Chiral anomalous transports from QFT

Defu Hou

Central China Normal University, Wuhan

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Outline

I. Introduction to anomalous transports

II. CME with non-constant Axial μ_5 & B

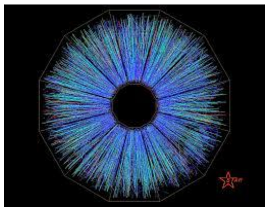
III. Subtlety of the Wigner function used for CME

IV Higher order corrections

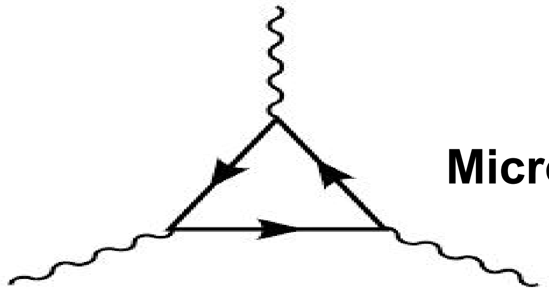
IV. Conclusion and outlook

Anomalous Transports

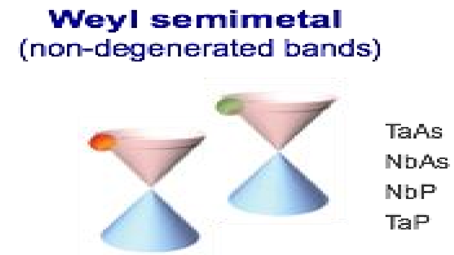
Micro-quantum anomaly + B/ Ω → macro-transport (CME/CVE)



Search in HIC



Micro



B **Ω**

$$\vec{J} = \sigma_5 \mu_5 \vec{B}$$

Macro

Nature Phys.12（2016）

Phys. Rev. X.5（2015）

Science350（2015）413

Astrophysics, cosmology

Kharzeev 2004, Kharzeev, Warringa, McLarren, Fukushima2008 … …

Induced currents in chirality background

- In linear response theory

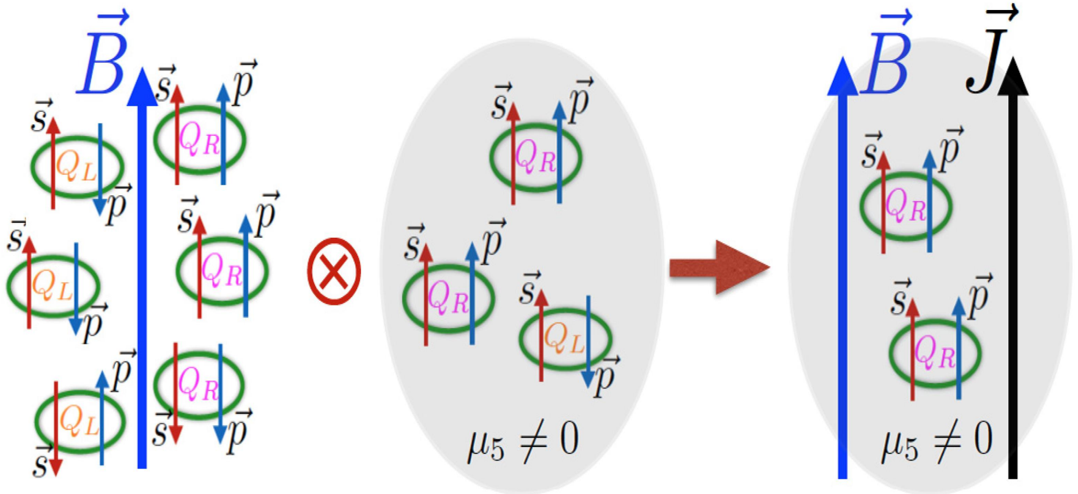
$$\vec{J} = \sigma^B \vec{B} + \sigma^V \vec{\omega}$$

Physics picture of CME

1. Quark spin locked by B or V
2. Chirality imbalance $\mu_5 \neq 0$
3. Momenta flip

Two conditions for CME:

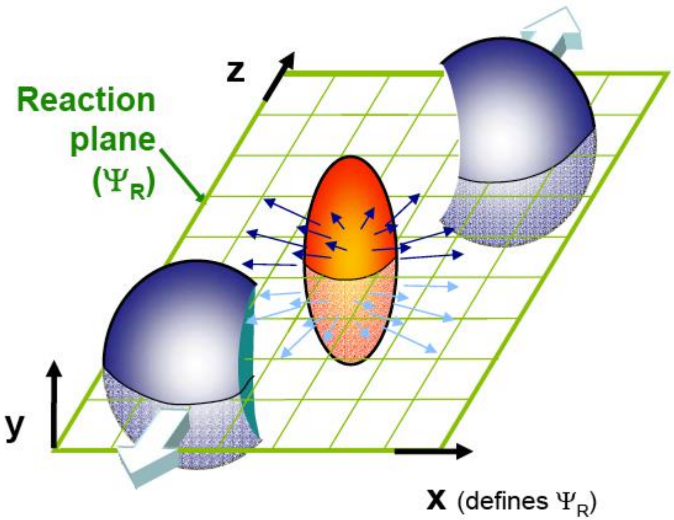
1. imbalance of left-handed & right-handed (anti)quarks (CP violation)
2. Strong magnetic field

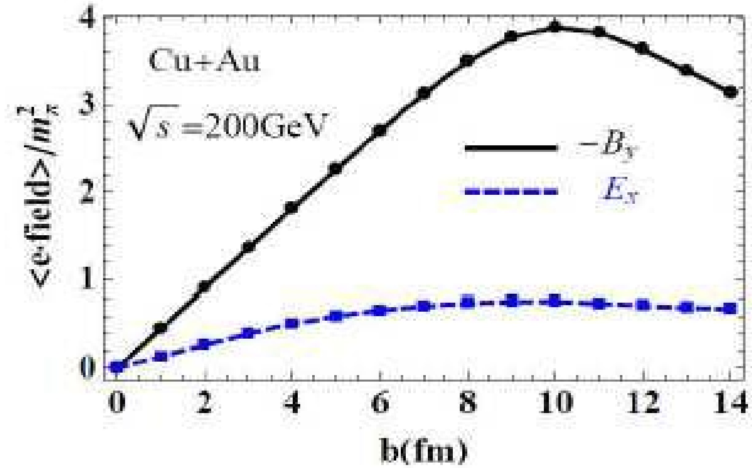


Picture of chiral magnetic effect(CME). This figure is taken from Kharzeev, Liao, Voloshin and Wang, PPNP 88(2016)1-28

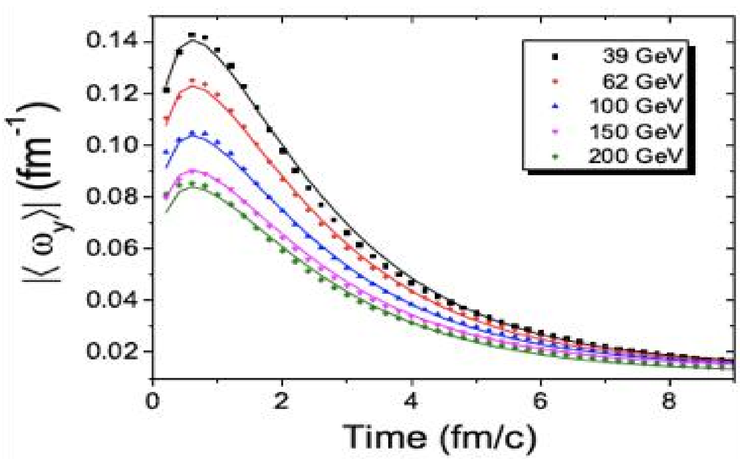
If we replace B with ω, similar effect will be observed, which is the so-called chiral vortical effect(CVE).

Strong EM Field/Rotation/ produced in HIC





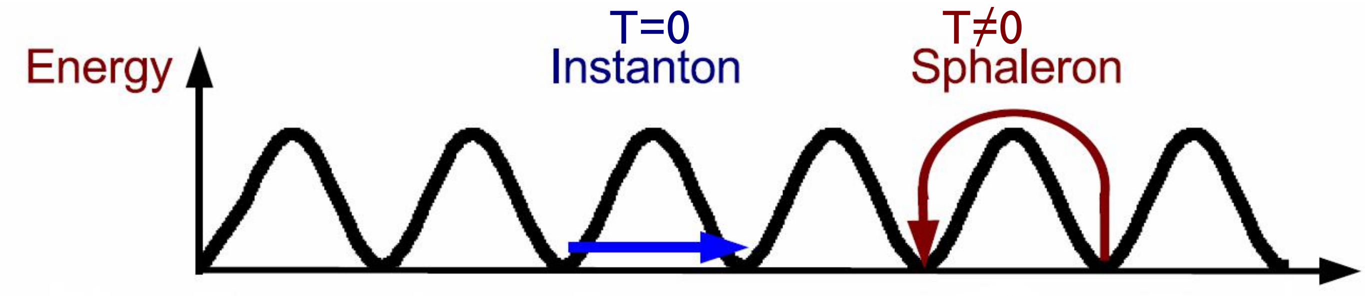
Deng, Huang, 2015



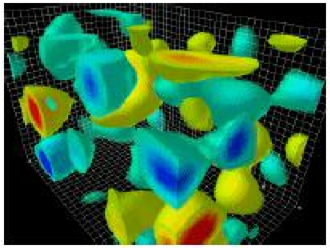
Jiang,Lin,Liao, 2016

* Net axial charge density $\mu_5 \neq 0$

Topological charge fluctuations of QCD in QGP



Axial anomaly



$$\Delta N_5 = -\frac{N_f g^2}{32\pi^2}\int d^4x \varepsilon_{\mu\nu\rho\lambda} F^l_{\mu\nu} F^l_{\rho\lambda} = n_W$$

$n_W =$ the wind number $\quad F^l_{\mu\nu} =$ QCD field strength

$$\partial_\mu j_5^\mu = -\frac{g^2 N_f}{8\pi^2} trG\tilde{G} + 2im\bar{\psi}\gamma^5\psi$$

Theoretical approaches:

--- Quantum Field theory

--- Holographic theory

--- Kinetic approach or hydrodaynamice

UV divergence demands regularization, IR behavior is crucuial

Adler-Bell-Jackiw Anomaly:

- Triangle diagram:

  Consider QED action

$$S_f[A,\psi,\bar{\psi}]$$

$$\|$$

$$S[A,\psi,\bar{\psi}] = -\frac{1}{4}\int d^4x F_{\mu\nu}F_{\mu\nu} - \int d^4x \bar{\psi}[\gamma_\mu(\partial_\mu - ieA_\mu) + m]\psi$$

Path integral

$$\int [dA]\,[d\psi d\bar{\psi}] e^{iS[A,\psi,\bar{\psi}]} = \int [dA] \exp\left[-\frac{i}{4}\int d^4x F_{\mu\nu}F_{\mu\nu} + i\Gamma[A]\right]$$

$$e^{i\Gamma[A]} = \int [d\psi d\bar{\psi}] e^{iS_f[A,\psi,\bar{\psi}]} = \det[\gamma_\mu(\partial_\mu - ieA_\mu) + m]$$

$\Gamma[A]$ = sum of diagrams with one fermion loop decorated by external photon vertices

Classical symmetry in the chiral limit $m = 0$:

$U_V(1)$: $\psi \to e^{-i\alpha}\psi \qquad \bar{\psi} \to \bar{\psi}e^{i\alpha}$

$U_A(1)$: $\psi \to e^{-i\alpha\gamma_5}\psi \qquad \bar{\psi} \to \bar{\psi}e^{-i\alpha\gamma_5}$

$$S[A,\psi,\bar{\psi}] \to S[A,\psi,\bar{\psi}]$$

Currents: $J_\mu = i\bar{\psi}\gamma_\mu\psi \qquad J_{5\mu} = i\bar{\psi}\gamma_\mu\gamma_5\psi$

$\partial_\mu J_\mu = 0 \implies$ electric charge conservation

$\partial_\mu J_{5\mu} = 0 \implies$ axial charge conservation

Quantum mechanical:

UV divergence demands regulators.

$U_V(1)$ has to be preserved because of gauge invariance.

$U_A(1)$ is explicitly broken by a gauge invariant regulator;

is not recovered when the regulator mass $M \to \infty \implies$ anomalous!

$$\partial_\mu J_\mu = 0 \quad \text{but} \quad \partial_\mu J_{5\mu} = \text{anomaly} \neq 0$$

Beyond chiral limit: $m \neq 0$:

Classical: $\partial_\mu J_\mu = 0$ $\partial_\mu J_{5\mu} = 2im\bar{\psi}\gamma_5\psi$

Quantum: $\partial_\mu J_\mu = 0$ $\partial_\mu J_{5\mu} = 2im\bar{\psi}\gamma_5\psi$ + anomaly

Axial current in an external EM field

$$< J_{5\mu}(x) >^A \equiv \frac{\int [d\psi d\bar{\psi}] e^{iS_1[A,\psi,\bar{\psi}]} J_{5\mu}(x)}{\int [d\psi d\bar{\psi}] e^{iS_1[A,\psi,\bar{\psi}]}}$$

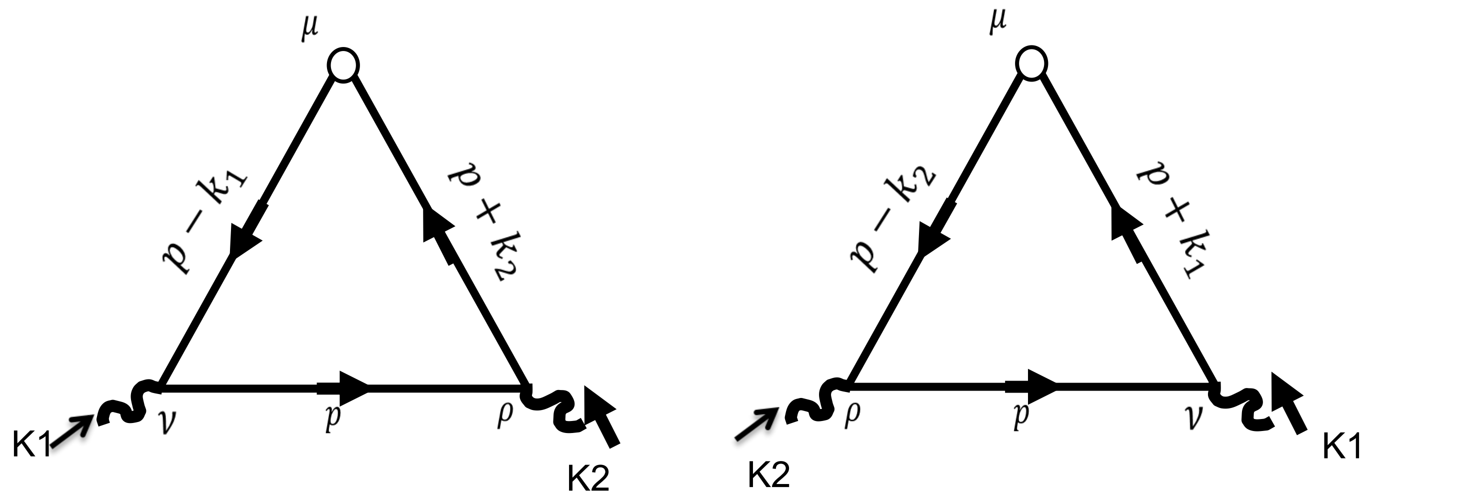
$$A_\mu(k) \equiv \int d^4x\, e^{-ik\cdot x} A_\mu(x)$$

$$= \frac{1}{2}\int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} e^{i(k_1+k_2)\cdot x} \Delta_{\mu\nu\rho}(k_1,k_2) A_\nu(k_1) A_\rho(k_2) + O(A^3)$$

$$\partial_\mu < J_{5\mu}(x) >^A =$$

$$\frac{i}{2}\int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} e^{i(k_1+k_2)\cdot x} (k_1+k_2)_\mu \Delta_{\mu\nu\rho}(k_1,k_2) A_\nu(k_1) A_\rho(k_2) + O(A^3)$$

- Triangle diagram:



$$\Delta_{\mu\nu\rho}(k_1,k_2|m) = -e^2 \int \frac{d^4p}{(2\pi)^4} \mathrm{tr}\gamma_5\gamma_\mu \left(\frac{1}{\not p + \not k_2 - m}\gamma_\rho \frac{1}{\not p - m}\gamma_\nu \frac{1}{\not p - \not k_1 - m} + \frac{1}{\not p + \not k_1 - m}\gamma_\nu \frac{1}{\not p - m}\gamma_\rho \frac{1}{\not p - \not k_2 - m} \right)$$

$$(k_1 + k_2)_\mu \Delta_{\mu\nu\rho}(k_1, k_2|m) = -ie^2 \int \frac{d^4p}{(2\pi)^4} \mathrm{tr}\gamma_5 (\not{k}_1 + \not{k}_2) \times$$

$$\left(\frac{1}{\not{p} + \not{k}_2 - m} \gamma_\rho \frac{1}{\not{p} - m} \gamma_\nu \frac{1}{\not{p} - \not{k}_1 - m} + \frac{1}{\not{p} + \not{k}_1 - m} \gamma_\nu \frac{1}{\not{p} - m} \gamma_\rho \frac{1}{\not{p} - \not{k}_2 - m} \right)$$

Using the identity

$$\frac{1}{\not{p} + \not{q} - m} \gamma_5 \not{q} \frac{1}{\not{p} - m} = -\gamma_5 \frac{1}{\not{p} - m} - \frac{1}{\not{p} + \not{q} - m} \gamma_5 - 2m \frac{1}{\not{p} + \not{q} - m} \gamma_5 \frac{1}{\not{p} - m}$$

We find

$$(k_1 + k_2)_\mu \Delta_{\mu\nu\rho}(k_1, k_2|m) = I_{\nu\rho}(m) + 2im\Delta_{\nu\rho}(m)$$

$$I_{\nu\rho}(m) = ie^2 \int \frac{d^4p}{(2\pi)^4} \mathrm{tr}\gamma_5 \left(-\gamma_\nu \frac{1}{\not p + \not k_2 - m} \gamma_\rho \frac{1}{\not p - m} + \gamma_\nu \frac{1}{\not p - m} \gamma_\rho \frac{1}{\not p - \not k_2 - m} \right.$$
$$\left. -\gamma_\rho \frac{1}{\not p - m} \gamma_\nu \frac{1}{\not p - \not k_1 - m} + \gamma_\rho \frac{1}{\not p + \not k_1 - m} \gamma_\nu \frac{1}{\not p - m} \right)$$

The 1st (3nd) term differ from 2nd (4th) term by a shift of integration momentum, but each integral is linearly divergent $\Longrightarrow I_{\nu\rho}(m) \neq 0$.

$$\Delta_{\nu\rho}(m) = e^2 \int \frac{d^4p}{(2\pi)^4} \mathrm{tr}\gamma_5 \left(\frac{1}{\not p + \not k_2 - m} \gamma_\rho \frac{1}{\not p - m} \gamma_\nu \frac{1}{\not p - \not k_1 - m} \right.$$
$$\left. + \frac{1}{\not p + \not k_1 - m} \gamma_\nu \frac{1}{\not p - m} \gamma_\rho \frac{1}{\not p - \not k_2 - m} \right)$$

Convergent integral

Pauli-Villars Regularization

Pauli-Villars regularization: Preserve the vector current conservation

$$\Delta^R_{\mu\nu\rho}(k_1,k_2|m) \equiv \lim_{M\to\infty}\left[\Delta_{\mu\nu\rho}(k_1,k_2|m) - \Delta_{\mu\nu\rho}(k_1,k_2|M)\right]$$

$$= \lim_{M\to\infty}\left[I_{\nu\rho}(m) - I_{\nu\rho}(M)\right] + 2i \lim_{M\to\infty}\left[m\Delta_{\nu\rho}(m) - M\Delta_{\nu\rho}(M)\right]$$

Shift integration momentum becomes legitimate!

$$\lim_{M\to\infty}\left[I_{\nu\rho}(m) - I_{\nu\rho}(M)\right] = \lim_{M\to\infty}\int \frac{d^4p}{(2\pi)^4}\left[\mathcal{J}_{\nu\rho}(p|m) - \mathcal{J}_{\nu\rho}(p|M)\right] = 0$$

$$(k_1+k_2)_\mu \Delta^R_{\mu\nu\rho}(k_1,k_2|m) = 2im\Delta_{\nu\rho}(m) - 2i\lim_{M\to\infty} M\Delta_{\nu\rho}(M)$$

Naïve Ward identity

Anomalous Ward identity

$$(k_1+k_2)_\mu \Delta^R_{\mu\nu\rho}(k_1,k_2|m) = 2im\Delta_{\nu\rho}(m) + \frac{e^2}{2\pi^2}\epsilon_{\nu\rho\alpha\beta}k_{1\alpha}k_{2\beta}$$

Anomaly!

- <u>Triangle diagram:</u>

$$-2iM\Delta_{\nu\rho}(M) = 2ie^2M\int\frac{d^4p}{(2\pi)^4}\mathrm{tr}\gamma_5\left\{\frac{(\not p+\not k_2+M)\gamma_\rho(\not p+M)\gamma_\nu(\not p-\not k_1+M)}{[(p+k_2)^2+M^2](p^2+M^2)[(p-k_1)^2+M^2]}\right.$$

$$\left.+(k_1\leftrightarrow k_2,\nu\leftrightarrow\rho)\right\}$$

$$= 4ie^2M\int_0^1 dx\int_0^{1-x}dy\int\frac{d^4p}{(2\pi)^4}\mathrm{tr}\gamma_5\left\{\frac{(\not p+\not k_2+M)\gamma_\rho(\not p+M)\gamma_\nu(\not p-\not k_1+M)}{[(p+k_2)^2y+(p-k_1)^2x+p^2(1-x-y)+M^2]^3}\right.$$

$$\left.+(k_1\leftrightarrow k_2,\nu\leftrightarrow\rho)\right\}$$

$$= 4ie^2M^2\int_0^1 dx\int_0^{1-x}dy\int\frac{d^4l}{(2\pi)^4}\frac{N_{\rho\nu}(l,k_1,k_2)+N_{\nu\rho}(l,k_2,k_1)}{[l^2+M^2+k_1^2x+k_2^2y-(k_1x-k_2y)^2]^3}$$

$$N_{\rho\nu}(l,k_1,k_2)\equiv\mathrm{tr}\gamma_5\gamma_\rho(\not l+\not k_1x-\not k_2y)\gamma_\nu[\not l-\not k_1(1-x)-\not k_2y]$$

Working out the trace

$$-2iM\Delta_{\nu\rho}(M) = \frac{e^2M^2}{\pi^2}\epsilon_{\nu\rho\alpha\beta}k_{1\alpha}k_{2\beta}\int_0^1 dx\int_0^{1-x}dy\frac{1}{M^2+k_1^2x+k_2^2y-(k_1x-k_2y)^2}$$

$$\longrightarrow\frac{e^2}{2\pi^2}\epsilon_{\nu\rho\alpha\beta}k_{1\alpha}k_{2\beta}\qquad\text{as } M\to\infty$$

Coordinate space:

$$\partial_\mu J_{5\mu} = 2im\bar{\psi}\gamma_5\psi + \frac{ie^2}{16\pi^2}\epsilon_{\mu\nu\rho\lambda}F_{\mu\nu}F_{\rho\lambda}$$

Generalization to QCD+QED

$$\partial_\mu J_{5\mu} = 2im\bar{\psi}\gamma_5\psi + i\frac{N_f g^2}{32\pi^2}\epsilon_{\mu\nu\rho\lambda}F^l_{\mu\nu}F^l_{\rho\lambda} + \frac{i\eta e^2}{16\pi^2}\epsilon_{\mu\nu\rho\lambda}F_{\mu\nu}F_{\rho\lambda}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$$F^l_{\mu\nu} = \partial_\mu A^l_\nu - \partial_\nu A^l_\mu + gf^{lmn}A^m_\mu A^n_\nu$$

color-flavor factor

$$\eta = N_c \sum_f q_f^2$$

UV divergence ⟹ Anomaly

⟹ Anomaly is **independent of temperature and chemical potential.**

- <u>Other approches:</u>

Point-splitting: *Schwinger*

$$J_{5\mu}(x) = \lim_{\delta\to 0} J_{5\mu}(x,\delta)$$

$$J_{5\mu}(x,\delta) \equiv iU(x_+,x_-)\bar{\psi}(x_+)\gamma_\mu\gamma_5\psi(x_-)$$

$$U(x_+,x_-) = \exp\left[ie\int_{x_-}^{x_+} d\xi_\rho\, A_\rho(\xi)\right] \quad x_\pm = x \pm \frac{\delta}{2}$$

Maintain gauge invariance

$$< J_{5\mu}(x,\delta) >^A = -iU(x_+,x_-)\mathrm{tr}\gamma_\mu\gamma_5 S_A(x_-,x_+)$$

where $S_A(x_-,x_+)$ = Dirac propagator in an external EM field

$$-\gamma_\mu(\partial_\mu - ieA_\mu)S_A(x_-,x_+) = i\delta^4(x-y) \qquad m = 0$$

$$S_A(x_-,x_+) = -i < x \left|\frac{1}{\gamma_\mu(\partial_\mu - ieA_\mu)}\right| y >

$$\partial_\mu U(x_+, x_-) = ie\delta_\rho \partial_\mu A_\rho(x) + O(\delta^2)$$

$$-\partial_\mu S_A(x_-, x_+) = \left[ie\delta_\rho \partial_\rho A_\mu(x) + O(\delta^3)\right]$$

$$\partial_\mu < J_{5\mu}(x,\delta) >^A = eF_{\mu\rho}(x)\delta_\rho \mathrm{tr}\gamma_\mu\gamma_5 S_A(x_-, x_+)$$

$$= -e^2 F_{\mu\rho}(x)\delta_\rho \int d^4y\, \mathrm{tr}\gamma_\mu\gamma_5 S_F(x_- - y)\gamma_\nu A_\nu(y) S_F(y - x_+)$$

$$= -\frac{e^2}{16\pi^2} e^2 F_{\mu\rho}(x)\delta_\rho \epsilon_{\mu\alpha\nu\beta} \int d^4y \left[\frac{\partial}{\partial x_{-\alpha}} \frac{1}{(x_- - y)^2}\right] \frac{1}{(y - x_+)^2} A_\beta(y)$$

$$= \frac{ie^2}{4\pi^2\delta^2} F_{\mu\rho} F_{\beta\nu} \epsilon_{\mu\alpha\nu\beta} \delta_\rho \delta_\alpha = \frac{ie^2}{16\pi^2} \epsilon_{\mu\nu\rho\lambda} F_{\mu\nu} F_{\rho\lambda}$$

where Schouten identity

$$\epsilon_{\mu\alpha\nu\beta}\delta_\rho + \epsilon_{\rho\mu\alpha\nu}\delta_\beta + \epsilon_{\beta\rho\mu\alpha}\delta_\nu + \epsilon_{\nu\beta\rho\mu}\delta_\alpha + \epsilon_{\alpha\nu\beta\rho}\delta_\mu = 0$$

has been employed.

Change of the path integral measure: *Fujikawa*

$$S_1[A,\psi,\bar{\psi}] = i\int d^4x\bar{\psi}\not{D}\psi \qquad \not{D} = -i\gamma_\mu(\partial_\mu - ieA_\mu)$$

$$Z[A] = \int [d\psi d\bar{\psi}]e^{iS_1[A,\psi,\bar{\psi}]}$$

$$\psi(x) \to e^{-i\alpha(x)\gamma_5}\psi(x) \qquad \bar{\psi}(x) \to \bar{\psi}(x)e^{-i\alpha(x)\gamma_5}$$

$$\frac{\delta\psi_\alpha(x)}{\delta\psi_\beta(x')} = \delta_{\alpha\beta}e^{-i\alpha(x)\gamma_5}\delta(x-x') \equiv (\mathcal{U}^A)_{\alpha\beta}(x,x')$$

$$\frac{\delta\bar{\psi}_\alpha(x)}{\delta\bar{\psi}_\beta(x')} = \delta_{\alpha\beta}e^{-i\alpha(x)\gamma_5}\delta(x-x') \equiv (\mathcal{U}^A)_{\alpha\beta}(x,x')$$

$$S_1[A,\psi,\bar{\psi}] \to S_1[A,\psi,\bar{\psi}] - \int d^4x\alpha\,\partial_\mu J_{5\mu}$$

$$[d\psi d\bar{\psi}] \to [d\psi d\bar{\psi}](\det\mathcal{U}^A)^2 \equiv [d\psi d\bar{\psi}]e^{i\int d^4x\alpha d}$$

$$d = i\delta^4(0)\mathrm{tr}\gamma_5 = \infty \times 0$$ Need regularization!

$$d \equiv i\lim_{\Lambda\to\infty}\mathrm{Tr}\gamma_5 f(\Lambda^{-2}\not{D}^2) = \frac{ie^2}{16\pi^2}\epsilon_{\mu\nu\rho\lambda}F_{\mu\nu}F_{\rho\lambda}$$

$$Z[A] \to \int[d\psi d\bar{\psi}]\,e^{iS_1[A,\psi,\bar{\psi}]+\int d^4x\alpha[d-\partial_\mu J_{5\mu}]} = Z[A]$$

$$\Longrightarrow \qquad \partial_\mu < J_{5\mu}(x) >^A = \frac{ie^2}{16\pi^2}\epsilon_{\mu\nu\rho\lambda}F_{\mu\nu}F_{\rho\lambda}$$

In contrast: $\psi \to e^{-i\alpha}\psi \qquad \bar{\psi} \to \bar{\psi}e^{i\alpha}$ for $U_V(1)$

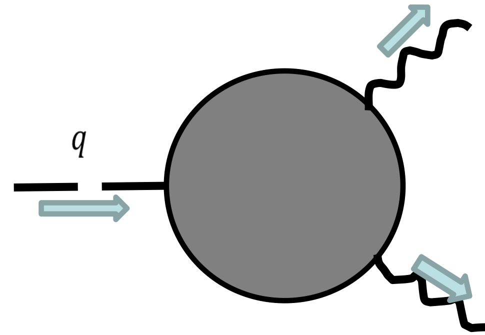
$[d\psi d\bar{\psi}] \to [d\psi d\bar{\psi}] \quad \Longrightarrow$ No anomaly

Applications:

- $\underline{\pi^0 \longrightarrow 2\gamma}$

  Decay amplitude

$$\mathcal{T} = \lim_{q^2 \to -m_\pi^2} \mathcal{F}(q^2)$$



$$\mathcal{F}(q^2) = (q^2 + m_\pi^2) e^2 \varepsilon_{1\mu} \varepsilon_{2\nu} \int d^4x d^4y < 0 \left| T J_\mu(x) J_\nu(y) \pi(0) \right| 0 > e^{i(k_1 \cdot x + k_2 \cdot y)}$$

Naïve PCAC relation $\partial_\mu J_{5\mu} = m_\pi^2 f_\pi \pi$

$$\mathcal{F}(q^2) = \frac{q^2 + m_\pi^2}{f_\pi m_\pi^2} \varepsilon_{1\mu} \varepsilon_{2\nu} q_\rho T_{\mu\nu\rho}$$

$$T_{\mu\nu\rho} \equiv -ie^2 \int d^4x d^4y < 0 \left| T J_\mu(x) J_\nu(y) J_{5\rho}(0) \right| 0 > e^{i(k_1 \cdot x + k_2 \cdot y)}$$

- $\pi^0 \longrightarrow 2\gamma$ (cont.):

Electric current conservation + Bose symmetry

$$q_\rho T_{\mu\nu\rho} = \epsilon_{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} \frac{q^2 F(q^2)}{q^2 + m_\pi^2}$$

where we assumed that π^0 is the only low-lying pole

$$\mathcal{F}(q^2) = \frac{q^2 + m_\pi^2}{f_\pi m_\pi^2} \varepsilon_{1\mu} \varepsilon_{2\nu} q_\rho T_{\mu\nu\rho} = \frac{1}{f_\pi m_\pi^2} \varepsilon_{1\mu} \varepsilon_{2\nu} \epsilon_{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} q^2 F(q^2)$$

$$\Longrightarrow \quad \mathcal{F}(0) = 0$$

Low π^0 mass $\quad \mathcal{F}(m_\pi^2) \cong \mathcal{F}(0) = 0 \quad$ disagree with experiments.

Modified PCAC relation

$$\partial_\mu J_{5\mu} = m_\pi^2 f_\pi \pi + \frac{ie^2}{32\pi^2} \epsilon_{\mu\nu\rho\lambda} F_{\mu\nu} F_{\rho\lambda}$$

$\Longrightarrow$ Decay width $\cong$ 7.63 eV

Experimental value $= (7.31 \pm 1.5)$eV

- <u>Chiral magnetic effect:</u>

QCD+QED Lagrangian with ordinary and chiral chemical potentials

$$\mathcal{L}[A,\psi,\bar{\psi}] = -\frac{1}{4}F^l_{\mu\nu}F^l_{\mu\nu} - \frac{1}{4}F_{\mu\nu}F_{\mu\nu} - \bar{\psi}\gamma_\mu\left(\partial_\mu - igA^l_\mu T^l - ieA_\mu\right)\psi$$
$$+\mu\psi^\dagger\psi + \mu_5\psi^\dagger\gamma_5\psi + J_\mu A_\mu$$
$$+\text{gauge fixing terms and counter terms}$$

Both μ and μ_5 can be functions of space and time.

Generating functional of connected Green functions of photons

$$Z[J] = \int [dA^l][dA][d\psi d\bar{\psi}]\, e^{i\int d^4x\mathcal{L}[A,\psi,\bar{\psi}]}$$

$\int dt$ may follow a closed time path to handle the non-equilibrium case.

$$\mathcal{A}_\mu(x) = -i\frac{\delta \ln Z}{\delta J_\mu(x)}$$

- *Chiral magnetic effect (cont.):*

Quantum effective action:

$$\Gamma[\mathcal{A}] = -i \ln Z[J] - \int d^4x\, J_\mu \mathcal{A}_\mu$$

$$J_\mu(x) = \frac{\delta\Gamma}{\delta\mathcal{A}_\mu(x)} = \int \frac{d^4q}{(2\pi)^4} e^{iq\cdot x} J_\mu(q) = J_\mu^{(0)} + J_\mu^{(1)} + \cdots$$

$$J_\mu^{(0)} = O(\mathcal{A}), \qquad J_\mu^{(1)} = O(\mu_5\mathcal{A}), \qquad \ldots$$

$$J_\mu^{(1)}(q) = -i \int \frac{d^4k}{(2\pi)^4} \Delta_{4\mu\nu}(-q, q-k|0)\mu_5(k)\mathcal{A}_\nu(q-k)$$

$$\mathcal{A}_\mu(q) = \int d^4x\, e^{-iq\cdot x}\mathcal{A}_\mu(x) \quad \mu_5(k) = \int d^4x\, e^{-iq\cdot x}\mu_5(x)$$

- *Chiral magnetic effect (cont.):*

Consider $\mathcal{A}_\mu(x) = \left(\vec{\mathcal{A}}(\vec{r}), 0\right)$, $\mu_5(x) = \mu_5 e^{-i\omega t}$

$$\mu_5(k) = (2\pi)^4 \mu_5 \delta^3\left(\vec{k}\right)\delta(k_0 - \omega)$$

$$J_i(q) = -i\mu_5 \Delta_{4ij}(-q, q-k|0)\mathcal{A}_j(q-k)$$

$$k = \left(\vec{0}, i\omega\right) \quad q = (\vec{q}, i\omega)$$

Anomalous Ward identity

$$i\omega\Delta_{4ij}(-q, q-k|0) = -\frac{e^2}{2\pi^2}\epsilon_{ij\alpha\beta}q_\alpha(q-k)_\beta = i\frac{e^2}{2\pi^2}\omega\epsilon_{ijk}q_k$$

$$\Delta_{4ij}(-q, q-k|0) = \frac{e^2}{2\pi^2}\epsilon_{ijk}q_k$$

In the limit ω→0 $\quad J_i(q) = -i\frac{e^2\mu_5}{2\pi^2}\epsilon_{ijk}q_k\mathcal{A}_j(q)$

$$\vec{J}(\vec{r}) = \frac{e^2\mu_5}{2\pi^2}\vec{B}(\vec{r}) \qquad \vec{B} = \vec{\nabla}\times\vec{\mathcal{A}}$$

Miscellaneous Topics:

- Other chiral anomalies:

  Non-Abelian gauge anomalies:

$$S_1[A,\psi,\bar{\psi}] = i\int d^4x(\bar{\psi}_L \not{D}_L \psi_L + \bar{\psi}_R \not{D}_R \psi_R)$$

$$\not{D}_L = \not{\partial} - i\not{A}^l T_L^l \qquad \not{D}_R = \not{\partial} - i\not{A}^l T_R^l$$

$$J_\mu = i\left(\bar{\psi}_L \gamma_\mu T_L \psi_L + \bar{\psi}_R \gamma_\mu T_R \psi_R\right)$$

(T_L, T_R) = generators of gauge transformation of left and right fermions

$$D_\mu J_\mu^a \equiv \partial_\mu J_\mu^a + f^{abc} A_\mu^b J_\mu^c$$

Classical $\qquad D_\mu J_\mu^a = 0$

Quantum $\qquad D_\mu J_\mu^a = \kappa \epsilon_{\mu\nu\rho\lambda} d^{abc} F_{\mu\nu}^b F_{\rho\lambda}^c$

$$d^{abc} = \frac{1}{2}\left(\mathrm{tr} T_R^a \{T_R^b, T_R^c\} - \mathrm{tr} T_L^a \{T_L^b, T_L^c\}\right)$$

3rd Casmir

- *Other chiral anomalies (cont.):*

Chiral anomaly in curved space:

$$D^\mu J^a_\mu = \kappa\epsilon^{\mu\nu\rho\lambda}\left(d^{abc}F^b_{\mu\nu}F^c_{\rho\lambda} + \kappa' b^a R^\alpha{}_{\beta\mu\nu}R^\beta{}_{\alpha\rho\lambda}\right)$$

$$D^\mu J^a_\mu \equiv \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}J^a_\mu\right) + f^{abc}A^b_\mu J^c_\mu$$

$$R^\alpha{}_{\beta\mu\nu} = \text{Riemann tensor}$$

$$d^{abc} = \frac{1}{2}\left(\mathrm{tr}T^a_R\{T^b_R, T^c_R\} - \mathrm{tr}T^a_L\{T^b_L, T^c_L\}\right)$$

$$b^a = \mathrm{tr}T^a_R - \mathrm{tr}T^a_L$$

Gauge-gravity mixed anomaly

Pure gravitational anomaly: *Alvarez-Gaume & Witten*

$$D^\mu T_{\mu\nu} \neq 0$$

Exists only in $d = 4k + 2$ $(2,6,10,\ldots)$ dimensions

- Anomaly cancellation:

Benign anomaly: Anomaly of global symmetry

$\Longrightarrow$ Interesting physics, e.g., CME

Bad anomaly: Anomaly of gauge symmetry

$\Longrightarrow$ Jeopardize unitary and renormalizability

$\Longrightarrow$ **Has to be cancelled!**

Example 1: Electroweak theory, gauge group $= SU(2) \times U(1)$

$$d^{abc} = \frac{1}{2}\left(\mathrm{tr} T_R^a \{T_R^b, T_R^c\} - \mathrm{tr} T_L^a \{T_L^b, T_L^c\}\right) = 0$$

$$b^a = \mathrm{tr} T_R^a - \mathrm{tr} T_L^a = 0$$

$\Longrightarrow$ **Anomaly free!**

Example 2: Supersymmetric Yang-Mills in $d = 10$

Anomalies to be cancelled: **gauge, mixed and gravity**

Anomaly free gauge group: $SO(32)$ and $\boldsymbol{E_8} \times \boldsymbol{E_8}$

⇩

Superstring

Linear response & anomalous transports

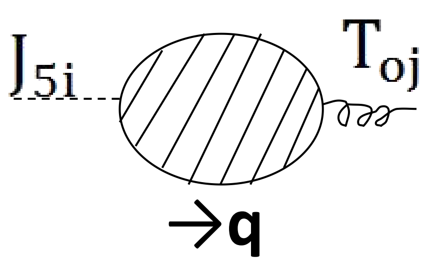
Under B & vorticity $\boldsymbol{\omega} = \nabla \times \mathbf{v}$

$$\vec{J}_{\rm em} = \sigma^{B}_{\rm em}\vec{B} + \sigma^{V}_{\rm em}\vec{\omega},$$
$$\vec{J}_{\rm b} = \sigma^{B}_{\rm b}\vec{B} + \sigma^{V}_{\rm b}\vec{\omega},$$
$$\vec{J}_{5} = \sigma^{B}_{5}\vec{B} + \sigma^{V}_{5}\vec{\omega},$$

Son & Surowka

$\boldsymbol{\sigma}^{\rm B}_{\rm em}, \boldsymbol{\sigma}^{\rm B}_{\rm b}, \boldsymbol{\sigma}^{\rm B}_{5} \rightarrow \text{CME}$ $\boldsymbol{\sigma}^{\rm v}_{\rm em},\ \boldsymbol{\sigma}^{\rm v}_{\rm b},\ \boldsymbol{\sigma}^{\rm v}_{5} \rightarrow \text{CVE}$

Kubo formula：



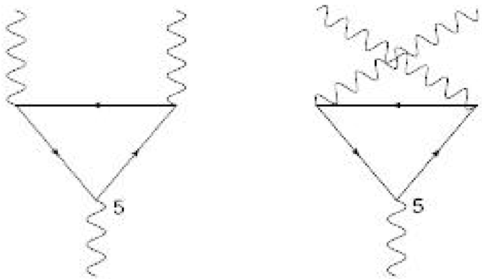
$\xrightarrow{\mathbf{q}\to 0}$ $\sigma^{\rm v}_{5}\varepsilon_{ijk}q_{k}$

The relation of CME current to chiral anomaly

The CME current

$$J_i(p) = \eta\mu_5 K_{ij}(p) A_j(p) + \mathcal{O}(\mu_5^3)$$

- In terms of the AVV three point function $\Lambda_{\mu\nu\rho}(Q_1, Q_2)$



$$Q_1 = (\mathbf{q}, i(\omega + \frac{k_0}{2})),$$

$$Q_2 = (-\mathbf{q}, i(-\omega + \frac{k_0}{2}))$$

- the coefficient

$$K_{ij}(q) = \Lambda_{ij4}(q, -q) = -i \lim_{k_0 \to 0} \frac{1}{k_0} (Q_1 + Q_2)_\rho \Lambda_{ij\rho}(Q_1, Q_2)$$

- the chiral anomaly

$$(Q_1 + Q_2)_\rho \Lambda_{\mu\nu\rho}(Q_1, Q_2) = -i \frac{e^2}{2\pi^2} \epsilon_{\mu\nu\alpha\beta} Q_{1\alpha} Q_{2\beta}$$

It follows that

$$K_{ij}(q) = i\frac{e^2}{2\pi^2}\epsilon_{ikj}q_k$$

Then the CME current

$$\mathbf{J} = \frac{e^2}{2\pi^2}\mu_5\mathbf{B} \tag{1}$$

There are, however, two shortcomings in the above establishment

1. distinction between chiral anomaly at the operator level and its matrix element

   only the former one is free from radiative corrections.

2. the constant μ_5 limit in eq.(1) becomes subtle at finite temperature

$$\lim_{k_0\to 0}\lim_{\mathbf{k}\to 0} \neq \lim_{\mathbf{k}\to 0}\lim_{k_0\to 0} \tag{2}$$

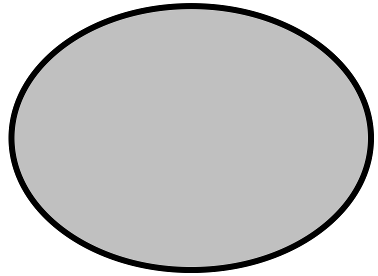
note that in the limiting process $\lim_{\mathbf{k}\to 0}\lim_{k_0\to 0}$*, the relation of CME current to chiral anomaly becomes unclear.*

Hou, Ren, Liu JHEP 05(2011)046

$$\mu_5\left(\mathbf{k}, k_0\right)$$

$$\Downarrow$$

$$\mathbf{J}\left(\mathbf{q}+\frac{1}{2}\mathbf{k}\ ,\omega+\frac{1}{2}k_0\right) \Leftarrow$$



$$\Leftarrow \mathbf{B}\left(\mathbf{q}-\frac{1}{2}\mathbf{k}, \omega-\frac{k_0}{2}\right)$$

Constant μ_5 *, non-constant* **B:** $\mathbf{k} = k_0 = 0$

$$\text{limit}_{\mathbf{q}\to 0}\text{limit}_{\omega\to 0} \Rightarrow \ \mathbf{J} = \eta\frac{e^2}{2\pi^2}\mu_5\mathbf{B}$$

$$\text{limit}_{\omega\to 0}\text{limit}_{q\to 0} \Rightarrow \ \mathbf{J} = \frac{1}{3}\times\eta\frac{e^2}{2\pi^2}\mu_5\mathbf{B}$$

Artifact of one-loop approximation. The ambiguity disappears with higher order corrections. (*Satow & Yee*)

Constant ***B****, non-constant* $\mu_5(\mathbf{k}, k_0)$

$$\Downarrow$$

$$\text{limit}_{\mathbf{k}\to 0}\text{limit}_{k_0\to 0} \Rightarrow \qquad \mathbf{J}=0$$

$$\text{limit}_{k_0\to 0}\text{limit}_{\mathbf{k}\to 0} \Rightarrow \qquad \mathbf{J}=\eta\frac{e^2}{2\pi^2}\mu_5\mathbf{B}$$

Follows from the EM gauge invariance and the non-renormalization of the axial anomaly. Valid to all orders!

_*with T=0 and* $\mu=0$*: relativistic invariance requires the two limit orders are equivalent:*

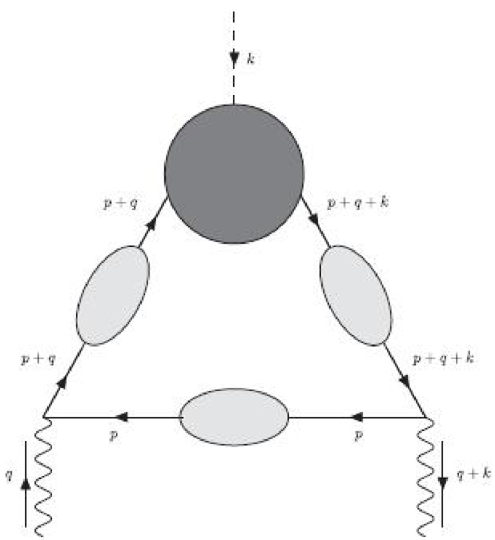
PHYSICAL REVIEW D **103**, 056004 (2021)

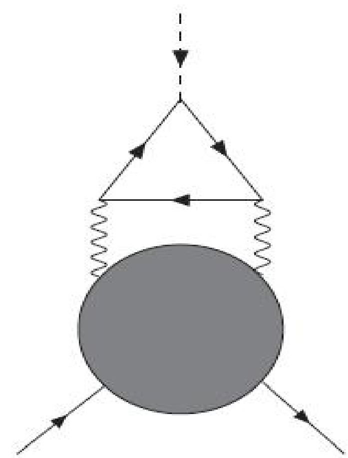
Noncommutativity of the static and homogeneous limit of the axial chemical potential in the chiral magnetic effect

Bo Feng,[1] De-fu Hou,[2,*] Hai-cang Ren,[2,3,†] and Shuai Yuan[1]

$$\lim_{\mathbf{k}\to 0}\lim_{k^0\to 0}\mathcal{G}^{ij0}(q,k)=0,$$

$$\lim_{k^0\to 0}\lim_{\mathbf{k}\to 0}\mathcal{G}^{ij0}(q,k)=i\epsilon^{ijk}q^k,$$





CME from regulated Wigner function

a robust regularization scheme has to be introduced to the underlying field theory before defining the wigner function. e.g. PV scheme

$$L = -\overline{\psi}\gamma_{\mu}(\partial_{\mu} - ieA_{\mu} - i\gamma_5 A_{5\mu})\psi$$

$$J_{\mu}(x) = i\int \frac{d^4 p}{(2\pi)^4} \mathrm{tr}\, W(x, p)\gamma_{\mu}$$
$$= i \lim_{y \to 0} U(x_+, x_-) < \overline{\psi}(x_+)\gamma_{\mu}\psi(x_-) >$$

$$J_{\mu}(x) = -ie\frac{1}{2}\left[\mathrm{Tr}\gamma_{\mu}\mathcal{S}_0(x,x) - \sum_s C_s \mathrm{Tr}\gamma_{\mu}\mathcal{S}_s(x,x)\right]$$

- **Closed time path Green function formation**

*** A fermion propagator**

$$S_{CTP}(x,y)=\begin{pmatrix} S_{11}(x,y) & S_{12}(x,y) \\ S_{21}(x,y) & S_{22}(x,y) \end{pmatrix}$$

$$S_{11}(x,y)_{\alpha\beta}=\left\langle T[\psi_{\alpha}(x)\overline{\psi}_{\beta}(y)]\right\rangle \quad S_{12}(x,y)_{\alpha\beta}=-\left\langle \overline{\psi}_{\beta}(y)\psi_{\alpha}(x)\right\rangle$$

$$S_{21}(x,y)_{\alpha\beta}=\left\langle \psi_{\alpha}(x)\overline{\psi}_{\beta}(y)\right\rangle \quad S_{22}(x,y)_{\alpha\beta}=\left\langle \widetilde{T}[\psi_{\alpha}(x)\overline{\psi}_{\beta}(y)]\right\rangle$$

T: time ordering $\qquad \widetilde{T}$: anti-time ordering

*** The electric current**

$$J_{\mu}(x,y)=\begin{cases} -ie\,\mathrm{tr}\,S_{11}(x_{-},x_{+})\gamma_{\mu}=J^{1}_{\mu}(x,y) & y_0\geq 0 \\ -ie\,\mathrm{tr}\,S_{22}(x_{-},x_{+})\gamma_{\mu}=J^{2}_{\mu}(x,y) & y_0<0 \end{cases}$$

* Expansion to the linear order in A_μ and $A_{5\mu}$

full propagator:

free propagator

$$S_{ab}(x_-, x_+)$$
$$= S_{ab}(x_-, x_+) - \sum_c \int d^4 z S_{ac}(x_- - z)\gamma^c_{\rho 5} S_{cb}(z - x_+) A_{5\rho}(z)$$
$$- e\sum_c \int d^4 z S_{ac}(x_- - z)\gamma^c_{\rho} S_{cb}(z - x_+) A_{\rho}(z)$$
$$+ e\sum_{cd} \int d^4 z_1 \int d^4 z_2 S_{ad}(x_- - z_2)\gamma^d_{\lambda 5} S_{dc}(z_2 - z_1)\gamma^c_{\rho} S_{ca}(z_1 - x_+) A_{\rho}(z_1) A_{5\lambda}(z_2)$$
$$+ e\sum_{cd} \int d^4 z_1 \int d^4 z_2 S_{ac}(x_- - z_2)\gamma^c_{\rho} S_{cd}(z_2 - z_1)\gamma^d_{\lambda 5} S_{da}(z_1 - x_+) A_{\rho}(z_2) A_{5\lambda}(z_1)$$

with $\gamma^1_\mu = \gamma_\mu, \quad \gamma^2_\mu = -\gamma_\mu, \quad \gamma^1_{\mu 5} = \gamma_\mu\gamma_5, \quad \gamma^2_{\mu 5} = -\gamma_\mu\gamma_5$

gauge link:

$$U(x_-, x_+) = 1 + ie\int_{x_-}^{x_+} d\xi_\upsilon A_\upsilon(\xi) + O(A^2)$$

$$J_\mu(x) = e^2 \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} e^{i(q_1+q_2)\cdot x} \Lambda_{\mu\rho\lambda}(q_1, q_2) A_\rho(q_1) A_{5\lambda}(q_2)$$

Wu,Hou, Ren, PRD 2017

gives CME current :

$$\lim_{q_{20}\to 0} \lim_{\vec{q}_2 \to 0} \Lambda_{ij4}(q_1, q_2) = -\frac{1}{2\pi^2} \epsilon_{ikj} q_{1k}$$

CME current canceled at thermal equilibrium.

$$\lim_{\vec{q}_2 \to 0} \lim_{q_{20}\to 0} \Lambda_{ij4}(q_1, q_2) = \frac{2f(0)-1}{2\pi^2} \epsilon_{ikj} q_{1k} + O(q_1^2)$$

Higher order correction to CVE

Field Theoretic Formulation:

QED Lagrangian density

$$\mathcal{L} = -\frac{1}{4e^2} V^{\mu\nu} V_{\mu\nu} - i\bar{\psi}\gamma^{\mu} D_{\mu}\psi + \frac{1}{2} h^{\mu\nu} T_{\mu\nu} + A^{\mu} J_{5\mu}$$

$$V_{\mu\nu} = \partial_{\mu} V_{\nu} - \partial_{\nu} V_{\mu}$$

$$D_{\mu} = \partial_{\mu} - iV_{\mu}$$

$$T_{\mu\nu} = V_{\mu}{}^{\rho} V_{\nu\rho} - \frac{1}{4} g_{\mu\nu} V^{\rho\lambda} V_{\rho\lambda} + \frac{1}{4}\left(-D_{\mu}\bar{\psi}\gamma_{\nu}\psi - D_{\nu}\bar{\psi}\gamma_{\mu}\psi + \bar{\psi}\gamma_{\mu} D_{\nu}\psi + \bar{\psi}\gamma_{\nu} D_{\mu}\psi\right)$$

$$J_5^{\mu} = i\bar{\psi}\gamma_{\mu}\gamma_5\psi$$

Anomalous Ward identity

$$\partial_{\mu} J_5^{\mu} = \frac{e^2}{16\pi^2\sqrt{-g}} \epsilon^{\mu\nu\rho\lambda} V_{\mu\nu} V_{\rho\lambda}$$

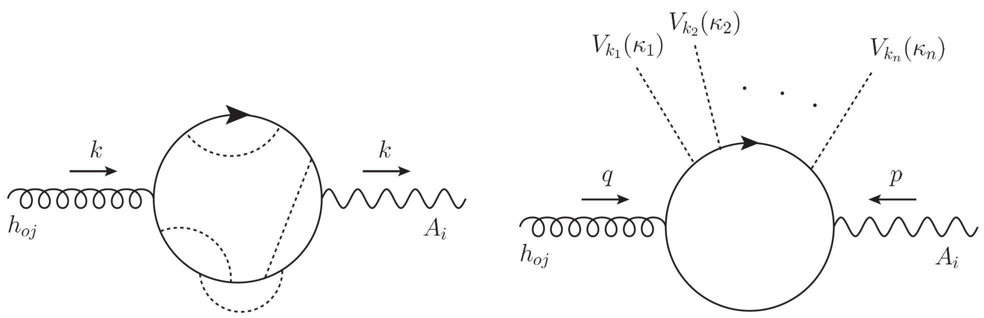
Kubo formula for CVE

$$\mathcal{G}_{ij}(\vec{q}) = -\int_0^{\infty} dt \int d^3\vec{r}\, e^{-i\vec{q}\cdot\vec{r}} \frac{\mathrm{Tr} e^{-\beta H}\left[J_{5i}(\vec{r},t), T_{0j}(0,0)\right]}{\mathrm{Tr} e^{-\beta H}} \underset{\vec{q}\to 0}{\longrightarrow} \sigma_V \epsilon_{ijk} q_k$$

Two-point correlation function

Golkar and Son, arXiv:1207.5806 *JHEP02(2015)16*

- An argument on the non-renormalization of CVE



(a) A generic diagram. (b) n-scalar effective vertex.

Conclusions:

All higher order diagrams beyond one loop are order of k^2, which does not contribute to the CVE.

Notice:

Both axial current and stress tensor are attached to the scalar loop.

Gauge invariance

$$p^i \Gamma^{(n)}_{ij}(p, q, \kappa_1, \cdots, \kappa_n) = 0,$$
$$q^j \Gamma^{(n)}_{ij}(p, q, \kappa_1, \cdots, \kappa_n) = 0.$$

Coleman-Hill ⇒

$$\Gamma^{(n)}_{ijk_1\cdots k_n}(p, q, \kappa_1, \cdots, \kappa_n) = \mathcal{O}(pq)$$

$\sim\mathcal{O}(k^2)$

Structure of CVE

- General form of CVE conductivity

$$\sigma_5^V = \frac{\mu_5^2}{2\pi^2} + cT^2$$

with $$c = \frac{1}{12}$$

Son&Surowka，PRL. 103, 191601
Neiman &Y. Oz, JHEP 03 (2011)

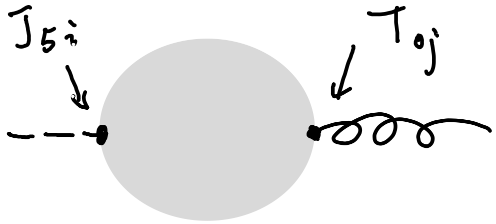
Non-renormalization suggested by

- holography
- hydrodynamics
- But how about in QFT?

- **CVE in quantum field theory**

$$G_{ij}(Q) = \sigma_5^V \epsilon_{ijk} q_k$$

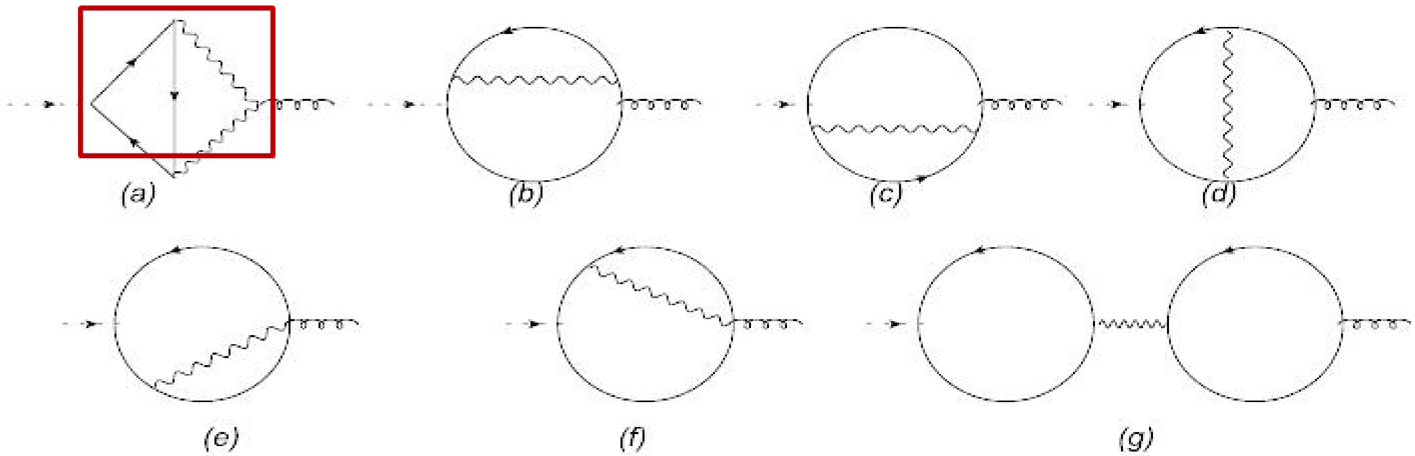
$$= -\int_0^\infty dt \int d\vec{r} e^{-i\vec{q}\cdot\vec{r}} \frac{\mathrm{Tr}\{e^{-\beta H}[J_{5i}(\vec{r},t), T_{0j}(0,0)]\}}{\mathrm{Tr} e^{-\beta H}}$$



Generic diagram for CVE related correction

Are there any correction from higher orders ?

Golkar and Son, arXiv:1207.5806 , JHEP02(2015)169 No (Yes)



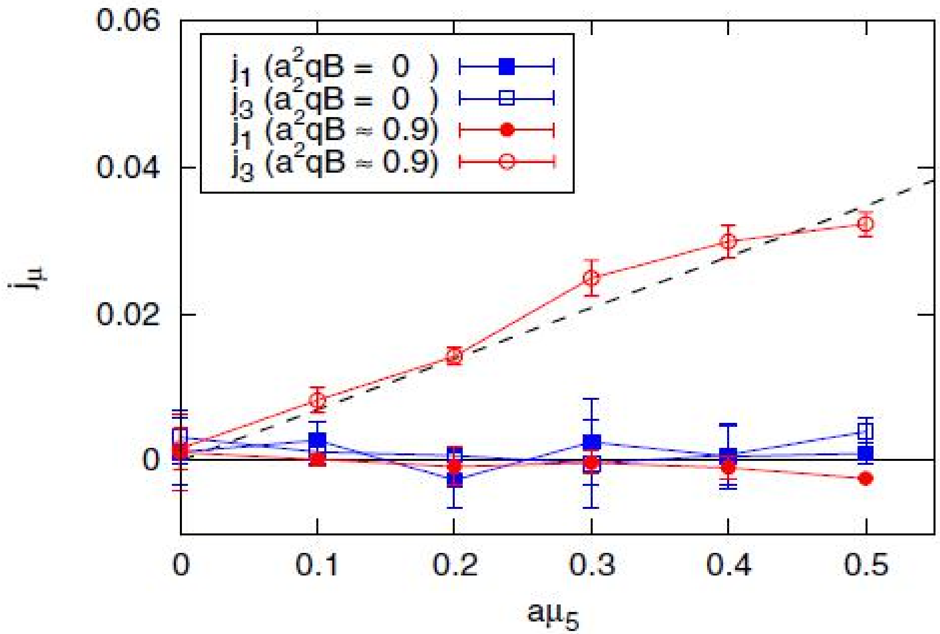
$$c = \frac{1}{12} + \frac{N_c^2 - 1}{2N_c}\frac{g_0^2}{48\pi^2} \xrightarrow[N_c \to \infty]{} \frac{1}{12} + \frac{\lambda}{96\pi^2}$$

$$c = \frac{1}{12} + \frac{e_0^2}{48\pi^2}$$

$$\xi_5 = \frac{\mu_5^2}{2\pi^2} + cT^2$$

Hou,Liu,Ren, PRD86 (2012) 121703®

CME on Lattice



Yamamoto,PRL(2011)

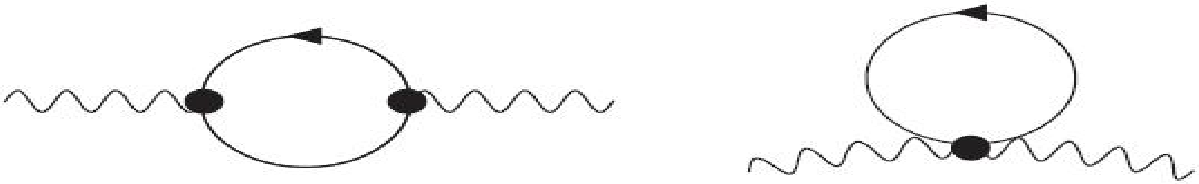
using lattice QCD with Wilson term

Karsten and Smit (1981)

$$
\begin{aligned}
I = & -\sum_x \sum_\mu \frac{1}{2a} \left[\bar{\psi}(x) \left(\frac{1}{i}\gamma_\mu - r \right) U_\mu(x) \psi(x + a_\mu) \right. \\
& \left. - \bar{\psi}(x + a_\mu) \left(\frac{1}{i}\gamma_\mu + r \right) U_\mu^\dagger(x) \psi(x) \right] \\
& - \sum_x M \bar{\psi}(x) \psi(x) + \cdots
\end{aligned}
$$

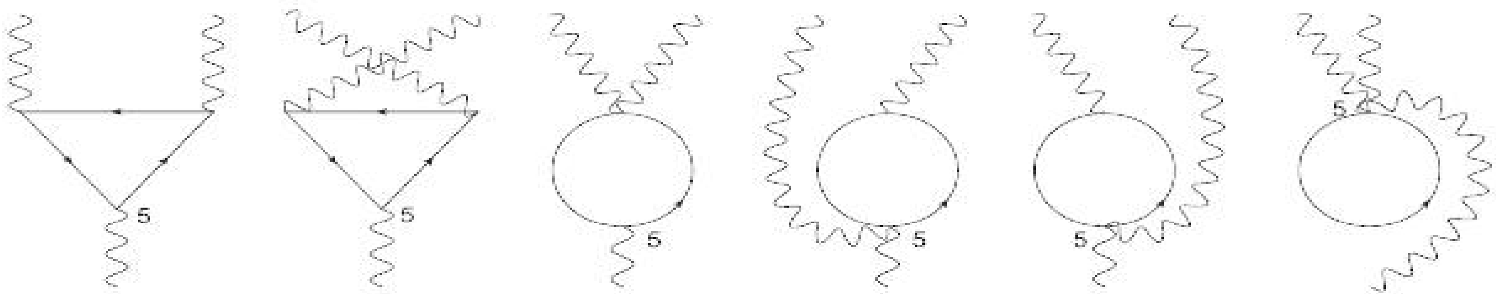
Feng,Hou, Liu, Ren, Wu, PRD95,(2017)

One-loop contributions to $\Pi_{\mu\nu}$.



(a) (b)

One-loop triangle diagrams corresponding to $\Pi^{(1)}_{\mu\nu}(p)$.



$$J_i(p) = -\Pi_{ij}(p) A_j(p)$$

One-loop self-energy on lattice of size $N_s^3 \times N_t$

$$\Pi_{ij}^{(1)}(p) = \mathcal{I} \sum_k \epsilon_{ikj} p_k + \mathcal{O}(a)$$

CME vanishes at continu. limit .

At zero temperature

$$\begin{aligned}\Pi_{ij}(q) &\equiv \Lambda_{ij4}(q) \\ &= -\lim_{q_4 \to 0} \frac{1}{q_4} \sum_\rho \frac{2}{a} \sin \frac{1}{2} a (Q_1 + Q_2)_\rho \Lambda_{ij\rho}(Q_1, Q_2)\end{aligned}$$

$$\Pi_{ij}(q) = \frac{e^2}{2\pi^2} \sum_k \epsilon_{ijk} q_k$$

- numerical calculations

Feng,Hou, Liu, Ren, Wu, PRD95,(2017)

Lattice size	$\mathcal{I}$
$N_s = 6, N_t = 4$	1.347×10^{-2}
$N_s = 12, N_t = 4$	2.439×10^{-4}
$N_s = 20, N_t = 4$	8.886×10^{-7}
$N_s = 50, N_t = 8$	4.512×10^{-9}

- analytical calculations(In the limit $N_s \to \infty$)

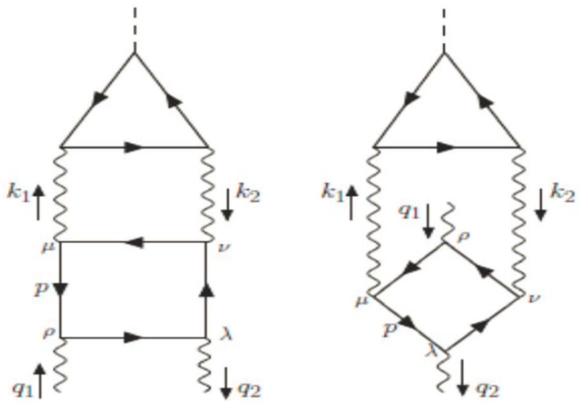
$$\mathcal{I} = 12 \frac{1}{N_t} \sum_{l_4} \int \frac{d^3\mathbf{l}}{(2\pi)^3} \frac{\mathcal{N}(l)}{\left[\sin^2 l + \mathcal{M}^2(l)\right]^3} = 0$$

3-loop radiation correction to CME

- the kernel of CME current

$$K_{ij}(\mathbf{q}) = i\frac{e^2}{2\pi^2}F_s\left(\frac{|\mathbf{q}|}{T}\right)\epsilon_{ikj}q_j$$

1. In low temperature limit($T << |\mathbf{q}|$): $F_s(|\mathbf{q}|/T) \to 1 - \frac{3e^4}{64\pi^4}\ln\frac{\Lambda^2}{q^2}$
2. At finite temperature(T>$|\mathbf{q}|$): for $\lim_{Q_0\to 0}\lim_{\mathbf{Q}\to 0}$, $F_s(|\mathbf{q}|/T) \to 1$
   for $\lim_{\mathbf{Q}\to 0}\lim_{Q_0\to 0}$, $F_s(|\mathbf{q}|/T) \to 0$



If the two internal photons are replaced by gluons

$$F_s(|\mathbf{q}|/T) \to 1 - \frac{3g^4}{32\pi^4}\log\frac{\Lambda^2}{q^2}.$$

Feng, Hou, Ren PRD99 (2019)

Chiral anomaly at operator level and its matrix element

- The operator equation of the anomaly

$$\partial_\mu j^5_\mu = 2imj^5 + i\frac{\alpha_0}{4\pi}\epsilon_{\rho\sigma\lambda\nu}F_{\rho\sigma}F_{\lambda\nu}$$

the coefficient of the anomalous term is exactly $\alpha_0/4\pi$ and *does not involve an unknown power series in the coupling constant coming from higher orders in perturbation theory.* Adler and Bardeen (1969')

- The matrix element between the vacuum and a state with two photons of momenta Q_1, Q_2

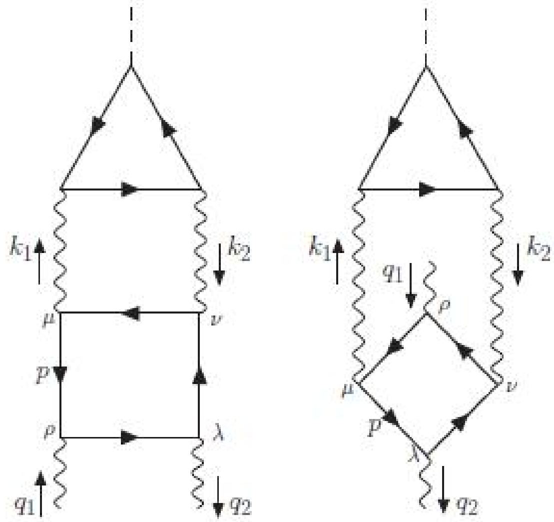
$$(Q_1+Q_2)_\mu\Lambda_{\mu\rho\lambda}(Q_1,Q_2) = -i\left[2mG\left(\frac{Q_1^2}{m^2},\frac{Q_2^2}{m^2},\frac{Q_1\cdot Q_2}{m^2}\right)+H\left(\frac{Q_1^2}{m^2},\frac{Q_2^2}{m^2},\frac{Q_1\cdot Q_2}{m^2}\right)\right] \times \epsilon_{\rho\lambda\alpha\beta}Q_{1\alpha}Q_{2\beta}$$

in low energy limit

$$2mG(0,0,0)+H(0,0,0)=0, \quad H(0,0,0)=\frac{2\alpha}{\pi}$$

- For massless fermions, the low energy kinematic point cannot be attained, the matrix elements receive radiative corrections.

Radiative corrections from photon-photon rescattering



Photon rescattering contribution to the AVV function
Ansel'm and Ioganson (1989')

- The anomalous Ward identity

$$(Q_1 + Q_2)_\rho \Lambda_{\mu\nu\rho}(Q_1, Q_2) = -i\frac{e^2}{2\pi^2}\epsilon_{\mu\nu\alpha\beta}Q_{1\alpha}Q_{2\beta} \times \left(1 - \frac{3e^4}{64\pi^4}\ln\frac{\Lambda^2}{k^2}\right)$$

- The kernel of CME current becomes

$$K_{ij}(q) = i\frac{e^2}{2\pi^2}\mu_5\epsilon_{ikj}q_k\left(1 - \frac{3e^4}{64\pi^4}\ln\frac{\Lambda^2}{k^2}\right)$$

- Likewisely, the same diagrams with two internal photons replaced by two gluons may also contribute to CME.

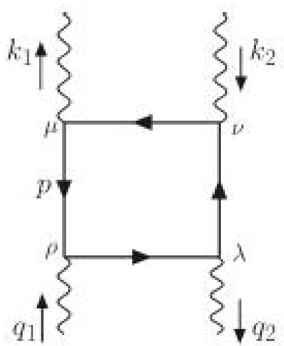
Radiative corrections at finite temperature

At finite temperature, the results correspond to the two limits

$$\lim_{Q_0\to 0}\lim_{\mathbf{Q}\to 0}, \qquad \lim_{\mathbf{Q}\to 0}\lim_{Q_0\to 0}$$

may be different.

- The order $\lim_{Q_0\to 0}\lim_{\mathbf{Q}\to 0}$
  1. The AVV triangle in real-time formulation(e.g., CTP) is diagonal, i.e., only 111 and 222 components survive.
  2. For static external momenta, the real-time formulation reduces to Mastubara one.

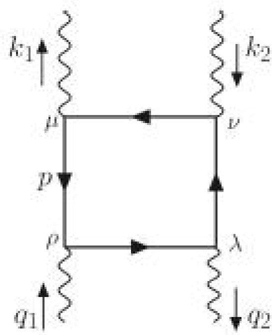


The photon box is free from IR singularity, thus

$$\Gamma_{mnij}(0,0;k) = \frac{\partial}{\partial q_l}\Gamma_{mnij}(q,q;k)\Big|_{q=0} = 0$$

the three-loop diagrams would be at order of $\mathcal{O}(\mathbf{q}^2)$ and thus do not contribute to CME.

- The order $\lim_{\mathbf{Q}\to 0} \lim_{Q_0 \to 0}$
  1. The AVV triangle may not be diagonal W.R.T the CTP indices.
  2. For static external momenta, the real-time formulation reduces to Mastubara one.

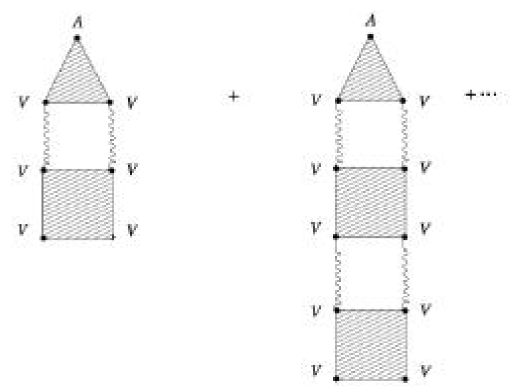


$$\Gamma_{mnij}(0,0;k) = \frac{\partial}{\partial q_l}\Gamma_{mnij}(q,q;k)\Big|_{q=0} = 0$$

$$\Gamma_{mnij}(q,q;0) = \frac{\partial}{\partial k_l}\Gamma_{mnij}(q,q;k)\Big|_{k=0} = 0$$

Thus, the amplitude of the box

$$\Gamma \simeq \mathcal{O}(\mathbf{q}^2\mathbf{k}^2)$$



Then Coleman-Hill theorem(for 3D QED) can apply and the chain diagrams can be ruled out.
Coleman and Hill (1985')

V. Concluding Remarks

- The zero P & E limits of μ_5 do not commute which is robust against Higer order correction

- While the CSE is expected in RHIC, its magnitude may not reach the ideal value $\mathbf{J} = \eta \frac{e^2}{2\pi^2}\mu_5 \mathbf{B}$ because of inhomogeneity

- Nonrenormalization is true for most but not for all anomal. transp. coefs . We obtained 2-loop correction to CVE coef.

- Naive Wigner function can not be applicable to the case with non-constant. The problem stems from axial anomaly . The PV RWF leads consistent results.

. We examine the issues raised here with lattice formulation we obtained the same results as that in continuous case with QFT and WF method .

. Radiation corrections to CME up to 3-loop massless QED (QCD) are derived at zero T and non-zero T

References:

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Thank you very much for your attention!

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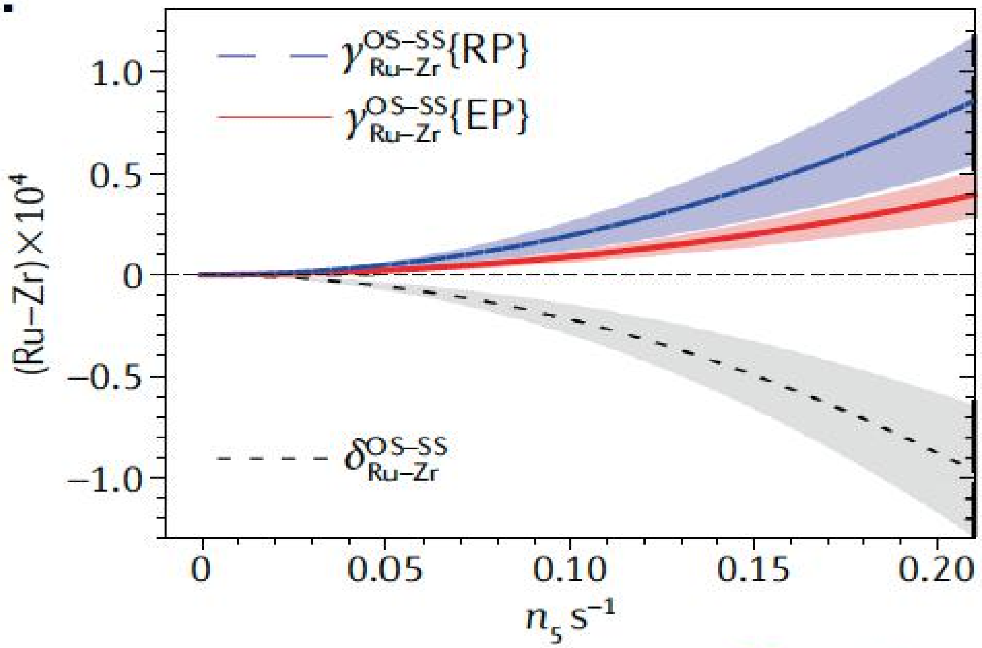
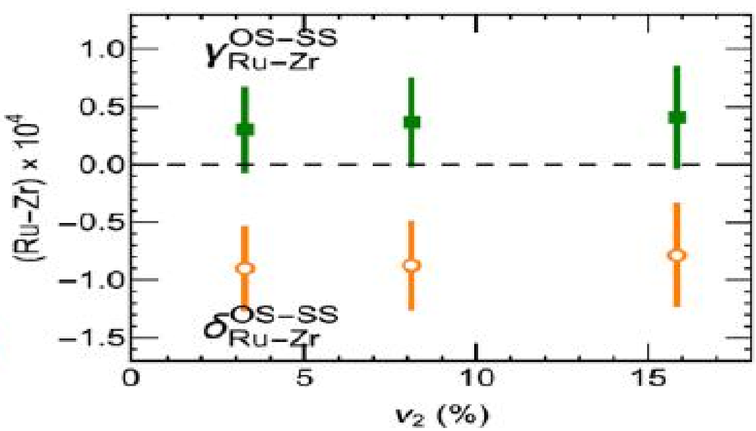
Signatures of Chiral Magnetic Effect in the Collisions of Isobars

Shuzhe Shi,[1] Hui Zhang,[2,3,4] Defu Hou,[2,*] and Jinfeng Liao[5,†]



$$\zeta^{RP}_{isobar} \equiv \frac{\gamma^{OS-SS}_{Ru-Zr}\Big|_{RP}}{\delta^{OS-SS}_{Ru-Zr}\Big|_{RP}} \simeq -(0.90 \pm 0.45)$$

$$\zeta^{EP}_{isobar} \equiv \frac{\gamma^{OS-SS}_{Ru-Zr}\Big|_{EP}}{\delta^{OS-SS}_{Ru-Zr}\Big|_{EP}} \simeq -(0.41 \pm 0.27)$$



Stay tuned!

Fluct. & dissip. of axial charge from massive quark

DF Hou, S. Lin, PRD98, (2018)

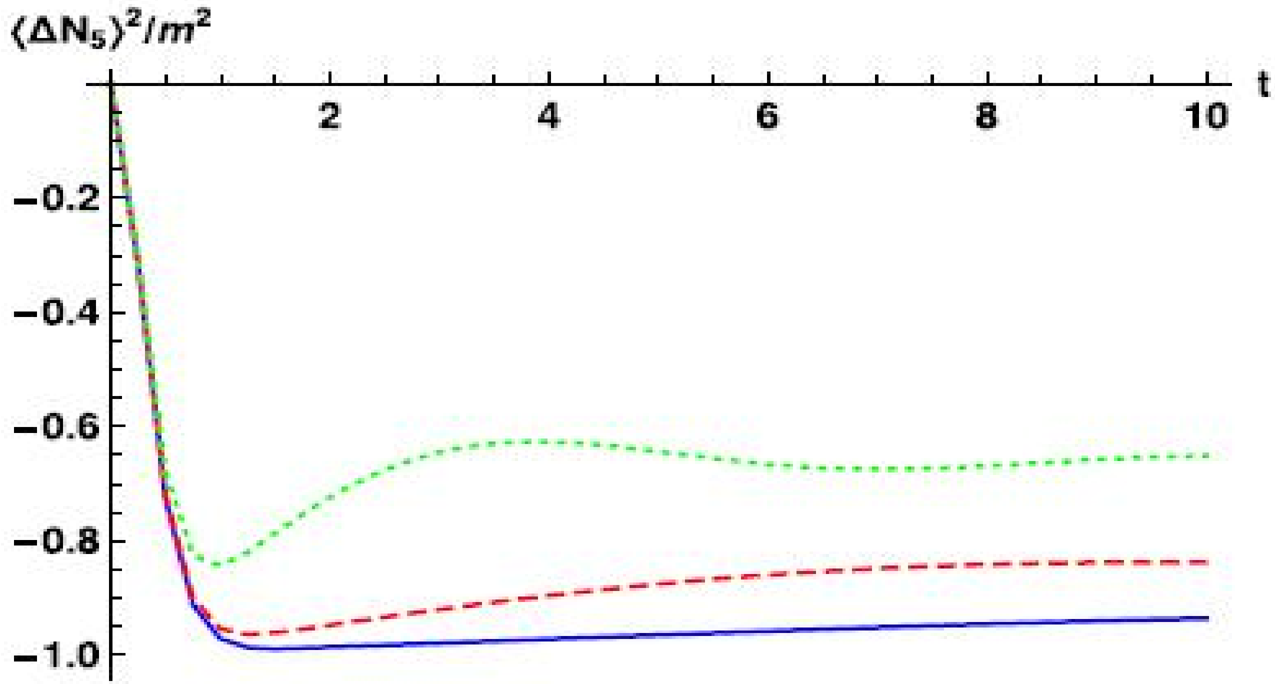
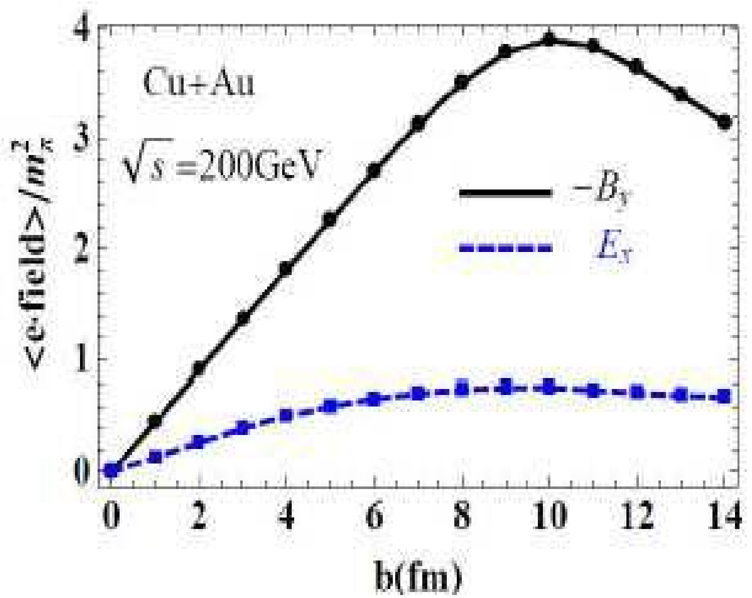
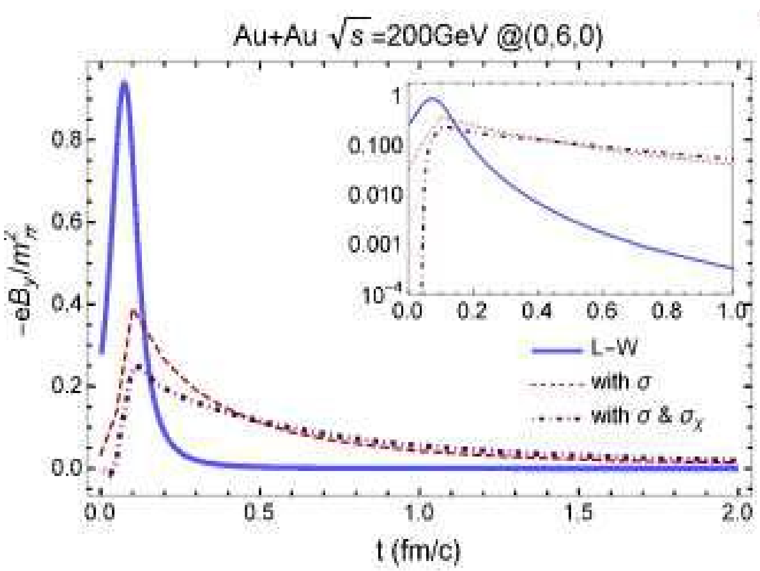


FIG. 2. Contributions from intrinsic fluctuation $\langle \Delta N_5(t)^2 \rangle / m^2$ for different masses: blue solid line for $m = 1/10$, red dashed line for $m = 1/5$, and green dotted line for $m = 1/2$. The unit is set by $T = 1$. The fluctuation is characterized by an initial rise followed by oscillatory decay to asymptotic value. The case with a larger mass shows more rapid convergence to the asymptotic value.

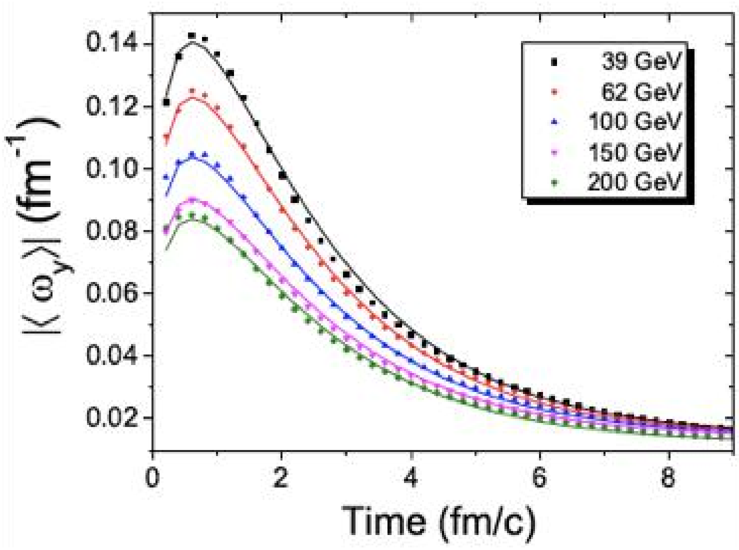
Strong EM Field/Rotation/polarization produced in HIC



Deng, Huang, 2015



Li, Sheng, Wang 2016



Jiang,Lin,Liao, 2016

Phenomenological implications of the subtleties regarding the order of limits

Axial charge generated via toplogical fluctuations dictated by the stochastic Eq with a white noise

$$\left(\frac{\partial}{\partial t} - D\nabla^2 + \frac{1}{\tau}\right) n_5 = g(x)$$

In Momentum sapce

$$n_5(k) = \frac{g(k)}{-ik_0 + D\vec{k}^2 - \frac{1}{\tau}}$$

Corresponding an axial potential

$$A_{5\mu}(k) = -i\delta_{\mu 4}\frac{n_5(k)}{\chi(k)}$$

Average current vanishes, the correlation funct. $< J_i(x) J_j(y) >$ **is dominated by diffusion pole**

$$-iq_{20} + D\vec{q}_2^{\,2} + \frac{1}{\tau} = 0$$

$$\frac{|q_{20}|}{|\vec{q}_2|} = D|\vec{q}_2| + \frac{1}{|\vec{q}_2|\tau} \geq \sqrt{\frac{D}{\tau}}.$$

If $\sqrt{D/\tau} >> 1$ **the homog. \mu_5 is a good approximation and classic form of CME current emerges ---Noneq. Phenom.**

Towards equilibrium, $\tau \to \infty,$ $\frac{|q_{20}|}{|\vec{q}_2|} = D|\vec{q}_2| \sim \frac{D}{|\vec{x} - \vec{y}|} \to 0$

Inverse limit-order prevails, and CME current disappears,