原子核结构与中高能重离子碰撞交叉学科理论讲习班

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原子核低激发态的简单配对结构

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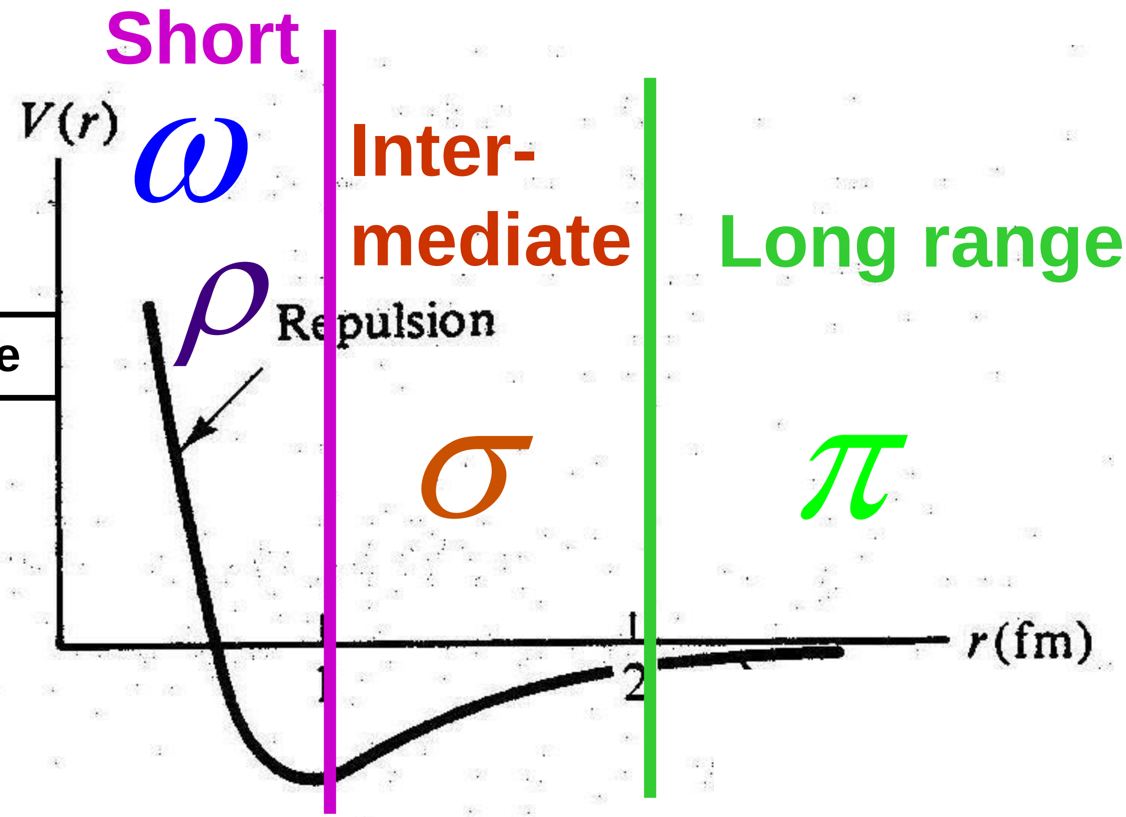
Outline

- WHY 低能量核子配对是原子核低激发态的关键组元?
- WHAT 准自旋算符 & 单j壳系统的辛弱数理论?
- WHAT 单j壳系统不变本征态 & 其解析配对波函数?
- WHAT BCS理论?
- WHAT 壳模型配对近似理论 & 一维配对态?

低能量核子配对
是原子核低激发态的关键组元

Realistic nuclear force

Central force

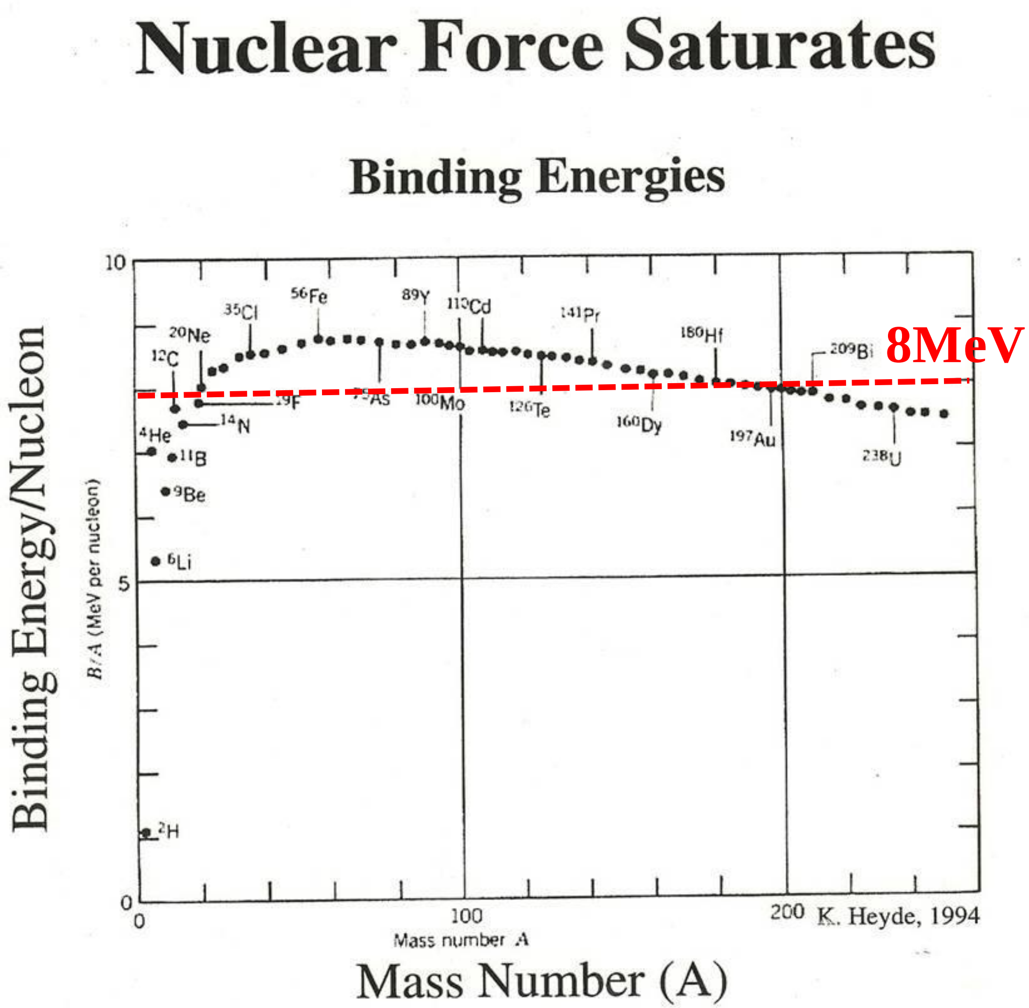


Tensor force π ρ

Spin-orbit force ω σ

Prof. R. Machleidt's PPT

Effective nuclear force



质子之间有很强的库仑排斥力，稳固的原子核系统是由核子间的强相互作用造成的。

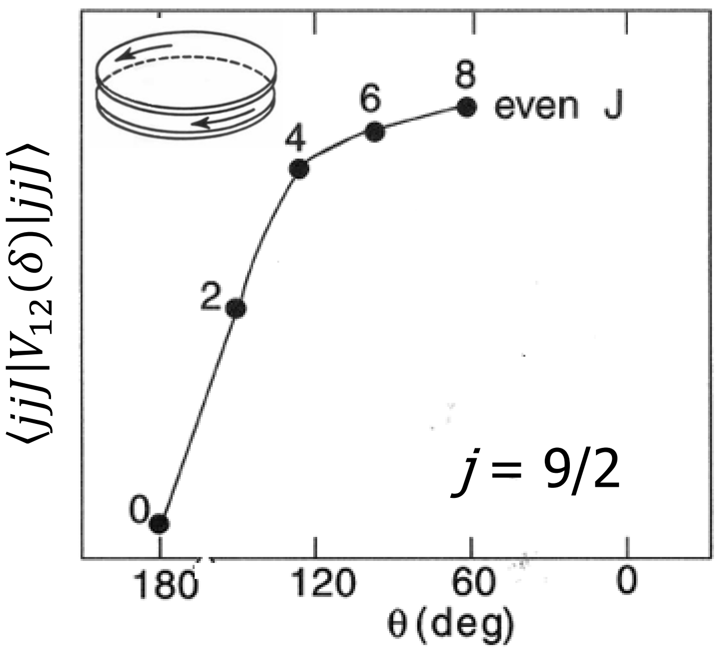
原子核的总结合能近似地正比于质量数，而不是质量数的平方。

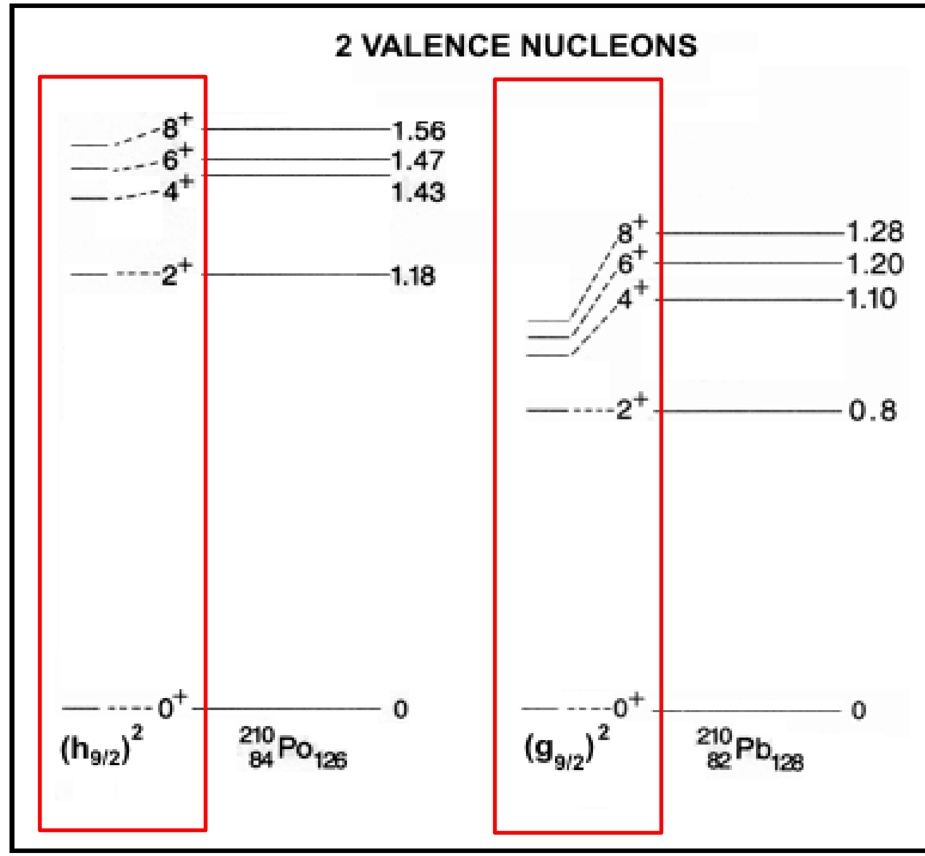
这说明<u>核子间有效相互作用</u>的**主导成分是短程吸引的**.

A simplification: Zero-range δ interaction

Attractive δ interaction

$$V_{12}(\delta) = -V_0\,\delta(\vec{r}_1 - \vec{r}_2)$$
$$= \frac{-V_0}{r_1 - r_2}\,\delta(r_1 - r_2)\,\delta(\cos\theta_1 - \cos\theta_2)\,\delta(\Phi_1 - \Phi_2)$$

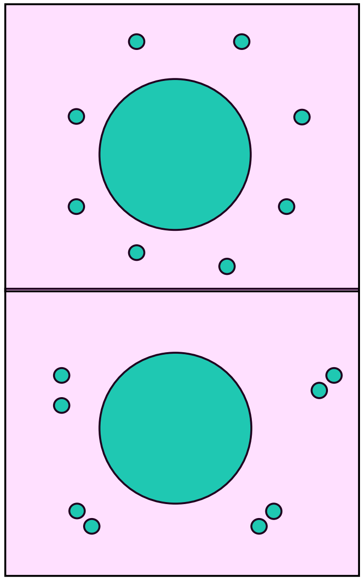




- Many-body basis states (m-scheme):

$$\left|\phi\right\rangle = a^{+}_{j_1 m_1} a^{+}_{j_2 m_2} \cdots a^{+}_{j_n m_n} \left|0\right\rangle$$

Pair truncation – nucleon-pair approximation



- Many-body basis states (m-scheme):

$$|\phi\rangle = a^{+}_{j_1 m_1} a^{+}_{j_2 m_2} \cdots a^{+}_{j_n m_n} |0\rangle$$

- Nucleon pairs

$$A^{(r)+} = \sum_{j_1 j_2} y(j_1 j_2 r) A^{(r)+}(j_1 j_2), \quad A^{(r)+}(j_1 j_2) = (a^{+}_{j_1} \times a^{+}_{j_2})^{(r)}$$

Pair basis states (J-scheme):

$$|\varphi\rangle = \left(\cdots \left(\left(A^{(r_1)+} \times A^{(r_2)+} \right)^{(J_2)} \times A^{(r_3)+} \right)^{(J_3)} \cdots \times A^{(r_N)+} \right)^{(J_N)}_{M_N} |0\rangle$$

J. Q. Chen, Nucl. Phys. A 562, 218 (1993).
J. Q. Chen, Nucl. Phys. A 626, 686 (1997).
Y. M. Zhao et al., Phys. Rev. C 62, 014304 (2000).
Y. M. Zhao and A. Arima, Physics Reports 545, 1 (2014).

Other pair truncation schemes

- Many-body basis states (m-scheme):

$$|\phi\rangle = a^{+}_{j_1 m_1} a^{+}_{j_2 m_2} \cdots a^{+}_{j_n m_n} |0\rangle$$

- Nucleon pairs

$$A^{(r)+} = \sum_{j_1 j_2} y(j_1 j_2 r) A^{(r)+}(j_1 j_2), \quad A^{(r)+}(j_1 j_2) = (a^{+}_{j_1} \times a^{+}_{j_2})^{(r)}$$

$r = 0$, S pair $\qquad r \neq 0$, non-S pairs

- S-pair approximation

(generalized) seniority scheme, BCS, …..

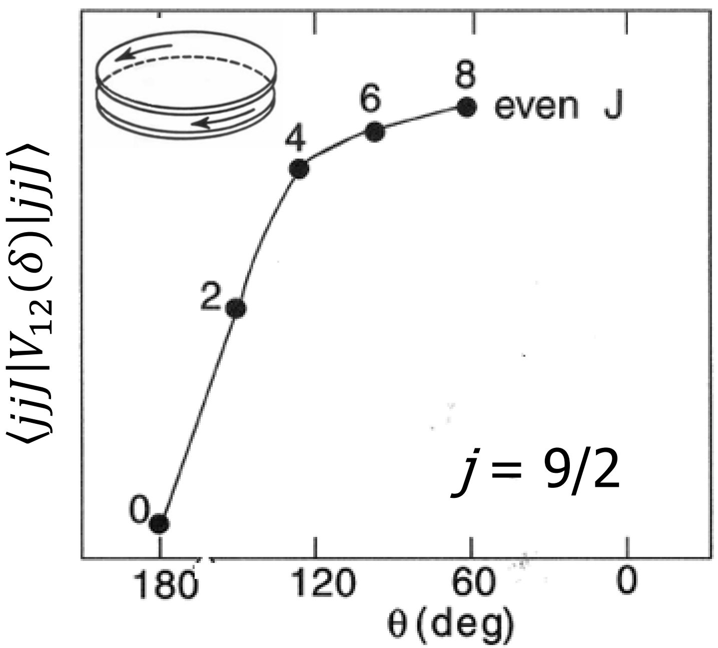
准自旋算符，

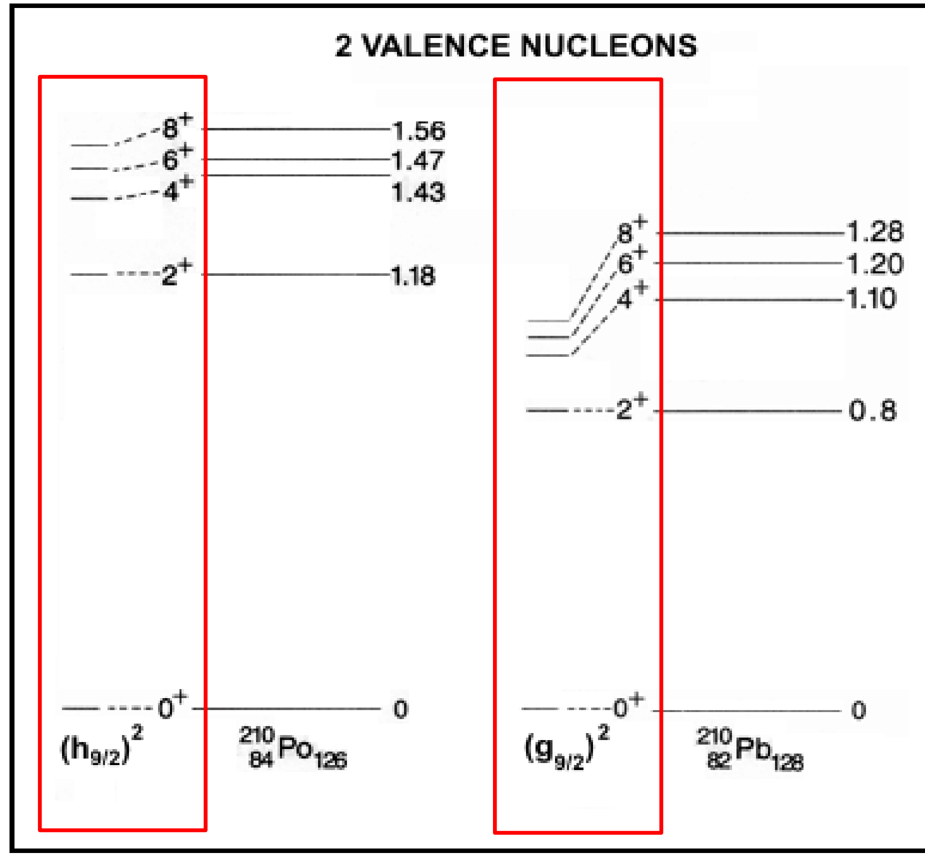
单j壳系统的辛弱数理论

A simplification: Zero-range δ interaction

Attractive δ interaction

$$V_{12}(\delta) = -V_0\,\delta(\vec{r}_1 - \vec{r}_2)$$
$$= \frac{-V_0}{r_1 - r_2}\,\delta(r_1 - r_2)\,\delta(\cos\theta_1 - \cos\theta_2)\,\delta(\Phi_1 - \Phi_2)$$

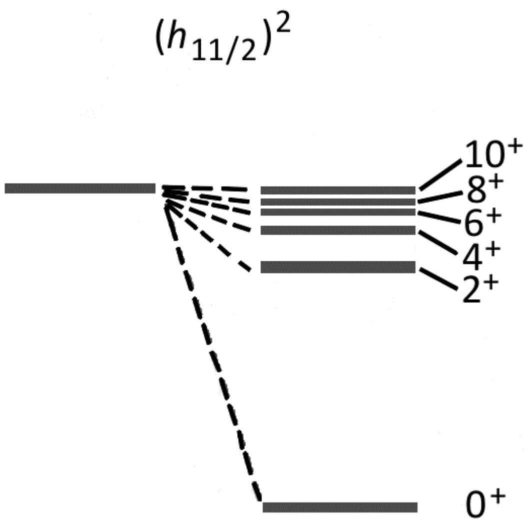




Further simplification: monopole pairing interaction

δ interaction

$$\left\langle j^2 JM \right| V_{pair} \left| j^2 JM \right\rangle = -\frac{G}{2}(2j+1)\delta_{J,0}\delta_{M,0}$$



S-pair operators

- $S_j^+ = \sqrt{\frac{2j+1}{2}} A^+(j^2 J = 0, M = 0) = \frac{1}{2} \sum (-1)^{j-m} a_{jm}^+ a_{j,-m}^+$

  $S_j^- = (S_j^+)^+$

  $2S_j^0 = \sum_m a_{jm}^+ a_{jm} - \frac{2j+1}{2}$

- $[S_j^+, S_j^-] = 2S_j^0 \qquad [S_j^0, S_j^+] = S_j^+ \qquad [S_j^0, S_j^-] = -S_j^-$

回顾：角动量的一般定义

如果满足对易式

$$\left[J_x, J_y\right] = i\hbar J_z \qquad \left[J_y, J_z\right] = i\hbar J_x \qquad \left[J_z, J_x\right] = i\hbar J_y$$

则$\vec{J}$为角动量算符

$$\left[\boldsymbol{J}^2, J_i\right] = 0 \qquad (i \equiv x, y, z)$$

定义角动量升、降算符

$$\left[J_+, J_-\right] = 2\hbar J_z \qquad \left[J_z, J_+\right] = \hbar J_+ \qquad \left[J_z, J_-\right] = -\hbar J_-$$

$$\left[\vec{J}^2, J_+\right] = \left[\vec{J}^2, J_-\right] = 0$$

Quasi-spin operators

- $$S_j^+ = \sqrt{\frac{2j+1}{2}} A^+(j^2 J = 0, M = 0) = \frac{1}{2}\sum (-1)^{j-m} a_{jm}^+ a_{j,-m}^+$$

$$S_j^- = (S_j^+)^+$$

$$2S_j^0 = \sum_m a_{jm}^+ a_{jm} - \frac{2j+1}{2}$$

- $[S_j^+, S_j^-] = 2S_j^0$ $\quad [S_j^0, S_j^+] = S_j^+$ $\quad [S_j^0, S_j^-] = -S_j^-$

$$[J_+, J_-] = 2\hbar J_z \qquad [J_z, J_+] = \hbar J_+ \qquad [J_z, J_-] = -\hbar J_-$$

Eigenstates of quasi-spin operators

- $\left| s, s^0 \right\rangle, \quad s^0 = s, s-1, \ldots, -s+1, -s$

$$S_j^0 \left| s, s^0 \right\rangle = s^0 \left| s, s^0 \right\rangle$$

$$\vec{S}_j^2 \left| s, s^0 \right\rangle = s(s+1) \left| s, s^0 \right\rangle$$

Quasi-spin operators & monopole pairing interaction

- $\left\langle j^2 JM \middle| V_{pair} \middle| j^2 JM \right\rangle = -\frac{G}{2}(2j+1)\delta_{J,0}\delta_{M,0}$
- $\widehat{V}_{pair} = -\frac{G}{2}\widehat{P}$ $\qquad\qquad \widehat{P} = 2S_j^+ S_j^-$

$$S_j^+ S_j^- = \vec{S}_j^2 - S_j^0(S_j^0 - 1)$$

Seniority quantum number

- $\left| s, s^0 \right\rangle, \quad s^0 = s, s-1, \ldots, -s+1, -s$

- $\left| \nu, n \right\rangle$

$$\boxed{S_j^- \left| \nu, n = \nu \right\rangle = 0}$$

particle number n vs. s^0

- $\left| s, s^0 \right\rangle = \left| \nu, n \right\rangle$

$$2S_j^0 = \sum_m a_{jm}^{+} a_{jm} - \frac{2j+1}{2}$$

$$s^0 = \frac{1}{2}\left(n - \frac{2j+1}{2}\right)$$

seniority number v vs. s

- $|s, s^0\rangle = |\nu, n\rangle$
- $S_j^+ S_j^- |\nu, n=\nu\rangle = 0$

$$S_j^+ S_j^- = \vec{S}_j^2 - S_j^0(S_j^0 - 1)$$

$$s^0 = \frac{1}{2}(n - \frac{2j+1}{2})$$

$$\Downarrow$$

$$s(s+1) - \frac{1}{4}(\frac{2j+1}{2} - \nu)(\frac{2j+5}{2} - \nu) = 0$$

$$\therefore \quad s = \frac{1}{2}(\frac{2j+1}{2} - \nu)$$

- $\left|s, s^0\right\rangle, \quad s^0 = s, s-1, \ldots, -s+1, -s$

- $\left|s, s^0\right\rangle = \left|\nu, n\right\rangle$

$$s^0 = \frac{1}{2}\left(n - \frac{2j+1}{2}\right)$$

$$s = \frac{1}{2}\left(\frac{2j+1}{2} - \nu\right)$$

- $\nu_{\max} = \frac{2j+1}{2}$

- $\left|n - \frac{2j+1}{2}\right| \leq \frac{2j+1}{2} - \nu$

Eigen energies

- $\hat{V}_{pair} = -\frac{G}{2}\hat{P}$ $\qquad \hat{P} = 2S_j^+ S_j^-$

$$S_j^+ S_j^- = \vec{S}_j^2 - S_j^0 (S_j^0 - 1)$$

- $P(\nu, n) = 2s(s+1) - 2s^0(s^0 - 1)$

$$s = \frac{1}{2}\left(\frac{2j+1}{2} - \nu\right) \qquad s^0 = \frac{1}{2}\left(n - \frac{2j+1}{2}\right)$$

$$\Downarrow$$

$$= \frac{n-\nu}{2}(2j+3-n-\nu)$$

部分动力学对称性，

单j壳系统不变本征态的解析波函数

monopole pairing interaction

$$\left\langle j^2 JM \right| V_{pair} \left| j^2 JM \right\rangle = -\frac{G}{2}(2j+1)\delta_{J,0}\delta_{M,0}$$

Seniority-conserving TBME

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Partially Solvable Pair-Coupling Models with Seniority-Conserving Interactions

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Given a single j-shell Hamiltonian, the algebraic conditions for conservation of seniority are derived from a quasispin tensor decomposition of the two-nucleon interaction. This makes it possible to construct useful solvable and partially solvable shell-model Hamiltonians with eigenstates classified by a spectrum generating algebra. Applications are made to the low-lying energy levels of the $N = 50$ nuclear isotones.

$$\text{for } j = 9/2, \quad 65V^2 - 315V^4 + 403V^6 - 153V^8 = 0, \tag{17}$$

$$\text{for } j = 11/2, \quad 1020V^2 - 3519V^4 + 637V^6 + 4403V^8 - 2541V^{10} = 0, \tag{18}$$

$$\text{for } j = 13/2, \quad 1615V^2 - 4275V^4 - 1456V^6 + 3196V^8 + 5145V^{10} - 4225V^{12} = 0. \tag{19}$$

$$V^J = \langle jjJ | V | jjJ \rangle_{nas}$$

Eigenstates of ANY two-body interactions - (1)

PHYSICAL REVIEW C **73**, 044302 (2006)

Seniority conservation and seniority violation in the $g_{9/2}$ shell

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The $g_{9/2}$ shell of identical particles is the first one for which one can have seniority-mixing effects. We consider three interactions: A delta interaction that conserves seniority, a quadrupole-quadrupole ($Q \cdot Q$) interaction that does not, and a third one consisting of two-body matrix elements taken from experiment (^{98}Cd) that also leads to some seniority mixing. We deal with proton holes relative to a $Z = 50$, $N = 50$ core. One surprising result is that, for a four-particle system with total angular momentum $I = 4$, there is one state with seniority $v = 4$ that is an eigenstate of *any* two-body interaction—seniority conserving or not. The other two states are mixtures of $v = 2$ and $v = 4$ for the seniority-mixing interactions. The same thing holds true for $I = 6$. Another point of interest is that, in the single-j-shell approximation, the splittings $\Delta E = E(I_{\mathrm{max}}) - E(I_{\mathrm{min}})$ are the same for three and five particles with a seniority conserving interaction (a well-known result), but are equal and opposite for a $Q \cdot Q$ interaction. We also fit the spectra with a combination of the delta and $Q \cdot Q$ interactions. The $Z = 40$, $N = 40$ core plus $g_{9/2}$ neutrons (Zr isotopes) is also considered, although it is recognized that the core is deformed.

Partial Conservation of Seniority and Nuclear Isomerism

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$$E[(9/2)^4, v = 4, J = 4] = \frac{68}{33}\nu_2 + \nu_4 + \frac{13}{15}\nu_6 + \frac{114}{55}\nu_8,$$

$$\nu_J = \langle jjJ | V | jjJ \rangle_{nas}$$

$$E[(9/2)^4, v = 4, J = 6] = \frac{19}{11}\nu_2 + \frac{12}{13}\nu_4 + \nu_6 + \frac{336}{143}\nu_8.$$

$$|(9/2)^4, v = 4, J = 4\rangle = \sqrt{\frac{2363}{1570}}|(9/2)^4[22], v = 4, J = 4\rangle - \sqrt{\frac{65}{5338}}|(9/2)^4[24], v = 4, J = 4\rangle,$$

$$|(9/2)^4, v = 4, J = 6\rangle = \sqrt{\frac{1\,620\,896}{635\,341}}|(9/2)^4[24], v = 4, J = 6\rangle - \sqrt{\frac{5725}{635\,341}}|(9/2)^4[44], v = 4, J = 6\rangle.$$

Eigenstates of ANY two-body interactions – (2)

- $s^0 = \frac{1}{2}(n - \frac{2j+1}{2})$ $\qquad$ $s = \frac{1}{2}(\frac{2j+1}{2} - \nu)$
- $V = V^{k=0} + E_0 S_j^0 + V^{k=2} + C$ $\qquad$ *Ref: Chapter 19 of Talmi's book*
- $\left\langle s, s^0 \middle| V_{\kappa=0}^{(k)} \middle| s+1, s^0 \right\rangle$

Eigenstates of ANY two-body interactions - (2)

- $s^0 = \frac{1}{2}(n - \frac{2j+1}{2})$ $\qquad$ $s = \frac{1}{2}(\frac{2j+1}{2} - \nu)$

- $V = V^{k=0} + E_0 S_j^0 + V^{k=2} + C$

- $\left\langle s, s^0 \middle| V_{\kappa=0}^{(k)} \middle| s+1, s^0 \right\rangle$

- Wigner-Eckart 定理（Edmond 约定）

$$\left\langle s, s^0 \middle| V_{\kappa=0}^{(k=2)} \middle| s+1, s^0 \right\rangle = (-)^{s-s^0} \begin{pmatrix} s & 2 & s+1 \\ -s^0 & 0 & s^0 \end{pmatrix} \left(s \,\|\, V^{(k=2)} \,\|\, s+1 \right)$$

if $n = \frac{2j+1}{2}$, $s^0 = 0$

Nuclear Physics A 846 (2010) 31–39

Solvability of eigenvalues in j^n configurations

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Eigenvalues of eigenstates in j^n configurations (n identical nucleons in the j-orbit) are functions of two-body energies. In some cases they are linear combinations of two-body energies whose coefficients are independent of the interaction and are rational non-negative numbers. It is shown here that a state which is an eigenstate of *any* two-body interaction has this *solvability* property. This includes, in particular, any state with spin J if there are no other states with this J in the j^n configuration. It is also shown that eigenstates with solvable eigenvalues have definite seniority v and thus, exhibit partial dynamical symmetry. Most of the derivations apply also to states of j^n nucleons with $T < n/2$.

PHYSICAL REVIEW C **98**, 061303(R) (2018)

Rapid Communications

Partial seniority conservation and solvability of single-j systems

Yibin Qian[1,2,*] and Chong Qi[2,†]
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[2]*Department of Physics, KTH Royal Institute of Technology, 10691 Stockholm, Sweden*

The seniority symmetry is known to be partially conserved in two special cases in the $(9/2)^4$ system, which can lead to striking features in the corresponding structure and electromagnetic transition properties. However, it is still quite difficult to derive those kind of solvable states, in general, especially towards higher-j orbits. We have developed a novel and effective way to confront this challenge by starting from the m scheme and making use of the angular momentum projection method. It also allows us to explore another special family of seniority conserving states in the midshell besides the specific case of the $(9/2)^4$ configuration. Moreover, we have studied systemically all states in single-j systems up to $j = 15/2$ and derived the analytic expressions for the eigenvalues of all solvable states with a focus on those in $j = 9/2$ and $j = 11/2$. Such studies can also be useful for the experimental search of relevant states and for the understanding of their electromagnetic transition properties.

PHYSICAL REVIEW C **98**, 061303(R) (2018)

Rapid Communications

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Configuration	J	v	Energy
$(7/2)^4$	2	2	$\frac{1}{2}V_0 + \frac{11}{6}V_2 + \frac{3}{2}V_4 + \frac{13}{6}V_6$
	2	4	$V_2 + \frac{42}{11}V_4 + \frac{13}{11}V_6$
	4	2	$\frac{1}{2}V_0 + \frac{5}{6}V_2 + \frac{5}{2}V_4 + \frac{13}{6}V_6$
	4	4	$\frac{7}{3}V_2 + V_4 + \frac{8}{3}V_6$
$(9/2)^5$	5/2	3	$\frac{2}{5}V_0 + \frac{85}{66}V_2 + \frac{371}{110}V_4 + \frac{559}{330}V_6 + \frac{357}{110}V_8$
	5/2	5	$\frac{89}{33}V_2 + \frac{278}{143}V_4 + \frac{67}{33}V_6 + \frac{476}{143}V_8$
	7/2	3	$\frac{2}{5}V_0 + \frac{155}{66}V_2 + \frac{2373}{1430}V_4 + \frac{346}{165}V_6 + \frac{2499}{715}V_8$
	7/2	5	$\frac{59}{66}V_2 + \frac{885}{286}V_4 + \frac{622}{165}V_6 + \frac{1603}{715}V_8$
	9/2	3	$\frac{2}{5}V_0 + \frac{28}{33}V_2 + \frac{1884}{715}V_4 + \frac{562}{165}V_6 + \frac{1938}{715}V_8$
	11/2	3	$\frac{2}{5}V_0 + \frac{50}{33}V_2 + \frac{73}{55}V_4 + \frac{1267}{330}V_6 + \frac{321}{110}V_8$
	11/2	5	$\frac{139}{66}V_2 + \frac{565}{286}V_4 + \frac{392}{165}V_6 + \frac{2533}{715}V_8$
	13/2	3	$\frac{2}{5}V_0 + \frac{50}{33}V_2 + \frac{123}{55}V_4 + \frac{302}{165}V_6 + \frac{221}{55}V_8$
	13/2	5	$\frac{8}{11}V_2 + \frac{393}{143}V_4 + \frac{40}{11}V_6 + \frac{413}{143}V_8$
	15/2	3	$\frac{2}{5}V_0 + \frac{5}{11}V_2 + \frac{2274}{715}V_4 + \frac{223}{110}V_6 + \frac{5631}{1430}V_8$
	15/2	5	$\frac{15}{11}V_2 + \frac{300}{143}V_4 + \frac{63}{22}V_6 + \frac{1051}{286}V_8$
	17/2	3	$\frac{2}{5}V_0 + \frac{32}{33}V_2 + \frac{829}{715}V_4 + \frac{632}{165}V_6 + \frac{2603}{715}V_8$
	17/2	5	$\frac{62}{33}V_2 + \frac{215}{143}V_4 + \frac{362}{165}V_6 + \frac{3163}{715}V_8$

Configuration	J	v	Energy
$(11/2)^6$	0	6	$\frac{2025}{1001}V_2 + \frac{1566}{1001}V_4 + \frac{61}{11}V_6 + \frac{4556}{2717}V_8 + \frac{1035}{247}V_{10}$
	3	4	$\frac{1}{3}V_0 + \frac{640}{429}V_2 + \frac{1187}{572}V_4 + \frac{7489}{2244}V_6 + \frac{7879}{1716}V_8 + \frac{2803}{884}V_{10}$
	5	6	$\frac{129}{91}V_2 + \frac{10487}{4004}V_4 + \frac{149}{44}V_6 + \frac{3863}{988}V_8 + \frac{3623}{988}V_{10}$
	11	4	$\frac{1}{3}V_0 + \frac{3670}{3003}V_2 + \frac{6267}{4004}V_4 + \frac{5683}{2244}V_6 + \frac{157147}{32604}V_8 + \frac{76035}{16796}V_{10}$
	11	6	$\frac{1725}{1001}V_2 + \frac{6645}{4004}V_4 + \frac{139}{44}V_6 + \frac{33347}{10868}V_8 + \frac{5325}{988}V_{10}$
	13	4	$\frac{1}{3}V_0 + \frac{1420}{3003}V_2 + \frac{4231}{2002}V_4 + \frac{3379}{1122}V_6 + \frac{55411}{16302}V_8 + \frac{47615}{8398}V_{10}$
	13	6	$\frac{925}{1001}V_2 + \frac{9005}{4004}V_4 + \frac{257}{132}V_6 + \frac{153661}{32604}V_8 + \frac{5105}{988}V_{10}$
	14	4	$\frac{1}{3}V_0 + \frac{2890}{3003}V_2 + \frac{1879}{2002}V_4 + \frac{2861}{1122}V_6 + \frac{80387}{16302}V_8 + \frac{44381}{8398}V_{10}$
	14	6	$\frac{1569}{1001}V_2 + \frac{3043}{2002}V_4 + \frac{811}{374}V_6 + \frac{19267}{5434}V_8 + \frac{52055}{8398}V_{10}$

Analytic w.f. of eigenstate of any TB interaction

PHYSICAL REVIEW C **100**, 014318 (2019)

Nucleon-pair wave functions in a single-*j* shell

Yi-Yuan Cheng,[1,*] Jia-Jie Shen,[2] Guan-Jian Fu,[3] Xian-Rong Zhou,[1] Yu-Min Zhao,[4,5,†] and Akito Arima[6,4]

- construct analytic expressions for states of definite seniority numbers v = 3, 4, 5 in terms of nucleon-pair basis states

- derive exact wave functions for a few eigenstates of *any* two-body interactions in the midshells of j = 7/2, 9/2, 11/2

Analytic w.f. for seniority = 3,4,5

- n=v state (alpha) satisfying below commutator

$$A^{\dagger(0)}\tilde{A}^{(0)}\alpha^{\dagger}|0\rangle = [A^{\dagger(0)}\tilde{A}^{(0)}, \alpha^{\dagger}]|0\rangle = 0$$

$$S_j^{\dagger} = \frac{\sqrt{2j+1}}{2} A^{\dagger(0)} \qquad S_j^{-} = \frac{\sqrt{2j+1}}{2} A^{(0)} = \frac{\sqrt{2j+1}}{2} A^{(0)}$$

- Seniority-ν state of ν-particle $\left|\alpha\right\rangle = \left| j^{\nu}, \nu, J, M \right\rangle$

Seniority-ν state of n-particle $\propto (S_j^{\dagger})^{(n-\nu)/2} \left| j^{\nu}, \nu, J, M \right\rangle$

Seniority-3

$$\begin{cases} \underbrace{A^{\dagger(0)}A^{\dagger(0)}\cdots A^{\dagger(0)}}_{N-1}[(a_j^\dagger \times A^{\dagger r})^J + \frac{4\hat{r}}{4j-2}A^{\dagger(0)}a_j^\dagger]|0\rangle & \text{if } J = j, \\ \underbrace{A^{\dagger(0)}A^{\dagger(0)}\cdots A^{\dagger(0)}}_{N-1}(a_j^\dagger \times A^{\dagger r})^J|0\rangle & \text{otherwise,} \end{cases}$$

Seniority-4

$$\begin{cases} \underbrace{A^{\dagger(0)}\cdots A^{\dagger(0)}}_{N-2}[(A^{\dagger r} \times A^{\dagger r})^{(0)} + \frac{2\hat{r}}{2j-1}A^{\dagger(0)}A^{\dagger(0)}]|0\rangle & \text{if } J = 0, \\ \underbrace{A^{\dagger(0)}\cdots A^{\dagger(0)}}_{N-2}[(A^{\dagger r_1} \times A^{\dagger r_2})^J - \frac{4\hat{r}_1\hat{r}_2\hat{j}}{2j-3}\begin{Bmatrix} j & j & r_1 \\ r_2 & J & j \end{Bmatrix}A^{\dagger(0)}A^{\dagger J}]|0\rangle & \text{if } J = 2,\ldots,2j-1, \\ \underbrace{A^{\dagger(0)}\cdots A^{\dagger(0)}}_{N-2}(A^{\dagger r_1} \times A^{\dagger r_2})^J|0\rangle & \text{otherwise.} \end{cases}$$

Seniority-5

$$\underbrace{A^{\dagger(0)}A^{\dagger(0)}\cdots A^{\dagger(0)}}_{N-2}\Bigg\{[(a_j^\dagger \times A^{\dagger r_1})^{J_1} \times A^{\dagger r_2}]^J - \sum_{\{r_1'\}}[\lambda_1(r_1')]A^{\dagger(0)}(a_j^\dagger \times A^{\dagger r_1'})^J - \lambda_2 A^{\dagger(0)}(a_j^\dagger \times A^{\dagger r_1})^J - \lambda_3 A^{\dagger(0)}(a_j^\dagger \times A^{\dagger r_2})^J\Bigg\}|0\rangle, \text{ if } J \neq j$$

$$\lambda_1(r_1') = (-)^{j+J}\frac{8\hat{j}\hat{r}_1\hat{r}_2\hat{r}_1'\hat{J}_1}{4j-10}\begin{Bmatrix} j & r_1 & J_1 \\ r_2 & J & r_1' \end{Bmatrix}\begin{Bmatrix} j & j & r_1 \\ r_2 & r_1' & j \end{Bmatrix}$$

$$\lambda_2 = (-)^{j+J_1+1}\frac{4\hat{j}\hat{r}_2\hat{J}_1}{4j-10}\begin{Bmatrix} j & r_1 & J_1 \\ J & r_2 & j \end{Bmatrix} \qquad \lambda_3 = -\delta_{j,J_1}\frac{4\hat{r}_1}{4j-10}.$$

$$\underbrace{A^{\dagger(0)}A^{\dagger(0)}\cdots A^{\dagger(0)}}_{N-2}\Bigg\{[(a_j^\dagger \times A^{\dagger r_1})^{J_1} \times A^{\dagger r_2}]^j - \sum_{\{r_1'\}}(1-\delta_{r_1',0})[\lambda_1(r_1')]A^{\dagger(0)}(a_j^\dagger \times A^{\dagger r_1'})^j - \lambda_2 A^{\dagger(0)}(a_j^\dagger \times A^{\dagger r_1})^j - \lambda_3 A^{\dagger(0)}(a_j^\dagger \times A^{\dagger r_2})^j + \lambda_4 A^{\dagger(0)}A^{\dagger(0)}a_j^\dagger\Bigg\}|0\rangle, \quad \text{if } J = j$$

$$\lambda_4 = \frac{4\hat{r}_1\hat{r}_2}{(4j-10)(2j-3)}\Bigg(\hat{j}\hat{J}_1\sum_{\{r_1'\}}[2(1-\delta_{r_1',0})(2r_1'+1) + \frac{1}{2}\delta_{r_1',0}(2j+1)] \times \begin{Bmatrix} j & r_1 & J_1 \\ r_2 & j & r_1' \end{Bmatrix}\begin{Bmatrix} j & j & r_1 \\ r_2 & r_1' & j \end{Bmatrix} + (-)^{j+J_1}\hat{j}\hat{J}_1\begin{Bmatrix} j & r_1 & J_1 \\ j & r_2 & j \end{Bmatrix} + \delta_{j,J_1}\Bigg).$$

$|r_1, r_2, \ldots, r_N; J_2, \ldots, J\rangle$ to denote $((A^{\dagger r_1} \times A^{\dagger r_2})^{J_2} \cdots \times A^{\dagger r_N})^J |0\rangle$

$|j, r_1, r_2, \ldots, r_N; J_1, J_2, \ldots, J\rangle$ to denote $(((a_j^\dagger \times A^{\dagger r_1})^{J_1} \times A^{\dagger r_2})^{J_2} \cdots \times A^{\dagger r_N})^J |0\rangle$

	J	v	$\lvert\alpha\rangle$	$\mathcal{N}^2$
$(7/2)^4$	2	2	$\lvert 2,0;2\rangle$	2
	2	4	$\lvert 2,2;2\rangle - \frac{8\sqrt{6}}{21}\lvert 2,0;2\rangle$	$\frac{132}{49}$
	4	2	$\lvert 4,0;4\rangle$	2
	4	4	$\lvert 2,2;4\rangle + \frac{2\sqrt{110}}{21}\lvert 4,0;4\rangle$	$\frac{1300}{441}$
$(9/2)^5$	5/2	3	$\lvert \frac{9}{2},2,0;\frac{5}{2},\frac{5}{2}\rangle$	$\frac{4}{3}$
	5/2	5	$\lvert \frac{9}{2},2,2;\frac{5}{2},\frac{5}{2}\rangle + \frac{5\sqrt{231}}{132}\lvert \frac{9}{2},2,0;\frac{5}{2},\frac{5}{2}\rangle + \frac{\sqrt{455}}{28}\lvert \frac{9}{2},4,0;\frac{5}{2},\frac{5}{2}\rangle$	$\frac{2080}{693}$
	7/2	3	$\lvert \frac{9}{2},2,0;\frac{7}{2},\frac{7}{2}\rangle$	$\frac{416}{165}$
	7/2	5	$\lvert \frac{9}{2},2,2;\frac{5}{2},\frac{7}{2}\rangle + \frac{78\sqrt{105}}{1512}\lvert \frac{9}{2},4,0;\frac{7}{2},\frac{7}{2}\rangle$	$\frac{442}{693}$
	9/2	3	$\lvert \frac{9}{2},2,0;\frac{9}{2},\frac{9}{2}\rangle + \frac{\sqrt{5}}{4}\lvert \frac{9}{2},0,0;\frac{9}{2},\frac{9}{2}\rangle$	$\frac{26}{165}$
	11/2	3	$\lvert \frac{9}{2},2,0;\frac{11}{2},\frac{11}{2}\rangle$	$\frac{136}{165}$
	11/2	5	$\lvert \frac{9}{2},2,2;\frac{7}{2},\frac{11}{2}\rangle - \frac{\sqrt{91}}{22}\lvert \frac{9}{2},2,0;\frac{11}{2},\frac{11}{2}\rangle + \frac{13\sqrt{1890}}{1188}\lvert \frac{9}{2},4,0;\frac{11}{2},\frac{11}{2}\rangle$	$\frac{96824}{35937}$
	13/2	3	$\lvert \frac{9}{2},2,0;\frac{13}{2},\frac{13}{2}\rangle$	$\frac{16}{11}$
	13/2	5	$\lvert \frac{9}{2},2,2;\frac{9}{2},\frac{13}{2}\rangle + \frac{4\sqrt{5}}{11}\lvert \frac{9}{2},2,0;\frac{13}{2},\frac{13}{2}\rangle + \frac{5\sqrt{78}}{66}\lvert \frac{9}{2},4,0;\frac{13}{2},\frac{13}{2}\rangle$	$\frac{68}{1331}$
	15/2	3	$\lvert \frac{9}{2},4,0;\frac{15}{2},\frac{15}{2}\rangle$	$\frac{456}{715}$
	15/2	5	$\lvert \frac{9}{2},2,2;\frac{11}{2},\frac{15}{2}\rangle + \frac{\sqrt{13090}}{132}\lvert \frac{9}{2},4,0;\frac{15}{2},\frac{15}{2}\rangle$	$\frac{2125}{4719}$
	17/2	3	$\lvert \frac{9}{2},4,0;\frac{17}{2},\frac{17}{2}\rangle$	$\frac{200}{143}$
	17/2	5	$\lvert \frac{9}{2},2,2;\frac{13}{2},\frac{17}{2}\rangle + \frac{\sqrt{5005}}{66}\lvert \frac{9}{2},4,0;\frac{17}{2},\frac{17}{2}\rangle$	$\frac{266}{99}$
$(11/2)^6$	3	4	$\lvert 2,4,0;3,3\rangle$	$\frac{680}{429}$
	11	4	$\lvert 2,10,0;11,11\rangle$	$\frac{200}{231}$
	11	6	$\lvert 2,2,8;4,11\rangle - 18\sqrt{\frac{2}{1729}}\lvert 2,10,0;11,11\rangle$	2.596
	13	4	$\lvert 4,10,0;13,13\rangle$	$\frac{280}{429}$
	13	6	$\lvert 2,2,10;4,13\rangle + \frac{24}{7}\sqrt{\frac{15}{143}}\lvert 4,10,0;13,13\rangle$	0.729
	14	4	$\lvert 4,10,0;14,14\rangle$	$\frac{868}{429}$
	14	6	$\lvert 2,2,10;4,14\rangle + \frac{24}{7}\sqrt{\frac{15}{143}}\lvert 4,10,0;14,14\rangle$	4.426

非简并多j壳，*BCS*理论

monopole pairing interaction:
non-degenerate multiple-j case

$$H = \sum_{j,m} \varepsilon_j a^\dagger_{jm} a_{jm} - G \sum_{j,j'} S^\dagger_j S^-_{j'}$$

BCS theory for nucleons in non-degenerate multiple-j shells

- A BCS trial wave function

$$|BCS\rangle = \prod_{j,m>0} [u_j + (-1)^{j-m} v_j a^\dagger_{jm} a^\dagger_{j-m}]|0>$$

Not conserving the particle number

BCS theory for nucleons in non-degenerate multiple-j shells

- A BCS trial wave function

$$|BCS\rangle = \prod_{j,m>0} [u_j + (-1)^{j-m} v_j a^\dagger_{jm} a^\dagger_{j-m}]|0>$$

Not conserving the particle number

- |BCS> is a linear combination of $\boxed{\left(\prod_j (S_j^\dagger)^{n_j/2}\right)|0\rangle}$, $n_j \in \{0, 2, \ldots, 2j+1\}$

BCS theory for nucleons in non-degenerate multiple-j shells

- $$\langle BCS | BCS \rangle = \prod_j (u_j^2 + v_j^2)^{(2j+1)/2}$$

  If $u_j^2 + v_j^2 = 1$ for each j， BCS wf is normalized

BCS theory for nucleons in non-degenerate multiple-j shells

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- If $u_j^2 + v_j^2 = 1$ for each j, one has

  $$\langle BCS | a_{jm}^\dagger a_{jm} | BCS \rangle = v_j^2$$

  probability interpretation of u_j^2 and v_j^2

Gap equations

- Determine the BCS trial wave function by variation

$$\delta \left\langle BCS \middle| \widehat{H} \middle| BCS \right\rangle = 0$$

$$H = \sum_{j,m} \varepsilon_j a^{\dagger}_{jm} a_{jm} - G \sum_{j,j'} S^{\dagger}_{j} S^{-}_{j'}$$

Gap equations

- Determine the BCS trial wave function by variation

$$\delta \langle BCS | \widehat{H} | BCS \rangle = 0$$

$$H = \sum_{j,m} \varepsilon_j a_{jm}^{\dagger} a_{jm} - G \sum_{j,j'} S_j^{\dagger} S_{j'}^{-}$$

- Variation with the constraint

$$\langle BCS | \widehat{N} | BCS \rangle = \sum_j (2j+1) v_j^2 = n$$

- Finally,

$$\boxed{\delta \langle BCS | \widehat{H} - \lambda \widehat{N} | BCS \rangle = 0}$$

Quasi-particles.
Bogolyubov-Valatin transformation

- The BCS quasi-particle operators α are defined as

$$\begin{aligned} \alpha^\dagger_{jm} &= u_j a^\dagger_{jm} - (-1)^{j-m} v_j a_{j-m} \\ \alpha_{jm} &= u_j a_{jm} - (-1)^{j-m} v_j a^\dagger_{j-m} \end{aligned}$$

and they satisfy

$$\{\alpha_i, \alpha^\dagger_j\} = \delta_{i,j}, \quad \{\alpha_i, \alpha_j\} = 0, \quad \{\alpha^\dagger_i, \alpha^\dagger_j\} = 0$$

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- Particle operators can also be expressed in terms of q.p. operators

$$\begin{aligned} a^\dagger_{jm} &= u_j \alpha^\dagger_{jm} + (-1)^{j-m} v_j \alpha_{j-m} \\ a_{jm} &= u_j \alpha_{jm} + (-1)^{j-m} v_j \alpha^\dagger_{j-m} \end{aligned}$$

BCS wave function = quasi-particle VACUUM

$$\alpha_{jm} \left| BCS \right\rangle = 0 \qquad \left| BCS \right\rangle = \left| \tilde{0} \right\rangle$$

0qp state: $\left| \tilde{0} \right\rangle$

2qp state: in terms of $\alpha^{\dagger}_{j1m1} \alpha^{\dagger}_{j2m2} \left| \tilde{0} \right\rangle$

4qp state: in terms of $\alpha^{\dagger}_{j1m1} \alpha^{\dagger}_{j2m2} \alpha^{\dagger}_{j3m3} \alpha^{\dagger}_{j4m4} \left| \tilde{0} \right\rangle$

BCS wave function = quasi-particle VACUUM

$$\alpha_{jm} \left| BCS \right\rangle = 0 \qquad\qquad \left| BCS \right\rangle = \left| \tilde{0} \right\rangle$$

- Express $\widehat{H}^{qp}(\lambda) = \widehat{H} - \lambda \widehat{N}$ in terms of quasi-particle operators
- Do normal ordering

$$\begin{aligned} H^{qp}(\lambda) &= H^{00} + H^{11} + H^{02} + H^{20} \\ &+ H^{22} + H^{13} + H^{31} + H^{04} + H^{40} \end{aligned}$$

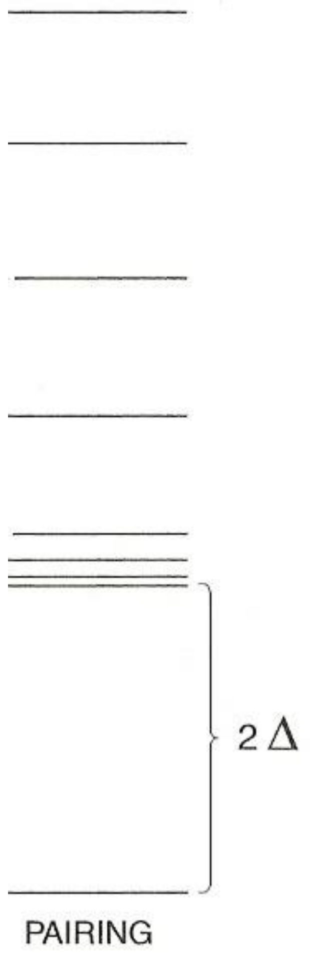
Quasi-particle states

If only considering $H^{00} + H^{11}$

$$\langle BCS | \widehat{H} - \lambda \widehat{N} | BCS \rangle = H^{00}$$

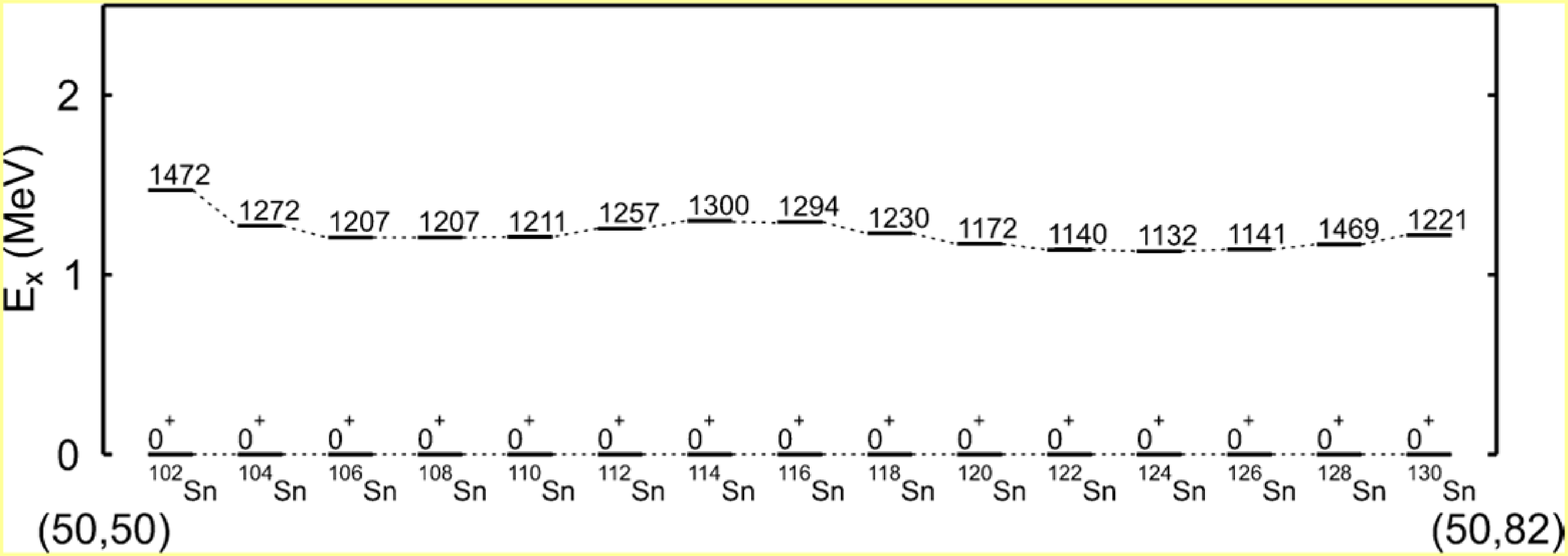
$$\begin{cases} H^{11} = \sum_{j,m} E_j \alpha^\dagger_{jm} \alpha_{jm} \\ E_j = \sqrt{(\varepsilon_j - \lambda)^2 + \Delta^2} \\ \Delta = \frac{G}{2} \sum_j (2j+1) u_j v_j \end{cases}$$

$$E_x(2\mathrm{qp}) \geq 2\Delta$$



Evidence for pairing correlations in nuclei

(iii) The excitation energy of the first excited 2^+ state in nuclei remains remarkably constant over large intervals of neutron (proton) numbers.



Prof. R. Casten's PPT

壳模型配对近似，

近球形核的一维配对态

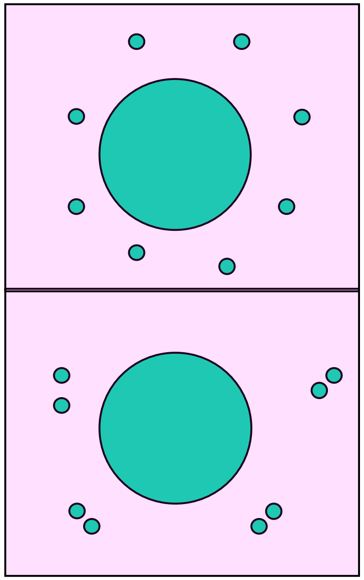
Nucleon pair approximation to the SM

- Many-body basis states (m-scheme):

$$|\phi\rangle = a^{+}_{j_1 m_1} a^{+}_{j_2 m_2} \cdots a^{+}_{j_n m_n} |0\rangle$$

- Nucleon pairs

$$A^{(r)+} = \sum_{j_1 j_2} y(j_1 j_2 r) A^{(r)+}(j_1 j_2), \quad A^{(r)+}(j_1 j_2) = (a^{+}_{j_1} \times a^{+}_{j_2})^{(r)}$$



Pair basis states (J-scheme):

$$|\varphi\rangle = \left(\cdots \left(\left(A^{(r_1)+} \times A^{(r_2)+} \right)^{(J_2)} \times A^{(r_3)+} \right)^{(J_3)} \cdots \times A^{(r_N)+} \right)^{(J_N)}_{M_N} |0\rangle$$

J. Q. Chen, Nucl. Phys. A 562, 218 (1993).
J. Q. Chen, Nucl. Phys. A 626, 686 (1997).
Y. M. Zhao et al., Phys. Rev. C 62, 014304 (2000).
Y. M. Zhao and A. Arima, Physics Reports 545, 1 (2014).

Nucleon pair approximation to the SM

选择低能量的配对构造配对组态空间

↓

计算哈密顿量在配对组态空间的矩阵元

↓

对角化哈密顿量矩阵得到本征能量&本征波函数

↓

计算可观测量

One-dimensional pair wave functions

PHYSICAL REVIEW C **94**, 024321 (2016)

Nucleon-pair states of even-even Sn isotopes based on realistic effective interactions

Y. Y. Cheng,[1] C. Qi,[2] Y. M. Zhao,[1,3,*] and A. Arima[1,4]

$$H_\sigma = \sum_j \varepsilon_j n_{j\sigma} + \sum_{j_1 \le j_2} \sum_{j_3 \le j_4} \sum_J \frac{V_{JT=1}(j_1 j_2 j_3 j_4)}{\sqrt{(1+\delta_{j_1 j_2})(1+\delta_{j_3 j_4})}} \hat{J} \left(A_\sigma^{J+}(j_1 j_2) \times \tilde{A}_\sigma^{J}(j_3 j_4) \right)^{(0)}$$

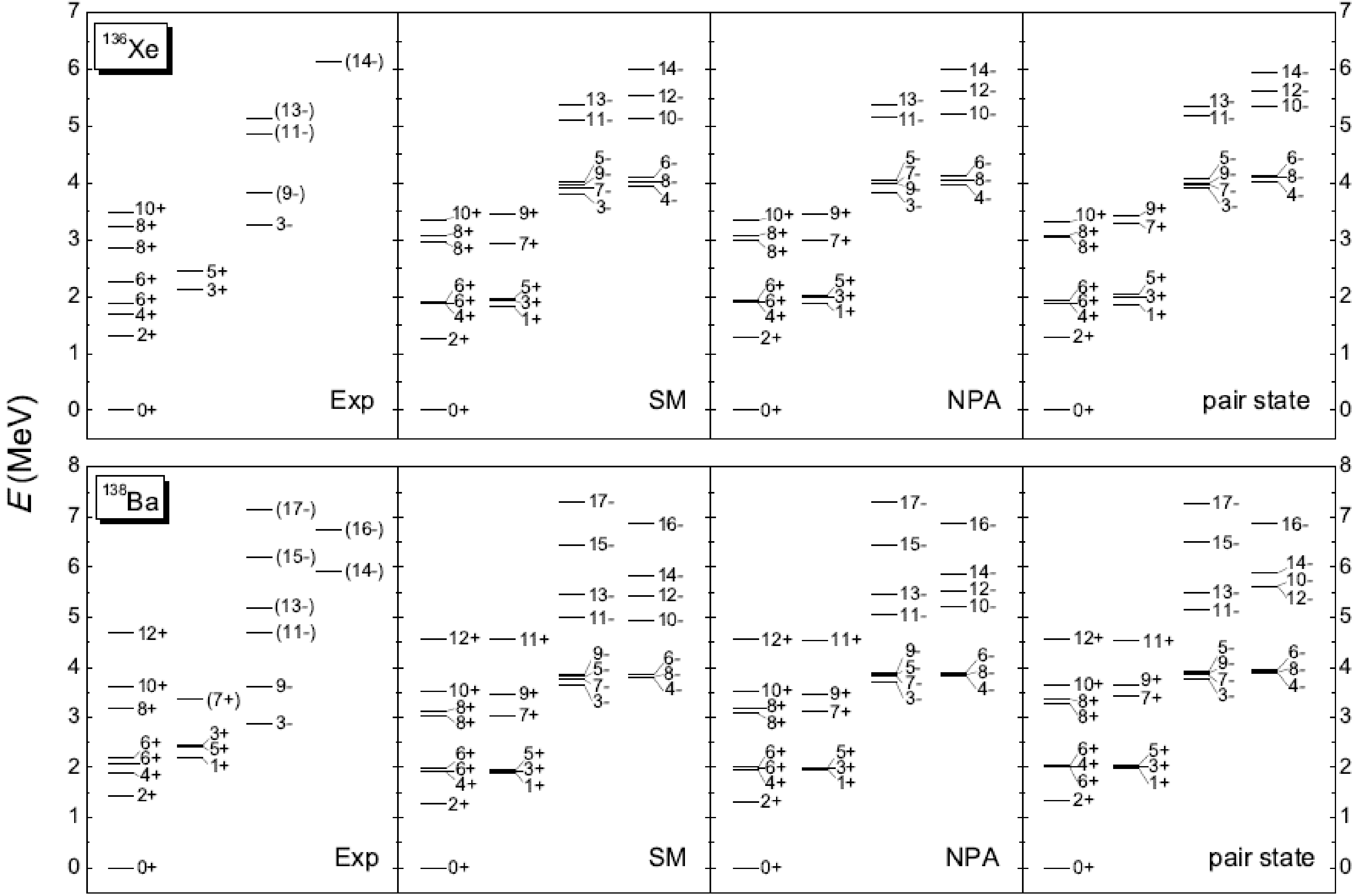
PHYSICAL REVIEW C **94**, 024307 (2016)

Nucleon-pair states of even-even $N = 82$ isotones

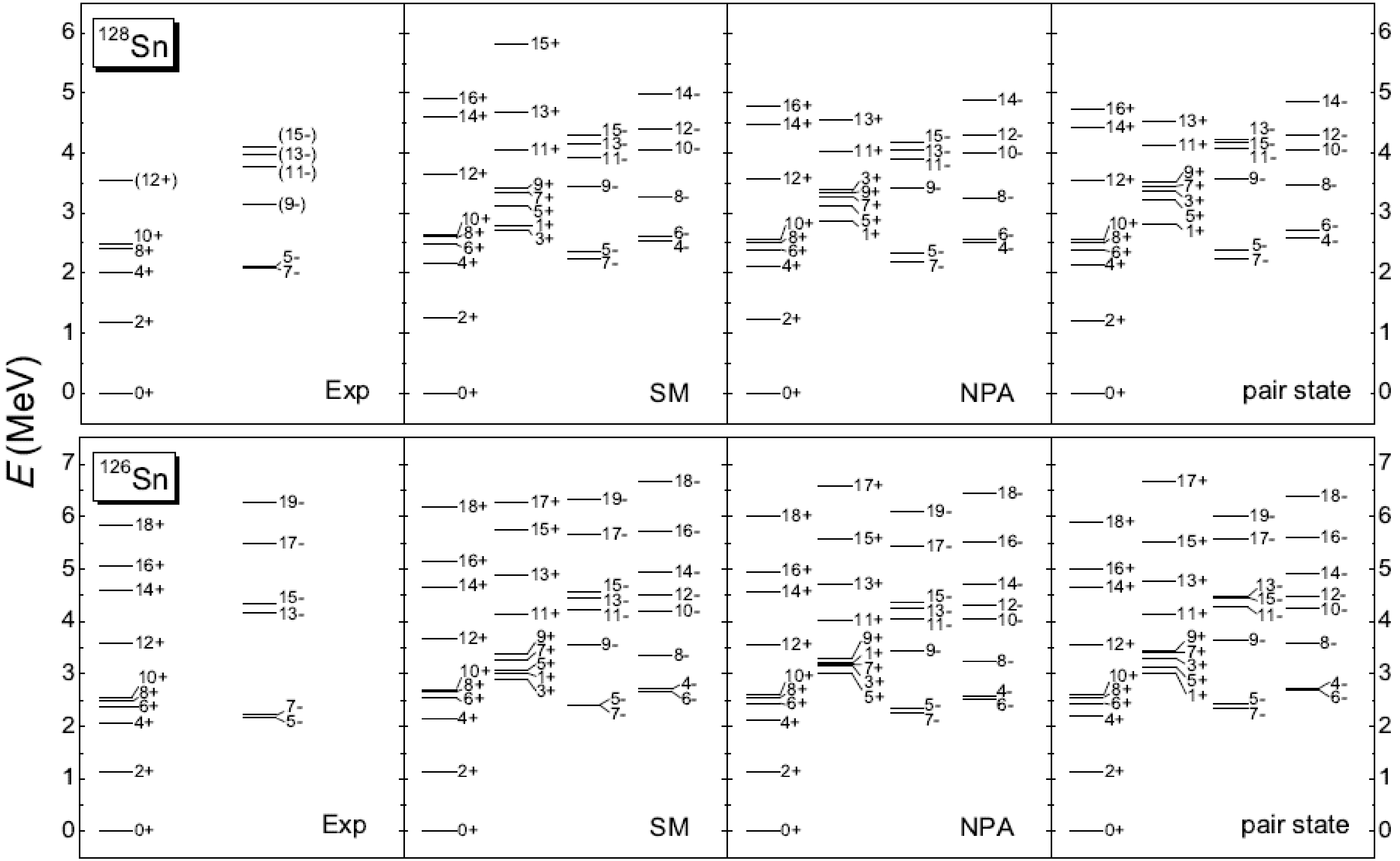
Y. Y. Cheng,[1] Y. M. Zhao,[1,2,*] and A. Arima[1,3]

$$H_\sigma = \sum_j \varepsilon_j n_{j\sigma} + \sum_{s=0,2} G_s \hat{s} \left(A_\sigma^{s+} \times \tilde{A}_\sigma^{s} \right)^{(0)} + \sum_{t=2,3} \kappa_t \hat{t} \left(Q_\sigma^t \times Q_\sigma^t \right)^{(0)}$$

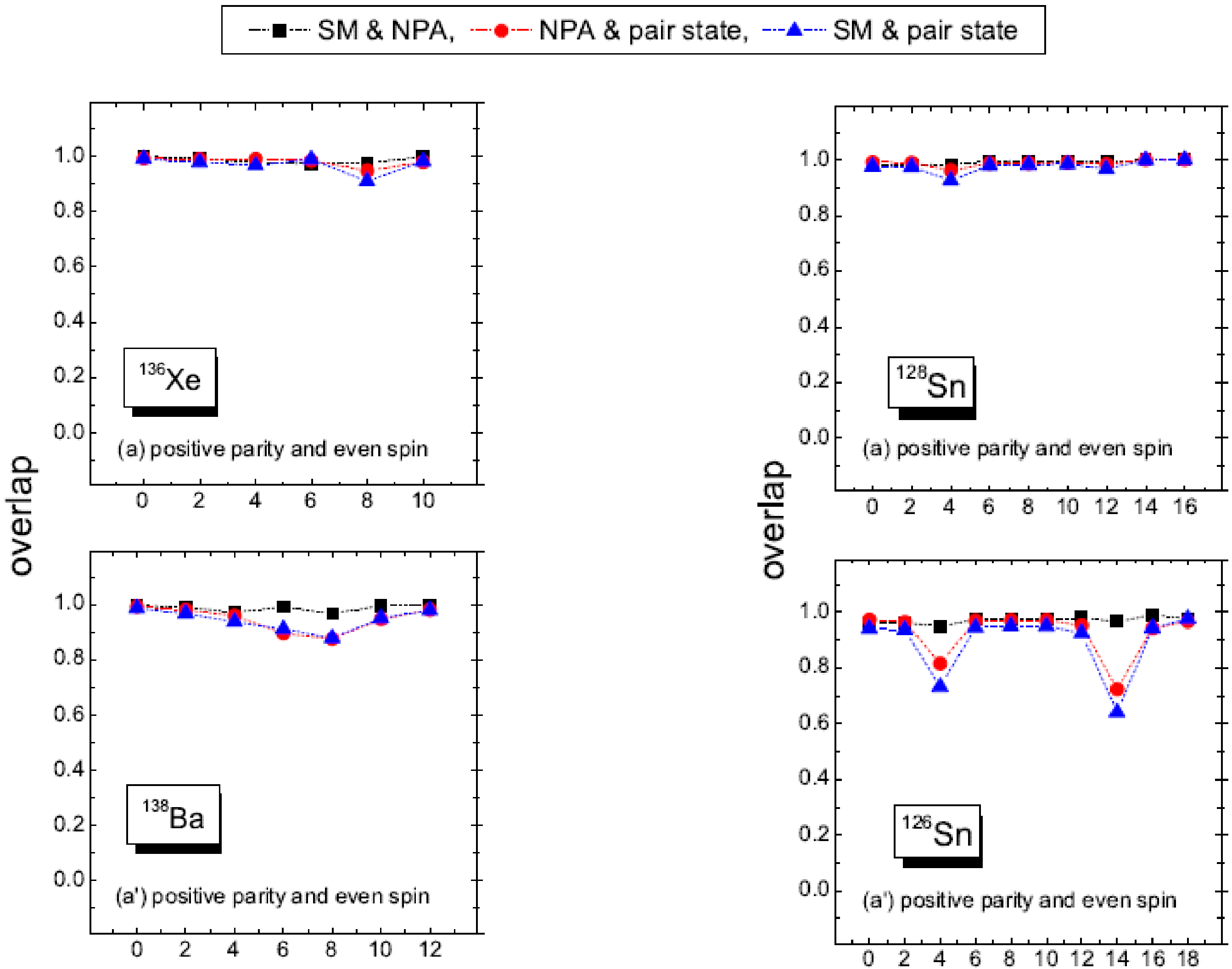
One-dimensional pair wave functions



One-dimensional pair wave functions



One-dimensional pair wave functions

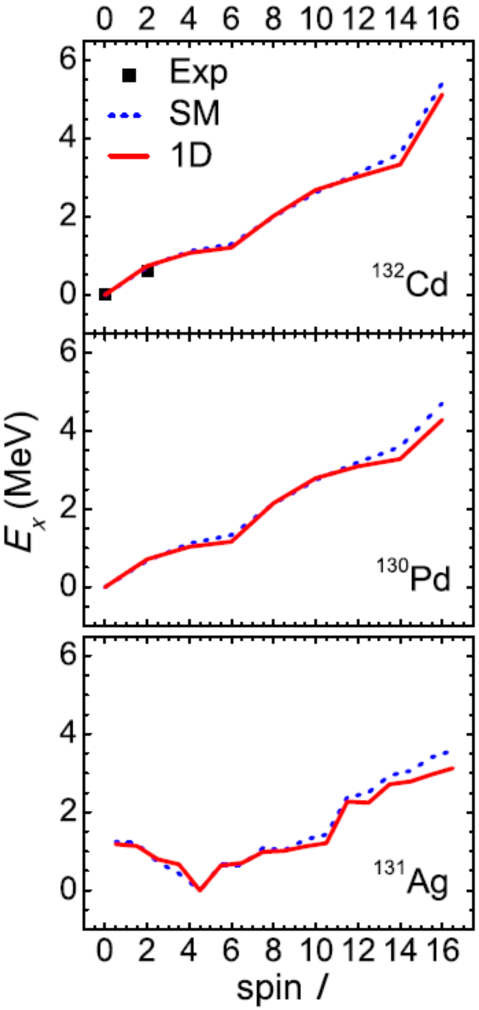


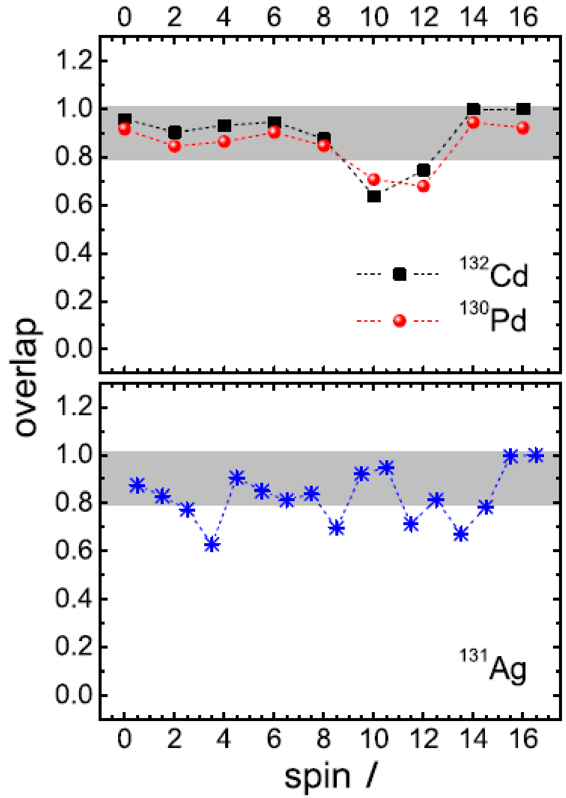
SM w.f. is well represented by the one-dimensional, optimized pair basis state !

$$\left(\cdots\left(\left(A^{(r_1)+}\times A^{(r_2)+}\right)^{(J_2)}\times A^{(r_3)+}\right)^{(J_3)}\cdots\times A^{(r_N)+}\right)^{(J_N)}_{M_N}|0\rangle$$

Applicable also to neutron-rich Open-Shell's

Calculated with jj46 effective interaction Of C.X.Yuan





	I^{P}	Configuration
^{132}Cd	0^+	$\lvert S\rangle_\pi \otimes \lvert S\rangle_\nu$
	2^+	$\lvert S\rangle_\pi \otimes \lvert D\rangle_\nu$
	4^+	$\lvert S\rangle_\pi \otimes \lvert G\rangle_\nu$
	6^+	$\lvert S\rangle_\pi \otimes \lvert I\rangle_\nu$
	8^+	$\lvert K\rangle_\pi \otimes \lvert S\rangle_\nu$
	10^+	$\lvert K\rangle_\pi \otimes \lvert D\rangle_\nu$
	12^+	$\lvert K\rangle_\pi \otimes \lvert G\rangle_\nu$
	14^+	$\lvert K\rangle_\pi \otimes \lvert I\rangle_\nu$
	16^+	$\lvert K\rangle_\pi \otimes \lvert K\rangle_\nu$
^{130}Pd	0^+	$\lvert SS;0\rangle_\pi \otimes \lvert S\rangle_\nu$
	2^+	$\lvert SS;0\rangle_\pi \otimes \lvert D\rangle_\nu$
	4^+	$\lvert SS;0\rangle_\pi \otimes \lvert G\rangle_\nu$
	6^+	$\lvert SS;0\rangle_\pi \otimes \lvert I\rangle_\nu$
	8^+	$\lvert SK;8\rangle_\pi \otimes \lvert S\rangle_\nu$
	10^+	$\lvert SK;8\rangle_\pi \otimes \lvert D\rangle_\nu$
	12^+	$\lvert SK;8\rangle_\pi \otimes \lvert G\rangle_\nu$
	14^+	$\lvert SK;8\rangle_\pi \otimes \lvert I\rangle_\nu$
	16^+	$\lvert DK;10\rangle_\pi \otimes \lvert I\rangle_\nu$

YYC, H.Wang, J.J.Shen et al., PRC 100, 024321

Nucleon pair approximation to the SM

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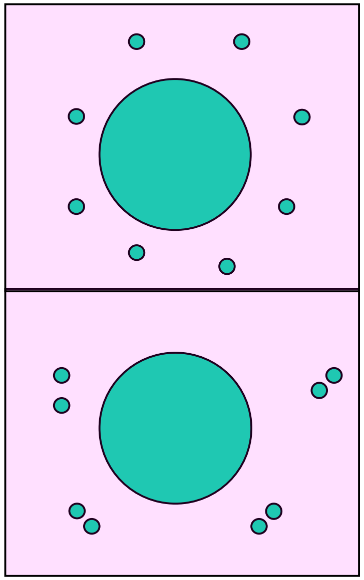

Pair basis states (J-scheme):

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J. Q. Chen, Nucl. Phys. A 562, 218 (1993).
J. Q. Chen, Nucl. Phys. A 626, 686 (1997).
Y. M. Zhao et al., Phys. Rev. C 62, 014304 (2000).
Y. M. Zhao and A. Arima, Physics Reports 545, 1 (2014).

Pair structure for spherical nuclei

- $$S^\dagger = \sum_j y(jj0)(a_j^\dagger \times a_j^\dagger)^{(0)} = \sum_j y(jj0)S_j^\dagger.$$

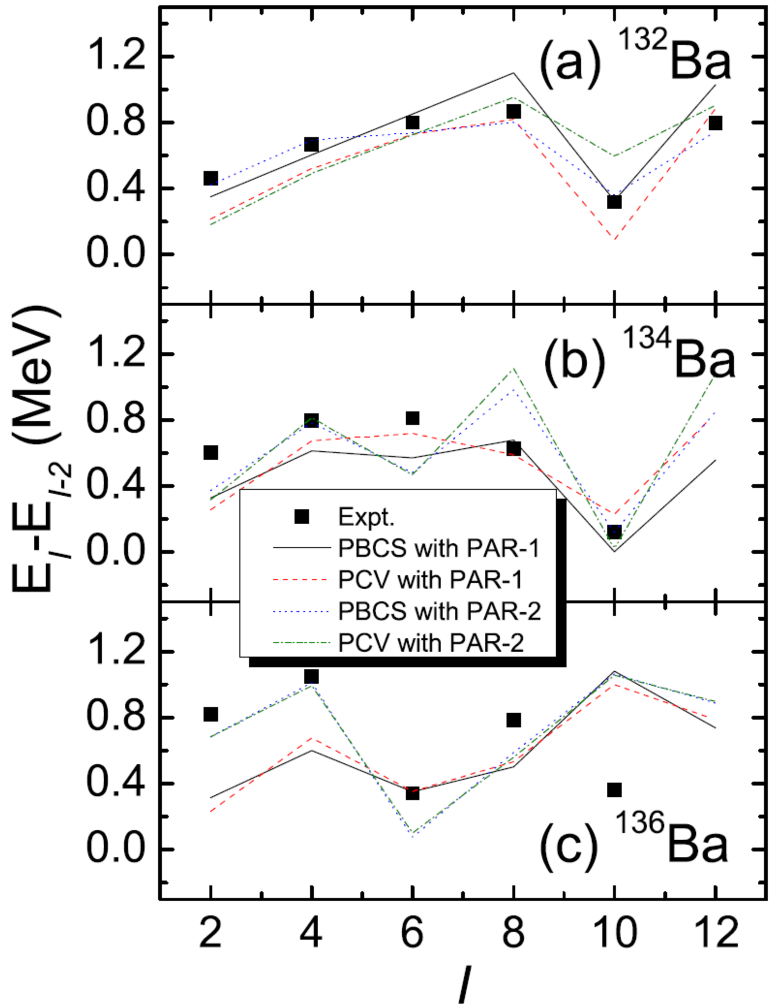
$$\delta \frac{\langle S^N|H|S^N\rangle}{\langle S^N|S^N\rangle} = 0.$$

- $$(S^\dagger)^{(N-1)} \sum_{j_1 j_2} c(j_1 j_2) A^{r\dagger}(j_1 j_2)$$

PHYSICAL REVIEW C **102**, 024310 (2020)

Variational approach for pair optimization in the nucleon pair approximation

Y. Lei (雷杨),[1,*] H. Jiang (姜慧),[2] and S. Pittel[3]



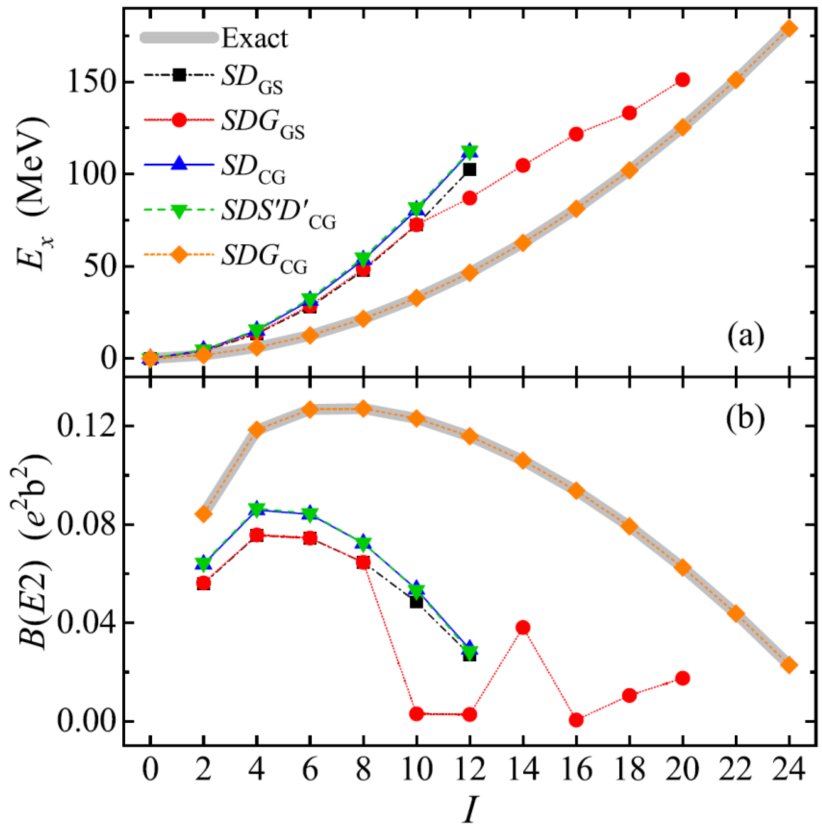
transitional nuclei

PHYSICAL REVIEW C **103**, L021302 (2021)

Letter

Nucleon-pair coupling scheme in Elliott's SU(3) model

G. J. Fu,[1,*] Calvin W. Johnson,[2,†] P. Van Isacker,[3,‡] and Zhongzhou Ren[1,§]



quadrupole deformed system

Summary

- 低能量核子配对是原子核低激发态的关键组元
- 准自旋算符，单j壳系统的辛弱数理论（only S pair）
- 部分动力学对称性，单j壳系统不变本征态的解析配对波函数（in terms of S pair and non-S pairs）
- 非简并多j壳，BCS理论（only S pair）
- 非简并多j壳，一维配对态（in terms of S pair and non-S pairs）

苏东坡 (宋朝)

横看成岭侧成峰

远近高低各不同

The low-lying states are very simple in terms of nucleon pair basis; they are very complicated if we look at them in terms of other basis.

感谢大家的聆听！

欢迎批评指正！

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