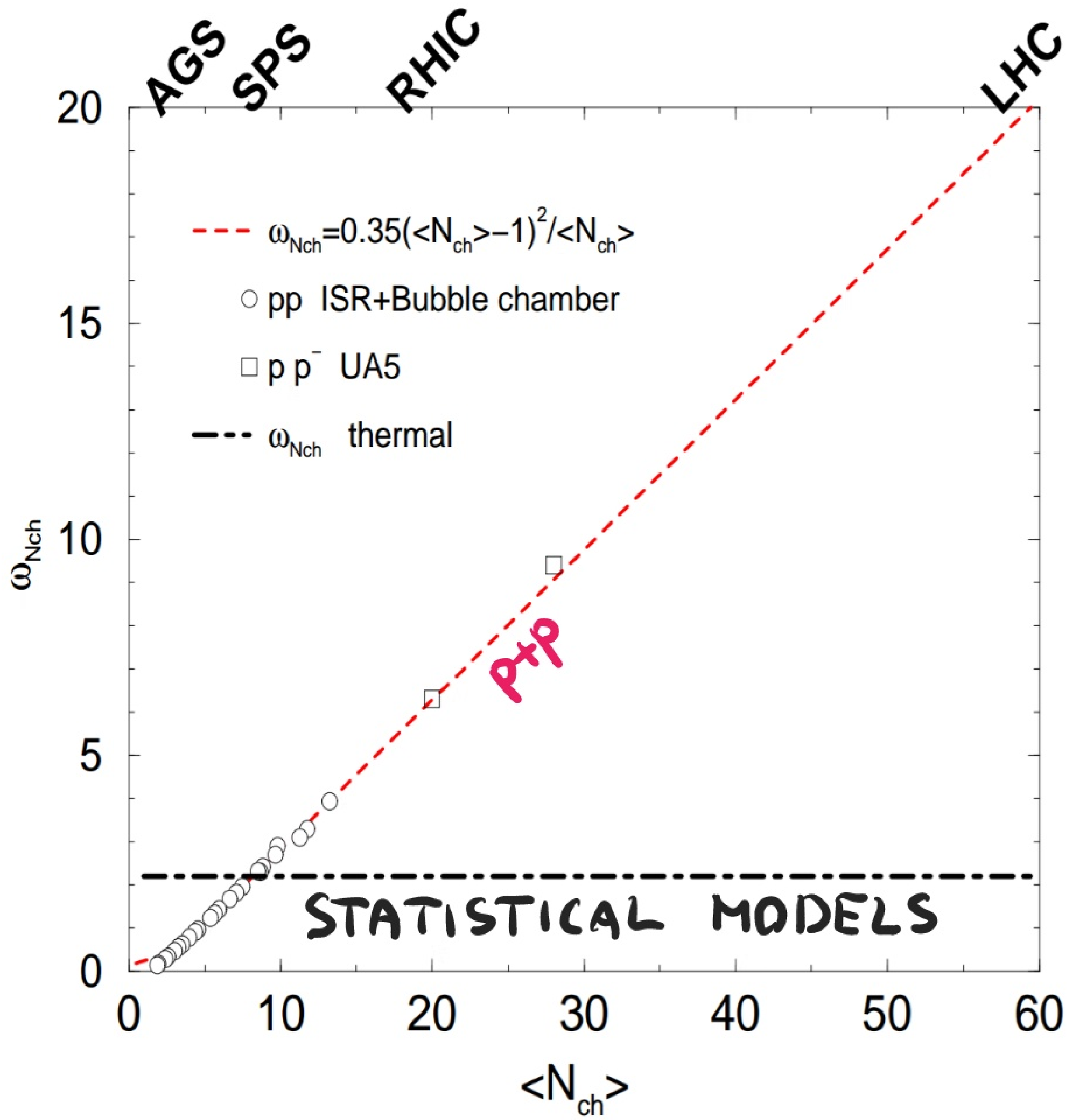
FLUCTUATIONS AT CERN SPS

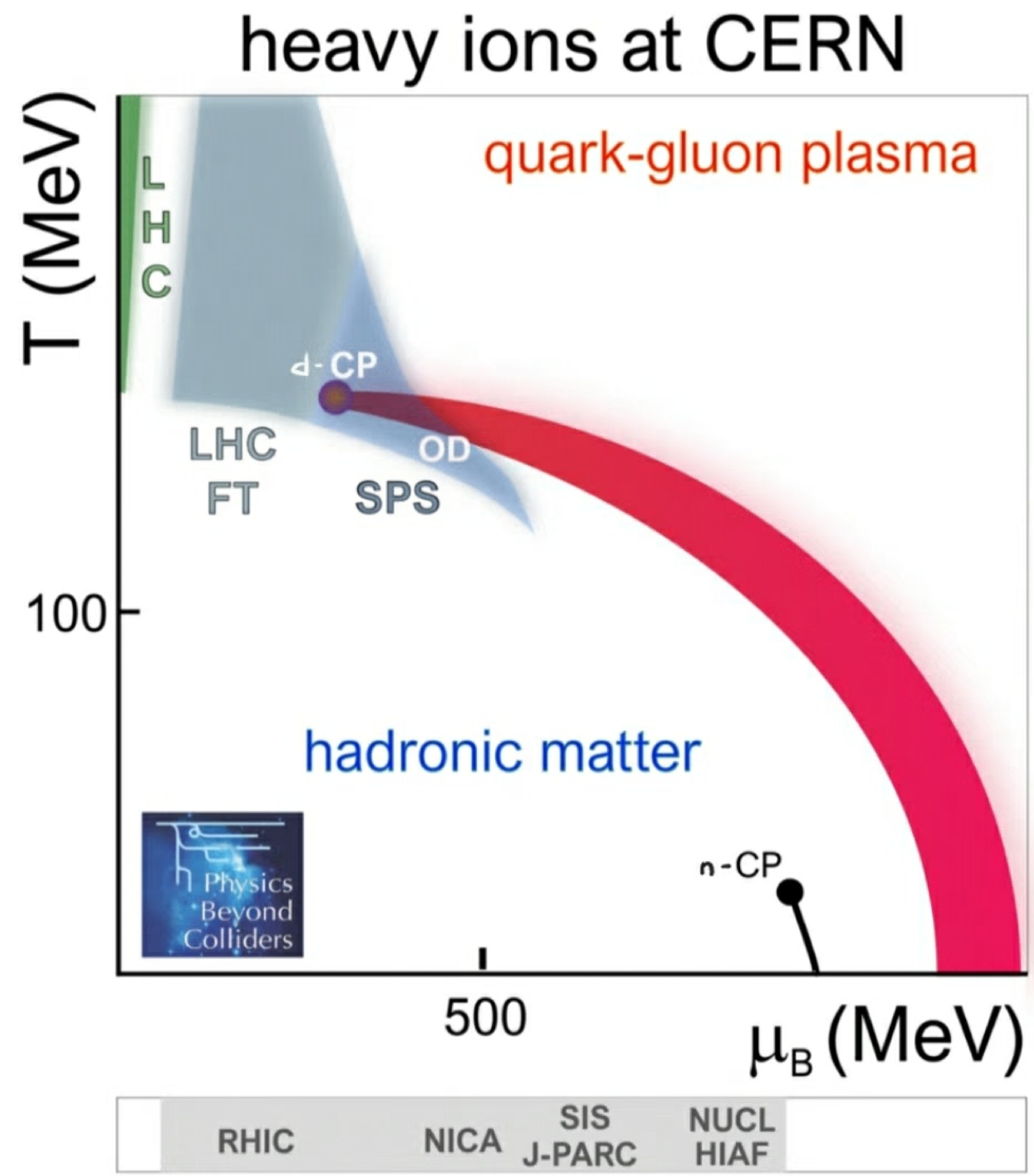
M. GAZDZICKI, FRANKFURT, KIELCE

- MULTIPLICITY FLUCTUATIONS
  (FROM p+p TO $e^+ + e^-$ TO A+A)

- CRITICAL-POINT SEARCH
  (INTERMITTENCY)

WHAT IS SPECIAL AT CERN SPS ?

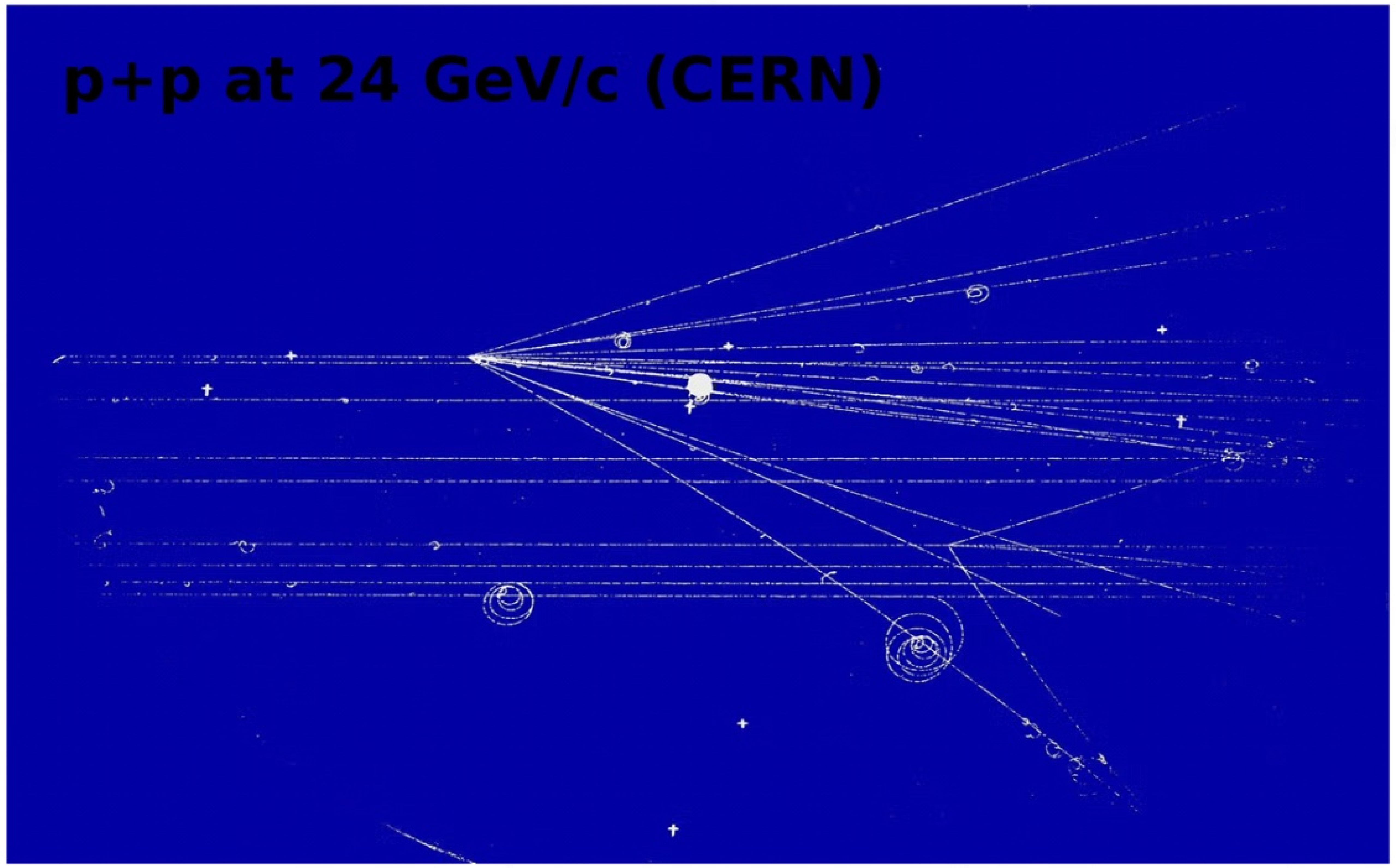


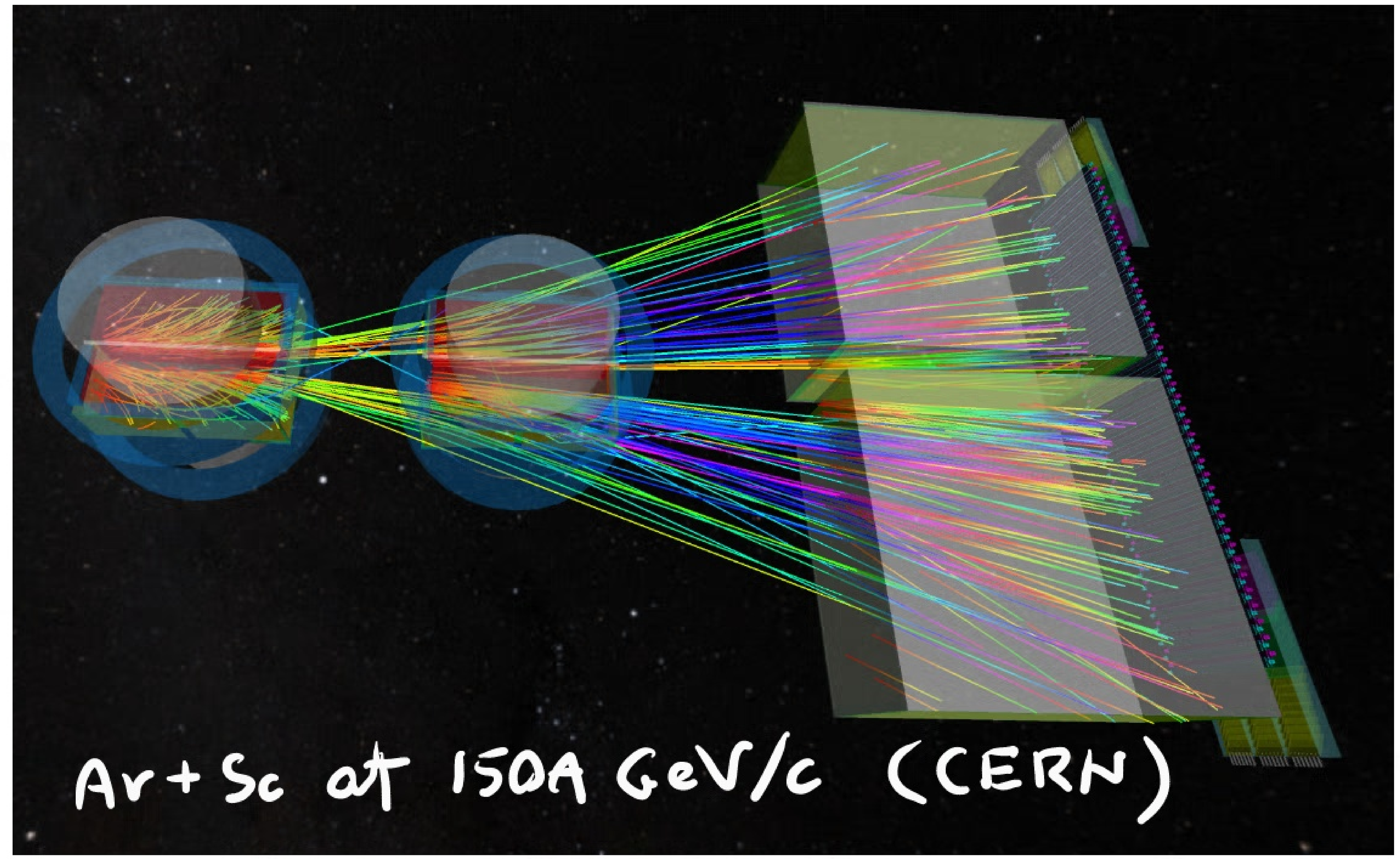


(1) HEISELBERG, PHYS. REPT. 351 (2001) 161

CERN PHYSICS BEYOND COLLIDERS

MULTIPLICITY FLUCTUATIONS
(FROM p+p TO $e^+ + e^-$ TO A+A)





\+ FULL ACCEPTANCE
\- LOW STATISTICS
\- NO PID

\+ HIGH STATISTICS
\+ PID
\- LIMITED ACCEPTANCE

NOTATION

N_{ch} – NUMBER OF ALL CHARGED PARTICLES PRODUCED IN A COLLISION

$N = 0, 1, 2, 3 \ldots$ – MULTIPLICITY VARIABLE

THAN FROM ELECTRIC CHARGE CONSERVATION FOLLOWS:

$$N_{ch} = 2 + 2 \cdot N \quad \text{FOR } p+p$$

$$N_{ch} = 0 + 2 \cdot N \quad \text{FOR } e^{+}+e^{-}$$

INITIAL (NET) CHARGE

NUMBER OF + AND – PRODUCED PARTICLES HAS TO BE EQUAL TO OBEY THE CHARGE CONSERVATION LAWS:

(NET) INITIAL CHARGE = (NET) FINAL CHARGE

NOTATION

$P(N)$ – PROBABILITY DISTRIBUTION OF N

D_k – k-th DISPERSION OF $P(N)$

$$D_k \equiv \langle (N - \langle N \rangle)^k \rangle^{1/k}$$

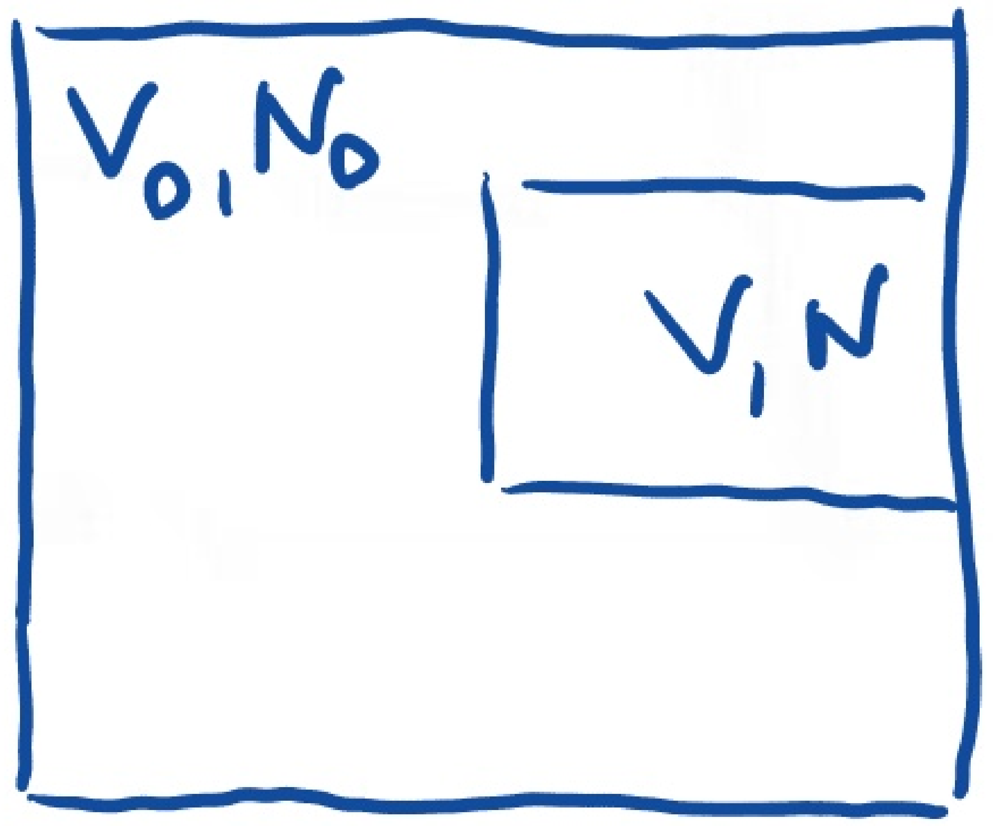
E.G. $D_2 \equiv \langle (N - \langle N \rangle)^2 \rangle^{1/2} = (\mathrm{Var}[N])^{1/2}$

$\omega[N] \equiv \mathrm{Var}[N] / \langle N \rangle$ ~ SCALED VARIANCE

$\sqrt{s}$ – CENTER OF MASS ENERGY OF $p+p$ OR $e^+ + e^-$ COLLISIONS

REFERENCE MULTIPLICITY DISTRIBUTIONS

BINOMINAL DISTRIBUTION



N_0 PARTICLES IS CREATED INDEPENDENTLY IN VOLUME V_0 ACCORDING TO THE CONSTANT PDF $f(\vec{r}) = 1/V_0$

THEN PROBABILITY $P(N)$ TO FIND N PARTICLES IN SUBVOLUME V IS GIVEN BY THE BINOMINAL DISTRIBUTION

$$P_B(N) = \binom{N_0}{N} q^N (1-q)^{N_0-N} = \frac{N_0!}{N!(N_0-N)!} q^N (1-q)^{N_0-N}$$

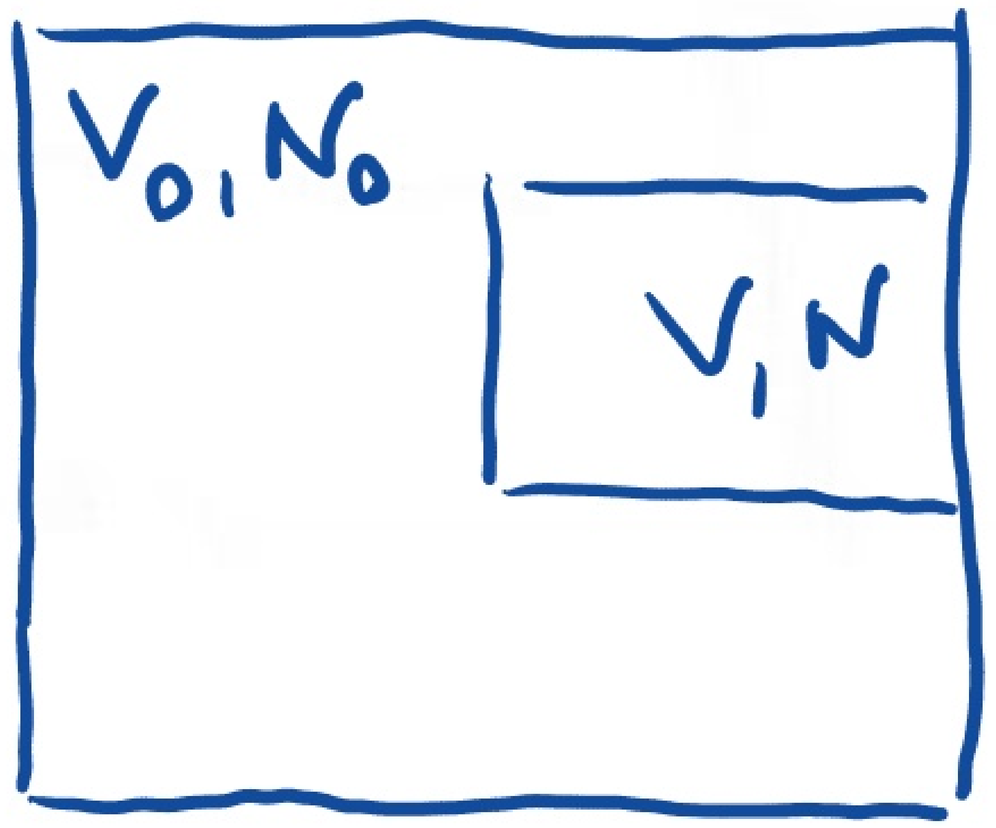
WHERE $\quad q = V/V_0, \quad N \leq N_0$

$$\langle N \rangle = q \cdot N_0, \qquad Var[N] = q(1-q) \cdot N_0, \rightarrow \omega[N] = (1-q) \in [0,1]$$

$\Rightarrow$ CE, MCE

REFERENCE MULTIPLICITY DISTRIBUTIONS

POISSON DISTRIBUTION



CONSIDER THE "THERMODYNAMICAL LIMIT" OF BINOMIAL DISTRIBUTION

$V_0 \to \infty$ AND $N_0 \to \infty$ WITH $N_0/V_0 =$ CONST

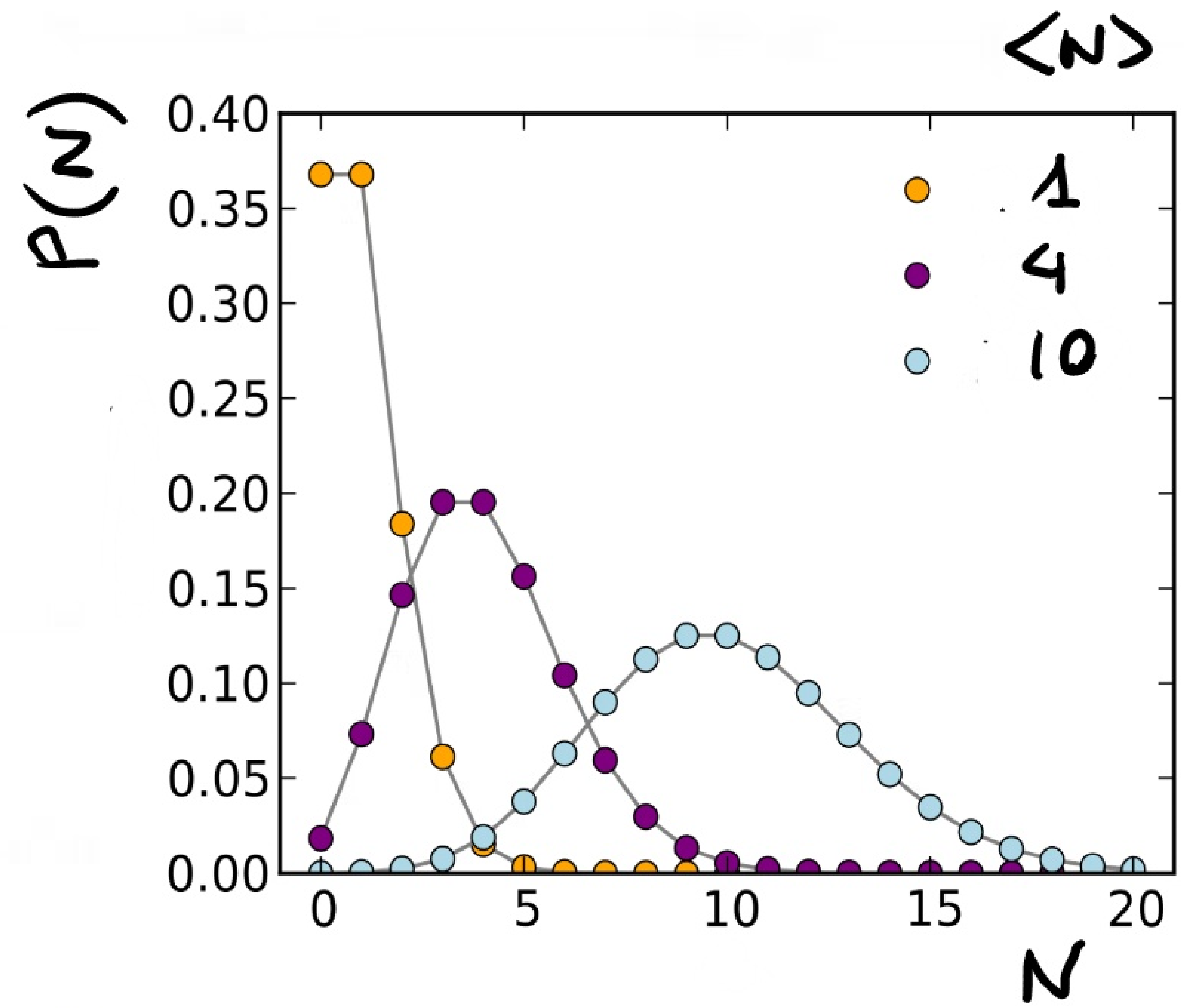
FOR $V =$ CONST ONE GETS

$$P_B(N) \to \text{POISSON}(N) = \langle N\rangle^N e^{-\langle N\rangle}/N!$$

$$\text{Var}[N] = \langle N\rangle \to \omega[N]=1$$

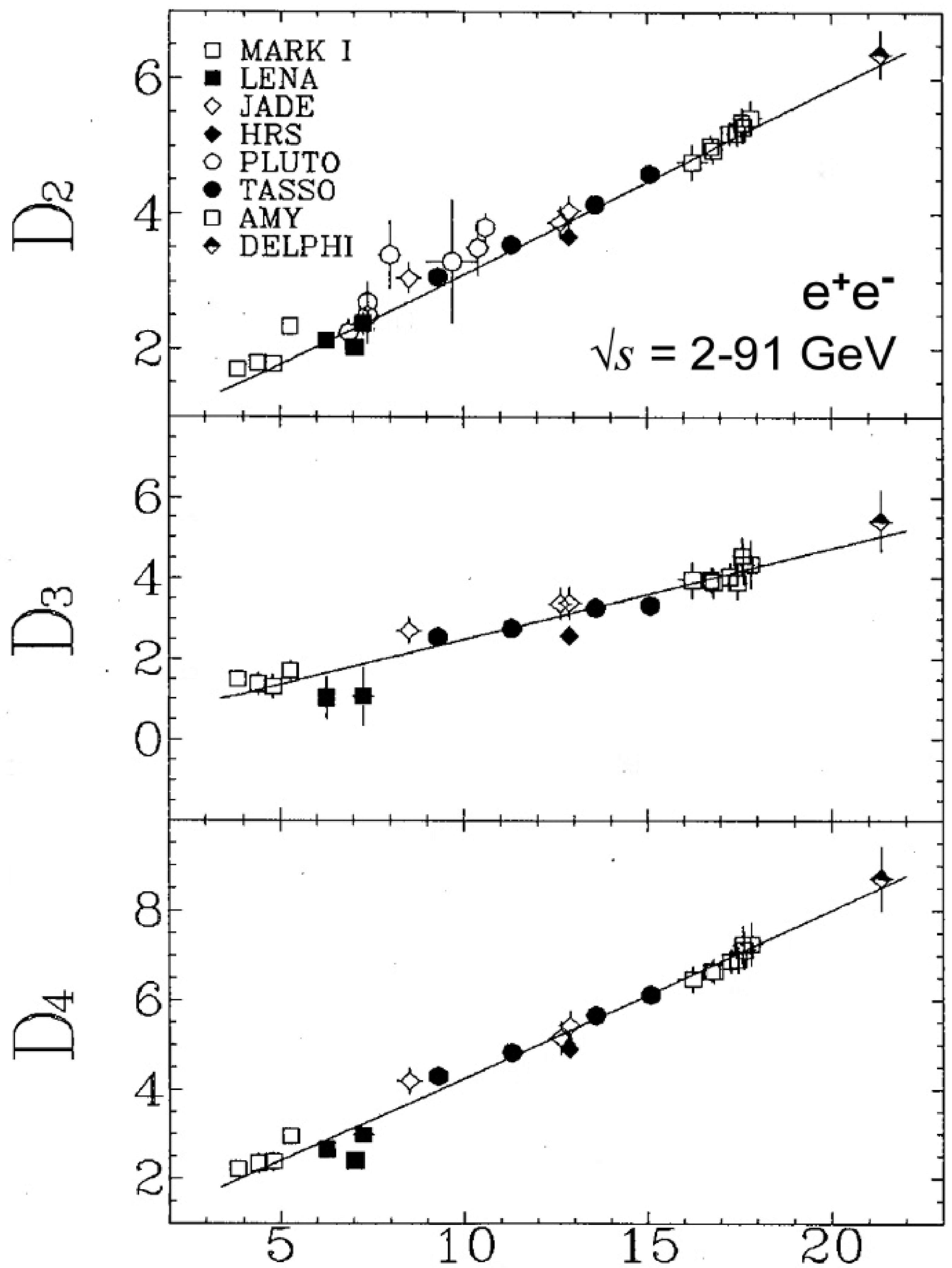
$$\langle (N-\langle N\rangle)^3\rangle = \langle N\rangle$$

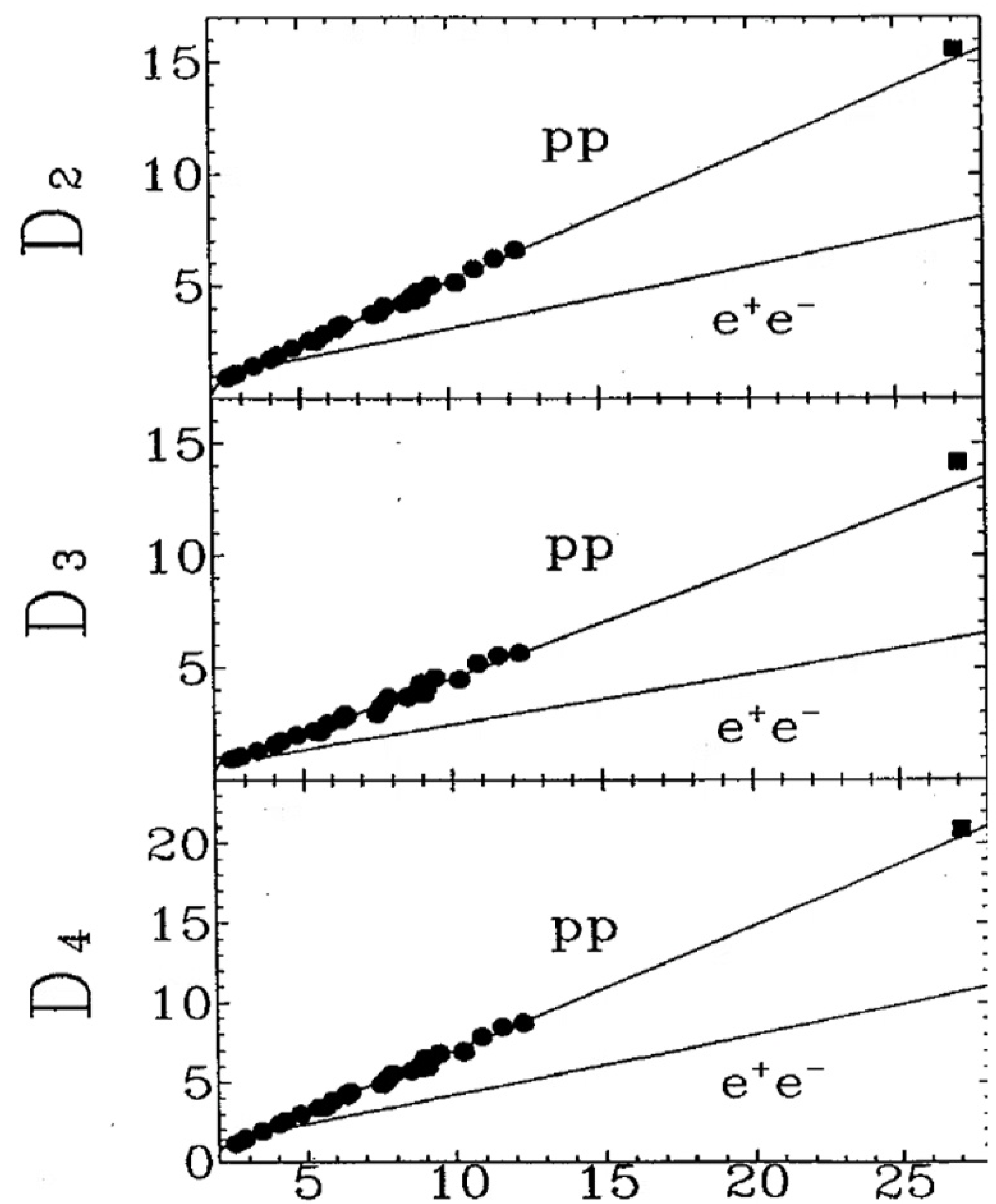
$$\langle (N-\langle N\rangle)^4\rangle = \langle N\rangle(1+3\langle N\rangle)$$



$\Rightarrow$ GCE

BASIC OBSERVATIONS: WROBLEŃSKI LAW (1973)





$D_2 \sim \langle N \rangle$

$D_3 \sim \langle N \rangle$

$D_4 \sim \langle N \rangle$

$\vdots$

7 WROBLEWSKI ACTA PHYS. POL. B4 (1973) 857

WROBLEWSKI LAW:

$$D_2 \sim \langle N \rangle$$

$$Var[N] \sim \langle N \rangle^2$$

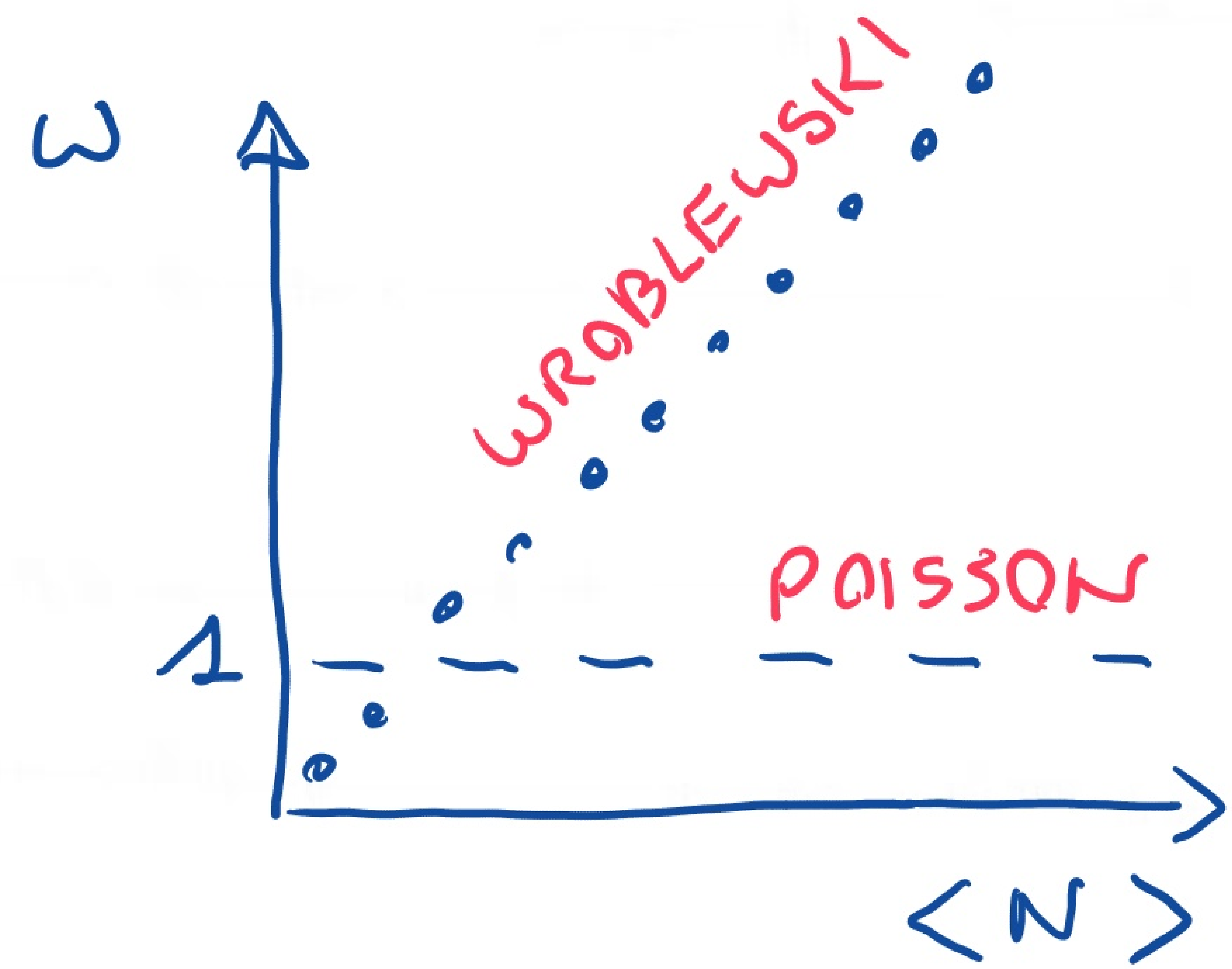
$$\omega \equiv \frac{Var[N]}{\langle N \rangle} \sim \langle N \rangle$$

POISSON:

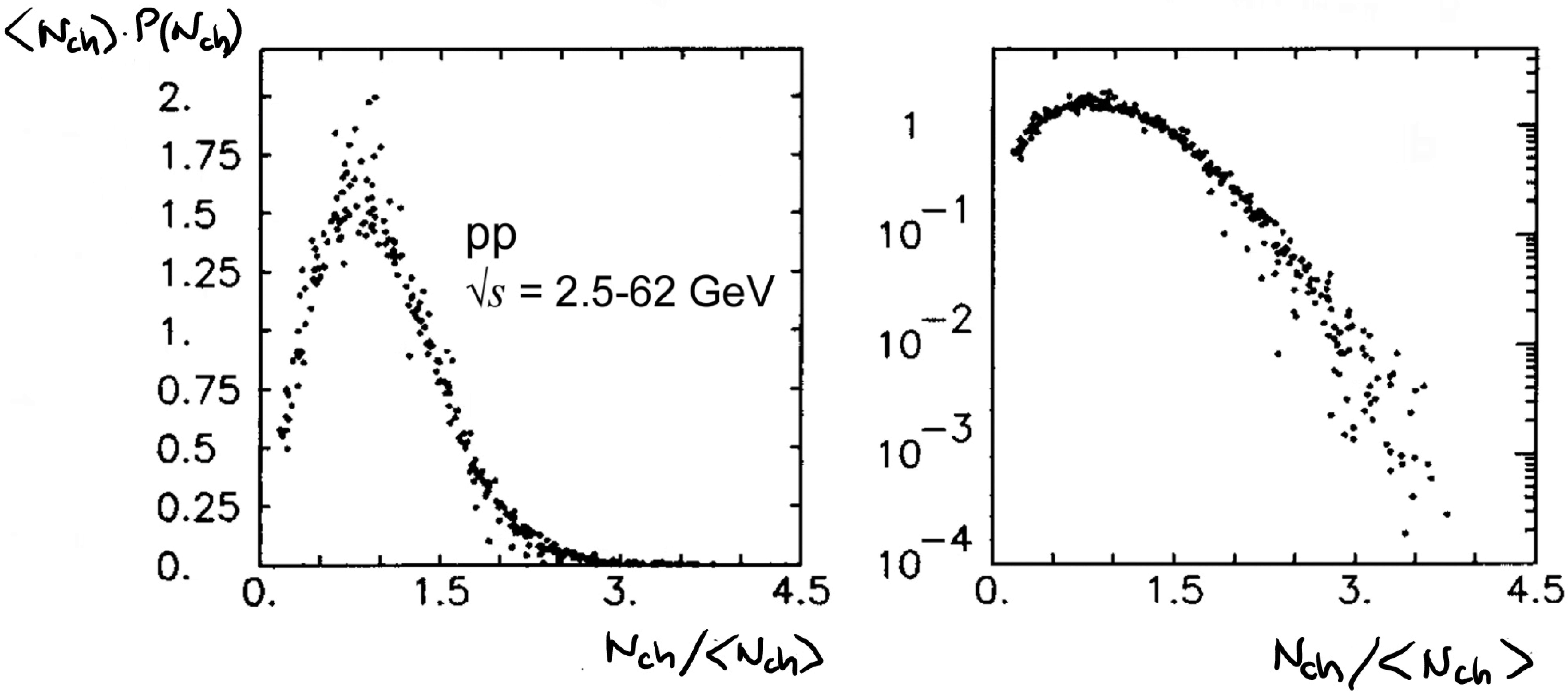
$$D_2 \sim \sqrt{\langle N \rangle}$$

$$Var[N] \sim \langle N \rangle$$

$$\omega = 1$$



BASIC OBSERVATIONS: KNO-SCALING (1972)



THE KNO-SCALING QUANTITIES: $\langle N\rangle P(N)$ vs $N/\langle N\rangle$

KOBA, NIELSEN, OLESEN, NUCL. PHYS. B40 (1972) 317

KNO - SCALING: MULTIPLICITY DISTRIBUTIONS PLOTTED IN THE KNO QUANTITIES ($\langle N \rangle P(N)$ vs $N/\langle N \rangle$) ARE INDEPENDENT OF COLLISION ENERGY

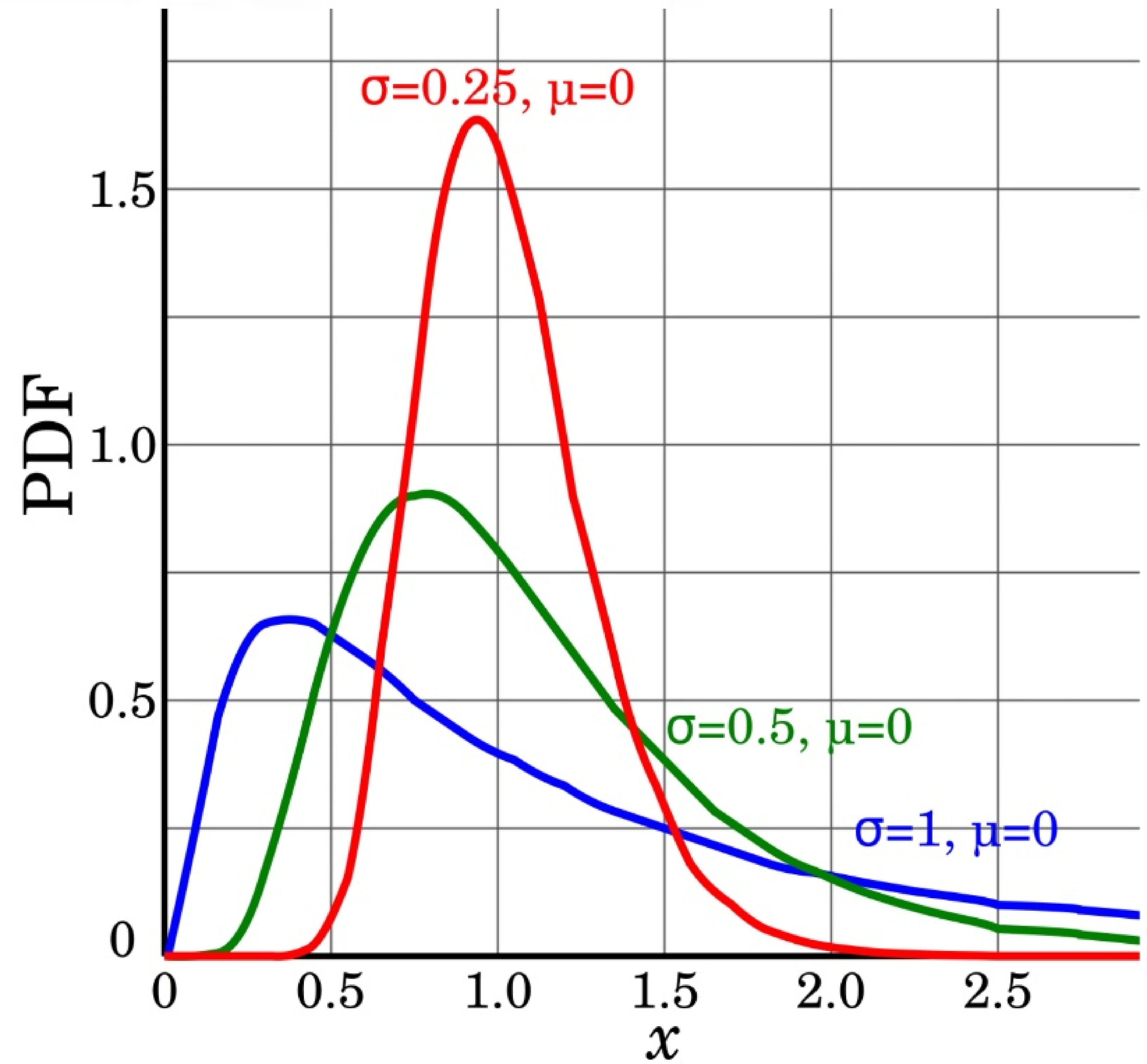
THEN $P(N)$ FOR DIFFERENT $\sqrt{s}$ CAN BE DESCRIBED BY A SINGLE $\sqrt{s}$ INDEPENDENT FUNCTION AS

$$P(N) = \frac{1}{\langle N \rangle} \cdot \psi_{KNO}\left(N/\langle N \rangle\right)$$

THE KNO SCALING IS APPROXIMATELY OBEYED BY $p+p$ AND $e^+ + e^-$ INTERACTIONS

BUT $\psi^{pp}_{KNO} \neq \psi^{ee}_{KNO}$

≈ LOGNORMAL SHAPE OF SCALING FUNCTION



$$\psi(z) = \frac{c_0}{\sqrt{2\pi}\,\sigma}\,\frac{1}{z+c}\,\exp\left(-\frac{[\ln(z+c)-\mu]^2}{2\sigma^2}\right)$$

$$(z = N/\langle N\rangle)$$

$\ln(z+c)$ IS DISTRIBUTED NORMALLY

SZWED, WROCHNA, WROBLEWSKI
MODERN PHYSICS LETT. A5 (1990) 1851

WHY LOGNORMAL MULTIPLICITY DISTRIBUTION?

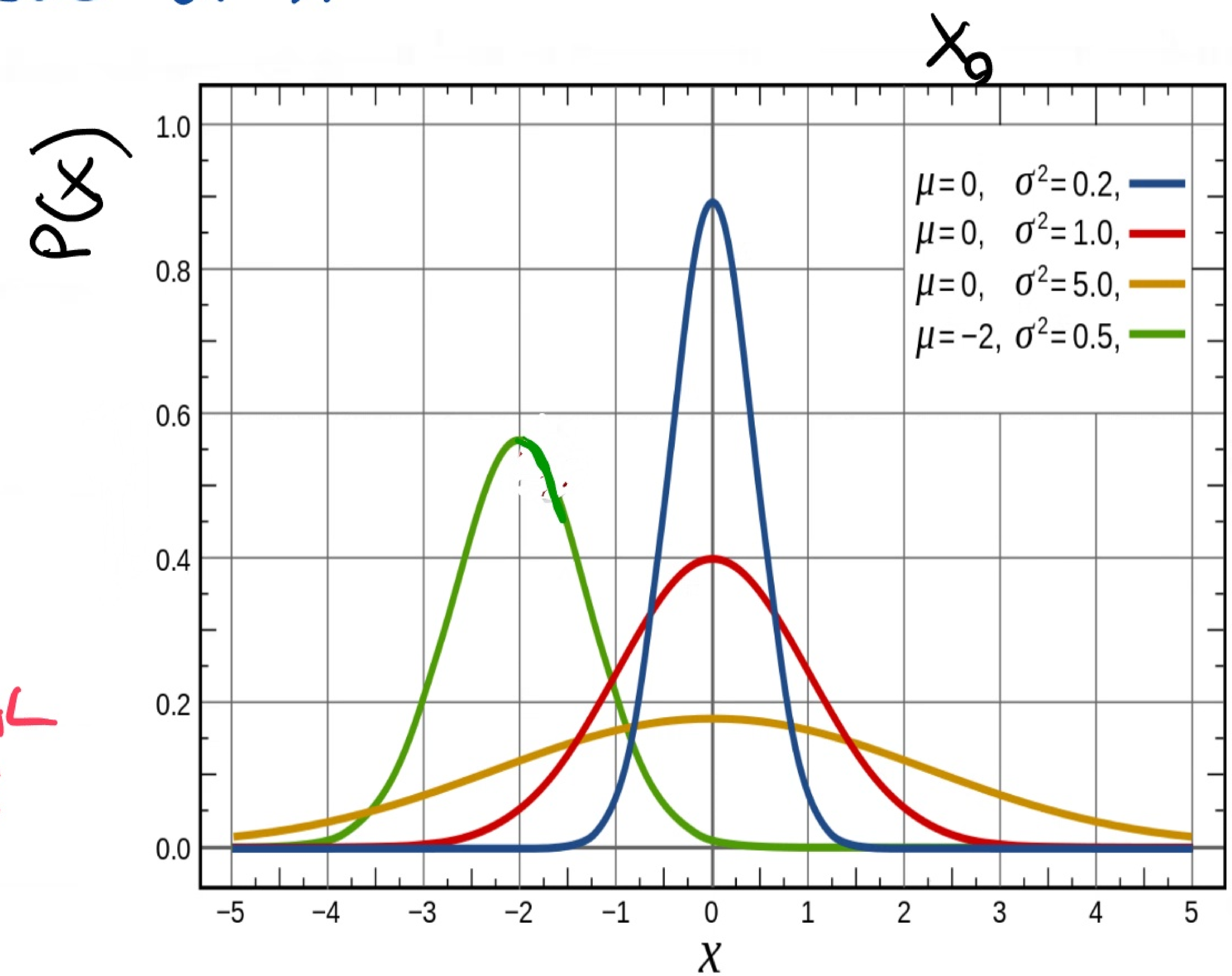
NORMAL (GAUSSIAN) DISTRIBUTION APPEARS IN ADDITATIVE PROCESSES, WHERE CHANGES ΔX OF A RANDOM VARIABLE X ARE INDEPENDENT OF THE VALUE OF X

$$X_1 = X_0 + \Delta X_1$$
$$X_2 = X_1 + \Delta X_2$$
$$X_3 = X_2 + \Delta X_3$$
$$\vdots$$

$$X_k = X_{k-1} + \Delta X_k = X_0 + \sum_{i=1}^{k} \Delta x_i$$

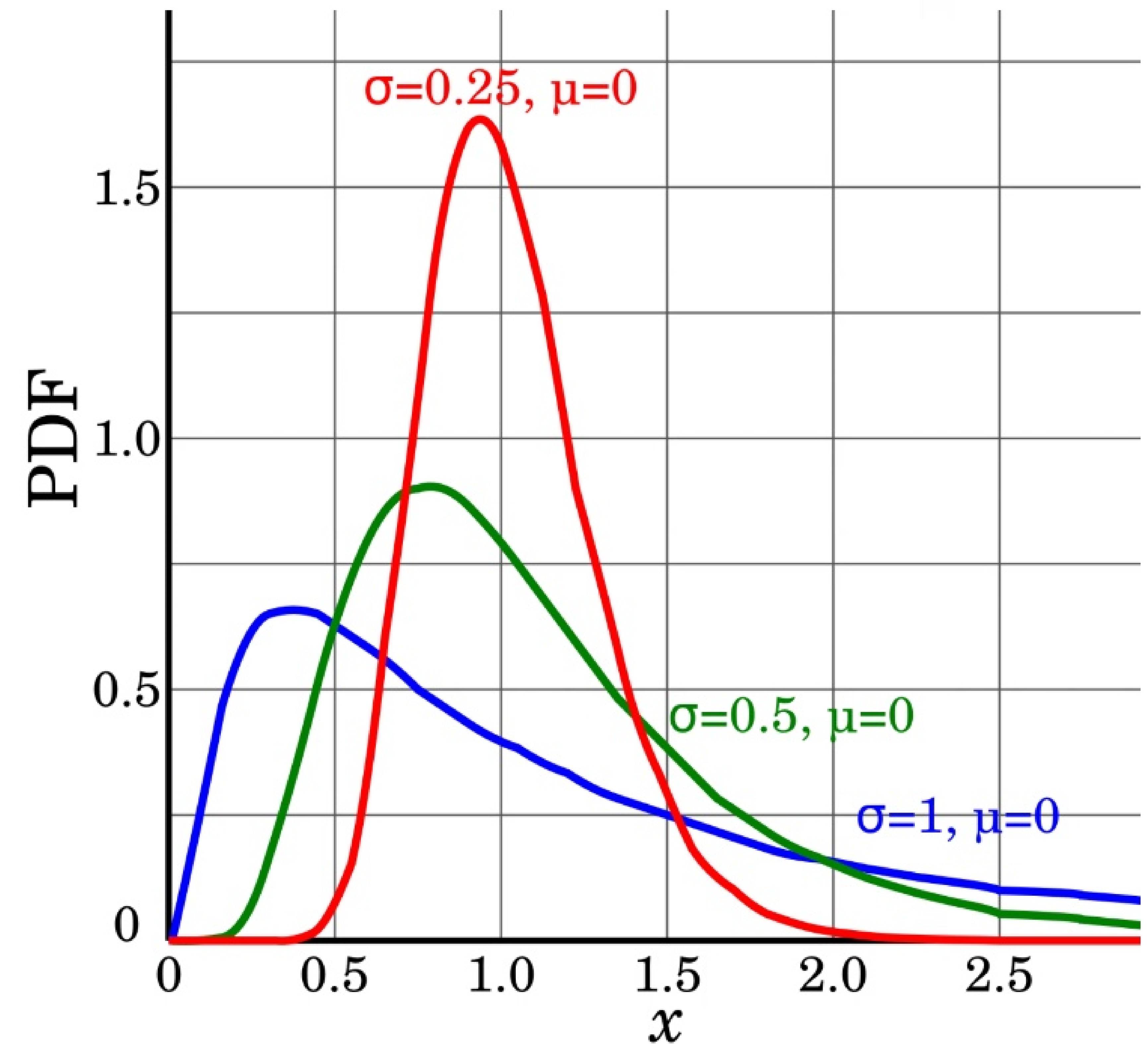
CENTRAL LIMIT THEOREM SAYS THAT DISTRIBUTION OF X IS NORMAL REGARDLES OF DISTRIBUTION OF ΔX

$$P(X) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-x_0)^2}{2\sigma^2}}$$



FOR MULTIPLICATIVE PROCESSES, ΔX IS PROPORTIONAL TO THE VALUE OF X

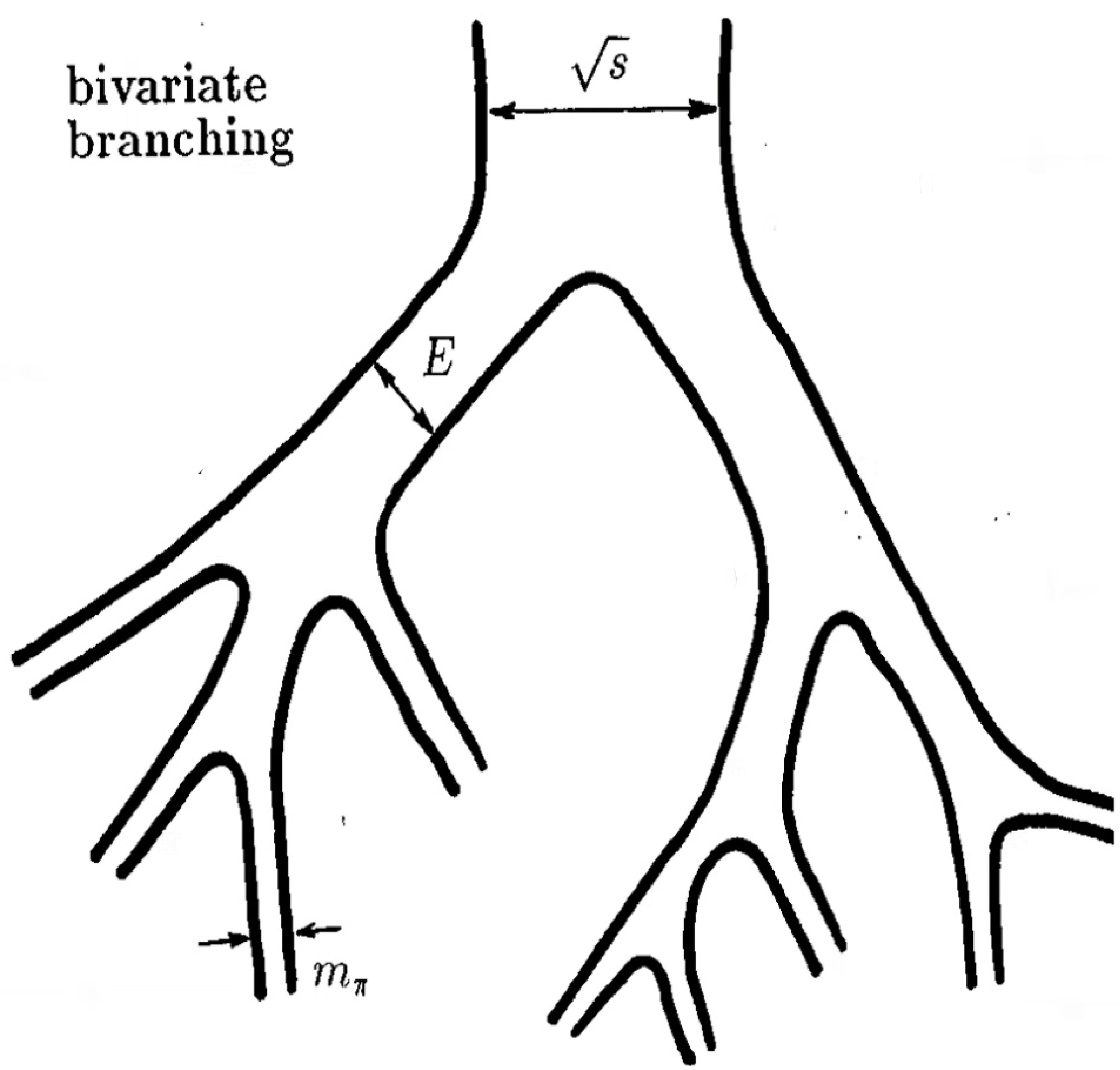
$$X_1 = X_0 + \underbrace{\varepsilon_1 X_0}_{\Delta X} = X_0(1+\varepsilon_1)$$
$$X_2 = X_1 + \varepsilon_2 X_1 = X_1(1+\varepsilon_2)$$
$$X_3 = X_2 + \varepsilon_3 X_2 = X_2(1+\varepsilon_3)$$
$$\vdots$$
$$X_k = X_{k-1} + \varepsilon_k X_{k-1} = X_0 \prod_{i=1}^{k}(1+\varepsilon_i)$$



$$\ln(X_k/X_0) = \sum_{i=1}^{k} \ln(1+\varepsilon_i)$$

HAS NORMAL DISTRIBUTION AND THUS X_k THE LOGNORMAL ONE

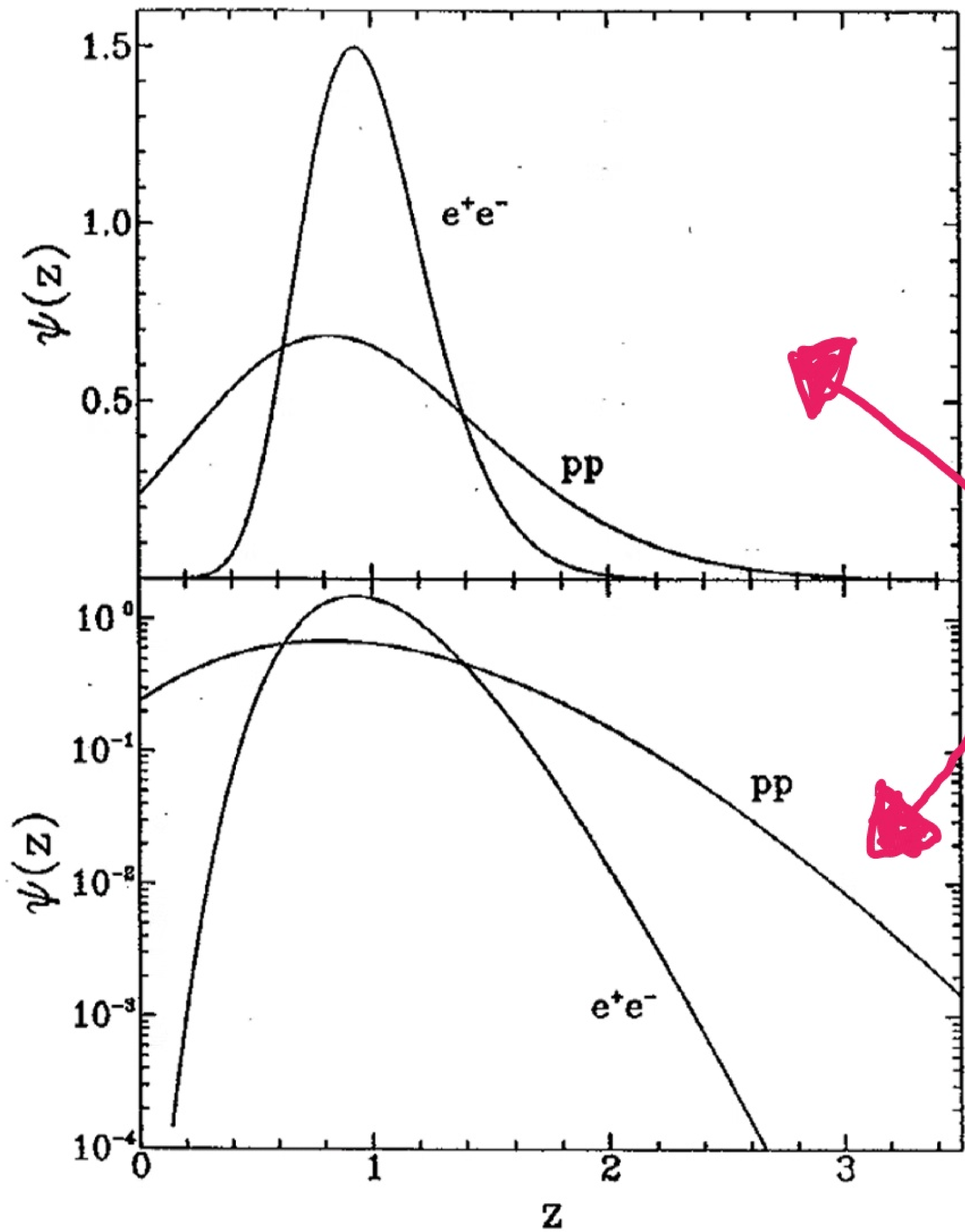
PARTICLE PRODUCTION AS BRANCHING PROCESS



STOCHASTIC BRANCHING OF STRINGS FROM QCD-INSPIRED MODELS

MAY SERVE AS AN EXPLANATION OF THE LOGNORMAL SCALING PROPERTIES OF $P(N)$ IN e^+e^- AND $p+p$ INTERACTIONS

FROM $e^+ + e^-$ AND $p+p$



THE QUARK-GLUON STRUCTURE OF PROTON

MULTI-PARTICLE PRODUCTION IN $p+p$
MORE COMPLICATED THAN IN $e^+ + e^-$

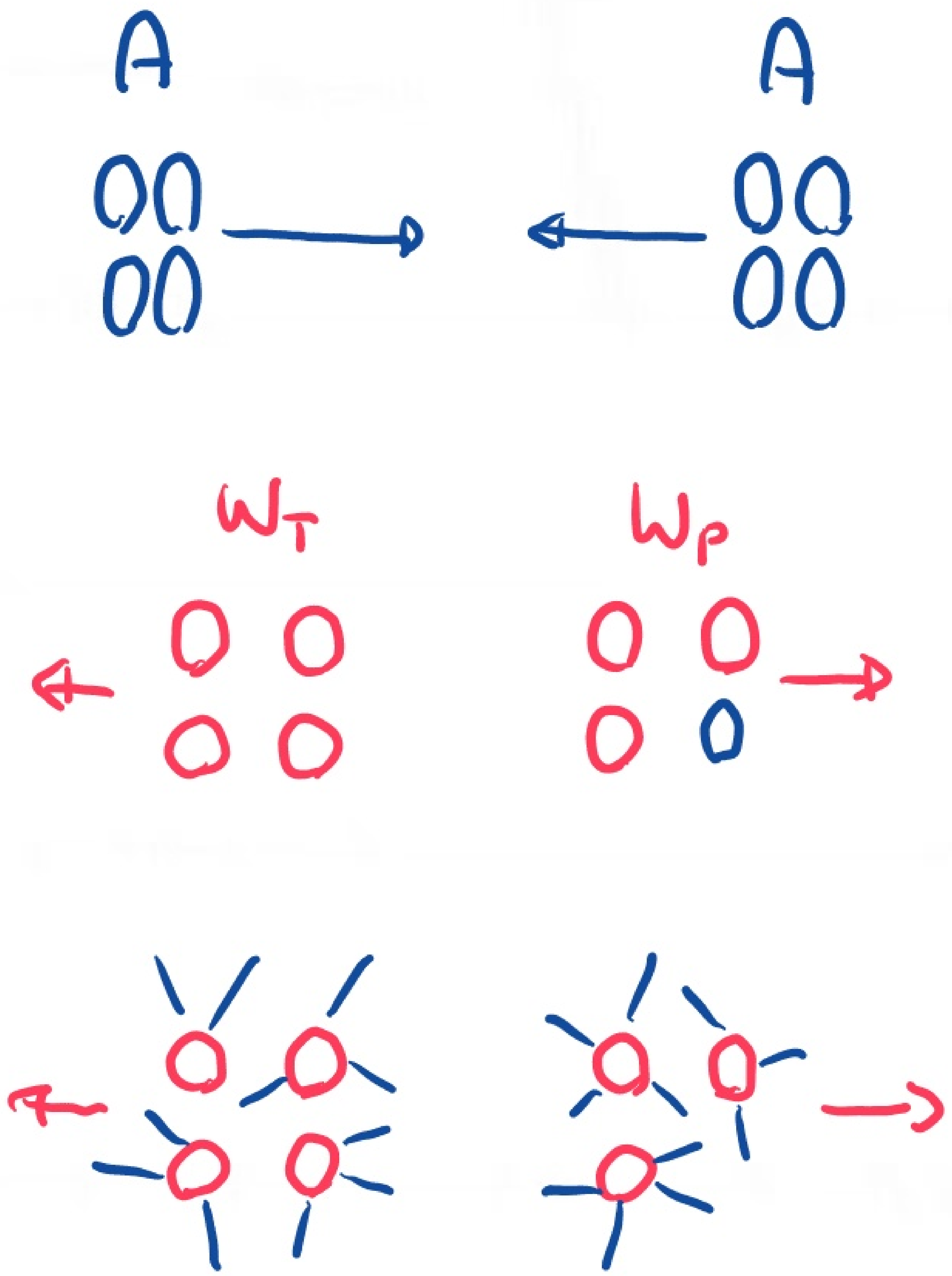
$p+p$ INTERACTIONS AS SUPPERPOSITION
OF $e^+ + e^-$ - LIKE PROCESSES:

$$\psi_{ee} \longrightarrow \psi_{pp}$$

MG, SZWED, WROCHNA, WROBLEWSKI
MODERN PHYS. LETT. A6 (1991) 981

FROM p+p TO A+A

WOUNDED NUCLEON MODEL



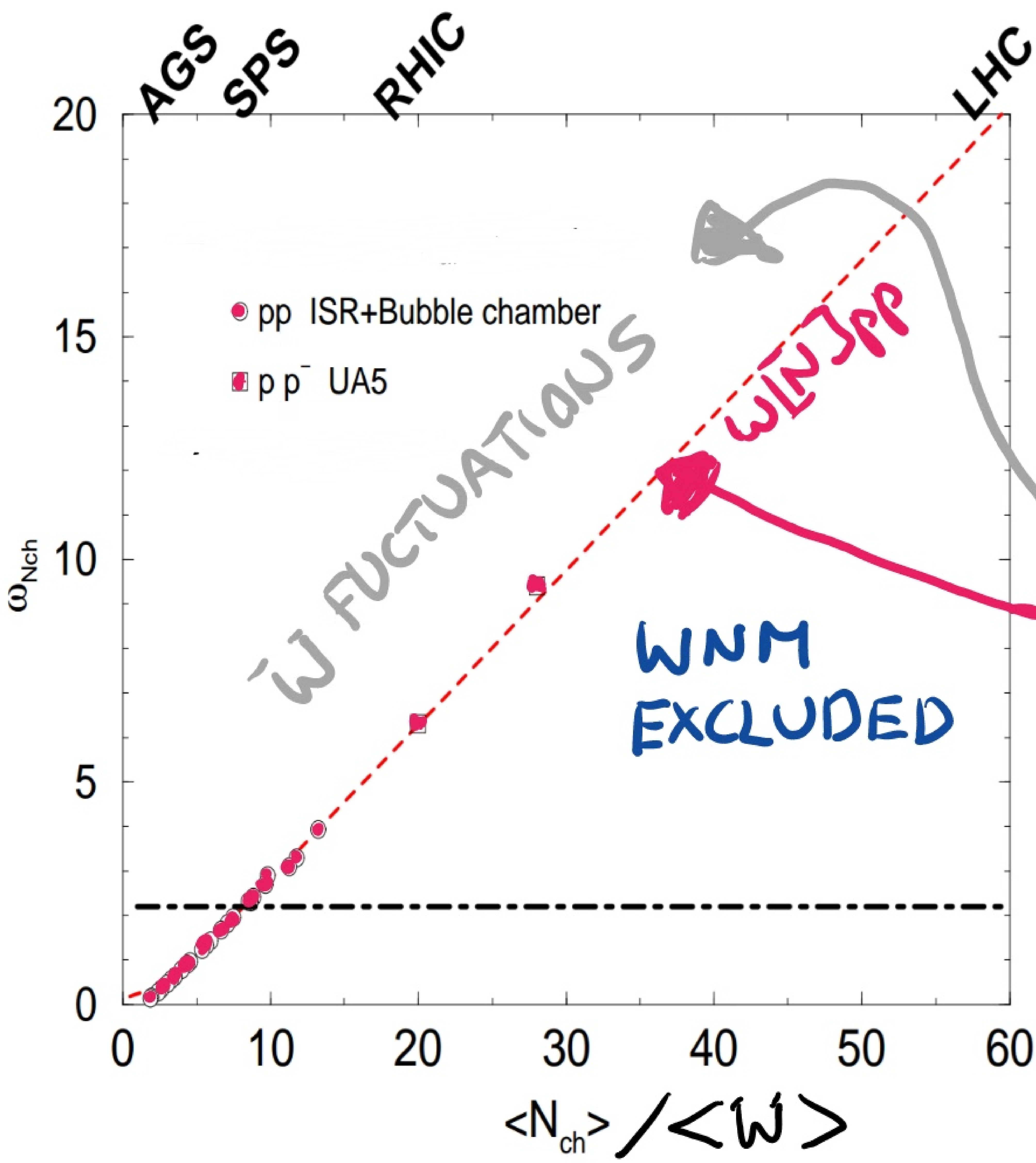
NUMBERS OF PROJECTILE (W_P) AND TARGET (W_T) NUCLEONS THAT INTERACTED INELASTICALLY (WOUNDED NUCLEONS) ARE CALCULATED ASSUMING STRAIGHT TRAJECTORIES ($W = W_P + W_T$).

WOUNDED NUCLEONS IN $A_P + A_T$ COLLISIONS HAVE PROPERTIES THE SAME AS WOUNDED NUCLEONS IN N+N ($\approx$ p+p) INTERACTIONS AT THE SAME $\sqrt{s_{NN}}$.

$$N = \sum_{i=1}^{W} n_i$$

BIAŁAS, BLESZYNSKI, CZYZ, NUCL. PHYS. B111 (1976) 461

WNM PREDICTIONS FOR $\omega[N]$



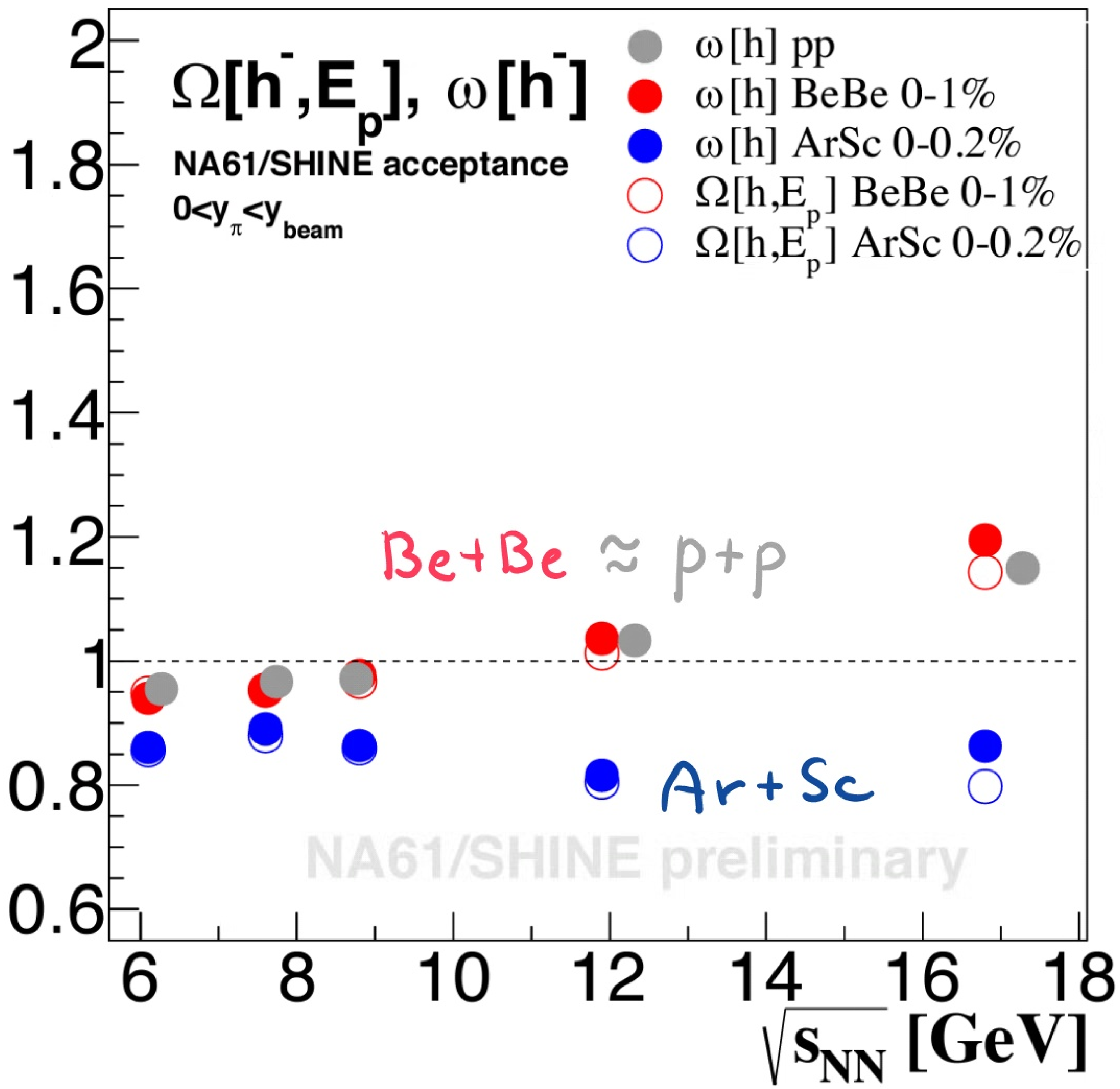
$$\langle N\rangle = \langle n\rangle \cdot \langle \bar{W}\rangle$$

$$\mathrm{Var}[N] = \mathrm{Var}[n] \cdot \langle \bar{W}\rangle + \langle n\rangle^2 \cdot \mathrm{Var}[\bar{W}]$$

$$\omega[N] \equiv \frac{\mathrm{Var}[N]}{\langle N\rangle} = \omega[n] + \langle n\rangle\, \omega[\bar{W}] =$$

$$\approx \omega[N]_{pp} + \langle N\rangle / \langle W\rangle\, \omega[\bar{W}]$$

PREDICTIONS VS A+A DATA
(LIMITED ACCEPTANCE)



Be+Be ≈ p+p

Ar+Sc < p+p

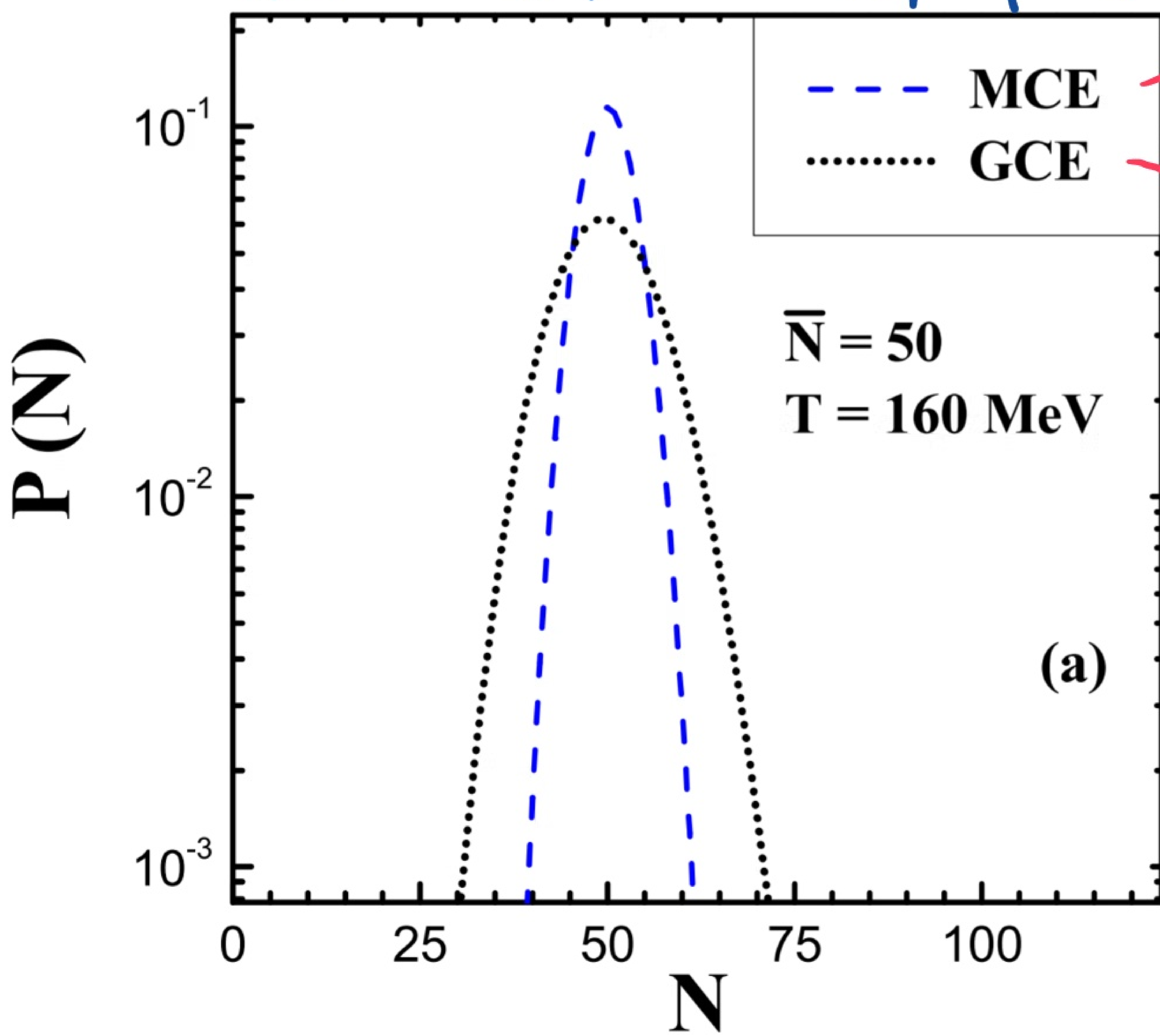
(IN THE WNM EXCLUDED DOMAIN)

Ar+Sc < 1

NARROWER THAN POISSON

Ar+Sc → STATISTICAL MODEL IN MICROCANONICAL FORMULATION

$\langle E \rangle = E_0, \quad m = 0, \quad q = 0$



$\langle N \rangle \to \infty$: GAUSS

$\omega[N] = 1/4$

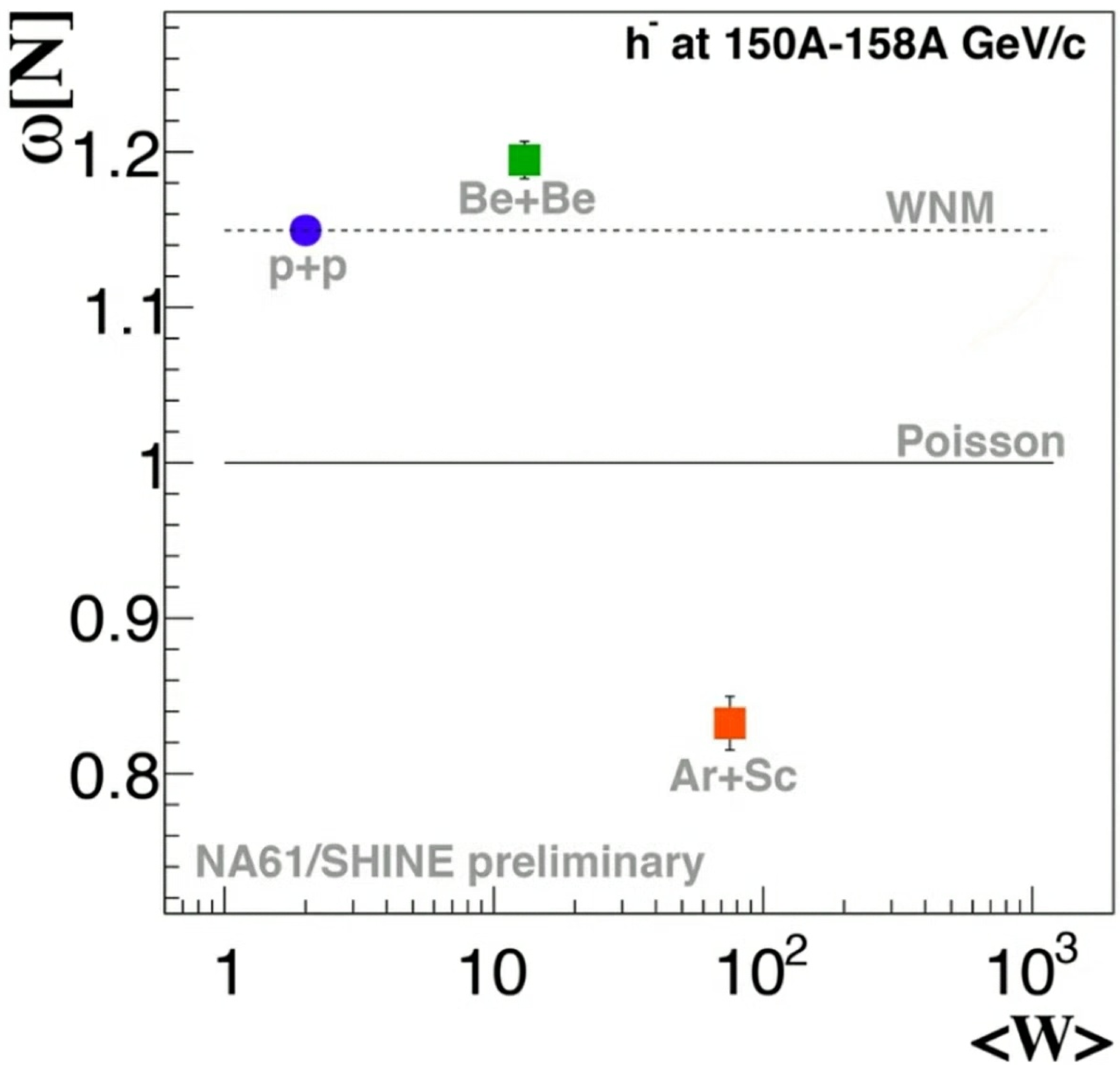
POISSON

($\langle N \rangle \to \infty$: GAUSS)

$\omega[N] = 1$

BEGUN, GORENSTEIN, ZOZULYA, PRC 71 (2005) 054904
PRC 72 (2005) 014902

ONSET OF FIREBALL

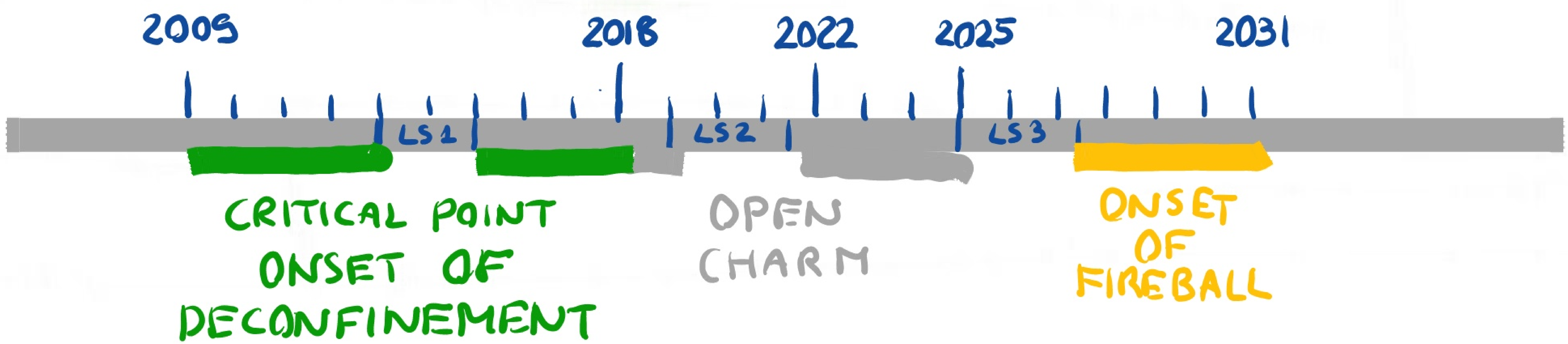


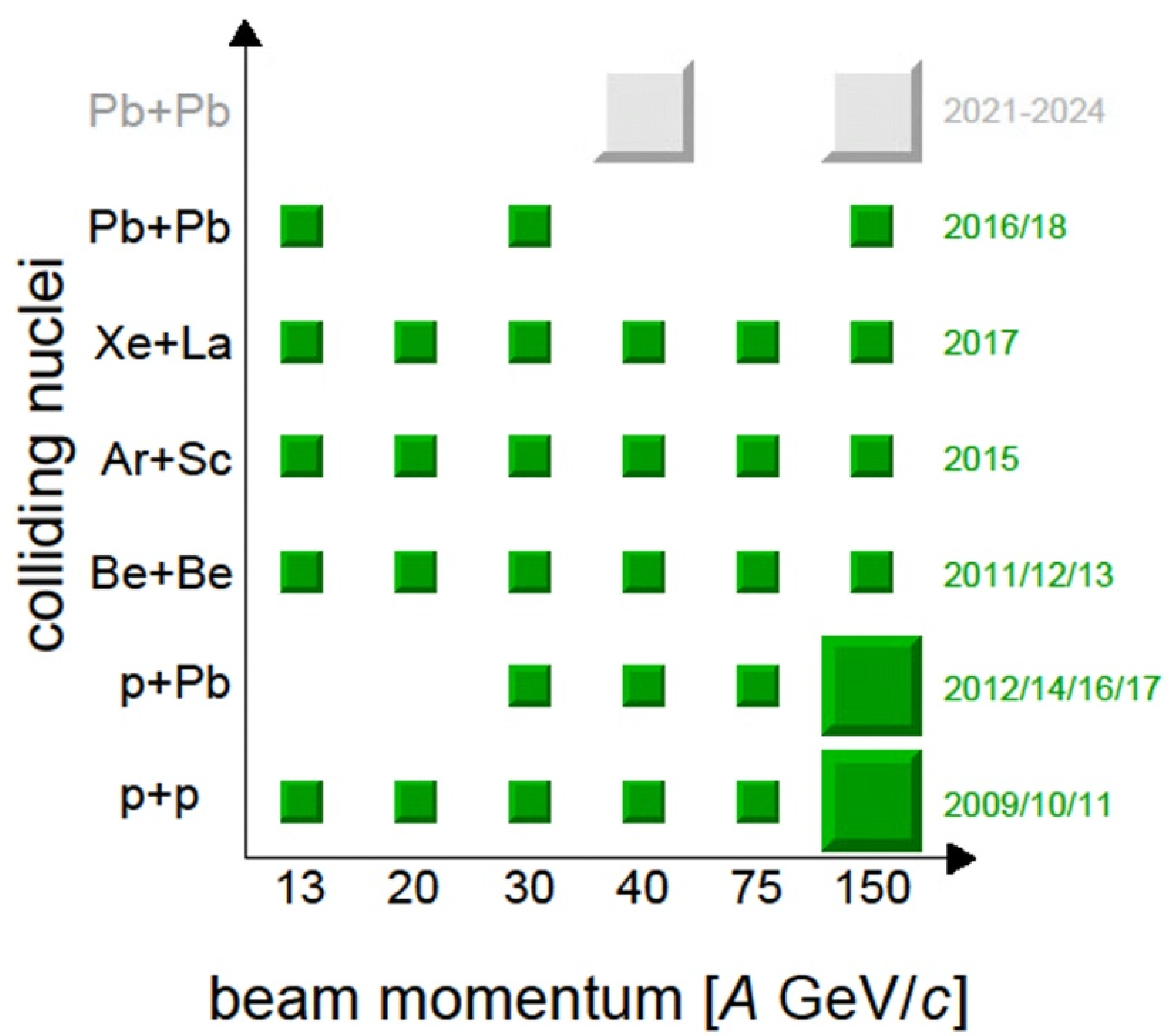
SCALE INVARIANT BRANCHING

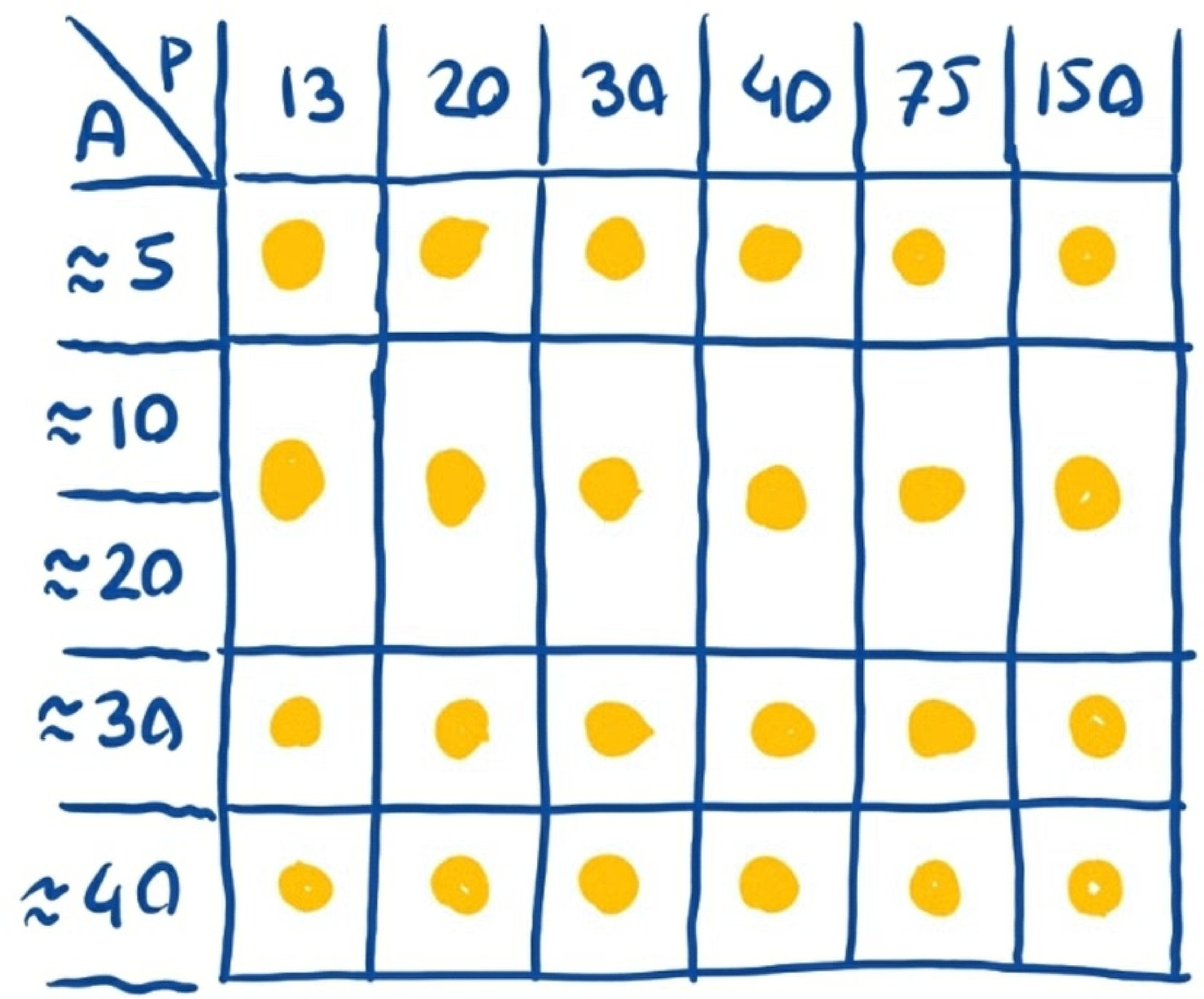
ONSET OF FIREBALL

MICROCANONICAL ENSEMBLE

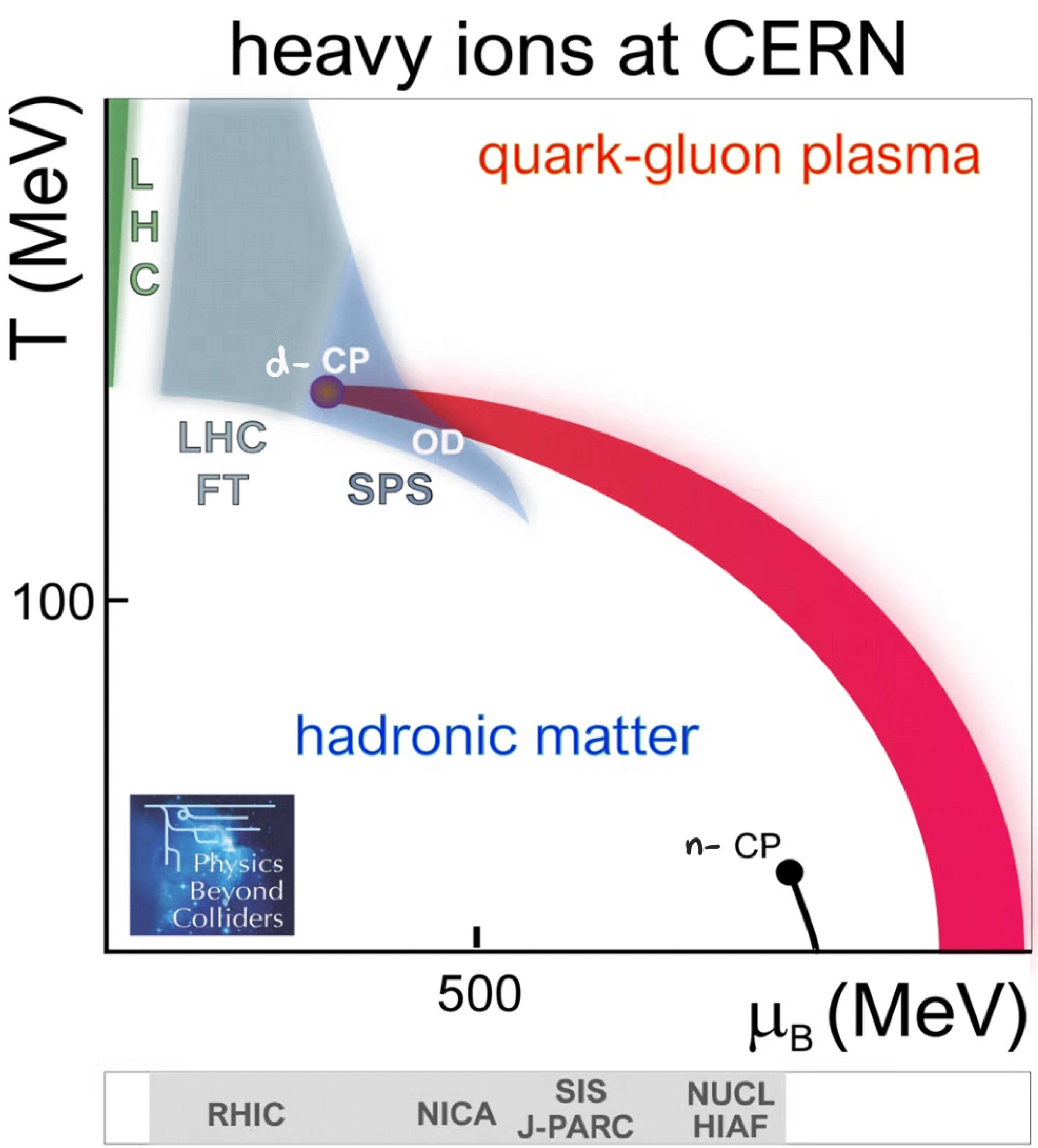
NA61/SHINE DATA TAKING FOR PHYSICS OF STRONG INTERACTIONS







CRITICAL-POINT SEARCH
(INTERMITTENCY)



DECONFINEMENT CRITICAL POINT

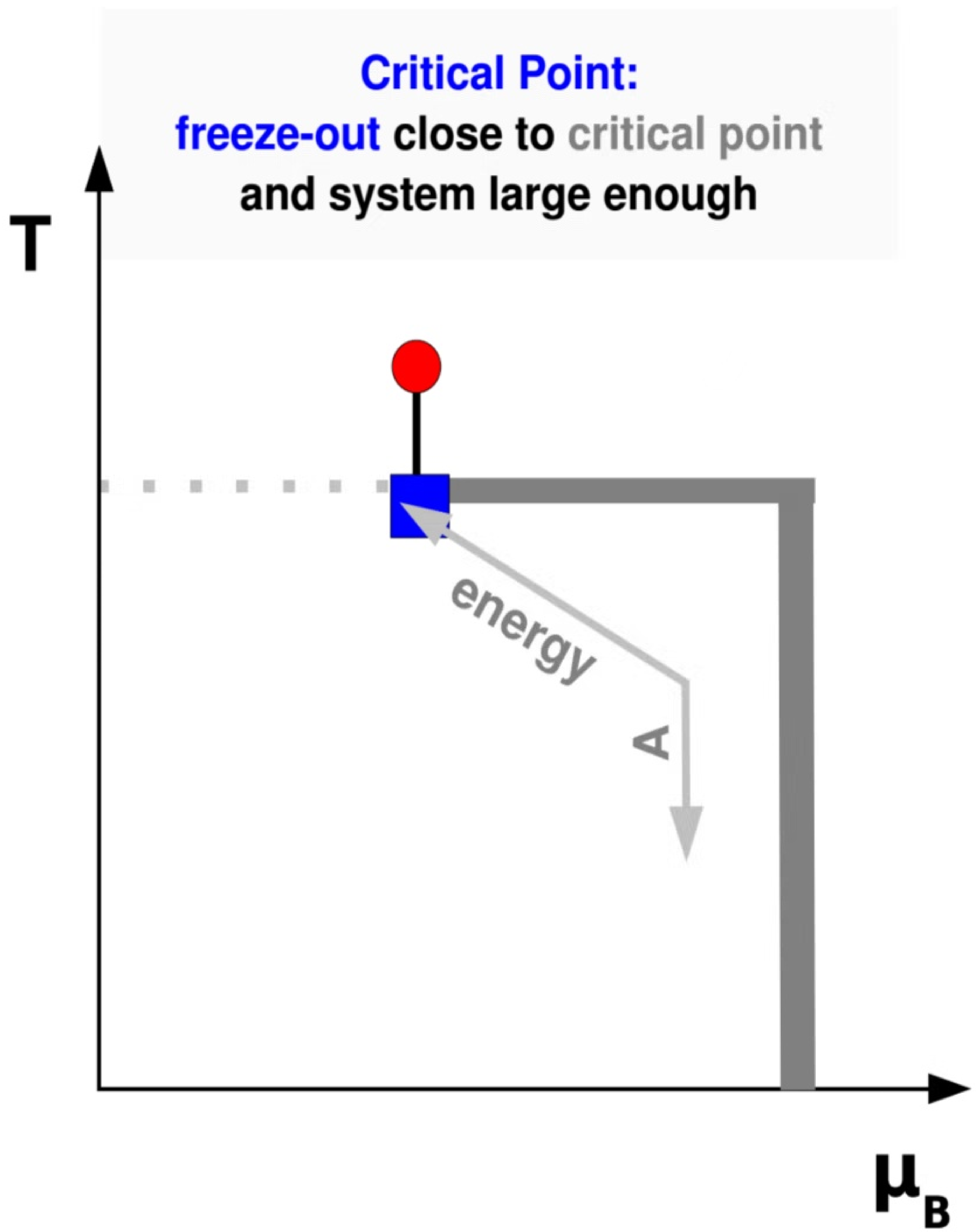
- HYPOTHETICAL END POINT OF FIRST ORDER TRANSITION LINE (QGP-HM) THAT HAS PROPERTIES OF SECOND ORDER TRANSITION

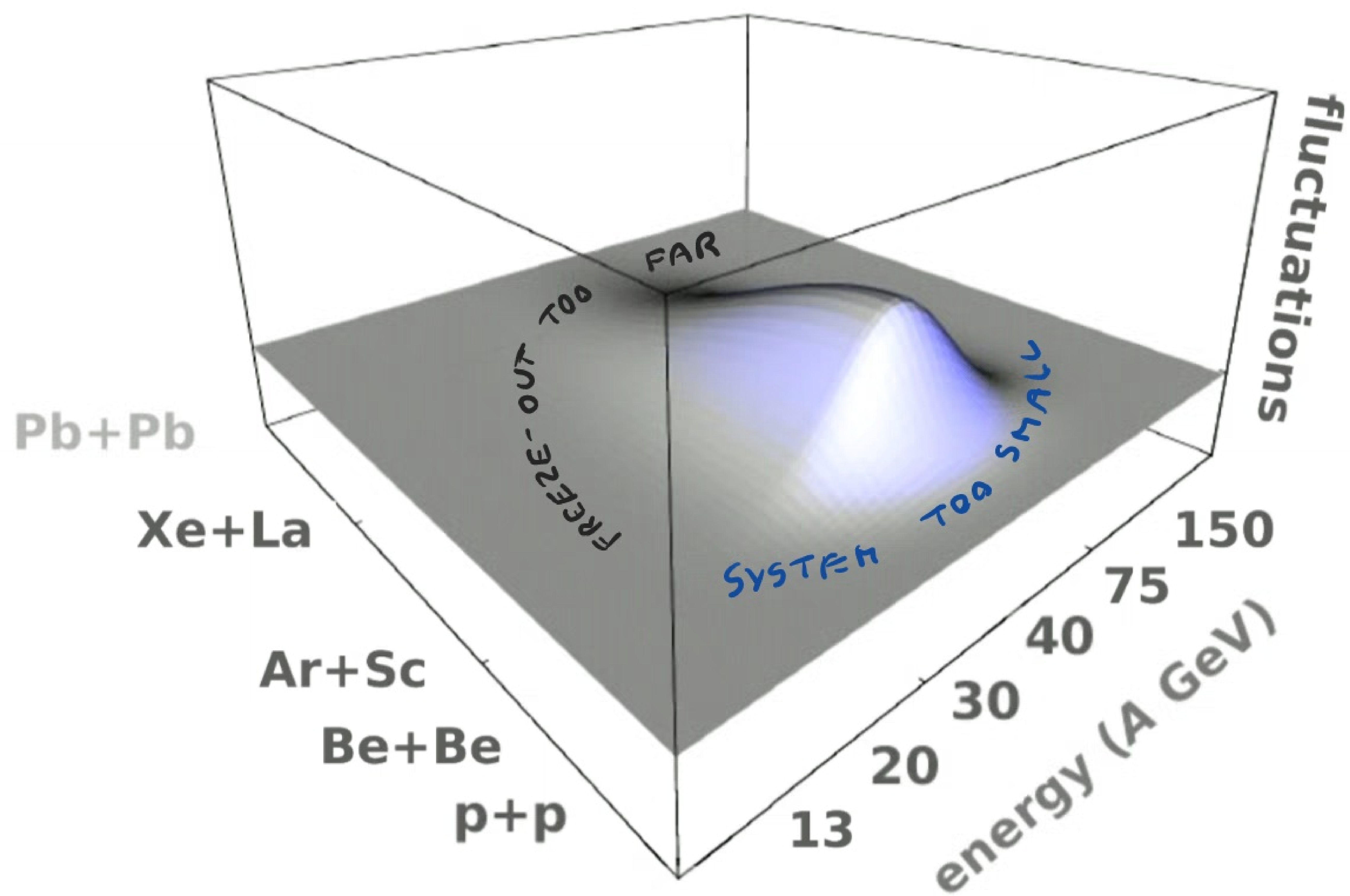
ASAKAWA, YAZAKI
NP A504 (89) 668

BARDUCCI ET AL.
PL B231 (89) 463

SEARCH FOR DECONFINEMENT-CP AT SPS

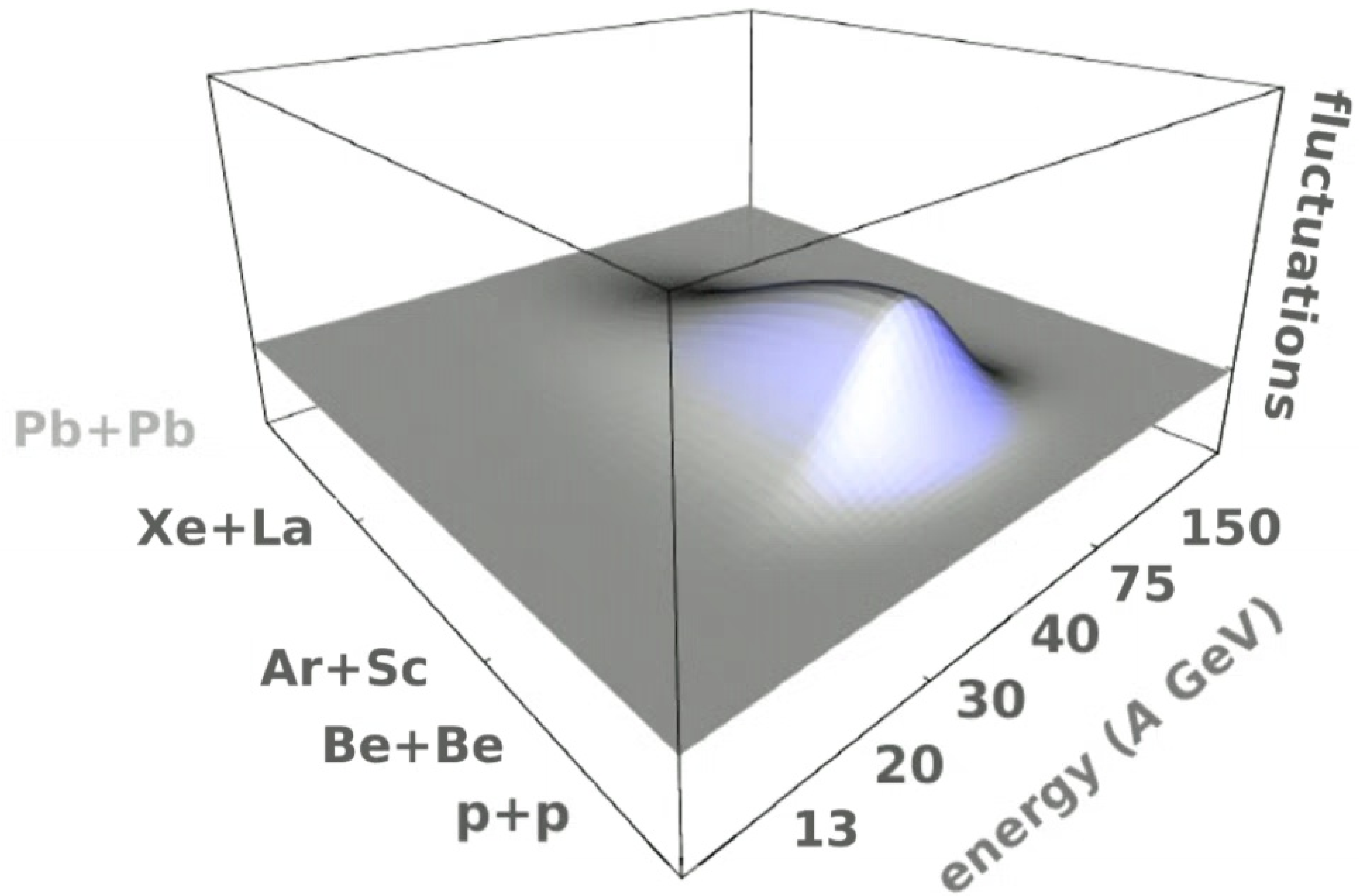
≡ SCAN IN $\sqrt{s_{NN}}$ AND A TO LOCATE FREEZE-OUT POINT CLOSE TO CP

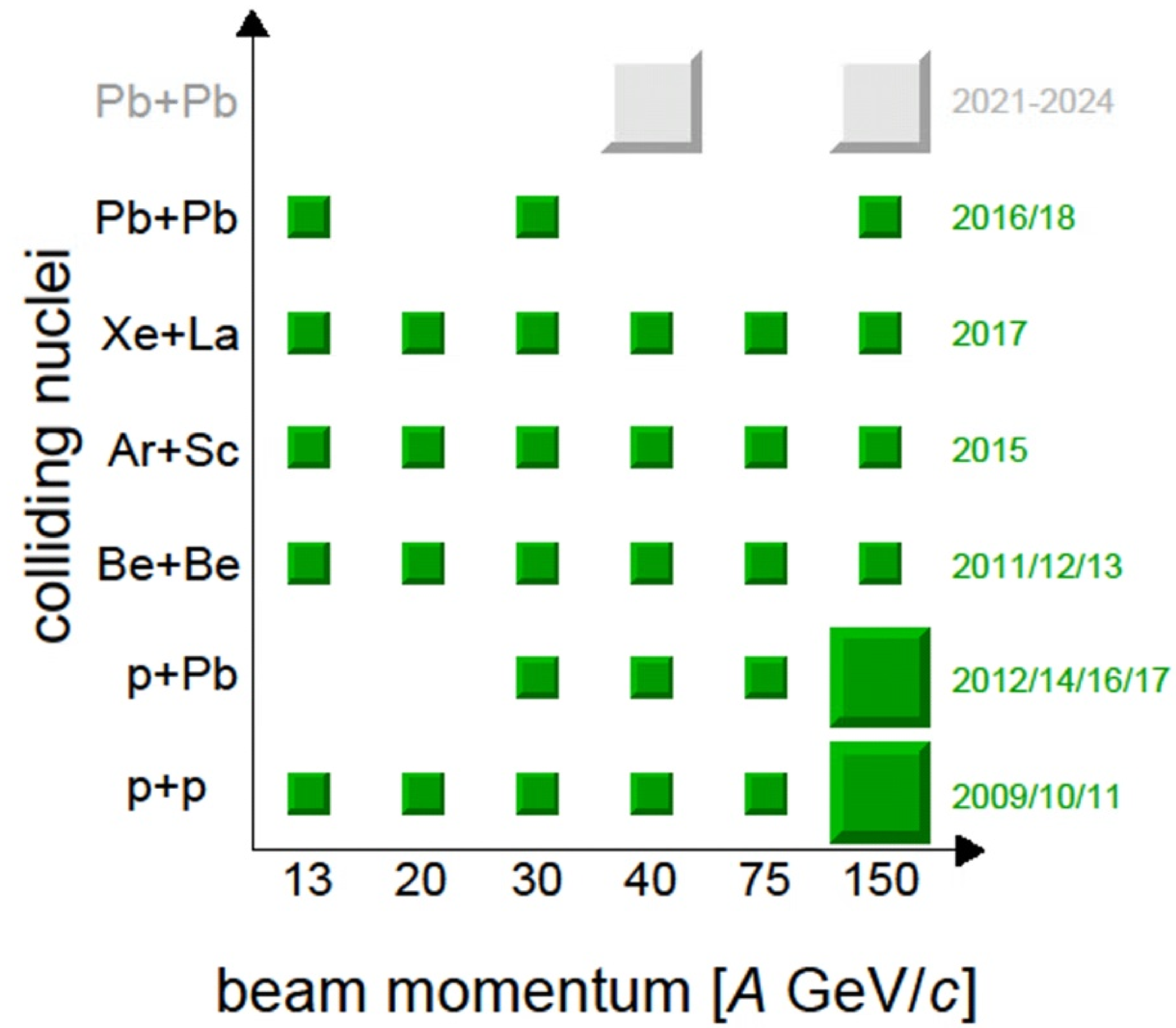




FLUCTUATIONS VS $\sqrt{s_{NN}}$ AND A

CP ⇒ "FLUCTUATION HILL" ⟶ NA61/SHINE 2D SCAN





d-CP AND ITS INTERMITTENCY SIGNAL

d-CP

↓

SECOND ORDER PT

↓

SCALE INVARIANCE

↓

POWER-LAW CORRELATION FUNCTION

↓

CHARACTERISTIC DEPENDECE OF FLUCTUATIONS ON BIN SIZE

↓

INTERMITTENCY ANALYSIS

1988 – 1991
d-CP GOLDEN YEARS

WOSIEK
APP B19 (88) 863

SATZ
NP B326 (89) 613

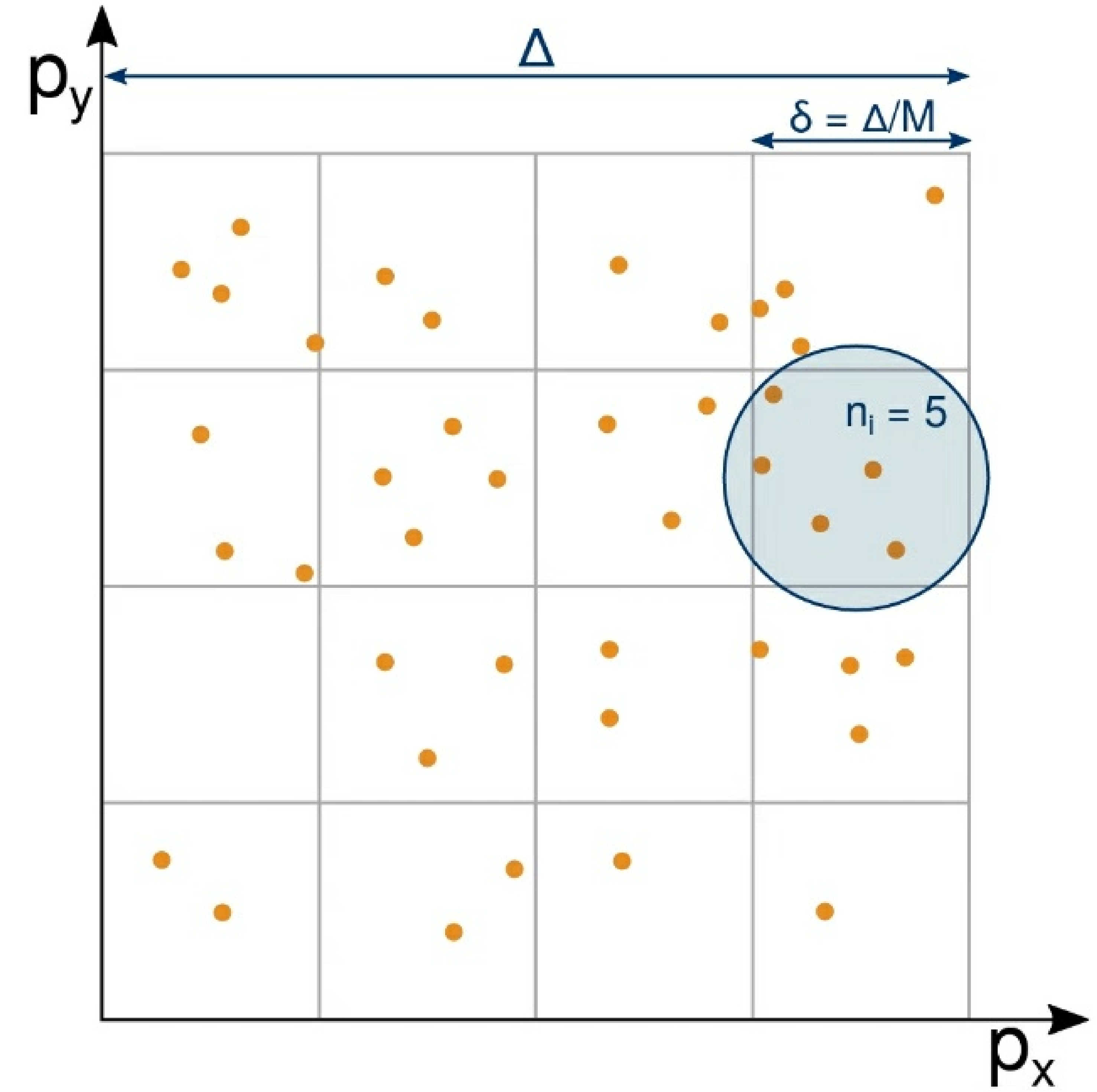
ASAKAWA, YAZAKI
NP A504 (89) 668

ANTONIOU ET AL.
PL B245 (90) 619

BARDUCCI ET AL.
PL B231 (89) 463

BIALAS, HWA
PL B253 (91) 436

INTERMITTENCY ANALYSIS



SCALED FACTORIAL MOMENTS:
(EXAMPLE FOR 2D SPACE)

$$F_r(M) \equiv \left\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i(n_i-1)\dots(n_i-r+1) \right\rangle \Big/ \left\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i \right\rangle^r$$

(MEAN NUMBER OF r- PARTICLE SETS IN BINS OF SIZE δ) / (MEAN MULTIPLICITY IN BINS OF SIZE δ)^r

n_i – PARTICLE IN BIN i

$\langle \dots \rangle$ – AVERAGING OVER EVENTS

CRITICAL POINT AND FACTORIAL MOMENTS

FOR $\langle n_i \rangle = \text{CONST}(i)$ (UNIFORM SINGLE PARTICLE DISTRIBUTION)

ONE GETS:

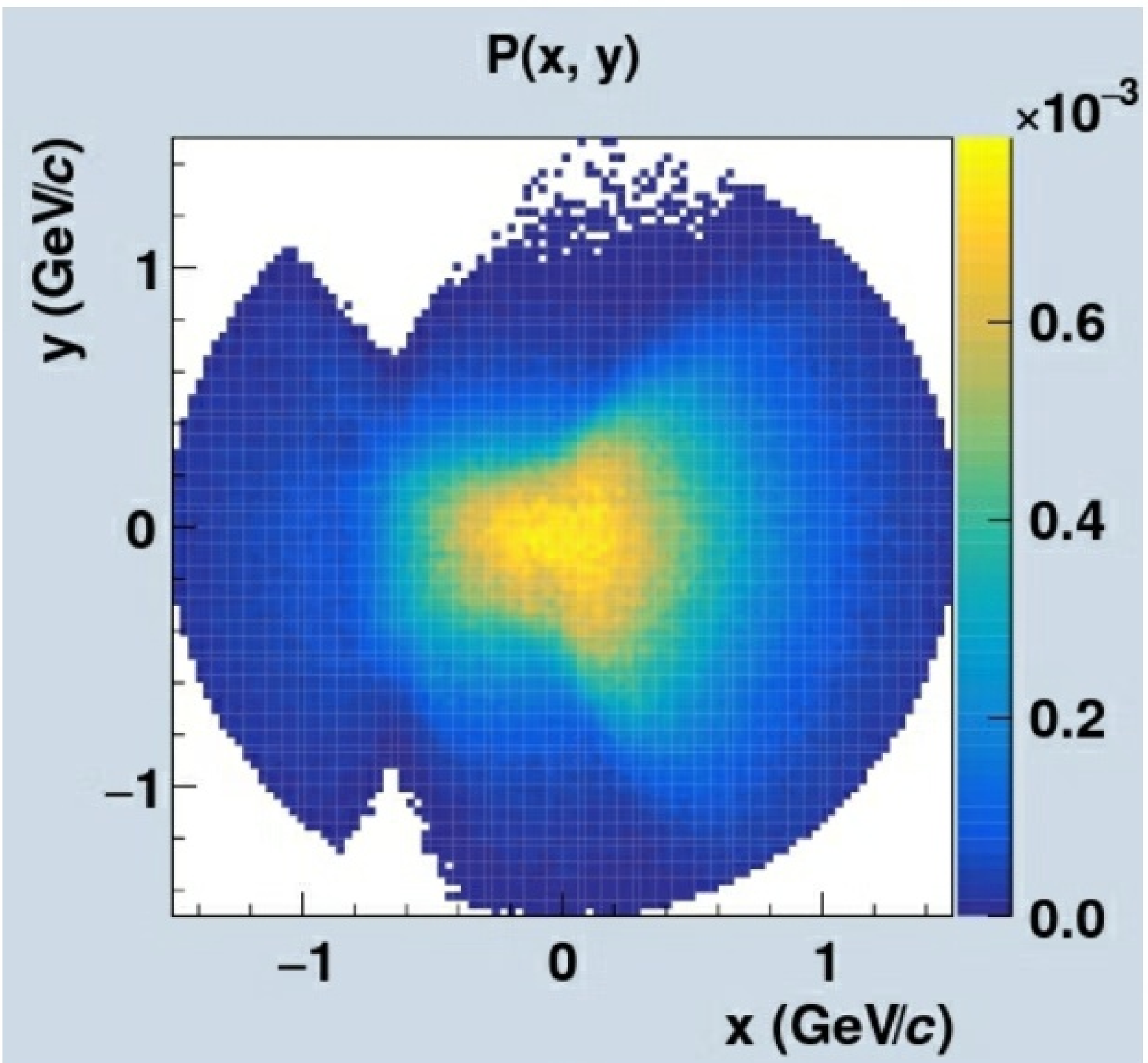
$$F_r(M) \sim \left(M^2 \right)^{\phi_r}$$

WITH THE ANOMALOUS FRACTAL DIMENSIONS:

$$d_r = \phi_r / (r-1) = \text{CONST}(r)$$

FROM NON-UNIFORM TO UNIFORM SINGLE PARTICLE PDF

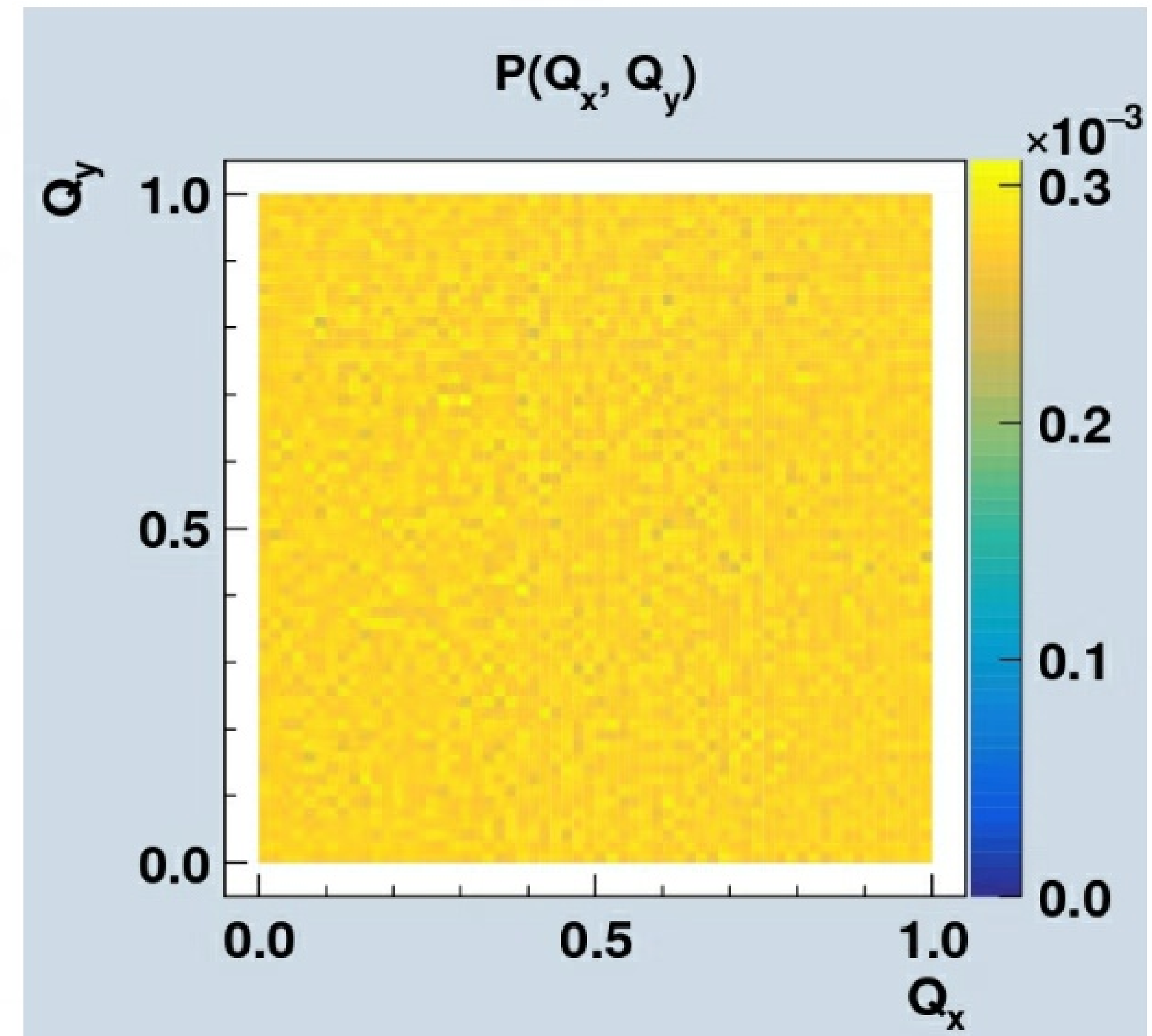
QUMULATIVE TRANSFORMATION



$$P(x) = \int_Y P(x,y)\,dy$$

$$Q_x = \int_{x_{MIN}}^{x} P(x)\,dx$$

$$Q_y(x) = \int_{y_{MIN}}^{y} P(x,y)\,dy \,/\, P(x)$$



BIALAS, MG
PL B252 (1990) 483

PROPERTIES OF CP - INTERMITTENCY SIGNAL

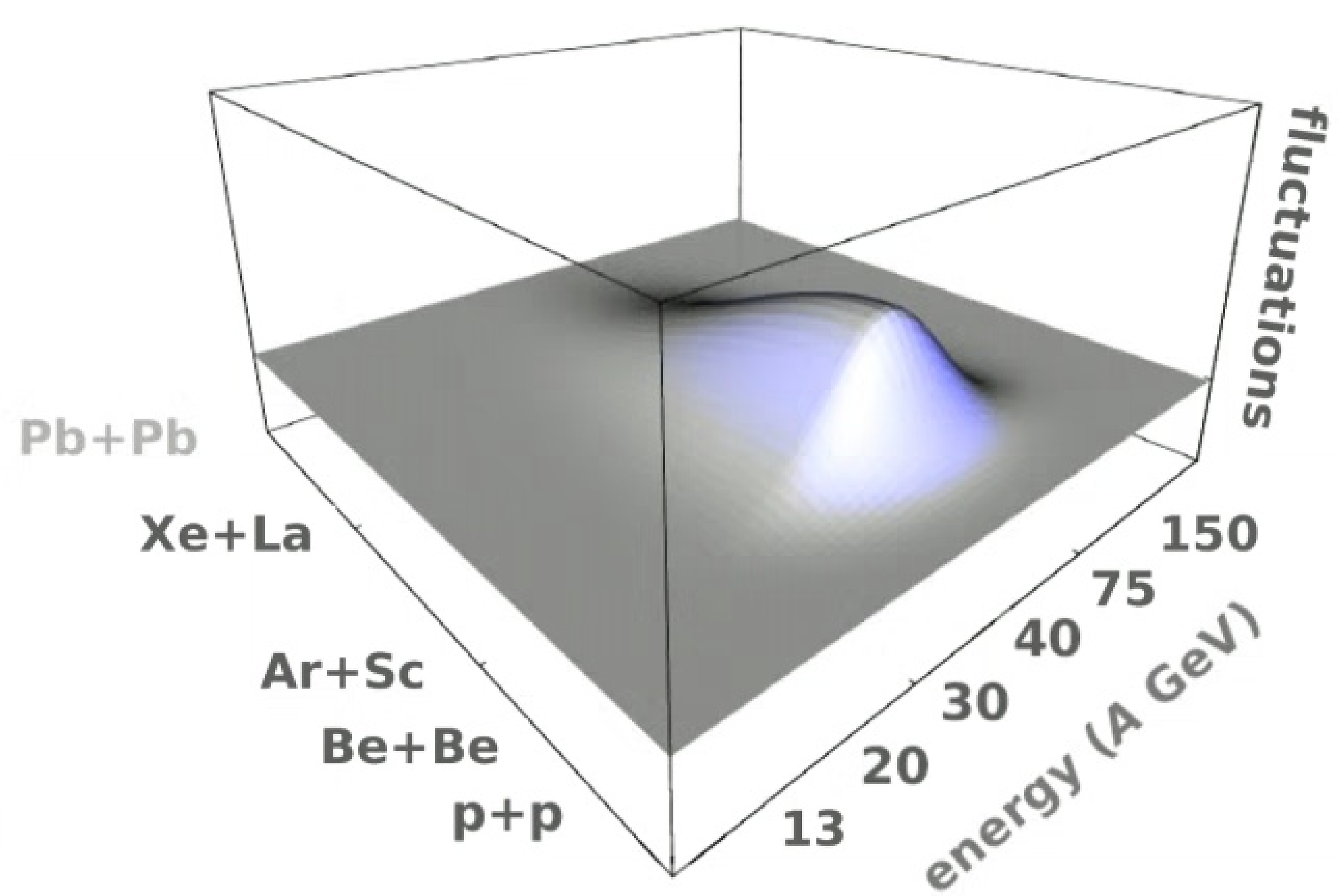
POWER LAW DEPENDENCE OF $F_r(M)$:

$$F_r(M) \sim (M^2)^{\phi_r}$$

ANOMALOUS FRACTAL DIMENSIONS INDEPENDENT OF ORDER

$$d_r = \phi_r/(r-1) = \mathrm{CONST}(r)$$

SIGNAL IN RESTRICTED (COLLISION ENERGY)-(SYSTEM SIZE) REGION



SIGNAL INSENSITIVE TO VOLUME FLUCTUATIONS

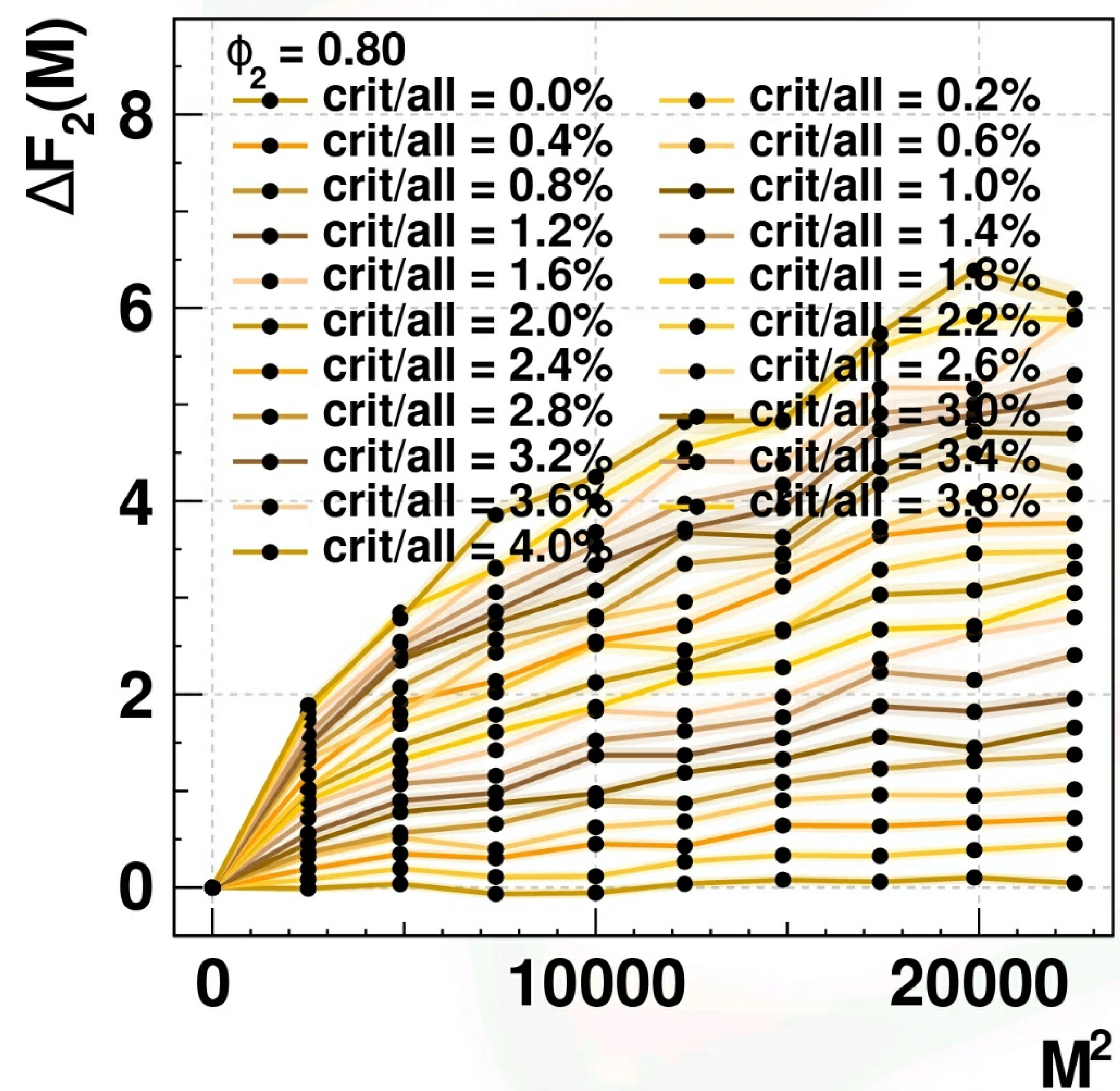
CP-TOY MODEL AND ITS PARAMETERS

Uncorrelated particles (background)

- $\rho_B(p_T) = p_T \cdot e^{-6p_T}$
- $\phi = \text{Uniform}(-\pi, +\pi)$
- $y^{\text{proton}}_{\text{LAB}} = \text{Uniform}(-0.75, 0.75) + y^{\text{beam}}_{\text{CMS}}$
- $p_x = p_T\cos(\phi)$
  $p_y = p_T\sin(\phi)$
  $p_z = m_T\sinh(y^{\text{proton}}_{\text{LAB}})$

Correlated pairs (signal)

- $$\rho_S(p_{T,1}, p_{T,2}) = \rho_B(p_{T,1}) \cdot \rho_B(p_{T,2}) \cdot \left[|\Delta p_x|^{\phi} + \epsilon\right]^{-1} \cdot \left[|\Delta p_y|^{\phi} + \epsilon\right]^{-1}$$

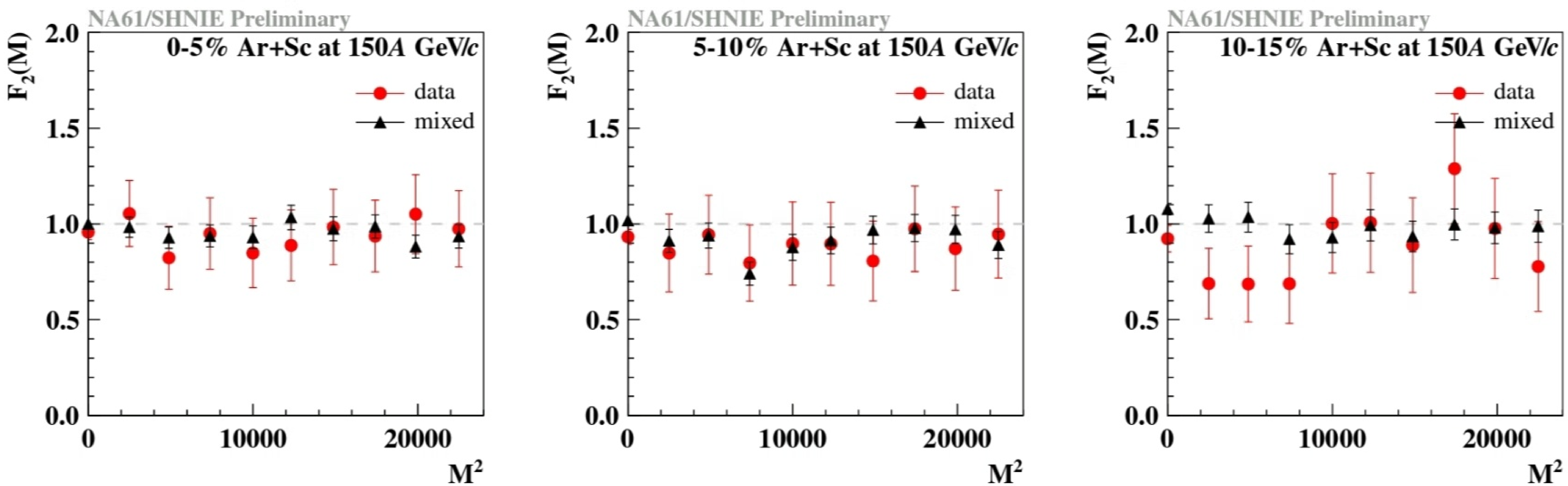


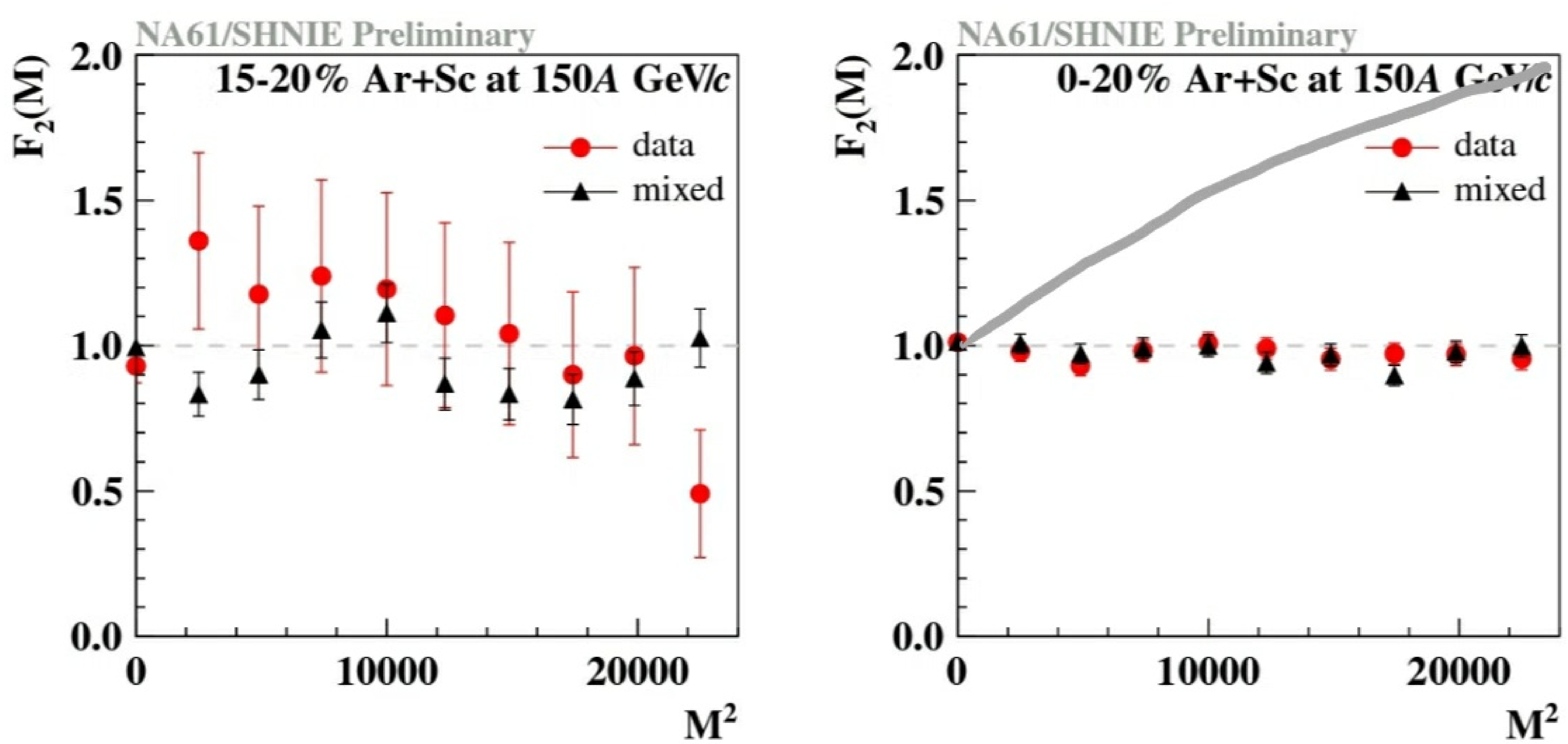
$\Delta F_2 = F_2(M) - F_2(1)$

CZOPOWICZ, PRIVATE COMM.

EXAMPLE RESULTS

PROTONS IN Ar+Sc COLLISIONS AT 150A GeV/c

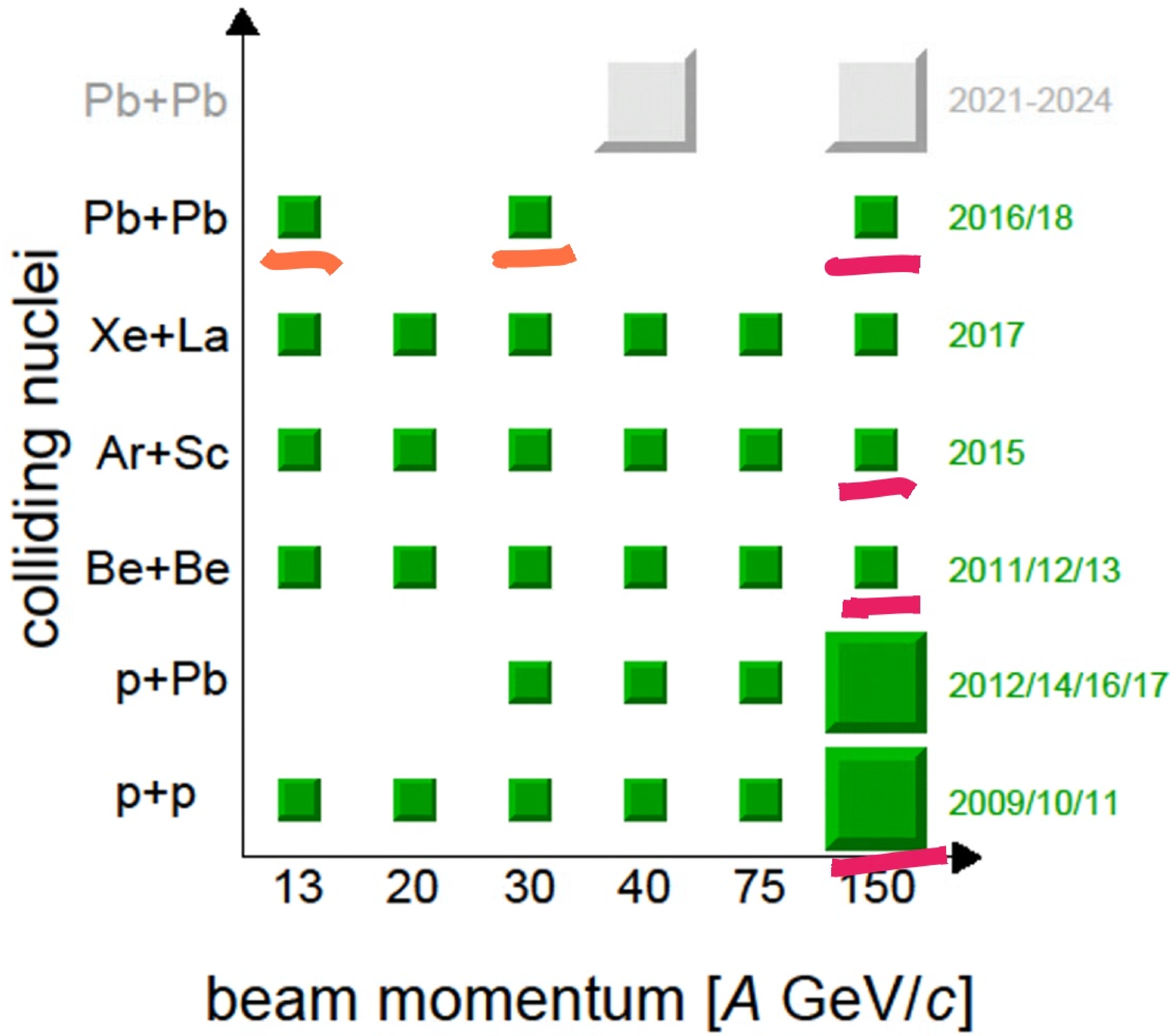




TOY MODEL: $\varphi_2 = 0.8$, $R \approx 1\%$
HIGH RESOLUTION OF NA61/SHINE DATA

NO SIGNAL OBSERVED YET

CRITICAL POINT SEARCH – INTERMITTENCY



RESULTS EXIST (NA61/NA49)

ONGOING ANALYSIS

NO SIGNAL OBSERVED YET

STILL MANY REACTIONS TO BE ANALYZED

FLUCTUATIONS AT CERN SPS

SUMMARY

- MULTIPLICITY FLUCTUATIONS

  $e^+ + e^-$, $p+p$, $Be+Be$ – SCALE INVARIAT BRANCHING

  ↓ ONSET OF FIREBALL

  $Ar+Sc$, $Pb+Pb$ – STAT. MODEL IN CE/MCE

- CRITICAL-POINT SEARCH: INTERMITTENCY

  NO SIGNAL OBSERVED YET

  STILL MANY REACTIONS TO BE ANALYZED