# QCD理论及其连续场论方法概述

刘玉鑫

北京大学物理学院 理论物理研究所

## Outline

Ⅰ. Degrees of Freedom

Ⅱ. Outline of the QCD Theory

Ⅲ. Fundamental Properties

Ⅳ. Examples of Applications

Ⅴ. Summary & Remarks

原子核结构与中高能重离子碰撞交叉学科理论讲习班，湖州师范学院，
2021/07/15

# I. The Degrees of Freedom

## 1. Ingredients of Strong Interaction Matter

e-p scattering experiments

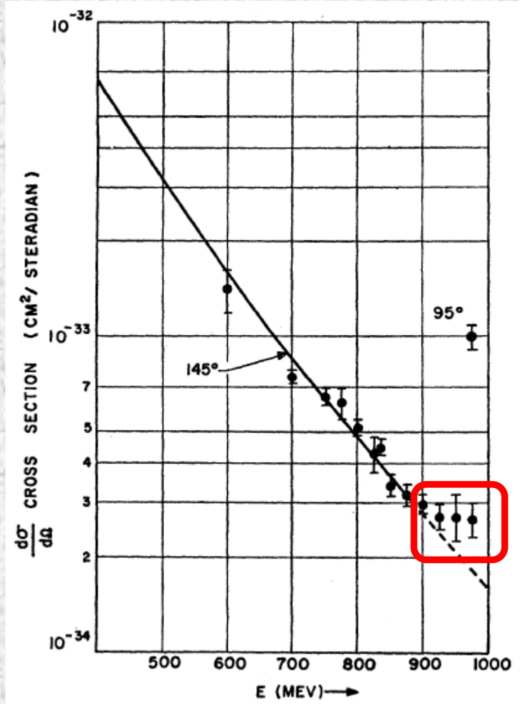
(PR 98, 217(1955); PRL 5, 263(1960); PR 124, 1623(1961); etc)

reveal that proton is **not point particle** but **composite one**.

➔Hadrons: composite particles.

Compositing particle: partons;

➔ quarks and gluons.




➔ Strong interaction matter in early universe is quark-gluon matter, even QGP.

# 2. Intrinsic property of partons

## ♣ Overview

| 代次 | 味道 ($F$) | 裸质量 ($m_0$) | 重子数 ($B$) | 电荷 ($Q$ (e)) | 超荷 ($Y$) | 同位旋三分量 ($I_3$) | 粲数 ($C$) | 奇异数 ($S$) | 顶数 ($T$) | 底数 ($B'$) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 上(u) | $2.3^{+0.7}_{-0.5}$ MeV | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{1}{2}$ | 0 | 0 | 0 | 0 |
| 1 | 下(d) | $4.8^{+0.7}_{-0.3}$ MeV | $\frac{1}{3}$ | $-\frac{1}{3}$ | $\frac{1}{3}$ | $-\frac{1}{2}$ | 0 | 0 | 0 | 0 |
| 2 | 粲(c) | $1.275 \pm 0.025$ GeV | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ | 0 | 1 | 0 | 0 | 0 |
| 2 | 奇异(s) | $95 \pm 5$ MeV | $\frac{1}{3}$ | $-\frac{1}{3}$ | $-\frac{2}{3}$ | 0 | 0 | $-1$ | 0 | 0 |
| 3 | 顶(t) | $173.5 \pm 0.6 \pm 0.8$ GeV | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ | 0 | 0 | 0 | 1 | 0 |
| 3 | 底(b) | $4.18 \pm 0.03$ GeV | $\frac{1}{3}$ | $-\frac{1}{3}$ | $\frac{1}{3}$ | 0 | 0 | 0 | 0 | $-1$ |

spin ½, charge 2/3 or -1/3, strangeness, current mass, etc.

Gell-Mann-Nishijima Relation $Q = I_3 + \frac{1}{2}(Y + C + B' + T).$

- spin symmetry: SU(2);
- approximate flavor symmetry: SU($N_f$); $N_f$ , $\overline{N_f}$ ;
- Color symmetry: SU(3); q: 3 , anti-q: $\bar{3}$; g: 8 .

# ♠ 色自由度概念的建立

实验发现存在 $\Delta^{++}(3/2)$ 和 $\Delta^{-}(3/2)$ 粒子，

推广核子的夸克模型知：

其夸克结构分别为 $u_{1/2}u_{1/2}u_{1/2}$，$d_{1/2}d_{1/2}d_{1/2}$，

违背泡利原理！ ➔ 存在其它自由度！

格林伯格para-statistics (Phys. Today 68, 33 (2015))；

实验测量 $e^+e^-$ 衰变分支比 $$R = \frac{\sigma(\mathrm{e^+e^-} \to \mathrm{hadrons})}{\sigma(\mathrm{e^+e^-} \to \mu^+\mu^-)} = \frac{e^2 \sum_q {Q_q}^2}{e^2} = \sum_q {Q_q}^2,$$

➔考虑u、d、s、c、b夸克：$R = 11/3$，与上式显示结果不一致！

➔ para-factor = 3， ➔ 色 ➔ 色SU(3)对称性。

夸克处于SU(3) 的基础表示 (1 0), 对应三原色:红 绿 蓝;

反夸克处于SU(3) 的基础表示 (0 1), 反红 反绿 反蓝;

胶子处于SU(3) 的伴随表示 (2 1) （8维表示）.

刘耀阳先生的工作的遗憾。

# II. Outline of the QCD Theory

## 1. 对称性的概念

- 系统状态在一些操作或变换下的不变性称为对称性;
  Noether定理：一个连续对称性对应一个守恒量。
  Wigner定理: 对称性变换作用在物理学量上, 观测量不变; …

- 理论载体： 群理论、代数理论;
  对称性有分立对称性和连续对称性,
  → 群有离散群和连续群（或李群）等之分,
  与李群相应有李代数。

# 2. Largest Symmetry of a Quantum Particle

(1) Quantum state: Wave function $\psi(q)$ ;
$|\psi(q)|^2$ describes the probability distribution of the particle at the state.

(2) Symmetry: $|\psi(q)|^2$ maintains its distribution under the operation, i.e., $\left|\hat{O}\psi(q)\right|^2 = |\psi(q)|^2$,
➔ the operation holds unitary symmetry.

(3) Realization of the Symmetry

Mathematically:
u($n$) algebra ➔ matrices: $\{E_{pp'}|p,p' = 1,2,\cdots,n\}$ ,
with $[E_{pp'},\ E_{qq'}] = \delta_{p'q}E_{pq'} - \delta_{pq'}E_{qp'}$ .

Physically: $E_{pp'} = a_p^{\dagger}a_{p'}$ , or $b_q^{\dagger}b_{q'}$ ,
➔ u($n$) can be realized with fermions or bosons.

# 3. Contraction of u($n$) Algebra & Dynamical Symmetry Breaking

## (1) U($n$) ⊃ O($n$)

Mathematically: o($n$) algebra → matrices:

$$\{I_{pp'} = E_{pp'} - E_{p'p} | p, p' = 1, 2, \cdots, n\},$$

with $\left[I_{pp'},\ I_{qq'}\right] = \delta_{pq'} I_{qp'} + \delta_{p'q} I_{pq'} - \delta_{pq} I_{p'q'} - \delta_{p'q'} I_{pq}$.

Physically: $I_{pq} = b_p^{\dagger} b_q - b_q^{\dagger} b_p$.

→ o($n$) can be realized with bosons, and there exists contraction u($n$) ⊃ o($n$).

## (2) Similarly, sp($n$) ($n$=even) can be realized with fermions, and the elements are

$$\Xi_{pq} = a_p^{\dagger} a_q + a_q^{\dagger} a_p \ .$$

There exists then contraction u($n$) ⊃ sp($n$).

# (3) Symmetry Breaking

There exists algebra contraction (group chain):

$u(n) \supset o(n)$ ,  for boson system;

$u(n) \supset sp(n)$ ,  for fermion system.

With the irreducible representation of the $u(n)$, $o(n)$ ($sp(n)$) being labelled as $\Lambda$, $\lambda$, the basis of the states are: $\{|\Lambda \lambda \cdots \rangle\}$.

Degeneracy is reduced by the $\lambda$, the symmetry is broken.

If the Lagrangian (Hamiltonian) involves terms of the Casimir operator(s) of the subgroup(s), it is the dynamical symmetry breaking (DSB).

If not, it is the spontaneous breaking (SSB).

# (4) Ex.: Classification of light-flavor hadrons

♣ Color Structure

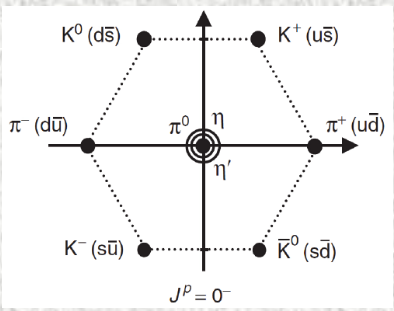
Hadron is colorless, corresponds to IRREP. [0].

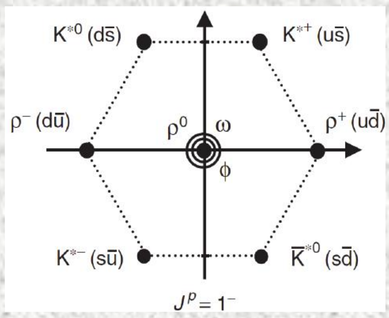
♣ Flavor Structure

Approximately $SU_F(3)$ symmetry,

- mesons, $(\bar{q}q)$,

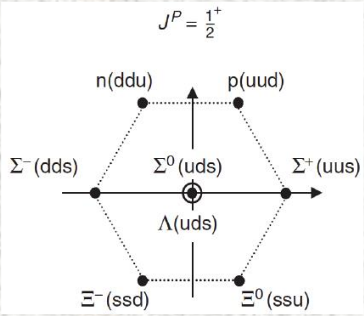
  $\bar{\mathbf{3}} \times \mathbf{3} = \mathbf{8} + \mathbf{1}$;

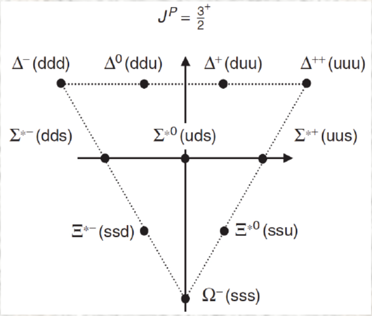



- baryons, $(qqq)$,

  $\mathbf{3} \times \mathbf{3} \times \mathbf{3} = \mathbf{10} + \mathbf{8} + \mathbf{8} + \mathbf{1}$;




♣ Mass Spectrum

# 4. Gauge Symmetry & the Framework of QCD

♠ **General expression of the gauge symmetry**

For the gauge symmetry with generator $\{\mathbb{X}^a\}$, it is usually written as $\mathrm{U} = e^{-i\theta^a X^a}$ .

♠ **Key point of gauge transformation**

gauge transformation is the derivative transformation that makes $\psi \to \psi' = U\psi$, but maintains $\mathcal{L}' = U\mathcal{L}U^{-1} = \mathcal{L}$ .

♠ **Gauge symmetry of the standard model**

QED : $U(1)$ ;

QCD : $SU_C(3)$ ;

Unified EW: $SU_L(2) \otimes U_Y(1)$ .

# ♠ 强相互作用具有 SU(3) 规范对称性

由 $$\partial_\mu\psi' = e^{i\Theta^a\lambda_a}(\partial_\mu + igA_\mu^a\lambda^a)\psi_k,$$

知，为使费米子场部分在 SU(3) 变换下保持不变，

微分算子 $\partial_\mu$ 应转变为协变微分算子：

$$D_\mu = \partial_\mu - igA_\mu^a\lambda^a,$$

其中 $g$ 为待定常量。

由于上述不变性即

$$U(x)\partial_\mu\psi + \left(\partial_\mu U(x)\right)\psi - igA'_\mu U(x)\psi = U(x)(\partial_\mu - igA_\mu)\psi,$$

亦即

$$A'_\mu U(x)\psi = (U(x)\,A_\mu - i\frac{1}{g}\partial_\mu U(x))\psi\,.$$

知，胶子场 $A_\mu$ 的变换应该是

$$A_\mu \longrightarrow A'_\mu = U(x)A_\mu U^{-1}(x) - i\frac{1}{g}(\partial_\mu U(x))U^{-1}(x)\,.$$

# ♠ 强相互作用具有 SU(3) 规范对称性

考虑 $\Theta^a = \epsilon^a \to 0$ 的无穷小变换，

类似电磁场张量 $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

的胶子场张量应该为

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \, .$$

总之，**强相互作用具有SU(3) 规范对称性**。

强相互作用系统的拉氏密度为

$$\mathcal{L}_S = \overline{\psi}_k (i\gamma^\mu D_\mu - m_k)\psi_k - \frac{1}{4} G^{\mu\nu} G_{\mu\nu} \, ,$$

其中协变微分算子为 $D_\mu = \partial_\mu - i g A^a_\mu \lambda^a$，

规范场场强张量为 $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \, .$

再经量子化即得QCD

(SU(3)的非阿贝尔性质使得它远较U(1)规范下的复杂，现常用Faddeev-Popov量子化方案)。具体由量子规范场理论表述。

# Ⅲ. Fundamental Properties

## 1. Chiral Symmetry & Its Breaking

♠ QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = \sum_{f=1}^{N_f} \bar{q}_f(x)(i\not{D} - m_f^0)q_f(x) - \frac{1}{4}G^a_{\mu\nu}(x)G^{\mu\nu}_a(x),$$

where $G^a_{\mu\nu}(x) = \partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) + g f^{abc} A^b_\mu(x) A^c_\nu(x)$

$\gamma_k = \begin{pmatrix} 0 & -i\sigma_k \\ i\sigma_k & 0 \end{pmatrix}$, $\gamma_4 = \begin{pmatrix} I_{2\times 2} & 0 \\ 0 & -I_{2\times 2} \end{pmatrix}$, $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$,

Chiral rotation operator $\mathcal{P}^{L,R} = \frac{1}{2}(1 \pm \gamma_5)$,

where

$\gamma_5 = -\gamma_1\gamma_2\gamma_3\gamma_4 = \begin{pmatrix} 0 & -I_{2\times 2} \\ -I_{2\times 2} & 0 \end{pmatrix}$, $\{\gamma_5, \gamma_\mu\} = 0 \ \ (\mu = 1, 2, 3, 4)$.

one has then

$q^{L,R} = \mathcal{P}^{L,R} q = \frac{1}{2}(1 \pm \gamma_5) q$, $\overline{q^{L,R}} = (\gamma_4 q^{L,R})^\dagger = \bar{q}\frac{1}{2}(1 \mp \gamma_5)$.

and in turn,

$$\bar{q}_f(x)\not{D}q_f(x) = \bar{q}^L_f(x)\not{D}q^L_f(x) + \bar{q}^R_f(x)\not{D}q^R_f(x) \,.$$

i.e., the kinetic energy part is invariant under the chiral transformation $e^{i\gamma_5\Theta}$.

The gauge field part is definitely invariant,

➔ the system holds chiral symmetry if $m_0^f \equiv 0$ .

However, $m^0_f\bar{q}_f(x)q_f(x) = m^0_f\left(\bar{q}^R_f(x)q^L_f(x) + \bar{q}^L_f(x)q^R_f(x)\right) \,,$

is not invariant!

➔ The chiral symmetry is broken, if $m_0^f \neq 0$ .

➔ Chiral symmetry breaking induces a mass for a quark.

# ♠ Ex.: the CS & CSB of two flavor system

## ♣ Chiral symmetry

### • Lagrangian

$$\mathcal{L}_{\rm QCD} = \bar{u}(x)(i\not{D} - m_u^0)u(x) + \bar{d}(x)(i\not{D} - m_d^0)d(x) - \frac{1}{4}G^a_{\mu\nu}(x)G_a^{\mu\nu}(x)\,.$$

### • Chiral symmetry

Isospin operators: $t_1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad t_2 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad t_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$

$\mathcal{L}$ of the system in chiral limit $(m_u^0 = m_d^0 = 0)$ is invariant under Flavor transformation

$$\begin{pmatrix} u \\ d \end{pmatrix} \longrightarrow \begin{pmatrix} u' \\ d' \end{pmatrix} = e^{i(t_k\Theta^{V,k} + \gamma_5 t_k \Theta^{A,k})}\begin{pmatrix} u \\ d \end{pmatrix} .$$

Decomposing the isospin operator as $\vec{t} = \vec{t}^L + \vec{t}^R\,,$

the $\vec{t}^L = \frac{1}{2}(1+\gamma_5)\vec{t}, \quad \vec{t}^R = \frac{1}{2}(1-\gamma_5)\vec{t}$ form two su(2) algebras,

$\mathcal{L}$ maintains invariant, ➔ $SU_L(2)\otimes SU_R(2)$ Sym.

Defining $\vec{x} = \vec{t}^L - \vec{t}^R = \gamma_5 \vec{t}\,,$

and noticing $[t_i,\ t_j] = i\varepsilon_{ijk} t_k\,, \quad [t_i,\ x_j] = i\varepsilon_{ijk} x_k\,, \quad [x_i,\ x_j] = i\varepsilon_{ijk} t_k\,,$ ,

➔ the $SU_L(2) \otimes SU_R(2)$ symmetry can be written in other form.

Taking $\vec{V}^\mu = i\bar{q}\gamma^\mu \vec{t} q\,, \qquad \vec{A}^\mu = i\bar{q}\gamma^\mu \gamma_5 \vec{t} q\,,$

one has $\partial_\mu \vec{V}^\mu = \partial_\mu \vec{A}^\mu = 0\,.$

Defining (charges) $\hat{T} = \int d^3 x V^4\,, \qquad \hat{X} = \int d^2 A^4$ ,

they behave $[T_i,\ T_j] = i\varepsilon_{ijk} T_k\,, \quad [T_i,\ X_j] = i\varepsilon_{ijk} X_k\,, \quad [X_i,\ X_j] = i\varepsilon_{ijk} T_k\,,$

the quark field transforms as

$$[\vec{T}\,,\ q] = -\vec{t} q\,, \qquad [\vec{X}\,,\ q] = -\vec{x} q\,.$$

➔ The chiral symmetry can be rewritten as

$$SU_L(2) \otimes SU_R(2) \cong SU_V(2) \otimes SU_{AV}(2)\,.$$

# • Chiral Symmetry Breaking

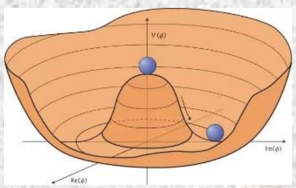
Above mentioned symmetry means that there exist chiral partner states with opposite parities.

In fact, $m_\pi \approx 138$ MeV, $\neq$ $m_\sigma \approx (400\sim550)$ MeV;

$m_\rho \approx 775$ MeV, $\neq$ $m_{a1} \approx 1230$ MeV .

➔ The chiral symmetry must be broken.

Since there is always $SU_V(2)$, the breaking should be $SU_L(2)\otimes SU_R(2) \supset SU_V(2)\otimes U_A(1)$ .



➔ in case $m_q^0 = 0$, Hadrons & Goldstone bosons;

in case $m_q^0 \neq 0$, Goldstone b. $\Rightarrow$ pseudo ones;

e.g., pion, GOR relation: $m_\pi^2 f_\pi^2 = -(m_u^0 + m_d^0)\langle \bar{q}q \rangle$ .

## ♣ Classification of the CSB

- **Explicit Breaking (ECSB)**

The current mass induced CSB.

- **Spontaneous/Dynamical Breaking (DCSB)**

If the Lagrangian maintains the symmetry but that of the vacuum is broken, the Br is SCSB.

e.g., $\mathcal{L} = \bar{\psi} i\gamma \cdot p\psi + \lambda(\bar{\psi}\psi\bar{\psi}\psi)\,,$

Self-energy → $M = \lambda Tr(S) = \lambda \int_0^{\Lambda^2} \frac{M}{p^2+M^2} dp^2 = \lambda M \Big[\Lambda^2 - M^2 \ln\big(1+\frac{\Lambda^2}{M^2}\big)\Big],$

i.e., $M(\Lambda^2 - \frac{1}{\lambda}) = M^3 \ln\Big(1+\frac{\Lambda^2}{M^2}\Big)\,.$

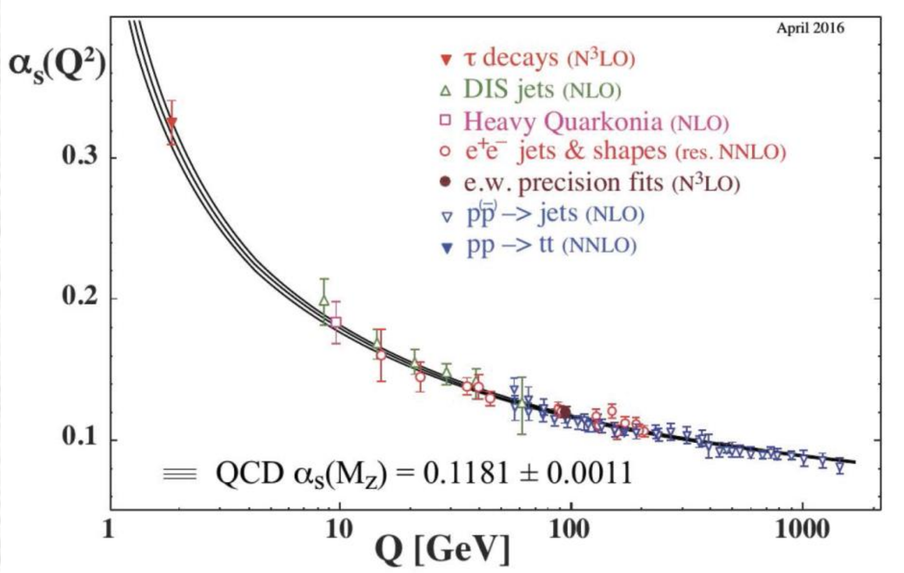
The Eq. has always solution $M \equiv 0$ ;

As $\lambda > \Lambda^2$, the equation has non-zero solution(s).

→ As the interaction is strong enough, the CS is broken simultaneously, in turn SCSB/DCSB.

# 2. Asymptotic Freedom

- Running Coupling Strength




# 3. Color Confinement

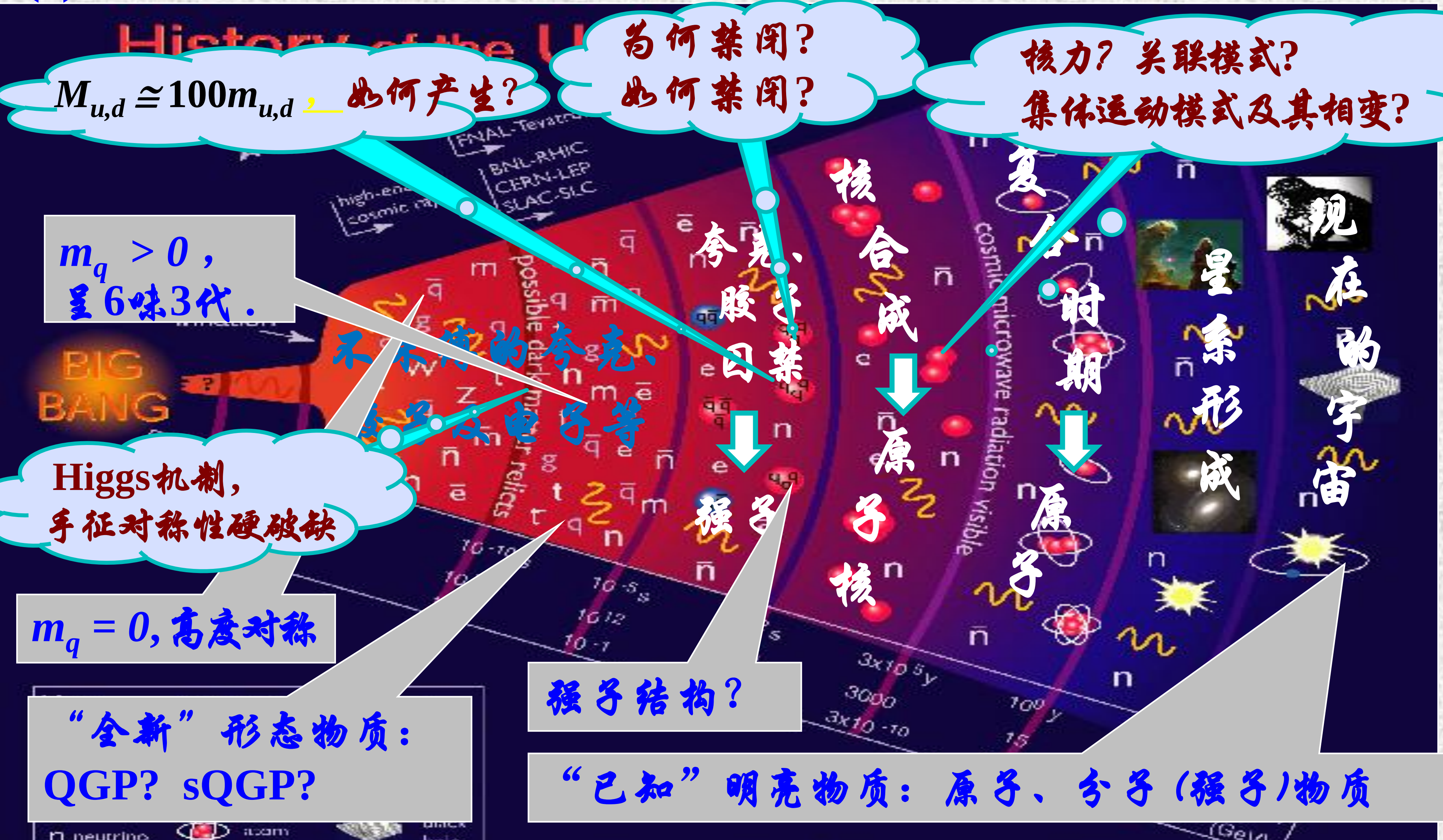
color degrees of freedom of the quarks & gluons can not be observed directly !

- No confinement, No Universe !
- Connection between the CC & the DCSB ?

# IV. 应用举例

## 1. 早期宇宙强作用物质的演化

### (1) 过程与图像



(2) 演化过程的物理描述

● 上述演化各阶段物质的性质各具特色，相差甚远；任何两阶段间的演化也各自千差万别。

♣ 如何具体描述是物理学人致力探索的瑰宝！

♠ 不均匀的状态中，可以由力学手段分离的组分相同、物理和化学性质相同的均匀部分的状态称为物质的相；外部条件不变情况下，由一相到另一相的演化称为相变。

♠ 强相互作用物质

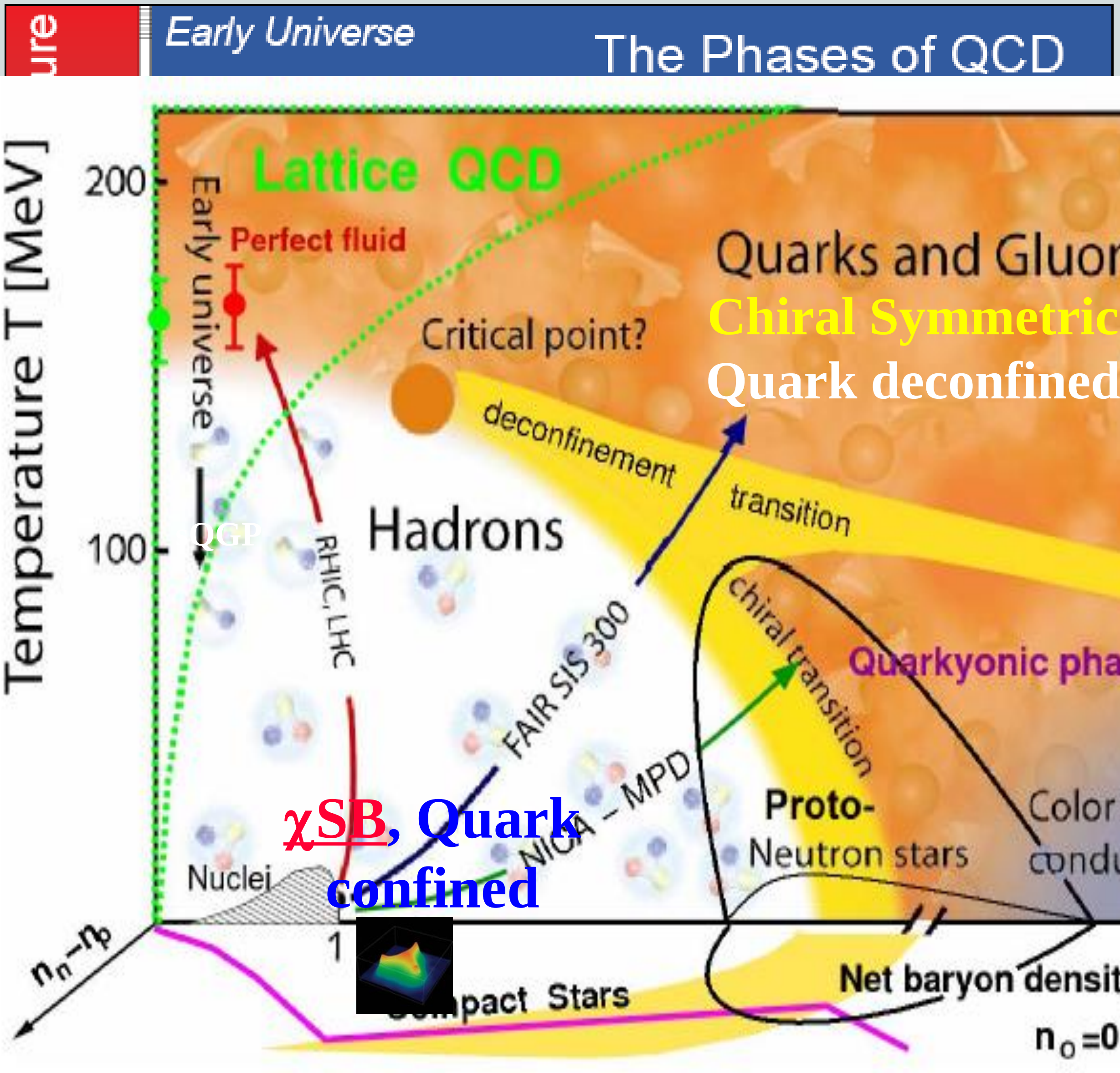
原子核形成的物质、强子形成的物质、夸克及胶子形成的物质、以及强子与夸克和胶子共存的系统都称为强相互作用物质。

➔ 上述演化过程可表述为强相互作用物质的相变！

➔ 相即确定对称性的态，相变即对称性破缺 or 恢复的过程！

➔ 早期宇宙强相互作用物质的演化即强作用系统的(手征)对称性动力学破缺和色禁闭过程！亦即QCD相变过程。

# →基本问题归结为QCD相变，研究正如火如荼




Schematic QCD phase diagram for nuclear matter. The solid lines show the phase boundaries for the indicated phases. The solid circle depicts the critical

**The Frontiers of Nuclear Science**

A LONG RANGE PLAN

December 2007

**涉及相变：**

禁闭（强子化）—退禁闭

手征对称性破缺 – 恢复

**影响QCD相变的因素：**

介质效应：温度，


密度（化学势）

有限尺度

内禀因素：流质量，

跑动耦合强度，

色味结构，•••

**研究方法：**

实验：RHIC、Ast-Obs.

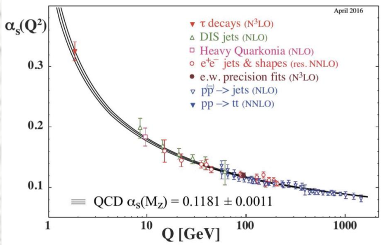
理论：离散场论、连续场论

计算：实现理论、模拟

# (3) Description of the DCSB

♠ Requirement for the theor. approach

- Phenomena related
  the DCSB & its Restoration ,
  the Confinement & Deconfinement ;
  Generally denoted as QCD phase transition.
- Energy scale: $10^2$ MeV ,
  typically nonperturbative !

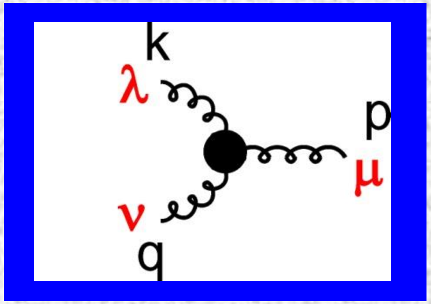



- The approaches should be the ones involving simultaneously the charters
  of the DCSB & its Restoration ,
  the Confinement & Deconfinement ;
  i.e., the NP QCD !

# ♠ Dyson-Schwinger Equations
# — A Nonperturbative QCD Approach

## (i) Outline of the DS Equations

### Slavnov-Taylor Identity

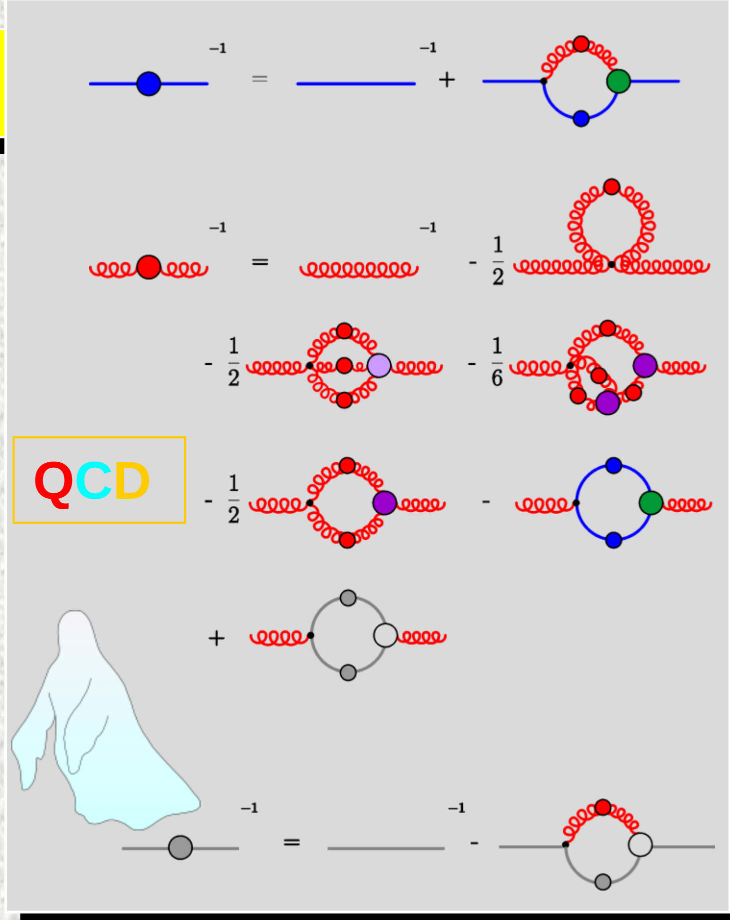



**axial gauges** BBZ

$$k_\lambda \, \Gamma^{\lambda\mu\nu}(k,p,q) = \Pi^{\mu\nu}(p) - \Pi^{\mu\nu}(q)$$

**covariant gauges**

$$k_\lambda \, \Gamma^{\lambda\mu\nu}(k,p,q) = H(k^2) \, \left[ G_{\mu,\sigma}(q,-k) \, \Pi^{T}_{\sigma,\nu}(p) - G_{\nu\sigma}(p,-k) \, \Pi^{T}_{\sigma\mu}(q) \right]$$

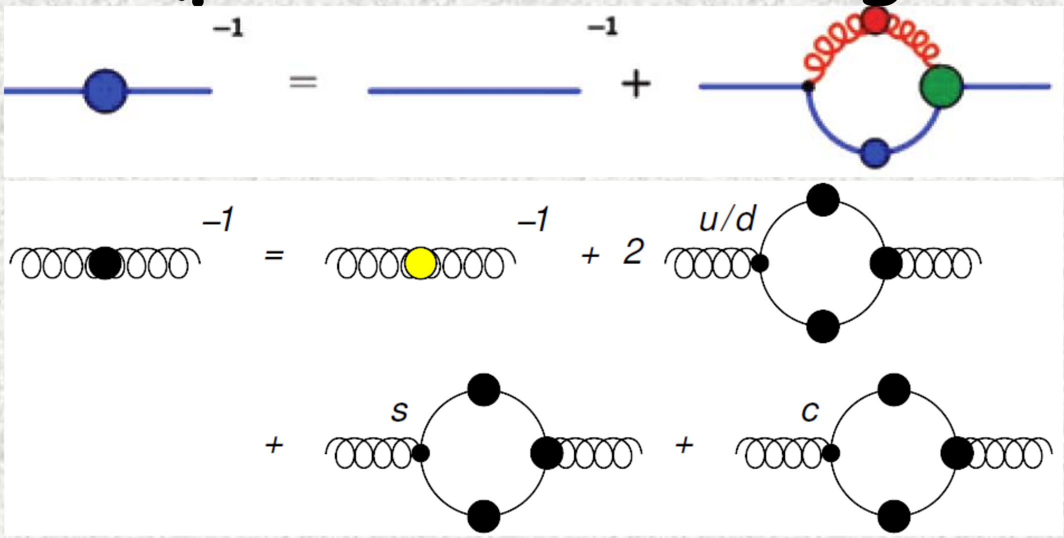




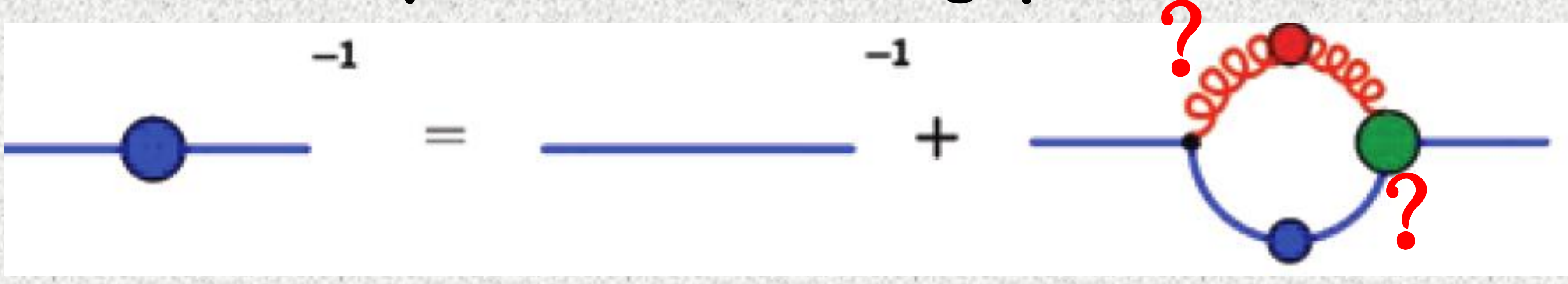
C. D. Roberts, et al, PPNP 33(1994), 477; R. Alkofer, et. al, Phys. Rep. 353(2001), 281;
A. Bashir, ···, LYX, Roberts, et al., CTP 58 (2012), 79; C.S. Fischer, PPNP 105(2019), 1;

# ✦ Overview of Solving the DSEs of QCD

- **Solving the coupled** quark, ghost and gluon **equations** (parts of the diagrams) :




- **Solving the truncated quark equation** with the symmetries being preserved.



# ✦ Expression of the quark gap equation

- Truncation: Preserving Symm. ➔ Quark Eq.

$$S^{-1}(p) = Z_2(-i\not{p} + Z_m m) + Z_1 g^2 \int \frac{d^4q}{(2\pi)^4} [t^a \gamma_\mu S(q) \Gamma^b_\nu(p,q) D^{ab}_{\mu\nu}(p-q)]$$

- Decomposition of the Lorentz Structure

➔ Quark Eq. in Vacuum :

$$S^{-1}(p) = i\not{p} A(p^2, \Lambda^2) + B(p^2, \Lambda^2)$$

with

$$\begin{cases} A(x) = 1 + \dfrac{1}{6\pi^3} \displaystyle\int dy \frac{yA(y)}{yA^2(y) + B^2(y)} \Theta_A(x,y) \\ B(x) = \dfrac{1}{2\pi^3} \displaystyle\int dy \frac{yB(y)}{yA^2(y) + B^2(y)} \Theta_B(x,y) \end{cases}$$

where $\Theta_A(x,y)$ & $\Theta_B(x,y)$ are functions of the vertex model & the gluon model.

# • Quark Eq. in Medium

## Matsubara Formalism

Temperature $T$ : → Matsubara Frequency

$$\omega_n = (2n+1)\pi T$$

Density $\rho$ : → Chemical Potential $\mu$

$$S^{-1}(p) \Longrightarrow S^{-1}(p, \omega_n, \mu)$$

Decomposition of the Lorentz Structure

$$S^{-1}(p) = i\gamma \cdot p A(p^2) + B(p^2), \Longrightarrow$$

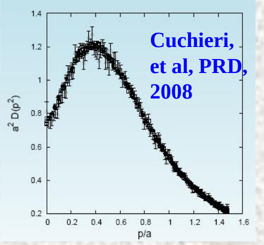
$$S^{-1}(p, \omega_n, \mu) = iA(p, \omega_n, \mu)\vec{\gamma} \cdot \vec{p} + iC(p, \mu)\gamma_4(\omega_n + i\mu) + B(\tilde{p}) + \cdots$$

# ✦ Models of the eff. gluon propagator

$$g^2 D_{\rho\sigma}(k) = 4\pi \frac{\mathcal{G}(k^2)}{k^2} \left(\delta_{\rho\sigma} - \frac{k_\rho k_\sigma}{k^2}\right)$$

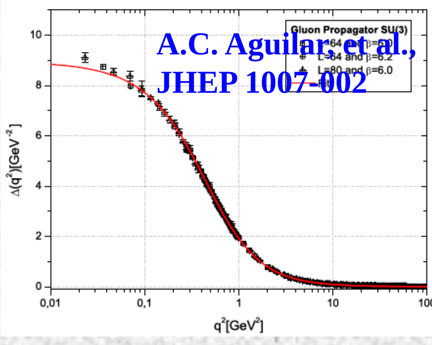
- **Commonly Used: Maris-Tandy Model (PRC 56, 3369)**

$$\frac{\mathcal{G}(t)}{t} = \frac{4\pi^2}{\omega^6} D\, t e^{-t/\omega^2} + \frac{8\pi^2 \gamma_m}{\ln\left[\tau + \left(1 + t/\Lambda_{QCD}^2\right)^2\right]} \frac{1 - \exp(-t/[4m_{\mathcal{F}}^2])}{t} \quad (3)$$




- **Recently Proposed: Infrared Constant Model**

( Qin, Chang, Liu, Roberts, Wilson,
Phys. Rev. C 84, 042202(R), (2011). )




Taking $t/\omega^2 = k^2/\omega^2 = 1$ in the coefficient of the above expression

- **Derivation and analysis in PRD 87, 085039 (2013) (Zwanziger) show that the one in 4-D should be infrared constant.**

# ✦ Models of quark-gluon interaction vertex

$$\Gamma^a_\mu(q,p) = t^a \Gamma_\mu(q,p)$$

- **Bare Ansatz**

$$\Gamma_\mu(q,p) = \gamma_\mu$$ (**Rainbow Approx.**)

- **Ball-Chiu (BC) Ansatz**

$$\Gamma^{BC}_\mu(p,q) = \frac{A(p^2)+A(q^2)}{2}\gamma_\mu + \frac{(p+q)_\mu}{p^2-q^2}\{[A(p^2)-A(q^2)]\frac{(\gamma\cdot p+\gamma\cdot q)}{2} - i[B(p^2)-B(q^2)]\}$$

Satisfying W-T Identity, L-C. restricted

- **Curtis-Pennington (CP) Ansatz**

$$\Gamma^{CP}_\mu(p,q) = \Gamma^{BC}_\mu(p,q) + \frac{1}{2}(A(p^2)-A(q^2))\frac{\gamma_\mu(p^2-q^2)-(k+p)_\mu\gamma\cdot(p+q)}{d(p,q)},$$

$$d(p,q) = \frac{(p^2-q^2)^2+[M^2(p^2)+M^2(q^2)]^2}{p^2+q^2}.$$

Satisfying Prod. Ren.

- **CLRQ** (**BC+ACM**, Chang, etc, PRL 106,072001('11); Qin, etc, PLB 722,384('1
C. Tang, F. Gao, & YXL, Phys. Rev. D 100, 056001 (2019) )

$$\Gamma^{\rm acm}_\mu(p_f,p_i) = \Gamma^{\rm acm_4}_\mu(p_f,p_i) + \Gamma^{\rm acm_5}_\mu(p_f,p_i),$$

# (ii) Describing the QCD PT via the DSE Approach

## ♠ Dynamical chiral symmetry breaking

$$\mathcal{M}(p) \simeq m_0 \, [\ln p/\Lambda_{QCD}]^d + C \, \frac{-\langle \bar{q} q \rangle}{p^2 \, [\ln p/\Lambda_{QCD}]^d}$$

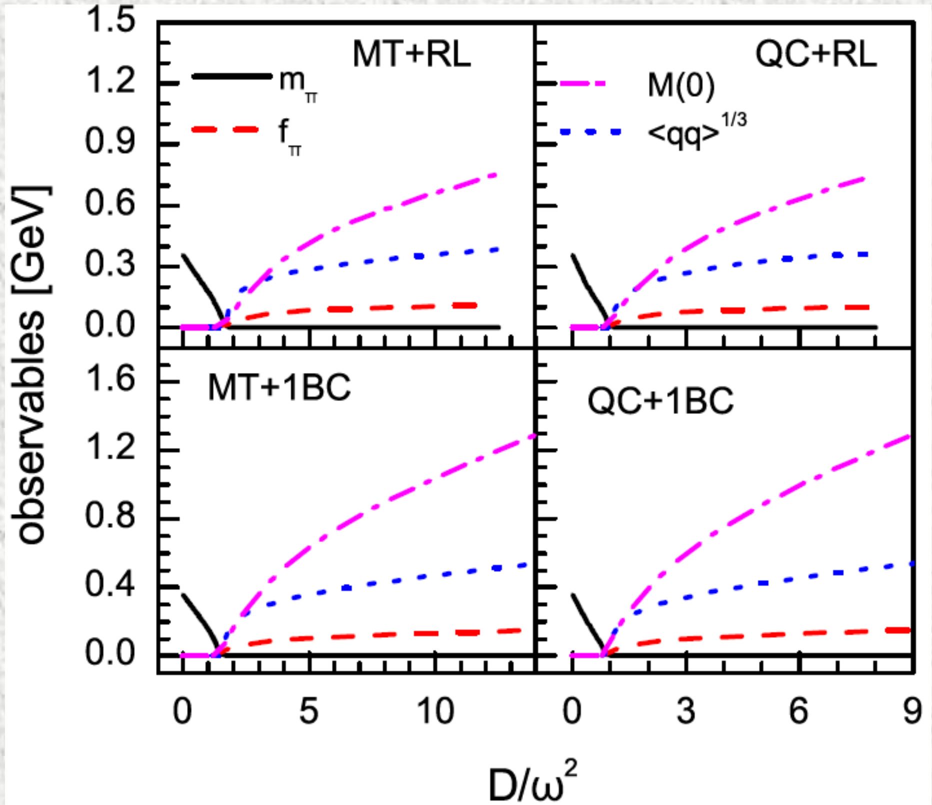
$\langle \bar{q} q \rangle \neq 0$ ➡ **DCSB**

**In DSE approach**

$$M(p^2) = \frac{B(p^2)}{A(p^2)}$$

**Numerical results in chiral limit**

➔ **Increasing the interaction strength induces the dynamical mass generation**

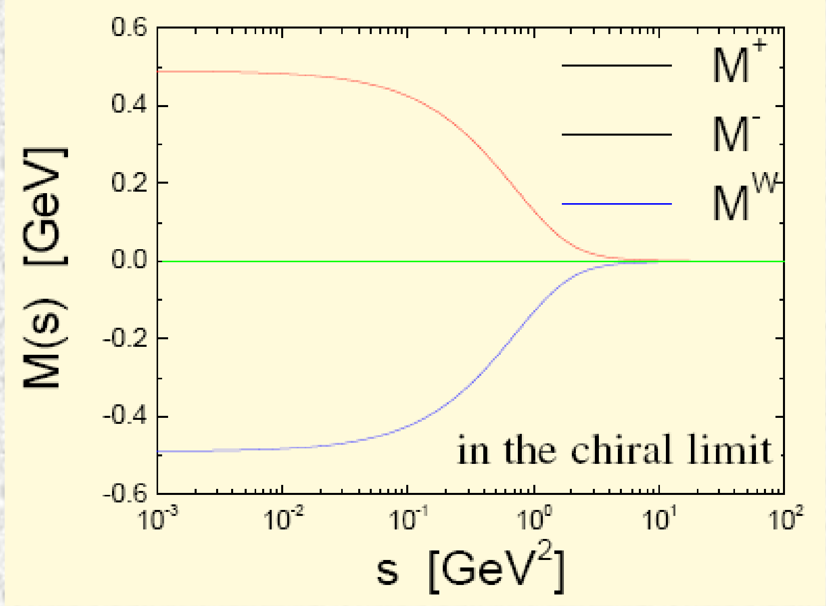


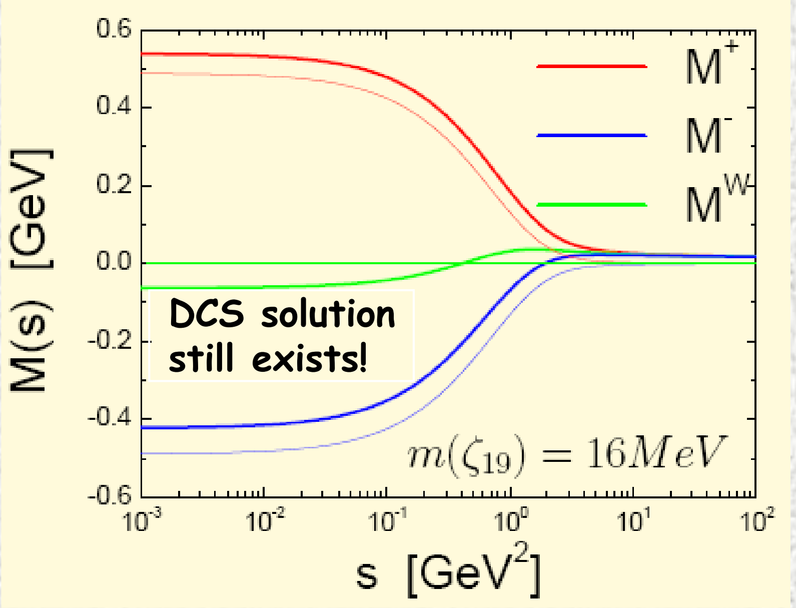

K.L. Wang, YXL, et al., PRD 86,114001('12);

# ✦ Dynamical Chiral Symmetry Breaking (DCSB) still exists beyond chiral limit

L. Chang, Y. X. Liu, C. D. Roberts, et al, arXiv: nucl-th/0605058;
R. Williams, C.S. Fischer, M.R. Pennington, arXiv: hep-ph/0612061;
K. L. Wang, Y. X. Liu, & C. D. Roberts, Phys. Rev. D 86, 114001 (2012).

Solutions of the DSE with MT model and QC model for the effective gluon propagator, bare & 1BC model for the quark-gluon interaction vertex :

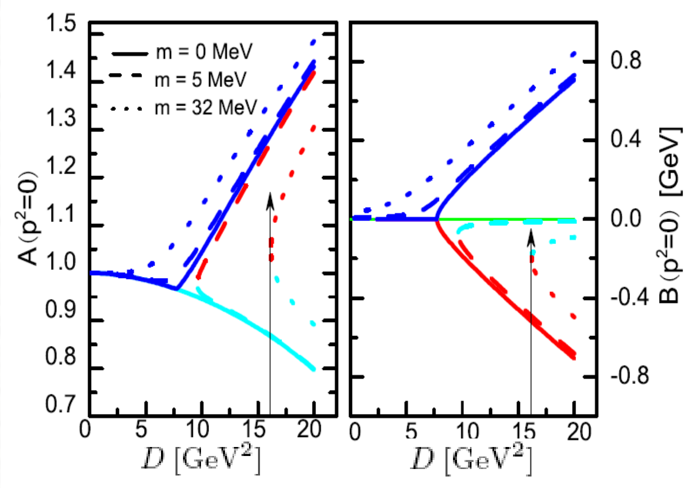


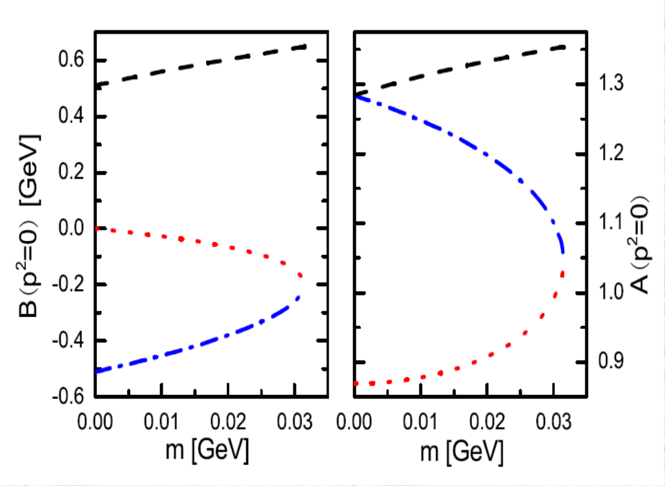




with $D = 16\ GeV^2$, $\omega = 0.4\ GeV$

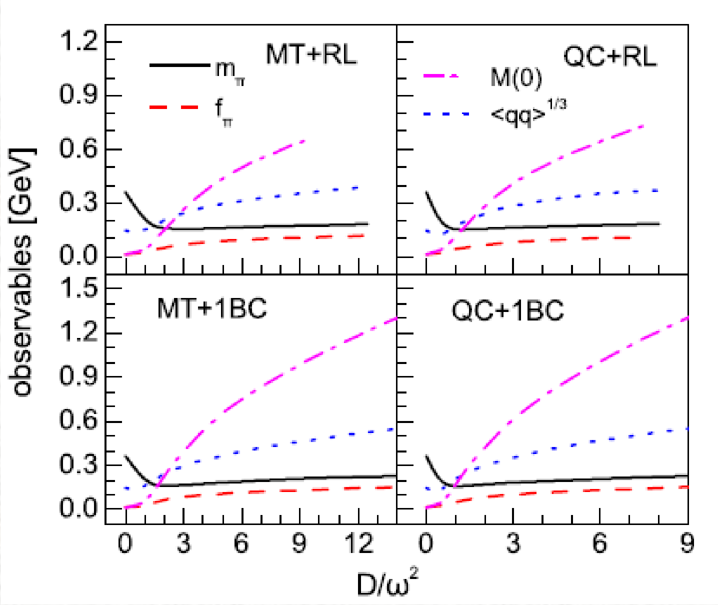
# ✦ DCSB still exists beyond chiral limit

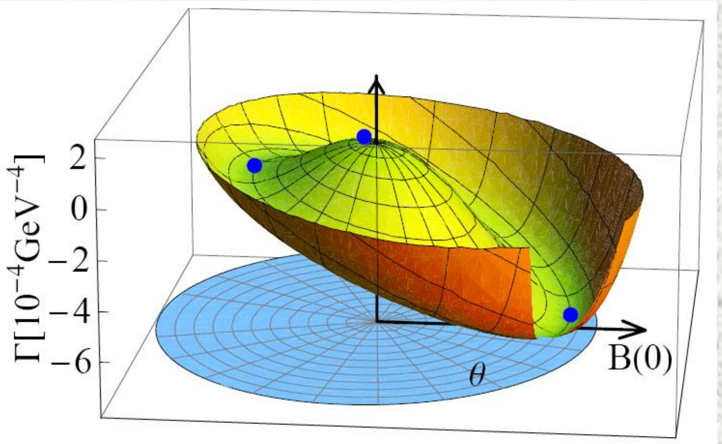
**Solutions continued**




**Conditions: Interaction strength large enough, m not very large.**

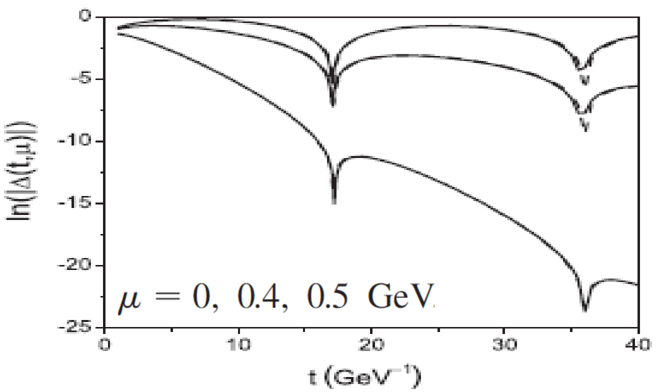


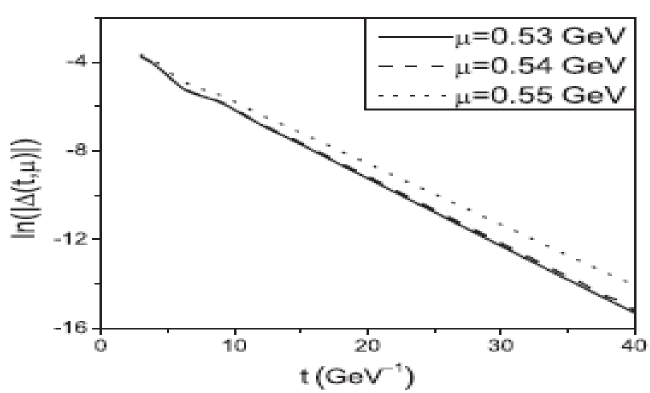




**2nd order Phase Transition shifts to crossover.**

## ♠ Confinement

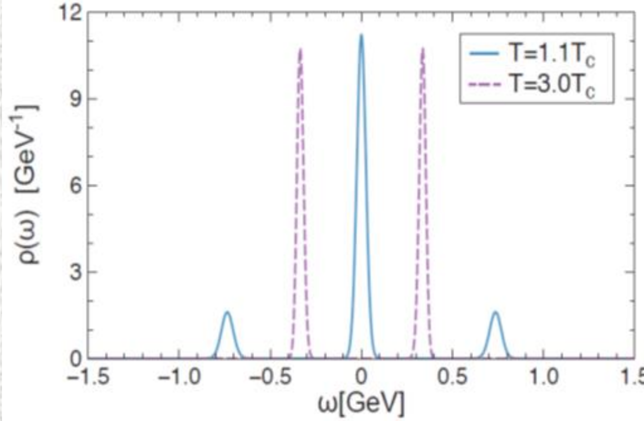
## Positivity of the spectral density function is violated at low T → quarks are confined.

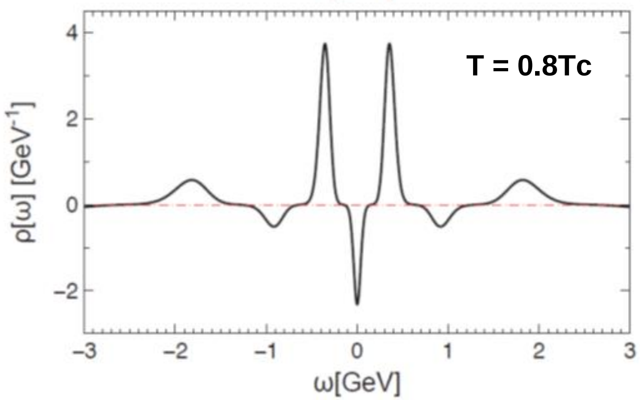



$$\Delta(\tau, \mu) = \int \frac{d^4 p}{(2\pi)^4} e^{i\vec{p}\cdot\vec{x} + i p_4 \tau} \delta(\vec{p}) \sigma_B(p; \mu).$$

H. Chen, YXL, et al., Phys. Rev. D 78, 116015 (2008)




S.X. Qin, D. Rischke, Phys. Rev. D 88, 056007 (2013)

$$S^R(\omega, \vec{p}) = S(i\omega_n, \vec{p})|_{i\omega_n \to \omega + i\epsilon}$$

$$S(i\omega_n, \vec{p}) = \int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi} \frac{\rho(\omega', \vec{p})}{i\omega_n - \omega'}$$

$$\rho(\omega, \vec{p}) = -i\vec{\gamma}\cdot\vec{p}\,\rho_v(\omega, \vec{p}^2) + \gamma_4 \omega\, \rho_e(\omega, \vec{p}^2) + \rho_s(\omega, \vec{p}^2)$$

**In MEM**

$$P[\rho|M(\alpha)] = \frac{1}{Z_S} e^{\alpha S[\rho, m]},$$

$$S[\rho, m] = \int_{-\infty}^{+\infty} d\omega \left[\rho(\omega) - m(\omega) - \rho(\omega) \ln \frac{\rho(\omega)}{m(\omega)}\right]$$

$$m(\omega) = m_0 \theta(\Lambda^2 - \omega^2).$$

# (iii) The QCD Phase Diagram

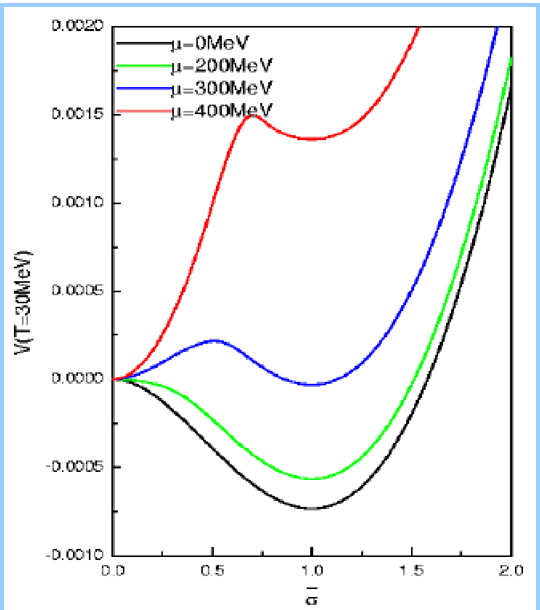
♠ **Criterion determining the phase boundary line & the position of the CEP**

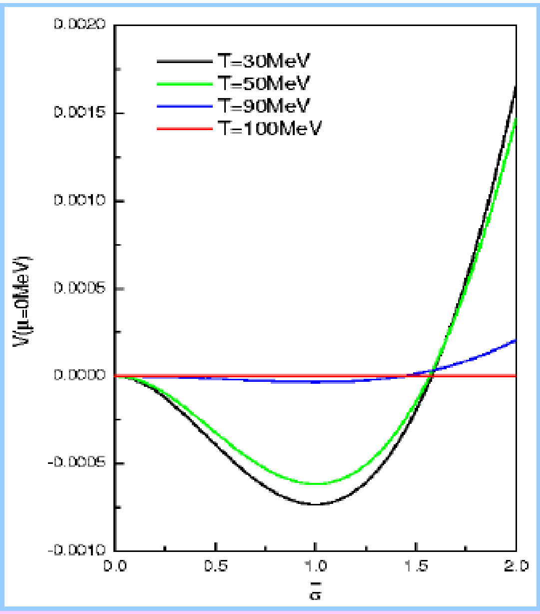
♣ **Conventional Criterion**

*Order Parameter: chiral cond.* $\langle \bar{q} q \rangle$!

$$\mathcal{M}(p) \simeq m_0 \left[\ln p/\Lambda_{QCD}\right]^d + C \frac{-\langle \bar{q} q \rangle}{p^2 \left[\ln p/\Lambda_{QCD}\right]^d}$$

**Procedure: Analyzing the ThDyn Potential**




Signature of PT: $\frac{\partial^2 \Omega}{\partial T^2}$, $\frac{\partial^2 \Omega}{\partial \mu^2}$, etc., change sign .

## Question:

In complete nonperturbation, one can not have the thermodynamic potential.

The conventional criterion fails.

One needs then new criterion!

# ♣ New Criterion: Chiral Susceptibility

- **Def.:** Response of the order parameter to control variables

$$\frac{\partial M}{\partial T},\ \frac{\partial M}{\partial \mu};\quad \frac{\partial \langle \bar{q}q \rangle}{\partial T},\ \frac{\partial \langle \bar{q}q \rangle}{\partial \mu};\quad \frac{\partial B}{\partial T},\ \frac{\partial B}{\partial \mu};\quad \frac{\partial B}{\partial m_0};$$

- **Simple Demonst. Equiv. of NewC to ConvC**

（刘玉鑫，《热学》，北京大学出版社，2016年第1版）

TD Potential: $\Omega(T,\eta) = \Omega_0(T) + \frac{1}{2}\alpha\eta^2 + \frac{1}{4}\beta(\eta^2)^2 + \frac{1}{6}\gamma(\eta^2)^3 + \cdots$

Stability Condition: $\frac{\partial \Omega}{\partial \eta} = \alpha\eta + \beta\eta^3 + \gamma\eta^5 = 0$

$$\frac{\partial^2 \Omega}{\partial \eta^2} = \alpha + 3\beta\eta^2 + 5\gamma\eta^4 > 0,\ St.;\ < 0,\ Unst..$$

Derivative of ext. cond. against control. var.:

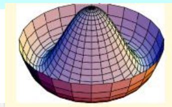
$$[\alpha + 3\beta\eta^2 + 5\gamma\eta^4]\left(\frac{\partial \eta}{\partial \varsigma}\right)_{\varsigma=\zeta_c} + \eta\left(\frac{\partial \alpha}{\partial \varsigma}\right)_{\varsigma=\zeta_c} + \eta^3\left(\frac{\partial \beta}{\partial \varsigma}\right)_{\varsigma=\zeta_c} + \eta^5\left(\frac{\partial \gamma}{\partial \varsigma}\right)_{\varsigma=\zeta_c} = 0$$

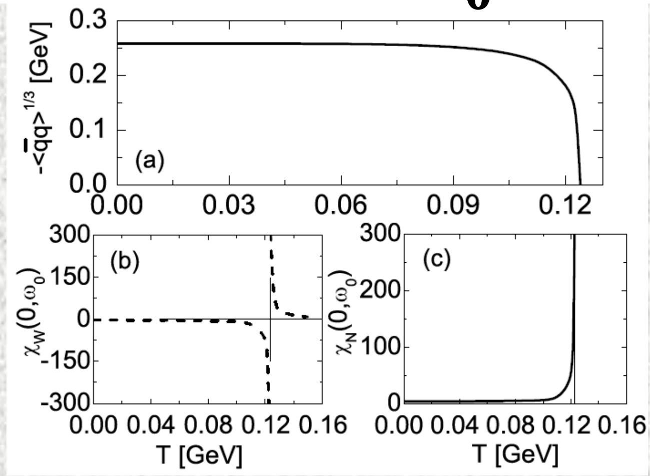
we have: $\chi = \left(\frac{\partial \eta}{\partial \varsigma}\right)_{\varsigma=\zeta_c} = -\frac{\eta\left(\frac{\partial \alpha}{\partial \varsigma}\right)_{\varsigma=\zeta_c} + \eta^3\left(\frac{\partial \beta}{\partial \varsigma}\right)_{\varsigma=\zeta_c} + \eta^5\left(\frac{\partial \gamma}{\partial \varsigma}\right)_{\varsigma=\zeta_c}}{\left(\frac{\partial^2 \Omega}{\partial \eta^2}\right)_{\frac{\partial \Omega}{\partial \eta}=0}}$

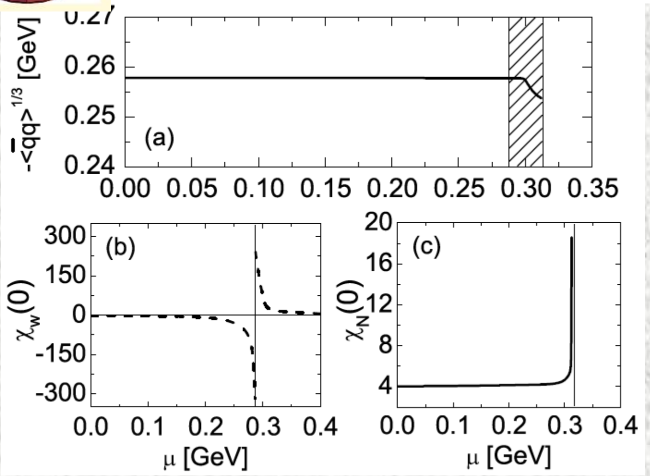
At field theory level, see Fei Gao, Y.X. Liu, Phys. Rev. D 94, 076009 (2016) .

# ♣Demonstration of the New Criterion

## In chiral limit（$m_0 = 0$）

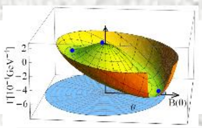




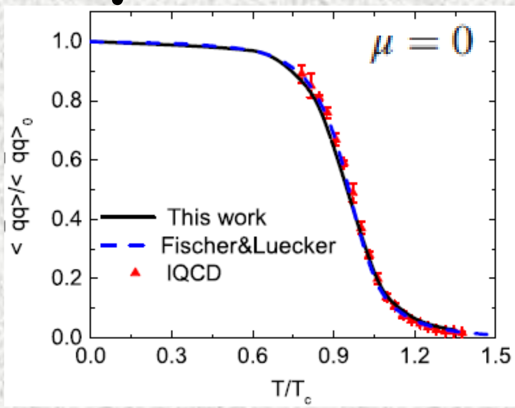



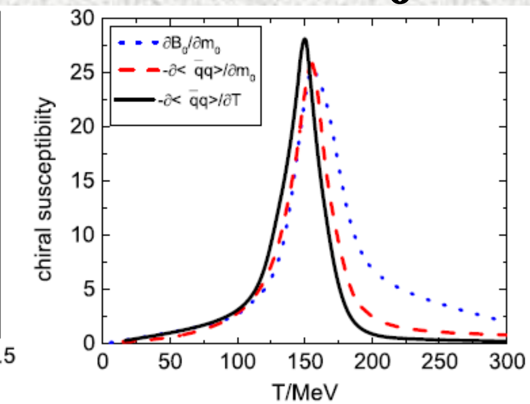

S.X. Qin, L. Chang, H. Chen, YXL, et al., Phys. Rev. Lett. 106, 172301 (2011).

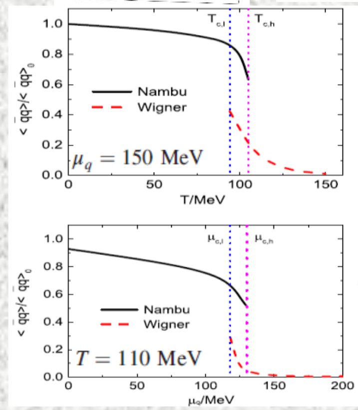
## Beyond chiral limit（$m_0 \neq 0$）

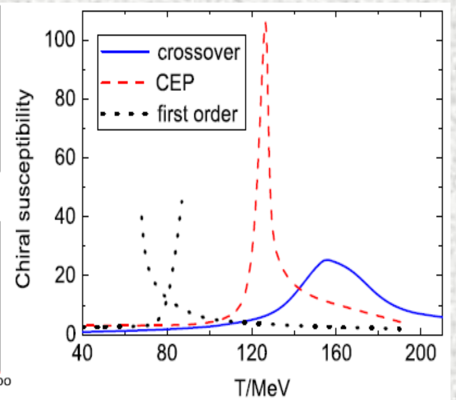


$$\langle\bar{q}q\rangle = \langle\bar{q}q\rangle_{m_0} - m_0\frac{\partial\langle\bar{q}q\rangle_{m_0}}{\partial m_0}$$




Fei Gao, Y.X. Liu, Phys. Rev. D 94, 076009 (2016) .

# ♣ Characteristic of the New Criterion

As 2nd order PT (Crossover) occurs, the $\chi$s of the two (DCS, DCSB) phases diverge (take maximum) at same states.

As 1st order PT takes place, $\chi$s of the two phases diverge at dif. states.

➔ the $\chi$ criterion can not only give the phase boundary, but also determine the position the CEP.

**For multi-flavor system,**

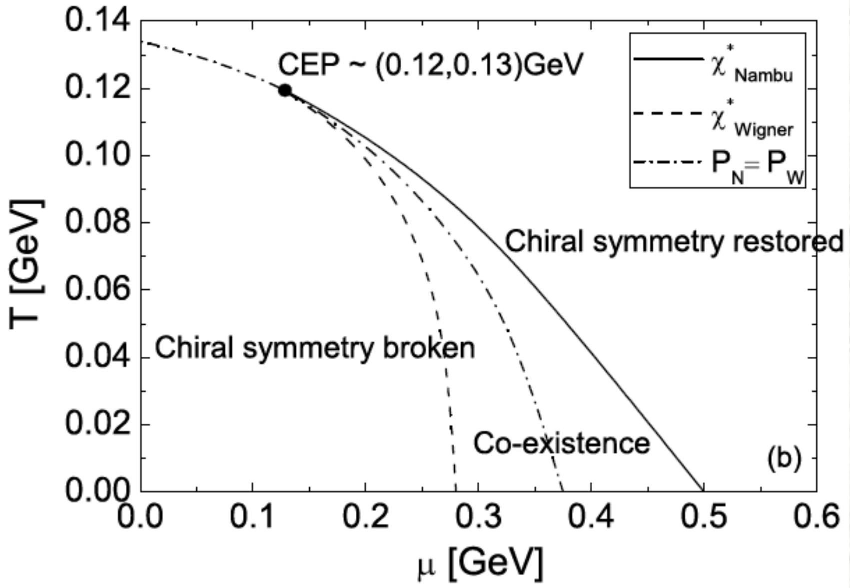
one should analyze the maximal eigenvalue of the susceptibility matrix (L.J. Jiang, YXL, et al., PRD 88, 016008), or the mixed susceptibility (F. Gao, YXL, PRD 94, 076009).

# ♠ Some Numerical Results

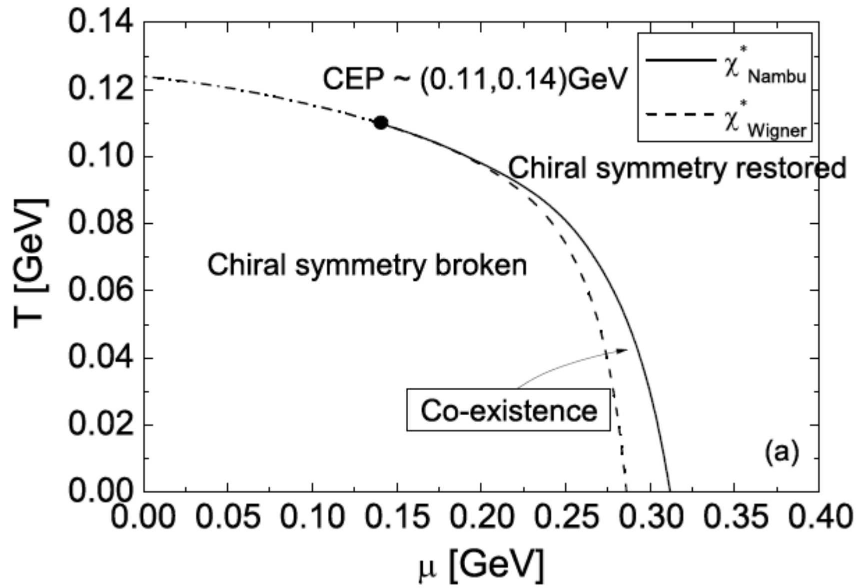
## ♠ QCD Phase Diagrams and the position of the CEP have been given in the DSE

- In chiral limit

With bare vertex
(ETP is available, the PB is shown as the dot-dashed line)




With Ball-Chiu vertex
(ETP is not available, but the coexistence region is obtained)

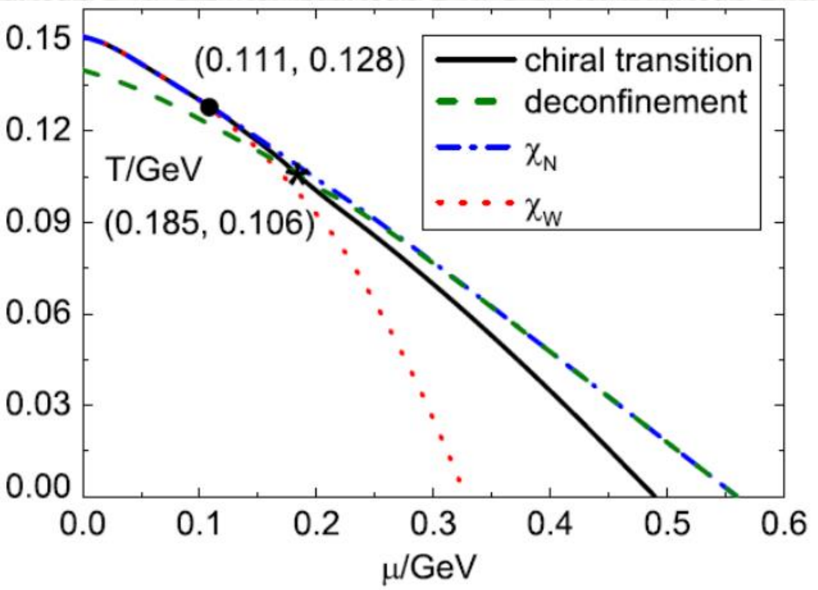



S.X. Qin, L. Chang, H. Chen, YXL, et al., Phys. Rev. Lett. 106, 172301 (2011).

# ♠ QCD Phase Diagrams and the position of the CEP have been given in the DSE

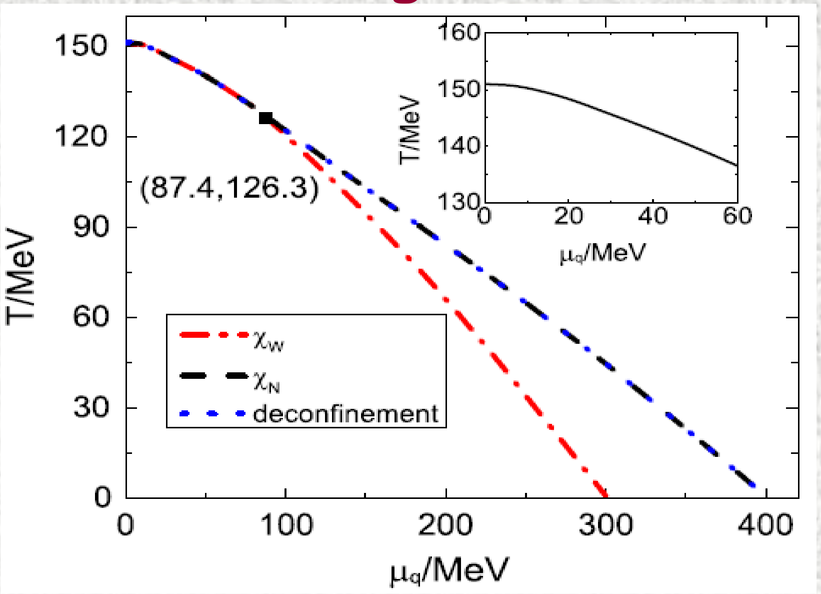
- **Beyond chiral limit**

**With bare vertex**
(ETP is available, the PB is shown as the dot-dashed line)




F. Gao, J. Chen, Y.X. Liu, et al., Phys. Rev. D 93, 094019 (2016).

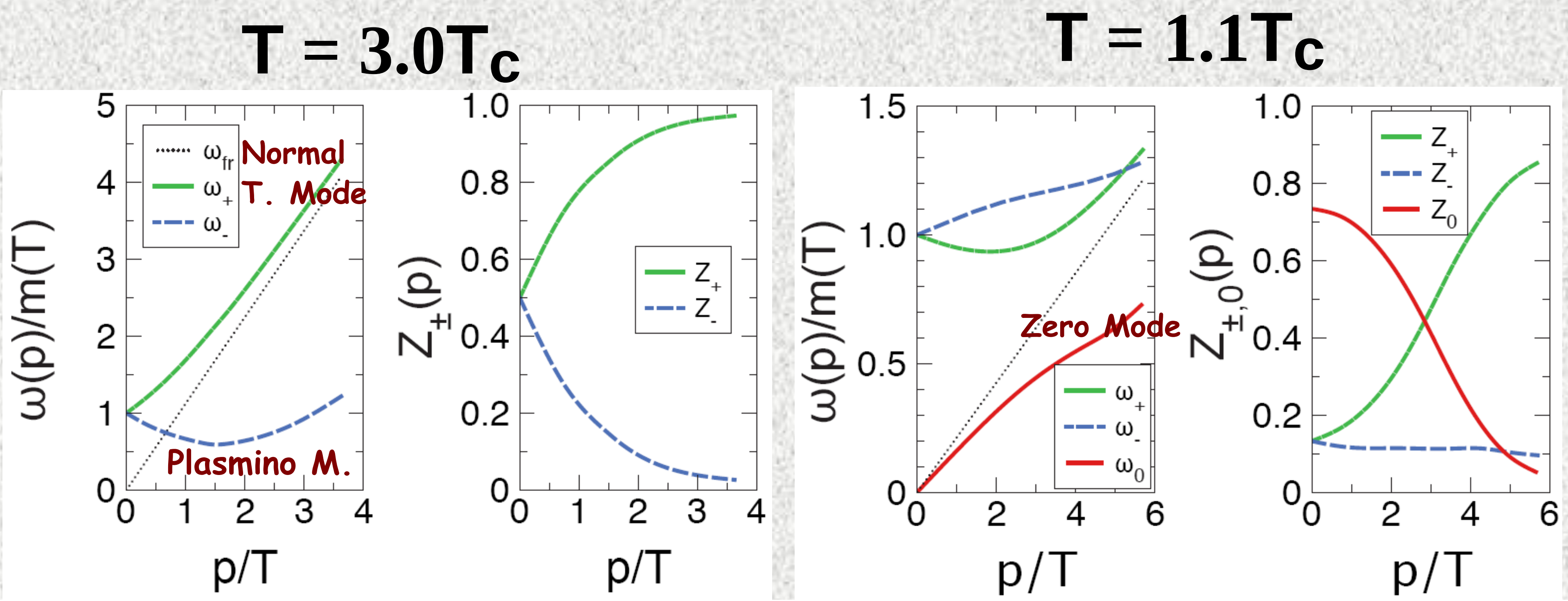
**With CLR vertex**
(ETP is not available, but the coexistence region is obtained)




F. Gao, Y.X. Liu, Phys. Rev. D 94, 076009 (2016).

## ♠ Taking the MEM to analyzing the spectral density function of the Quasi-particles' poles

## ➔ the matter at $T \in (1,1.5)Tc$ is in sQGP state



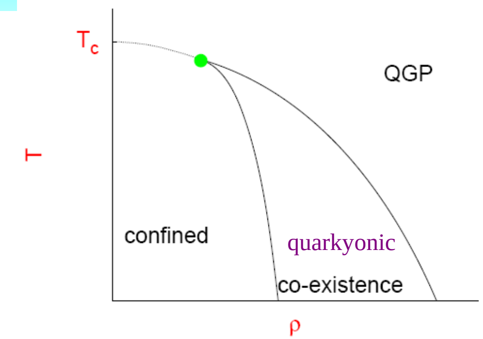

- **The zero mode exists at low momentum**($<7.0T_C$), **and is long-range correlation** ($\lambda \sim \omega^{-1} > \lambda_{FP}$) .

**S.X. Qin, L. Chang, Y.X. Liu, C.D. Roberts, PRD 84, 014017 (2011); F. Gao, S.X. Qin, Y.X. Liu, C.D. Roberts, PRD 89, 076009 (2014).**

# ♠ Relation between the Chiral PT & the Confinement-Deconfinement PT

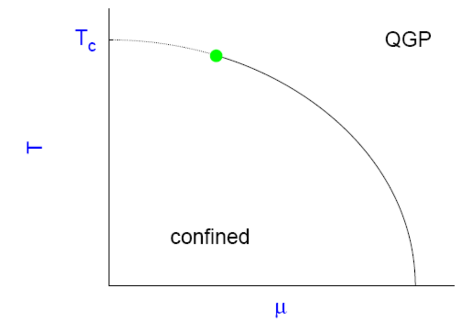
♣ **Lattice QCD Calculation**

de Forcrand, et al.,
Nucl. Phys. B Proc. Suppl. 153, 62 (2006); ···




and **General (large-$N_C$) Analysis**

McLerran, et al., NPA 796, 83 (‘07);
NPA 808, 117 (‘08);
NPA 824, 86 (‘09), ···




**claim that there exists a quarkyonic phase.**

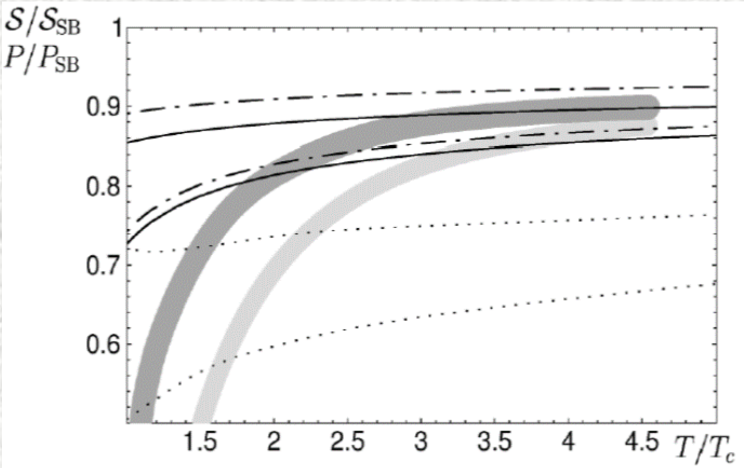
♣ **Coleman-Witten Theorem** (PRL 45, 100 (‘80)):

**Confinement coincides with DCSB !!**

♣ **Inconsistence really exists?!**
**Nature of the Quarkyonic Phase ?!**

# ♣ Hadronization Process

- For the system at **thermodynamic limit**,




J.-P. Blaizot, et al., PRL 83, 2906 (1999);

etc.

**The monotonic behavior manifests that the entropy density of the quark-gluon phase is always larger than that of hadron phase.**

(Nonaka, et al., Phys. Rev. C **71**, 051901R (2005) ; tec.,)

**---- Entropy Puzzle.**

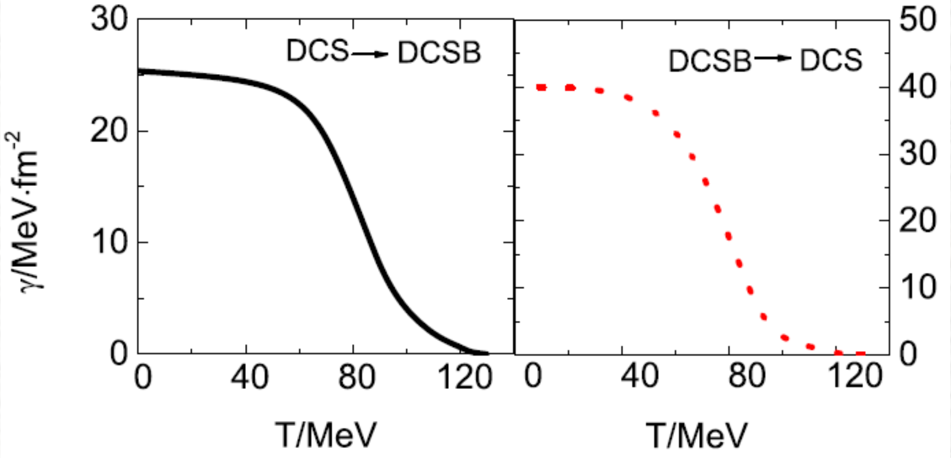
**In fact, there exists finite interface, which contributes to the entropy of the system.**

# ♣ Solving the entropy puzzle

- Interface tension between the DCS-unconf. phase and the DCSB-confined phase

$$\gamma(T) = \int_{n_L}^{n_H} \sqrt{\frac{C}{2}\Delta F_T(n)}dn \ .$$

**J. Randrup, PRC 79, 054911 (2009)**




Parameterized as $\gamma(T) = a + be^{(c/T + d/T^2)}$,

with parameters

| | $a/(\mathrm{MeV/fm^2})$ | $b/(\mathrm{MeV/fm^2})$ | $c/$ MeV | $d/\mathrm{GeV^2}$ |
|---|---|---|---|---|
| DCS → DCSB | 25.4 | −1.5 | 736 | −0.048 |
| DCSB → DCS | 40.0 | −8.1 | 399 | −0.025 |

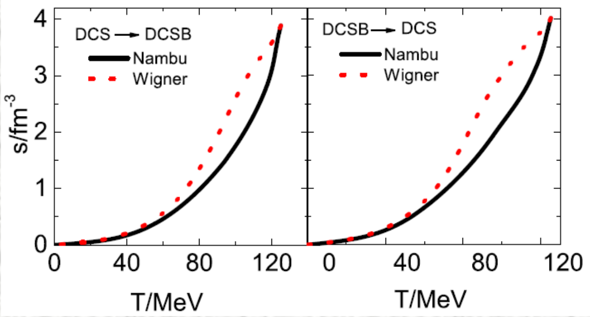
Fei Gao, & Yu-xin Liu， Phys. Rev. D 94, 094030 (2016)

# • Interface eff. in the hadronization Proc.

## Solving the entropy puzzle

**In thermodynamic limit**

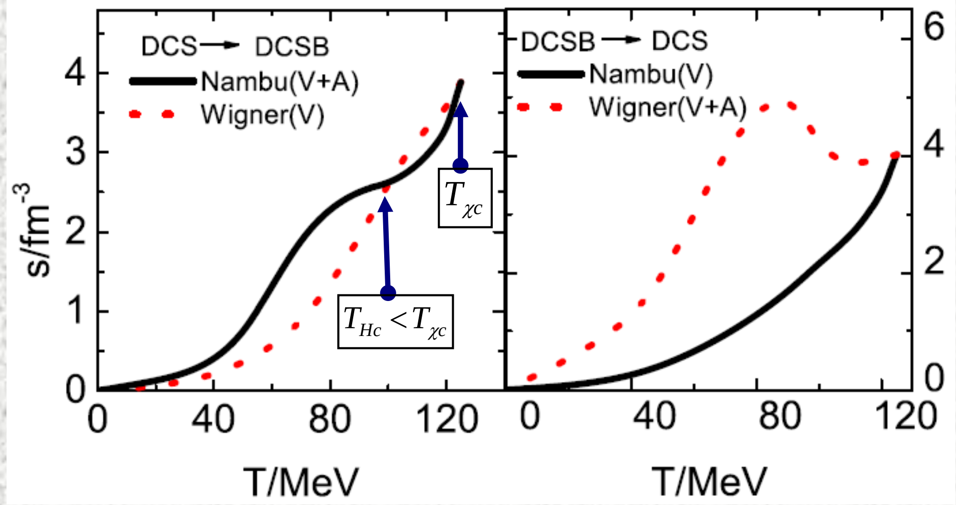
$$s_V = \frac{1}{T}(\epsilon + P - \mu n) = \frac{\partial P}{\partial T}.$$




**With the interface entropy density** $s_A = -(\frac{\partial \gamma}{\partial T})_{VA}$ **being included, we have**

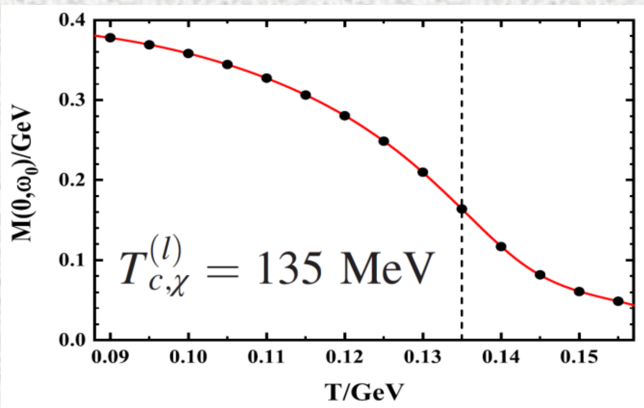
F. Gao, & Y.X. Liu, Phys. Rev. D 94, 094030 (2016);

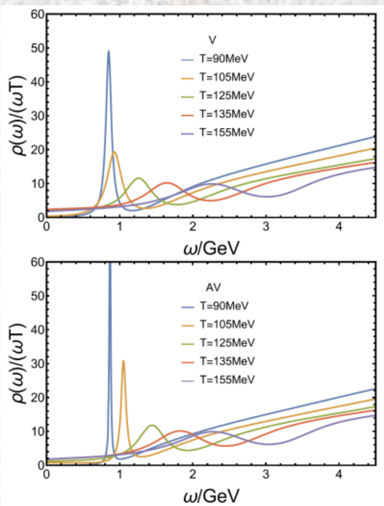
••••••

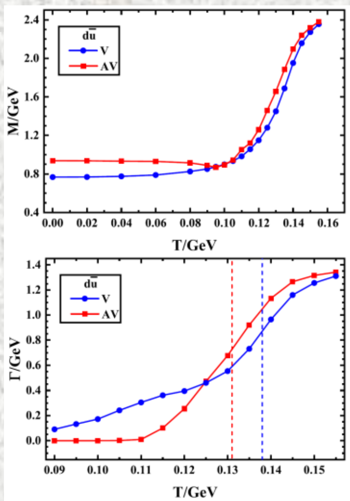


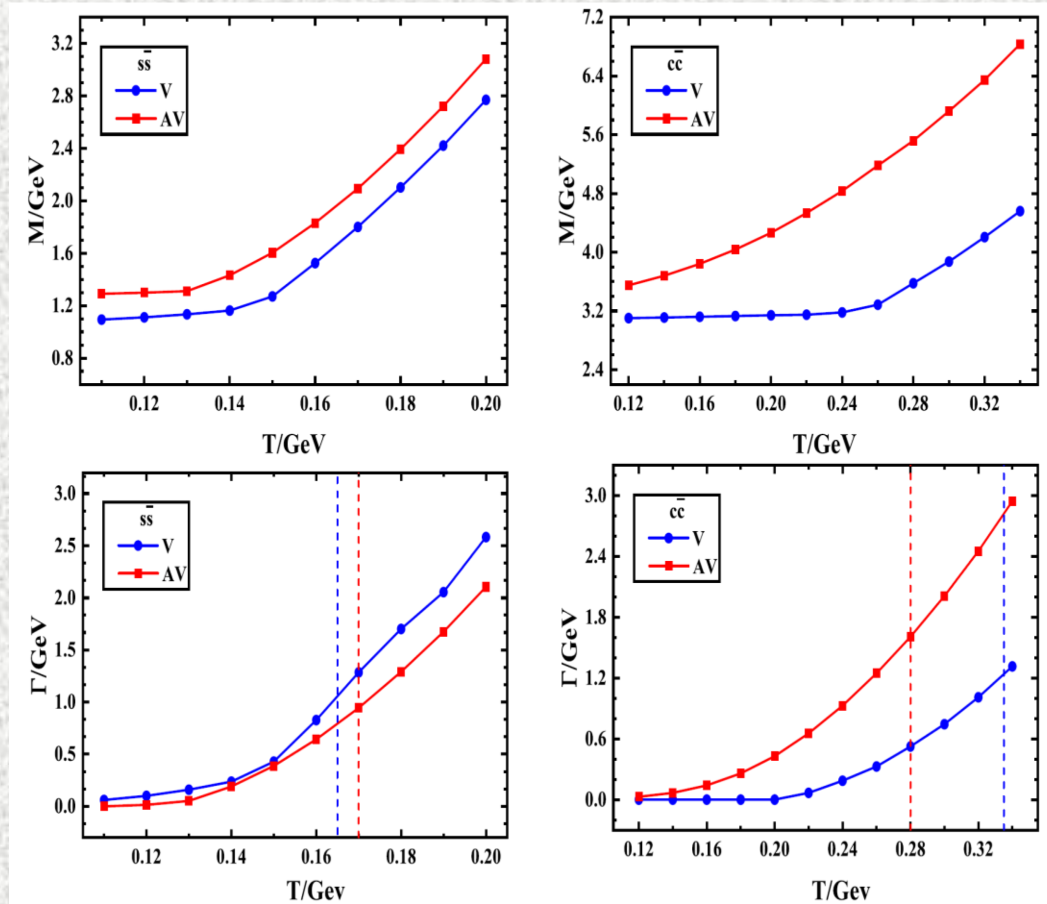

$T_{Hc} < T_{\chi c}$ means that the hadronization (confinement) temperature may really be different from the chiral pseudocritical temperature, which is just a demonstration of the Ultracold Phenomenon due to the interface.

# • Flavor-dependence of the dissociation Temperature & the relation with the $T_{c,\chi}$



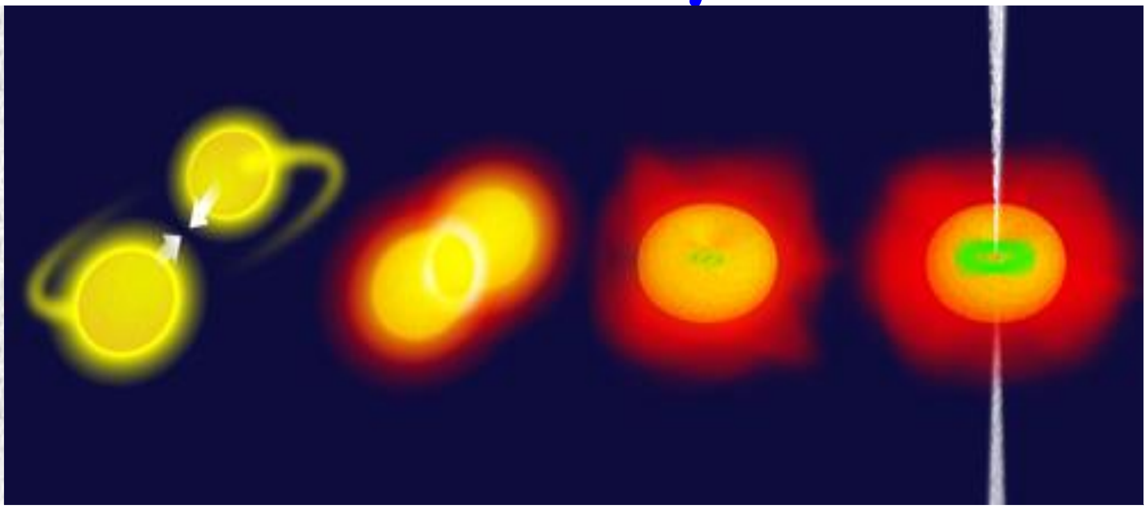




**Light flavor: coincident; Heavy flavor: $T_{c,d} > T_{c,\chi}$ !**

Ling-feng Chen, Si-xue Qin, and YXL, Phys. Rev. D 102, 054012 (2020)

# ♠ an Excellent Astronomic Observation Signal: Gravitational Mode Oscillation Frequency

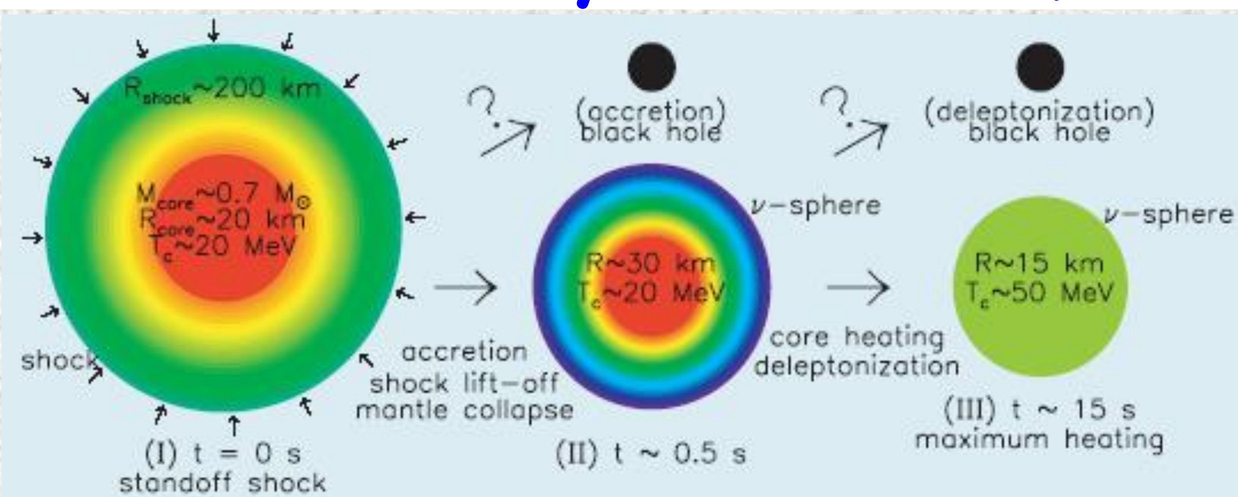
- **G-Wave in Binary Neutron Star Merger**

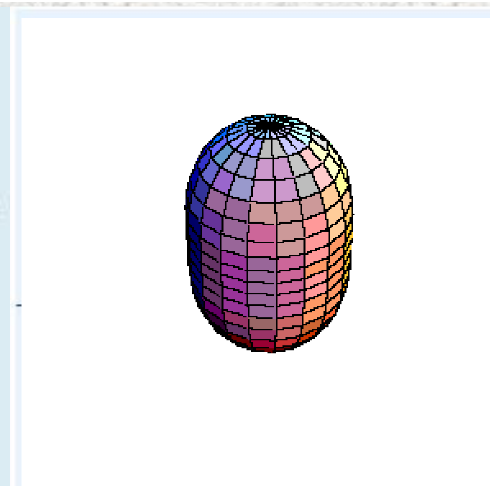


$F_{postmerger} \in (1.84, 3.73)$kHz, with width<200Hz, (PRD 86, 063001(2012))

$F_{spiral} < F_{postmerger}$

- **G-Wave in Newly Born NS/QS after the SNE**



# • Comparison of G-mode Oscillation Frequencies of the two kind nb Stars

Neutron Star: RMF, Quark Star: Bag Model

→ Frequency of the G-mode oscillation

| Radial order of $g$-mode | Neutron Star | | | Strange Quark Star | | |
|---|---|---|---|---|---|---|
| | $t=100$ | $t=200$ | $t=300$ | $t=100$ | $t=200$ | $t=300$ |
| $n=1$ | 717.6 | 774.6 | 780.3 | 82.3 | 78.0 | 63.1 |
| $n=2$ | 443.5 | 467.3 | 464.2 | 52.6 | 45.5 | 40.0 |
| $n=3$ | 323.8 | 339.0 | 337.5 | 35.3 | 30.8 | 27.8 |

W.J. Fu, H.Q. Wei, and Y.X. Liu, arXiv: 0810.1084, Phys. Rev. Lett. 101, 181102 (2008)

# • Comparison with other modes

Neutron Star: RMF, Quark Star: Bag Model

→ Frequencies of the f- & p-mode oscillations

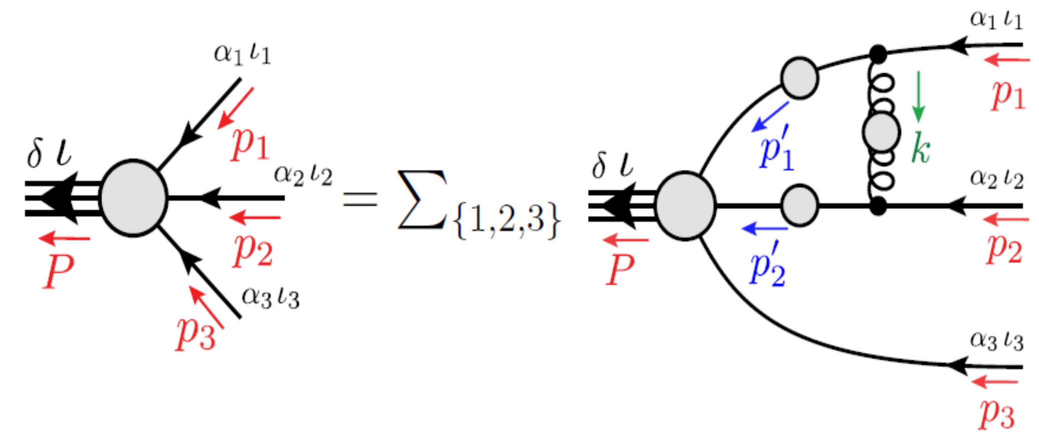
| Modes | Neutron Star | | | Strange Quark Star | | |
|---|---|---|---|---|---|---|
| | $t=100$ | $t=200$ | $t=300$ | $t=100$ | $t=200$ | $t=300$ |
| ${}_2f$ | 1103 | 1133 | 1176 | 2980 | 2997 | 3016 |
| ${}_2p_1$ | 2265 | 2426 | 2494 | 18282 | 17330 | 16792 |
| ${}_2p_2$ | 3780 | 4054 | 4179 | 28792 | 27288 | 26438 |
| ${}_2p_3$ | 5319 | 5702 | 5869 | 38988 | 36950 | 35798 |

♣ G-mode oscillation in quark star has very low freq. !

W.J. Fu, Z. Bai, Y.X. Liu, arXve:1701.00418

# 2. Hadron Mass Spectrum

## (1) Nucleon- Relativistic there body problem, Poincare covariant Faddeev Eq.




$$\Psi^{\alpha_1\alpha_2\alpha_3,\delta}_{\iota_1\iota_2\iota_3,\iota}(p_1, p_2, p_3) = \sum_{j=1,2,3} [\mathcal{K}SS\Psi]_j,$$

$$[\mathcal{K}SS\Psi]_3 = \int_{dk} \mathcal{K}^{\alpha_1\alpha_1',\alpha_2\alpha_2'}_{\iota_1\iota_1'\iota_2\iota_2'}(p_1, p_2; p_1', p_2') S^{\alpha_1'\alpha_1''}_{\iota_1'\iota_1''}(p_1') S^{\alpha_2'\alpha_2''}_{\iota_2'\iota_2''}(p_2') \Psi^{\alpha_1''\alpha_2''\alpha_3;\delta}_{\iota_1''\iota_2''\iota_3;\iota}(p_1', p_2', p_3),$$

$$\mathcal{K}_{\alpha_1\alpha_1',\alpha_2\alpha_2'} = \mathcal{G}_{\mu\nu}(k)[i\gamma_\mu]_{\alpha_1\alpha_1'}[i\gamma_\nu]_{\alpha_2\alpha_2'},$$

$$\mathcal{G}_{\mu\nu}(k) = \tilde{\mathcal{G}}(k^2)T_{\mu\nu}(k), \quad k^2T_{\mu\nu}(k) = k^2\delta_{\mu\nu} - k_\mu k_\nu.$$

$$\frac{1}{Z_2^2}\tilde{\mathcal{G}}(s) = \frac{8\pi^2}{\omega^4}De^{-s/\omega^2} + \frac{8\pi^2\gamma_m\mathcal{F}(s)}{\ln[\tau + (1 + s/\Lambda_{QCD}^2)^2]},$$

- **Numerical results in RL approximation**

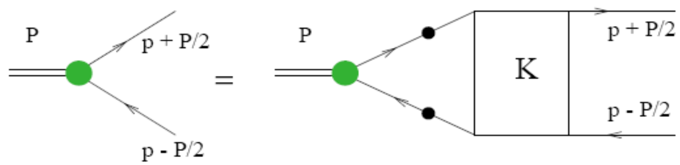
| Baryon | quarks | Empirical [90] | | | Herein | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $(0, \frac{1}{2}^+)_3$ | $(1, \frac{1}{2}^+)_4$ | $(0, \frac{1}{2}^-)_5$ | $(0, \frac{1}{2}^+)_6$ | $(1, \frac{1}{2}^+)_7$ | $(0, \frac{1}{2}^-)_8$ | $(1, \frac{1}{2}^+)_9^*$ | $(0, \frac{1}{2}^-)_{10}^*$ |
| $N$ | *uud* | 0.938 | 1.440 | 1.535 | 0.948 | 1.279 | 1.144 | 1.440 | 1.542 |
| $\Lambda$ | *uds* | 1.116 | 1.600 | 1.670 | 1.114 | 1.474 | 1.316 | 1.582 | 1.581 |
| $\Sigma$ | *uus* | 1.189 | 1.660 | 1.620 | 1.114 | 1.474 | 1.316 | 1.582 | 1.581 |
| $\Xi$ | *uss* | 1.315 | | | 1.279 | 1.670 | 1.487 | 1.723 | 1.620 |
| $\Lambda_c$ | *udc* | 2.286 | | 2.595 | 2.184 | 2.543 | 2.401 | 2.650 | 2.666 |
| $\Sigma_c$ | *uuc* | 2.455 | | | 2.184 | 2.543 | 2.401 | 2.650 | 2.666 |
| $\Lambda_b$ | *udb* | 5.619 | | 5.912 | 5.394 | 5.809 | 5.650 | 5.916 | 5.916 |
| $\Sigma_b$ | *uub* | 5.811 | | | 5.394 | 5.809 | 5.650 | 5.916 | 5.916 |
| $\Xi_c$ | *usc* | 2.468 | | 2.790 | 2.350 | 2.738 | 2.572 | 2.792 | 2.705 |
| $\Xi_c'$ | *usc* | 2.577 | | | 2.350 | 2.738 | 2.572 | 2.792 | 2.705 |
| $\Xi_{cc}$ | *ucc* | 3.621 | | | 3.421 | 3.807 | 3.657 | 3.861 | 3.790 |
| $\Xi_b$ | *usb* | 5.792 | | | 5.560 | 6.004 | 5.822 | 6.058 | 5.955 |
| $\Xi_b'$ | *usb* | 5.945 | | | 5.560 | 6.004 | 5.822 | 6.058 | 5.955 |
| $\Xi_{cb}$ | *ucb* | | | | 6.631 | 7.073 | 6.907 | 7.127 | 7.040 |
| $\Xi_{cb}'$ | *ucb* | | | | 6.631 | 7.073 | 6.907 | 7.127 | 7.040 |

S.X. Qin, C.D. Roberts, S.M. Schmidt, Few-body Syst. 60, 26 (2019); S.X. Qin, et al. Phys. Rev. D 97,114017('18); etc.

# (2) Meson-Poincare covariant BSE with DSE

Quantum field theory bound states: BSE

$$\Gamma_M(p;P) = \int_k^{\Lambda} K(p,k;P)\, S(k_+)\, \Gamma_M(k;P)\, S(k_-)$$




**L. Chang, C.D. Roberts, PRL 103, 081601 (2009); ……**

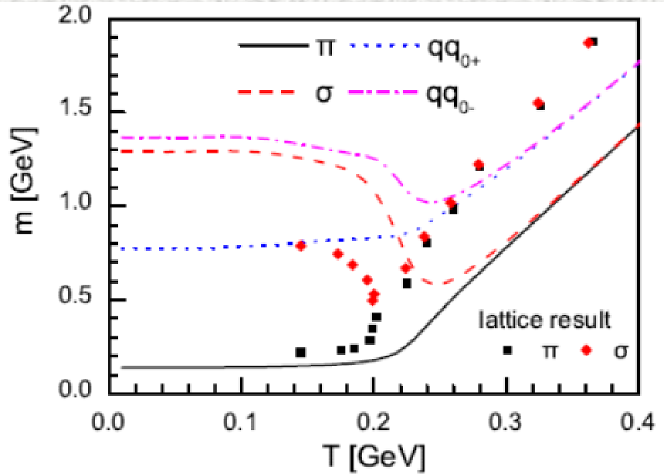
| Interaction | Eq. (6) | Eq. (8) | Eq. (8) | Eq. (8) | Eq. (8) |
|---|---|---|---|---|---|
| $(D\omega)^{1/3}$ | 0.72 | 0.8 | 0.8 | 0.8 | 0.8 |
| $\omega$ | 0.4 | 0.4 | 0.5 | 0.6 | 0.7 |
| $m_{u,d}^{\zeta}$ | 0.0037 | 0.0034 | 0.0034 | 0.0034 | 0.0034 |
| $m_s^{\zeta}$ | 0.084 | 0.082 | 0.082 | 0.082 | 0.082 |
| $A(0)$ | 1.58 | 2.07 | 1.70 | 1.38 | 1.16 |
| $M(0)$ | 0.50 | 0.62 | 0.52 | 0.42 | 0.29 |
| $m_\pi$ | 0.138 | 0.139 | 0.134 | 0.136 | 0.139 |
| $f_\pi$ | 0.093 | 0.094 | 0.093 | 0.090 | 0.081 |
| $\rho_\pi^{1/2}$ | 0.48 | 0.49 | 0.49 | 0.49 | 0.48 |
| $m_K$ | 0.496 | 0.496 | 0.495 | 0.497 | 0.503 |
| $f_K$ | 0.11 | 0.11 | 0.11 | 0.11 | 0.10 |
| $\rho_K^{1/2}$ | 0.54 | 0.55 | 0.55 | 0.55 | 0.55 |
| $m_\rho$ | 0.74 | 0.76 | 0.74 | 0.72 | 0.67 |
| $f_\rho$ | 0.15 | 0.14 | 0.15 | 0.14 | 0.12 |
| $m_\phi$ | 1.07 | 1.09 | 1.08 | 1.07 | 1.05 |
| $f_\phi$ | 0.18 | 0.19 | 0.19 | 0.19 | 0.18 |
| $m_\sigma$ | 0.67 | 0.67 | 0.65 | 0.59 | 0.46 |
| $\rho_\sigma^{1/2}$ | 0.52 | 0.53 | 0.53 | 0.51 | 0.48 |

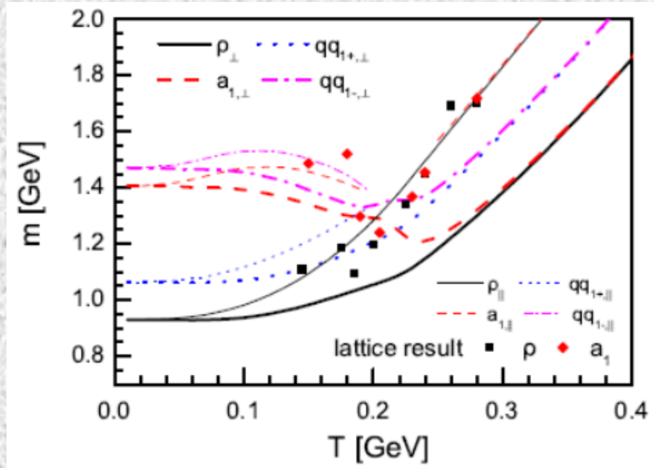
(S.X. Qin, et al., Phys. Rev. C 84, 042202(R) (2011) ,

| with CLR vertex | | Expt. | RL-Padé | RL-direct |
|---|---|---|---|---|
| $m_\pi$ | 0.138 | 0.138 | 0.138 | 0.137 |
| $m_\rho$ | $0.84 \pm 0.03$ | 0.777 | 0.754 | 0.758 |
| $m_\sigma$ | $1.13 \pm 0.01$ | $0.4 - 1.2$ | 0.645 | 0.645 |
| $m_{a_1}$ | $1.28 \pm 0.01$ | $1.24 \pm 0.04$ | 0.938 | 0.927 |
| $m_{b_1}$ | $1.24 \pm 0.10$ | $1.21 \pm 0.02$ | 0.904 | 0.912 |
| $m_{a_1} - m_\rho$ | $0.44 \pm 0.04$ | $0.46 \pm 0.04$ | 0.18 | 0.17 |
| $m_{b_1} - m_\rho$ | $0.40 \pm 0.14$ | $0.43 \pm 0.02$ | 0.15 | 0.15 |

( L. Chang, et al., Phys. Rev. C 85, 052201(R) (2012) )

# (3) Identifying the chiral phase transition with the screening masses of some hadrons

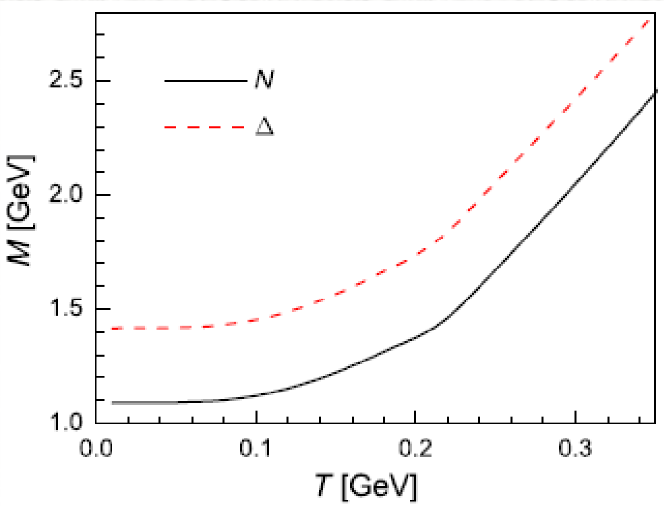



**GT Relation**

$$M_\sigma^2 = M_\pi^2 + 4M_q^2$$

➔ $M_\sigma \cong M_\pi$ can be a signal of the DCS.




$r_S \propto 1/M_S$ , when $r_S < r_{md}$ , the color gets deconfined.

**Hadron properties provide signals for not only the chiral phase transt. but also the confinement-deconfnmt. phase transition.**

Wei-jie Fu, and Yu-xin Liu, Phys. Rev. D 79, 074011 (2009);
K.L. Wang, Y.X. Liu, C.D. Roberts, Phys. Rev. D 87, 074038 (2013);

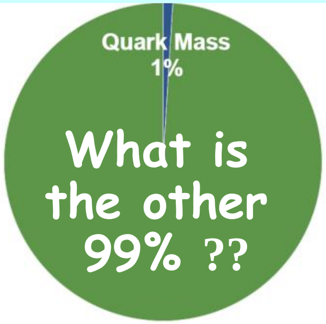
# IV. Summary & Remarks

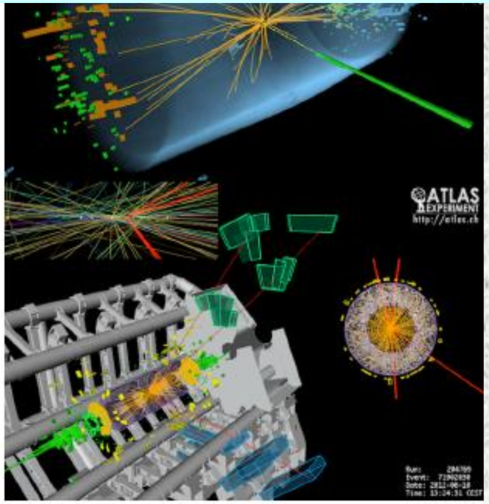
- ♠ The framework of QCD is surveyed,
- ♠ DS equations, — A continuum npQCD approach is presented.
- ♠ The observable mass generation & confinement are described with the DSE approach.
- ♠ Hadron mass spectra are described with the DSE approach.
- ♠ Continuum QCD has contributed great to fundamental issues of physics, and is promising! ……

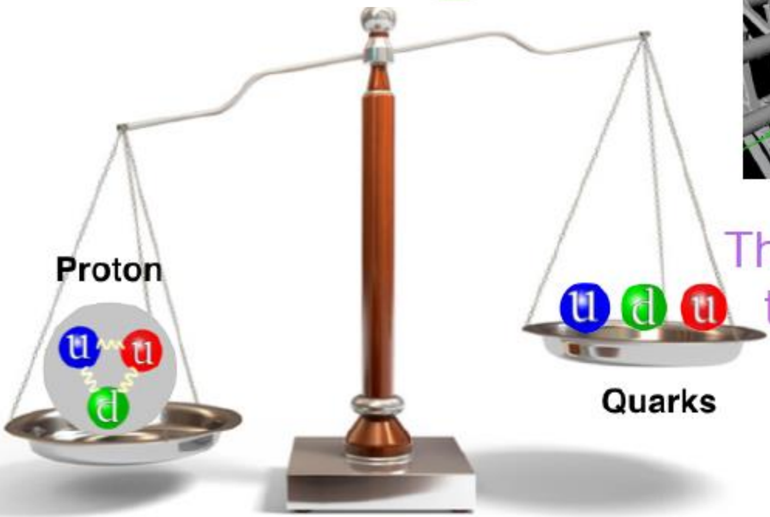
**Thanks !**

# ♠ The Nucleon Mass Crisis

How does the mass of nucleon arise ??




The Higgs boson make the u/d quark having masses (2GeV MS-bar):

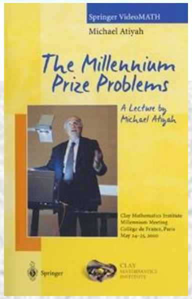
$m_u = 2.08(09)$ MeV
$m_d = 4.73(12)$ MeV

http://flag.unibe.ch/2019/Quark%20masses

But the mass of the proton is ***938.272046(21) MeV.***

**~100 times of the sum of the quark masses!**

“核子质量起源”与“夸克囚禁”共同构成一个世纪大奖问题！



# ♠ 电弱统一作用的规范对称性

- 弱相互作用

实验发现，原子核会释放出电子、等轻子— $\beta$ 衰变。

实际过程是 $n \longrightarrow p + e^{-} + \overline{\nu}_e$，等。

核子二重态— 同位旋二重态之间转变：

记中子 $|n\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$， 质子 $|p\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$，

同位旋生降算符为 $T_{\pm} = T_1 \pm iT_2 = \frac{1}{2}(\sigma_1 \pm i\sigma_2)$，

则 $|n\rangle = T_{+}|p\rangle$， $|p\rangle = T_{-}|n\rangle$ .

唯象理论：矢量流–轴矢流理论

$$\mathcal{L}_{eff} = -2\sqrt{2}G J_{\mu}^{\dagger}(x) J^{\mu}(x),$$

其中 $G = 1.165 GeV^{-2}$，常称为费米 $\beta$ 衰变常数，

$J_{\mu}$(带电流，矢量流与轴矢流的叠加) 为

$$J_{\mu} = \overline{\psi}^{f} \gamma^{\mu} T_{-} \psi^{f} .$$

# ♠ 电弱统一作用的规范对称性

- 弱相互作用

夸克层次上，$\beta$ 衰变过程为：

$$d \longrightarrow u + e^- + \bar{\nu}_e,$$ 等。

$\Lambda \longrightarrow p + \mu^- + \bar{\nu}_\mu$ 等则为

$$s \longrightarrow u + \mu^- + \bar{\nu}_\mu.$$

➔ d 实际是 d 与 s 的混合态。

记混合角为 $\Theta_C$ (Cabibbo角)，则有

$$d' = d\cos\Theta_C + s\sin\Theta_C,$$

弱衰变中的费米子场一般标记为：

$$\psi_L^f = \frac{1}{2}(1+\gamma_5)\left\{\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}, \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}, \begin{pmatrix}u\\ d'\end{pmatrix}\right\}.$$

这里的同位旋不同于描述核子的同位旋，称为弱同位旋。

由 $[T_+, T_-] = 2T_3 = 2\sigma_3$ 知，与带电流相应，还应有中性流。

# ♠ 电弱统一作用的规范对称性

- 弱相互作用

实验发现奇异数不守恒的中性流转变$(K^0(d\bar{s}) \leftrightarrow \bar{K}^0(\bar{d}s))$振幅远小于弱作用过程的转变振幅，

➔ s 实际是 s 与 d 的混合态，即有

$$s' = s\cos\Theta_C - d\sin\Theta_C,$$

并且存在重夸克 c，即有第4组费米子二重态$\begin{pmatrix} c \\ s' \end{pmatrix}$.

在目前的六味三代层次上，还有$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$，$\begin{pmatrix} t \\ b \end{pmatrix}$.

与前述弱作用流相应的弱荷算符为：

$$\hat{T}_i^L = \int J_i^4 d^3x,$$

其间有李乘积关系

$$\left[\hat{T}_u^L, \hat{T}_v^L\right] = i\varepsilon_{uvw}\hat{T}_w^L,$$

其中$\varepsilon_{uvw}$为反对称张量。总之有弱同位旋对称性。

# ♠ 电弱统一作用的规范对称性

- 弱电统一作用及其对称性

$\beta$衰变过程和弱中性流转变过程显然还与电荷相关。

推广 Gel-Mann-Nishijima 关系 $Y = Q - T_3$

至电荷、弱同位旋和弱荷，定义弱超荷算符 $\hat{Y} = \hat{Q} - \hat{T}_3^L$，

前述的左手轻子都为该算符的本征态，并且 $\left[\hat{T}_u^L, \hat{Y}\right] = 0$。

这表明弱电统一作用具有 $SU_L(2) \otimes U_Y(1)$ 对称性，

弱作用过程的弱荷算符和包含电荷的弱超荷算符为其生成元。

记相应的四个规范场为 $W^a (a = 1,2,3)$ 和 $B$，

它们与 $SU_L(2) \otimes U_Y(1)$ 的流间的耦合为 $gJ_\mu^a W_\mu^a + g' J_\mu^Y B_\mu$，

$W_\mu^{1,2}$ 线性组合的荷电规范玻色子 $W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \pm iW_\mu^2)$，

与光子场相应的 $W^3$ 与 $B$ 线性叠加的电中性的 $Z$ 玻色子，

并有温伯格角 $\Theta_W$ 表征其耦合 $(\tan\Theta_W = g'/g)$。 1982年证实。