RHIC-BES online seminar

Theoretical overview on both chirality and vorticity

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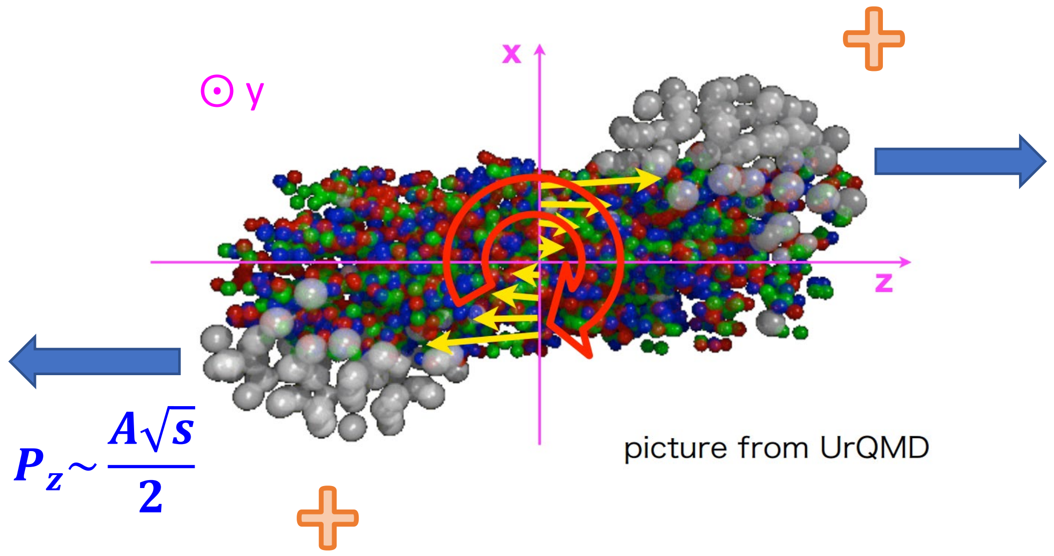
Outline

- Introduction: vorticity and magnetic field
- The isobar collisions for CME search
- Realistic evolution of magnetic field
- Deep-learning assisted CME search
- Global spin polarization of Λ and Ξ and Ω
- Puzzles in local spin polarization and spin alignment
- Summary

Introduction: B and ω in heavy-ion collisions

Vorticity and magnetic field in heavy-ion collisions



$$L_0 \sim \frac{Ab\sqrt{s}}{2} \sim 10^6 \hbar$$

Global angular momentum

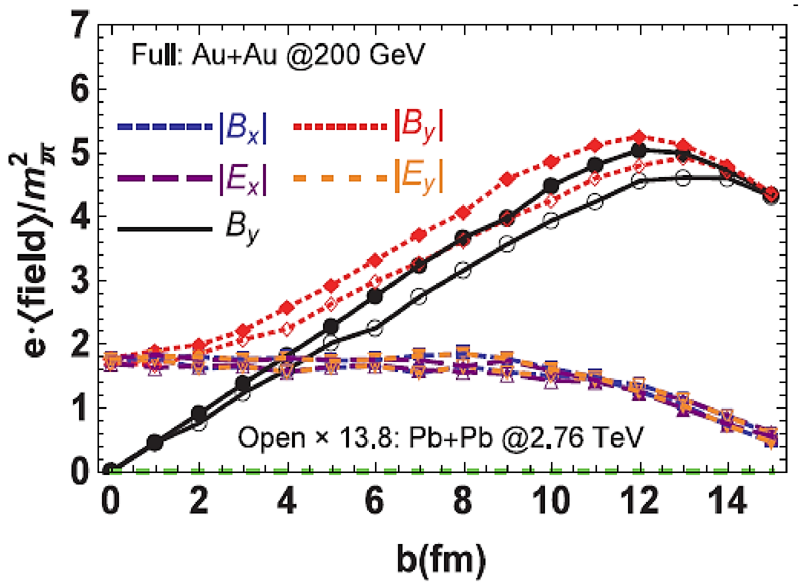
$$eB \sim \gamma \alpha_{\mathrm{EM}} \frac{Z}{b^2} \sim 10^{18}\ \mathrm{G}$$

Strong magnetic field

(RHIC Au+Au 200 GeV, b=10 fm)

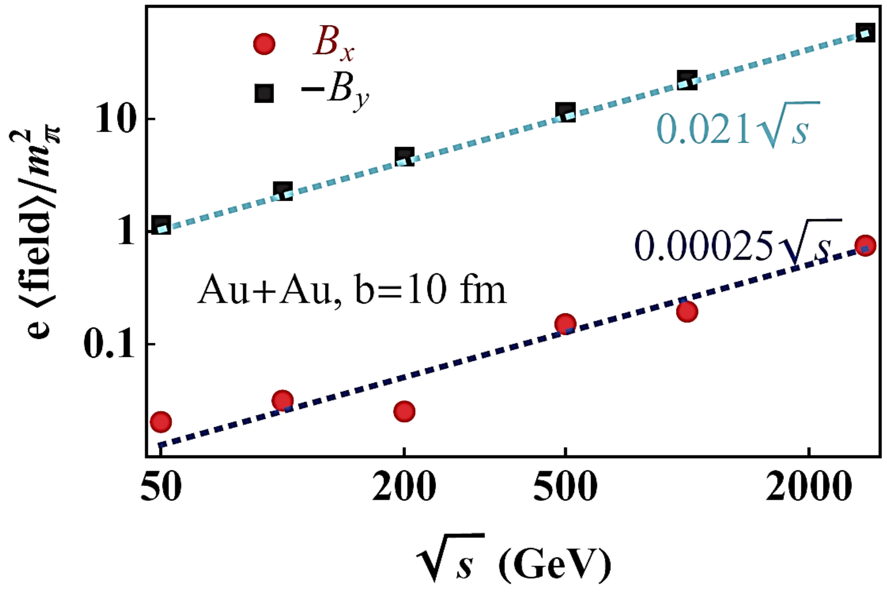
Initial magnetic field

Centrality dependence

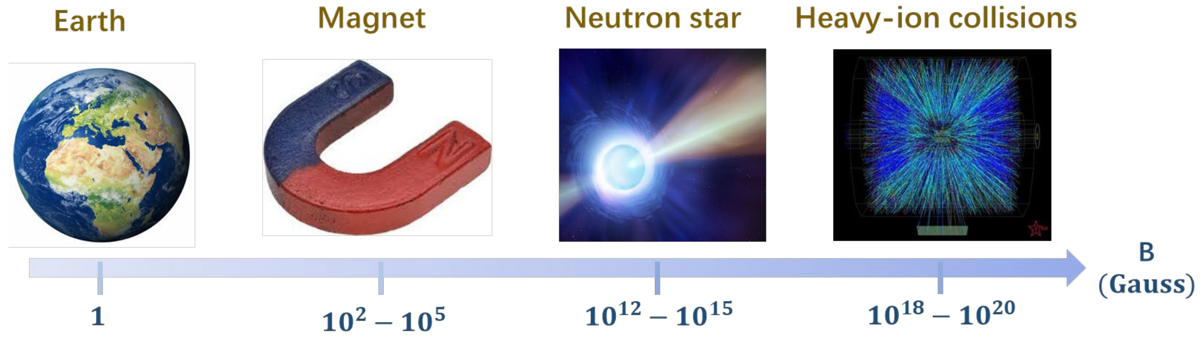


HIJING
(Deng-XGH PRC2012)

Energy dependence



- **Strongest magnetic field $\langle|B_y|\rangle \sim 10^{18-20} G$**

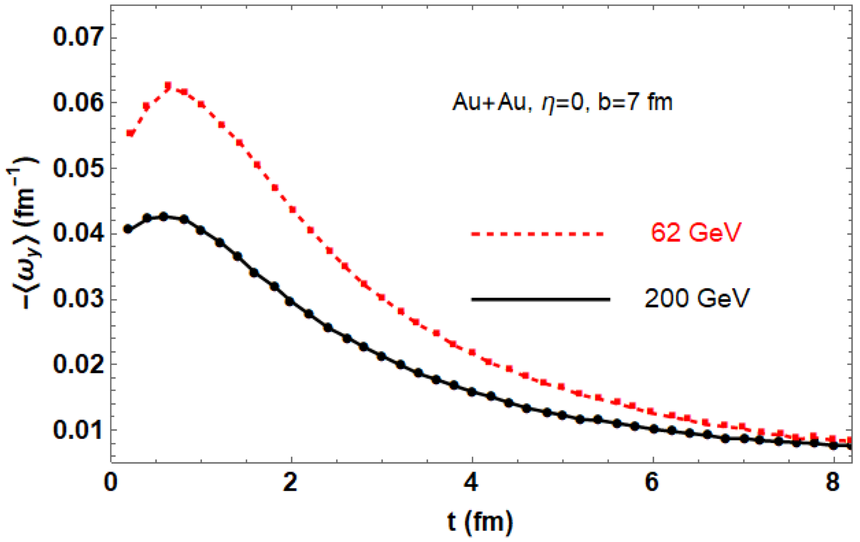


- **Unknown: realistic time evolution**

(Many similar calculations, e.g.: Skokov-Illarionov-Toneev 2009, Voronyuk etal 2011, Bzdak-Skokov 2011, Bloczynski etal 2012, Tuchin 2013, Feng etal 2013, Ma etal 2018, … …)

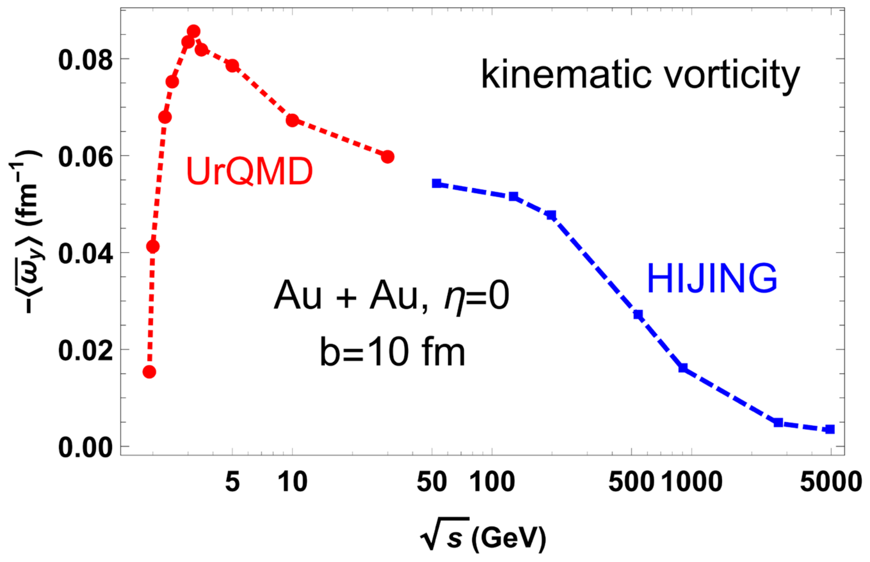
Vorticity by global angular momentum

Time dependence

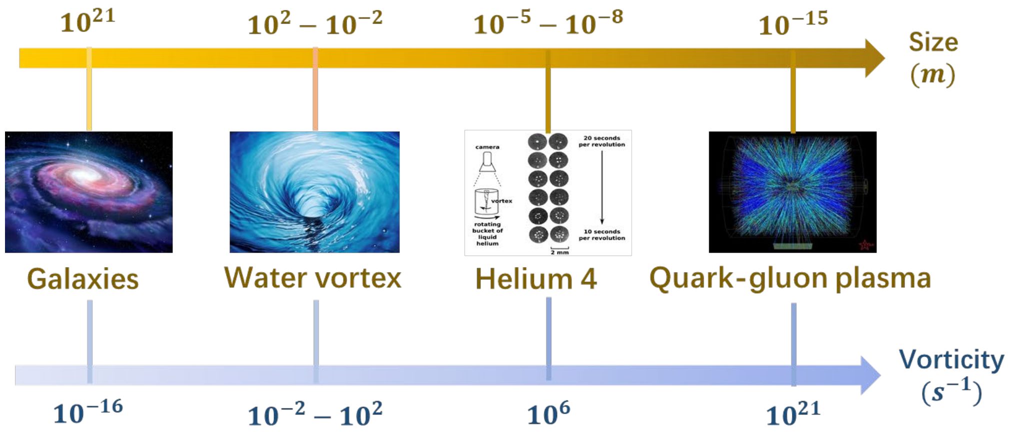


AMPT (Jiang-Lin-Liao PRC2016)

Energy dependence



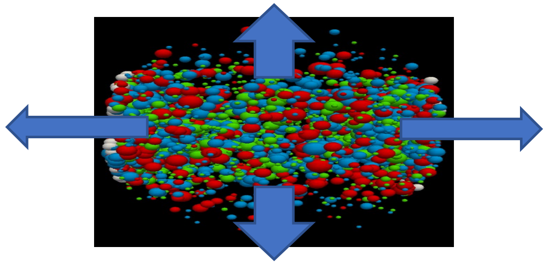
Deng-XGH-Ma-Zhang PRC2020 Deng-XGH PRC2016

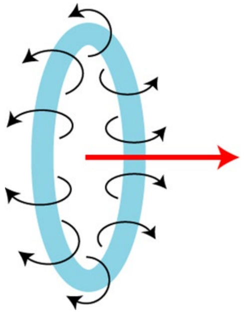


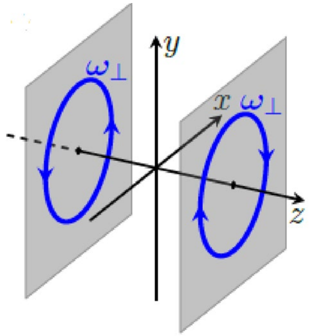
- **Most vortical fluid $\langle|\omega_y|\rangle \sim 10^{21} s^{-1}$**
- Relativistic suppression at high energy

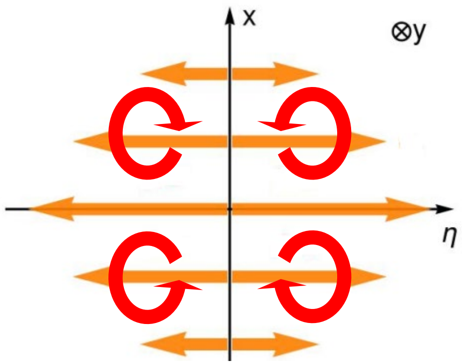
(See also: Becattini etal EPJC2015, Csernai etal PRC2013, PRC2014, Ivanov etal PRC2017, PRC2019, … …)

Vorticity by inhomogeneous expansion

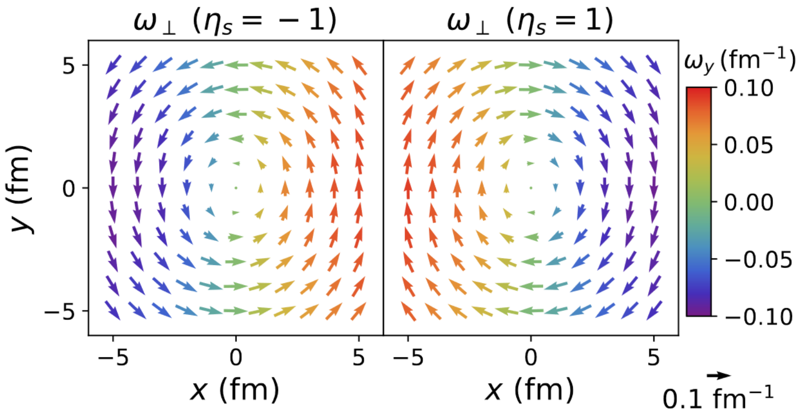




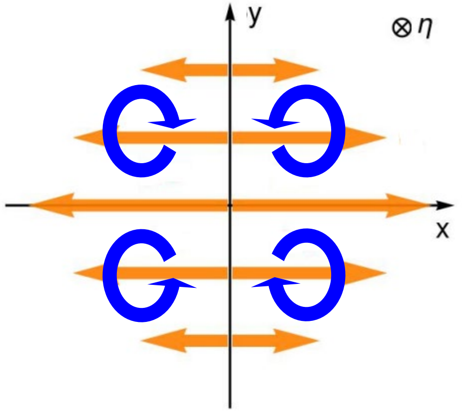




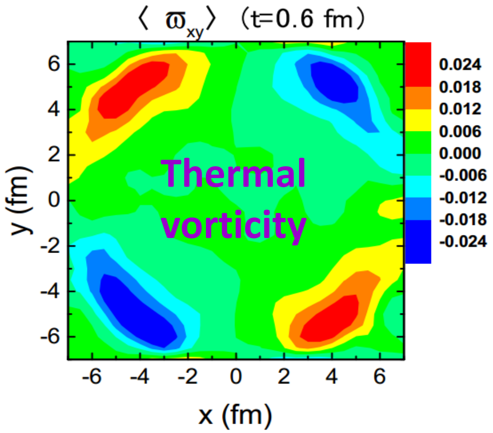
Transverse



Xia-Li-Wang PRC2017



Longitudinal

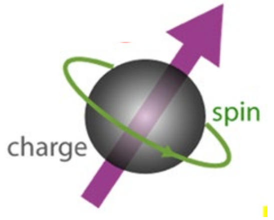


Wei-Deng-XGH PRC2019

(See also: Karpenko-Becattini EPJC2017, Csernai etal PRC2014, Teryaev-Usubov PRC2015, Ivanov-Soldatov PRC2018, … …)

Effect of B and ω: Spin polarization

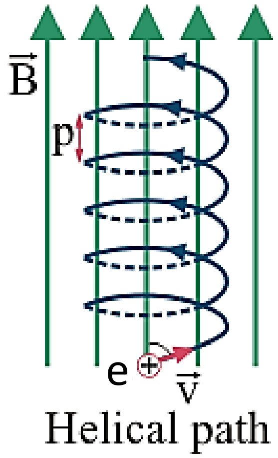
- A charged fermion in $\boldsymbol{B}$ and $\boldsymbol{\omega}$ fields: At rest



$$H = -\boldsymbol{\mu}_{\mathrm{B}} \cdot \boldsymbol{B} - \boldsymbol{S} \cdot \boldsymbol{\omega}$$

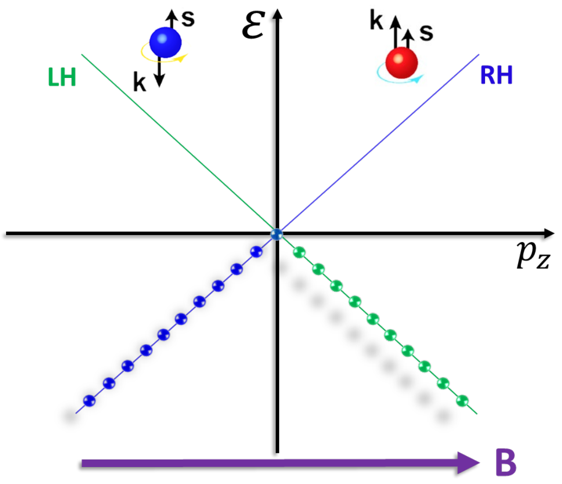
Spin polarization

- A charged fermion in $\boldsymbol{B}$ ~~and $\boldsymbol{\omega}$ fields~~: At motion



$$E_n^2 = p_z^2 + 2neB$$

$n = 0$



Chiral anomaly

Chiral magnetic effect

$$J_V = J_R + J_L = \frac{e\boldsymbol{B}}{2\pi^2}\mu_A$$

⋮

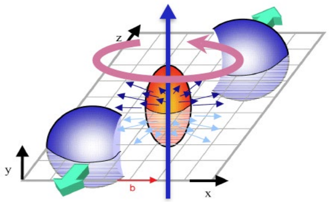
The CME and isobar

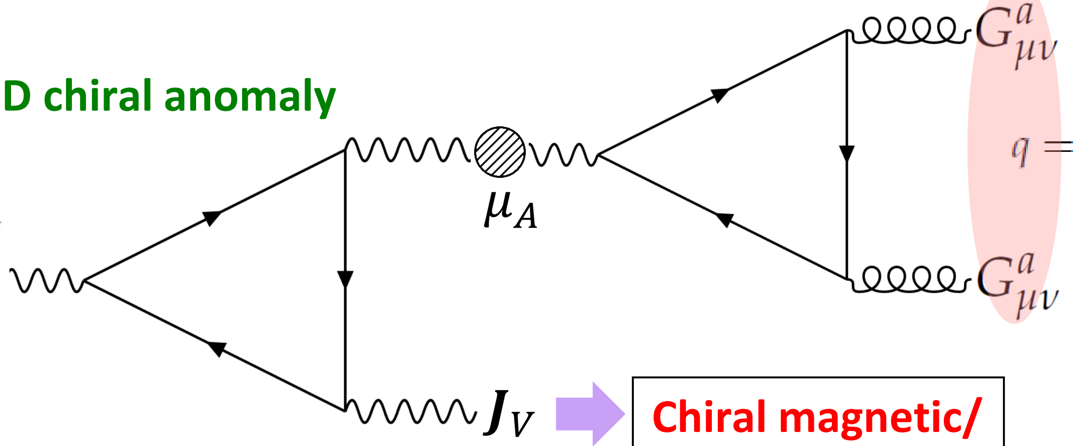
Probe QCD topological sector

QCD chiral anomaly

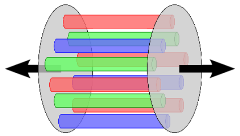
QED chiral anomaly

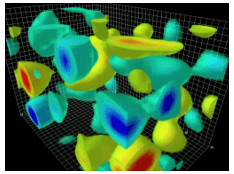
B,ω





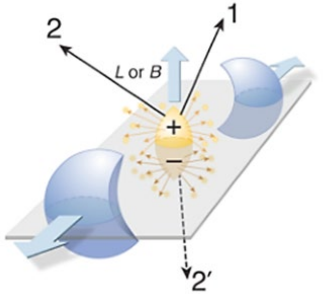
$$q = \frac{g^2}{32\pi^2}\int d^4x G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a$$





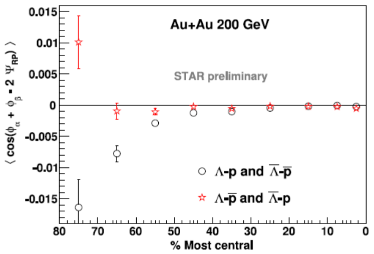
Chiral magnetic/ vortical effects

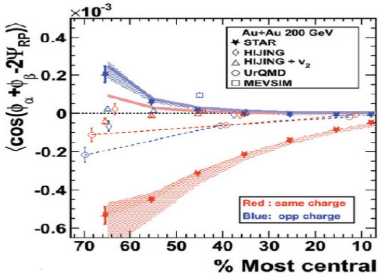
Initial state topological fluctuations



Observable: e.g. γ-correlator

(Voloshin 2004)

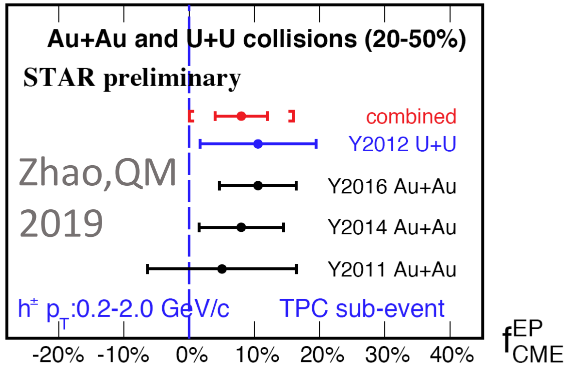




(STAR 2009, 2014)

Difficulties in observing CME

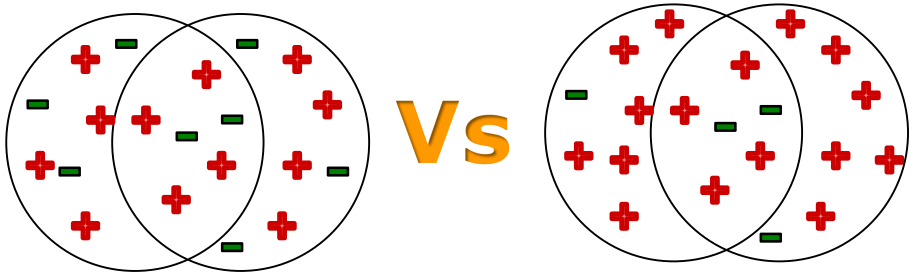
- Small signal versus big elliptic-flow related backgrounds



Averaged CME fraction = (8±4±8)%

One eccentric geometry gives two outcomes, B field and v_2. Difficult to disentangle them.

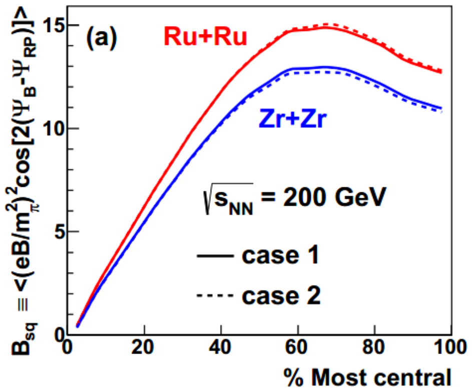
- Isobar collisions: fix v_2 but vary B field

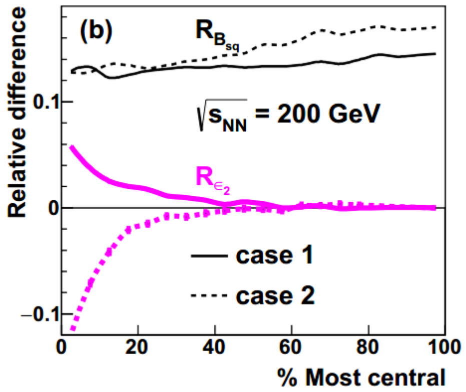


$_{40}Zr^{96} + _{40}Zr^{96}$ $_{44}Ru^{96} + _{44}Ru^{96}$

Difficulties in observing CME

- Isobar collisions: fix v_2 but vary B field

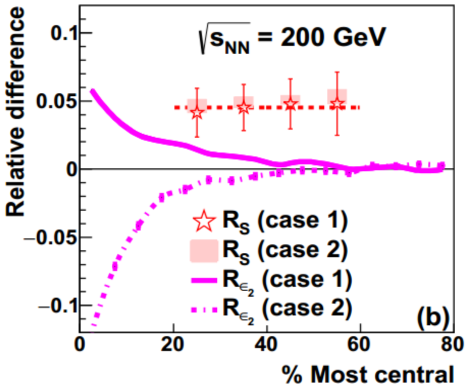


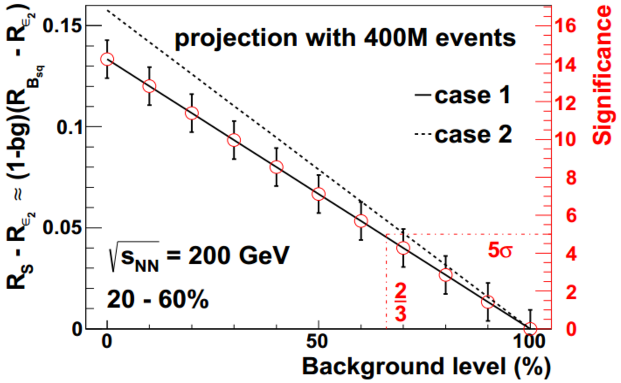


Relative difference R=2(Ru-Zr)/(Ru+Zr)

Centrality 20-60%:
sizable R for B: $\mathbf{R_{B_{sq}} \sim 10-20\%}$
small R for eccentricity: $\mathbf{R_{\varepsilon_2}} < \mathbf{2\%}$

- Signal versus background level





RHIC first run 2018 produces 3B events.
If bg level = 88%,
signal significance= 5σ
If bg level =93%,
signal significance= 3σ

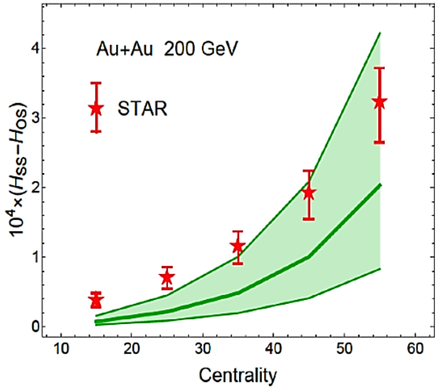
If background level = 67 %, 400M events give 5σ signal

(Deng-XGH-Ma-Wang 2016, 2018)

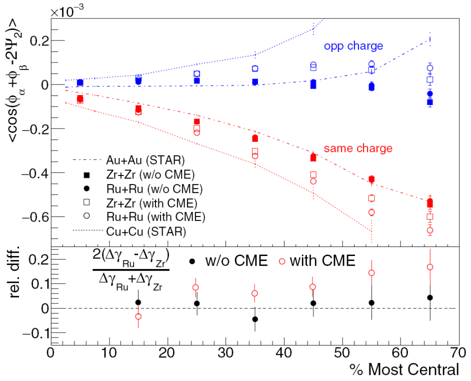
Evolution of B field

Difficulties in quantifying CME

- Quantifying CME in theory: hydrodynamic and transport models



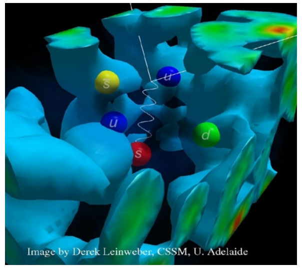
(AVFD: Liao etal 2018, 2019)

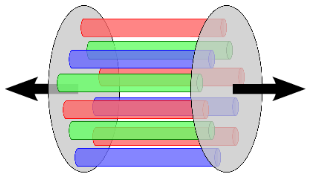


(AMPT: Ma-Zhang 2011; Deng-XGH-Ma-Wang 2018)

- Main theoretical uncertainties:

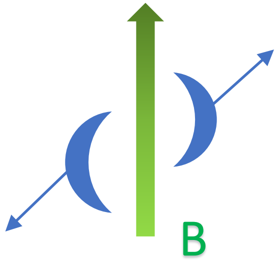
Initial axial charges



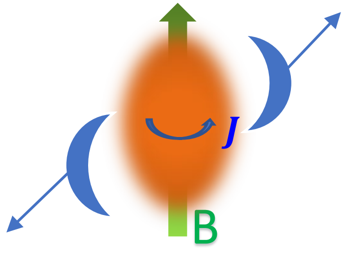


(Early attempts: Muller- Schlichting-Sharma 2016, Ruggieri etal 2019)

Realistic evolution of B field



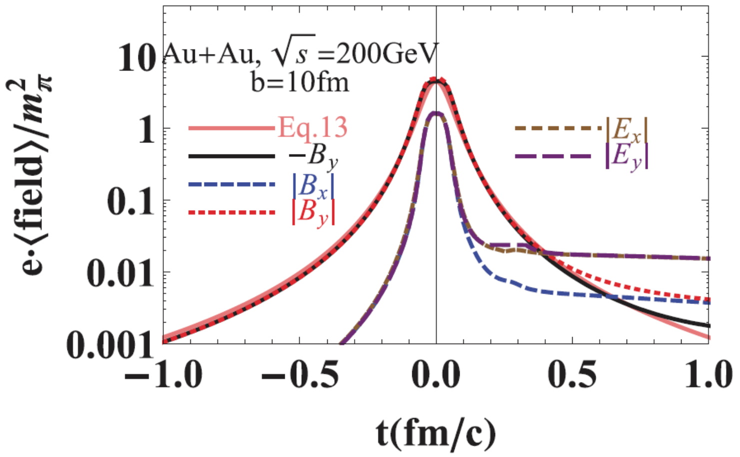
In vacuum:
moving charges



In conductor:
Faraday effect

Realistic evolution of B field

- If quark-gluon matter is insulating (Deng-XGH 2012; Hattori-XGH 2016; and many others)



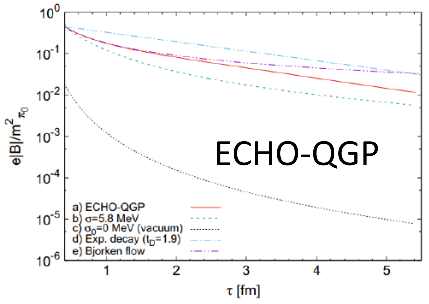
Well fitted by

$$\langle eB_y(t)\rangle \approx \frac{\langle eB_y(0)\rangle}{(1+t^2/t_B^2)^{3/2}}$$

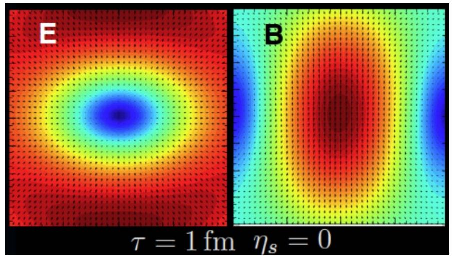
Life time of B field

$$t_B \approx R_A/(\gamma v_z) \approx \frac{2m_N}{\sqrt{s}} R_A$$

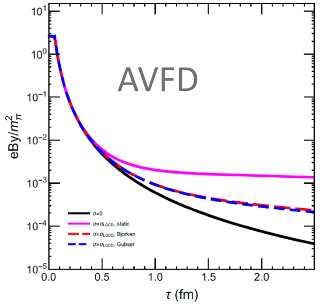
- In hydro stage: couple Maxwell with hydro equations



(Inghirami etal 2016)



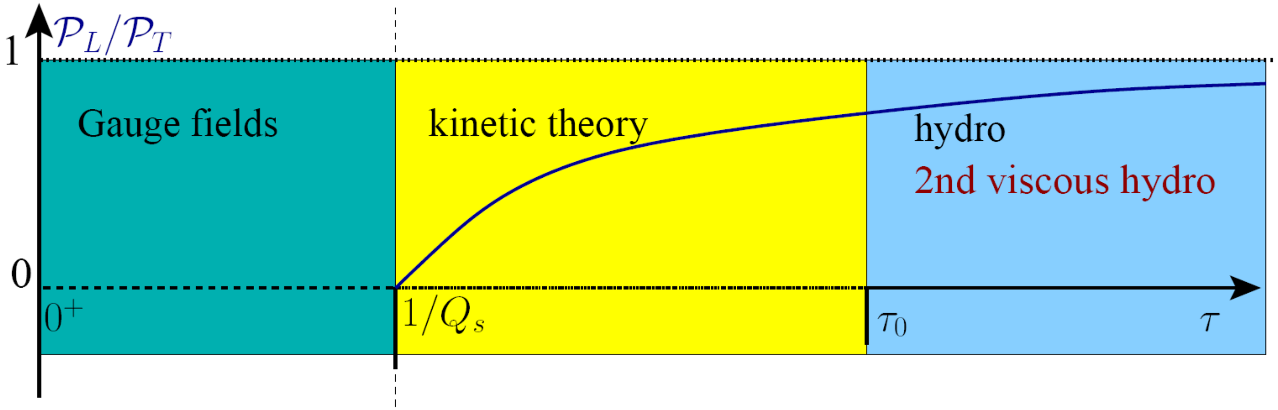
(Gursoy-Kharzeev-Rajagopal-Shen 2018)



(Huang-Kharzeev-Liao-Shi-She 2020)

Realistic evolution of B field

- But what is the pre-hydro evolution and the IC for hydro?



- We study the pre-hydro evolution for $t \sim Q_s^{-1} - \tau_0$
  by solving coupled Maxwell and Boltzmann equations

$$\begin{cases} [p^\mu \partial_\mu + eQ_a p_\mu F^{\mu\nu}\partial_{p^\nu}] f_a(t, \mathbf{x}, \mathbf{p}) = \mathcal{C}[f_a] \qquad a = q, \bar{q}, g \\ \partial_\mu F^{\mu\nu} = j^\nu \\ j^\mu = e \sum_F Q_F s_F \int \frac{d^3\mathbf{p}}{(2\pi)^3 E_p} p^\mu \left(f_q^F - f_{\bar{q}}^F\right) \end{cases}$$

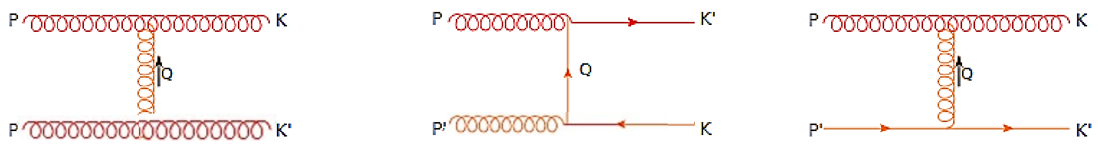
Initial condition for EM field: moving colliding nuclei in vacuum
Initial condition for q and g: CGC inspired distribution (Blaizot-Wu-Yan 2014)

Realistic evolution of B field

- For the collision kernel: 2-2 processes

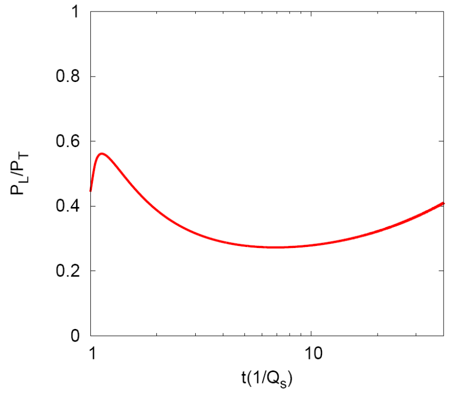
$$\mathcal{C}[f^a_{\mathbf{p}}] = \frac{1}{2E_p\nu_a}\sum_{b,c,d}\frac{1}{s_{cd}}\int\frac{d^3\mathbf{p}'}{(2\pi)^3 2E_{\mathbf{p}'}}\frac{d^3\mathbf{k}}{(2\pi)^3 2E_{\mathbf{k}}}\frac{d^3\mathbf{k}'}{(2\pi)^3 2E_{\mathbf{k}'}}$$
$$\times (2\pi)^4\delta^{(4)}(P+P'-K-K')|\mathcal{M}^{ab}_{cd}|^2$$
$$\times \left[f^c_{\mathbf{k}}f^d_{\mathbf{k}'}(1+\epsilon_a f^a_{\mathbf{p}})(1+\epsilon_b f^b_{\mathbf{p}'}) - f^a_{\mathbf{p}}f^b_{\mathbf{p}'}(1+\epsilon_c f^c_{\mathbf{k}})(1+\epsilon_d f^d_{\mathbf{k}'})\right]$$

$$|\mathcal{M}|^2 \ni gg \leftrightarrow q\bar{q},\ gq \leftrightarrow gq,\ g\bar{q} \leftrightarrow g\bar{q},\ gg \leftrightarrow gg$$



Plus s, u channels

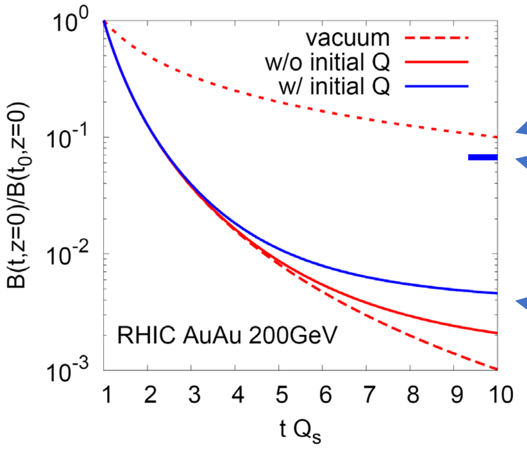
Under 2-2 scattering, the system evolves towards hydrodynamization



(Yan-XGH to appear)

Realistic evolution of B field

- The B field (In case of Bjorken longitudinal expansion)

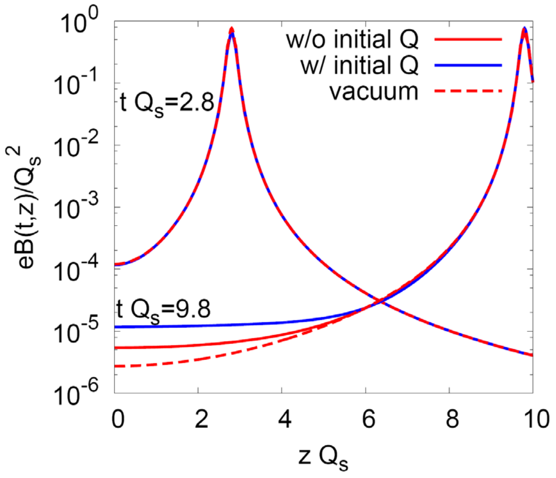


Ideal Bjorken MHD (Roy-Pu-Rezzola-Rischke 2015)

ECHO-QGP (Inghirami etal 2016)

Coupled Maxwell-Boltzmann equation (Yan-XGH to appear)

- Longitudinal distribution of B field

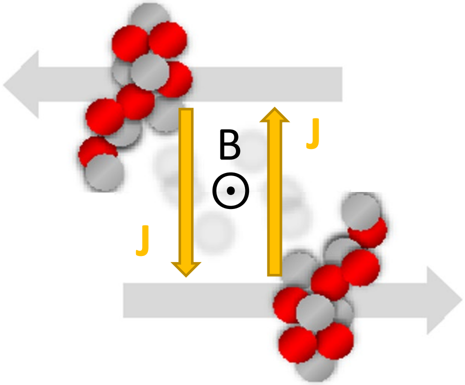


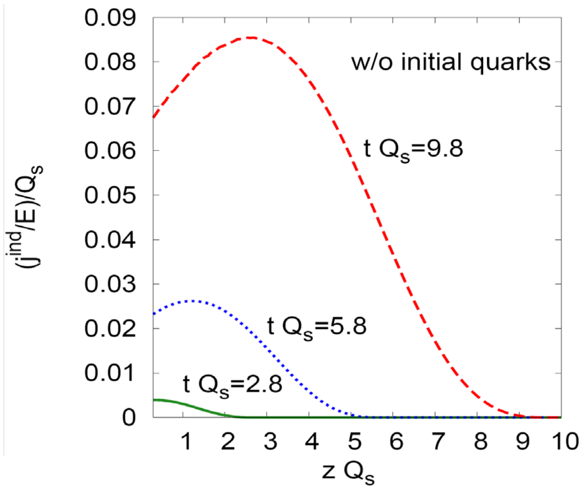
Background = B field by moving nucleus

Yan-XGH to appear

Realistic evolution of B field

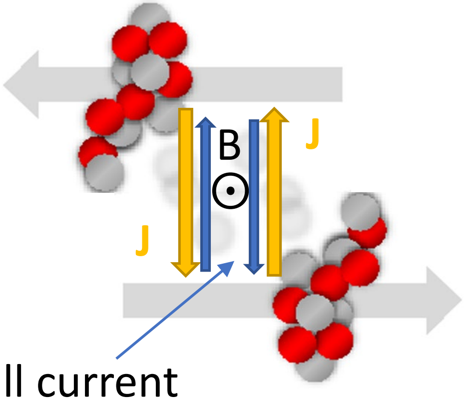
- The induced Faraday current

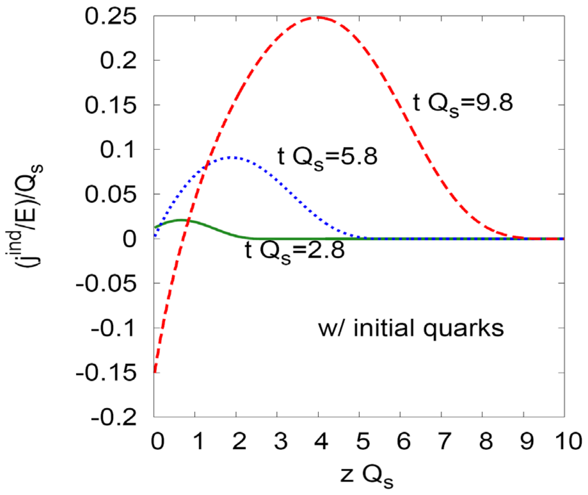




Large effective conductivity (comparable to LQCD at tQs=9.8)

- If put more charge carriers



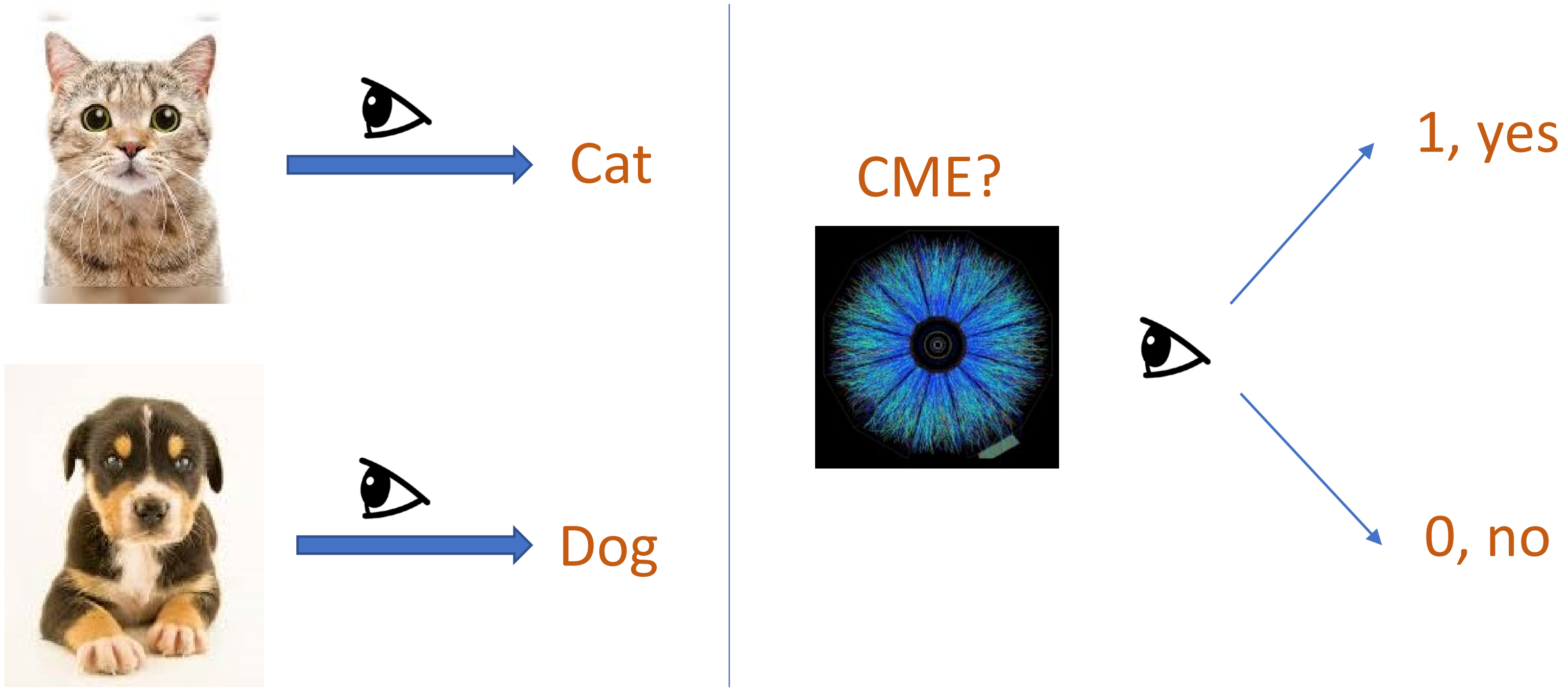


Direction of current near mid-rapidity depends on quark-production rate thus may depends on energy

Deep-learning and CME search

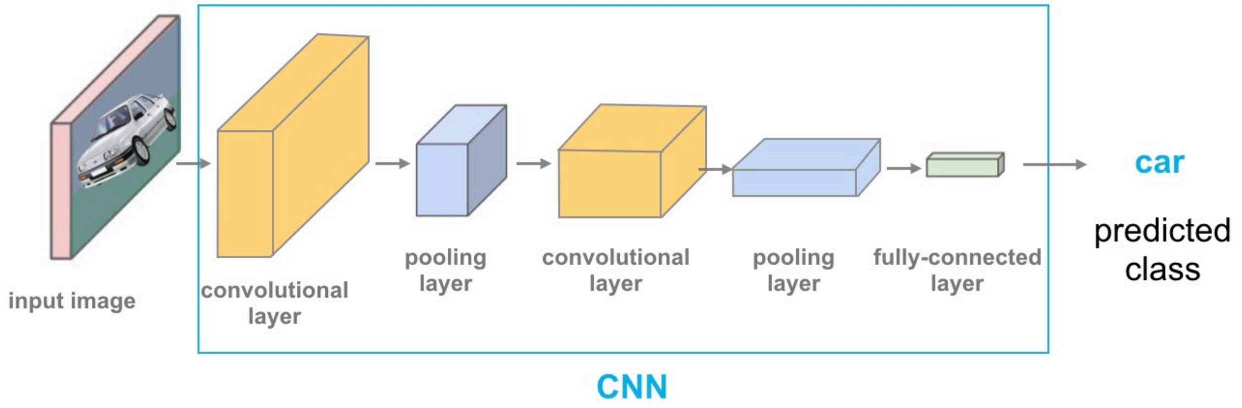
Deep-learning assisted CME search

- Recall the main challenge of CME search:
  Find a way to disentangle signal and elliptic-flow backgrounds
- Any designed observable is based on hadron distribution in momentum space.
  Why don't we just look at the distribution itself?

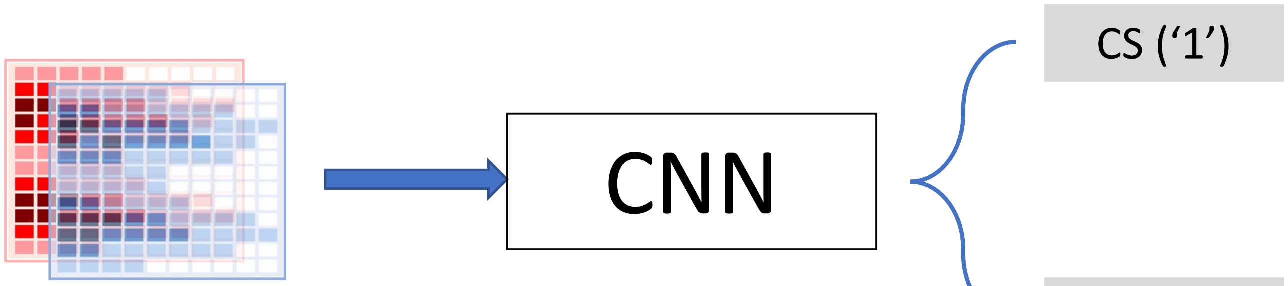


Deep-learning assisted CME search

- We train a machine to recognize initial charge separation (mimicking CME): Supervised learning
- We use Convolutional Neural Network (CNN) : good at pattern recognition of figures.



In our case: input = $\pi^{\pm}$ with $|Y| < 1$ projected on (p_x, p_y)-plane generated by AMPT



AMPT simulation (with or without initial charge separation)

Deep-learning assisted CME search

- Training set: 50000 events for each centrality and energy in blue

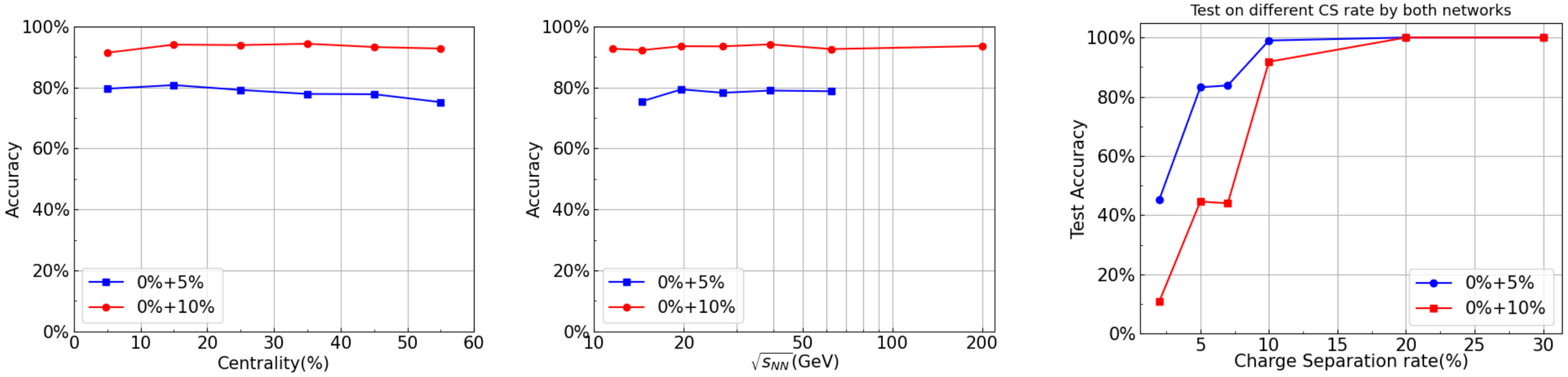
f		$\sqrt{s_{NN}}$ (GeV)						
Centrality		11.5	14.5	19.6	27	39	62.4	200
	0-10							
	10-20							
	20-30							
	30-40							
	40-50							
	50-60							

f = initial charge separation (CS) fraction

$f = 0$: No CME, Label '0'

$f = 5\%$ and 10%: With CME, Label '1'

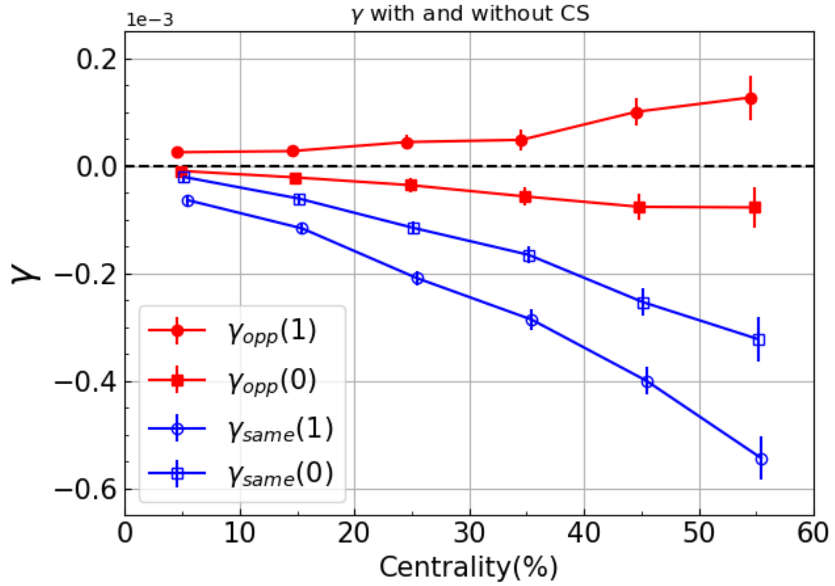
- Test set: All centrality and energy region in the above
- Robust, insensitive to centrality and energy. The machine learns key feature of charge separation.

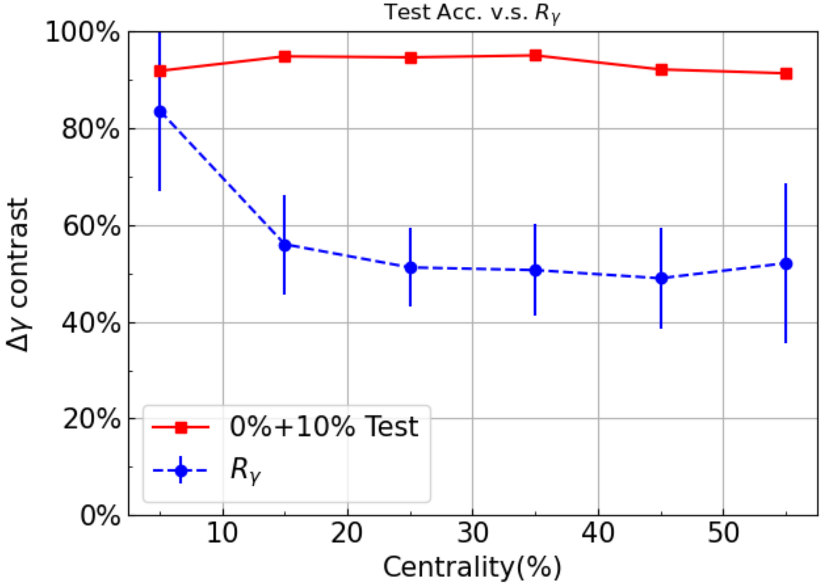


(Zhao-Zhou-XGH to appear)

Deep-learning assisted CME search

- Test: Comparing to γ-correlator with 10% charge separation (CS)





$$\gamma_{same} = \left\langle \cos\left(\phi_\alpha^{(\pm)} + \phi_\beta^{(\pm)} - 2\Phi_R\right)\right\rangle$$
$$\gamma_{opp} = \left\langle \cos\left(\phi_\alpha^{(\pm)} + \phi_\beta^{(\mp)} - 2\Phi_R\right)\right\rangle$$
$$\Delta\gamma = \gamma_{opp} - \gamma_{same}$$

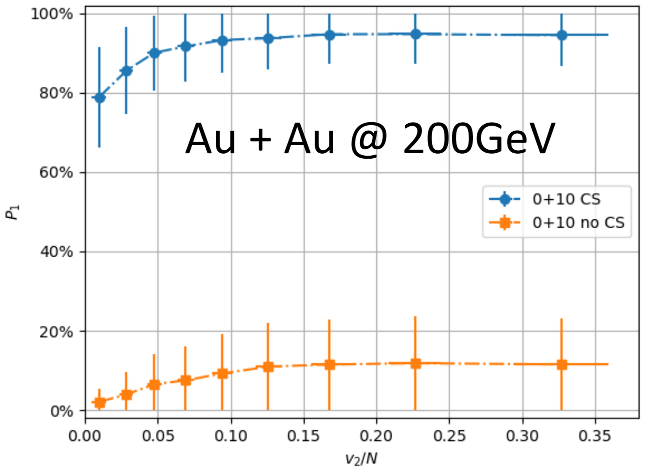
$$R_\gamma = \left| \frac{\langle\Delta\gamma(1)\rangle - \langle\Delta\gamma(0)\rangle}{\langle\Delta\gamma(1)\rangle + \langle\Delta\gamma(0)\rangle} \right|$$

(Zhao-Zhou-XGH to appear)

Deep-learning assisted CME search

- Test: dependence of elliptic flow (Not sensitive to elliptic-flow background)

  - P_1 = accuracy



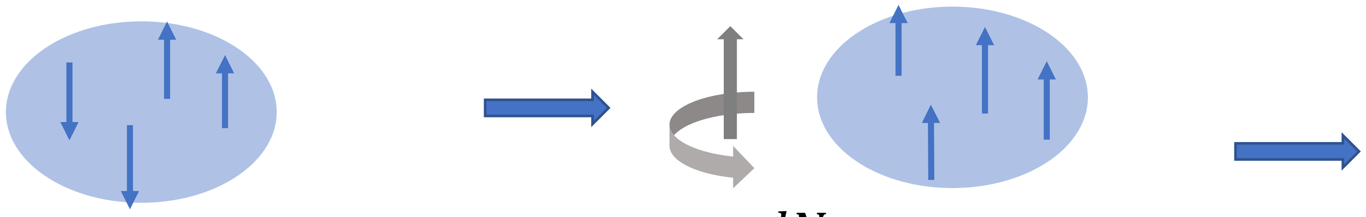
(Zhao-Zhou-XGH to appear)

- Future: Further optimize the machine.
  Understand what the feature the machine learns.
  Can be applied to real data?
  Isobar results?
  Train a machine for chiral magnetic wave search.

Spin polarization of hyperons

Spin polarization and thermal vorticity

- Early idea: Liang-Wang PRL2005; Voloshin 2004
- Vorticity interpretation (at thermal equilibrium)



$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} \sim \frac{\omega}{2T}$$

$$H = H_0 - \boldsymbol{\omega} \cdot \boldsymbol{S} \quad \longrightarrow \quad \frac{dN}{d\boldsymbol{p}} \sim e^{-(H_0 - \boldsymbol{\omega}\cdot\boldsymbol{S})/T}$$

- More rigorous derivation (Becattini etal 2013; Fang etal 2016; Liu etal 2020)

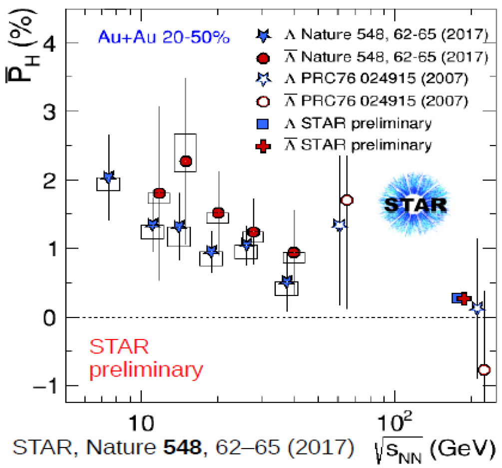
$$P^\mu(p) = \frac{1}{4E_p}\epsilon^{\mu\nu\rho\sigma}p_\nu \frac{\int d\Sigma_\lambda p^\lambda f'(x,p)\varpi_{\rho\sigma}(x)}{\int d\Sigma_\lambda p^\lambda f(x,p)} + O(\varpi^2)$$

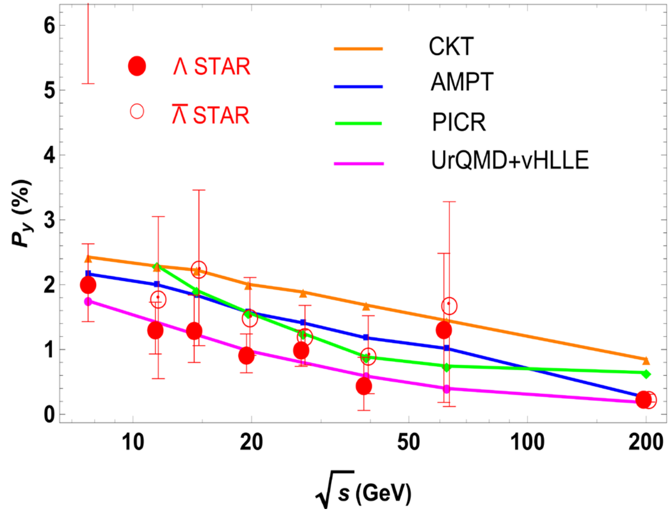
- Valid at global equilibrium. $f(x,p)$ is the distribution function (Fermi-Dirac)
- Thermal vorticity $\varpi_{\rho\sigma} = \left(\partial_\sigma \beta_\rho - \partial_\rho \beta_\sigma\right)/2$
- Spin polarization is enslaved to thermal vorticity, not dynamical
- Friendly for numerical simulation (a spin Cooper-Frye type formula)

Global Λ spin polarization

- The global polarization (i.e., integrated polarization over kinematics):

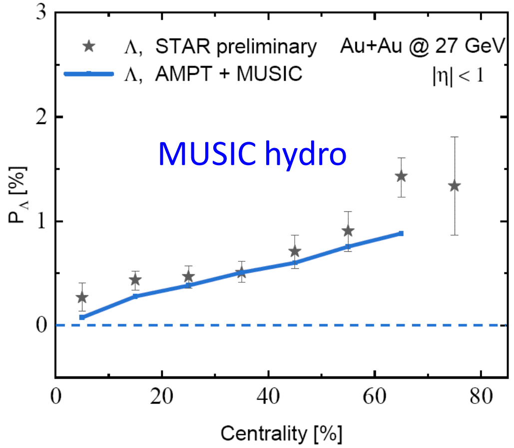
Experiment = Theory





Sun-Ko PRC2017; Wei-Deng-XGH PRC2019; Xie-Wang-Csernai PRC2017; Karpenko-Becattini EPJC2016

(Many similar results in literature)

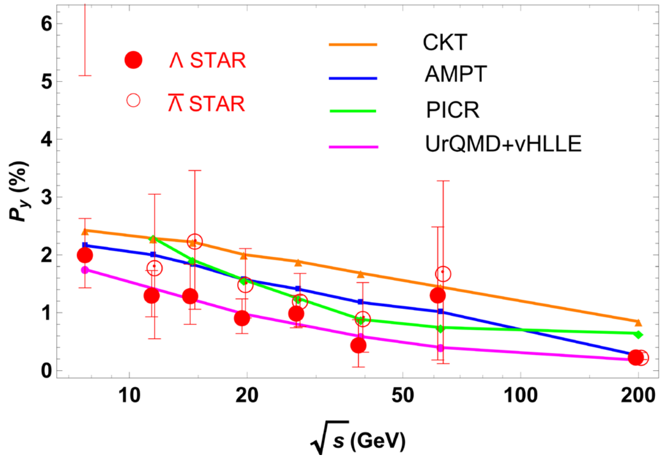


Fu-Xu-XGH-Song 2020

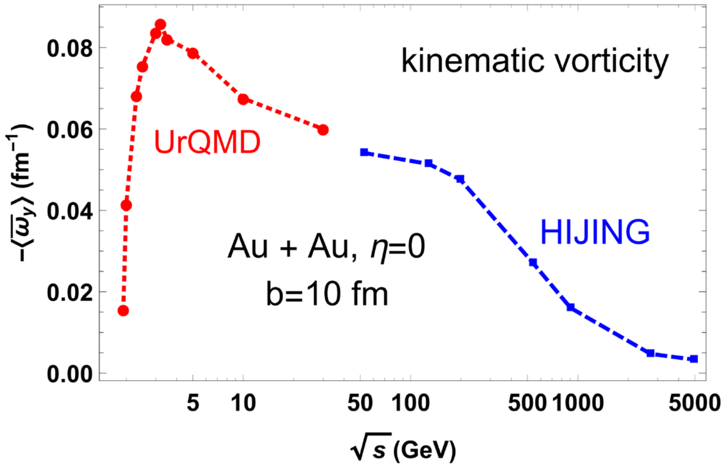
Vorticity interpretation of global Λ polarization works well!

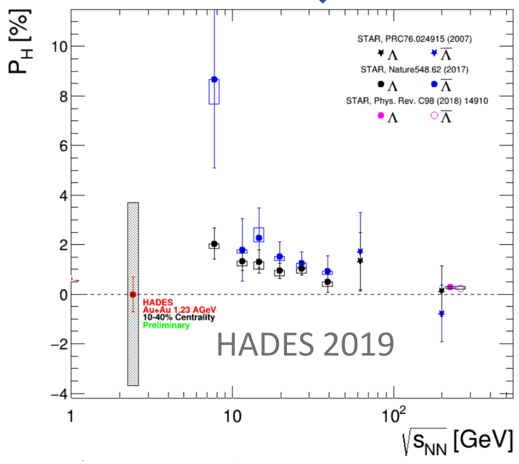
Global Λ spin polarization

- The global polarization: **Experiment = Theory**



VS





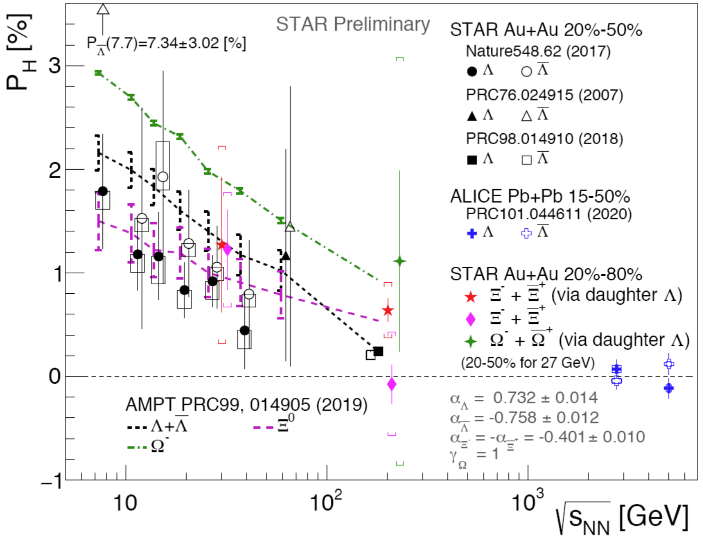
Need to study polarization at very low $\sqrt{s}$: NICA, FAIR, HIAF, BES II@RHIC?
Need an out-of-equilibrium theory to calculate.

Global Ξ and Ω spin polarization

- The global Ξ and Ω polarization are measured through their Λ decay

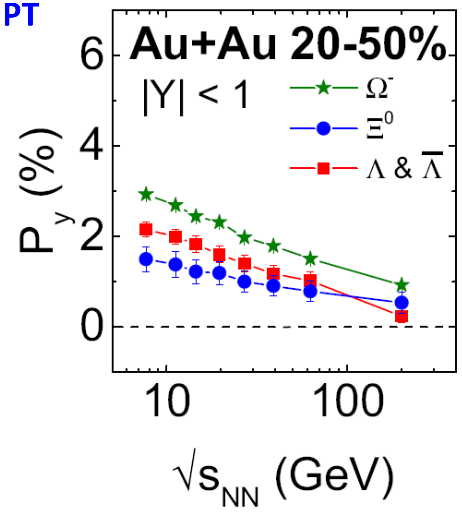
Experiment ≈ **Theory**

(Niida, talk at RHIC-AGS meeting 2020)



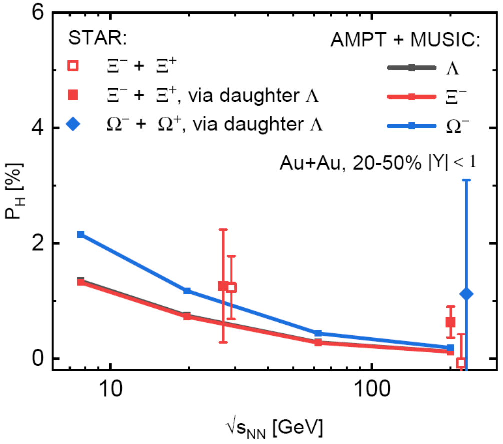
(Wei-Deng-XGH PRC99,014905(2019))

AMPT



(Fu-Xu-XGH-Song 2011.03740)

AMPT + Hydro



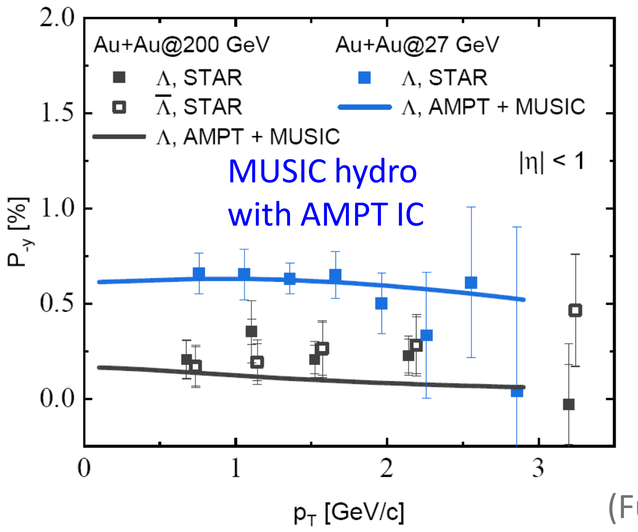
- Feed-down contribution to Ξ and Ω polarization
  - For Ξ, main decay channel $\Xi(1530) \to \Xi + \pi$ contributes about 40% of Ξ yield and 30% of Ξ polarization
  - For Ω, very small feed-down contribution

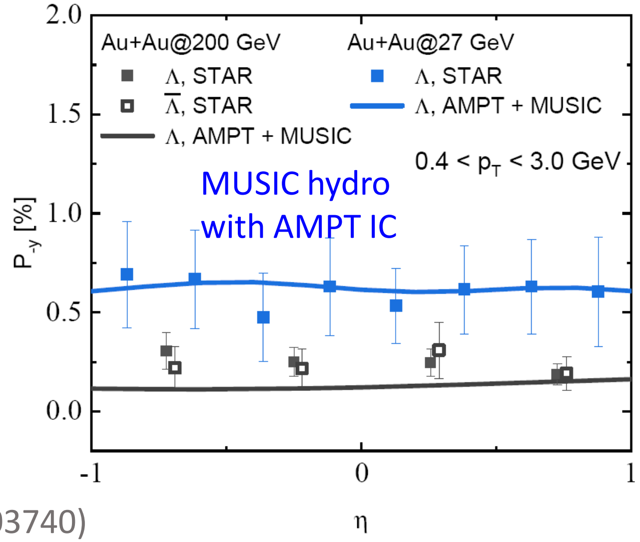
(Xia-Li-XGH-Huang PRC2019)

Vorticity interpretation of global to Ξ and Ω polarization works.

Differential Λ spin polarization

- The global Λ polarization reflects the total amount of angular momentum retained in the (-1,1) rapidity region. How is it distributed in e.g. p_T, η, and ϕ?

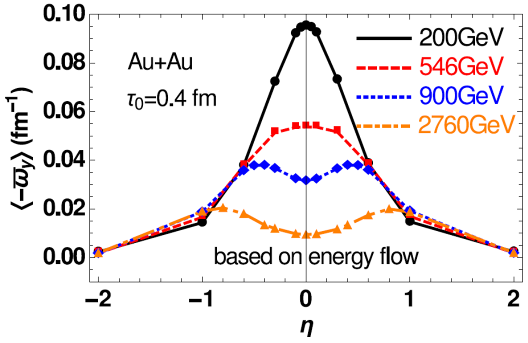




(Fu-Xu-XGH-Song 2011.03740)

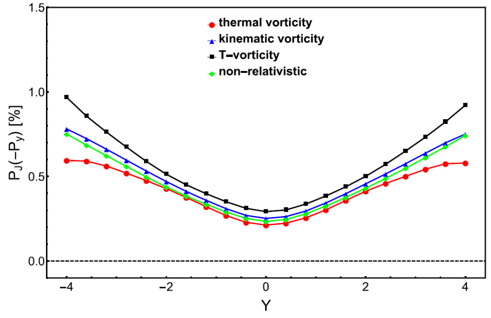
Would be interesting to look at very large rapidity?

Initial vorticity by HIJING



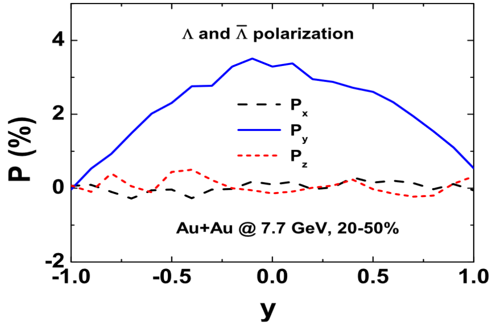
Deng-XGH PRC2016

Final polarization by hydro



Wu etal PRR2019

Final polarization by chiral kinetic theory



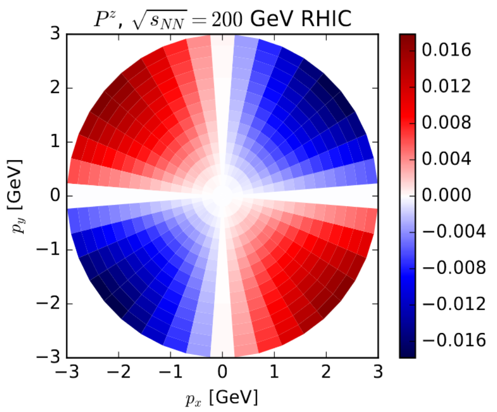
Sun-Ko PRC2017

Differential Λ spin polarization

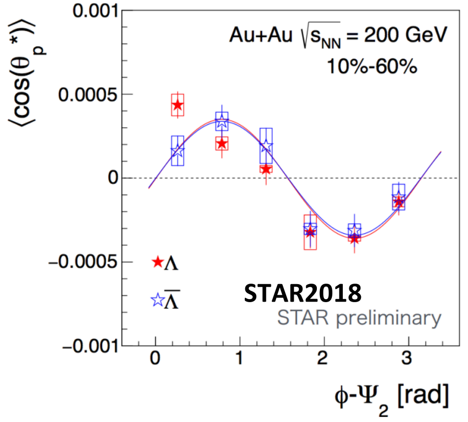
- The global Λ polarization reflects the total amount of angular momentum retained in the (-1,1) rapidity region. How is it distributed in e.g. p_T, η, and ϕ?
  - Spin harmonic flow: $$\frac{dP_{y,z}}{d\phi} \propto P_{y,z} + 2f_{2y,z}\sin(2\phi) + 2g_{2y,z}\cos(2\phi) + \cdots$$

1) longitudinal polarization vs ϕ

(Becattini-Karpenko PRL2018)

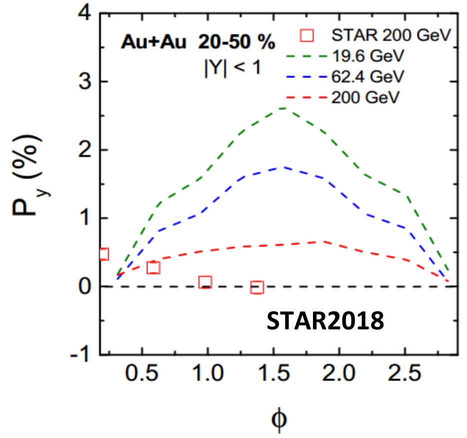


Vs



2) Transverse polarization vs ϕ

(Wei-Deng-XGH PRC(2019))



$f_{2z}^{\text{ther}} < 0$ $\quad$ $f_{2z}^{\text{exp}} > 0$ $\quad$ $g_{2y}^{\text{ther}} < 0,\ g_{2y}^{\text{exp}} > 0$

We have a spin “sign problem”!

Differential Λ spin polarization

Efforts to resolve the puzzles from theory side:

- Understand the vorticity (☺)
- Effect of feed-down decays (☺) (Xia-Li-XGH-Huang PRC2019, Becattini-Cao-Speranza EPJC2019)
  (Measured Λ may from decays of heavier particles)
- Go beyond equilibrium treatment (spin as a dynamic d.o.f)
  spin hydrodynamics (Florkowski-Friman-Jaiswal-Speranza PRC2017, Hattori etal PLB2019, …)
  spin kinetic theory (Gao-Liang 2019, Weickgenannt etal PRD2019, Hattori etal PRD2019, Wang etal PRD2019, Liu etal CPC2020, …)
- Initial condition
  (Initial polarization, initial flow, … …)
- Other possibilities
  (chiral vortical effect (Liu-Sun-Ko 2019), mesonic mean-field(Csernai-Kapusta-Welle PRC2019), other spin chemical potential (Wu etal PRR2019, Florkowski etal2019), contribution from gluons, … …)
- Other observables for vorticity and spin polarization
  Vector meson spin alignment (Liang-Wang 2005, STAR and ALICE 2019)
  Vorticity dependent hadron yield (ExHIC-P Collaboration PRC2020)

Spin "sign problem", though unsolved, inspires many theoretical developments about spin dynamics in and out of equilibrium!

Spin alignment of vector mesons

Global ϕ-spin alignment

- Vorticity can also polarize spin of vector mesons, e.g. ϕ
- Consider recombination $q + \bar{q} \to \phi$, the density matrix of q:

$$\rho^q = \frac{1}{2}\begin{pmatrix} 1+P_q & 0 \\ 0 & 1-P_q \end{pmatrix}$$

- The density matrix of ϕ is obtained from $\rho^q \otimes \rho^{\bar{q}}$ in basis of |↑↑), |↑↓)- |↓↑), and |↓↓)

$$\rho^V = \begin{pmatrix} \frac{(1+P_q)(1+P_{\bar{q}})}{3+P_q P_{\bar{q}}} & 0 & 0 \\ 0 & \frac{1-P_q P_{\bar{q}}}{3+P_q P_{\bar{q}}} & 0 \\ 0 & 0 & \frac{(1-P_q)(1-P_{\bar{q}})}{3+P_q P_{\bar{q}}} \end{pmatrix}$$

- Suppose $P_q = P_{\bar{q}}$,

$$\rho_{00}^{\rho(\mathrm{rec})} = \frac{1-P_q^2}{3+P_q^2}$$

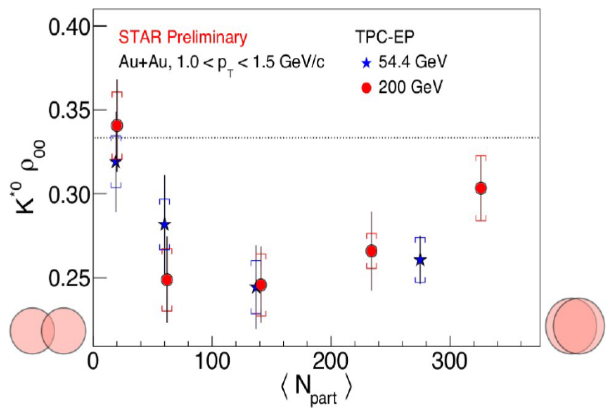
Liang-Wang 2005

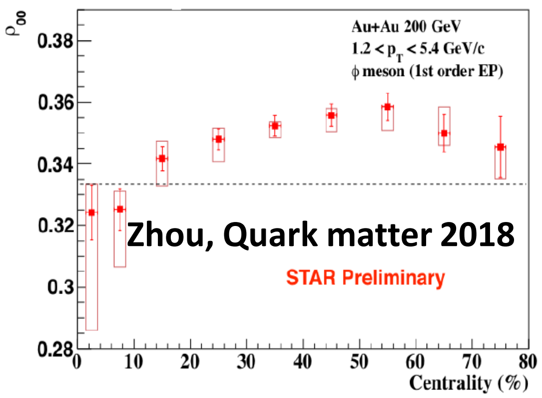
Because P is small, ρ_{00} should be slightly smaller than 1/3 !

Global ϕ-spin alignment

- Experimental results

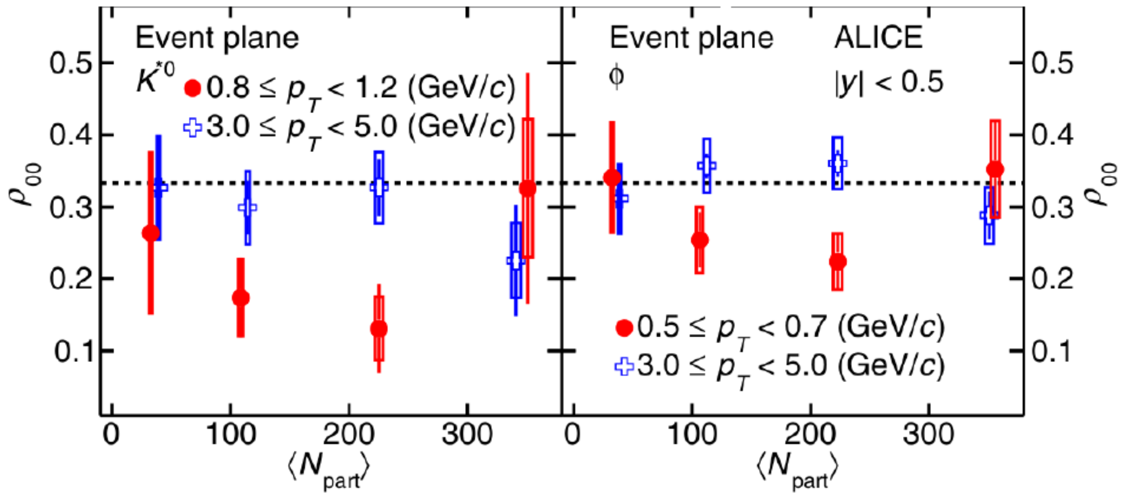
STAR





ALICE

(ALICE 2019)



Puzzle 1: for most centrality, ρ_{00} is far from 1/3.

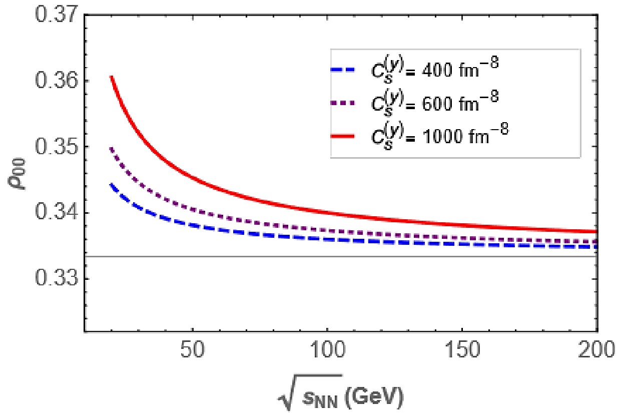
Magnetic field contribution?

Mesonic mean-field ?

Gluon contribution?

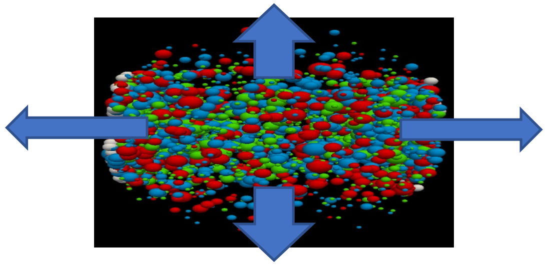
… …

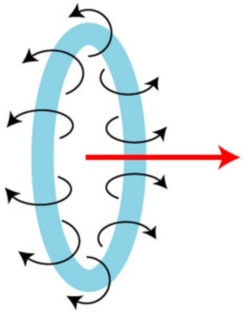
Mesonic strangeness field (Sheng-Oliva-Wang 2019)

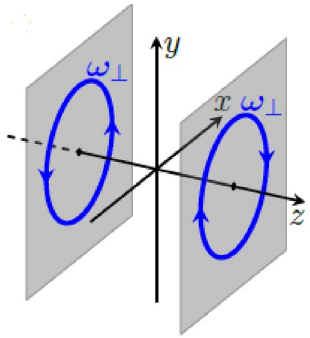


Puzzle 2: $\rho_{00} < \frac{1}{3}$ for central collisions.

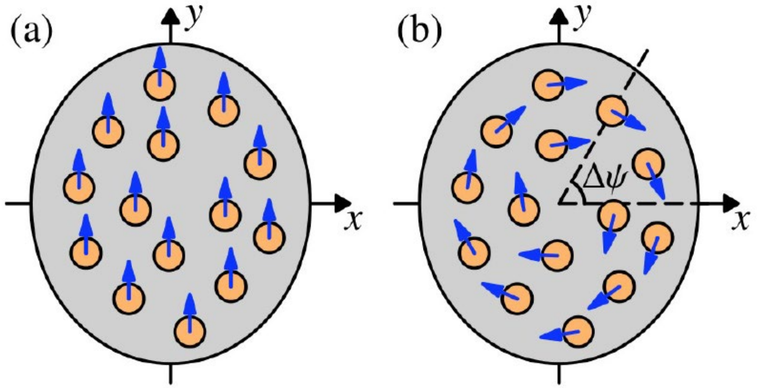
Local ϕ-spin alignment







Global polarization



Local polarization

$$\rho_{00} = \frac{1 - P_y^2}{3 + P_y^2}$$

$$\rho_{00} = \frac{1 - P_y^2 + P_x^2 + P_z^2}{3 + P^2}$$

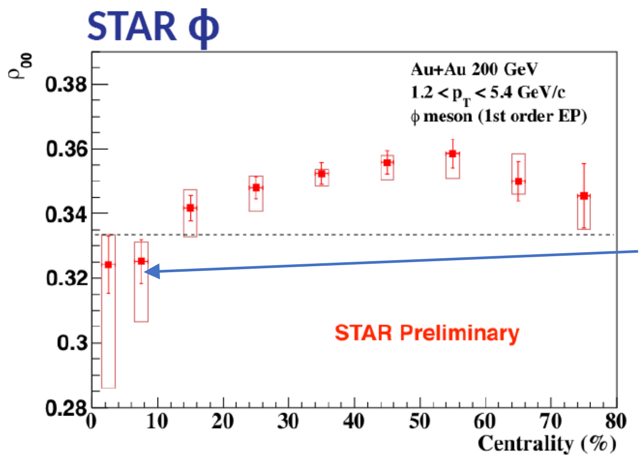
Xia-Li-XGH-Huang 2020

Local ϕ-spin alignment

- In central collisions, we can model quark polarization as

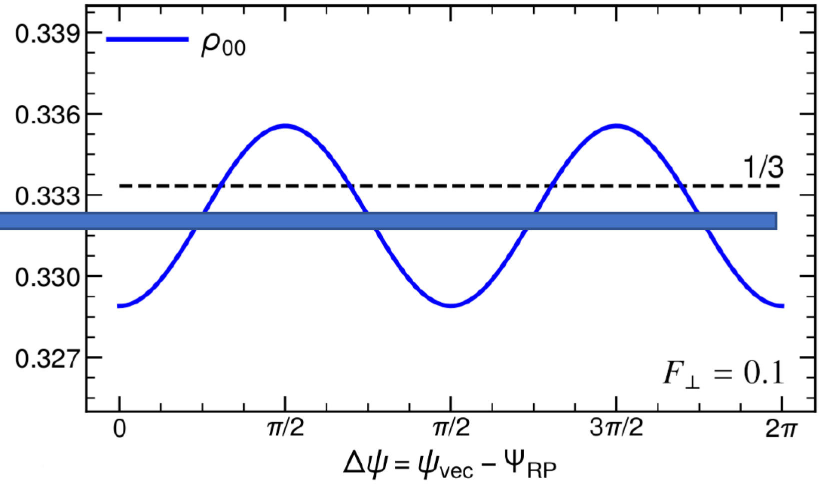
$$P_x^{q,\bar{q}}(\Delta\psi) = F_\perp \sin(\Delta\psi)$$
$$P_y^{q,\bar{q}}(\Delta\psi) = -F_\perp \cos(\Delta\psi)$$
$$P_z^{q,\bar{q}}(\Delta\psi) = 0$$

$$\rho_{00}(\Delta\psi) = \frac{1 - F_\perp^2 \cos(2\Delta\psi)}{3 + F_\perp^2} \approx \frac{1}{3} - \frac{F_\perp^2}{9} - \frac{F_\perp^2}{3}\cos(2\Delta\psi)$$



$$\langle\rho_{00}\rangle \approx \frac{1}{3} - \frac{F_\perp^2}{9}$$

Smaller than 1/3 !



Xia-Li-XGH-Huang 2020

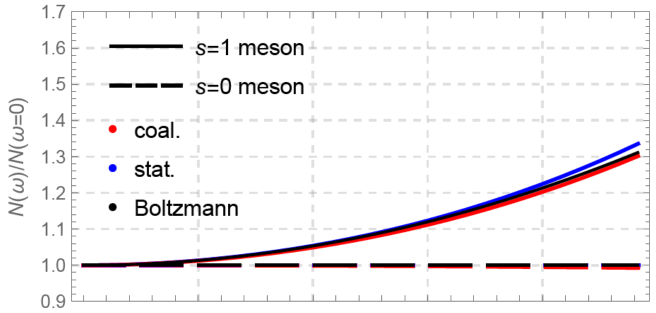
Spin dependent hadron yields

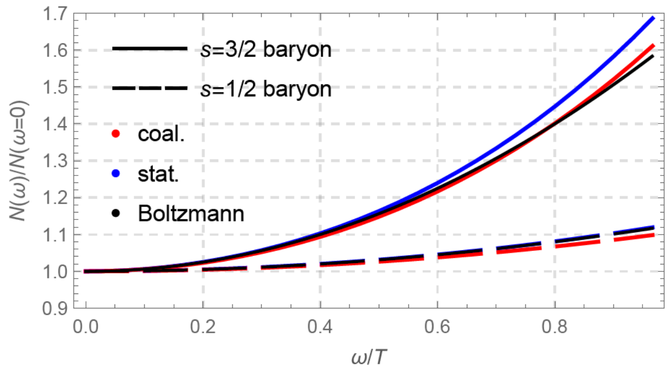
Vorticity is the "spin chemical potential" (ExHIC-P Collaboration 2002.10082)

$$E_{\mathrm{h}} = \sqrt{m_{\mathrm{h}}^2 + \boldsymbol{p}^2} - \boldsymbol{\mu}^{\mathrm{ch}} \cdot \boldsymbol{Q}_{\mathrm{h}} - \omega^{\mathrm{ch}} s_z$$

$$\Rightarrow \quad \frac{N^{\mathrm{stat/coal}}(\omega)}{N^{\mathrm{stat/coal}}(\omega=0)} \sim 1 + \frac{s(1+s)}{6}\left(\frac{\omega}{T}\right)^2$$

Naively, it is the same order as ρ_{00}, could be cross-check of vector spin alignment





Observable: ratio of e.g. $\frac{N_\phi}{N_K}$ or $\frac{N_\Omega}{N_\Xi}$ as function of centrality and energy

Summary

- Very interesting physics of chirality, vorticity, magnetic fields, and spin polarization!
- We study the pre-hydro evolution of B field, see the Faraday retaining effect for B field. This result may used as initial condition of hydro computation of B field.
- We train a CNN that can recognize the initial charge separation pattern (mimicking CME). The machine behaves robust against centrality, energy, and colliding systems.
- The global polarization of hyperons are well understood by global angular momentum through thermal vorticity. Local polarization is still a puzzle.
- Vector meson global spin alignment is too big to be understood via vorticity picture. But in central collisions, the local spin alignment could explain the smaller-than-1/3 spin alignment.

Thank you!

Chiral anomalies

- Quantumly, in external $U(1)$ gauge field and background geometry

$$\nabla_\mu J_A^\mu = -\frac{e^2}{8\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{1}{192\pi^2} R^{\alpha}{}_{\beta\mu\nu}\tilde{R}_{\alpha}{}^{\beta\mu\nu} + \frac{\Lambda^2}{16\pi^2}(2\tilde{R}_{\mu\nu}{}^{\mu\nu} - T_{\lambda}{}^{\mu\nu}\tilde{T}^{\lambda}{}_{\mu\nu})$$

ABJ anomaly — Gravitational anomaly — Nieh-Yan anomaly

- Macroscopic anomalous chiral transport phenomena
  - Chiral magnetic effect (CME): Axial imbalance + B field = vector current
    (Kharzeev 2004; Kharzeev-Fukushima-McLerran-Warringa 2007; …)
  - Chiral separation effect (CSE): vector imbalance + B field = axial current
    (Son-Zhitnitsky 2004; …)
  - Chiral vortical effect (CVE): Temperature + vorticity = vector/axial current
    (Erdmenger etal 2008; Banerjee etal 2008; Torabian-Yee 2009; …)
  - Chiral torsional effect (CTE): Temperature + torsion = vector/axial current
    (Khaidukov-Zubkov 2018; Imaki-Yamamoto 2019; Nissinen-Volovik 2019; …)
  - … …

Chiral anomalies

- Quantumly, in external $U(1)$ gauge field and background geometry

$$\nabla_\mu J^\mu_A = -\frac{e^2}{8\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{1}{192\pi^2} R^{\alpha}{}_{\beta\mu\nu}\tilde{R}_{\alpha}{}^{\beta\mu\nu} + \frac{\Lambda^2}{16\pi^2}\left(2\tilde{R}_{\mu\nu}{}^{\mu\nu} - T_{\lambda}{}^{\mu\nu}\tilde{T}^{\lambda}{}_{\mu\nu}\right)$$

ABJ anomaly | Gravitational anomaly | Nieh-Yan anomaly

- Macroscopic anomalous chiral transport phenomena