Nuclear Matter and Nuclear Structure II

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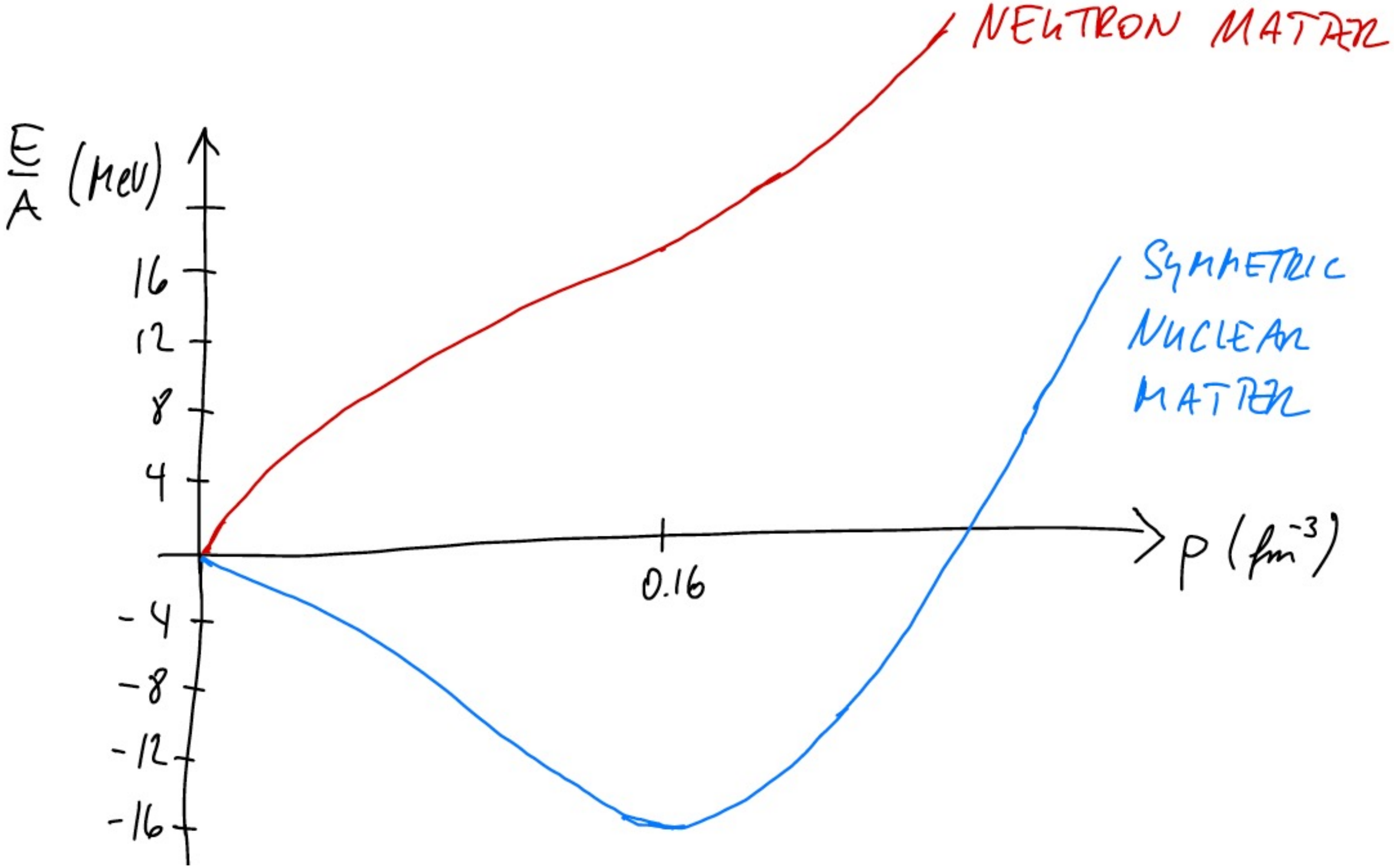
Work supported by the US Department of Energy

- Questions welcome
  - Please ask during breaks, or use slack later
- Please participate actively in the lectures
  - Research shows: You will learn better by active participation
- Thanks to all who asked questions in chat/slack, and to those who provided answers!

Nuclear Equation of State



Pure neutron matter: $A = N$

Symmetric matter: $N = Z$

Note: Coulomb force neglected; electrons not included

Let us discuss symmetric nuclear matter!

Computations of symmetric nuclear matter

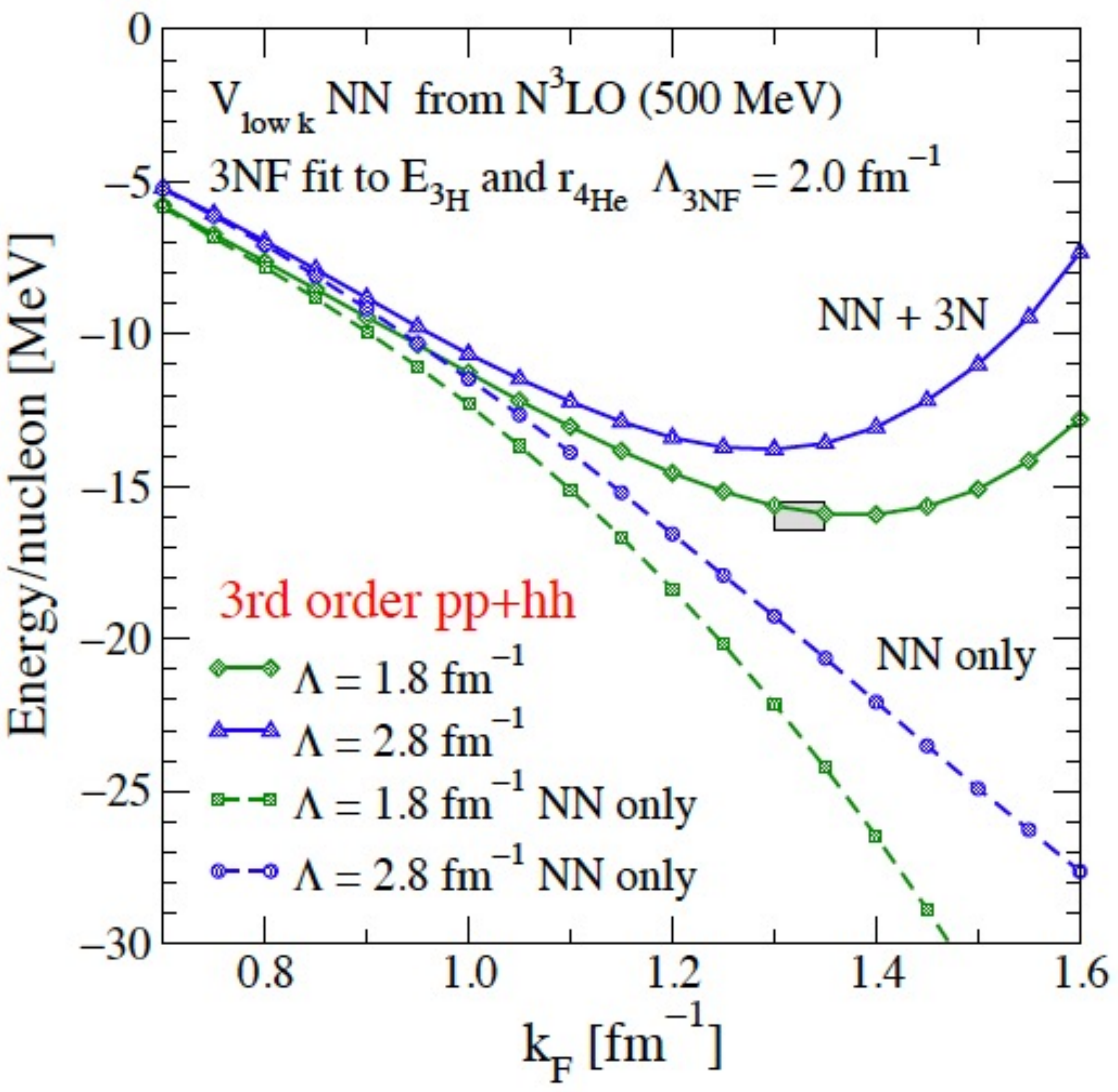
The empirical saturation point of nuclear matter is sufficiently well known

Saturation could be viewed as an emergent phenomenon

What is the saturation mechanism?

How well do interactions from effective field theory perform?

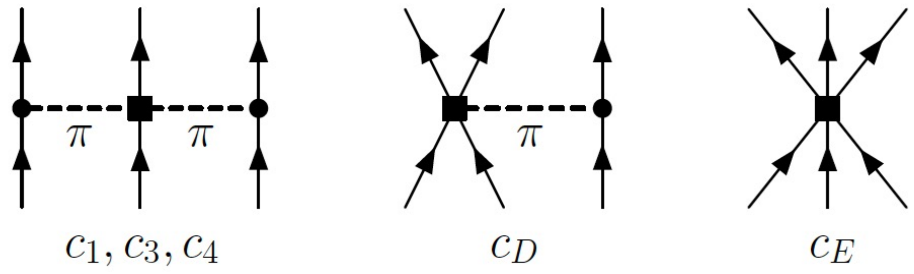
Saturation of symmetric nuclear matter



Nucleon-nucleon force from chiral EFT

Similarity renormalization group (SRG) transformation to lower momentum cutoff Λ

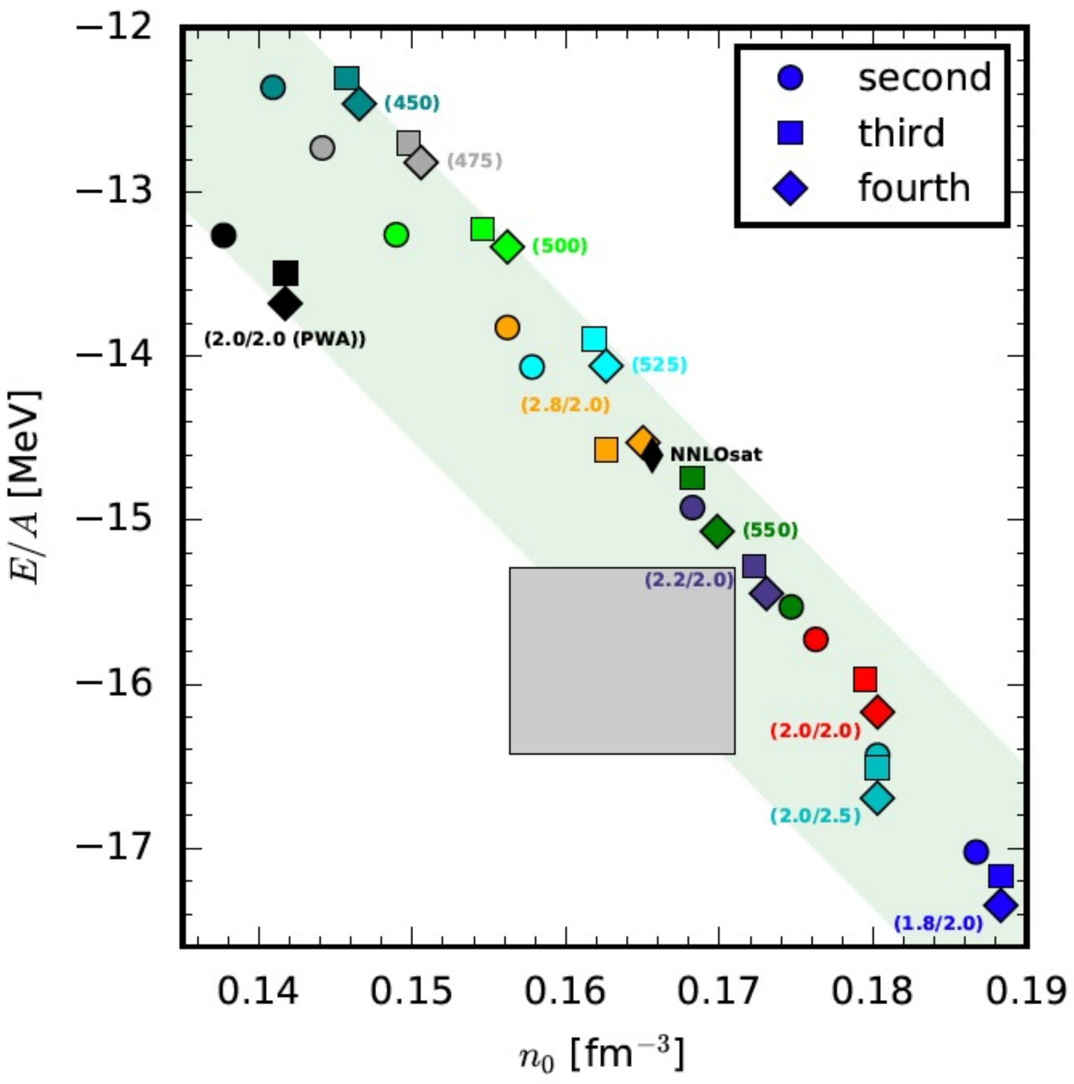
Adjust three-nucleon forces to A=3,4 nuclei



- 3N forces cause saturation
- Empirical saturation point (grey box) hard to meet

K. Hebeler, S. K. Bogner, R. J. Furnstahl, A. Nogga, A. Schwenk, Phys. Rev. C 83, 031301 (2011); arXiv:1012.3381

Saturation point of interactions from chiral effective field theory

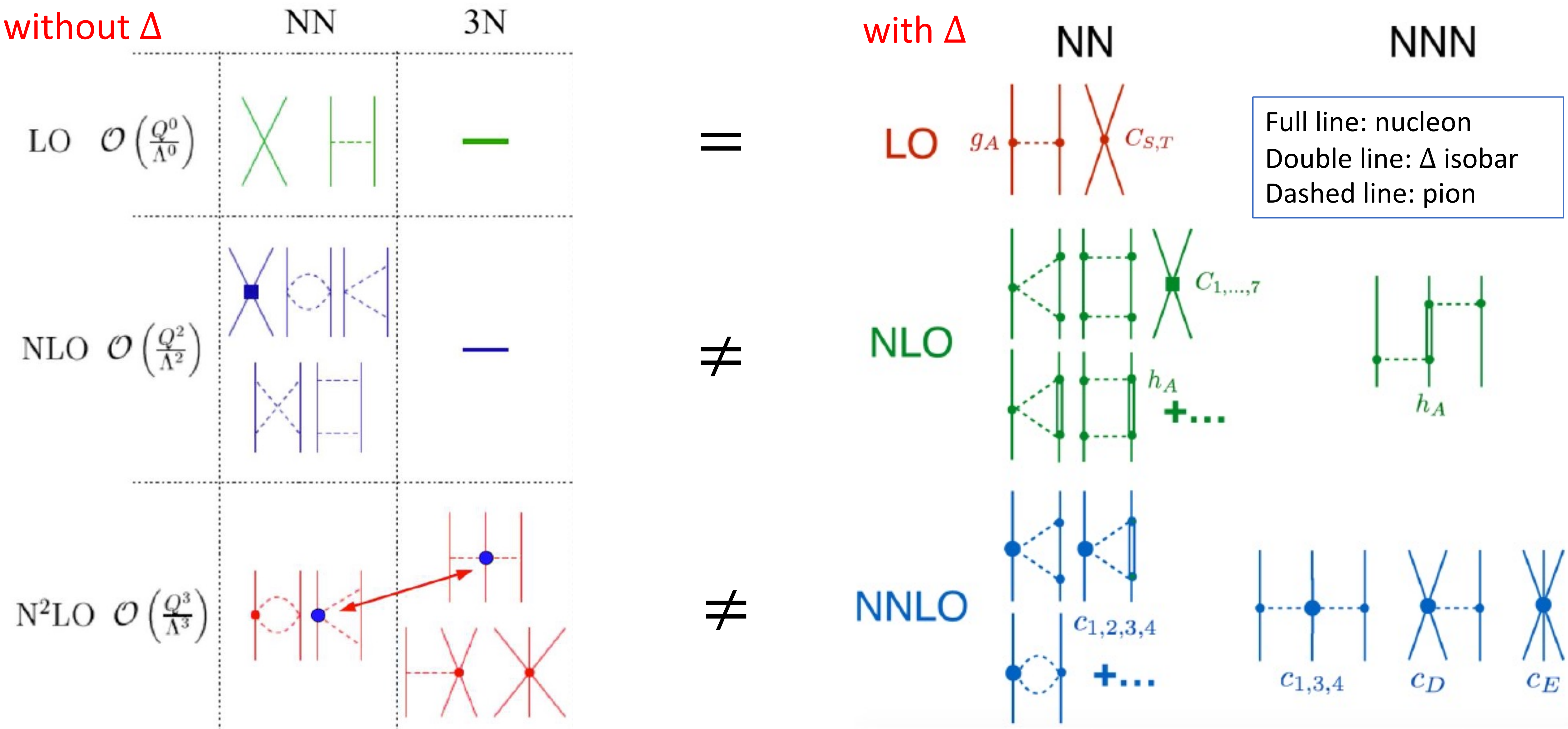


- The saturation points of current interactions from chiral EFT (different parameterizations, cutoffs, orders) exhibit a linear trend
- None agree with the empirical saturation point

C. Drischler, K. Hebeler, A. Schwenk, Phys. Rev. Lett. 122, 042501 (2019); arXiv:1710.08220

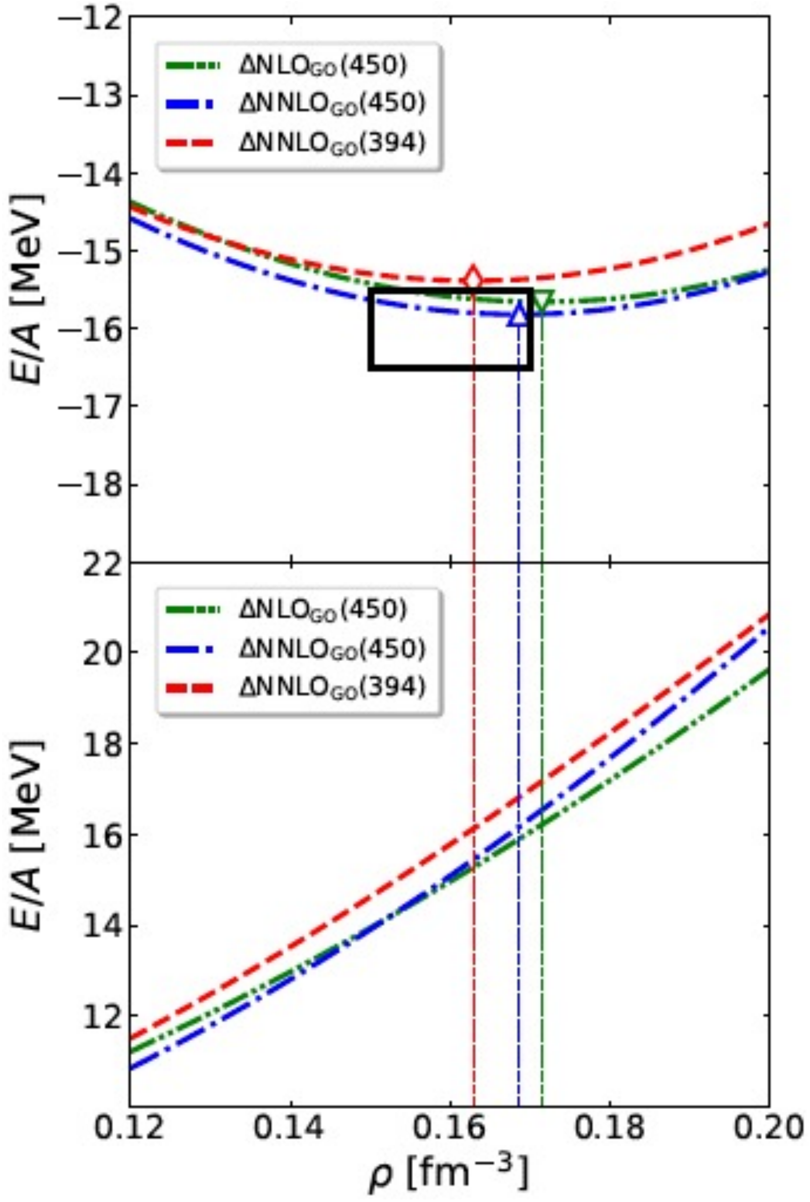
Interactions with Δ isobar intermediate states

Δ isobar = nucleon resonance (S=T=3/2) at 293 MeV of excitation energy



[Van Kolck (1994); Hemmert, Holstein, Kambor (1998); Kaiser, Gerstendörfer, Weise (1998); Krebs, Epelbaum, Meissner (2007)]

Nuclear equation of state with Δ degrees of freedom



Chiral potentials with Δ isobars fitted to light nuclei and nuclear matter (and informed by medium-mass nuclei) exhibit improved saturation properties

NNLO$_{GO}$(450): $\rho_0 = 0.169$ fm^{-3}, $E_{sym} = 32.0$ MeV, $L = 65$

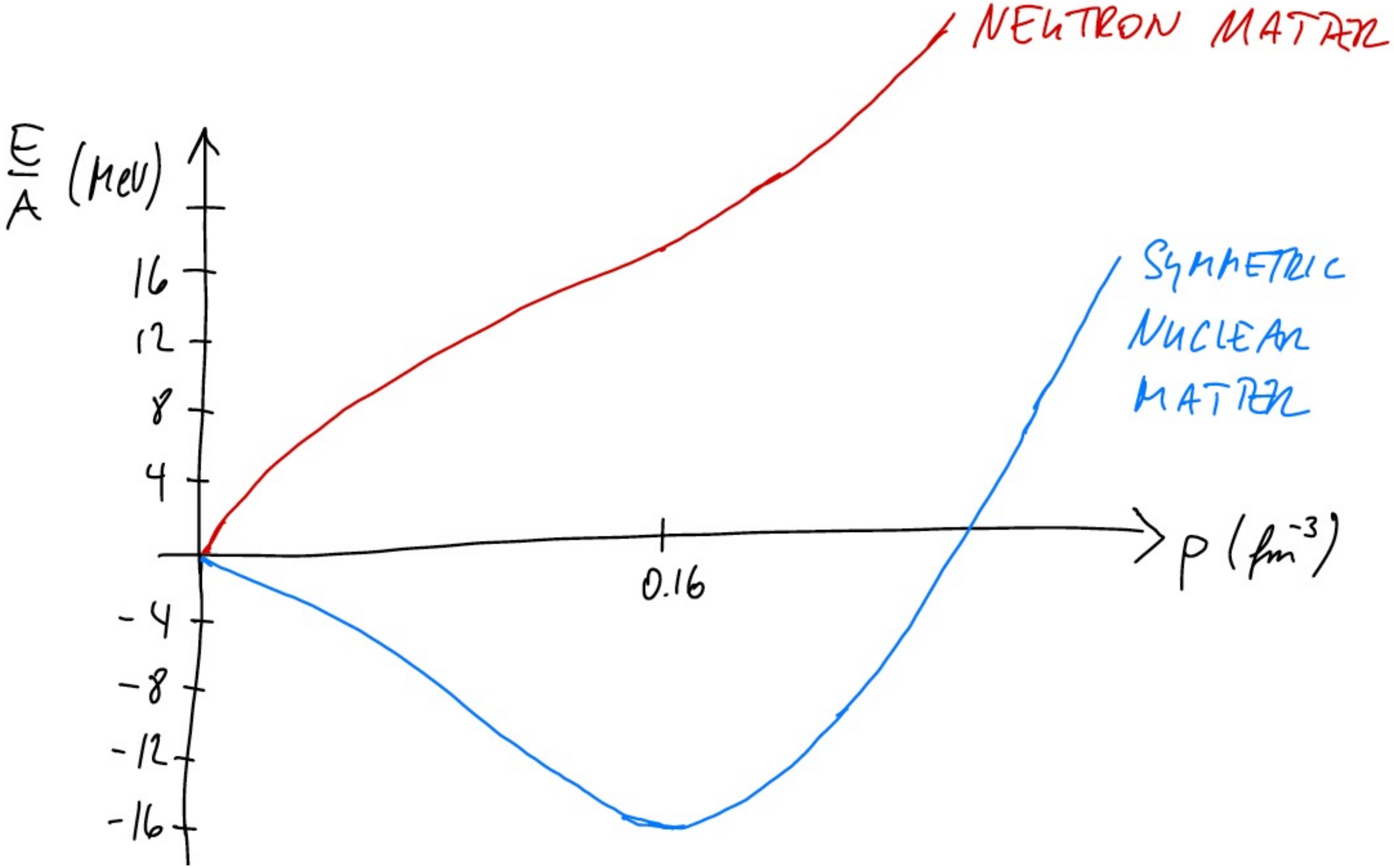
NNLO$_{GO}$(394): $\rho_0 = 0.163$ fm^{-3}, $E_{sym} = 31.5$ MeV, $L = 58$

Tension with PREX-II, though on symmetry energy E_{sym} & L

→ Other potentials with Δs: Maria Piarulli et al, PRC 2016; PRL 2018

W. G. Jiang, A. Ekström, C. Forssén, G. Hagen, G.R. Jansen, T. Papenbrock, Phys. Rev. C 102, 054301 (2020); arXiv:2006.16774

Nuclear equation of state at high densities



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Symmetric matter: $N = Z$

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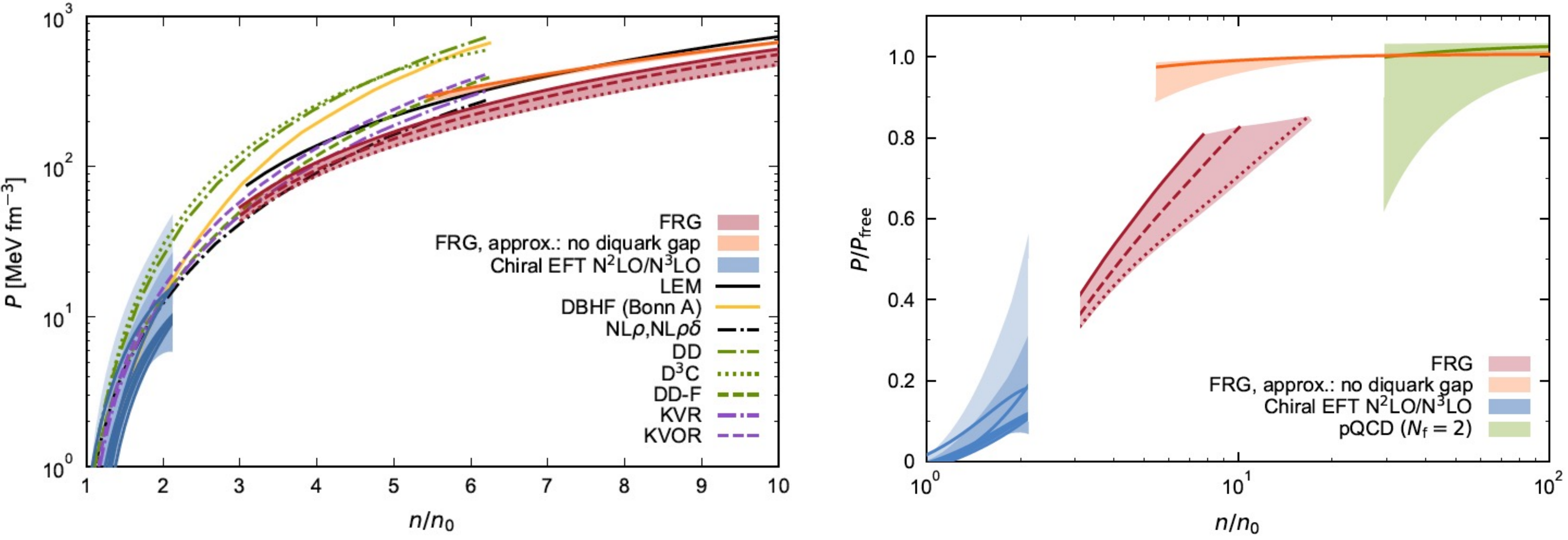
What's going on at high densities?

Pressure of symmetric nuclear matter over many scales



- At highest densities: perturbative QCD (pQCD)
- At high densities: Functional Renormalization Group (FRG); uncertainties from various transition scales
- At "low" densities: Chiral EFT; uncertainties from different orders in the Weinberg power counting
- Uncertainties still quite large

M. Leonhardt, M. Pospiech, B. Schallmo, J. Braun, C. Drischler, K. Hebeler, A. Schwenk, Phys. Rev. Lett. 125, 142502 (2020); arXiv:1907.05814

Summary nuclear matter

- Soft (i.e. low-momentum) potentials saturate because of repulsive 3N forces
  - easier-to-use than, and in contrast to, historical high-resolution “hard-core” 2N potentials
- Meeting the empirical saturation point challenging; potentials with Δ degrees of freedom perform somewhat better
- First steps taken to explore nuclear matter from low to high densities
  - links chiral effective field theory to perturbative QCD via functional renormalization group
  - uncertainties still appreciable

Nuclear Structure

Energy scales and relevant degrees of freedom

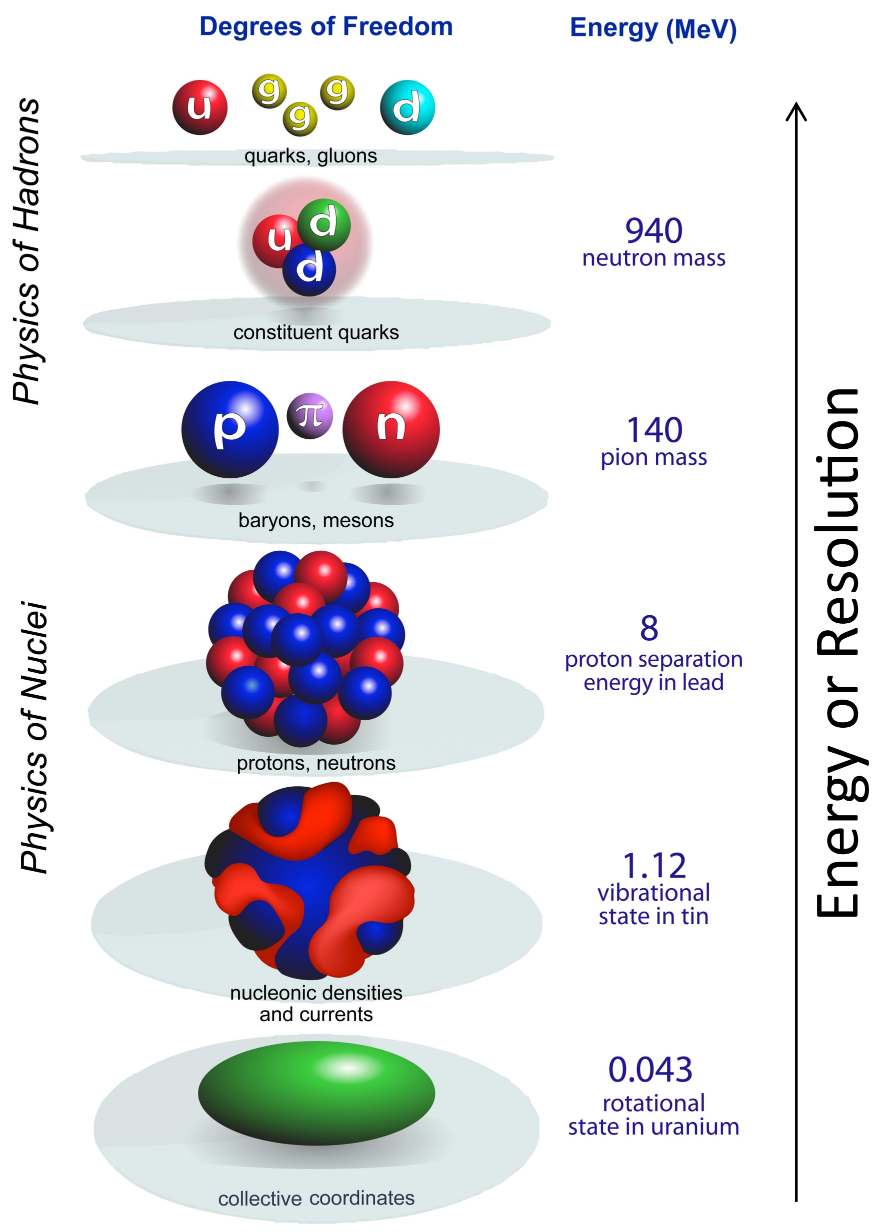
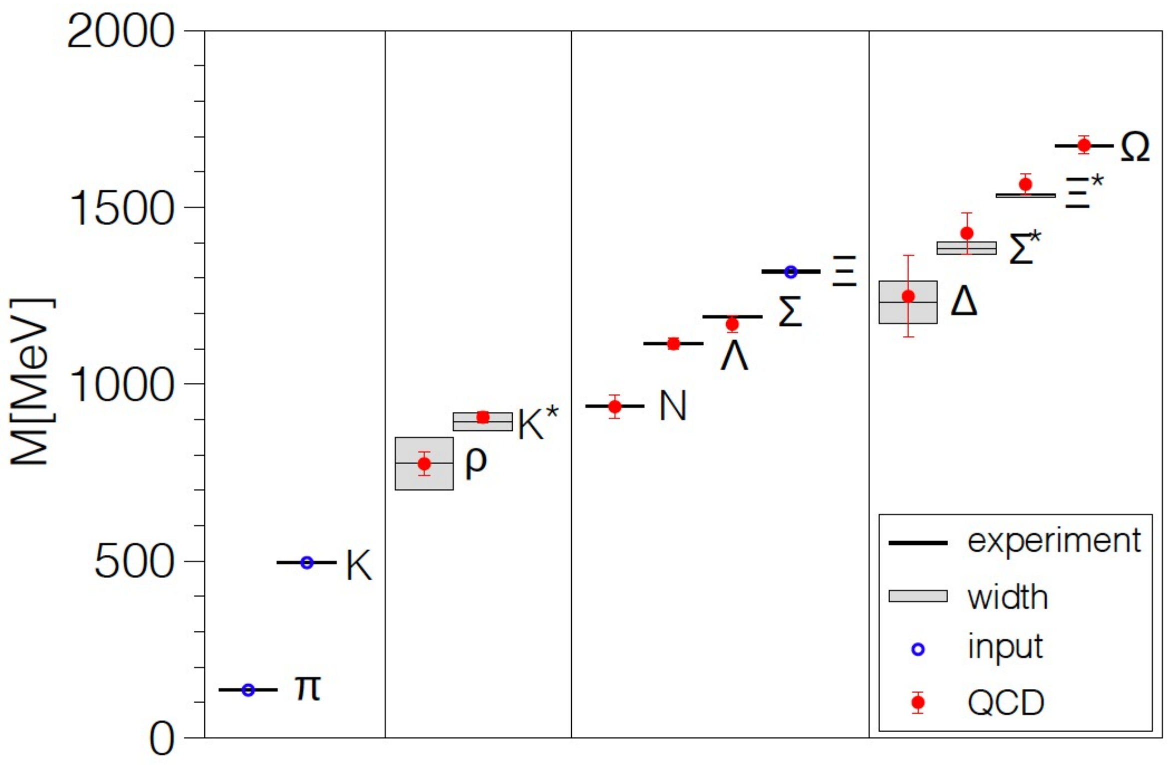


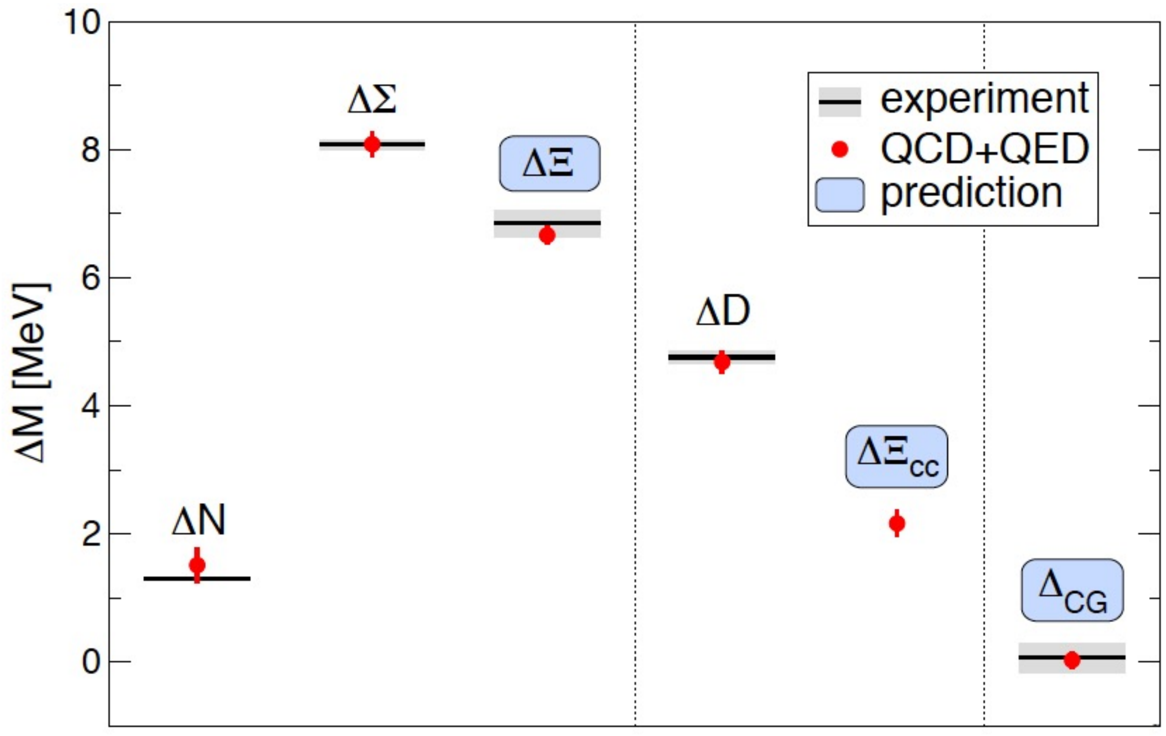
Fig.: Bertsch, Dean, Nazarewicz (2007)

- Physics of atomic nuclei spans several orders of magnitude
- Scales are well separated
- Which degrees of freedom are active depends on the resolution scale
- Many opportunities to construct effective field theories!

Precision computations from lattice QCD



Hadron mass spectrum from lattice QCD.
Dürr et al., Science (2009); arXiv:0906.3599



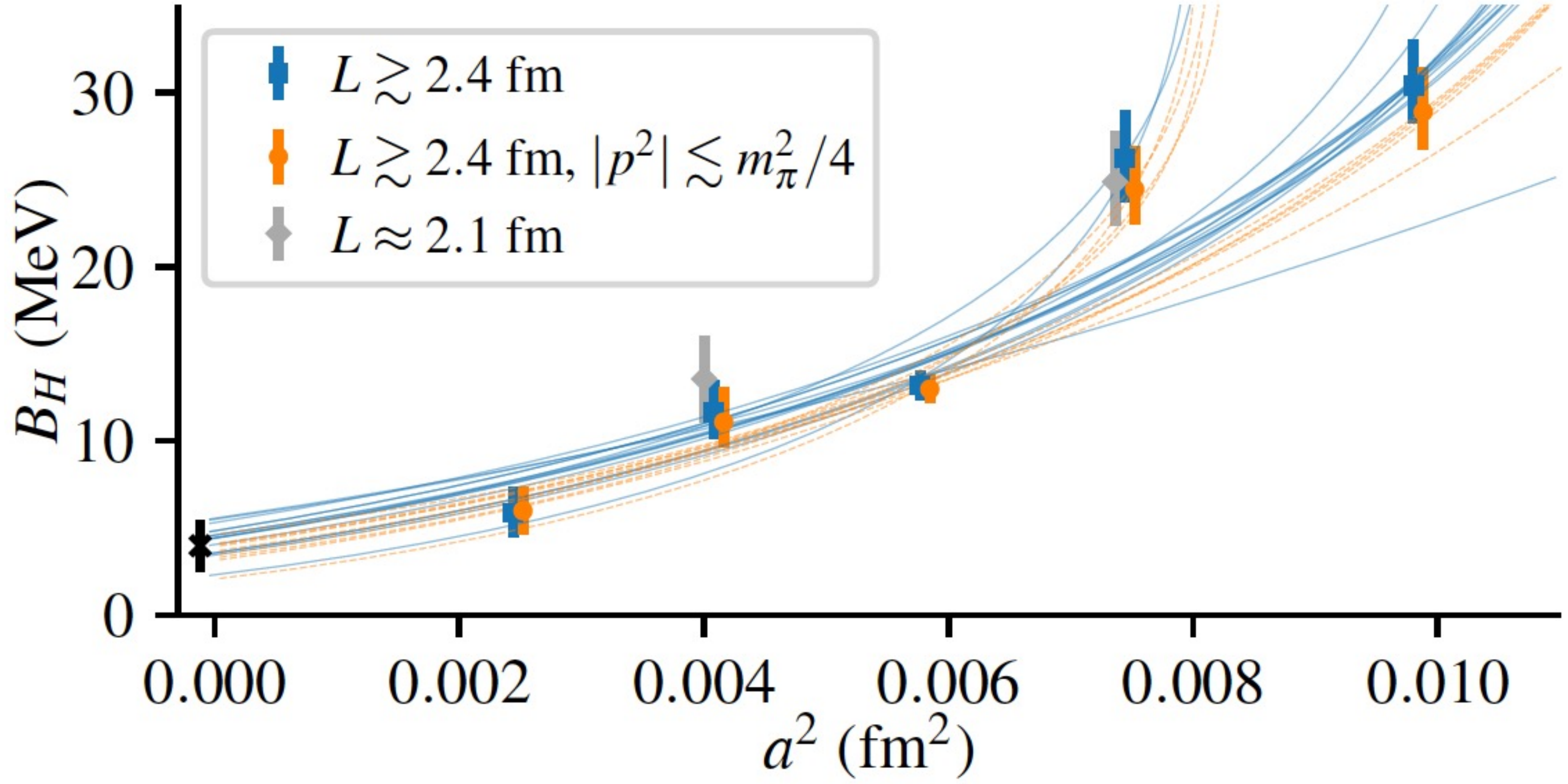
Proton-neutron mass splittings from lattice QCD & QED.
Borsanyi et al., Science (2015); arXiv:1406.4088

Lattice QCD very precise for hadrons, but what about nuclei as bound states of hadrons?

Towards Lattice QCD computations of hadron bound states

H-baryon, hypothetical six-quark bound state *uuddss*, computed at $m_\pi = m_K = 420$ MeV

$a =$ lattice spacing; $B_H =$ H-baryon binding energy



Challenges:

- Continuum limit ✓
- Physical meson masses ✗

$$B_H = 3.97 \pm 1.16 \pm 0.86 \text{ MeV}$$

Jeremy R. Green, Andrew D. Hanlon, Parikshit M. Junnarkar, Hartmut Wittig, arXiv: 2103.01054

Computing nuclei to QCD

The computation of light nuclei from lattice QCD is controversial, see discussion in [Drischler, Haxton, McElvain, Mereghetti, Nicholson, Vranas, Walker-Loud, arXiv:1910.07961]

It is not clear, whether nuclear binding increases with increasing pion mass [see, e.g., NPLQCD collaboration] or whether it decreases [see, e.g., HAL QCD collaboration].

Theorists are ready to match effective field theories to lattice QCD data, and compute nuclei as heavy as ^{40}Ca, see [Barnea et al, Phys. Rev. Lett. (2015); Contessi et al, Phys. Lett. B (2017); C. McIlroy et al Phys. Rev C (2018); Bansal *et al*., Phys. Rev. C 98, 054301 (2018)]

Enter effective field theories …

Energy scales and relevant degrees of freedom

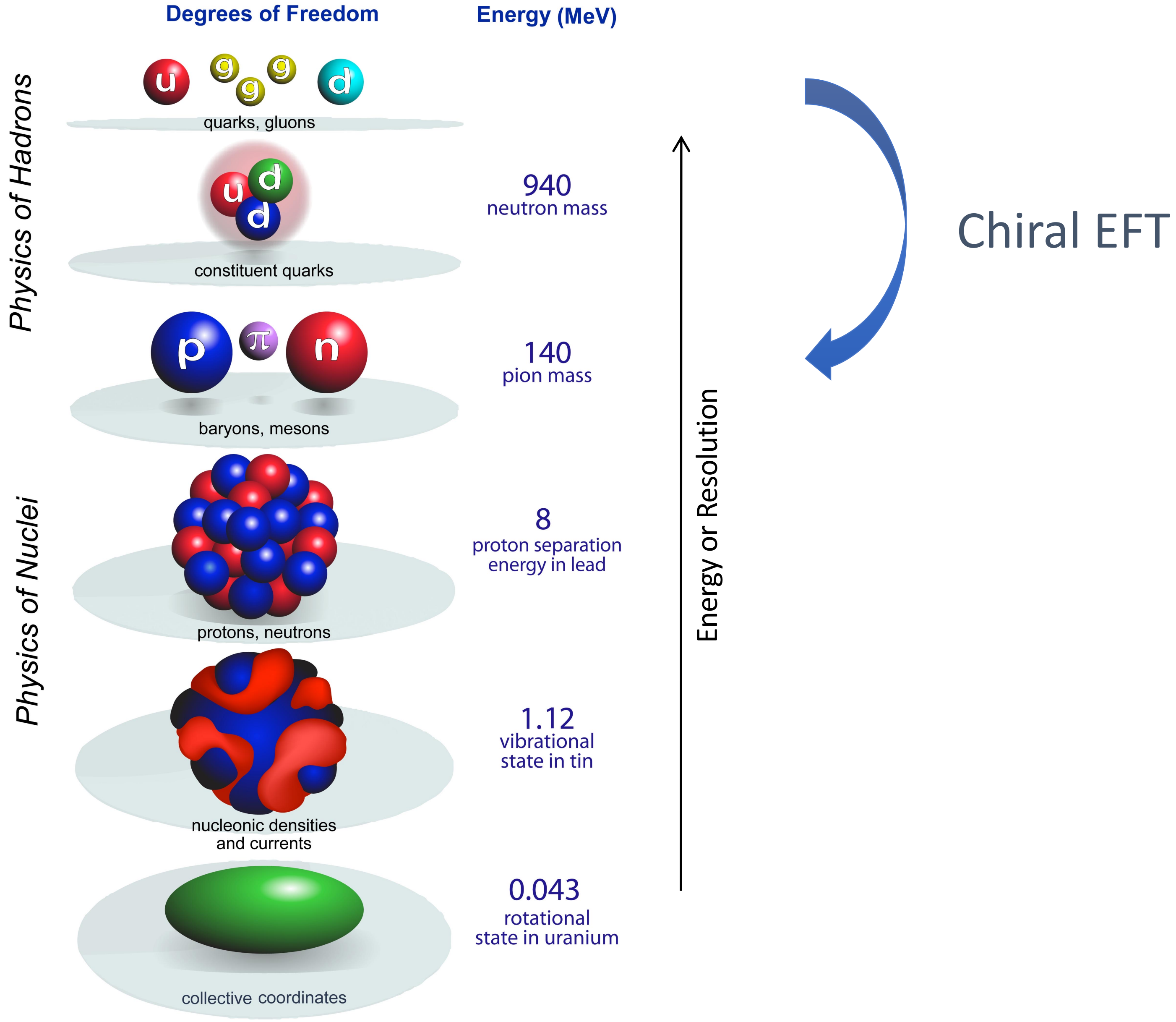
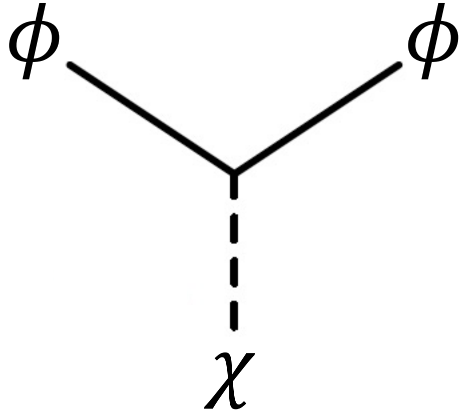


Fig.: Bertsch, Dean, Nazarewicz (2007)

EFT – Sophisticated "Fake it till you make it"

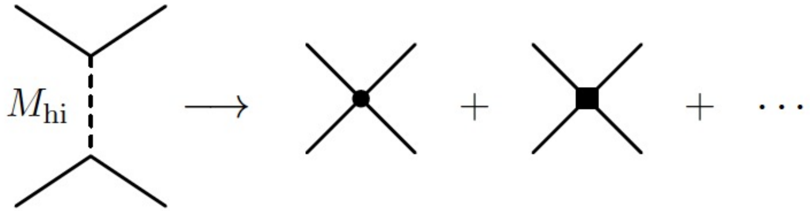
Fields ϕ, χ. Interaction via exchange of a heavy meson χ with mass M_{hi}



$$\mathscr{L}_{\text{int}} = g\left(\chi^\dagger \phi\phi + \phi^\dagger\phi^\dagger\chi\right)$$

Amplitude at small momenta $q \ll M_{hi}$ (introduce separation of scales)

$$T \sim \frac{g^2}{M_{\text{hi}}^2 - q^2} = \frac{g^2}{M_{\text{hi}}^2} + \frac{g^2\, q^2}{M_{\text{hi}}^4} + \cdots$$



Result: A systematic improvable theory, valid at low momenta $q \ll M_{hi}$, in powers of q/M_{hi}

$$\mathscr{L}_{\text{int}} = -\frac{C_0}{4}\left(\phi^\dagger\phi\right)^2 - \frac{C_2}{4}\left(\nabla(\phi^\dagger\phi)\right)^2 + \cdots$$

Nice pedagogical lectures on this topic: H.-W. Hammer, Sebastian König, arXiv:1610.02961

Chiral effective field theory

[Weinberg; van Kolck; Kaiser, Weise; Epelbaum, Gloeckle, Krebs, Meissner; Entem & Machleidt; Ekström, …]

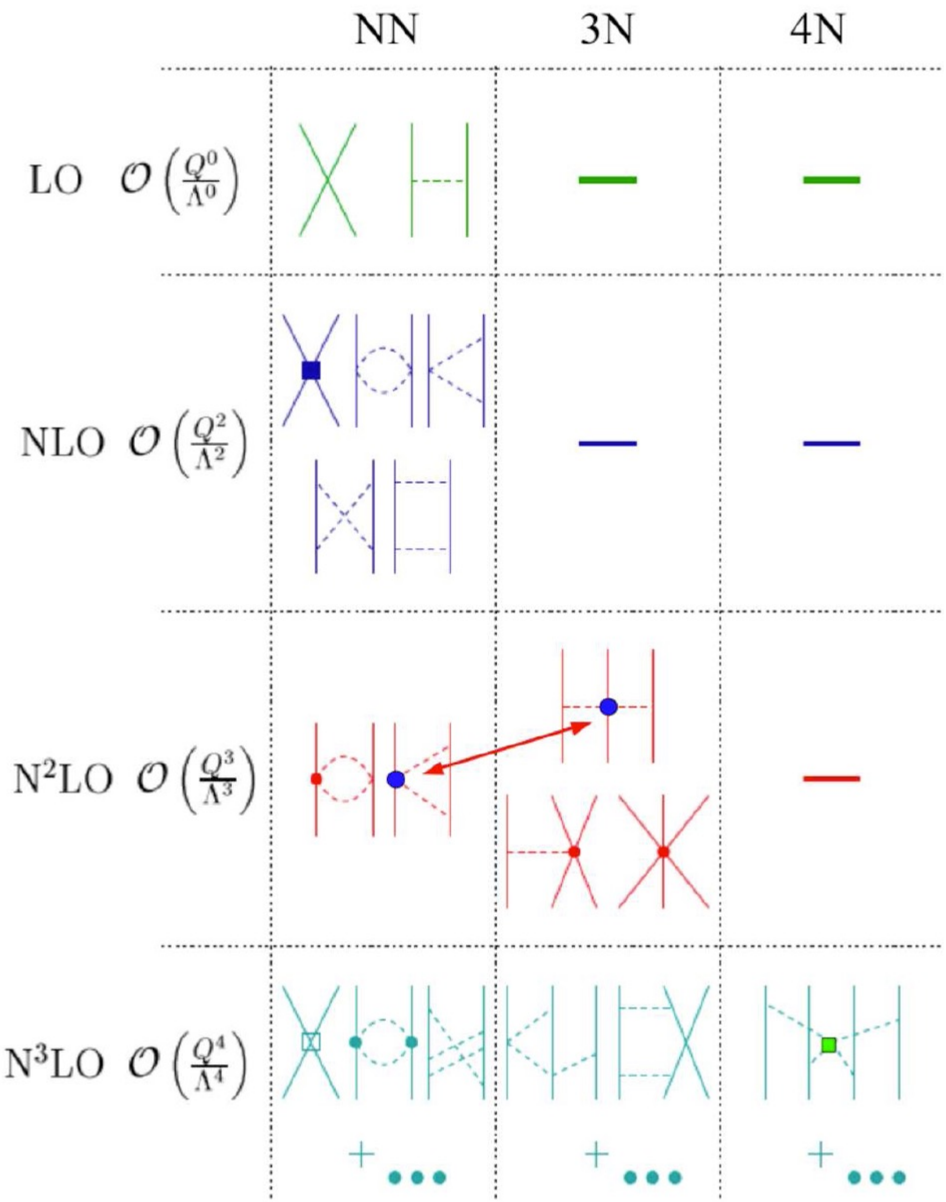
- The pion is the Nambu-Goldstone boson of the spontaneously broken chiral symmetry → Alejandro Ayala’s lecture
  - Severely constrains the form of the nucleon-pion interaction ☺
  - Interactions between Nambu-Goldstone bosons are weak ☺
  - Provides the connection to QCD via chiral perturbation theory
- Pion exchange constitutes the long-range part of the nuclear force
- Everything else (presumably unknown/short ranged) is captured by contact interactions and derivatives thereof
- Power counting orders contributions

One-pion exchange potential: $V(q) = -\frac{g_A^2}{4f_\pi^2}\frac{(\sigma_1\cdot q)\,(\sigma_2\cdot q)}{m_\pi^2+q^2}\tau_1\cdot\tau_2$

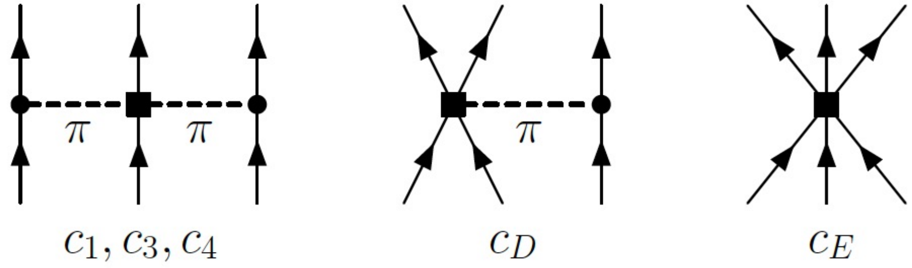
There are clouds in paradise (e.g. questions regarding the power counting), but these lectures will not dwell on them

Chiral effective field theory

[Weinberg; van Kolck; Epelbaum, Gloeckle, Krebs, Meissner; Entem & Machleidt; Ekström, …]



Q: Why three-nucleon forces?



Chiral effective field theory

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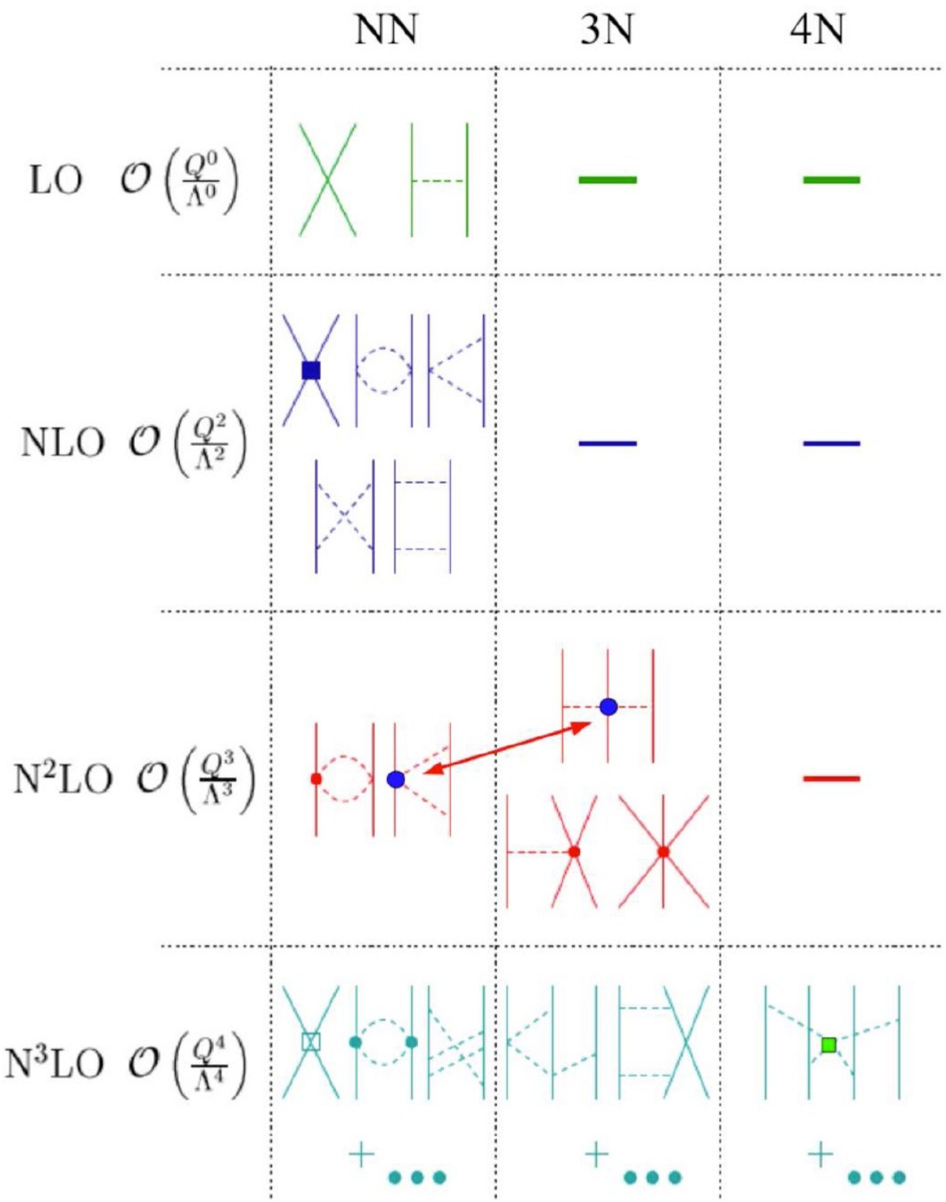

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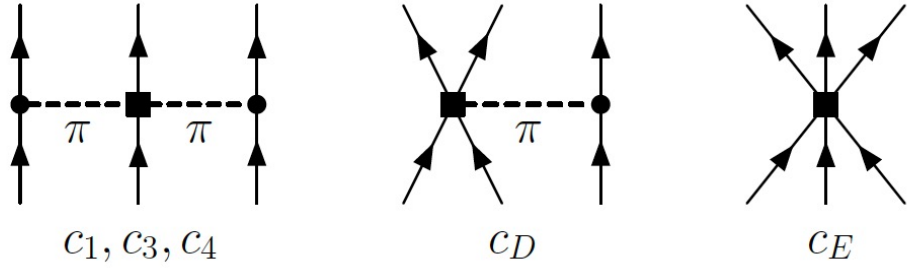


A1: In an EFT, one writes down everything that is allowed by symmetries and then orders according to a power counting

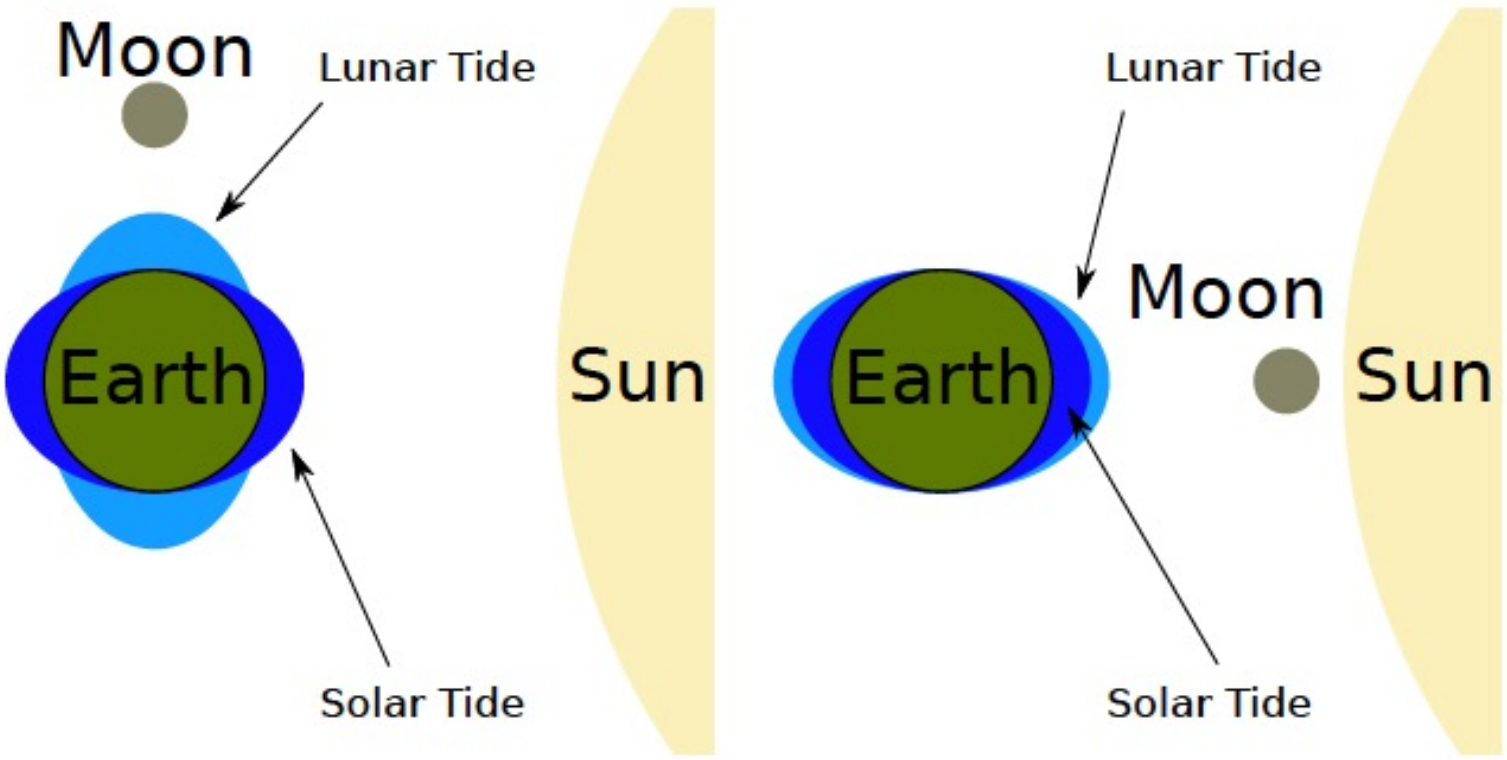
A2: Nucleons are composite particles, and many-body forces arise when treating them as point particles, i.e. when removing high-momentum "stiff" degrees of freedom

A3: all of the above

Understanding three-body forces

Let us aim at a description of the Earth-Moon-Sun system using point masses only, i.e. we have removed all "stiff" degrees of freedom corresponding to deformations of these finite objects.

What are the consequences?

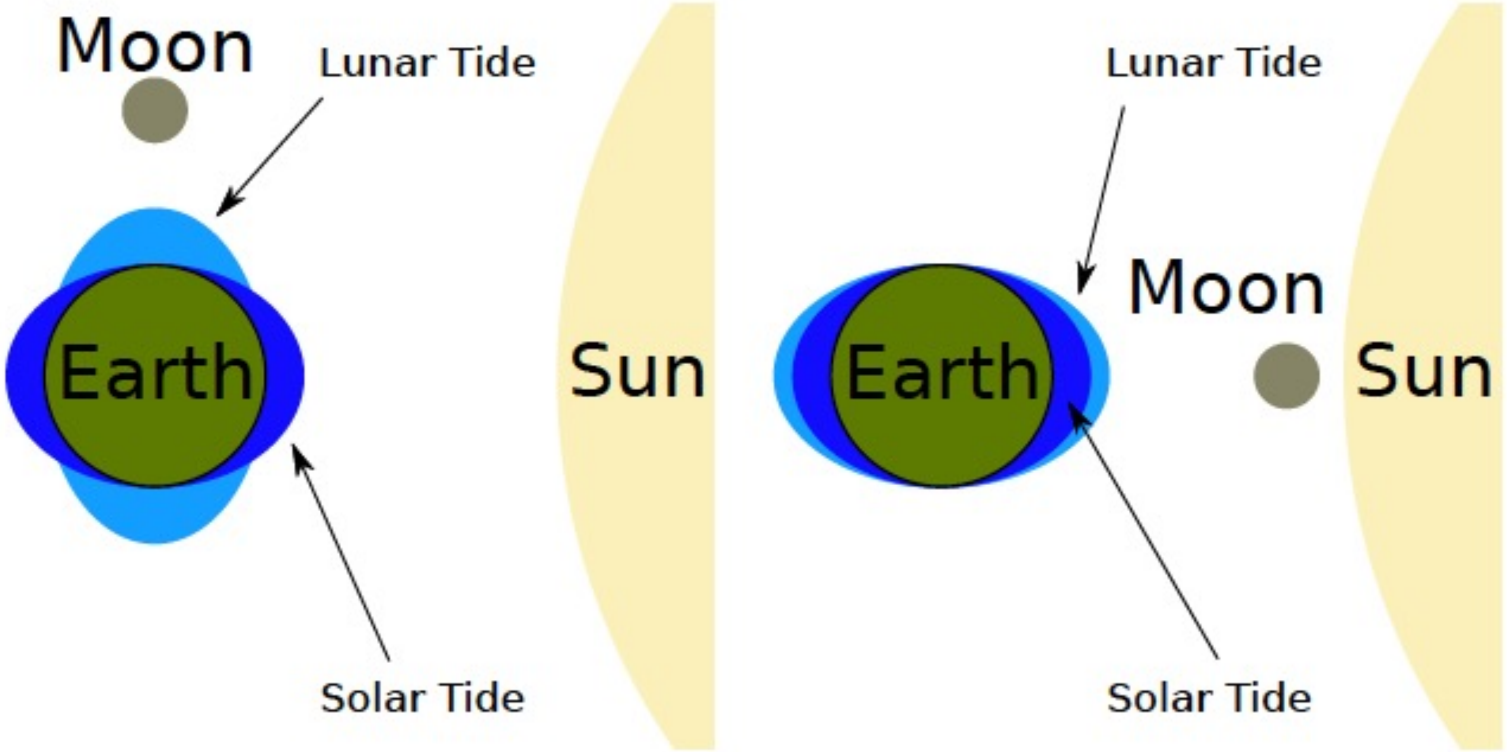


Understanding three-body forces

Let us aim at a description of the Earth-Moon-Sun system using point masses only, i.e. we have removed all "stiff" degrees of freedom corresponding to deformations of these finite objects.

What are the consequences?

1. Newton's 1/r potential law between two bodies gets modified due to a multipole expansion, i.e. $1/r^2$, $1/r^3$ terms enter.
2. Three-body forces arise due to the removal of degrees of freedom



Three nucleon forces

- How do 3NFs arise in nuclear physics?
- What are omitted degrees of freedom? Can you draw diagrams that explain the origin of three nucleon forces?

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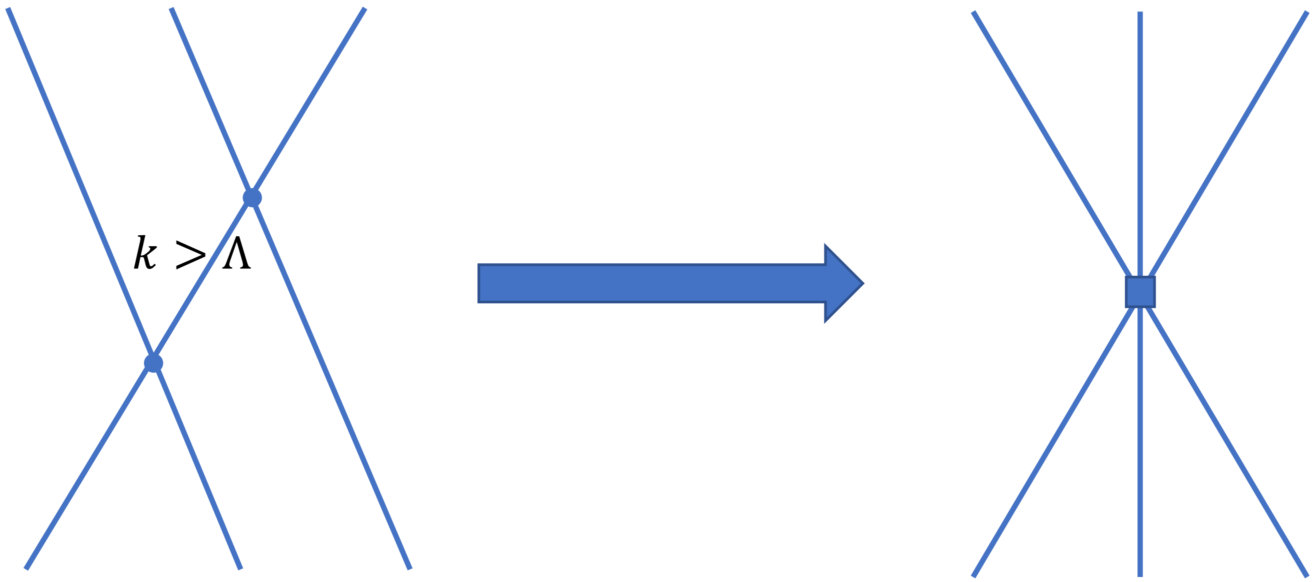
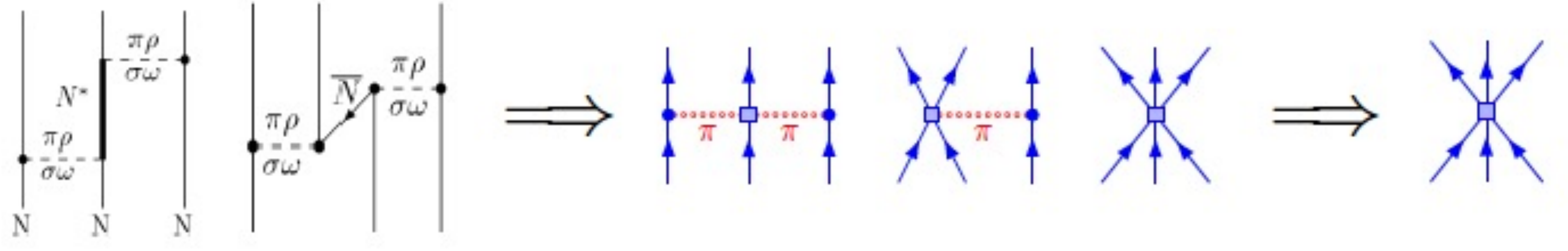


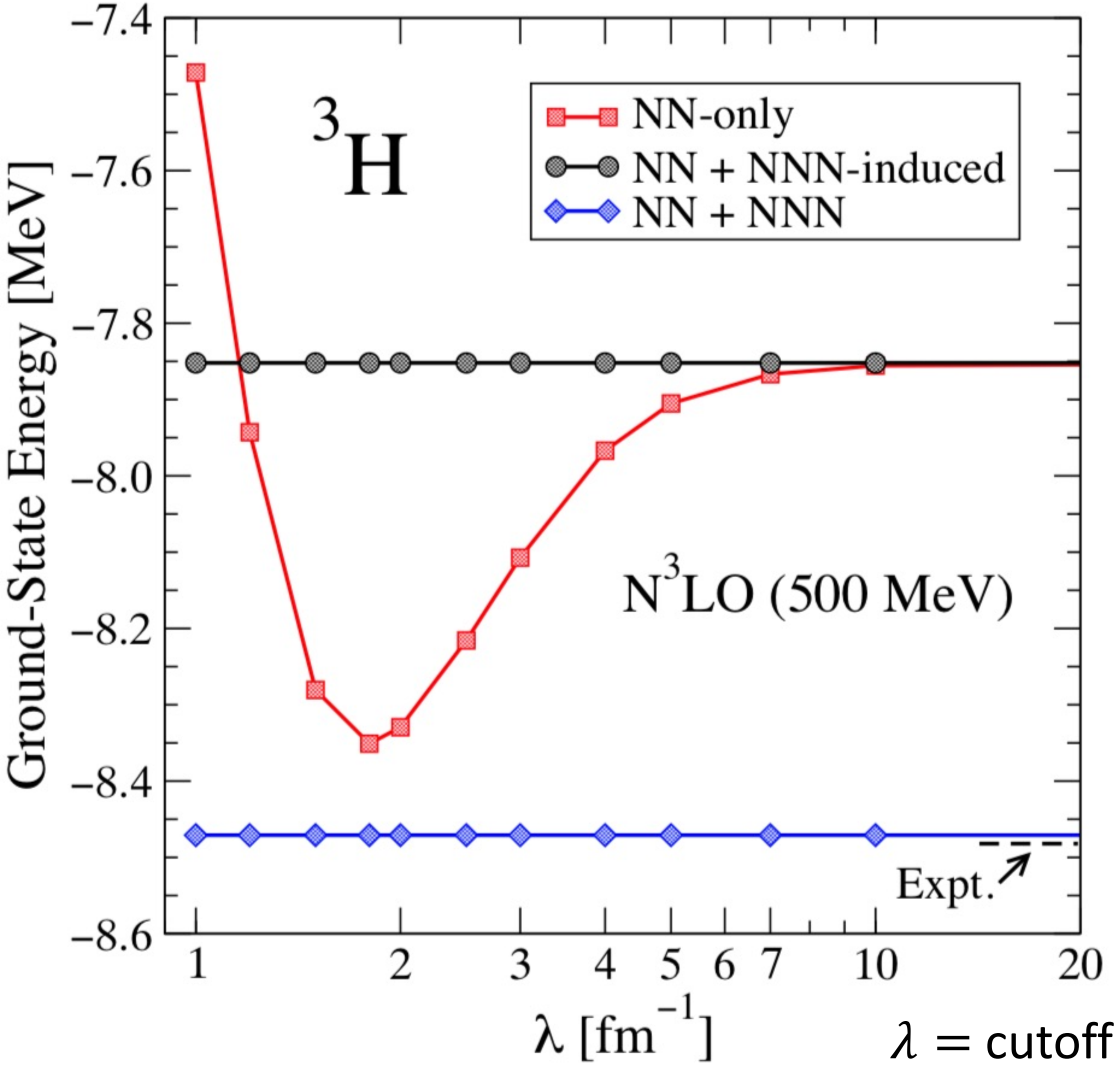
Removal (or omission) of high-energy degrees of freedom leads to new interactions.

3NFs in a theory with pions

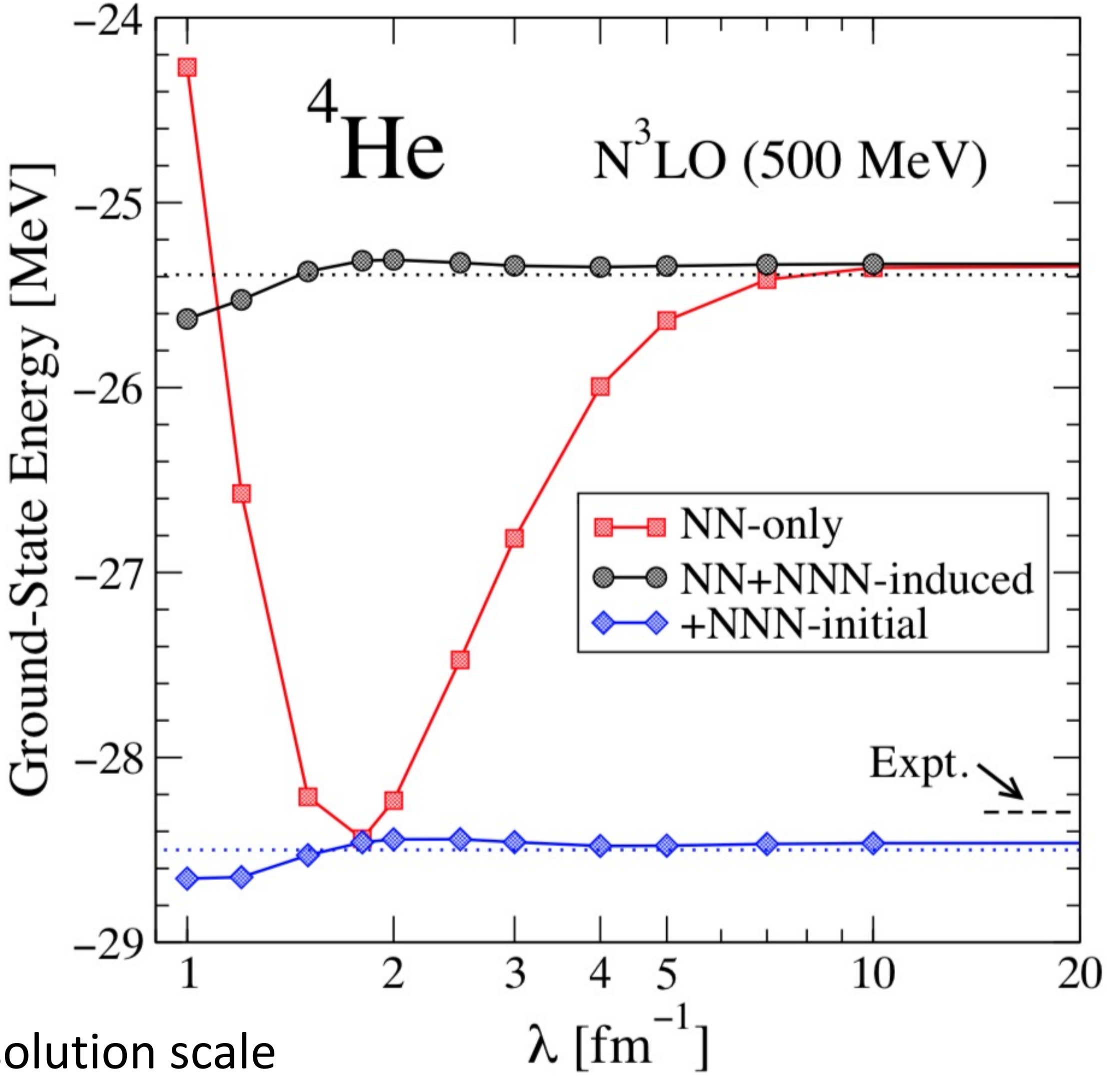


The essential rationale is:
Nuclei are extended objects, i.e. they have intrinsic degrees of freedom. They have excited states, can be deformed etc.
We treat nuclei as point particles, i.e. we neglect their intrinsic structure. While this is justified at low energies (low resolution), it comes with a price tag of 3NFs, 4NFs, …

RG Evolution of Nuclear Many-Body Forces



λ = cutoff or resolution scale



Jurgenson, Furnstahl, Navratil, Phys. Rev. Lett. 103, 082501 (2009); arXiv:0905.1873

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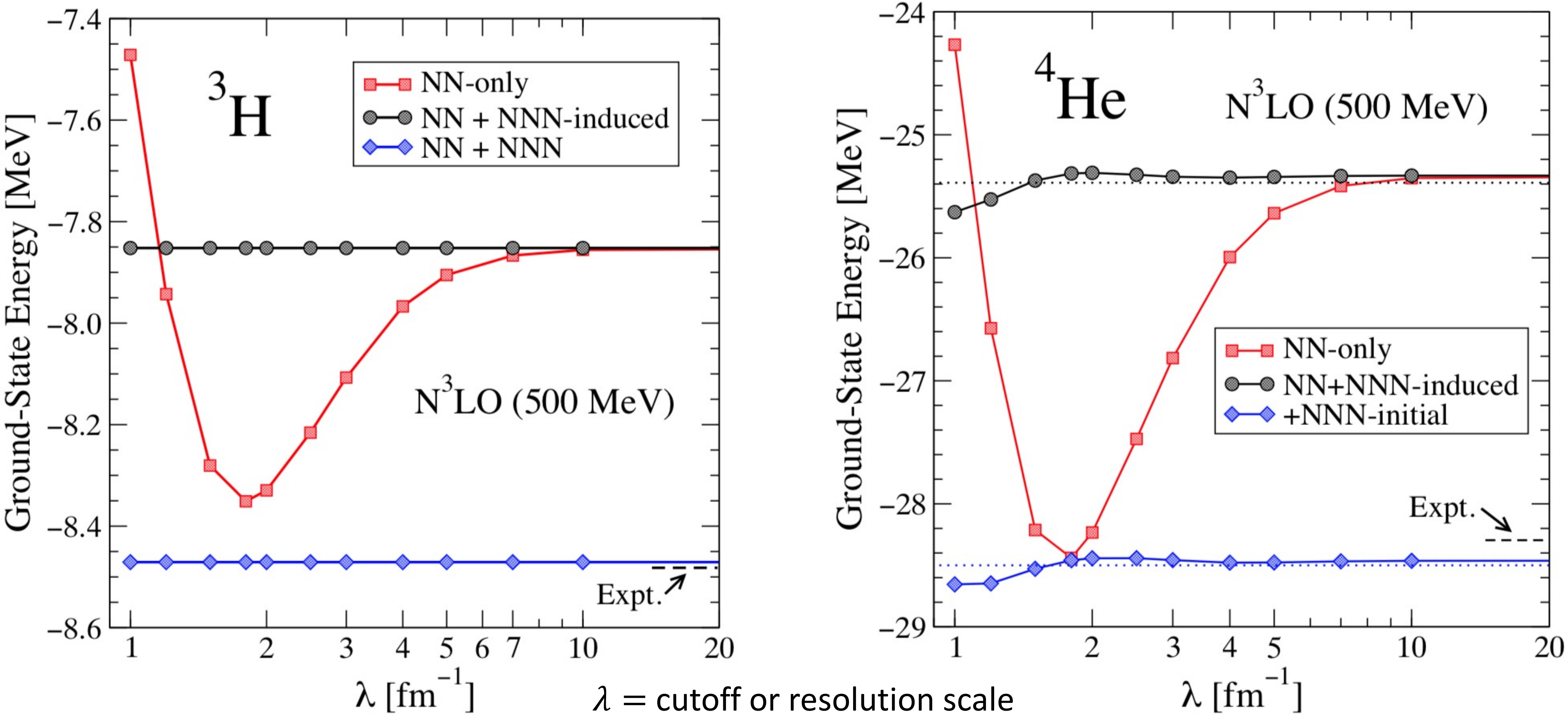

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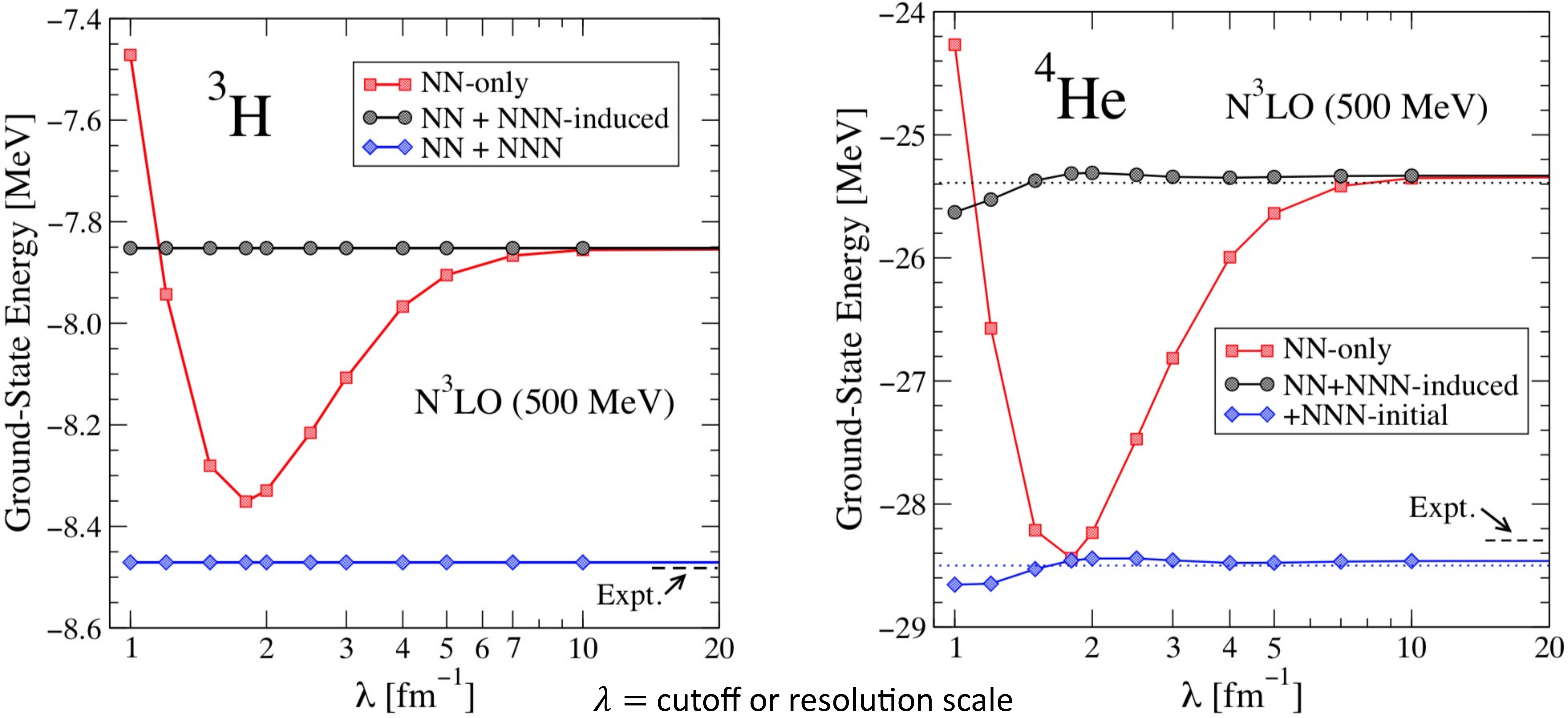
Q: How large are (omitted in this calculation) four-nucleon forces?

RG Evolution of Nuclear Many-Body Forces



λ = cutoff or resolution scale

Q: How large are (omitted in this calculation) four-nucleon forces?

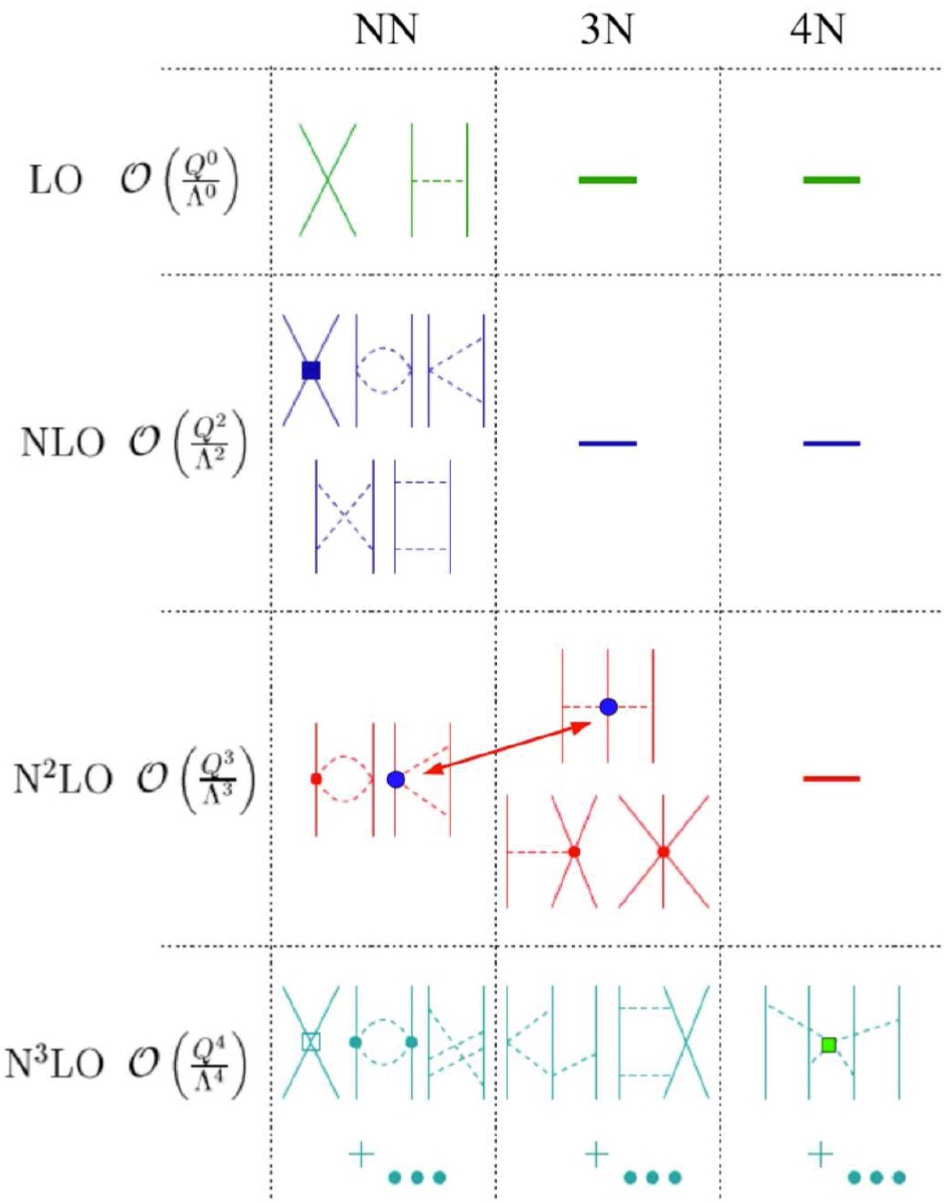
A: The four-body system does not meet data; difference is about 0.2 MeV ⟷ 1% of binding energy

Summary EFT Intro / three-nucleon forces

- Lattice QCD not yet there to compute nuclei
  - Even when that day arrives, the physical degrees of freedom are colorless hadrons
- Effective field theories can, in principle, be matched to QCD input
  - Meanwhile, we use data from nuclei
- Three-nucleon forces naturally arise as high-energy degrees of freedom are removed (“integrated out”)

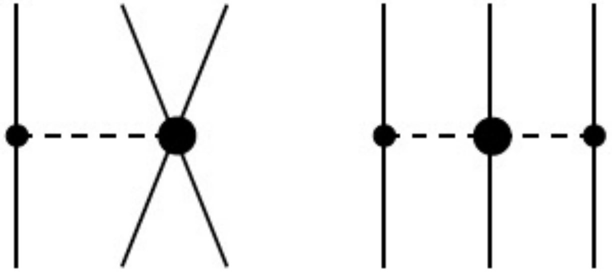
Chiral effective field theory: consistency of currents and interactions

[Weinberg; van Kolck; Epelbaum, Gloeckle, Krebs, Meissner; Entem & Machleidt; Ekström, …]



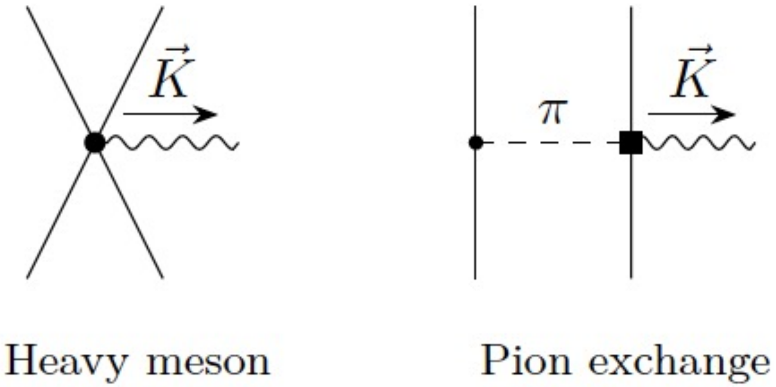
Effective field theories provide us with a consist formulation of

interactions



and

currents:



Three-body forces go hand in hand with two-body currents.

Consistency between Hamiltonians and currents
example: electromagnetic interactions

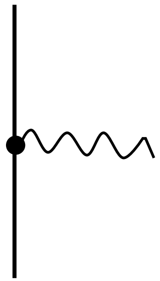
Heisenberg Eq. of motion $\frac{d\rho}{dt} = \frac{i}{\hbar}[H, \rho]$

Continuity equation $\frac{d\rho}{dt} = -\nabla \cdot j$

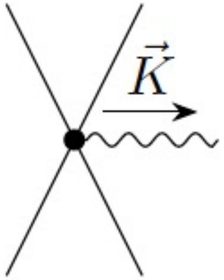
We see that Hamiltonians and currents must fulfill $\frac{i}{\hbar}[H, \rho] + \nabla \cdot j = 0$

As EFT Hamiltonians contain momentum-dependent interactions, this is a non-trivial constraint on the current operator

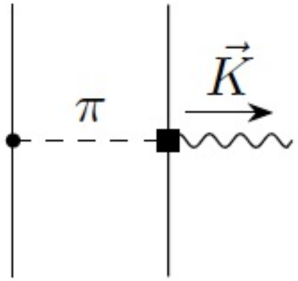
Leading order: 1-body current



Subleading corrections: 2-body currents a.k.a. “meson-exchange currents”



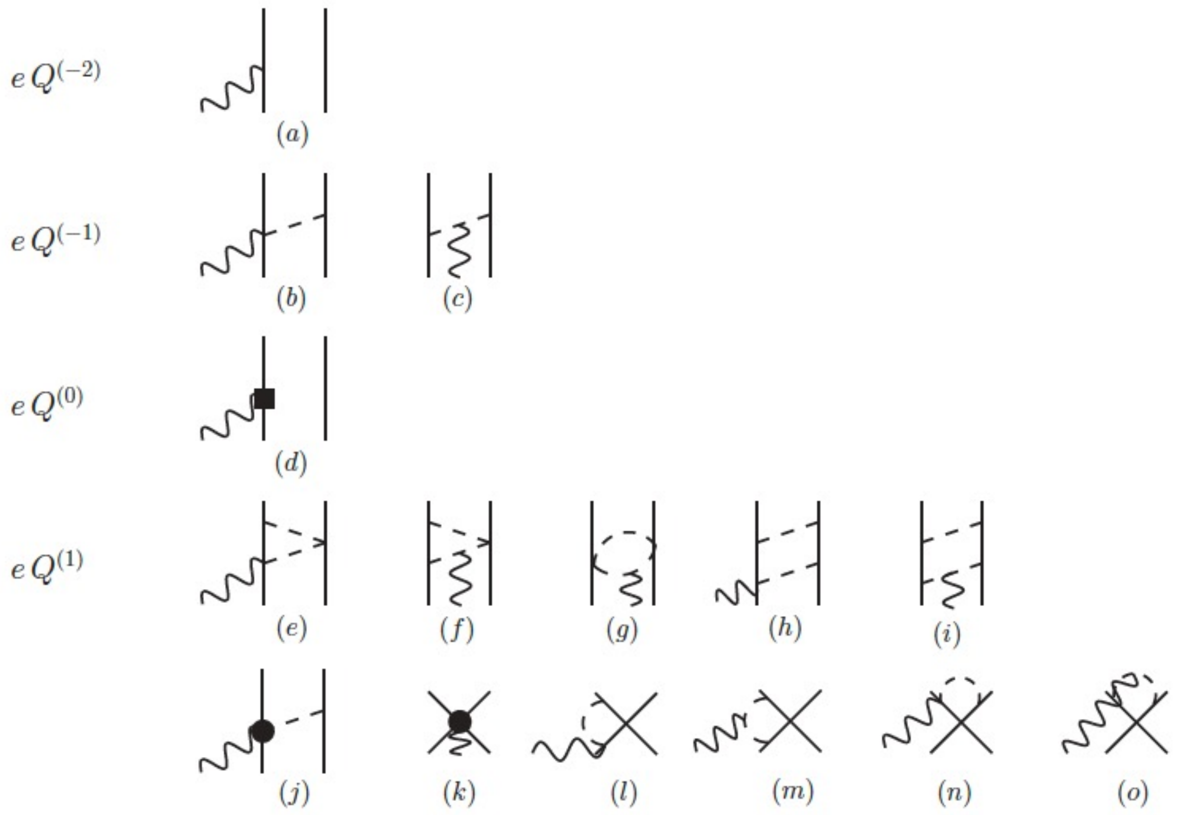
Heavy meson exchange
c_D

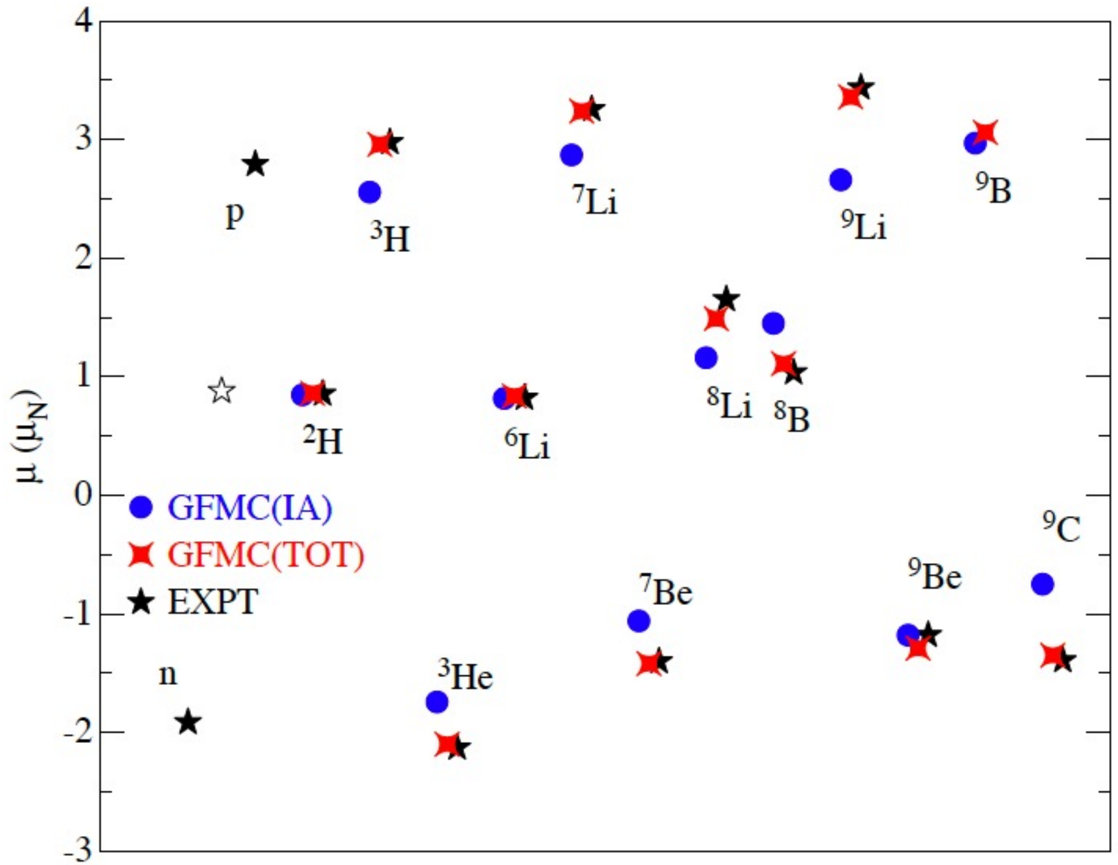


Pion exchange
$c_3,\ c_4$

Role of two-body currents: magnetic moments

The magnetic moment is a short-range operator, so we expect significant contributions from two-body currents



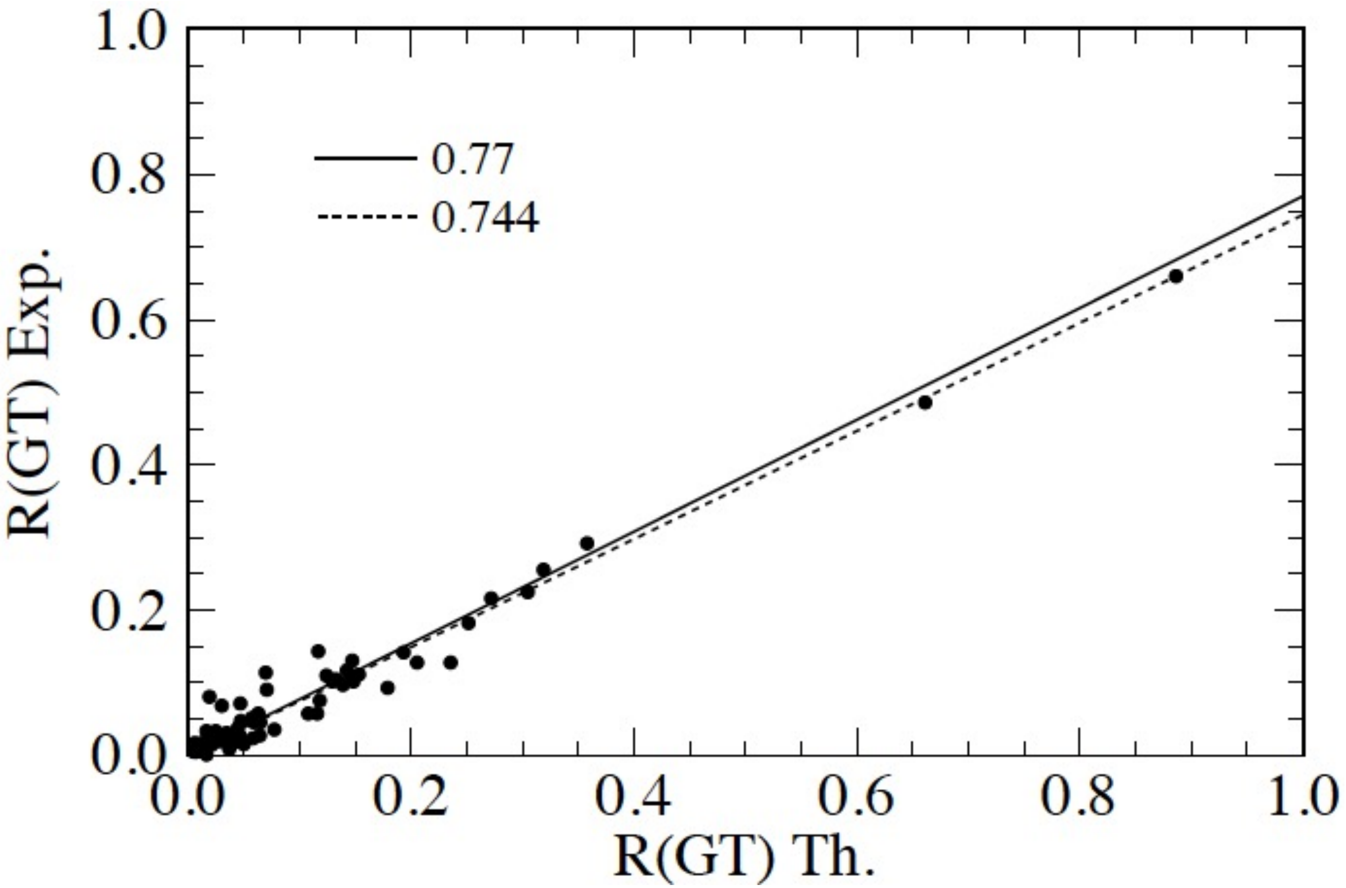


S. Pastore, Steven C. Pieper, R. Schiavilla, R. B. Wiringa, Phys. Rev. C 87, 035503 (2013); arXiv:1212.3375

Two-body currents solve 50-year-old puzzle of quenched β-decays

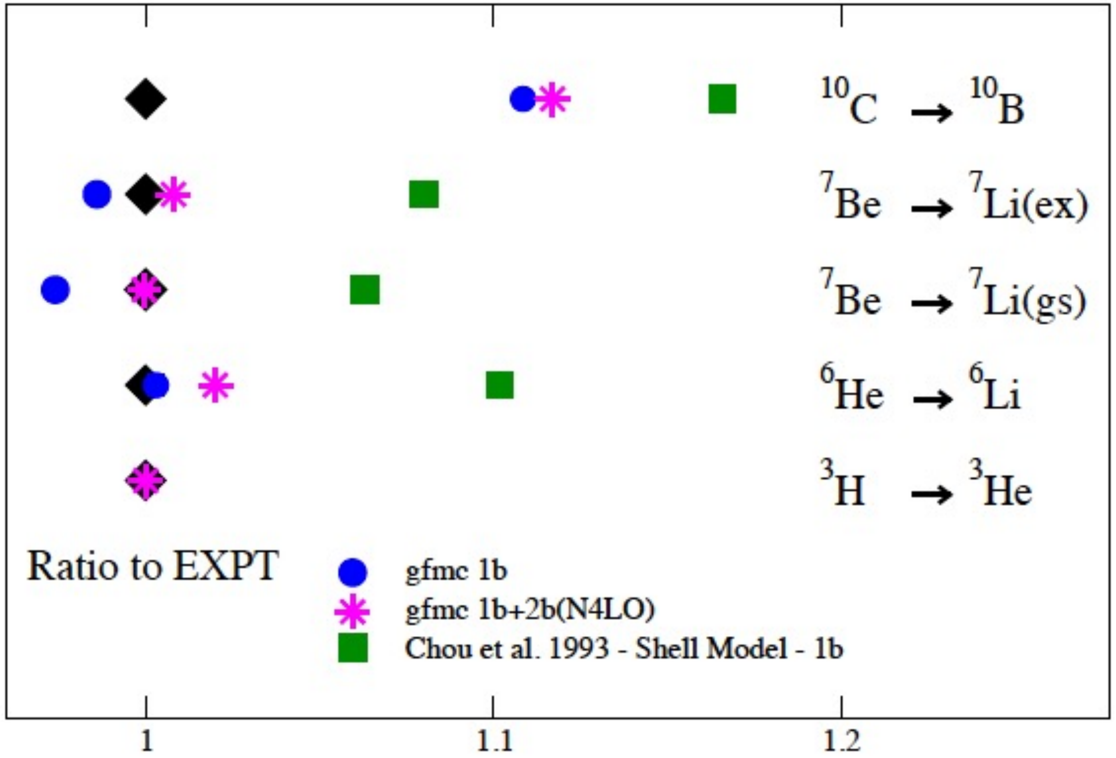
Puzzle: The strengths of Gamow-Teller transitions (operator $\propto g_A \vec{\sigma}\tau^{\pm}$) in nuclei are smaller ("quenched") than what is expected from the β-decay of the free neutron.

- Wilkinson (1973): quenching factor $q^2 \approx 0.90$ for nuclei with $A = 17 \ldots 21$
- Brown & Wildenthal (1985): quenching factor $q^2 \approx 0.77$ for nuclei with $A = 17 \ldots 40$
- Martinez-Pinedo et al. (1996): quenching factor $q^2 \approx 0.74$ for nuclei with $A = 40 \ldots 60$

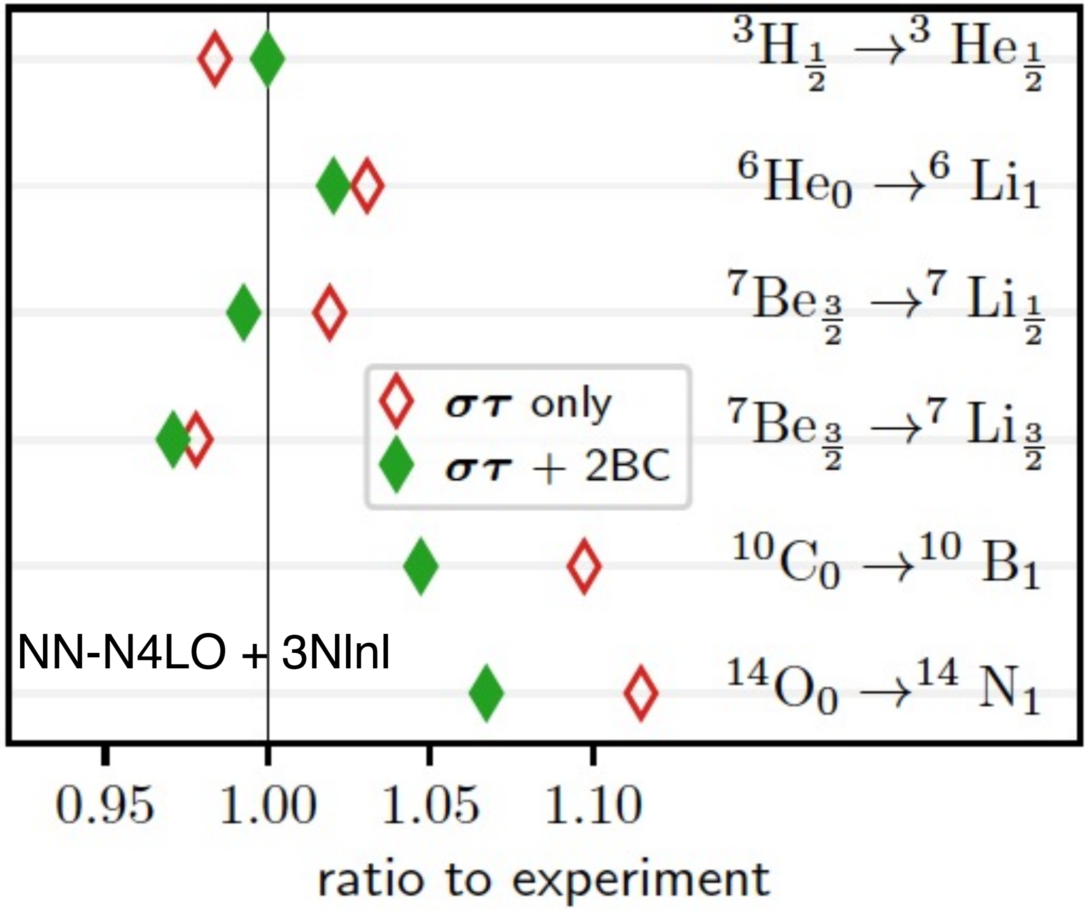


Martinez-Pinedo, Poves, Caurier, and Zuker, Phys. Rev. C **53**, R2602 (1996)

β decays in light nuclei, including two-body currents



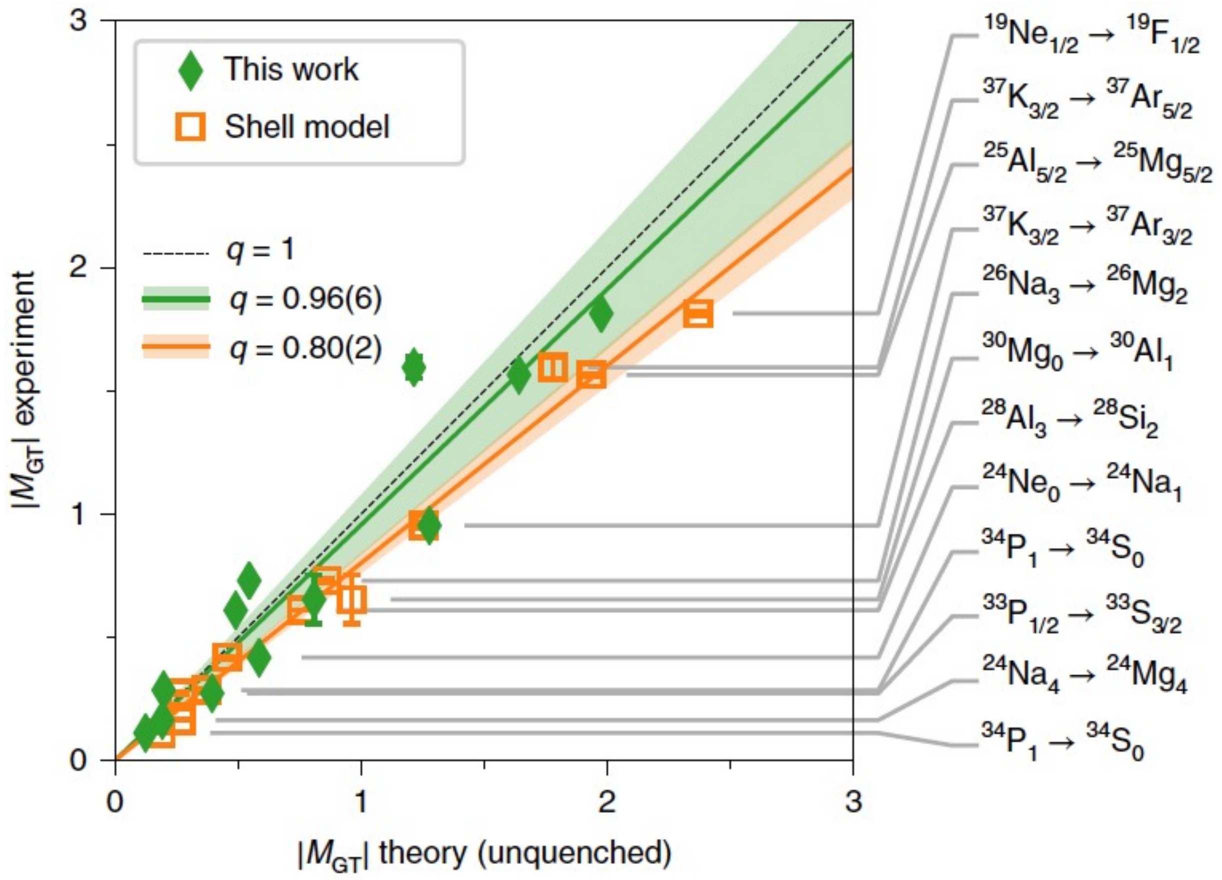
Pastore, Baroni, Carlson, Gandolfi, Pieper, Schiavilla & Wiringa, Phys. Rev. C (2018)

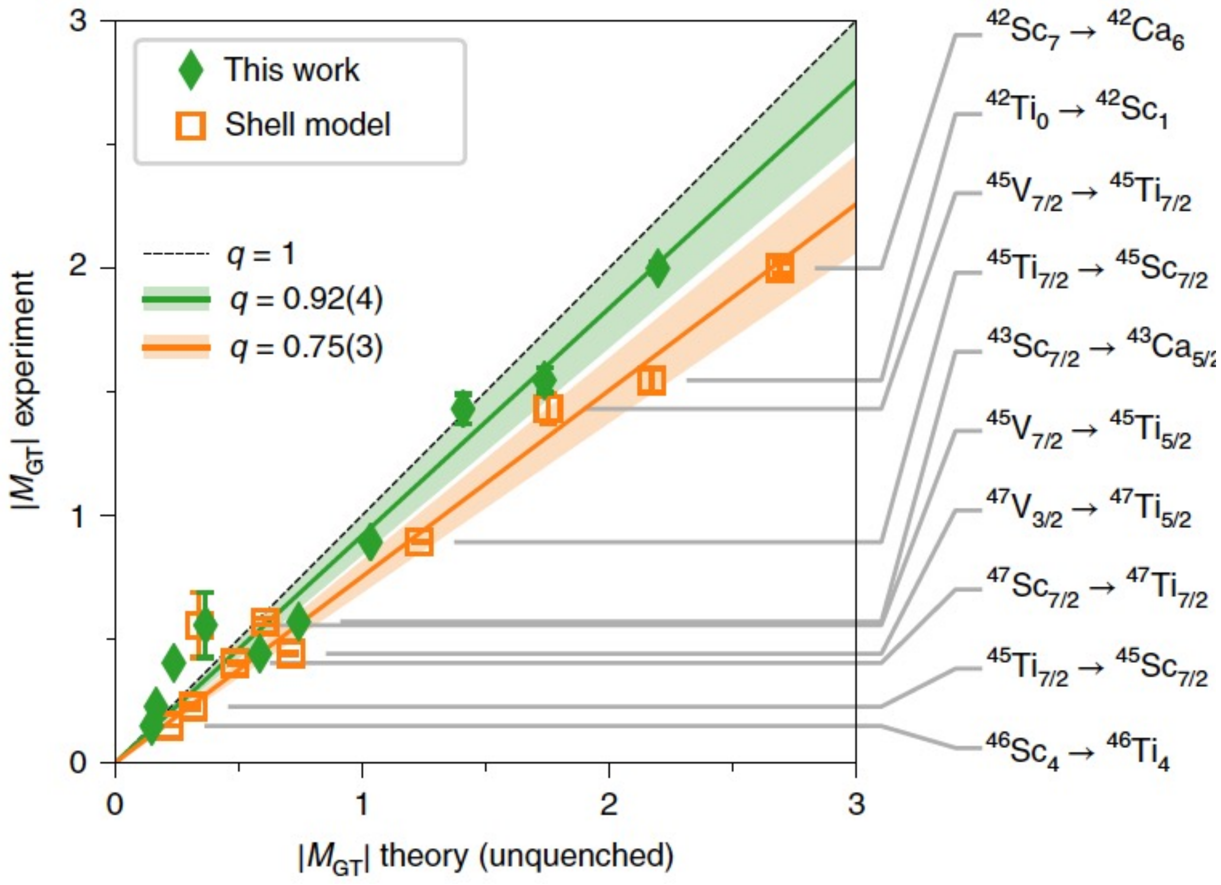


Gysbers, Hagen, Holt, Jansen, Morris, Navratil, TP, Quaglioni, Schwenk, Stroberg & Wendt, Nature Physics (2019)

In light nuclei, two-body currents play a smaller role;
some tension between quantum Monte Carlo and no-core shell model computations, though.

β decays in medium-mass nuclei, including two-body currents

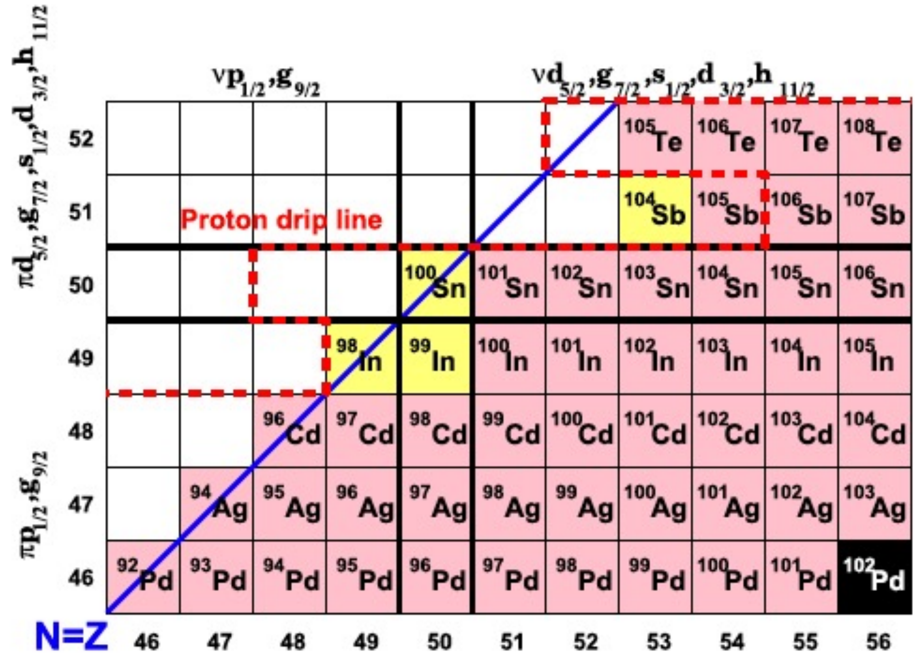




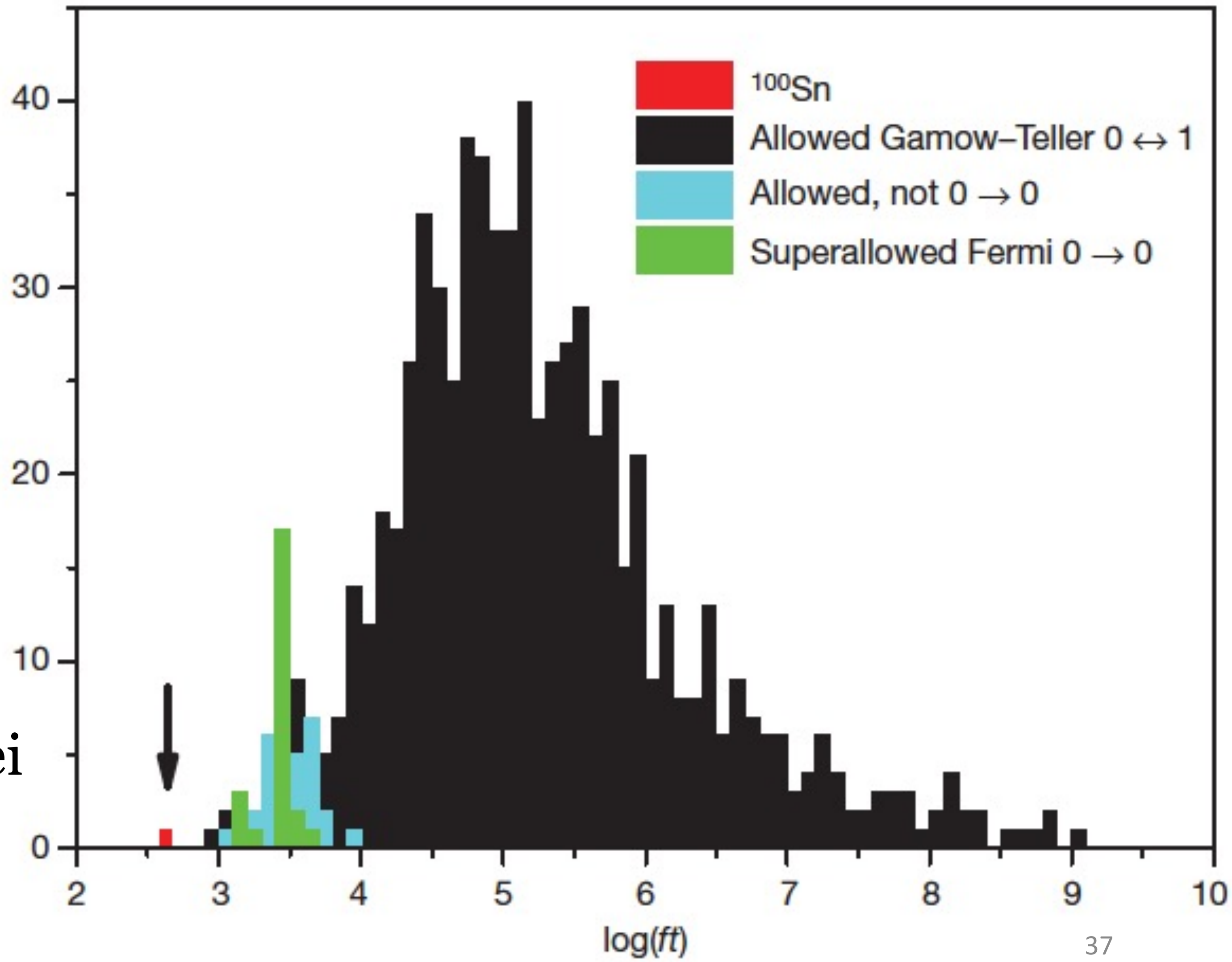
IMSRG computations with NN-N^{4}LO + 3Nlnl interaction

Gysbers, Hagen, Holt, Jansen, Morris, Navratil, TP, Quaglioni, Schwenk, Stroberg & Wendt, Nature Physics (2019); arXiv:1212.3375

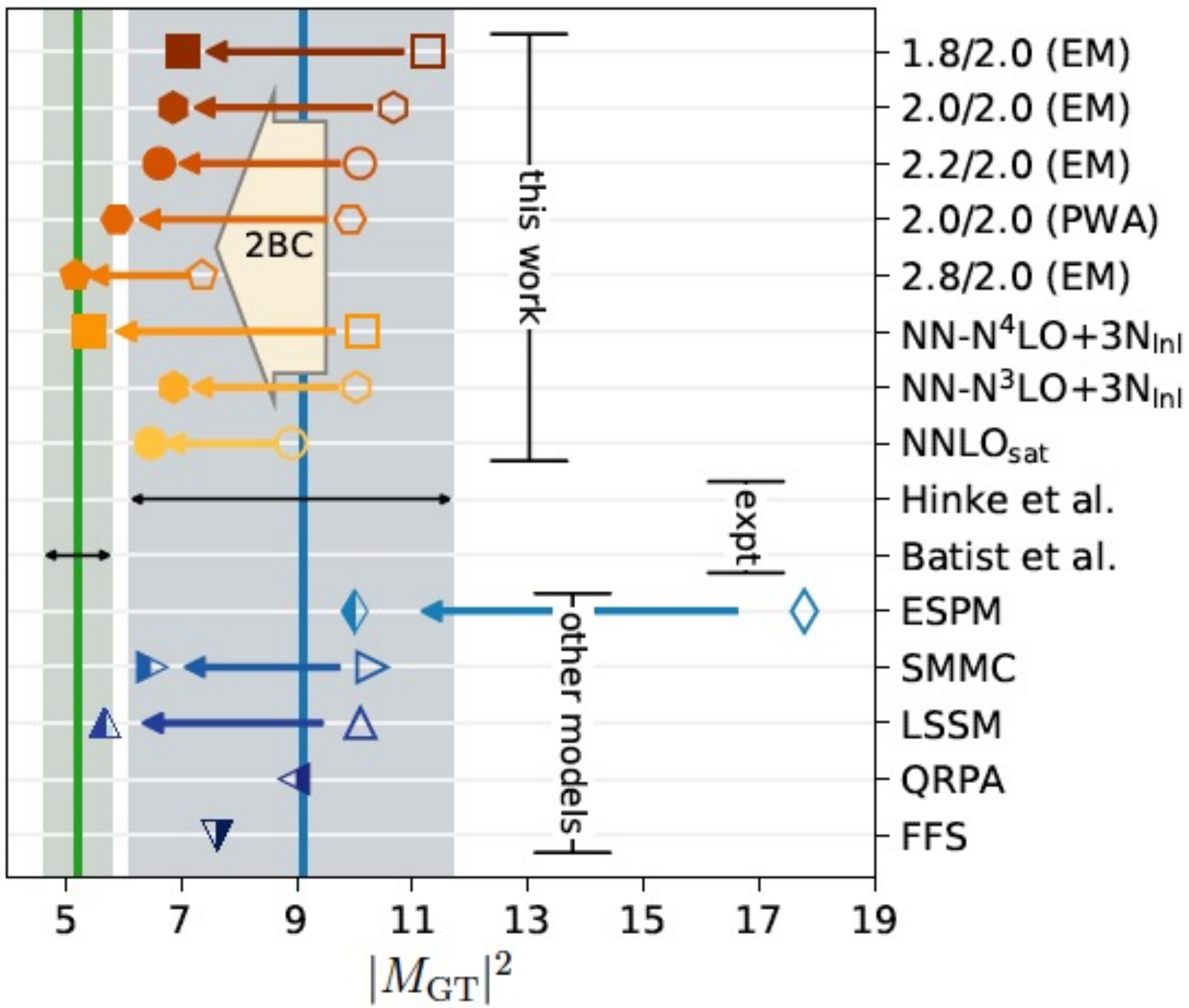
β decay of ^{100}Sn



^{100}Sn has strongest Gamow-Teller matrix-element strength of all nuclei [Hinke et al., Nature (2012)]



β decay of ^{100}Sn, including two-body currents



Coupled-cluster computations based on various potentials from chiral EFT

Open symbols: no two-body currents

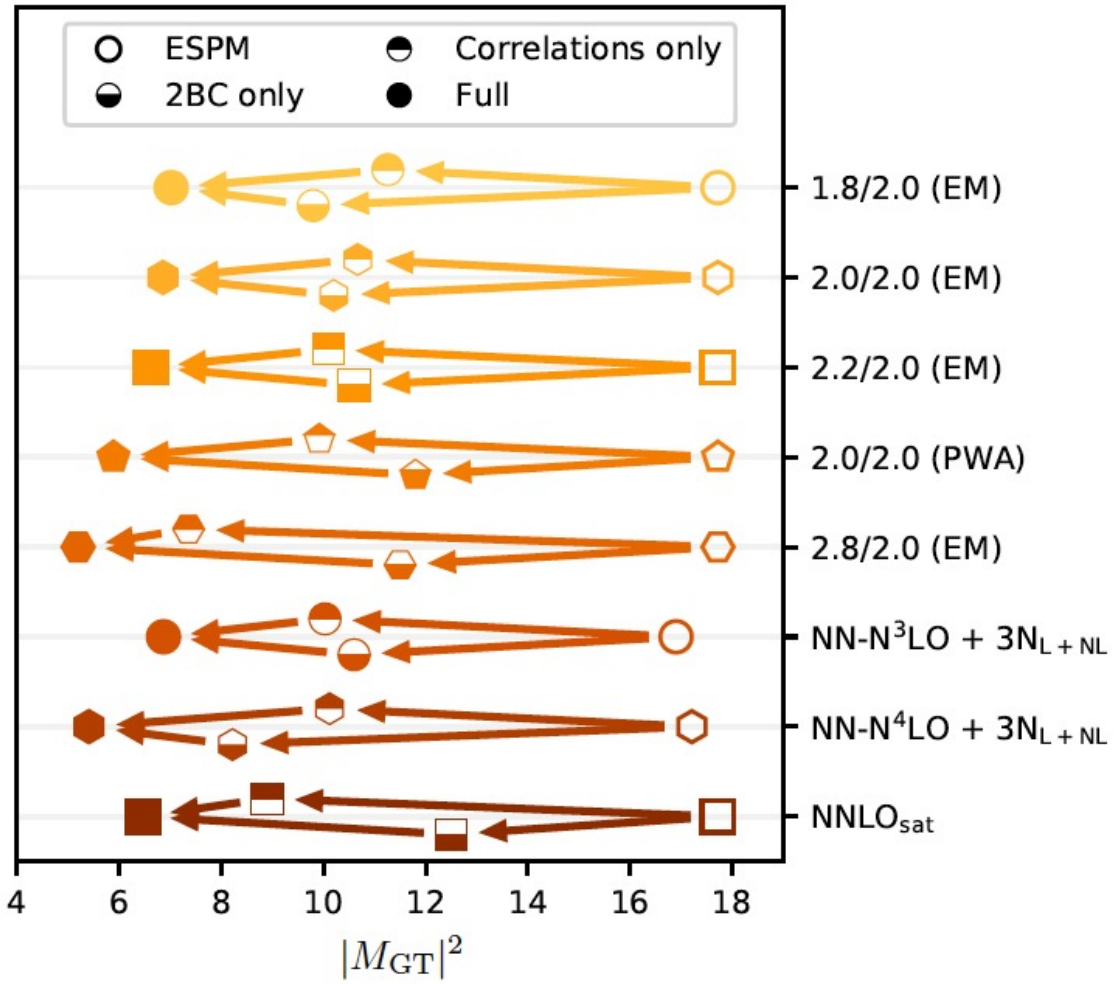
Full symbols: with two-body currents

Two-body currents reduce the systematic uncertainty from the set of chiral interactions.

Traditional models need quenching factors to describe data.
(open symbols: no quenching).

Gysbers, Hagen, Holt, Jansen, Morris, Navratil, TP, Quaglioni, Schwenk, Stroberg & Wendt, Nature Physics (2019); arXiv:1212.3375

Resolution-scale dependence of correlations and two-body currents in ^{100}Sn



- Starting from the extreme single-particle model (ESPM), the contributions from correlations and two-body currents depend on the order in which they are included
- The contributions of two-body currents also depend on the resolution (harder interactions yield stronger correlations and smaller two-body currents)

Gysbers, Hagen, Holt, Jansen, Morris, Navratil, TP, Quaglioni, Schwenk, Stroberg & Wendt, Nature Physics (2019); arXiv:1212.3375

Summary two-body currents

- Two-body currents (2BCs) naturally arise in theories with three-body forces
  - In chiral EFTs, these are subleading corrections
- 2BCs deliver visible contributions to nuclear magnetic moments
- 2BCs provide us with a solution to the long-standing puzzle of quenched β decays