INDIANA UNIVERSITY
BLOOMINGTON

Jefferson Lab

Fundamental Symmetries

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*Supported by NSF

Outline :

1. Introduction and Motivation

2. The Standard Model

3. Selected examples
   1. $\eta \rightarrow 3\pi$ and light quark mass ratio
   2. Anomalous magnetic moment of the muon
   3. Axial form factor of the nucleon and neutrino physics

4. Conclusion and outlook

1. Introduction and Motivation

1.1 The Standard Model

- Particle and Nuclear Physics
  - extract fundamental parameters of Nature on the smallest scale
  - test our understanding of Laws of Nature

1.1 Precise test of the Standard Model

- Particle and Nuclear Physics
  - extract fundamental parameters of Nature at Quantum Level
  - test our understanding of Laws of Nature

- In Chemistry our knowledge summarized by Mendeleev table of chemical elements

Group → / Period ↓	1	2	3		4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 H																		2 He
2	3 Li	4 Be												5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg												13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca	21 Sc		22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y		40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	57 La	*	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	89 Ac	**	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Nh	114 Fl	115 Mc	116 Lv	117 Ts	118 Og
				*	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	
				**	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr	

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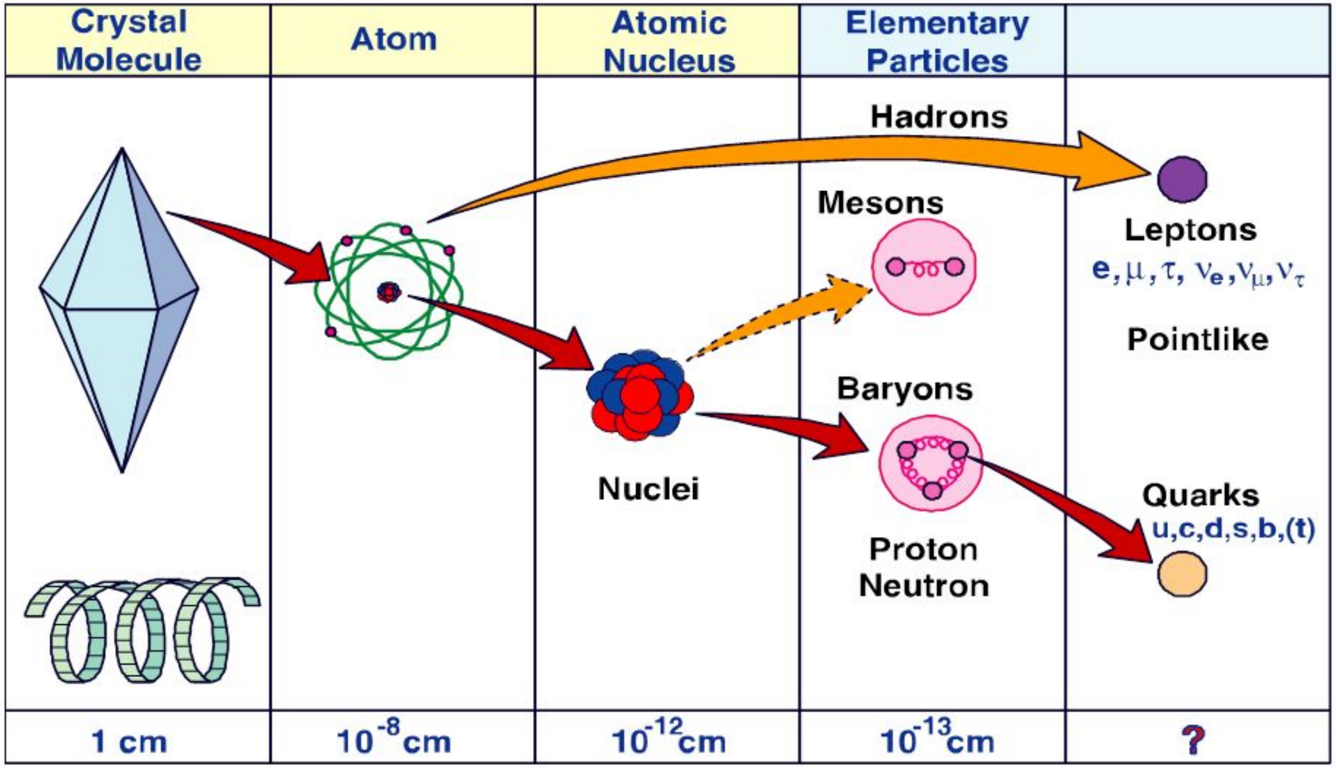
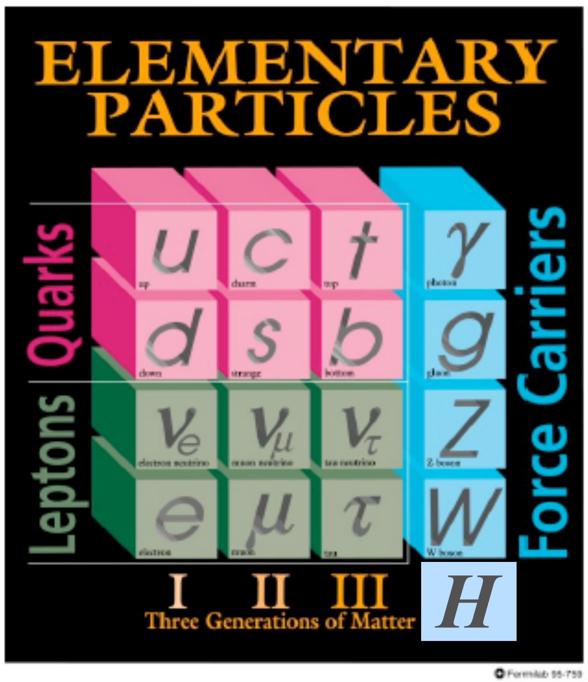
- In particle physics a simpler table made of leptons and quarks



1.1 The Standard Model

- In particle physics a simpler table made of leptons and quarks: the degrees of freedom



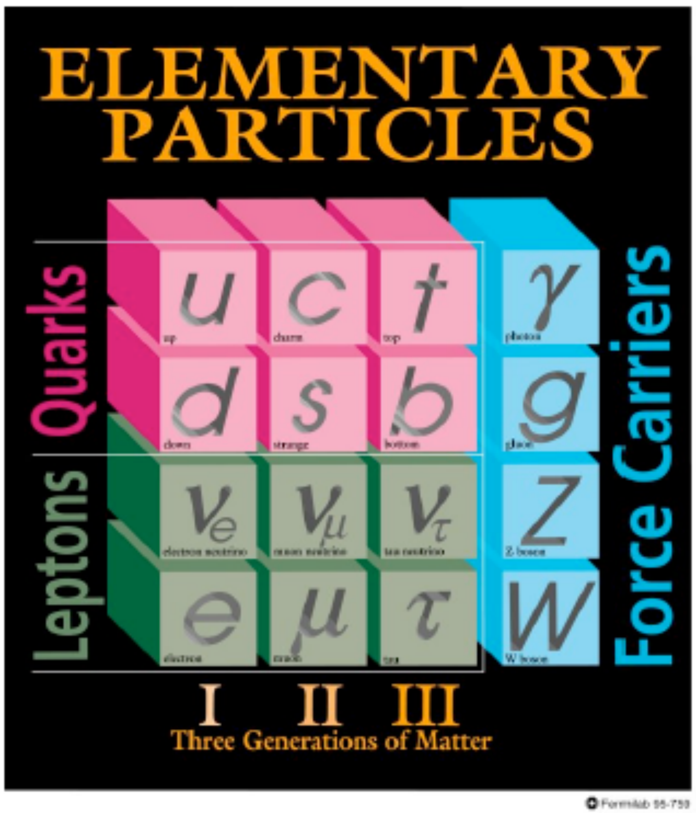
Charge				Int.
0	ν_e	ν_μ	ν_τ	W
-1	e	μ	τ	W,e
+2/3	u	c	t	W,e,S
-1/3	d	s	b	

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

- 3 forces: electromagnetic, weak and strong forces

1.1 The Standard Model



Governed by gauge symmetry principle

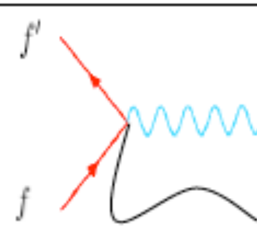
$$SU(3)_C \times \underbrace{SU(2)_{I_w} \times U(1)_Y}$$

Strong force

Unified Electro-weak interactions

Introduce massless gauge bosons (force carriers)

↓



A_μ

e, g_{strong} , g_{weak}

$$\mathcal{L}_I(x) \sim J_\mu(x)\, A^\mu(x)$$

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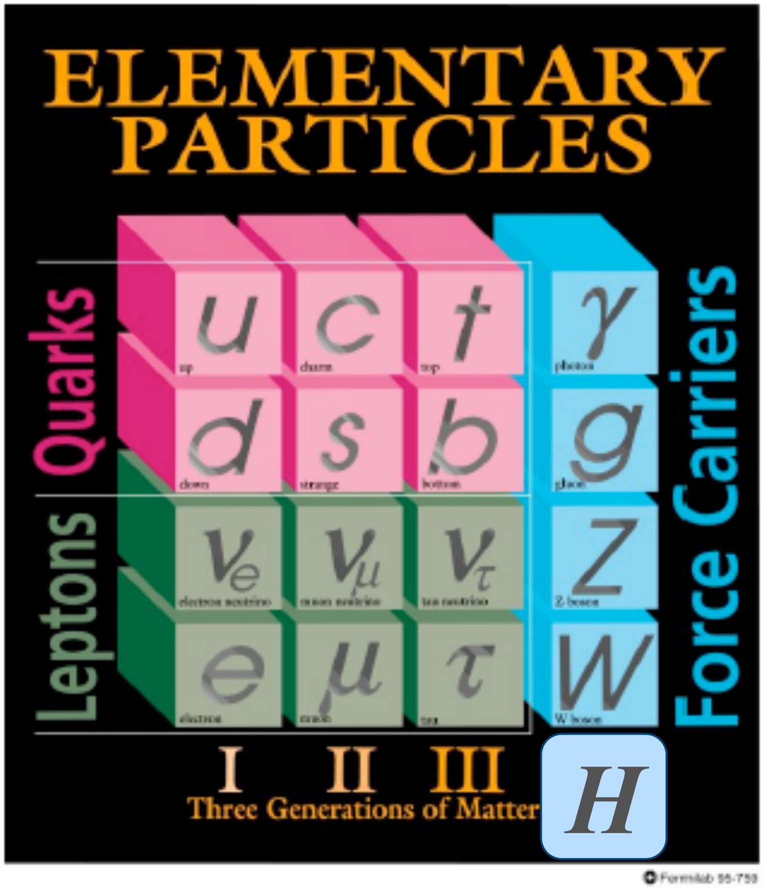
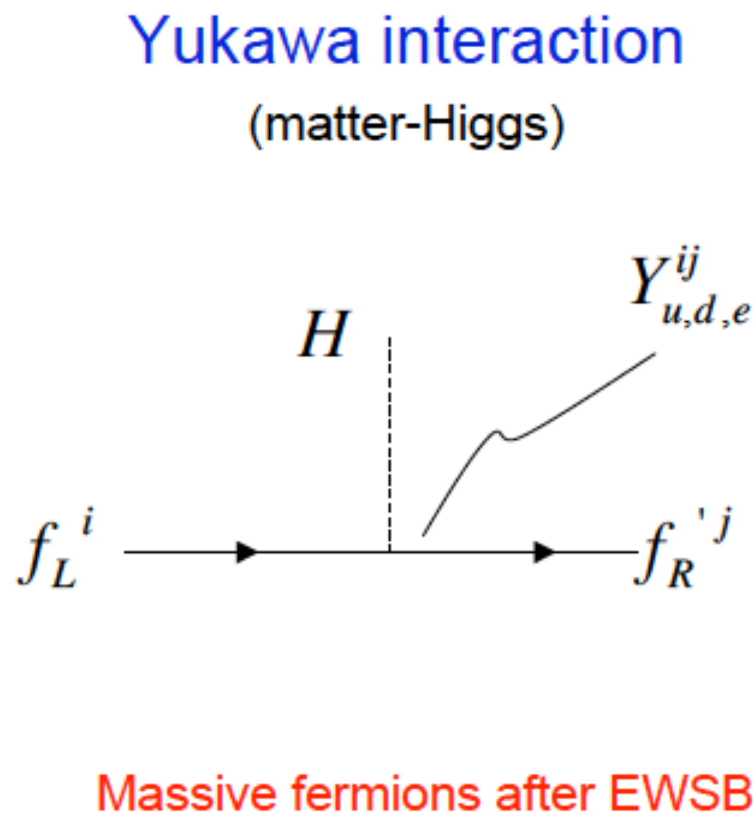
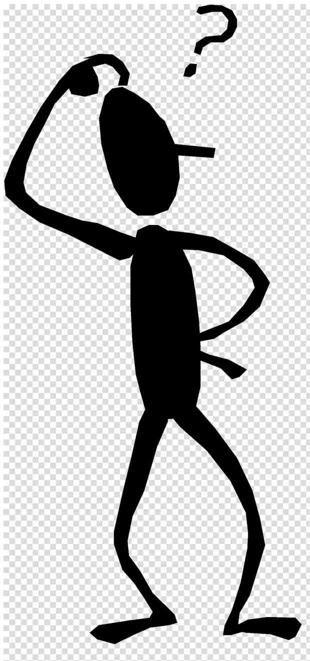




The mediators of weak interaction (W, Z) become massive through the Higgs Mechanism ⟹ one scalar particle remains in the spectrum: H

1.2 Challenges



- Searching physics beyond the Standard Model:
  - Are there new forces besides the 3 gauge groups?
  - Are there new particles?
  - A more profound understanding of the origin of this table?
  - Origin of matter/anti-matter asymmetry
  - Origin of dark matter

- One type of new physics already discovered: neutrino masses

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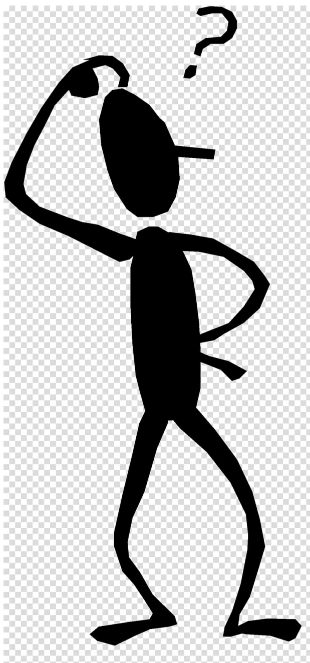

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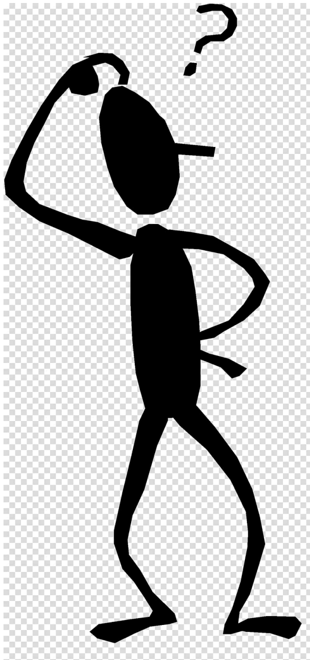

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- In this quest it is essential to have a *robust understanding* of *Hadronic Physics*

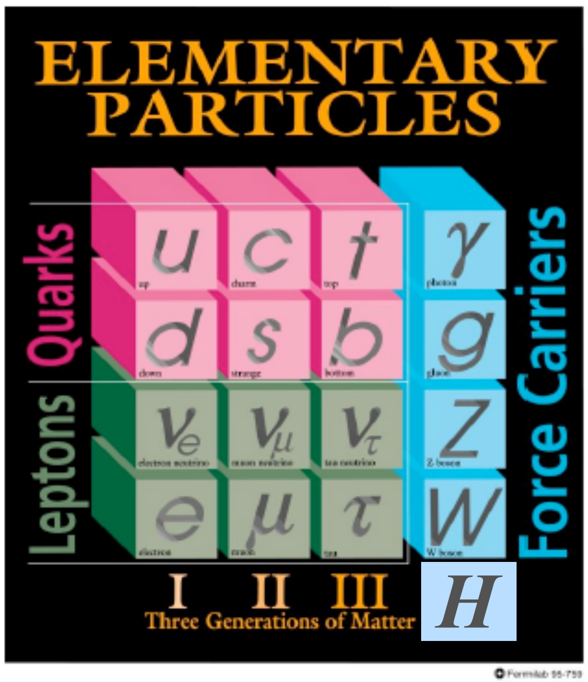
- This is true for quarks and leptons and even for neutrinos!

2. The Standard Model

See A. Pich, 1201.0537
Halzen & Martin, Quarks & Leptons

2.1 Introduction

- In particle physics a simpler table made of leptons and quarks: the degrees of freedom



Charge: 0, -1 (leptons); +2/3, -1/3 (quarks)

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Int.: w (neutrinos), w,e (charged leptons), w,e,s (quarks)

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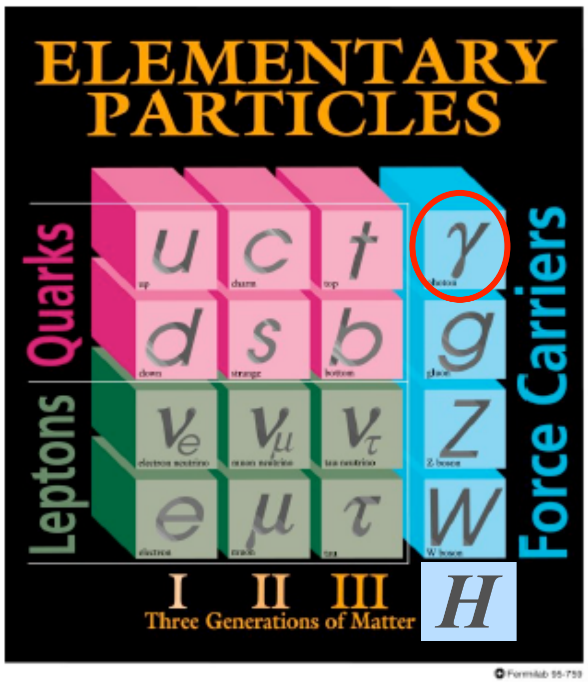

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Theoretical Formulation

- Lagrangian describing a free Dirac fermion:

$$\mathcal{L}_0 = i\bar{\psi}(x)\gamma^{\mu}\partial_{\mu}\psi(x) - m\bar{\psi}(x)\psi(x)$$

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- Gauge principle: global U(1) transformations → *local*, i.e., space-time dependent $\theta \to \theta(x)$

$$\partial_\mu \psi(x) \quad \stackrel{U(1)}{\longrightarrow} \quad \exp\{iQ\theta\}\ (\partial_\mu + iQ\,\partial_\mu\theta)\ \psi(x)$$

- $\mathcal{L}_0$ is no longer invariant !

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- $\mathcal{L}_0$ is no longer invariant ! ⇨ Add an extra piece to the Lagrangian
Introduce a new spin-1 (since $\partial_\mu\theta$ has a Lorentz index) field $A_\mu(x)$:

$$A_\mu(x) \quad \xrightarrow{U(1)} \quad A'_\mu(x) \ \equiv\ A_\mu(x) - \frac{1}{e}\,\partial_\mu\theta$$

Theoretical Formulation

- We define a covariant derivative

$$\partial_\mu \psi(x) \overset{U(1)}{\rightarrow} \color{red}{D_\mu}\psi(x) \equiv \left[\partial_\mu + ieQA_\mu\right]\psi(x)$$

which transforms as $\psi(x)$ itself

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$$= i\bar{\psi}(x)\gamma^\mu \partial_\mu \psi(x) - m\bar{\psi}(x)\psi(x) - eQA_\mu \bar{\psi}(x)\gamma^\mu \psi(x)$$

$$= \mathcal{L}_0 - eQA_\mu \bar{\psi}(x)\gamma^\mu \psi(x)$$

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Quantum Electrodynamics

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$$\mathcal{L}_{QED} = i\bar{\psi}(x)\gamma^\mu D_\mu \psi(x) - m\bar{\psi}(x)\psi(x) - \frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)$$

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$$

⇨ *Kinetic term for* A_μ

Quantum Electrodynamics

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- NB: A mass term for A_μ : $$\mathcal{L}_m = \frac{1}{2}m^2 A_{\mu}(x)A^{\mu}(x)$$

  is forbidden because it would violate the local U(1) gauge invariance

  ⇨ A_μ is predicted to be massless.

  Experimentally, $$m_\gamma < 1\times 10^{-18}\ \text{eV}$$ *Ryutov'07 PDG'21*

Anomalous magnetic moments

- $$\mathcal{L}_{QED} = i\bar{\psi}(x)\gamma^{\mu}D_{\mu}\psi(x) - m\bar{\psi}(x)\psi(x) - \frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)$$

- QED is a very successful quantum field theory

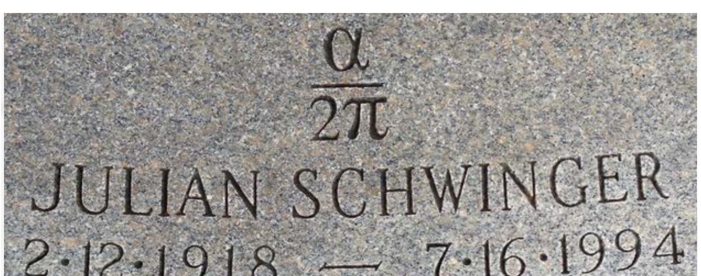
- The most stringent QED test comes from the high-precision measurements of the electron and muon *anomalous magnetic moments*:

$$a_l \equiv (g_l^{\gamma} - 2)/2 \qquad \text{with} \qquad \vec{\mu}_l \equiv g_l^{\gamma}\,(e/2m_l)\,\vec{S}_l$$

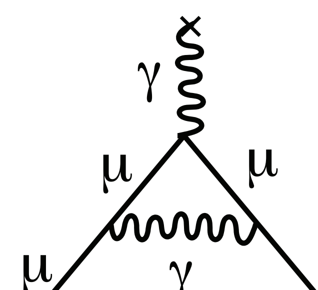
- g was predicted by Dirac to be 2

QED

- *Schwinger* computed the first order correction in 1948



$$a_e = \frac{\alpha}{2\pi} \approx 0.001\,161\,4$$



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- Experimentally $$a_e = (1\ 159\ 652\ 180.73 \pm 0.28) \cdot 10^{-12}$$

  Hanneke, Fogwell Hoogerheide, Gabrielse'11

  and $$a_\mu = (11\ 659\ 206.1 \pm 4.1) \cdot 10^{-10}$$

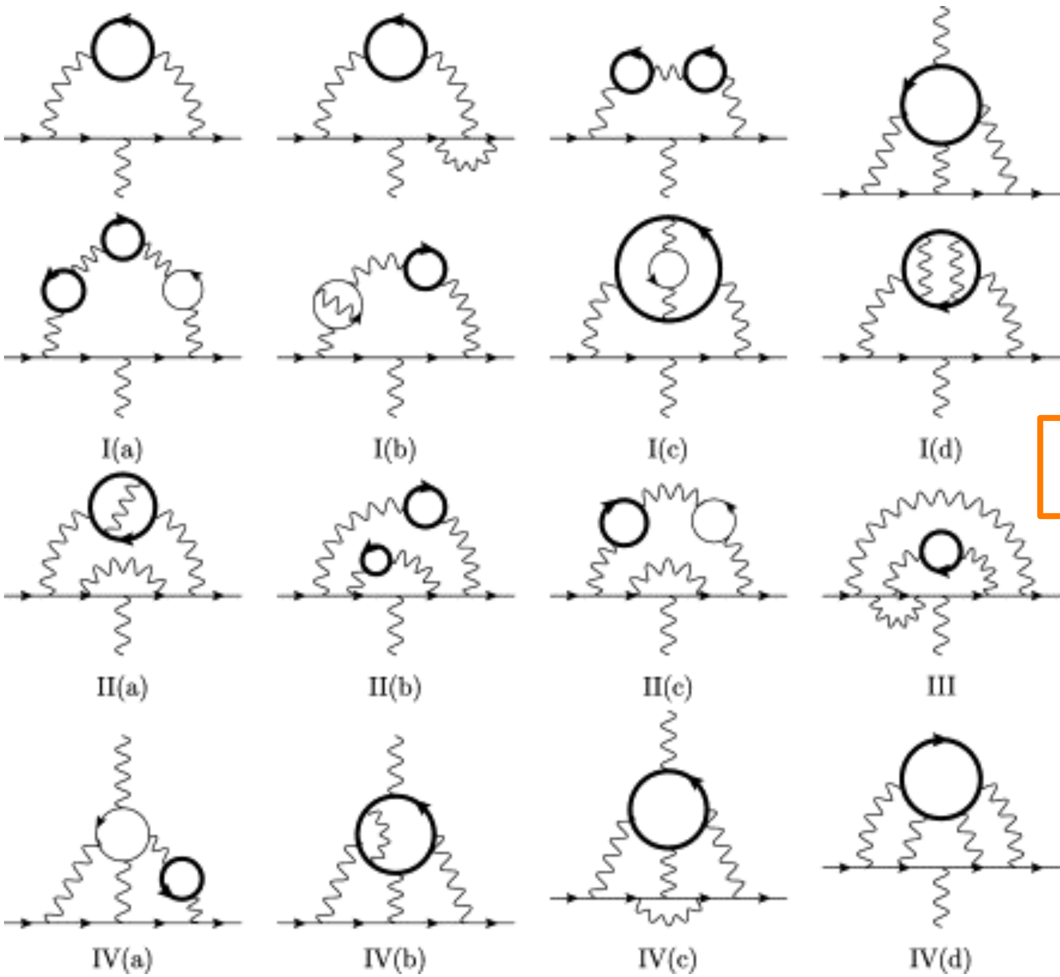
  E821, BNL'04 +
  Muon g-2, FNAL'21

- These are incredible levels of precision !

Anomalous magnetic moments

- To a measurable level, a_e arises entirely from *virtual electrons* and *photons*

  ⇨ fully known to $O(\alpha^4)$ and many $O(\alpha^5)$ corrections computed



Kinoshita & Nio, Aoyama et al.'03-12, Passera'05,'07, Laporta'93, Kataev'06, Kurz et al.'13, etc

$$a_e^{\text{theo}} = (1\ 159\ 652\ 181.643 \pm 0.764) \cdot 10^{-12}$$

and

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Impressive agreement on a_e between theory and experiment

⇨ *QED very successful theory to describe Nature.*

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- The theoretical error dominated by uncertainty on $\alpha_{QED} \equiv e^2/(4\pi)$

- Turning things around, a_e provides the most accurate determination of α_{QED}

⇨ $$\alpha^{-1} = 137.035\ 999\ 084\ \pm\ 0.000\ 000\ 051$$

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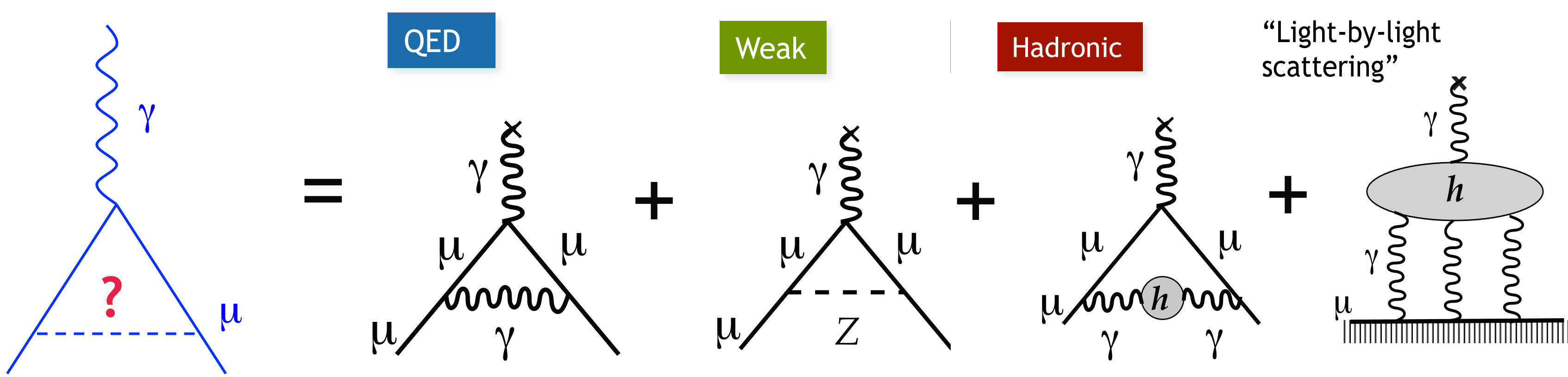
- On the muon side: a_μ is sensitive to small corrections from virtual heavier states; compared to a_e, they scale with the mass ratio $(m_\mu/m_e)^2$.



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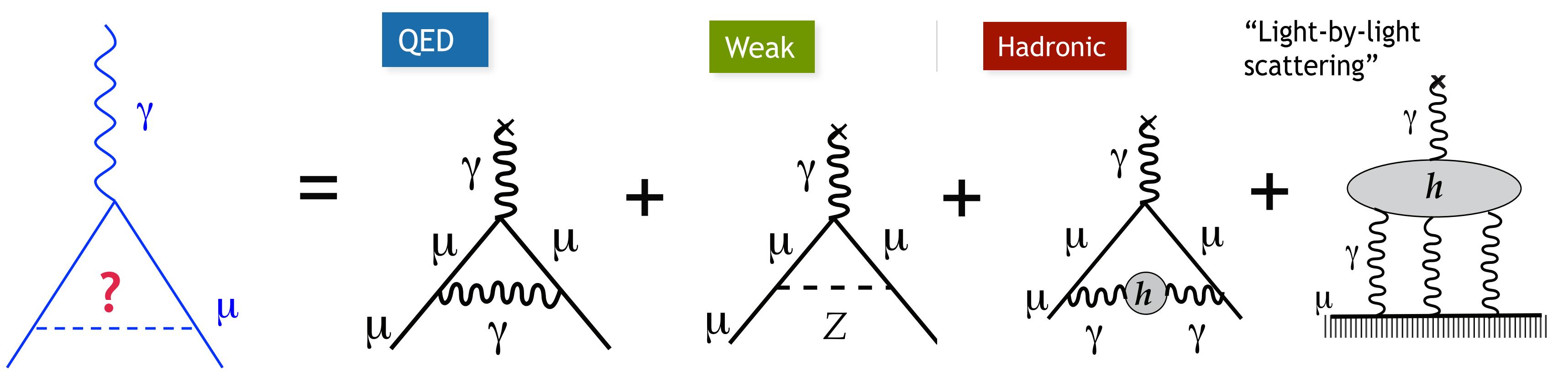

Test the whole Standard Model ⇨ See later

2.3 Electroweak Interactions

Weak Interactions: Introduction

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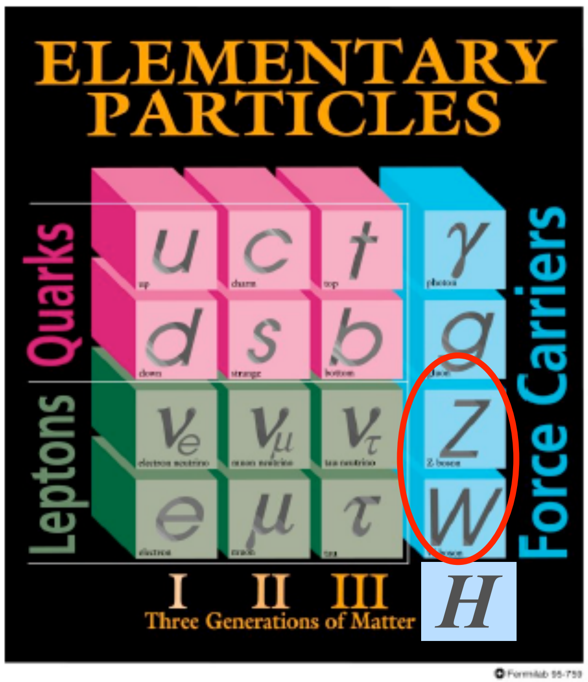

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Electroweak Interactions: Charged Currents

Experimentally: weak interaction exhibits interesting characteristics:

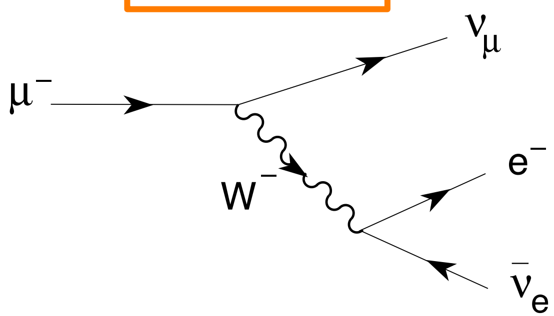
- **Charged Currents:** The interaction of quarks and leptons with the $W^\pm$ bosons:
  - W couples only to *left-handed fermions* and *right-handed antifermions*

Parity (P: left ↔ right)

⇨ Charge conjugation (C: particle ↔ antiparticle) *not conserved*

But *CP* is still a *good symmetry*.

$\mu^- \to e^- \bar{\nu}_e \nu_\mu$



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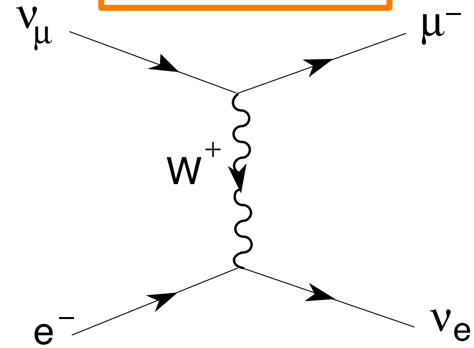

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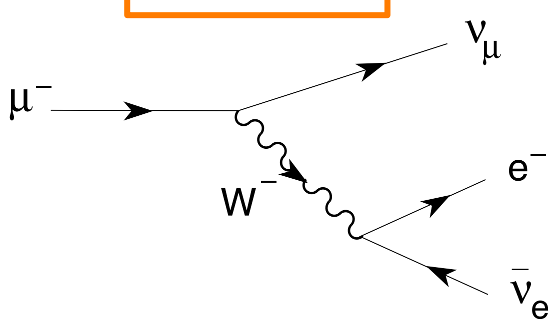

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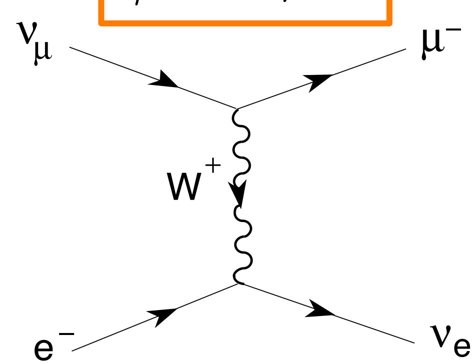


  - W couples only to fermionic doublets with g : universal coupling

$$\begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L , \quad \begin{pmatrix} q_u \\ q_d \end{pmatrix}_L$$

Electroweak Interactions: Charged Currents

Experimentally: electroweak interaction exhibits interesting characteristics:

- The doublet partners of the up, charm and top quarks appear to be mixtures of the three quarks with charge − 1/3
  ⇨ the weak eigenstates are different than the mass eigenstates:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

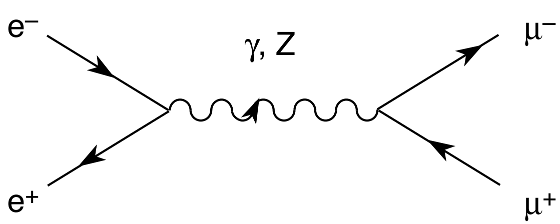
Weak Eigenstates   CKM Matrix   Mass Eigenstates

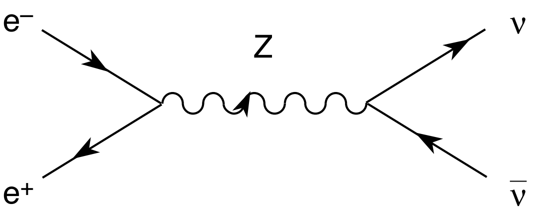
Unitary 3x3 Matrix, parametrizes rotation between mass and weak interaction eigenstates in Standard Model

Electroweak Interactions: Neutral Currents

Experimentally: electroweak interaction exhibits interesting characteristics:

- **Neutral Currents:** The interaction of quarks and leptons with the Z boson: or phtoton
  - All interacting vertices are flavour conserving.





  - The interactions depend on the fermion electric charge Q_f for em interactions Neutrinos do not have electromagnetic interactions (Q_ν = 0), but they have a non-zero coupling to the Z boson.
  - The Z couplings are different for left-handed and right-handed fermions. The neutrino coupling to the Z involves only left-handed chiralities.
  - There are three different light neutrino species.

Theoretical formulation

- Theory should give:
  - different properties for left- and right-handed fields;
  - left-handed fermions should appear in doublets
  - massive gauge bosons $W^{\pm}$ and Z in addition to the photon.

- Gauge group: $G \equiv SU(2)_L \otimes U(1)_Y$ with L for left-handed fermion

- Degrees of freedom:

$$\psi_1(x) \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L \text{ or } \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L , \qquad \psi_2(x) \equiv u_R \text{ or } \nu_{eR} , \qquad \psi_3(x) \equiv d_R \text{ or } e^-_R$$

- The free Lagrangian:

$$\mathcal{L}_0 = i\bar{u}(x)\gamma^\mu \partial_\mu u(x) + i\bar{d}(x)\gamma^\mu \partial_\mu d(x) = i\sum_{j=1}^{3} \bar{\psi}_j \gamma^\mu \partial_\mu \psi_j$$

Theoretical formulation

- $\mathcal{L}_0$ is invariant under *global* G transformations

$$\psi_1(x) \xrightarrow{G} \psi_1'(x) \equiv \exp\{iy_1\beta\}\, U_L\, \psi_1(x)\,,$$
$$\psi_2(x) \xrightarrow{G} \psi_2'(x) \equiv \exp\{iy_2\beta\}\, \psi_2(x)\,,$$
$$\psi_3(x) \xrightarrow{G} \psi_3'(x) \equiv \exp\{iy_3\beta\}\, \psi_3(x)\,,$$

with $U_L \equiv \exp\left\{i\,\frac{\sigma_i}{2}\,\alpha^i\right\}$

hypercharges

Pauli matrices

- Gauge principle: global G transformations → *local:* $\alpha_i = \alpha_i(x)$ and $\beta = \beta(x)$

- For $\mathcal{L}$ to be invariant introduction of covariant derivatives:

$$D_\mu\psi_1(x) \equiv \left[\partial_\mu + i\,g\,\widetilde{W}_\mu(x) + i\,g'\,y_1\,B_\mu(x)\right]\,\psi_1(x)\,,$$
$$D_\mu\psi_2(x) \equiv \left[\partial_\mu + i\,g'\,y_2\,B_\mu(x)\right]\,\psi_2(x)\,,$$
$$D_\mu\psi_3(x) \equiv \left[\partial_\mu + i\,g'\,y_3\,B_\mu(x)\right]\,\psi_3(x)\,,$$

with 4 gauge fields: $\widetilde{W}_\mu(x) \equiv \frac{\sigma_i}{2}\,W_\mu^i(x)$ and $B_\mu(x)$ corresponding to $W^{+/-}$, Z and γ

Theoretical formulation

- The covariant derivative transforms as the field itself dictating the transf. properties of W_μ (x) and B_μ(x)

$$B_\mu(x) \quad \xrightarrow{G} \quad B'_\mu(x) \equiv B_\mu(x) - \frac{1}{g'}\,\partial_\mu \beta(x),$$

$$\widetilde{W}_\mu \quad \xrightarrow{G} \quad \widetilde{W}'_\mu \equiv U_L(x)\,\widetilde{W}_\mu\, U_L^\dagger(x) + \frac{i}{g}\,\partial_\mu U_L(x)\, U_L^\dagger(x)$$

- The EW Lagrangian is:

$$\mathcal{L}_{EW} = \sum_{j=1}^{3} i\bar{\psi}_j(x)\gamma^\mu D_\mu \psi_j(x) - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^i_{\mu\nu} W_i^{\mu\nu}$$

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- At the moment the Lagrangian describes interactions between massless fermions and gauge bosons

Charged Current Interactions

$$\mathcal{L} \quad \longrightarrow \quad -g\,\overline{\psi}_1\gamma^\mu\widetilde{W}_\mu\psi_1 \;-\; g'\,B_\mu\sum_{j=1}^{3} y_j\,\overline{\psi}_j\gamma^\mu\psi_j$$

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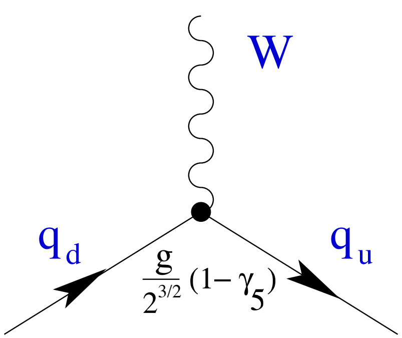
CC

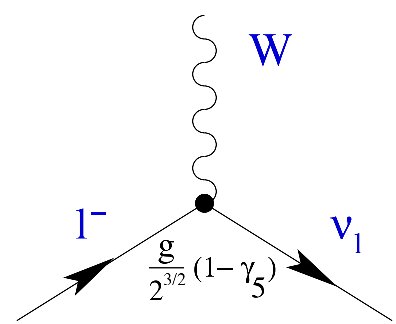
- **Charged Current Interactions**

$$\widetilde{W}_\mu = \frac{\sigma^i}{2}W^i_\mu = \frac{1}{2}\begin{pmatrix} W^3_\mu & \sqrt{2}\,W^\dagger_\mu \\ \sqrt{2}\,W_\mu & -W^3_\mu \end{pmatrix}$$

$$W_\mu \equiv (W^1_\mu + i\,W^2_\mu)/\sqrt{2}$$

$$\Rightarrow \quad \mathcal{L}_{\rm CC} = -\frac{g}{2\sqrt{2}}\left\{ W^\dagger_\mu\,\left[\bar{u}\gamma^\mu(1-\gamma_5)d + \bar{\nu}_e\gamma^\mu(1-\gamma_5)e\right] + \text{h.c.}\right\}$$





Neutral Current Interactions

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3rd component   NC

- **Neutral Current Interactions**
  Identify $W_{\mu 3}$ and B_μ. with the Z and the γ. But B_μ cannot be equal to γ.
  $y_1 = y_2 = y_3$ and $g'\,y_j = eQ_j$, *cannot* be simultaneously true

$$\Rightarrow \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} \equiv \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

$$\mathcal{L}_{\rm NC} = -\sum_j \overline{\psi}_j\,\gamma^\mu \left\{ A_\mu \left[g\,\frac{\sigma_3}{2}\,\sin\theta_W + g'\,y_j\,\cos\theta_W \right] + Z_\mu \left[g\,\frac{\sigma_3}{2}\,\cos\theta_W - g'\,y_j\,\sin\theta_W \right] \right\} \psi_j$$

To get QED from the A_μ piece, one needs to impose the conditions:

$$g\,\sin\theta_W = g'\,\cos\theta_W = e$$ and $$Y = Q - T_3$$

Neutral Current Interactions

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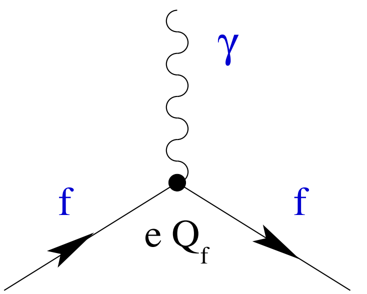
3rd component NC

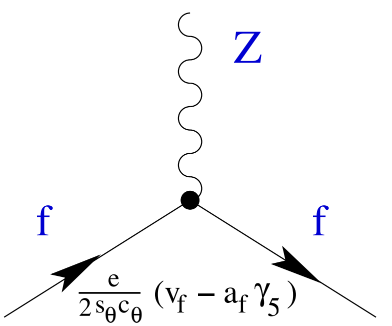
- **Neutral Current Interaction** $\mathcal{L}_{\rm NC} = \mathcal{L}_{\rm QED} + \mathcal{L}_{\rm NC}^{Z}$

$$\mathcal{L}_{\rm QED} = -e\,A_\mu \sum_j \overline{\psi}_j \gamma^\mu Q_j \psi_j$$



and

$$\mathcal{L}_{\rm NC}^{Z} = -\frac{e}{2\sin\theta_W \cos\theta_W} Z_\mu \sum_f \bar{f}\gamma^\mu (v_f - a_f\gamma_5)\, f$$



Mass generation: electroweak symmetry breaking

- As we have seen introducing a mass terms for the fermions and the gauge bosons *breaks* gauge symmetry and $\mathcal{L}$ is no longer invariant
- However in nature the gauge bosons as well as the fermions are massive:
  ⇨ Dilemma: *break* the gauge symmetry while having a *fully symmetric Lagrangian* to preserve renormalizability

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  ⇒ Dilemma: *break* the gauge symmetry while having a *fully symmetric Lagrangian* to preserve renormalizability

  ⇒ Obtained through *Spontaneous Symmetry Breaking*

  $\mathcal{L}$ is invariant under G but the ground state or vacuum is no longer invariant

Spontaneous symmetry breaking

- $\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi - V(\phi)$ with $V(\phi) = \mu^2 \phi^\dagger \phi + h\left(\phi^\dagger \phi\right)^2$

- $\mathcal{L}$ is invariant under global phase transformations U(1) of the scalar field

  $$\phi(x) \rightarrow \phi'(x) \equiv \exp\left(i\theta\right)\phi(x)$$

- In order to have a ground state the potential should be bounded from below, i.e., h > 0. 2 possibilities:

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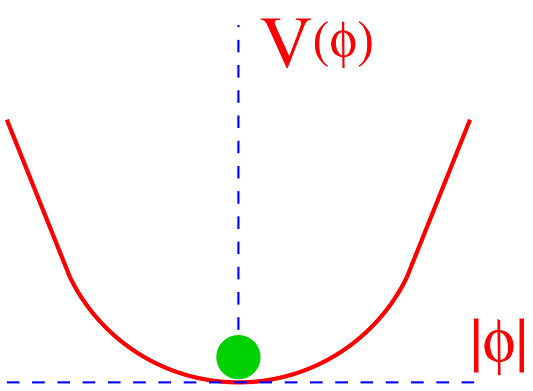
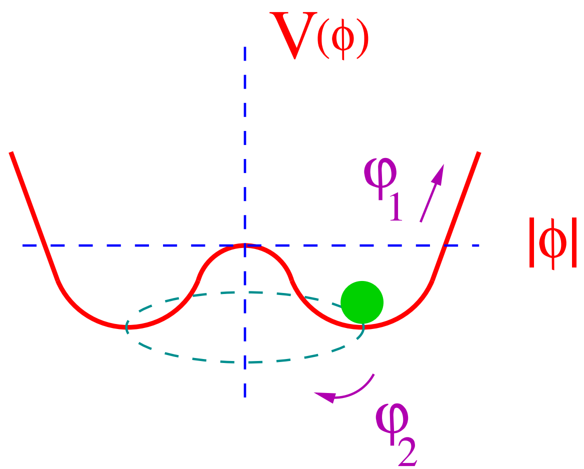




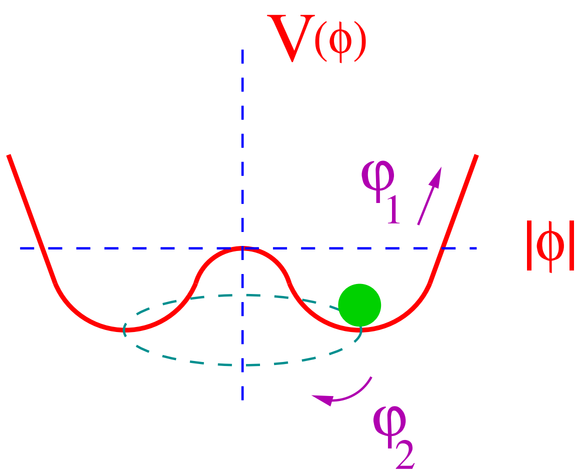
$\mu^2 > 0$: The potential has only the trivial minimum $\varphi = 0$. ⇨ A massive scalar particle with mass μ and quartic coupling h.

$\mu^2 < 0$: The minimum is obtained for

$$|\phi_0| = \sqrt{\frac{-\mu^2}{2h}} \equiv \frac{v}{\sqrt{2}} > 0$$

Spontaneous symmetry breaking

- Due to U(1) invariance of $\phi_0(x) = \frac{v}{\sqrt{2}} \exp\{i\theta\}$.
- By choosing a particular direction: θ = 0 as the ground state ⇨ the symmetry gets *spontaneously broken*.
- $\phi(x) \equiv \frac{1}{\sqrt{2}}\left[v + \varphi_1(x)\right]\exp\left(i\varphi_2(x)/v\right)$
- φ_2 excitations around a flat direction in the potential
  ⇨ states with the same energy as the chosen ground state.
  Those excitations do not cost any energy ⇨ correspond to *massless states*



$$V(\phi) = V(\phi_0) + \frac{1}{2} m^2_{\varphi_1} \varphi_1^2 + h v \varphi_1^3 + h \varphi_1^4$$

$$m^2_{\varphi_1} = -2\mu^2 > 0 \ , \ \boxed{m^2_{\varphi_2} = 0}$$

1 massless Goldstone Boson

Mass generation: electroweak SSB

- We introduce a $SU(2)_L$ doublet of complex scalar fields:

$$\phi(x) \equiv \begin{pmatrix} \phi^{(+)}(x) \\ \phi^{(0)}(x) \end{pmatrix} \qquad y_\phi = Q_\phi - T_3 = \frac{1}{2}$$

- $\mathcal{L}_S = (D_\mu\phi)^\dagger D^\mu\phi - \mu^2\phi^\dagger\phi - h\left(\phi^\dagger\phi\right)^2$ is invariant under $G \equiv SU(2)_L \otimes U(1)_Y$

$$D^\mu\phi = \left[\partial^\mu + i\,g\,\widetilde{W}^\mu + i\,g'\,y_\phi\,B^\mu\right]\phi\,,$$

- Degenerate Vacuum States: $\left|\langle 0|\phi^{(0)}|0\rangle\right| = \sqrt{\frac{-\mu^2}{2h}} \equiv \frac{v}{\sqrt{2}}$

- Spontaneous Symmetry Breaking:

$$\phi(x) = \exp\left\{i\,\frac{\sigma_i}{2}\,\theta^i(x)\right\}\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

4 real fields $\theta^i(x) + H(x)$

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4 real fields $\theta^i(x)$ + H(x)

- $SU(2)_L$ invariance ⇨ $\theta^i(x)$ can be gauged away

- 3 massless Goldstone bosons that are « eaten » to give masses to $W^{+/-}$ and Z

$$(D_\mu\phi)^\dagger D^\mu\phi \quad \xrightarrow{\theta^i=0} \quad \frac{1}{2}\,\partial_\mu H\,\partial^\mu H + (v+H)^2 \left\{ \frac{g^2}{4}\,W_\mu^\dagger W^\mu + \frac{g^2}{8\cos^2\theta_W}\,Z_\mu Z^\mu \right\}$$

⇨ *Massive Gauge Bosons*

$$M_Z\,\cos\theta_W \;=\; M_W \;=\; \frac{1}{2}\,v\,g$$

Mass generation: electroweak SSB

- $G \equiv SU(2)_L \otimes U(1)_Y$ $\xrightarrow{\text{SSB}}$ $U(1)_{EM}$

- Before SSB:
  - 3 massless $W^{\pm}$ and Z bosons, i.e., 3 × 2 = 6 d.o.f fields
  - 3 Goldstones $\theta^i(x)$
  - H(x)

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- After SSB:
  - 3 massives $W^{\pm}$ and Z bosons, i.e., 3 × 3 = 9 d.o.f fields
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- Higgs field remains in the spectrum

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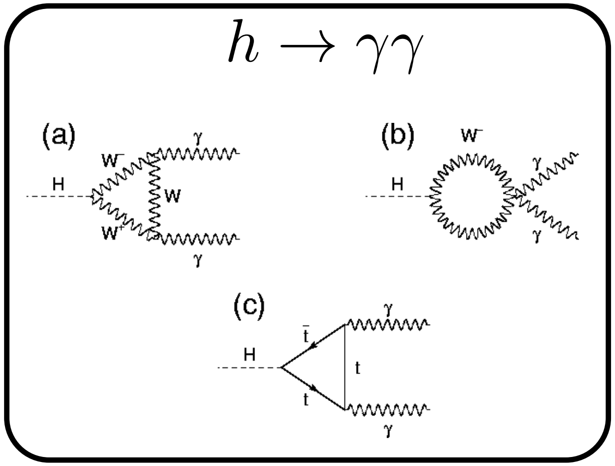
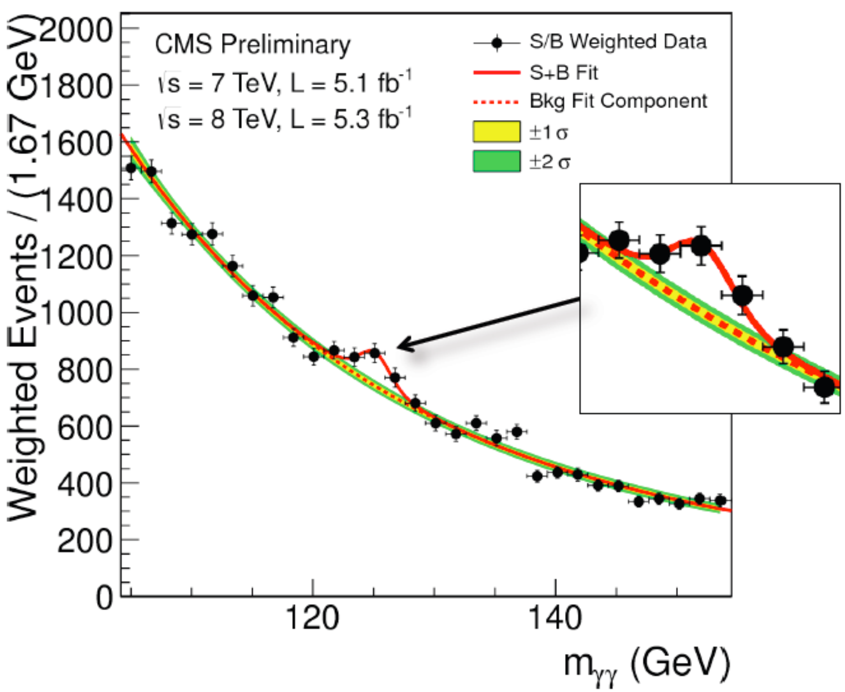
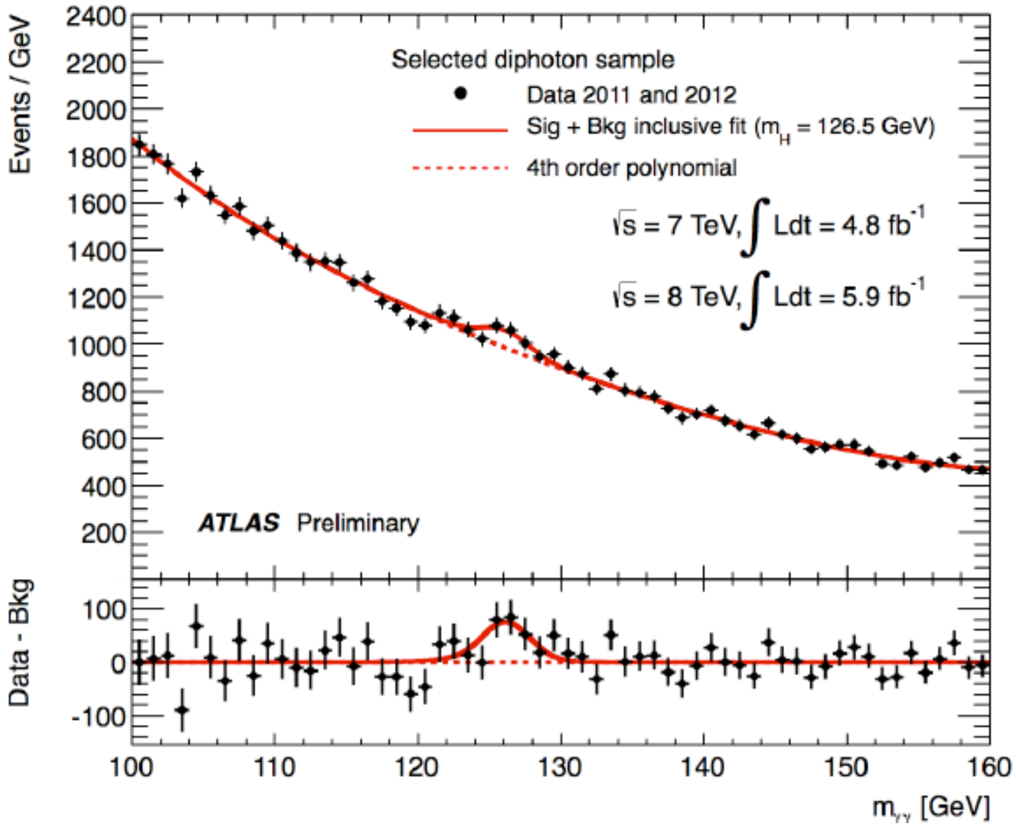
Higgs field

- Discovery of a 125 GeV scalar particle at LHC on July 4, 2012: Missing piece of the Standard Model







Fermion masses

- Yukawa Lagrangian:

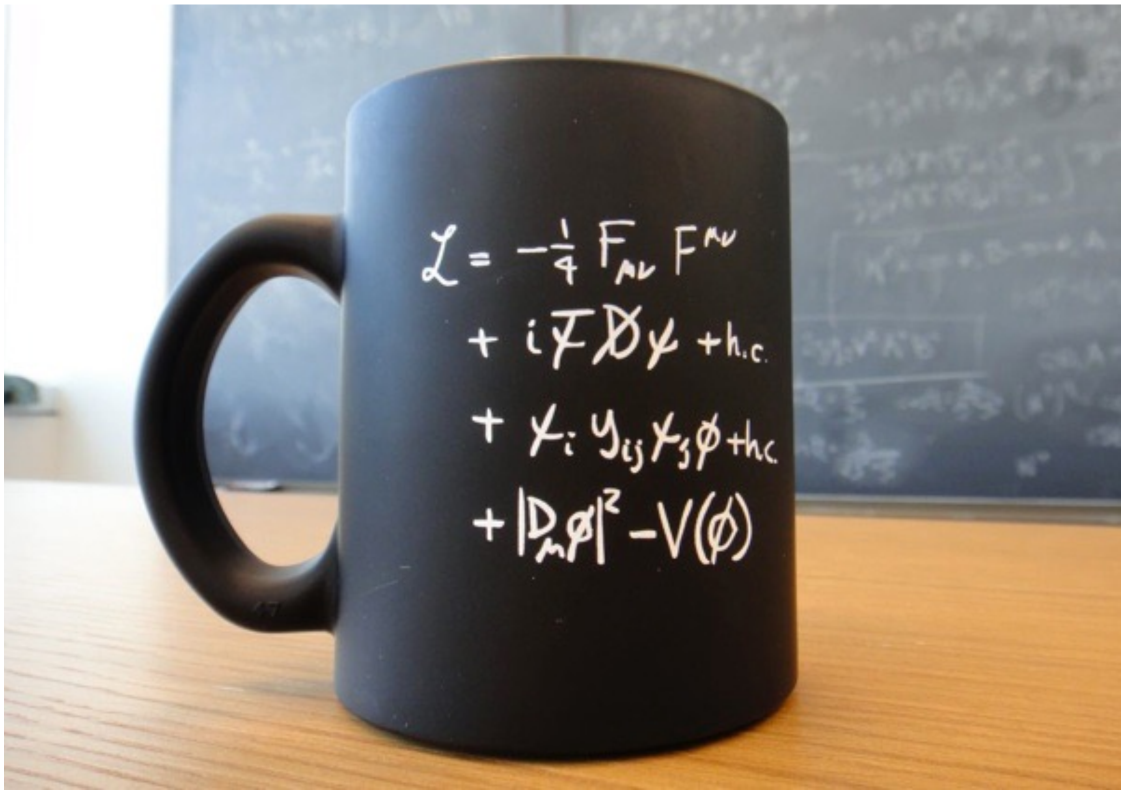
$$\mathcal{L}_Y = -c_1 \, (\bar{u}, \bar{d})_L \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} d_R - c_2 \, (\bar{u}, \bar{d})_L \begin{pmatrix} \phi^{(0)*} \\ -\phi^{(-)} \end{pmatrix} u_R - c_3 \, (\bar{\nu}_e, \bar{e})_L \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} e_R + \text{h.c.}$$

⇓ **SSB**

$$\mathcal{L}_Y = -\left(1 + \frac{H}{v}\right) \left\{ m_d \, \bar{d}d + m_u \, \bar{u}u + m_e \, \bar{e}e \right\}$$

⇨ *Massive Fermions*

Standard Model Lagrangian



Application of EW interactions

- Study of the process: $\nu_e + e^- \rightarrow \nu_e + e^-$

- Can it go through strong, EM, weak interactions?

- How many Feynman diagrams at tree level?

Application of EW interactions

- Study of the process: $\nu_e + e^- \rightarrow \nu_e + e^-$
- Involve leptons only ⇒ no strong interaction
- The neutrinos are electrically neutral ⇒ no EM interaction
  ⇒ Only Weak interactions !
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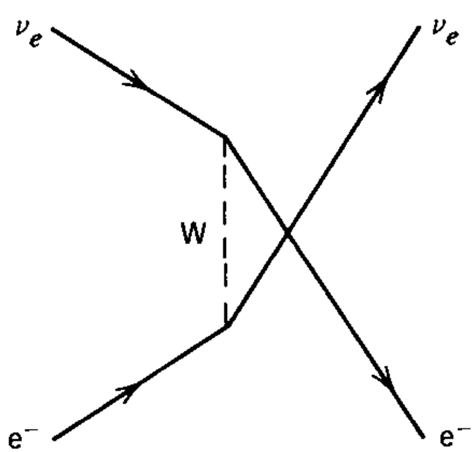
CC



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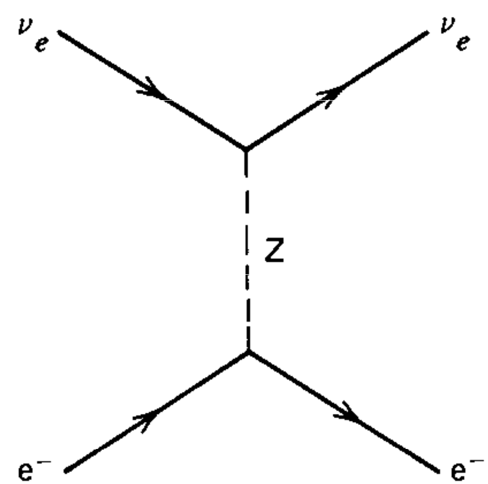
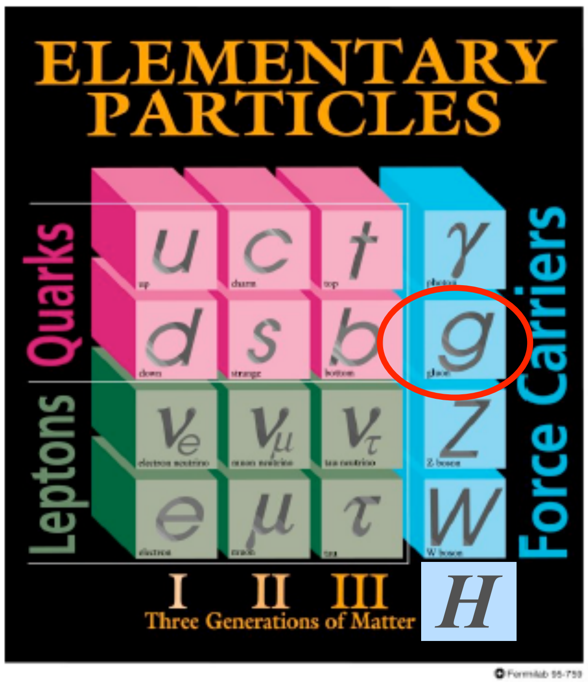
NC



2.4 Strong Interactions

- In particle physics a simpler table made of leptons and quarks: the degrees of freedom



Charge: 0, -1, +2/3, -1/3

$$\begin{pmatrix}\nu_e\\ e\end{pmatrix} \quad \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix} \quad \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}$$

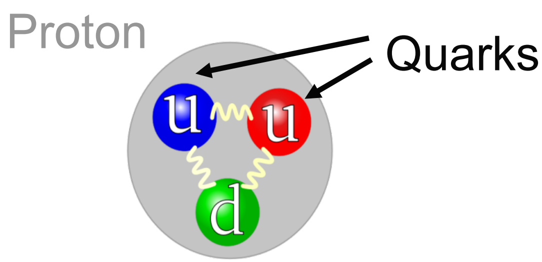
$$\begin{pmatrix}u\\ d\end{pmatrix} \quad \begin{pmatrix}c\\ s\end{pmatrix} \quad \begin{pmatrix}t\\ b\end{pmatrix}$$

Int.: w, w,e, w,e,s

- 3 forces: electromagnetic, weak and strong forces

Quark masses

- Let us consider the proton: it is not a fundamental particle, but a bound state of 3 quarks



Contrary to naïve expectation, most of its mass comes from *strong force*

Only 1% of its mass comes from the quark masses (Coupling of the quarks to the Higgs boson)

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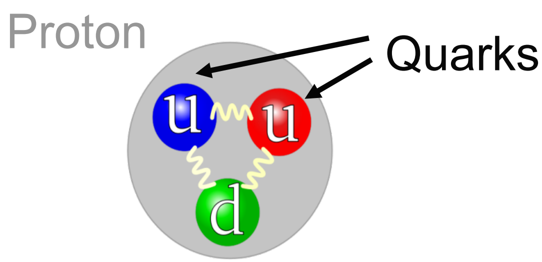

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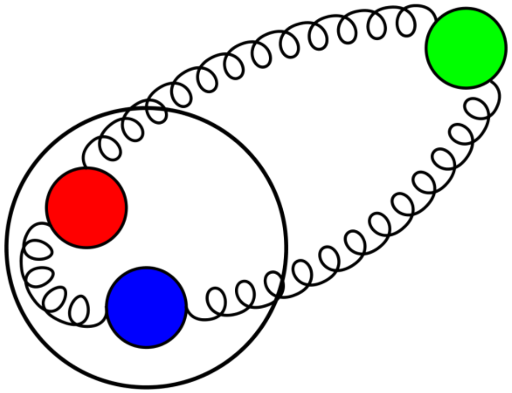
- How can we access the quark masses?

Strong interaction

- Problem: quarks and gluons are not free particles: they are bound inside hadrons



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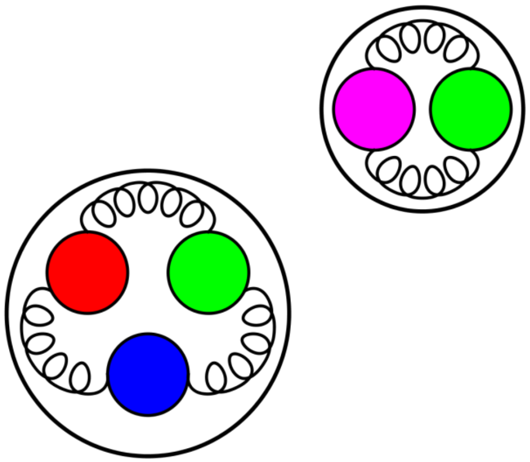

- Two properties:
  - Confinement
  - Asymptotic freedom : The interaction decreases at high energies
    Nobel Prize in 2004 for Frank Wilczek and David Gross and David Politzer

Quark masses

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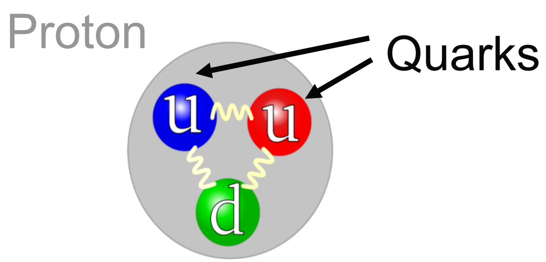

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- How can we access the quark masses?
- In principle a theory ⇒ Quantum ChromoDynamics

⇒

$$\mathcal{L}_{QCD} = -\frac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a + \sum_{k=1}^{N_F} \bar{q}_k \left(i\gamma^\mu D_\mu - m_k \right) q_k$$

Formulation of QCD

- SU(3)$_C$ QCD invariant Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4} G_a^{\mu\nu} G^a_{\mu\nu} + \sum_{k=1}^{N_F} \bar{q}_k \left(i\gamma^\mu D_\mu - m_k \right) q_k$$

- Different parts to describe the interactions

$$\mathcal{L}_{QCD} = -\frac{1}{4}\left(\partial^\mu G_a^\nu - \partial^\nu G_a^\mu\right)\left(\partial_\mu G_\nu^a - \partial_\nu G_\mu^a\right) + \sum_{k=1}^{N_F} \bar{q}_k \left(i\gamma^\mu \partial_\mu - m_k \right) q_k$$
$$+ g_S G_a^\mu \sum_{k=1}^{N_F} \bar{q}_k \gamma_\mu \left(\frac{\lambda_a}{2}\right) q_k$$
$$- \frac{g_S}{2} f^{abc} \left(\partial^\mu G_a^\nu - \partial^\nu G_a^\mu\right) G^b_\mu G^c_\nu - \frac{g_S^2}{4} f^{abc} f_{ade} G_b^\mu G_c^\nu G^d_\mu G^e_\nu$$

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Kinetic terms

$$+ g_S G_a^\mu \sum_{k=1}^{N_F} \bar{q}_k \gamma_\mu \left(\frac{\lambda_a}{2}\right) q_k$$

$$-\frac{g_S}{2} f^{abc} \left(\partial^\mu G_a^\nu - \partial^\nu G_a^\mu\right) G_\mu^b G_\nu^c - \frac{g_S^2}{4} f^{abc} f_{ade} G_b^\mu G_c^\nu G_\mu^d G_\nu^e$$

Formulation of QCD

- $SU(3)_C$ QCD invariant Lagrangian

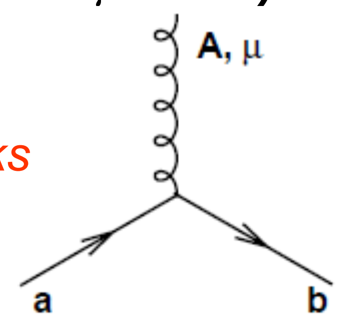
$$\Rightarrow \quad \boxed{\mathcal{L}_{QCD} = -\frac{1}{4} G_a^{\mu\nu} G^a_{\mu\nu} + \sum_{k=1}^{N_F} \bar{q}_k \left(i\gamma^\mu D_\mu - m_k \right) q_k}$$

- Different parts to describe the interactions

$$\mathcal{L}_{QCD} = -\frac{1}{4}\left(\partial^\mu G_a^\nu - \partial^\nu G_a^\mu\right)\left(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu\right) + \sum_{k=1}^{N_F} \bar{q}_k \left(i\gamma^\mu \partial_\mu - m_k \right) q_k$$

$$+ g_S G_a^\mu \sum_{k=1}^{N_F} \bar{q}_k \gamma_\mu \left(\frac{\lambda_a}{2} \right) q_k \quad \Rightarrow$$

Interaction quarks gluon



$$- \frac{g_S}{2} f^{abc} \left(\partial^\mu G_a^\nu - \partial^\nu G_a^\mu\right) G^b_\mu G^c_\nu - \frac{g_S^2}{4} f^{abc} f_{ade} G_b^\mu G_c^\nu G^d_\mu G^e_\nu$$

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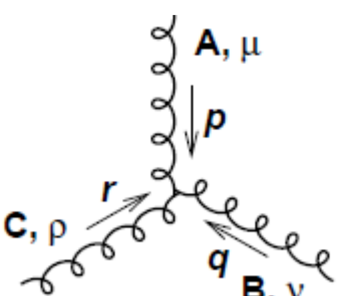

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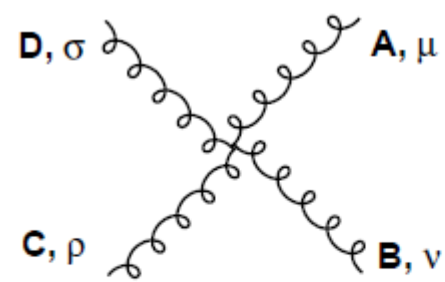

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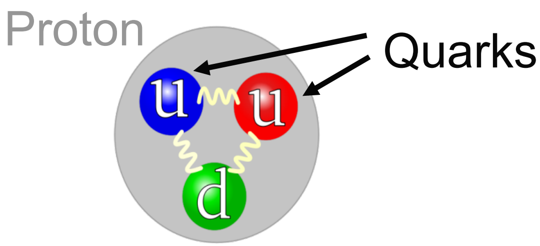
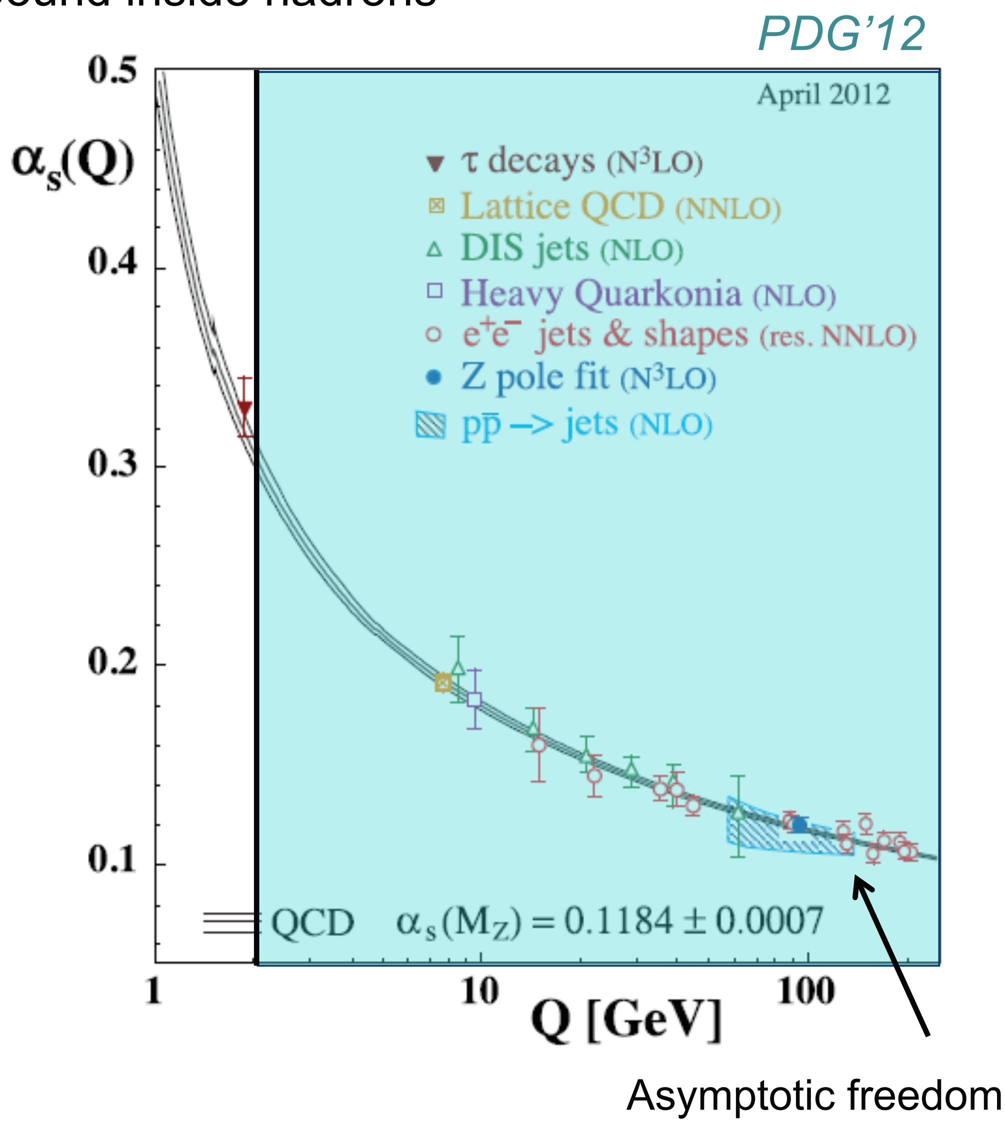
- One single universal coupling : $\alpha_S(\mu) = \frac{g_S^2(\mu)}{4\pi}$ *strong coupling constant*
- It is not a constant, depends on the *energy !*

Strong interaction

- Problem: quarks and gluons are bound inside hadrons





- High energies, short distance:
  α_S small ⇨ Asymptotic freedom

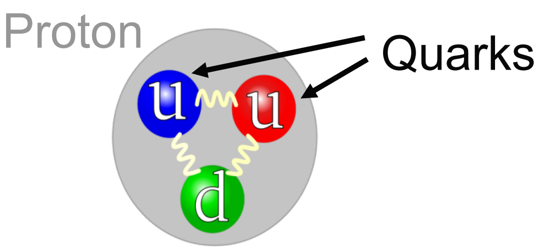
  Perturbative QCD
  Theory "easy" to solve

Order-by-order *expansion* in $\frac{\alpha_s(\mu)}{\pi}$

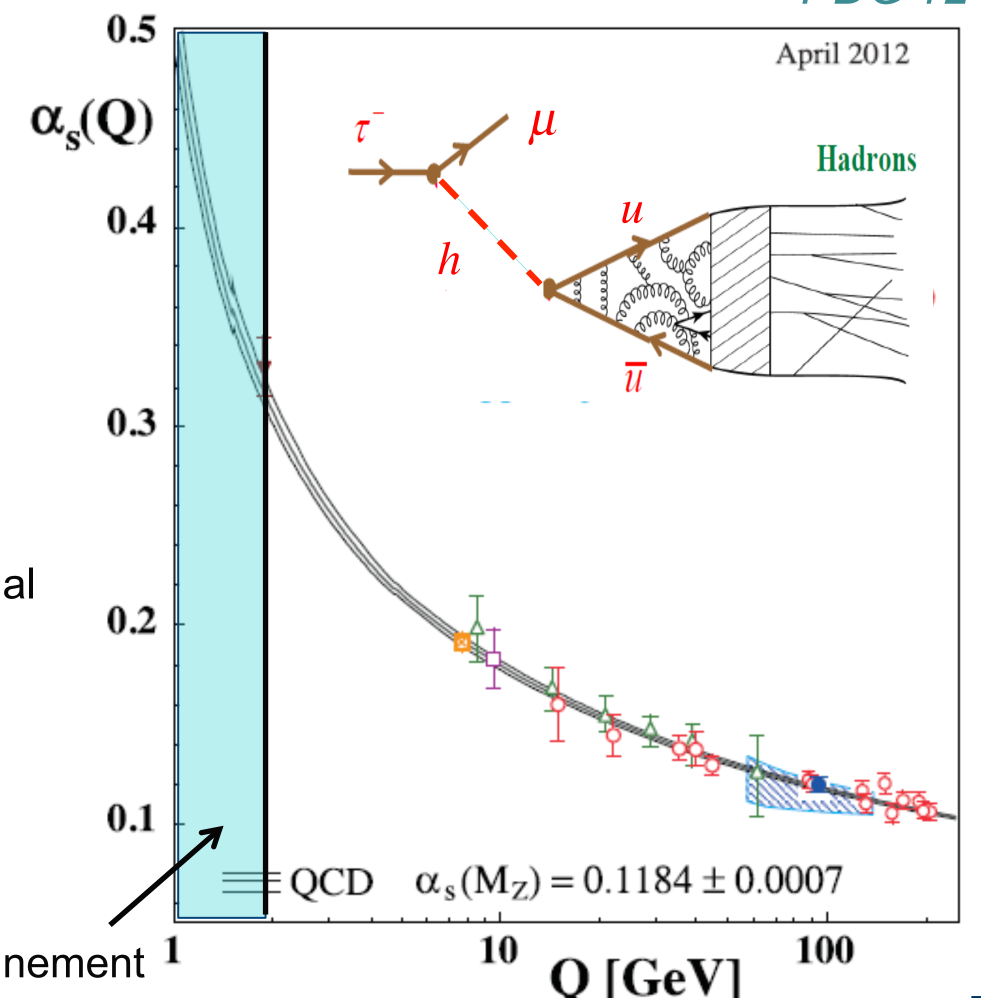
$$\sigma = \sigma_0 + \underbrace{\frac{\alpha_s}{\pi}\sigma_1}_{\text{small}} + \underbrace{\left(\frac{\alpha_s}{\pi}\right)^2\sigma_2}_{\text{smaller}} + \left(\frac{\alpha_s}{\pi}\right)^3\sigma_3 + \ldots$$

Strong interaction

- Looking for new physics in hadronic processes ⇨ not direct access to quarks due to confinement



PDG'12



Confinement

➢ Low energy (Q <~1 GeV), long distance: *α_S becomes large !*

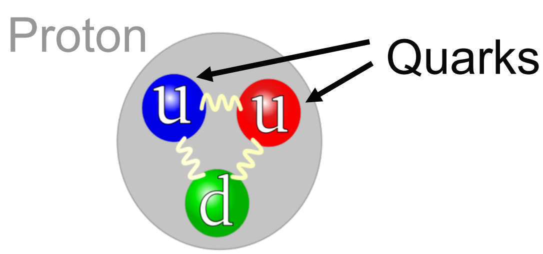
⇨ *Non-perturbative QCD*

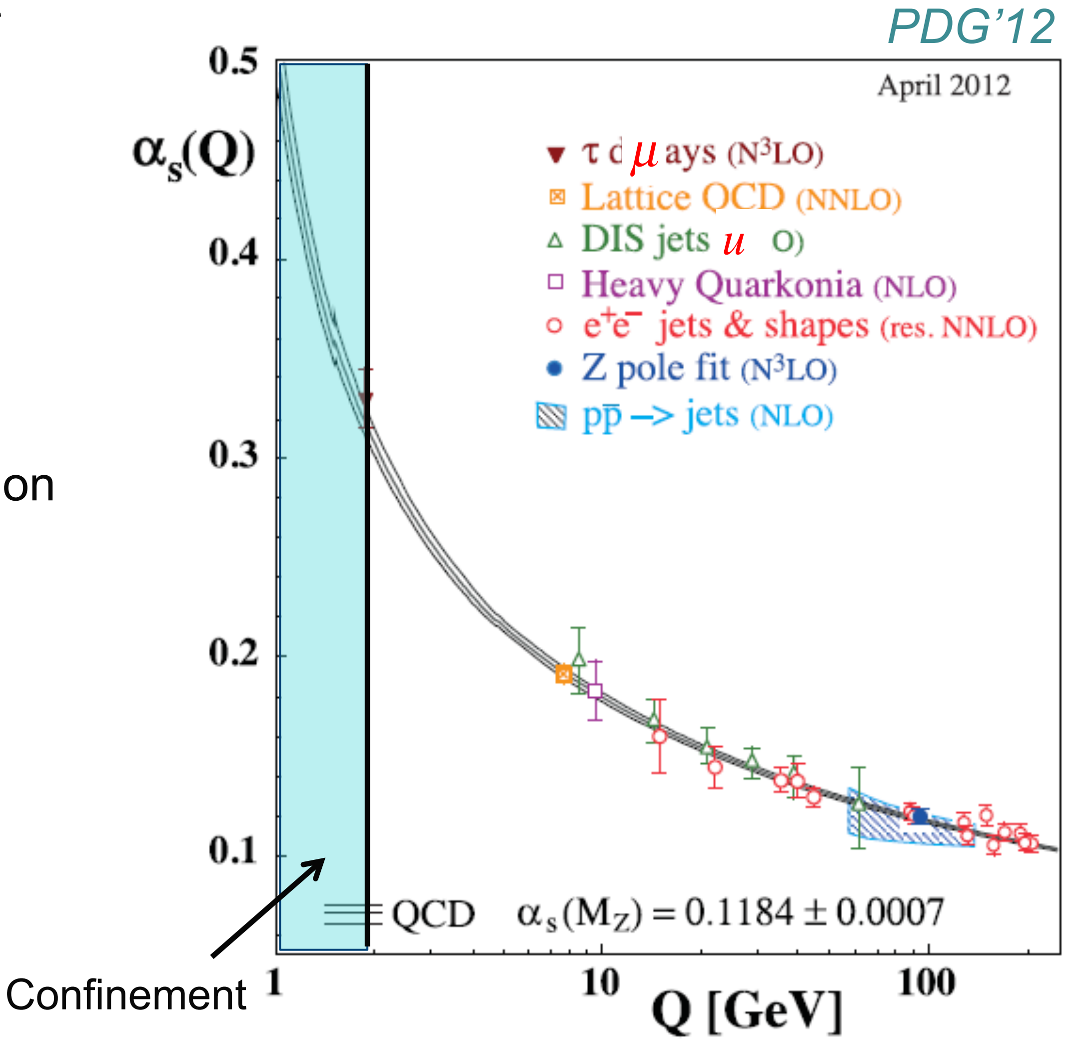
A perturbative expansion in the usual sense fails

⇨ Use of alternative approaches, expansions…

Strong interaction

- Looking for new physics in hadronic processes ⇨ not direct access to quarks due to confinement

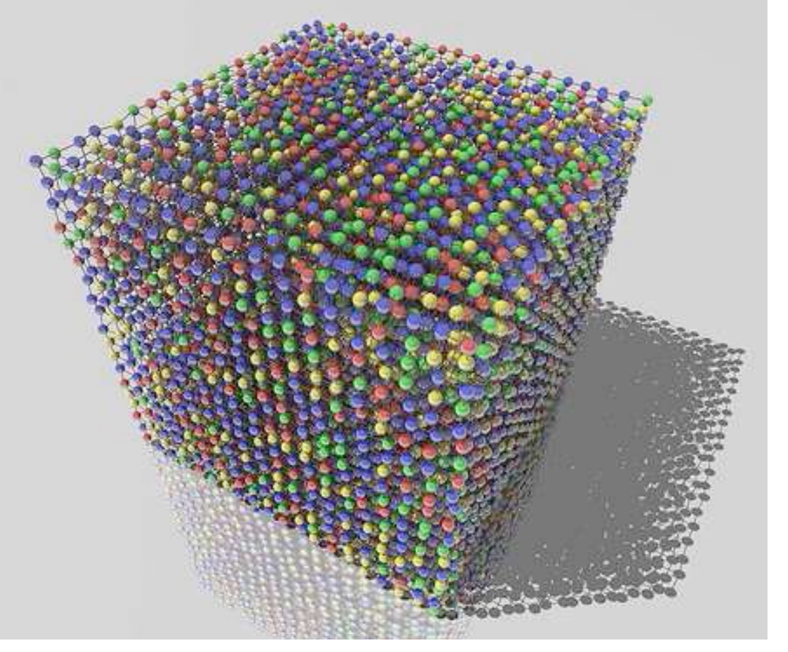




➢ Non-perturbative methods:
  – Numerical simulations on the lattice

Lattice QCD

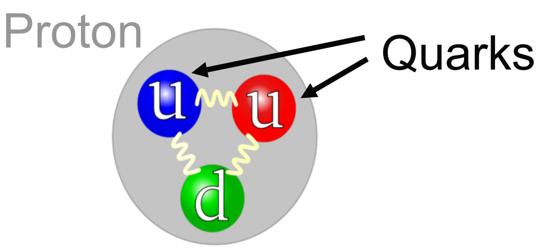
- Principle: Discretization of the space time and solve QCD on the lattice numerically
  - All quark and gluon fields of QCD on a 4D-lattice
  - Field configurations by Monte Carlo sampling



- Important subtleties due to the discretization, should come back to the continuum, formulation of the fermions on the lattice…

Strong interaction

- Looking for new physics in hadronic processes ➡ not direct access to quarks due to confinement



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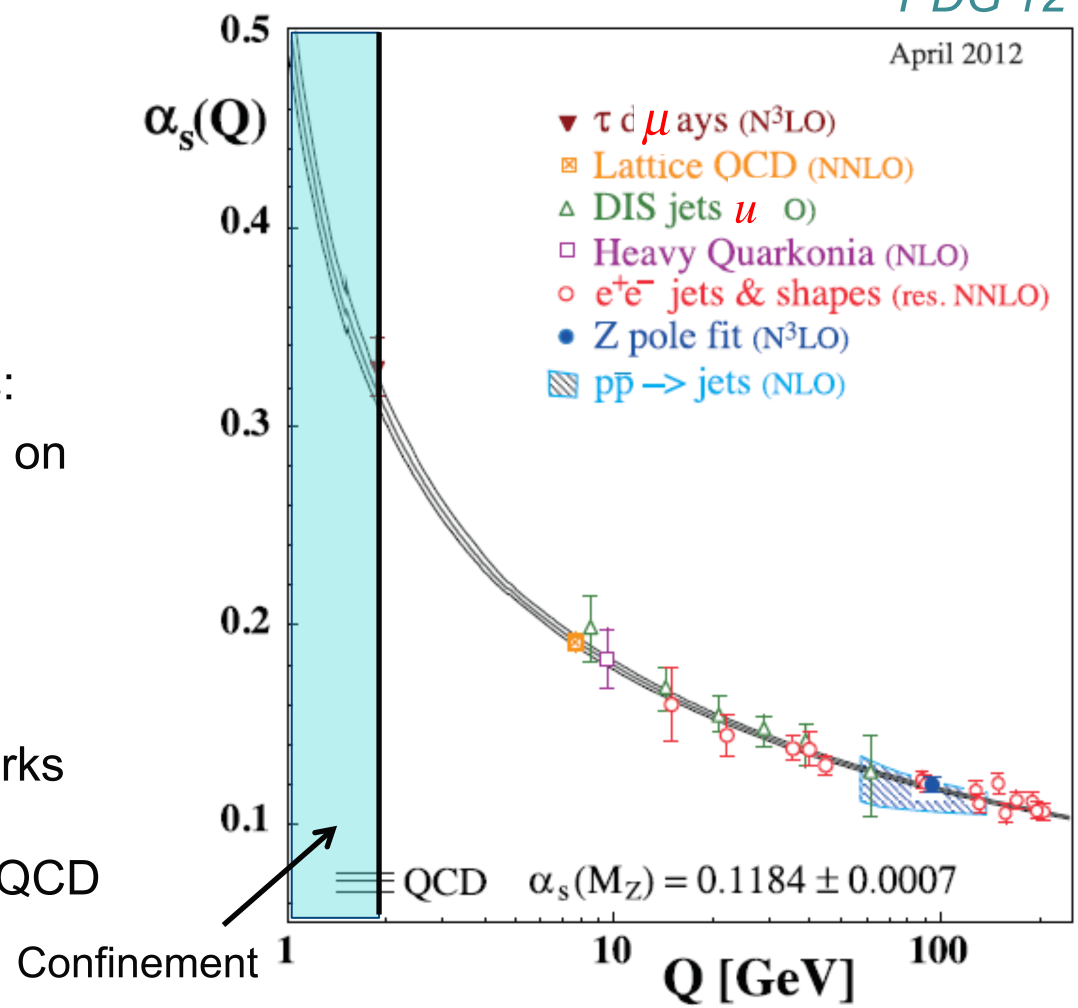
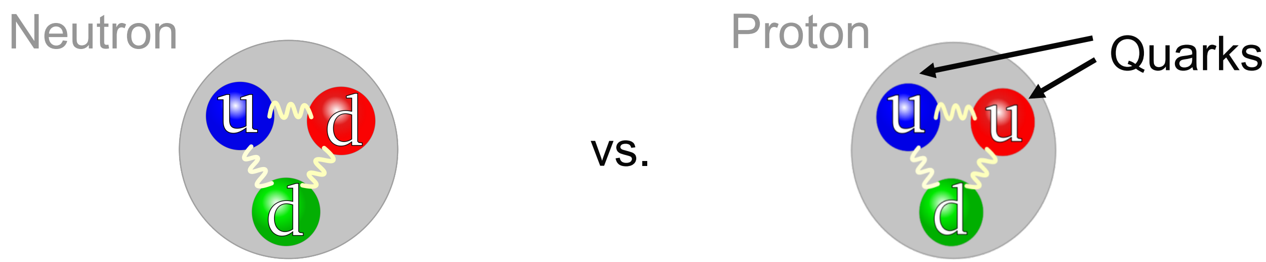


➢ Non-perturbative methods:
  – Numerical simulations on the lattice
  – Analytical methods:
    Effective field theory
    Ex: ChPT for light quarks
    Dispersion relations
    Synergies with lattice QCD

➡ *Hadronic Physics*

Quark masses



$M_n = 939.57$ MeV

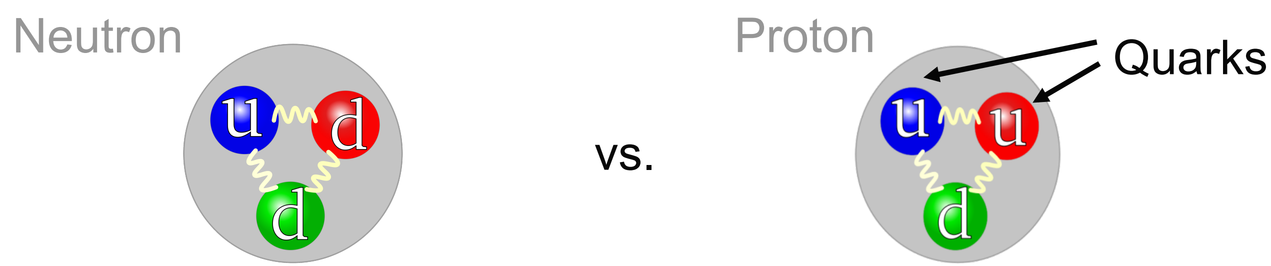
$M_p = 938.27$ MeV

- Strong force: If $m_u \sim m_d$: $M_n \sim M_p$ *isospin symmetry*

  Heisenberg'60

  Countless experiments have shown that strong force obeys isospin symmetry
  Results are the same if we interchange neutrons and protons (or up and down quarks)

Quark masses



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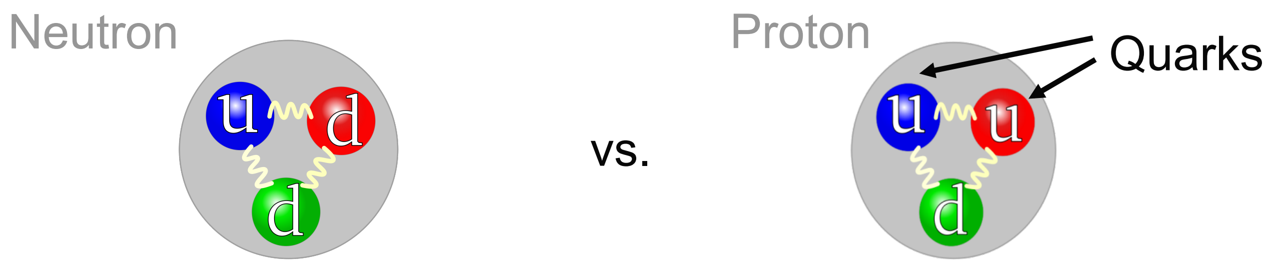

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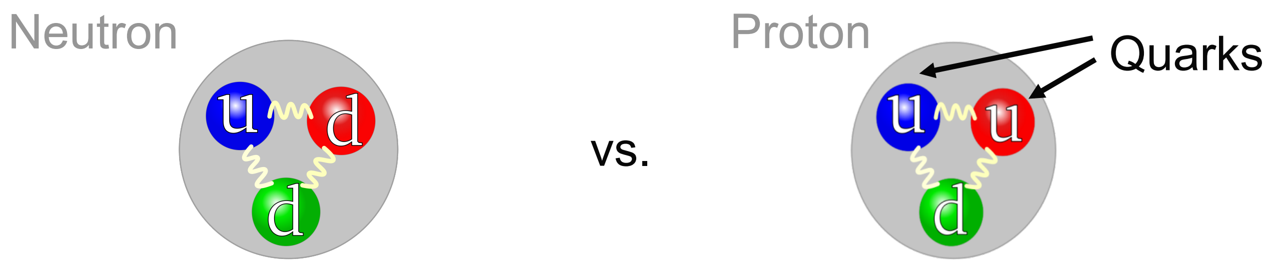
- Electromagnetic energy: one obvious difference between a neutron and a proton is their electric charges:

$Q_P = 1$ and $Q_n = 0$ Since $E_e \propto \frac{Q^2}{R}$ ⇨ $M_p > M_n$?

Quark masses



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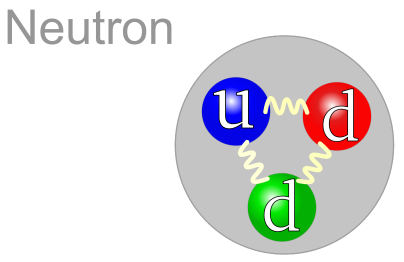
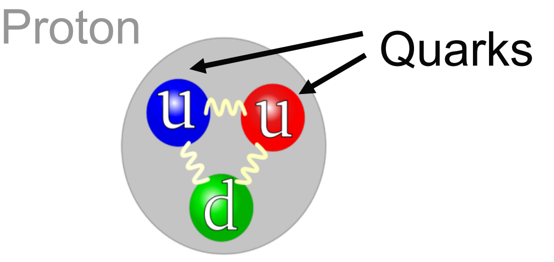
$Q_P = 1$ and $Q_n = 0$ Since $E_e \propto \frac{Q^2}{R}$ ⇨ $M_p > M_n$ **?**

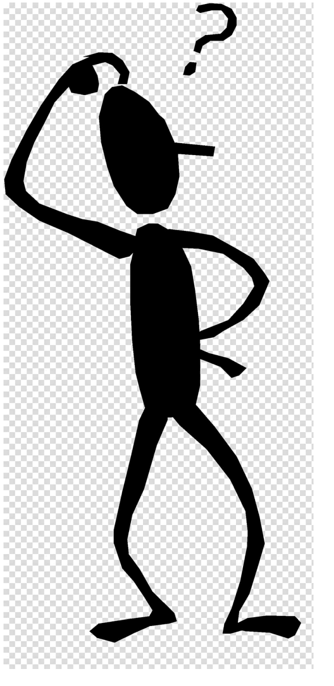
⇨ *Terrible consequences :* Proton would decay into neutrons and there will be no chemistry and we would not be there in this room!

Quark masses



vs.





- Strong force: If $m_u \sim m_d$: $M_n \sim M_p$ *isospin symmetry*

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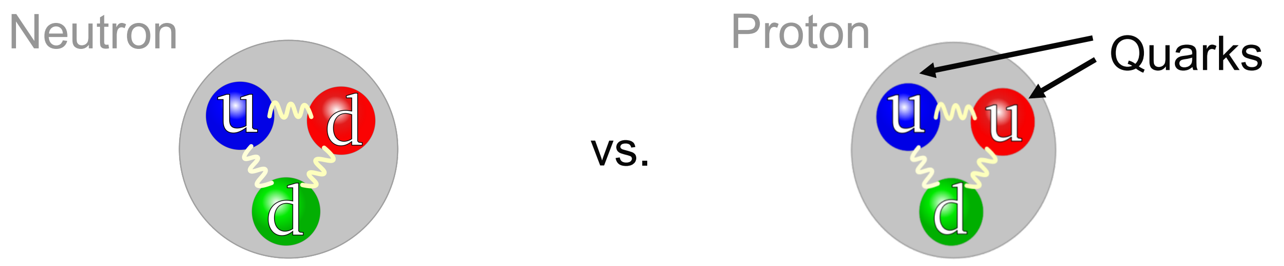
- This is not the case: *Why?*

Quark masses



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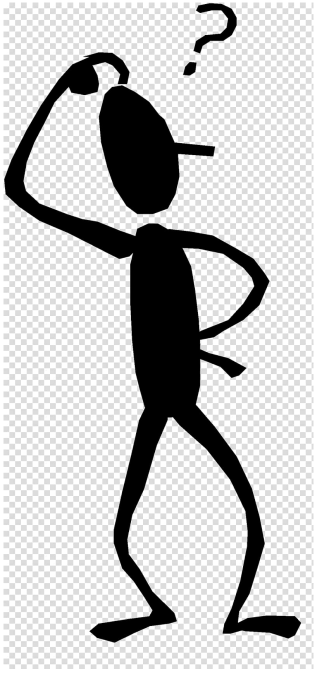
- This is not the case: *Why?*

- Another small effect in addition to e.m. force:

  different fundamental quark masses $m_d \neq m_u$

  ⇨ Different coupling to Higgs field



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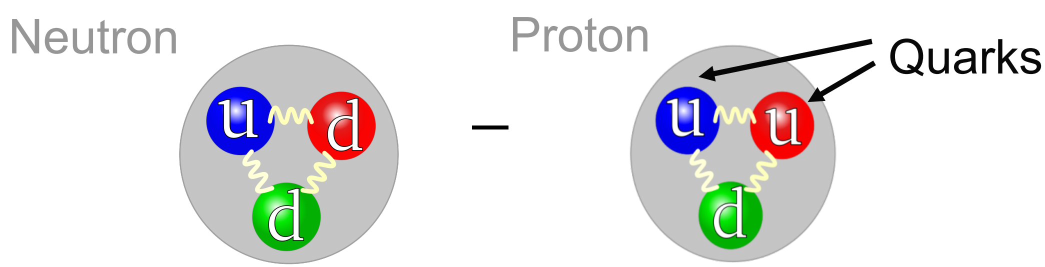

QUARKS

The *u*-, *d*-, and *s*-quark masses are estimates of so-called "current-quark masses," in a mass-independent subtraction scheme such as $\overline{\text{MS}}$ at a scale $\mu \approx 2$ GeV. The *c*- and *b*-quark masses are the "running" masses in the $\overline{\text{MS}}$ scheme. For the *b*-quark we also quote the 1S mass. These can be different from the heavy quark masses obtained in potential models.

Particle Data Group'18

$$m_d - m_u = 4.7 - 2.2 = 2.5 \text{ MeV}$$

u $\qquad I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

$m_u = 2.2^{+0.5}_{-0.4}$ MeV $\qquad$ Charge $= \frac{2}{3}\, e \qquad I_z = +\frac{1}{2}$

$m_u/m_d = 0.48^{+0.07}_{-0.08}$

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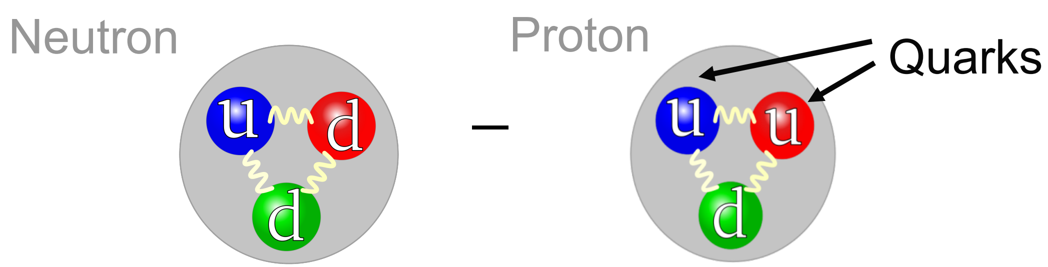
$m_s/m_d = 17$–22

$\overline{m} = (m_u + m_d)/2 = 3.5^{+0.5}_{-0.2}$ MeV

Quark mass difference more important than e.m. effect

Neutrons can decay in protons!

Quark masses



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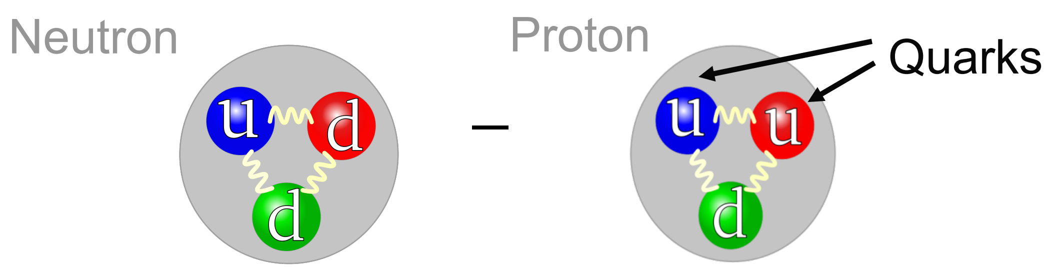
Neutron lifetime experiments

Quark masses



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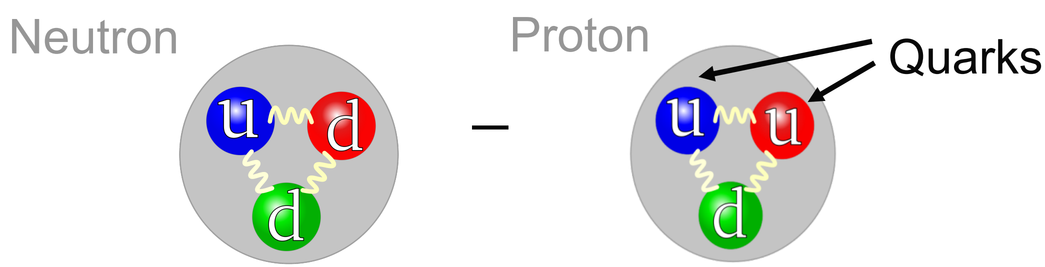
To determine these fundamental parameters need to know how to disentangle them from *QCD*

→ treat *strong interactions*

Quark masses



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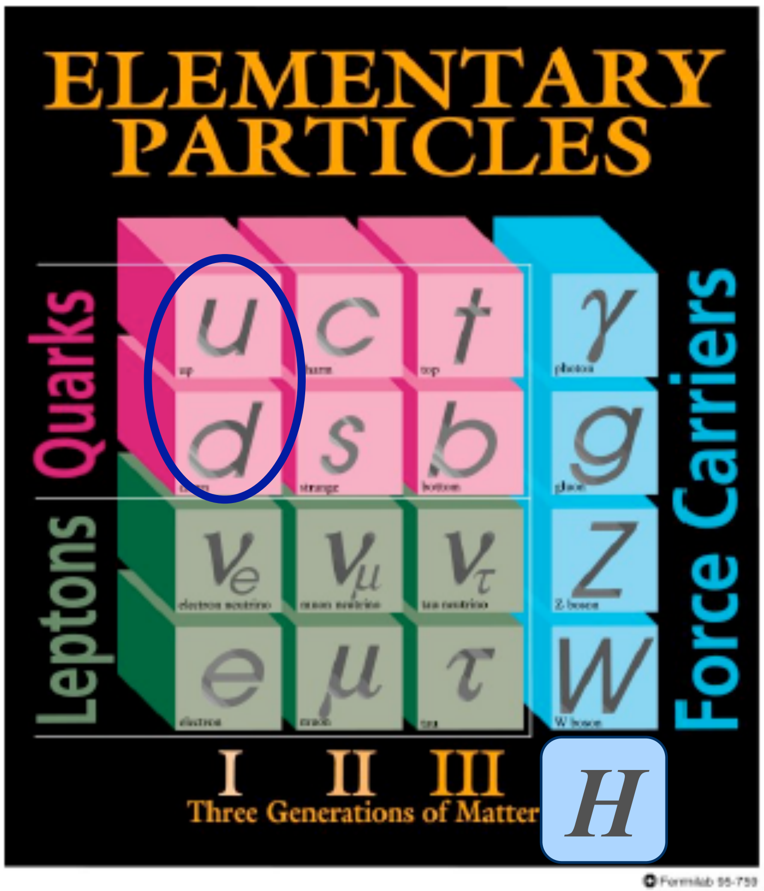
We will come back to the determination of quark mass difference later

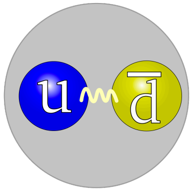
2.5 Success of the Standard Model and search for New Physics

Oscillations of Kaons

- Let us consider simplest hadrons: the mesons. They are quark-anti-quark bound states. They interact with strong, electromagnetic and weak forces

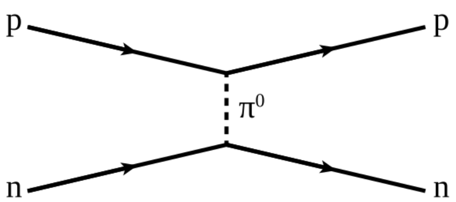


- The simplest one is the pion:



$$\pi^+ : u\bar{d} \quad , \quad \pi^0 : u\bar{u} \text{ or } d\bar{d}$$

$$\pi^- : \bar{u}d$$



The pions mediate strong force in nuclei
It is ubiquitous in hadronic collisions

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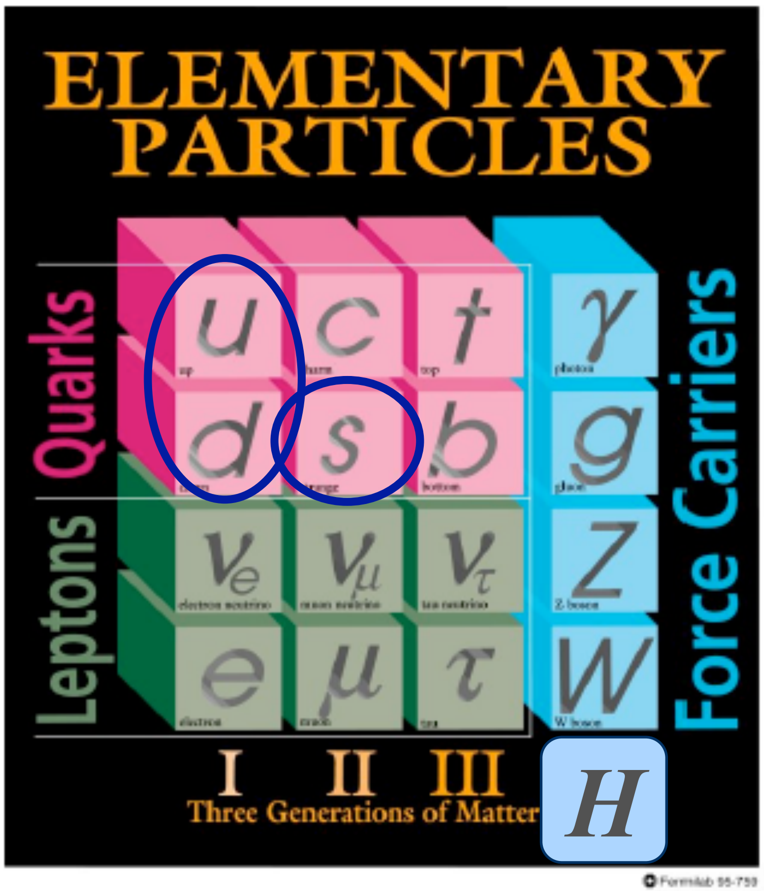

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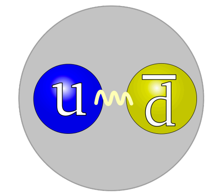

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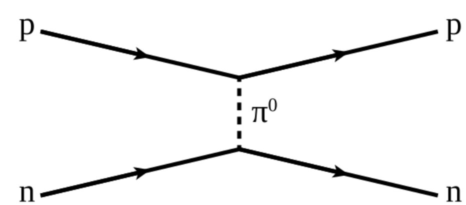
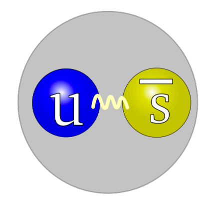


– The ones containing a s quark are the kaons



$$K^+ : u\bar{s}, \quad K^0 : d\bar{s}, \quad \bar{K}^0 : s\bar{d}$$

$$K^- : \bar{u}s$$

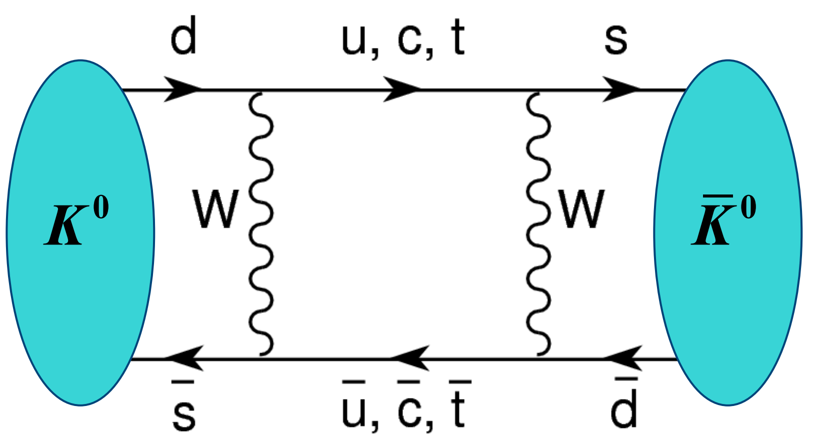
Discovered in *cosmic ray experiments*

Oscillations of Kaons

- Discovered in 1964 by Christenson, Cronin, Fitch and Turlay

  ⇒ Nobel Prize in 1980 for Cronin and Fitch

- Start with a K^0 ⇒ after some time it transforms into a $\bar{K}^0$



through weak interaction
Short distance effect

- The rate of this oscillation is suppressed but measurable in the Standard Model

  ⇒ goes through *weak interactions* $\sim G_F$ $G_F \simeq 1.17 \times 10^{-5}\ \text{GeV}^{-2}$

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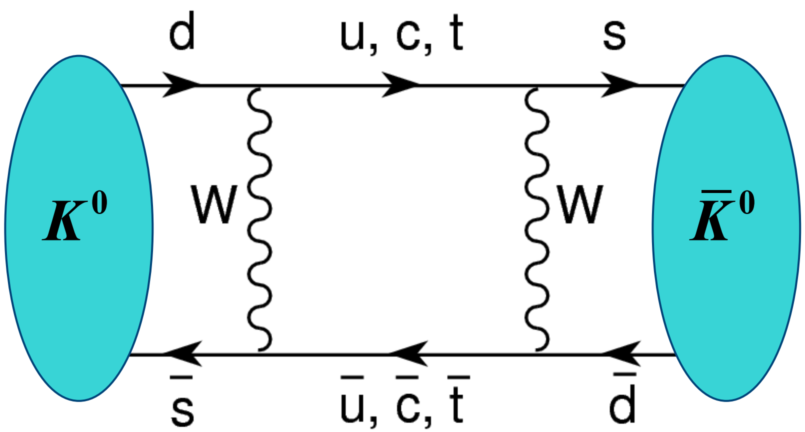


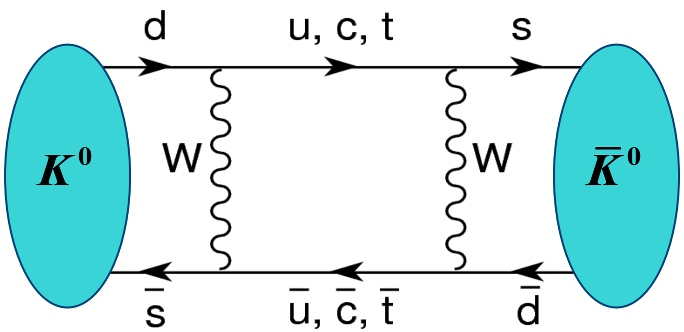
through weak interaction
Short distance effect

- The rate of this oscillation is very suppressed in the Standard Model

  ⇨ goes through *weak interactions* $\sim G_F$

- How can we understand the oscillation rate?

Oscillations of Kaons



- Process described using the bag parameter B_K
  Fundamental hadronic quantity proportional to matrix element
  ⇒ determined using *lattice QCD*

$$\langle \bar{K}^0 | \mathbf{H} | K^0 \rangle \sim \sum_{ij} \lambda_i \lambda_j \, S(r_i, r_j) \, \eta_{ij} \quad \langle O_{\Delta S=2} \rangle$$

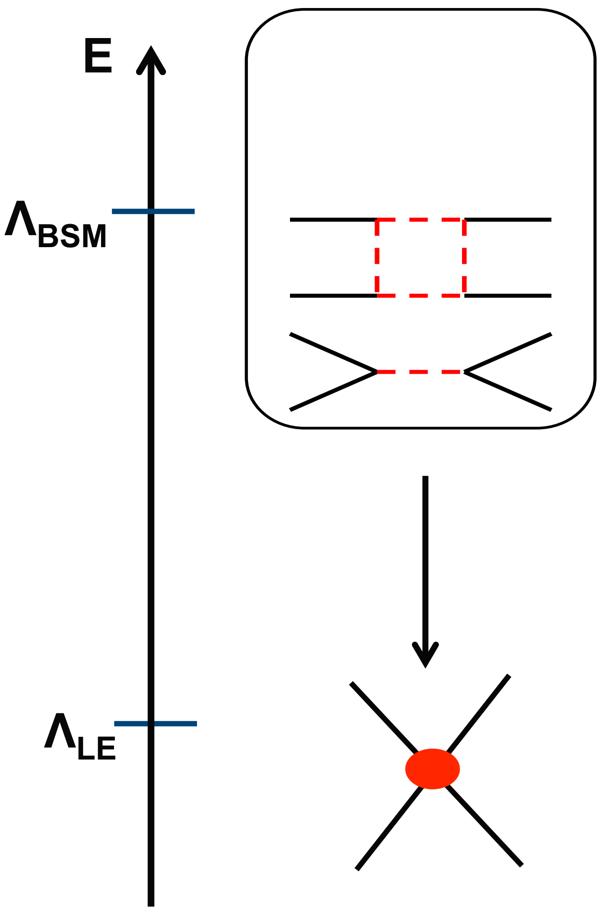
$$\langle O_{\Delta S=2} \rangle = \alpha_s(\mu)^{-2/9} \langle \bar{K}^0 | (\bar{s}_L \gamma^\alpha d_L)(\bar{s}_L \gamma_\alpha d_L) | K^0 \rangle \equiv \left(\frac{4}{3} M_K^2 f_K^2 \right) \hat{B}_K$$

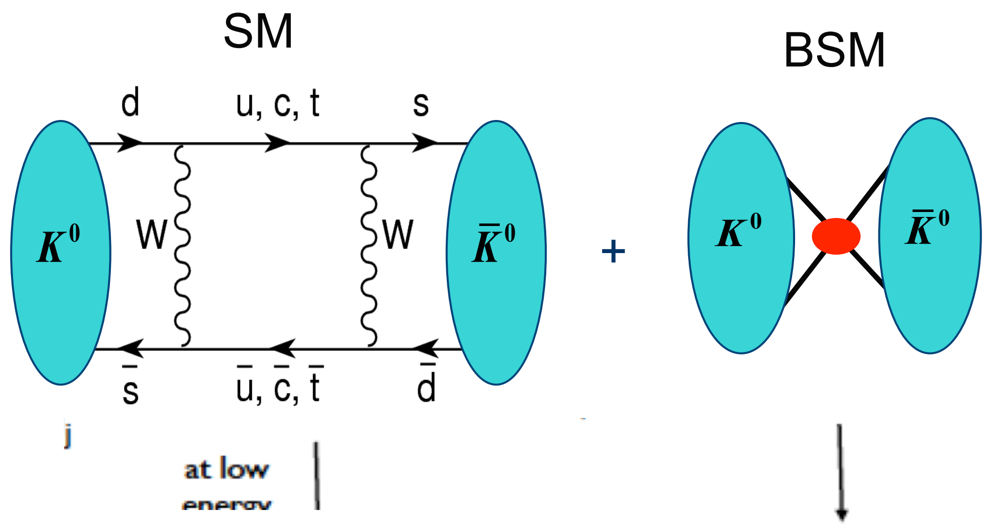
$$\lambda_i \equiv V_{id} V_{is}^* \quad ; \quad r_i \equiv m_i^2 / M_W^2 \quad (i = u, c, t)$$

Oscillations of Kaons

- Since process is suppressed in the Standard Model:

  ➡ very sensitive to *new physics*: new degrees of freedom and symmetries





$$G_F \, V_{ij} \, V_{ij}^{\dagger} \, \frac{m_{i,j}^2}{M_W^2}$$

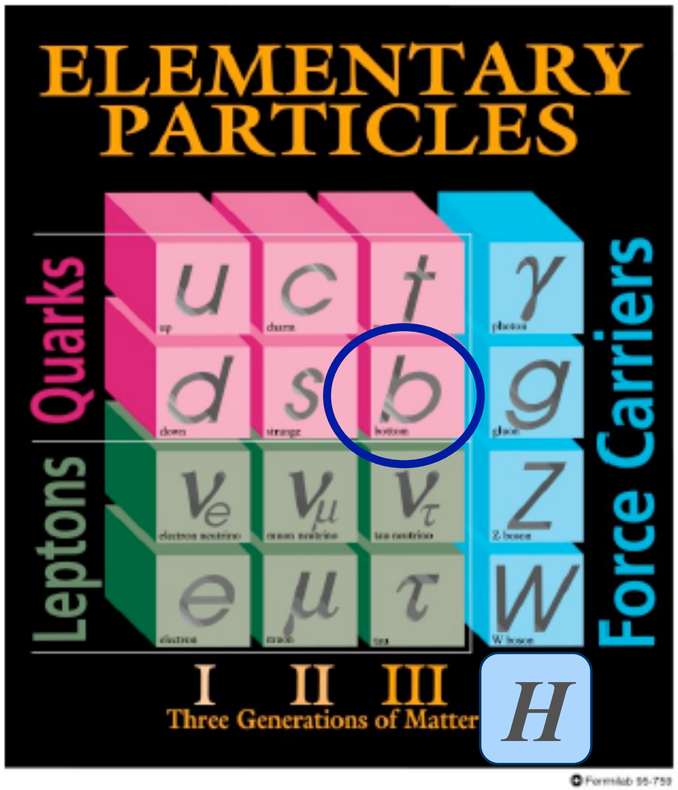
$$1/\Lambda^2$$

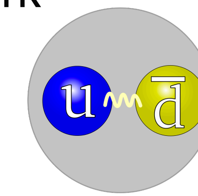
- If measured with very good precision provided the SM contribution is known

  ➡ *stringent constraints* on new physics models

Oscillations of B mesons

- Similar tests with other mesons ⇒ Beauty mesons contain a b-quark





$B^+ : u\bar{b}$, $B^0 : d\bar{b}$

$B^- : \bar{u}b$, $\bar{B}^0 : \bar{d}b$

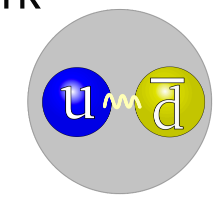
$B_s^0 : s\bar{b}$, $\bar{B}_s^0 : \bar{s}b$

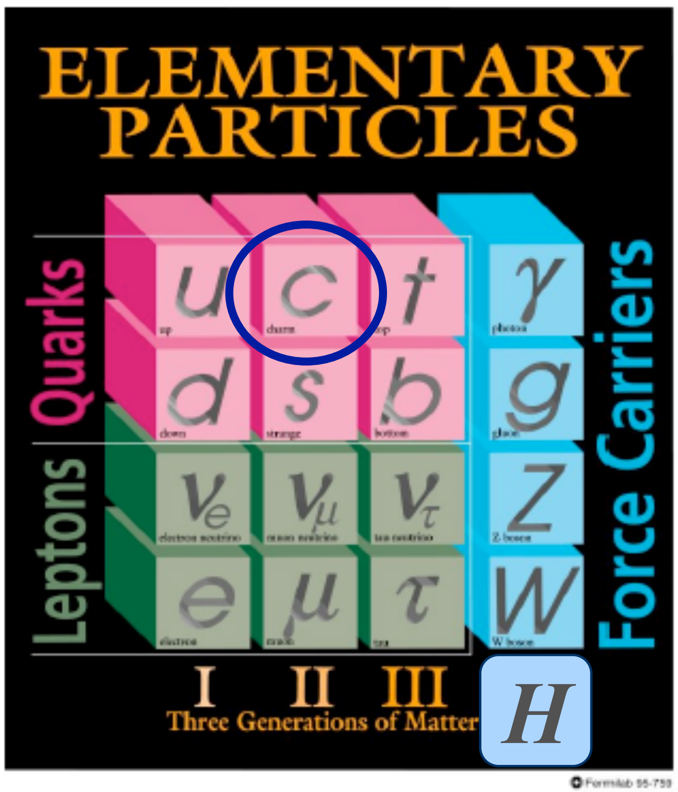
$B_c^0 : c\bar{b}$, $B_c^0 : \bar{c}b$

- B meson physics have been studied extensively at BaBar, Belle, CDF, D0@Tevatron and now Belle-II, LHCb, CMS and ATLAS@LHC

Oscillations of B mesons

- Similar tests with other mesons ⇨ Beauty mesons contain a b-quark





$$B^+ : u\bar{b} \quad , \qquad B^0 : d\bar{b}$$

$$B^- : \bar{u}b \quad , \qquad \bar{B}^0 : \bar{d}b$$

$$B_s^0 : s\bar{b} \quad , \qquad \bar{B}_s^0 : \bar{s}b$$

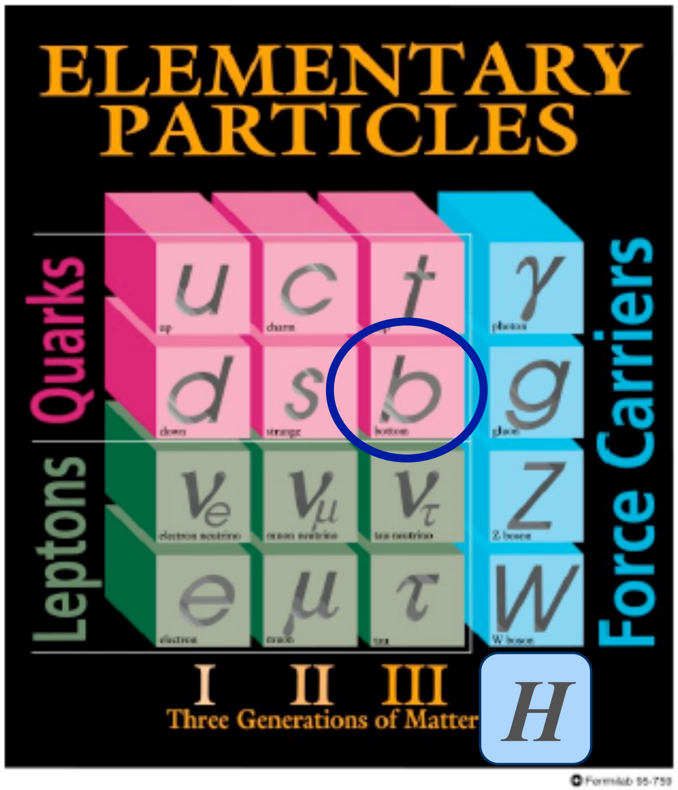
$$B_c^0 : c\bar{b} \quad , \qquad B_c^0 : \bar{c}b$$

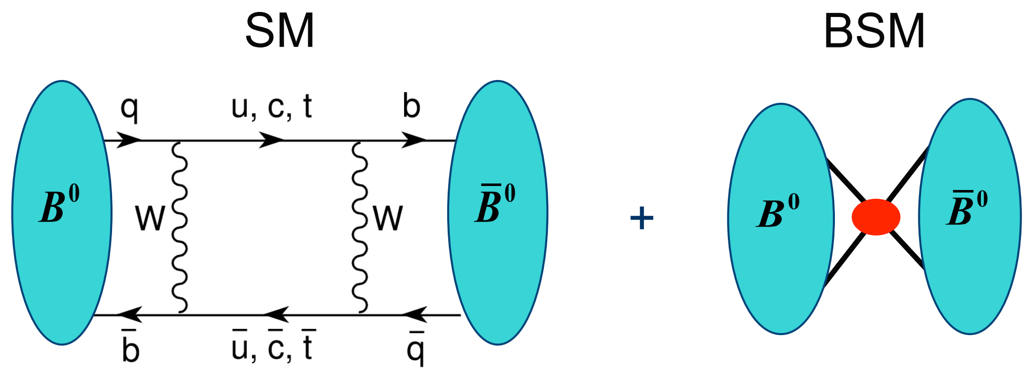
- B meson physics have been studied extensively at BaBar, Belle, CDF, D0@Tevatron and now Belle-II, LHCb, CMS and ATLAS@LHC

- Similar tests with D mesons

Oscillations of B mesons

- Similar tests with other mesons





- B-Bbar measured by *BaBar* and *Belle'01*
- Bs-Bsbar mixing observed by *CDF'06* and *LHCb'11*

CP violation in B decays *LHCb'13*

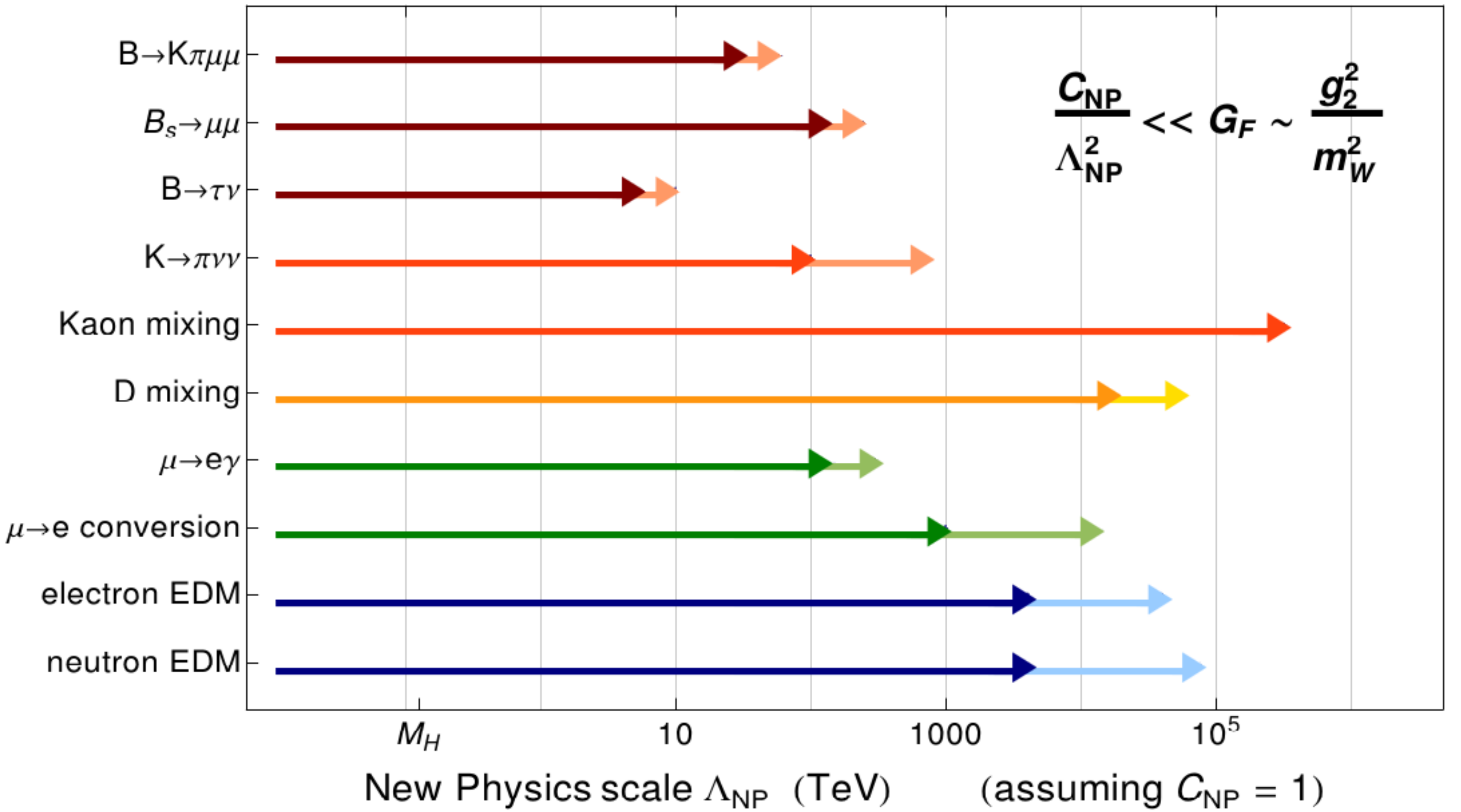
CP violation in D decays *LHCb'19*

- Stringent constraints on new physics models provided *hadronic* matrix elements known

New Physics and Flavour sector

- Very sensitive to New Physics

W. Altmannshofer



Anomalies in Flavour Physics

- Exciting discrepancies found recently:

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2 Accelerators Find Particles That May Break Known Laws of Physics

The LHC and the Belle experiment have found particle decay patterns that violate the Standard Model of particle physics, confirming earlier observations at the BaBar facility

By Clara Moskowitz | September 9, 2015 | Véalo en español

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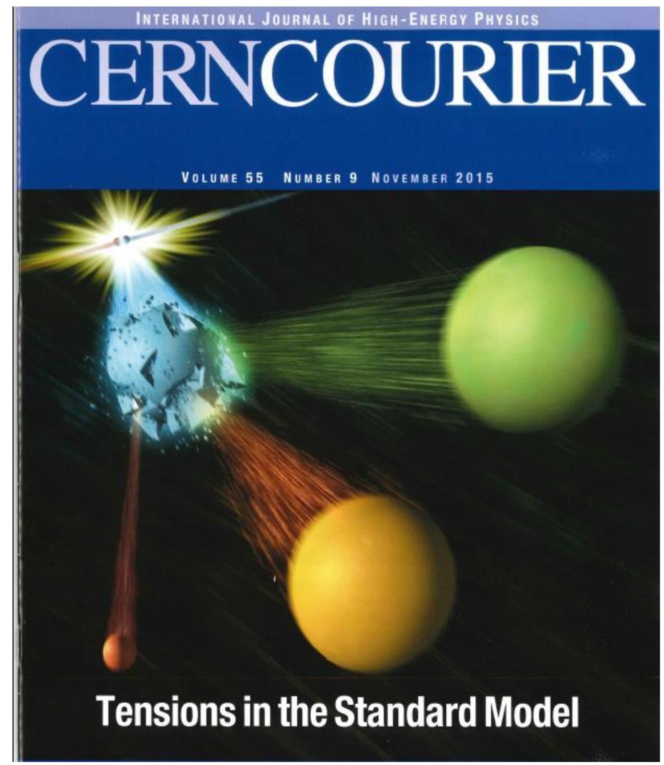
Democracy suffers a blow—in particle physics

Three independent B-meson experiments suggest that the charged leptons may not be so equal after all.

Steven K. Blau 17 September 2015



Anomalies in Flavour Physics

- These anomalies have generated a lot of excitement and theoretical papers to try to explain them using new physics models

- This requires a good understanding of hadronic physics
  see e.g. *Celis, Cirigliano, E.P., Phys.Rev. D89 (2014) 013008,*
  Phys.Rev. D89 (2014) no.9, 095014

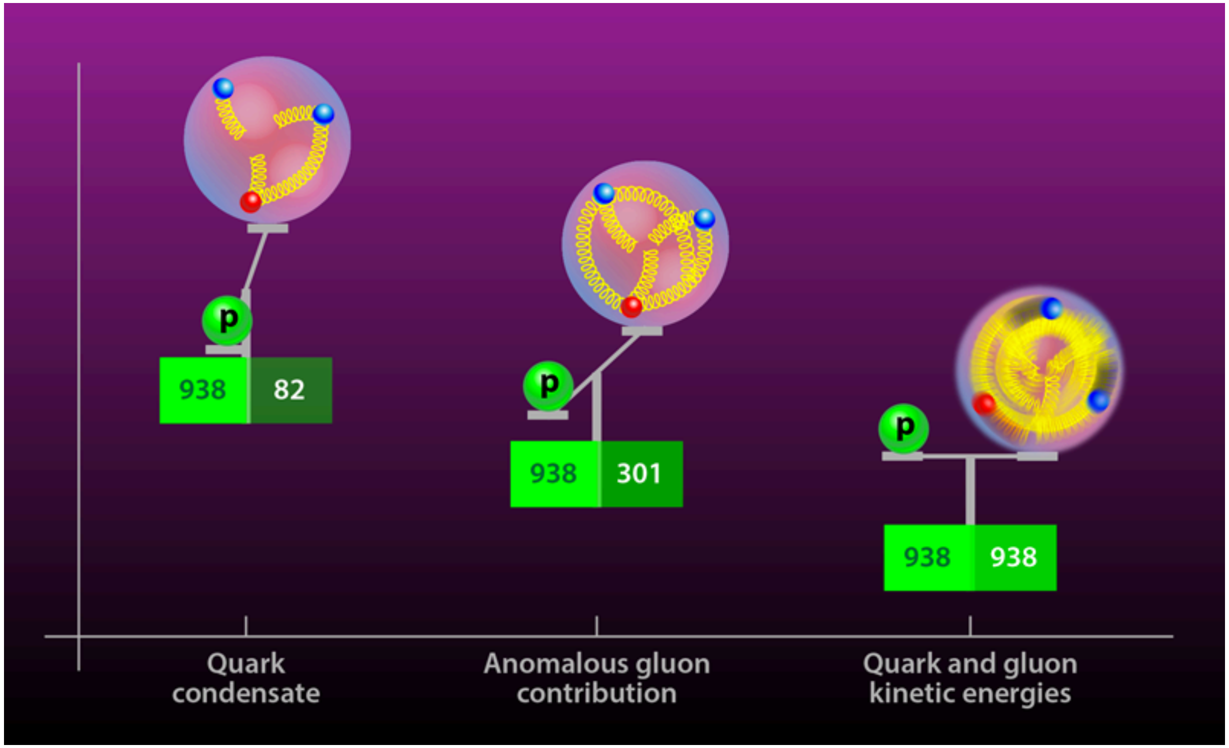
- New measurements are planned at ATLAS, CMS (dedicated B physics run) LHCb and Belle II

- Better precision within the next decade ⇒ match the level of precision theoretically with *hadronic physics*

3. Back up

Proton

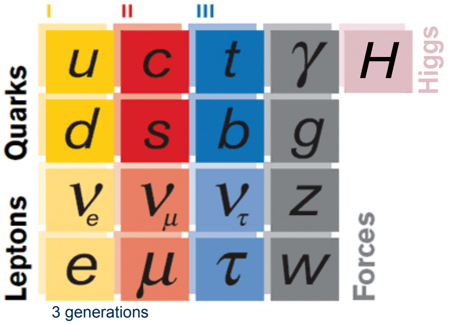
- Let us consider the proton: it is not a fundamental particle, it is made of 3 quarks



2.2 Flavour Physics

Description of the weak interactions:

$$\mathcal{L}_{EW} = \frac{g}{\sqrt{2}} W_\alpha^+ \left(\bar{D}_L V_{CKM} \gamma^\alpha U_L + \bar{e}_L \gamma^\alpha \nu_{e_L} + \bar{\mu}_L \gamma^\alpha \nu_{\mu_L} + \bar{\tau}_L \gamma^\alpha \nu_{\tau_L} \right) + \text{h.c.}$$



Probing the CKM mechanism

- The CKM Mechanism source of *Charge Parity Violation* in SM

- Unitary 3x3 Matrix, parametrizes rotation between mass and weak interaction eigenstates in Standard Model

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Weak Eigenstates CKM Matrix Mass Eigenstates

$$\sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

3.1 Probing the CKM mechanism

- The CKM Mechanism source of *Charge Parity Violation* in SM
- Unitary 3x3 Matrix, parametrizes rotation between mass and weak interaction eigenstates in Standard Model

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Weak Eigenstates CKM Matrix Mass Eigenstates

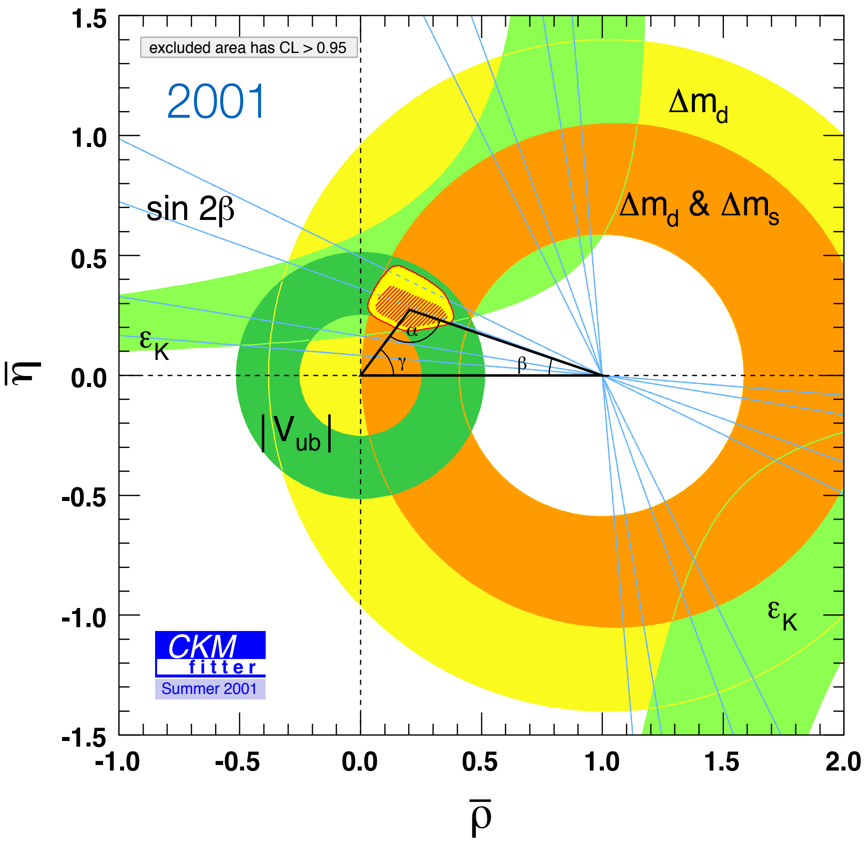
- Fully parametrized by **four** parameters if unitarity holds: three real parameters and *one complex phase* that if non-zero results in *CPV*
- Unitarity can be visualized using triangle equations, e.g.

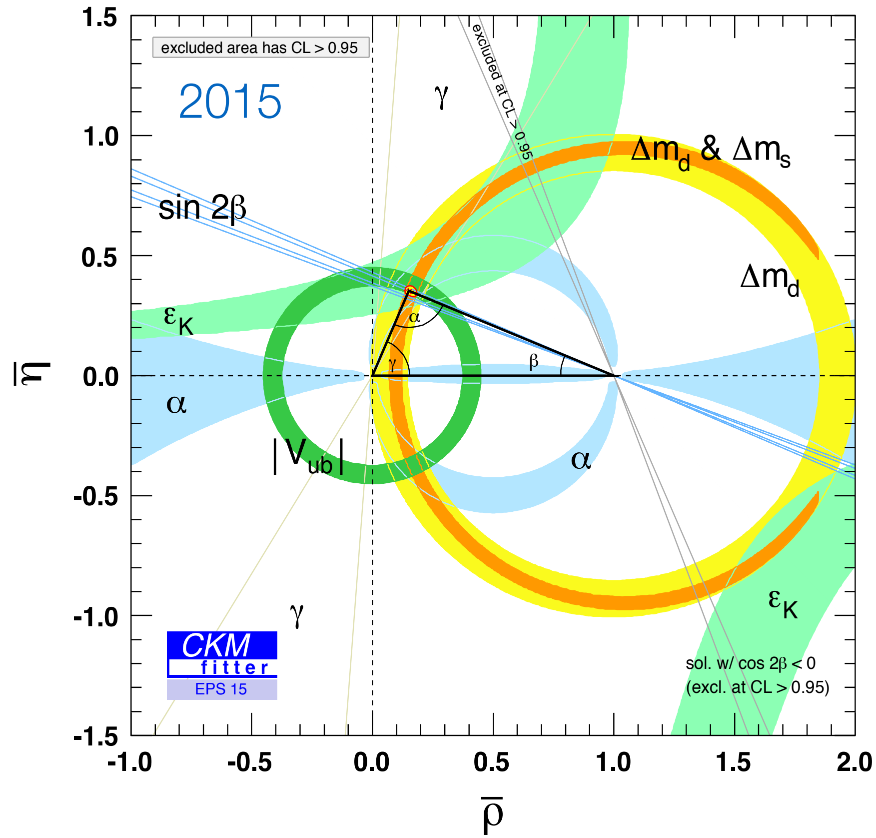
$$V_{CKM} V_{CKM}^{\dagger} = \mathbf{1} \quad \rightarrow \quad \boxed{V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0}$$

CKM picture over the years: from discovery to precision

Existence of *CPV* phase established in 2001 by BaBar & Belle

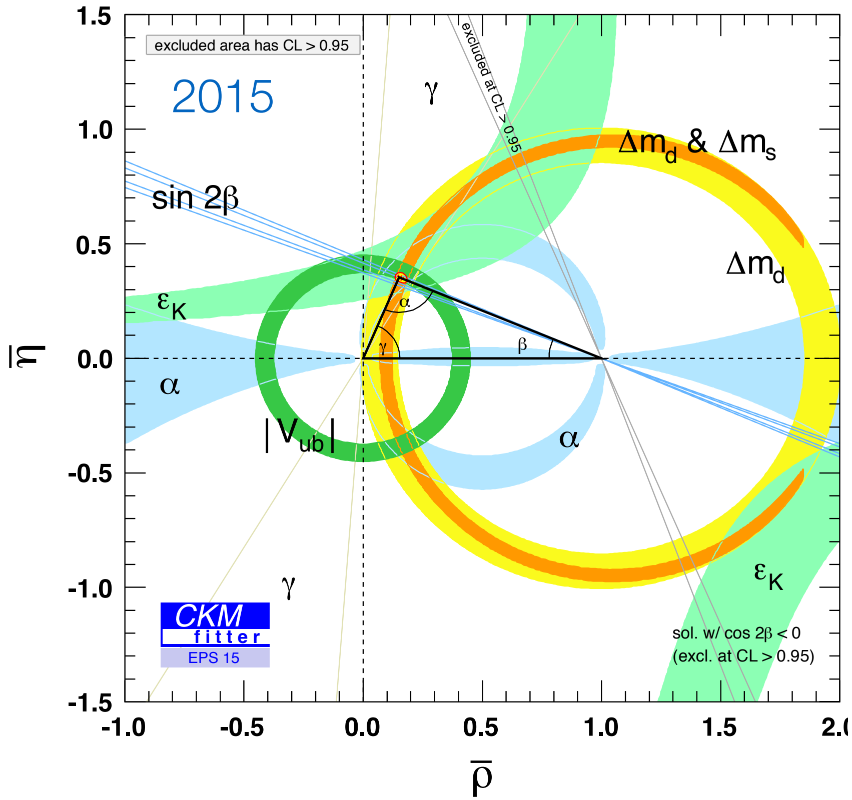
- Picture still holds 15 years later, constrained with remarkable precision
- But: still leaves room for new physics contributions





3.1 Probing the CKM mechanism

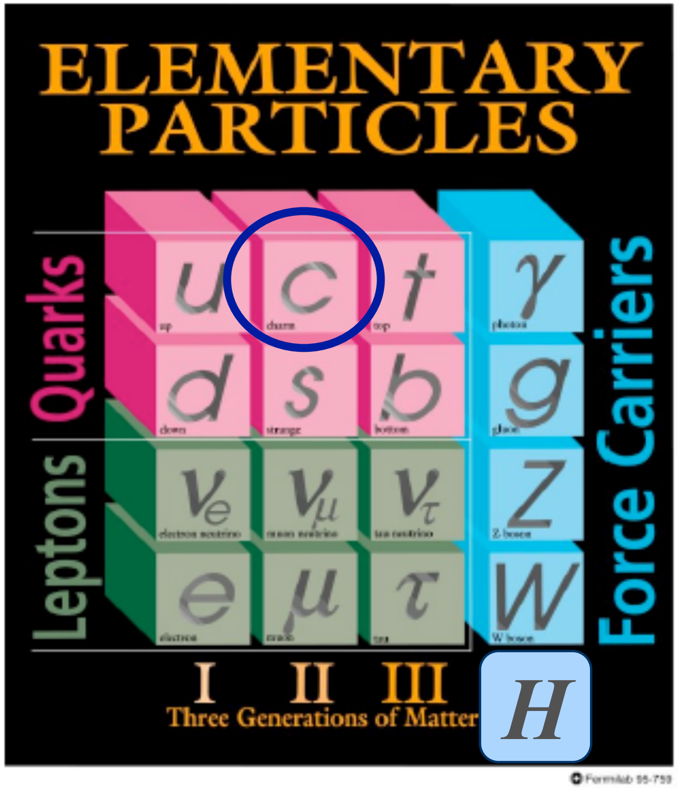
Input	World average 2016	Belle II (+LHCb) 2025
$\lvert V_{ub}\rvert(\text{semileptonic})[10^{-3}]$	$4.01 \pm 0.08 \pm 0.22$	± 0.10
$\lvert V_{cb}\rvert(\text{semileptonic})[10^{-3}]$	$41.00 \pm 0.33 \pm 0.74$	± 0.57
$\mathcal{B}(B \to \tau\nu)$	1.08 ± 0.21	± 0.04
$\sin 2\beta$	0.691 ± 0.017	± 0.008
$\gamma[^\circ]$	$73.2^{+6.3}_{-7.0}$	± 1.5 (± 1.0)
$\alpha[^\circ]$	$87.6^{+3.5}_{-3.3}$	± 1.0
Δm_d	0.510 ± 0.003	-
Δm_s	17.757 ± 0.021	-
$\mathcal{B}(B_s \to \mu\mu)$	$2.8^{+0.7}_{-0.6}$	(± 0.5)
f_{B_s}	$0.224 \pm 0.001 \pm 0.002$	0.001
B_{B_s}	$1.320 \pm 0.016 \pm 0.030$	0.010
f_{B_s}/f_{B_d}	$1.205 \pm 0.003 \pm 0.006$	0.005
B_{B_s}/B_{B_d}	$1.023 \pm 0.013 \pm 0.014$	0.005

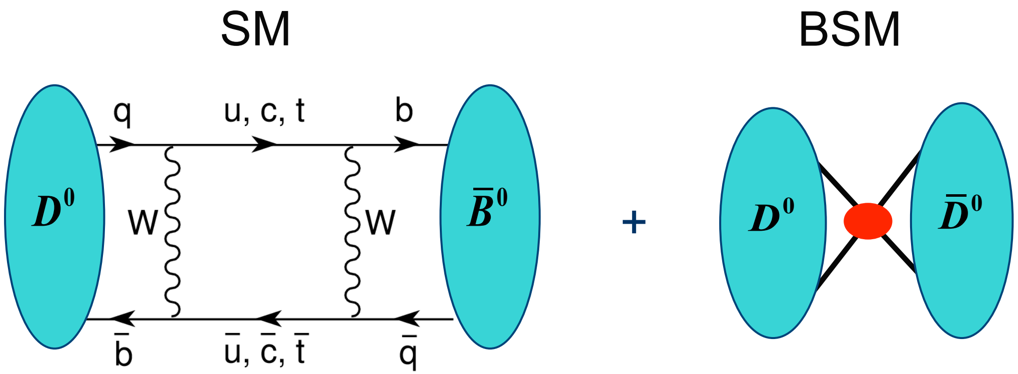


Expect substantial improvements to tree constraints!

2.2 Oscillations of Kaons

- Similar tests with other mesons





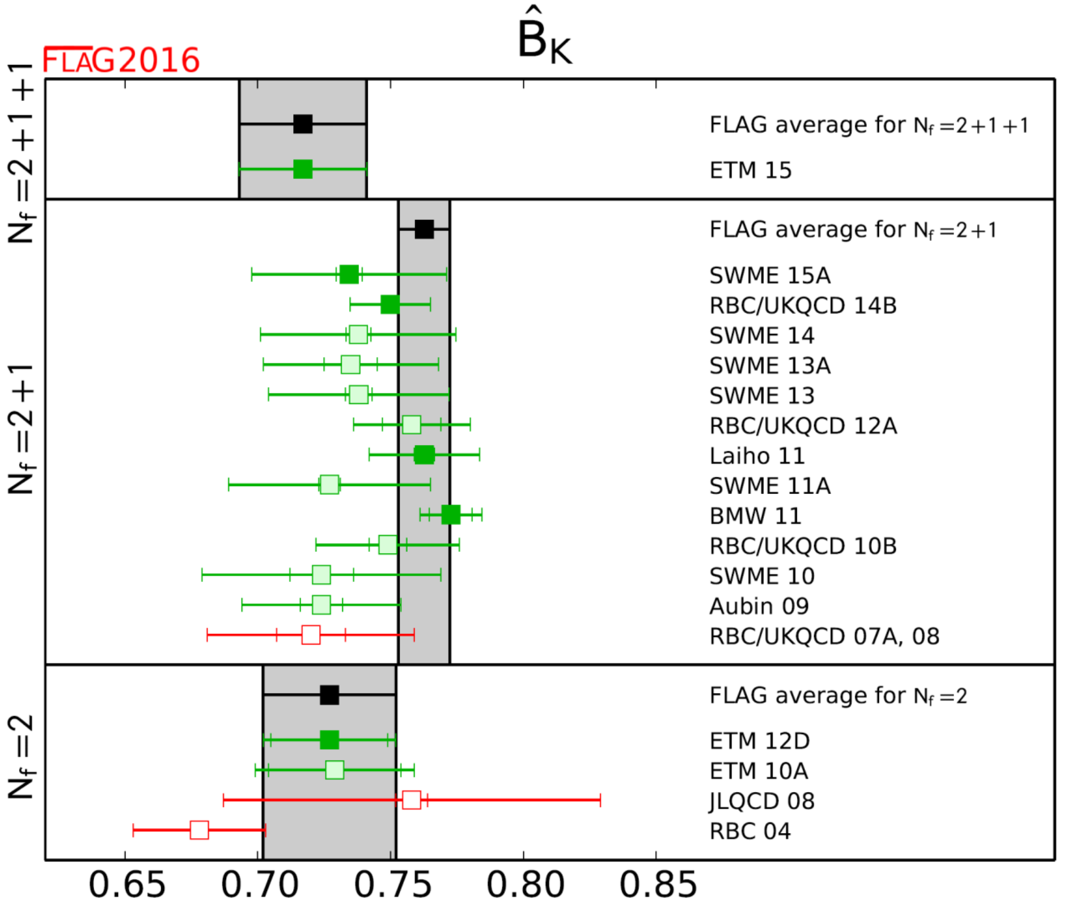
CDF, D0'06, LHCb'11

➡ CP violation in D decays *LHCb'19*

- Stringent constraints on new physics models provided *hadronic* matrix elements known

Lattice results for BK

$$B_K^{\overline{MS}}(2\,\text{GeV}) = 0.557 \pm 0.007 \quad , \quad \hat{B}_K = 0.763 \pm 0.010 \quad \left(N_f = 2+1\right)$$



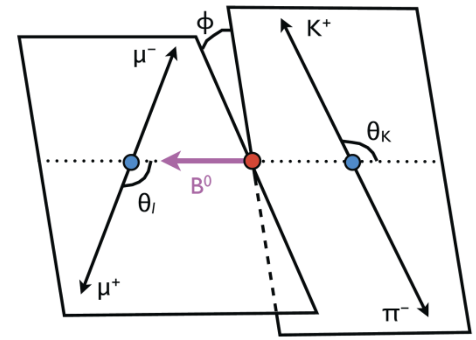
Flavianet Lattice Averaging Group

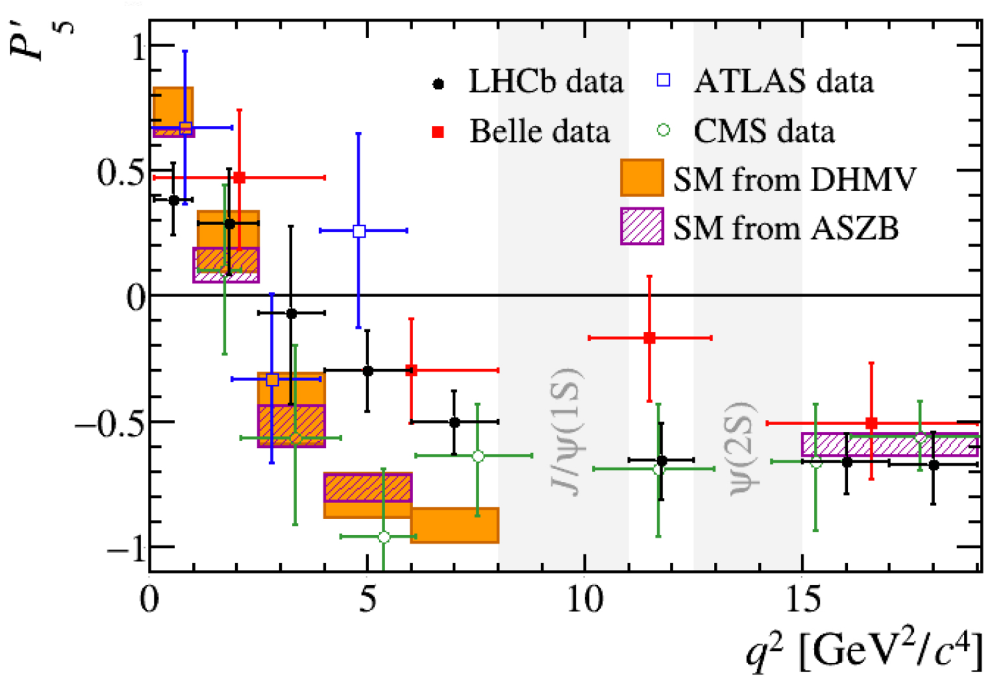
B → K*μ⁺μ⁻ → Kπμ⁺μ⁻

$$\frac{1}{\mathrm{d}\Gamma/dq^2}\frac{\mathrm{d}^4\Gamma}{\mathrm{d}\cos\theta_\ell\,\mathrm{d}\cos\theta_K\,\mathrm{d}\phi\,\mathrm{d}q^2} = \frac{9}{32\pi}\Big[\frac{3}{4}(1-F_\mathrm{L})\sin^2\theta_K + F_\mathrm{L}\cos^2\theta_K + \frac{1}{4}(1-F_\mathrm{L})\sin^2\theta_K\cos 2\theta_\ell$$
$$- F_\mathrm{L}\cos^2\theta_K\cos 2\theta_\ell + S_3\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi$$
$$+ S_4\sin 2\theta_K\sin 2\theta_\ell\cos\phi + S_5\sin 2\theta_K\sin\theta_\ell\cos\phi$$
$$+ S_6\sin^2\theta_K\cos\theta_\ell + S_7\sin 2\theta_K\sin\theta_\ell\sin\phi$$
$$+ S_8\sin 2\theta_K\sin 2\theta_\ell\sin\phi + S_9\sin^2\theta_K\sin^2\theta_\ell\sin 2\phi\Big]$$

$$q^2 = s_{\mu^+\mu^-}$$

$$P'_{i=4,5,6,8} = \frac{S_{j=4,5,7,8}}{\sqrt{F_L(1-F_L)}}$$





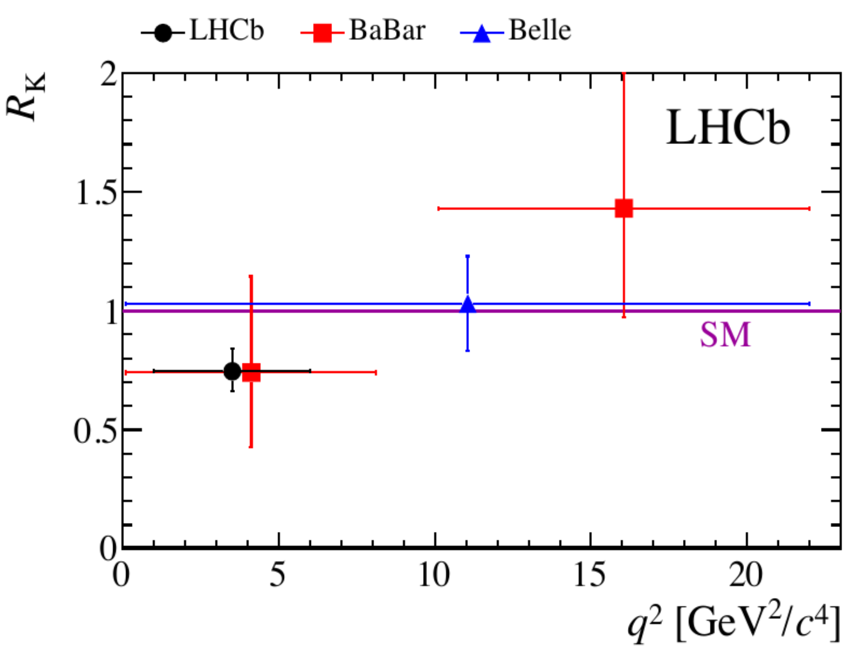
- Build an observable the less sensitive possible to hadronic uncertainties ⇨ P5'
  Only at LO

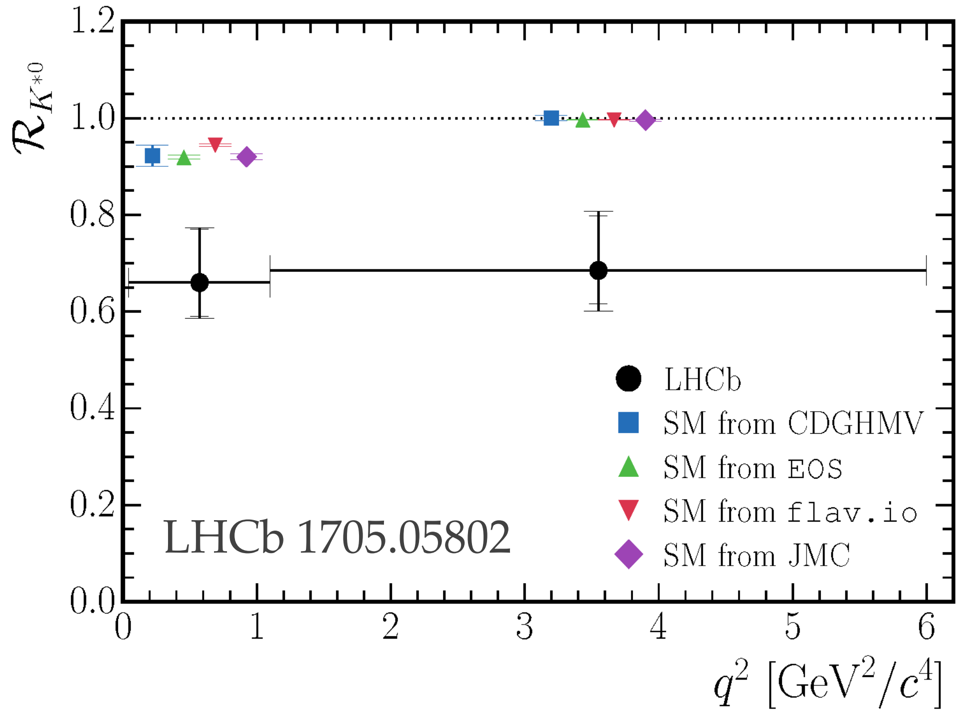
  DHMV: Descotes-Genon et al.'15
  ASZB:

- But new physics contributions involve *hadronic physics*!

R_K, R_{K*}

LHCb 1406.6482



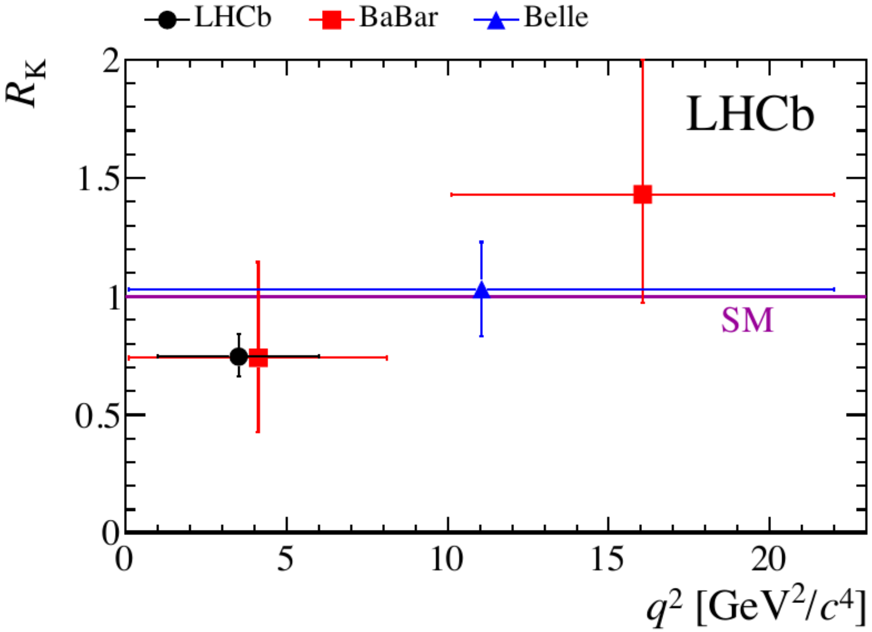


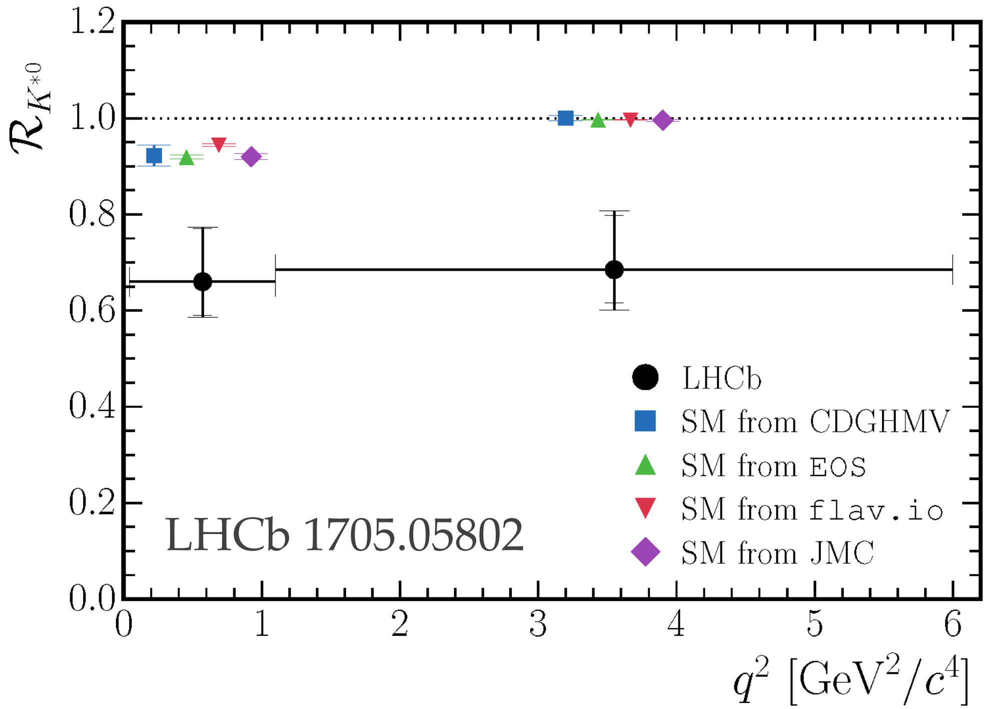
$$R_{K^{(*)}} = \frac{\Gamma(\bar{B} \to \bar{K}^{(*)}\mu^+\mu^-)}{\Gamma(\bar{B} \to \bar{K}^{(*)}e^+e^-)}$$

- Hadronic uncertainties cancel in the ratio

R_K, R_{K*}

LHCb 1406.6482





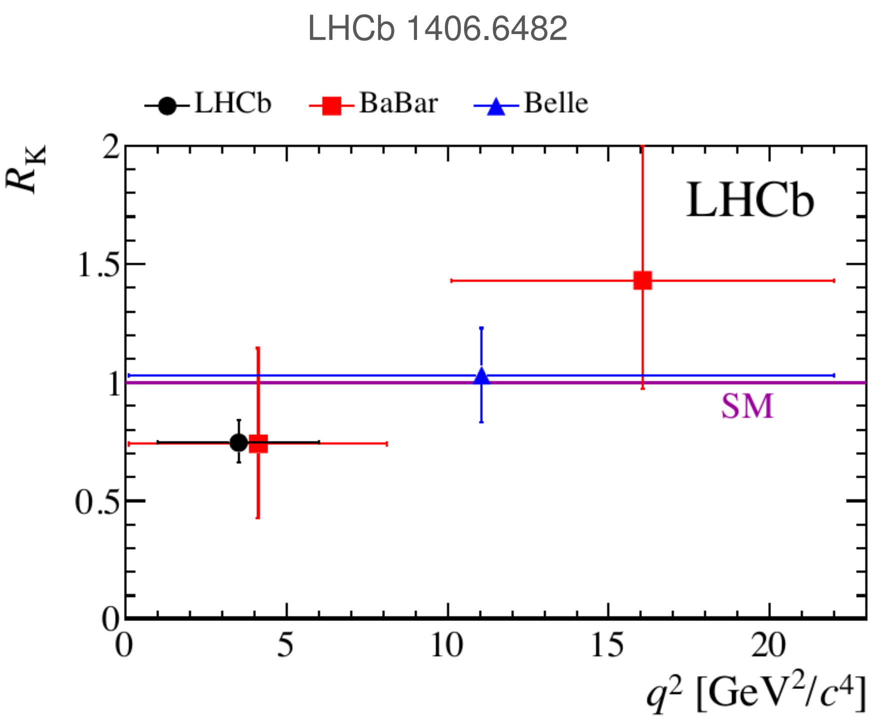
$$R_{K^{(*)}} = \frac{\Gamma(\bar{B} \to \bar{K}^{(*)}\mu^+\mu^-)}{\Gamma(\bar{B} \to \bar{K}^{(*)}e^+e^-)}$$

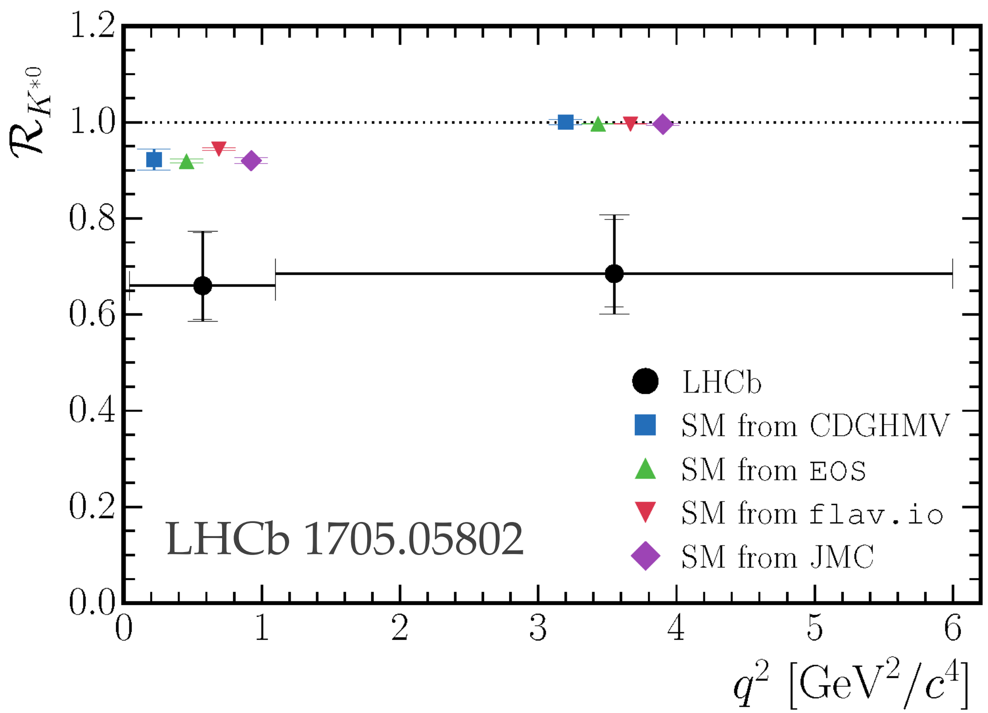
- Hadronic uncertainties cancel in the ratio
- Update from LHCb and Belle

❖ Original LHC*b* result (2.6σ):

$$R_K = 0.745^{+0.090}_{-0.074}\,(\text{stat}) \pm 0.036\,(\text{syst})$$

R_K, R_{K^*}





$$R_{K^{(*)}} = \frac{\Gamma(\bar{B} \to \bar{K}^{(*)}\mu^+\mu^-)}{\Gamma(\bar{B} \to \bar{K}^{(*)}e^+e^-)}$$

- Hadronic uncertainties cancel in the ratio
- Update from LHCb and Belle

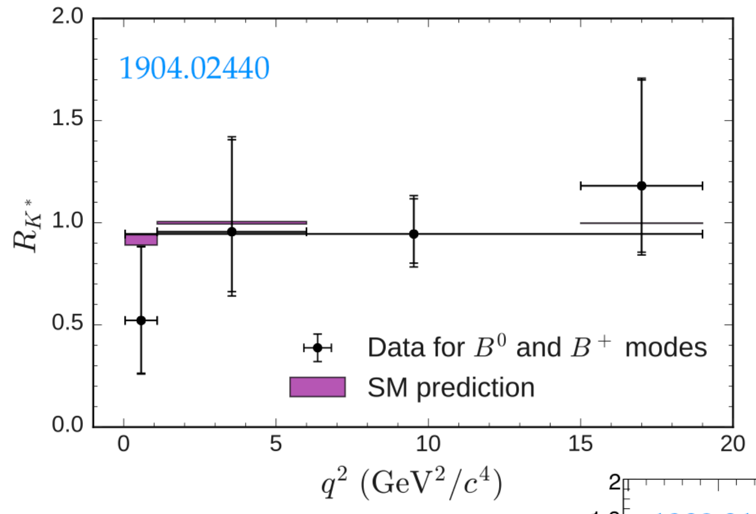
- Original LHC*b* result (2.6σ):

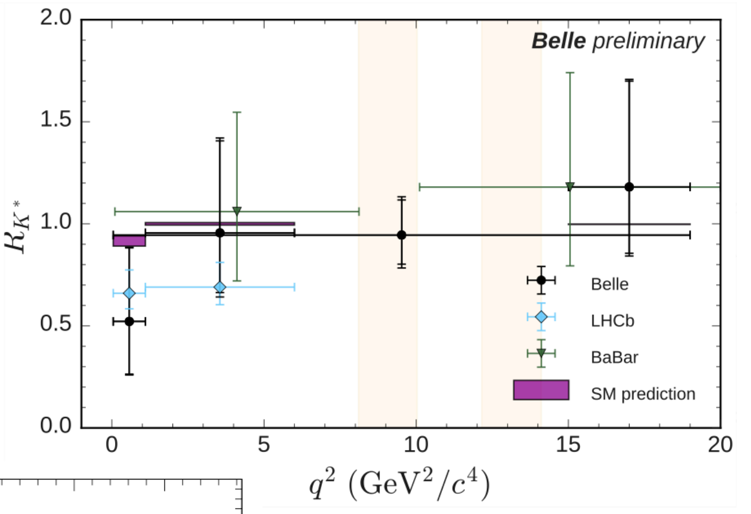
$$R_K = 0.745^{+0.090}_{-0.074}\,(\text{stat}) \pm 0.036\,(\text{syst})$$

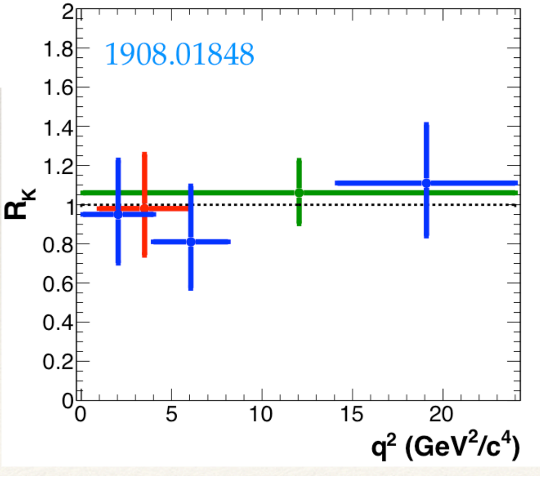
- New result including data until 2016 (2.5σ):

$$R_K = 0.846\,^{+0.060}_{-0.054}\,^{+0.016}_{-0.014}$$

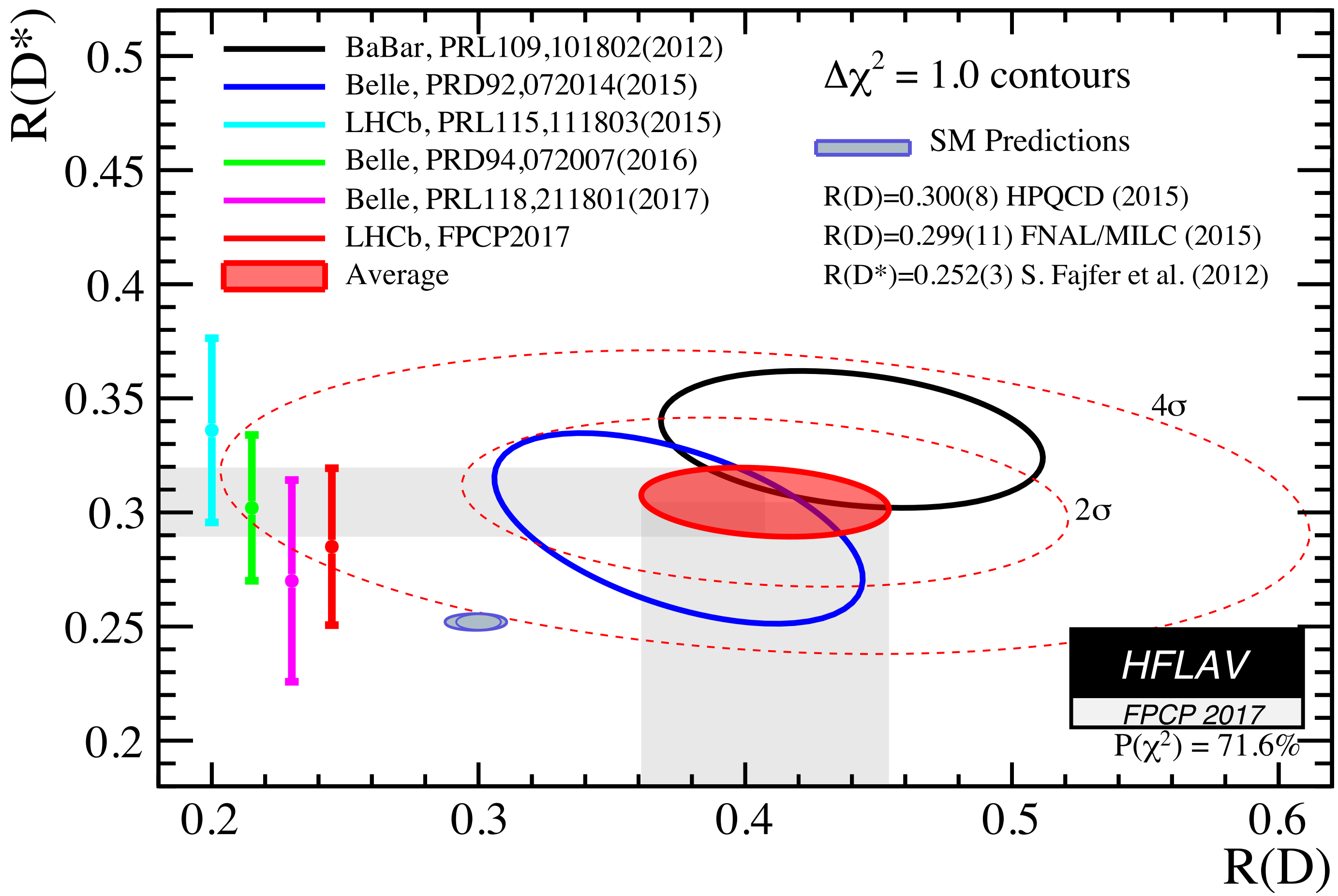
R_K, R_{K*}: Belle results





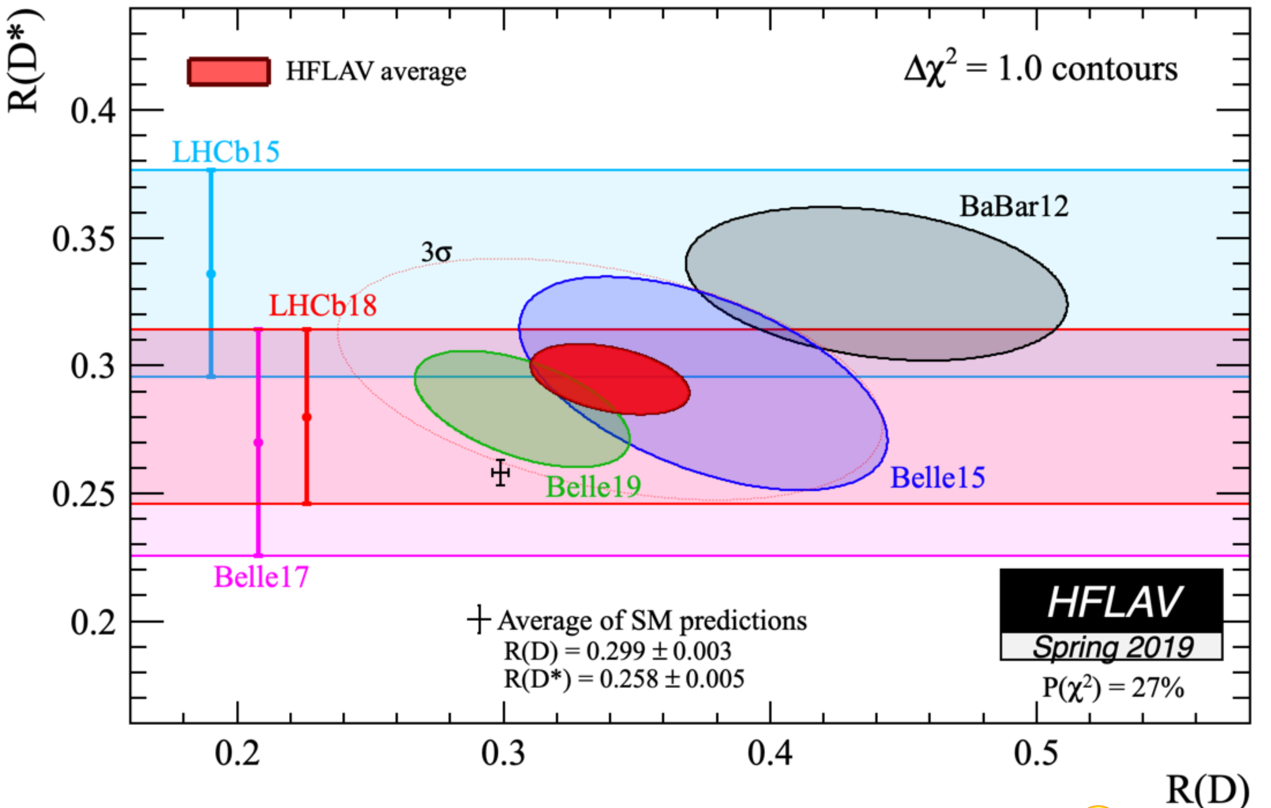


R_D, R_{D*}



$$R_{D^{(*)}} = \frac{\Gamma(\bar{B} \to D^{(*)}\tau\bar{\nu})}{\Gamma(\bar{B} \to D^{(*)}\ell\bar{\nu})}\ ; \quad \ell = e, \mu$$

R_D, R_{D*} : recent update from Belle



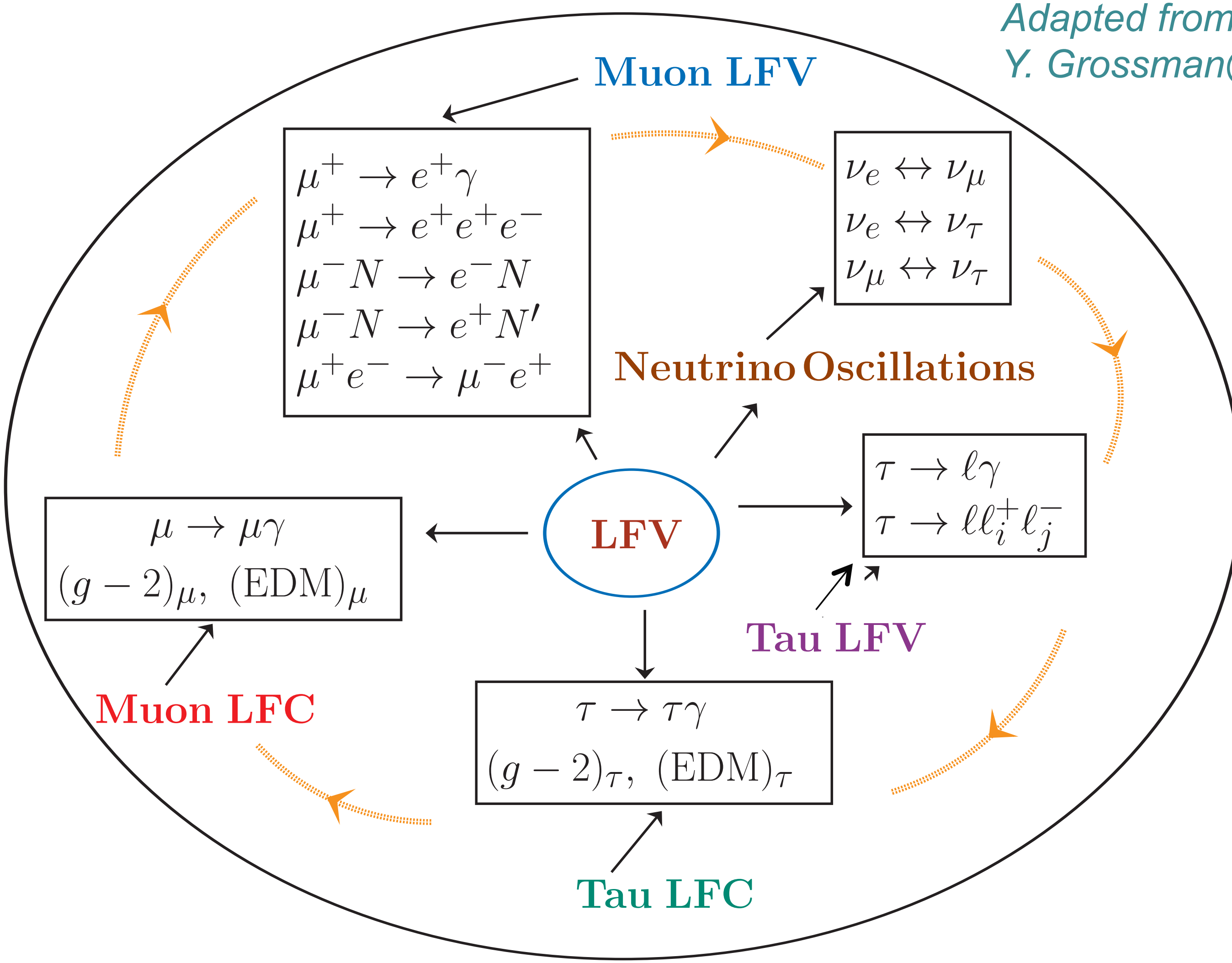
Significance reduced from 4.1 to 3.1σ 🙁

$$\mathcal{R}(D) = 0.307 \pm 0.037 \pm 0.016$$
$$\mathcal{R}(D^*) = 0.283 \pm 0.018 \pm 0.014$$

(Belle 2019: 1.2σ)

Leptons decays

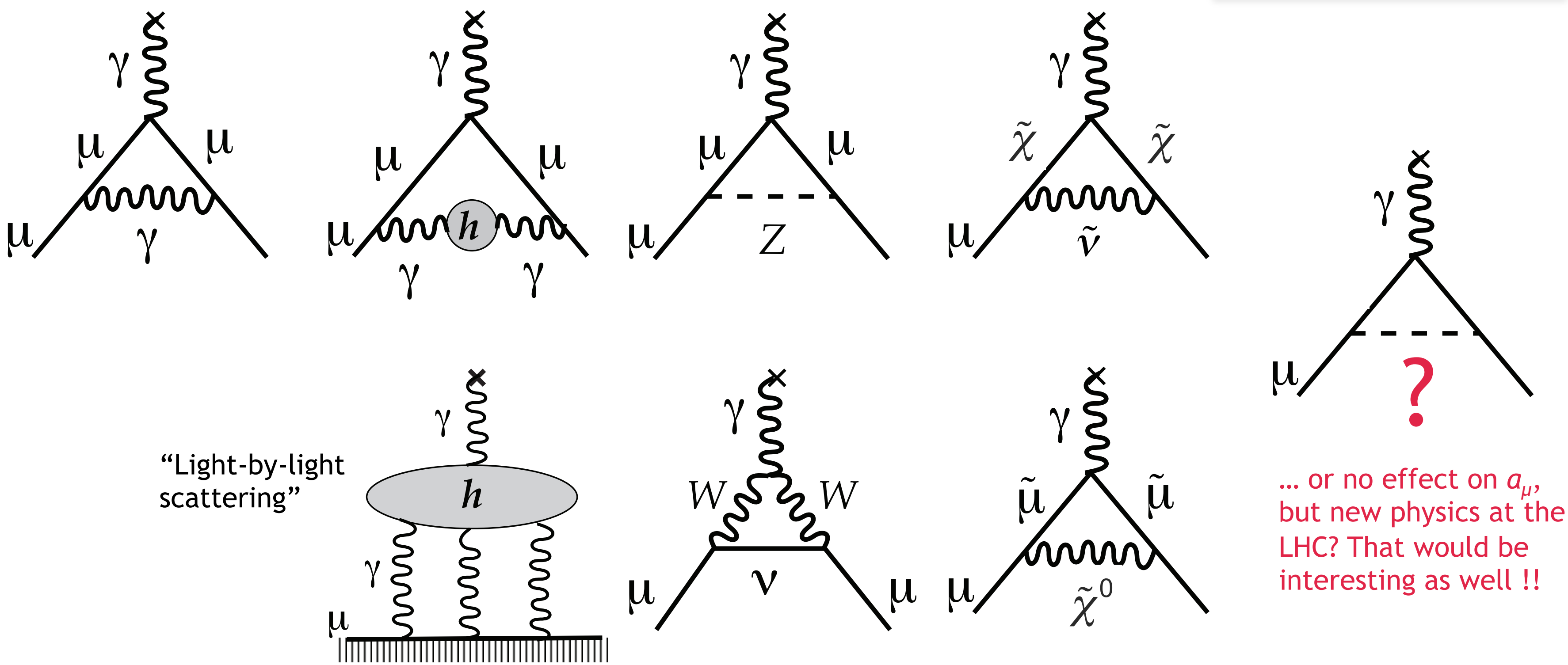
Adapted from Talk by Y. Grossman@CLFV2013



Contribution to $(g\text{-}2)_\mu$

Hoecker'11

QED | Hadronic | Weak | SUSY… ? | … or some unknown type of new physics ?



… or no effect on a_μ, but new physics at the LHC? That would be interesting as well !!

Need to compute the SM prediction with high precision! ⇒ *Not so easy!*
Hadrons enter virtually through loops!

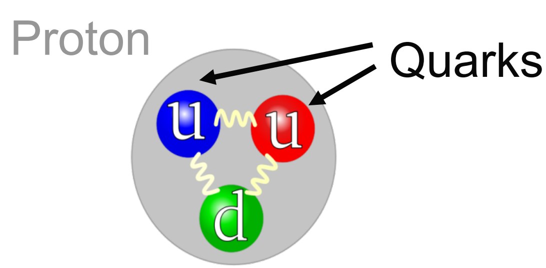
2.1 Quark masses

- Quark masses fundamental parameters of the *QCD Lagrangian*

$$\mathcal{L}_{QCD} = -\frac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a + \sum_{k=1}^{N_F} \bar{q}_k \left(i\gamma^\mu D_\mu - m_k \right) q_k$$

- No direct experimental access to quark masses due to *confinement*!

- Let us consider the proton: it is not a fundamental particle, but a bound state of 3 quarks



Contrary to naïve expectation, most of its mass comes from *strong force*

Only 1% of its mass comes from the quark masses (Coupling of the quarks to the Higgs boson)

2.1 Quark masses

- Quark masses fundamental parameters of the *QCD Lagrangian*

$$\mathcal{L}_{QCD} = -\frac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a + \sum_{k=1}^{N_F} \bar{q}_k \left(i\gamma^\mu D_\mu - m_k \right) q_k$$

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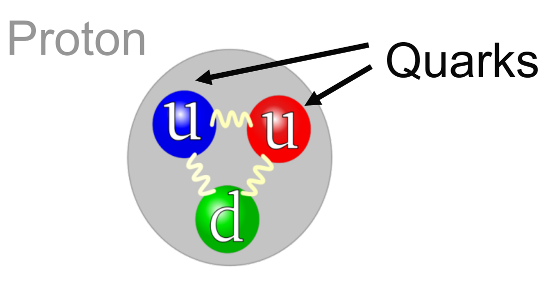


2.6 Why a new dispersive analysis?

- Several new ingredients:
  - New inputs available: extraction $\pi\pi$ phase shifts has improved

    Ananthanarayan et al'01, Colangelo et al'01

    Descotes-Genon et al'01

    Kaminsky et al'01, Garcia-Martin et al'09

  - New experimental programs, precise Dalitz plot measurements

    TAPS/CBall-MAMI (Mainz), WASA-Celsius (Uppsala), WASA-Cosy (Juelich)

    CBall-Brookhaven, CLAS, GlueX (JLab), KLOE I-II (Frascati)

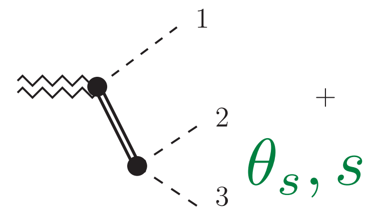
    BES III (Beijing)

  - Many improvements needed in view of very precise data: inclusion of
    - Electromagnetic effects ($\mathcal{O}(e^2m)$) *Ditsche, Kubis, Meissner'09*
    - Isospin breaking effects

2.7 Method

- S-channel partial wave decomposition

$$A_\lambda(s,t) = \sum_J^{\infty} (2J+1) d^J_{\lambda,0}(\theta_s) A_J(s)$$



- One truncates the partial wave expansion : ➡ Isobar approximation

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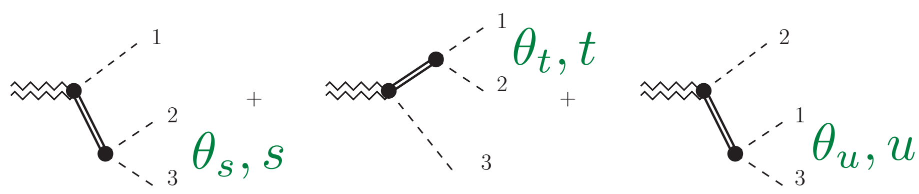

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➡ Improve to include final states interactions

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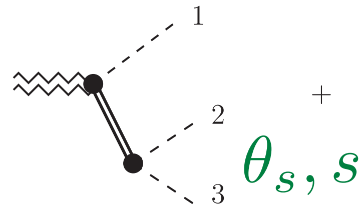

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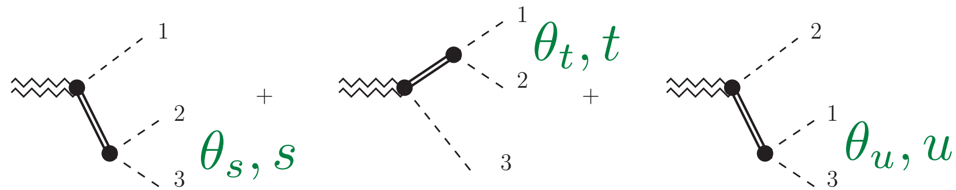

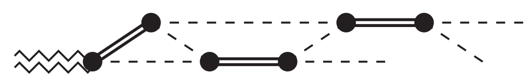

- Use a Khuri-Treiman approach or dispersive approach

  ⇨ Restore 3 body unitarity and take into account the final state interactions in a systematic way

2.8 Representation of the amplitude

- Decomposition of the amplitude as a function of isospin states

$$M(s,t,u) = M_0(s) + (s-u)M_1(t) + (s-t)M_1(u) + M_2(t) + M_2(u) - \frac{2}{3}M_2(s)$$

Fuchs, Sazdjian & Stern'93
Anisovich & Leutwyler'96

  - M_I isospin *I* rescattering in two particles
  - Amplitude in terms of S and P waves ⇨ exact up to NNLO ($\mathcal{O}(p^6)$)
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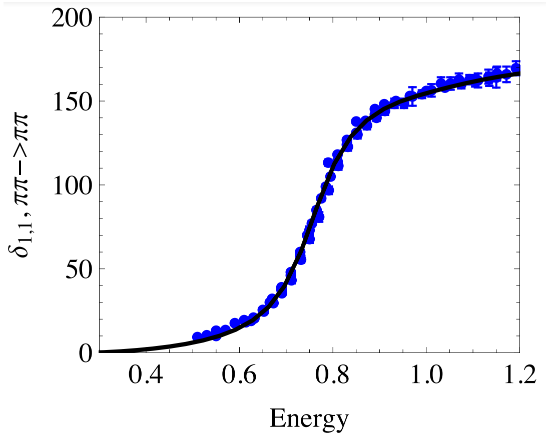
- Unitarity relation:

$\pi\pi \to \pi\pi$

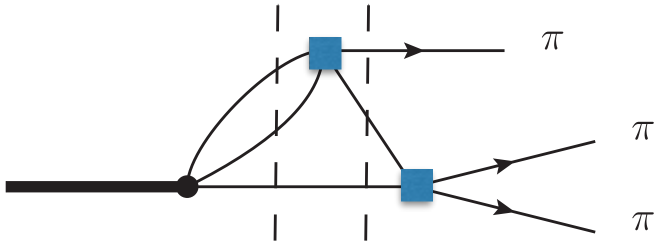
$$disc\left[M_\ell^I(s)\right] = \rho(s) t_\ell^*(s)\left(M_\ell^I(s) + \hat{M}_\ell^I(s)\right)$$

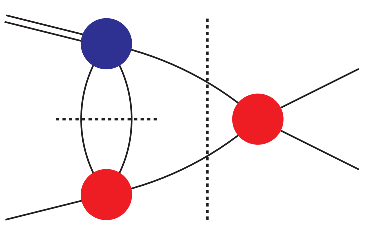
input

Roy analysis
Colangelo et al.'01



right-hand cut   left-hand cut





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- Relation of dispersion to reconstruct the amplitude everywhere:

$$M_I(s) = \Omega_I(s)\left(P_I(s) + \frac{s^n}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'^n}\frac{\sin\delta_I(s')\,\hat{M}_I(s')}{|\Omega_I(s')|(s'-s-i\varepsilon)}\right)$$

$$\left[\Omega_I(s) = \exp\left(\frac{s}{\pi}\int_{4M_\pi^2}^{\infty}ds'\frac{\delta_I(s')}{s'(s'-s-i\varepsilon)}\right)\right]$$

Omnès function

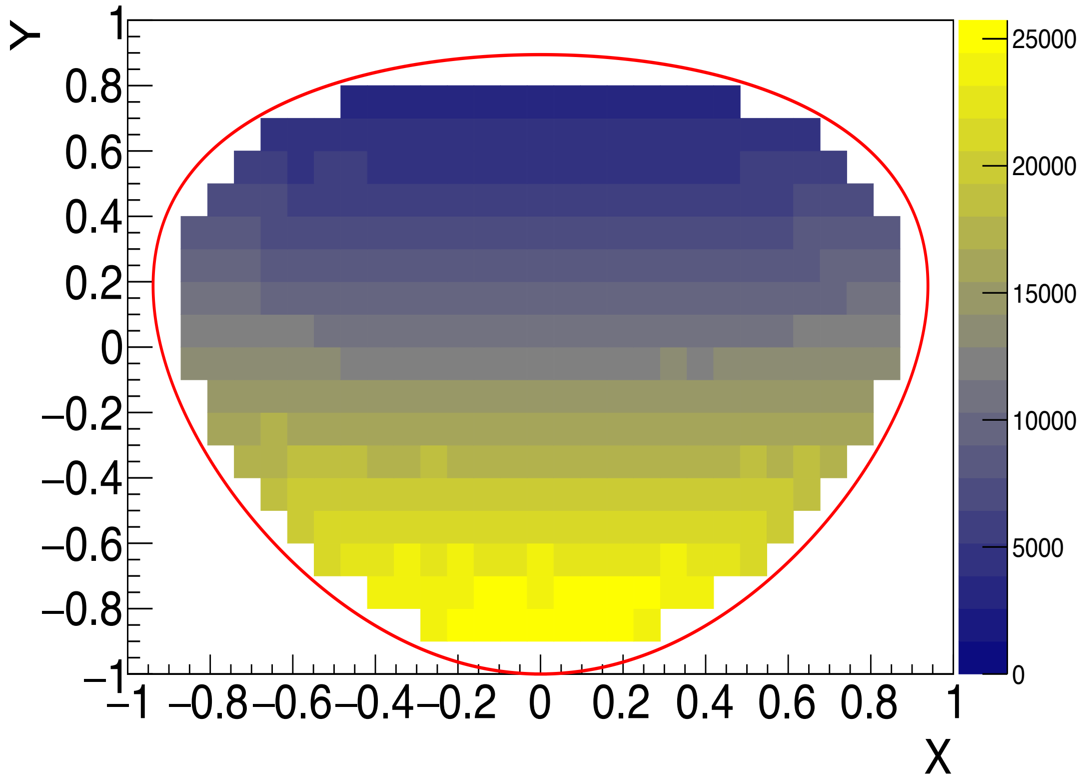
Gasser & Rusetsky'18

- $P_I(s)$ determined from a fit to NLO ChPT + experimental Dalitz plot

2.9 $\eta \to 3\pi$ Dalitz plot

- In the charged channel: experimental data from *WASA, KLOE, BESIII*

KLOE'16



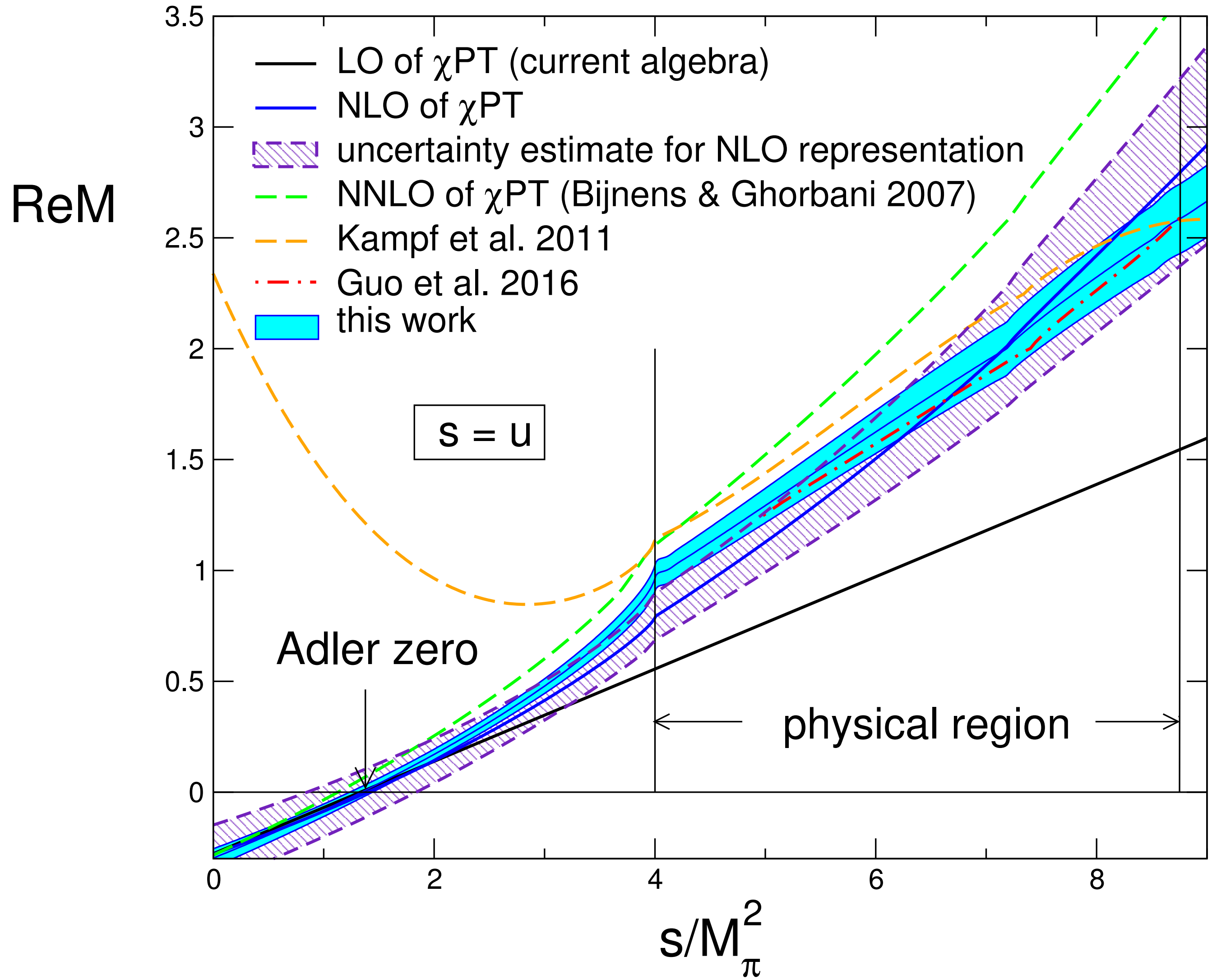
$$X = \sqrt{3}\,\frac{T_+ - T_-}{Q_c} = \frac{\sqrt{3}}{2M_\eta Q_c}(u-t)$$

$$Y = \frac{3T_0}{Q_c} - 1 = \frac{3}{2M_\eta Q_c}\left(\left(M_\eta - M_{\pi^0}\right)^2 - s\right) - 1$$

- New data expected from *CLAS* and *GlueX* with very different systematics

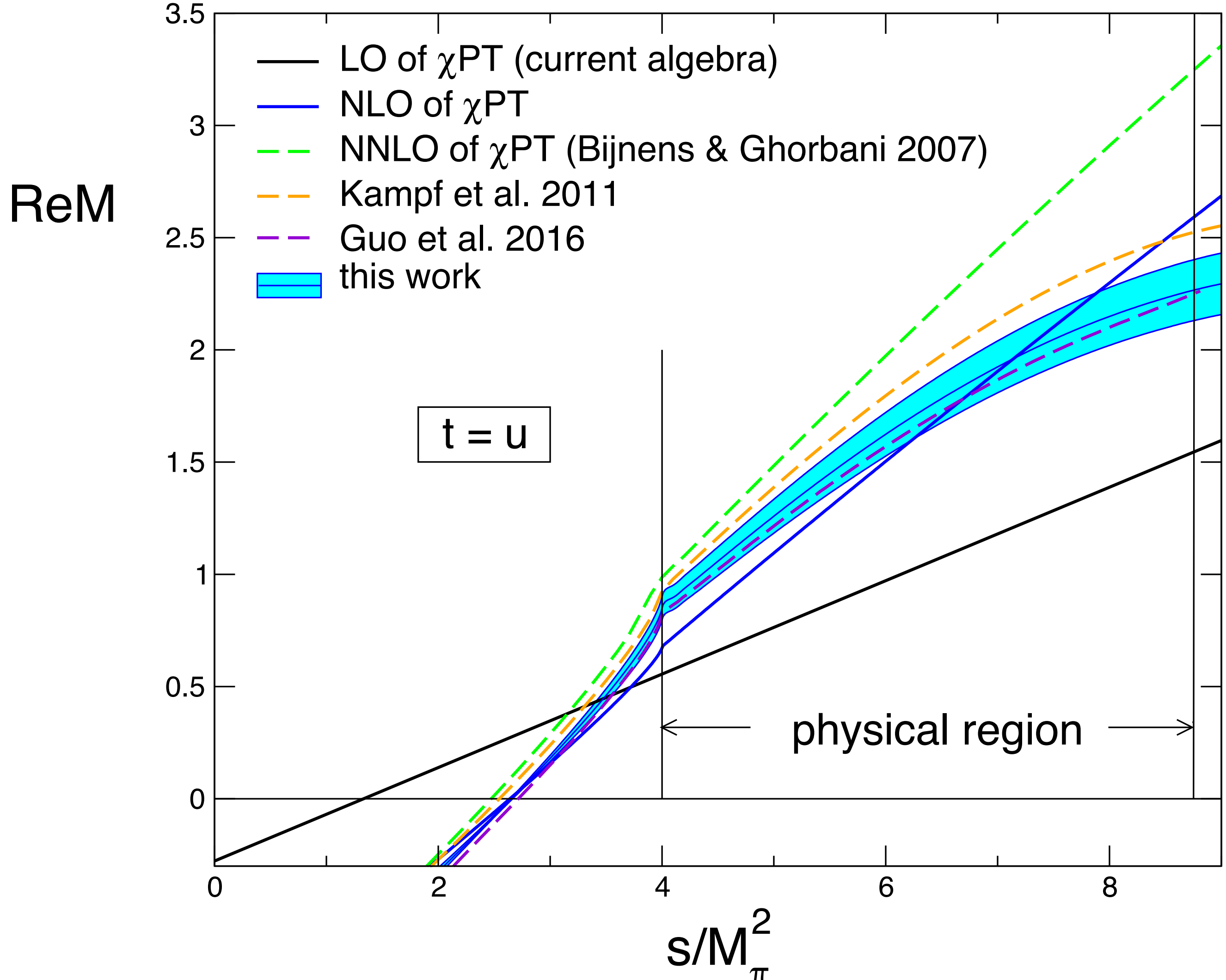
2.10 Results: Amplitude for $\eta \rightarrow \pi^+ \pi^- \pi^0$ decays

- The amplitude along the line s = u :



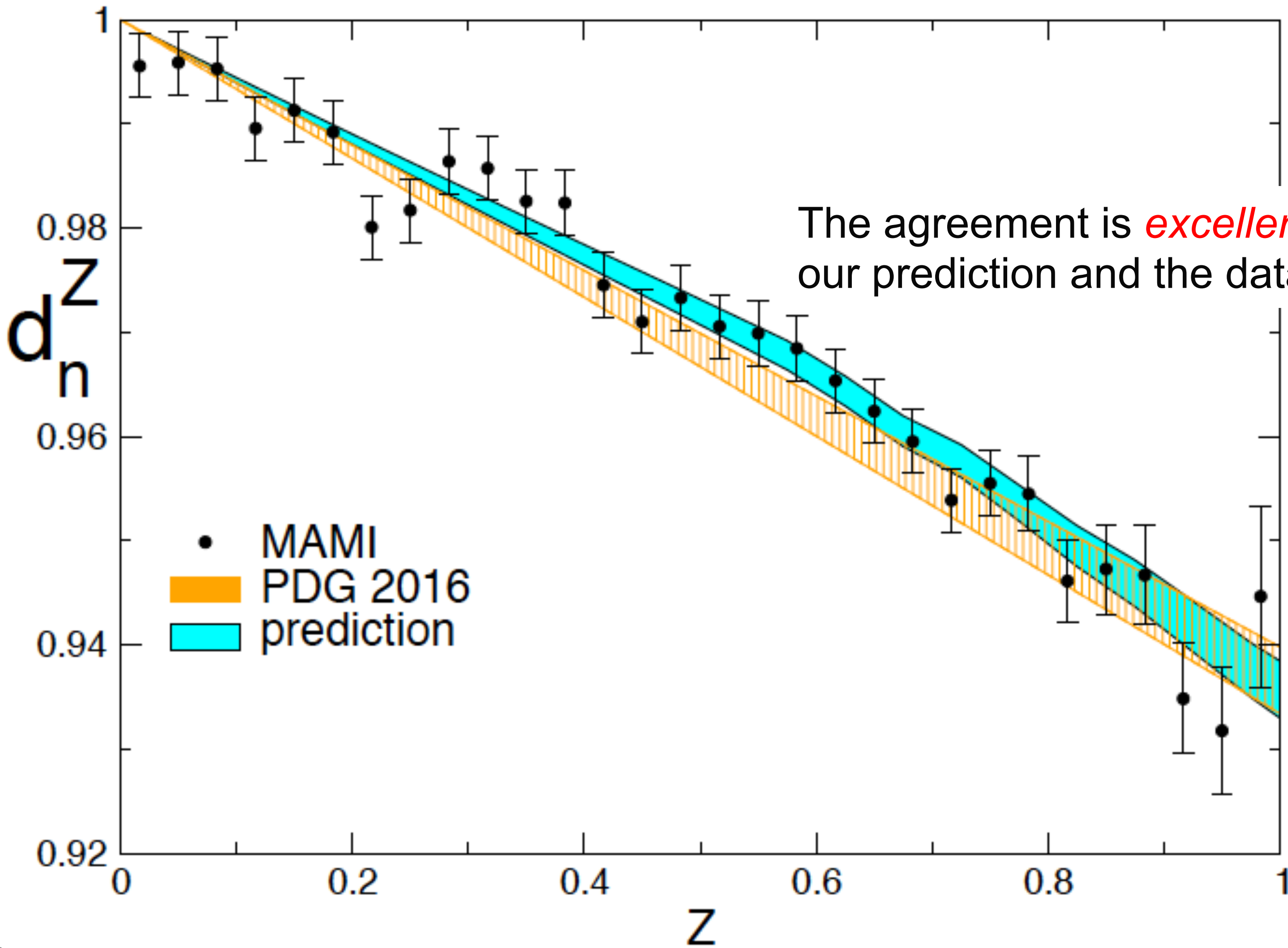
2.10 Results: Amplitude for $\eta \rightarrow \pi^+ \pi^- \pi^0$ decays

- The amplitude along the line t = u :



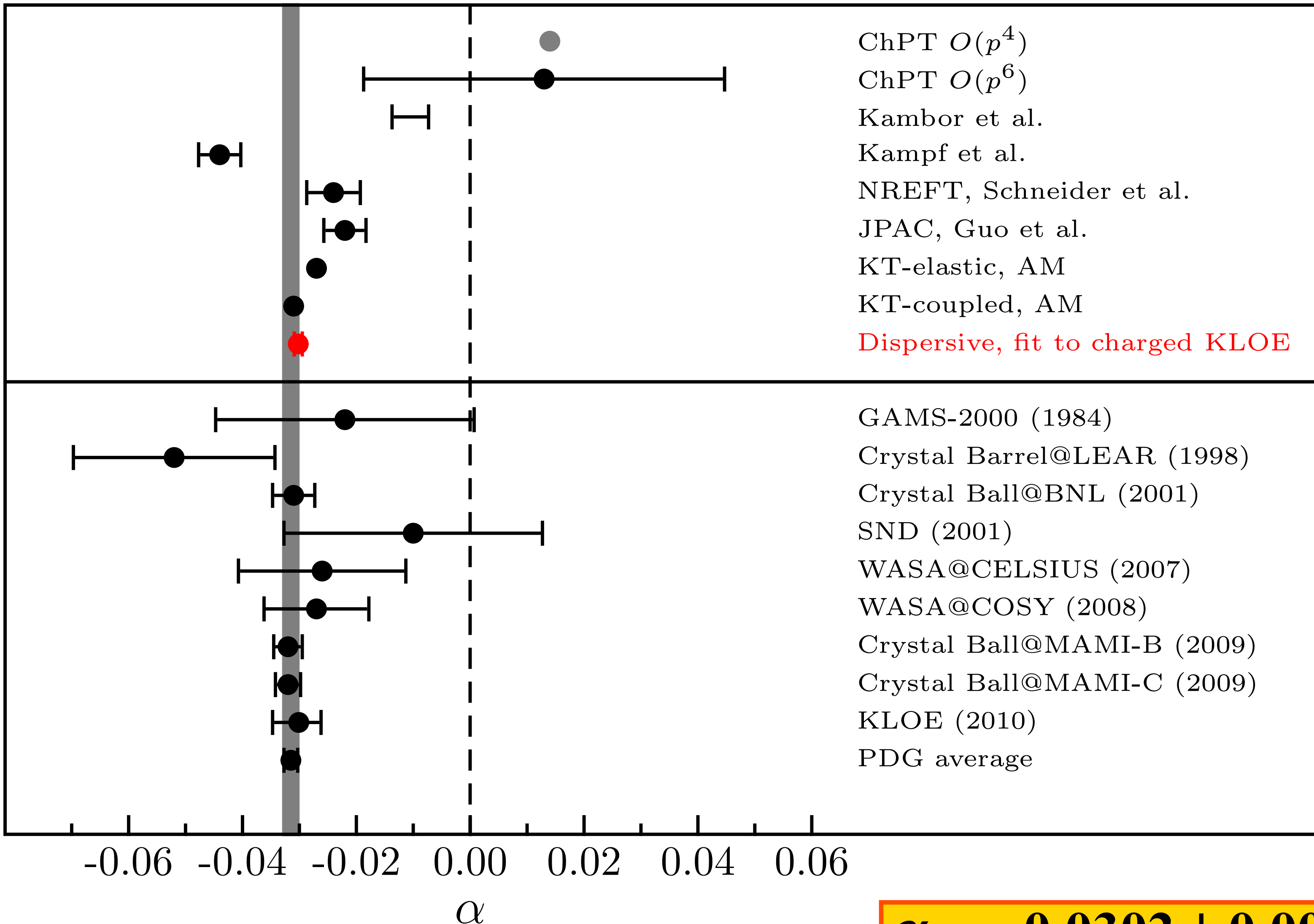
2.11 Z distribution for $\eta \to \pi^0 \pi^0 \pi^0$ decays

- The amplitude squared in the neutral channel is



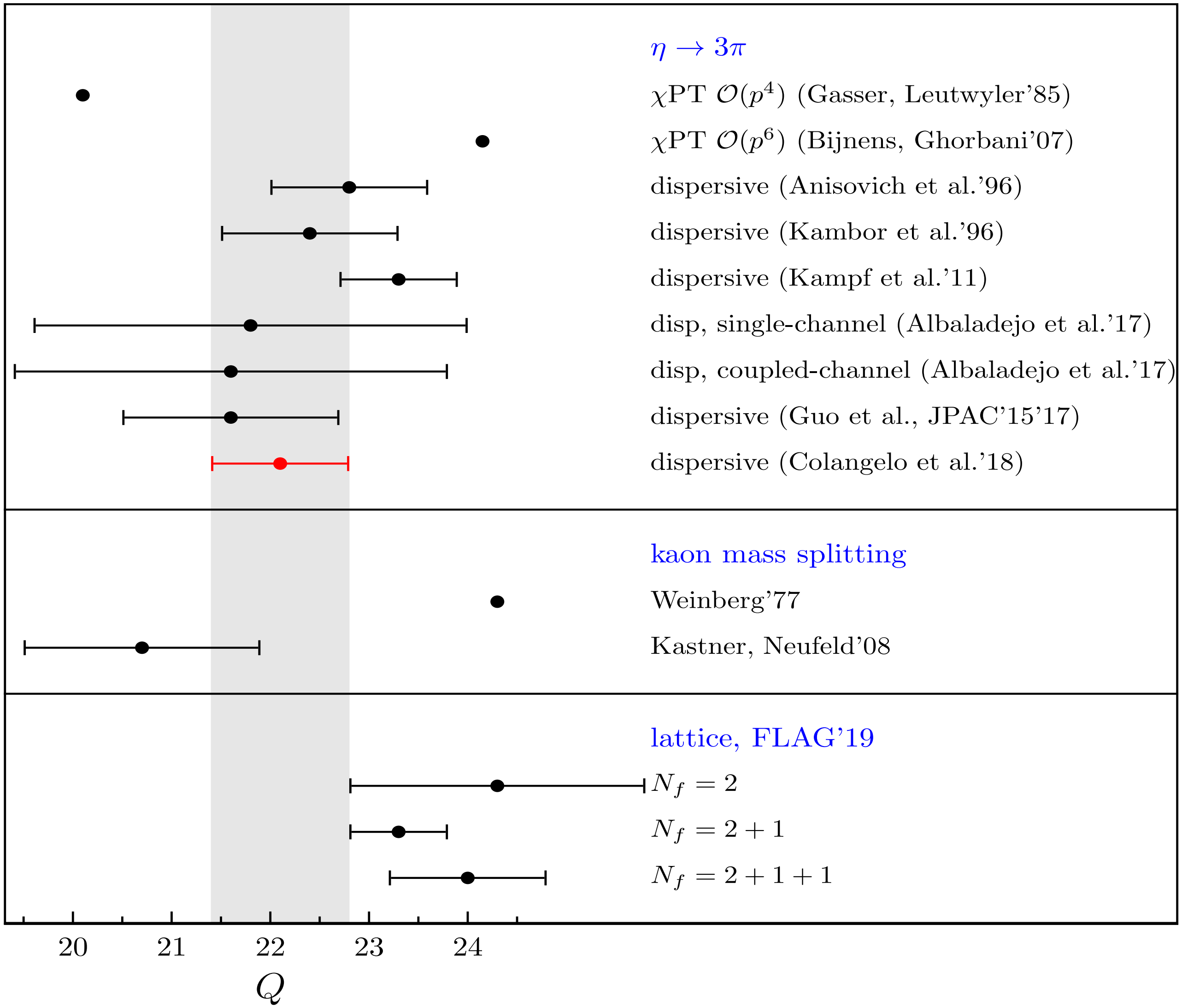
The agreement is *excellent* between our prediction and the data!

2.12 Comparison of results for α



$$\alpha = -0.0302 \pm 0.0011$$

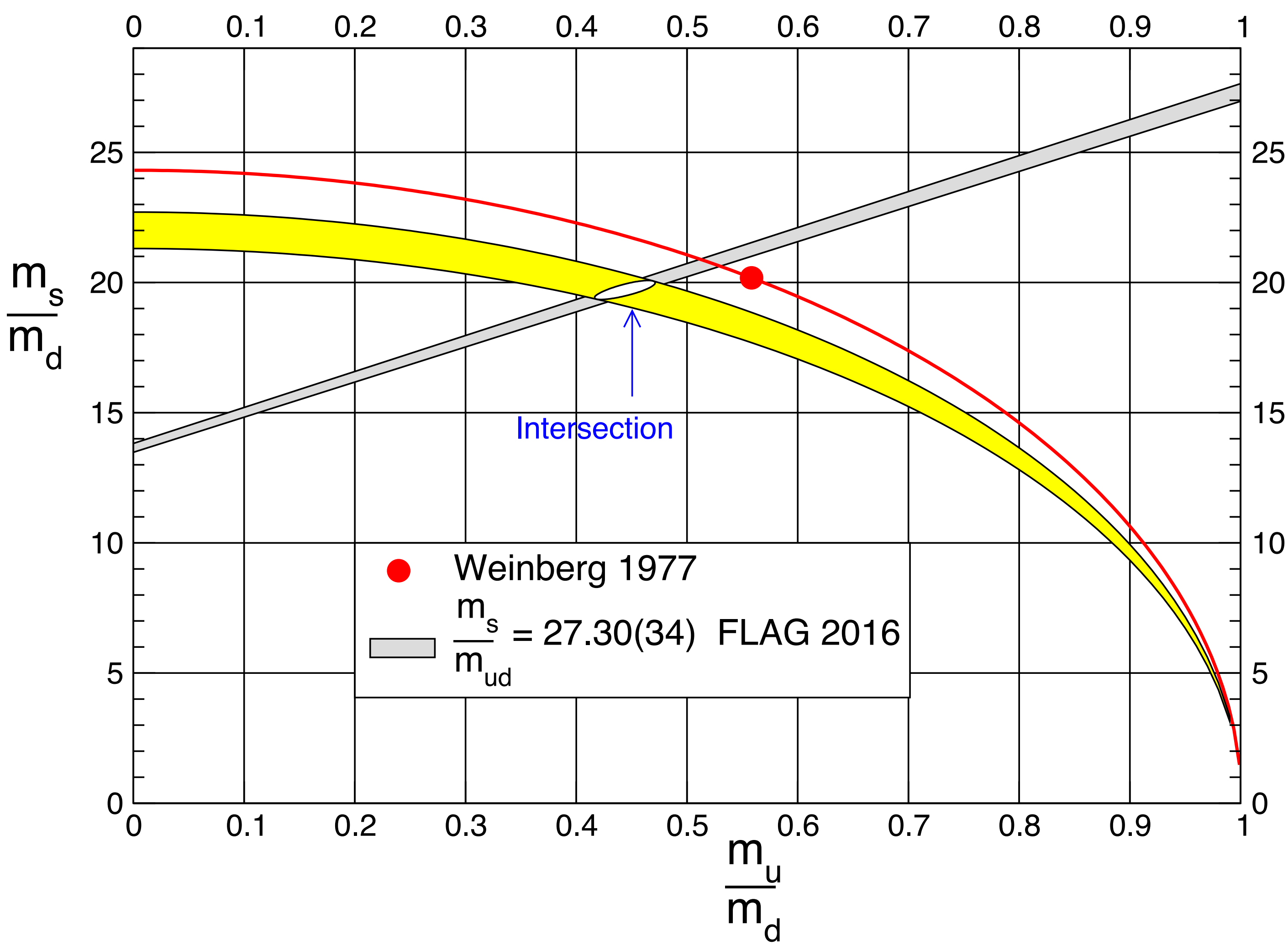
2.13 Quark mass ratio



$Q = 22.1 \pm 0.7$

- No systematics taken into account ⇨ collaboration with experimentalists

2.14 Light quark masses

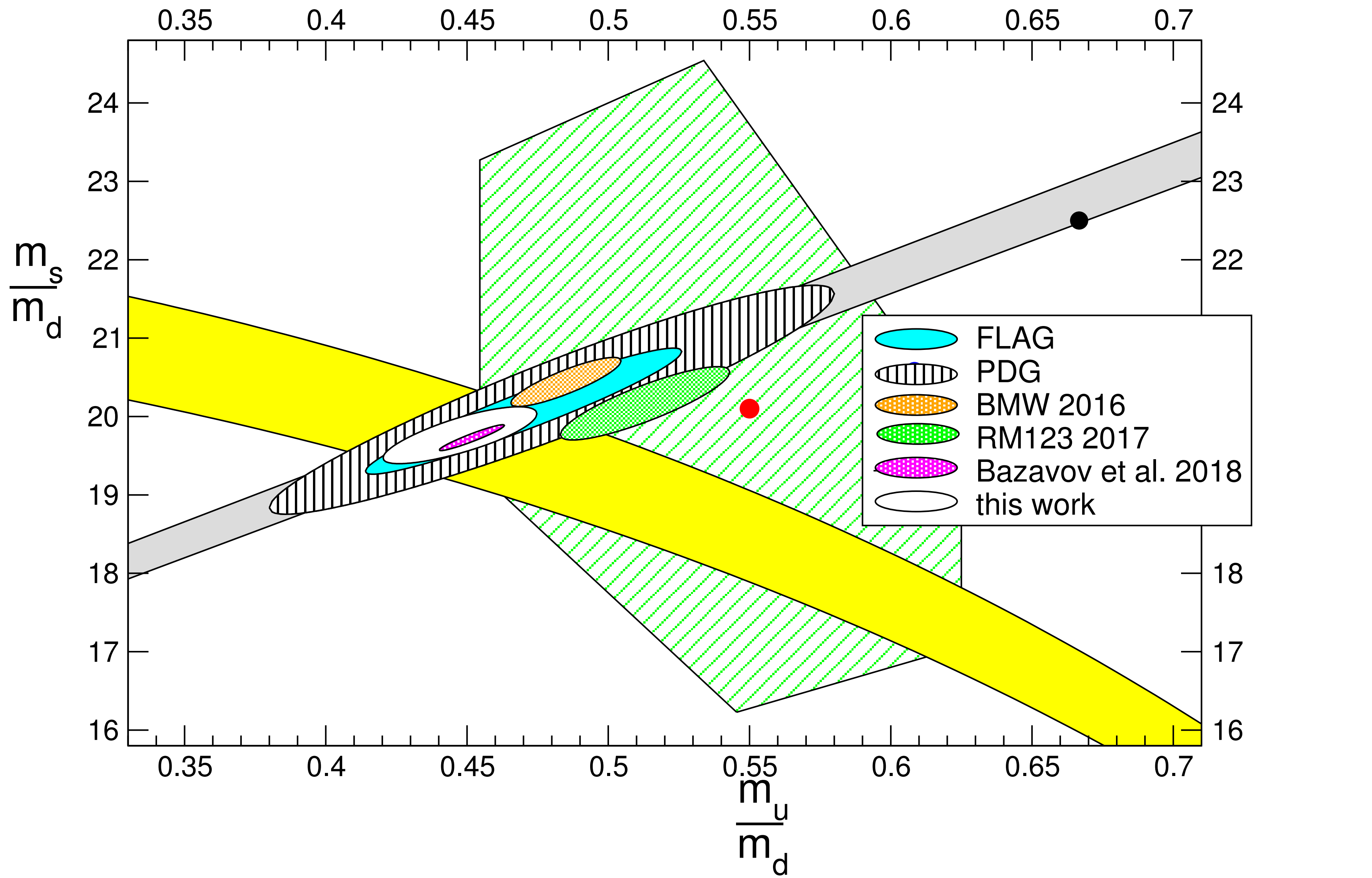


$Q = 22.1 \pm 0.7$

$\frac{m_u}{m_d} = 0.44 \pm 0.03$

- Smaller values for Q ⇨ smaller values for m_s/m_d and m_u/m_d than LO ChPT

2.14 Light quark masses

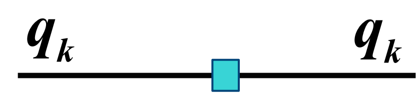


Formulation of QCD

Dynamics: The Lagrangien

- Build all the invariants under *SU(3)$_C$* with the quarks

⇨ $$\mathcal{L}_0 = \sum_{k=1}^{N_F} \bar{q}_k \left(i\gamma^\mu \partial_\mu - m_k \right) q_k$$

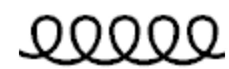


invariant under global SU(3)$_C$: $q_k^\alpha \to \left(q_k^\alpha\right)' = U^\alpha{}_\beta \, q_k^\beta$

with $U = \exp\left(-i g_S \frac{\lambda_a}{2} \theta_a \right)$ and λ_a the generators of SU(3)$_C$: $\left[\lambda^a, \lambda^b\right] = 2 i f^{abc} \lambda^c$

- Gauge the theory: SU(3)$_C$ → local ⇨ $\theta_a \to \theta_a(x)$

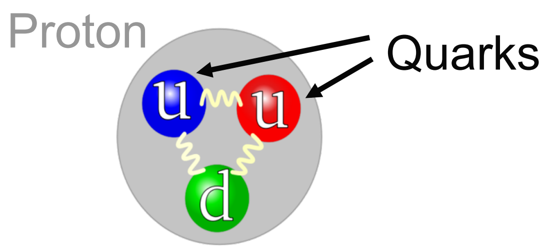
⇨ 8 different independent gauge fields: G_μ^a the *gluons*



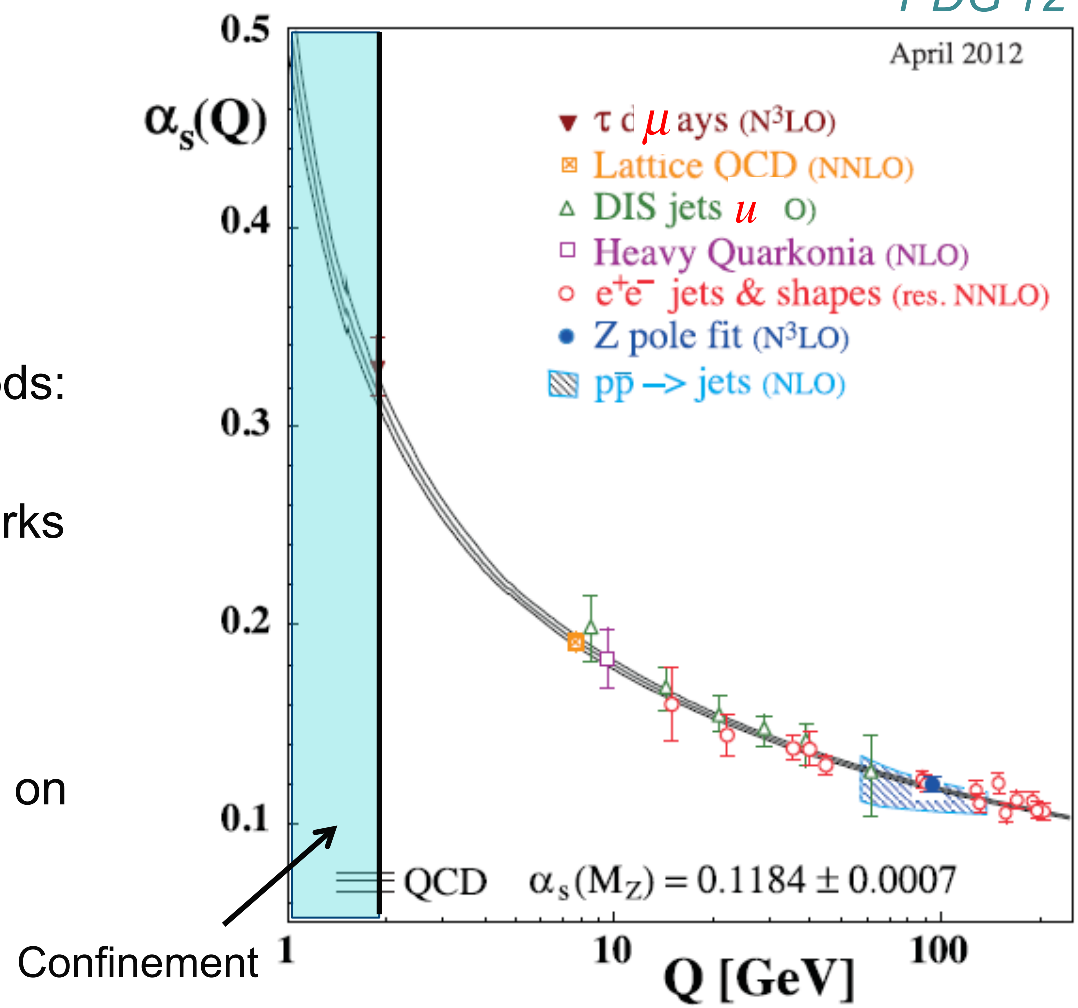
$$\partial_\mu q_k \to D_\mu q_k \equiv \Big[\partial_\mu - i g_s \underbrace{\frac{\lambda_a}{2} G_\mu^a(x)}_{G_\mu(x)} \Big] q_k$$

1.4 Strong interaction

- Looking for new physics in hadronic processes ⇨ not direct access to quarks due to confinement



PDG'12



➢ Model independent methods:
  - Effective field theory

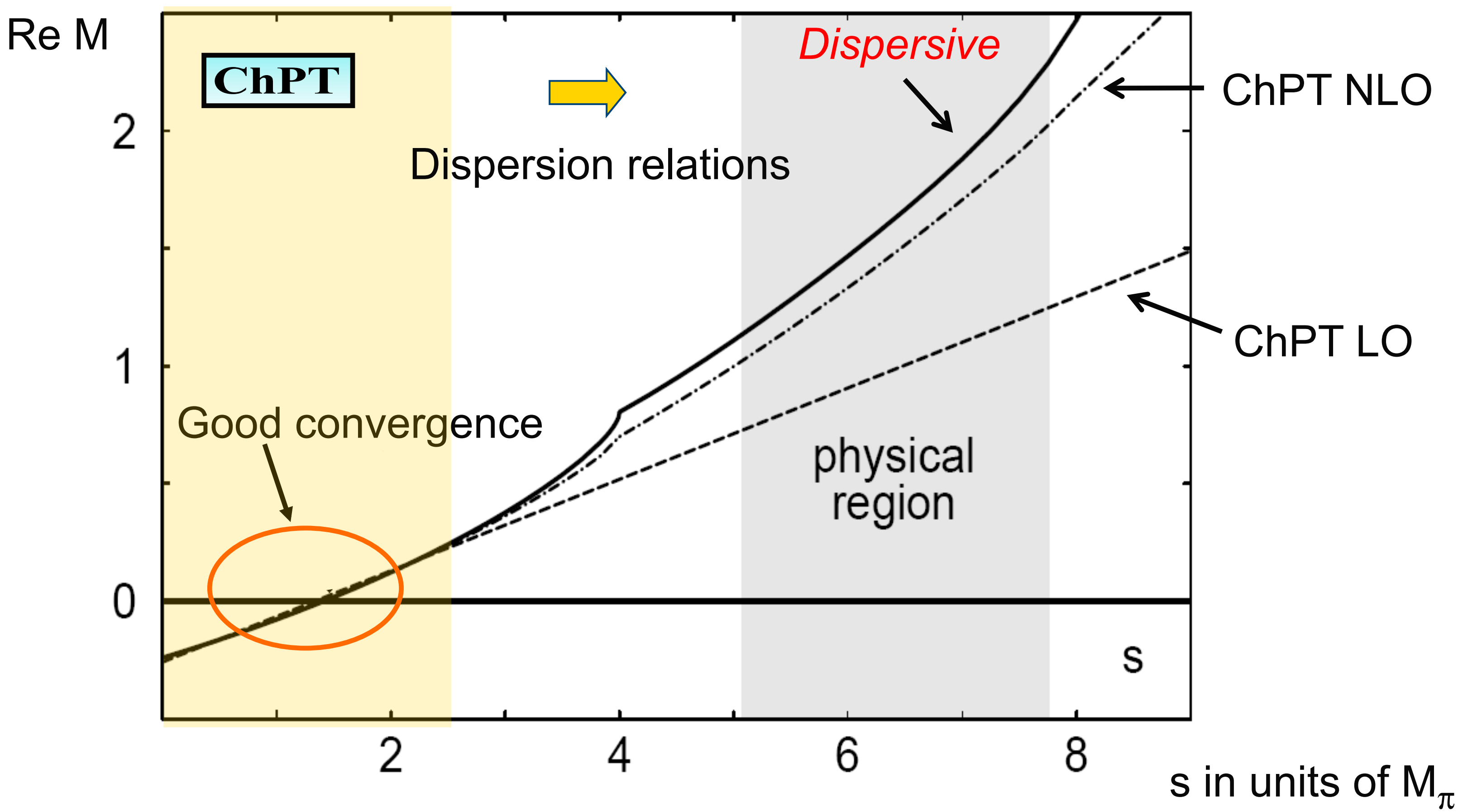
    Ex: ChPT for light quarks
  - Dispersion relations
  - Numerical simulations on the lattice

⇨ *Hadronic Physics*

Dispersive approach

- Dispersion Relations: extrapolate ChPT at higher energies

Anisovich & Leutwyler'96

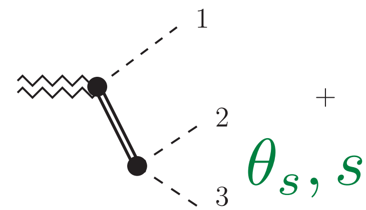


- Important corrections in the physical region taken care of by the *dispersive treatment!*

Method

- S-channel partial wave decomposition

$$A_\lambda(s,t) = \sum_J^{\infty}(2J+1)d^J_{\lambda,0}(\theta_s)A_J(s)$$



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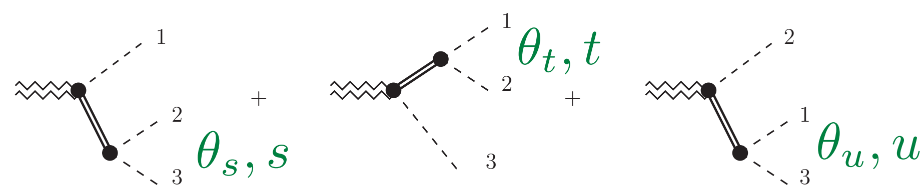

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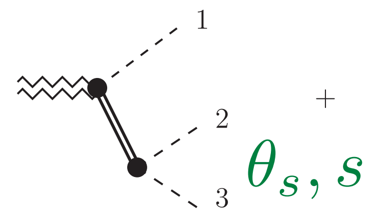

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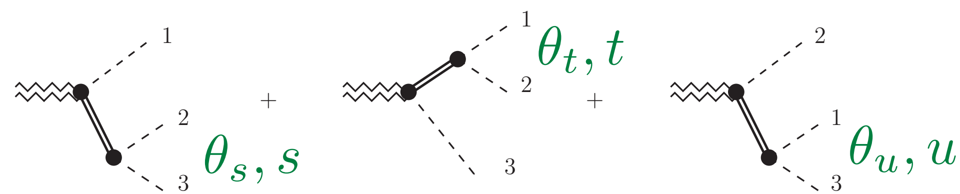

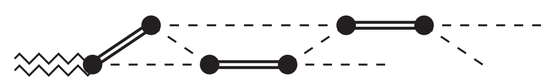

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$$\left[\Omega_I(s) = \exp\left(\frac{s}{\pi}\int_{4M_\pi^2}^{\infty}ds'\frac{\delta_I(s')}{s'(s'-s-i\varepsilon)}\right)\right]$$

Omnès function

Gasser & Rusetsky'18

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$\eta \rightarrow 3\pi$ Dalitz plot

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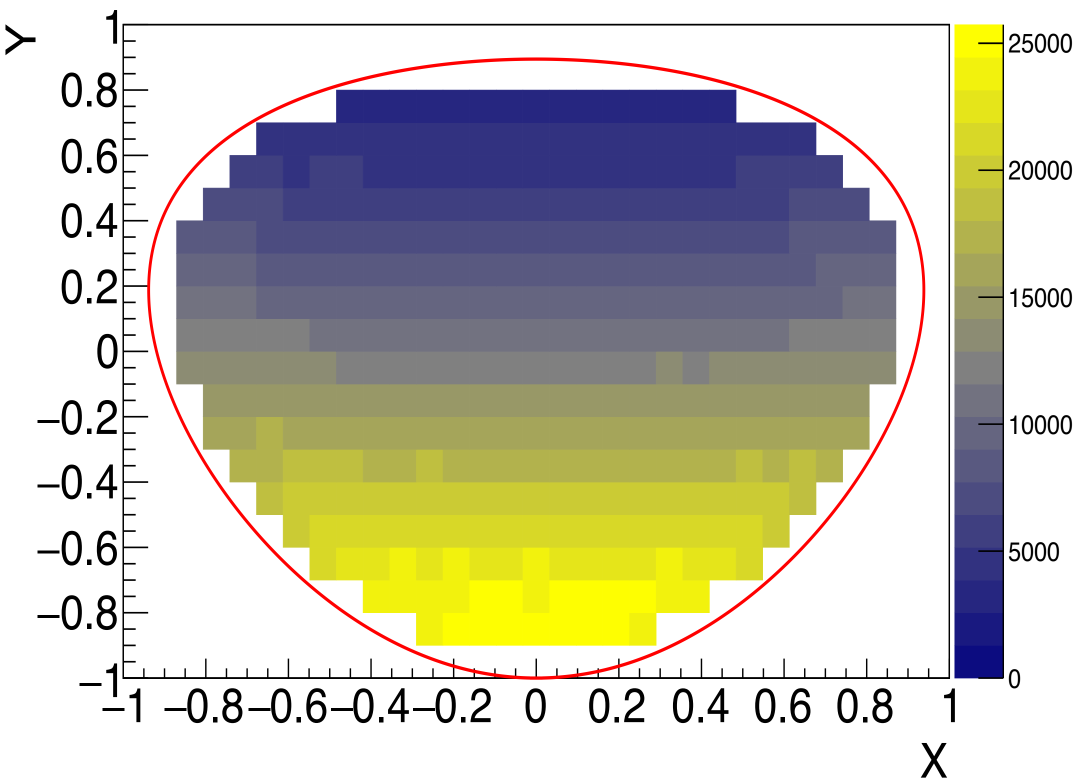
KLOE'16



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- New data expected from *CLAS* and *GlueX* with very different systematics

Which value of Q^2 impact neutrino data?

- The experimental results point towards a larger value of the axial form factor $M_A \sim 1.35$ GeV
- If true, the value of M_A saturates the cross section leaving little room for multi nucleon effects
- Is the dipole physically motivated?

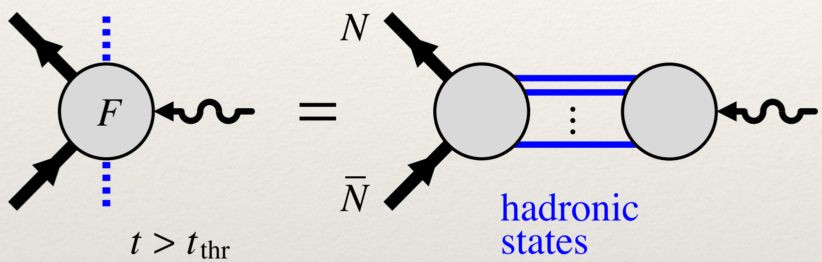
$$F_A(q^2) = \frac{F_A(0)}{\left(1 - \frac{q^2}{M_A^2}\right)^2}$$

The parametrisation has an impact on different q^2 dependence ranges on the neutrino data

Improving the Form Factor parametrization

- For intermediate energy region: Can try to use ***VMD***

  - ***Analytical structure*** of FF (e.g. F_1 or F_A)

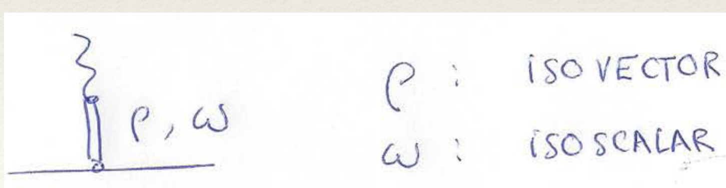


Photon or W sees proton through all hadronic states (with vector or axial-vector Quantum Number)

Isovector: $\pi\pi$ (incl. ρ), 4π, $K\bar{K}$, ...
Isoscalar: 3π (incl. ω), $K\bar{K}$ (incl. ϕ), ...

Processes in unphysical region $t < 4\, m_N^2$

  - Resonances (Vector Mesons)



For F_A (Axial Vector Mesons)

$a_1(1230)$ and $a_1'(1647)$

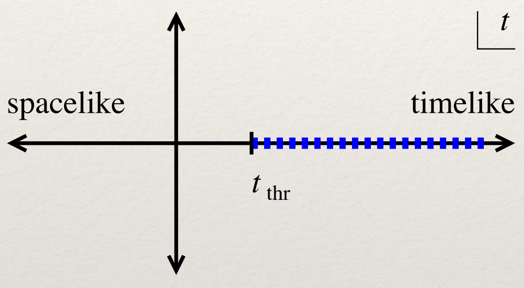
Masjuan et al.'12

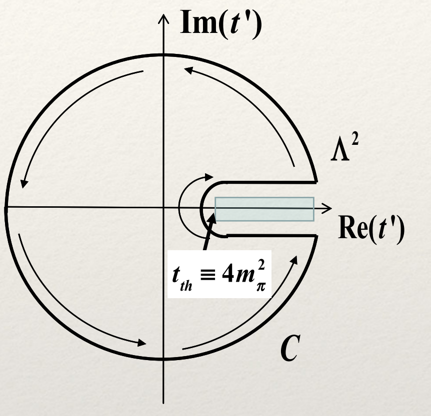
$$F_A(t) = g_A \frac{m_{a_1}^2\, m_{a_1'}^2}{(m_{a_1}^2 - t)(m_{a_1'}^2 - t)}$$

Improving the Form Factor parametrization

❖ For intermediate energy region: Can try to use ***VMD***, e.g. EM FF

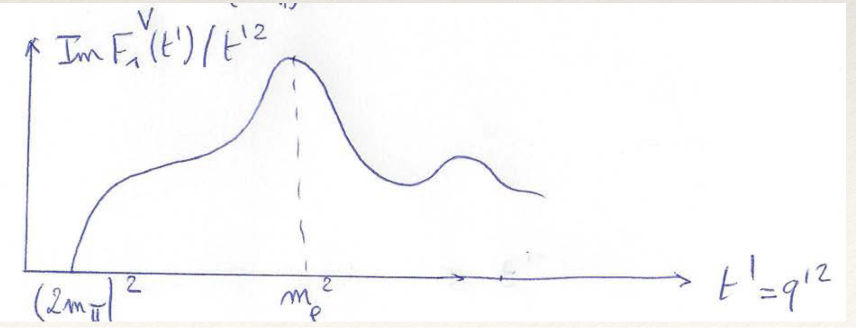
- ***Dispersion Relations***





- Use spectral function from theory or from experiment

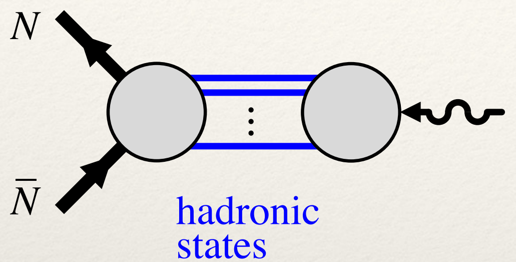
Frazer &Fulco'60, Hohler et al'75



$$F_i(t) = \int_{t_{\text{thr}}}^{\infty} \frac{dt'}{\pi} \frac{\text{Im}\, F_i(t')}{t' - t - i0}$$

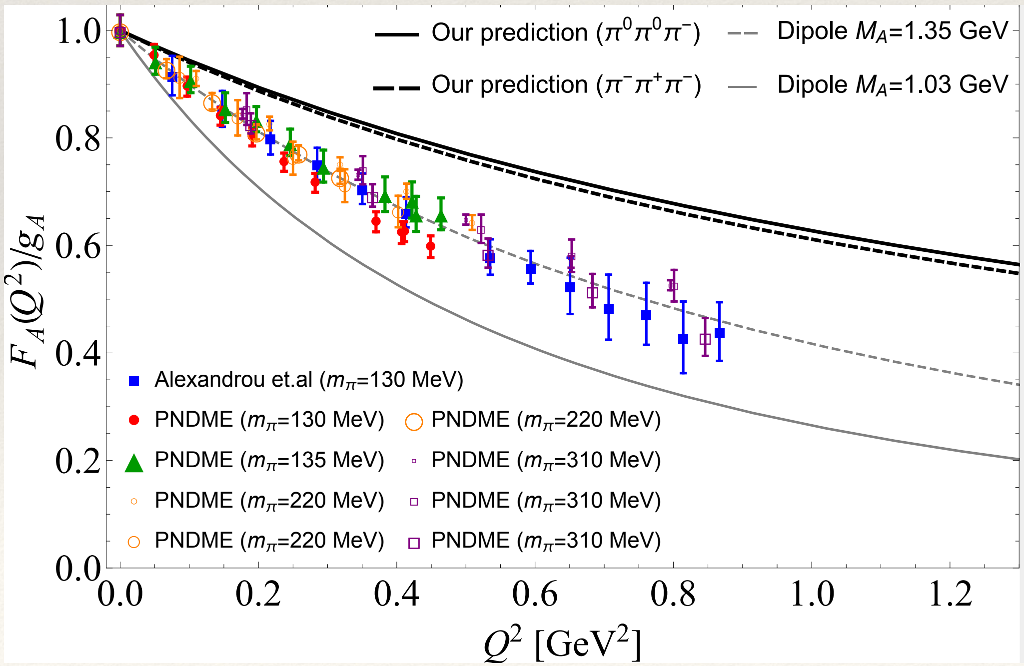
Improving the Form Factor parametrization

- How to connect to the nucleon?



- Take a constant g_A

$$F_A(q^2) = g_A \cdot f_{A\to 3\pi}\left(q^2\right)$$



→ Does not work!