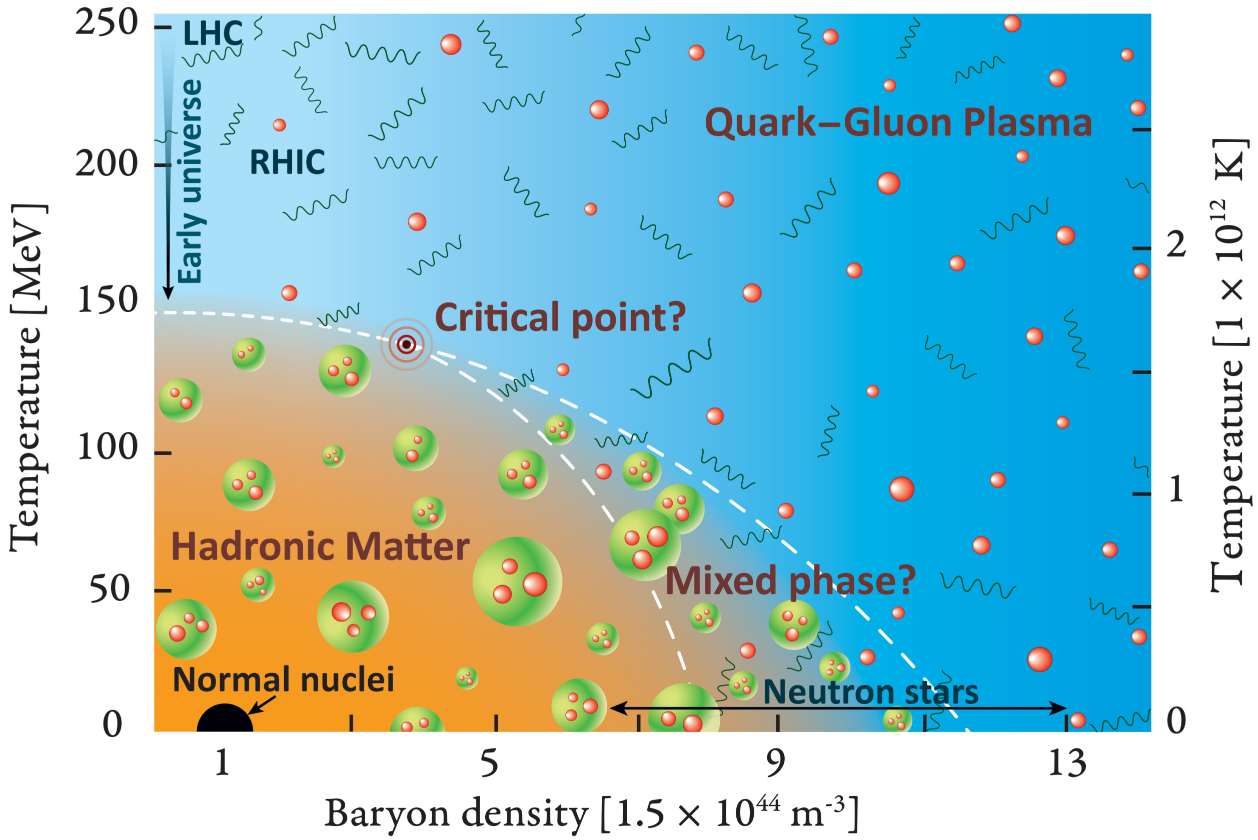
Nuclear Matter and Nuclear Structure



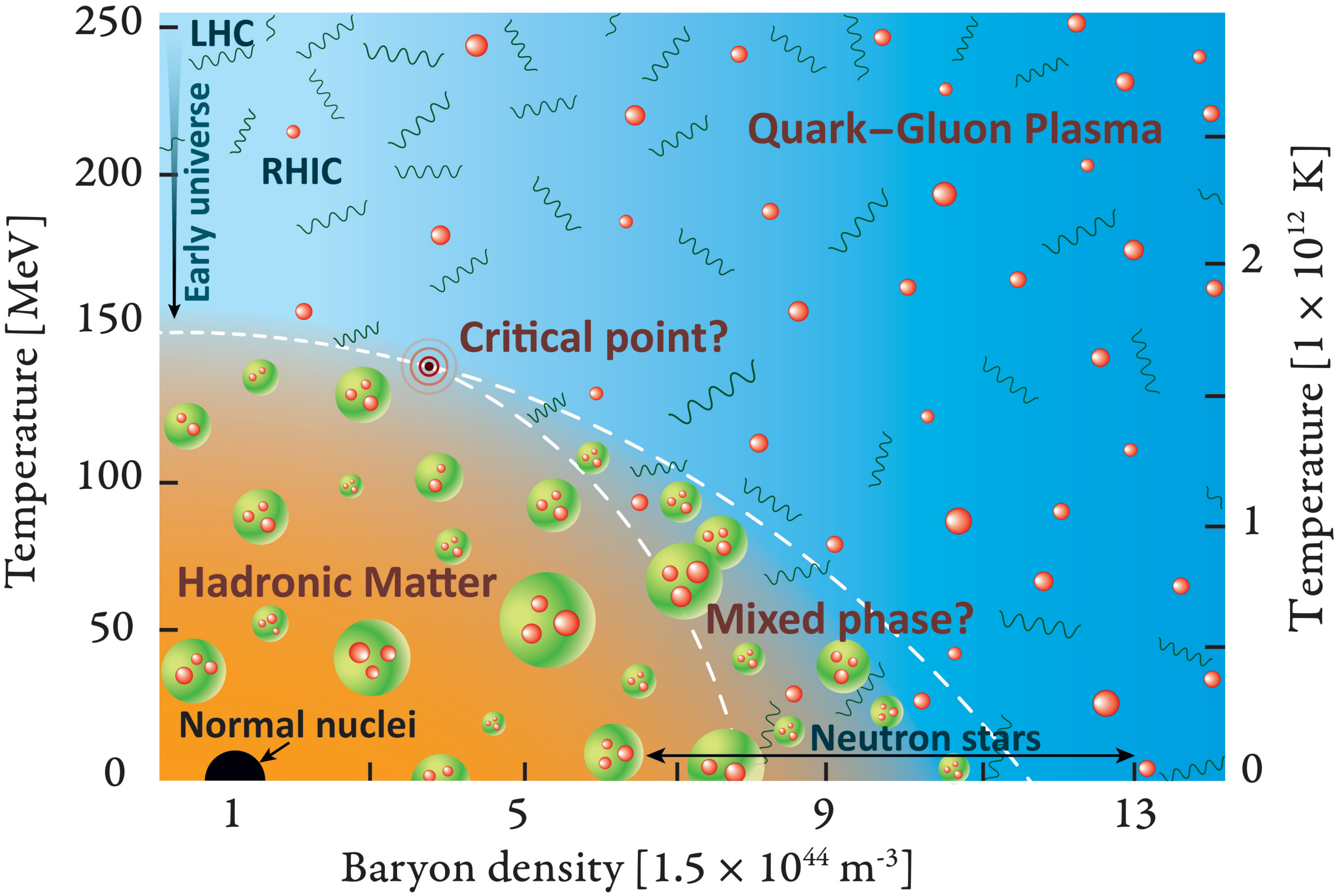
Thomas Papenbrock, The University of Tennessee, tpapenbr@utk.edu

(Virtual) National Nuclear Physics Summer School, UNAM / Indiana U, June 2021

Work supported by the US Department of Energy

- Questions welcome
  - Please ask during breaks, or use slack later
- Please participate actively in the lectures
  - Research shows: You will learn better by active participation

Phase diagram of nuclear matter



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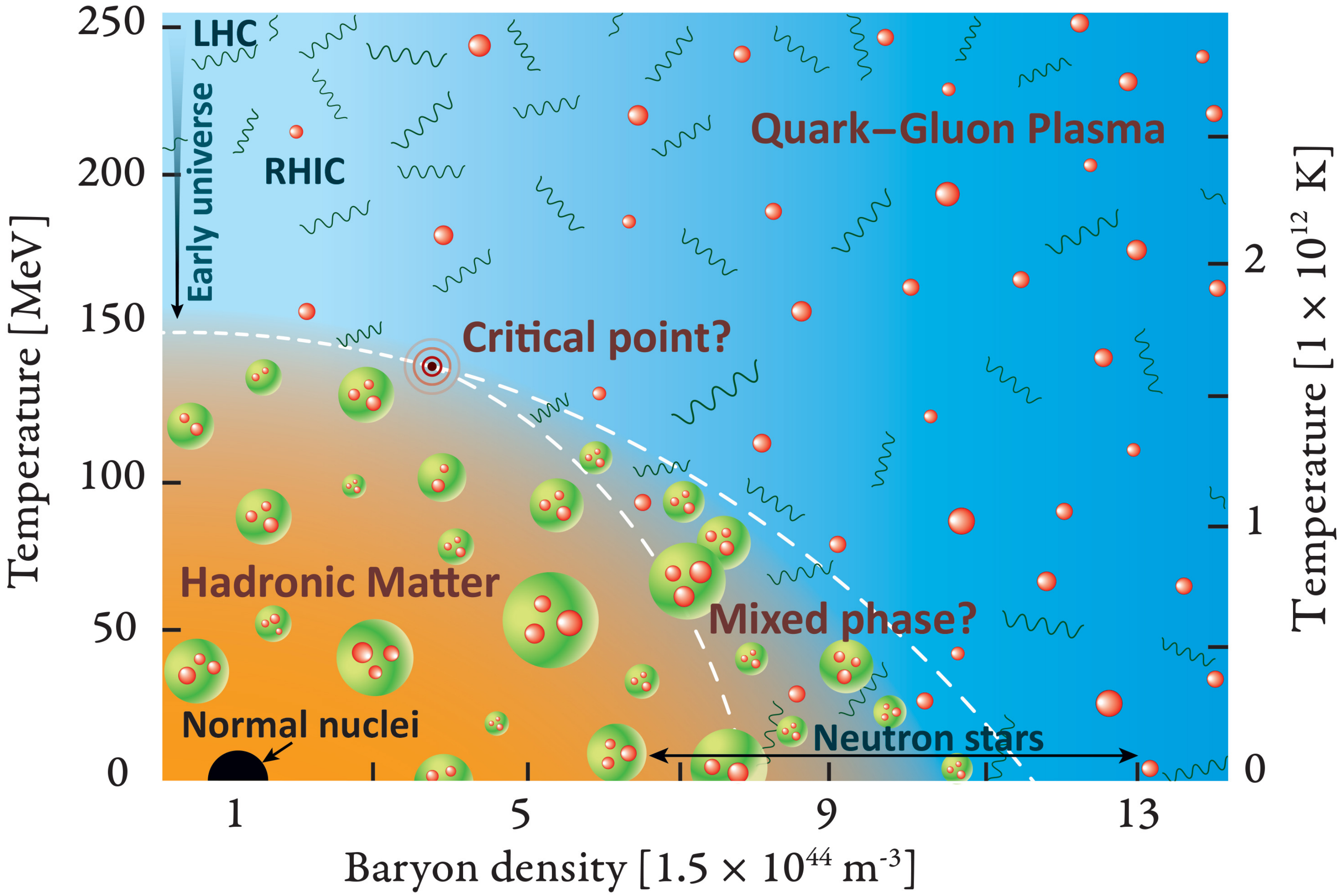
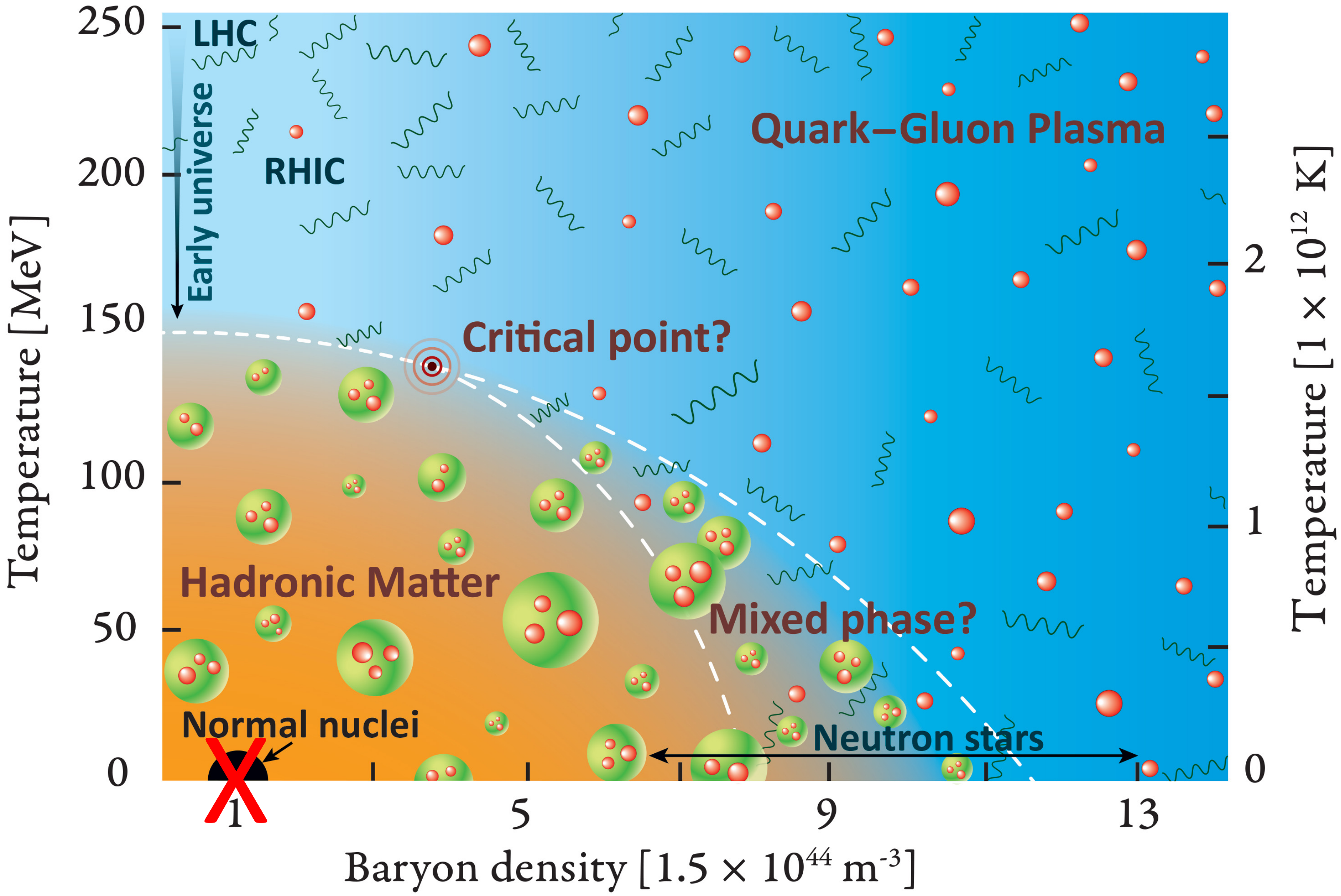


Locate on this phase diagram

1. Atomic nucleus
2. Antimatter nucleus
3. Interior of the sun
4. Black holes

Phase diagram of nuclear matter



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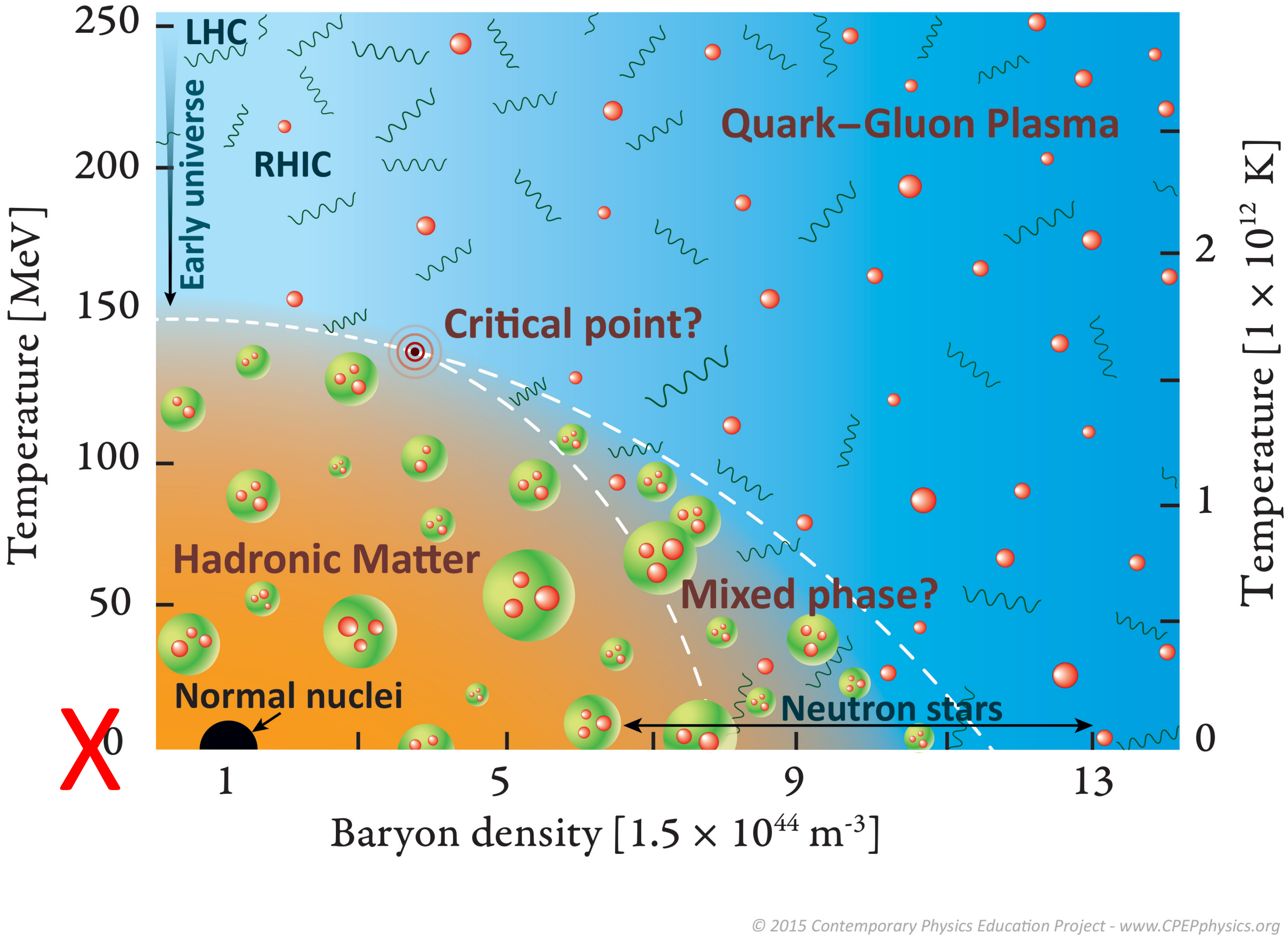

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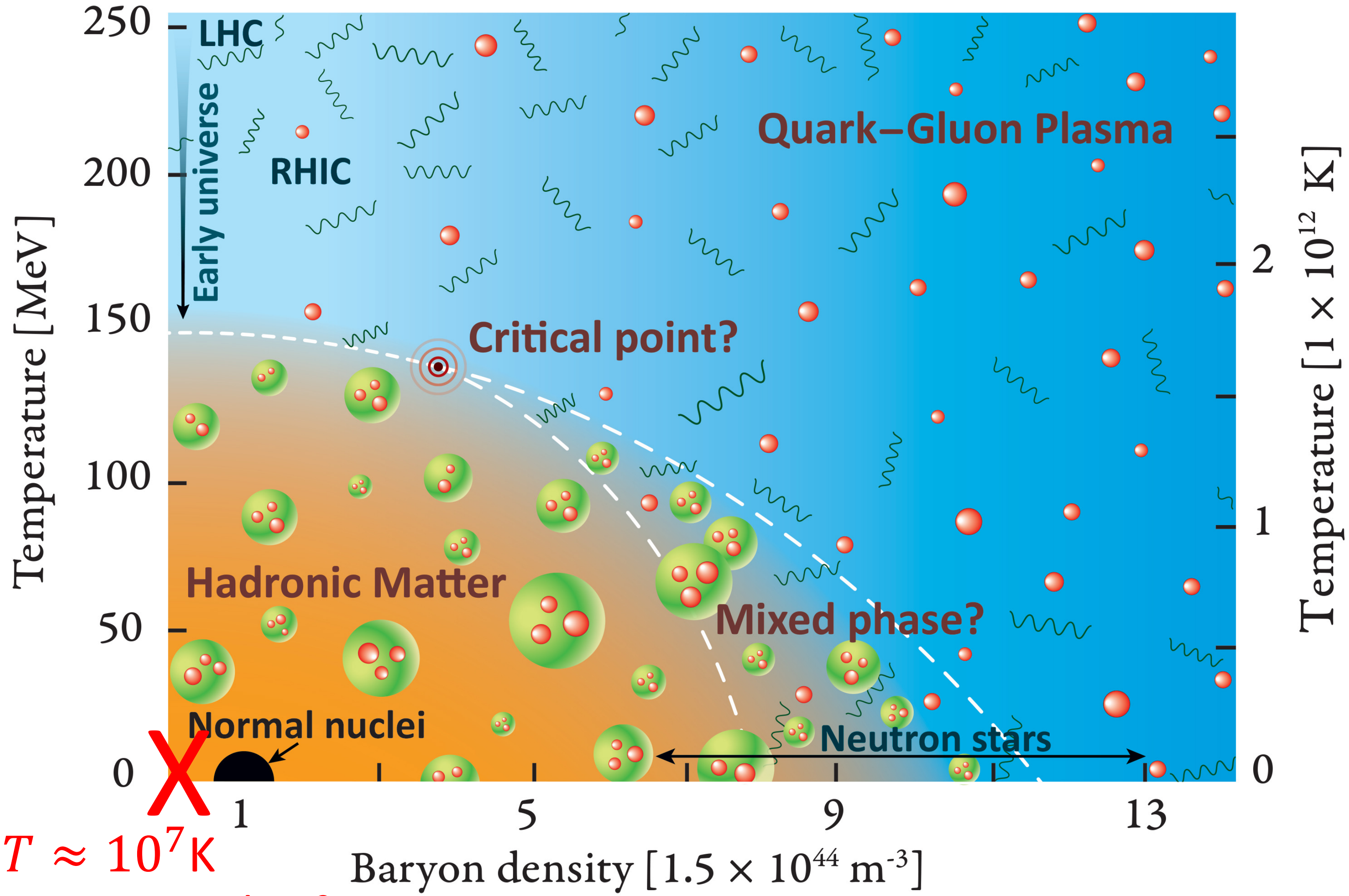

$T \approx 10^7$K

$\rho \approx 160$ g/cm^3

$\rho_b \approx 10^{32}$m^{-3}

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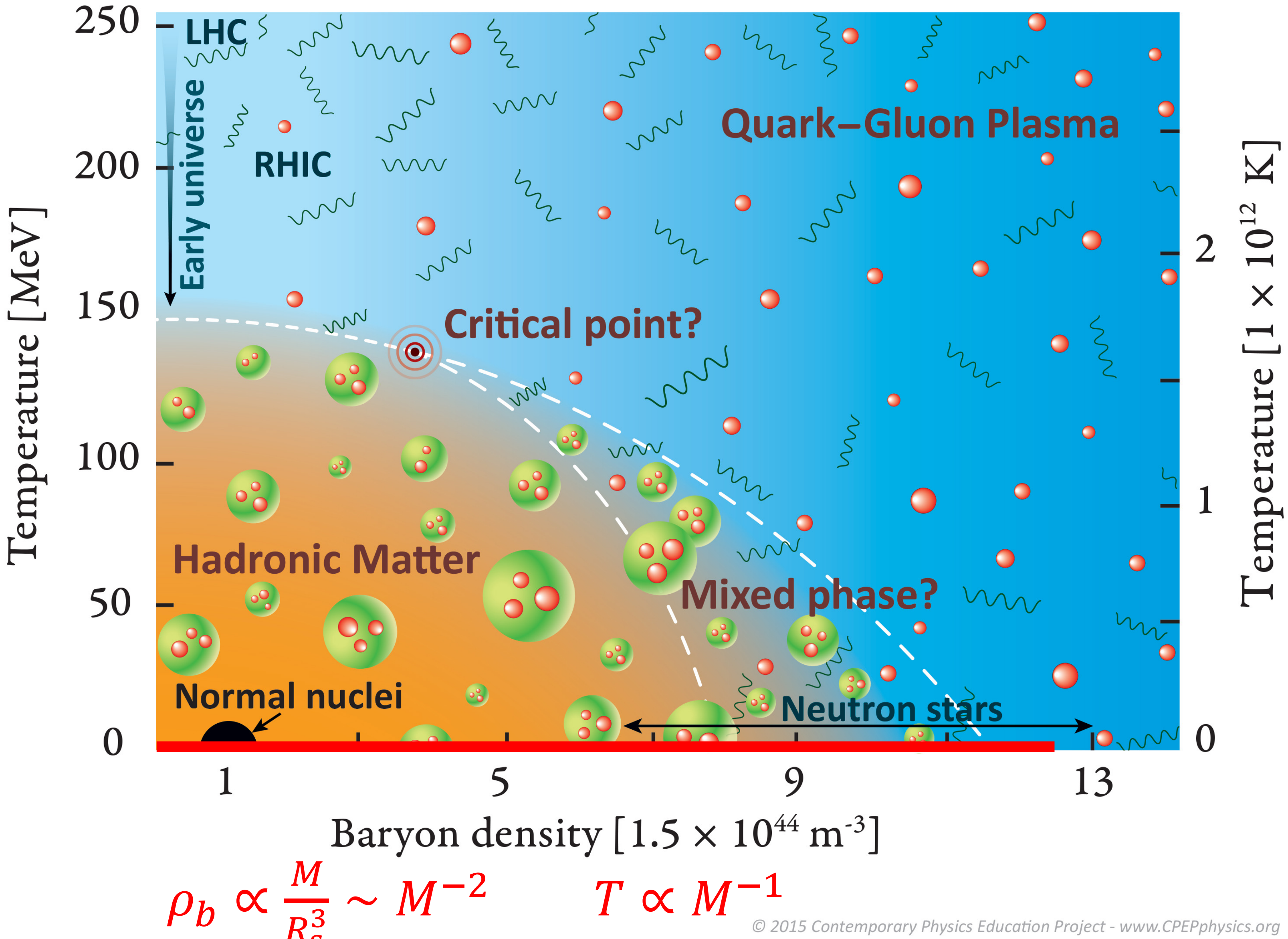
Phase diagram of nuclear matter



$\rho_b \propto \frac{M}{R_s^3} \sim M^{-2}$ $\qquad T \propto M^{-1}$

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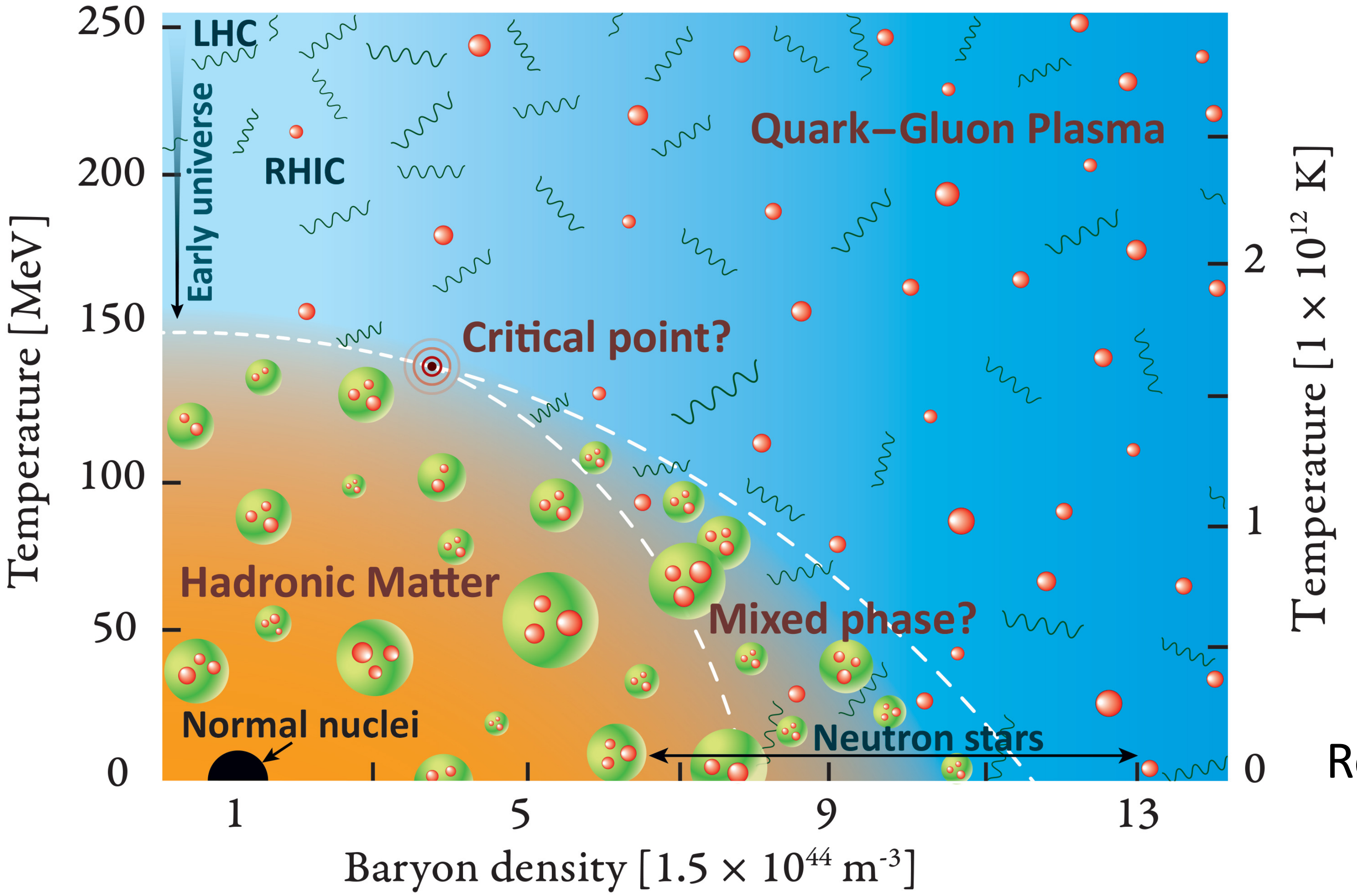
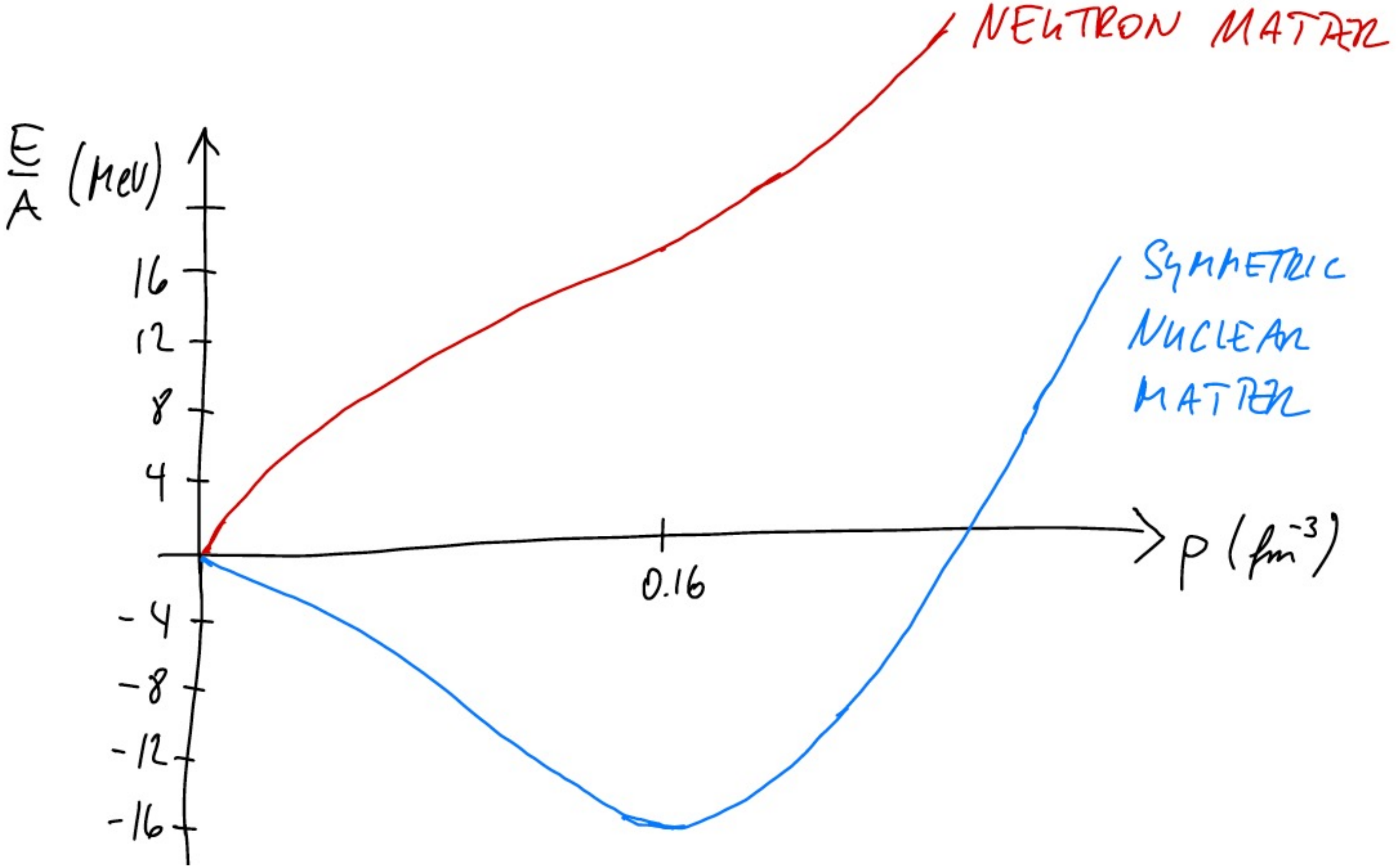
Phase diagram of nuclear matter



Remainder of these lectures

$T = 0$

Nuclear Equation of State
(energy per nucleon in infinite nuclear matter)

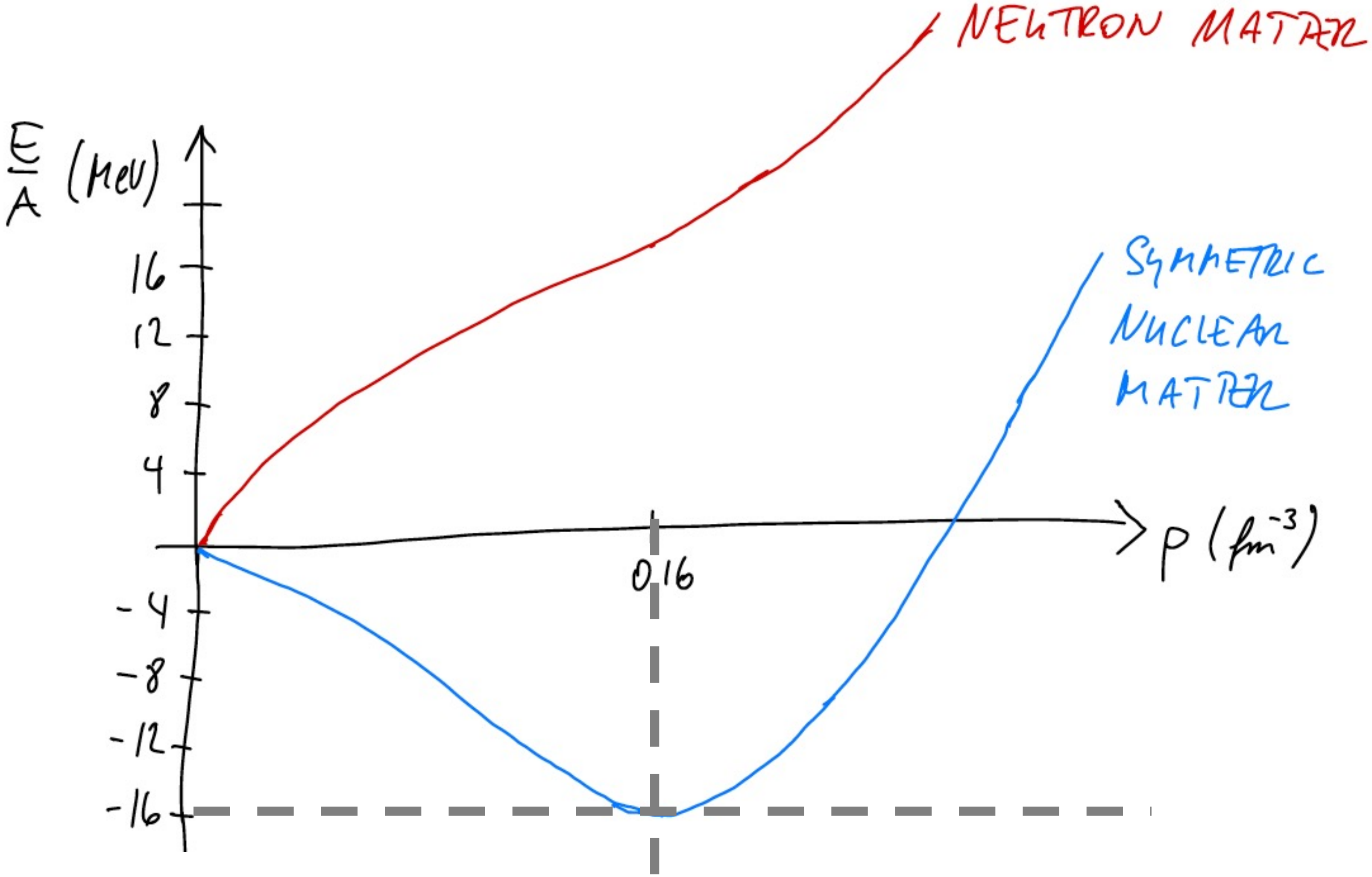


Pure neutron matter: $A = N$

Symmetric matter: $N = Z$

Note: Coulomb force neglected; electrons not included

Nuclear Equation of State



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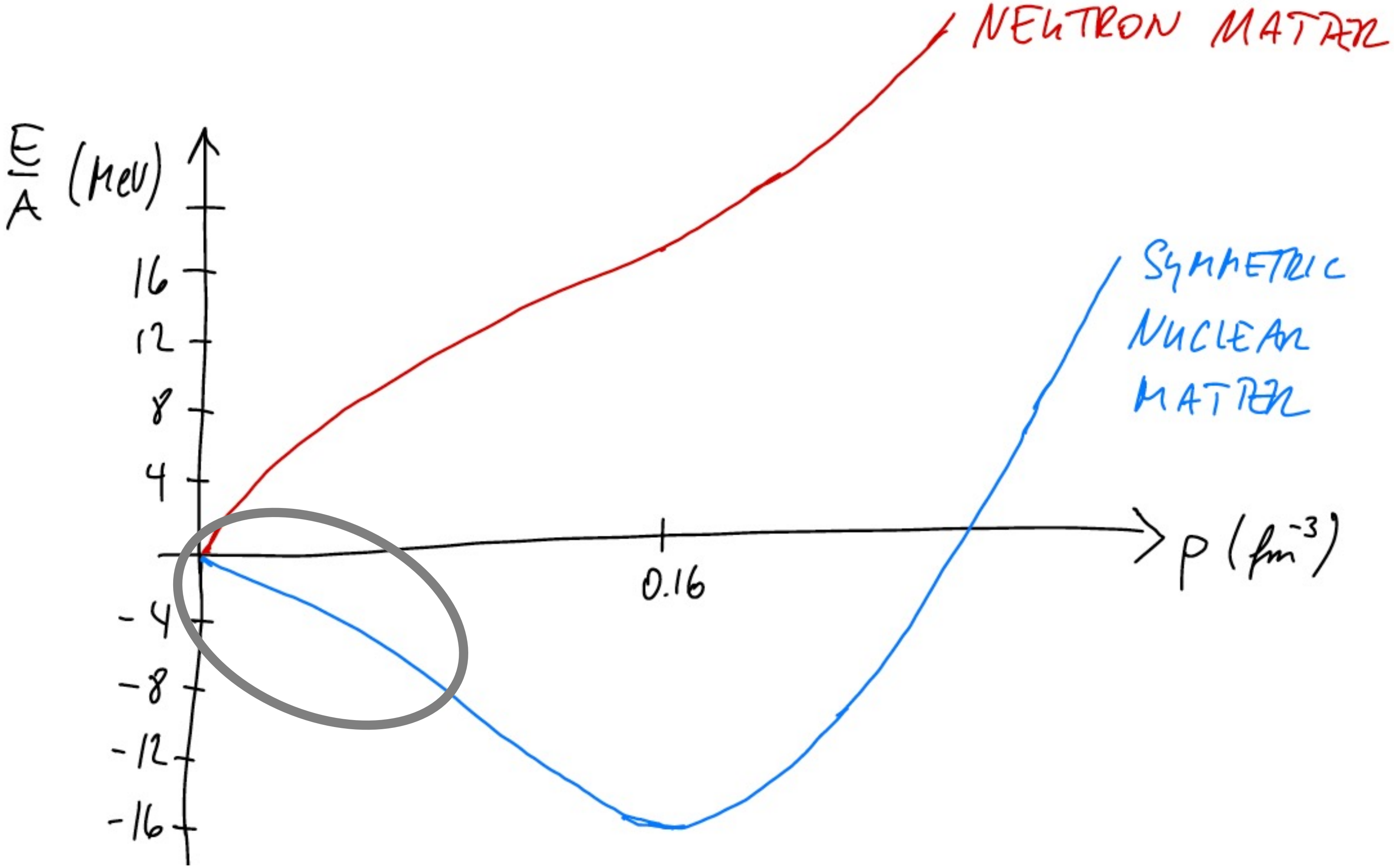
Saturation point of symmetric nuclear matter

$$\frac{E_{sat}}{N} \approx -16 \text{ MeV}$$

$$\rho_{sat} \approx 0.16 \text{ fm}^{-3}$$

Nuclear Equation of State



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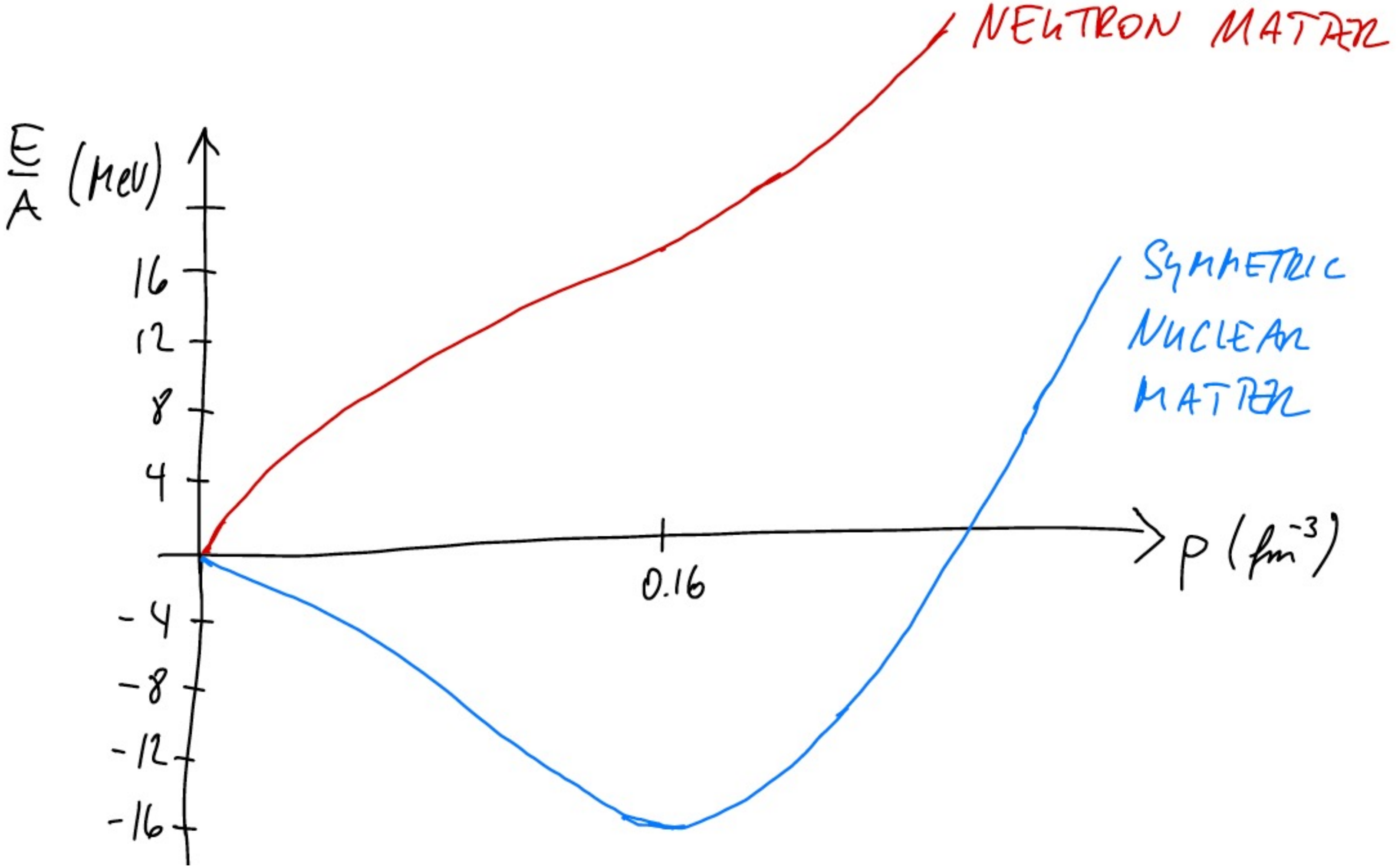
Q: What does it mean for the EoS of matter to have a negative slope, i.e. a negative pressure?

Nuclear Equation of State



Pure neutron matter: $A = N$

Symmetric matter: $N = Z$

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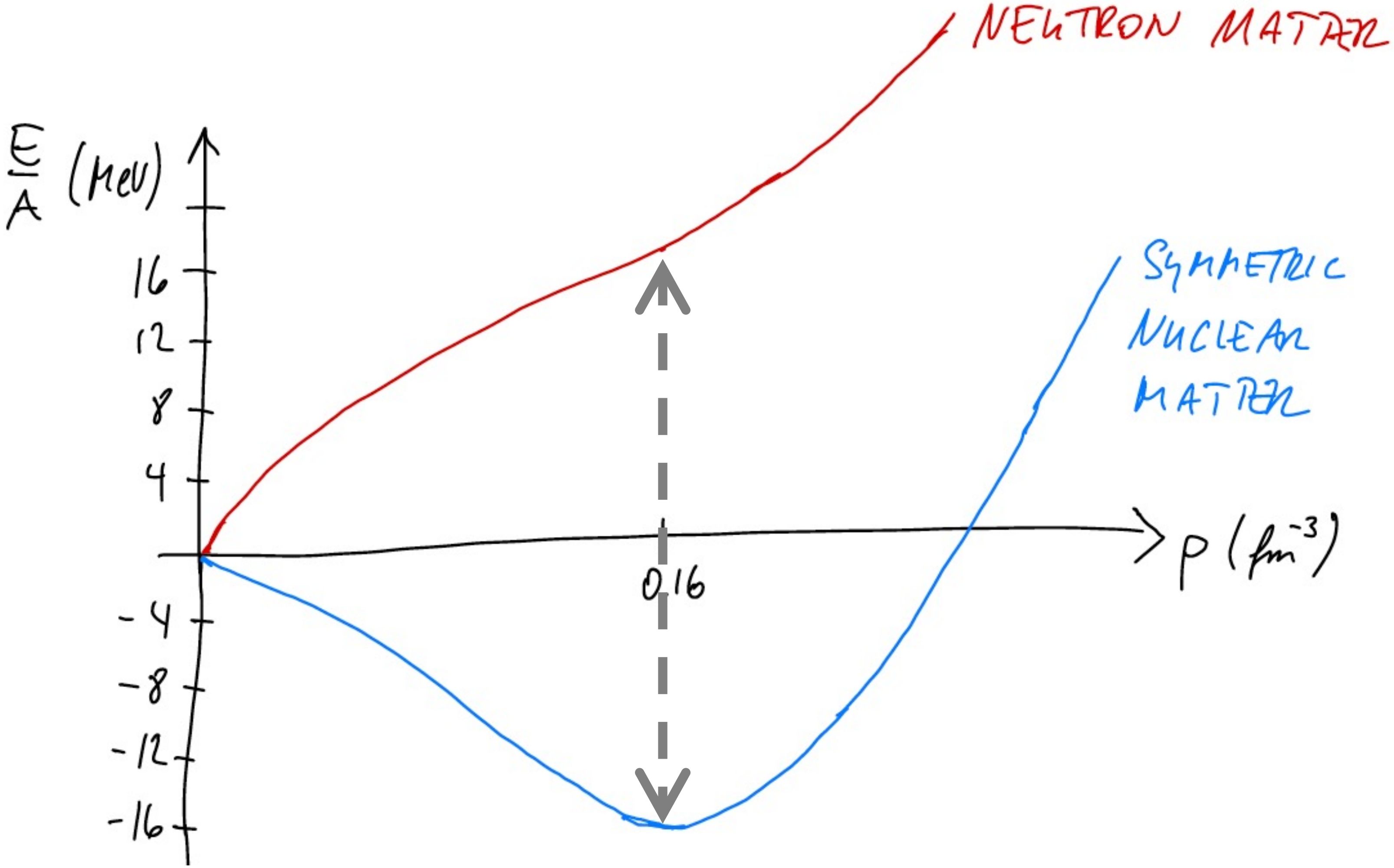
Q: What does it mean for the EoS of matter to have a negative slope, i.e. a negative pressure?

A: Such matter cannot be homogeneous but rather gain energy by clustering into drops of matter at saturation density

Nuclear Equation of State



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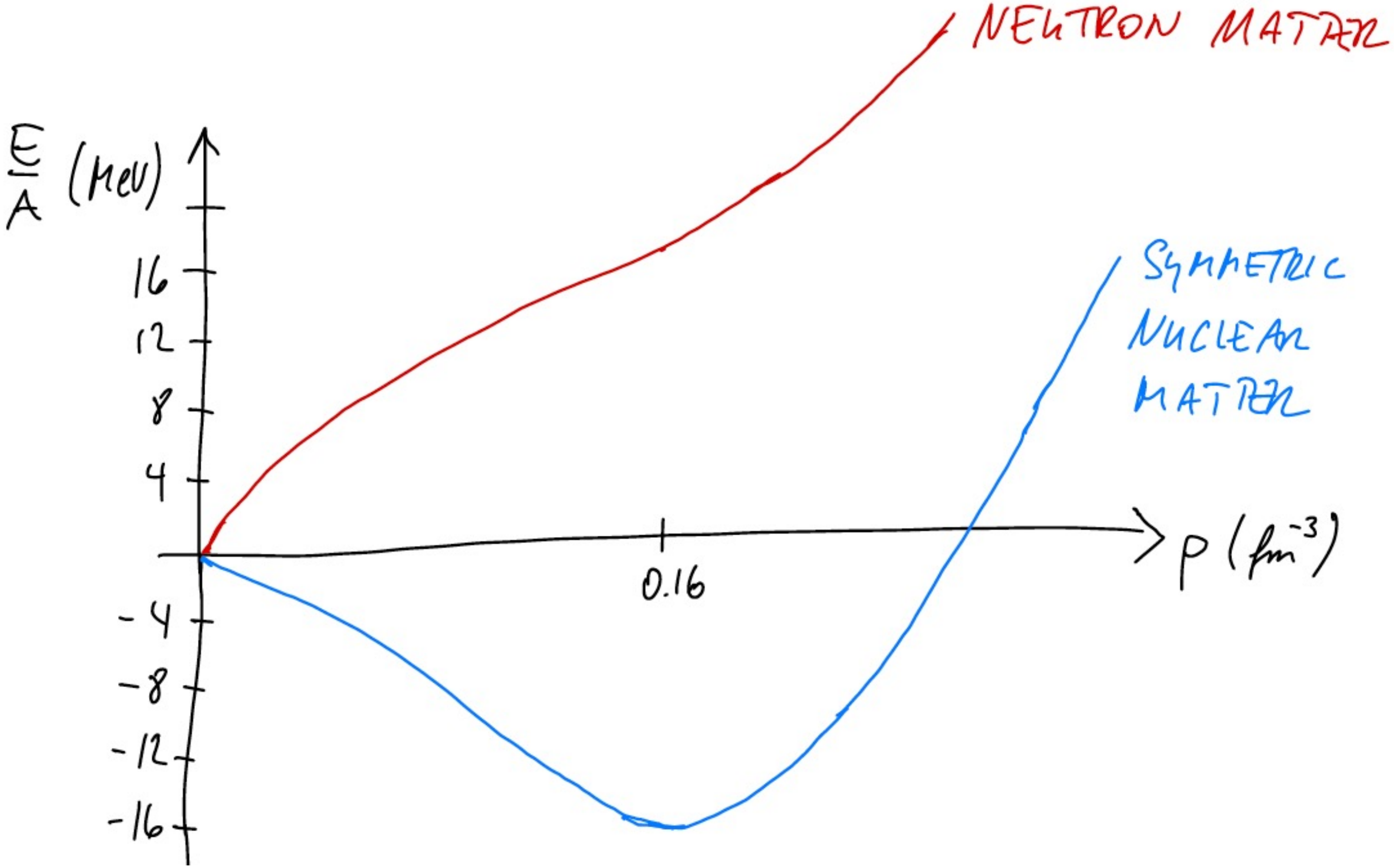
Symmetry energy: Difference between neutron matter and symmetric nuclear matter at saturation density

$$E_{sym} \approx 32 \text{ MeV}$$

Nuclear Equation of State



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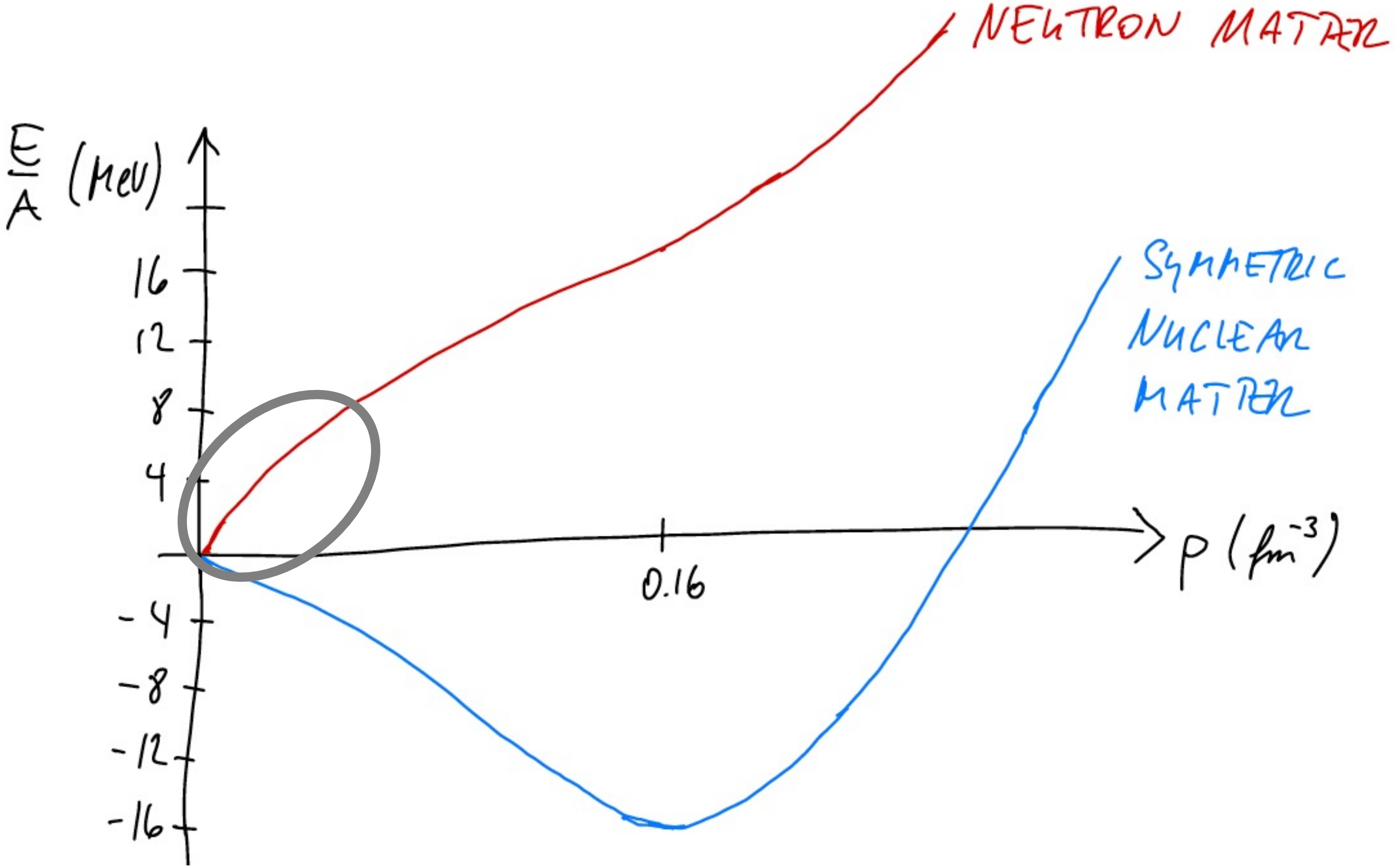
Regions where we think we know well what's going on are below about twice the saturation density

Nuclear Equation of State



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Let us discuss dilute neutron matter!

Dilute neutron matter

- At low densities, we do not need to know details of the nuclear interaction
- Let us get an overview of relevant scales

Q: What’s the relevant momentum scale?

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At saturation density $k_{F,sat} \approx 1.35$ fm^{-1}. We have $k_F \ll k_{F,sat}$.

The wavelengths involved in the scattering of two neutrons are much larger than the range of the nuclear interaction $R \sim \frac{1}{m_\pi} \approx 1.4$ fm, i.e. $k_F R \ll 1$

However: the neutron-neutron scattering length is also much larger than the range of the nuclear interaction: $a_s \approx -24$ fm. In the regime of interest, $k_F |a_s| \gg 1$

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Let us consider the unitary limit $a_s \to -\infty$, i.e. two neutrons have a zero-energy bound state

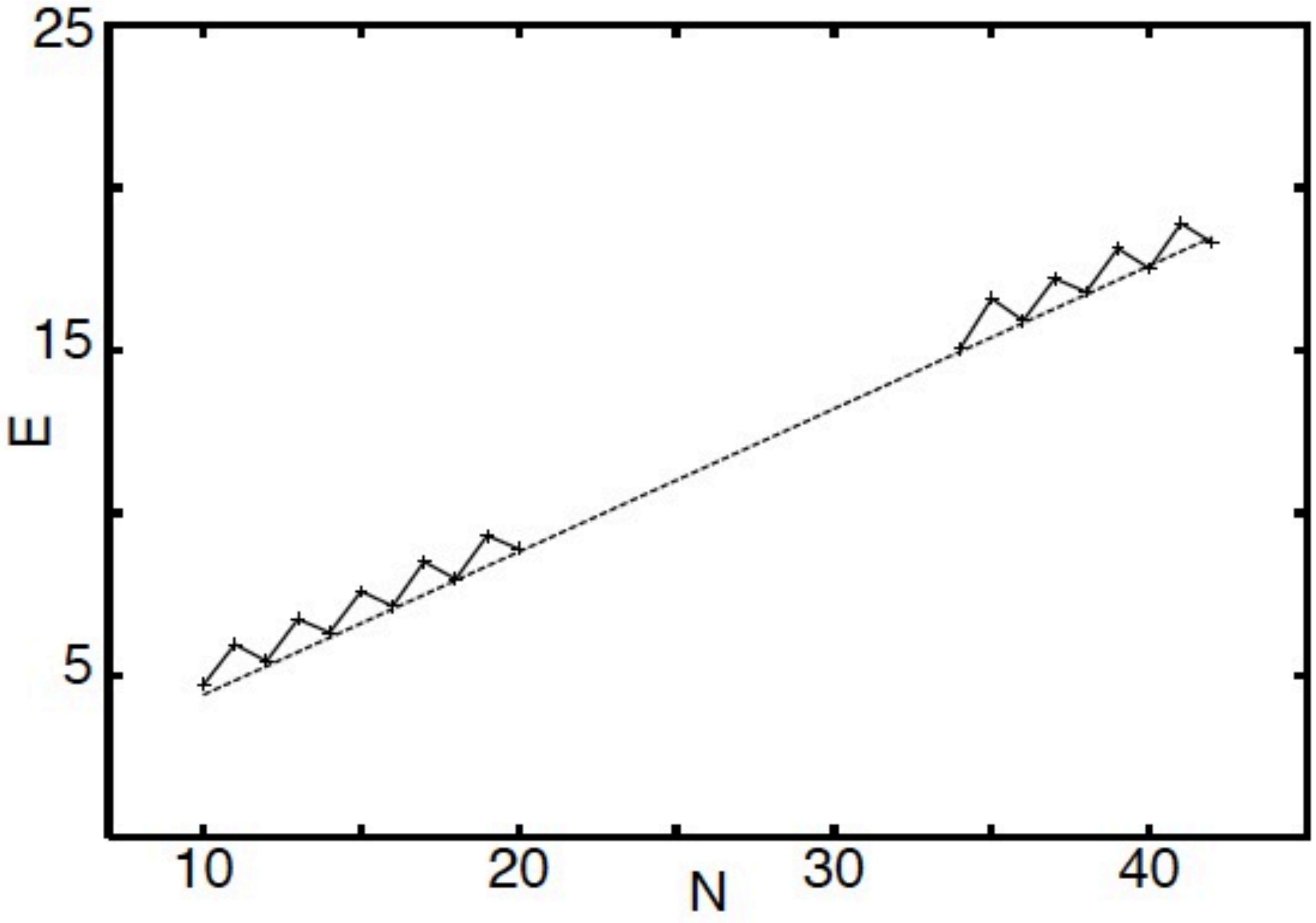
Dilute neutron matter as a unitary Fermi gas

In the unitary limit $k_F|a_s| = \infty$, there is no dimensionfull parameter left. This makes this a universal system, valid whenever the interaction has (approximately) zero range and infinite scattering length.

Bertsch (1990's): The energy of the unitary Fermi gas must be proportional to that of the free Fermi gas, $E_\infty = \xi E_{free}$

What is the size of the Bertsch parameter ξ?

Dilute neutron matter as a unitary Fermi gas

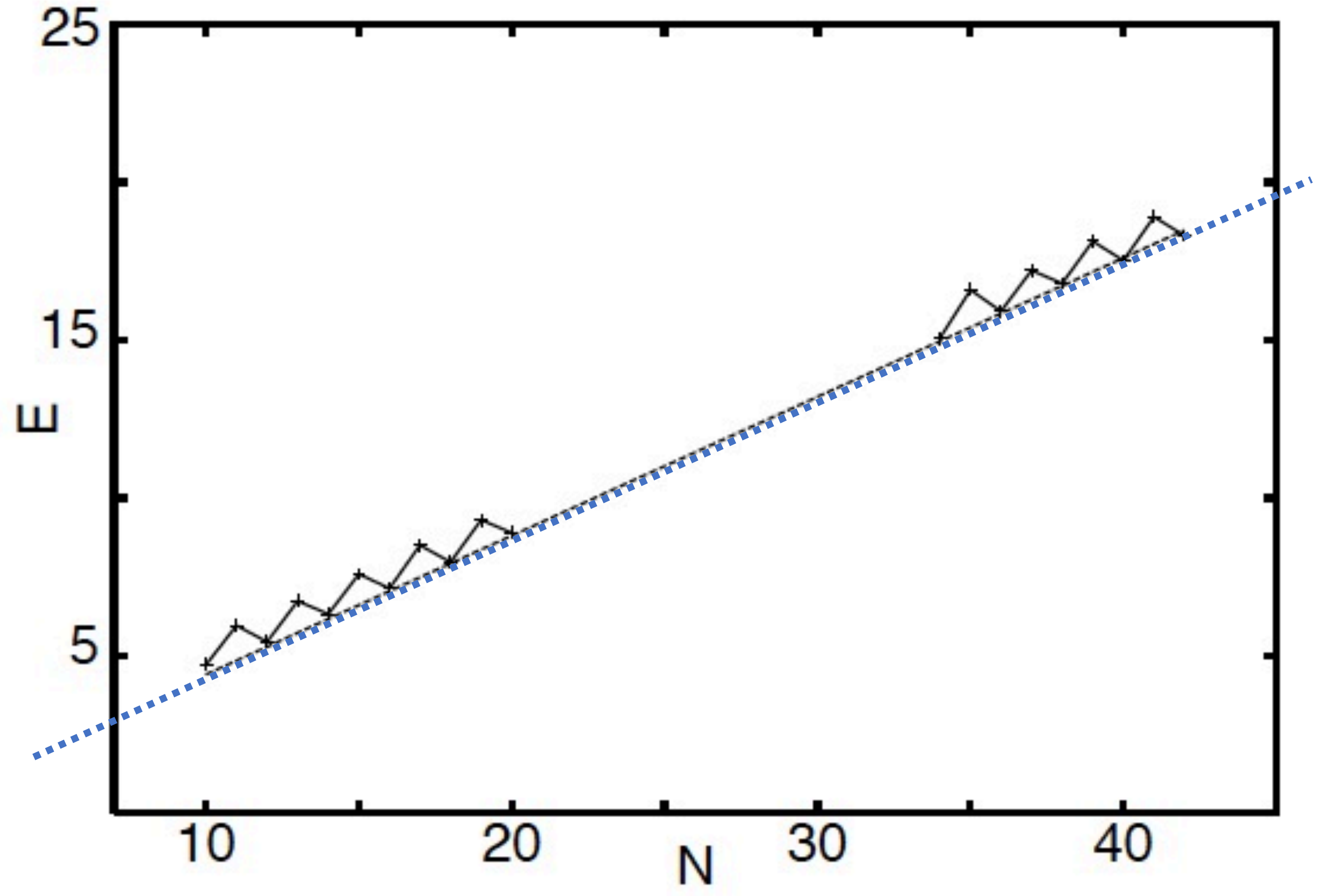


Shown is the total energy E of a system of N fermions in units of $E_{FG} = \frac{3}{10}\frac{k_F^2}{m}$, i.e. the energy per particle of a free Fermi gas.

Q: From this figure, what is an estimate of the Bertsch parameter ξ?

[J. Carlson, S. Y. Chang, V. R. Pandharipande, K. E. Schmidt, Phys. Rev. Lett. 91, 50401 (2003); arXiv:physics/0303094].

Dilute neutron matter as a unitary Fermi gas



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Q: From this figure, what is an estimate of the Bertsch parameter ξ?

A: Carlson et al found $\xi \approx 0.44 \pm 0.01$

(More precise values available, see below)

[J. Carlson, S. Y. Chang, V. R. Pandharipande, K. E. Schmidt, Phys. Rev. Lett. 91, 50401 (2003); arXiv:physics/0303094].

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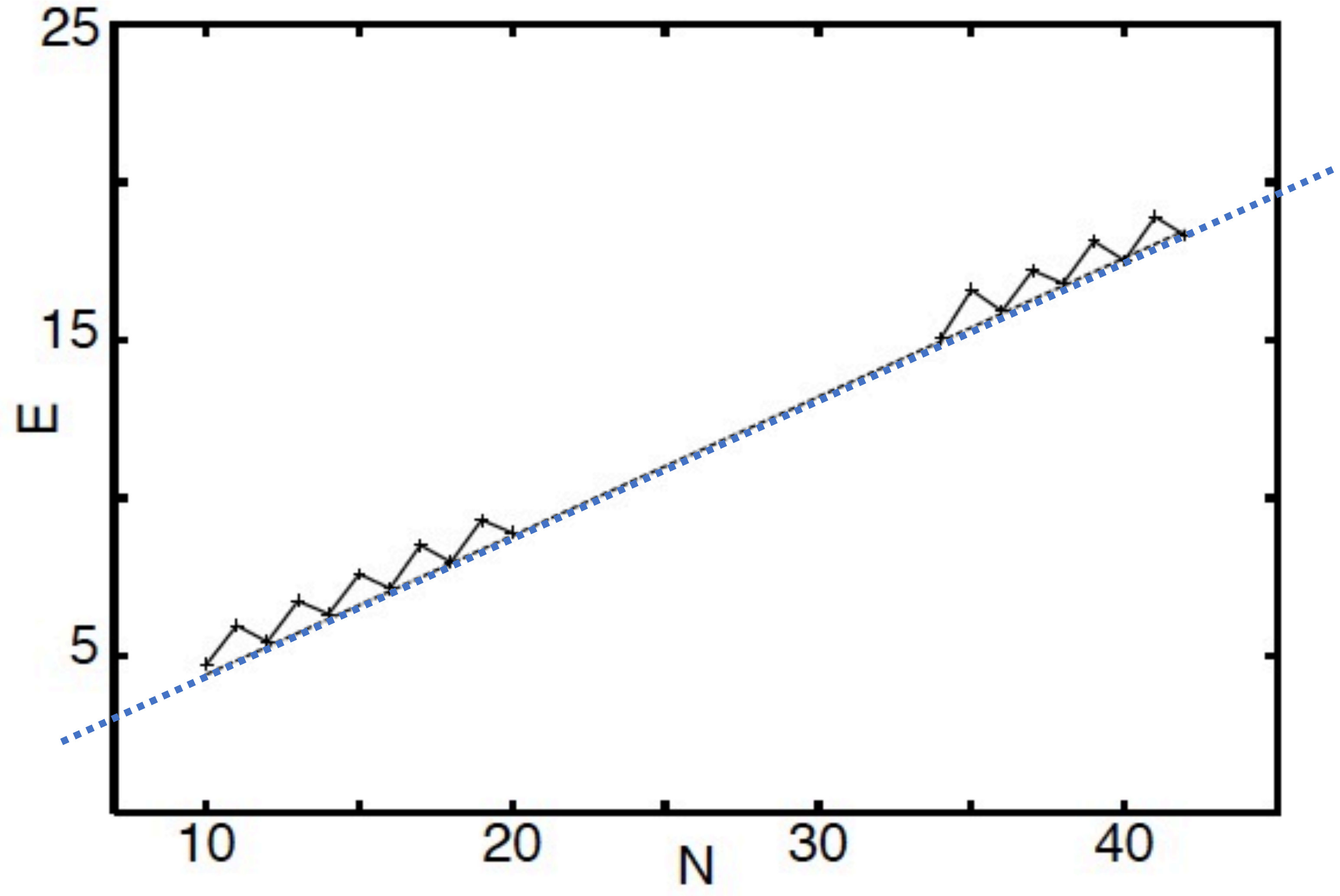

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We also see an odd-even staggering, indication of pairing and nuclear superfluidity.

Reminder: A spin-1/2 fermion system with an attractive s-wave interaction is a BCS superconductor

BCS gap: The empirical pairing gap is $\Delta(N) = E(N) - \frac{1}{2}[E(N+1) + E(N-1)]$.

Task: Estimate Δ

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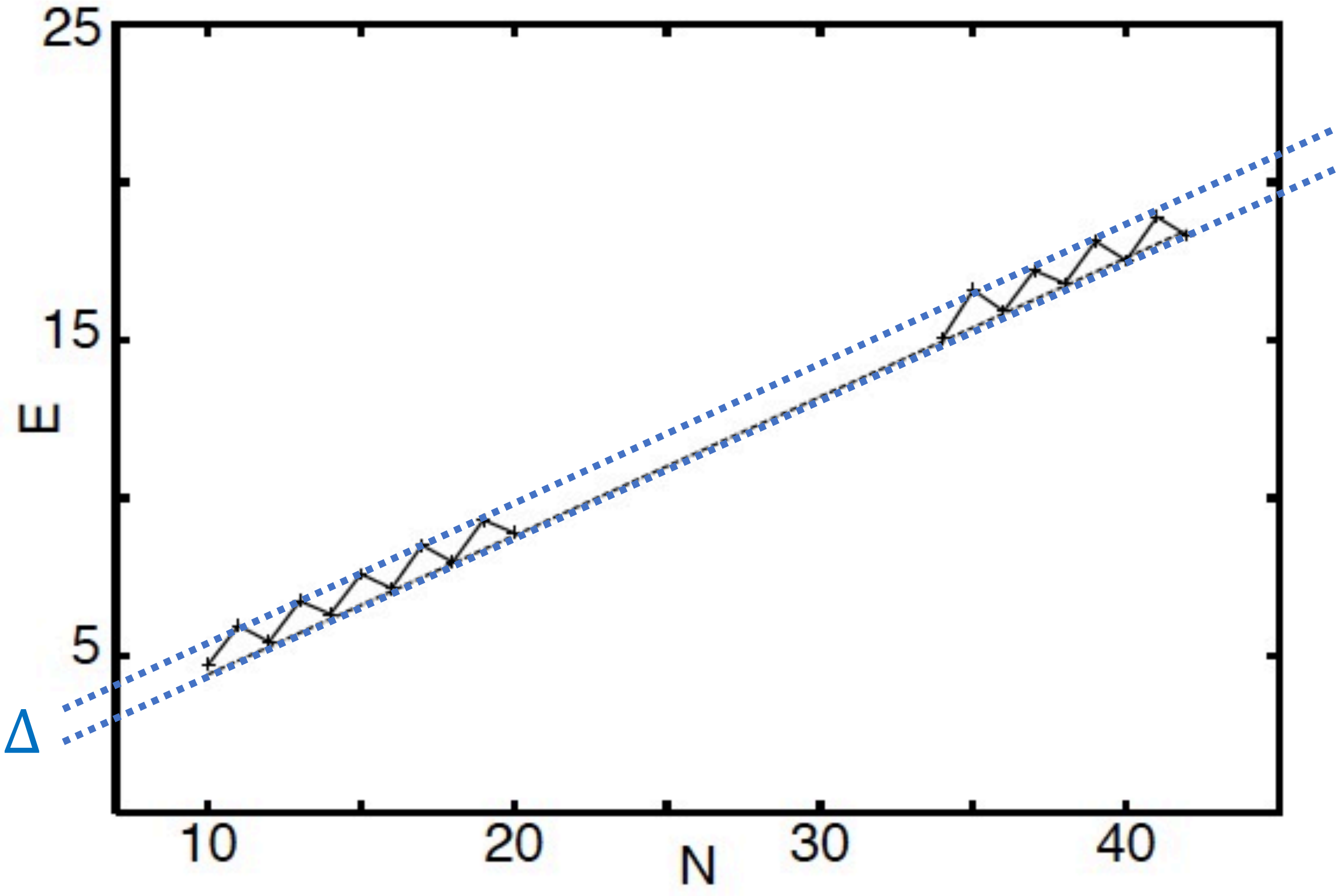

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A: $\Delta \approx E_{free}$

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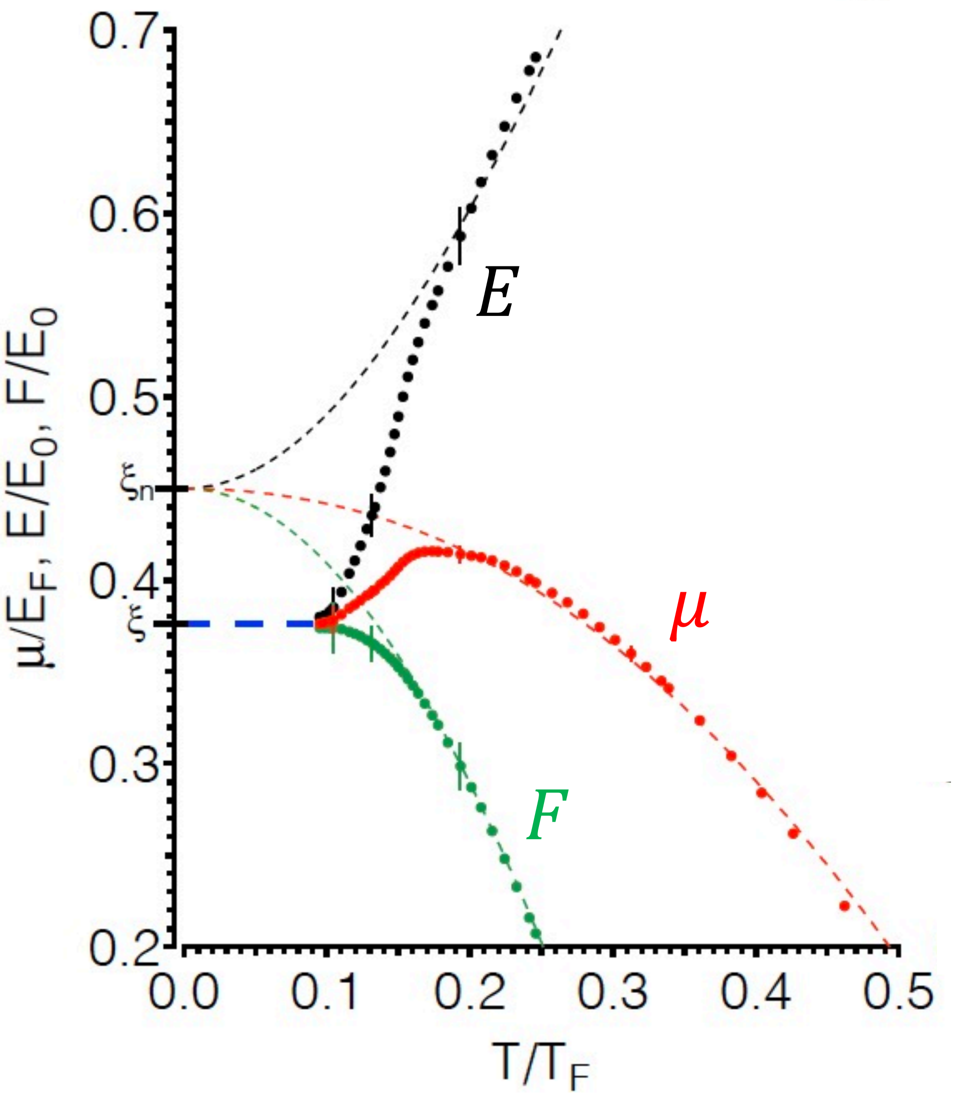
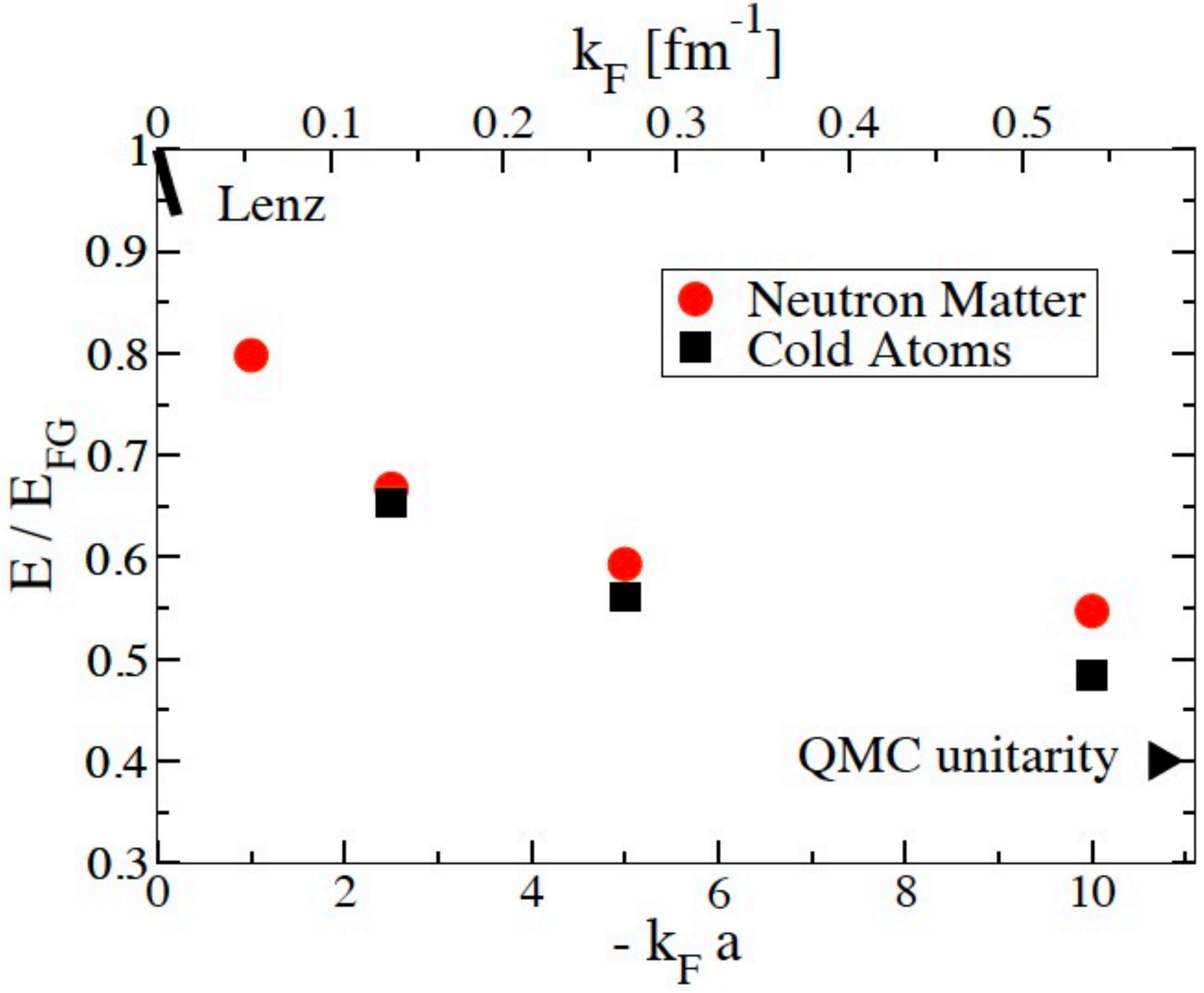
Measurement of the Bertsch parameter using ultracold atom gases

$$\xi = 0.376(5)$$



M. J. H. Ku, A. T. Sommer, L. W. Cheuk, M. W. Zwierlein, Science 335, 563 (2012); arXiv:1110.3309

Neutron matter at low densities



A. Gezerlis & J. Carlson, Phys. Rev. C 77, 032801 (2008), arxiv:0711.3006

Energy of neutron matter, in units of the Fermi-gas energy, as a function of Fermi momentum. ($a \approx -24$ fm)

Summary unitary Fermi gas / dilute neutron matter

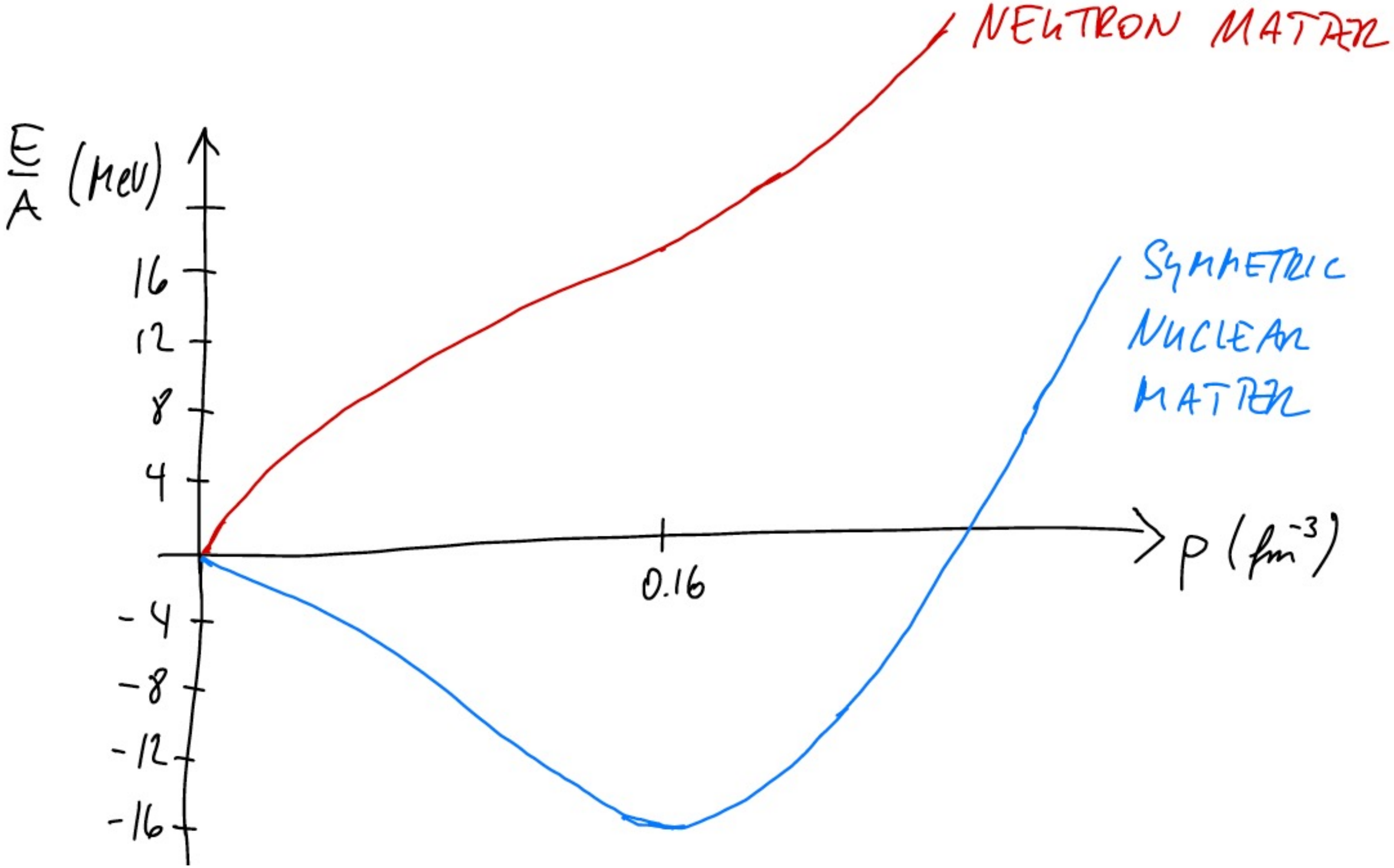
- Attractive short range interactions with a zero-energy bound state yield an energy $E = \xi E_{FG}$, with $\xi \approx 0.4$

- The system is a BCS superconductor with a large pairing gap $\Delta \sim E_{FG}$

- Compare to nuclei: $\Delta \approx 1 - 2$ MeV, $k_F \approx 1.35$ fm^{-1}, $\frac{\Delta}{E_F} \approx \frac{1-2}{38} \ll 1$

- Unitary Fermi gas / dilute neutron matter are the strongest BCS superconductors

Nuclear Equation of State



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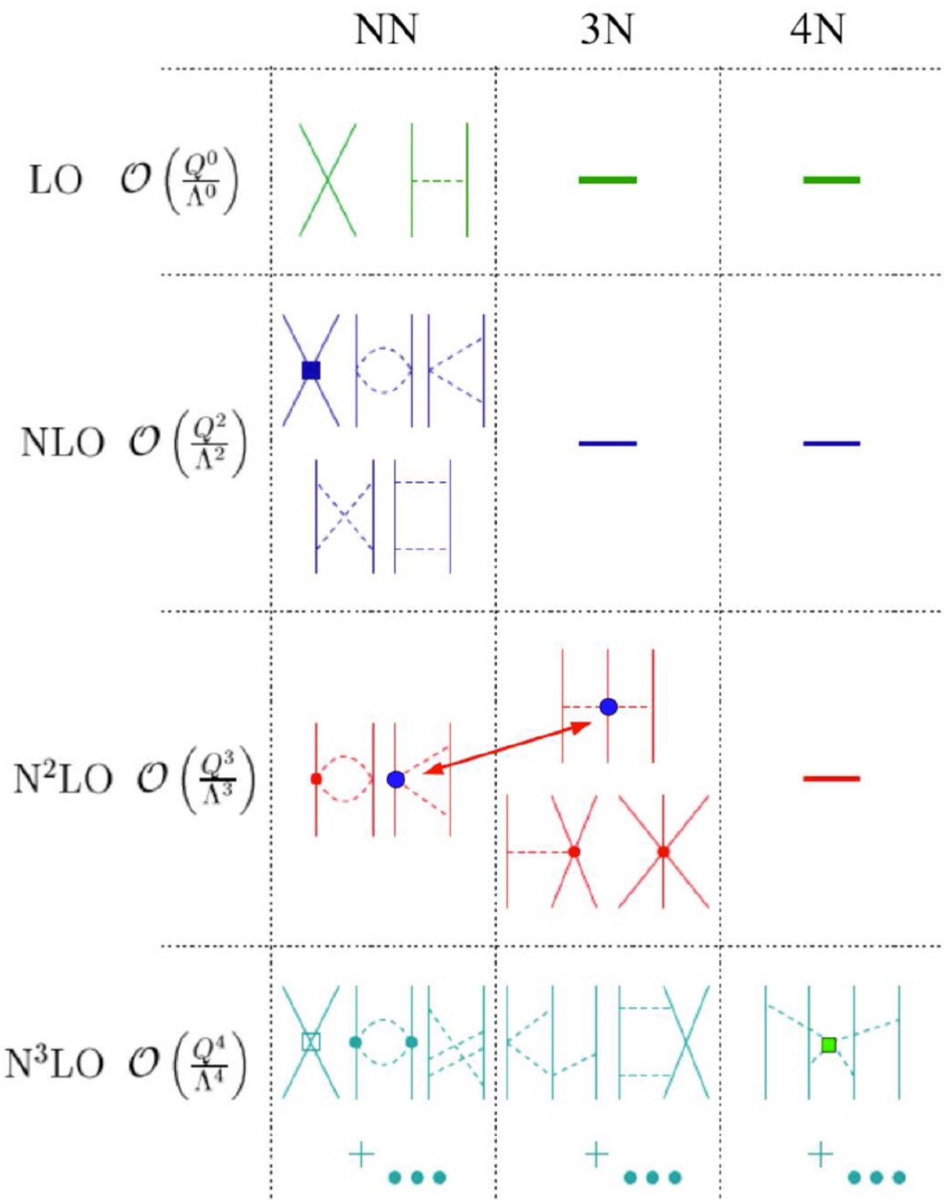
Let us discuss neutron matter!

Nuclear interactions from chiral effective field theory

[Weinberg; van Kolck; Epelbaum, Gloeckle, Krebs, Meissner; Entem & Machleidt; Ekström, …]



Interactions between two (NN), three (3N), and four (4N) nucleons, ordered according to the Weinberg power counting

Full line: nucleon
Dashed line: pion

NN forces are dominant in the Weinberg power counting with 3N forces entering at next-to-next-to-leading order (NNLO)

Pion-nucleon constants (blue circles) from pion-nucleon scattering

Q: What enters at leading order (LO) ?

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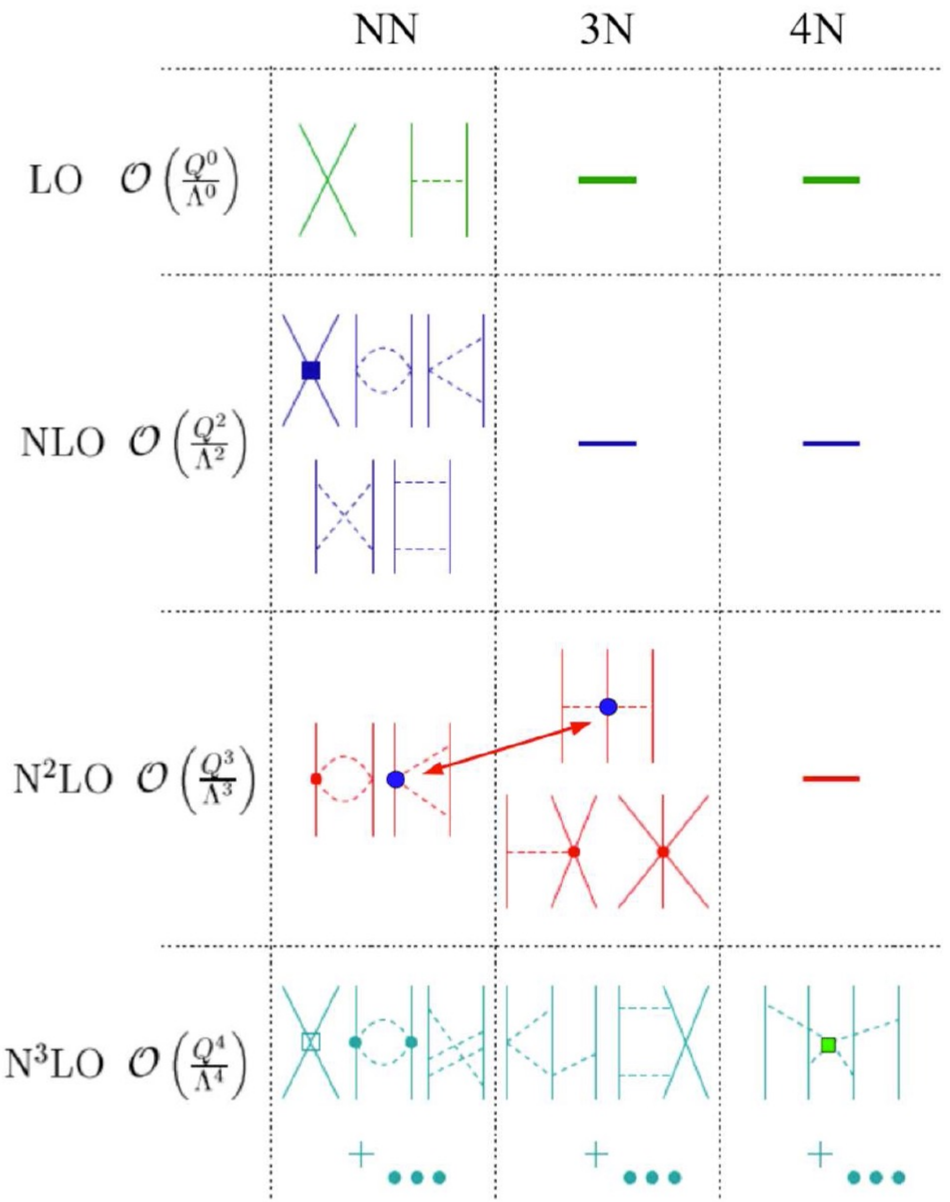

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Q: What enters at leading order (LO) ?
A: One-pion exchange and contact interactions

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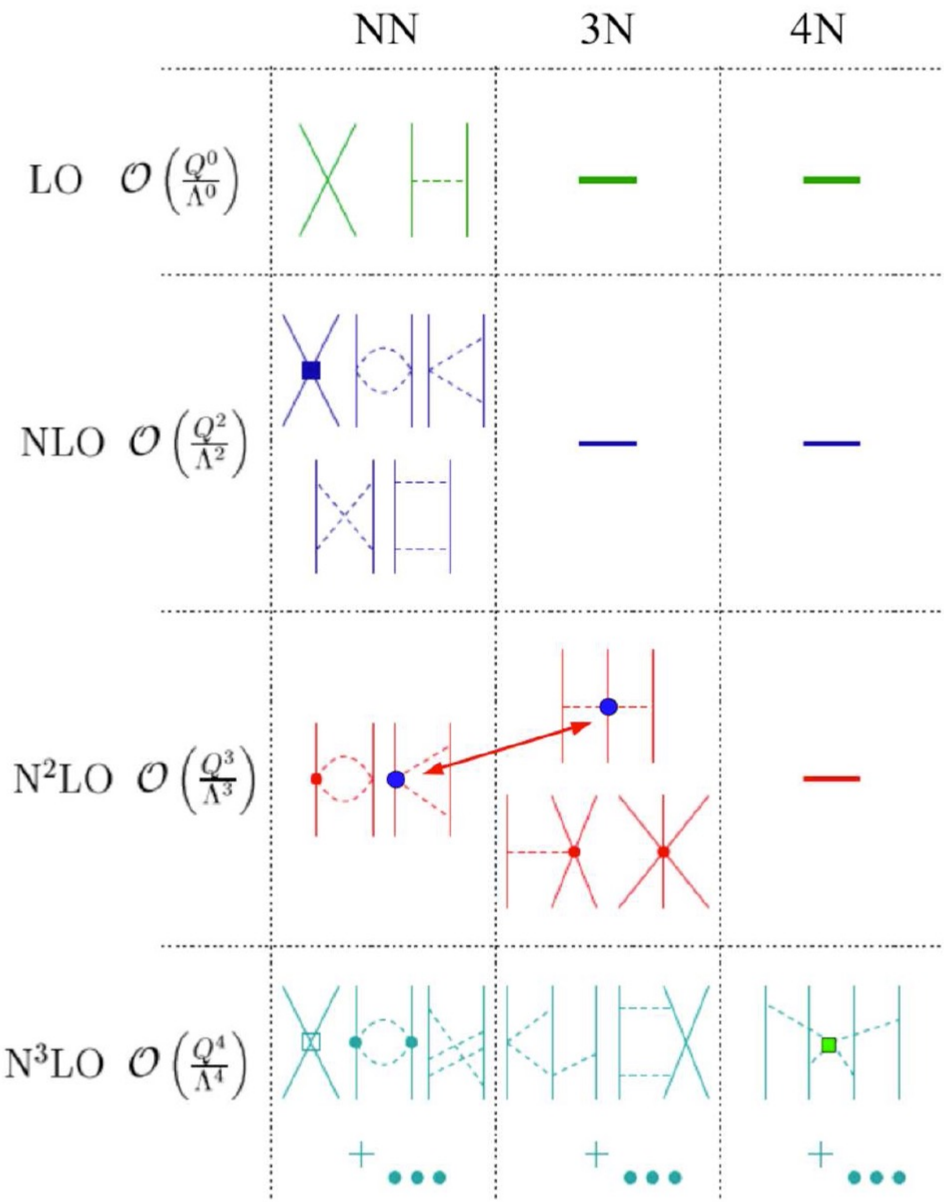

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Typical cutoffs are $\Lambda \approx 2$ fm^{-1}
Q: Up to which densities can we use such potentials?

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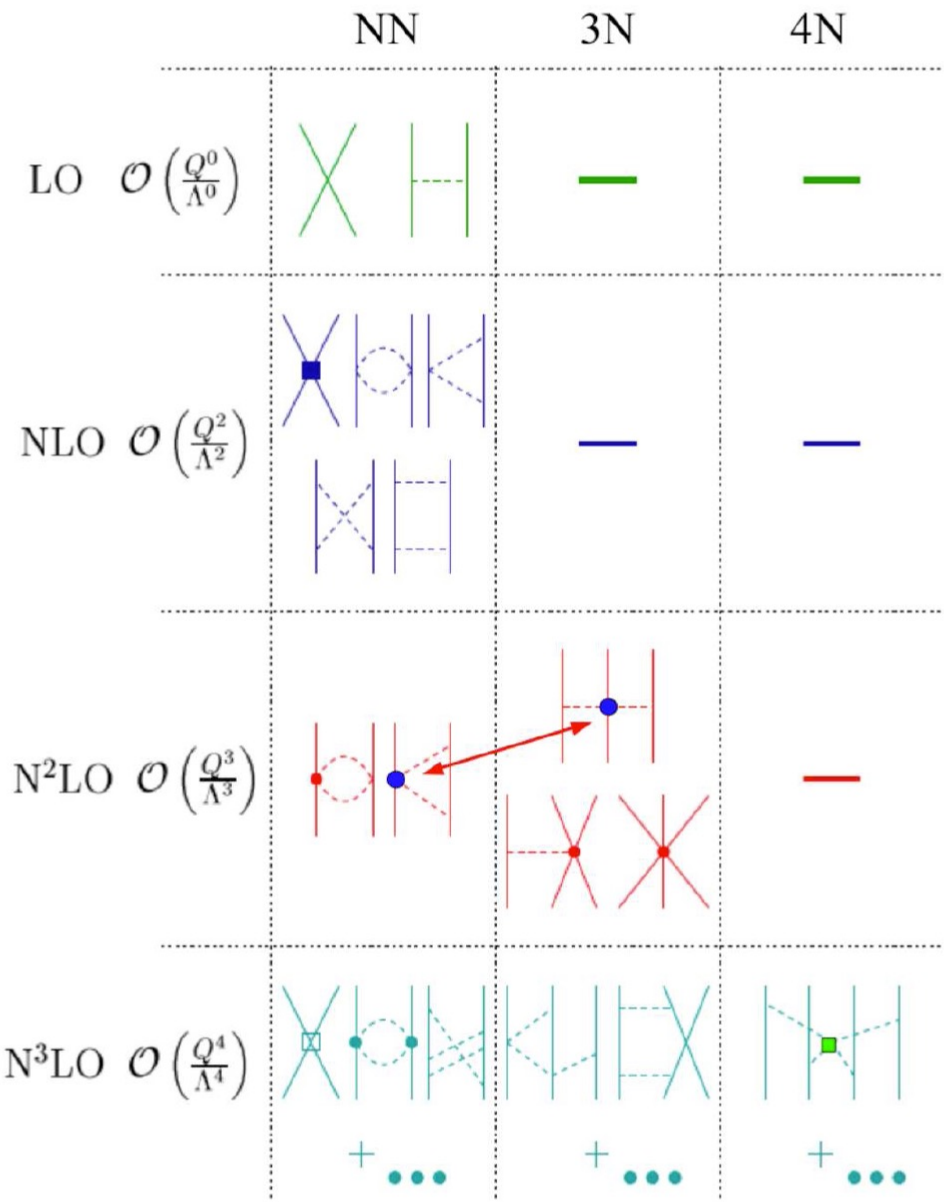

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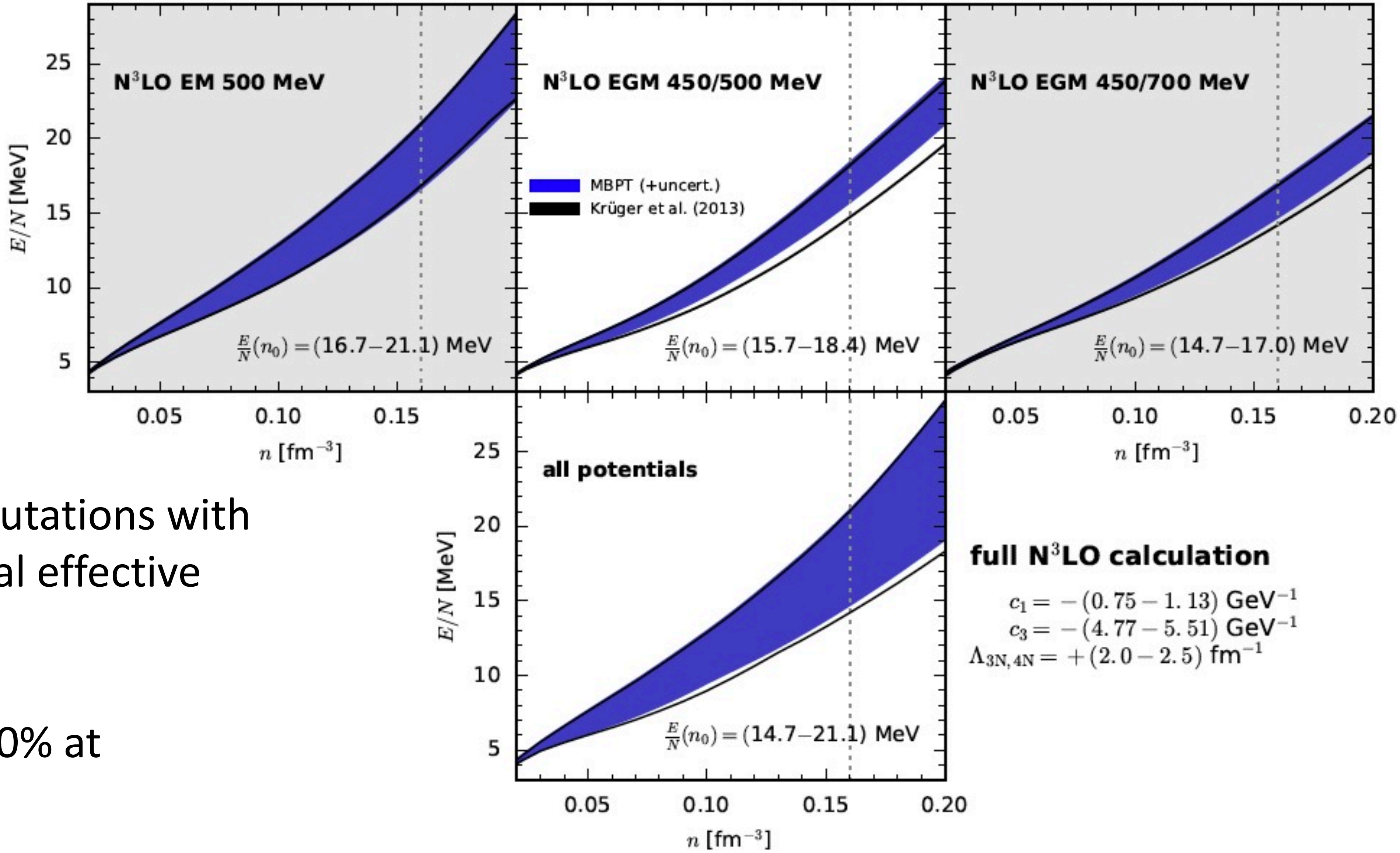
Typical cutoffs are $\Lambda \approx 2\ \text{fm}^{-1}$
Q: Up to which densities can we use such potentials?
A: $\frac{\rho}{\rho_0} = \frac{\Lambda^3}{k_F^3} \sim 3$

Neutron matter



full N^{3}LO calculation

$c_1 = -(0.75 - 1.13)$ GeV^{-1}
$c_3 = -(4.77 - 5.51)$ GeV^{-1}
$\Lambda_{3N,4N} = +(2.0 - 2.5)$ fm^{-1}

Neutron matter computations with interactions from chiral effective field theory (EFT)

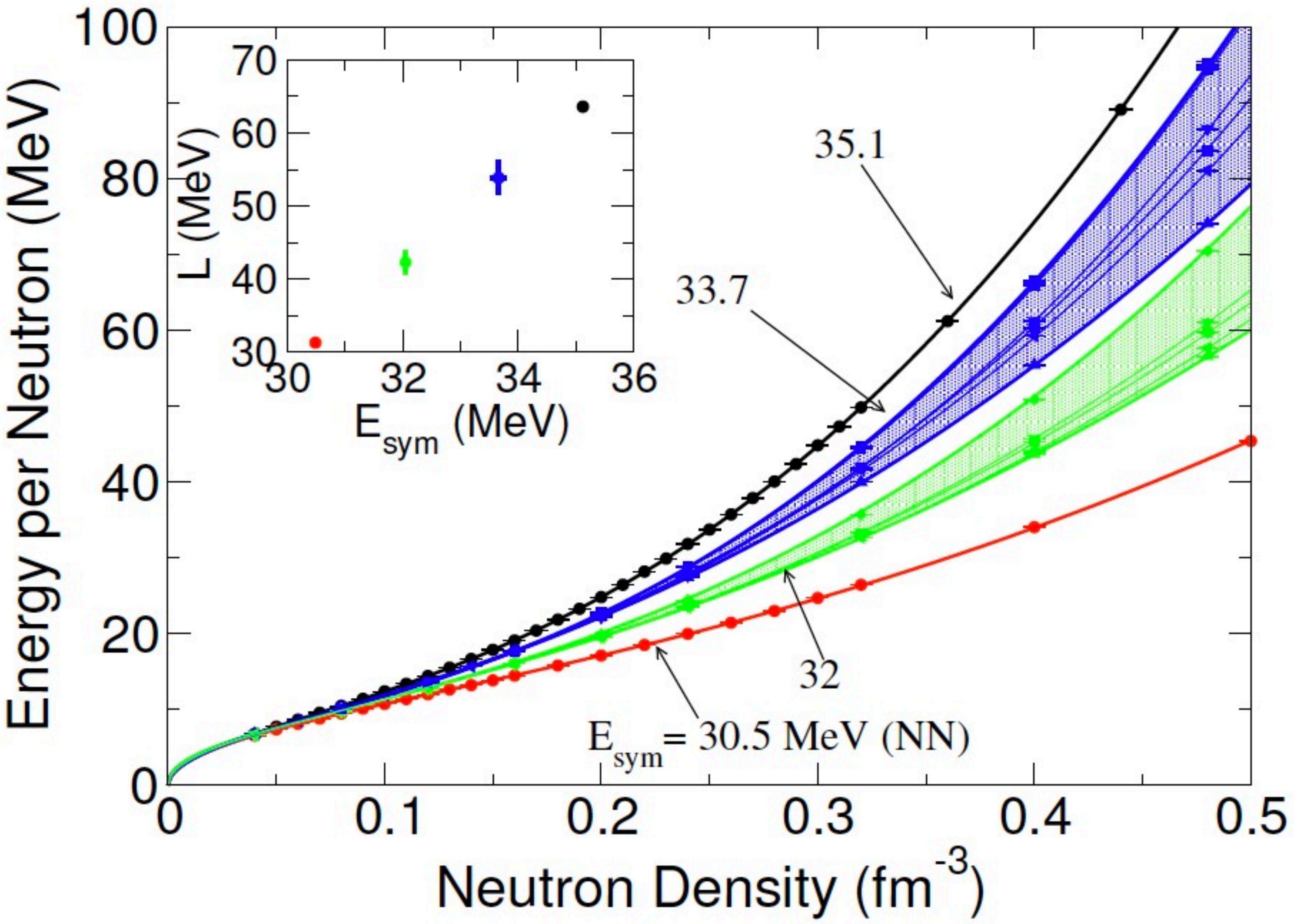
Uncertainties about 30% at saturation density

C. Drischler, A. Carbone, K. Hebeler, A. Schwenk, Phys. Rev. C 94, 054307 (2016); arXiv:1608.05615

Neutron matter as a function of the symmetry energy

$$E_{\mathrm{sym}}(\rho) = E_{\mathrm{PNM}}(\rho) - E_{\mathrm{SNM}}(\rho)$$

$$E_{\mathrm{sym}}(\rho) = E_{\mathrm{sym}} + \frac{L}{3}\frac{\rho - \rho_0}{\rho_0}$$



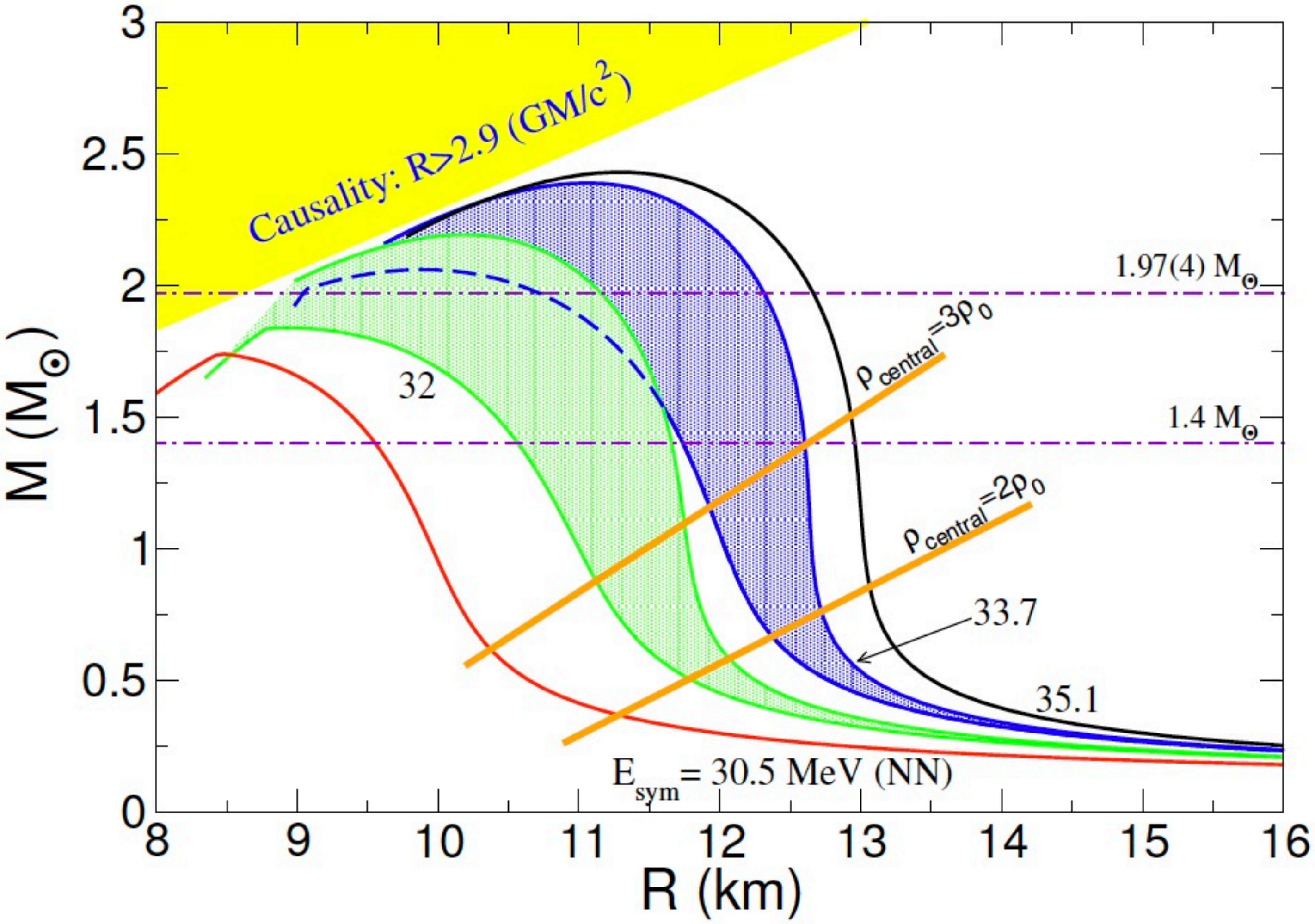
Understanding of neutron EOS at multiples of saturation densities is still poorly constrained

S. Gandolfi, J. Carlson, Sanjay Reddy,
Phys. Rev. C 85, 032801 (2012); arXiv:1101.1921

Neutron stars as a function of the symmetry energy

Neutron-star mass-radius relationship uniquely determined by neutron EOS

Figure shows that the mass-radius relationship is sensitive to the precise value of the symmetry energy



S. Gandolfi, J. Carlson, Sanjay Reddy,
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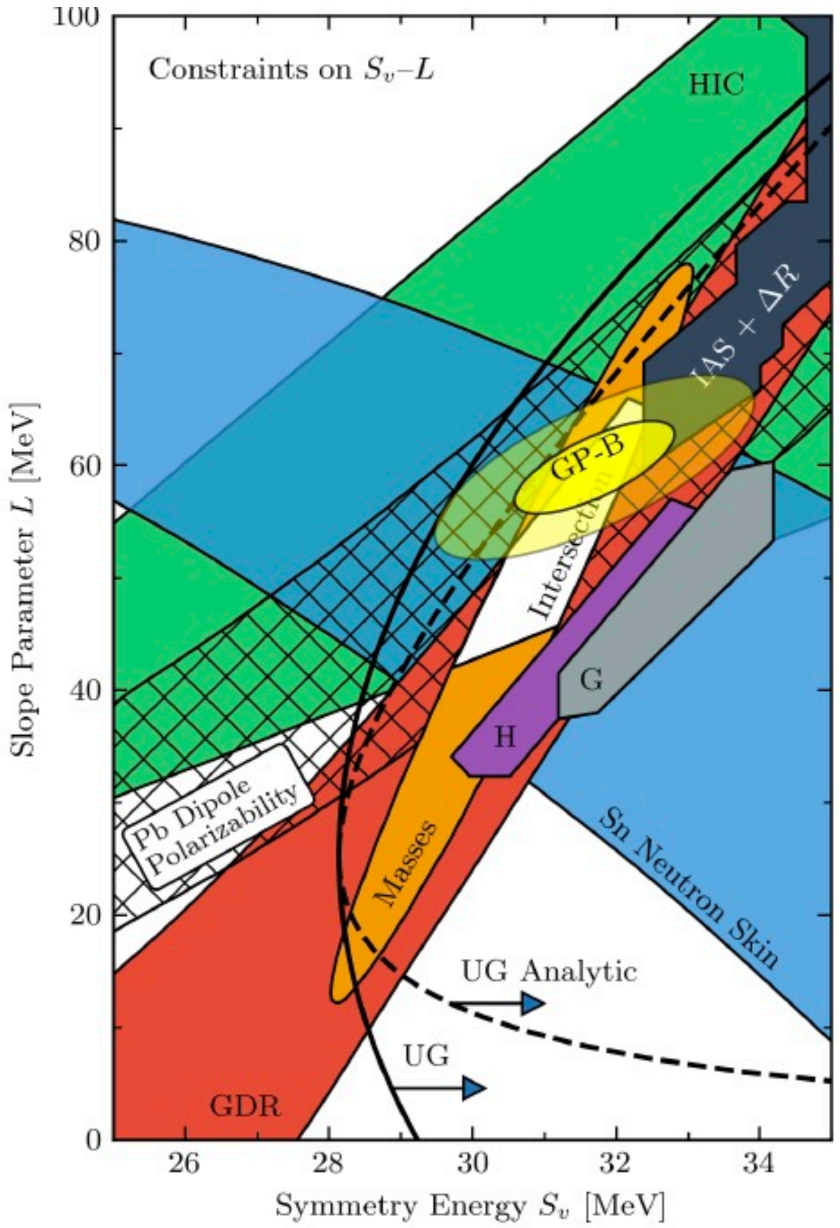
Constraining the symmetry energy



$$\frac{E}{N}(n) - \frac{E}{A}(n) \equiv S_v + \frac{L}{3}\left(\frac{n - n_0}{n_0}\right) + \ldots$$

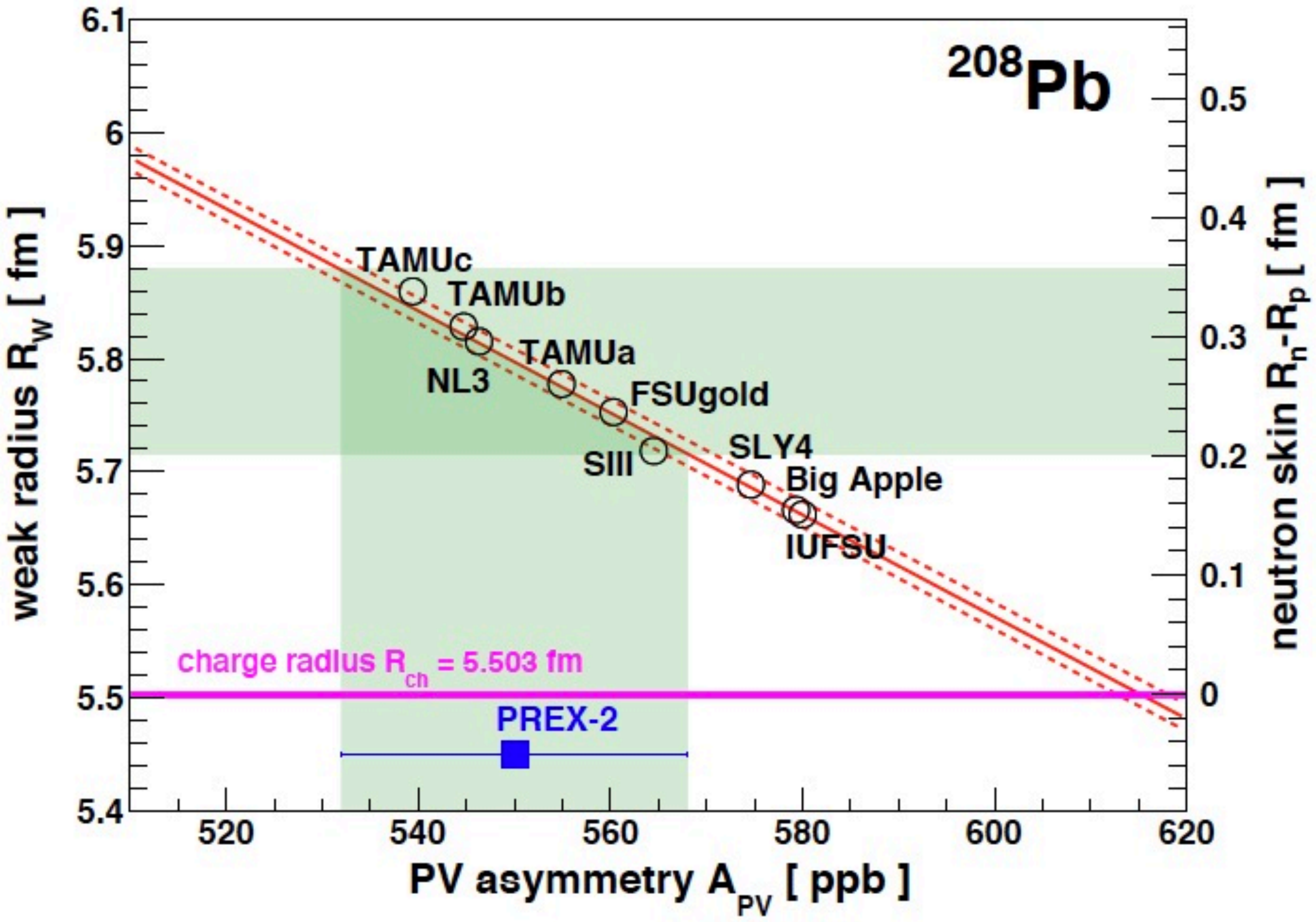
GP-B: Gaussian Processes & Bayesian analysis
H: computations using chiral potentials [Hebeler et al]
G: Quantum Monte Carlo computations[Gandolfi et al]
HIC: Heavy-ion collisions
GDR: Giant dipole resonances
Sn: Neutron-skin data

In 2020, it seemed the symmetry energy was cornered …

C. Drischler, R. J. Furnstahl, J. A. Melendez, D. R. Phillips, Phys. Rev. Lett. 125, 202702 (2020); arXiv:2004.07232
See also Drischler, Hebeler & Schwenk, Phys Rev Lett (2019)

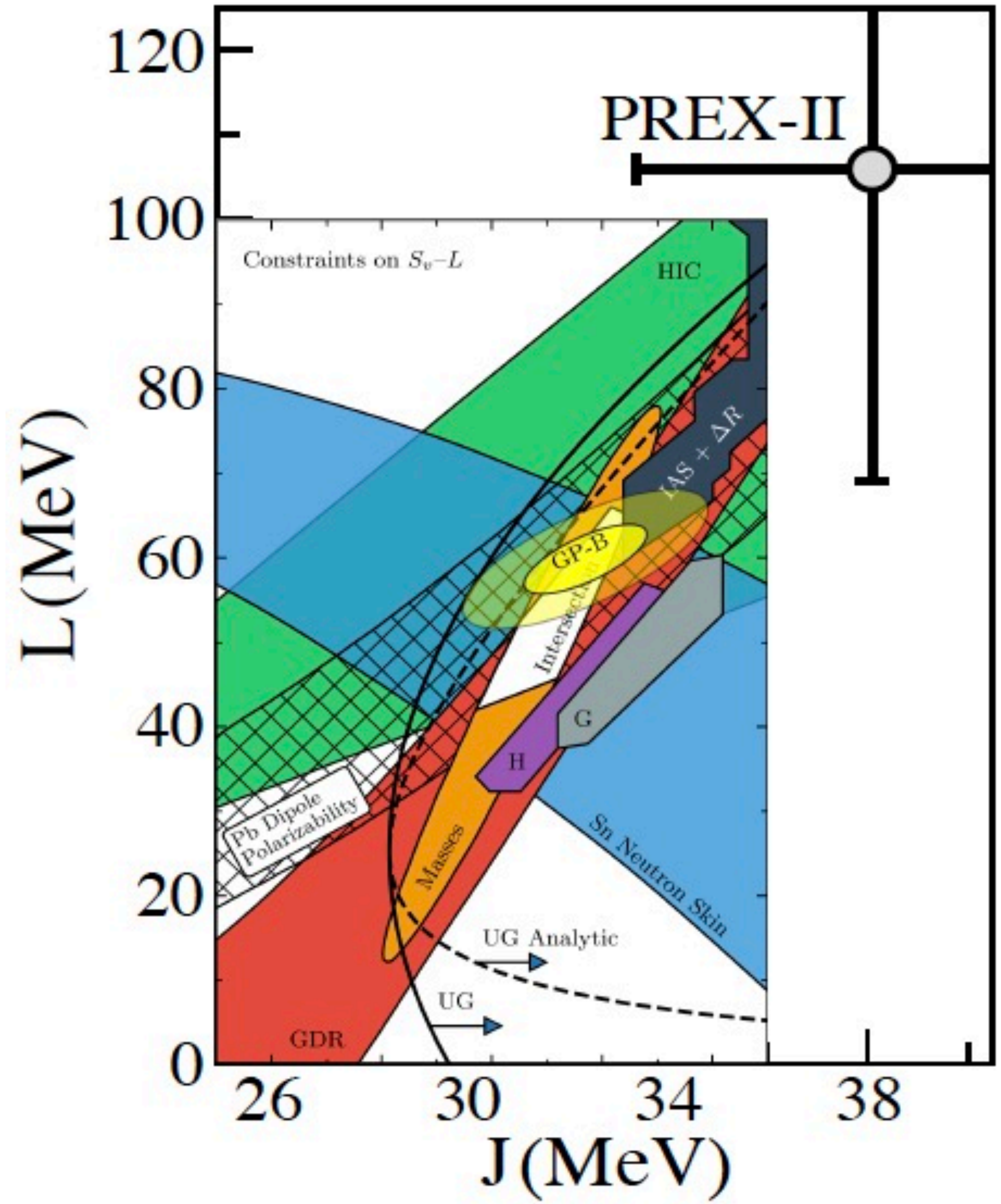
Parity-violating electron scattering of ^{208}Pb

Parity-violating electron scattering via Z-boson exchange
Z-boson couples (almost) exclusively to neutrons; weak radius R_W is the result
Density-functional theory maps out neutron skin



PREX-2: D. Adhikari et al., Phys. Rev. Lett. 126, 172502 (2021); arXiv:2102.10767

Constraining the symmetry energy?



$$\mathcal{S}(\rho) = J + L\frac{(\rho - \rho_0)}{3\rho_0} + \ldots$$

The recent measurement of the asymmetry in parity-violating electron-scattering of ^{208}Pb, combined with a model-dependent extraction of the symmetry energy is somewhat puzzling …

PREX-II: D. Adhikari et al., arXiv:2102.10767; Phys. Rev. Lett. 126, 172502 (2021)

Brendan T. Reed, F. J. Fattoyev, C. J. Horowitz, J. Piekarewicz, Phys. Rev. Lett. 126, 172503 (2021); arXiv:2101.03193

Summary neutron matter around saturation density

- EOS can be computed with uncertainties starting with interactions from chiral effective field theory
- Uncertainty is about 30% at saturation density; much larger at multiples thereof
- PREX-2 results challenge nuclear models; tension at the 1-sigma level … stay tuned
- Complementary results from gravitational waves of neutron-star mergers and neutron-star radius measurements (NICER) also constrain neutron equation of state