量子动力论

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我将讨论以下非平庸问题:

- 自旋分布
- 超出准粒子
- 驰豫时间近似
- 局域与整体平衡

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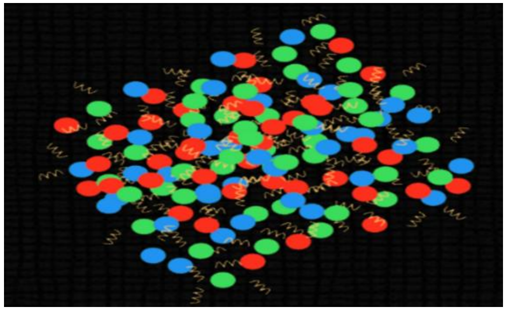
Introduction

Classical Boltzmann equation



$$\left(p^{\mu}\partial_{\mu}^{x} + F^{\mu}\partial_{\mu}^{p}\right)f(x,p) = C$$

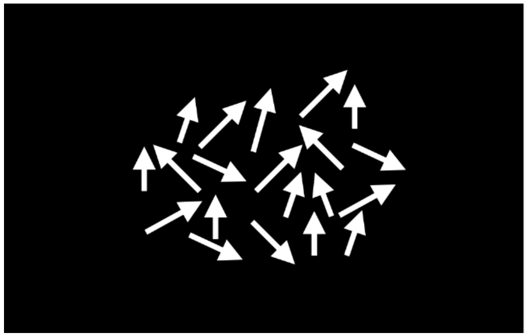
1) A single distribution (particle number) f

2) Quasi-particle (on-shell) approximation:

$$(p^2 - m^2)f(x,p) = 0 \rightarrow \; f(x,\vec{p}|p_0 = \pm\sqrt{\vec{p}^2 + m^2})$$

Spin

Spin is a pure quantum quantity. Many quantum anomalies in science are induced by spin, for instance the CME and CVE in high-energy nuclear collisions..



	Schroedinger equation (without spin)	**Dirac equation (with spin)**
Wave function:	scalar $\psi(x)$	spinor $\psi(x) = \begin{pmatrix}\psi_1(x)\\ \psi_2(x)\\ \psi_3(x)\\ \psi_4(x)\end{pmatrix}$
Probability:	$\psi(x)\psi^+(x)$	$\psi(x)\bar{\psi}(x), \quad a\ 4\times 4\ matrix$
Distribution in phase space:	$f(x,p) = \int d^4y e^{ipy}\psi\left(x+\frac{y}{2}\right)\psi^+\left(x-\frac{y}{2}\right)$	$W(x,p) = \int d^4y e^{ipy}\,\psi\left(x+\frac{y}{2}\right)\bar{\psi}\left(x-\frac{y}{2}\right)$

Wigner function, a matrix, 16 distributions

Beyond quasi-particle

Particles in medium are in general not quasi-particles (off-shell effect), especially in high energy physics.

1) On-shell → Off-shell:

$$(p^2 - m^2)f(x,p) = 0 \rightarrow [(p^2 - m^2) + \hbar\mathcal{A}(p)\,]W(x,p) = 0$$

2) Physics distributions $W(x,\vec{p})$ *are defined in* ***7D*** *phase space*

→ Equal-time formalism

$$W(x,\vec{p}) = \int dp_0 W(x,p)$$

Question:

How to self-consistently and completely construct a quantum kinetic theory, including spin and off-shell effect ?

General formalism:

Covariant & equal-time kinetic equations

Wigner functions

- *Covariant Wigner operator for fermion field interacting with a gauge field:*

$$\widehat{W}(x,p) = \int d^4y e^{ipy} \hat{\psi}(x+\frac{y}{2}) e^{iq\int_{-1/2}^{1/2} ds \hat{A}(x+sy)y} \hat{\bar{\psi}}(x-\frac{y}{2})$$

gauge link $e^{iq\int_{-1/2}^{1/2} ds \hat{A}(x+sy)y}$ *to guarantee gauge invariance*

Wigner function

$$W(x,p) = \langle \widehat{W}(x,p) \rangle \qquad (W = \widehat{W} \text{ in quantum mechanics}), \qquad 4\times 4 \text{ matrix}$$

- *Dyson-Schwinger equation for quantum fields* $\hat{\psi}$ *or Dirac equation for wave function* ψ

→ kinetic equations for $W(x,p)$

QED: D.Vasak, M.Gyulassy and H.-Th.Elze, Ann. Phys. 173, 462(1987)
QCD: H.-Th.Elze and U.Heinz, Phys. Rep. 183, 81(1989)

- *Problem:*

Initial $W(x,p)$ *is related to the fields* $\hat{\psi}(x)$ *and* $\hat{A}(x)$ *at all times (due to* $\int_{-\infty}^{\infty} dy_0$*)*
→ In general the covariant kinetic equation cannot be solved as an initial value problem !

- *Physics distributions are defined in* ***7D*** *phase space → Equal-time Wigner function*

$$W_0(x,\vec{p}) = \int d^3\vec{y} e^{-i\vec{p}\cdot\vec{y}} \left\langle \hat{\psi}(x+\vec{y}/2) e^{-iq\int_{-1/2}^{1/2} ds \hat{\vec{A}}(x+s\vec{y})\cdot\vec{y}} \hat{\psi}^+(x-\vec{y}/2) \right\rangle$$

Dirac-Heisenberg-Wigner equation

I.Bialynicki-Birula, P.Gornicki and J.Rafelski, PRD44, 1825(1991)

- *Fermions in external electromagnetic field (quantum mechanics system)*

$$\left(i\gamma^{\mu}\mathcal{D}_{\mu}-m\right)\psi(x)=0$$

→ *DHW transport equation:*

$$D_t W_0=-\frac{1}{2}\vec{D}\cdot\{\rho_1\vec{\sigma},W_0\}-\frac{i}{\hbar}\left[\rho_1\vec{\sigma}\cdot\vec{P}+\rho_3 m,W_0\right]$$

$$D_t=\partial_t+q\int_{-1/2}^{1/2}ds\vec{E}(\vec{x}+is\hbar\vec{\nabla}_p)\cdot\vec{\nabla}_p,$$
$$\vec{D}=\vec{\nabla}+q\int_{-1/2}^{1/2}ds\vec{B}(\vec{x}+is\hbar\vec{\nabla}_p)\times\vec{\nabla}_p$$
$$\vec{P}=\vec{p}-iq\hbar\int_{-1/2}^{1/2}dss\vec{B}(\vec{x}+is\hbar\vec{\nabla}_p)\times\vec{\nabla}_p$$

- *However, $W_0(x,\vec{p})=\int dp_0 W(x,p)\gamma_0$ is not equivalent to $W(x,p)$, some thing is missed!*

PZ and U.Heinz, Ann.Phys.245, 311(1996)

We must consider all the energy moments

$$W_n(x,\vec{p})=\int dp_0 p_0^n W(x,p)\gamma_0 \qquad (n=0,1,2,\ldots).$$

- *Only when particles are quasi-particles (on-shell, $p^2-m^2=0,\ p_0=\pm\sqrt{m^2+\vec{p}^2}$),*

$$W_n\,(x,\vec{p})=E_p^n W_0(x,\vec{p}),$$

$W_0(x,\vec{p})$ is enough to describe the system.

From covariant to equal-time kinetic equations I

PZ and U.Heinz, PRD57, 6525(1998)

以电磁场中的量子力学系统为例。

1) Covariant kinetic equations

$$(\gamma^\mu K_\mu - m)W = 0$$

$$K_\mu = \Pi_\mu + \frac{i\hbar}{2} D_\mu \qquad \Pi_\mu = p_\mu - iq\hbar \int_{-1/2}^{1/2} ds\, s F_{\mu\nu}(x - i\hbar s \partial_p)\partial_p^\nu$$

$$D_\mu = \partial_\mu - q\int_{-\frac{1}{2}}^{\frac{1}{2}} ds F_{\mu\nu}(x - i\hbar s\partial_p)\partial_p^\nu$$

Constraint (with p_μ) and transport (with ∂_μ) equations

$$\begin{cases} [\gamma^\mu(K_\mu + K_\mu^+) - 2m]W(x,p) = 0 \\ \gamma^\mu(K_\mu - K_\mu^+)W(x,p) = 0 \end{cases}$$

2) Equal-time hierarchy for $W_n(x,\vec{p})$

p_0-integrating the covariant equations:

$$\begin{cases} Transport\ equations\ for\ W_0(x,\vec{p}) & \rightarrow\ DHW\ equation \\ Constraint\ equation\ for\ W_1(x,\vec{p}) \end{cases}$$

p_0-integrating $p_0 \cdot$ (covariant equations):

$$\begin{cases} Transport\ equations\ for\ W_1(x,\vec{p}) \\ Constraint\ equation\ for\ W_2(x,\vec{p}) \end{cases}$$

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From covariant to equal-time kinetic equations II

PZ and U.Heinz, PRD57, 6525(1998)

3) *Spin decomposition*

$$W_0(x,\vec{p}) = \frac{1}{4}[f_0 + \gamma^5 f_1 - i\gamma^0\gamma^5 f_2 + \gamma^0 f_3 + \gamma^5\gamma^0\vec{\gamma}\cdot\vec{g}_0 + \gamma^0\vec{\gamma}\cdot\vec{g}_1 - i\vec{\gamma}\cdot\vec{g}_2 - \gamma^5\vec{\gamma}\cdot\vec{g}_3]$$

D.Vasak, M.Gyulassy and H.-Th.Elze, Ann. Phys. 173, 462(1987)

Conservation laws → Physics of the spin components

f_0 : *number density*, $\vec{g}_0$: *spin density*

f_1 : *helicity density* , f_2 : *topologic charge density*, f_3 : *mass density*

$\vec{g}_1$: *number current*, $\vec{g}_3$: *magnetic moment*

16 constraint equations + 16 transport equations for the 16 distributions

4) *Truncating the hierarchy*

spin 1/2 particles: W_0 *and* W_1 *form a closed subgroup*

spin 0 particles: W_0, W_1 *and* W_2 *form a closed subgroup*

Quantum mechanics systems:
Spin interaction with external fields

Spin interaction with electromagnetic field: $\hbar^0$

PZ and U.Heinz, PRD53, 2096(1996)

$$W_0(x,\vec{p}) = W_0^{(0)}(x,\vec{p}) + \hbar W_0^{(1)}(x,\vec{p}) + \cdots$$

- Constraint equations → only 4 independent components:
  number density f_0 and spin density $\vec{g}_0$

- Boltzmann equation for number density f_0:

$$\left(D_t + \frac{\vec{p}}{E_p} \cdot \vec{D}\right) f_0 = 0$$

$$D_t = \partial_t + q\vec{E} \cdot \vec{\nabla}_p, \qquad \vec{D} = \vec{\nabla} + q\vec{B} \times \vec{\nabla}_p$$

- Bargmann-Michel-Telegdi equation for spin density $\vec{g}_0$:

$$\left(D_t + \frac{\vec{p}}{E_p} \cdot \vec{D}\right) \vec{g}_0 = \frac{q}{E_p^2}\left[\vec{p} \times \left(\vec{E} \times \vec{g}_0\right) - E_p \vec{B} \times \vec{g}_0\right]$$

the phase-space version of the Bargmann-Michel-Telegdi equation to describe spin precession in electromagnetic fields

V.Bargmann, L.Michel and V.Telegdi, PRL2, 435(1959)

Spin interaction with electromagnetic field: $\hbar^1$

A.Huang, S.Shi, Y.Jiang, J.Liao and PZ, PRD98, 036010(2018)

CME: a chirality imbalance induced electric current in external magnetic field, a probe of nontrivial topology of QCD.

M.Stephanov and Y.Yin, PRL109, 162001 (2012)
D.Son and N.Yamamoto. PRD, 87, 85016(2013)
J.Chen, S.Pu, Q.Wang and X.Wang, PRL110, 26230
Y.Hidaka, S.Pu and D.Yang. PRD95, 091901(2017)
Wu, Hou, Ren, PRD 96 (2017)096015
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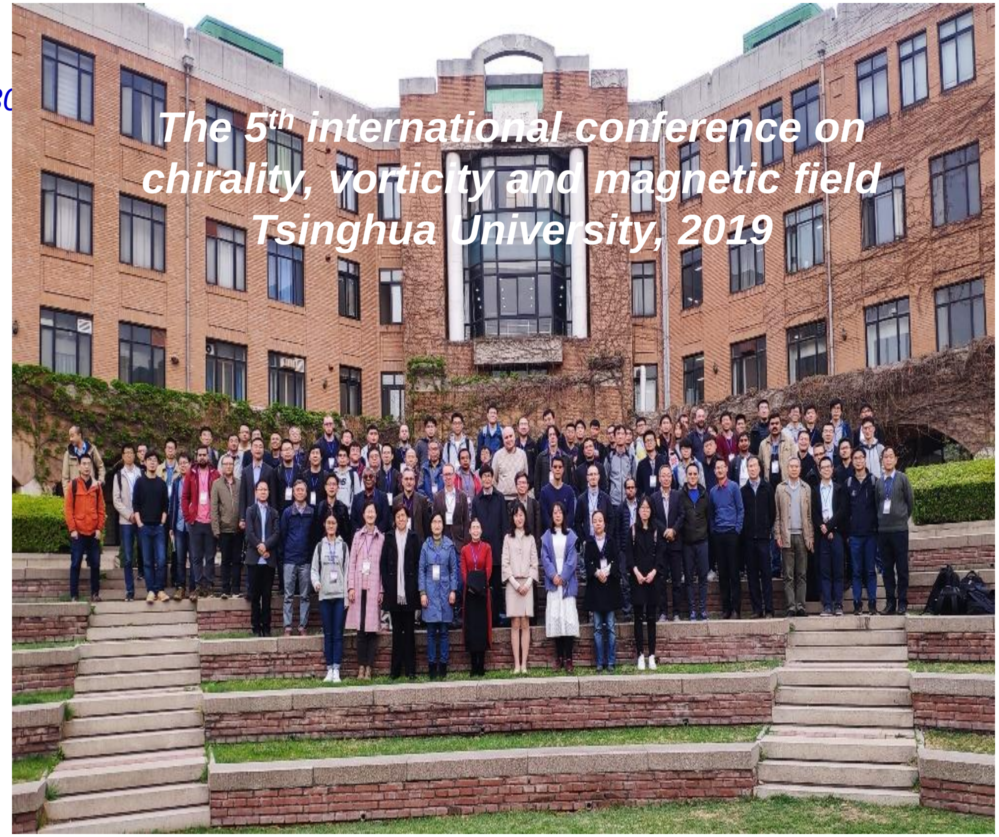


- transport equation for chiral fermions to the first order in $\hbar$:

$$f_\pm = f_0 \pm f_1$$

$$\begin{cases} (\partial_t + \dot{\vec{x}} \cdot \vec{\nabla} + \dot{\vec{p}} \cdot \vec{\nabla}_p) f_\pm = -\dfrac{f_\pm - f_\pm^{th}}{\tau} \\ \dot{\vec{x}} = \dfrac{1}{1 + q\vec{B} \cdot \vec{b}_\pm} \dfrac{\vec{p}}{|\vec{p}|} (1 + 2q\vec{B} \cdot \vec{b}_\pm) \\ \dot{\vec{p}} = \dfrac{1}{1 + q\vec{B} \cdot \vec{b}_\pm} q \dfrac{\vec{p}}{|\vec{p}|} \times \vec{B}, \qquad \vec{b}_\pm = \pm \dfrac{\vec{p}}{|\vec{p}|^3} \end{cases}$$

- Mass correction

$$\partial_t f_\chi + \dot{\vec{x}} \cdot \vec{\nabla} f_\chi + \dot{\vec{p}} \cdot \vec{\nabla}_p f_\chi = \frac{\hbar m}{\sqrt{G}} A[\vec{g}_0]$$

Z.Wang and PZ, PRD100, 014015(2019)

Charge separation

A.Huang, Y.Jiang, S.Shi, J.Liao and PZ, PLB777, 177(2018)

■ Analytic solution of the chiral transport equation

$$f_{\pm}(x,\vec{p}) = f_{\pm}(x_0,\vec{p}_0)e^{-\frac{t-t_0}{\tau}} + \frac{1}{\tau}\int_{t_0}^{t} f_{\pm}^{eq}(x',\vec{p}')e^{-\frac{t-t'}{\tau}}dt'$$

$\vec{B} = B(t)\vec{e}_y$

initial imbalance $f_{\pm}\sim 1 \pm \lambda$

■ Charge separation in nuclear collisions

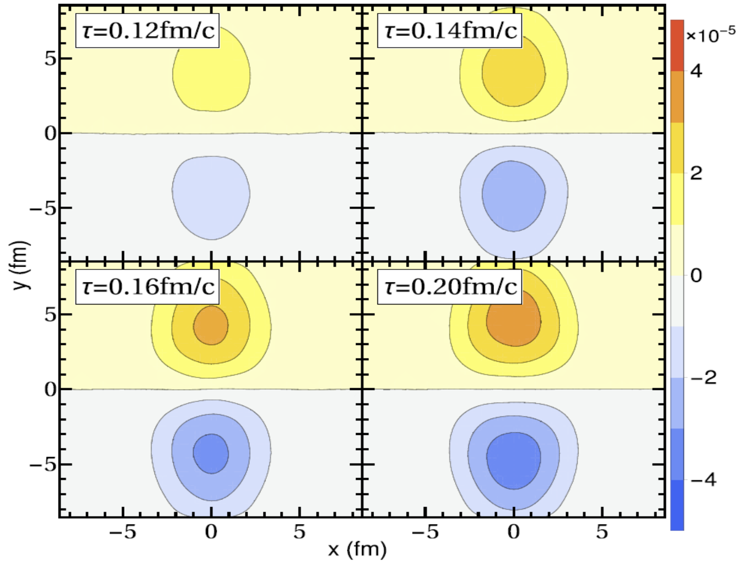


Fig. 3. Net charge density n^Q (normalized by ξQ_s^3) on the $x-y$ plane at different time (computed with FD initial distribution, ECHO magnetic field, $\tau_R = 0.1$ fm/c and $\lambda_5 = 0.2$).

Spin interaction with (external) rotational field

S.Chen, Z.Wang and PZ, arXiv: 2101.07596
Y.Jiang, X.Huang and J.Liao, Phys. Rev. D92, 071501(2015)
Y.Jiang, X.Guo and PZ, Chapter 6 of Strongly Interacting Matter under Rotation, Springer, 2021

- Dirac equation in rotational frame,

$$[i\gamma^\mu \partial_\mu + \gamma_0 \boldsymbol{\omega} \cdot \boldsymbol{J} - m]\, \psi = 0.$$

- Covariant kinetic equation

$$\left[\gamma^\mu K_\mu + \frac{\hbar}{2}\gamma^5 \gamma^\mu \omega_\mu - m\right] W(x,p) = 0,$$

- Equal-time transport equations at order $\hbar^0$

$$\left[\partial_t + \left(\pm\frac{\boldsymbol{p}}{\epsilon_p} + \boldsymbol{x}\times\boldsymbol{\omega}\right)\cdot\boldsymbol{\nabla} - (\boldsymbol{\omega}\times\boldsymbol{p})\cdot\boldsymbol{\nabla}_p\right] f_0^{(0)\pm} = 0,$$
$$\left[\partial_t + \left(\pm\frac{\boldsymbol{p}}{\epsilon_p} + \boldsymbol{x}\times\boldsymbol{\omega}\right)\cdot\boldsymbol{\nabla} - (\boldsymbol{\omega}\times\boldsymbol{p})\cdot\boldsymbol{\nabla}_p\right] \mathbf{g}_0^{(0)\pm} = -\boldsymbol{\omega}\times\mathbf{g}_0^{(0)\pm}.$$

- Equal-time (off-shell) transport equations at order $\hbar^1$

$$\left[\partial_t + \left(\pm\frac{\boldsymbol{p}}{\epsilon_p} + \boldsymbol{x}\times\boldsymbol{\omega}\right)\cdot\boldsymbol{\nabla} - (\boldsymbol{\omega}\times\boldsymbol{p})\cdot\boldsymbol{\nabla}_p\right] f_0^{(1)\pm} = 0,$$
$$\left[\partial_t + \left(\pm\frac{\boldsymbol{p}}{\epsilon_p} + \boldsymbol{x}\times\boldsymbol{\omega}\right)\cdot\boldsymbol{\nabla} - (\boldsymbol{\omega}\times\boldsymbol{p})\cdot\boldsymbol{\nabla}_p\right] \mathbf{g}_0^{(1)\pm} = -\boldsymbol{\omega}\times\mathbf{g}_0^{(1)} - \frac{1}{2\epsilon_p^4}\boldsymbol{p}\times(\boldsymbol{p}\times\boldsymbol{\omega})(\boldsymbol{p}\cdot\boldsymbol{\nabla})f_0^{(0)\pm}.$$

J/ψ polarization, S.Chen, Z.wang and PZ, in progress

Quantum field systems:

Spin interaction among particles (collision term)

Li, Yee, PRD100 (2019), 056022
Ayala, PLB.801 (2020) 135169
Kapusta, Rrapaj, Rudaz, PRC101 (2020), 024907
Chen, Son, Stephanov, PRL115 (2015) 021601
Zhang, Fang, Q.Wang, X.Wang, PRC100 (2019), 064904
Yang, Hattori, Hidaka, JHEP 2020 (2020) 070, arXiv:2002.02612
Weickgenannt, Speranza, Sheng, Wang, Rischke, arXiv:2005.01506, arXiv: 2103.10636
……

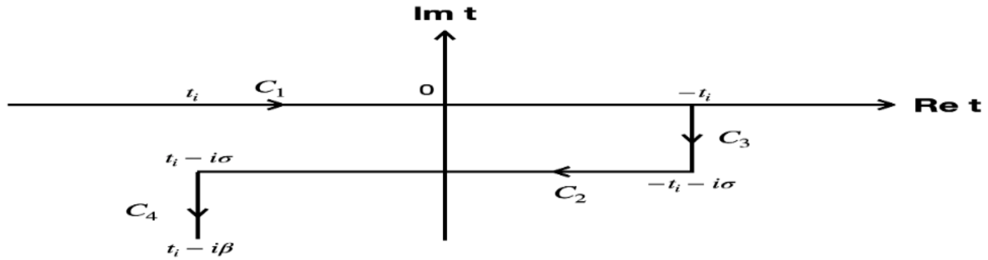
Kadanoff-Baym equations

Dyson-Schwinger equation

$$S(x,y) = S^0(x,y) + \int d^4z d^4w S^0(x,w)\Sigma(w,z)S(z,y)$$

Considering the time order in $W(x,p) = \int d^4y e^{ipy}\, \psi\left(x+\frac{y}{2}\right)\bar{\psi}\left(x-\frac{y}{2}\right)$

→ Schwinger-Keldish time contour



$S^<(x,p) = W(x,p),$ $\qquad$ $S^>(x,p)$

Constraint and transport equations with collision terms

$$\{(\gamma^\mu p_\mu - m), S^<\} + \frac{i\hbar}{2}[\gamma^\mu, \nabla_\mu S^<] = \frac{i\hbar}{2}([\Sigma^<, S^>]_* - [\Sigma^>, S^<]_*)$$

$$[(\gamma^\mu p_\mu - m), S^<] + \frac{i\hbar}{2}\{\gamma^\mu, \nabla_\mu S^<\} = \frac{i\hbar}{2}(\{\Sigma^<, S^>\}_* - \{\Sigma^>, S^<\}_*)$$

$$A * B = AB + \frac{i\hbar}{2}[AB]_{P.B.} + \mathcal{O}(\hbar^2)$$

including spin: Yang, Hattori, Hidaka, JHEP 2020 (2020) 070, arXiv:2002.02612

- Spin decomposition for Wigner function, self-energy and collision term

General collision terms

Z.Wang, X.Guo and P.Zhuang, [arXiv:2009.10930 [hep-th]]

- **0th order transport**

$$p\cdot\nabla\mathcal{V}^{(0)}_\mu = m\widehat{\Sigma^{(0)}_S\mathcal{V}^{(0)}_\mu} + p^\nu\widehat{\Sigma^{(0)}_{V\nu}\mathcal{V}^{(0)}_\mu} + \frac{m}{2}\epsilon_{\alpha\beta\lambda\mu}\widehat{\Sigma^{(0)\alpha\beta}_T\mathcal{A}^{(0)\lambda}} - \frac{p_\nu}{m}\epsilon_{\alpha\mu\beta\lambda}p^\beta\widehat{\Sigma^{(0)\alpha\nu}_T\mathcal{A}^{(0)\lambda}} - p_\mu\widehat{\Sigma^{(0)\nu}_A\mathcal{A}^{(0)}_\nu},$$

$$p\cdot\nabla\mathcal{A}^{(0)}_\mu = m\widehat{\Sigma^{(0)}_S\mathcal{A}^{(0)}_\mu} + p^\nu\widehat{\Sigma^{(0)}_{V\nu}\mathcal{A}^{(0)}_\mu} + \frac{m}{2}\epsilon_{\alpha\beta\lambda\mu}\widehat{\Sigma^{(0)\alpha\beta}_T\mathcal{V}^{(0)\lambda}} + \widehat{\Sigma^{(0)}_{A\mu}p^\nu\mathcal{V}^{(0)}_\nu} - p_\mu\widehat{\Sigma^{(0)}_{A\nu}\mathcal{V}^{(0)\nu}},$$

$$\widehat{FG} = \bar{F}G - F\bar{G}$$

Local collision term
Dynamical effect ,
e.g. diffusion effect

- **1st order transport**

$$\begin{aligned}
p\cdot\nabla\mathcal{V}^{(1)}_\mu =& + m\widehat{\Sigma^{(0)}_S\mathcal{V}^{(1)}_\mu} + p^\nu\widehat{\Sigma^{(0)}_{V\nu}\mathcal{V}^{(1)}_\mu} - p_\mu\widehat{\Sigma^{(0)\nu}_A\mathcal{A}^{(1)}_\nu} - \frac{p_\nu}{m}\epsilon_{\rho\sigma\alpha\mu}p^\rho\widehat{\Sigma^{(0)\alpha\nu}_T\mathcal{A}^{(1)\sigma}} + \frac{m}{2}\epsilon_{\sigma\nu\lambda\mu}\widehat{\Sigma^{(0)\sigma\nu}_T\mathcal{A}^{(1)\lambda}} \\
& + m\widehat{\Sigma^{(1)}_S\mathcal{V}^{(0)}_\mu} + p^\nu\widehat{\Sigma^{(1)}_{V\nu}\mathcal{V}^{(0)}_\mu} - p_\mu\widehat{\Sigma^{(1)\nu}_A\mathcal{A}^{(0)}_\nu} - \frac{p_\nu}{m}\epsilon_{\alpha\mu\beta\lambda}p^\beta\widehat{\Sigma^{(1)\alpha\nu}_T\mathcal{A}^{(0)\lambda}} + \frac{m}{2}\epsilon_{\sigma\nu\lambda\mu}\widehat{\Sigma^{(1)\sigma\nu}_T\mathcal{A}^{(0)\lambda}} \\
& + \frac{1}{2m}p^\nu[\widehat{\Sigma^{(0)}_{T\mu\nu}, (p^\alpha\mathcal{V}^{(0)}_\alpha)}]_{\text{P.B.}} - \frac{m}{2}[\widehat{\Sigma^{(0)}_{T\mu\nu}, \mathcal{V}^{(0)\nu}}]_{\text{P.B.}} + \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}p^\nu[\widehat{\Sigma^{(0)\alpha}_A, \mathcal{V}^{(0)\beta}}]_{\text{P.B.}} \\
& - \frac{1}{2m}p_\nu\Sigma^{(0)}_{T\alpha\mu}\widehat{\nabla^{[\alpha}\mathcal{V}^{(0)\nu]}} + \frac{1}{2m}p_\nu\widehat{\Sigma^{\alpha\nu(0)}_T\nabla_{[\alpha}\mathcal{V}^{(0)}_{\mu]}} + \frac{1}{2}\epsilon_{\beta\nu\lambda\mu}\Sigma^{(0)\beta}_A\widehat{\nabla^\nu\mathcal{V}^{(0)\lambda}} \\
& + \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\widehat{(\nabla^\alpha\Sigma^{(0)\nu}_V)\mathcal{A}^{(0)\beta}} - \frac{1}{2m}p_\mu\widehat{(\nabla^\nu\Sigma^{(0)}_P)\mathcal{A}^{(0)}_\nu} - \frac{1}{2m}\widehat{(p^\nu\nabla_\nu\Sigma^{(0)}_P)\mathcal{A}^{(0)}_\mu} + \frac{p^\nu}{2m}\epsilon_{\mu\nu\alpha\beta}\widehat{(\nabla^\alpha\Sigma^{(0)}_S)\mathcal{A}^{(0)\beta}},
\end{aligned}$$

$$\begin{aligned}
p\cdot\nabla\mathcal{A}^{(1)}_\mu =& + m\widehat{\Sigma^{(0)}_S\mathcal{A}^{(1)}_\mu} + p^\nu\widehat{\Sigma^{(0)}_{V\nu}\mathcal{A}^{(1)}_\mu} + p^\nu\widehat{\Sigma^{(0)}_{A\mu}\mathcal{V}^{(1)}_\nu} + \frac{m}{2}\epsilon_{\alpha\beta\lambda\mu}\widehat{\Sigma^{(0)\alpha\beta}_T\mathcal{V}^{(1)\lambda}} - p_\mu\widehat{\Sigma^{(0)}_{A\nu}\mathcal{V}^{(1)\nu}} \\
& + m\widehat{\Sigma^{(1)}_S\mathcal{A}^{(0)}_\mu} + p^\nu\widehat{\Sigma^{(1)}_{V\nu}\mathcal{A}^{(0)}_\mu} + p^\nu\widehat{\Sigma^{(1)}_{A\mu}\mathcal{V}^{(0)}_\nu} + \frac{m}{2}\epsilon_{\alpha\beta\lambda\mu}\widehat{\Sigma^{(1)\alpha\beta}_T\mathcal{V}^{(0)\lambda}} - p_\mu\widehat{\Sigma^{(1)}_{A\nu}\mathcal{V}^{(0)\nu}} \\
& - \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\widehat{(\nabla^\sigma\Sigma^{(0)\nu}_V)\mathcal{V}^{(0)\rho}} - \frac{m}{2}[\widehat{\Sigma^{(0)}_P, \mathcal{V}^{(0)}_\mu}]_{\text{P.B.}} + \frac{1}{2m}p_\mu[\widehat{\Sigma^{(0)}_P, (p^\nu\mathcal{V}^{(0)}_\nu)}]_{\text{P.B.}} \\
& + \frac{1}{2}\epsilon_{\mu\sigma\nu\rho}\nabla^\sigma\widehat{\Sigma^{(0)\nu}_A\mathcal{A}^{(0)\rho}} + \frac{1}{2}\epsilon_{\nu\mu\alpha\beta}[\widehat{\Sigma^{(0)\nu}_A, (p^\alpha\mathcal{A}^{(0)\beta})}]_{\text{P.B.}} - \frac{m}{2}[\widehat{\Sigma^{(0)}_{T\mu\nu}, \mathcal{A}^{(0)\nu}}]_{\text{P.B.}} + \frac{p_\mu}{2m}[\widehat{\Sigma^{(0)}_{T\rho\nu}, (p^\rho\mathcal{A}^{(0)\nu})}]_{\text{P.B.}} \\
& - \frac{1}{2m}p_\sigma\nabla^\sigma(\widehat{\Sigma^{(0)}_{T\mu\nu}\mathcal{A}^{(0)\nu}}) + \frac{1}{2m}p^\nu\nabla^\sigma(\widehat{\Sigma^{(0)}_{T\mu\nu}\mathcal{A}^{(0)}_\sigma}) + \frac{1}{2m}p_\mu\nabla^\sigma(\widehat{\Sigma^{(0)}_{T\sigma\nu}\mathcal{A}^{(0)\nu}}) - \frac{1}{2m}p^\nu\nabla^\sigma(\widehat{\Sigma^{(0)}_{T\sigma\nu}\mathcal{A}^{(0)}_\mu}).
\end{aligned}$$

- **Nonlocal collision term**
- **Related to spatial derivatives**
- **Correlated transport of V&A**
- **Polarization can be generated in a initially unpolarized syste**

the interaction needs to be specified to calculate the off-diagonal self-energy $\Sigma^>$ & $\Sigma^<$

Near equilibrium state

- *Collision is the driving force for the system to go from non-equilibrium state to equilibrium state..*

- *Local equilibrium state is determined by detailed balance principle (loss term and gain term cancel to each other).*

- *Global equilibrium state is determined by detailed balance + Killing condition*

$$\partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0, \qquad \beta_\mu = u_\mu / T$$

- *Near-equilibrium state can be described by relaxation time approximation.*

Spin distribution in equilibrium state

Z.Wang, X.Guo and P.Zhuang, [arXiv:2009.10930 [hep-th]]

Detailed balance → Zeroth-order spin distribution:

$$A_\mu^{(0)}(x,p) = 0$$

Detailed balance + Killing condition

$$A_\mu^{(1)} = -\frac{1}{(2\pi)^3 4E_p}\epsilon_{\mu\nu\sigma\rho}p^\nu\partial^\sigma\beta^\rho f_V'(p)$$

- *Spin is generated from the curl of the medium* $\sim\epsilon_{\mu\nu\sigma\rho}p^\nu\partial^\sigma\beta^\rho$
- *Spin can reach only global equilibrium, very different from the number distribution!*

Anderson-Witting relaxation time approximation (RTA)

Classical RTA:

$$p^\mu \partial_\mu f = -\frac{\delta f}{\tau}, \qquad \delta f = f - f_{eq}$$

Quantum RTA (A-W RTA):

$$(\gamma^\mu p_\mu - m)W + \frac{i\hbar}{2}\gamma^\mu \partial_\mu W = -\frac{i\hbar}{2}\gamma^\mu u_\mu \frac{\delta W}{\tau}, \qquad \delta W = W - W_{eq}$$

A single relaxation time τ $\rightarrow$

Different degrees of freedom (spin components) have the same thermalization time.

Is it true?

RTA from Kadanoff-Baym equations

Z.Wang and P.Zhuang, [arXiv:2105.00915 [hep-ph]].

Calculating relaxation times for different degrees of freedom with the Kadanoff-Baym equations.

Method:

1) Quantum kinetic equations in near equilibrium state:

$$W = W_{eq} + \delta W$$

2) Expanding collision terms around the equilibrium state:

$$C(\Sigma, W) = C'(\Sigma_{eq}, W_{eq})\delta W$$

Note: $C\left(\Sigma_{eq}, W_{eq}\right) = 0 \; in \; equilibrium \; state$

Kadanoff-Baym RTA:

$$\begin{cases} p^\mu \partial_\mu f_V = -\Gamma_0 \delta f_V \\ p^\mu \partial_\mu \vec{A} = -\Gamma_0 \delta \vec{A} + \vec{\Gamma}_s \cdot (\hbar \vec{\omega} \delta f_V) \end{cases}$$

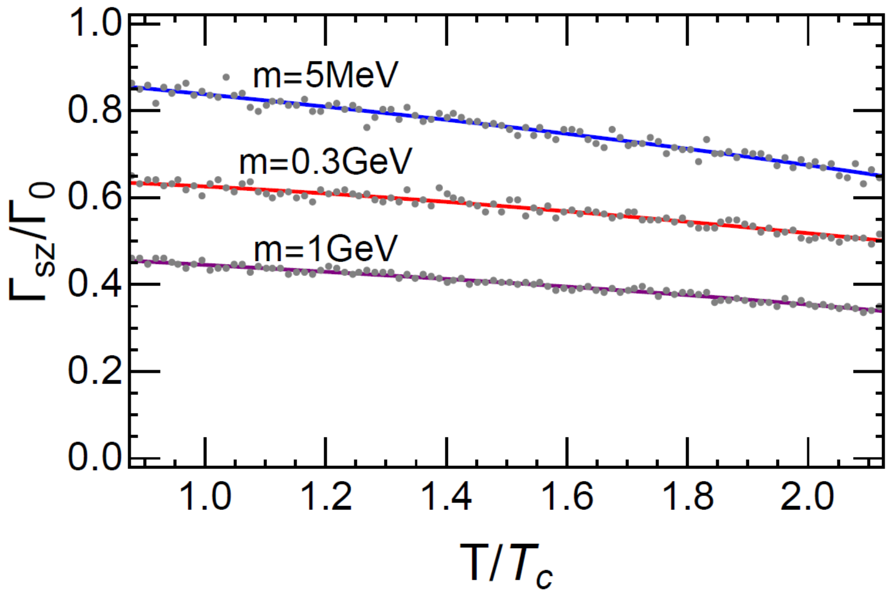
$$\frac{\Gamma_{si}}{\Gamma_0} = \frac{\tau_0}{\tau_{si}} = \frac{2m}{\omega}\frac{\omega_{si}}{\omega_0}$$

Different thermalization time scales

Z.Wang and P.Zhuang, [arXiv:2105.00915 [hep-ph]].

NJL model:

$$\mathcal{L} = \bar{\psi}\left(i\hbar\gamma^{\mu}\partial_{\mu} - m_0\right)\psi + G(\bar{\psi}\psi)^2$$



charge is thermalized earlier and spin is thermalized later.

Summary

- *A self-consistent way to go from quantum field theory (quantum mechanics) to quantum kinetic theory in Wigner function formalism.*

- *Going beyond quasi-particle approximation → equal-time kinetic hierarchy.*

- *Spin effect at first order in ħ.*

- *Spin polarization can be generated via medium vorticity (spin-orbital coupling).*

- *Quantum relaxation time approximation*

- *Spin is thermalized later in comparison with charge.*

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谢谢大家！

I learned a lot from working together with them !

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