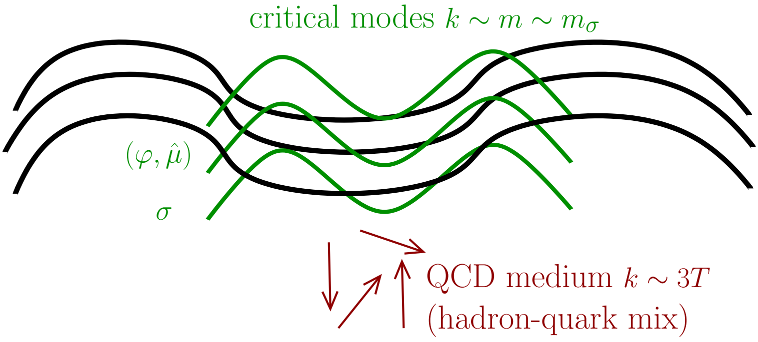
How do these superfluid modes contribute to pressure and viscosities, and the diffusion rate of isovector charge ?

Picture for $T \sim T_c$

- Work in the regime:

$$k \ll m \sim m_\sigma \ll \pi T_C \sim \pi \Lambda_{QCD}$$

equilbriated hydro modes $k \ll m$



How do these superfluid modes contribute to pressure and viscosities, and the diffusion rate of isovector charge ?

The pressure from soft pion modes:

- Use 3D dimensionally reduced chiral perturbation theory:

$$Z_{QCD} = \underbrace{e^{\beta p_0(T,\hat{\mu})V}}_{\text{from hard modes } p\sim T} \times \underbrace{\int^{\Lambda} [D\varphi] \exp\left(-\beta \int d^3\boldsymbol{x}\mathcal{L}_{\rm eff}\right)}_{\text{from soft modes } p \sim m_\pi}$$

where using $U = e^{i\varphi_a \tau_a}$

$$\mathcal{L}_{\rm eff} \simeq \frac{f^2(T)}{4}\,{\rm Tr}\,\vec{\nabla}U\cdot\vec{\nabla}U^\dagger + \frac{f^2m^2(T)}{2}{\rm Re\,Tr}\,U \quad \Rightarrow \quad \frac{f^2}{2}(\nabla\varphi_a)^2 + \frac{f^2m^2}{2}\varphi_a^2$$

The parameters have universal dependence near the $O(4)$ critical point:

$$f^2m^2 \propto m_q \left\langle \bar{\psi}\psi \right\rangle \propto m_q t^\beta \qquad\qquad t \equiv (T-T_c)/T_c$$

$$f^2 \propto t^{\nu(d-2)}$$

Can compute $f^2(T)$ and $m^2(T)$ the real world lattice QCD with precision!

Stress and Current for Superfluids:

Son; Jensen et al 1203.3556, Bhattacharya et al

The pressure in the presence of the phase is p_φ

$$p_\varphi(T, \nabla\varphi, \varphi^2) = p_0(T) + \frac{1}{2}\hat{\chi}\hat{\mu}^2 - \frac{f^2}{2}\left((\nabla\varphi)^2 + m^2\varphi^2\right)$$

- Derive the ideal stress and current from pressure $W = \int d^4x\sqrt{g}p_\varphi$

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta W}{\delta g_{\mu\nu}} = (e_\varphi + p_\varphi)u^\mu u^\nu + \eta^{\mu\nu}p_\varphi + \underbrace{f^2\partial^\mu\varphi\partial^\nu\varphi}_{\text{super fluid stress}}$$

$$\hat{J}^\mu_a = \frac{1}{\sqrt{-g}}\frac{\partial W}{\partial A_\mu} = \underbrace{\hat{n}_a u^\mu}_{\text{normal fluid}} + \underbrace{f^2\partial^\mu\varphi_a}_{\text{super fluid current}}$$

- In an extension of the formalism coming from $[\bar{q}_R q_L, H - \mu N] = 0$

$$\underbrace{-u^\mu\partial_\mu\varphi_a = \hat{\mu}_a}_{\text{Josephson constraint}} \quad \text{and} \quad \underbrace{\partial_\mu\hat{J}^\mu_a = f^2m^2\varphi_a}_{\text{PCAC}}$$

The Josephson constraint:

$\langle \bar{q}_R q_L \rangle$ is stable

- The phase U is related to $\hat{\mu}$, since $\Sigma \equiv \bar{q}_R q_L = \sigma U$ is stationary:

$$[\Sigma,\ H - \mu_L \cdot Q_L - \mu_R \cdot Q_R] = 0$$

using the transformation properties e.g. $[\Sigma, Q_L^a] = -it^a\Sigma$, we find:

$$\underbrace{i\partial_t U U^\dagger}_{\text{(minus) deriv of phase}} \quad = \quad \underbrace{\mu_L - U\mu_R U^\dagger}_{\text{the axial chem } \hat{\mu}}$$

- In linearized form:

$$\underbrace{-\partial_t \varphi = \hat{\mu}}_{\text{Josephson constraint}}$$

Dissipative corrections to EOM

Son, Stephanov hep-ph/020422 + a bit by us

$$\underbrace{\partial_t \hat{J}^0 + \nabla\cdot\hat{\boldsymbol{J}} = f^2 m^2 \varphi}_{\text{PCAC}} \qquad \text{and} \qquad \underbrace{-\partial_t \varphi_a = \hat{\mu}_a}_{\text{Josephn's constraint}}$$

- Then expand the current in gradients

$$\hat{\boldsymbol{J}} = \underbrace{f^2\nabla\varphi}_{\text{ideal current}} - \underbrace{\lambda_0\,\nabla\hat{\mu}}_{\text{axial conductivity}} + \underbrace{\vec{\xi}_J}_{\text{noise}}$$

and the josephson constraint

$$-\partial_t\varphi = \underbrace{\hat{\mu}}_{\text{ideal fluid}} + \underbrace{-\kappa_2\,\nabla^2\varphi + \kappa_1\,m^2\varphi}_{\text{visc correction}} + \underbrace{\xi_S}_{\text{noise}}$$

- To reach the equilibrium fluctuations we must have:

$$\left\langle \xi_J^i \xi_J^j \right\rangle = 2T\lambda_0\,\delta^{ij}\delta^4(x-x') \qquad \langle \xi_S \xi_S \rangle = 2T\lambda_m\,\delta^4(x-x')$$

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$$-\partial_t\varphi = \underbrace{\hat{\mu}}_{\text{ideal fluid}} + \underbrace{\lambda_m(-\partial_i(f^2\partial^i\varphi) + f^2 m^2\varphi)}_{\text{visc correction}} + \underbrace{\xi_S}_{\text{noise}}$$

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Long wavelength pion (superfluid) modes: Son, Stephanov hep-ph/020422 + a bit by us

- Linearizing the equation of motion $\varphi = Ce^{-i\omega t + i\boldsymbol{q}\cdot\boldsymbol{x}}$ one finds

$$\varphi(t, \boldsymbol{q}) = Ce^{-(\Gamma/2)t}e^{-i\omega_q t} \quad \Leftarrow \text{This is second sound!}$$

- The quasi-particle energy is:

$$\omega_q^2 \equiv v_0^2(q^2 + m^2) \qquad\qquad v_0^2(T) \equiv \frac{f^2}{\hat{\chi}} \quad \Leftarrow \text{pion velocity}$$

- The damping rate is set by two diffusion coefficients, D_A and D_m:

$$\Gamma \equiv D_A q^2 + D_m m^2$$

$$D_A = (\lambda_0/\hat{\chi}) + f^2\lambda_m \quad \Leftarrow \text{Axial charge diffusion coefficient}$$

$$D_m = f^2\lambda_m \quad \Leftarrow \text{Axial damping coefficient}$$

Interpretation of the pion velocity

Son, Stephanov hep-ph/020422

- The ideal massless pion equation of motion: $(f_t^2 \equiv \hat{\chi}$ and $f_s^2 \equiv f^2)$

$$\underbrace{-\partial_t(\hat{\chi}\partial_t\varphi)}_{\partial_t(\text{axial-chrg})} + \underbrace{\partial_x(f^2\partial_x\varphi)}_{\partial_x(\text{super-current})} = 0$$

The conserved charge is

$$J^0 = \underbrace{\hat{\chi}\partial^t\varphi}_{\text{total axial-chrg}} = \underbrace{f^2\partial^t\varphi}_{\text{super component}} + \underbrace{\Delta\hat{\chi}\,\partial^t\varphi}_{\text{normal component}}$$

So the pion velocity has a simple interpretation

$$v_0^2 \equiv \frac{f^2}{\hat{\chi}} = \frac{f^2}{f^2 + \Delta\hat{\chi}} = \frac{\text{super}}{\text{super} + \text{normal}}$$

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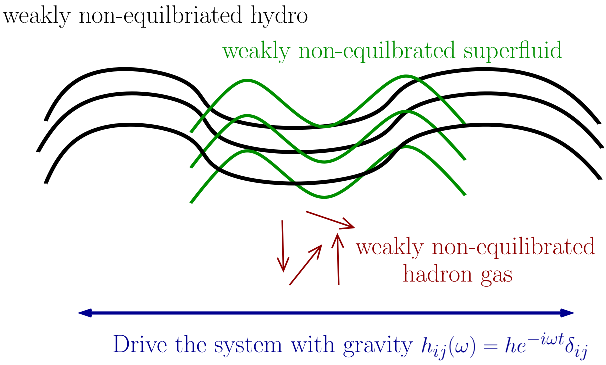
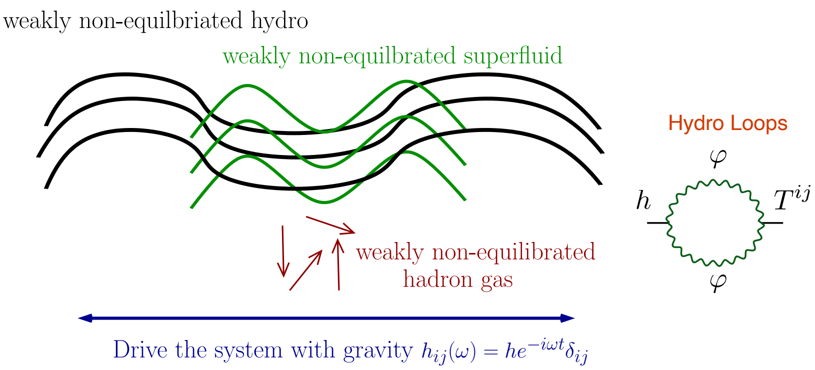


Superfluid fluctns and shorter are *absorbed* into the transport coefficients

$$\underbrace{\frac{1}{3}\left\langle \delta T^{i}_{\ i} \right\rangle}_{\text{non-equil. stress}} \quad = +i\frac{3}{2}\omega h\zeta = -\zeta \underbrace{\nabla \cdot u}_{\text{cov-d}}$$



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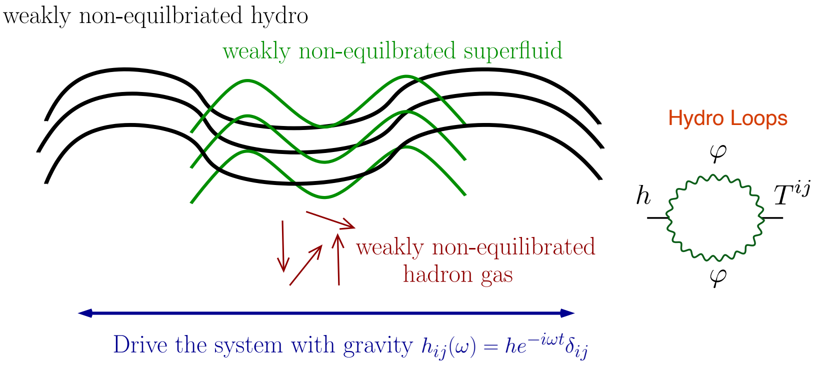

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Let's evolve the phase-space density of the stochastic superfluid with (hydro) kinetics

Developing hydro-kinetics – linearized stochastic hydro

1. Evolve fields of linearized super fluid hydro:

$$\phi_a(\boldsymbol{k}) \equiv \left(\varphi(\boldsymbol{k}), \hat{J}^0\right)$$

2. The stochastic EOM are matrix versions of Brownian motion:

$$\frac{d\phi_a(\boldsymbol{k})}{dt} = \underbrace{\mathcal{L}_{ab}(\boldsymbol{k})}_{\text{ideal}}\,\phi_b(\boldsymbol{k}) + \underbrace{\mathcal{D}_{ab}\phi_b}_{\text{damping}} + \underbrace{\xi_a}_{\text{noise}}$$

3. Break up the equations into eigen modes of $\mathcal{L}_{ab}$, and analyze:

    - The eigenmodes of the superfluid equations are the propagating pions:

$$\hat{\phi}_\pm = \frac{\hat{J}^0 \pm i\omega_k\,\hat{\chi}\,\varphi}{\sqrt{2}} \qquad \text{with} \qquad \omega_{k,\pm} = \underbrace{\pm(v_0^2\,(k^2+m^2))^{1/2}}_{\text{eigen-vals}}$$

The hydro-kinetic equations

1. Find the evolution of the phase-space density of 2nd sound modes:

$$\hat{W}_{++}(t,\boldsymbol{x},\boldsymbol{q}) \equiv \int d\boldsymbol{y}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{y}} \left\langle \hat{\phi}^*_+(t,\boldsymbol{x}+\boldsymbol{y}/2)\hat{\phi}_+(t,\boldsymbol{x}-\boldsymbol{y}/2) \right\rangle$$

   The phase space density is $W_{++} \equiv \chi\omega_q f_\pi(t,\boldsymbol{x},\boldsymbol{q})$.

2. The phase-space distribution evolution follows the Boltzmann eqn:

$$\left(\partial_t + \frac{\partial\omega_q}{\partial\boldsymbol{q}}\cdot\frac{\partial f_\pi}{\partial\boldsymbol{x}} - \frac{\partial\omega_q}{\partial\boldsymbol{x}}\cdot\frac{\partial f_\pi}{\partial\boldsymbol{q}}\right) = \underbrace{-(D_A q^2 + D_m m^2)\left[f_\pi - \frac{T}{\omega_q}\right]}_{\text{damping to equilibrium}}$$

3. The particles stream with effective 4D Hamiltonian An,Basar,Yee,Stephanov

$$\mathcal{H} = \tfrac{1}{2}G^{\mu\nu}q_\mu q_\nu + \tfrac{1}{2}f^2 m^2 \qquad \underbrace{G^{\mu\nu} = -u^\mu u^\nu + v_0^2\Delta^{\mu\nu}}_{\text{fluid metric}}$$

   i.e. $\dot{q}_\mu = -\partial\mathcal{H}/\partial x^\mu$ etc. with particles onshell $\mathcal{H}=0$.

Final kinetic theory expression for bulk viscosity

1. Expression

$$\zeta = \zeta^{(0)}(\Lambda) + \int^{\Lambda} \frac{d^3q}{(2\pi)^3} \frac{T}{D_A\, q^2 + D_m\, m^2} \left[\frac{\boldsymbol{q}}{\omega_q} \cdot \frac{\partial \omega_q}{\partial \boldsymbol{q}} - \frac{c_s^2}{\omega_q} \frac{\partial(\beta\omega_q)}{\partial \beta} \right]^2$$

2. Definitions characterizing the dispersion curve, $m_p^2 \equiv v_0^2 m^2$

$$\underbrace{\tilde{v}_0^2 = v_0^2 - T^2 \frac{\partial v_0^2}{\partial T^2}}_{\text{thermal velocity (euclidean!)}} \qquad \text{and} \qquad \underbrace{\tilde{m}_p^2 = m_p^2 - T^2 \frac{\partial m_p^2}{\partial T^2}}_{\text{thermal mass (euclidean!)}}$$

3. Find, with $r \equiv \sqrt{D_m/D_A}$, the first correction to the chiral limit:

$$\zeta = \zeta^{(0)} + \frac{3Tm}{8\pi D_A} \left[\left(\frac{c_s^2}{1+r} \frac{\tilde{m}_p^2}{m_p^2} - \frac{1+2r}{1+r} \left(\tfrac{1}{3} - c_s^2 \frac{\tilde{v}_0^2}{v_0^2} \right) \right)^2 - (4+2r) \left(\tfrac{1}{3} - c_s^2 \frac{\tilde{v}_0^2}{v_0^2} \right)^2 \right]$$

Final formulas:

(in terms of superfluid parameters D_A and $r = \sqrt{D_m/D_A}$)

$r = \sqrt{D_m/D_A}$ is an order one number:

$r^2 = 3/4$ in χPT.

$$\zeta \simeq \zeta^{(0)} + \frac{3Tm}{8\pi D_A}\left(\left(\frac{c_s^2}{1+r}\frac{\tilde{m}_p^2}{m_p^2} - \frac{1+2r}{1+r}\left(\frac{1}{3} - c_s^2\frac{\tilde{v}_0^2}{v_0^2}\right)\right)^2 - (4+2r)\left(\frac{1}{3} - c_s^2\frac{\tilde{v}_0^2}{v_0^2}\right)^2\right)$$

$$\eta = \eta^{(0)} - \frac{Tm}{40\pi D_A}\left[\frac{2r^3+4r^2+6r+3}{(1+r)^2}\right]$$

$$\sigma_I = \frac{T}{12\pi m D_A}\left[\frac{1+2r}{(1+r)^2}\right] + \sigma_I^{(0)}$$

These show how chiral phase flucts modify transport coeffs:

What happens near the $O(4)$ critical point when σ is active?

$$\underbrace{\Sigma \equiv \langle \bar{q}_R q_L \rangle}_{\text{order parameter}} = \underbrace{\sigma}_{\text{amplitude}} \times \underbrace{U}_{\text{phase}}$$

The pressure from the critical modes:

- The eff. Lagrangian near the critical point is a Landau-Ginzburg type

$$Z_{QCD} = \underbrace{e^{\beta p_0(T,\hat{\mu})V}}_{\text{from hard modes } p \sim T} \times \underbrace{\int^{\Lambda} [D\varphi] \exp\left(-\beta \int d^3\boldsymbol{x} \mathcal{L}_{\rm eff} \right)}_{\text{from soft modes } p \sim m_c}$$

where using $\Sigma = \sigma e^{i\varphi_a \tau_a}$, where σ and φ now fluctuate:

$$p_\Sigma = p_0(T) + \frac{\chi_0}{2}{\rm tr}(\mu_L^2 + \mu_R^2) - \left(\frac{\rm tr}{4} \partial_i \Sigma \partial_i \Sigma^\dagger + \frac{m_0^2(T)}{2}\sigma^2 + \frac{\lambda}{4}\sigma^4 \right) + \frac{H}{4}{\rm tr}(\Sigma + \Sigma^\dagger)$$

with $m_0 \propto (T - T_c)/T_c$.

We will work with a mean field approximation

$$\Sigma \simeq \underbrace{\bar{\sigma}}_{\text{mean field}} + \underbrace{\delta\sigma + i\bar{\sigma}\vec{\varphi}\cdot\vec{\tau}}_{\text{flucts}}$$

Mean field approximation

- The mean order parameter, or EOS, takes takes the scaling form

$$\bar{\sigma} = h^{1/3} f_G(z) \qquad z = t h^{-2/3}$$

where $h = H/\lambda$ and $t = m_0^2(T)/\lambda$ with $f_G(0) = 1$.

- The action for the quadratic fluctuations takes the forms

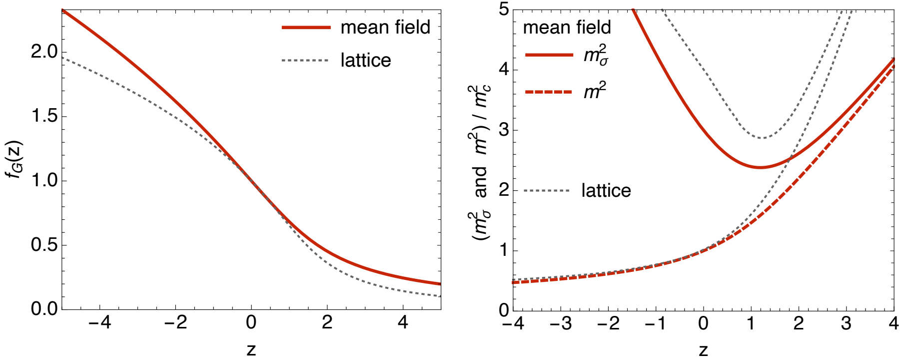
$$p_\Sigma = p_0(T) + \frac{1}{2}\chi_0 \mathrm{tr}(\mu_L^2 + \mu_R^2) - \frac{1}{2}\left(\nabla\delta\sigma \cdot \nabla\delta\sigma + m_\sigma^2 \delta\sigma^2\right) - \frac{1}{2}\chi_0 v^2(T)\left(\nabla\varphi^a \cdot \nabla\varphi^a + m^2 \varphi^a\varphi^a\right)$$

Relating the pion $m^2(z)$ and sigma screening masses to the EOS, e.g.

$$\underbrace{m^2(z)}_{\text{pion screening mass}} = \frac{H}{\bar{\sigma}(z)} = \frac{m_c^2}{f_G(z)}$$

Screening masses and magnetic EOS compared to lattice

Lattice: Engels, Vogt 0911.1939. Engels, Karsch 1105.0584



General trends are reproduced by mean field analysis

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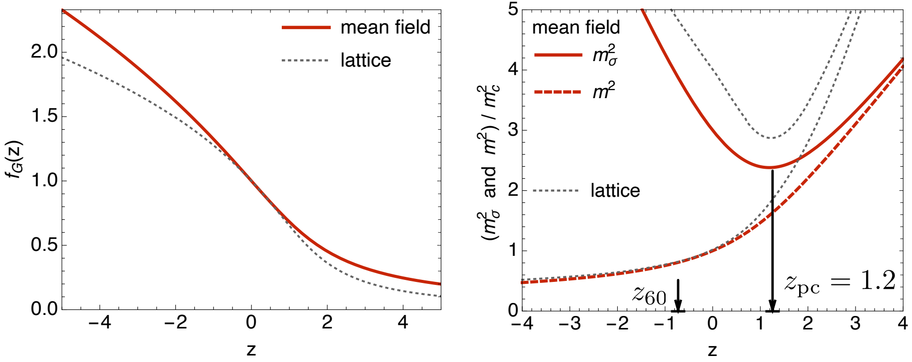


General trends are reproduced by mean field analysis

Stochastic hydrodynamic equations

Wilczek-Rajagopal hep-ph/9210253

Given the pressure, we go find the hydro equations for $\Sigma = \sigma e^{i\varphi_a \tau_a}$

- For example: the equation of the phase is, with $\vec{L} \equiv -i\nabla U U^\dagger$

$$\underbrace{(\tfrac{-i}{2}\partial_t U U^\dagger)}_{\text{phase deriv}} = \underbrace{-\tfrac{1}{2}(\mu_L - U\mu_R U^\dagger)}_{\text{joseph constraint}} + \underbrace{\frac{D_m}{\sigma^2}\left[\nabla\cdot(\tfrac{\sigma^2}{2}\vec{L}) + \frac{H\sigma}{4} i(U - U^\dagger)\right]}_{\text{viscous correction}} + \xi$$

And are coupled to the partially conserved currents, e.g.

$$\partial_\mu J^\mu_L = -i\frac{H}{8}\left(\Sigma - \Sigma^\dagger\right) \qquad \text{with} \qquad \vec{J}_L = \tfrac{\sigma^2}{4}\vec{L} + \underbrace{\lambda_0 \vec{\nabla}\mu_L}_{\text{diffusion}} + \underbrace{\xi}_{\text{noise}}$$

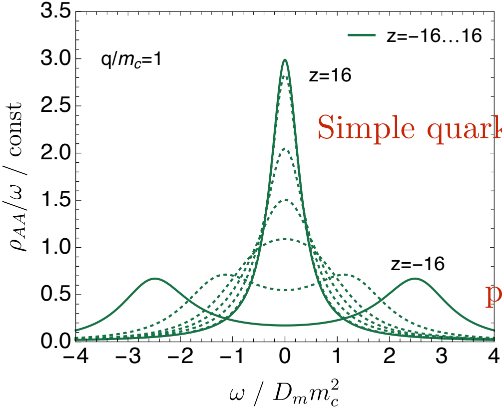
and the pion contribution to the current

$$\sigma^2 \vec{L} = \frac{i}{2}\underbrace{(\Sigma\vec{\nabla}\Sigma^\dagger - \vec{\nabla}\Sigma\Sigma^\dagger)}_{\text{pion current}}$$

- And a similar equation for σ, e.g. $\partial_t \sigma = D_m\left(\delta S/\delta\sigma\right) + \xi$.

Iso-axial denisty-density correlator and pions from stochastic EOM:

$$\frac{\rho_{AA}(\omega, \boldsymbol{q})}{\omega} = \frac{1}{T}\int_{-\infty}^{\infty} d^4x\, e^{-i\omega t + i\boldsymbol{q}\cdot\boldsymbol{x}} \left\langle \hat{J}_a^0(x)\hat{J}_a^0(0) \right\rangle$$

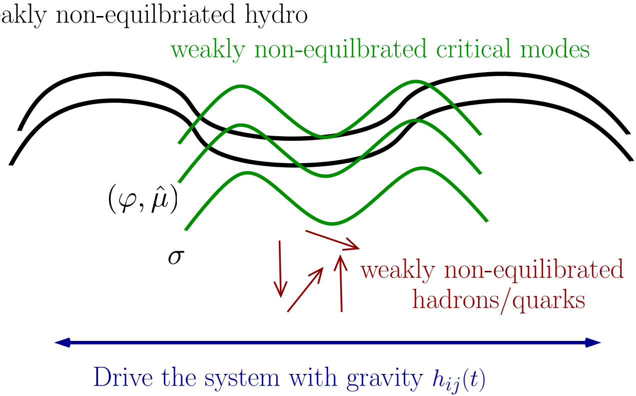


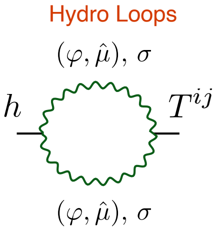
Simple quark diffusion for $T \gg T_c$

pions for $T \ll T_c$

Can see the transition from QGP to propagating pions from the EOM.

Hydro loops:

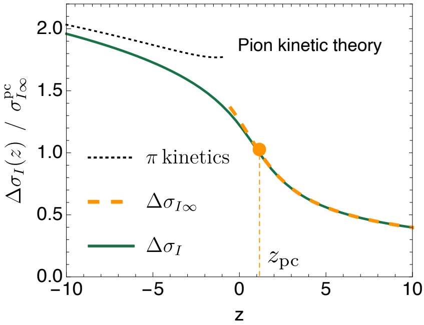




The retarded propagators are given by the linearized stochastic EOM.
Integrate out $(\varphi, \hat{\mu})$, σ to find its influence on hydro

The conductivity through T_c

$$\sigma = \sigma_{\rm reg} + \underbrace{\Delta\sigma_I}_{\text{crit. part}}$$



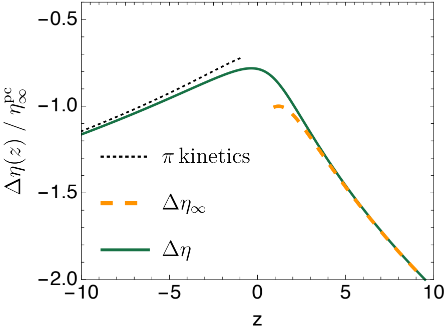
$$\sigma^{\rm pc}_{I\infty} \equiv \left.\frac{T}{16\pi D_m m}\right|_{z=z_{\rm pc}}$$

Estimate of the absolute magnitude for $\sigma^{\rm pc}_{I\infty}$ with $T_{\rm pc} \simeq 155\,{\rm MeV}$

$$\Delta D_I = \left(\frac{\Delta\sigma_I}{\chi}\right) = \frac{0.50}{2\pi T} \times \left(\frac{1.3}{m_{\rm pc}/T}\right)\left(\frac{0.4}{\chi_Q/T^2}\right)\left(\frac{3.0}{2\pi T D_m}\right)$$

The shear through T_c

$$\eta = \eta_{\rm reg} + \underbrace{\Delta\eta}_{\text{crit. part}}$$



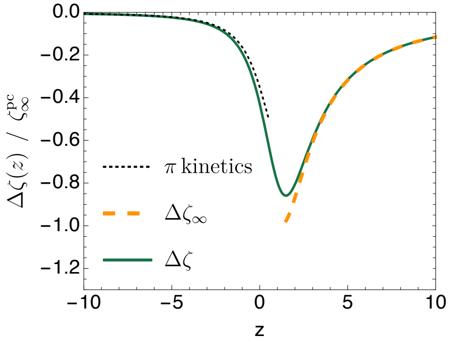
$$\eta_\infty^{\rm pc} = -\left.\frac{T m_\sigma}{8\pi D_m}\right|_{z=z_{\rm pc}}$$

Estimate of the absolute magnitude for $\eta_\infty^{\rm pc}$ with $T_{\rm pc} \simeq 155\,{\rm MeV}$

$$4\pi\eta/s = -0.30 \times \left(\frac{5.3}{s/T^3}\right)\left(\frac{m_\sigma/T}{1.6}\right)\left(\frac{3.0}{2\pi T D_m}\right)$$

The bulk through T_c

$$\zeta = \zeta_{\rm reg} + \underbrace{\Delta\zeta}_{\text{crit. part}}$$



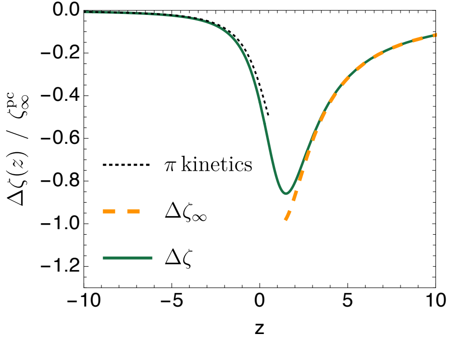
$$\zeta_\infty^{\rm pc} = -\frac{140}{32}\cdot\frac{Tm_\sigma}{\pi D_m}\left(\frac{c_s^2\tilde{m}_p^2}{m_p^2}\right)^2\Bigg|_{z=z_{\rm pc}}$$

Estimate of the absolute magnitude for $\zeta_\infty^{\rm pc}$ with $T_{\rm pc} \simeq 155\,{\rm MeV}$

$$\frac{4\pi\zeta}{s} = -3.5\times\left(\frac{c_s^2}{0.2}\right)^2\left(\frac{d\ln\langle\bar{q}q\rangle/d\ln T^2}{2.8}\right)^2\left(\frac{5.4}{s/T^3}\right)\left(\frac{m_\sigma/T}{1.6}\right)\left(\frac{3.0}{2\pi TD_m}\right)$$

The bulk through T_c

$$\zeta = \zeta_{\rm reg} + \underbrace{\Delta\zeta}_{\text{crit. part}}$$



$$\zeta_\infty^{\rm pc} = -\frac{140}{32}\cdot\frac{T m_\sigma}{\pi D_m}\left(\frac{c_s^2\tilde{m}_p^2}{m_p^2}\right)^2\Bigg|_{z=z_{\rm pc}}$$

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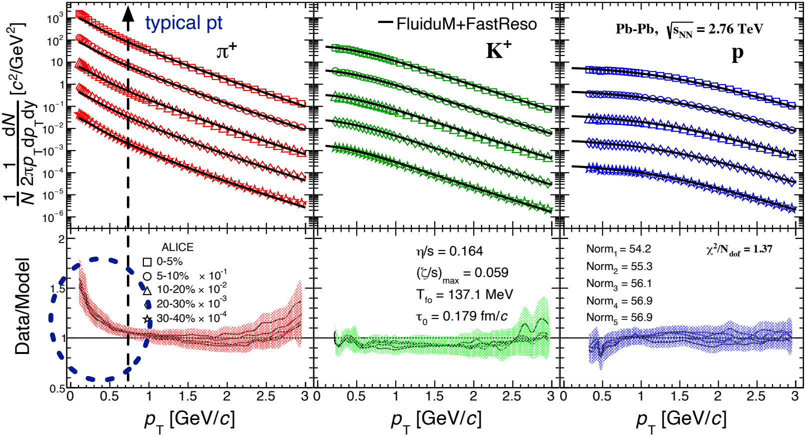
Evidence for the chiral crossover in the heavy ion data?

A recent ordinary hydro fit from Devetak et al 1909.10485



Because the pions are the Goldstones expect an enhancement at low p_T

$$n(\omega_q) = \frac{1}{e^{vq/T} - 1} \simeq \frac{T}{vq} \Rightarrow \infty,$$

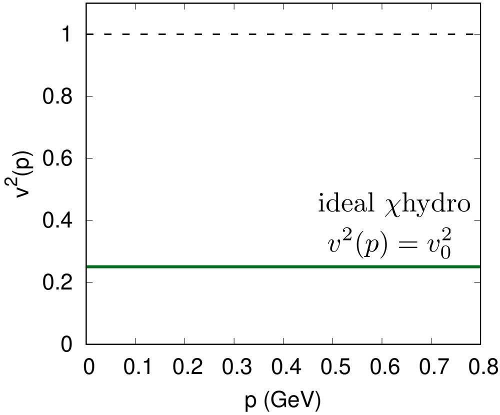
Since at T_c the velocity $\Rightarrow 0$!

Estimate of dispersion curve

The velocity $v_0 \equiv v(0)$ and pole mass $m_p^2 = v^2 m^2$ scale with $\langle \bar{q} q \rangle$

$$E_{\boldsymbol{p}}^2 = v^2(p) p^2 + m_p^2(p)$$

and are reduced for $T \simeq T_{\rm pc}$.



Cut off the χhydro for:

$$p \sim \Lambda = \pi T_{\rm pc}$$

Estimate of dispersion curve

The velocity $v_0 \equiv v(0)$ and pole mass $m_p^2 = v^2 m^2$ scale with $\langle \bar{q} q \rangle$

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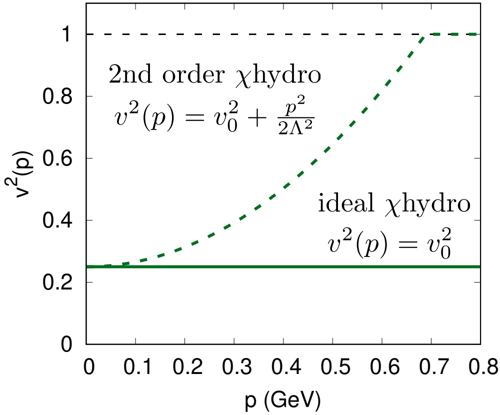

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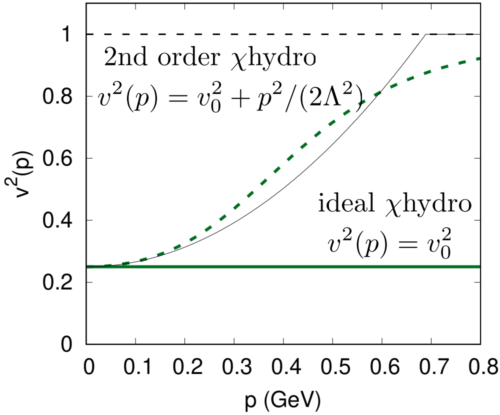


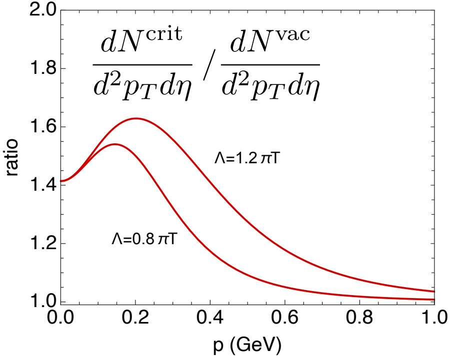
Cut off the χhydro for:

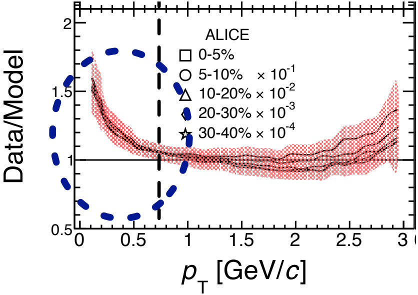
$$p \sim \Lambda = \pi T_{\rm pc}$$

Estimate of soft pion enhancement at $T_{\rm pc} \simeq 155\,{\rm MeV}$

With the modified dispersion curve $E_{\boldsymbol{p}}$ predict the yields

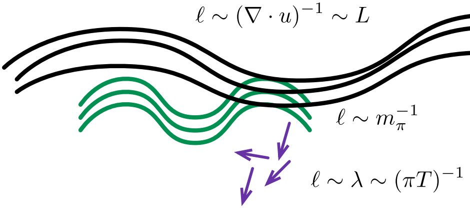
$$n(E_p) = \frac{1}{e^{E_p/T} - 1}$$





Encouraging, but this is just direct pions for a bath at rest.
Nevertheless, I think this is it!

Summary



1. Wrote down the appropriate $SU_L(2) \times SU_R(2)$ superfluid theory

   Included non-linear forms, viscous and mass corrections, noise etc.

2. Determined how the ordinary transport parameters depend on m_π

   Solved the driven kinetic equation, or integrating out the hydro loops

3. Determined a kinetic equation for soft pions from the hydro EOM

   This can be used for the real world! It is on the right track!

4. Developed the $O(4)$ scaling theory in hydro with $\Sigma = \sigma e^{i\varphi}$ field

Thank you and stay safe!