QCD Phase Transition

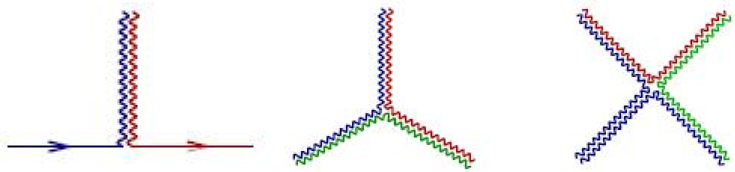
Mei HUANG（黄梅）

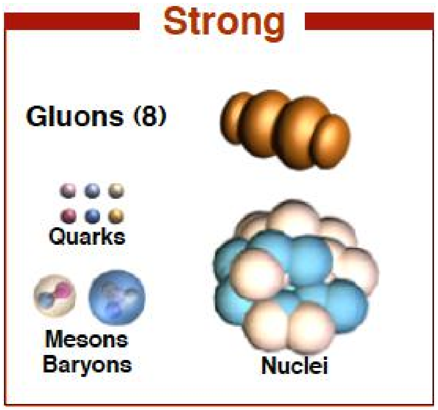
University of Chinese Academy of Sciences

“湖州暑期讲习班”湖州师范学院，2021年7月18日

强耦合夸克物质和QCD相结构

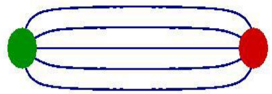
量子色动力学(QCD)： 描述强相互作用的基本理论



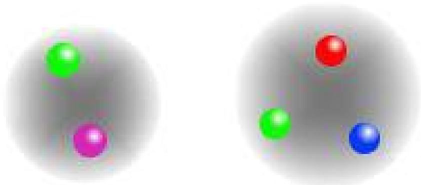


强耦合区

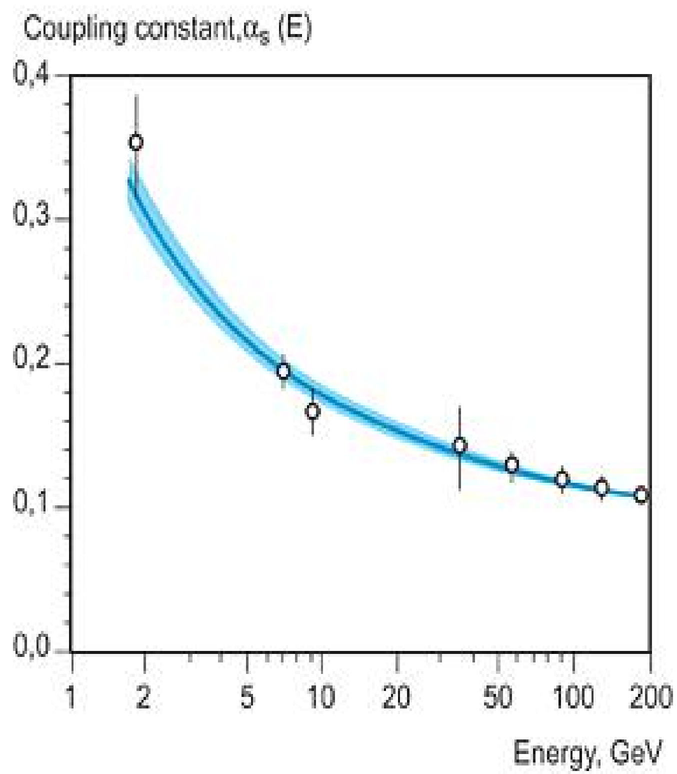
色禁闭



手征对称性自发破缺



质量起源：
宇宙可见物质99%



弱耦合区

渐近自由

"for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics"

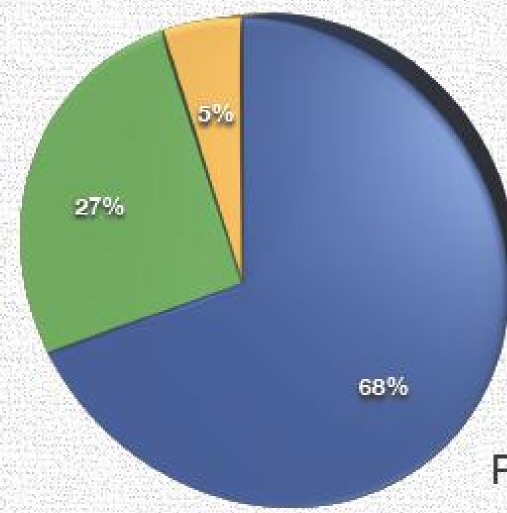
Nobel prize 2008

Nobel Prize 2004

What makes up the mass of the visible universe?

Our Universe:

- 68% Dark energy
- 27% Dark matter
- 5% Visible matter



Planck 2013 Results

- What makes up the mass of the **visible universe?**

Atomic mass (visible matter): 99.9% from nuclear mass

Nuclear Mass: all of it from **nucleon mass**

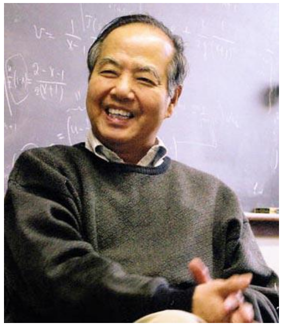
Nucleon mass? ➔ energy of massless *gluons* and almost massless up & down quarks

Gluon & quark interactions & dynamics make up the entire mass of the visible universe!

➔ *"Mass without mass" – John Wheeler*

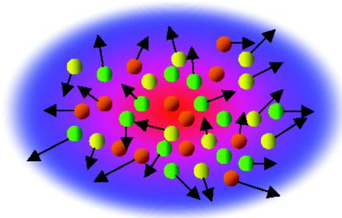
Abhay Deshpande

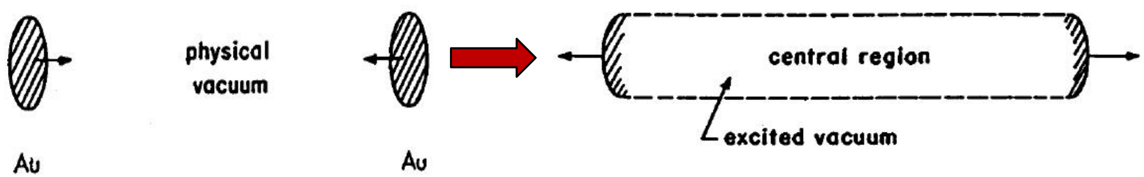
研究领域：强相互作用物质性质和相结构



重离子碰撞实验：激发真空

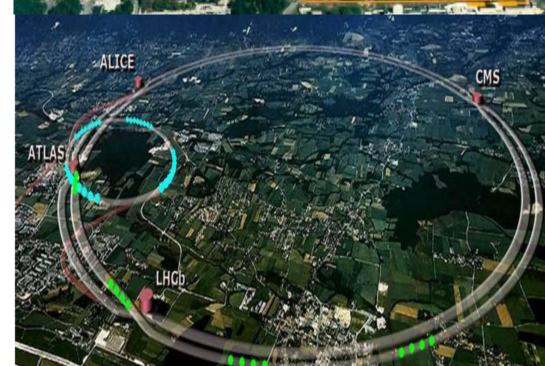
夸克-胶子等离子体





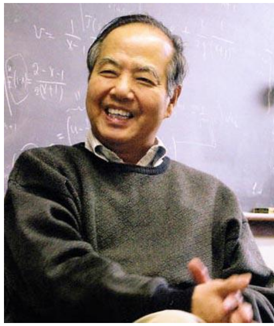
重离子碰撞实验（高温&高密）：

1，RHIC@BNL 美国布鲁克海文国家实验室
2，ALICE@CERN 欧洲核子研究中心
3，FAIR@GSI 德国亥姆霍兹重离子研究中心
4，NICA@DUBNA，俄国杜布纳联合核子研究所
5，CSR@兰州，中国科学院近代物理研究所
6，HIAF@惠州，中国科学院近代物理研究所

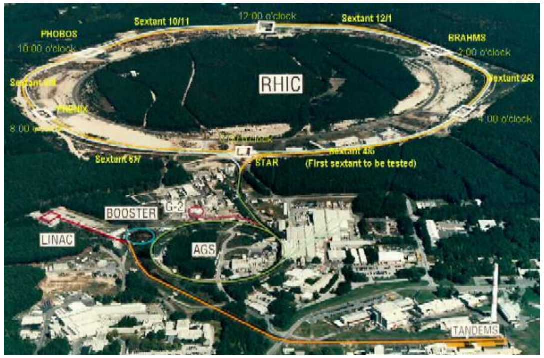


研究领域：强耦合夸克物质和QCD相结构

通过激发真空和压缩核物质可以得到手征恢复和退禁闭的夸克物质

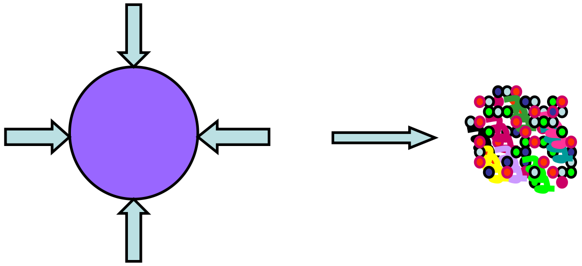


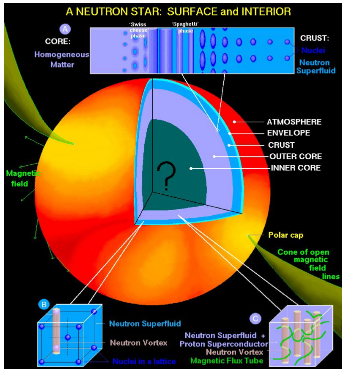
- 1970’李政道提出用重离子碰撞激发真空研究QCD相变
  - RHIC@BNL 美国布鲁克海文国家实验室
  - LHC@CERN 欧洲核子研究中心
  - FAIR@GSI 德国亥姆霍兹重离子研究中心
  - NICA@DUBNA，俄罗斯杜布纳联合核子研究所
  - CSR 、HIAF @兰州，中国科学院近代物理研究所



- 自然界可能存在退禁闭夸克物质的地方是致密星体内部（低温@高密）

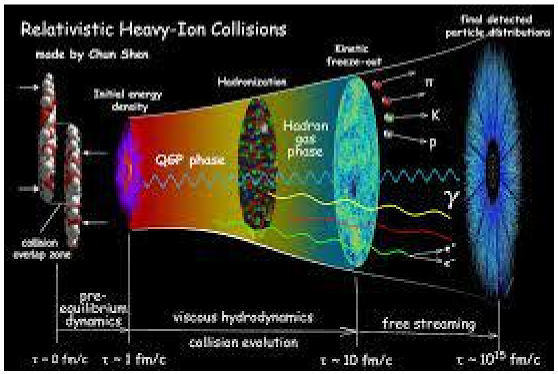
费米面 + 夸克之间吸引力
形成BCS色超导态

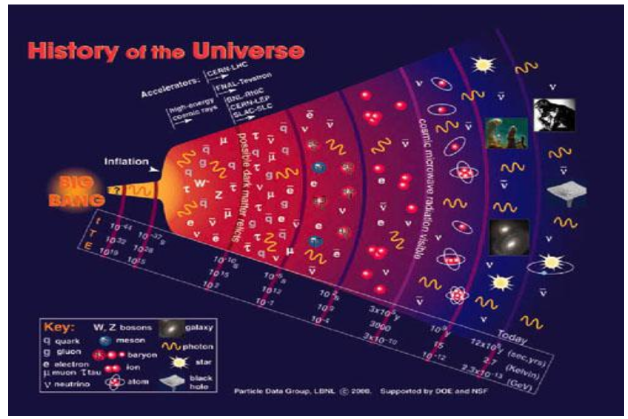


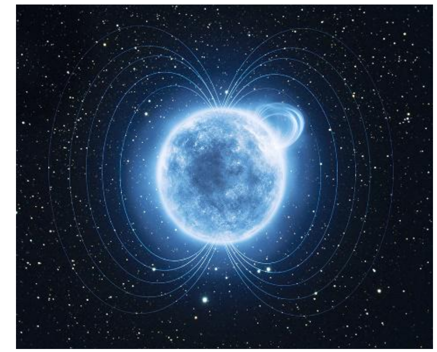


QCD matter under extreme conditions

$$\mathrm{T}, \mu_B, \mathbf{B}, \mathbf{E}\cdot\mathbf{B}, \boldsymbol{\omega}, \mu_{\mathrm{I}}, \boldsymbol{L}$$







LHC,RHIC,FAIR,NICA,HIAF

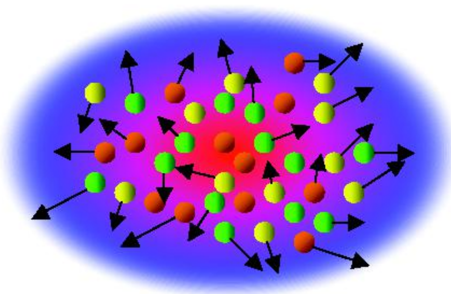
Early universe

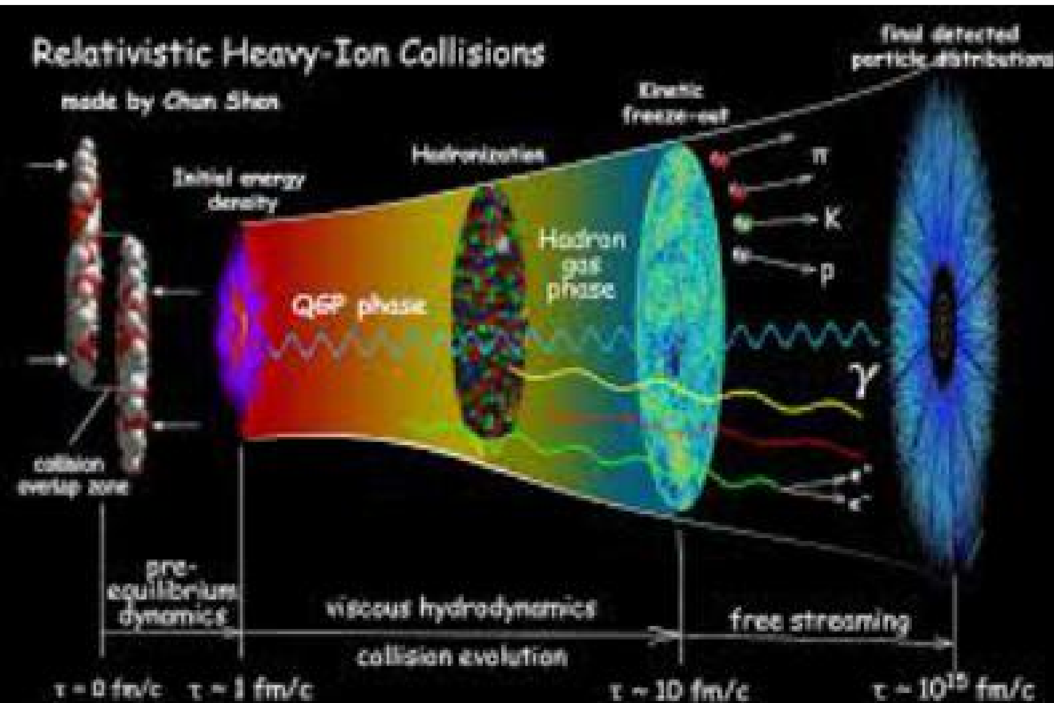
Neutron star

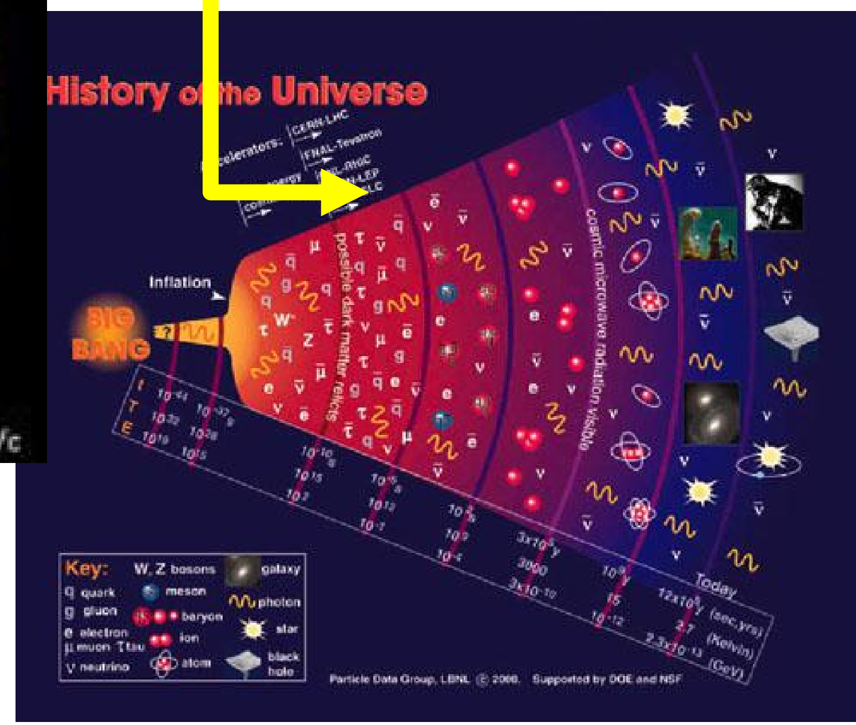
Neutron star merge→BH

QCD phase transition at early universe

相对论性重离子碰撞实验产生手征恢复和退禁闭的高温夸克-胶子等离子体，回到宇宙早期百万分之几秒。

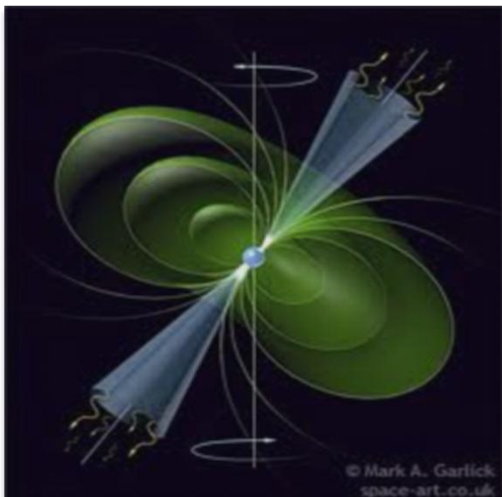


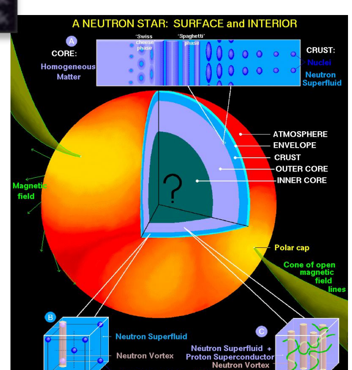


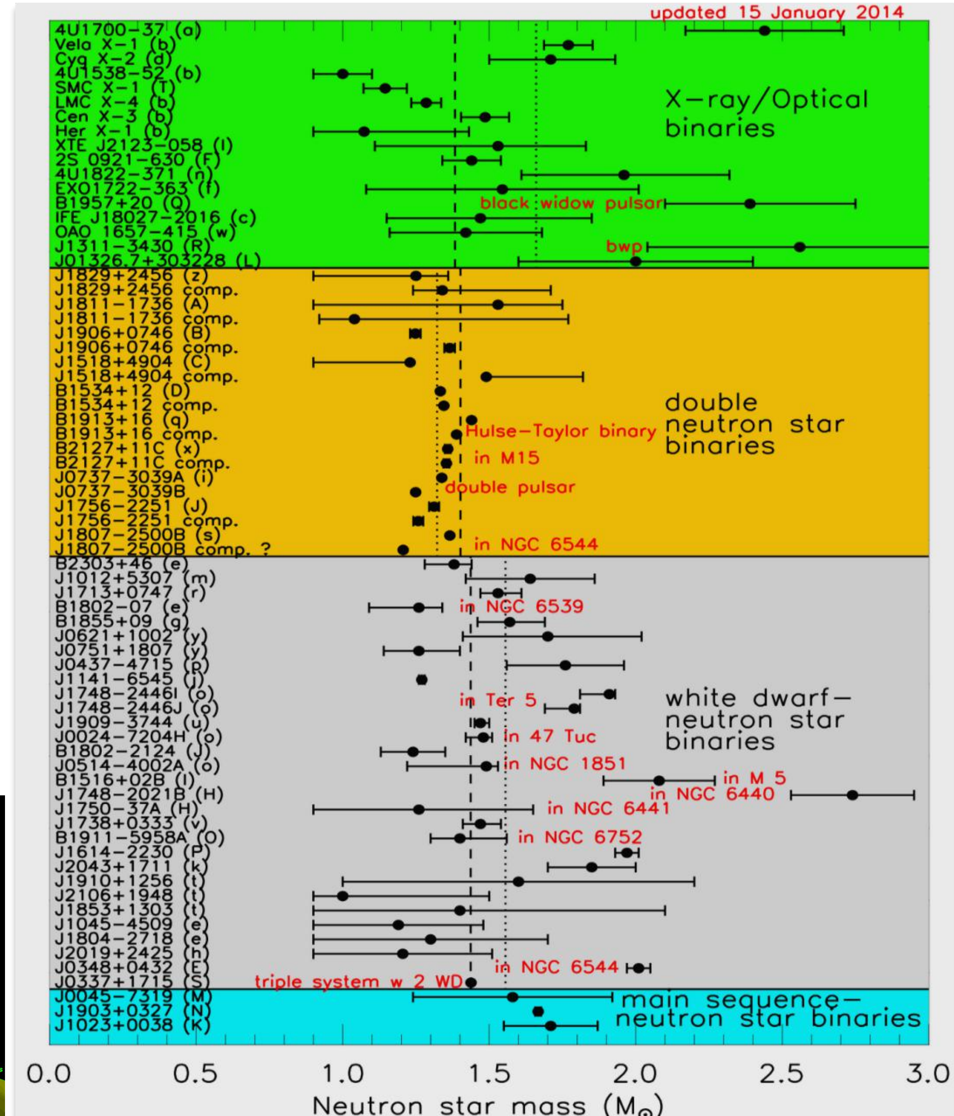


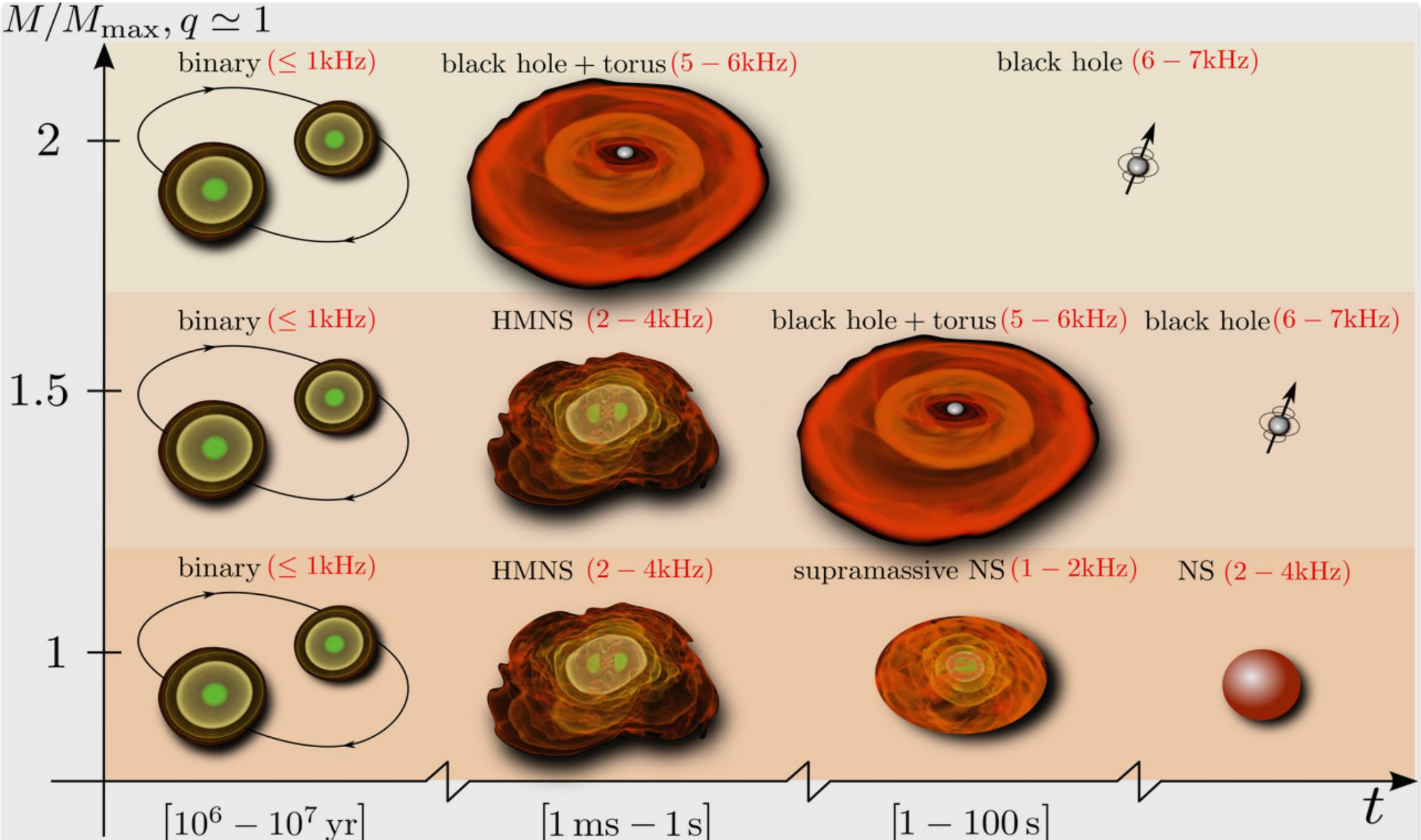
QCD phase transition in compact star

radius ~ 10 km,
mass ~ 1-2 Sun masses,
large magnetic fields ~ 10^{11} Tesla,
high rotation (up to 716 Hz)

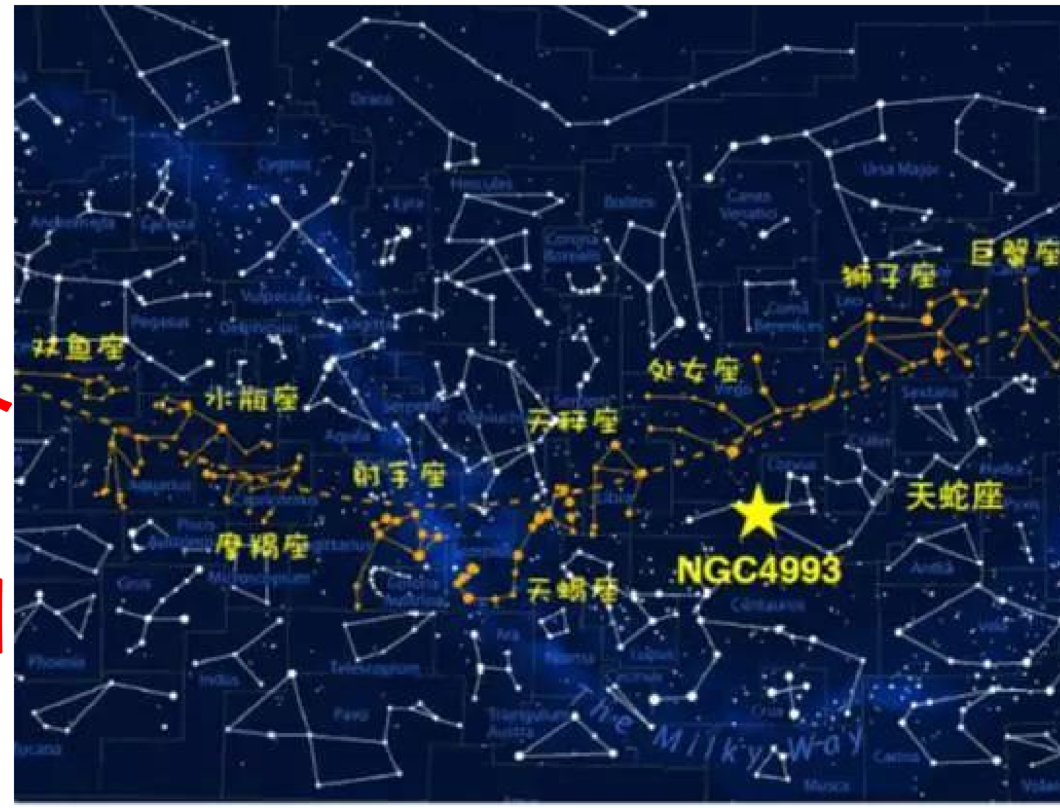







$M/M_{\rm max}, q \simeq 1$
binary (≤ 1kHz)
black hole + torus (5 – 6kHz)
black hole (6 – 7kHz)
2
binary (≤ 1kHz)
HMNS (2 – 4kHz)
black hole + torus (5 – 6kHz)
black hole (6 – 7kHz)
1.5
binary (≤ 1kHz)
HMNS (2 – 4kHz)
supramassive NS (1 – 2kHz)
NS (2 – 4kHz)
1
$[10^6 - 10^7\,{\rm yr}]$
$[1\,{\rm ms} - 1\,{\rm s}]$
$[1 - 100\,{\rm s}]$
t

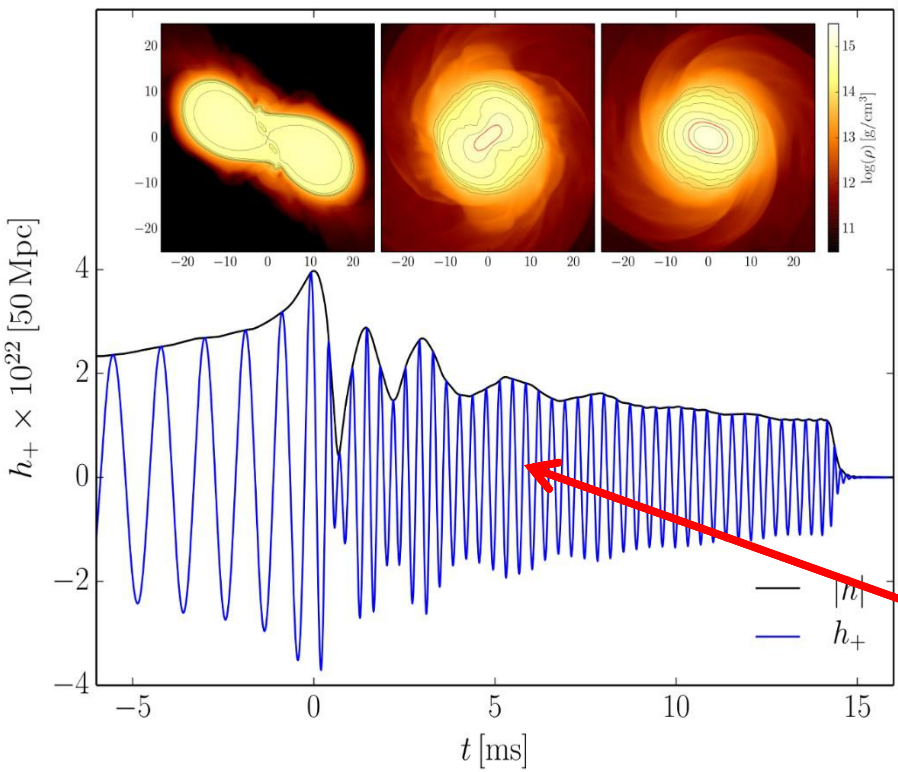
继2015年9月14日LIGO首次发现双黑洞并合产生的引力波，2017年10月16日LIGO和VIRGO联合举办新闻发布会报道了2017年8月17日首次探测到双中子星合并产生引力波事件（GW170817）



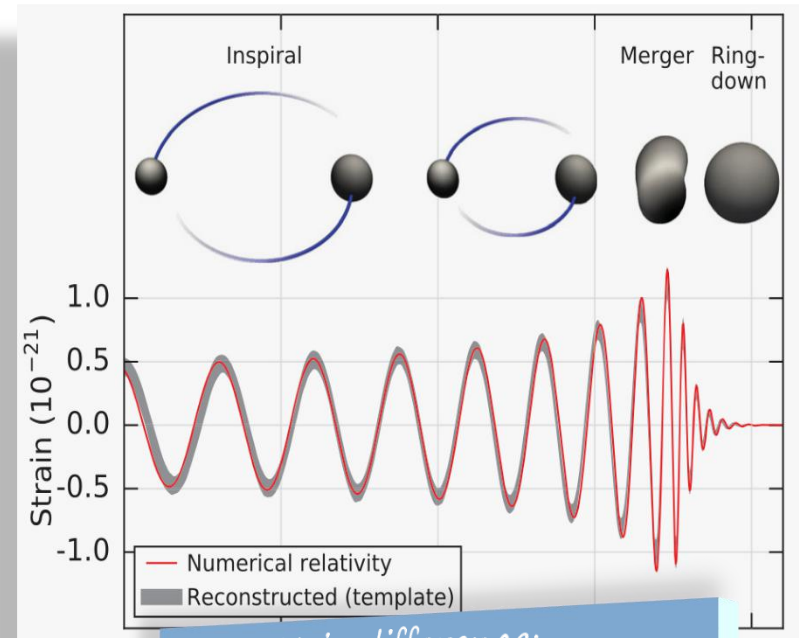
该星系距离地球1亿3千万光年

Gravitational Waves from Neutron Star Mergers

Neutron Star Collision (Simulation)



Collision of two Black Holes



Main difference:

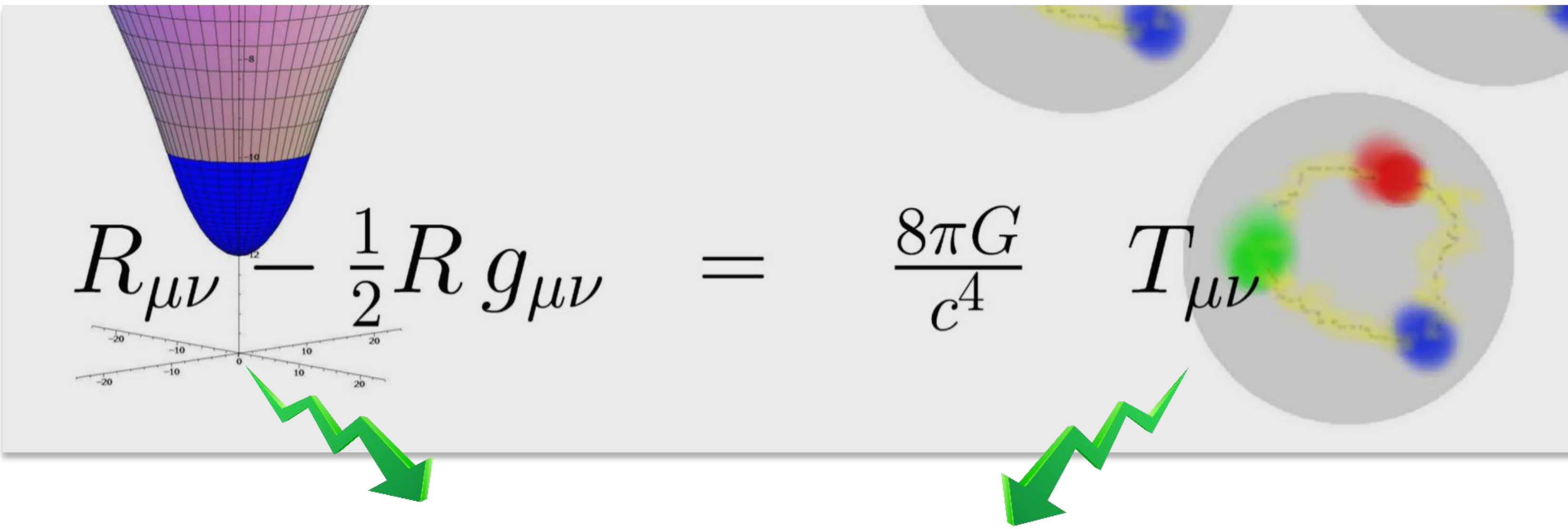
In binary neutron star mergers a Post-Merger Phase often exists

M. Hanauske@Frankfurt Uni., 2017

Gravitational Waves from Neutron Star Mergers

The Einstein-Equation

100 years ago, Albert Einstein presented the main equation of General Relativity:



$$R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Spacetime curvature
Properties of the Spacetime metric

Mass, Energy and Momentum of the System
Equation of State of elementary matter
(density, temperature)

研究领域：强耦合夸克物质和QCD相结构

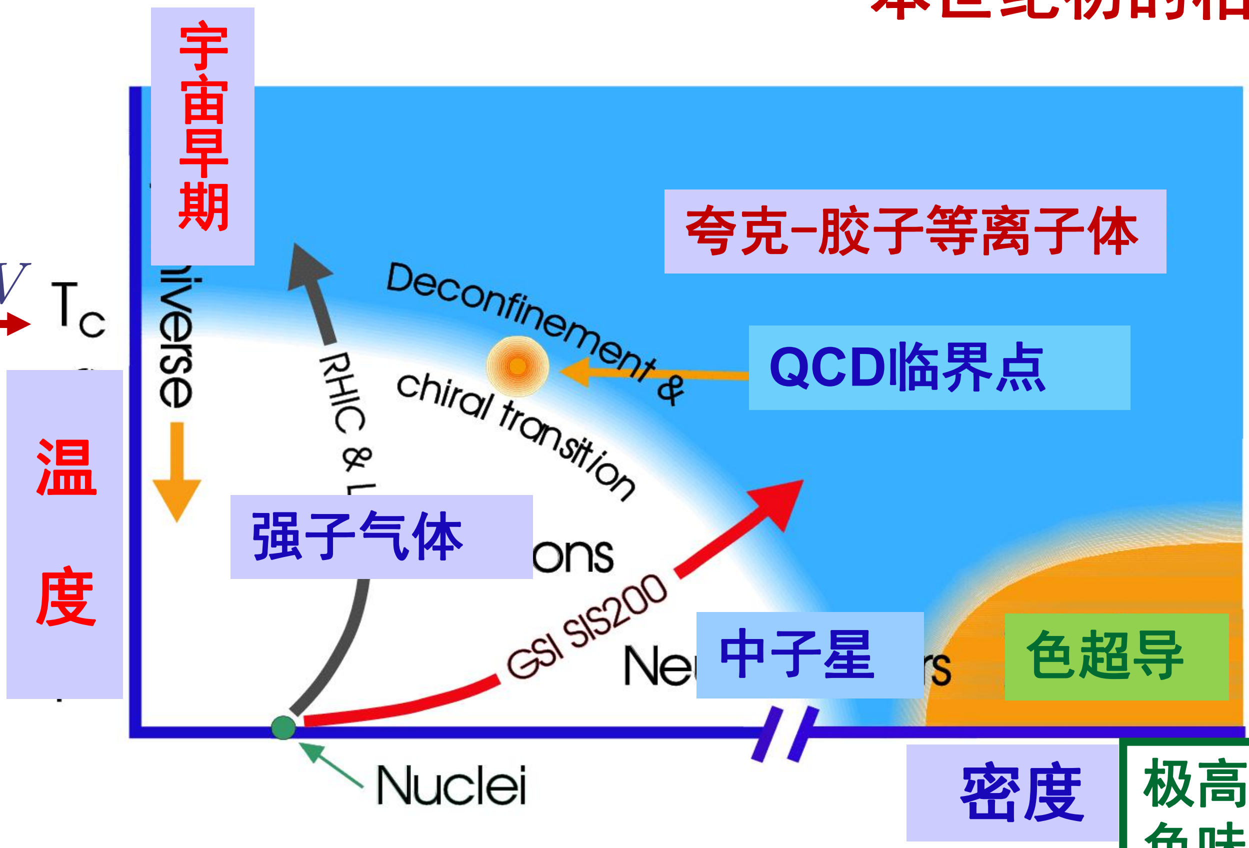
QCD相变和相结构跟宇宙的演化和致密星体密切相关，是高能核物理研究的前沿领域和核心课题，具有重要的科学意义。

本世纪初的相图！

极高温（微扰）夸克-胶子气体

$T_c = 156MeV$

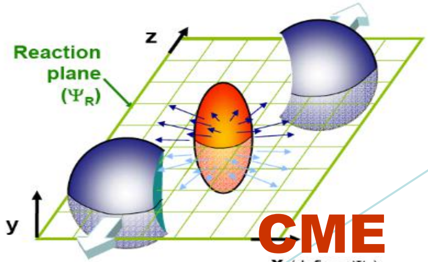
非微扰格点QCD 零密有限温

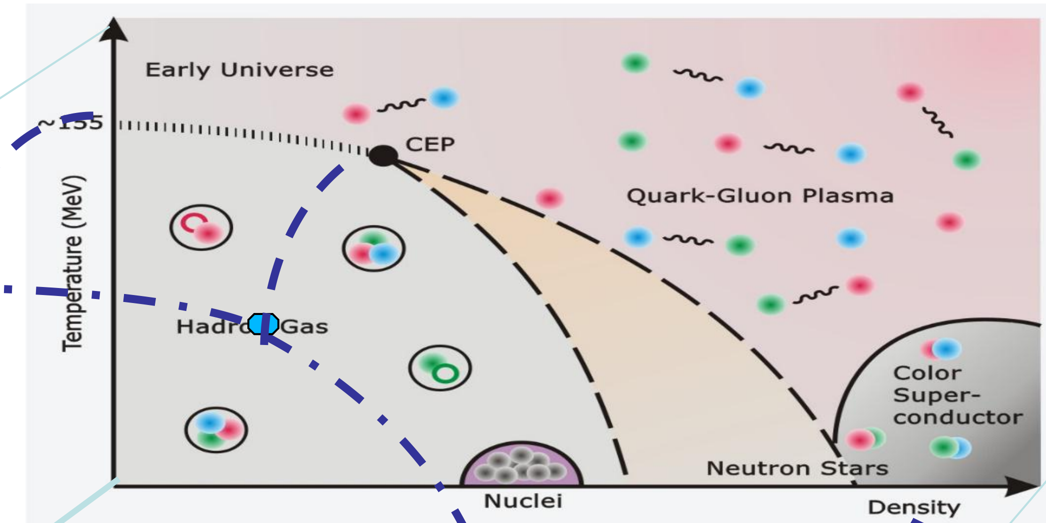


极高密（微扰）色味锁定超导态

Explored QCD phase diagram

by theorists





CME

CVE

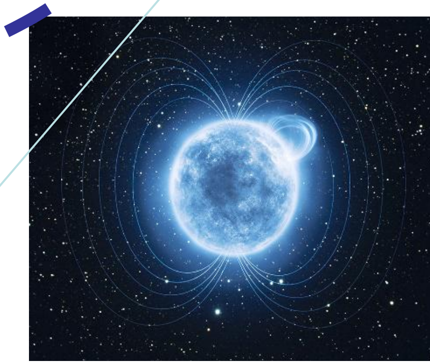
Inverse Magnetic Catalysis

Charged rho condensation

Neutral/charged pion condensation

(Inverse) Magnetic Catalysis

Magnetar

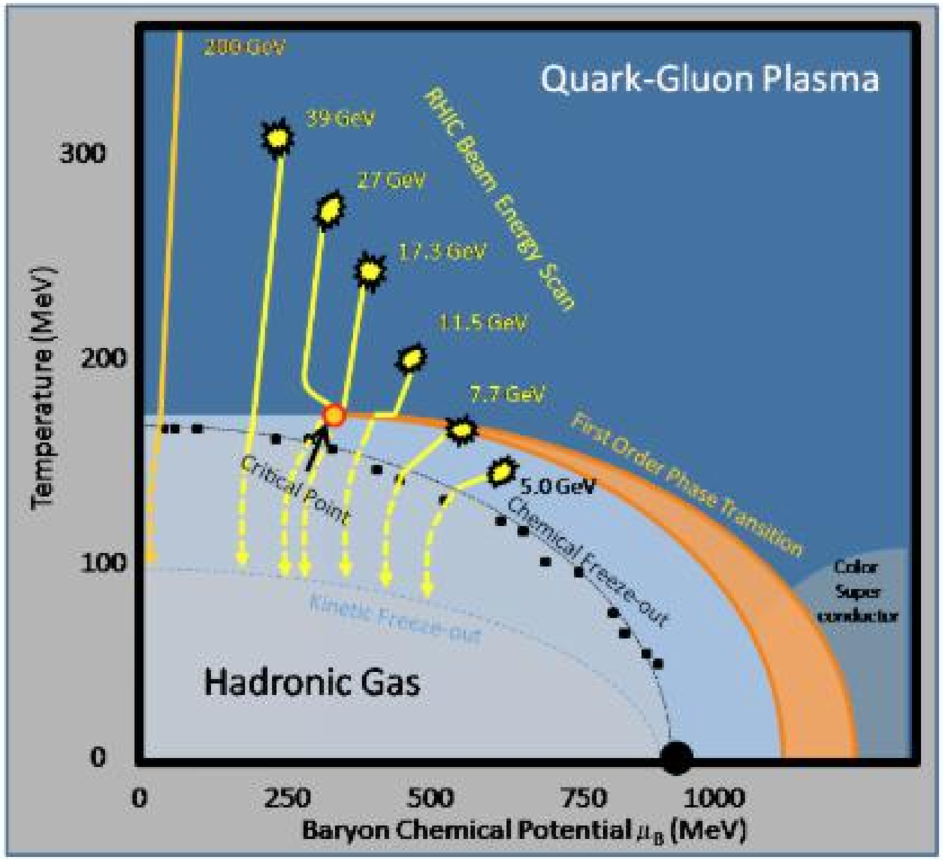


$$\mathbf{B}, \mathbf{E}\cdot\mathbf{B}, \boldsymbol{\omega}, \mu_{\mathrm{I}}, \boldsymbol{L}$$

1，CEP

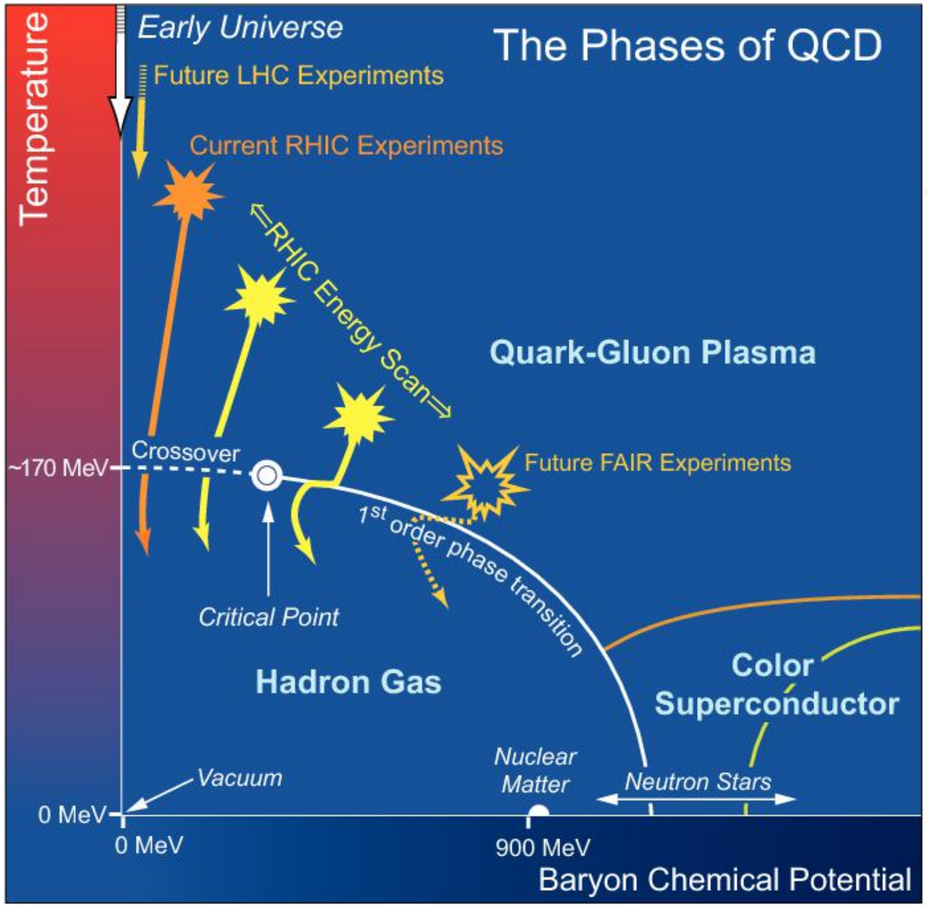
2，QCD matter under magnetic field and rotation

Ⅰ. CEP

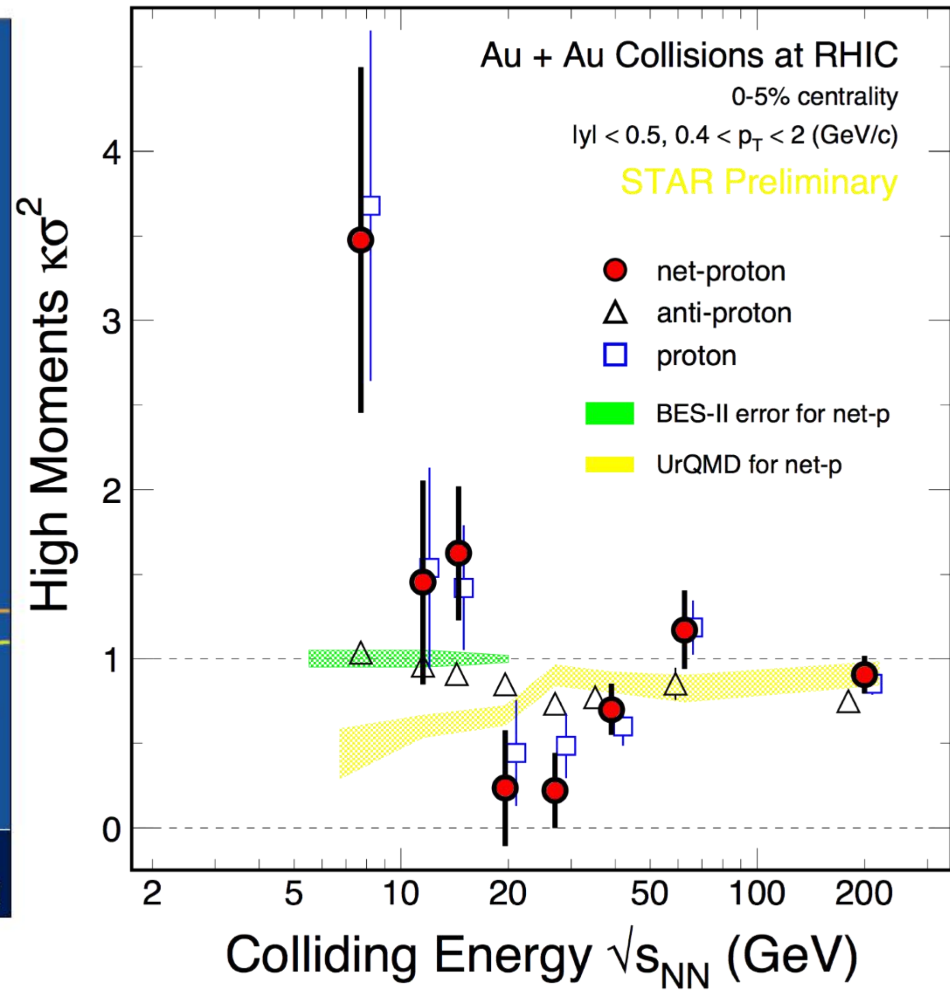


- ❑ BES @ RHIC
- ❑ NICA @Dubna
- ❑ CBM@FAIR
- ❑ HIAF@IMP

寻找CEP；

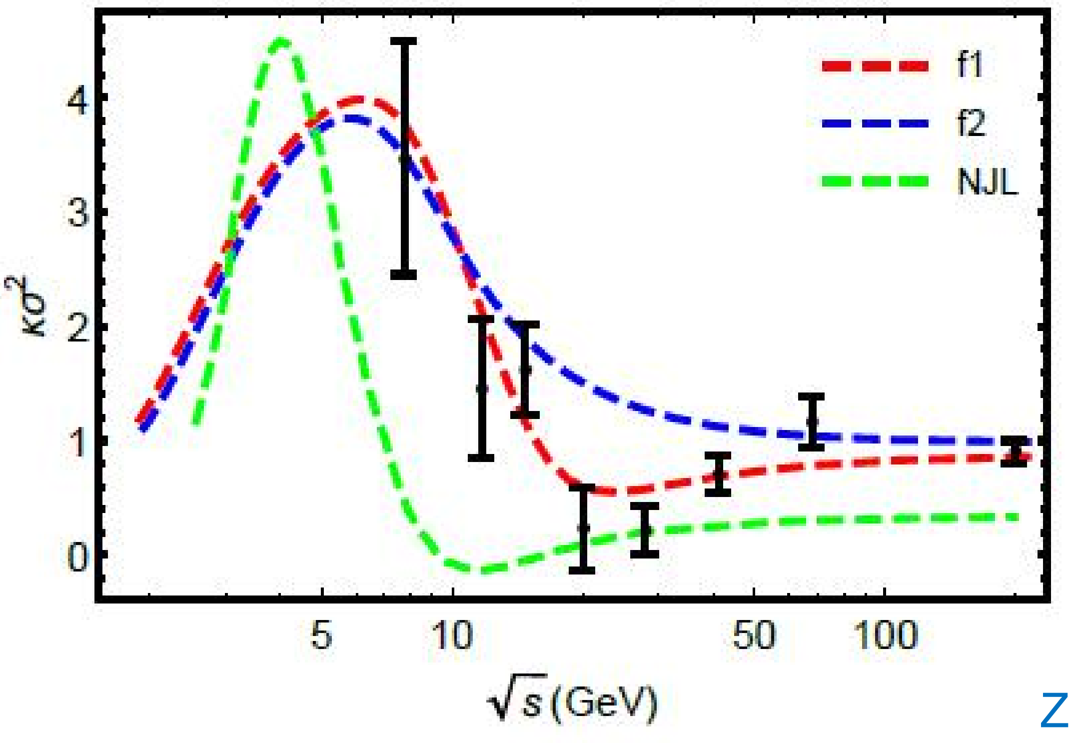


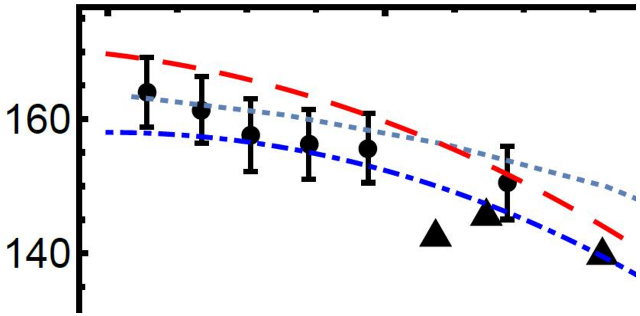
Temperature
Early Universe
The Phases of QCD
Future LHC Experiments
Current RHIC Experiments
⇐RHIC Energy Scan⇒
Quark-Gluon Plasma
~170 MeV
Crossover
Future FAIR Experiments
1st order phase transition
Critical Point
Hadron Gas
Color Superconductor
Nuclear Matter
Vacuum
Neutron Stars
0 MeV
0 MeV
900 MeV
Baryon Chemical Potential



Au + Au Collisions at RHIC
0-5% centrality
|y| < 0.5, 0.4 < p_T < 2 (GeV/c)
STAR Preliminary
net-proton
anti-proton
proton
BES-II error for net-p
UrQMD for net-p
High Moments $\kappa\sigma^2$
0
1
2
3
4
2
5
10
20
50
100
200
Colliding Energy $\sqrt{s_{NN}}$ (GeV)

Dip structure





f1 cross the phase boundary while f2 not!

Z.B Li, K.Xu,X.Y.Wang, M.Huang
arXiv:1801.09215

The dip structure is sensitive to the relation between the freeze-out line and the phase boundary !

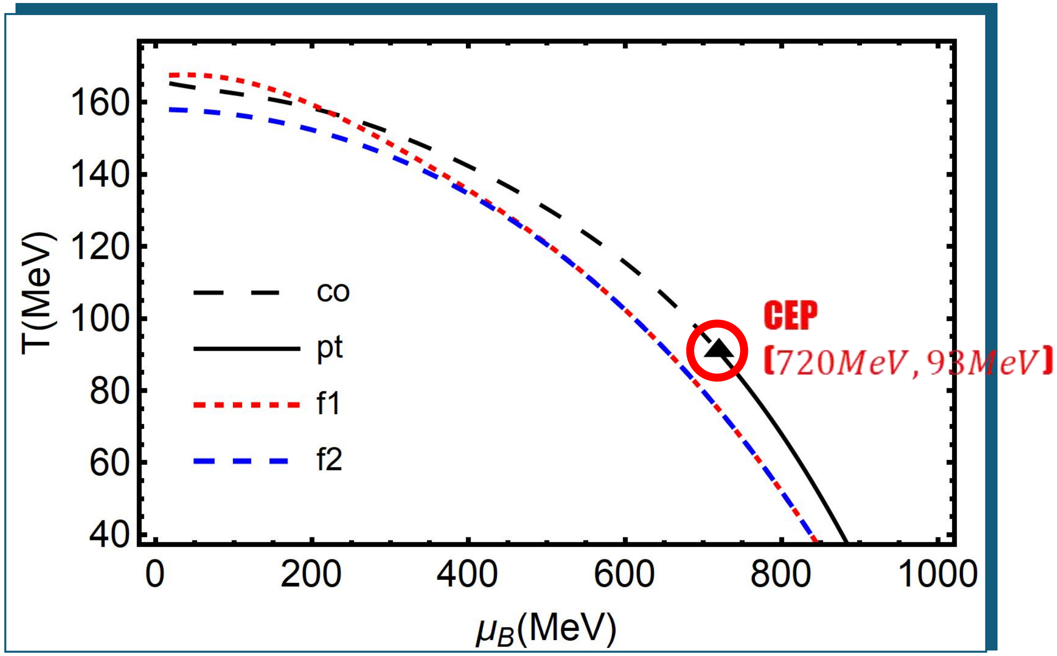
沿着冷却线的$\kappa\sigma^2$

$$\chi_n^B = \frac{\partial^n}{\partial(\mu_B/T)^n}\frac{p}{T^4}, \qquad \kappa\sigma^2 = \frac{\chi_4^B}{\chi_2^B}$$

对相变行为敏感

Baryon number susceptibility

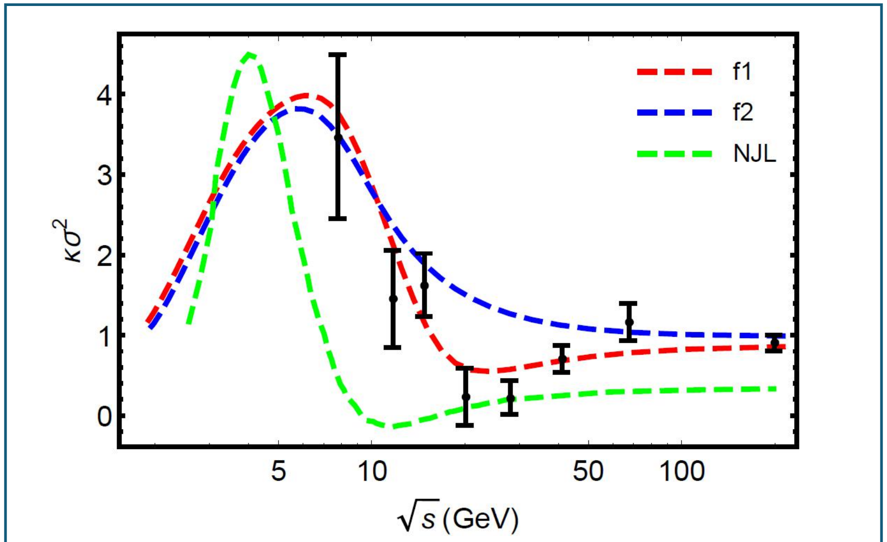
$$f_1: T(\mu) = 0.158 - 0.14\mu^2 - 0.04\mu^4 - 0.01e^{-\frac{\mu - 0.067}{0.05}}$$



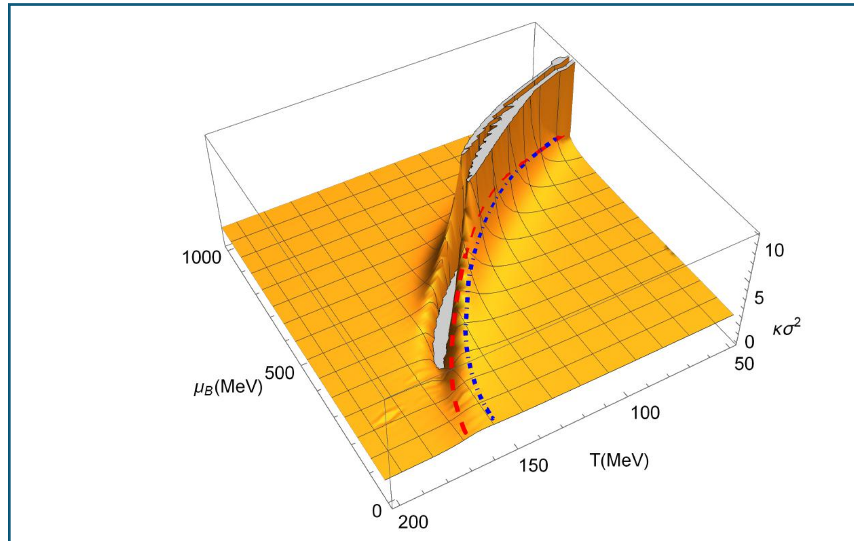
$$f_2: T(\mu) = 0.158 - 0.14\mu^2 - 0.04\mu^4$$

$$\mu_B(\sqrt{s}) = \frac{1.477}{1 + 0.343\sqrt{s}}$$

沿着冷却线的$\kappa\sigma^2$



$\kappa\sigma^2$ 与对撞能量



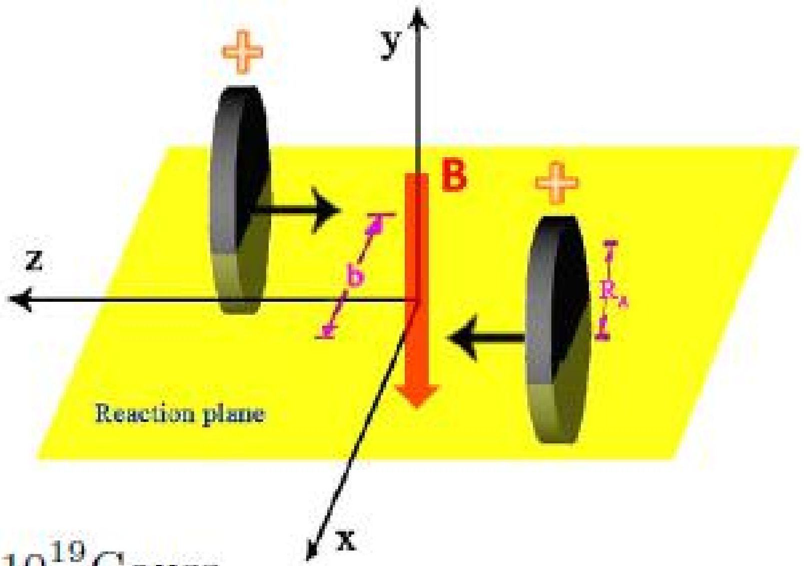
$\kappa\sigma^2$ 在 $T-\mu B$ 图上的

冷却线穿过相边界→$\kappa\sigma^2$下凹结构

冷却线擦过**CEP**→$\kappa\sigma^2$峰结构

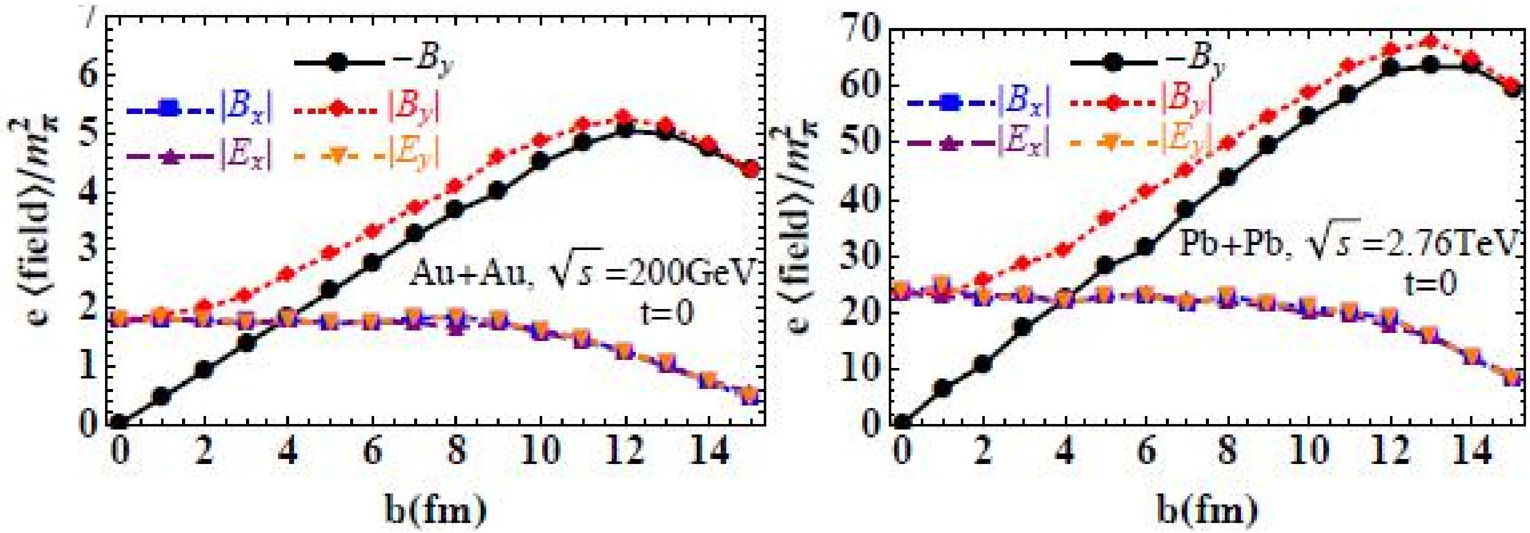
II.QCD matter under strong magnetic fields

Non-central HIC



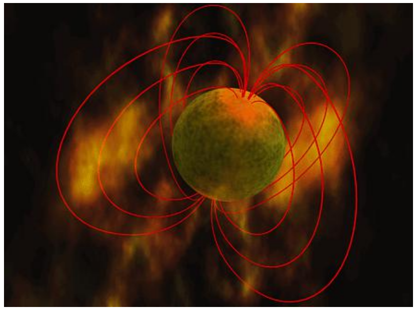
Biot-Savart law

$$-eB_y \sim 2 \times \gamma \frac{e^2}{4\pi} Z v_z \left(\frac{2}{b}\right)^2 \approx 40 m_\pi^2 \sim 10^{19}\text{Gauss}$$

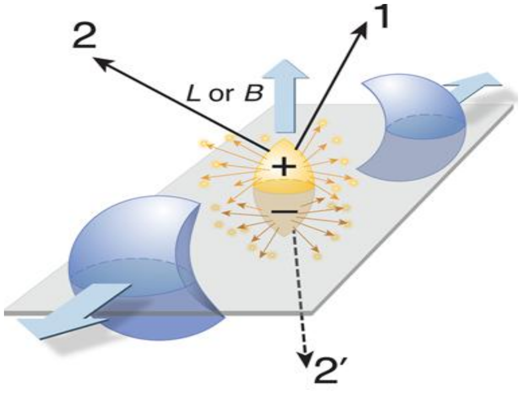


Weitian Deng, Xuguang Huang

MAGNETIC FIELDS



- Inside *compact stars*
  - **10^{10} to 10^{15} Gauss**



- Non-central HIC
  - **10^{18} to 10^{19} Gauss**
- *Early Universe*
  - **up to 10^{24} Gauss**

$$
\begin{aligned}
1\,\mathrm{MeV}^2 &= 1.44 \times 10^{13}\ \mathrm{Gauss} \\
m_\pi^2 &\sim 2.8 \times 10^{17}\ \mathrm{Gauss}
\end{aligned}
$$

Chiral phase transition in strong magnetic fields

Magnetic catalysis at zero temperature

S.P. Klevansky and R. H. Lemmer ('89); H. Suganuma and T. Tatsumi ('91);
V. P. Gusynin, V. A. Miransky and I. A. Shovkovy ('94, '95, '96,...)

$$\mathscr{L} = \bar{\Psi} i\gamma^{\mu} D_{\mu}\Psi + \frac{G}{2}\left[(\bar{\Psi}\Psi)^2 + (\bar{\Psi} i\gamma^5\Psi)^2\right]$$

$$D_{\mu} = \partial_{\mu} - ieA_{\mu}^{\text{ext}} \qquad \mathbf{A}^{\text{ext}} = (0, Bx^1, 0)$$

$$m = G\,\mathrm{tr}\left[S(x,x)\right] \approx \frac{G\,m}{(2\pi)^2}\left(\Lambda^2 + |eB|\ln\frac{|eB|}{\pi m^2} + O(m^2)\right)$$

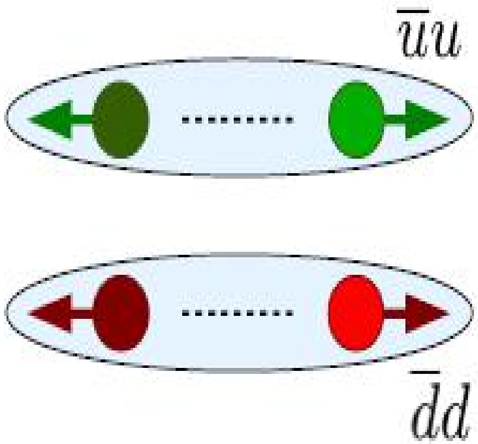
$$m \propto \exp\left(-\frac{2\pi^2}{G|eB|}\right)$$

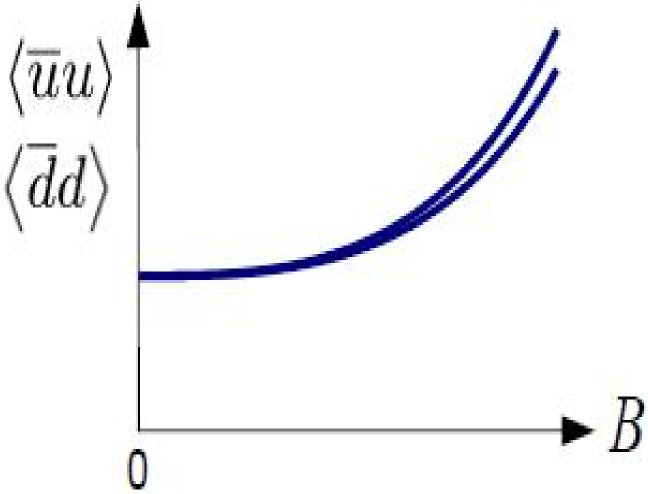
nonzero mass for arbitrary small G

Magnetic catalysis at zero temperature

S.P. Klevansky and R. H. Lemmer ('89); H. Suganuma and T. Tatsumi ('91);
V. P. Gusynin, V. A. Miransky and I. A. Shovkovy ('94, '95, '96,...)

attractive channel: spin-0 flavor-diagonal states

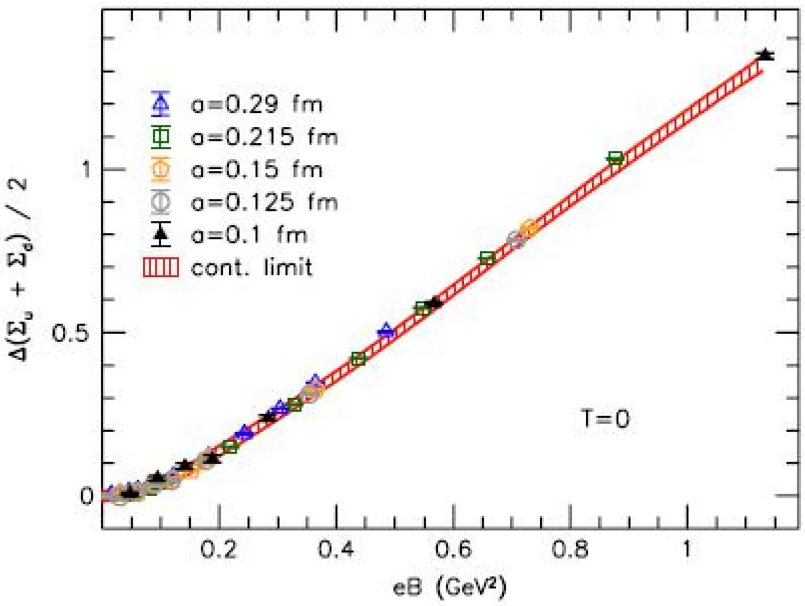


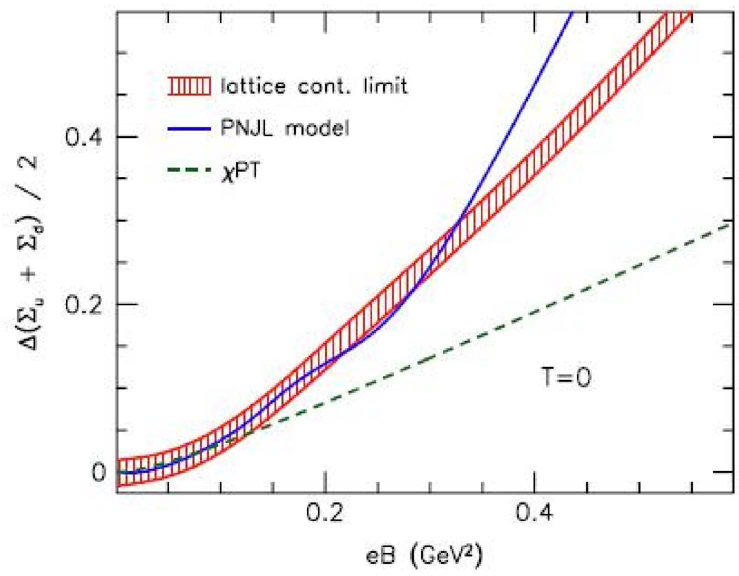


enhances chiral symmetry breaking

Magnetic catalysis at zero temperature

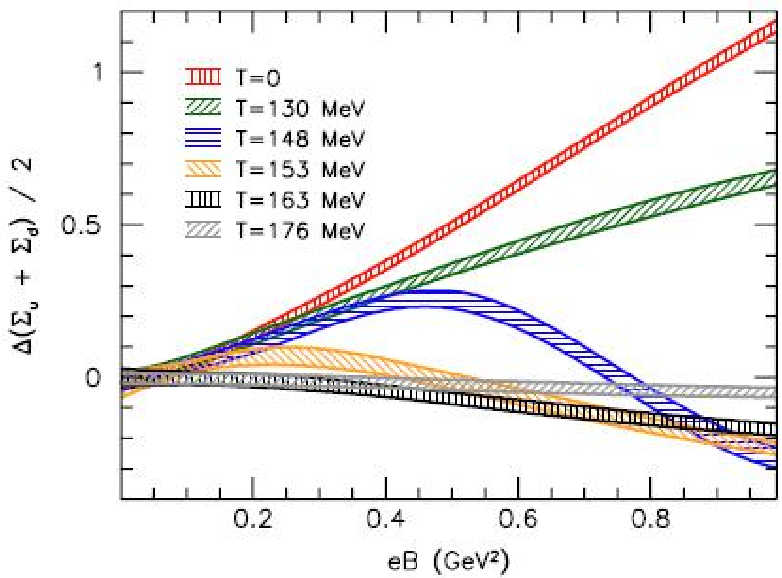
Bali et.al. arXiv:1206.4205 [hep-lat]

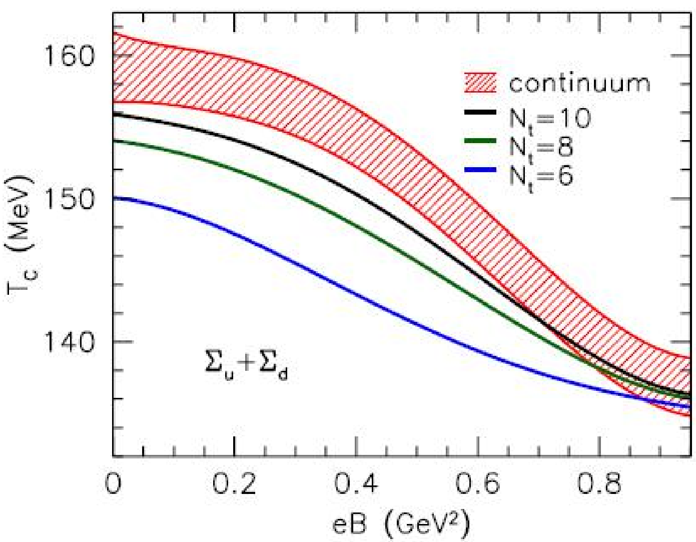




Inverse Magnetic catalysis at nonzero temperature

Bali et.al. arXiv:1206.4205 [hep-lat]





Surprise !!!

Some important information is missing in our understanding of chiral phase transition, which is enhanced by magnetic field!

How to understand inverse magnetic catalysis ?

1) Magnetic inhibition K. Fukushima, Y. Hidaka, PRL 110, 031601 (2013)

Contribution from neutral pions

2) Contribution from sea quarks

Bruckmann et.al. arXiv:1303.3972

3) Polyakov holomoly

Nowak et.al. arXiv:1304.6020

4) Chirality imbalance

Sphaleron transition

Jingyi Chao, Pengcheng Chu, MH, arXiv:1305.1100, PRD88(2013)

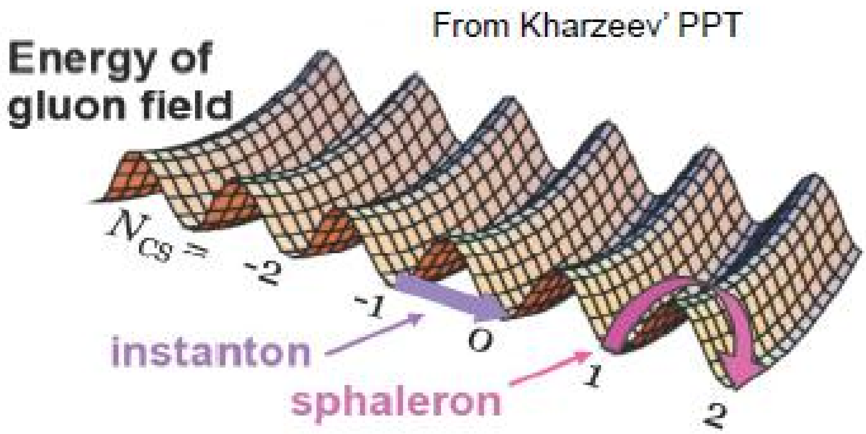
Instanton-anti-instanton pairing condensate

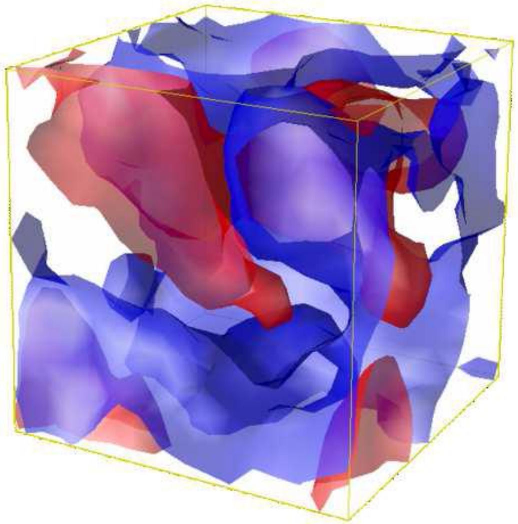
Lang Yu, Hao Liu, MH, arXiv:1404.6969, PRD90(2014)

Theta vacuum, instantons and sphalerons

QCD vacuum has non-trivial topological structure characterized by an integer valued Chern-Simons number

Buividovich et al. arXiv:1111.6733





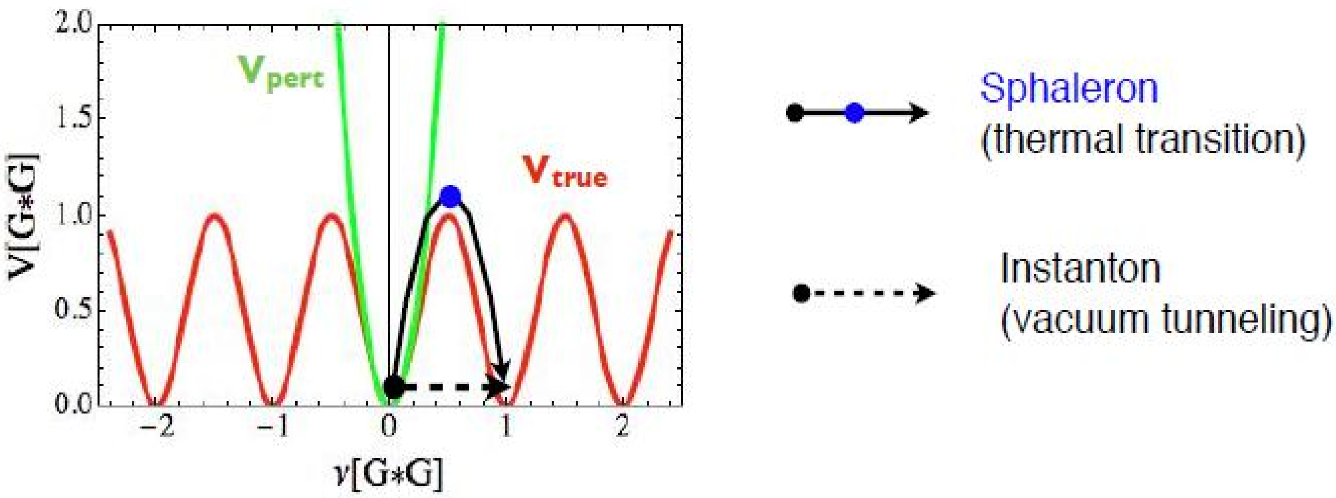
$$\Delta N_{cs} = \frac{g^2}{32\pi^2}\int d^4x \, \mathrm{Tr}[F_{a\mu\nu}\tilde{F}^{a\mu\nu}]$$

Induce chirality imbalance:

$$(N_R - N_L)_{t=+\infty} - (N_R - N_L)_{t=-\infty} = -2N_f\Delta N_{cs}$$

Theta vacuum, instanton and sphaleron:

QCD vacuum has non-trivial topological structure characterized by an integer valued Chern-Simons number N_{cs}



$$\Delta N_{cs} = \frac{g^2}{32\pi^2} \int d^4x \, \mathrm{Tr}[F_{a\mu\nu} \tilde{F}^{a\mu\nu}]$$

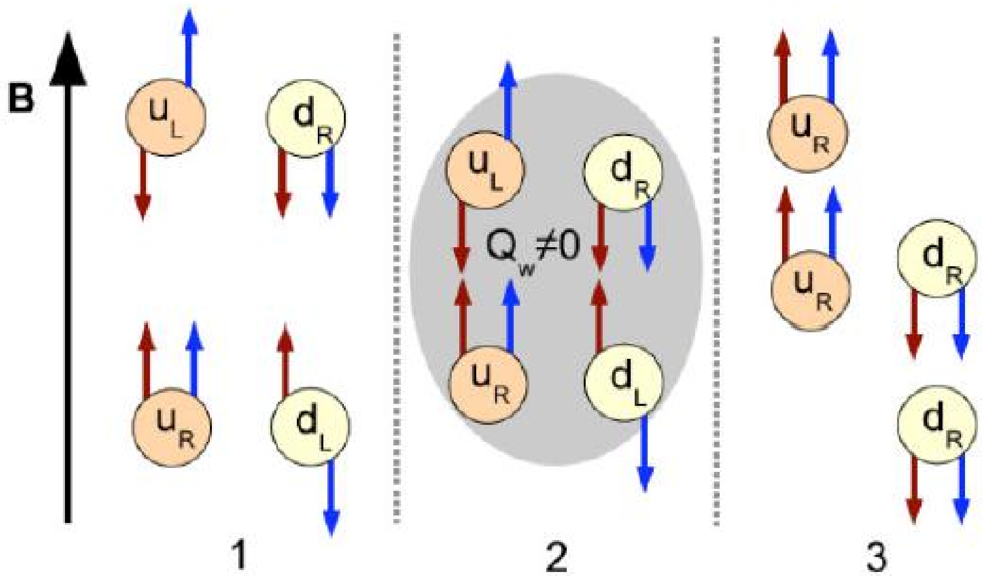
Induce chiral imbalance:

$$(N_R - N_L)_{t=+\infty} - (N_R - N_L)_{t=-\infty} = -2N_f \Delta N_{cs}$$

Chiral Magnetic Effect

Fukushima,Kharzeev,Warringa 2008

Visual picture:



P-invariant plasma of quarks (and gluons)

Sphaleron (instanton) = topological charge

P-odd plasma of quarks (and gluons)

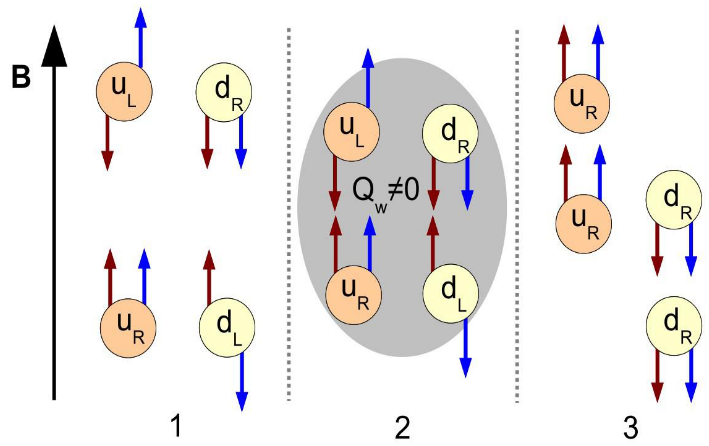
Red: momentum
Blue: spin

Electric charges:
u-quark: q=+2e/3
d-quark: q= - e/3

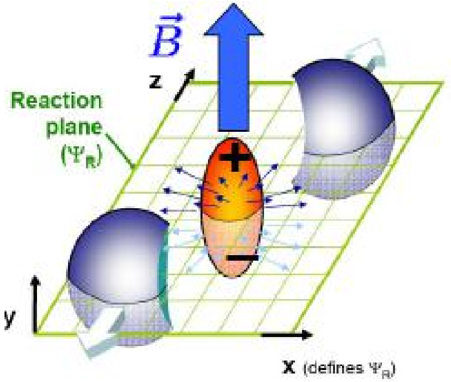
Role of topology:
$u_L \rightarrow u_R$
$d_L \rightarrow d_R$

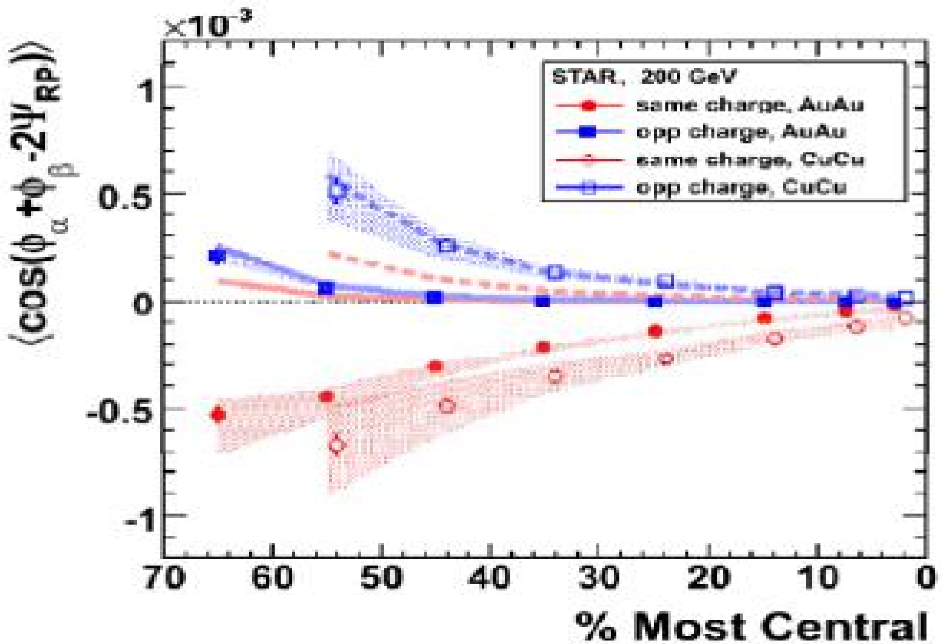
Chirality imbalance & Chiral Magnetic Effect

Fukushima,Kharzeev,Warringa 2008



CME ⟷ **Charge Separation**





STAR Collaboration PRL103(2009)251601

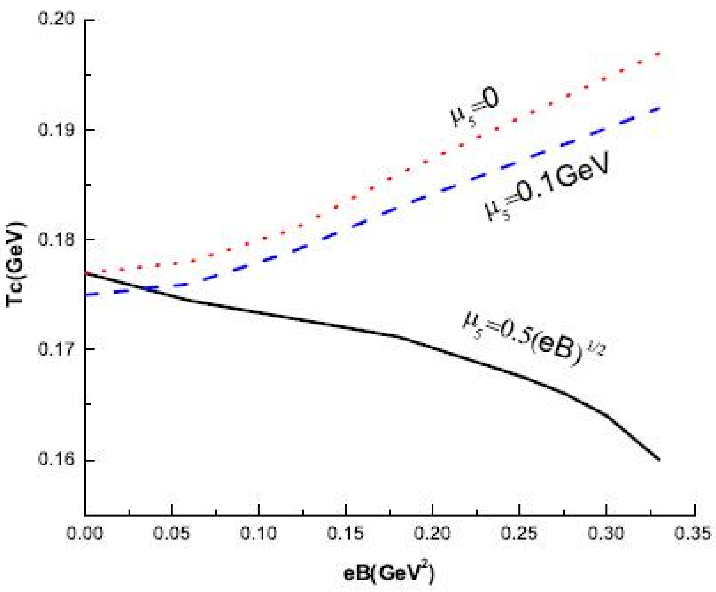
See Qun Wang's lecture

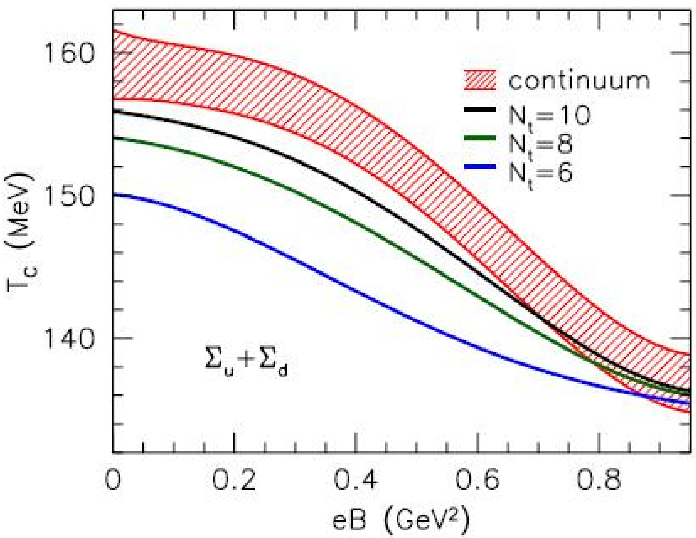
Chiral phase transition induced by chiral anomaly

$$\mathcal{L} = \bar{\psi}\left(i\gamma_\mu D^\mu + \mu\gamma^0 + \mu_5\gamma^0\gamma^5\right)\psi + G\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma^5\tau\psi\right)^2\right],$$

$$\Omega = \frac{\sigma^2}{4G} - N_c\sum_{f=u,d}\frac{|q_f B|}{2\pi}\sum_{s,k}\alpha_{sk}\int_{-\infty}^{\infty}\frac{dp_z}{2\pi}\omega_s(p) - TN_c\sum_{f=u,d}\frac{|q_f B|}{2\pi}\sum_{s,k}\alpha_{sk}\int_{-\infty}^{\infty}\frac{dp_z}{2\pi} \times \left(\ln[1+e^{-\beta(\omega_s+\mu)}] + \ln[1+e^{-(\beta\omega_s-\mu)}]\right).$$

Jingyi Chao, Pengcheng Chu, Mei Huang, arXiv:1305.1100

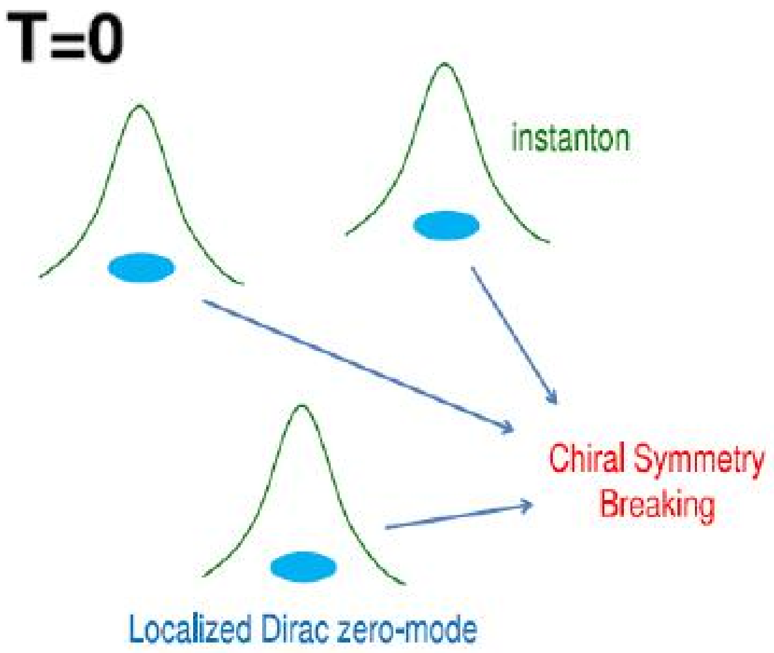


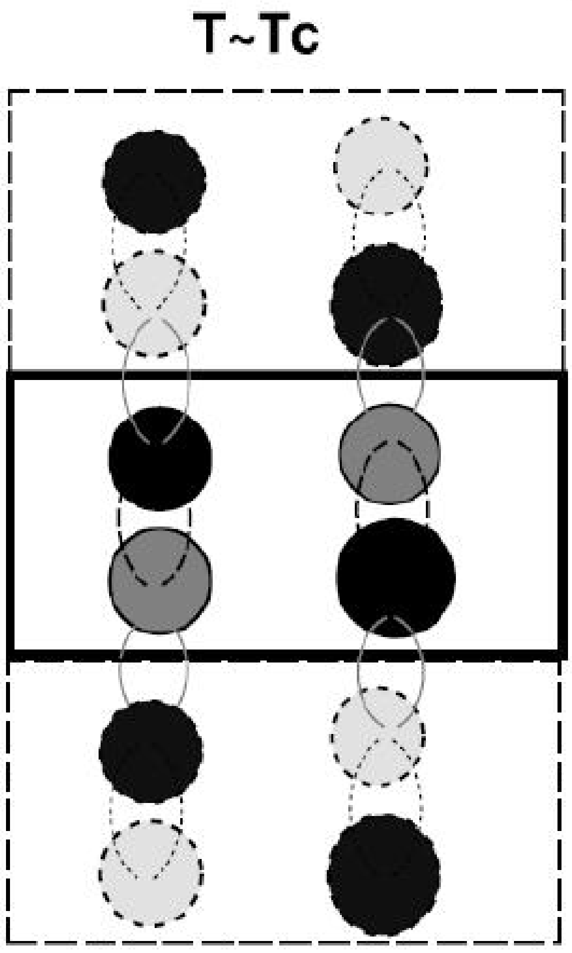


Inverse magnetic catalysis induced by instanton-anti-instanton molecule pairing

Chiral symmetry breaking and restoration from intantons

$$\langle 0|\bar{\psi}\psi|0\rangle \propto \rho(\lambda \to 0)$$





isolated instantons

instanton-anti-instanton molecule pairing

Chirality imbalance induced by instanton anti-instanton molecule pairing:

T. Schafer, E. V. Shuryak and J. J. M. Verbaarschot, Phys. Rev. D **51**, 1267 (1995) [hep-ph/9406210].

$$\mathcal{L}_{mol\ sym} = G\left\{ \frac{2}{N_c^2}\left[(\bar\psi\tau^a\psi)^2 - (\bar\psi\tau^a\gamma^5\psi)^2 \right] - \frac{1}{2N_c^2}\left[(\bar\psi\tau^a\gamma^\mu\psi)^2 + (\bar\psi\tau^a\gamma^\mu\gamma^5\psi)^2 \right] + \frac{2}{N_c^2}(\bar\psi\gamma^\mu\gamma^5\psi)^2 \right\} + \mathcal{L}_8 \ ,$$

$T \gtrsim T_c$:

$$G_S = \frac{2G}{N_c^2},\ G_V = \frac{G}{2N_c^2},\ G_A = -\frac{3G}{2N_c^2}$$

$$\mathcal{L} = \bar\psi i\gamma_\mu D^\mu\psi + G_S\left[(\bar\psi\psi)^2 + (\bar\psi i\gamma^5\boldsymbol{\tau}\psi)^2 \right] - G_V(\bar\psi\gamma^\mu\psi)^2 - G_A(\bar\psi\gamma^\mu\gamma^5\psi)^2 \ .$$

Mean-field approximation:

$$\mathcal{L} = -\frac{\sigma^2}{4G_S} + \frac{\tilde{\mu}_5^2}{4G_A} + \bar{\psi}\left(i\gamma_\mu D^\mu - \sigma + \tilde{\mu}_5\gamma^0\gamma^5\right)\psi$$

$$\sigma = -2G_S\langle\bar{\psi}\psi\rangle \qquad \tilde{\mu}_5 = -2G_A\langle\bar{\psi}\gamma^0\gamma^5\psi\rangle$$

$$r_A = G_A/G_S$$

$$\begin{aligned}\Omega = &\frac{\sigma^2}{4G_S} - \frac{\tilde{\mu}_5^2}{4G_A} \\ &-N_c \sum_{f=u,d}\frac{|q_f B|}{2\pi}\sum_{s,k}\alpha_{sk}\int_{-\infty}^{\infty}\frac{dp_z}{2\pi}\, f_\Lambda^2(p)\,\omega_{sk}(p) \\ &-2N_c T\sum_{f=u,d}\frac{|q_f B|}{2\pi}\sum_{s,k}\alpha_{sk}\int_{-\infty}^{\infty}\frac{dp_z}{2\pi} \qquad (16) \\ &\times\ln\left(1+e^{-\beta\omega_{sk}}\right),\end{aligned}$$

Inverse magnetic catalysis

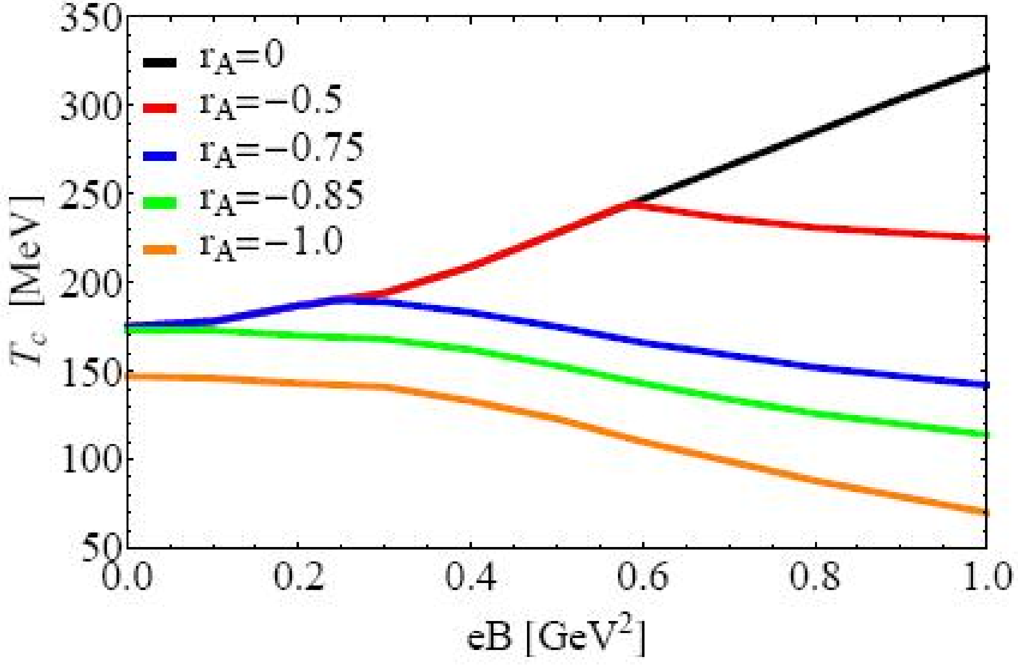


FIG. 2. (Color online) T_c as a function of eB for r_A=0,-0.5, -0.75, -0.85 and -1.0.

Lang Yu, Hao Liu, MH, arXiv:1404.6969, PRD90,074009(2014)

Vacuum superconductor

Vacuum Superconductor

•M. N. Chernodub, Phys. Rev. Lett. 106 (2011) 142003 [arXiv:1101.0117 [hep-ph]]

-Energy of relativistic particle in the external magnetic field B:

$$\varepsilon^2_{n,s_z}(p_z) = p_z^2 + (2n - 2\mathrm{sgn}(q)s_z + 1)|qB| + m^2$$

nonnegative integer number

the momentum along the external magnetic field

projection of spin on the direction of magnetic field

-Masses of ρ mesons and π in magnetic field:

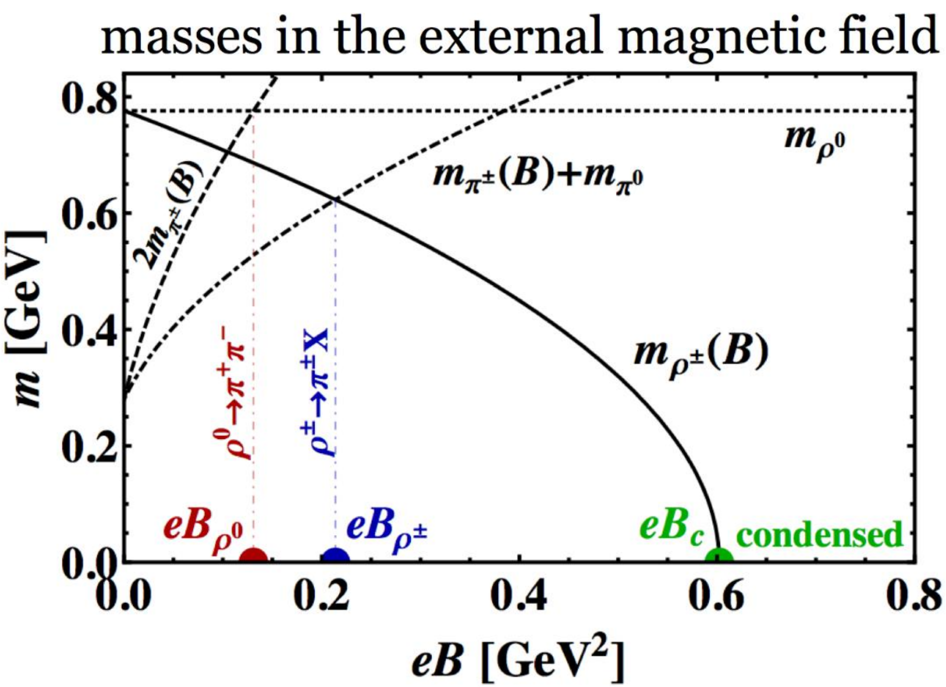
$$\mathrm{m}^2_{\pi^\pm}(\mathrm{B}) = \mathrm{m}^2_{\pi^\pm} + \mathrm{eB}$$ **becomes larger**

$$\mathrm{m}^2_{\rho^\pm}(\mathrm{B}) = \mathrm{m}^2_{\rho^\pm} - \mathrm{eB}$$ **becomes lighter**

where $\mathrm{m}_{\rho_\pm} = 768\mathrm{MeV}$, $\mathrm{m}_{\pi_\pm} = 140\ \mathrm{MeV}$

The charged rho becomes massless and condensate at a critical magnetic fields : $\mathrm{eB}_c = \mathrm{m}_{\rho^\pm}^2$

M. N. Chernodub, Phys. Rev. Lett. 106 (2011) 142003 [arXiv:1101.0117 [hep-ph]]



The pions become heavier while the charged vector mesons become lighter in the external magnetic field

The $\rho^\pm \to \pi^\pm\pi^0$ **decay stops at a critical eB**

Vacuum Superconductor?

- A point particle model for the charged rho :

$$\mathrm{eB}_c = \mathrm{m}^2_{\rho^\pm}$$

- NJL Model (LLL): $\mathrm{eB}_c > 1\ \mathrm{GeV}^2$

M. N. Chernodub, Phys. Rev. Lett. 106 (2011) 142003 [arXiv:1101.0117 [hep-ph]]

- NJL Model: $\mathrm{eB}_c = 0.978\ m_q^2$

M. Frasca, JHEP 1311, 099 (2013) [arXiv:1309.3966 [hep-ph]]

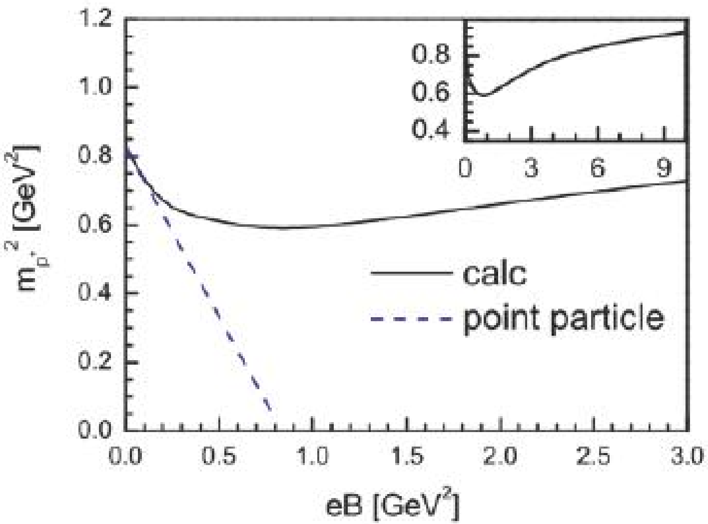
Vacuum Superconductor?

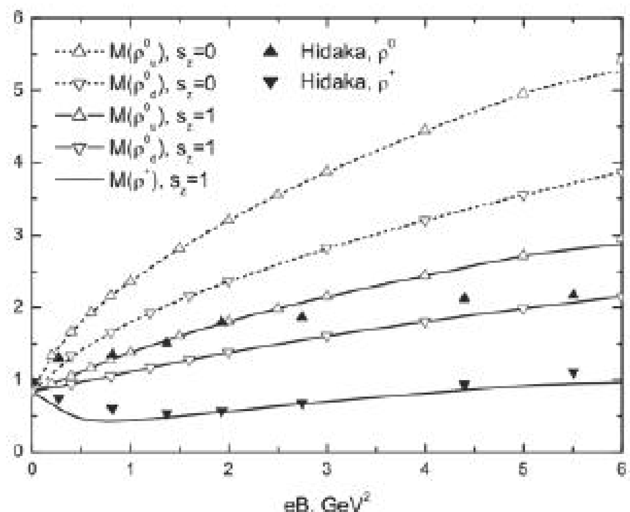
- **DSE and BSE:**

  Kunlun Wang PhD thesis

- **Quark-antiquark Green Function and effective Hamiltonian (LLL)**

  M. A. Andreichikov, B. O. Kerbikov, V. D. Orlovsky and Y. . A. Simonov, Phys. Rev. D 87, no. 9, 094029 (2013) [arXiv:1304.2533 [hep-ph]]

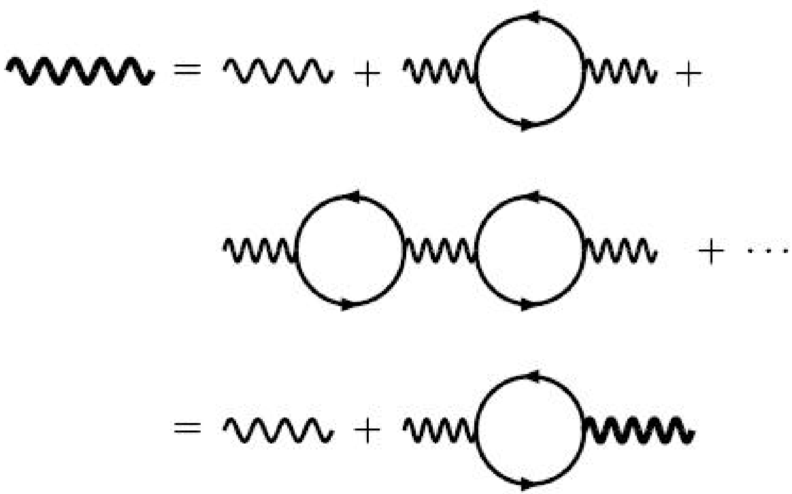


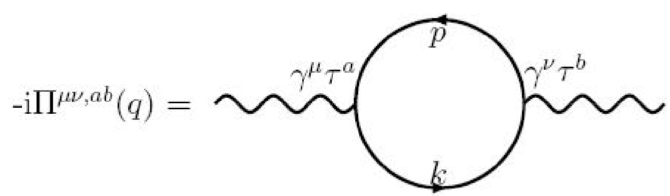


The masses of the systems in GeV as a functions of eB

Charged and neutral vector meson in NJL model

$$\mathcal{L} = \bar{\psi}(i\not{D} - \hat{m})\psi + G_S\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\vec{\tau}\psi)^2\right]$$
$$-G_V\left[(\bar{\psi}\gamma^\mu\tau^a\psi)^2 + (\bar{\psi}\gamma^\mu\gamma^5\tau^a\psi)^2\right]$$
$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}.$$





Charged vector meson in vacuum

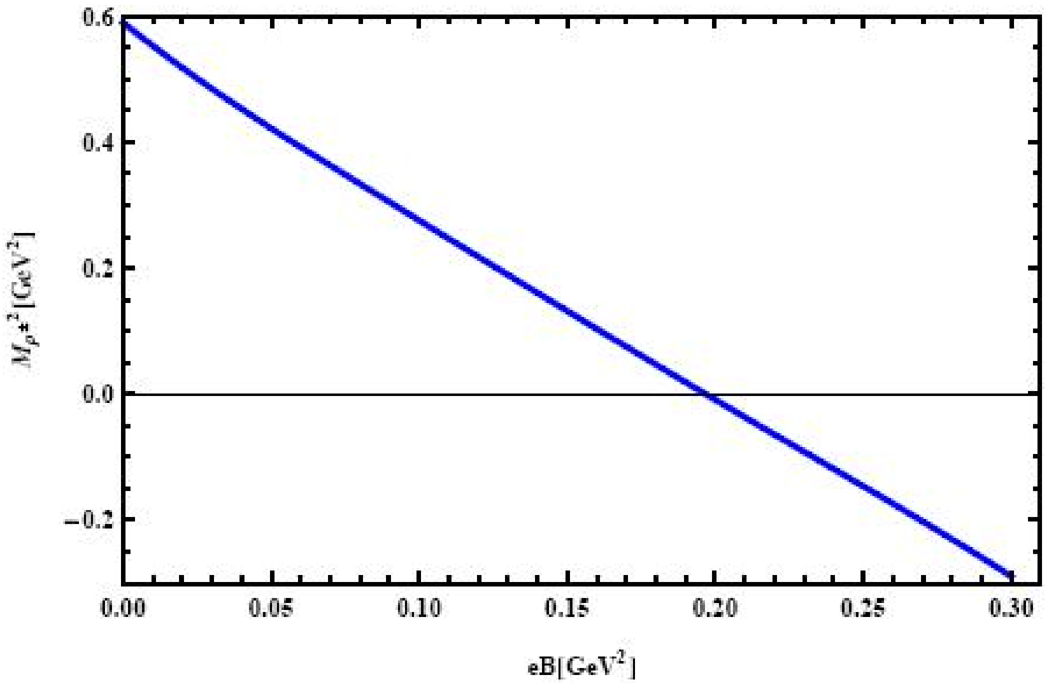


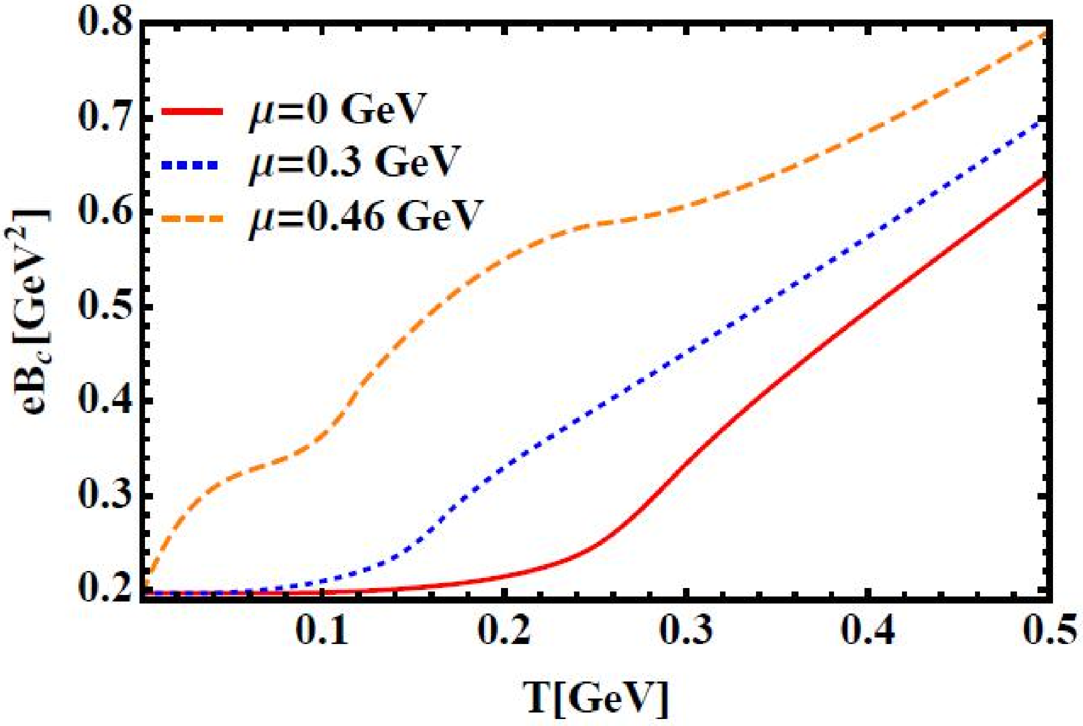
FIG. 4: The mass square of charged $\rho^{\pm}$ with spin component $s_z = \pm 1$ as a function of eB.

$$eB_c \simeq 0.2\mathrm{GeV}^2$$!

Hao Liu, Lang Yu, MH, arXiv:1408.1318

Charged vector meson at finite temperature

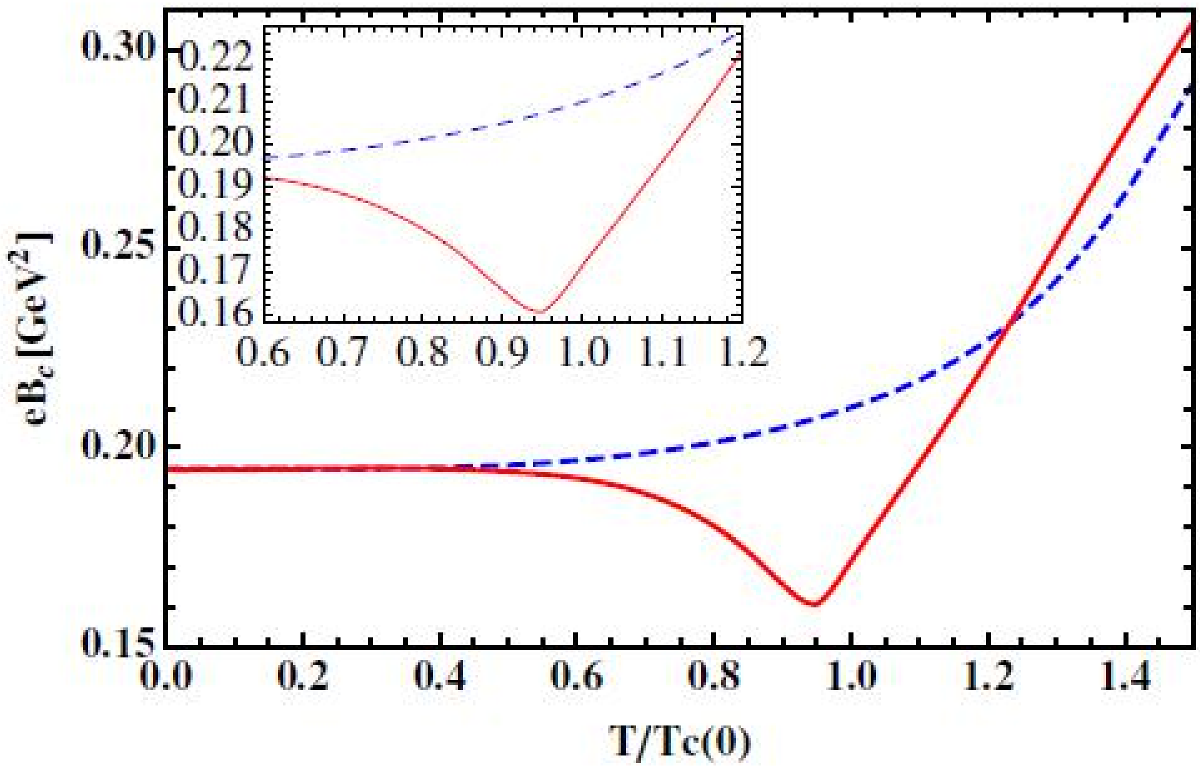
Charged vector meson can condense at high T!



Hao Liu, Lang Yu, MH, 2015

Charged vector meson at finite temperature

Charged vector meson can condense at high T with IMC!



Hao Liu, Lang Yu, M.Chernodub, MH, 2016

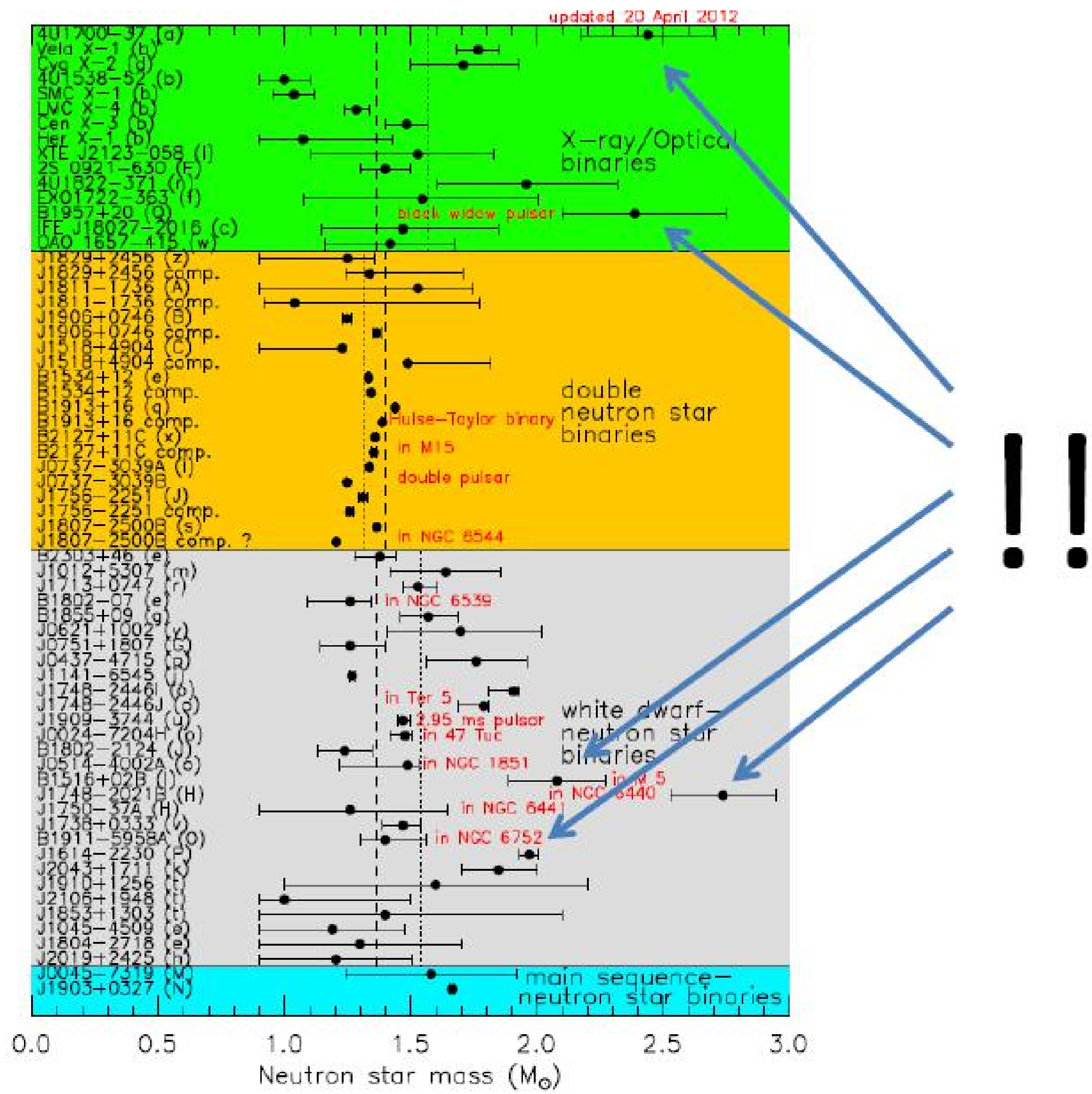
Magnetar

Pengcheng Chu, Xin Wang, Liewen Chen, MH, arXiv:1409.6154

2 solar mass Neutron star

A stiff (hard) equation of state is needed:

Quark matter has soft EoS ! Excluded inside neutron star?



Strange quark matter under magnetic field

$$\mathcal{L} = \mathcal{L}_q + \mathcal{L}_e - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$\mathcal{L}_q = \bar{\psi}_f [\gamma_\mu (i\partial^\mu - q_f A^\mu_{ext}) - \hat{m}_c] \psi_f + \mathcal{L}_4 + \mathcal{L}_6$$

$$\mathcal{L}_4 = \mathcal{L}_S + \mathcal{L}_V + \mathcal{L}_{I,V}$$

$$\mathcal{L}_S = G_S \sum_{a=0}^{8} [(\bar{\psi}_f \lambda_a \psi_f)^2 + (\bar{\psi}_f i\gamma_5 \lambda_a \psi_f)^2],$$

$$\mathcal{L}_V = -G_V \sum_{a=0}^{8} [(\bar{\psi} \gamma^\mu \lambda^a \psi)^2 + (\bar{\psi} i\gamma^\mu \gamma_5 \lambda^a \psi)^2]$$

Pengcheng Chu, Xin Wang, Liewen Chen, MH, arXiv:1409.6154

Strange quark matter under magnetic field

$$
\begin{aligned}
p_q &= -2G_S(\sigma_u^2+\sigma_d^2+\sigma_s^2)+4K\sigma_u\sigma_d\sigma_s \\
&+ 2G_V(n_u^2+n_d^2+n_s^2)+G_{IV}(n_u-n_d)^2 \\
&+ (\theta_u+\theta_d+\theta_s)
\end{aligned}
$$

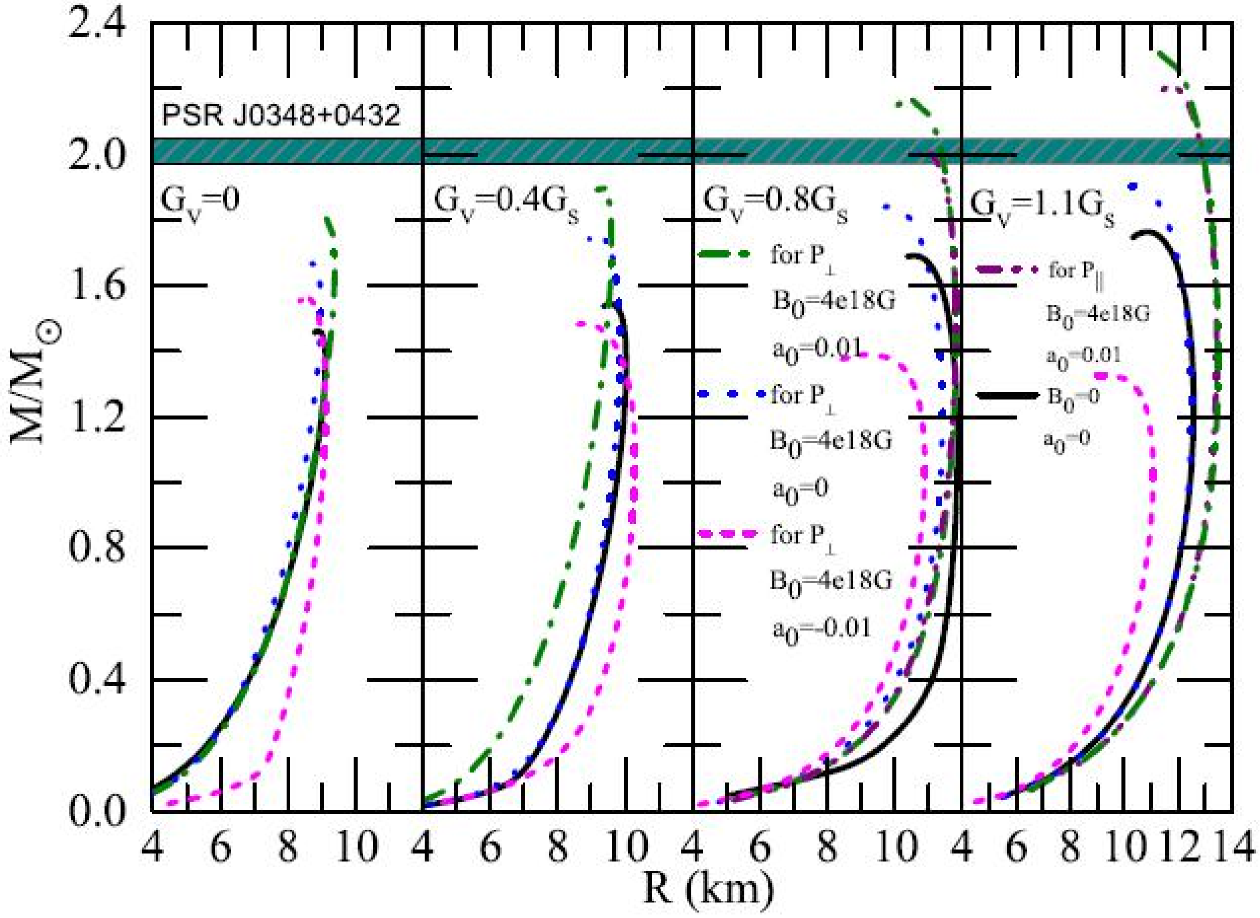
$$
\begin{aligned}
p_l &= \sum_{k=0}^{k_{l\,max}} \alpha_k \frac{(|q_l|BN_c)}{4\pi^2}\Big\{\mu_l\sqrt{\mu_l^2-s_l(k,B)^2} \\
&- s_l(k,B)^2\ln\Big[\frac{\mu_l+\sqrt{\mu_l^2-s_l(k,B)^2}}{s_l(k,B)}\Big]\Big\}.
\end{aligned}
$$

Pressure contribution from magnetized gluons

$$
p_g(T=0,\mu;eB)=a_0(\mu^2 eB+\mu^4)
$$

Pengcheng Chu, Xin Wang, Liewen Chen, MH, arXiv:1409.6154

Mass of quark magnetar



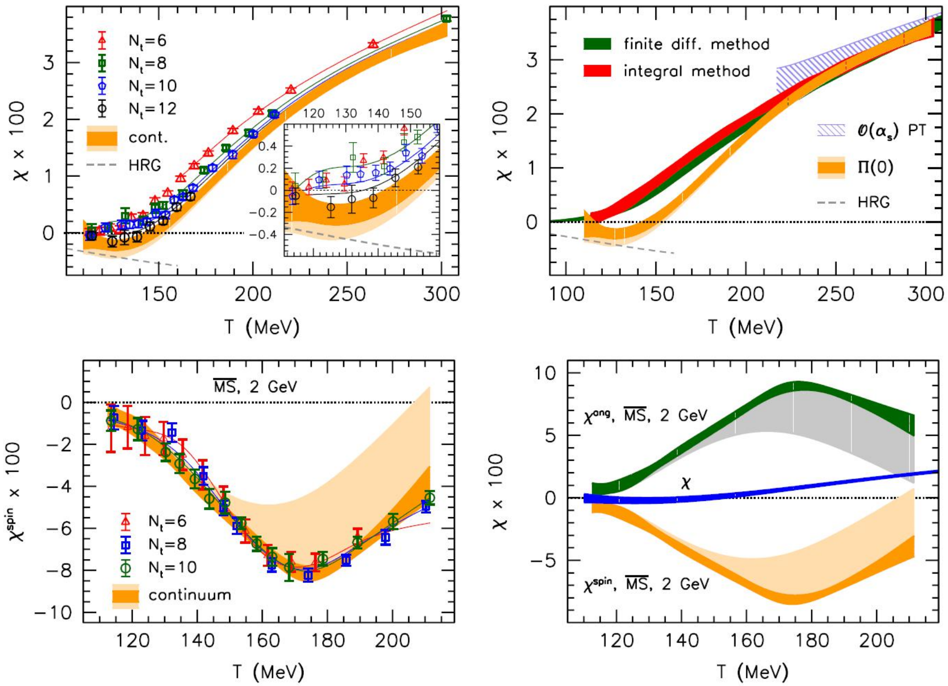
Pengcheng Chu, Xin Wang, Liewen Chen, MH, arXiv:1409.6154

Problems not solved …

Magnetic susceptibility: $\chi_b = -\frac{\partial^2}{\partial(e\ldots}$

$\chi = \chi_h(\mathrm{T}) - \chi_h(\mathrm{T}$

Latest Lattice paper: 2004.08778



Diamagnetism at low temperature while strong paramagnetism at high temperature

Lattic,PRD:1209.6015

Spin **angular**

$$\xi_f = \frac{q_f/e}{2m_f}\left(\frac{\partial\langle\bar\psi_f\sigma_{xy}\psi_f\rangle}{\partial(eB)} + \frac{\partial\langle\bar\psi_f L_{xy}\psi_f\rangle}{\partial(eB)}\right)\Bigg|_{eB=0},$$

$$\xi^S = \sum_f \frac{(q_f/e)^2}{2m_f}\tau_f, \qquad \xi^L = \sum_f \frac{q_f/e}{2m_f}\frac{\partial\langle\bar\psi_f L_{xy}\psi_f\rangle}{\partial(eB)},$$

$$\langle\bar\psi_f\sigma_{xy}\psi_f\rangle = q_f B\cdot\langle\bar\psi_f\psi_f\rangle\cdot\chi_f \equiv q_f B\cdot\tau_f,$$

τ_f tensor coefficient

Lattic,PRD:1209.6015

$$\frac{\partial\log Z}{\partial B} = \sum_f\left\langle \mathrm{tr}\frac{1}{\not{D}_f + m_f}\frac{\partial\not{D}_f}{\partial B}\right\rangle. \qquad \text{(A2)}$$

We manipulate this using $\mathrm{tr}\,\partial\not{D}_f/\partial B \propto \mathrm{tr}\,\gamma_\mu = 0$ and the cyclicity of the trace:

$$\begin{aligned}\frac{\partial\log Z}{\partial B} &= \sum_f\frac{1}{m_f}\left\langle \mathrm{tr}\left(\frac{m_f}{\not{D}_f + m_f} - 1\right)\frac{\partial\not{D}_f}{\partial B}\right\rangle \\ &= -\sum_f\frac{1}{m_f}\left\langle \mathrm{tr}\frac{1}{\not{D}_f + m_f}\not{D}_f\frac{\partial\not{D}_f}{\partial B}\right\rangle \\ &= -\frac{1}{2}\sum_f\frac{1}{m_f}\left\langle \mathrm{tr}\frac{1}{\not{D}_f + m_f}\frac{\partial\not{D}_f^2}{\partial B}\right\rangle.\end{aligned} \qquad \text{(A3)}$$

Puzzle:

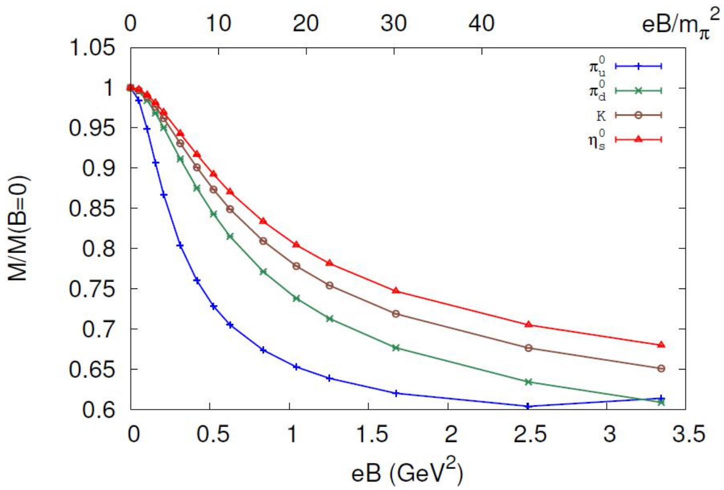
Latest Lattice paper: 2004.08778

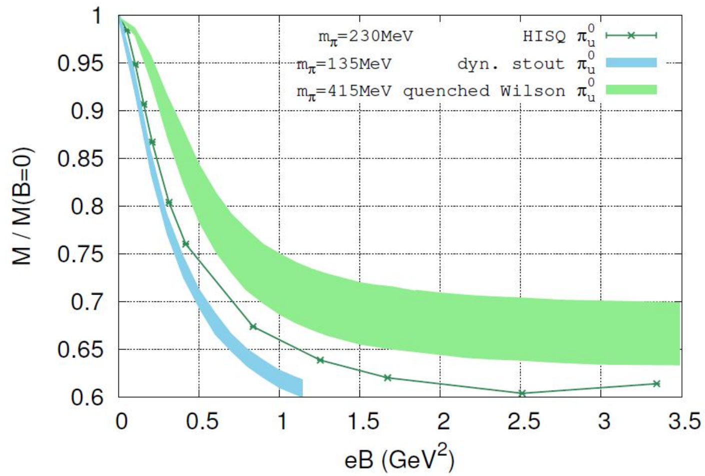
$$-\frac{\partial f}{\partial B} = \frac{T}{V}\sum_f\left\langle \mathrm{tr}\frac{1}{\not{D}_f + m_f}\frac{\partial\not{D}_f}{\partial B}\right\rangle = \frac{T}{2V}\sum_f\left\langle \mathrm{tr}\frac{1}{(\not{D}_f + m_f)\not{D}_f}\frac{\partial\not{D}_f^2}{\partial B}\right\rangle,$$

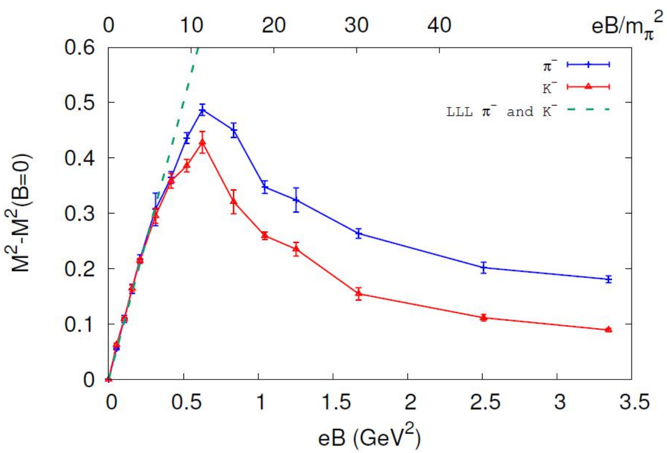
Meson masses in external magnetic fields with HISQ fermions

arXiv:2001.05322v1 [hep-lat]

Heng-Tong Ding[1], Sheng-Tai Li [2,1], Swagato Mukherjee[3], Akio Tomiya[4], Xiao-Dan Wang[*1 †]





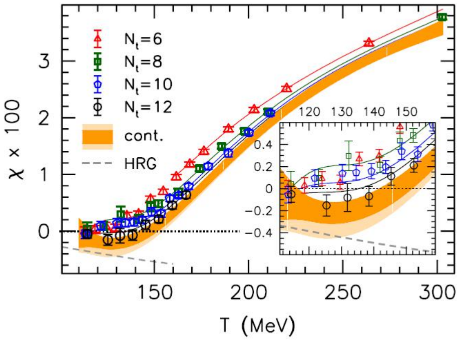


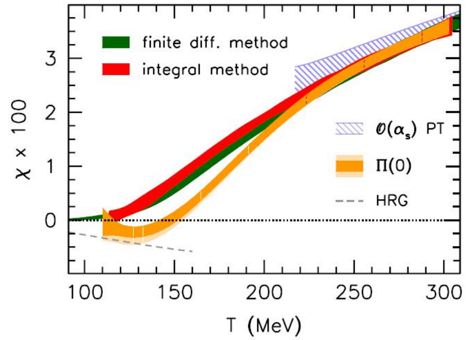
Neutral and charged pion masses spectra

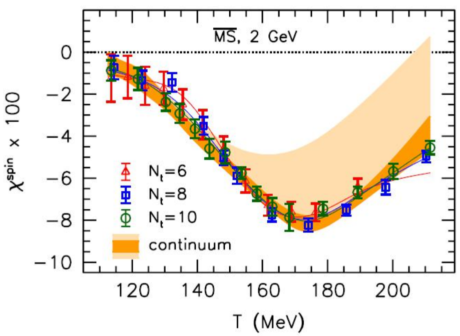
Negative Magnetic susceptibility:

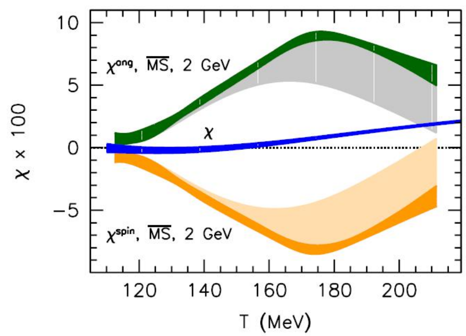
$$\chi_b = -\frac{\partial^2 \ldots}{\ldots}$$

Latest Lattice paper: 2004.08778









Diamagnetism at low temperature while strong paramagnetism at high temperature

Lattic,PRD:1209.6015

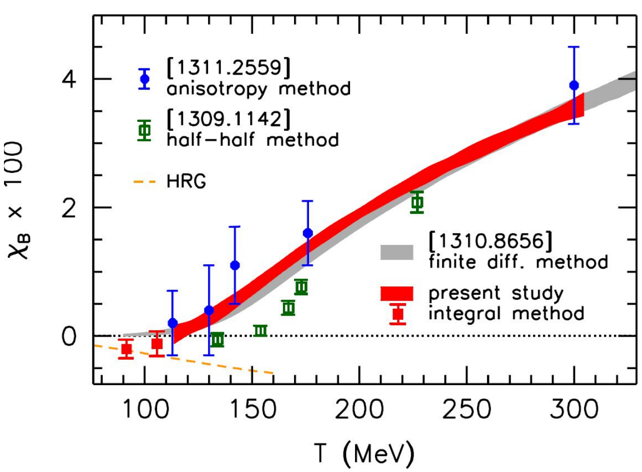
Spin angular

$$\xi_f = \frac{q_f/e}{2m_f}\left(\frac{\partial\langle\bar{\psi}_f\sigma_{xy}\psi_f\rangle}{\partial(eB)} + \frac{\partial\langle\bar{\psi}_f L_{xy}\psi_f\rangle}{\partial(eB)}\right)\Bigg|_{eB=0},$$

$$\xi^S = \sum_f \frac{(q_f/e)^2}{2m_f}\tau_f, \qquad \xi^L = \sum_f \frac{q_f/e}{2m_f}\frac{\partial\langle\bar{\psi}_f L_{xy}\psi_f\rangle}{\partial(eB)},$$

$$\langle\bar{\psi}_f\sigma_{xy}\psi_f\rangle = q_f B\cdot\langle\bar{\psi}_f\psi_f\rangle\cdot\chi_f \equiv q_f B\cdot\tau_f,$$

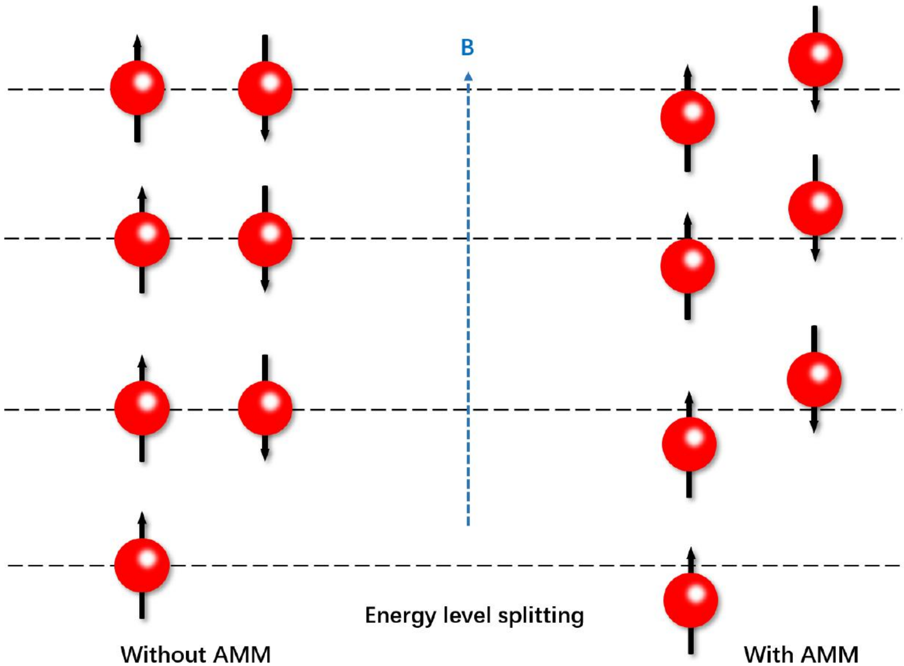
τ_f tensor coefficient



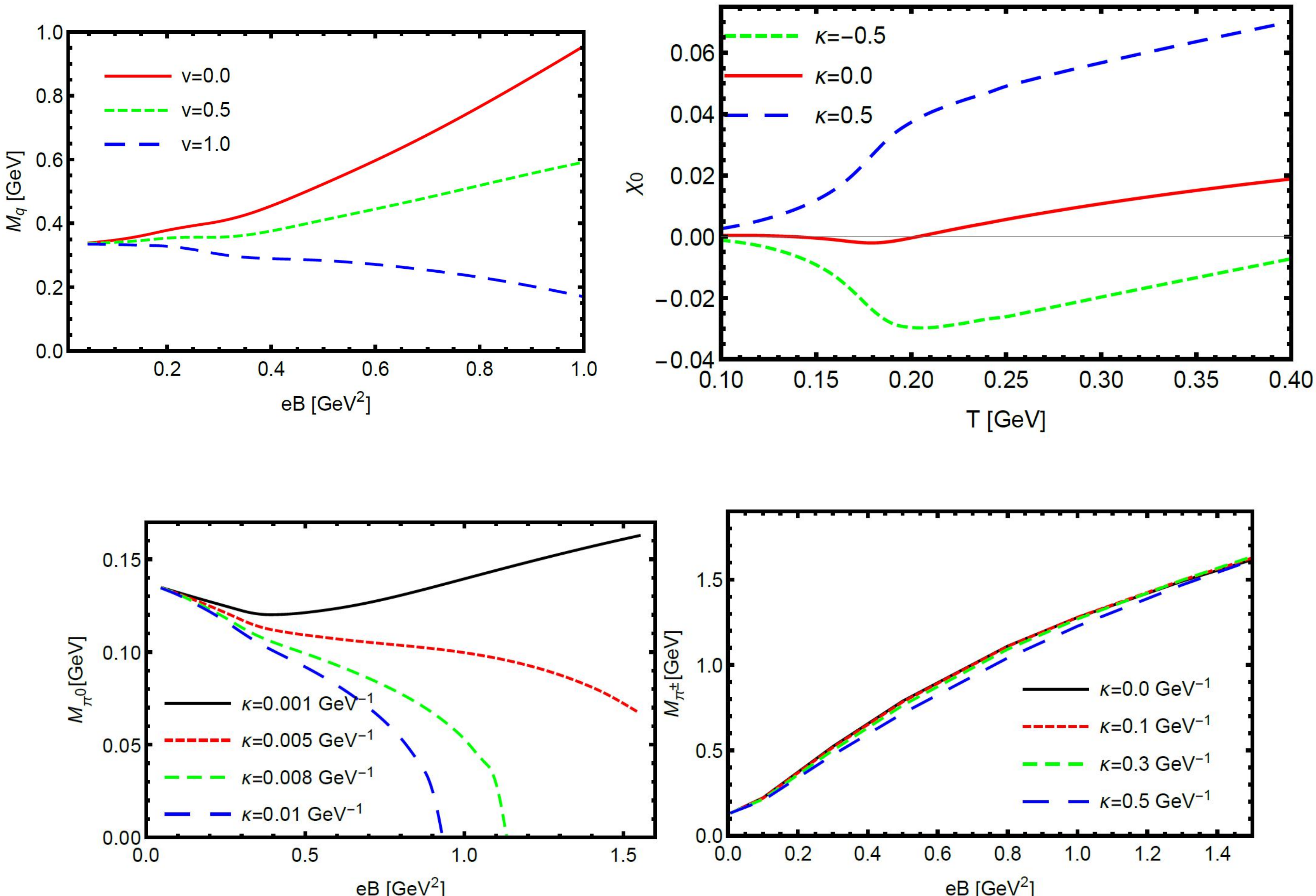
Lattice:1406.0269

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m_0 + \kappa_f q_f F_{\mu\nu}\sigma^{\mu\nu})\psi + G_S\left\{(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\vec{\tau}\psi)^2\right\}$$

$$E_k^2 = p_z^2 + \{\sqrt{M^2 + (2k+1-s\xi)|qB|} - s\kappa qB\}^2$$



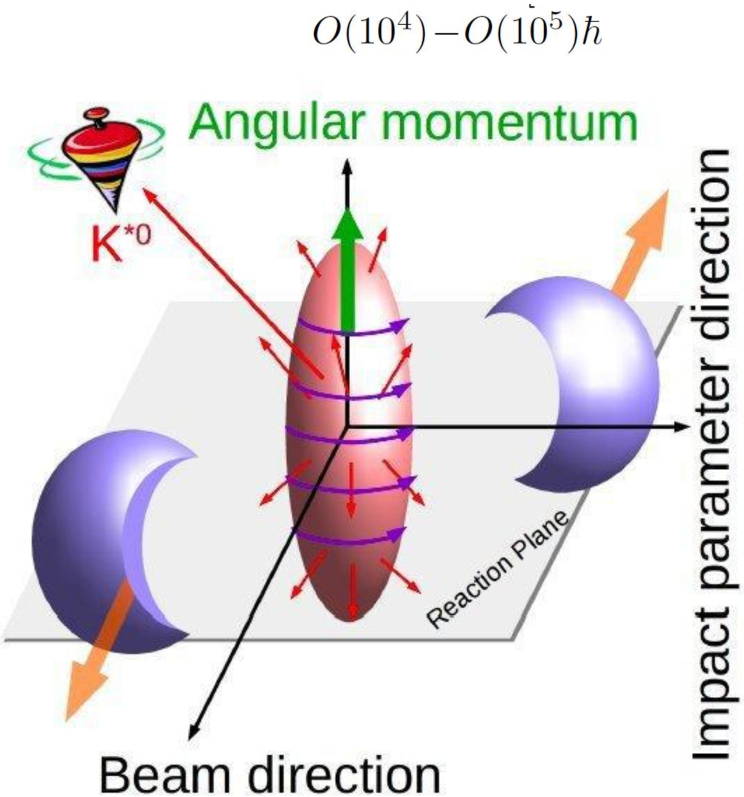
Kun Xu, Jingyi Chao, Mei Huang, 2007.13122, PRD to appear

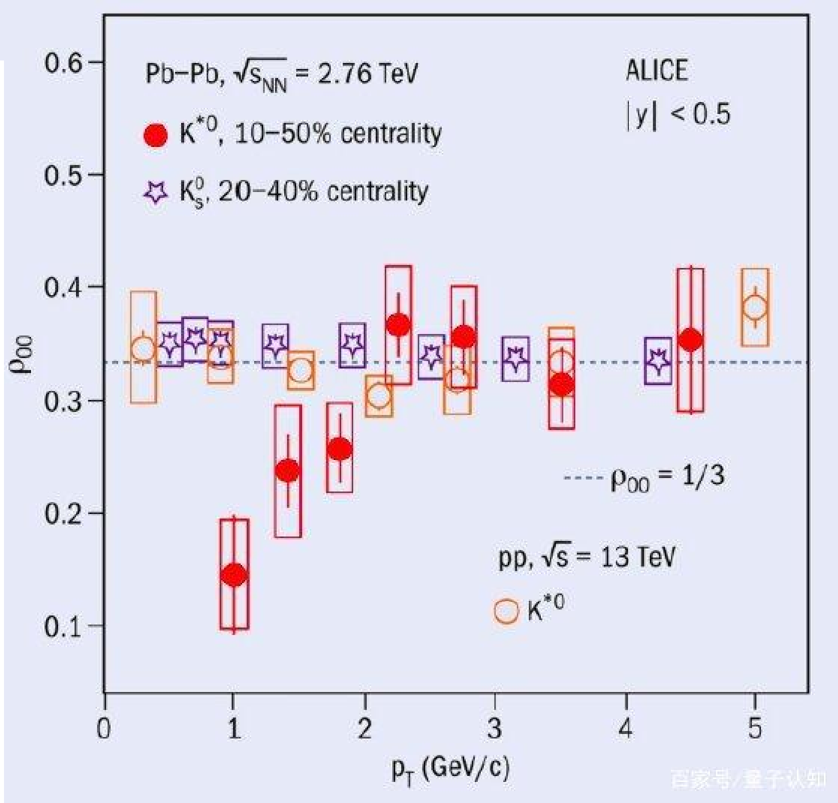


v=0.0
v=0.5
v=1.0
M_q [GeV]
eB [GeV^2]
κ=−0.5
κ=0.0
κ=0.5
χ_0
T [GeV]
κ=0.001 GeV^{-1}
κ=0.005 GeV^{-1}
κ=0.008 GeV^{-1}
κ=0.01 GeV^{-1}
M_{π^0}[GeV]
eB [GeV^2]
κ=0.0 GeV^{-1}
κ=0.1 GeV^{-1}
κ=0.3 GeV^{-1}
κ=0.5 GeV^{-1}
$M_{\pi^\pm}$[GeV]
eB [GeV^2]

II.QCD matter under rotation

I. Introduction





Evidence of Spin-Orbital Angular Momentum Interactions in Relativistic Heavy-Ion Collisions, Physical Review Letters (2020). DOI: 10.1103/PhysRevLett.125.012301

II. Chiral dynamics under rotation

Xinyang Wang, Minghua Wei, Zhibing Li, M.H.
Phys.Rev.D 99 (2019) 1, 016018,e-Print: 1808.01931

Minghua Wei, Ying Jiang, M.H. to appear

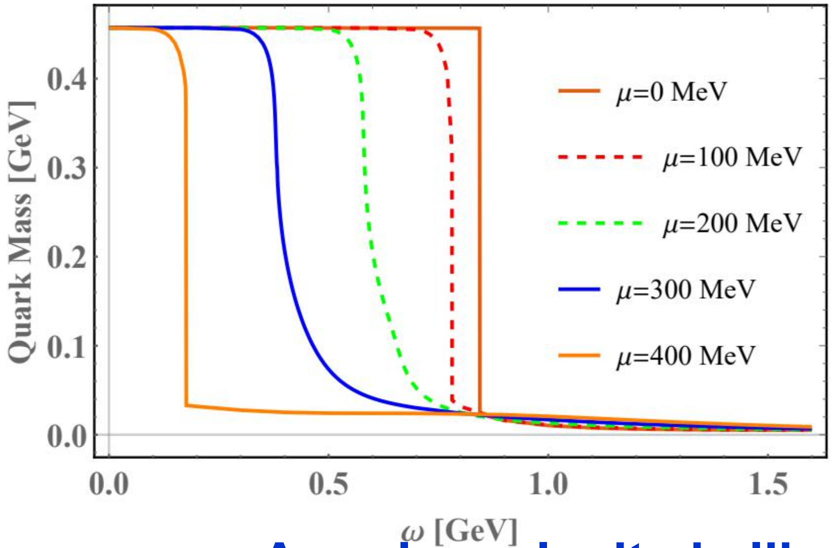
Chial dynamics under rotation from NJL model

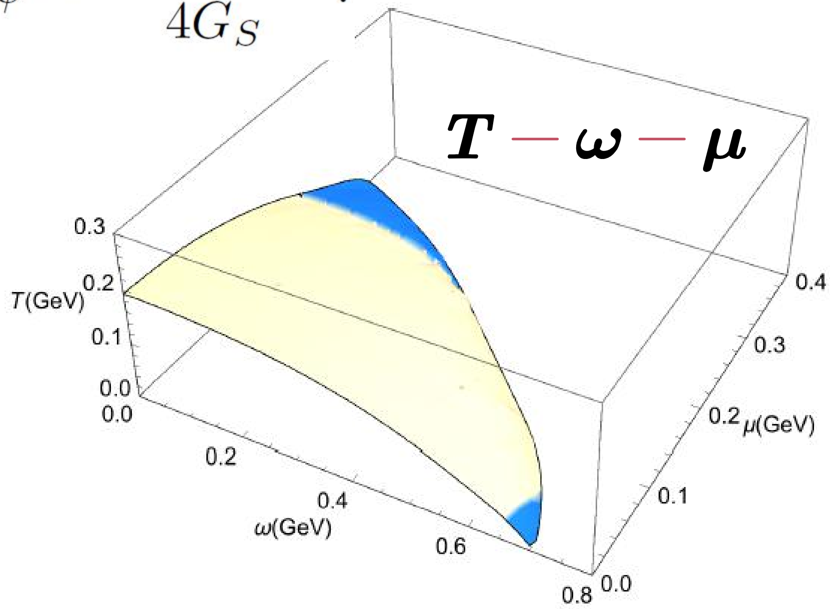
Yin Jiang, Jinfeng Liao PRL2015

$$\mathcal{L} = \bar{\psi}[i\bar{\gamma}^\mu(\partial_\mu + \Gamma_\mu) - m]\psi + G_S[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] - G_V[(\bar{\psi}\gamma_\mu\psi)^2 + (\bar{\psi}\gamma_\mu\gamma_5\psi)^2].$$

$$\Gamma_\mu = \frac{1}{4}\times\frac{1}{2}[\gamma^a,\gamma^b]\Gamma_{ab\mu} \qquad \Gamma_{ab\mu} = \eta_{ac}\left(e^c_\sigma G^\sigma_{\mu\nu} e^\nu_b - e^\nu_b \partial_\mu e^c_\nu\right)$$

$$\mathcal{L} = \bar{\psi}[i\gamma^\mu(\partial_\mu + \gamma^0\omega\hat{J}_z) - M]\psi - \mu\psi^\dagger\psi - \frac{(M-m)^2}{4G_S}.$$



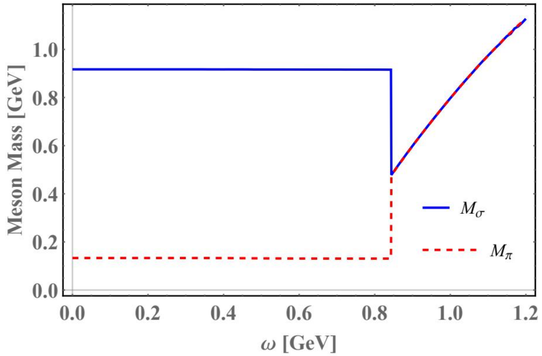


Angular velocity is like the chemical potential,
1st order phase transition in two corners!

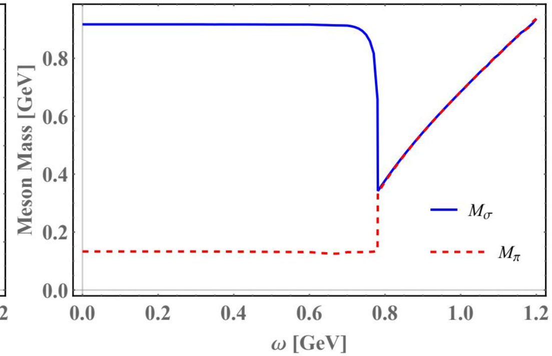
Minghua Wei, Ying Jiang,
M.H. to appear

Xinyang Wang, Minghua Wei,
Zhibin Li, Mei Huang PRD2019

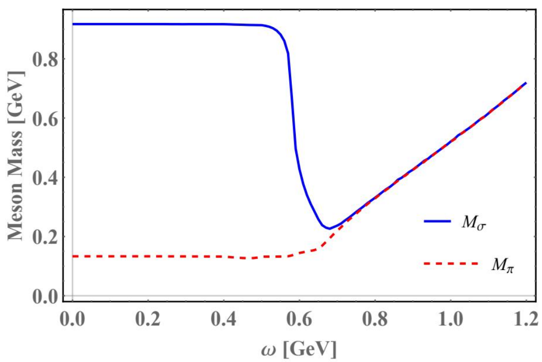
Scalar meson masses as functions of angular velocity



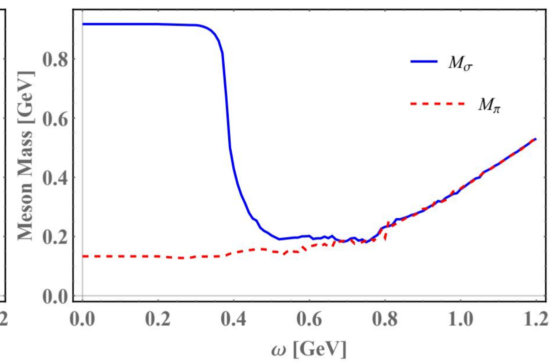
(a) scalar meson mass as a function of angular velocity at $\mu = 0MeV$



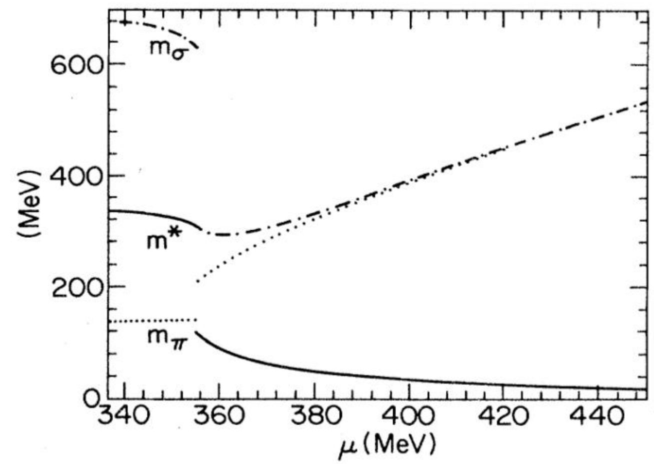
(b) scalar meson mass as a function of angular velocity at $\mu = 100MeV$



(c) scalar meson mass as a function of angular velocity at $\mu = 200MeV$



(d) scalar meson mass as a function of angular velocity at $\mu = 300MeV$

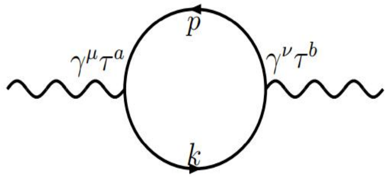


Minghua Wei, Ying Jiang, M.H. to appear

The effect of rotation on the scalar meson mass is similar to that of chemical potential !

Vector meson masses as functions of angular velocity

$$\Pi^{\mu\nu,ab}(q) = -i\int d^4\tilde{r}Tr_{sfc}[i\gamma^\mu\tau^a S(0;\tilde{r})i\gamma^\nu\tau^b S(\tilde{r};0)]e^{q\cdot\tilde{r}}$$

$-i\Pi^{\mu\nu,ab}(q) =$ 

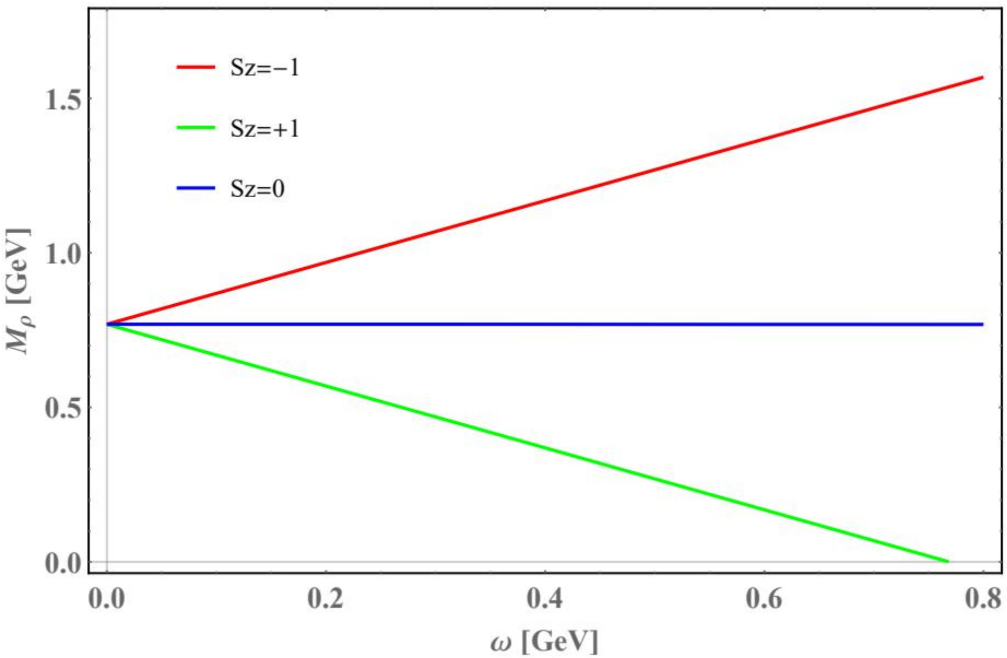
$$D_\rho^{\mu\nu}(q^2) = D_1(q^2)P_1^{\mu\nu} + D_2(q^2)P_2^{\mu\nu} + D_3(q^2)L^{\mu\nu} + D_4(q^2)u^\mu u^\nu$$

$$P_1^{\mu\nu} = -\epsilon_1^\mu\epsilon_1^\nu, (S_z = -1 \text{ for } \rho \text{ meson })$$
$$P_2^{\mu\nu} = -\epsilon_2^\mu\epsilon_2^\nu, (S_z = +1 \text{ for } \rho \text{ meson })$$
$$L^{\mu\nu} = -b^\mu b^\nu, (S_z = 0 \text{ for } \rho \text{ meson })$$

$$1 + 2G_V A_i^2 = 0$$

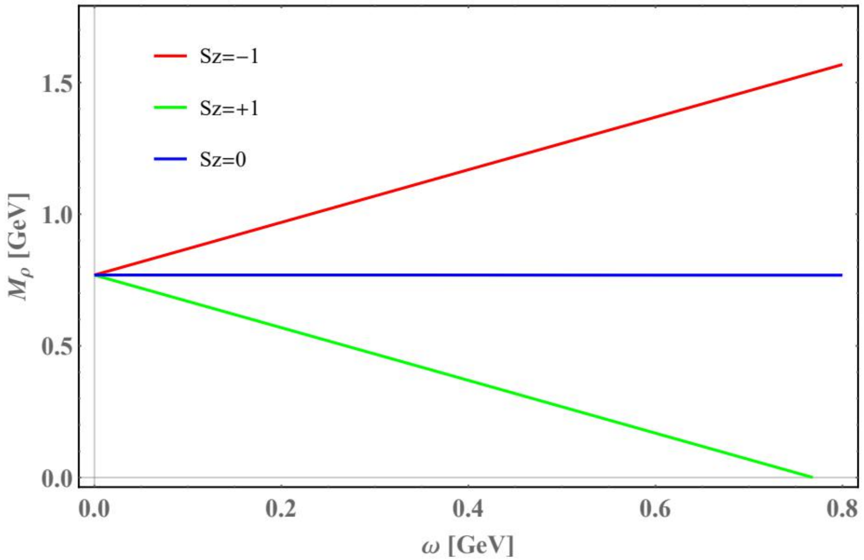
$$A_1^2 = -(\Pi_{11} - i\Pi_{12}), (S_z = -1 \text{ for } \rho \text{ meson })$$
$$A_2^2 = -\Pi_{11} - i\Pi_{12}, (S_z = +1 \text{ for } \rho \text{ meson })$$
$$A_3^2 = \Pi_{33}, (S_z = 0 \text{ for } \rho \text{ meson })$$

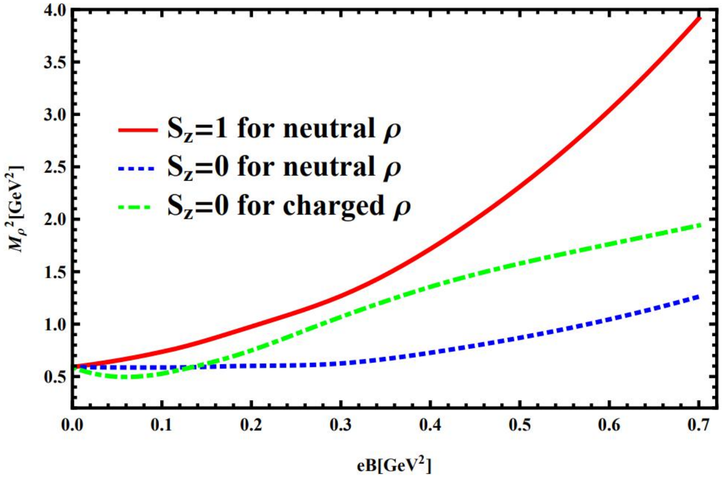
Zeeman splitting effect for different spin component!



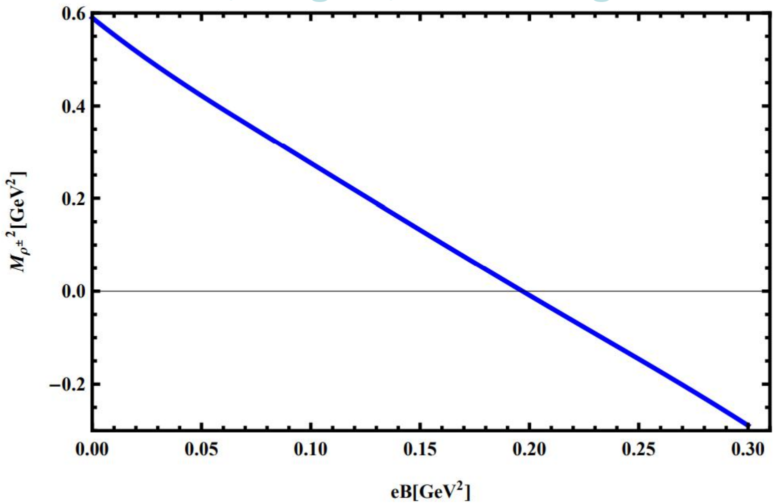
Minghua Wei, Ying Jiang, M.H. to appear

Vector meson masses as functions of angular velocity



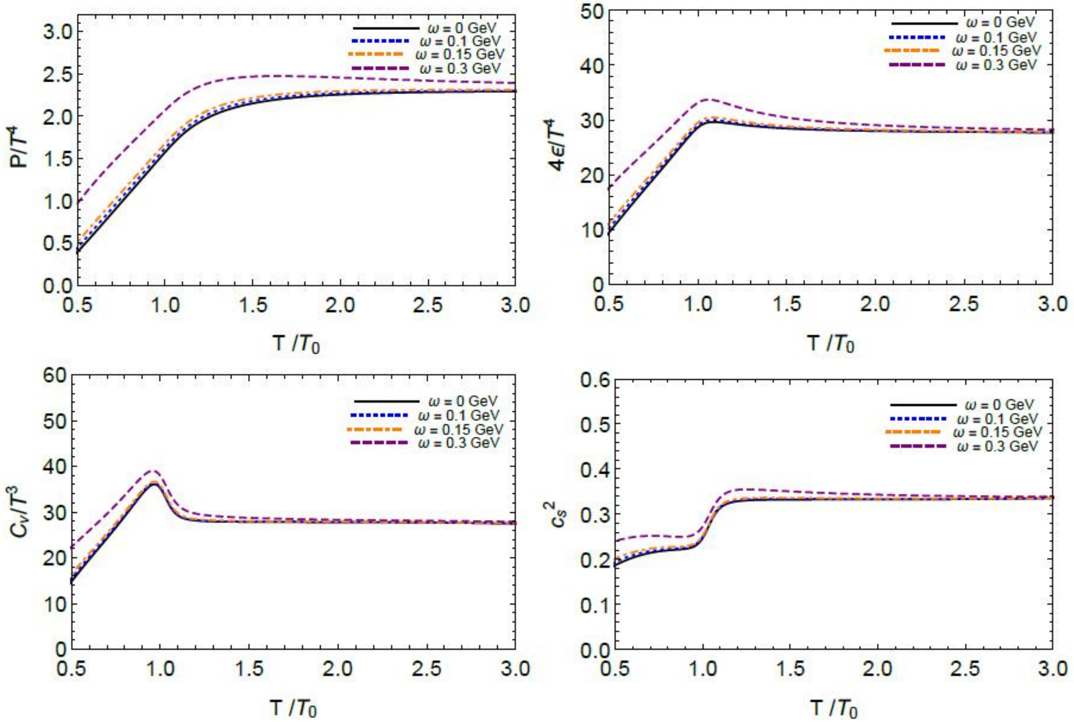


Hao Liu,Lang Yu,Mei Huang PRD2014



The effect of rotation on spin component of vector meson is similar to that of the magnetic field on charged vector mesons !

Enhancement of thermodynamical properties under rotation



Xinyang Wang, Minghua Wei, Zhibin Li, Mei Huang PRD2019

III. Gluodynamics under rotation

Xun Chen, Lin Zhang, Danning Li,
Defu Hou, M.H. arXiv: 2010.14478

Gluons are spin-1 particles, should be more sensitive to rotation!
No good 4D effective theory for gluodynamics, we use dynamical holographic QCD model!

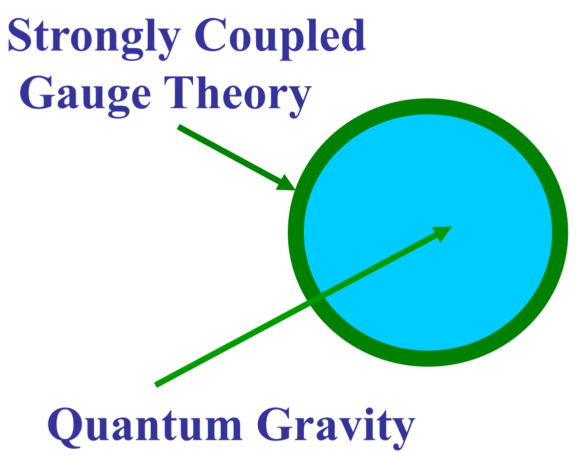
Holographic Duality: Gravity/QFT

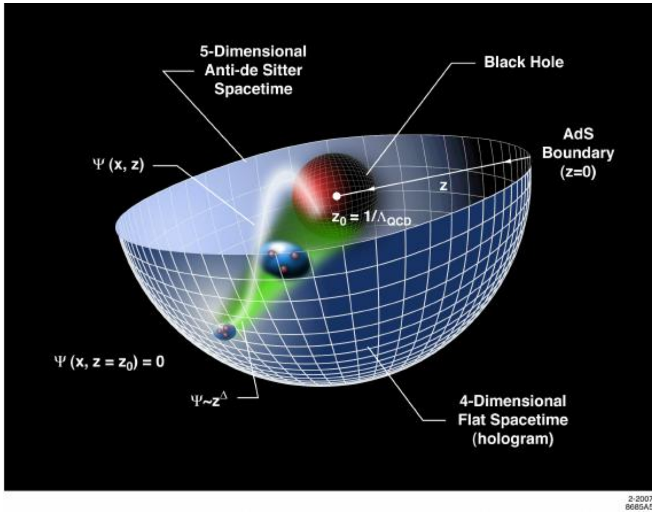
AdS/CFT : Original discovery of duality

Supersymmetry and conformality are required for AdS/CFT.

J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998)

Holographic Duality: (d+1)-Gravity/ (d)-QFT



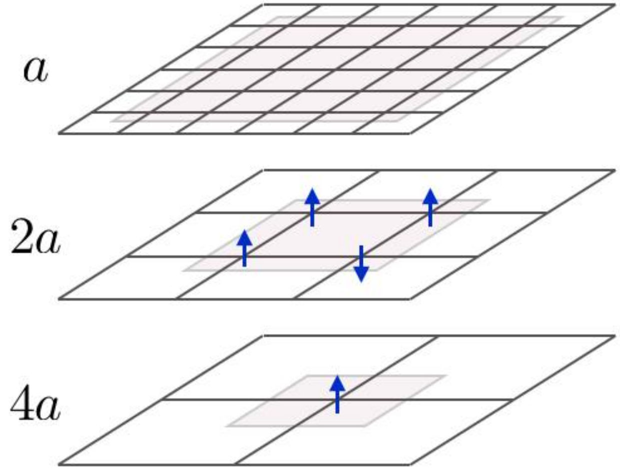


Holographic Duality & RG flow

Coarse graining spins on a lattice: Kadanoff and Wilson

$$H = \sum_{x,i} J_i(x) \mathcal{O}^i(x)$$

J(x): coupling constant or source for the operator



$$H = \sum_i J_i(x, a)\, \mathcal{O}^i(x)$$

$$H = \sum_i J_i(x, 2a)\, \mathcal{O}^i(x)$$

$$H = \sum_i J_i(x, 4a)\, \mathcal{O}^i(x)$$

$$u \frac{\partial}{\partial u} J_i(x, u) = \beta_i(J_j(x, u), u)$$

A.Adams, L.D.Carr, T.Shaefer, J.E.Thomas
arXiv:1205.5180

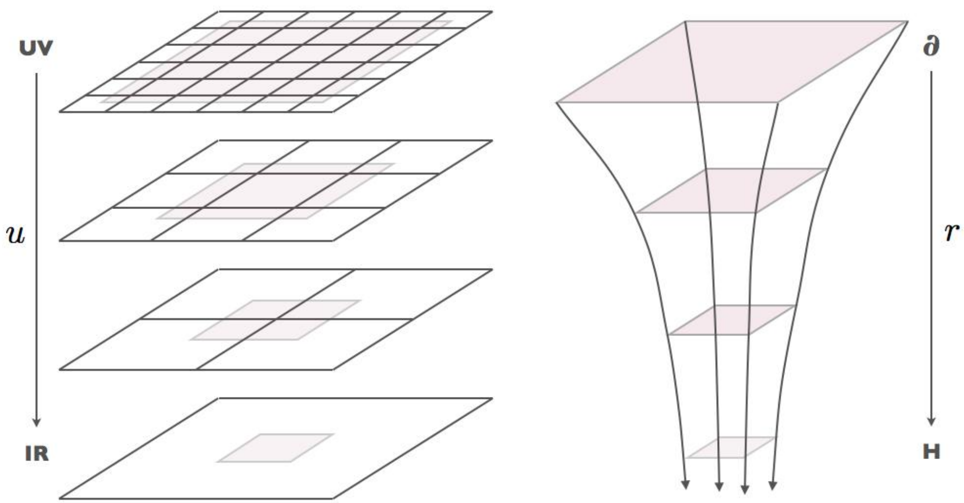
Dynamical holographic QCD ! **Graviton-dilaton-scalar system**

QFT on lattice equivalent to GR problem from Gravity

RG or energy scale → an extra spatial dimension

Coupling constant —promote→ dynamical filed

$$J_i|_{UV} = \Phi_i|_\partial$$



A AdS_5 **Dynamical**

Bulk field/Operator correspondence

$\Phi(z)$	$\mathrm{Tr}\langle G^2\rangle$ $\langle g^2A^2\rangle$
$\chi(z)$	$\langle\bar{q}q\rangle$

From UV to IR

Deformation of AdS_5

Glueball spectra:

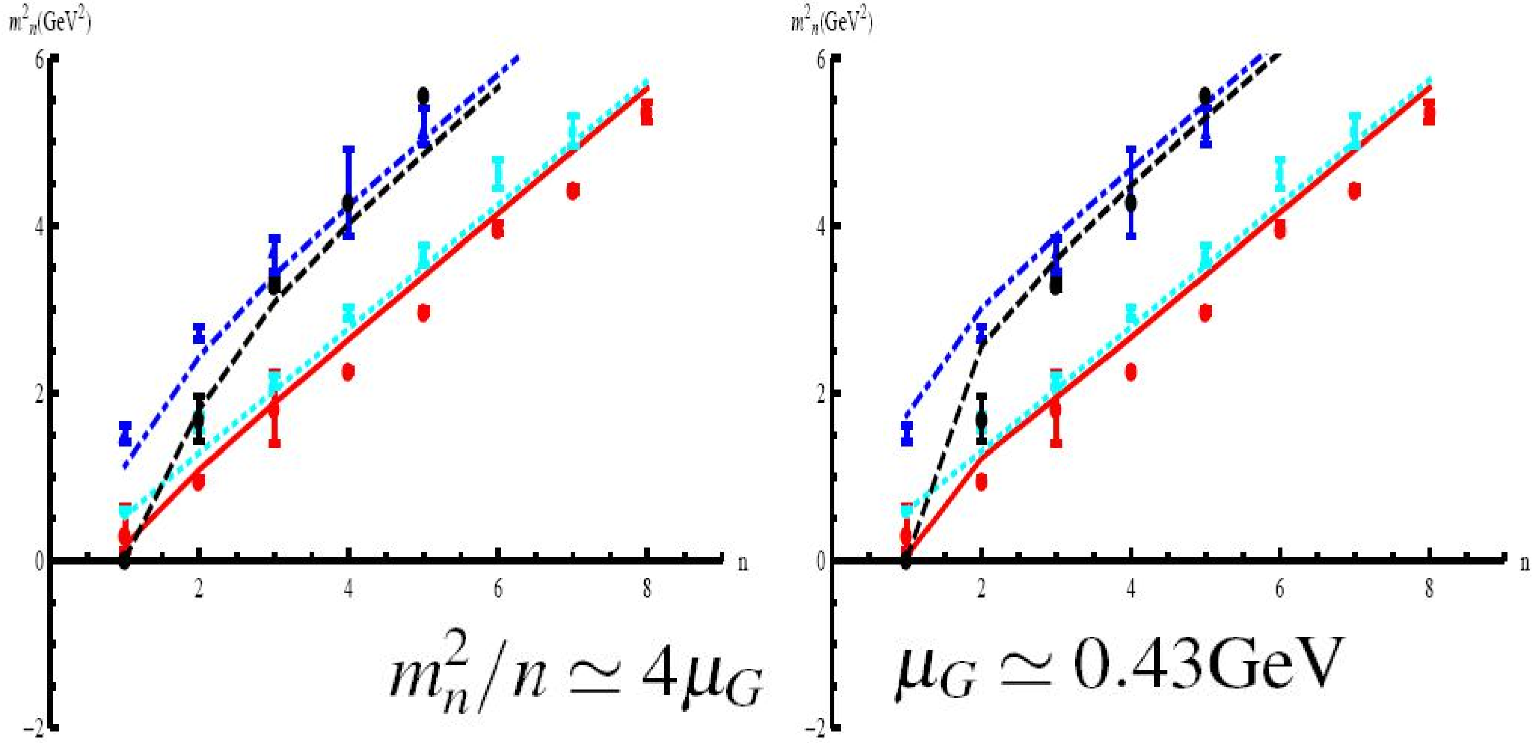
Yidian Chen, M.H., arXiv: 1511.07018

J^{PC}	LQCD	Flux tube model	QCDSR	MDSM
0^{++}	1.475-1.73	1.52	1.5	1.593
0^{*++}	2.67-2.83	2.75	–	2.618
0^{**++}	3.37	–	–	3.311
0^{***++}	3.99	–	–	3.877
0^{-+}	2.59	2.79	2.05	2.606
0^{*-+}	3.64	–	–	3.317
0^{--}	5.166	2.79	3.81	3.817
0^{+-}	4.74	2.79	4.57	3.04
$0^{++\S}$	–	–	3.1	2.667
1^{+-}	2.94	2.25	–	2.954
1^{--}	3.85	–	–	3.44
2^{++}	2.4	2.84	2	2.203
2^{-+}	3.1	2.84	–	3.161
2^{*-+}	3.89	–	–	3.703
2^{+-}	4.14	2.84	6.06	2.786
2^{--}	3.93	2.84	–	3.619

Odderon

Produced hadron spectra compared with data

D.N. Li, M.H., JHEP2013, arXiv:1303.6929



(Mod IA)

(Mod IB)

Ground states: chiral symmetry breaking

Excitation states: linear confinemnt

Strongly coupled QGP

Equation of state & transport properties

D.N. Li, S. He, M.H., Q. S. Yan, arXiv:1103.5389, JHEP2011

Danning Li, Jinfeng Liao, M.H. arXiv:1401.2035, PRD2014

Danning Li, Song He, M.H. arXiv:1411.5332,JHEP2015

Phase transition and EOS

5D graviton action:

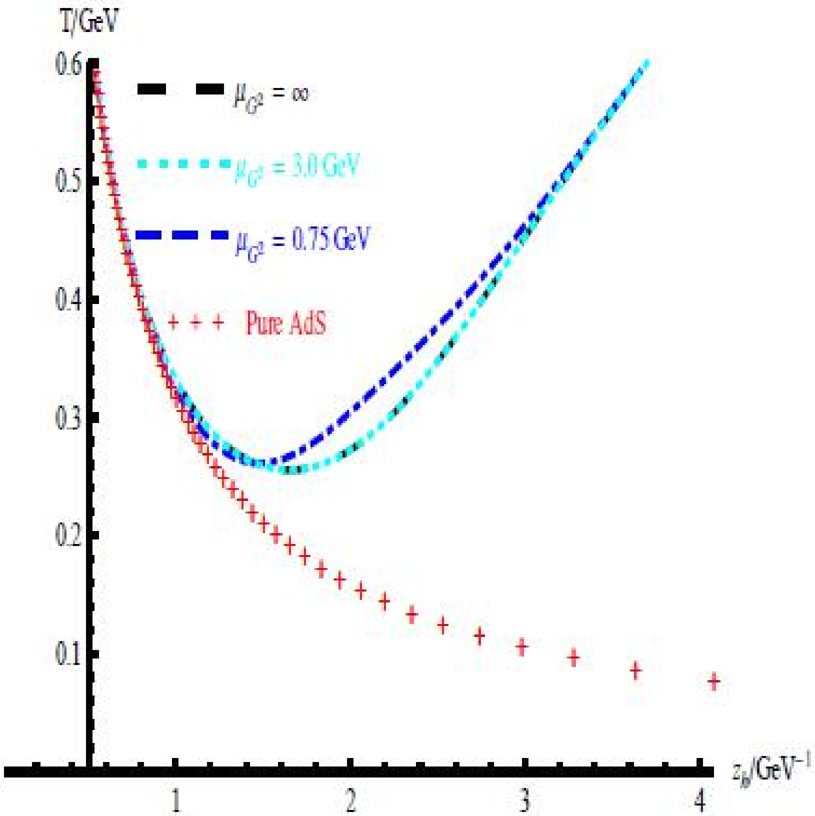
$$S_{5D} = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g^E}\left(R - \frac{4}{3}\partial_\mu\phi\partial^\mu\phi - V_E(\phi)\right)$$

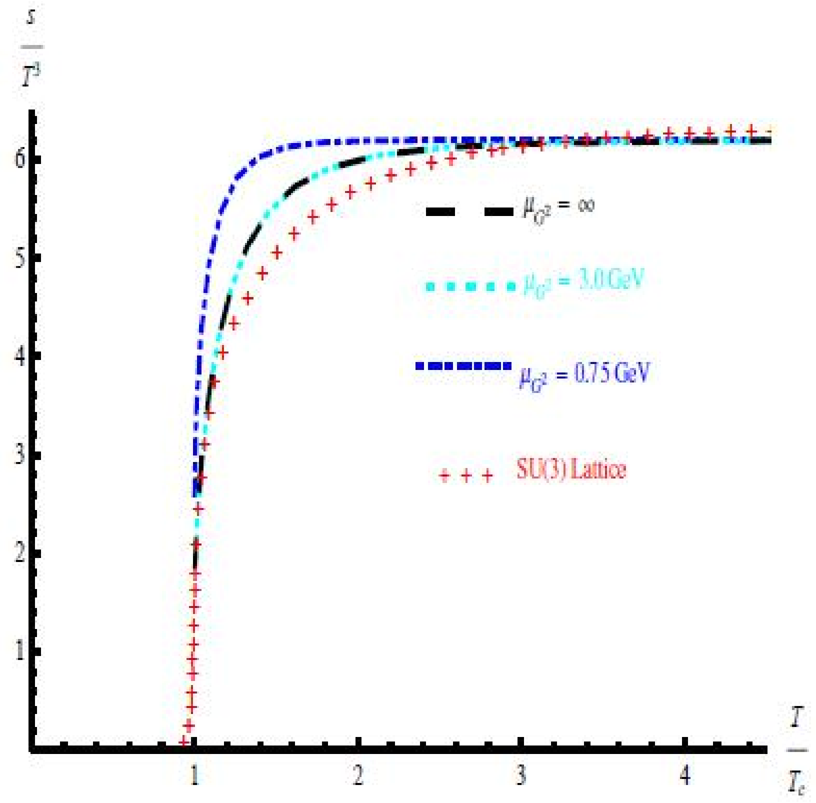
$$ds_S^2 = \frac{L^2 e^{2A_s}}{z^2}\left(-f(z)dt^2 + \frac{dz^2}{f(z)} + dx^i dx^i\right),$$

Metric structure, blackhole, Dilaton field and Dilaton potential should be solved self-consistently from the Einstein equations.

$$T = \frac{|f'(z_h)|}{4\pi}.$$

$$s = \frac{A_{area}}{4G_5V_3} = \frac{L^3}{4G_5}\left(\frac{e^{A_s-\frac{2}{3}\phi}}{z}\right)^3.$$



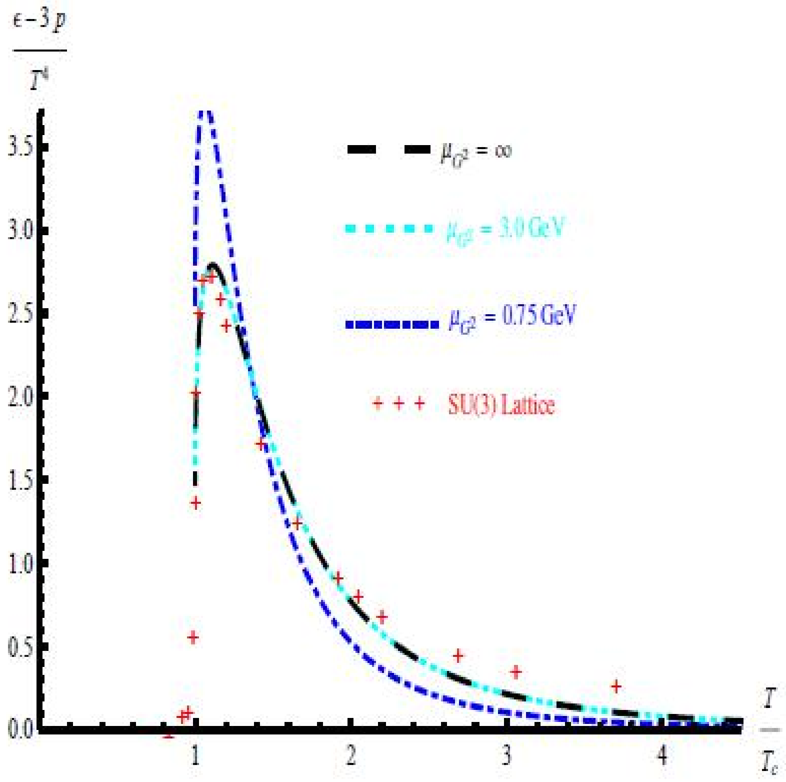


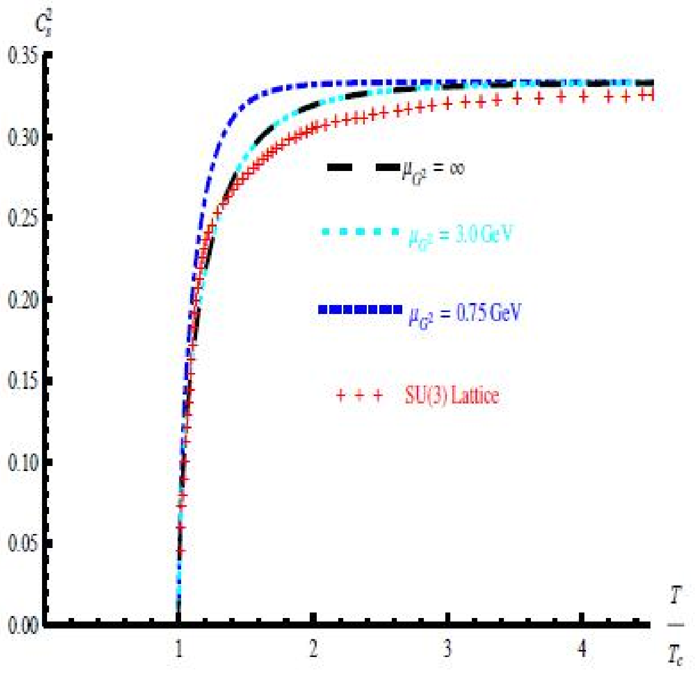
Hawking-Page phase transition

Danning Li, Jinfeng Liao, M.H. arXiv:1401.2035, PRD2014

Trace anomaly

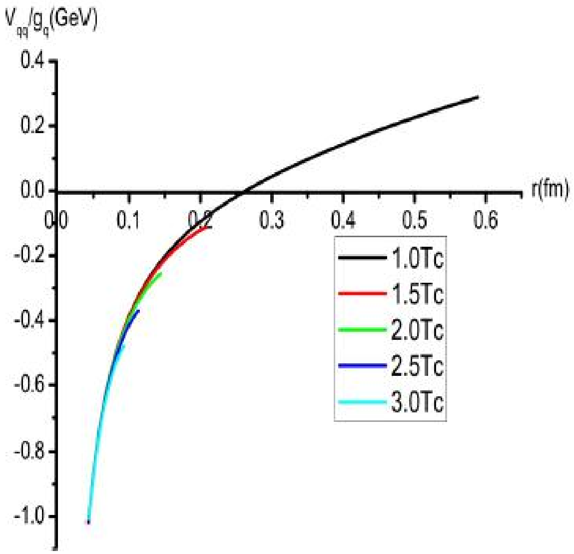
$$c_s^2 = \frac{d \log T}{d \log s} = \frac{s}{T ds/dT},$$



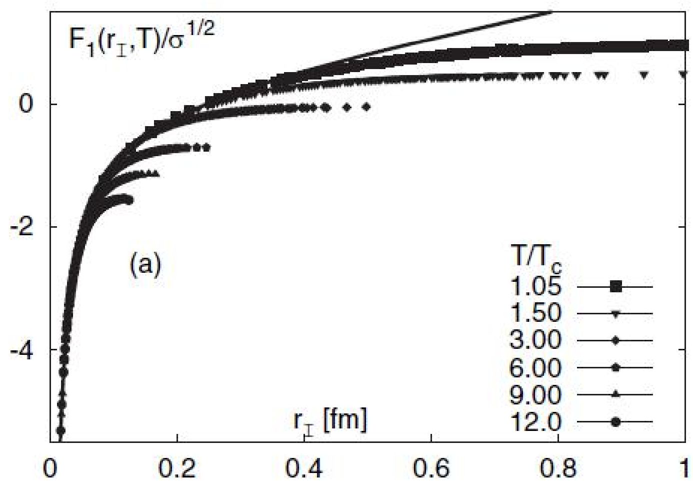


Danning Li, Jinfeng Liao, M.H. arXiv:1401.2035, PRD2014

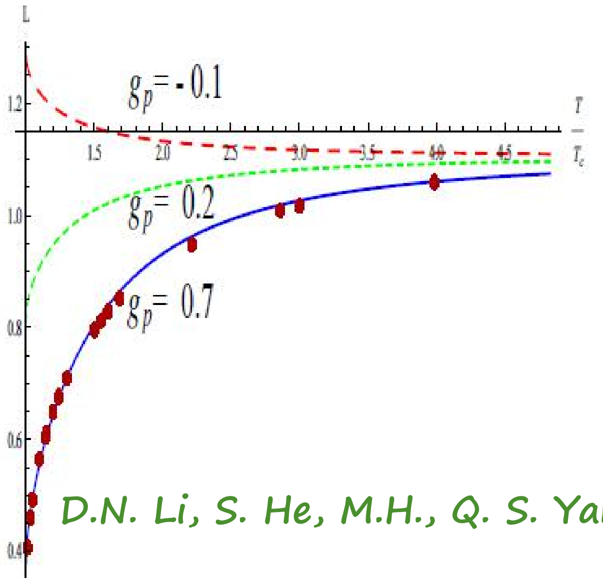
Electric screening



Heavy quark potential

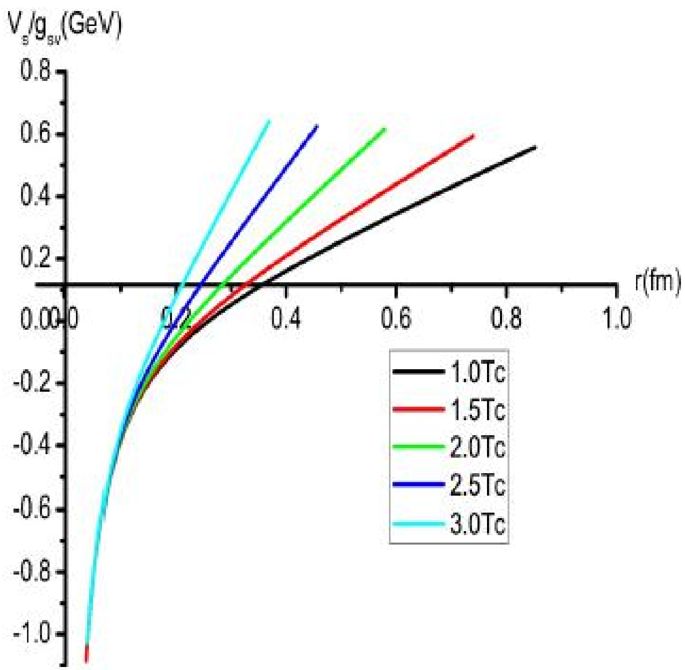


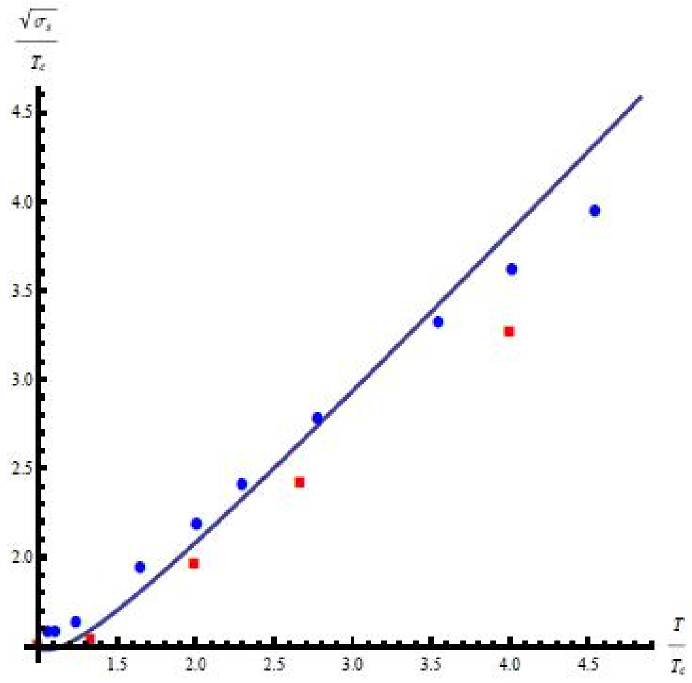
Polyakov loop: color electric deconfinement



D.N. Li, S. He, M.H., Q. S. Yan, arXiv:1103.5389, JHEP2

Magnetic screening and magnetic confinement





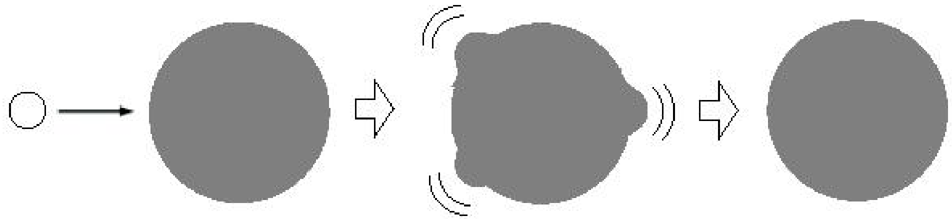
spatial Wilson loop **spatial string tension**

D.N. Li, S. He, M.H., Q. S. Yan, arXiv:1103.5389, JHEP2011

Temperature dependent transport properties reflect phase transitions?

shear/bulk viscosity,
Jet quenching parameter
Electric conductivity

Shear viscosity from AdS/CFT



shear viscosity ⇔ absorption cross section of graviton

$$\eta = \pi N^2 T^3 / 8$$

entropy ⇔ horizon area

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

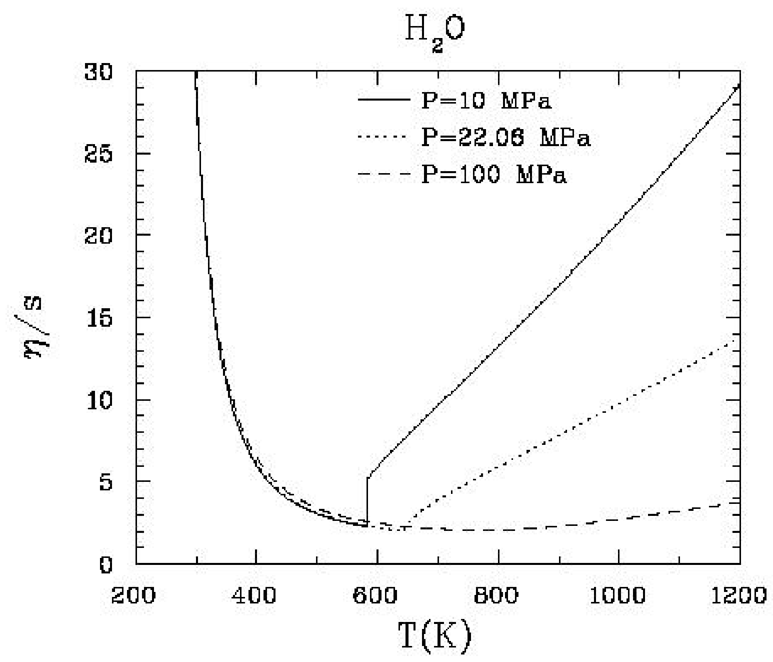
$$s = \pi^2 N^2 T^3 / 2$$

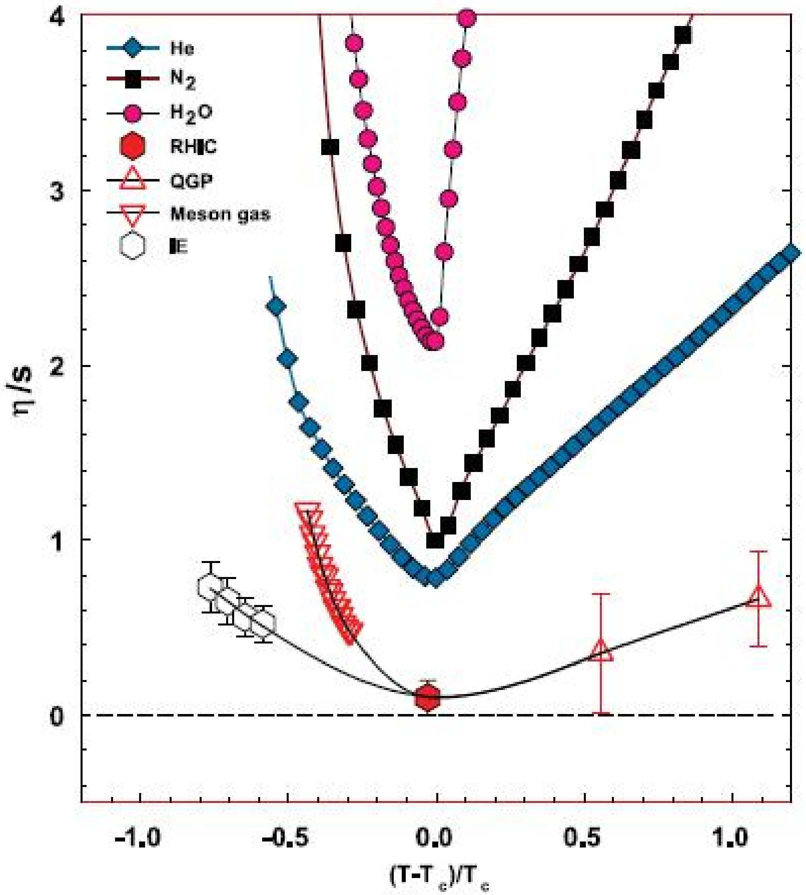
Minimum bound

Kovtun - Son - Starinets (2004)

Shear viscosity over entropy density:

minimum near phase transition

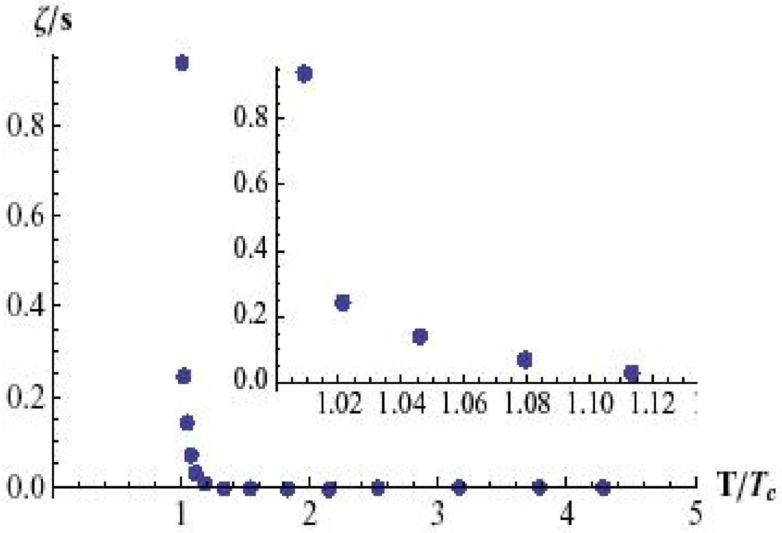




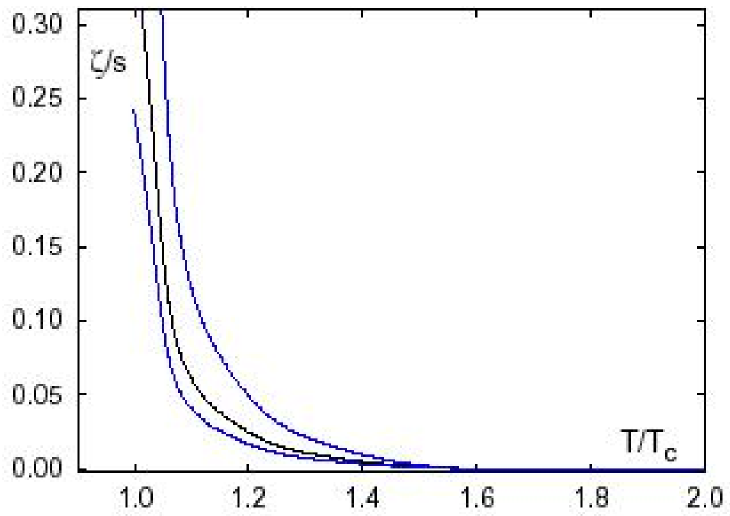
Csernai et.al. Phys.Rev.Lett.97:152303,2006

Lacey et al., PRL 98:092301,2007

Bulk viscosity over entropy density: LQCD
sharply rising near phase transition



Pure gluodynamics



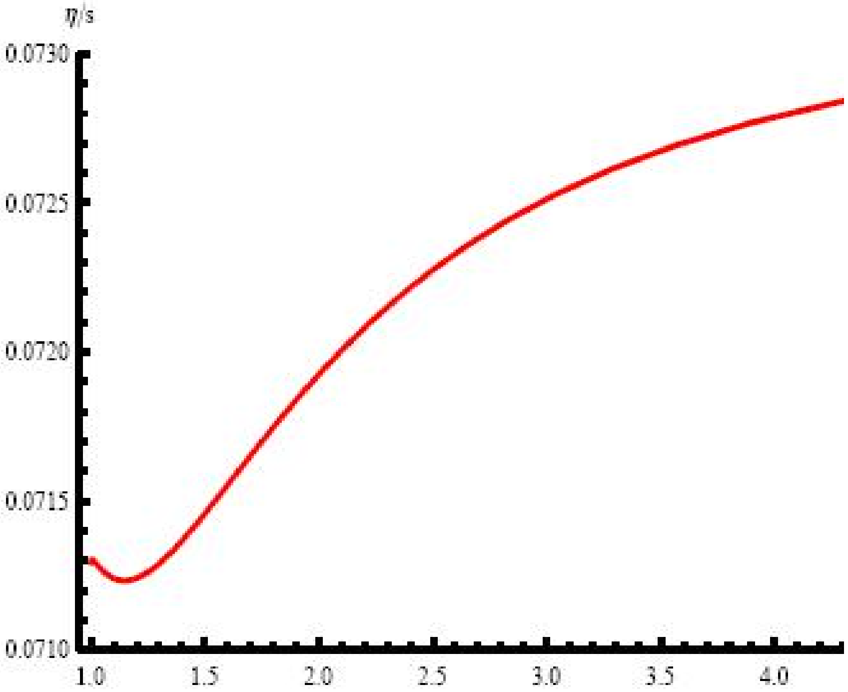
2-flavor case

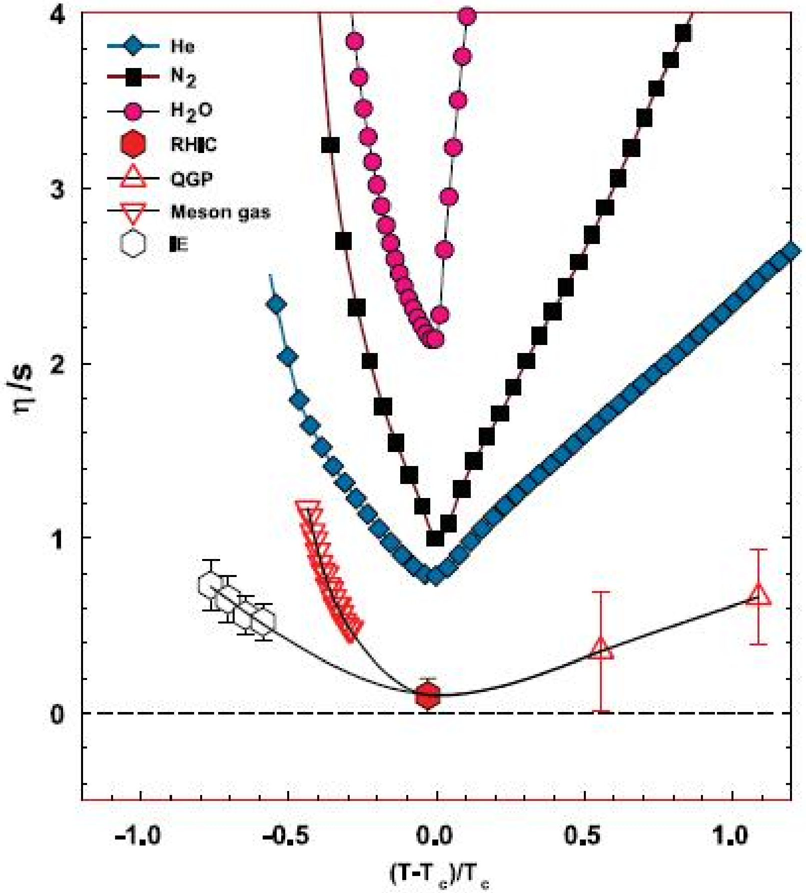
$$\zeta = \frac{1}{9\,\omega_0}\left\{T^5\frac{\partial}{\partial T}\frac{(\epsilon_T - 3p_T)}{T^4} + 16|\epsilon_v|\right\}$$

Dmitri Kharzeev, Kirill Tuchin arXiv:0705.4280 [hep-ph],
F.Karsch, Dmitri Kharzeev, Kirill Tuchin arXiv:0711.0914 [hep-ph],
Harvey Meyer arXiv:0710.3717 [hep-ph],

Shear viscosity from dynamical hQCD

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left[R - \frac{4}{3}(\nabla\phi)^2 + V(\phi) + \ell^2\beta e^{\sqrt{\frac{2}{3}}\gamma\phi} R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho}\right]$$

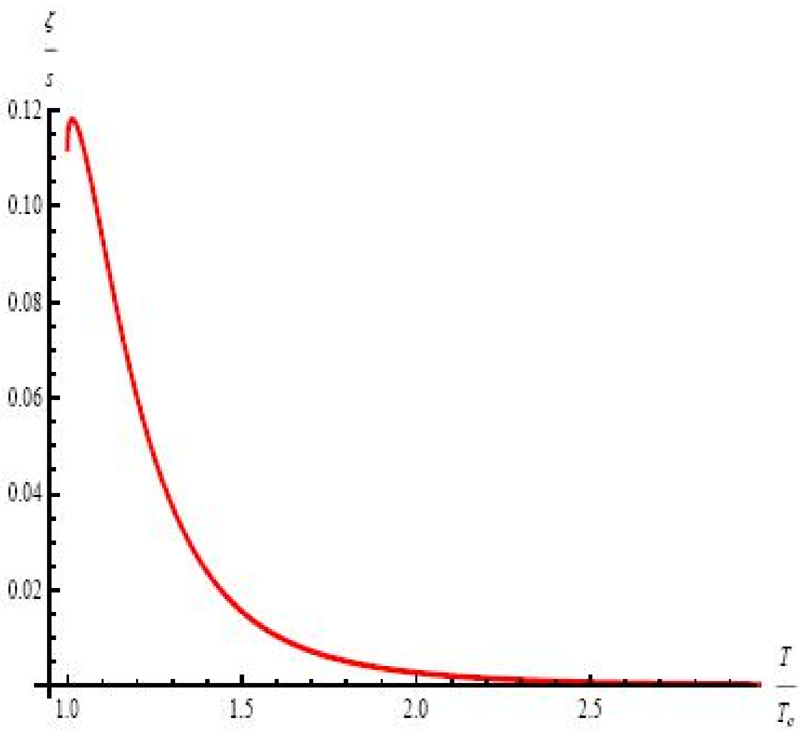


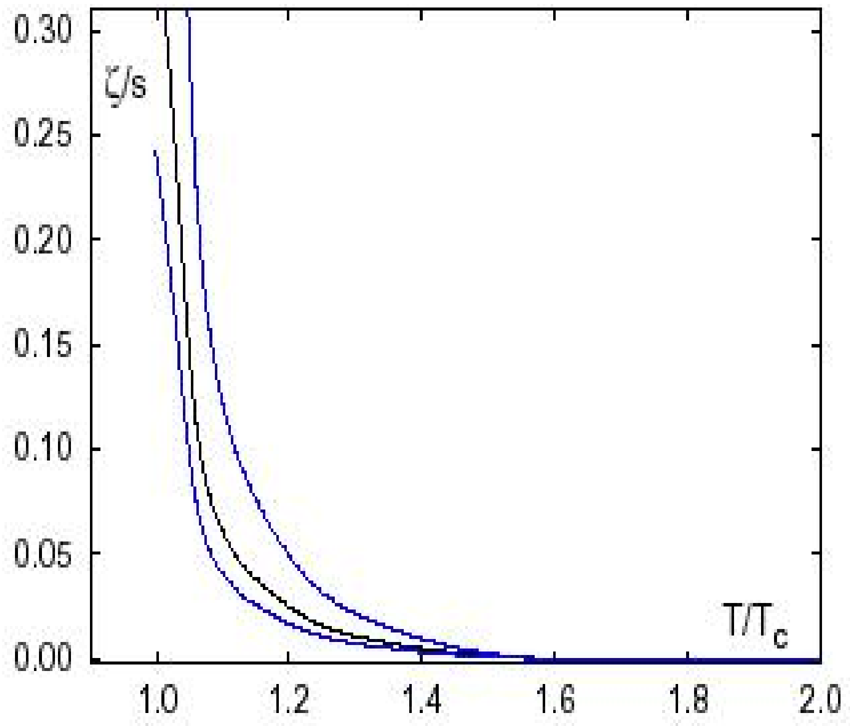


Danning Li, Song He, M.H. JHEP2015

Lacey et al., PRL 98:092301,2007

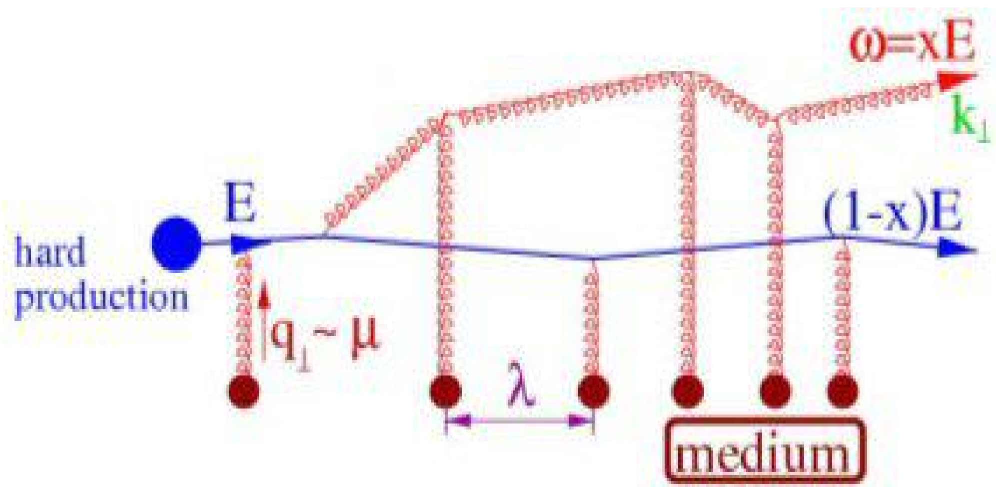
Bulk viscosity from dynamical hQCD

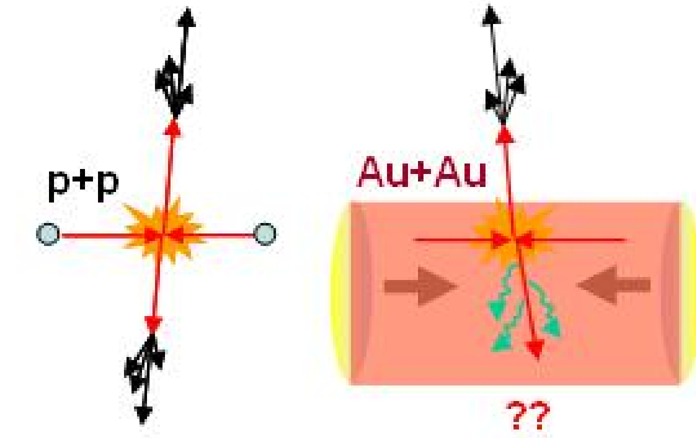




Danning Li, Song He, M.H. JHEP2015 Dmitri Kharzeev, Kirill Tuchin arXiv:0705.4280,

Jet quenching parameter





$$\Delta E \approx -\frac{\alpha_S}{2\pi} N_C \, \hat{q} \, L^2$$

Baier, Dokshitzer, Mueller, Peigne, Schiff (1996):

$\hat{q}$: reflects the ability of the medium to "quench" jets.

$$\hat{q} = \frac{\langle k_T^2 \rangle}{L} \approx \frac{\mu^2}{\lambda}$$

μ : Debye mass λ : mean free path

$\hat{q}$ of $\mathcal{N}$=4 SYM theory

BDMPS transport coefficient reads: $\lambda = g_{YM}^2 N_c$

$$\hat{q}_{SYM} = \frac{\pi^{3/2}\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{5}{4}\right)}\sqrt{\lambda}T^3 \approx 26.69\sqrt{\alpha_{SYM} N_c}T^3$$

- Take: $N_C = 3, \alpha_s = \frac{1}{2}, T = 300 \ MeV$

$$\hat{q}_{SYM} = 4.5 \,\mathrm{GeV}^2/\mathrm{fm}.$$

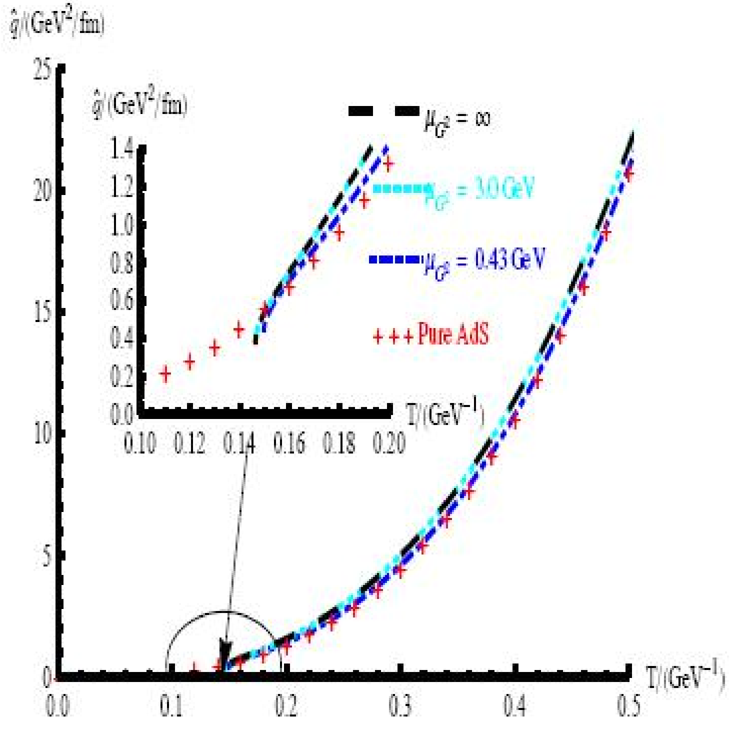
- Experimental estimates: 1-15 GeV^2/fm

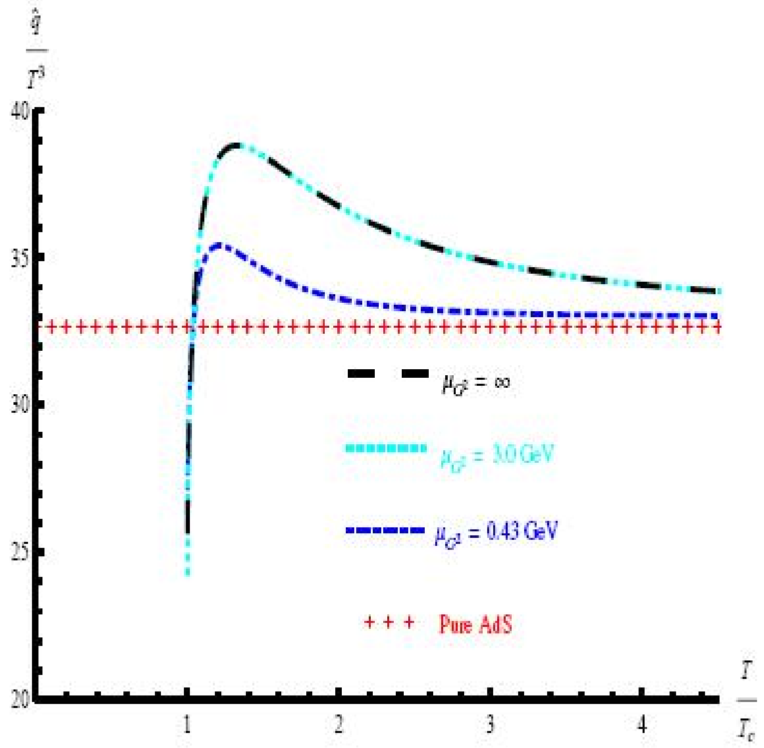
$$\frac{\eta}{s} \geq \frac{3T^3}{2\hat{q}}$$

Majumder, Muller & XNW 2007

Jet quenching from dynamical hQCD

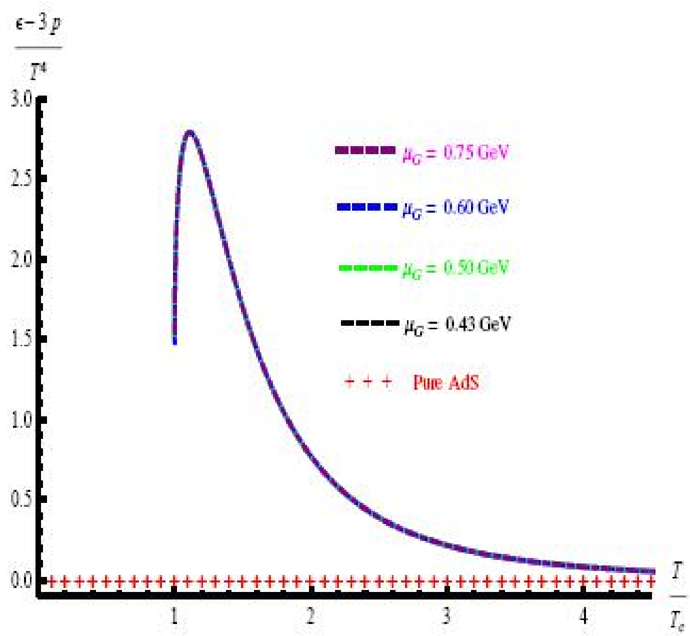
Danning Li, Jinfeng Liao, M.H. PRD2014

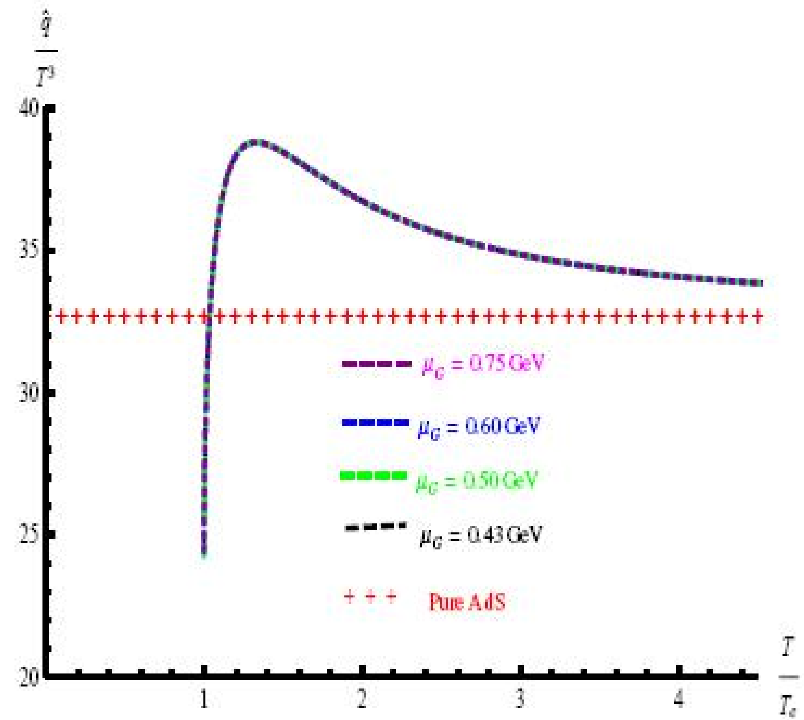




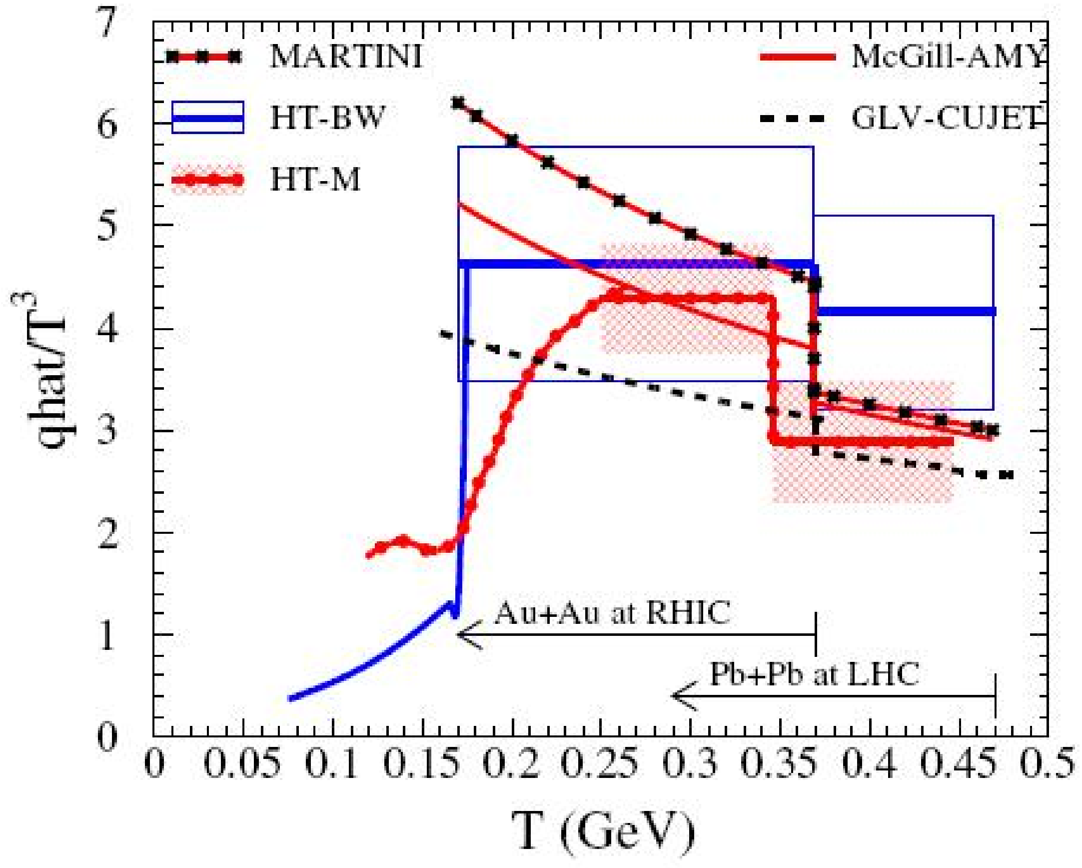
Jet quenching characterizing phase transition!

Danning Li, Jinfeng Liao, M.H. PRD2014

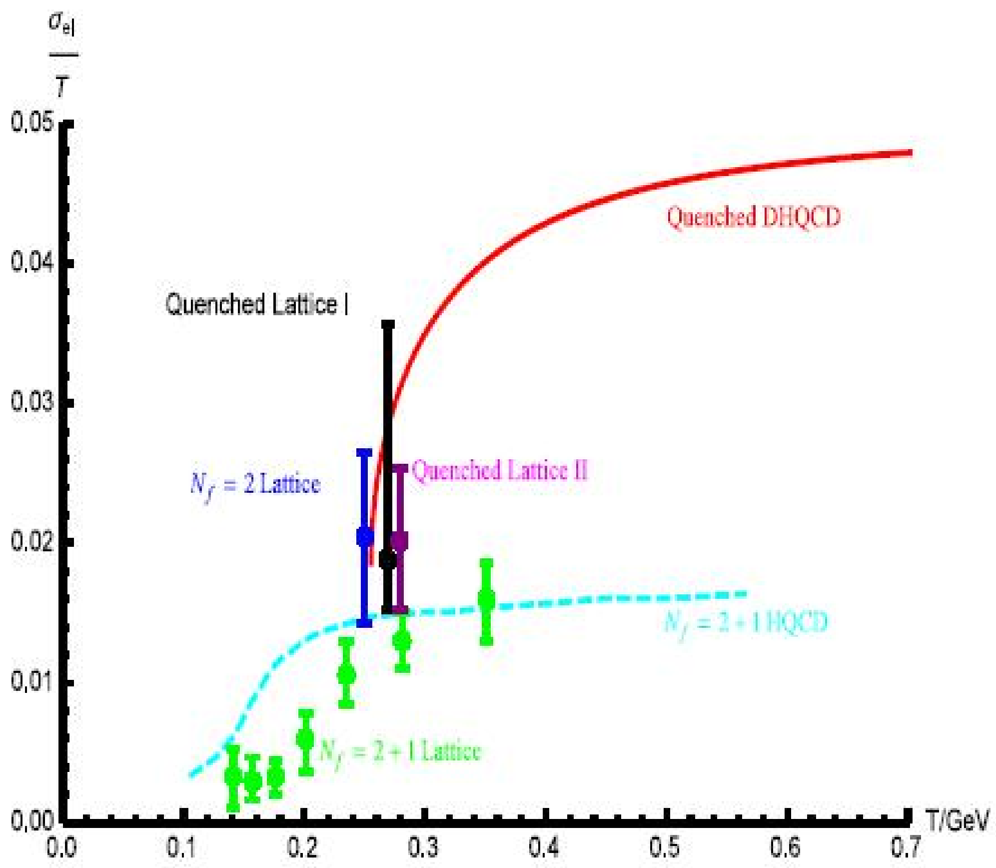




Temperature dependence of jet quenching parameter
[Jet Collaboration] arXiv:1312.5003



Electric conductivity



Dynamical holographic QCD   Graviton-dilaton-scalar system

D.N. Li, M.H., JHEP2013, arXiv:1303.6929

Gluonic background: Graviton-dilaton coupling

$$S_G = \frac{1}{16\pi G_5}\int d^5x\sqrt{g_s}e^{-2\Phi}\left(R + 4\partial_M\Phi\partial^M\Phi - V_G(\Phi)\right)$$

Flavor background:

Action for light hadrons

5D linear sigma model (KKSS model)

$$S_M = -\int d^5x\sqrt{g_s}e^{-\Phi}Tr(|DX|^2 + V_X(X^+X,\Phi) + \frac{1}{4g_5^2}(F_L^2 + F_R^2))$$

Full Dynamics:

$$S = S_G + S_M$$

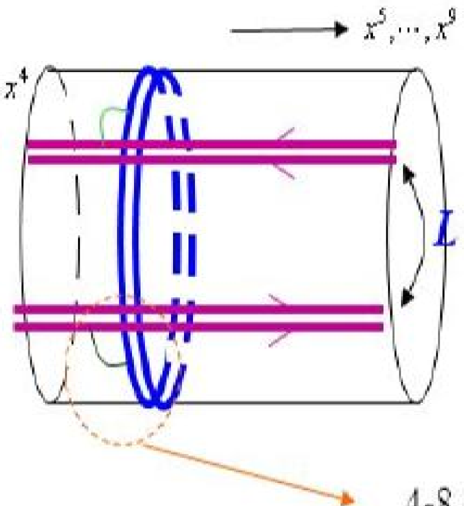
Interplay between gluodynamics and quark dynamics!!!

Dynamical holographic QCD   Graviton-dilaton-scalar system

	Gluodynamics	Quark dynamics
DhQCD	Dilaton background	Flavor background
SS:D4-D8 D3-D7	Dp brane: D4, D3	Dq brane: D8, D7
PNJL	Polyakov-loop potential	NJL model

Interplay between gluodynamics and quark dynamics!!!

Comparing with the Witten-Sakai-Sugimoto model



	0	1	2	3	4	5	6	7	8	9
N_c D4	o	o	o	o	o					
N_f $D8 - \overline{D8}$	o	o	o	o		o	o	o	o	o

4-8 open strings give chiral (from D8) and anti-chiral (from anti-D8) fermions in the fundamental representation.

Comparing with the Polyakov-loop NJL model

Quark dynamics:

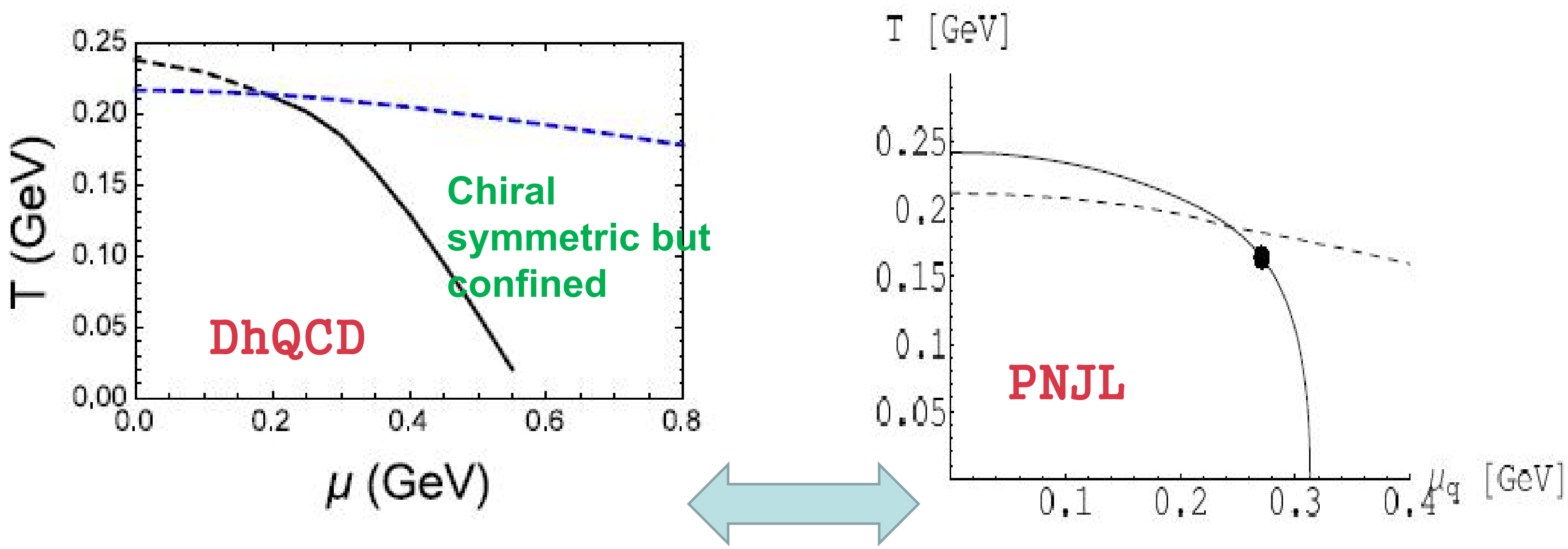
$$\mathcal{L}_{\rm NJL} = \bar{\psi}(i\gamma_\mu\partial^\mu - m)\psi + G_S[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] - G_V[(\bar{\psi}\gamma_\mu\psi)^2 + (\bar{\psi}\gamma_\mu\gamma_5\psi)^2]$$

Gluon "dynamics": Polyakov-loop effective potential

$$\frac{\mathcal{U}(\Phi,\bar{\Phi},T)}{T^4} = -\frac{a(T)}{2}\bar{\Phi}\Phi + b(T)\ln[1 - 6\bar{\Phi}\Phi + 4(\bar{\Phi}^3 + \Phi^3) - 3(\bar{\Phi}\Phi)^2]$$

$$\begin{aligned}\Omega_{PNJL} = {} & \mathcal{U}(\Phi,\bar{\Phi},T) - 2N_c\sum_{i=u,d}\int_0^\Lambda \frac{d^3p}{(2\pi)^3}[E_i] + G_S(\sigma_u+\sigma_d)^2 - G_V(\rho_u+\rho_d)^2 \\ & -2T\sum_{i=u,d}\int\frac{d^3p}{(2\pi)^3}[\ln(1+3\Phi e^{-\beta(E_i-\tilde{\mu}_i)} + 3\bar{\Phi}e^{-2\beta(E_i-\tilde{\mu}_i)} + e^{-3\beta(E_i-\tilde{\mu}_i)})] \\ & -2T\sum_{i=u,d}\int\frac{d^3p}{(2\pi)^3}[\ln(1+3\bar{\Phi} e^{-\beta(E_i+\tilde{\mu}_i)} + 3\Phi e^{-2\beta(E_i+\tilde{\mu}_i)} + e^{-3\beta(E_i+\tilde{\mu}_i)})]\end{aligned}$$

Quarkyonic phase in quenched DhQCD



Xun Chen, Danning Li, Defu Hou, M.H, arXiv:1908.02000

Sasaki, Friman, Redlich, hep-ph/0611147

4D effective theory mainly investigate chiral phase transition, HQCD can handle gluodynamics

Einstein-Maxwell-Dilaton system

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}[R - \frac{h(\phi)}{4}F^2 - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)].$$

$$ds^2 = \frac{e^{2A_e(z)}}{z^2}[-F(z)dt^2 + \frac{1}{F(z)}dz^2 + d\vec{x}^2]$$

$$A_e(z) = -\frac{3}{4}\ln(az^2+1) + \frac{1}{2}\ln(bz^3+1) - \frac{3}{4}\ln(dz^4+1)$$

$$h(z) = e^{-cz^2 - A_e(z)}.$$

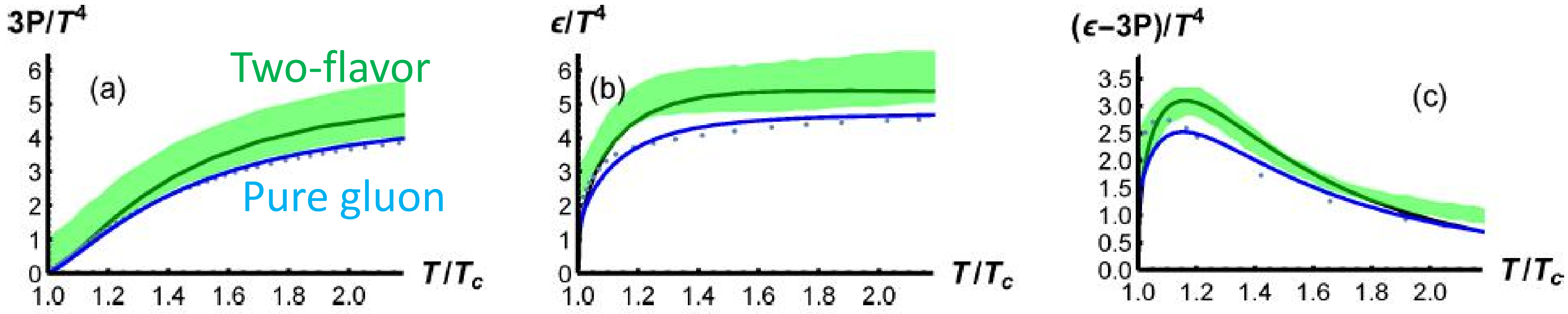
D. Dudal and S. Mahapatra, "Thermal entropy of a quark-antiquark pair above and below deconfinement from a dynamical holographic QCD model," Phys. Rev. D **96** (2017) no.12, 126010 [arXiv:1708.06995 [hep-th]].

$$t \to \frac{1}{\sqrt{1-\omega^2}}(t + \omega L\phi), \phi \to \frac{1}{\sqrt{1-\omega^2}}(\phi + \frac{\omega}{L}t),$$

ω is a dimensionless angular velocity parameter ranging from 0 to 1

Xun Chen, Lin Zhang, Danning Li, Defu Hou, M.H. arXiv: 2010.14478

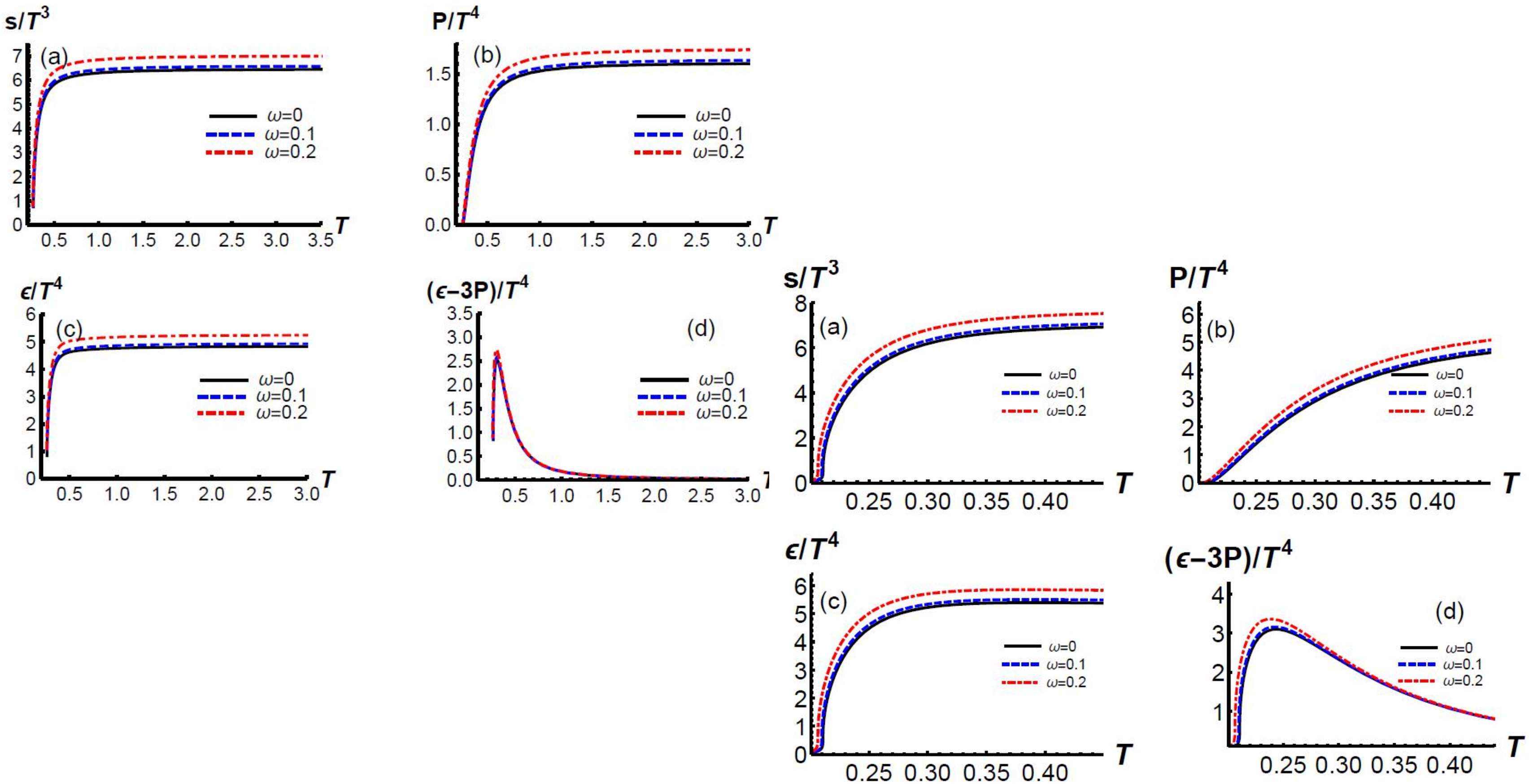
Fit parameters from lattice QCD results for pure gluon system and 2-flavor system



	c	a	b	d	G_5	T_c
$N_f = 2$	-0.227	0.01	0.045	0.035	1.1	211MeV
$N_f = 0$	1.16	0.075	0.12	0.075	1.2	265MeV

Xun Chen, Lin Zhang, Danning Li,Defu Hou, M.H. arXiv: 2010.14478

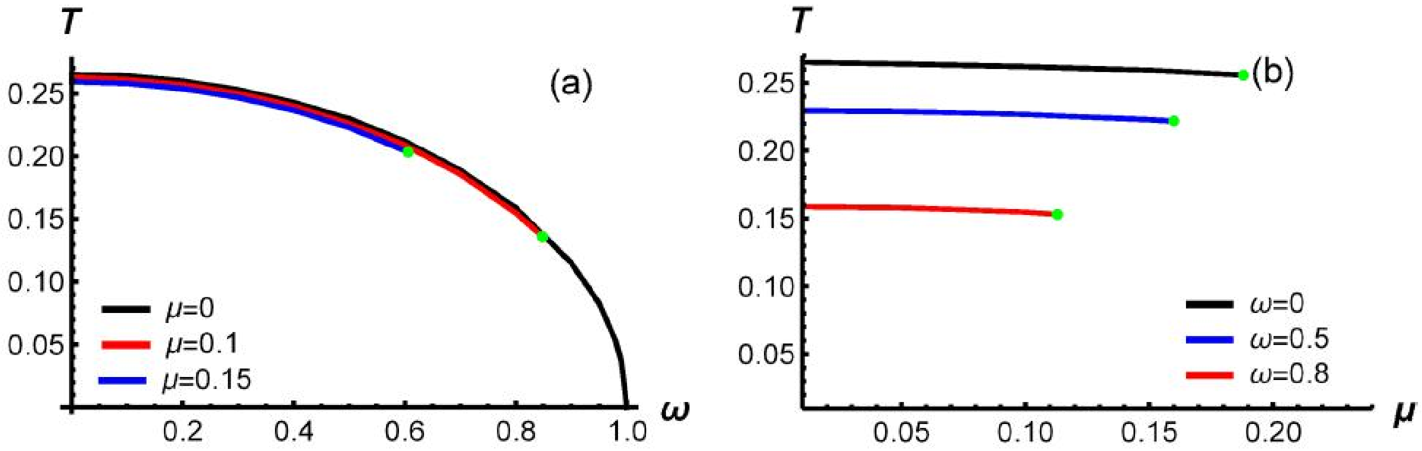
Enhancement of thermodynamical properties under rotation



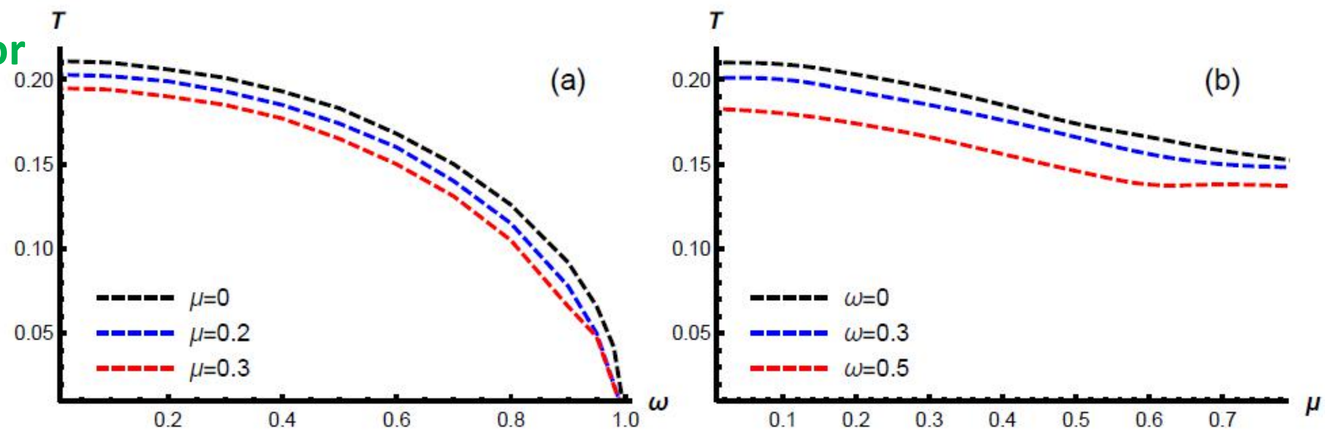
Xun Chen, Lin Zhang, Danning Li,
Defu Hou, M.H. arXiv: 2010.14478

Deconfinement phase transition under rotation

Pure gluon

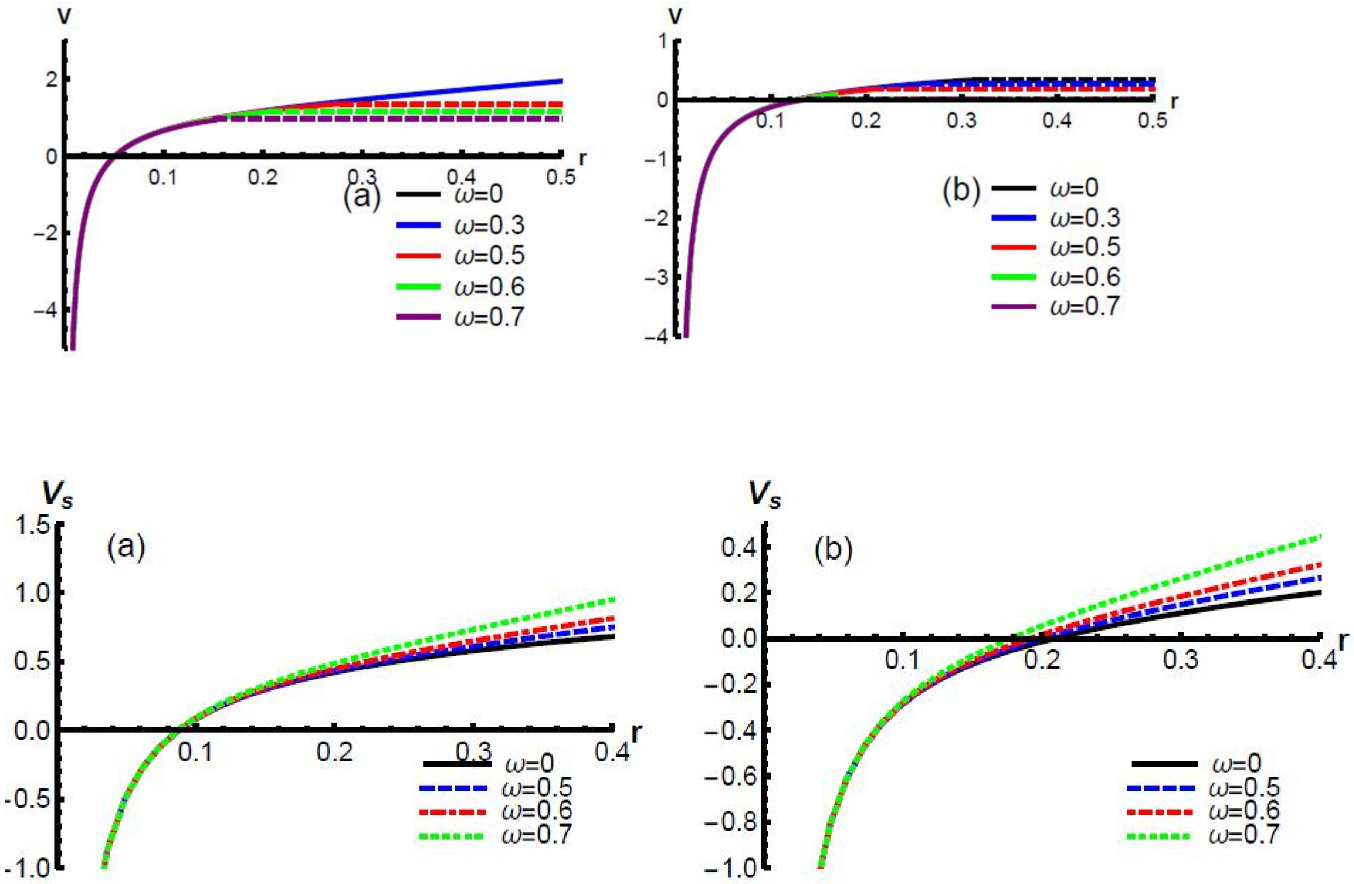


Two-flavor



Xun Chen, Lin Zhang, Danning Li, Defu Hou, M.H. arXiv: 2010.14478

Heavy quark potential, spatial Wilson loop and Polyakov-loop under rotation



Xun Chen, Lin Zhang, Danning Li, Defu Hou, M.H. arXiv: 2010.14478

Heavy quark potential, spatial Wilson loop and Polyakov-loop under rotation

Pure gluon

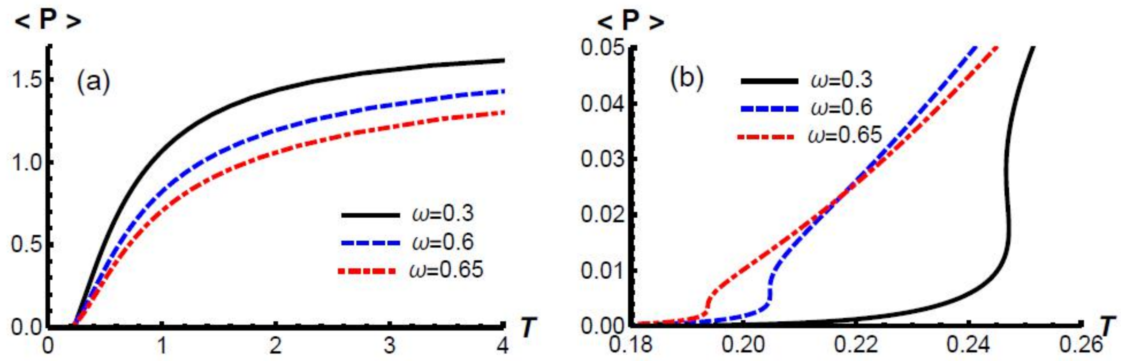


Figure 9. (a)In pure gluon system, the expectation value of a single Polyakov loop as a function of T at $\mu = 0.15GeV$ for different angular velocities of $\omega = 0$(solid black line), $\omega = 0.6$(dashed blue line) and $\omega = 0.65$(dot-dashed red line). (b) An enlarged view of (a).The unit for T, μ is in GeV.

Two-flavor

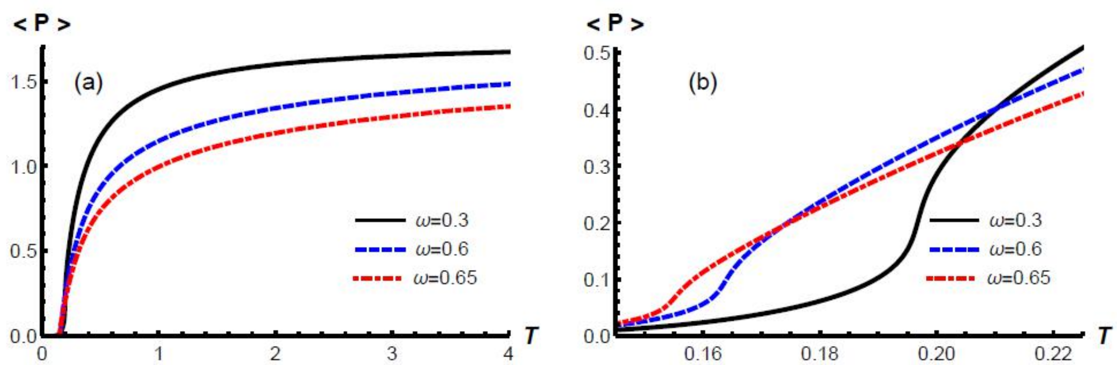
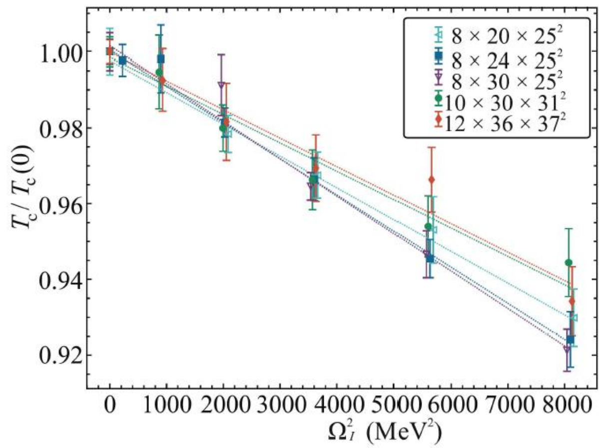


Figure 10. (a) In two-flavor system, the expectation value of a single Polyakov loop as a function of T at $\mu = 0.15GeV$ for different angular velocities of $\omega = 0$(solid black line), $\omega = 0.6$(dashed blue line) and $\omega = 0.65$(dot-dashed red line). (b) An enlarged view of (a).The unit for T, μ is in GeV.

Xun Chen, Lin Zhang, Danning Li, Defu Hou, M.H. arXiv: 2010.14478

Deconfinement phase transition under rotation from lattice

Results from V. V. Braguta, A. Yu. Kotov, D. D. Kuznedelev, A. A. Roenko, "Study of the Confinement/Deconfinement Phase Transition in Rotating Lattice SU(3) Gluodynamics", Pisma Zh.Eksp.Teor.Fiz. 112 (2020) 1, 9-16.



$$\frac{T_c(\Omega)}{T_c(0)} = 1 - C_2\Omega_I^2,$$

$$\frac{T_c(\Omega)}{T_c(0)} = 1 + C_2\Omega^2.$$

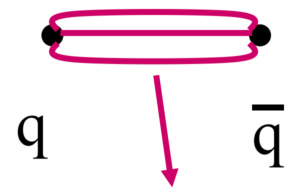
IV. Summary

Properties of QCD matter under strong magnetic field and rotation are not fully understood yet!

Thanks for your attention!

QCD and string theory: 1968-1974

2, String model & confinement



$V = T\ R$

Flux tubes of color field = glue

Dual superconductor picture

Type-II superconductor

Abrikosov vortex in U(1) theory

A.A.Abrikosov, Soviet Phys.JTEP 5, 1174(1957)

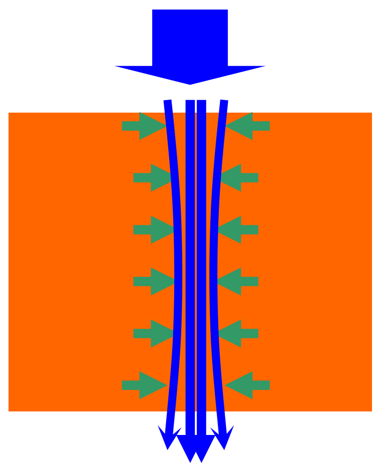
Color flux tube in QCD

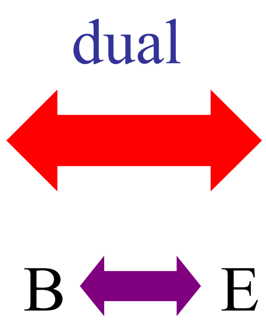
Y.Nambu, PRD.122,4262(1974)

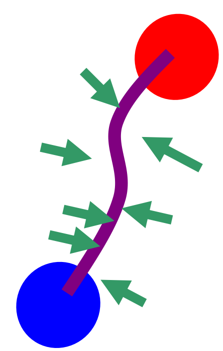
'‘t Hooft , Nucl.Phys.B190.455(1981)

Mandelstam, Phys.Rep.C23.245(1976)

electric
Cooper-pair
condensation







magnetic monopole
condensation

squeeze
magnetic field

squeeze
color electric flux

QCD and string theory: 1968-1974

3, Effective theory in terms of strings

t' Hooft '74

t' Hooft large Nc limit

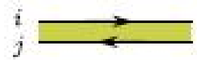
take Nc colors instead of 3, SU(Nc)

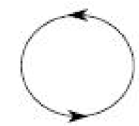
$$S = \frac{1}{4\, g_{\rm YM}^2} \int \mathrm{d}^4x \ \mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$

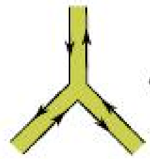
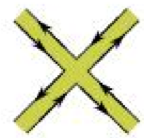
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$$

$$(A_\mu)_{ij} = A_\mu^a \ (T^a)_{ij}$$

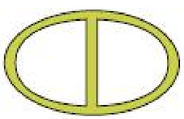
QCD and string theory: 1968-1974

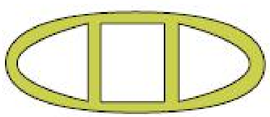
Gluon propagator  $\sim g^2_{\rm YM}$

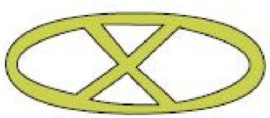
 $\sim N_c$

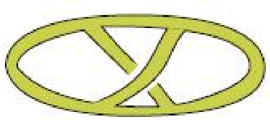
Interactions  $\sim \frac{1}{g^2_{\rm YM}}$  $\sim \frac{1}{g^2_{\rm YM}}$

't Hooft coupling $\lambda = g^2_{\rm YM} N_c$

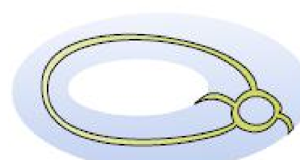
 $\sim (g^2_{\rm YM})^{3-2}\, N_c^3 = \lambda\, N_c^2$

 $\sim (g^2_{\rm YM})^{6-4}\, N_c^4 = \lambda^2\, N_c^2$

 $\sim (g^2_{\rm YM})^{8-5}\, N_c^5 = \lambda^3\, N_c^2$

 $\sim (g^2_{\rm YM})^{6-4}\, N_c^2 = \lambda^2$

Planar diagram most dominant



Non-planar diagram 1/Nc^2 suppressed

QCD and string theory: 1968-1974

QCD at low energies, when the coupling is large, dual of a weakly coupled string theory

Vacuum-to-vacuum amplitude in large Nc gauge theory

$$\log Z = \sum_{h=0}^{\infty} N_c^{2-2h} f_h(\lambda) = N_c^2 f_0(\lambda) + f_1(\lambda) + \frac{1}{N_c^2} f_2(\lambda) + \cdots ,$$

Vacuum-to-vacuum amplitude in string theory

$$\mathcal{A} = \sum_{h=0}^{\infty} g_s^{2h-2} F_h(\alpha') = \frac{1}{g_s^2} F_0(\alpha') + F_1(\alpha') + g_s^2 F_2(\alpha') + \cdots ,$$

where g_s is the string coupling, $2\pi\alpha'$ is the inverse string tension, and $F_h(\alpha')$ is the contribution of 2d surfaces with h holes.

The string coupling constant gs is of order 1/Nc,

Closed strings would be glueballs.
Open strings would be the mesons.

QCD and string theory: 1968-1974

Problems:

1) Strings do not make sense in 4 (flat) dimensions

Trying to quantize a string in four dimension leads to tacyons.

2) Strings always include a graviton, ie., a particle with m=0, s=2

For this reason strings are normally studied as a model for quantum gravity.

QCD and string theory: 1974-1997

QCD: pQCD is confirmed by DIS
non-perturbative QCD region, challenging in describing hadrons in terms of quark and gluon DOF.

String theory: trying to make itself a theory of everything.

QCD and string theory: 1997-Now

$\mathcal{N}=4$ SU(Nc) Yang-Mills theory	=	String theory on $AdS_5 \times S^5$

AdS/CFT Polyakov 1997 Maldacena 1997

$\mathcal{N}=4$ super-Yang-Mills

$$S = \frac{1}{g_{\rm YM}^2}\int \mathrm{d}^4x\ \mathrm{Tr}\left(\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(D_\mu\phi_{ij})^2 + \frac{1}{2}\bar{\chi}_i \not{D}\chi_i - \frac{1}{2}\bar{\chi}_i[\phi_{ij},\chi_j] - \frac{1}{4}[\phi_{ij},\phi_{kl}][\phi_{ij},\phi_{kl}]\right)$$

with 6 adjoint scalars $\phi^{(ij)}$, a gauge field A_μ and 4 chiral adjoint fermions χ_i.

$AdS_5 \times S^5$ metric

$$ds^2 = ds^2_{AdS_5} + R^2 d\Omega_5^2\ ,$$

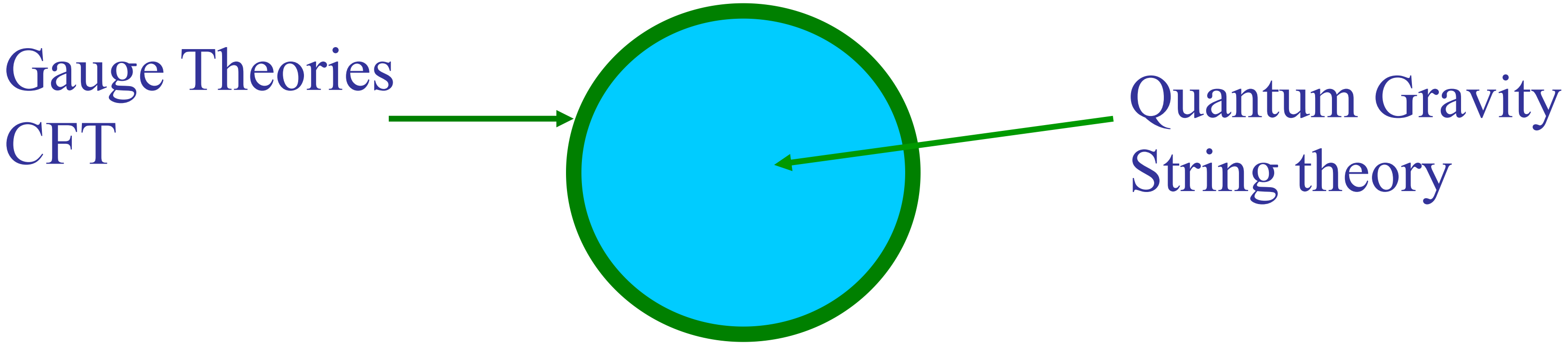
$$ds^2_{AdS_5} = \frac{R^2}{z^2}\left(-dt^2 + d\vec{x}^2 + dz^2\right)$$

QCD and string theory: 1997-Now

The precise duality relationship is

$$\left\langle e^{\int d^4x \phi_0(\vec{x})\mathcal{O}(\vec{x})} \right\rangle_{\mathrm{CFT}} = \mathcal{Z}_{\mathrm{string}} \left[\phi\left(\vec{x}, z\right) |_{z=0} \equiv \phi_0\left(\vec{x}\right) \right].$$

$$\frac{\delta^n Z_{\mathrm{string}}}{\delta\phi_0(t_1, \mathrm{x}_1) \cdots \delta\phi_0(t_n, \mathrm{x}_n)} = \left\langle T\, \mathcal{O}(t_1, \mathrm{x}_1) \cdots \mathcal{O}(t_n, \mathrm{x}_n) \right\rangle_{\text{field theory}}$$



QCD and string theory: 1997-Now

AdS/CFT conjecture

$AdS_5 \times S^5$ ⟷ $N = 4$ SYM theory

If it's true for any gauge theory

String theory, quantum gravity ⟷ Non-Abelian gauge theory

Then what's the dual string theory of QCD?

? ⟷ **QCD**

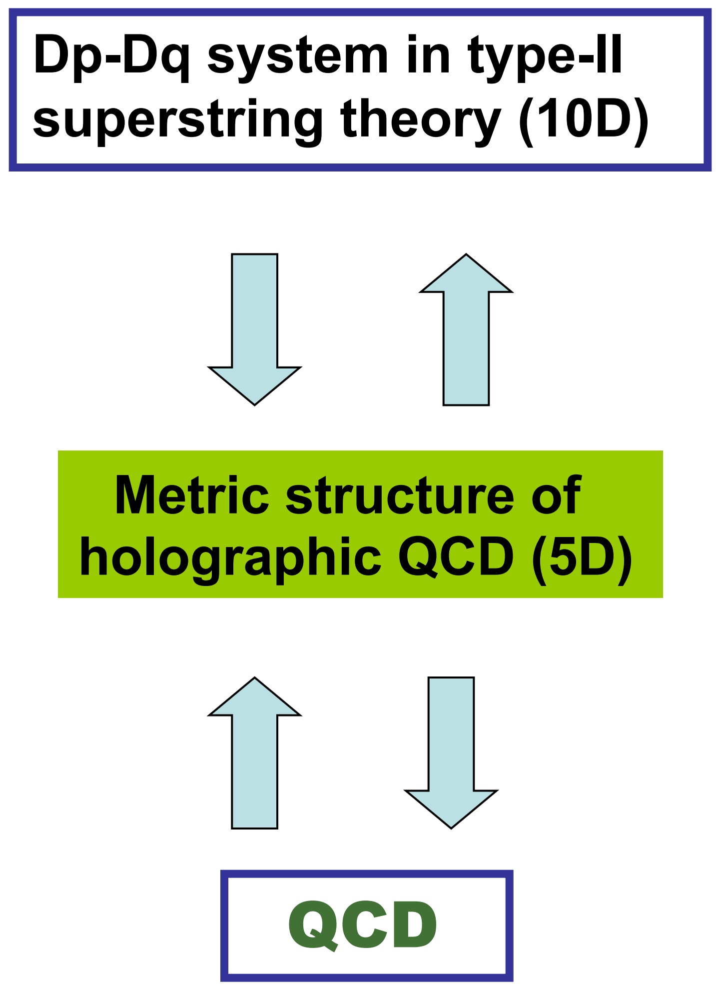
Nonconformal

Question: Is it possible to find a string theory dual to QCD?

QCD and string theory: 1997-Now

QCD is not a conformal theory, then what's the dual string theory of QCD?

Leave the task of deriving the holographic QCD model to string theorists



What we can do: extract a workable holographic QCD model from the real world

Dynamical hQCD model
----- 5D effective QCD model

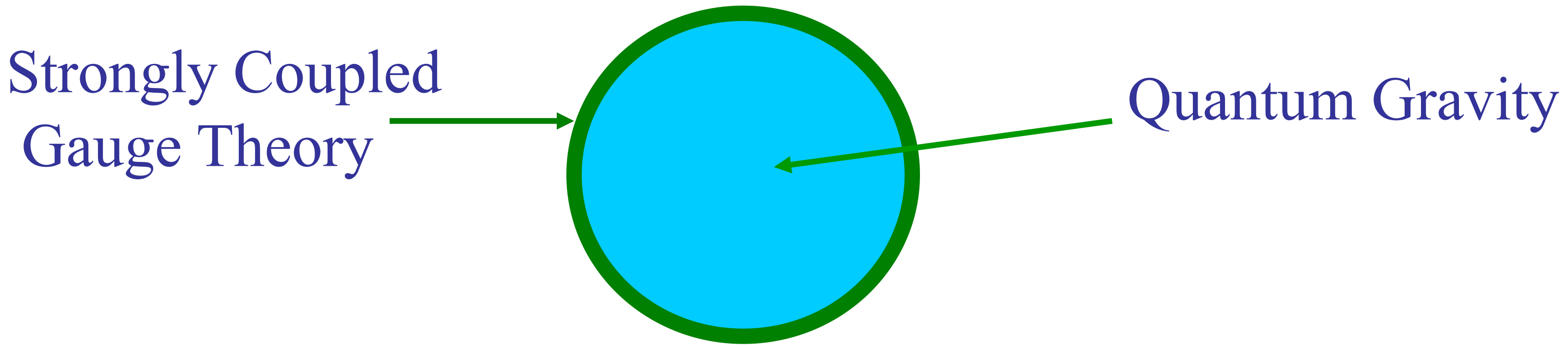
Holographic Duality: Gravity/QFT

AdS/CFT : Original discovery of duality

J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998)

Supersymmetry and conformality are required for AdS/CFT.

Holographic Duality: (d+1)-Gravity/ (d)-QFT



Holographic Duality: (d+1)-Gravity/ (d)-QFT

Holography & Emergent critical phenomena:

When system is strongly coupled, new weakly-coupled degrees of freedom dynamically emerge.

The emergent fields live in a dynamical spacetime with an extra spatial dimension.

The extra dimension plays the role of energy scale in QFT, with motion along the extra dimension representing a change of scale, or renormalization group (RG) flow.

arXiv:1205.5180

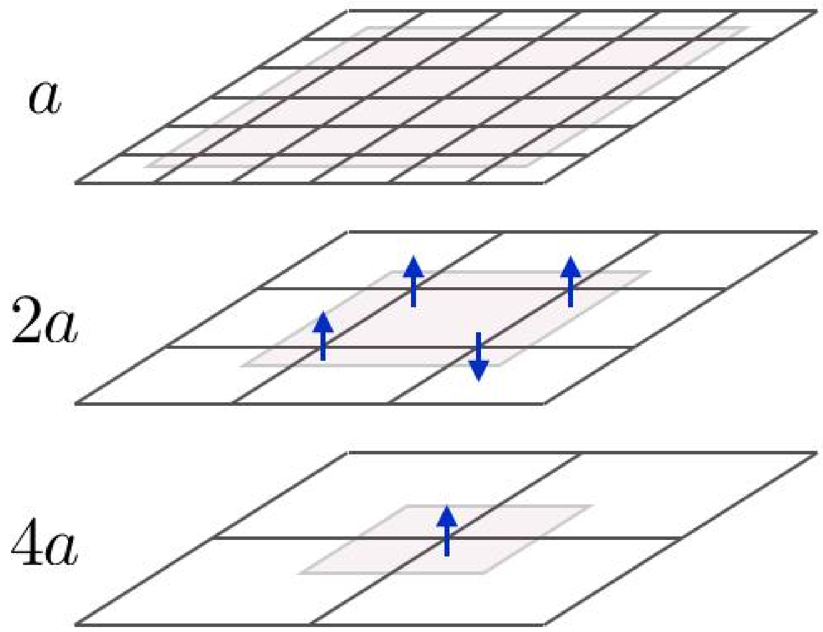
Allan Adams,[1] Lincoln D. Carr,[2,3] Thomas Schäfer,[4] Peter Steinberg[5] and John E. Thomas[4]

Holographic Duality & RG flow

Coarse graining spins on a lattice: Kadanoff and Wilson

$$H = \sum_{x,i} J_i(x) \mathcal{O}^i(x)$$

J(x): coupling constant or source for the operator



$$H = \sum_{i} J_i(x, a)\, \mathcal{O}^i(x)$$

$$H = \sum_{i} J_i(x, 2a)\, \mathcal{O}^i(x)$$

$$H = \sum_{i} J_i(x, 4a)\, \mathcal{O}^i(x)$$

$$u\frac{\partial}{\partial u} J_i(x, u) = \beta_i(J_j(x, u), u)$$

arXiv:1205.5180

Holographic Duality & RG flow

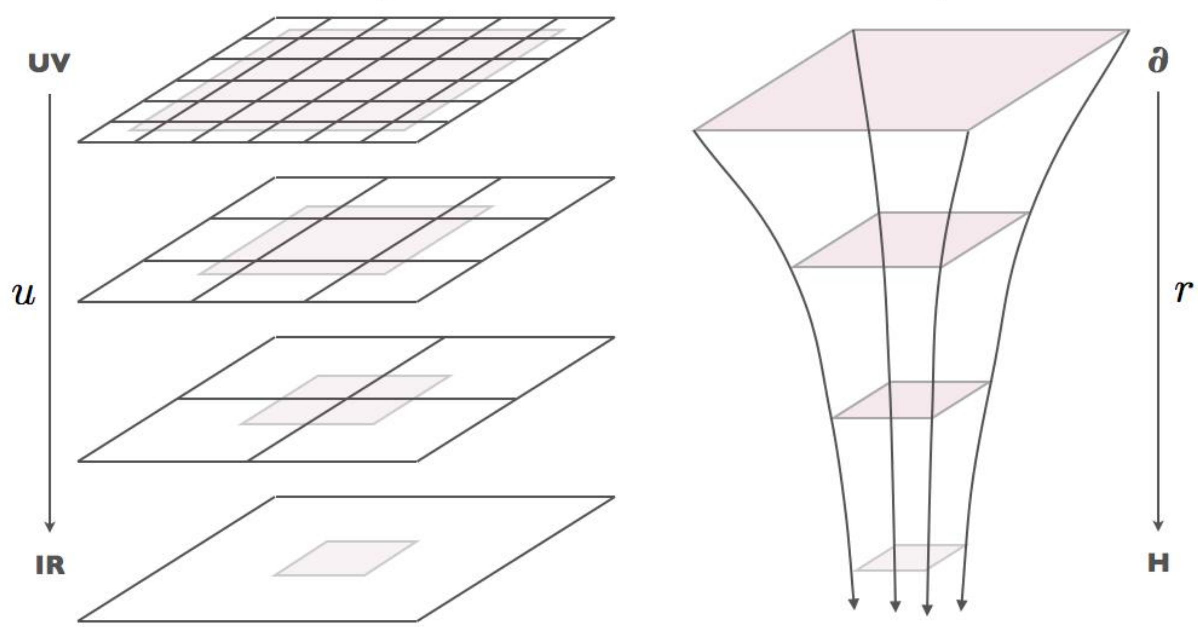
QFT on lattice equivalent to GR problem from Gravity

RG scale -> an extra spatial dimension

Coupling constant -> dynamical filed

arXiv:1205.5180

$$J_i|_{UV} = \Phi_i|_\partial$$



The extra dimension plays the role of energy scale in QFT, with motion along the extra dimension representing a change of scale, or renormalization group (RG) flow.

A systematic framework: Graviton-dilaton system

$$S_G = \frac{1}{16\pi G_5}\int d^5x\sqrt{g_s}e^{-2\Phi}\left(R_s + 4\partial_M\Phi\partial^M\Phi - V_G^s(\Phi)\right)$$

N=4 Super YM

conformal

AdS$_5$

$$ds^2 = \frac{L^2}{z^2}\left(dt^2 + d\vec{x}^2 + dz^2\right)$$

$$V_E(\phi) = -\frac{12}{L^2}$$

QCD

nonconformal

deformed AdS$_5$

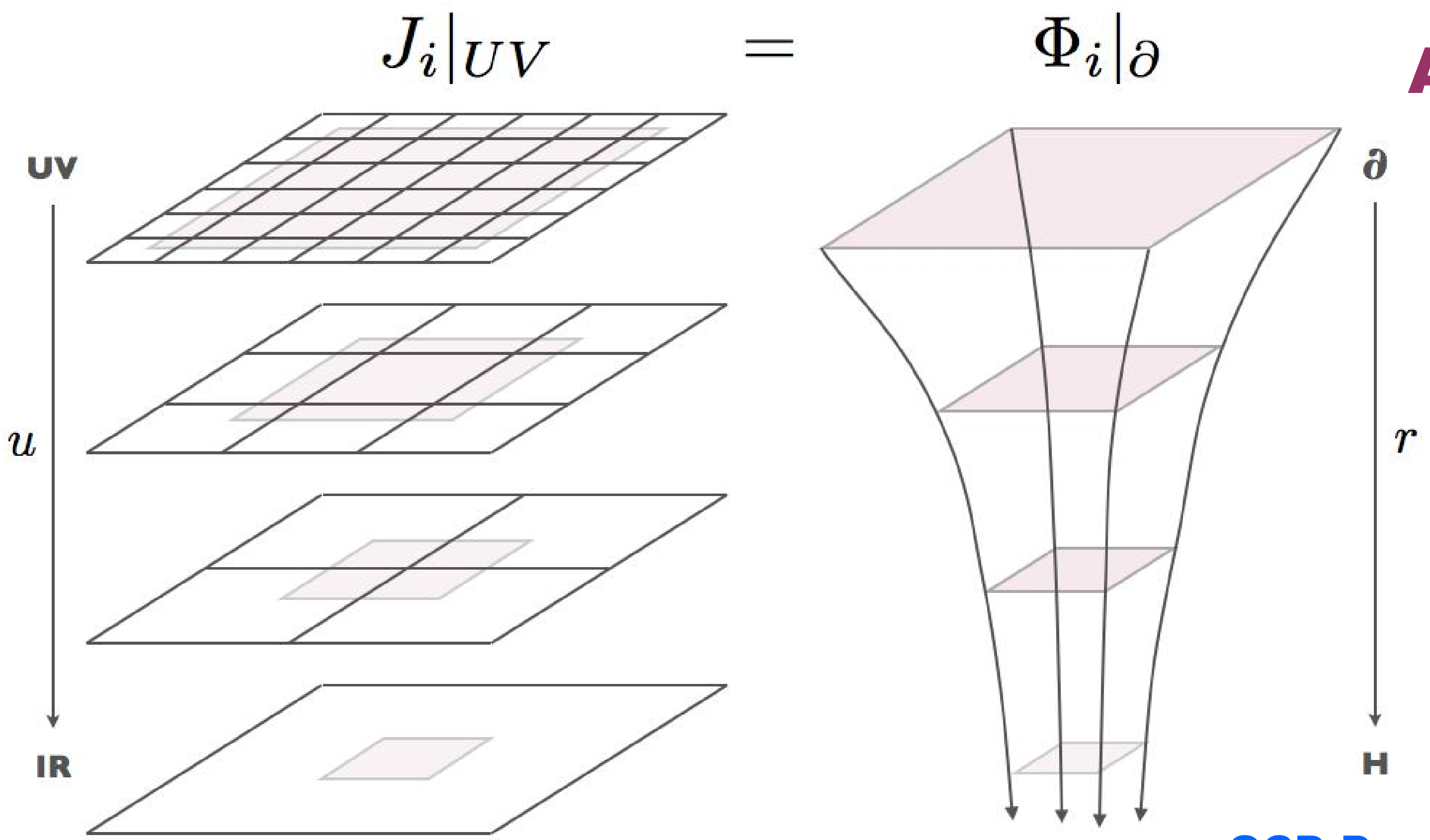
$$ds^2 = \frac{h(z)L^2}{z^2}\left(dt^2 + d\vec{x}^2 + dz^2\right)$$

Dilaton field breaks conformal symmetry

Input: QCD dynamics at IR

Solve: Metric structure, dilaton potential

Dynamical hQCD & RG



A AdS_5

From UV to IR

QCD Dynamics at IR

deformed AdS_5

The goal is to describe

Hadron spectra

chiral symmetry breaking

& linear confinement

Phase transitions

equation of state

Transport properties

in one systematic framework

Hadron spectra:

Glueball spectra
Light-flavor meson spectra

D.N. Li, M.H., JHEP2013, arXiv:1303.6929

Yidian Chen, M.H., arXiv: 1511.07018

Can AdS_5 metric describe hadron spectra?

L. Da Rold and A. Pomarol, Nucl. Phys. B **721**, 79 (2005)
J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. Lett. **95**, 261602 (2005)

$$ds^2 = \frac{1}{z^2}(-dz^2 + dx^\mu dx_\mu), \qquad 0 < z \le z_m.$$

5D hadron action

$$S = \int d^5x \sqrt{g}\, \mathrm{Tr}\left\{ |DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \right\}$$

TABLE I: Operators/fields of the model

4D: $\mathcal{O}(x)$	5D: $\phi(x,z)$	p	Δ	$(m_5)^2$
$\bar{q}_L \gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R \gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3

$(\Delta - p)(\Delta + p - 4)$

Lowest excitations: 80-90% agreement

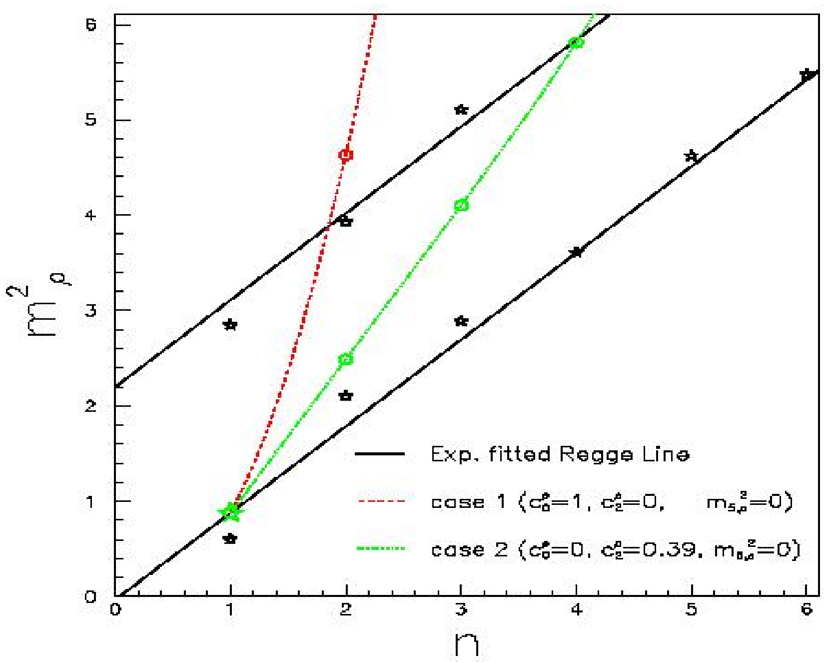
Observable	Measured (MeV)	Model A (MeV)	Model B (MeV)
m_π	139.6 ± 0.0004 [8]	139.6^*	141
m_ρ	775.8 ± 0.5 [8]	775.8^*	832
m_{a_1}	1230 ± 40 [8]	1363	1220
f_π	92.4 ± 0.35 [8]	92.4^*	84.0
$F_\rho^{1/2}$	345 ± 8 [15]	329	353
$F_{a_1}^{1/2}$	433 ± 13 [6]	486	440
$g_{\rho\pi\pi}$	6.03 ± 0.07 [8]	4.48	5.29

$z_m = 1/(323\ \text{MeV})$ $z_m = 1/(346\ \text{MeV})$

J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. Lett. **95**, 261602 (2005)

However, no Regge behavior in the hard-wall AdS_5 model !

m_n^2 grow as n^2.



How to improve AdS$_5$ metric?

A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. D **74**, 015005 (2006)

soft-wall AdS$_5$ model or KKSS model

$$g_{MN}\, dx^M dx^N = e^{2A(z)}(dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu)$$

$$I = \int d^5x \sqrt{g}\, e^{-\Phi}\, \mathcal{L},$$

$$A = -\log z,\ \Phi = z^2$$

Introduce a dilaton field to restore Regge behavior

$$M_{n,S}^2 = 4n + 4S$$

Pure gluon system: Gluonic background

D.N. Li, M.H., JHEP2013, arXiv:1303.6929

$$\mathscr{L}_G = -\frac{1}{4} G^a_{\mu\nu}(x) G^{\mu\nu,a}(x),$$

IR: Gluon condensate $\mathrm{Tr}\langle G^2 \rangle$

Effective gluon mass $\langle g^2 A^2 \rangle$

5D action: graviton-dilaton

$$S_G = \frac{1}{16\pi G_5} \int d^5x \sqrt{g_s} e^{-2\Phi} \left(R_s + 4\partial_M \Phi \partial^M \Phi - V^s_G(\Phi) \right)$$

$\mathrm{Tr}\langle G^2 \rangle$ $\langle g^2 A^2 \rangle$ **dual to** $\Phi(z)$

$$\Phi(z) = \mu_G^2 z^2 \tanh(\mu_{G^2}^4 z^2 / \mu_G^2)$$

$$\Phi(z) \overset{z\to 0}{\rightarrow} \mu_{G^2}^4 z^4, \qquad \Phi(z) \overset{z\to\infty}{\rightarrow} \mu_G^2 z^2$$

Dimension-2 gluon condensate & linear confinement

$\langle g^2 A^2 \rangle$

F.V. Gubarev, L. Stodolsky and V.I. Zakharov

Phys. Rev. Lett. 86, 2220-2222 (2001)

R. Akhoury and V.I. Zakharov

Phys. Lett. B 438, 165-172 (1998)

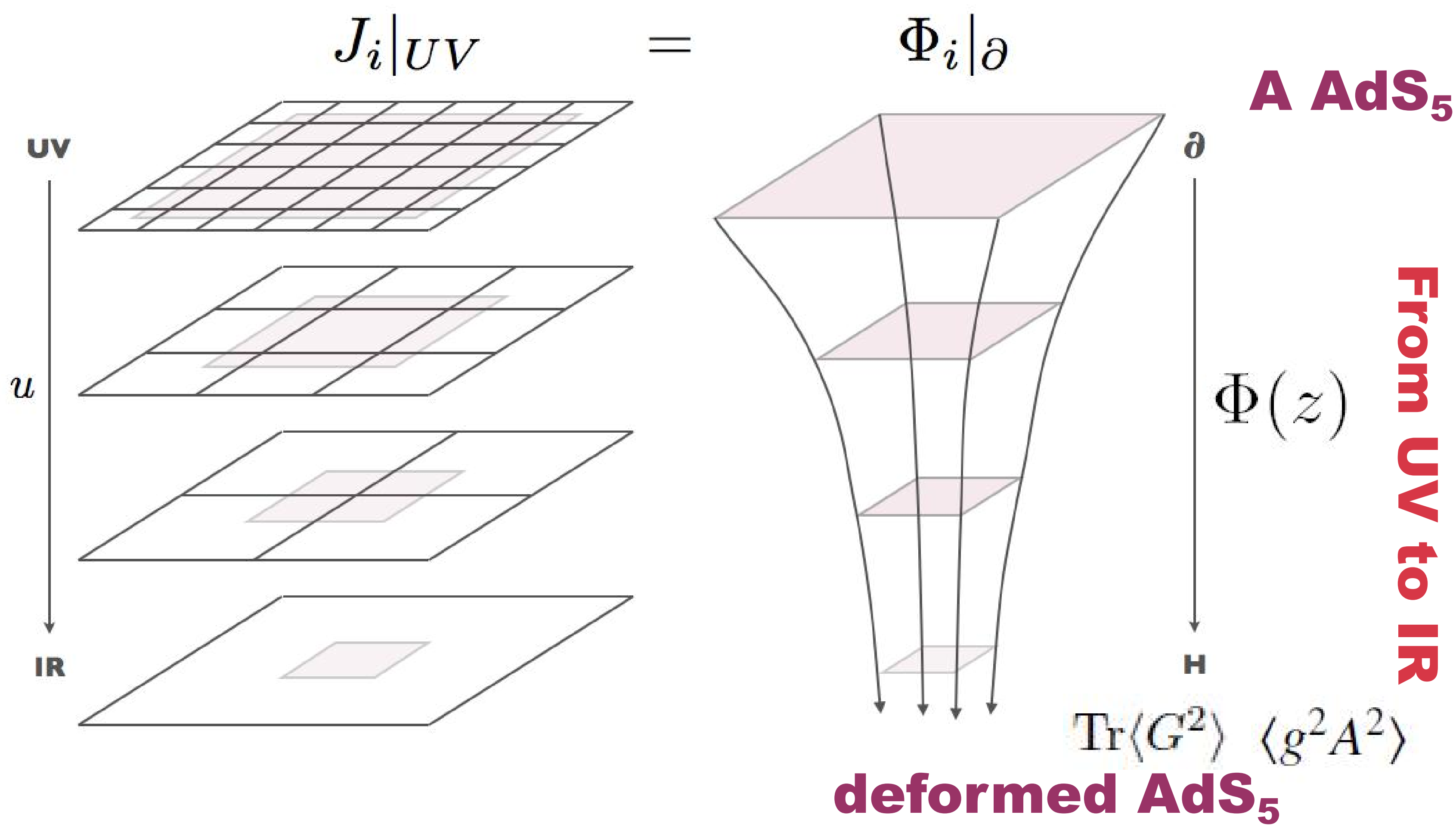
K. I. Kondo, Phys. Lett. B **514**, 335 (2001)

$$\alpha_s(Q^2) = \alpha_s(Q^2)_{pert}\left[1 + \frac{g_R^2\langle \mathscr{A}_\mu^2\rangle_R}{4(N_c^2-1)}\frac{9}{Q^2} + O(\alpha)\right]$$

$$V(r) = -C_F\frac{\alpha_s(r)}{r} + \sigma_s r \qquad \boxed{\sigma_s \cong g_R^2\langle \mathscr{A}_\mu^2\rangle_R.}$$

Recent progress: Paris group and Belgium group

Graviton-dilaton system



$$g^s_{MN} = b_s^2(z)(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu), \quad b_s(z) \equiv e^{A_s(z)}$$

Holographic Duality: Dictionary

Boundary QFT

Bulk Gravity

Local operator $\mathcal{O}_i(x)$

Bulk field $\Phi_i(x, r)$

$$\Delta(d - \Delta) = m^2 L^2$$

Strongly coupled

Semi-classical

$$Z_{\mathrm{QFT}}[J_i] \;=\; Z_{\mathrm{QG}}[\Phi[J_i]]$$

$$Z_{\mathrm{QFT}}[J] \;\simeq\; e^{-I_{\mathrm{GR}}[\Phi[J]]}$$

$$\langle \mathcal{O}_1(x_1) \ldots \mathcal{O}_n(x_n) \rangle = \frac{\delta^n I_{\mathrm{GR}}[\Phi[J_i]]}{\delta J_1(x_1) \ldots \delta J_n(x_n)} \Big|_{J_i=0}$$

Two-gluon and tri-gluon Glueball spectra:

Yidian Chen, M.H., arXiv: 1511.07018

$$M_5^2 = (\Delta - f)(\Delta + f - 4)$$

J^{PC}	$4D : \mathcal{O}(x)$	Δ	f	M_5^2	
0^{++}	$Tr(G^2)$	4	0	0	
0^{--}	$Tr(\tilde{G}\{D_{\mu_1}D_{\mu_2}G, G\})$	8	0	32	tri-gluon
0^{-+}	$Tr(G\tilde{G})$	4	0	0	
$1^{\pm-}$	$Tr(G\{G, G\})$	6	1	15	tri-gluon
2^{++}	$Tr(G_{\mu\alpha}G_{\alpha\nu} - \frac{1}{4}\delta_{\mu\nu}G^2)$	4	2	4	
2^{++}	$E_i^a E_j^a - B_i^a B_j^a - trace$	4	2	4	
2^{-+}	$E_i^a B_j^a + B_i^a E_j^a - trace$	4	2	4	
$2^{\pm-}$	$Tr(G\{G, G\})$	6	2	16	tri-gluon

Two-gluon and tri-gluon Glueball spectra:

C. -F. Qiao and L. Tang, "Finding the 0^{--} Glueball," Phys. Rev. Lett. **113**, 221601 (2014).

C. F. Qiao and L. Tang, arXiv:1509.00305 [hep-ph].

Tri-gluon glueball

$$j^A_{0^{--}} \sim d^{abc}[g^t_{\alpha\beta}\tilde{G}^a_{\mu\nu}][\partial_\alpha\partial_\beta G^b_{\nu\rho}][G^c_{\rho\mu}],$$
$$j^B_{0^{--}} \sim d^{abc}[g^t_{\alpha\beta}G^a_{\mu\nu}][\partial_\alpha\partial_\beta \tilde{G}^b_{\nu\rho}][G^c_{\rho\mu}],$$
$$j^C_{0^{--}} \sim d^{abc}[g^t_{\alpha\beta}G^a_{\mu\nu}][\partial_\alpha\partial_\beta G^b_{\nu\rho}][\tilde{G}^c_{\rho\mu}],$$
$$j^D_{0^{--}} \sim d^{abc}[g^t_{\alpha\beta}\tilde{G}^a_{\mu\nu}][\partial_\alpha\partial_\beta \tilde{G}^b_{\nu\rho}][\tilde{G}^c_{\rho\mu}],$$

$$j^{2^{+-},A}_{\mu\alpha}(x) = g_s^3 d^{abc}[G^a_{\mu\nu}(x)][G^b_{\nu\rho}(x)][G^c_{\rho\alpha}(x)]\,,$$
$$j^{2^{+-},B}_{\mu\alpha}(x) = g_s^3 d^{abc}[G^a_{\mu\nu}(x)][\tilde{G}^b_{\nu\rho}(x)][\tilde{G}^c_{\rho\alpha}(x)]\,,$$
$$j^{2^{+-},C}_{\mu\alpha}(x) = g_s^3 d^{abc}[\tilde{G}^a_{\mu\nu}(x)][G^b_{\nu\rho}(x)][\tilde{G}^c_{\rho\alpha}(x)]\,,$$
$$j^{2^{+-},D}_{\mu\alpha}(x) = g_s^3 d^{abc}[\tilde{G}^a_{\mu\nu}(x)][\tilde{G}^b_{\nu\rho}(x)][G^c_{\rho\alpha}(x)]\,.$$

Glueball

Yidian Chen, M.H., arXiv: 1511.07018

Excitations from gluonic background

$$S_{\mathcal{G}} = -\frac{1}{2}\int d^5x\sqrt{g_s}e^{-p\Phi}\left(\partial_M \mathcal{G} \partial^M \mathcal{G} + M^2_{\mathcal{G},5}(z)\mathcal{G}^2\right),$$

$$S_V = -\frac{1}{2}\int d^5x\sqrt{g_s}e^{-p\Phi}\left(\frac{1}{2}F^{MN}F_{MN} + M^2_{\mathcal{V},5}(z)\mathcal{V}^2\right),$$

$$S_T = -\frac{1}{2}\int d^5x\sqrt{g_s}e^{-p\Phi}\left(\nabla_L h_{MN}\nabla^L h^{MN} - 2\nabla_L h^{LM}\nabla^N h_{NM} + 2\nabla_M h^{MN}\nabla_N h \right.$$
$$\left. -\nabla_M h \nabla^M h + M^2_{h,5}(z)(h^{MN}h_{MN} - h^2)\right)$$

$M_5^2(z) = M_5^2 e^{-2\Phi/3}$, $p=1$ for even parity and $p=-1$ for odd parity.

Glueball

Yidian Chen, M.H., arXiv: 1511.07018

EOM:

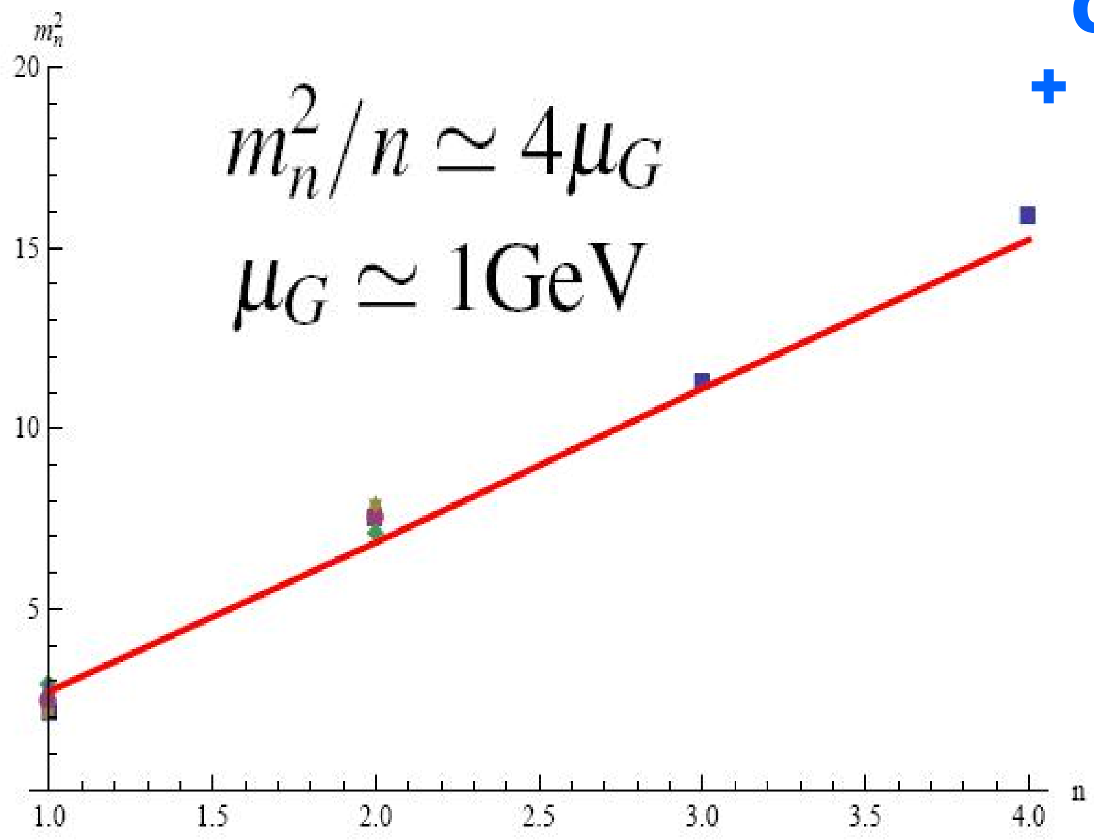
$$-\mathscr{A}_n'' + V_{\mathscr{A}}\mathscr{A}_n = m^2_{\mathscr{A},n}\mathscr{A}_n,$$

$$V_{\mathscr{A}} = \frac{cA_s'' - p\Phi''}{2} + \frac{(cA_s' - p\Phi')^2}{4} + e^{2A_s - \frac{2}{3}\Phi} M^2_{\mathscr{A},5},$$

Only one parameter determined from the Regge slope of the scalar glueball spectra:

$$\mu_G = 1\mathrm{GeV}$$

Scalar glueball D.N. Li, M.H., JHEP2013, arXiv:1303.6929



Ground state + Regge slope !

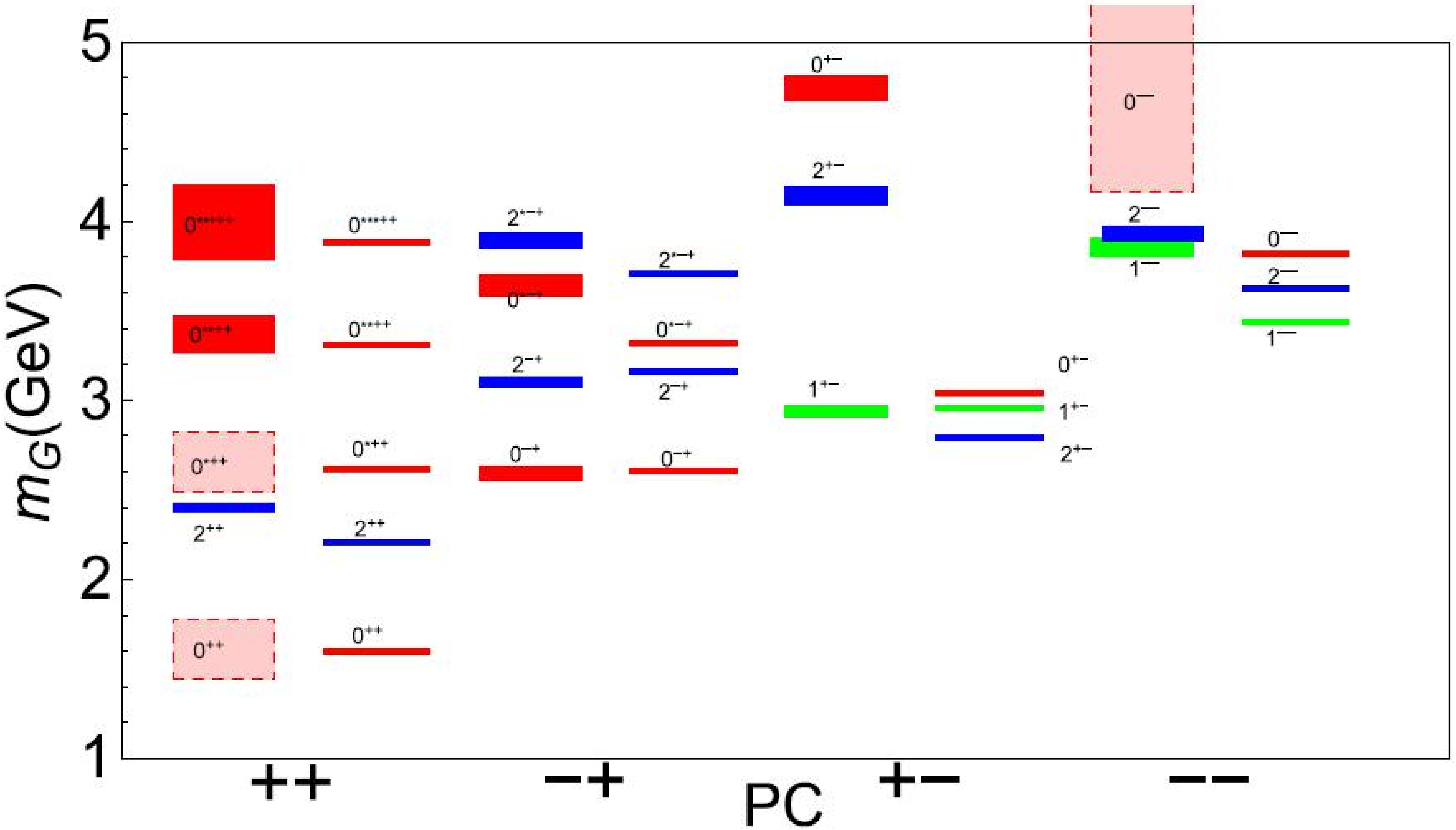
hep-lat/0508002.

[hep-lat/0510074].

[hep-lat/0103027].

[hep-lat/9901004]

Glueball spectra: Yidian Chen, M.H., arXiv: 1511.07018



Agree well with lattice result except three trigluon glueball 0^{--} , 0^{+-} and 2^{+-}

Glueball spectra: **Yidian Chen, M.H., arXiv: 1511.07018**

J^{PC}	LQCD	Flux tube model	QCDSR	MDSM
0^{++}	1.475-1.73	1.52	1.5	1.593
0^{*++}	2.67-2.83	2.75	–	2.618
0^{**++}	3.37	–	–	3.311
0^{***++}	3.99	–	–	3.877
0^{-+}	2.59	2.79	2.05	2.606
0^{*-+}	3.64	–	–	3.317
0^{--}	5.166	2.79	3.81	3.817
0^{+-}	4.74	2.79	4.57	3.04
$0^{++\S}$	–	–	3.1	2.667
1^{+-}	2.94	2.25	–	2.954
1^{--}	3.85	–	–	3.44
2^{++}	2.4	2.84	2	2.203
2^{-+}	3.1	2.84	–	3.161
2^{*-+}	3.89	–	–	3.703
2^{+-}	4.14	2.84	6.06	2.786
2^{--}	3.93	2.84	–	3.619

All two-gluon and tri-gluon glueball spectra agree well with lattice result except three trigluon glueballs

0^{--} , 0^{+-} and 2^{+-}

These three trigluon glueballs

0^{--} , 0^{+-} and 2^{+-}

are dominated by three-gluon condensate.

Our model only considered two-gluon condensate.

Add flavor dynamics

D.N. Li, M.H., JHEP2013, arXiv:1303.6929

Action for pure gluon system: Graviton-dilaton coupling

$$S_G = \frac{1}{16\pi G_5}\int d^5x\sqrt{g_s}e^{-2\Phi}\left(R + 4\partial_M\Phi\partial^M\Phi - V_G(\Phi)\right)$$

Gluonic background

Action for light hadrons: KKSS model

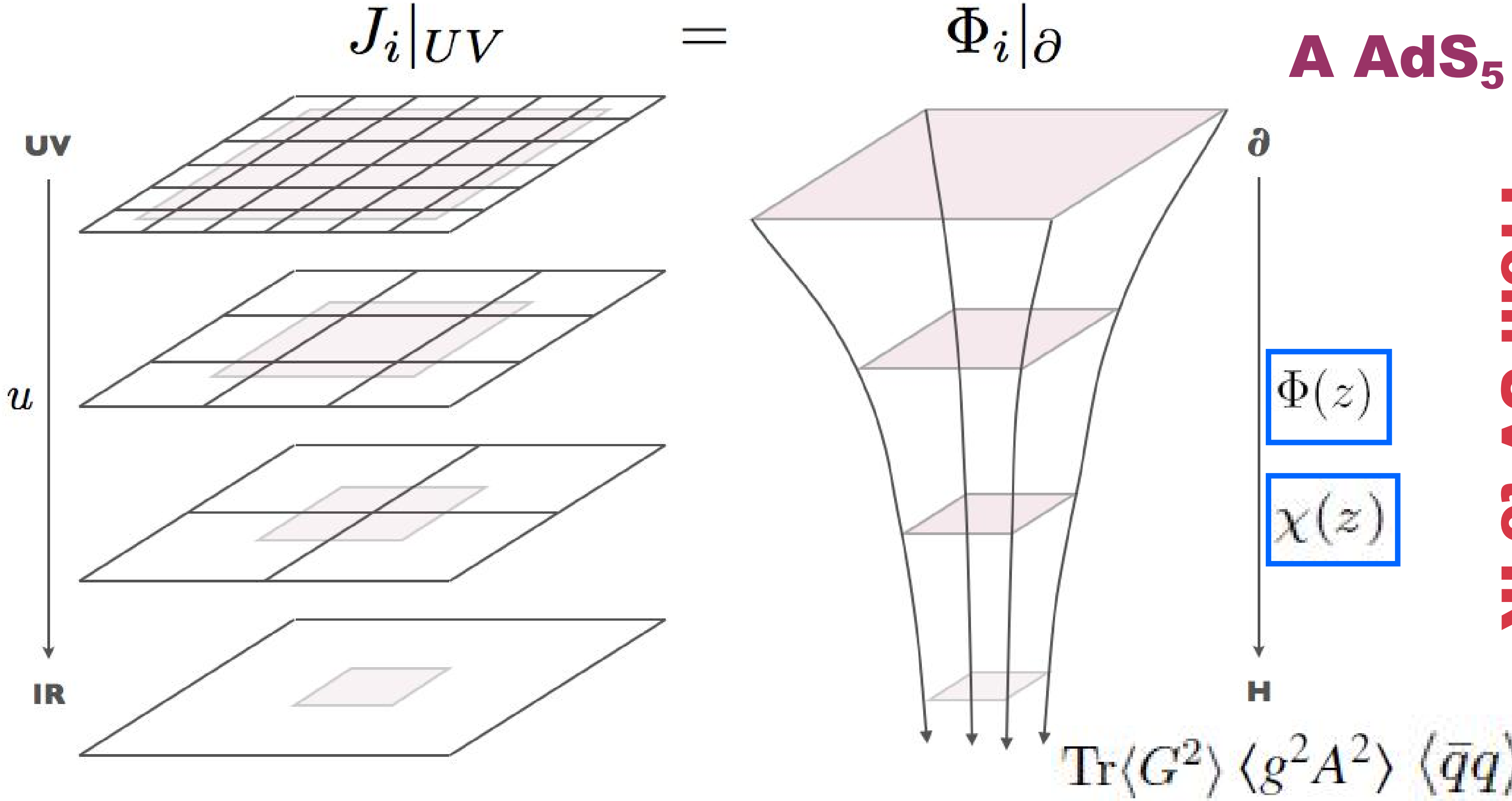
$$S_{KKSS} = -\int d^5x\sqrt{g_s}e^{-\Phi}Tr(|DX|^2 + V_X(X^+X,\Phi) + \frac{1}{4g_5^2}(F_L^2 + F_R^2))$$

5D linear sigma model

Total action:

$$S = S_G + \frac{N_f}{N_c}S_{KKSS}$$

Graviton-dilaton-scalar system

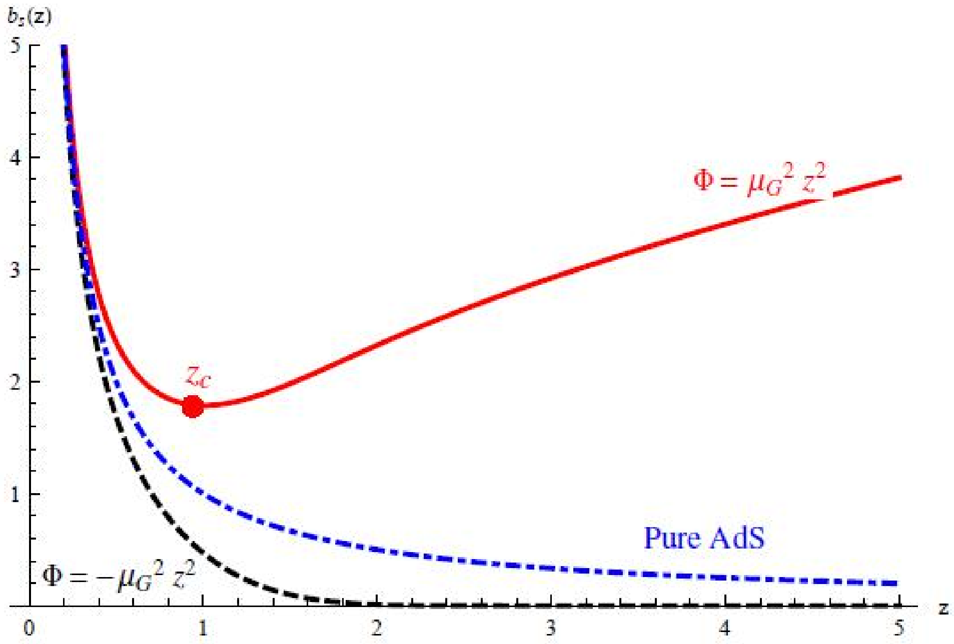


A AdS_5

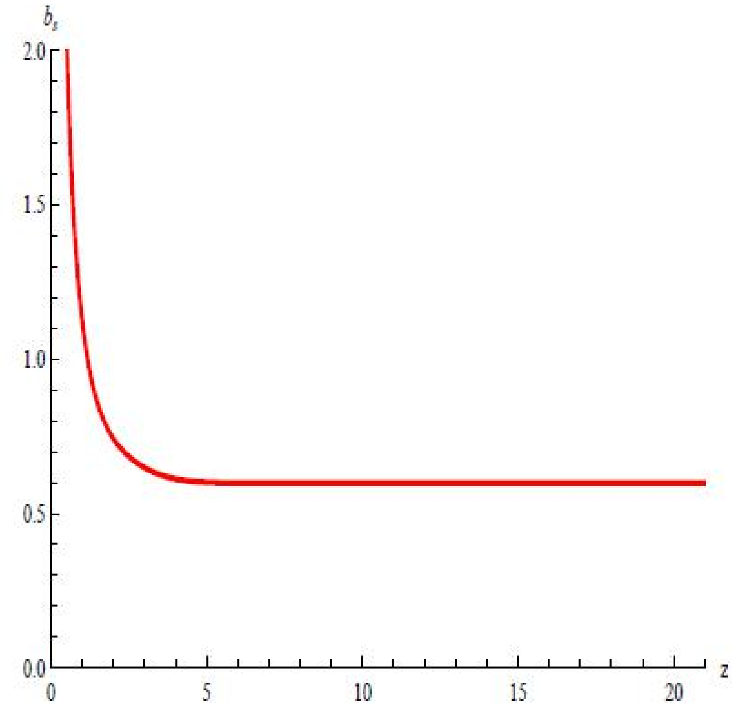
From UV to IR

deformed AdS_5

Quenched background



Unquenched background



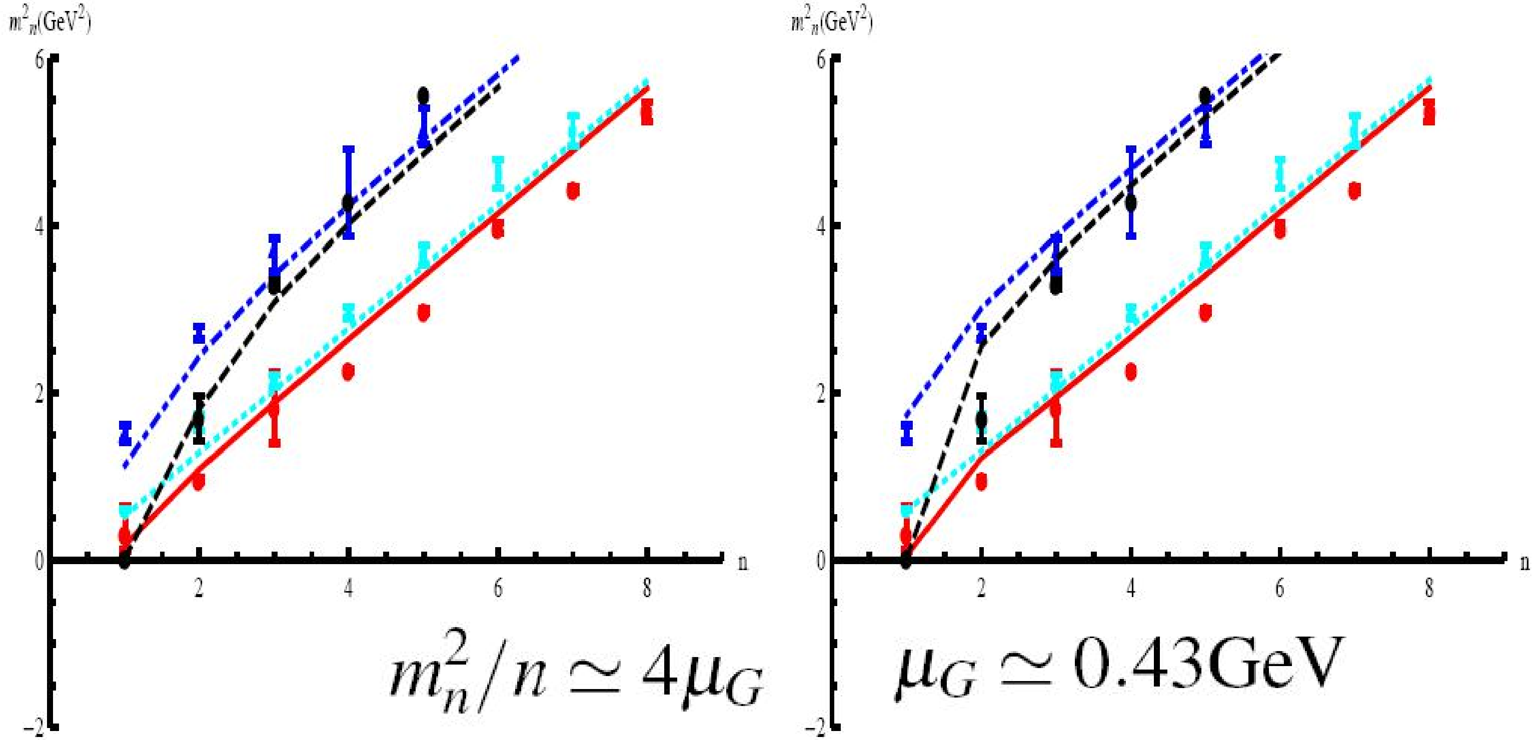
$$-A_s'' + A_s'^2 + \frac{2}{3}\Phi'' - \frac{4}{3}A_s'\Phi' - \frac{\lambda}{6}e^{\Phi}\chi'^2 = 0,$$

$$\Phi'' + (3A_s' - 2\Phi')\Phi' - \frac{3\lambda}{16}e^{\Phi}\chi'^2 - \frac{3}{8}e^{2A_s - \frac{4}{3}\Phi}\partial_\Phi\left(V_G(\Phi) + \lambda e^{\frac{7}{3}\Phi}V_C(\chi,\Phi)\right) = 0,$$

$$\chi'' + (3A_s' - \Phi')\chi' - e^{2A_s}V_{C,\chi}(\chi,\Phi) = 0.$$

Produced hadron spectra compared with data

D.N. Li, M.H., JHEP2013, arXiv:1303.6929



(Mod IA) (Mod IB)

Ground states: chiral symmetry breaking

Excitation states: linear confinemnt