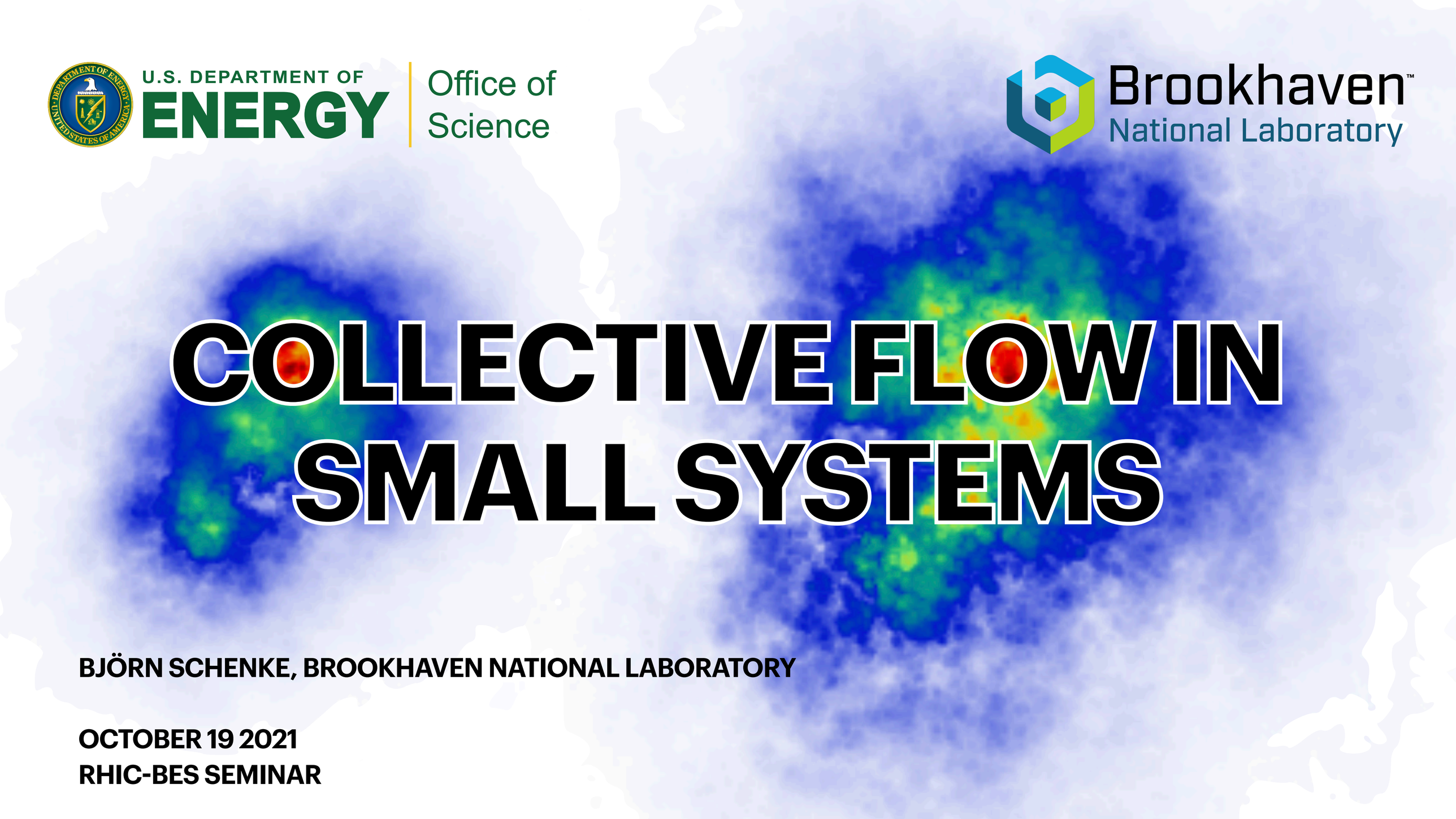


U.S. DEPARTMENT OF ENERGY | Office of Science

Brookhaven National Laboratory

# COLLECTIVE FLOW IN SMALL SYSTEMS

BJÖRN SCHENKE, BROOKHAVEN NATIONAL LABORATORY

OCTOBER 19 2021
RHIC-BES SEMINAR

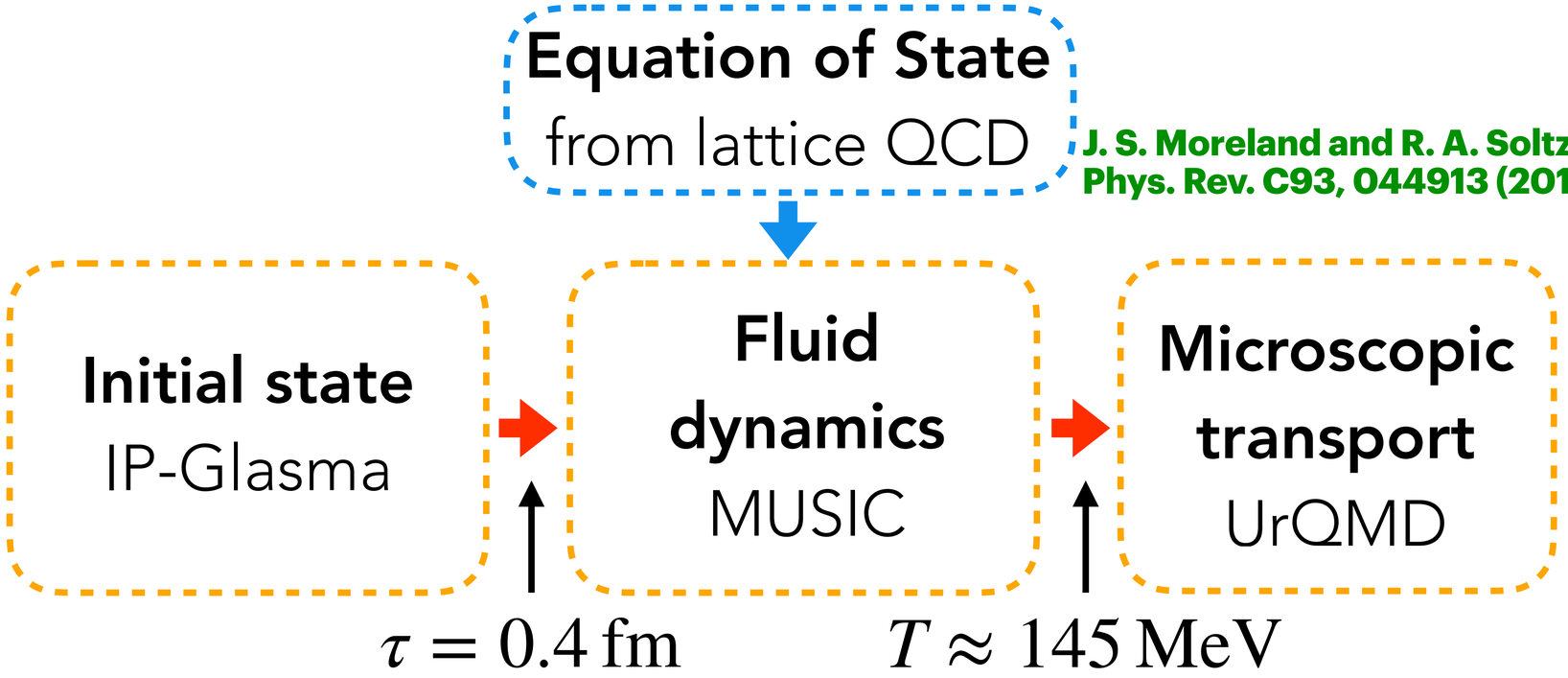
# BULK DYNAMICS OF HEAVY ION AND SMALL SYSTEM COLLISIONS WITHIN A FRAMEWORK BUILT AROUND FLUID DYNAMICS

- **PART I: Very High Energy: Boost invariant calculations**
- **PART II: Corrections to the boost invariant framework**
- **PART III: Fully 3+1D calculations, suitable for the BES**

# PART I: VERY HIGH ENERGY

- Use the CGC to compute the initial energy momentum tensor
- Assume boost invariance and do 2+1D hydrodynamics

# THE HYBRID FRAMEWORK




## Described in detail in

**B. Schenke, C. Shen, P. Tribedy, Phys. Rev. C 102 (2020) 4, 044905**

**"Running the gamut of high energy nuclear collisions"**

The term gamut was adopted from the field of **music**, where in middle age Latin "gamut" meant the entire range of musical notes of which musical melodies are composed

- **Exactly match $T^{\mu\nu}$ when switching from one part to the next**

  **B. Schenke, P. Tribedy, and R. Venugopalan, Phys. Rev. Lett. 108, 252301 (2012)**
  **B. Schenke, S. Jeon, and C. Gale, Phys. Rev. Lett. 106, 042301 (2011)**
  **S. A. Bass et al., Prog. Part. Nucl. Phys. 41, 255 (1998); M. Bleicher et al., J. Phys. G25, 1859 (1999)**

# INITIAL STATE GEOMETRY

**Nucleus:**

**Sample nucleons from Woods Saxon distribution**

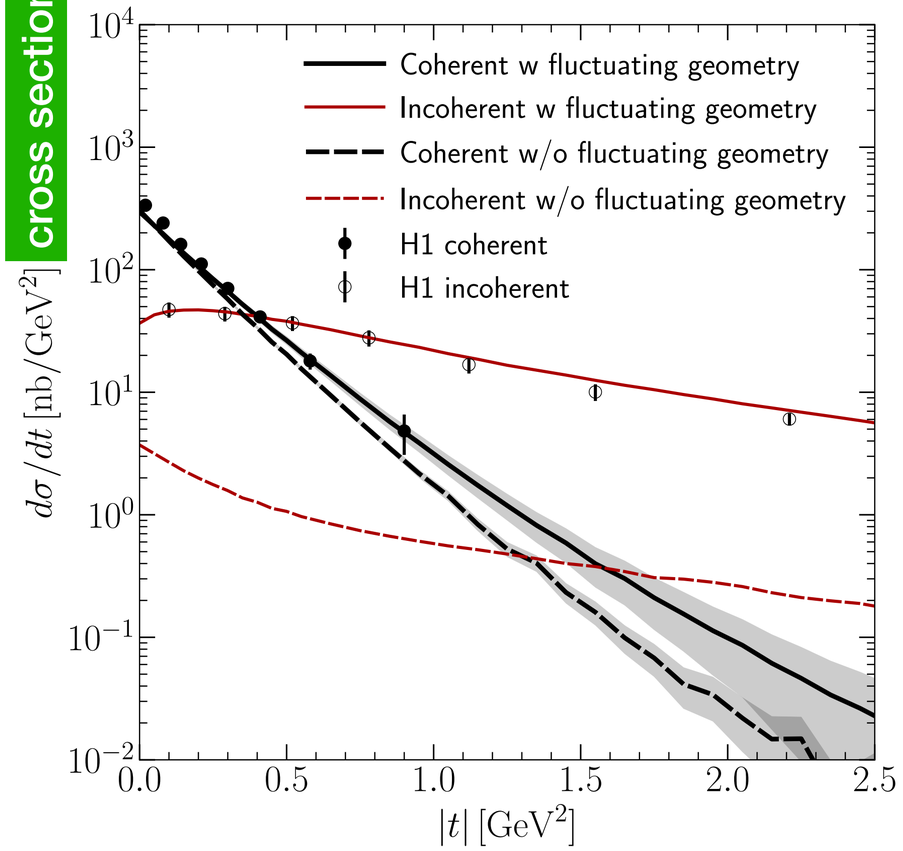
**Nucleon:**

- **consider substructure**
- **constrain parameters with HERA data**

**Exclusive diffractive J/Ψ production in e+p:**
**Incoherent x-sec sensitive to fluctuations**

H. Mäntysaari, B. Schenke, Phys. Rev. Lett. 117 (2016) 052301
Phys.Rev. D94 (2016) 034042
also see:
S. Schlichting, B. Schenke, Phys.Lett. B739 (2014) 313-319
H. Mäntysaari, Rep. Prog. Phys. 83 082201 (2020)
B. Schenke, Rep. Prog. Phys. 84 082301 (2021)

cross section




H1 Collaboration, Eur. Phys. J. C73 (2013) no. 6 2466

(transverse momentum transfer)$^2$

# INITIAL STATE GEOMETRY

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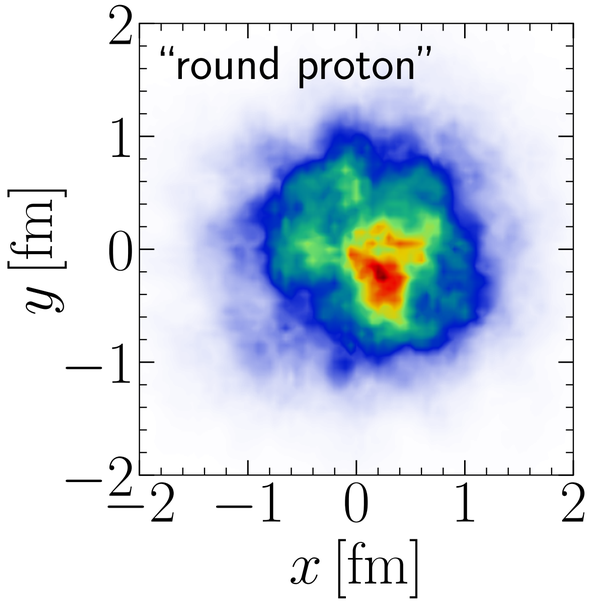


**Exclusive diffractive J/Ψ production in e+p:**
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H. Mäntysaari, B. Schenke, Phys. Rev. Lett. 117 (2016) 052301
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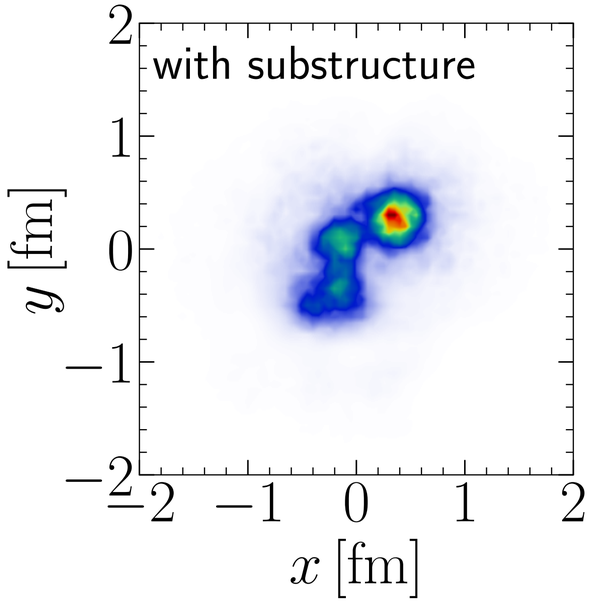
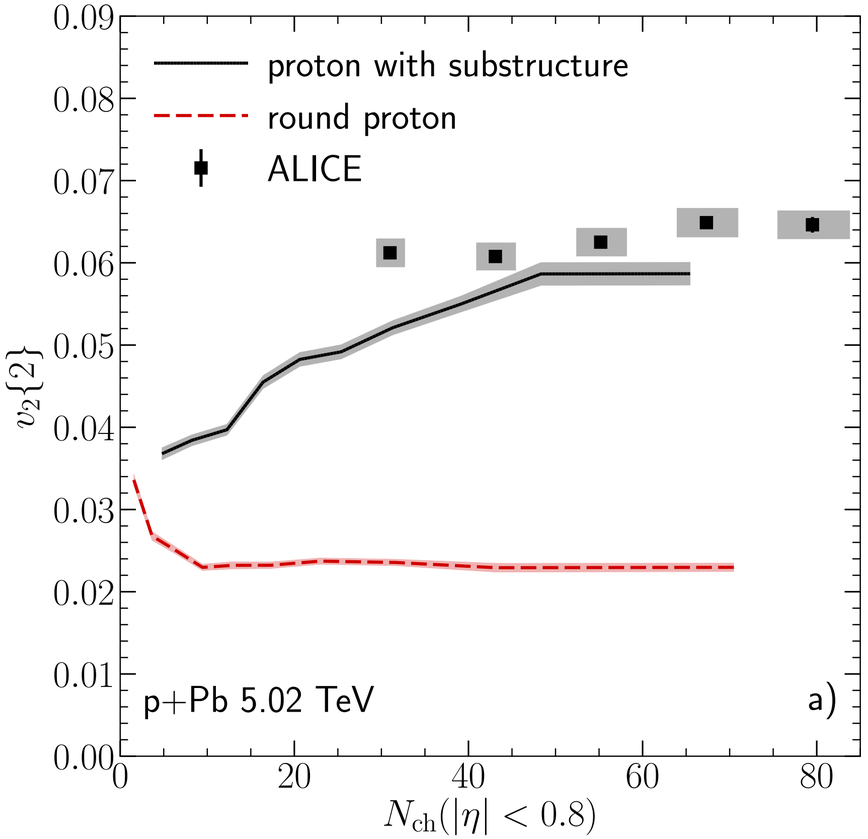
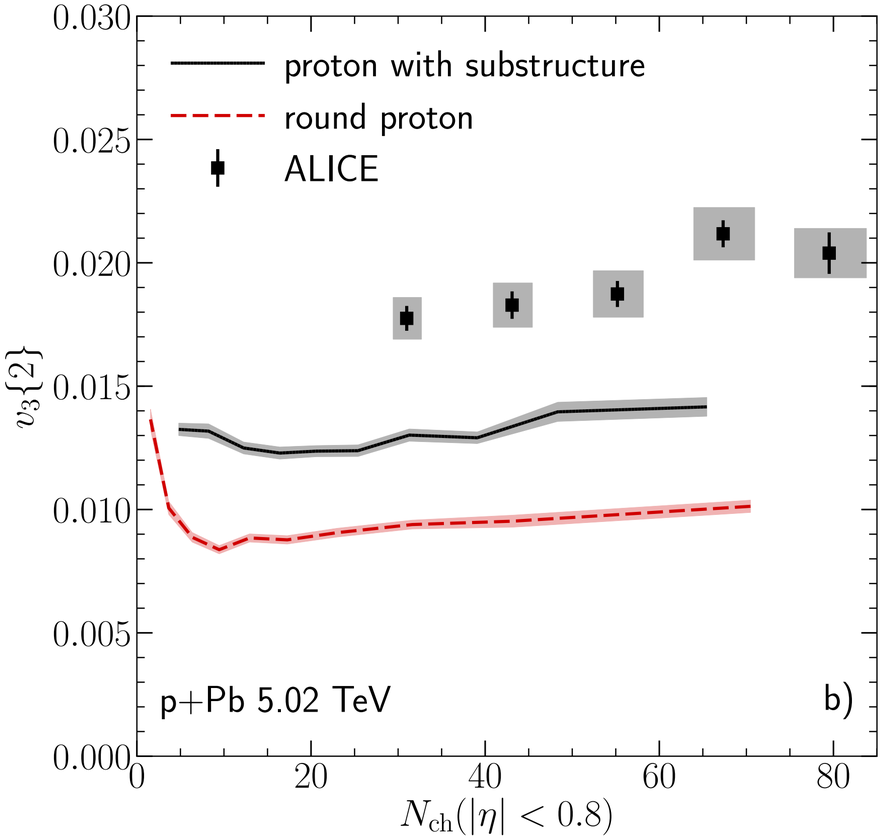
B. Schenke, Rep. Prog. Phys. 84 082301 (2021)

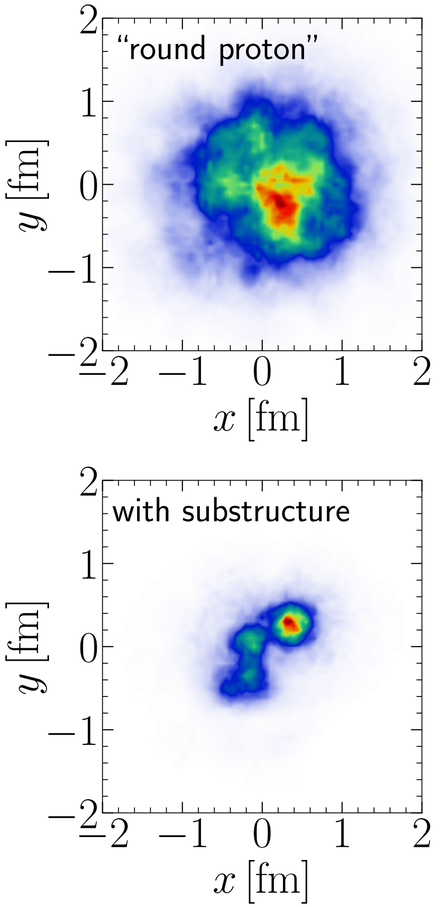


# NUCLEON SUBSTRUCTURE

**Substructure is also needed to describe the anisotropic flow in p+A collisions with IP-Glasma**




**B. Schenke, Rep. Prog. Phys. 84 082301 (2021)**

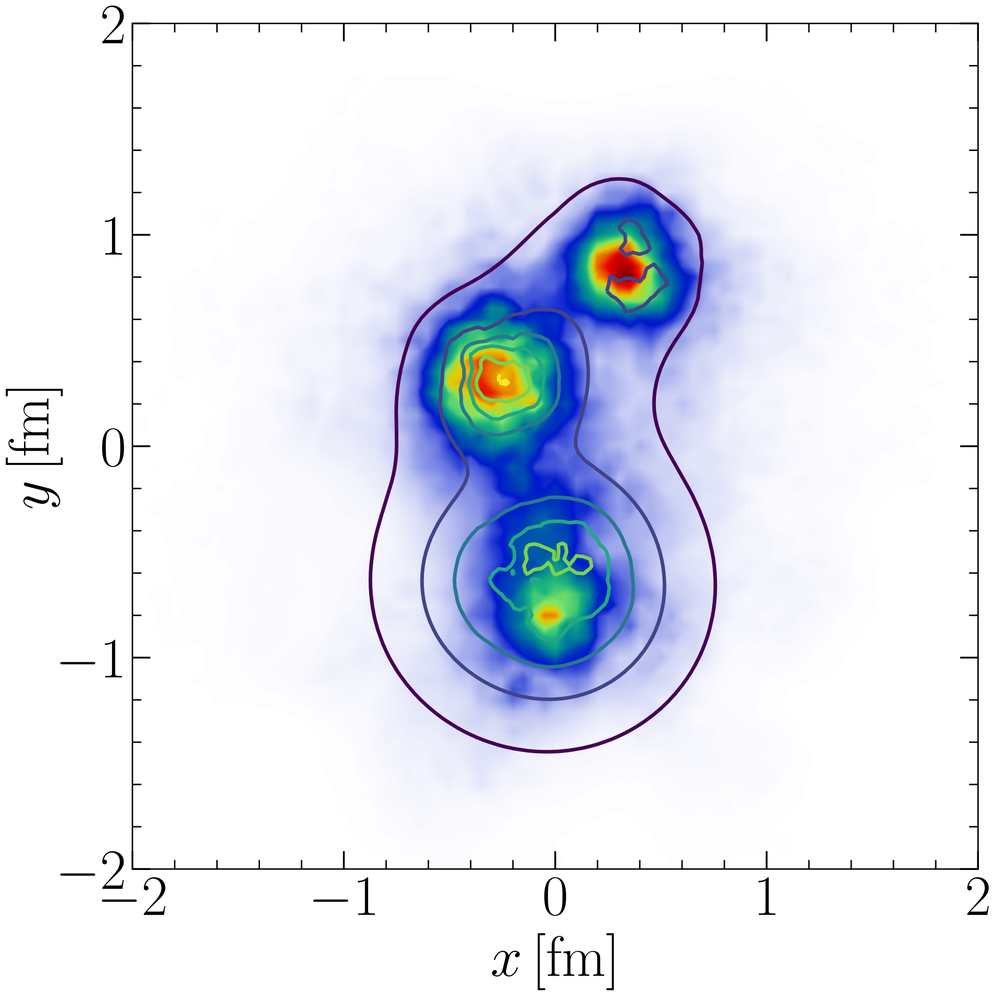
# FIREBALL SHAPE ~ PROTON SHAPE

**The shape of the overlap region in p+A collisions resembles the proton's shape**

**Color map:** **Energy density distribution (arbitrary units)**

**Contour lines:** **Shape of the projectile proton (quantified using a measure of the gluon density in the proton)**

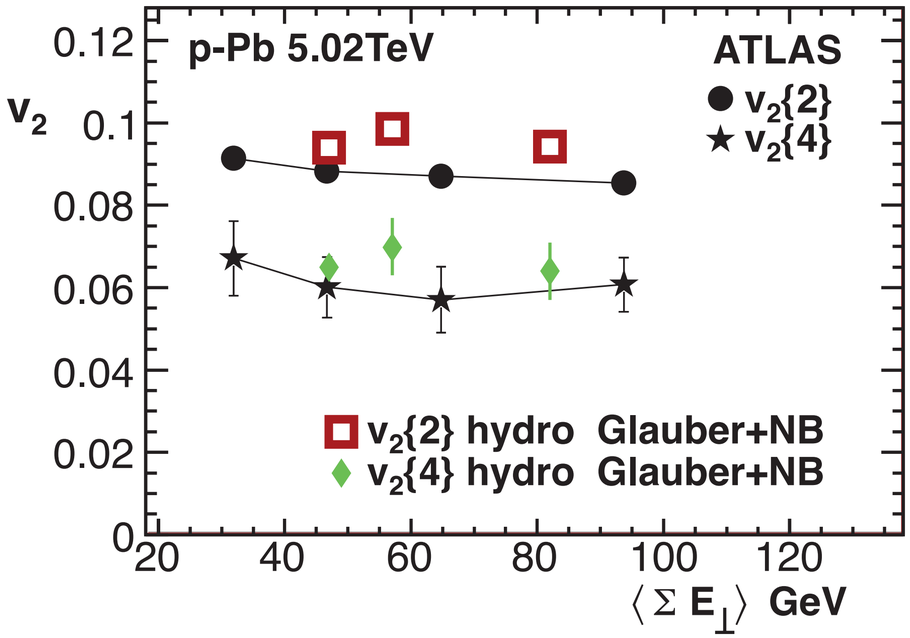
B. Schenke, Rep. Prog. Phys. 84 082301 (2021)



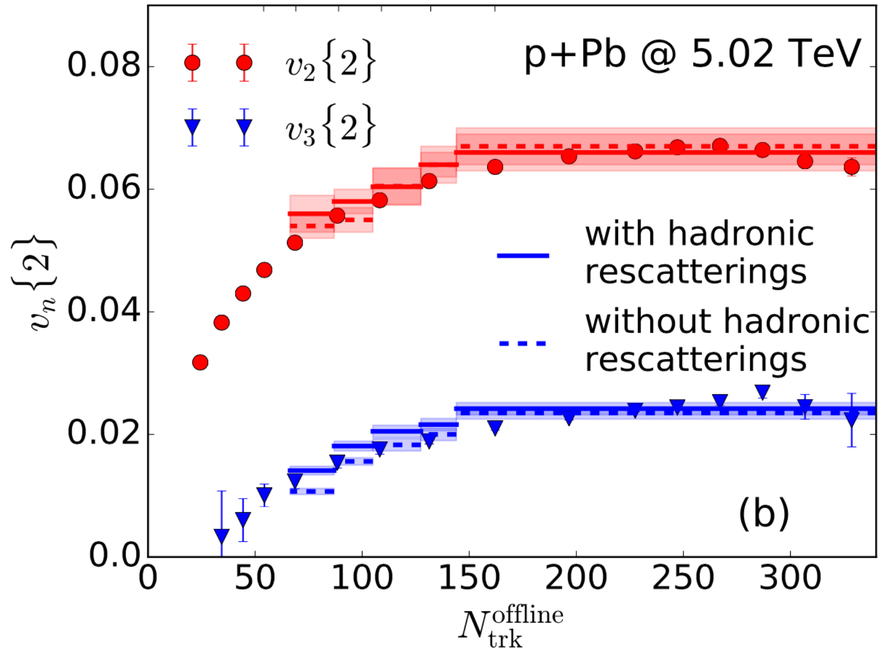
# NUCLEON SUBSTRUCTURE

**What about MC-Glauber (-like) models? They seem to get it right with just nucleons**

**ATLAS Coll. PLB725 (2013) 60-78**




**CMS Coll. PLB724, 213–240 (2013)**




**Bozek, Broniowski, PRC88 (2013) 014903**

**Shen, Paquet, Denicol, Jeon, Gale, PRC95 (2017) 014906**

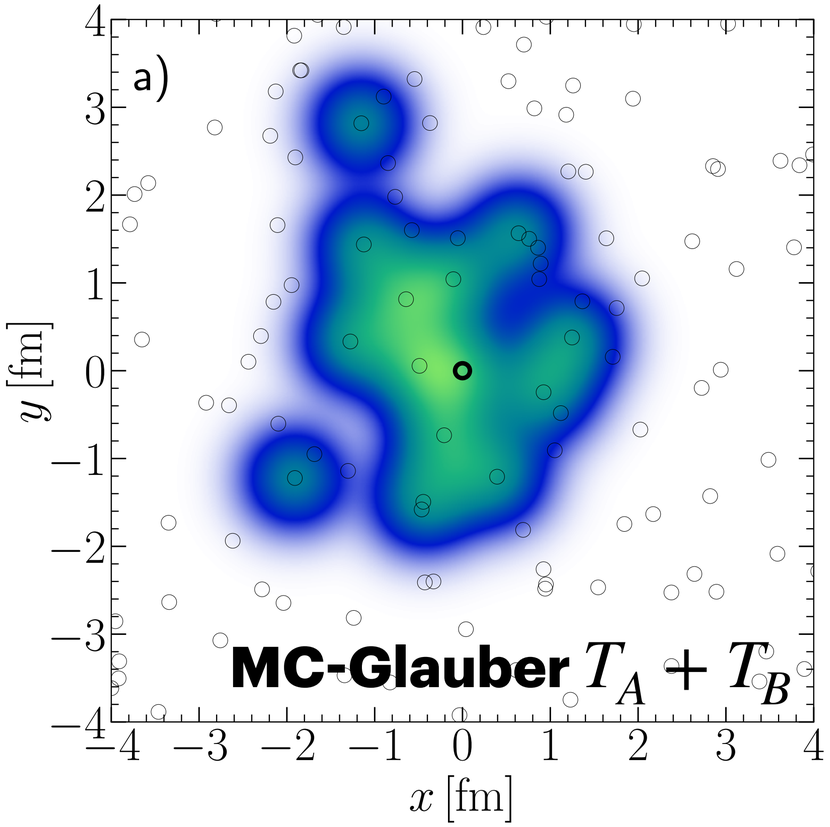
Also see: Kozlov, Luzum, Denicol, Jeon, Gale; Werner, Beicher, Guiot, Karpenko, Pierog; Romatschke; Kalaydzhyan, Shuryak, Zahed; Ghosh, Muhuri, Nayak, Varma; Qin, Mueller; Bozek, Broniowski, Torrieri; Habich, Miller, Romatschke, Xiang; T. Hirano, K. Kawaguchi, K. Murase; …

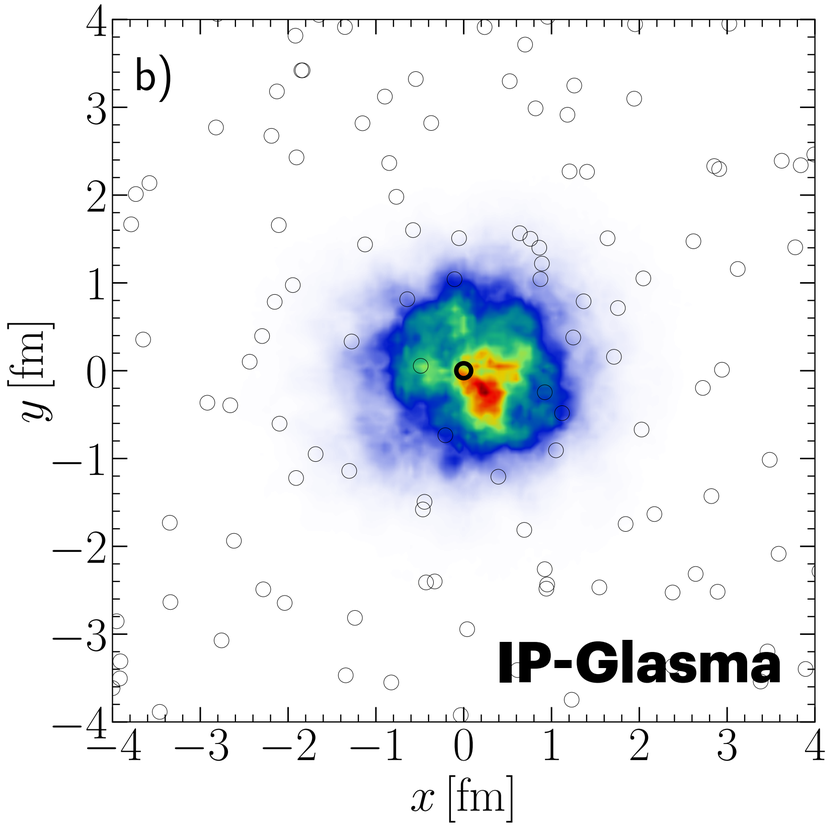
# MC-GLAUBER VS. IP-GLASMA

**Proton position: thick circle; Nucleon positions in Pb: thin circles**

**Same configurations: Compare MC-Glauber $T_A + T_B$ to IP-Glasma with round nucleon**

B. Schenke, Rep. Prog. Phys. 84 082301 (2021)



# HOW SHOULD THE ENERGY DEPOSITION GO?

**Energy (or entropy) deposition $\sim (T_A T_B)^q$ is preferred:**

- **Bayesian analysis: Trento model prefers initial transverse entropy density to behave as $\sim \sqrt{T_A T_B}$ and also prefers a nucleon substructure**
  J.S. Moreland, J.E. Bernhard, and S.A. Bass, Phys. Rev. C 92 (2015) 011901
  G. Nijs, W. van der Schee, U. Gürsoy, R. Snellings, Phys.Rev.Lett. 126 (2021), Phys.Rev.C 103 (2021) 5, 054909
  JETSCAPE Collaboration, Phys.Rev.C 103 (2021) 5, 054904
- **AdS/CFT based calculations also result in such a relation**
  P. Romatschke, J.D. Hogg, JHEP 04 (2013) 048
- **IP-Glasma results in the initial energy density $\sim T_A T_B$**
- **$T_A + T_B$ disfavored by centrality dependence of $v_2$ in A+A** G. Giacalone, J. Noronha-Hostler, J.-Y. Ollitrault, Phys. Rev. C 95, 054910 (2017)

So, $T_A + T_B$ prescription with nucleons, that works for $v_n$ in p+A, is generally disfavored

$\sim T_A T_B$ for round proton leads to too small fluctuations (eccentricities)

$\rightarrow$ subnucleon fluctuations required

# TRANSPORT COEFFICIENTS

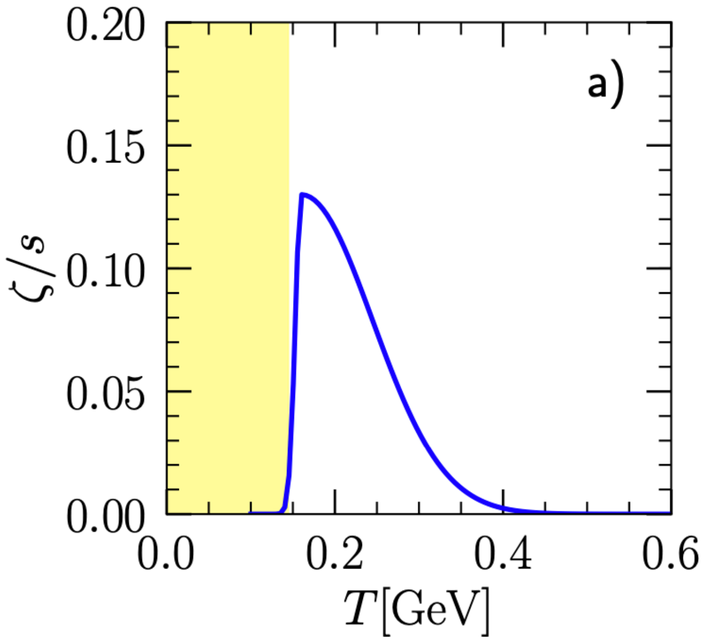
B. Schenke, C. Shen, P. Tribedy, Phys. Rev. C 102 (2020) 4, 044905

## Transport coefficients:

### Shear viscosity:

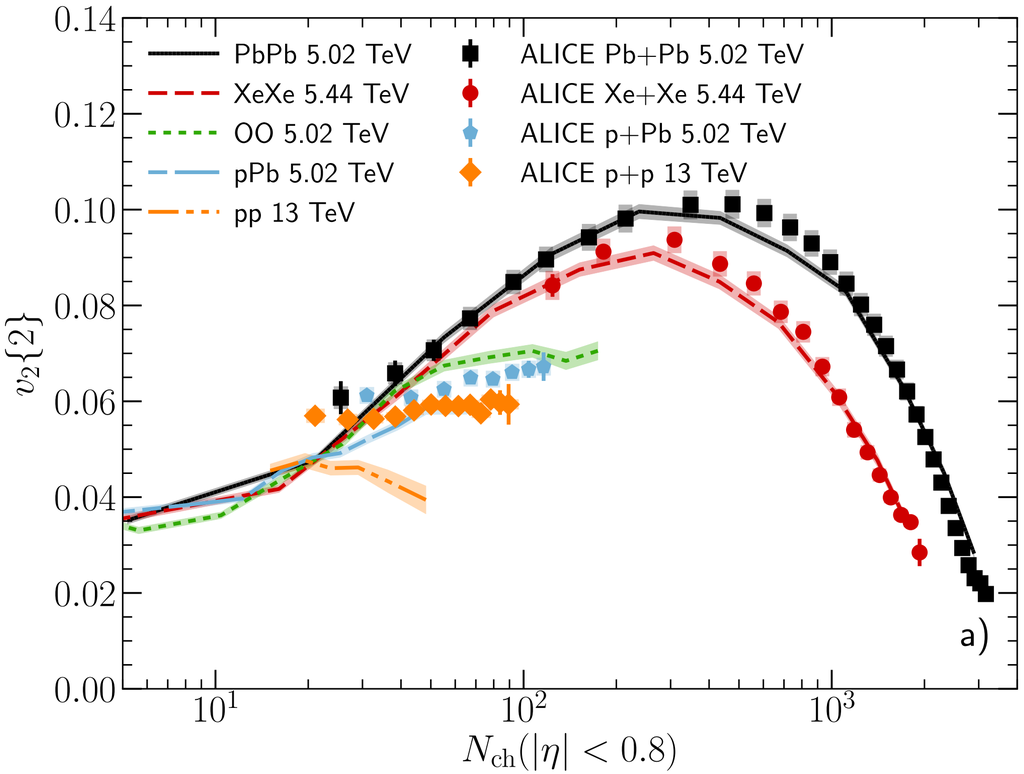
$$\eta/s = 0.12$$

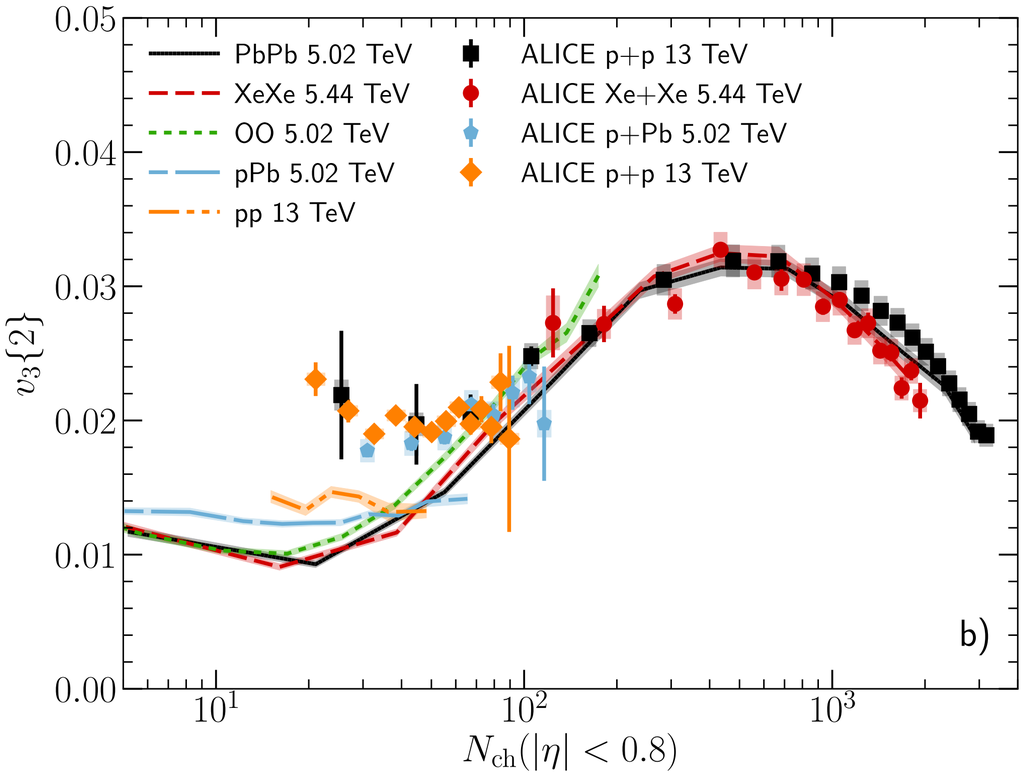
### Bulk viscosity:



# SOME HYBRID MODEL RESULTS

**B. Schenke, C. Shen, P. Tribedy, Phys. Rev. C 102 (2020) 4, 044905**



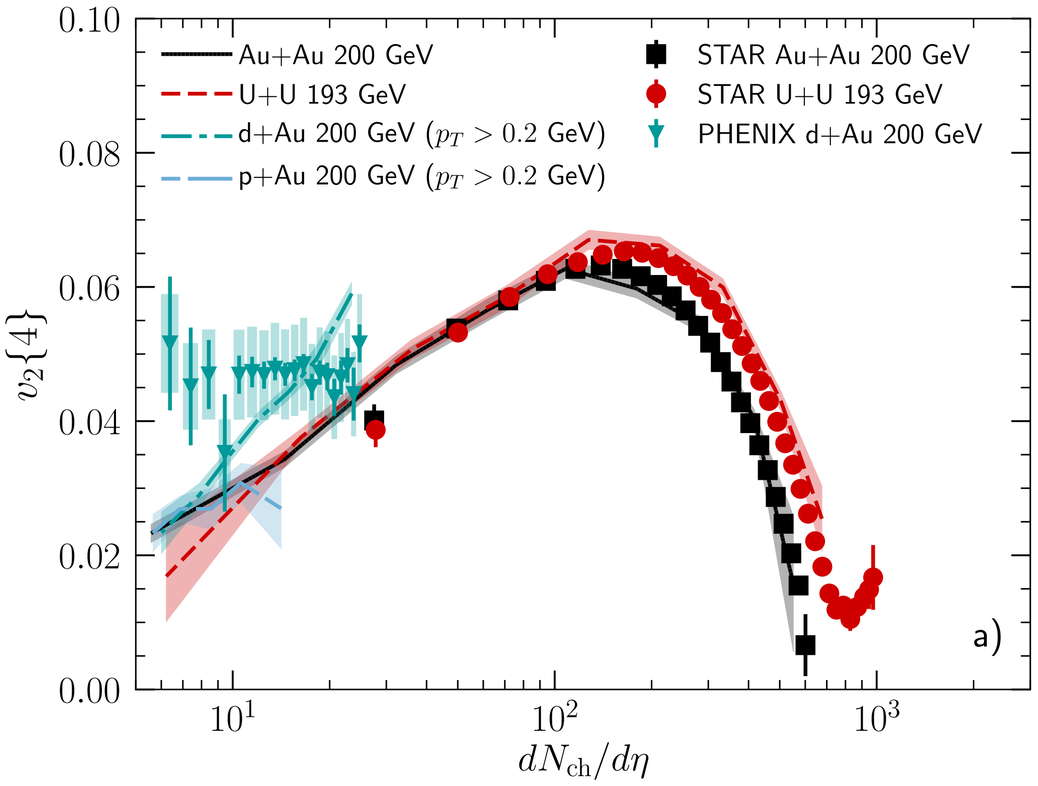



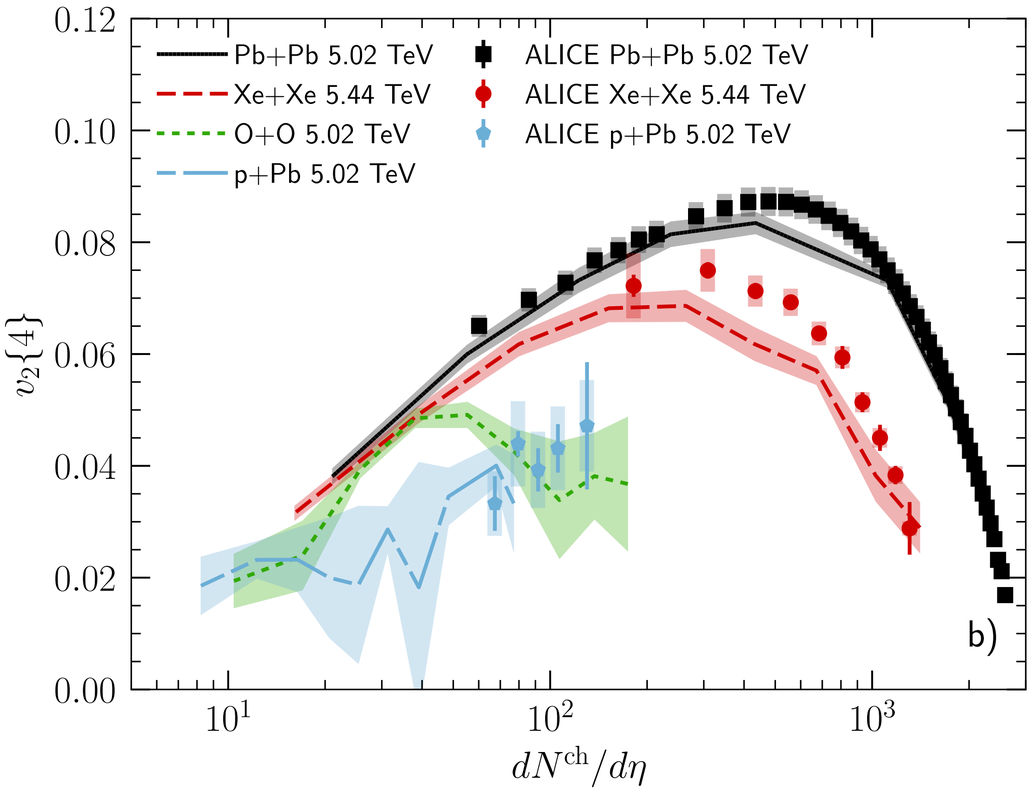

**Parameters constrained only by HERA data (proton shape) and Au+Au data at RHIC**

**Underestimate $v_n$ in p+p and $v_3$ in p+Pb**

# 4-PARTICLE CUMULANTS

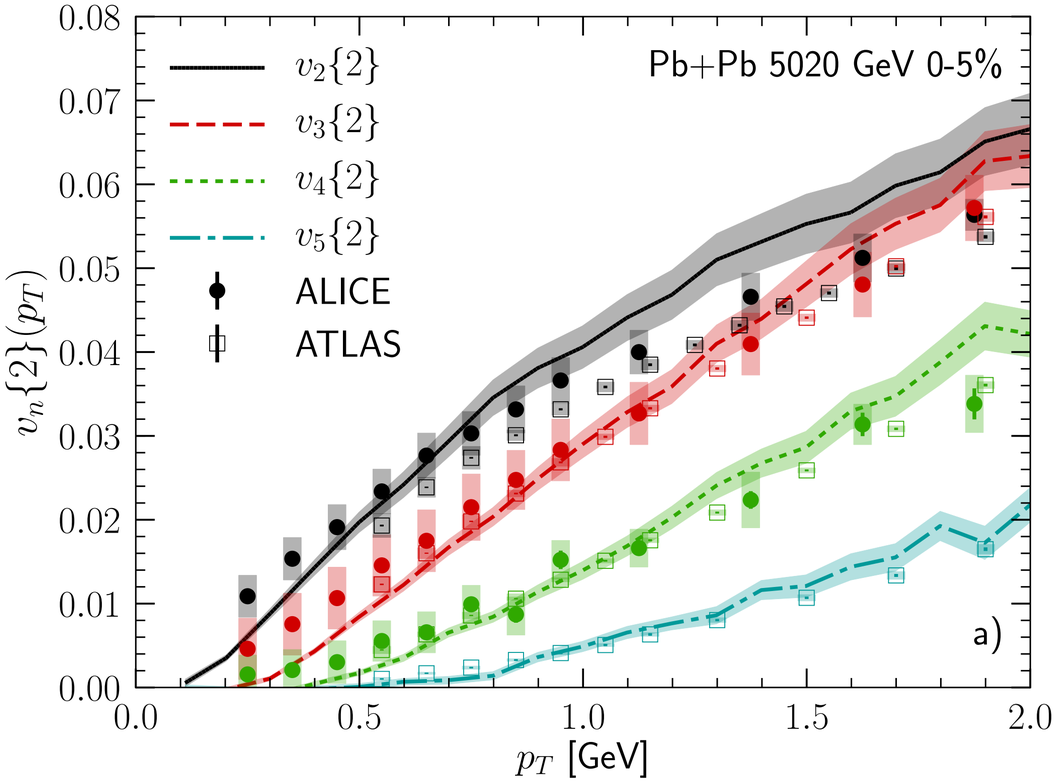
**B. Schenke, C. Shen, P. Tribedy, Phys. Rev. C 102 (2020) 4, 044905**

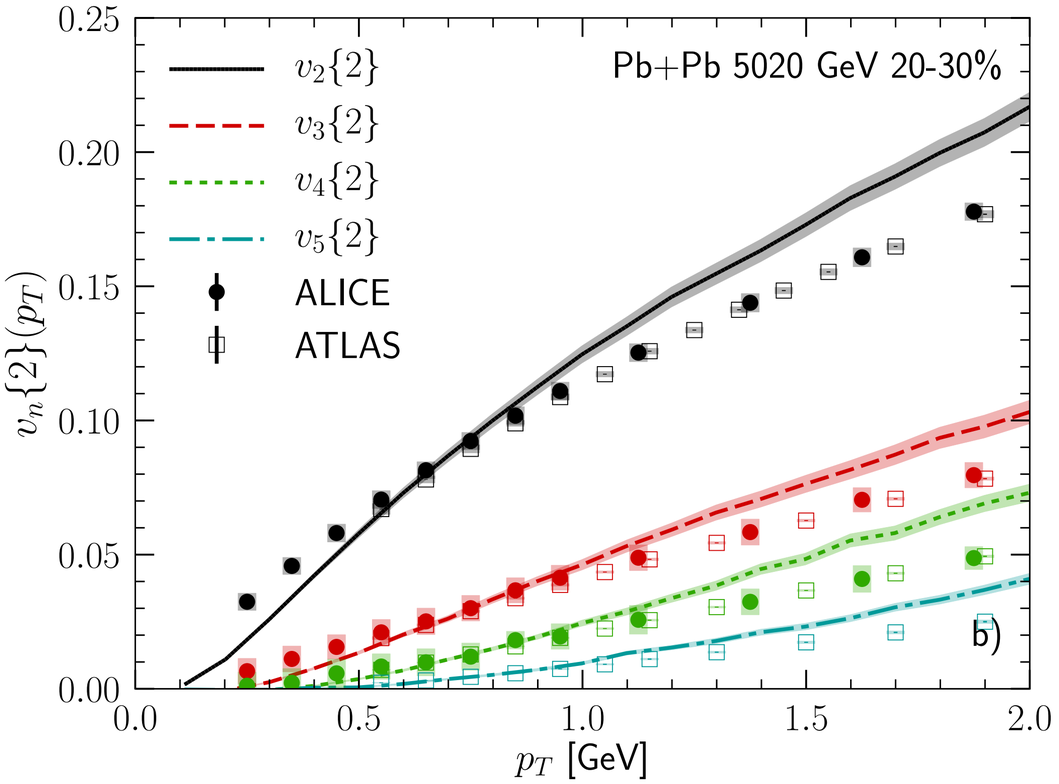


# TRANSVERSE MOMENTUM DEPENDENCE

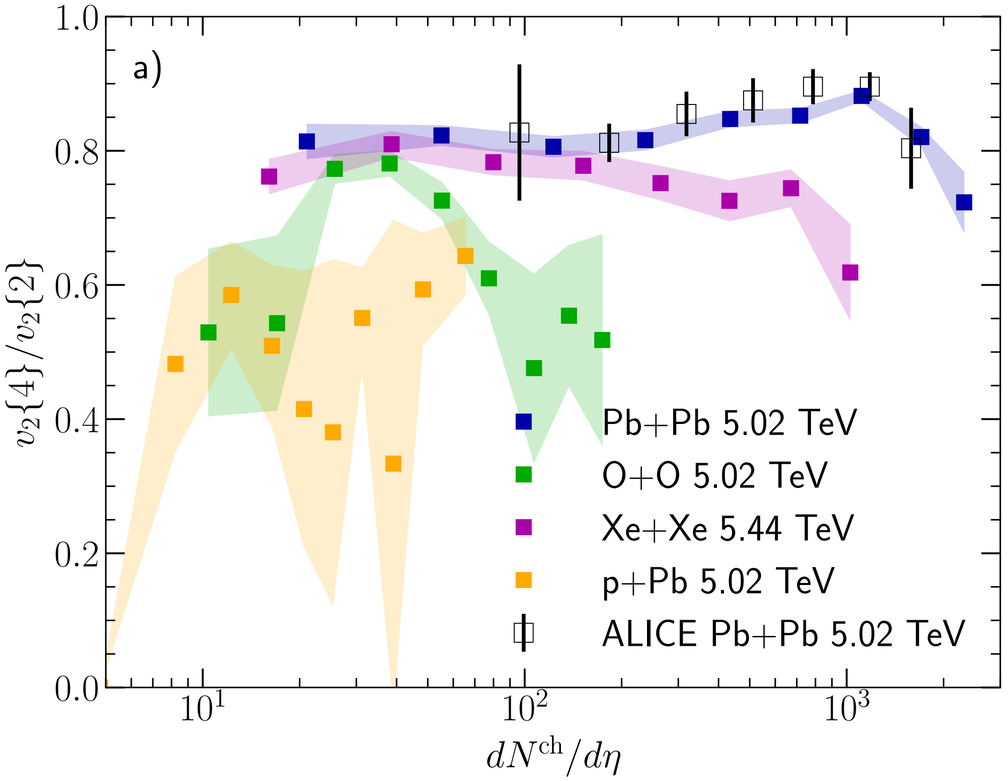
**B. Schenke, C. Shen, P. Tribedy, Phys. Rev. C 102 (2020) 4, 044905**

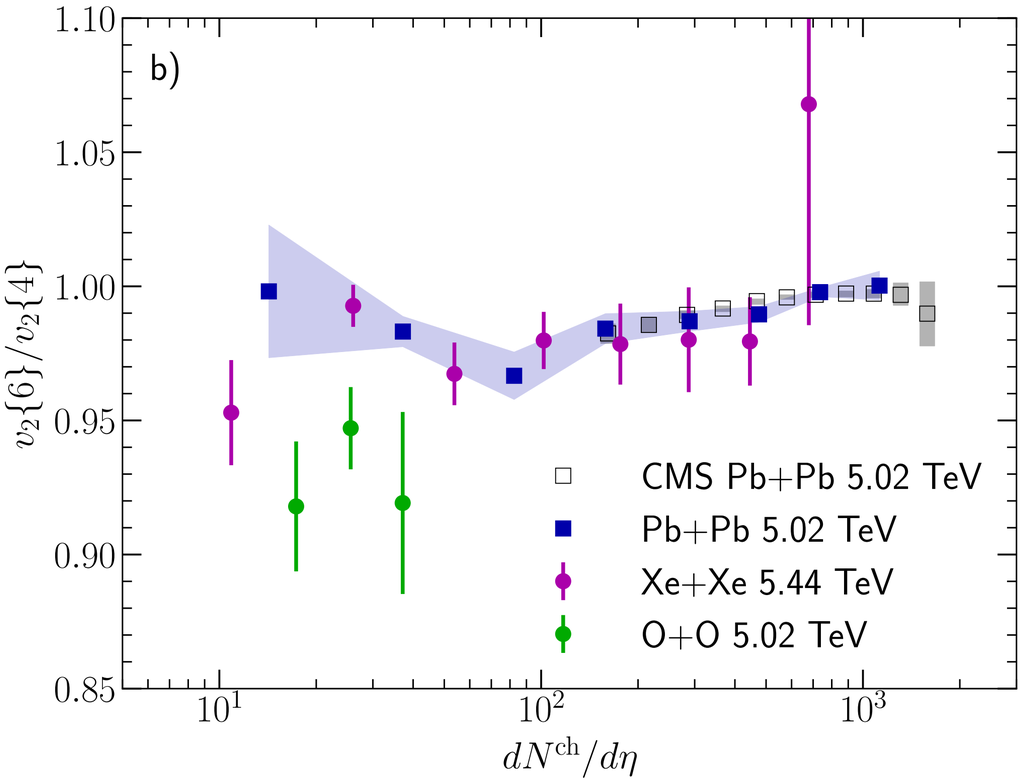


# CUMULANT RATIOS

**B. Schenke, C. Shen, P. Tribedy, Phys. Rev. C 102 (2020) 4, 044905**



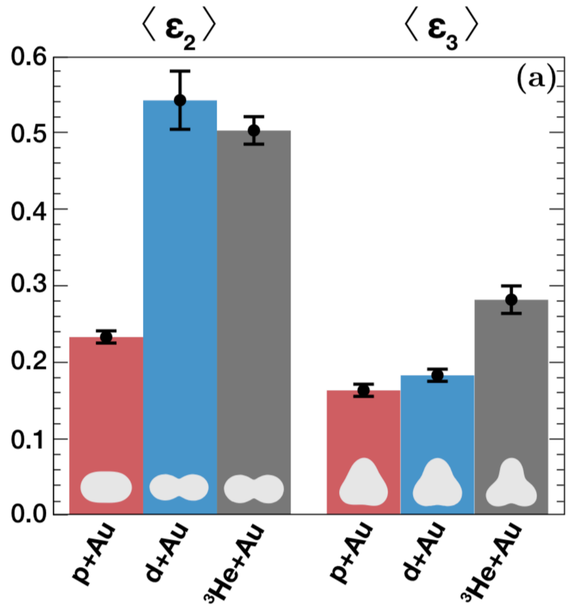



# RHIC SYSTEM SCAN

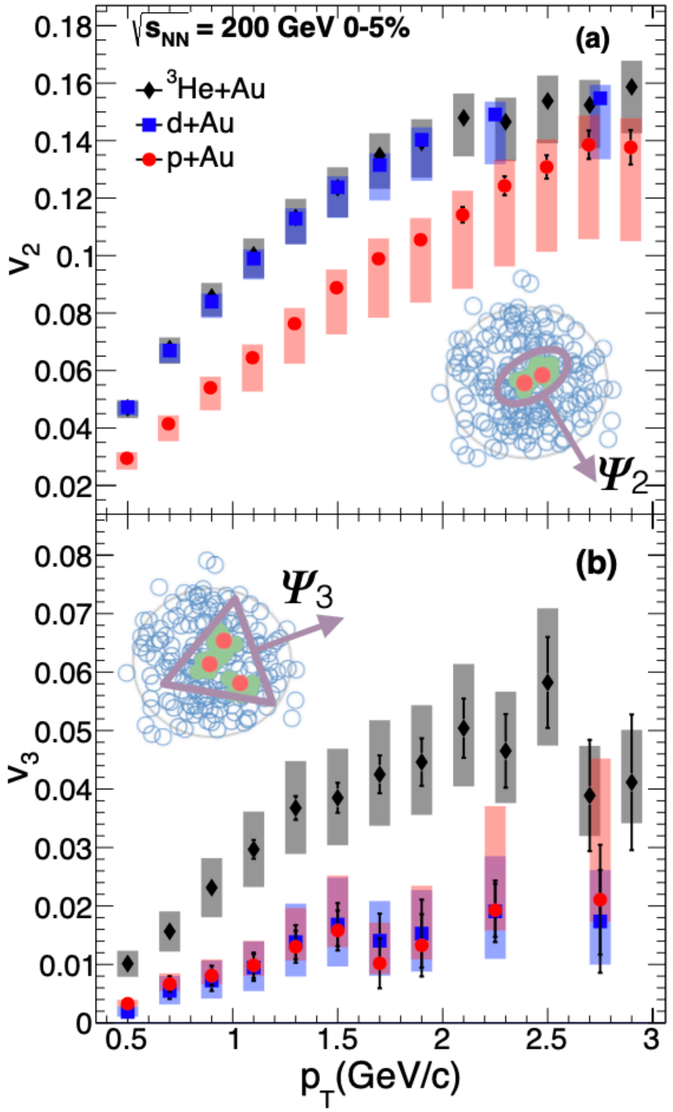
**PHENIX Collaboration, Nature Phys. 15 (2019) no.3, 214-220**

Idea: Engineer the geometry using different projectiles: p, d, $^3$He

Simple model expectation: (using nucleon degrees of freedom)




Results confirmed expectation:

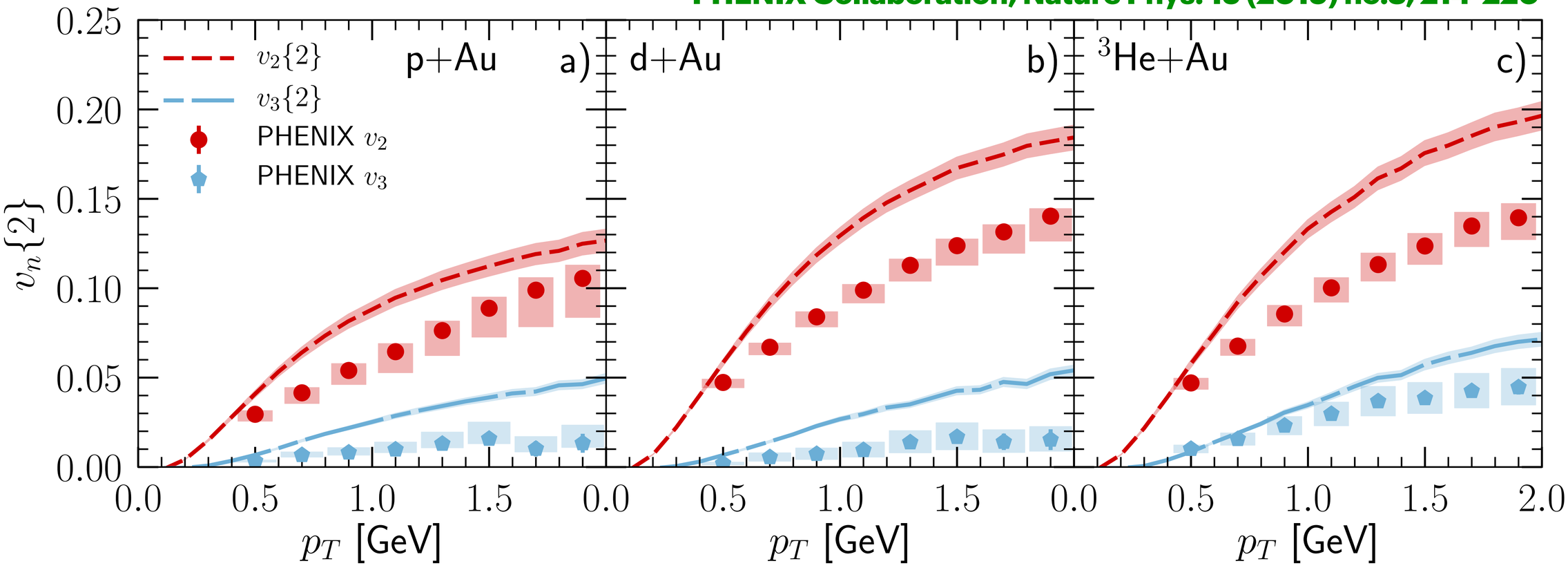


# RHIC SYSTEM SCAN

**B. Schenke, C. Shen, P. Tribedy, Phys. Rev. C 102 (2020) 4, 044905**

Model results. Again, no fine tuning after fitting Au+Au data

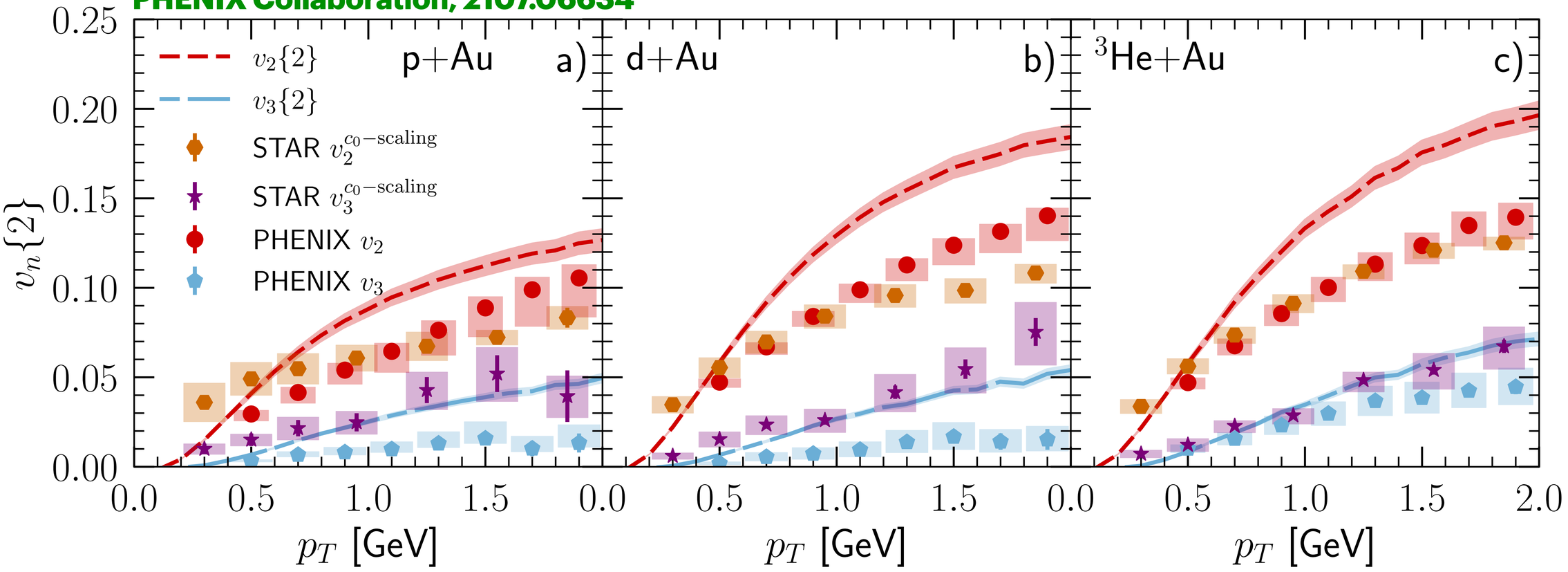
**PHENIX Collaboration, Nature Phys. 15 (2019) no.3, 214-220**



# PHENIX VS. STAR

- Task force issued a report on the origin of the differences
  https://indico.bnl.gov/event/11308/contributions/47820/attachments/35369/57704/Dunlop_PAC_2021.pdf
- PHENIX sees large changes in $v_3$ when changing η

PHENIX Collaboration, 2107.06634




PHENIX Collaboration, Nature Phys. 15 (2019) no.3, 214-220, STAR data: New at QM2019, Shengli Huang IS 2021

B. Schenke, C. Shen, P. Tribedy, Phys. Rev. C 102 (2020) 4, 044905

# STAR vs. PHENIX: Slide from P. Tribedy

## Strong acceptance dependence of (raw) harmonic coefficients

PHENIX (3x2PC,EP)
$v_2(^3He+Au) \sim v_2(d+Au) > v_2(p+Au)$ ☺
$v_3(^3He+Au) > v_3(d+Au) \sim v_3(p+Au)$ ☺

PHENIX collab, Nature Phys. 15, 214–220 (2019)

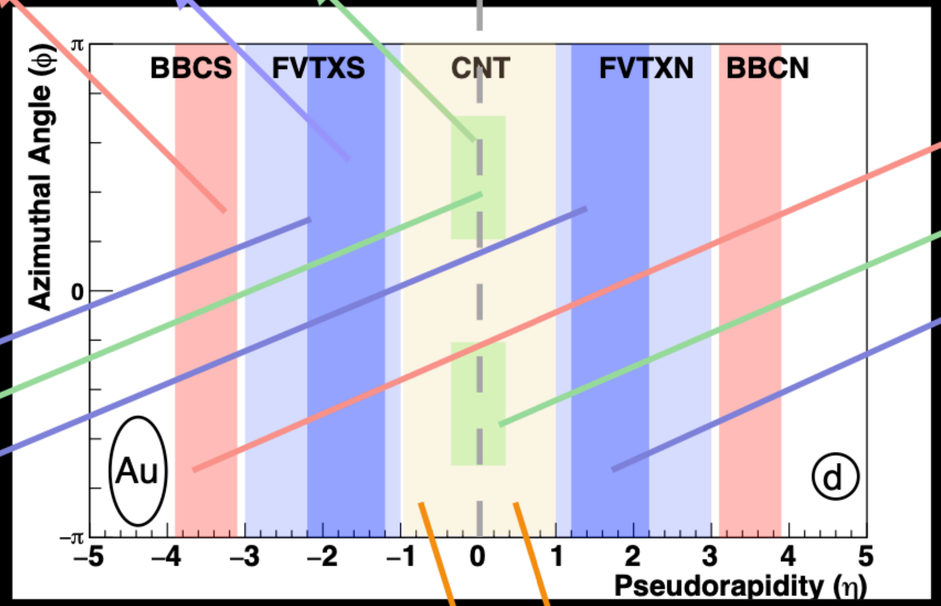
CGC+Hydro (Boost invariant)
$v_2(^3He+Au) \sim v_2(d+Au) > v_2(p+Au)$ ☺
$v_3(^3He+Au) \sim v_3(d+Au) \sim v_3(p+Au)$ ☹

Schenke, Shen, PT, Phys.Lett.B 803 (2020) 135322

PHENIX (3x2PC)
$v_2(p+Au) \geq v_2(d+Au) \geq v_2(^3He+Au)$ ☹

$v_3(^3He+Au) > 0$
$v_3(d+Au) = ?$
$v_3(p+Au) = ?$ ☹

PHENIX collab., arXiv: 2107.06634v1,
Nagle et. al, arXiv:2107.07287




PHENIX (3x2PC)
$v_2(p+Au) \geq v_2(d+Au) \geq v_2(^3He+Au)$ ☹

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$v_3(d+Au) = ?$
$v_3(p+Au) = ?$ ☹

PHENIX collab., arXiv: 2107.06634v1
Nagle et. al, arXiv:2107.07287

☺ Shape engineering works
☹ Doesn't work (or can't be checked)

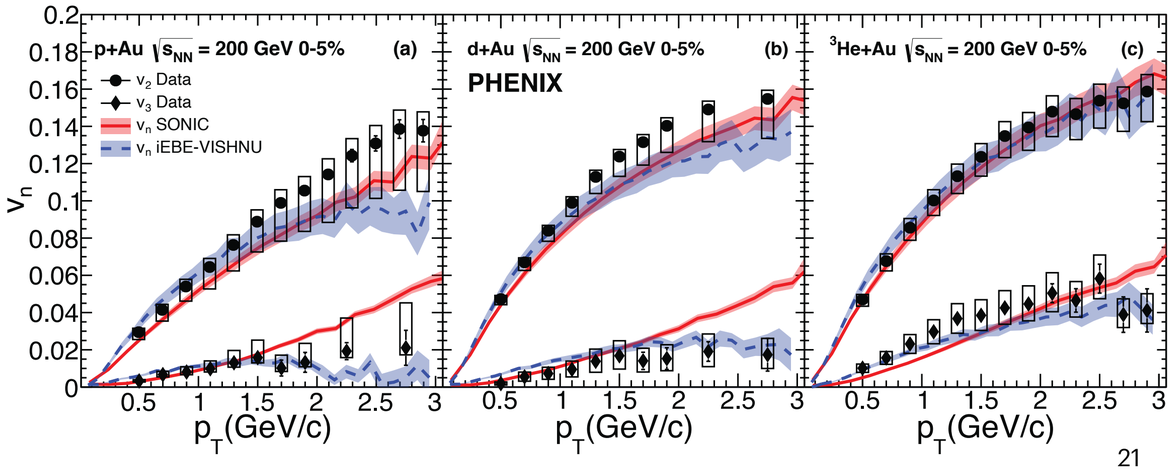
STAR TPC (2PC)
$v_2(^3He+Au) \sim v_2(d+Au) > v_2(p+Au)$ ☺
$v_3(^3He+Au) \sim v_3(d+Au) \sim v_3(p+Au)$ ☹

Shengli Huang, IS 2021

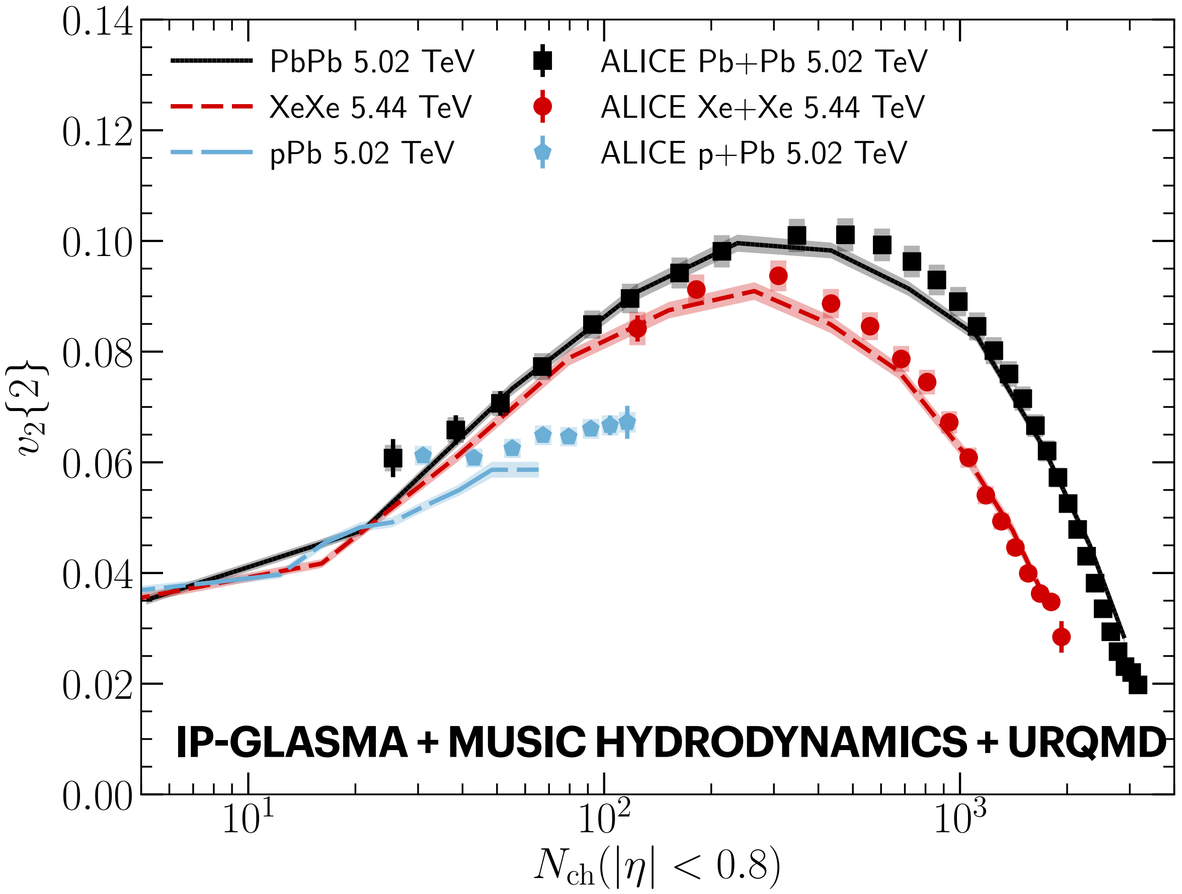
# RAPIDITY DEPENDENCE

- Task force issued a report on the origin of the differences
  https://indico.bnl.gov/event/11308/contributions/47820/attachments/35369/57704/Dunlop_PAC_2021.pdf
- PHENIX sees large changes in $v_3$ when changing η
- Strong indication that boost invariant calculations **cannot** be compared to the PHENIX data **PHENIX Collaboration, Nature Phys. 15 (2019) no.3, 214-220,**
- That includes ours **B. Schenke, C. Shen, P. Tribedy, Phys. Rev. C 102 (2020) 4, 044905**
  **B. Schenke, C. Shen, P. Tribedy, Phys.Lett.B 803 (2020) 135322**
- But also SONIC
  and iEbE-VISHNU results

**M. Habich, J.L. Nagle, P. Romatschke**
**Eur. Phys. J. C 75, 15 (2015)**
**C. Shen, J.-F.,Paquet, G. S. Denicol, S. Jeon, S., C. Gale**
**Phys. Rev. C 95, 014906 (2017).**



# ORIGINS OF MOMENTUM ANISOTROPY




**Anisotropic flow in heavy ion and high multiplicity small system collisions is driven by final state response to the initial geometry**

B. Schenke, C. Shen, P. Tribedy, Phys.Rev.C 102 (2020) 044905
ALICE Collaboration, Phys.Rev.Lett. 123 (2019) 142301

# ORIGINS OF MOMENTUM ANISOTROPY

**The initial momentum distribution is already anisotropic**

**The Color Glass Condensate predicts anisotropic particle production because of**

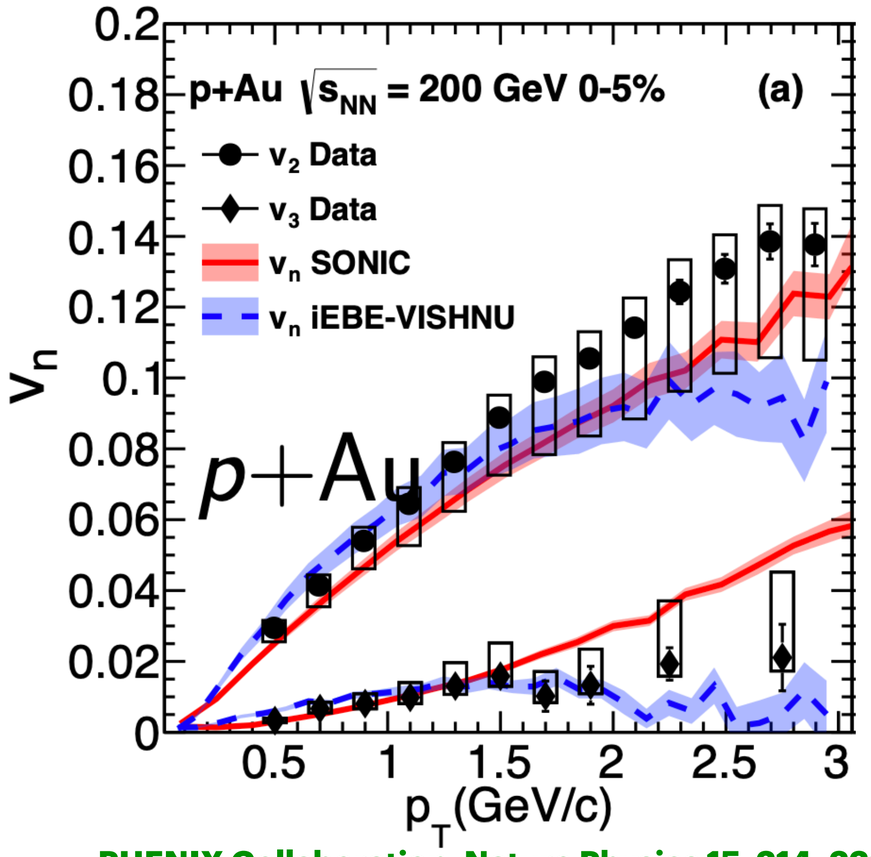
1. **Local anisotropies in the color fields**
2. **Local density gradients**
3. **Quantum interference effects**


Gelis,Lappi Venugopalan PRD 78 054020 (2008), PRD 79 094017 (2009)
Dumitru, Gelis, McLerran,Venugopalan NPA810, 91 (2008)
Dumitru, Jalilian-Marian PRD 81 094015 (2010)
Dusling, Venugopalan PRD 87 (2013)
A. Dumitru, A.V. Giannini, Nucl.Phys.A933 (2014) 212
V. Skokov. Phys.Rev.D91 (2015) 054014
T. Lappi, B. Schenke, S. Schlichting, R. Venugopalan, JHEP 1601 (2016) 061
Kovner, Skokov, Phys.Rev. D98 (2018) no.1, 014004
...

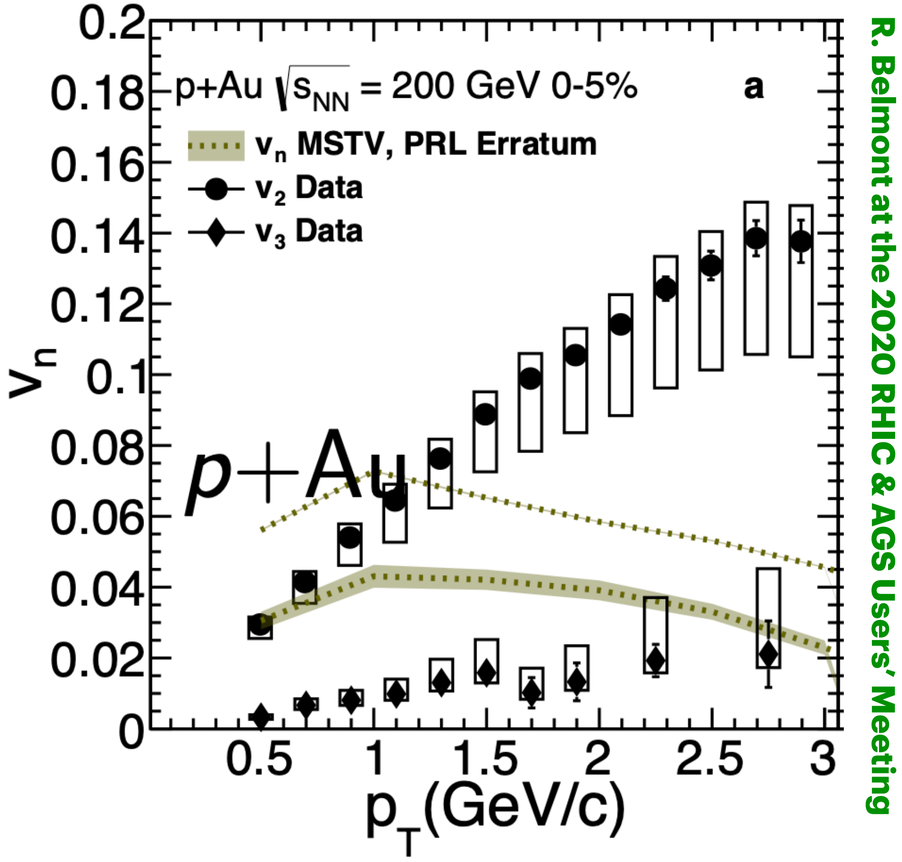
# FINAL STATE EFFECTS ARE NEEDED

**Qualitative features of the data are not well described by initial state anisotropy *alone***

**GEOMETRY RESPONSE ONLY**




**INITIAL MOMENTUM ANISOTROPY ONLY**




R. Belmont at the 2020 RHIC & AGS Users' Meeting

PHENIX Collaboration, Nature Physics 15, 214–220 (2019)

M. Mace, V. V. Skokov, P. Tribedy, R. Venugopalan , Phys. Rev. Lett. 123, 039901 (Erratum) (2019)

# BUT INITIAL STATE EFFECTS ARE THERE

**Our calculation using the IP-Glasma initial state and hydrodynamics includes both effects**

**Initial state anisotropies are significant and can affect the final result at low multiplicity**

$$Q_\varepsilon = \frac{\mathrm{Re}\langle \mathscr{E} V_2^* \rangle}{\sqrt{\langle |\mathscr{E}|^2 \rangle \langle |V_2|^2 \rangle}}$$

**CORRELATION OF THE FINAL ELLIPTIC FLOW $V_2$ WITH**

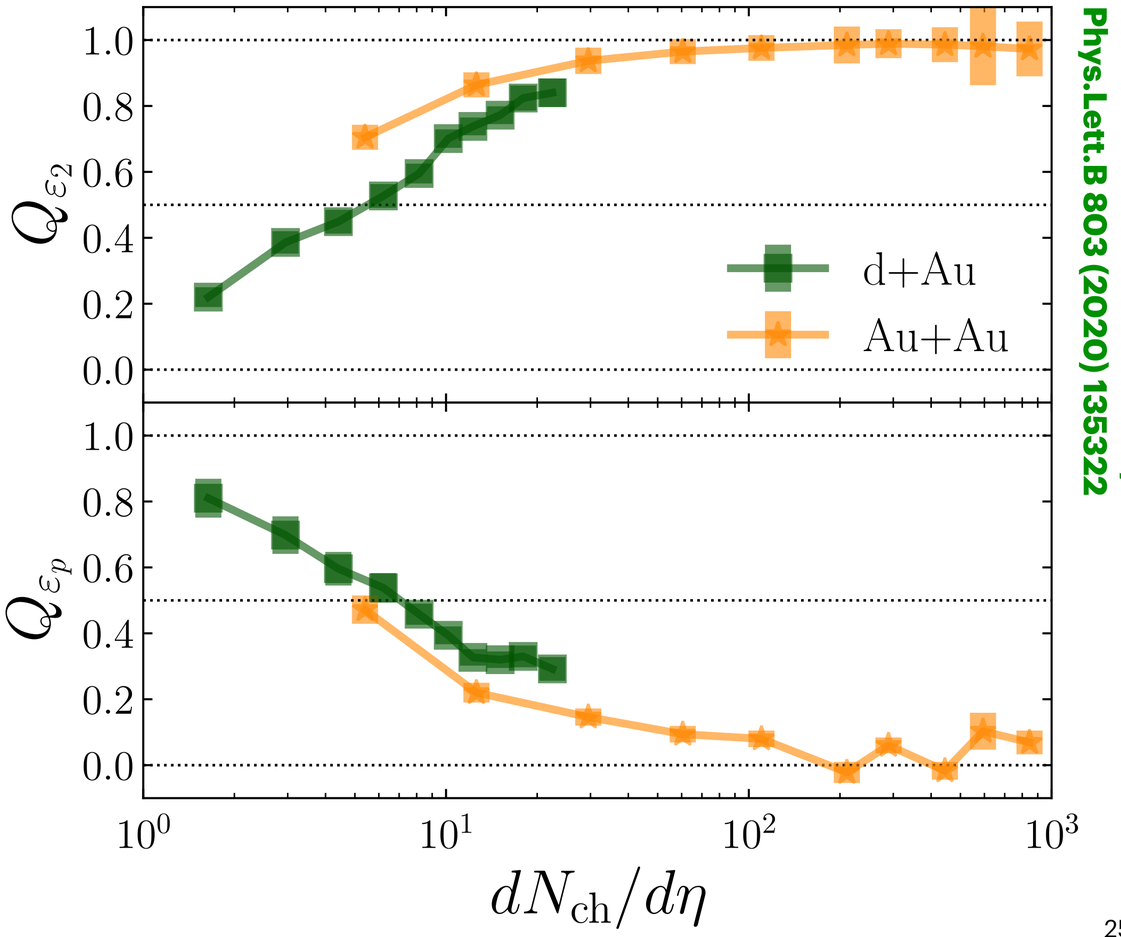
**THE GEOMETRIC ELLIPTICITY**

$$\mathscr{E}_2 = \varepsilon_2 e^{i2\psi_2} = \frac{\langle x^2 - y^2 \rangle + i\langle 2xy \rangle}{\langle x^2 + y^2 \rangle}$$

**AND**

**THE INITIAL MOMENTUM ANISOTROPY**

$$\mathscr{E}_p = \varepsilon_p e^{i2\psi_2^p} = \frac{\langle T^{xx} - T^{yy} \rangle + i\langle 2T^{xy} \rangle}{\langle T^{xx} + T^{yy} \rangle}$$




B. Schenke, C. Shen, P. Tribedy
Phys.Lett.B 803 (2020) 135322

# HOW TO DISTINGUISH THE SOURCE OF ANISOTROPY

P. Bozek, Phys. Rev. C 93, 044908 (2016); B. Schenke, C. Shen, D. Teaney, Phys. Rev. C 102, 034905 (2020)
G. Giacalone, B. Schenke, C. Shen, Phys. Rev. Lett. 125 (2020) 19, 192301

Use the correlation of mean transverse momentum $[p_T]$ and $v_2^2$ at fixed multiplicity:

$$\hat{\rho}(v_n^2, [p_T]) = \frac{\langle \hat{\delta} v_n^2 \, \hat{\delta}[p_T] \rangle}{\langle (\hat{\delta} v_n^2)^2 \rangle \langle (\hat{\delta}[p_T])^2 \rangle}$$

$$\hat{\delta O} \equiv \delta O - \frac{\langle \delta O \delta N \rangle}{\sigma_N^2} \delta N$$

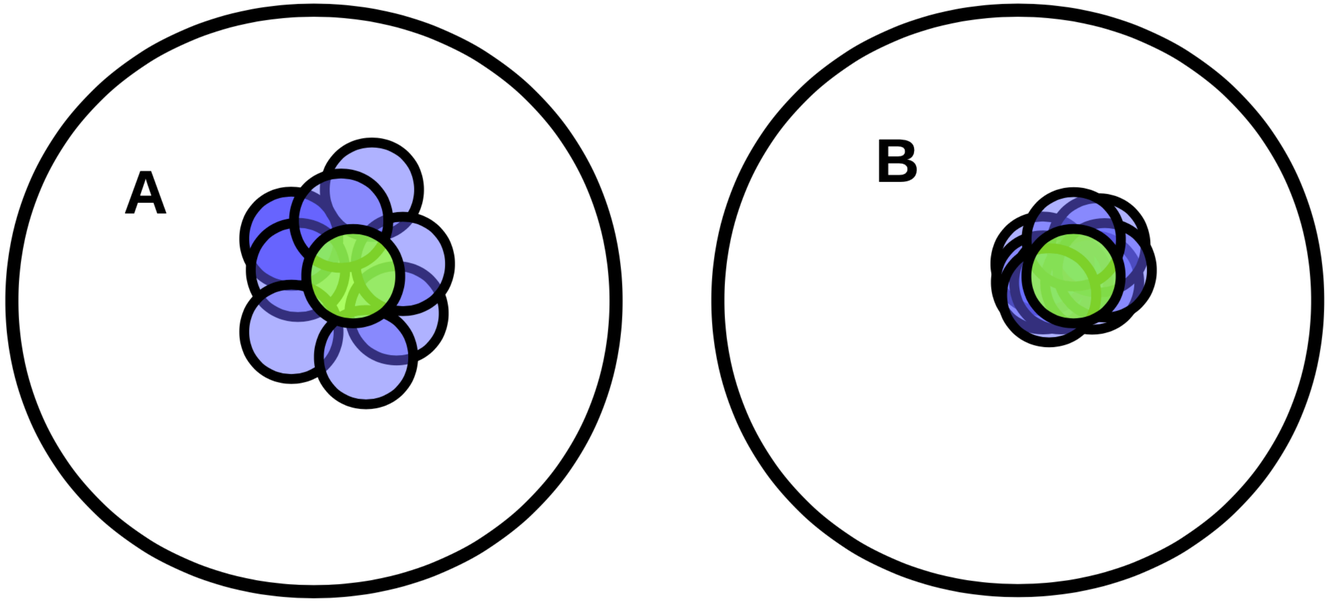
A. Olszewski, W. Broniowski, Phys. Rev. C96, 054903 (2017)

It can help to identify the source of the azimuthal anisotropy in the experimental data

The two origins of $v_2$ have very distinct predictions for this correlator

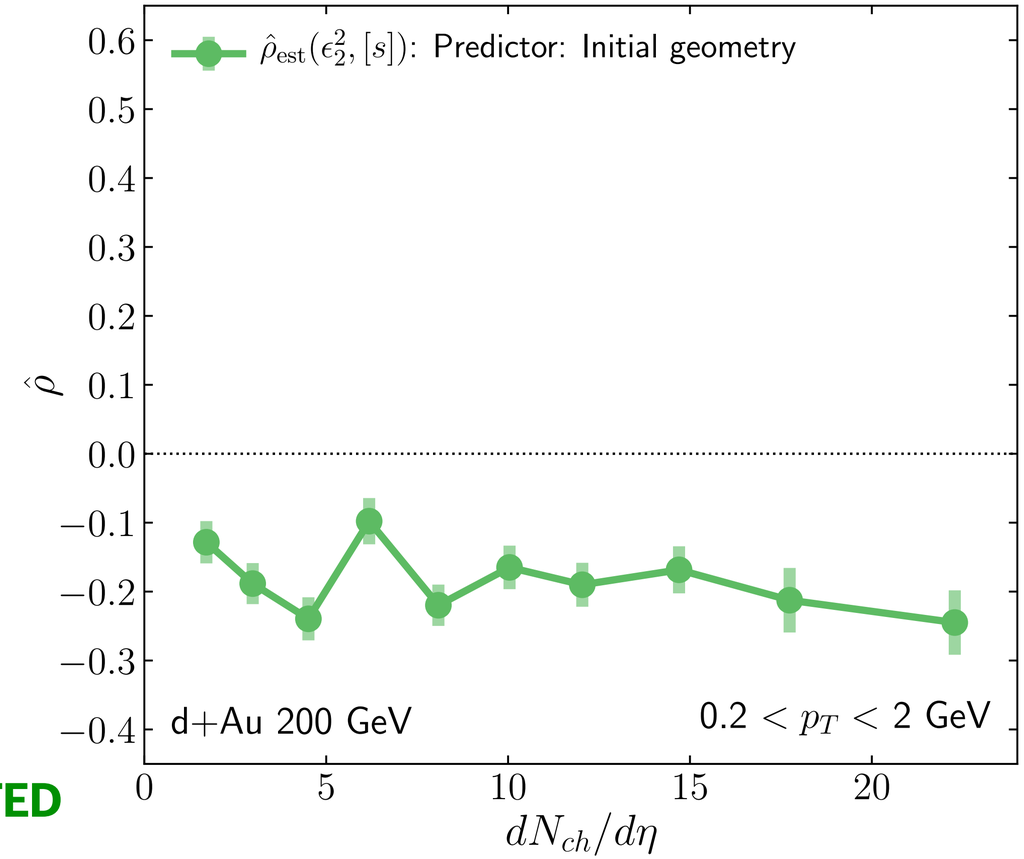
# SMALL SYSTEMS: CORRELATION FROM GEOMETRY

**G. Giacalone, B. Schenke, C. Shen, Phys. Rev. Lett. 125 (2020) 19, 192301**




$$R(A) > R(B)$$
$$\langle p_T \rangle(A) < \langle p_T \rangle(B)$$
$$\varepsilon_2(A) > \varepsilon_2(B)$$

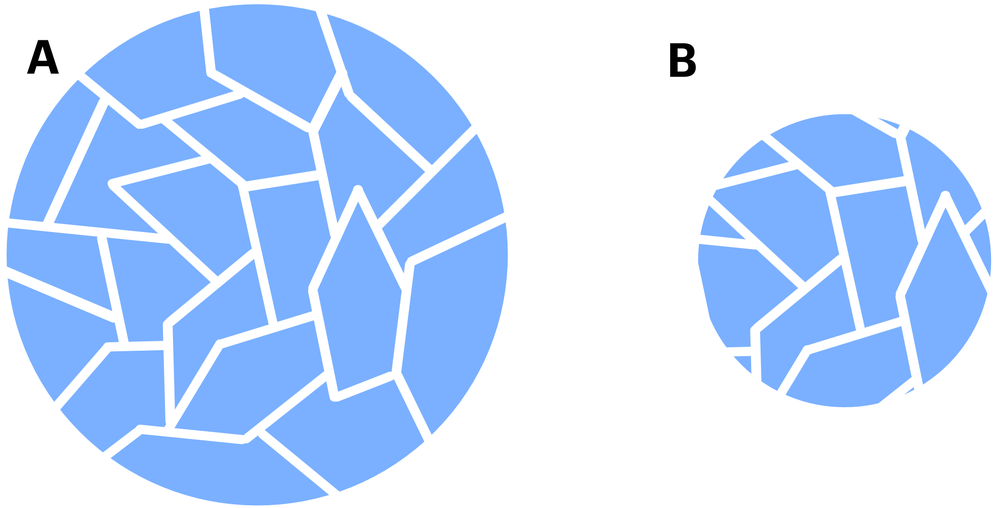
} $v_2$ **AND** $\langle p_T \rangle$ **ARE ANTI-CORRELATED**



# CORRELATION FROM INITIAL MOMENTUM ANISOTROPY

**G. Giacalone, B. Schenke, C. Shen, Phys. Rev. Lett. 125 (2020) 19, 192301**

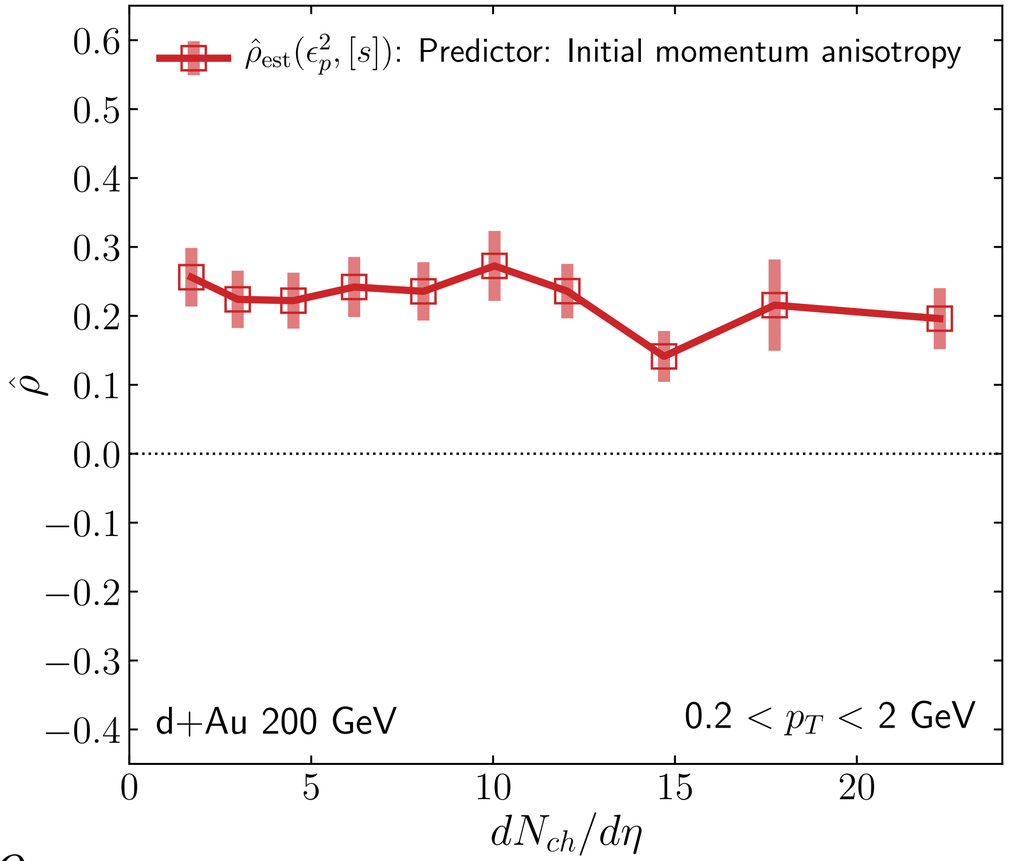
**COLOR DOMAIN CARTOON (PARTICLES PRODUCED FROM ONE DOMAIN ARE CORRELATED):**




$$R(A) > R(B)$$
$$\langle p_T \rangle(A) < \langle p_T \rangle(B)$$
$$\varepsilon_p(A) < \varepsilon_p(B)$$

} $v_2$ **AND** $\langle p_T \rangle$ **ARE CORRELATED**

**BECAUSE A HAS MORE DOMAINS**




**NOTE: Multiplicity is dominated by projectile $Q_s$**
**At fixed multiplicity, for the target: $Q_s(A) \approx Q_s(B) \rightarrow$ domain sizes $1/Q_s$ are the same**
**$\langle p_T \rangle$ assumed to be still driven by the geometry**

# WHICH ONE SURVIVES?

G. Giacalone, B. Schenke, C. Shen, Phys. Rev. Lett. 125 (2020) 19, 192301

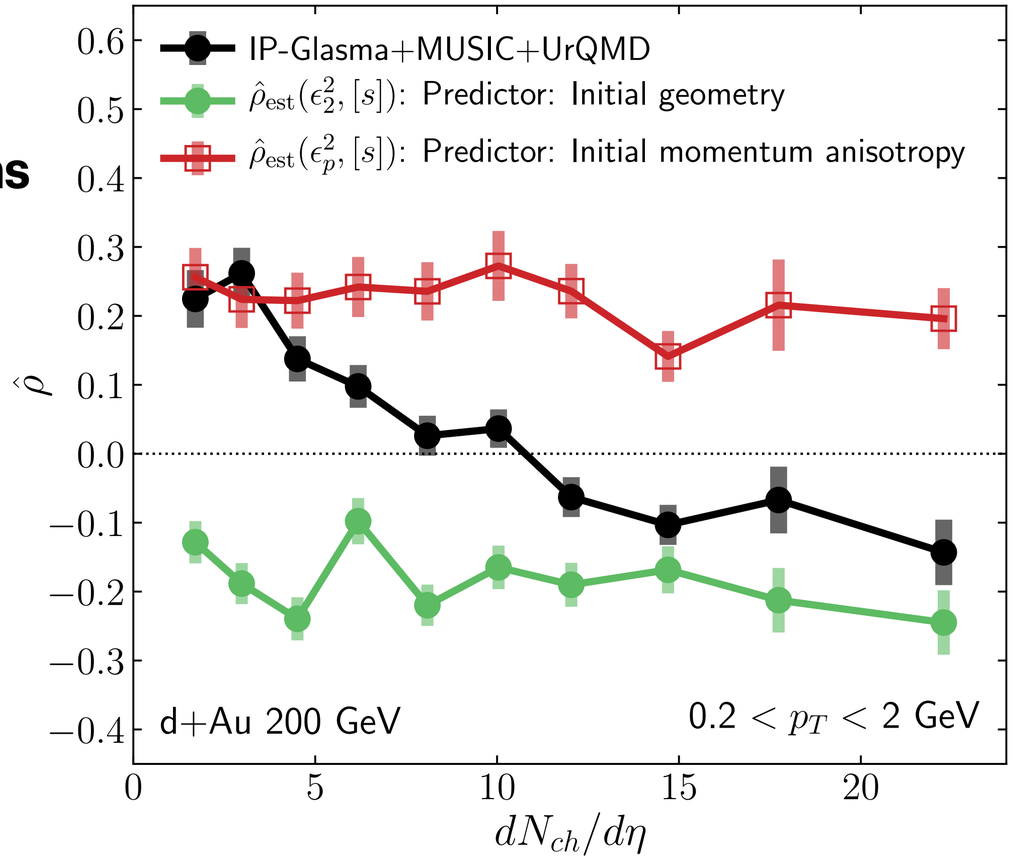
**IT DEPENDS ON MULTIPLICITY**

**We predict a sign change of the $\rho$ correlator with multiplicity in p/d+Au collisions at RHIC and p+Pb collisions at LHC around $dN_{\rm ch}/d\eta \approx 10$**

**Below $dN_{\rm ch}/d\eta \approx 10$**
**initial momentum anisotropy dominates**

**Above $dN_{\rm ch}/d\eta \approx 10$**
**final state response to the initial geometry dominates**

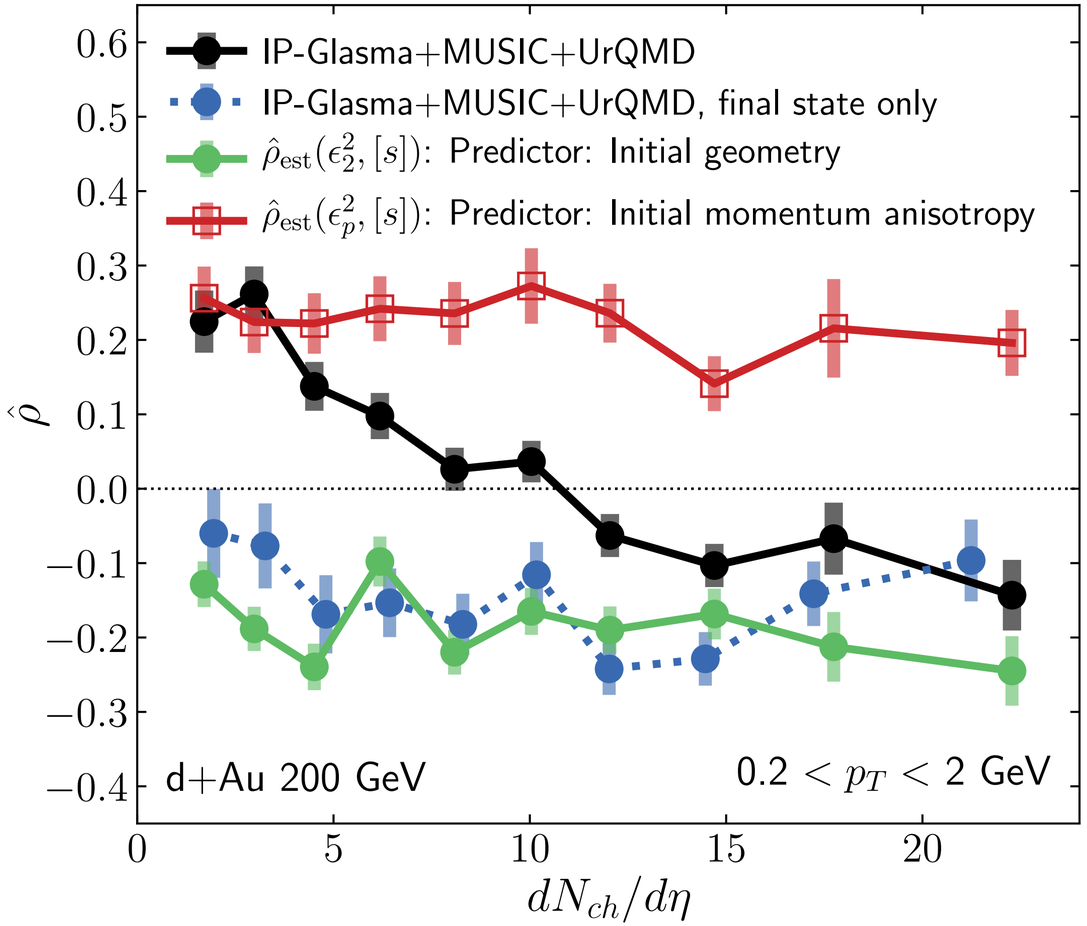
**The observable moves from one predictor to the other**



# CHECK: REMOVE THE INITIAL STATE MOMENTUM ANISOTROPY

G. Giacalone, B. Schenke, C. Shen, Phys. Rev. Lett. 125 (2020) 19, 192301

**With no initial momentum anisotropy, our result follows the geometry predictor for all $dN_{\rm ch}/d\eta$ as expected**



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G. Giacalone, B. Schenke, C. Shen, Phys. Rev. Lett. 125 (2020) 19, 192301

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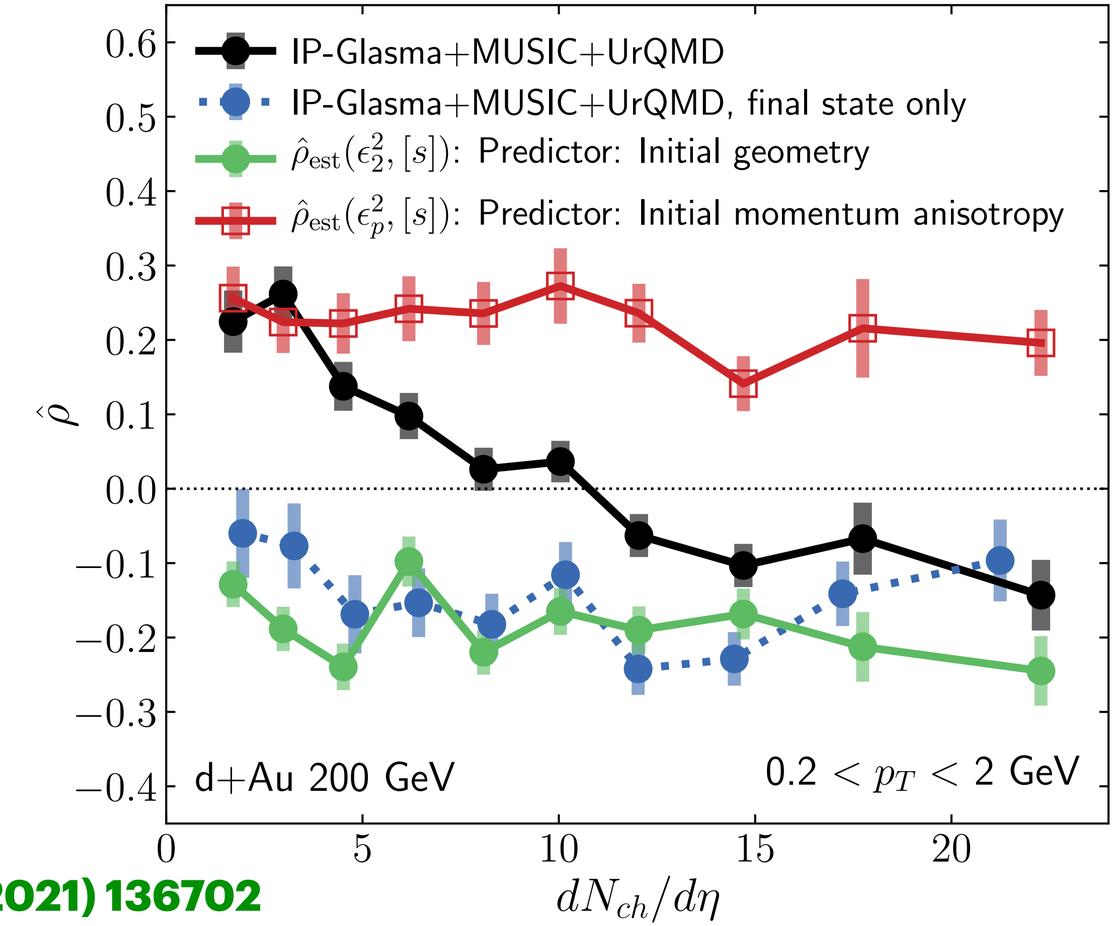



Is it detectable?

Very low multiplicity

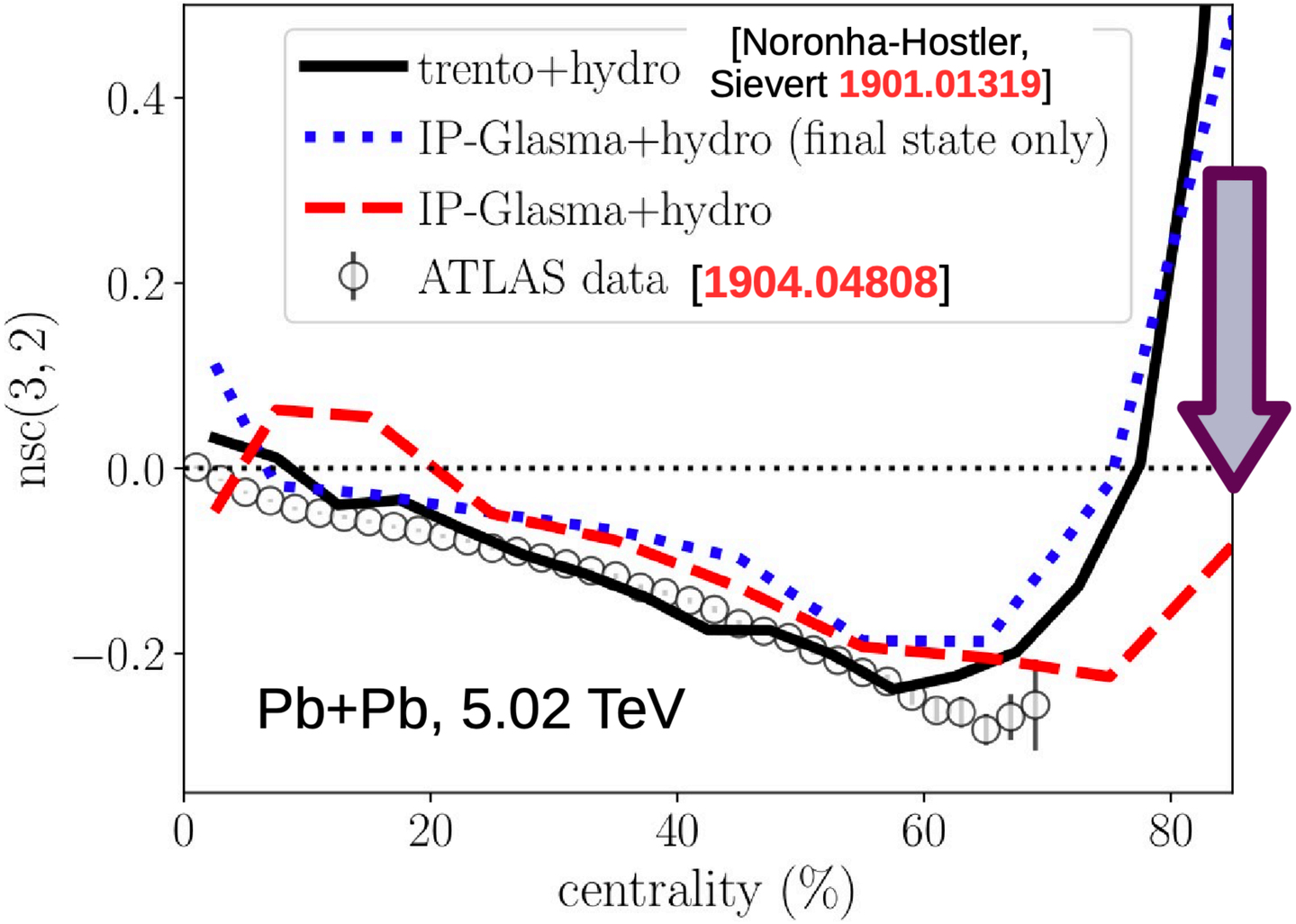
Non-flow can mimic the signal

C. Zhang, A. Behera, S. Bhatta, J. Jia, Phys.Lett.B 822 (2021) 136702

S. H. Lim, J. L. Nagle, Phys.Rev.C 103 (2021) 6, 064906

# CORRELATION BETWEEN $v_2^2$ AND $v_3^2$

**G. Giacalone, B. Schenke, C. Shen, in preparation**



# BEYOND BOOST-INVARIANCE

**What is the rapidity dependence of the initial momentum space anisotropy?**

**Does it survive the experimentally applied gap?**

# PART II: GOING BEYOND BOOST INVARIANCE

- Use the CGC including JIMWLK evolution
- Do not assume boost invariance

# BEYOND BOOST-INVARIANCE

**work in progress with Pragya Singh and Sören Schlichting**

## Employ JIMWLK small-x evolution to the proton and nucleus

**as in B. Schenke, S. Schlichting, Phys.Lett.B 739 (2014) 313-319 and Phys.Rev.C 94 (2016) 4, 044907**

Functional Langevin equation

$$V_{\mathbf{x}_\perp}(Y+dY) = \exp\left\{-i\frac{\sqrt{\alpha_s dY}}{\pi}\int_{\mathbf{z}_\perp} K_{\mathbf{x}_\perp-\mathbf{z}_\perp}\cdot\left(V_{\mathbf{z}_\perp}\boldsymbol{\xi}_{\mathbf{z}_\perp}V^\dagger_{\mathbf{z}_\perp}\right)\right\} \times V_{\mathbf{x}_\perp}(Y)\exp\left\{i\frac{\sqrt{\alpha_s dY}}{\pi}\int_{\mathbf{z}_\perp} K_{\mathbf{x}_\perp-\mathbf{z}_\perp}\cdot\boldsymbol{\xi}_{\mathbf{z}_\perp}\right\}, \quad (7)$$

IR regularized JIMWLK Kernel

$$K_{\mathbf{x}_\perp-\mathbf{z}_\perp} = m|\mathbf{x}_\perp-\mathbf{z}_\perp|\,K_1(m|\mathbf{x}_\perp-\mathbf{z}_\perp|)\,\frac{\mathbf{x}_\perp-\mathbf{z}_\perp}{(\mathbf{x}_\perp-\mathbf{z}_\perp)^2}$$

## Compute the energy momentum tensor in different rapidity bins after Yang-Mills evolution

## Study the correlation function of the geometry and initial momentum anisotropy as a function of rapidity

# EVOLUTION OF THE PROTON

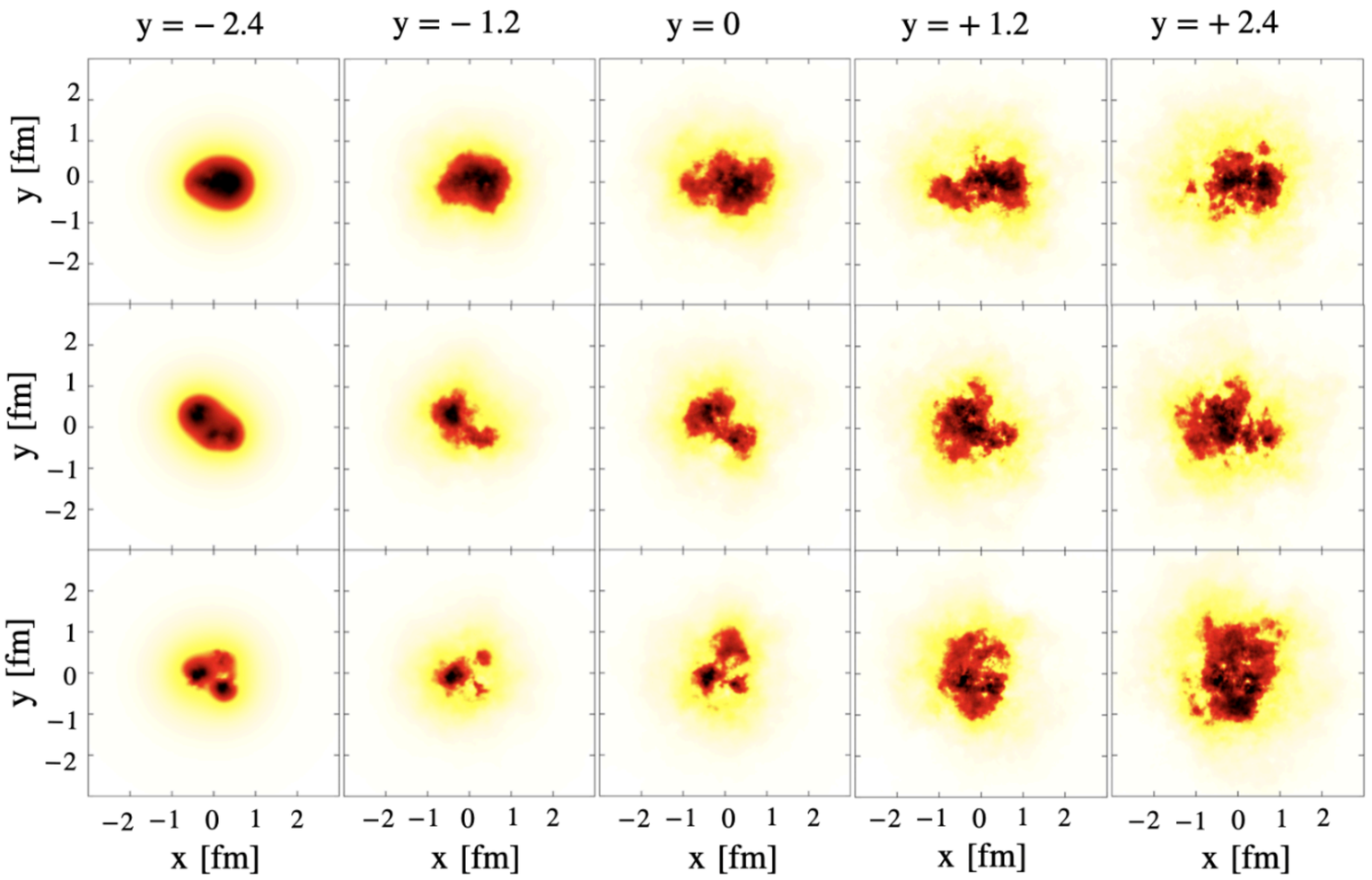
**work in progress with Pragya Singh and Sören Schlichting**

3 EXAMPLES

PROTON 1

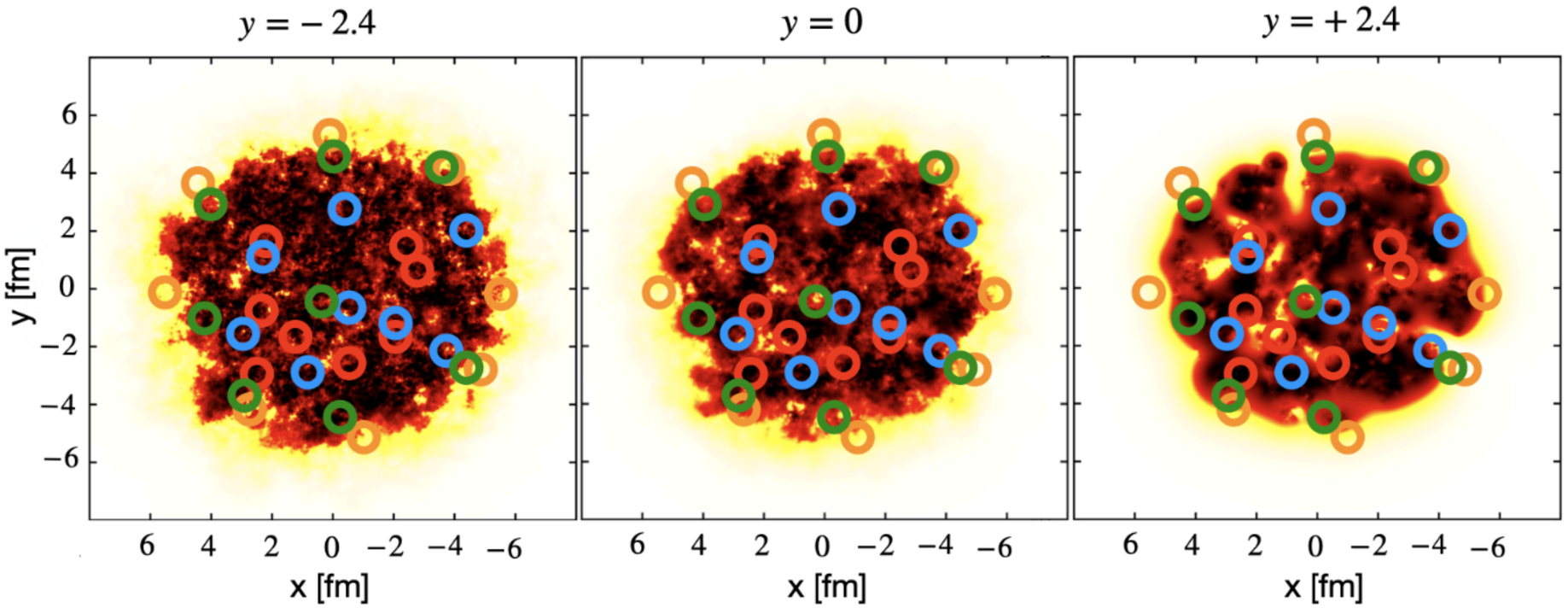
PROTON 2

PROTON 3



# EVOLUTION OF THE NUCLEUS

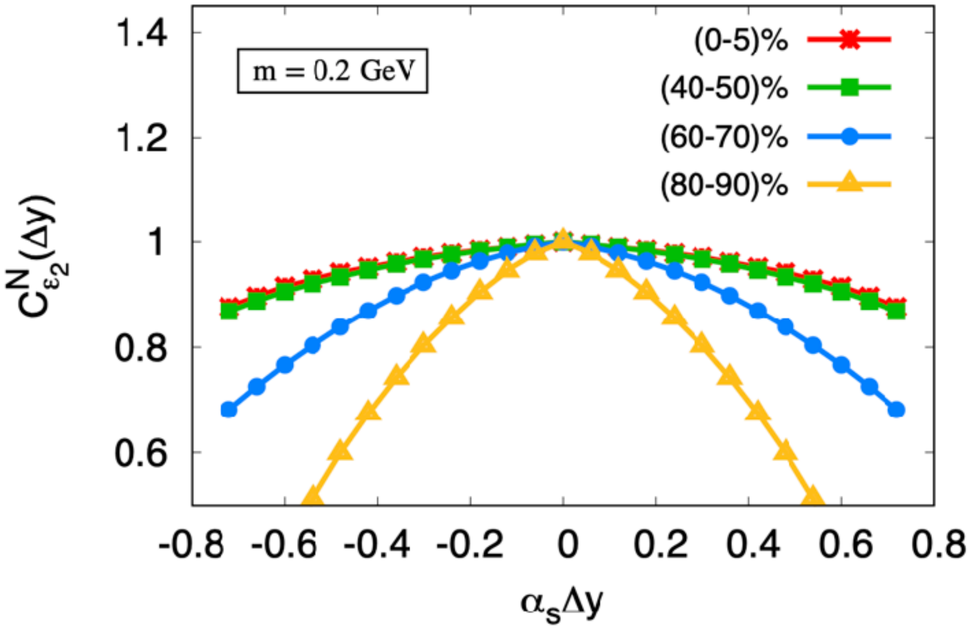
**work in progress with Pragya Singh and Sören Schlichting**

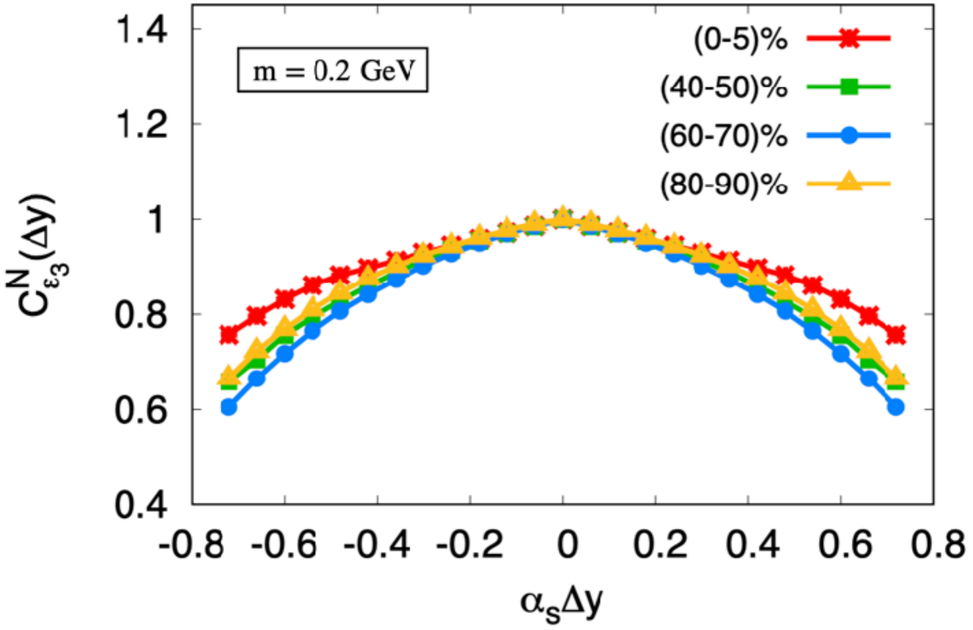



**Dots indicate places where the proton hit in a given event in centrality class 0-5% (blue), 40-50% (blue), 60-70% (green), 80-90% (orange)**
**Centrality is determined at mid-rapidity**

# CORRELATION OF ECCENTRICITY

work in progress with Pragya Singh and Sören Schlichting




**The geometry, quantified here with $\varepsilon_2$ and $\varepsilon_3$, decorrelates fairly slowly, at least for not too peripheral events**

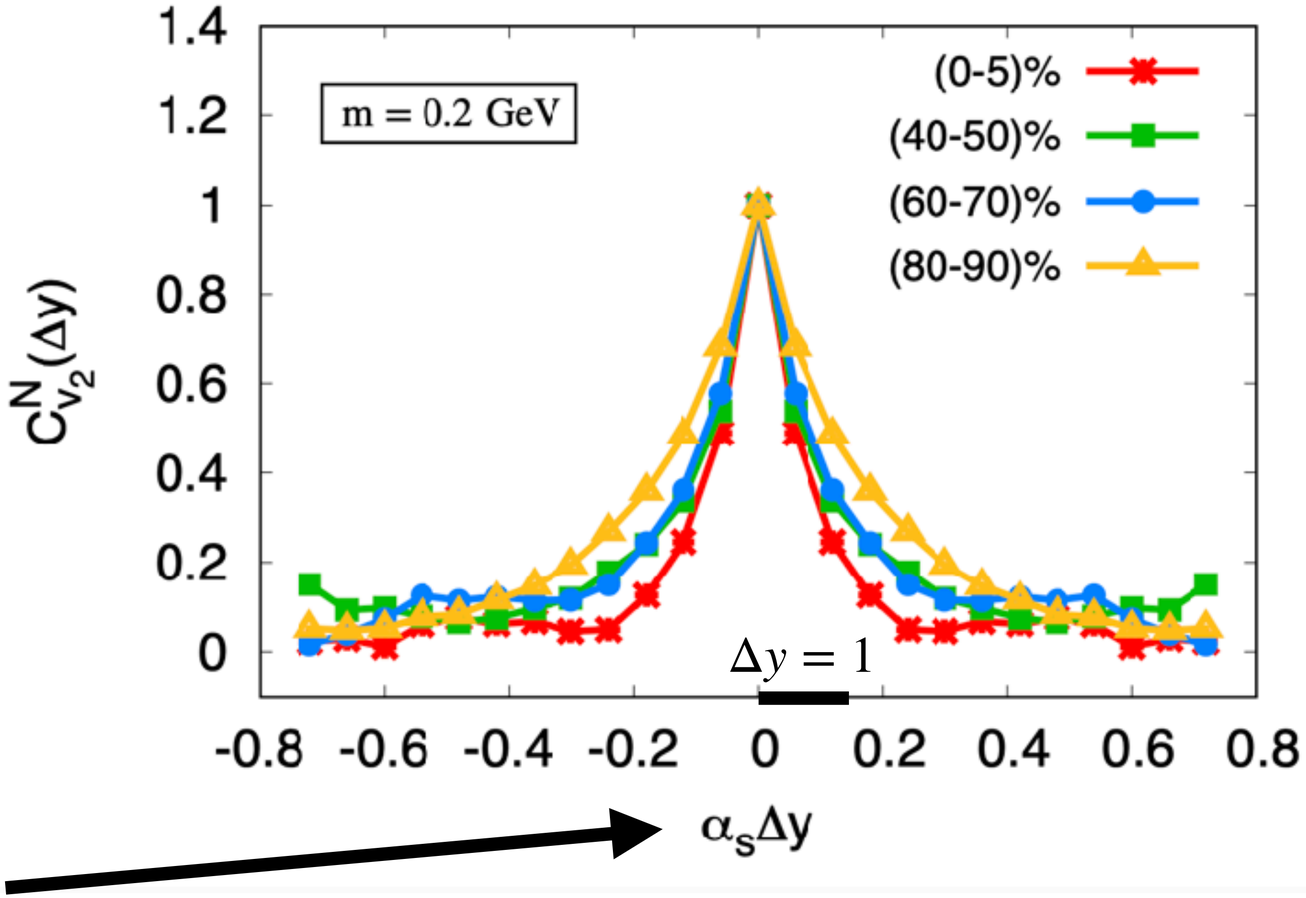
$$C^N_{\mathcal{O}}(\eta_1,\eta_2) = \frac{\left\langle \mathrm{Re}\left(\mathcal{O}(\eta_1)\mathcal{O}^*(\eta_2)\right)\right\rangle}{\sqrt{\langle\,|\,\mathcal{O}(\eta_1)\,|^2\,\rangle\langle\,|\,\mathcal{O}(\eta_2)\,|^2\,\rangle}}$$

# CORRELATION OF INITIAL MOMENTUM ANISOTROPY

work in progress with Pragya Singh and Sören Schlichting

The initial momentum anisotropy, $\varepsilon_p$, or equivalently the initial anisotropy of the gluons, decorrelates comparatively quickly

Plug in $\alpha_s = 0.15$, which works well for the rapidity dependence of the charged hadron distribution

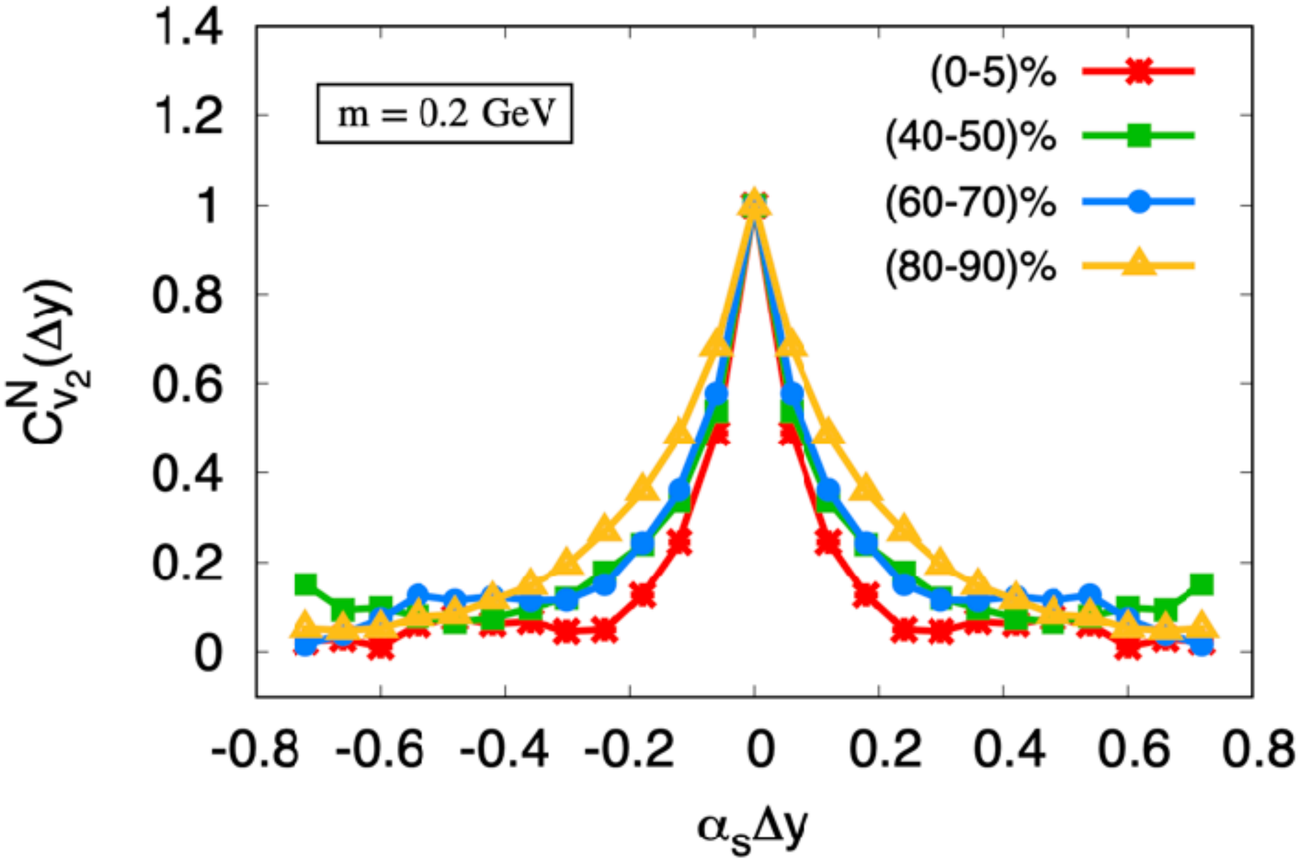


# CORRELATION OF INITIAL MOMENTUM ANISOTROPY

work in progress with Pragya Singh and Sören Schlichting

So this is a bad sign for the initial momentum anisotropy

The $\rho$ correlator is measured using different rapidity bins for $[p_T]$ and the two particles used for $v_2$ . Over the rapidity range used, the signal from initial state momentum anisotropy would have decayed significantly.



# PART III: FULL 3+1D HYDRO FOR LOWER ENERGIES

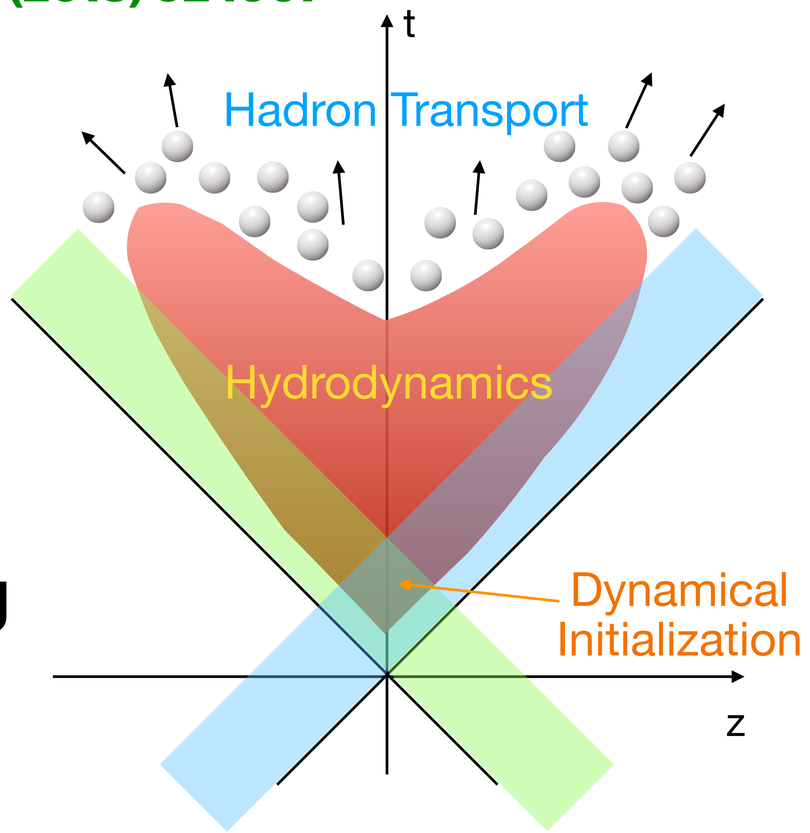
- Do not use CGC but dynamical MC-Glauber + string model
- Do full 3+1D hydrodynamic calculations

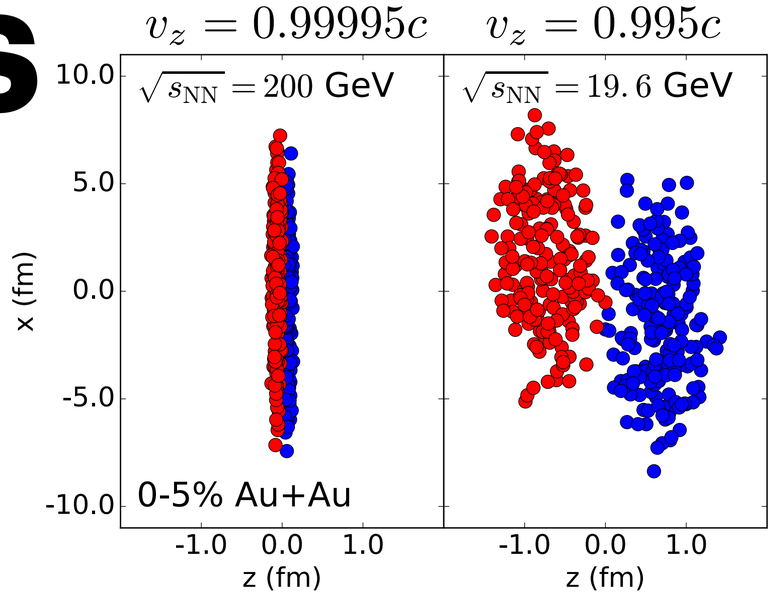
# PART III: LOWER ENERGIES

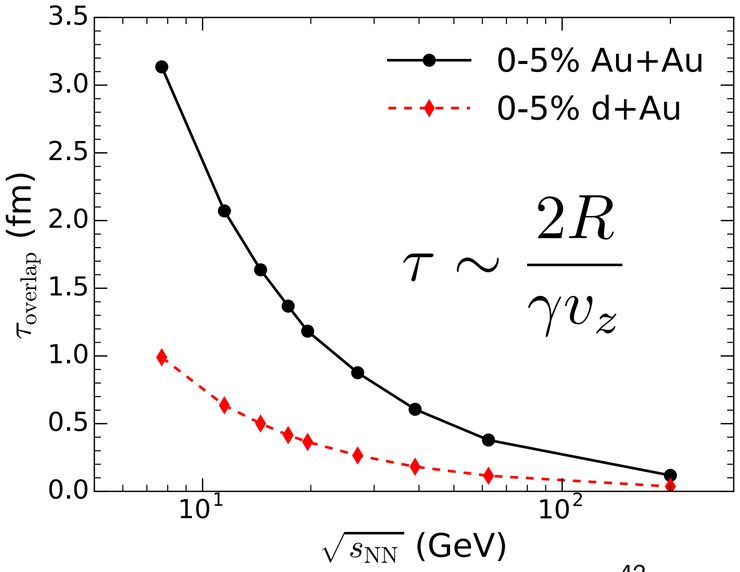
C. Shen and B. Schenke, Phys.Rev. C97 (2018) 024907

**For lower energies, the CGC has very limited validity**

**Model the initial state within a dynamical string deceleration model**

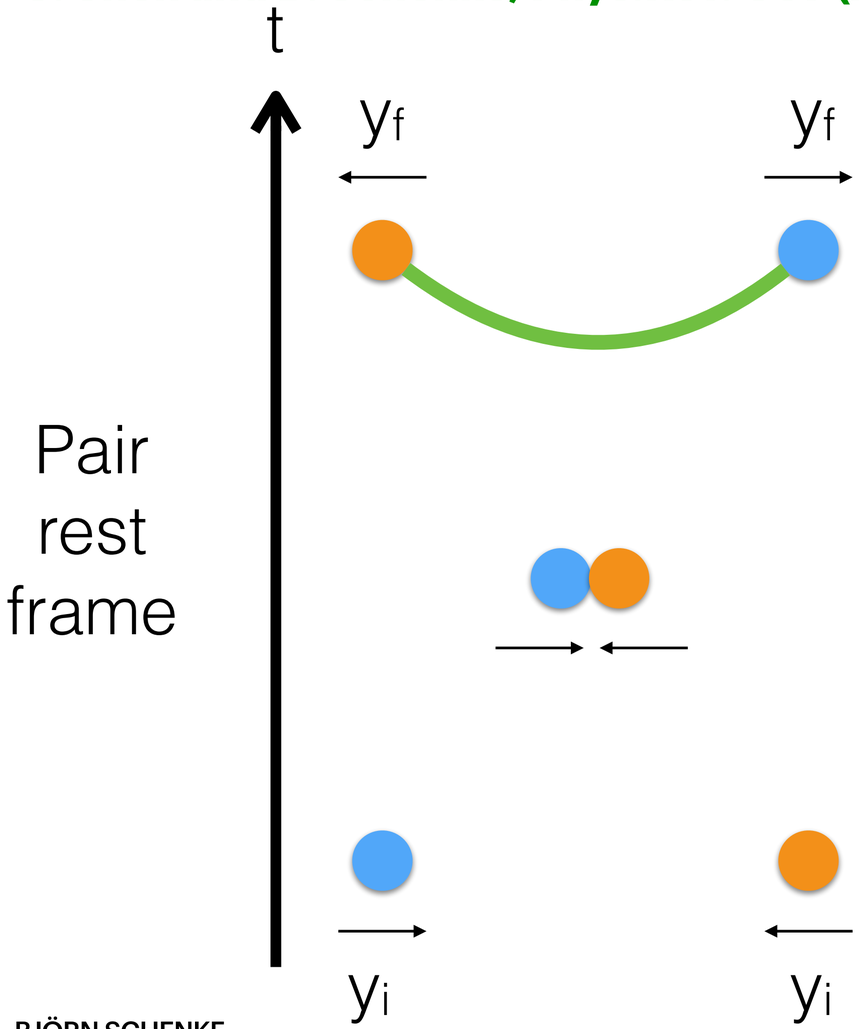


# 3D MC-GLAUBER + STRING MODEL

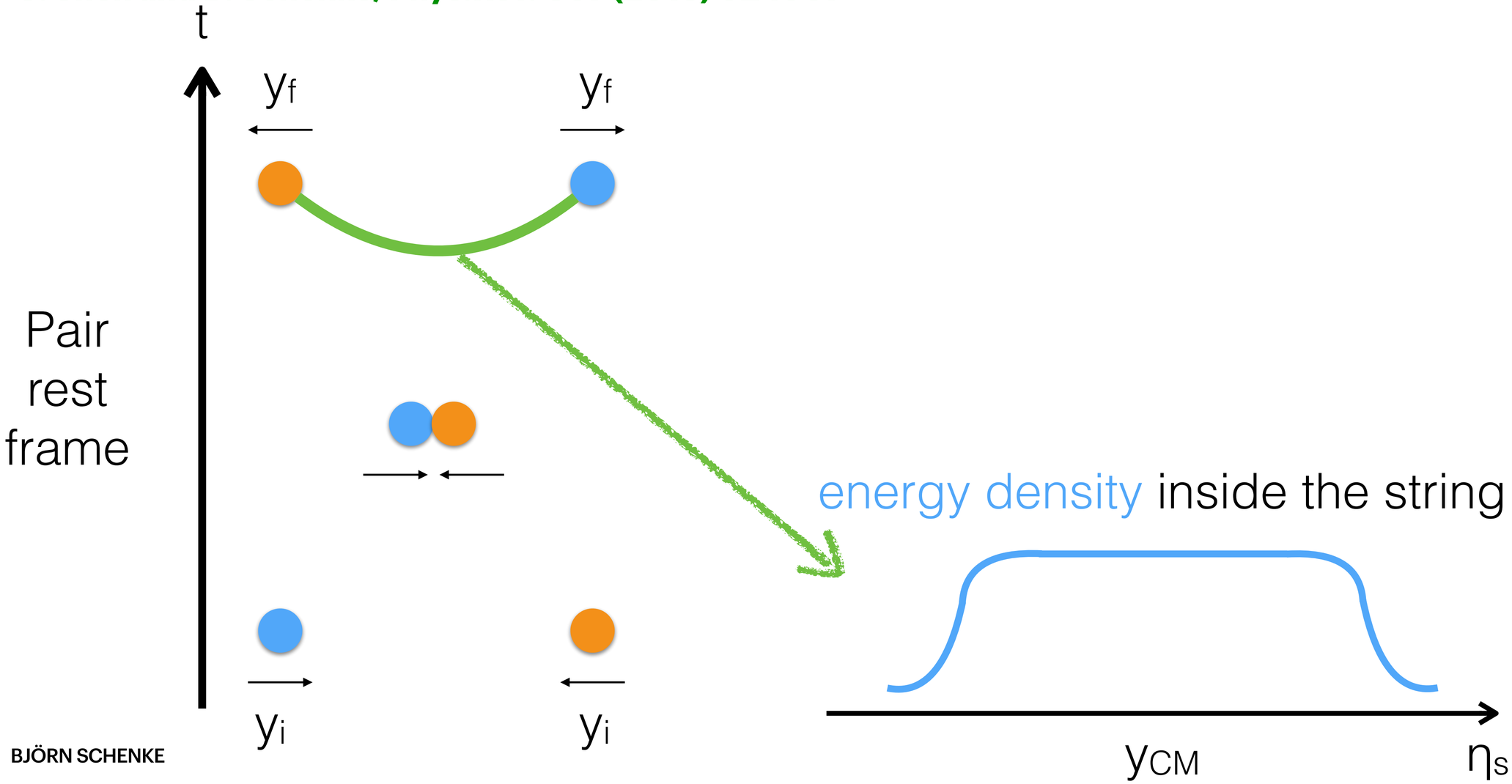
**C. Shen and B. Schenke, Phys.Rev. C97 (2018) 024907**




- Collision geometry is determined by MC-Glauber model
- 3 valence quarks are sampled from PDF and randomly picked to lose energy during a collision $\left(\sum_i x_i \leq 1\right)$
- Incoming quarks are decelerated with a classical string tension,

# 3D MC-GLAUBER + STRING MODEL

**C. Shen and B. Schenke, Phys.Rev. C97 (2018) 024907**



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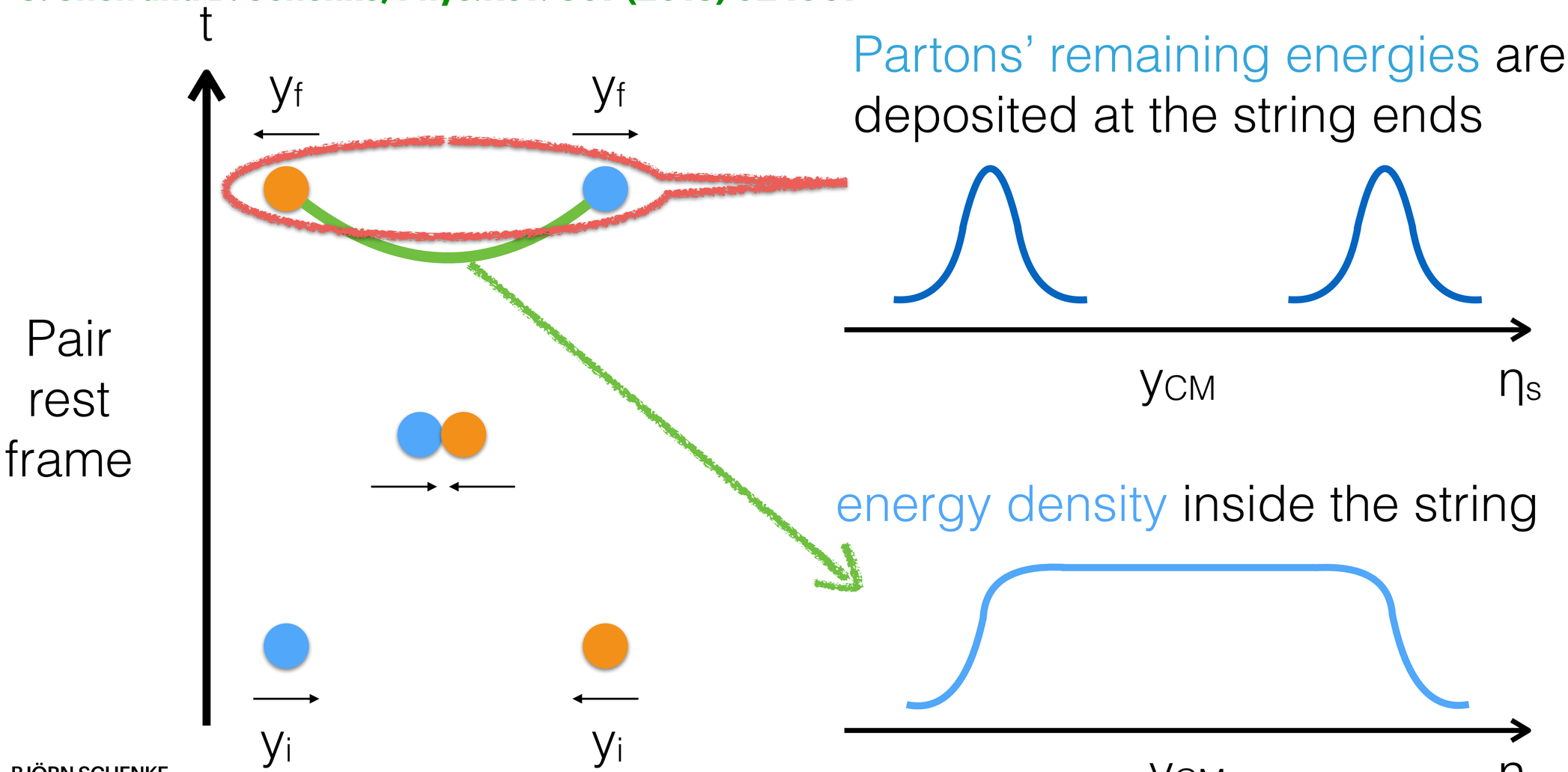
**C. Shen and B. Schenke, Phys.Rev. C97 (2018) 024907**



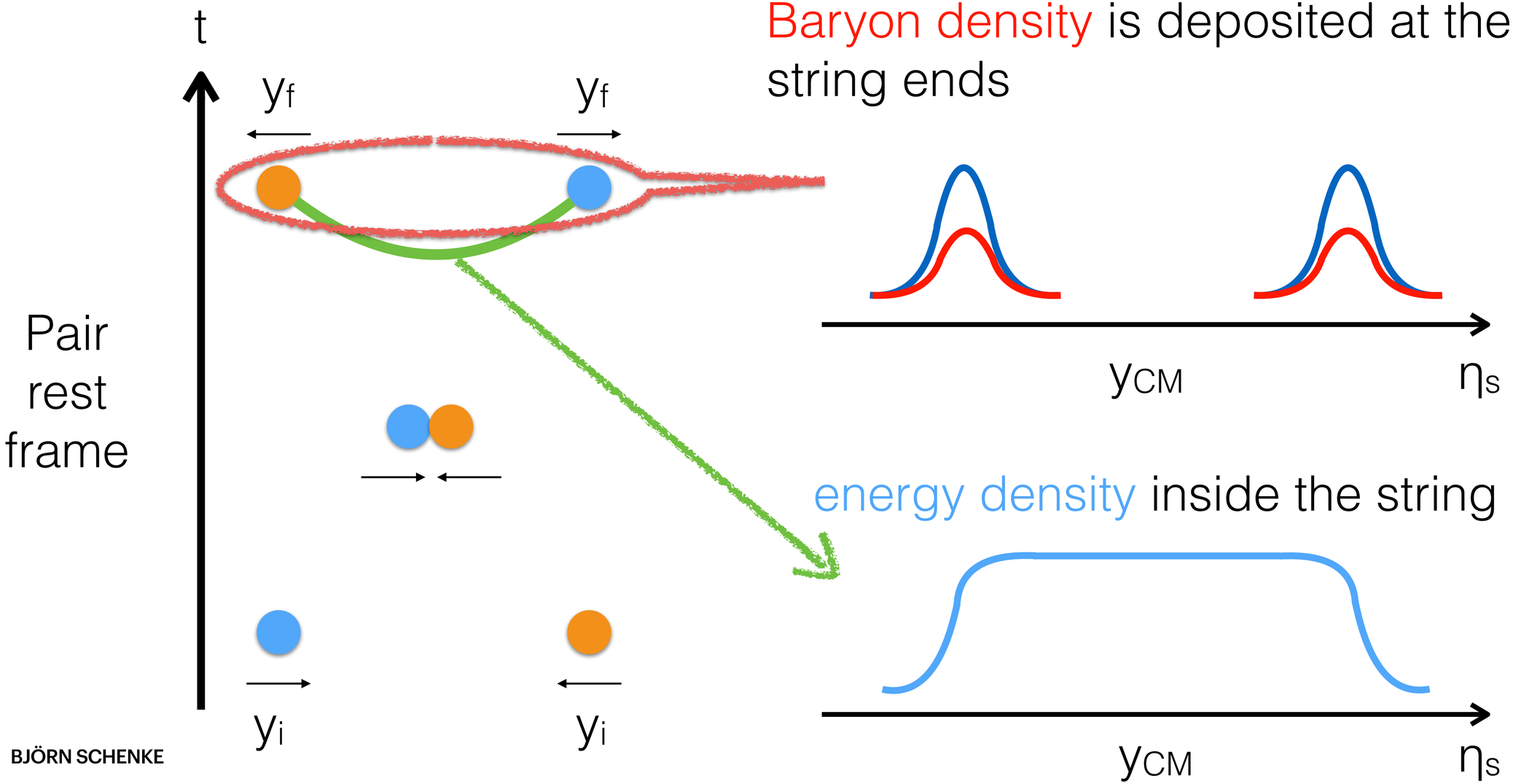

Partons' remaining energies are deposited at the string ends




energy density inside the string

# 3D MC-GLAUBER + STRING MODEL

**C. Shen and B. Schenke, Phys.Rev. C97 (2018) 024907**



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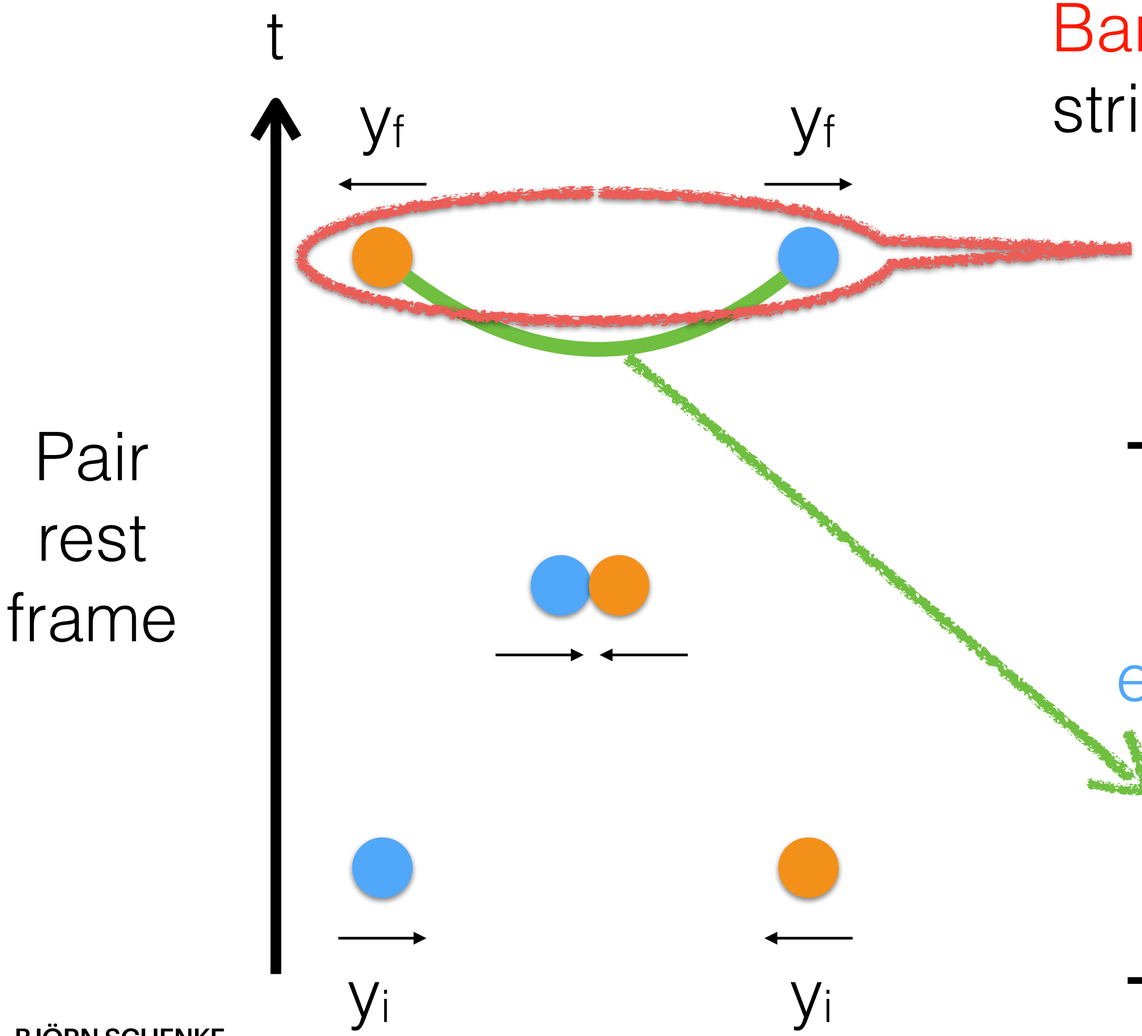
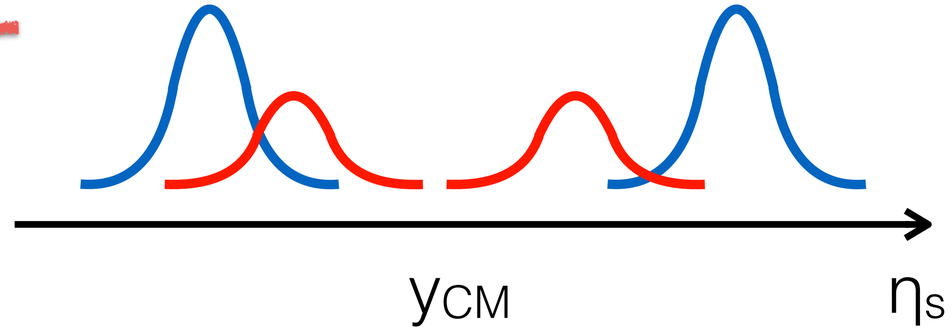
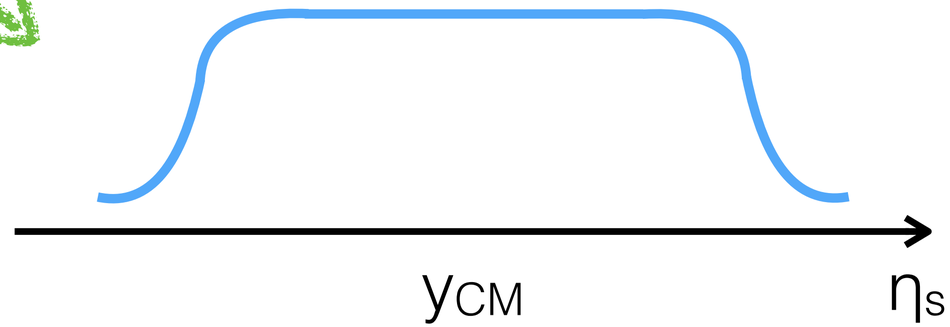



Baryon density is deposited at the string ends or string junctions

**D. Kharzeev, Phys. Lett. B 378, 238 (1996)**




energy density inside the string



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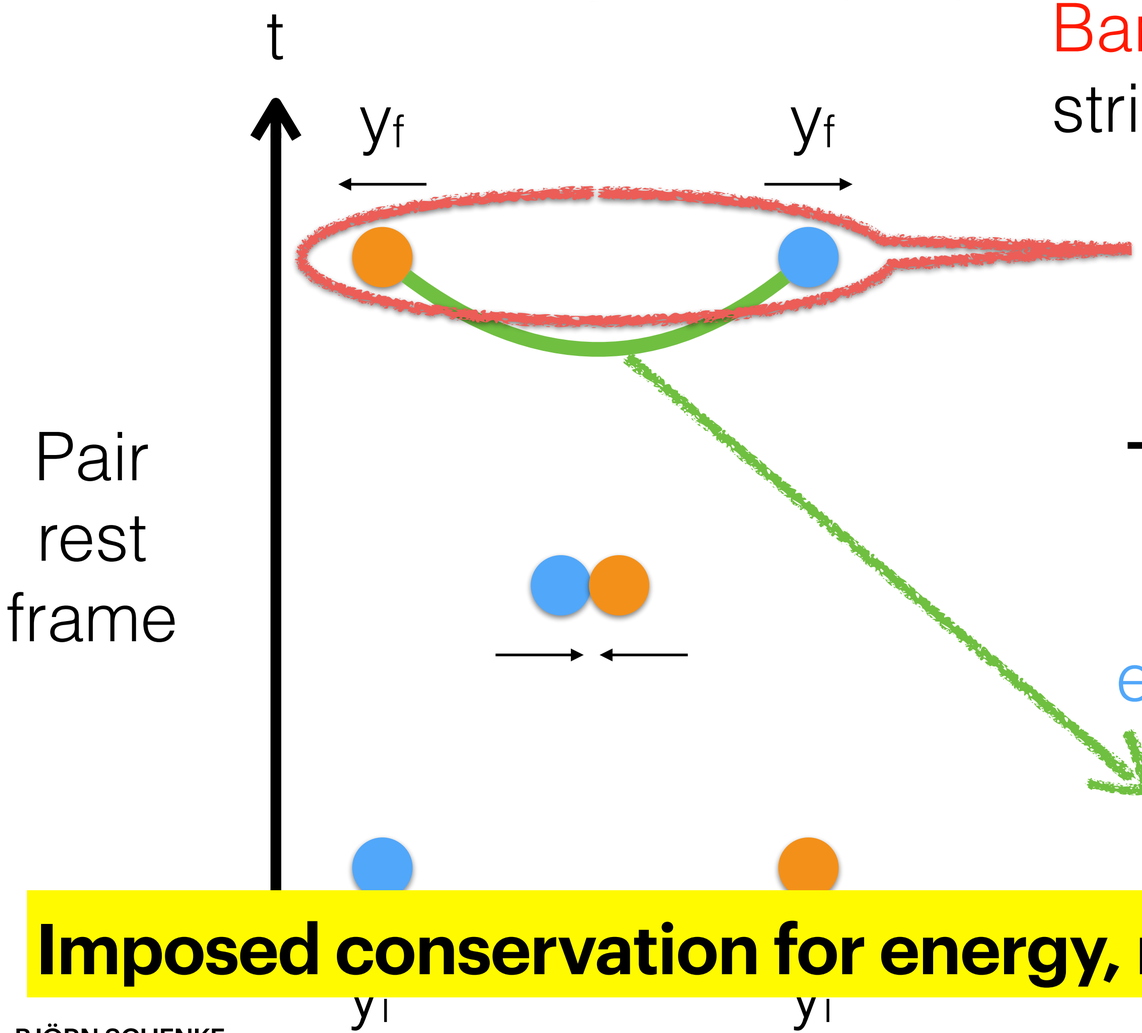


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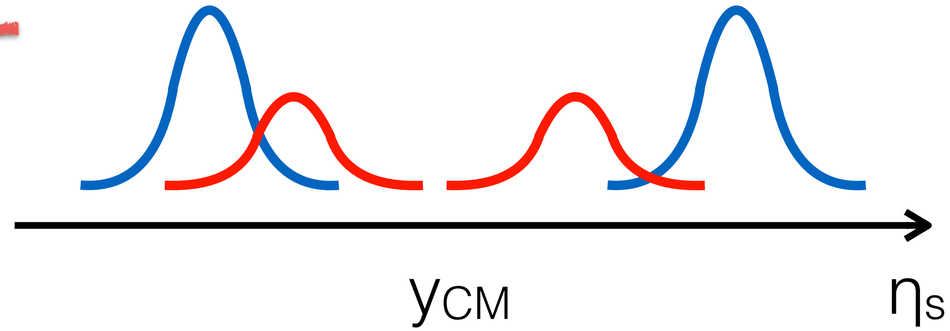
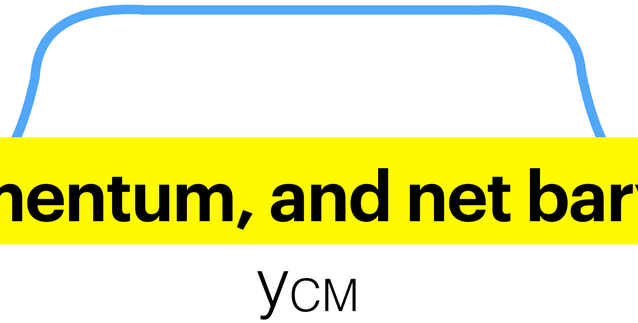


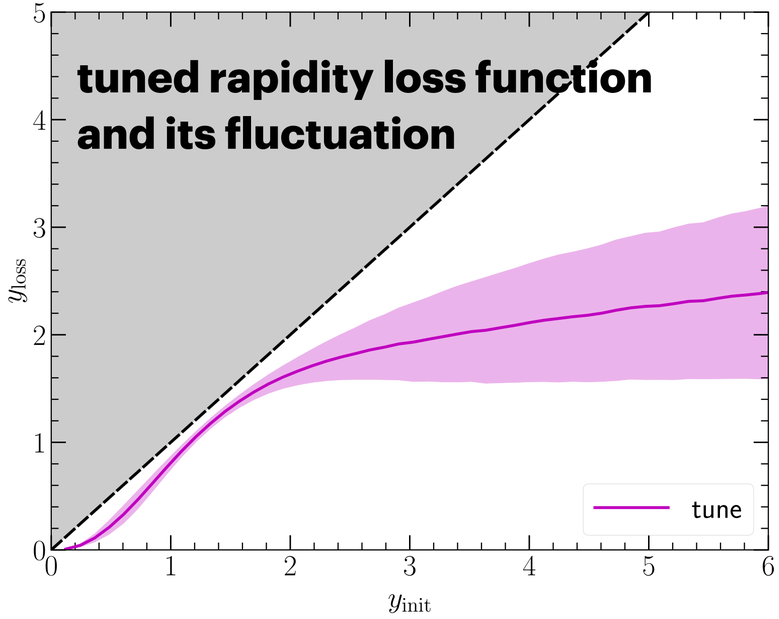

energy density inside the string

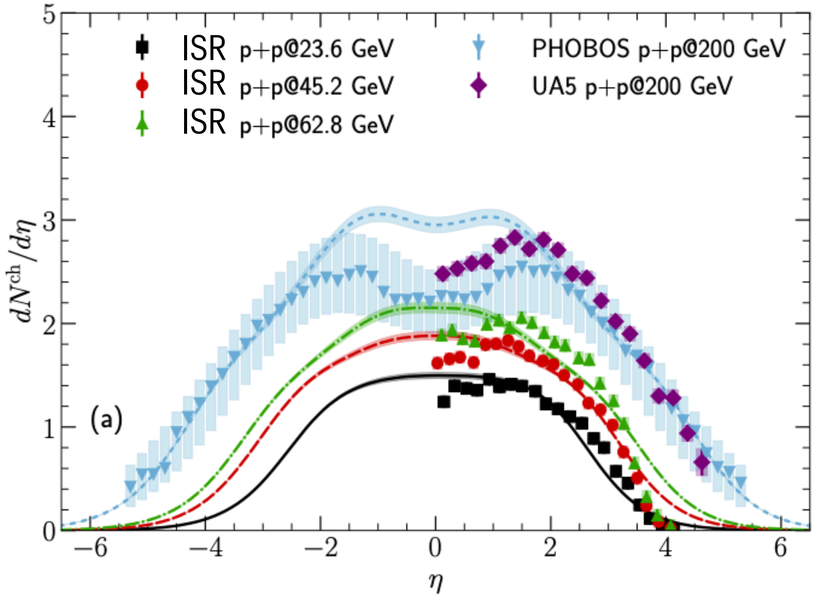


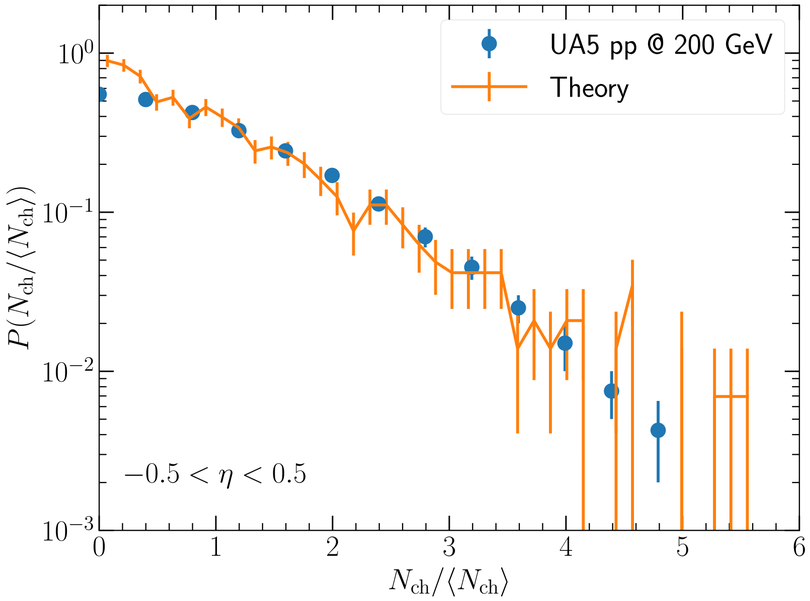

**Imposed conservation for energy, momentum, and net baryon density**

# 3D MC-GLAUBER + STRING MODEL+MUSIC

**C. Shen and B. Schenke, in preparation**




- **Feed energy and momentum of the strings dynamically into hydrodynamics via source terms**
- **Calibrated with minimum bias pp measurements at mid-rapidity and their multiplicity distributions**

# HYDRODYNAMICS AND EOS

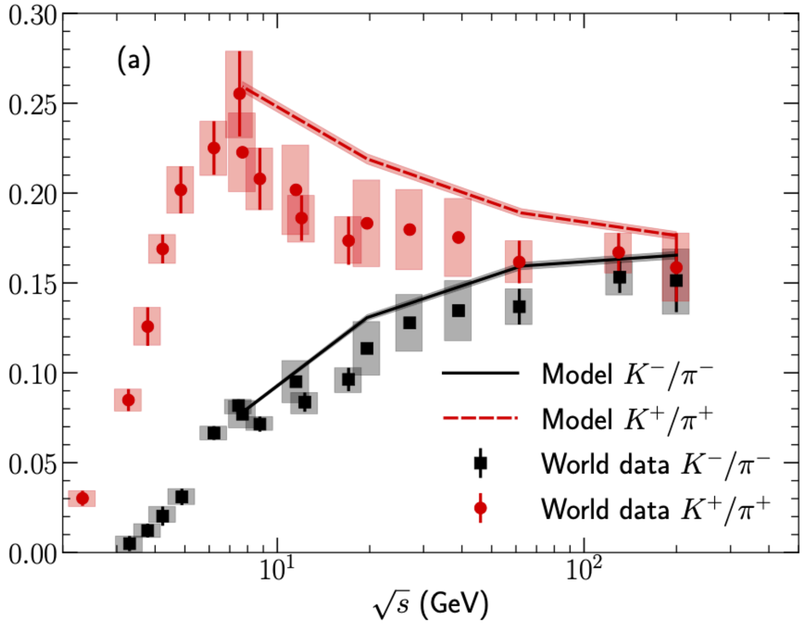
- **For the equation of state we use NEOS with finite $\mu_B, \mu_S, \mu_Q$**

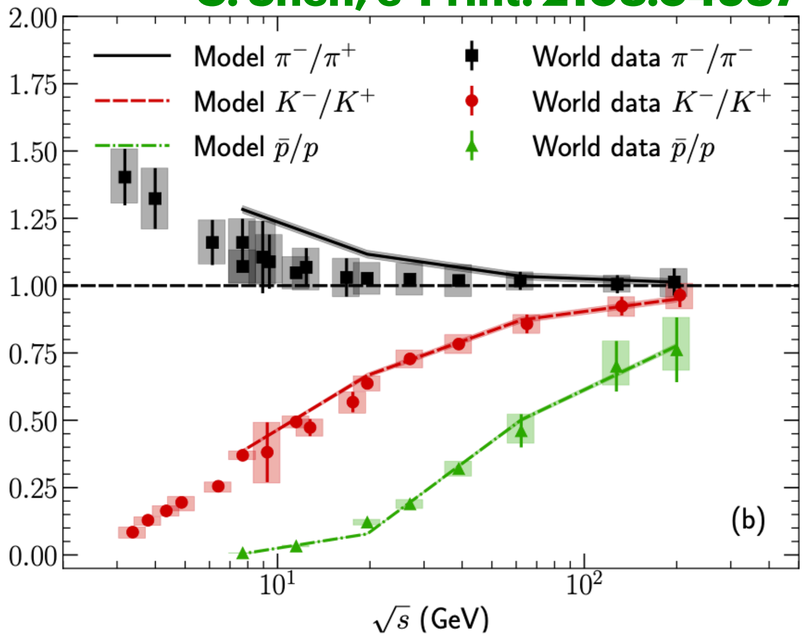
  **A. Monnai, B. Schenke, C. Shen, Phys. Rev. C 100, 024907 (2019)**

  **A. Monnai, B. Schenke, C. Shen, Int.J.Mod.Phys.A 36 (2021) 07, 2130007**

  **And choose $n_S = 0$ and $n_Q = 0.4 n_B$ for Au+Au collisions:**

**C. Shen, e-Print: 2108.04987**

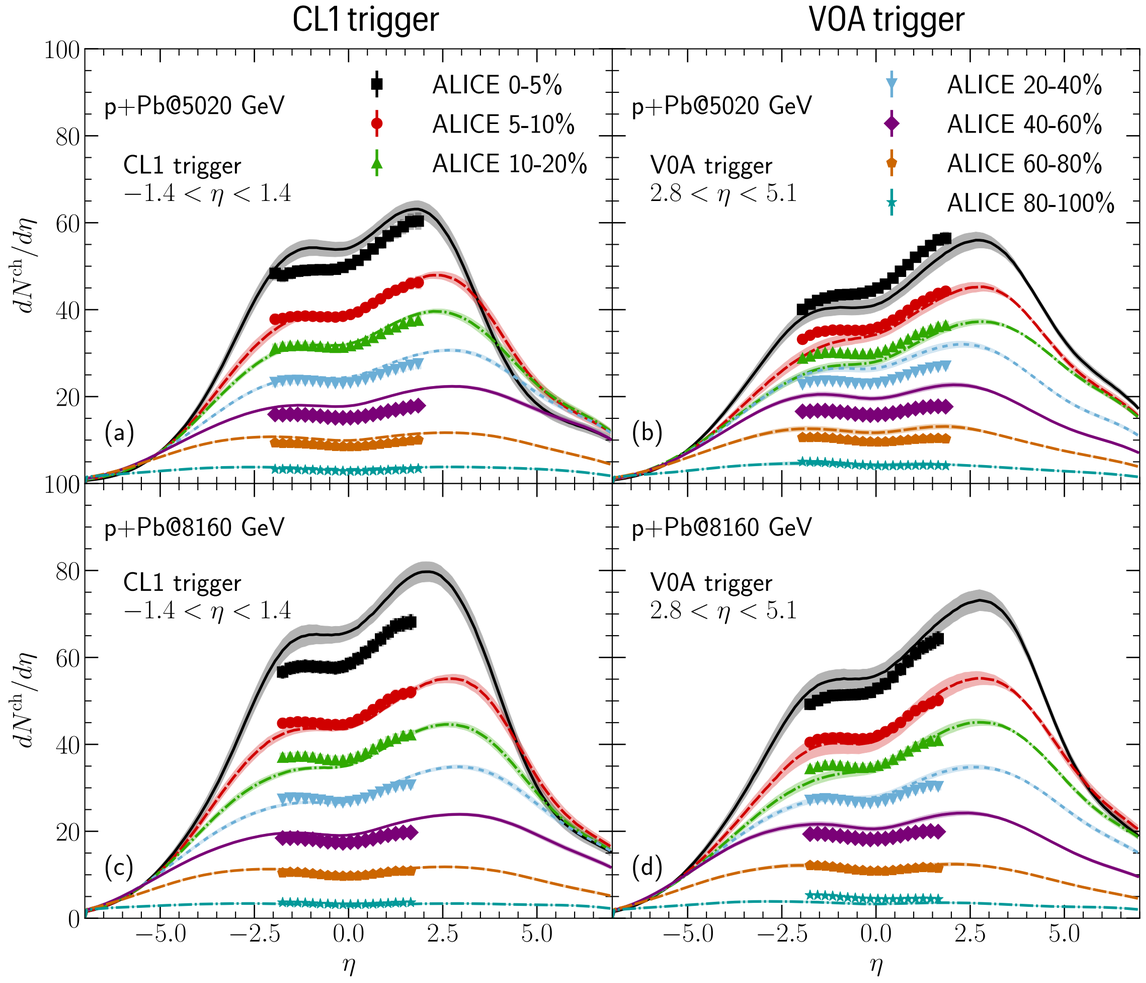


# CENTRALITY DEFINITION

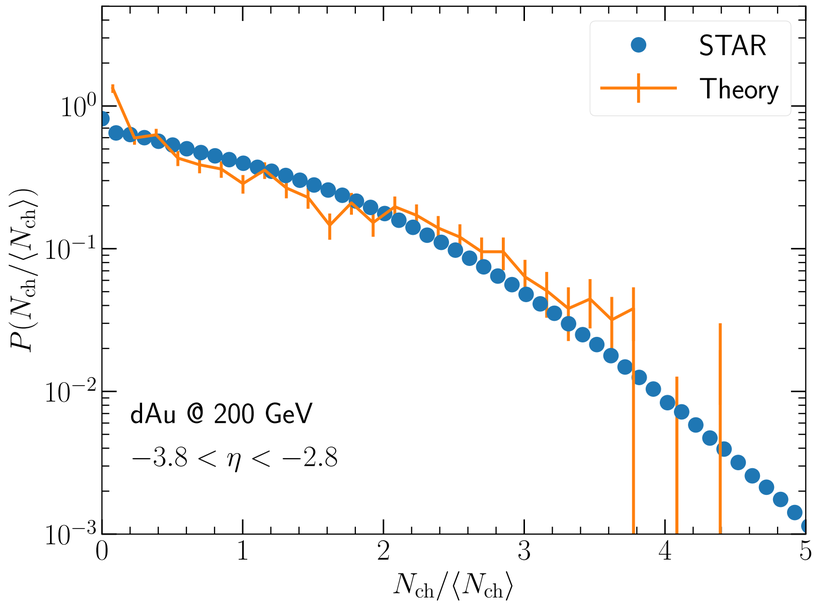
**C. Shen and B. Schenke, in preparation**

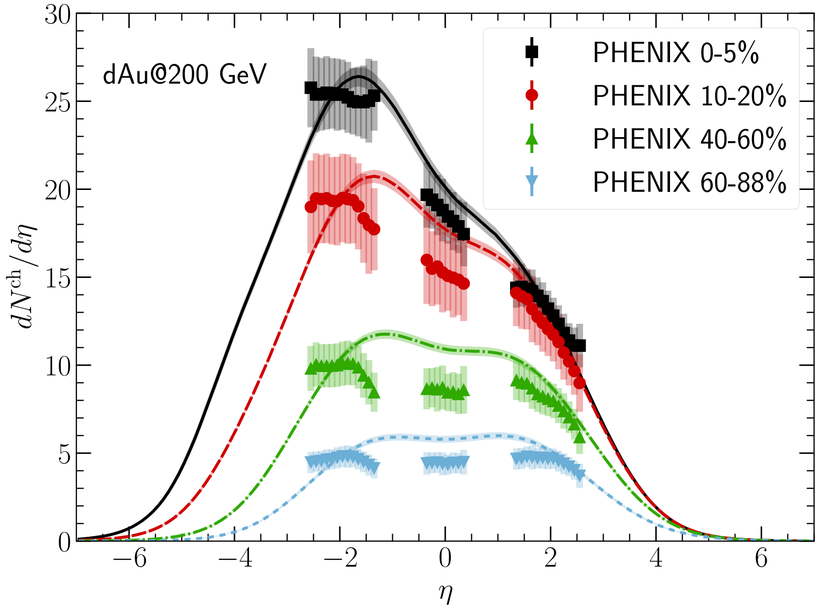
- **We can now determine centrality in the same rapidity bins as the experiments**
- **It does make a difference for the shape of the rapidity distribution**

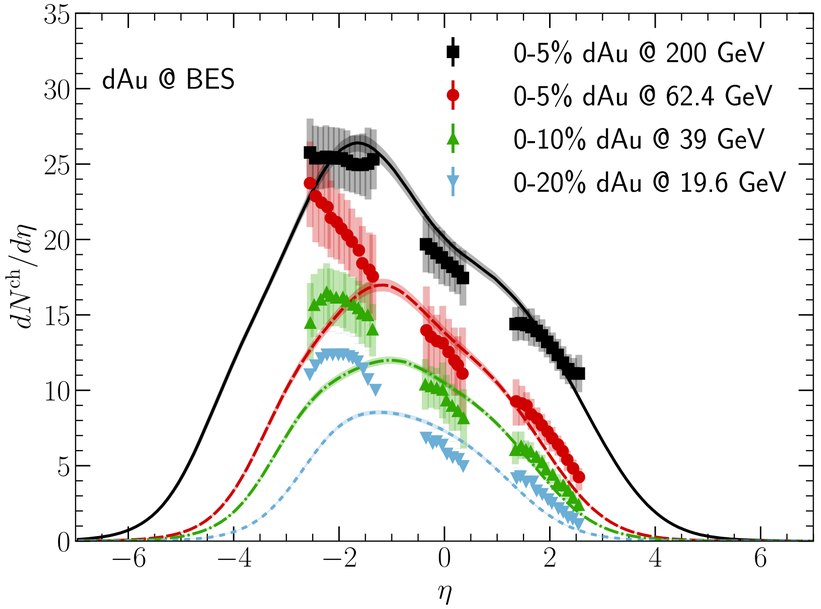


# MULTIPLICITY DISTRIBUTIONS

**C. Shen and B. Schenke, in preparation**



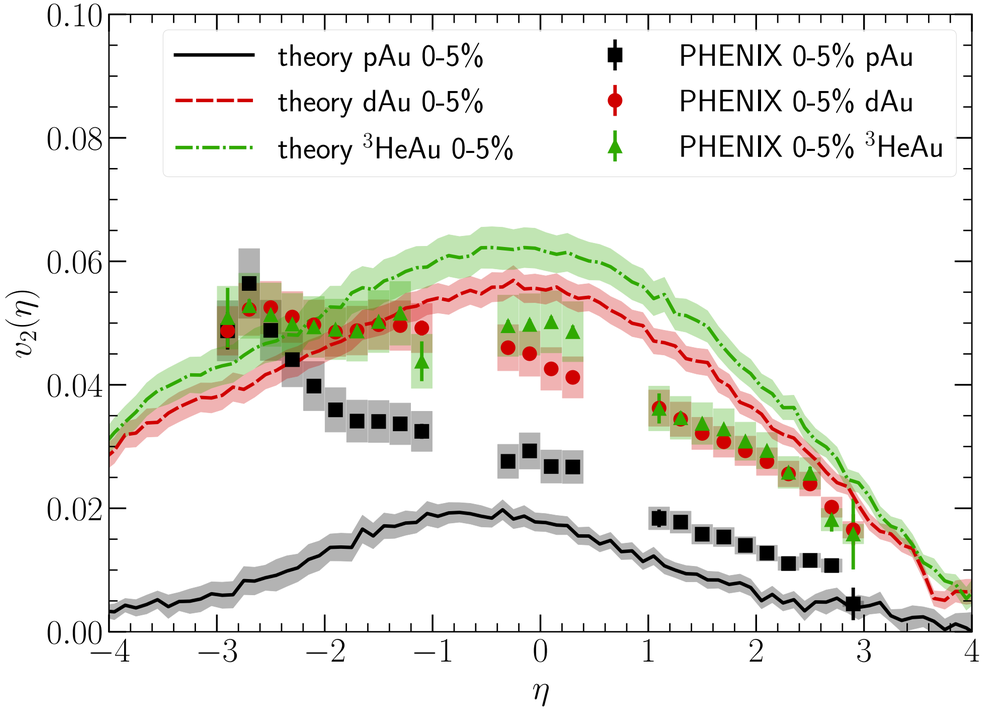




- **Model reproduces the STAR multiplicity distribution in d+Au collisions at 200 GeV**
- **The predicted charged hadron rapidity distribution agrees well with the PHENIX measurements from central to peripheral collisions**
- **The role of spectators at forward rapidity needs further investigation**

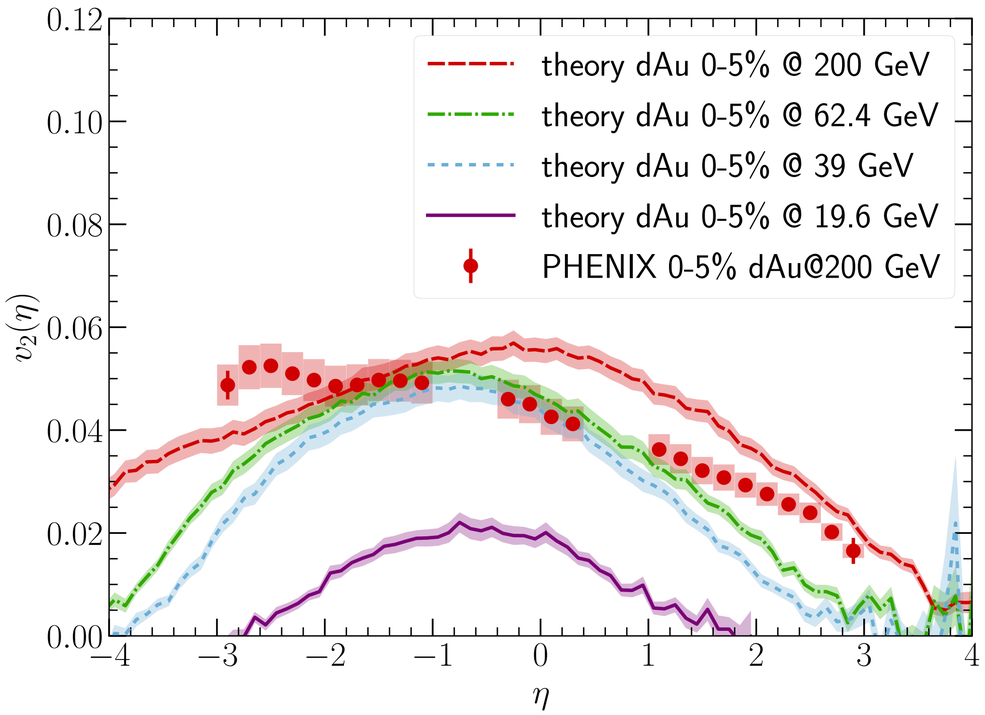
# ANISOTROPIC FLOW

C. Shen and B. Schenke, in preparation

## System dependence




## Energy dependence




- **Discrepancies could come from different method being used in the experiment**
- **Biggest deviation from the data in p+Au in the Au going direction (non-flow?)**
- **PHENIX event plane method with event plane at *y* in [-3.9, -3.1]**

# SUMMARY

- High energies: Boost-invariant hybrid model with IP-Glasma initial state and sub-nucleon fluctuations can describe bulk properties over a wide range of collision systems
- Especially for small systems, observables that use large rapidity gaps cannot be compared to boost invariant calculations
- IP-Glasma+JIMWLK evolution: Geometry decorrelates with rapidity (faster for low multiplicity); initial state anisotropy decorrelates more quickly (faster for high multiplicity)
- Low energies: 3+1D dynamic initial state and 3+1D hydrodynamics needed: Allows the calculation of rapidity dependent observables and use of the experimental rapidity gaps and proper centrality definitions. Need these calculations to be ready for BESII data.

# CODES - ALL PUBLIC

- The iEBE-MUSIC framework integrates individual physical models which describe different stages of relativistic heavy-ion collisions. This work uses v1.0 of this framework, which can be downloaded from
  https://github.com/chunshen1987/iEBE-MUSIC/releases

- The official code repository of the IP-Glasma initial conditions is
  https://github.com/schenke/ipglasma . Here we used v1.0:
  https://github.com/schenke/ipglasma/releases

- MUSIC is the numerical implementation of (3+1)D relativistic viscous hydrodynamic simulations for high energy heavy-ion collisions. Its official website is
  http://www.physics.mcgill.ca/music . This work uses the version 3.0 of MUSIC, which can be downloaded from
  https://github.com/MUSIC-fluid/MUSIC/releases

- The iSS code package is an open-source particle sampler based on the Cooper-Frye freeze-out prescription. It converts fluid cells to particle samples. This work uses v1.0 of the iSS, which can be downloaded from
  https://github.com/chunshen1987/iSS/releases

- We use the official UrQMD v3.4 and set it up to run as the afterburner mode
  https://bitbucket.org/Chunshen1987/urqmd_afterburner/src/master/

- The hadronic afterburner toolkit is a code package which performs analysis of particle spectra, flow observables, and their correlations using the outputs from hadronic transport models. This work uses v1.0 of the code which can be downloaded from
  https://github.com/chunshen1987/hadronic_afterburner_toolkit/releases

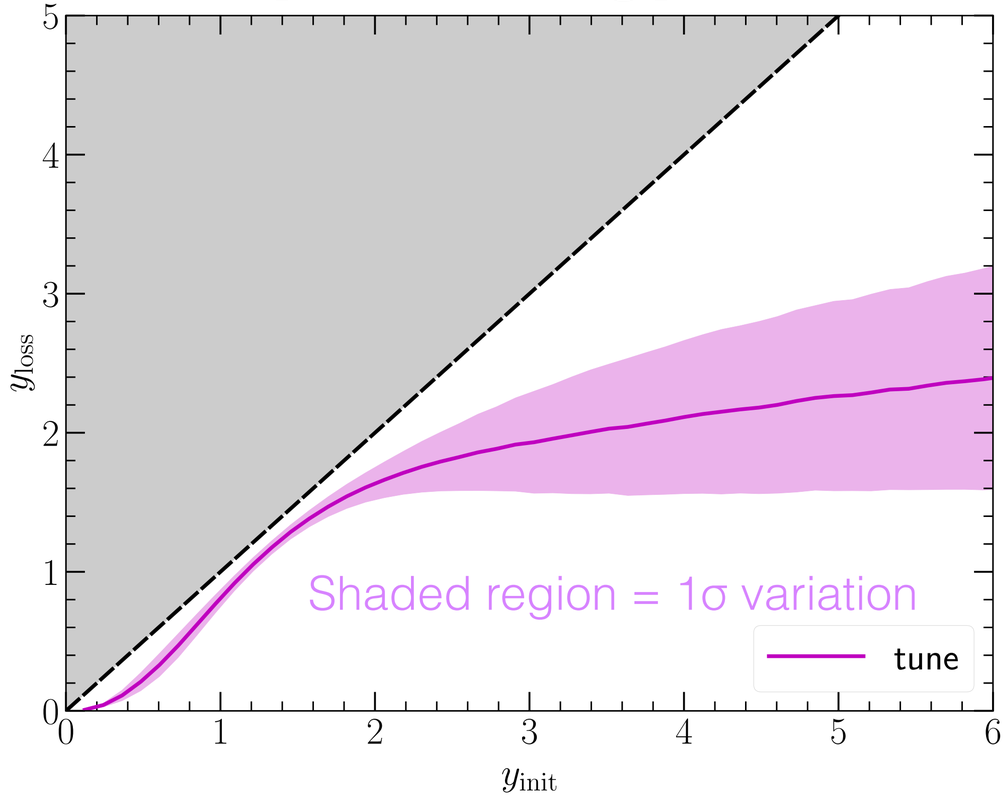
- The NEOS equation of state v0.11:
  https://sites.google.com/view/qcdneos/

# BACKUP

# 3D MC-GLAUBER + STRING MODEL

**C. Shen and B. Schenke, in preparation**

## Parametrize the valence quark energy loss




$$\langle y_{\rm loss} \rangle = A y_{\rm init}^{\alpha_2} [\tanh(y_{\rm init})]^{\alpha_1 - \alpha_2}$$

- $A$: the slope
- At small y: $\langle y_{\rm loss} \rangle \propto y_{\rm init}^{\alpha_1}$
- At large y: $\langle y_{\rm loss} \rangle \propto y_{\rm init}^{\alpha_2}$
- Std of $y_{\rm loss}$ fluctuations: $\sigma_y$ ($y_{\rm loss} \in [0, y_{\rm init}]$)

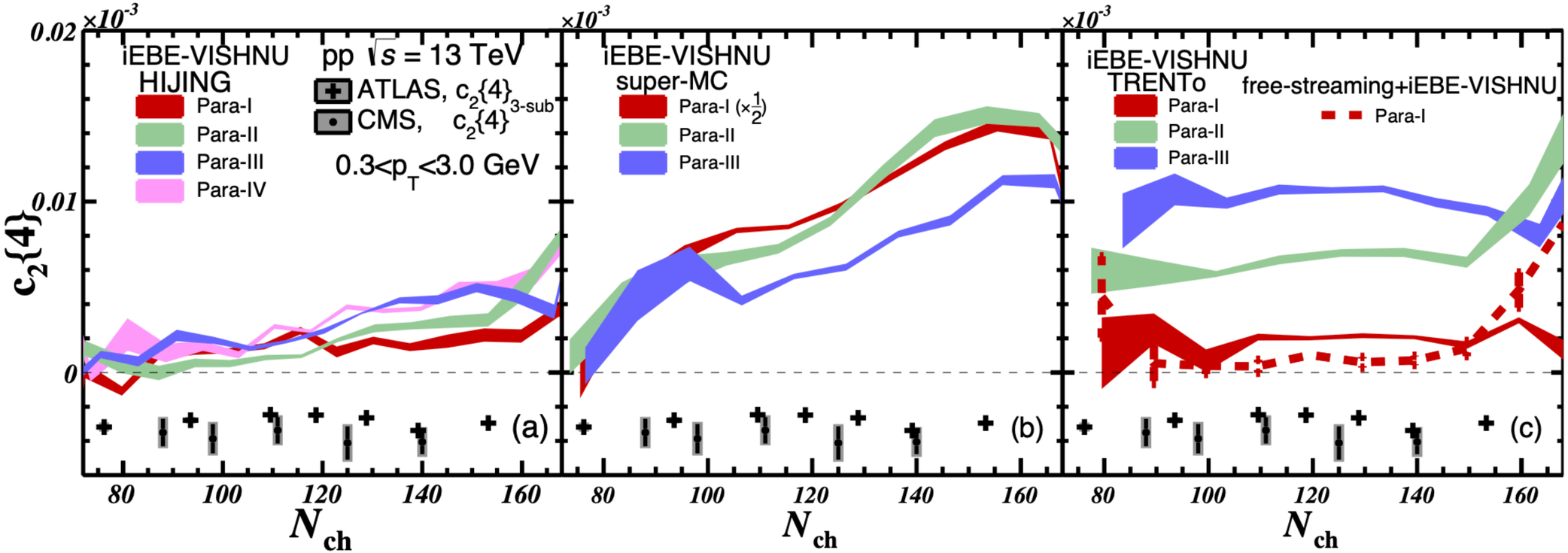
...motivated by the baryon stopping extracted by the BRAHMS Collaboration

**I. C. Arsene et al. (BRAHMS), Phys. Lett. B677, 267–271 (2009)**

# A WORD ON p+p

Multi-particle single and mixed harmonics cumulants can not be described by hydrodynamics with HIJING, super-MC or TRENTo initial conditions, even for the signs in a few cases

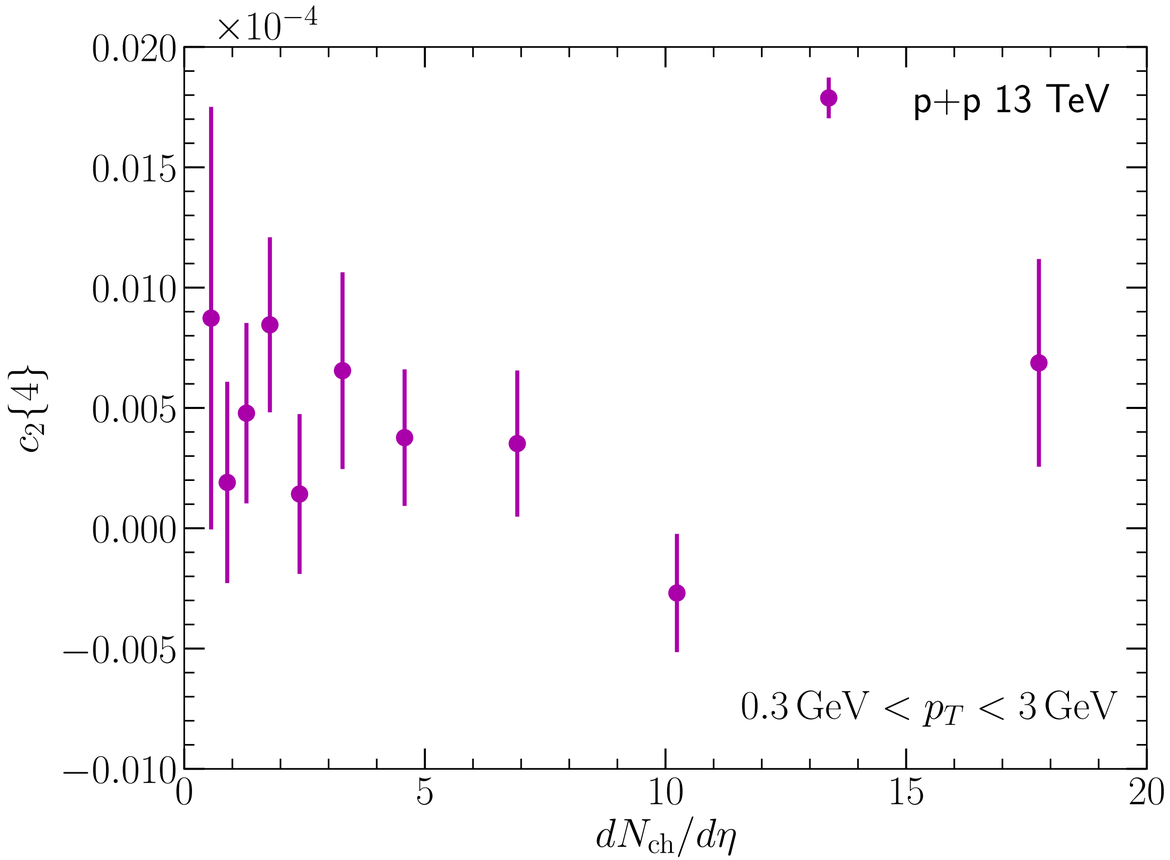
**Y. Zhou, W. Zhao, K. Murase, H. Song, Nucl.Phys.A 1005 (2021) 121908**




So, it is very hard to generate initial conditions that produce negative $c_2\{4\}$ in p+p collisions

# A WORD ON p+p: IP-GLASMA + MUSIC + URQMD

**preliminary**




Errors still large, but looks positive as well. However, we do not have more central results yet. Need to “trigger” on even more high multiplicity events. Ongoing…

# THE MODEL: IP-GLASMA

**B. Schenke, C. Shen, P. Tribedy, Phys. Rev. C 102 (2020) 4, 044905**

$$\rho(r,\theta) = \frac{\rho_0}{1+\exp[(r-R'(\theta))/a]},$$

with $R'(\theta) = R[1+\beta_2 Y_2^0(\theta)+\beta_4 Y_4^0(\theta)]$, and $\rho_0$ the nuclear density at the center of the nucleus. $R$ is the radius parameter, $a$ the skin depth.

| Nucleus | $R$ [fm] | $a$ [fm] | $\beta_2$ | $\beta_4$ |
|---|---|---|---|---|
| $^{238}$U | 6.81 | 0.55 | 0.28 | 0.093 |
| $^{208}$Pb | 6.62 | 0.546 | 0 | 0 |
| $^{197}$Au | 6.37 | 0.535 | -0.13 | -0.03 |
| $^{129}$Xe | 5.42 | 0.57 | 0.162 | -0.003 |
| $^{96}$Ru | 5.085 | 0.46 | 0.158 | 0 |
| $^{96}$Zr | 5.02 | 0.46 | 0 | 0 |

Sample nucleons, determine thickness functions, then compute $x = x(\mathbf{b}_\perp) = Q_s(x,\mathbf{b}_\perp)/\sqrt{s_{\rm NN}}$

Using $g^2\mu_i(x,\mathbf{b}_\perp) = cQ_s^i(x,\mathbf{b}_\perp)$ sample color charges with zero mean and variance: $\langle \rho_i^a(\mathbf{b}_\perp)\rho_i^b(\mathbf{x}_\perp)\rangle = g^2\mu_i^2(x,\mathbf{b}_\perp)\delta^{ab}\delta^{(2)}(\mathbf{b}_\perp-\mathbf{x}_\perp)$

Sampled color charges yield currents moving on the light cone $J_i^\nu = \delta^{\nu\pm}\rho_i(x^\mp,\mathbf{x}_\perp) = \delta^{\nu\pm}\rho_i^a(x^\mp,\mathbf{x}_\perp)t^a$

which are sources in the Yang-Mills equations $[D_\mu, F^{\mu\nu}] = J^\nu$

# THE MODEL: IP-GLASMA

**B. Schenke, C. Shen, P. Tribedy, Phys. Rev. C 102 (2020) 4, 044905**

Yang-Mills equations $\left[D_\mu, F^{\mu\nu}\right] = J^\nu$

are solved in Lorenz gauge to yield $A_i^\pm = -\frac{\rho_i(x^\mp, \mathbf{x}_\perp)}{\boldsymbol{\nabla}_\perp^2 - m^2}$ with m=0.2 GeV

The field from both nuclei in Fock-Schwinger gauge reads

$$A^j(\mathbf{x}_\perp) = \theta(x^-)\theta(-x^+)\frac{i}{g}V_P(\mathbf{x}_\perp)\partial^j V_P^\dagger(\mathbf{x}_\perp) + \theta(x^+)\theta(-x^-)\frac{i}{g}V_T(\mathbf{x}_\perp)\partial^j V_T^\dagger(\mathbf{x}_\perp)$$

where $V_i(\mathbf{x}_\perp) = P\exp\left(-ig\int dx^\mp \frac{\rho_i(x^\mp, \mathbf{x}_\perp)}{\boldsymbol{\nabla}_\perp^2 - m^2}\right)$

The fields in the forward light-cone are then

$$A^j = A_P^j + A_T^j\,,$$
$$A^\eta = -\frac{ig}{2}\left[A_{Pj}, A_T^j\right],$$
$$\partial_\tau A^j = 0\,,$$
$$\partial_\tau A^\eta = 0\,.$$

# THE MODEL: IP-GLASMA

**B. Schenke, C. Shen, P. Tribedy, Phys. Rev. C 102 (2020) 4, 044905**

Yang-Mills evolution in the forward light cone is given by the discretized equations of motion ( $\pi = E^{\eta}$, $\phi = A_{\eta}$ )

$$\dot{U}_j = i\frac{g^2}{\tau}E^j U_j \quad \text{(no sum over } j\text{)} \tag{29}$$

$$\dot{\phi} = \tau\pi \tag{30}$$

$$\dot{E}^1 = \frac{i\tau}{2g^2}[U_{(1,2)} + U_{(1,-2)} - U^\dagger_{(1,2)} - U^\dagger_{(1,-2)} - T_1] + \frac{i}{\tau}[\tilde{\phi}_1, \phi] \tag{31}$$

$$\dot{E}^2 = \frac{i\tau}{2g^2}[U_{(2,1)} + U_{(2,-1)} - U^\dagger_{(2,1)} - U^\dagger_{(2,-1)} - T_2] + \frac{i}{\tau}[\tilde{\phi}_2, \phi] \tag{32}$$

$$\dot{\pi} = \frac{1}{\tau}\sum_j \left[\tilde{\phi}_j + \tilde{\phi}_{-j} - 2\phi\right], \tag{33}$$

One can then compute $T^{\mu\nu}$. The $\tau\tau$ component for example reads

$$\begin{aligned}
T^{\tau\tau}(s) = \frac{g^2}{2\tau^2}\mathrm{Tr}[&(E^1)^2(s) + (E^1)^2(s+\hat{e}_2) \\
&+ (E^2)^2(s) + (E^2)^2(s+\hat{e}_1)] \\
+ \frac{1}{2\tau^2}\mathrm{Tr}\big[&\left(\phi(s) - \tilde{\phi}_1(s)\right)^2 \\
&+ \left(\phi(s+\hat{e}_2) - \tilde{\phi}_1(s+\hat{e}_2)\right)^2 \\
&+ \left(\phi(s) - \tilde{\phi}_2(s)\right)^2 \\
&+ \left(\phi(s+\hat{e}_1) - \tilde{\phi}_2(s+\hat{e}_1)\right)^2\big] \\
+ \frac{1}{4}\mathrm{Tr}\big[&\pi^2(s) + \pi^2(s+\hat{e}_1) \\
&+ \pi^2(s+\hat{e}_2) + \pi^2(s+\hat{e}_1+\hat{e}_2)\big] \\
+ \frac{2}{g^2}&\left(N_c - \mathrm{ReTr}[U_{(1,2)}(s)]\right).
\end{aligned}$$

# THE MODEL: MUSIC

**B. Schenke, C. Shen, P. Tribedy, Phys. Rev. C 102 (2020) 4, 044905**

$T^{\mu\nu}$ is matched to hydrodynamics, which solves $\partial_\mu T^{\mu\nu} = 0$

We decompose $T^{\mu\nu} = e u^\mu u^\nu - (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$

We use the DNMR formalism that has constitutive relations

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\,\theta - \delta_{\Pi\Pi}\Pi\,\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}$$
$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\,\sigma^{\mu\nu} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \varphi_7\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} - \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \lambda_{\pi\Pi}\Pi\,\sigma^{\mu\nu}\,,$$

G. S. Denicol, H. Niemi, E. Molnar, and D. H. Rischke, Phys. Rev. **D85**, 114047 (2012), [Erratum: Phys. Rev.D91,no.3,039902(2015)], arXiv:1202.4551 [nucl-th].
G. S. Denicol, S. Jeon, and C. Gale, Phys. Rev. **C90**, 024912 (2014), arXiv:1403.0962 [nucl-th].

$$\sigma^{\mu\nu} = \frac{1}{2}\left[\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}(\nabla_\alpha u^\alpha)\right]$$

$$\tau_\pi = \frac{5\,\eta}{e + P}\,,$$

$$\tau_\Pi = \frac{\zeta}{15\,\left(\frac{1}{3} - c_s^2\right)^2 (e + P)}$$

| $\tau_{\pi\pi}[\tau_\pi]$ | $\delta_{\pi\pi}[\tau_\pi]$ | $\varphi_7 P$ | $\lambda_{\pi\Pi}[\tau_\pi]$ | $\lambda_{\Pi\pi}[\tau_\Pi]$ | $\delta_{\Pi\Pi}[\tau_\Pi]$ |
|---|---|---|---|---|---|
| $10/7$ | $4/3$ | $9/70$ | $6/5$ | $8(1/3 - c_s^2)/5$ | $2/3$ |

# THE MODEL: PARTICLE SAMPLING

**B. Schenke, C. Shen, P. Tribedy, Phys. Rev. C 102 (2020) 4, 044905**

When converting to particles, we use viscous corrections

$$\delta f^i_{\rm shear}(x^\mu,p^\mu) = f^i_0(1\pm f^i_0)\frac{\pi^{\mu\nu}p_\mu p_\nu}{2T^2(e+P)}\,,$$

and

$$\delta f^i_{\rm bulk}(x^\mu,p^\mu) = f^i_0(1\pm f^i_0)\left(-\frac{\Pi}{\hat{\zeta}}\right)\frac{(p\cdot u)}{T} \times\left[\frac{m_i^2}{3(p\cdot u)^2}-\left(\frac{1}{3}-c_s^2\right)\right]$$

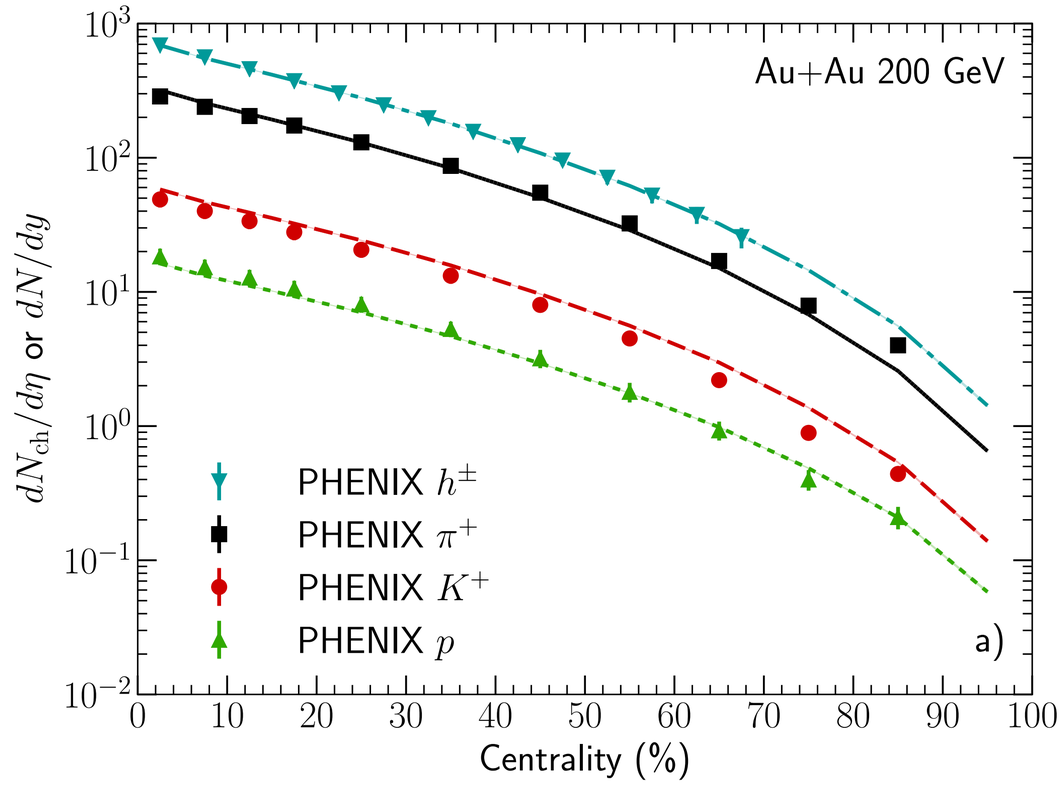
where

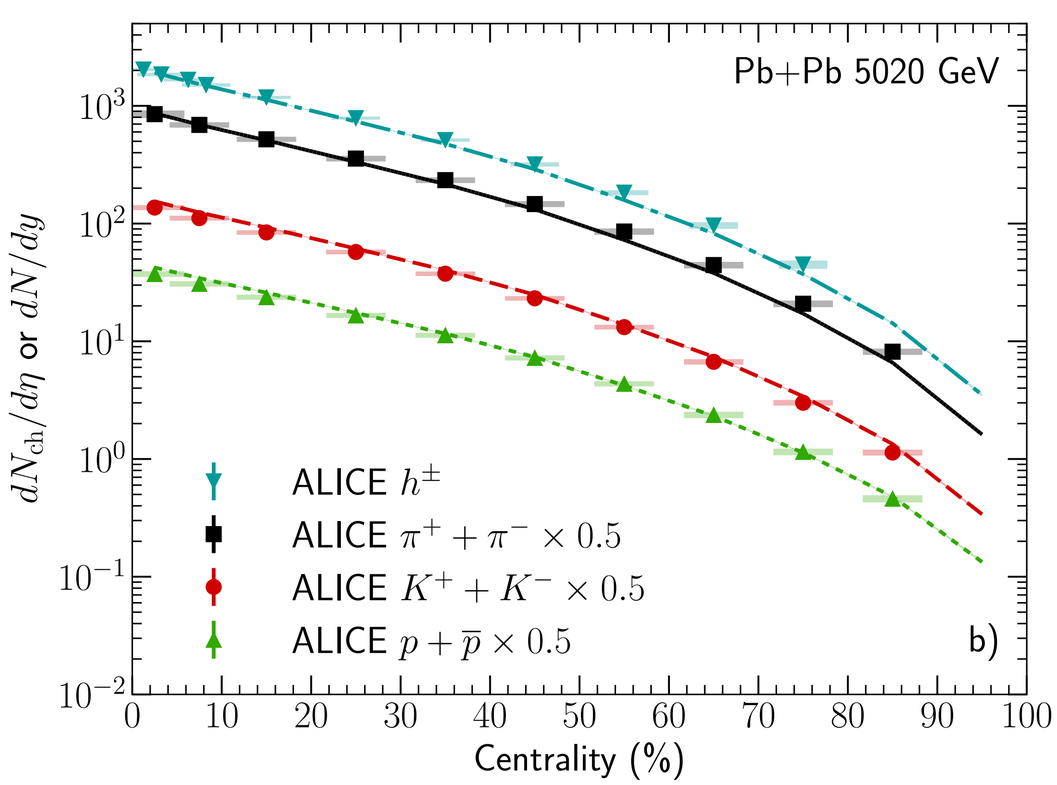
$$\hat{\zeta} = \frac{1}{3T}\sum_i \frac{g_i}{(2\pi)^3}m_i^2\int\frac{d^3k}{E}f^i_0(1\pm f^i_0) \times E\left[\frac{m_i^2}{3E^2}-\left(\frac{1}{3}-c_s^2\right)\right]$$

Should the value of any $\delta f$ be larger than that of the equilibrium distribution, we set the distribution to zero,

# BACKUP: Model validation

**B. Schenke, C. Shen, P. Tribedy, Phys. Rev. C 102 (2020) 4, 044905**



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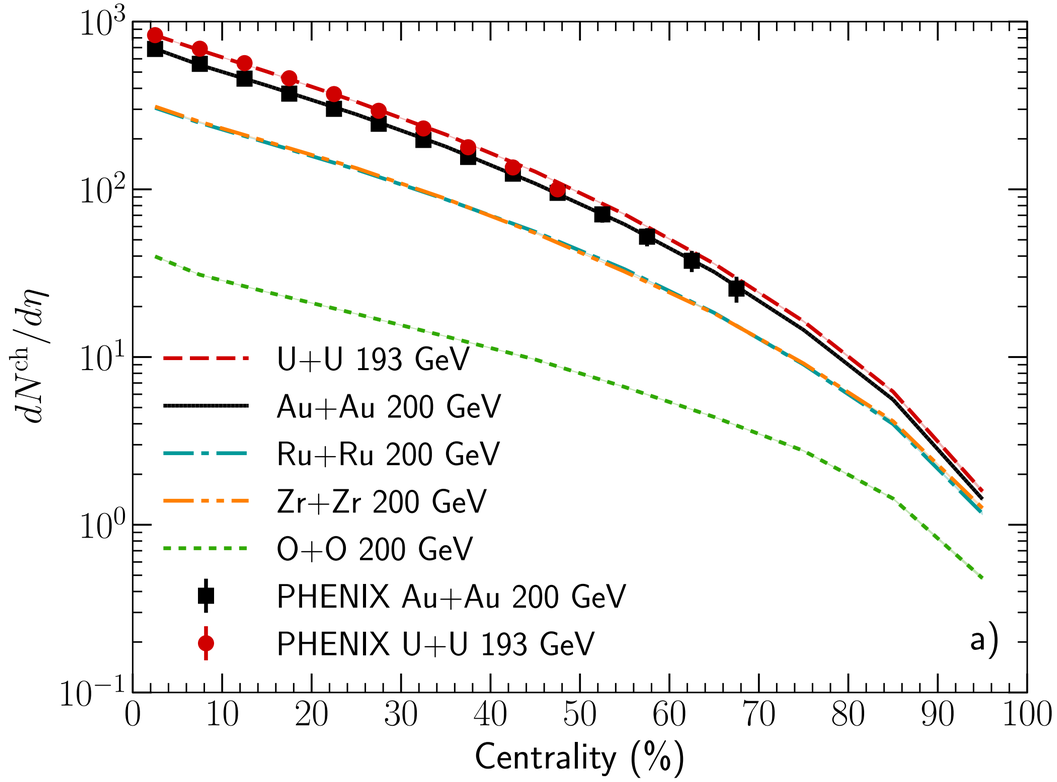


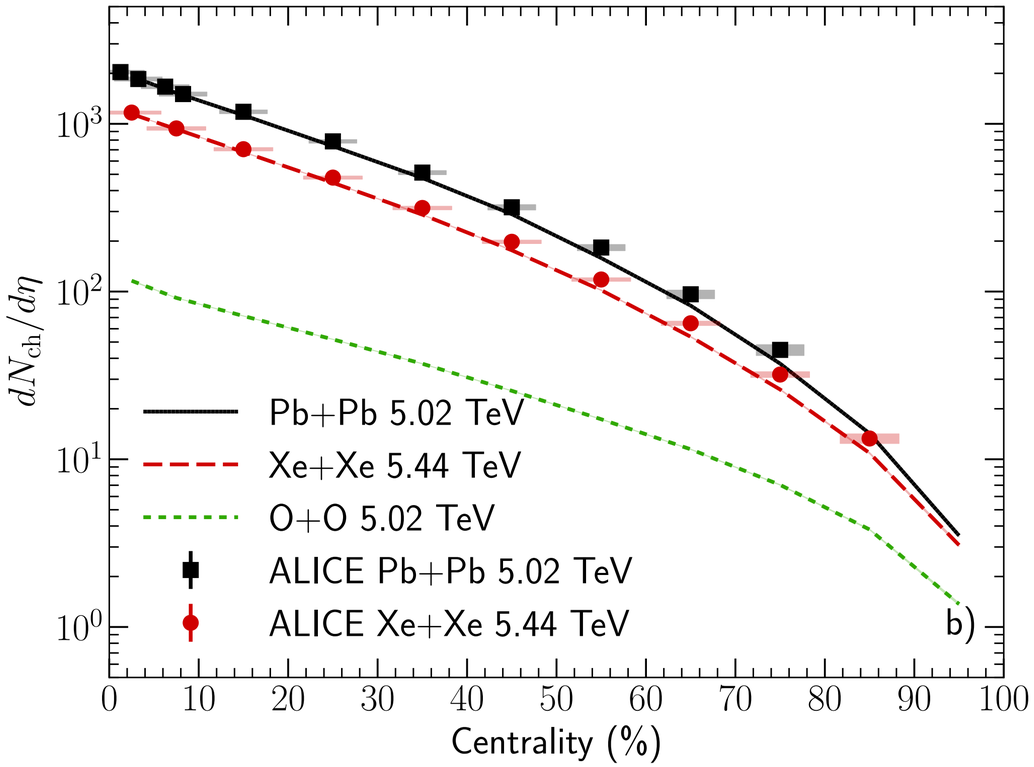

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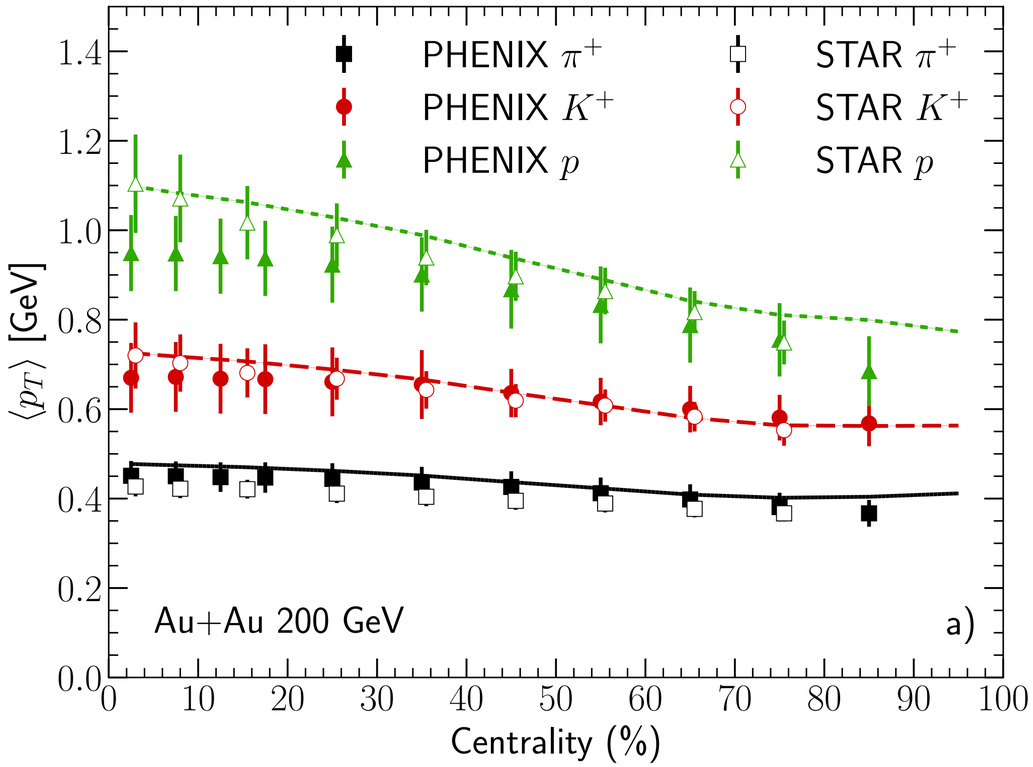


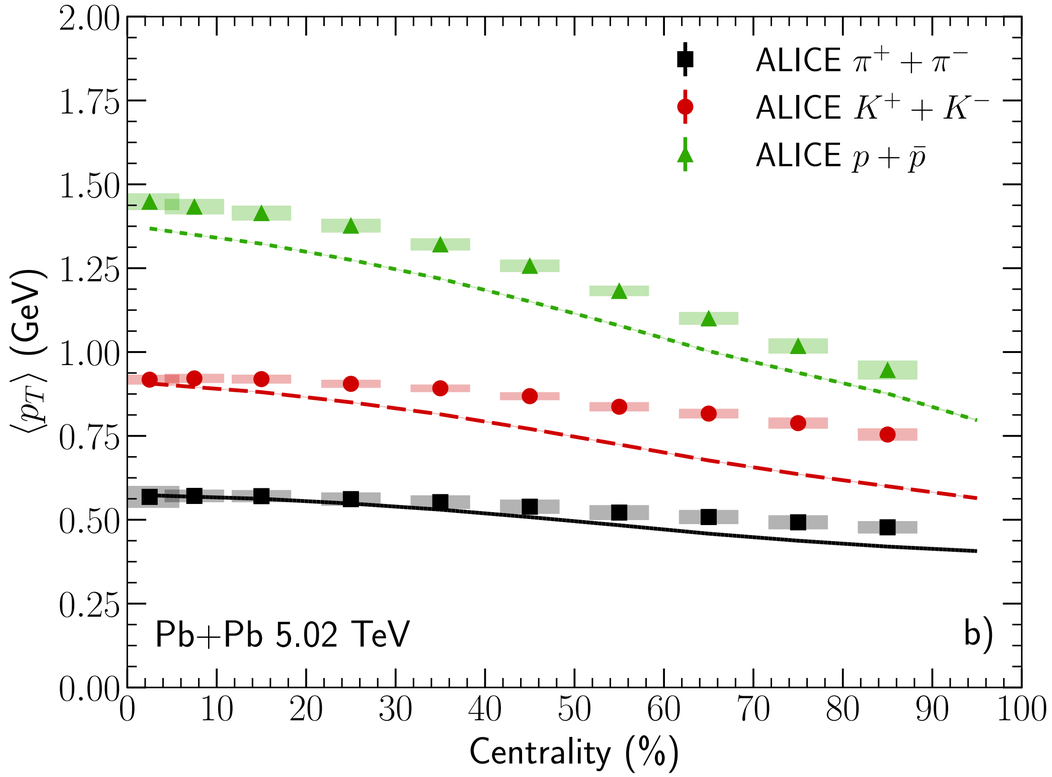

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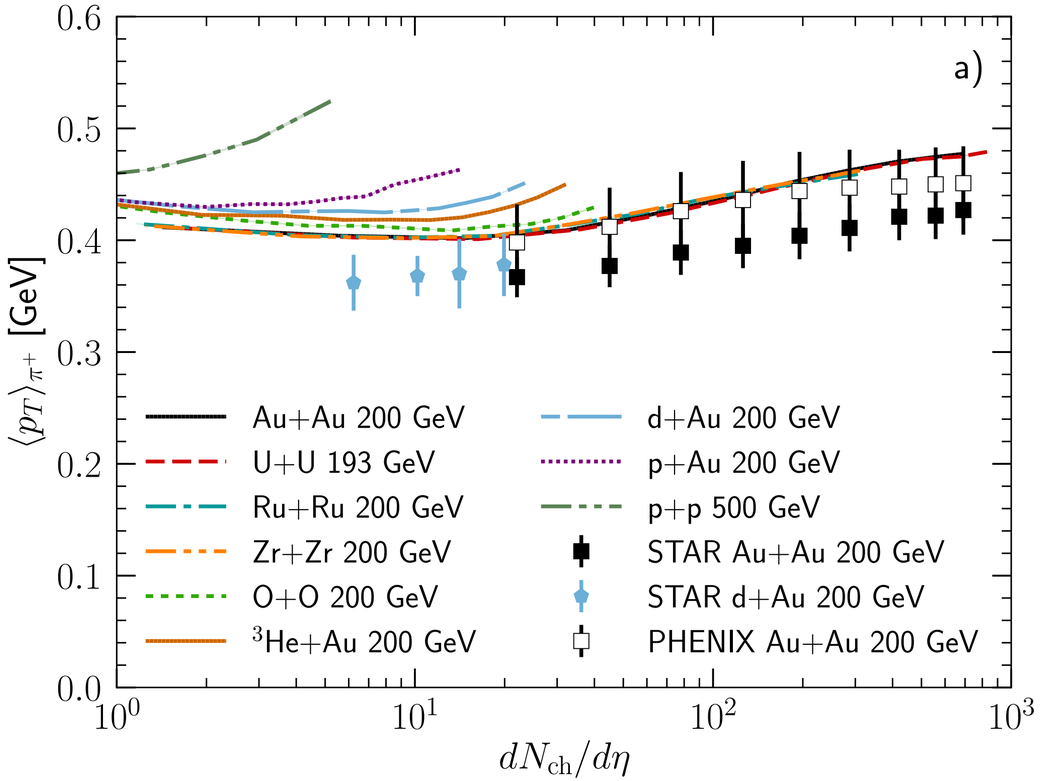


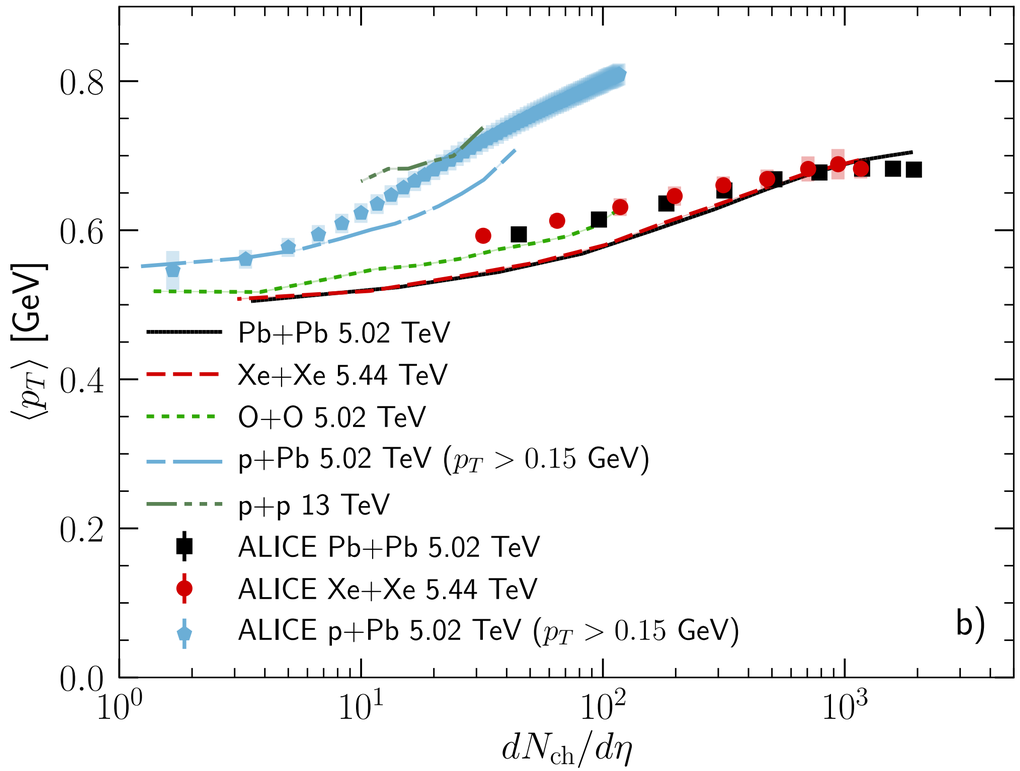

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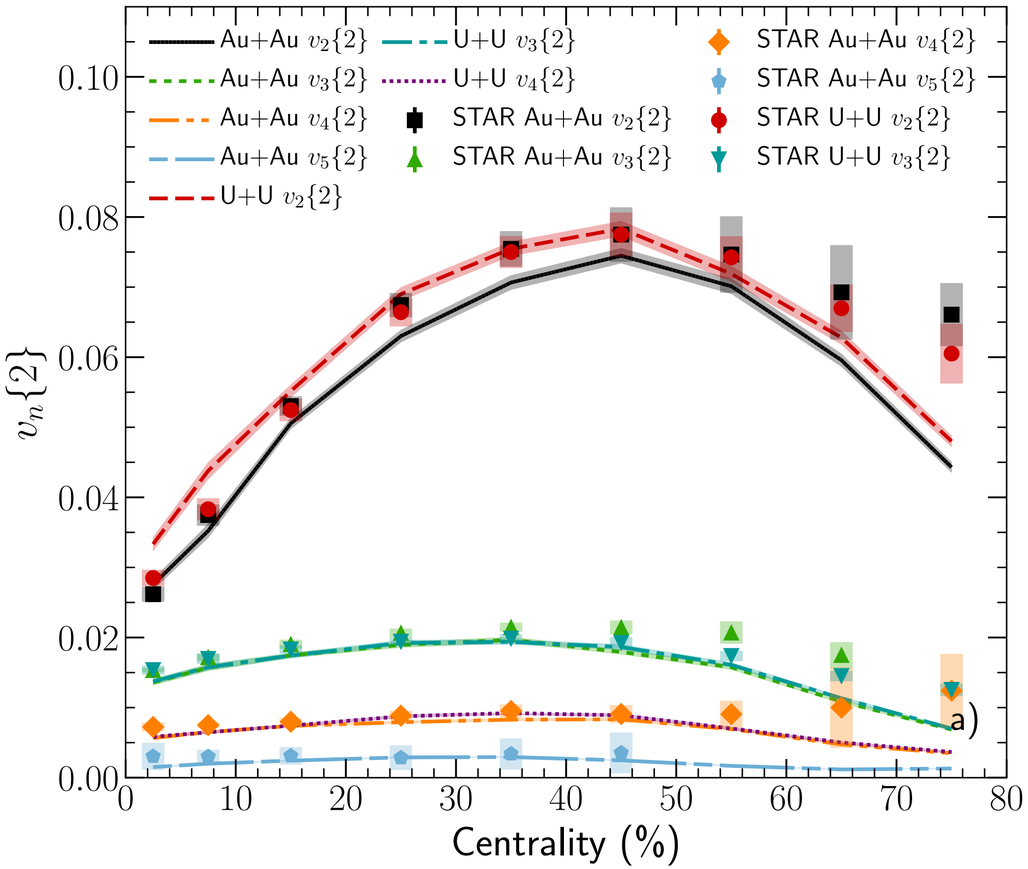


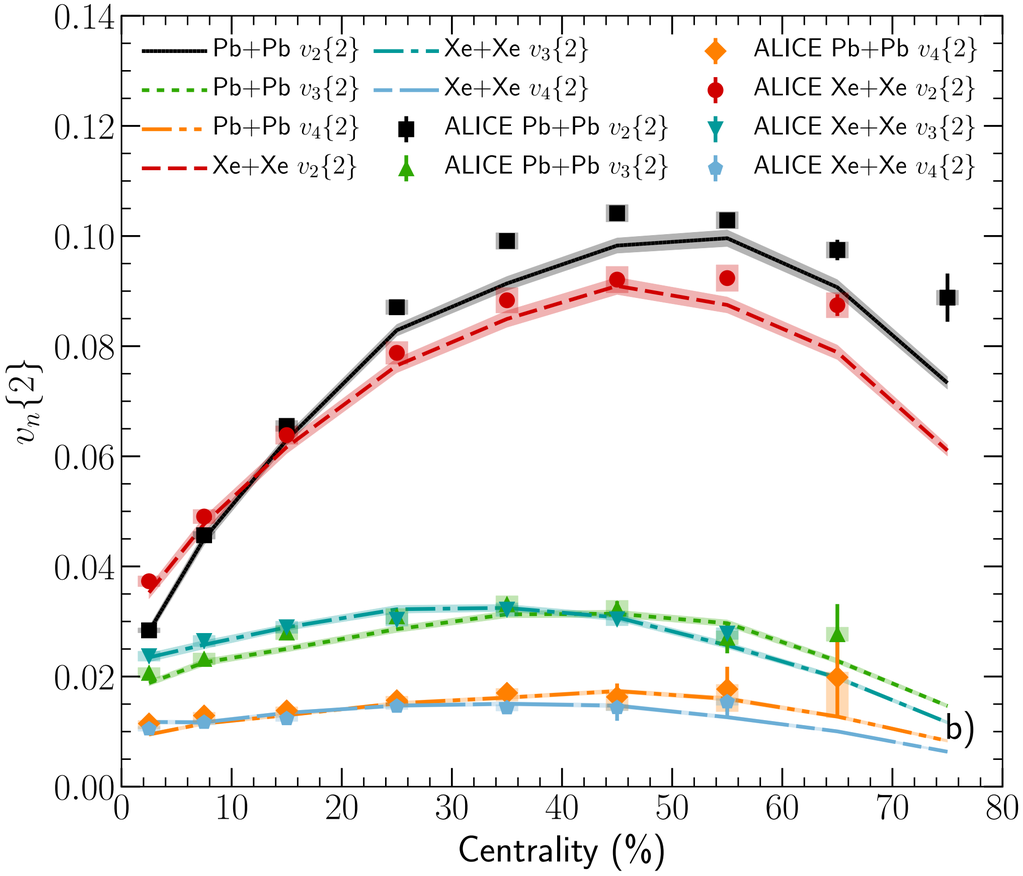

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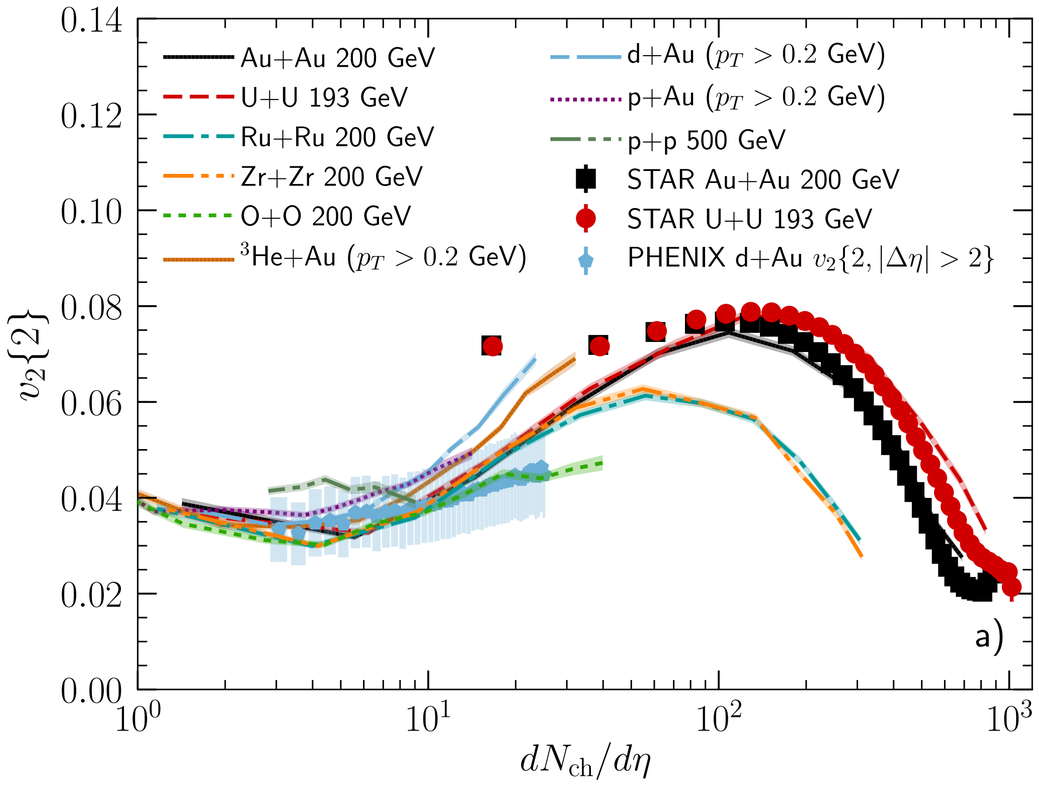


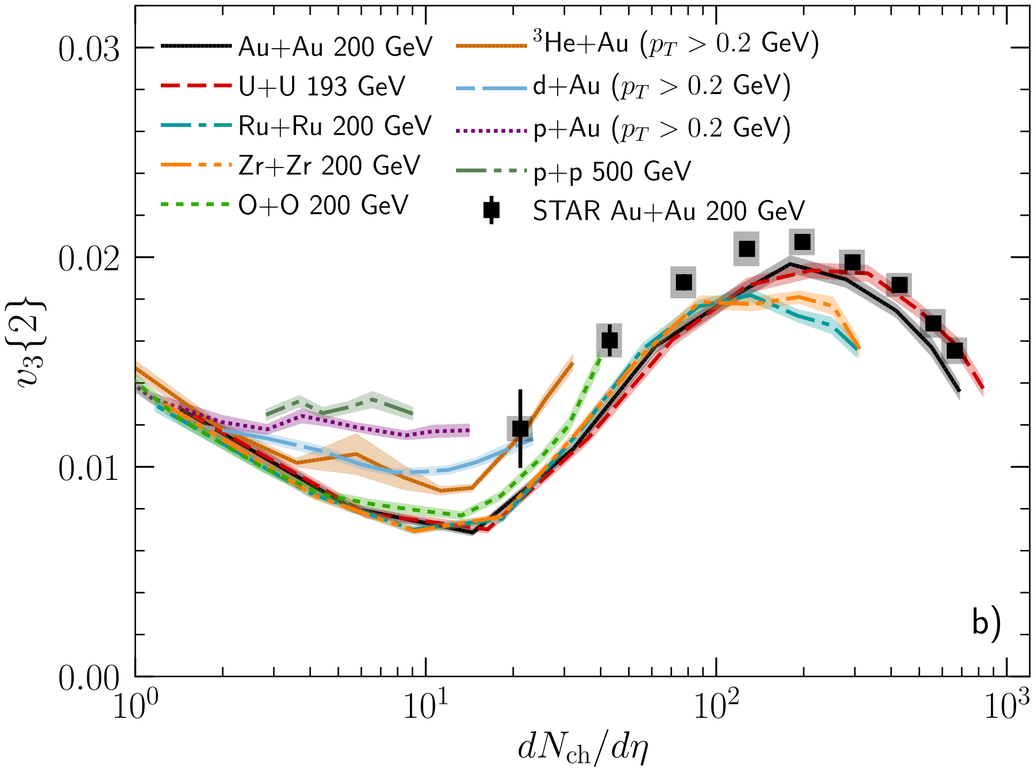

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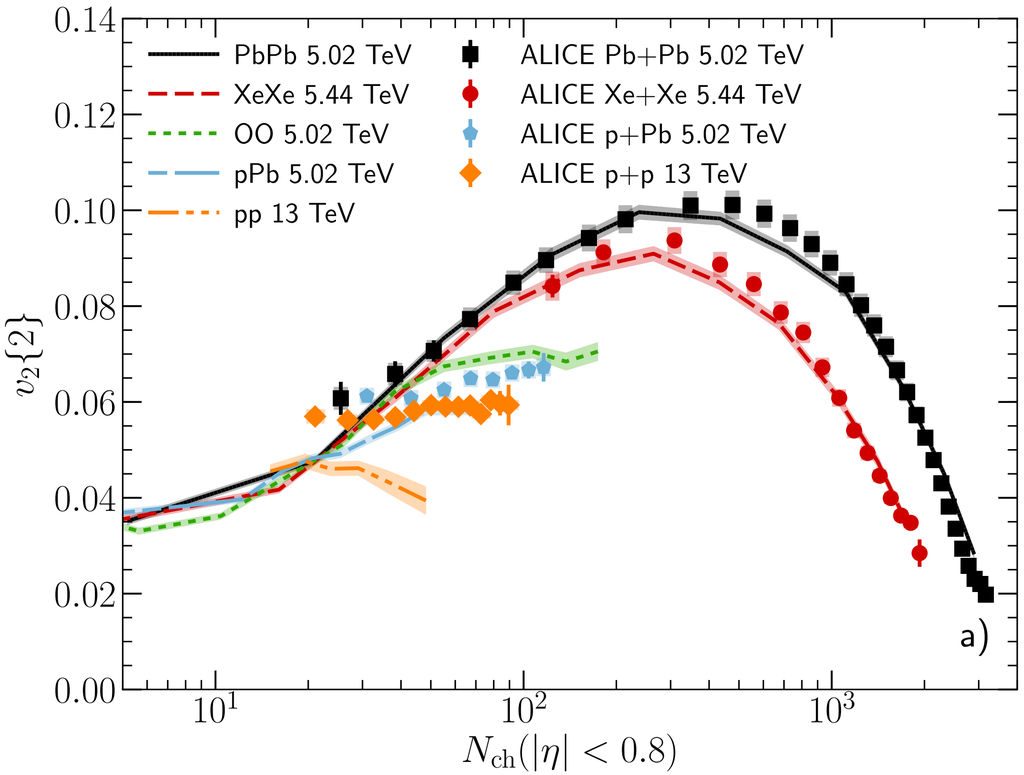


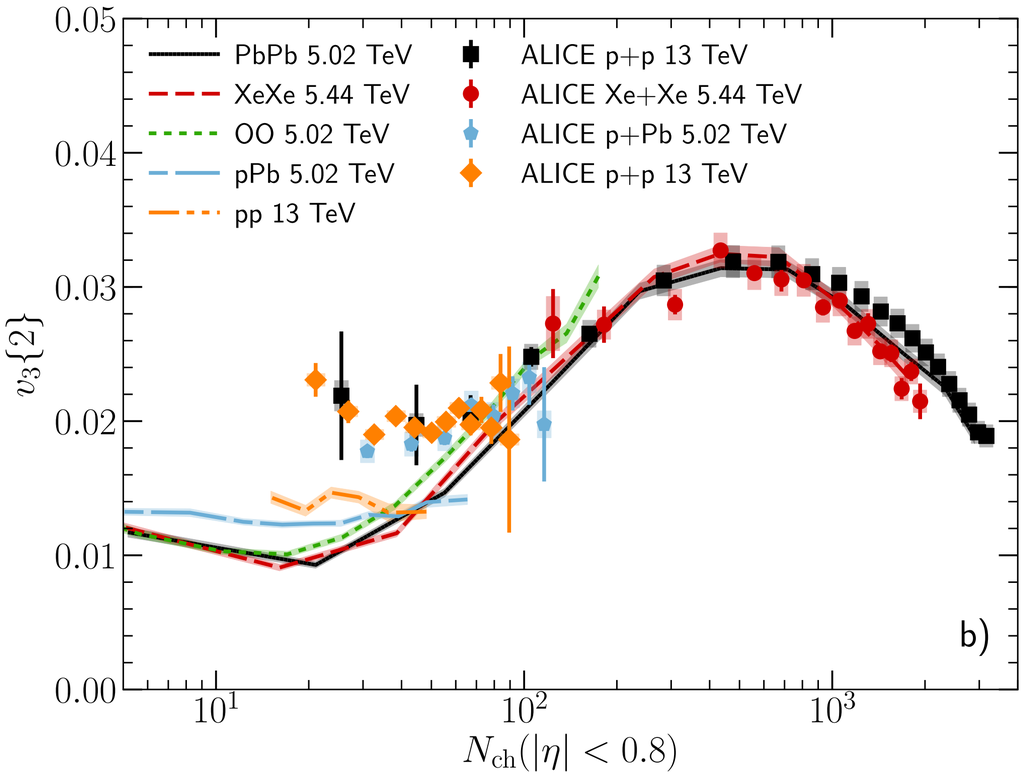

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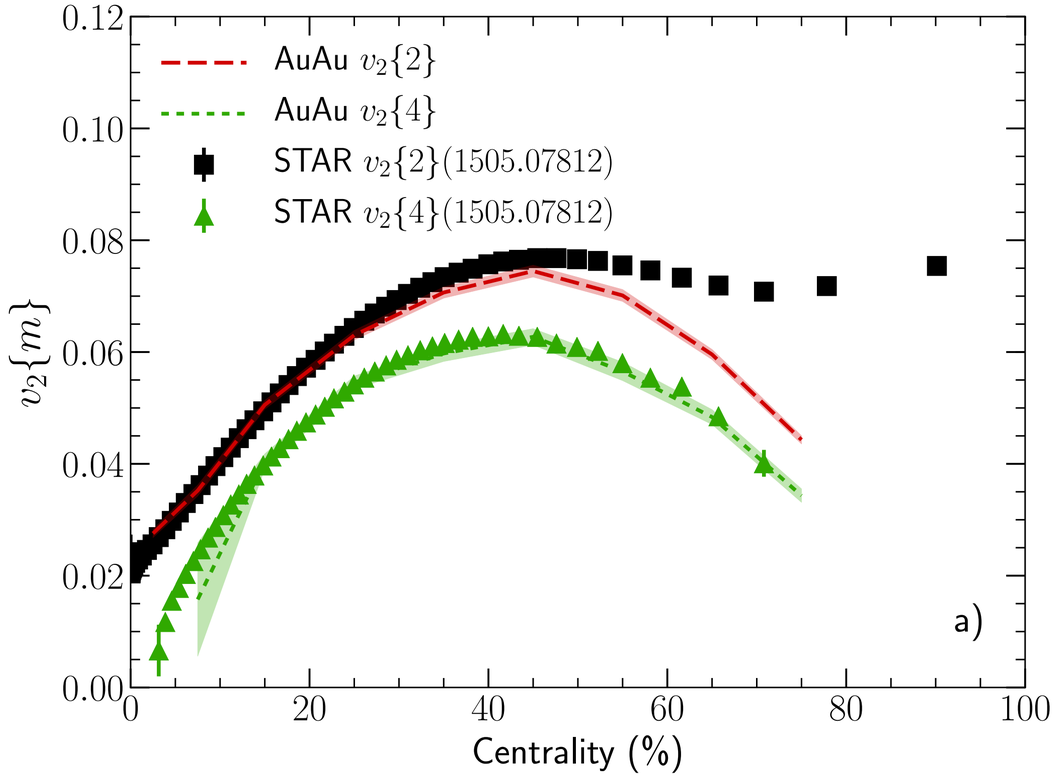
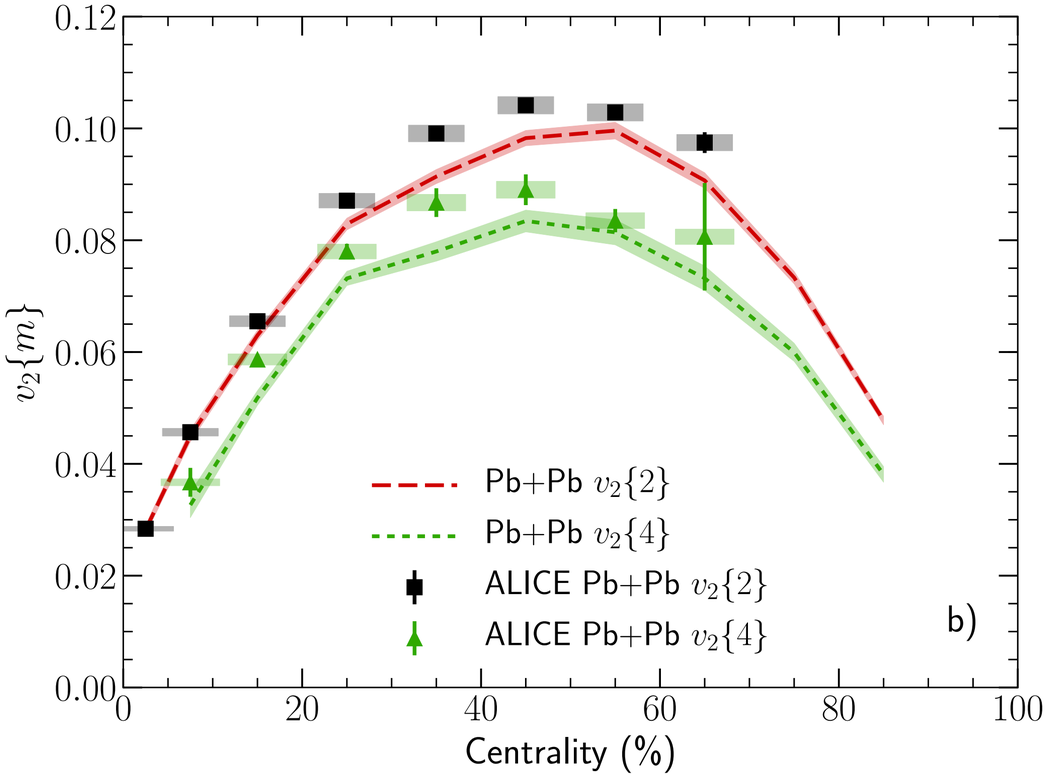
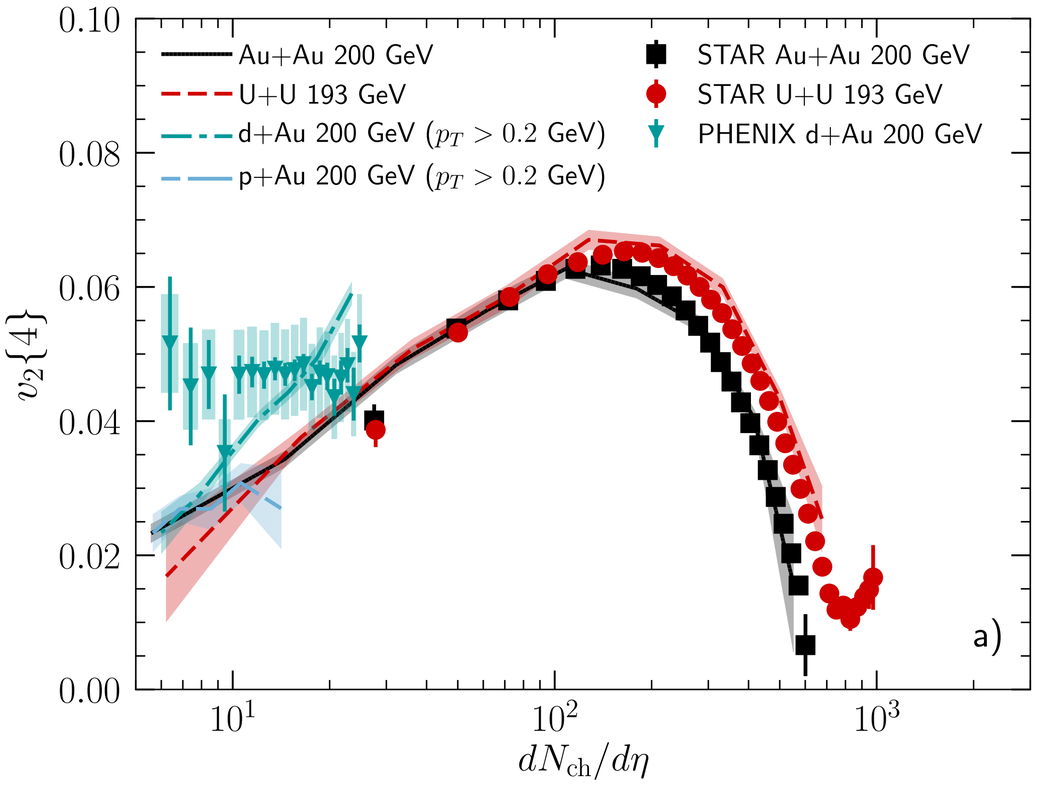
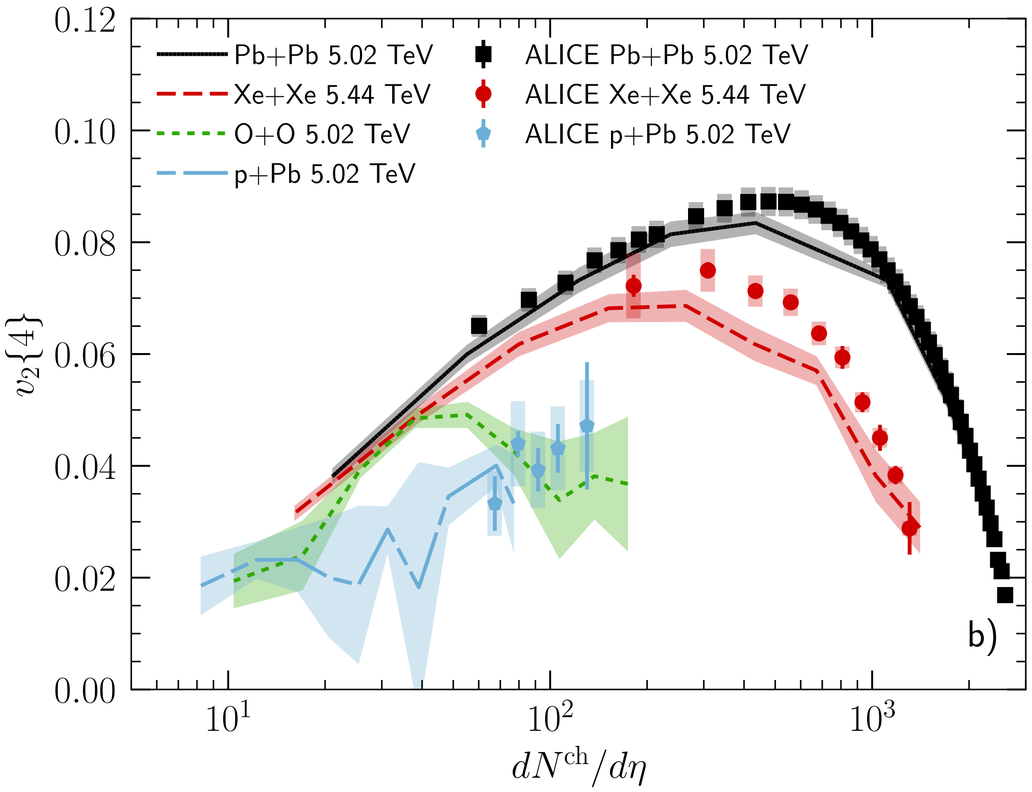


# BACKUP: Model validation

**B. Schenke, C. Shen, P. Tribedy, Phys. Rev. C 102 (2020) 4, 044905**



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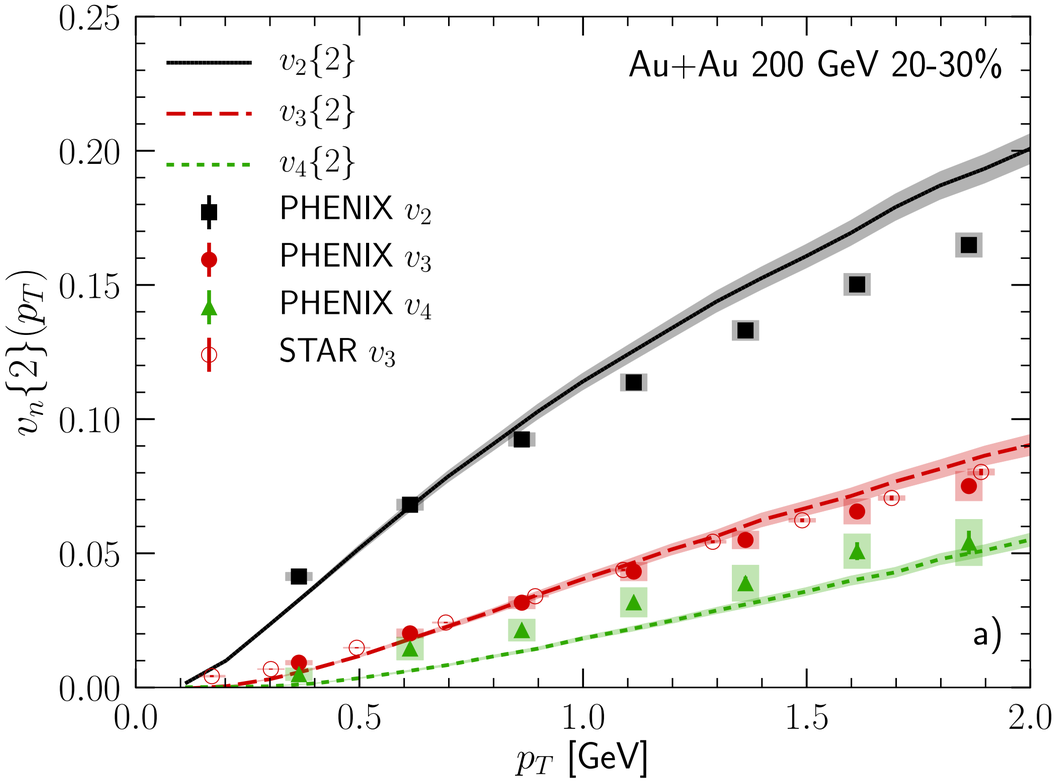


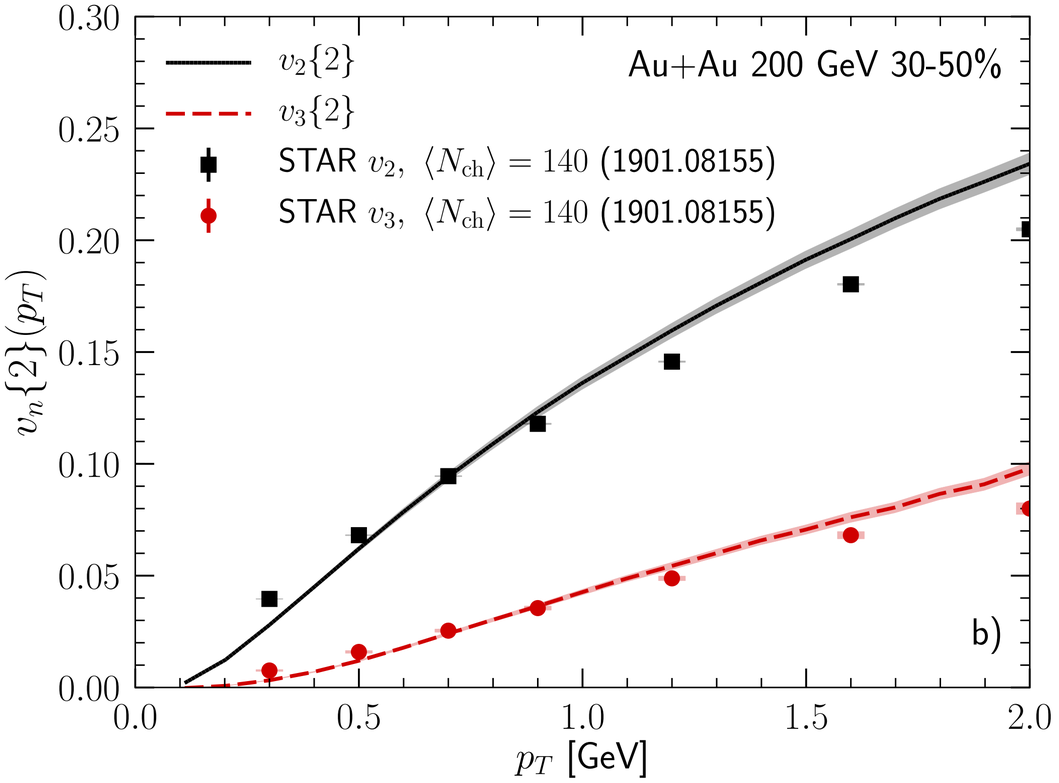

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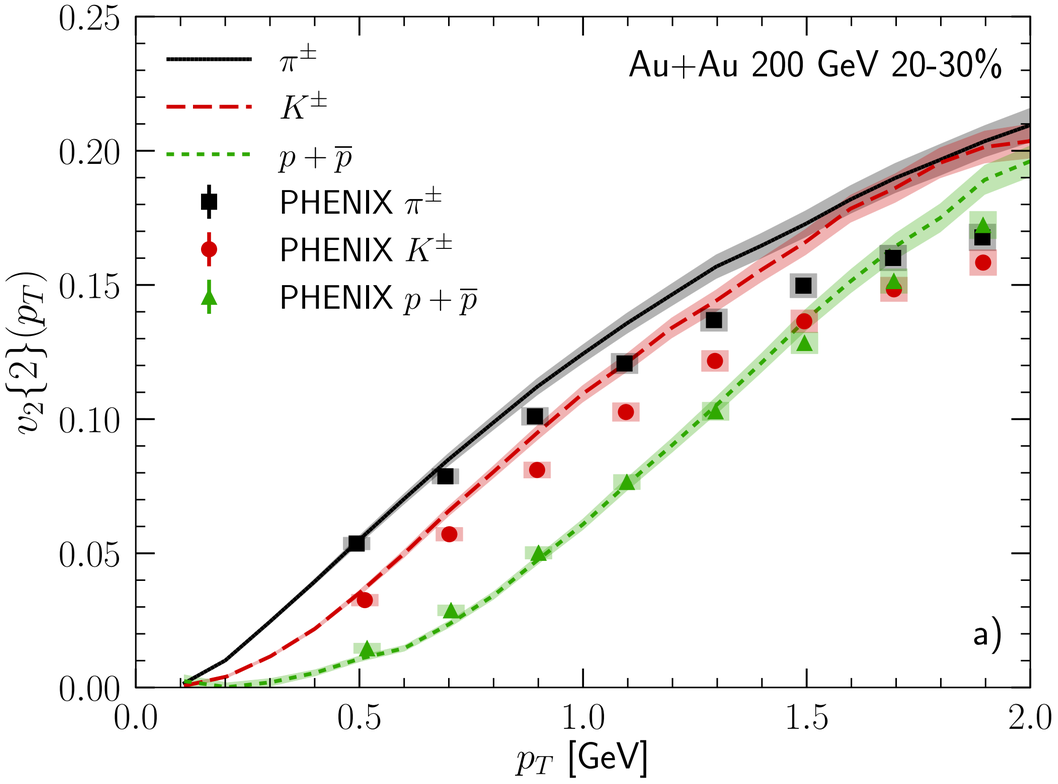


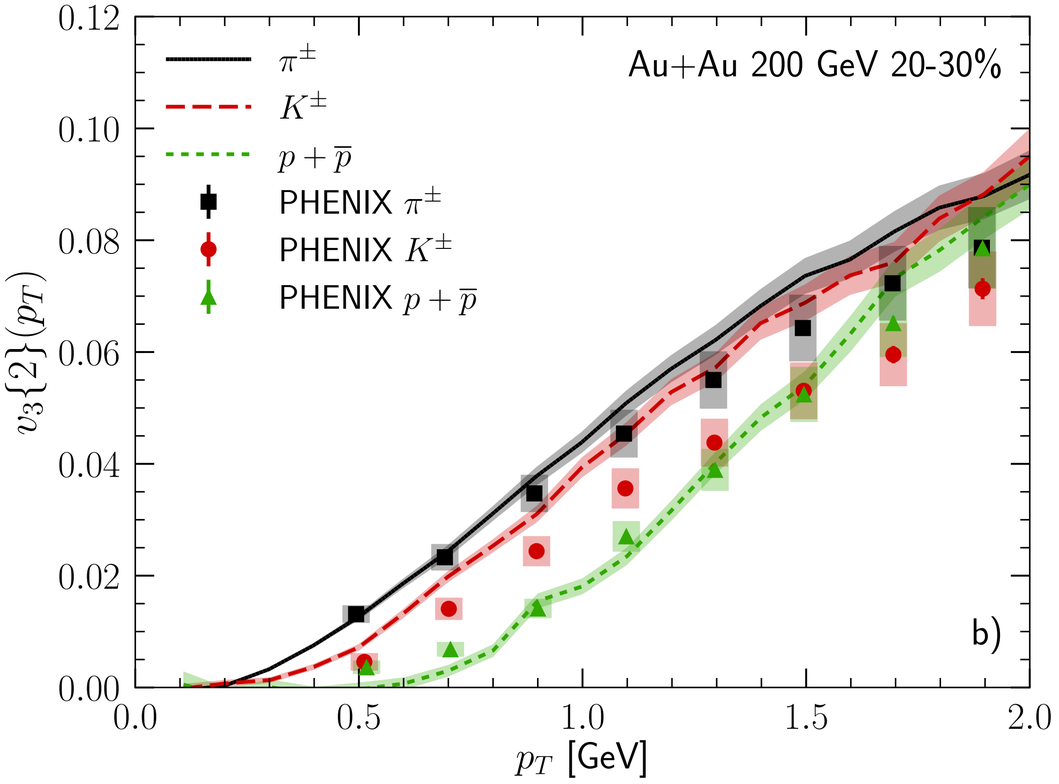

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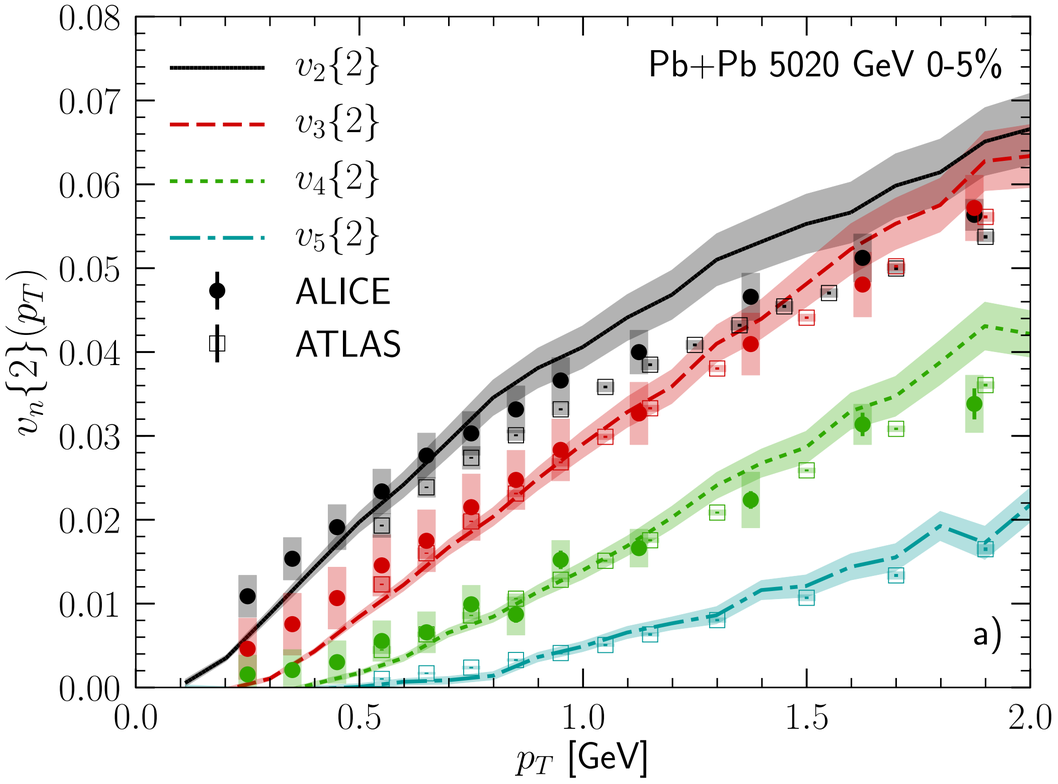


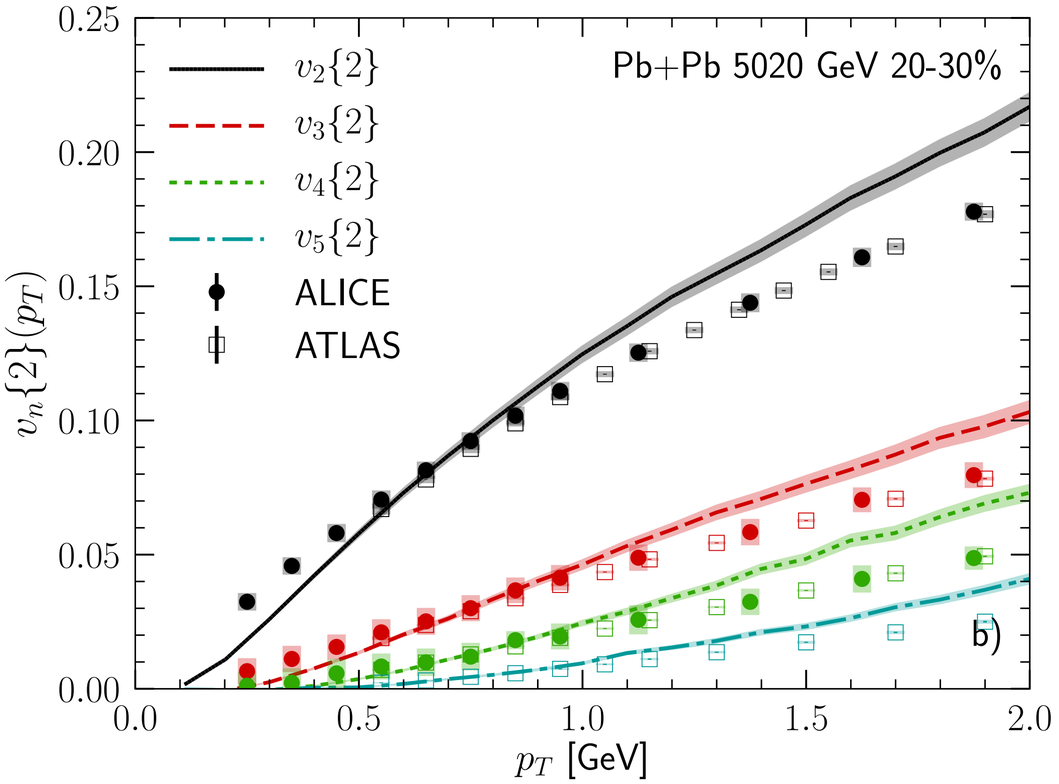

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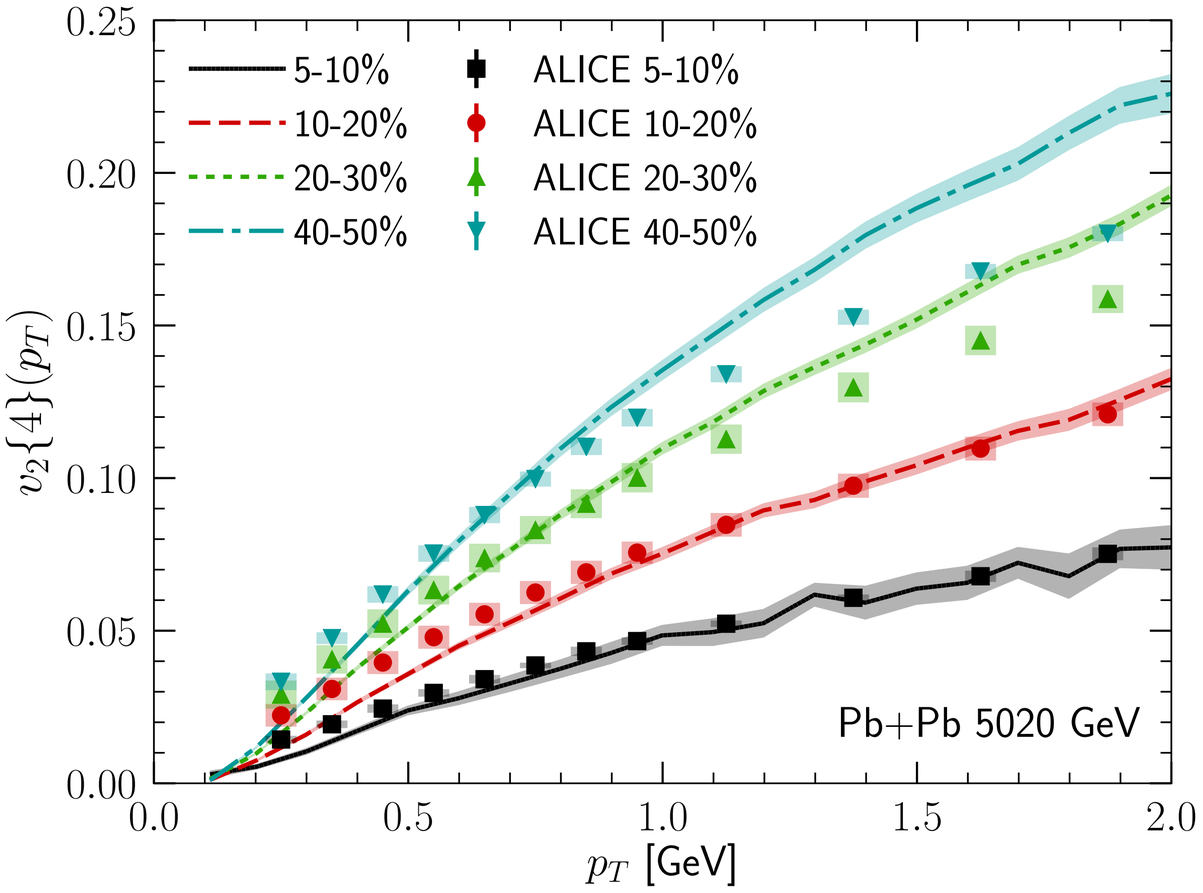

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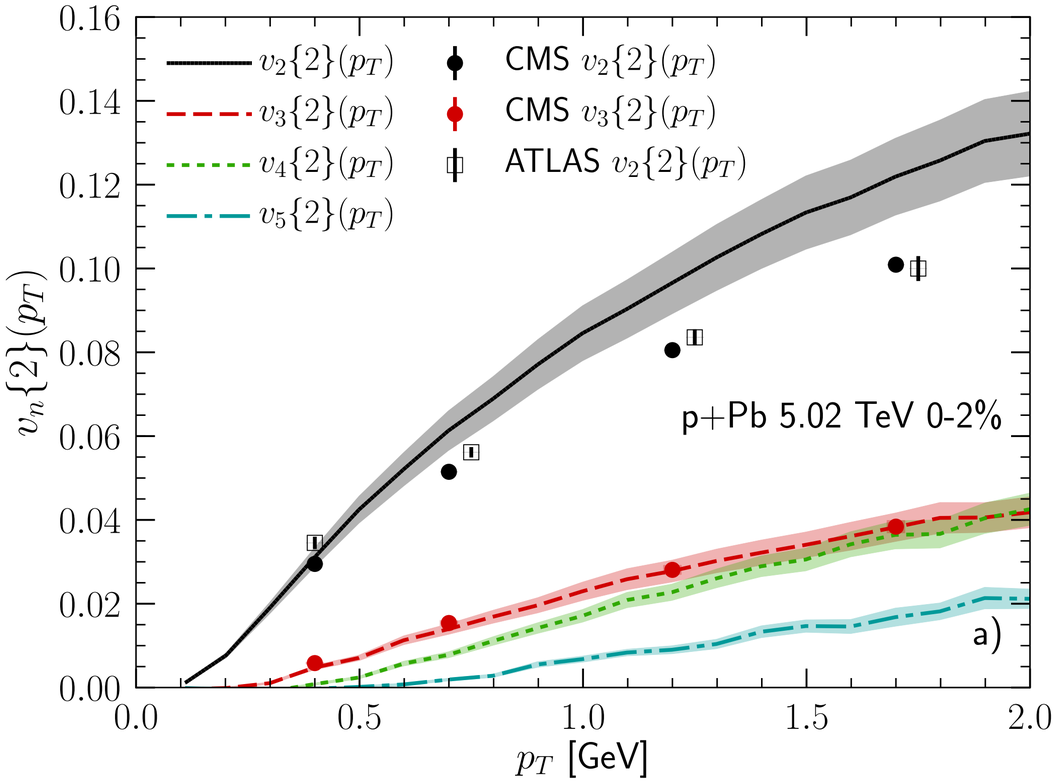


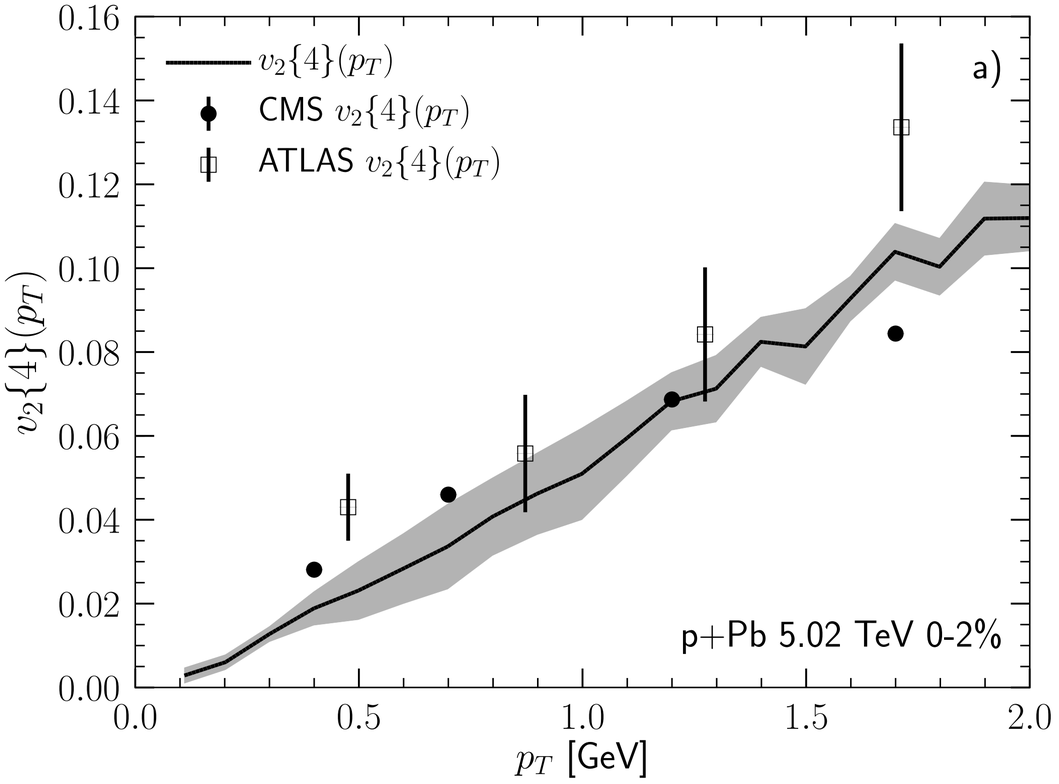

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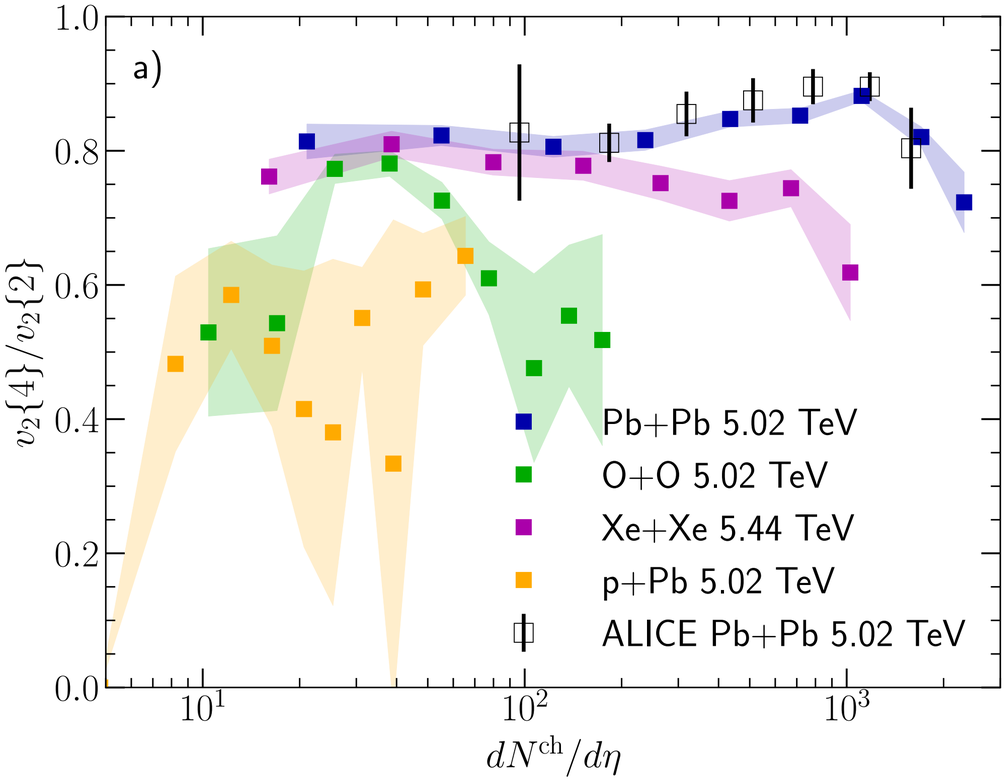
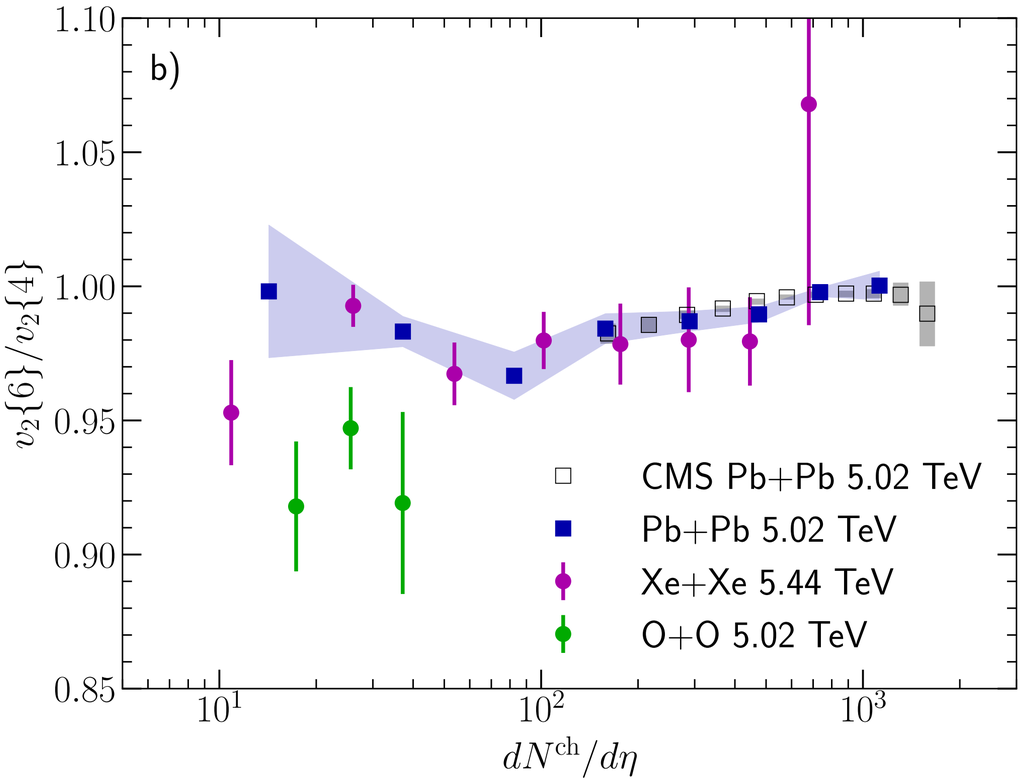
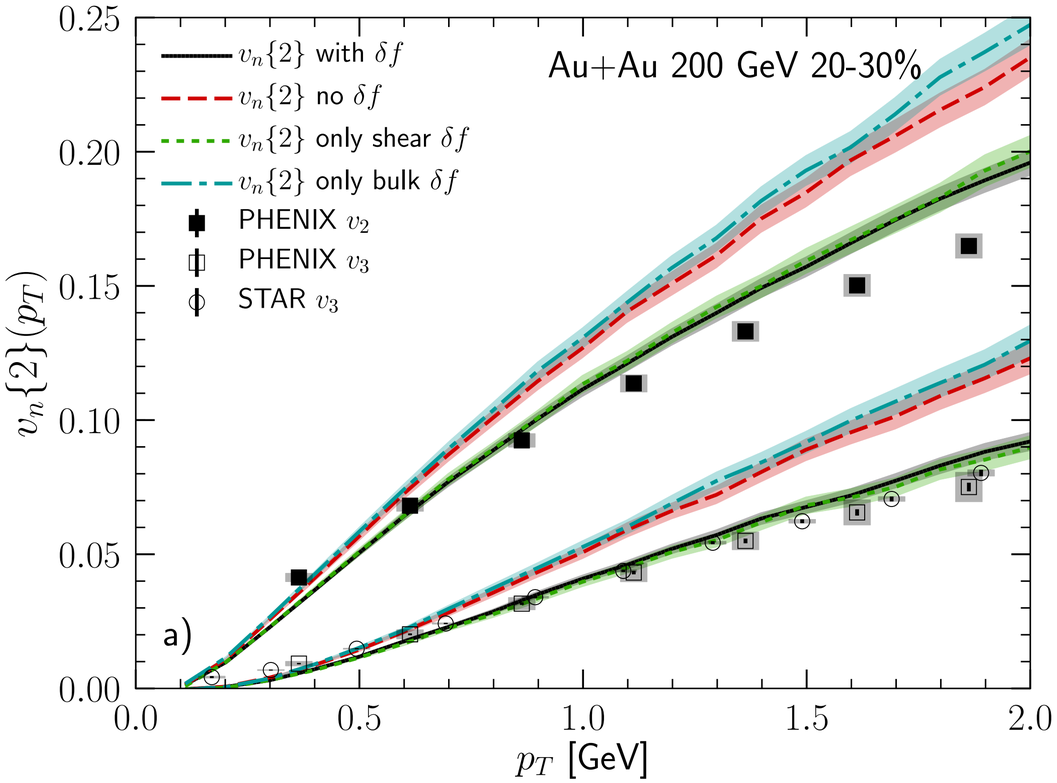
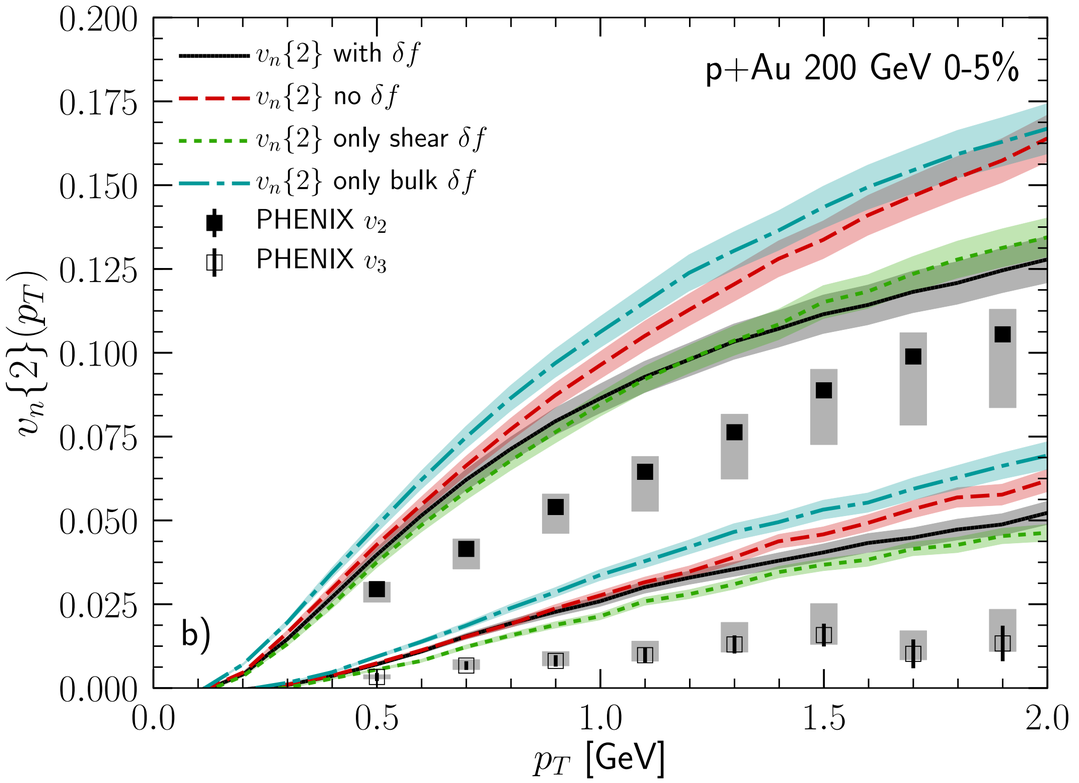


# BACKUP: Effects of $\delta f$

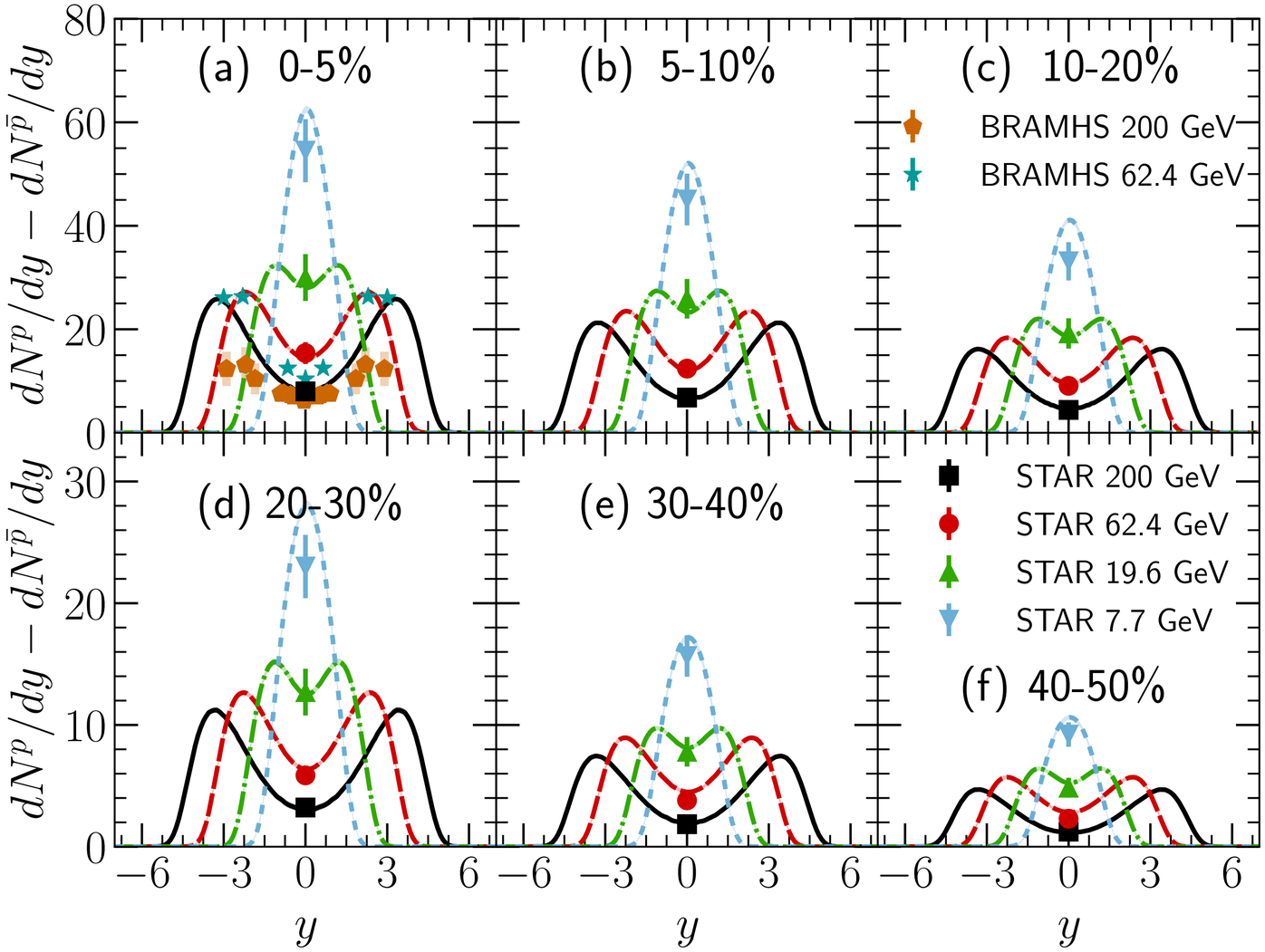
**B. Schenke, C. Shen, P. Tribedy, Phys. Rev. C 102 (2020) 4, 044905**



# NET-BARYON DISTRIBUTIONS

**C. Shen and B. Schenke, in preparation**



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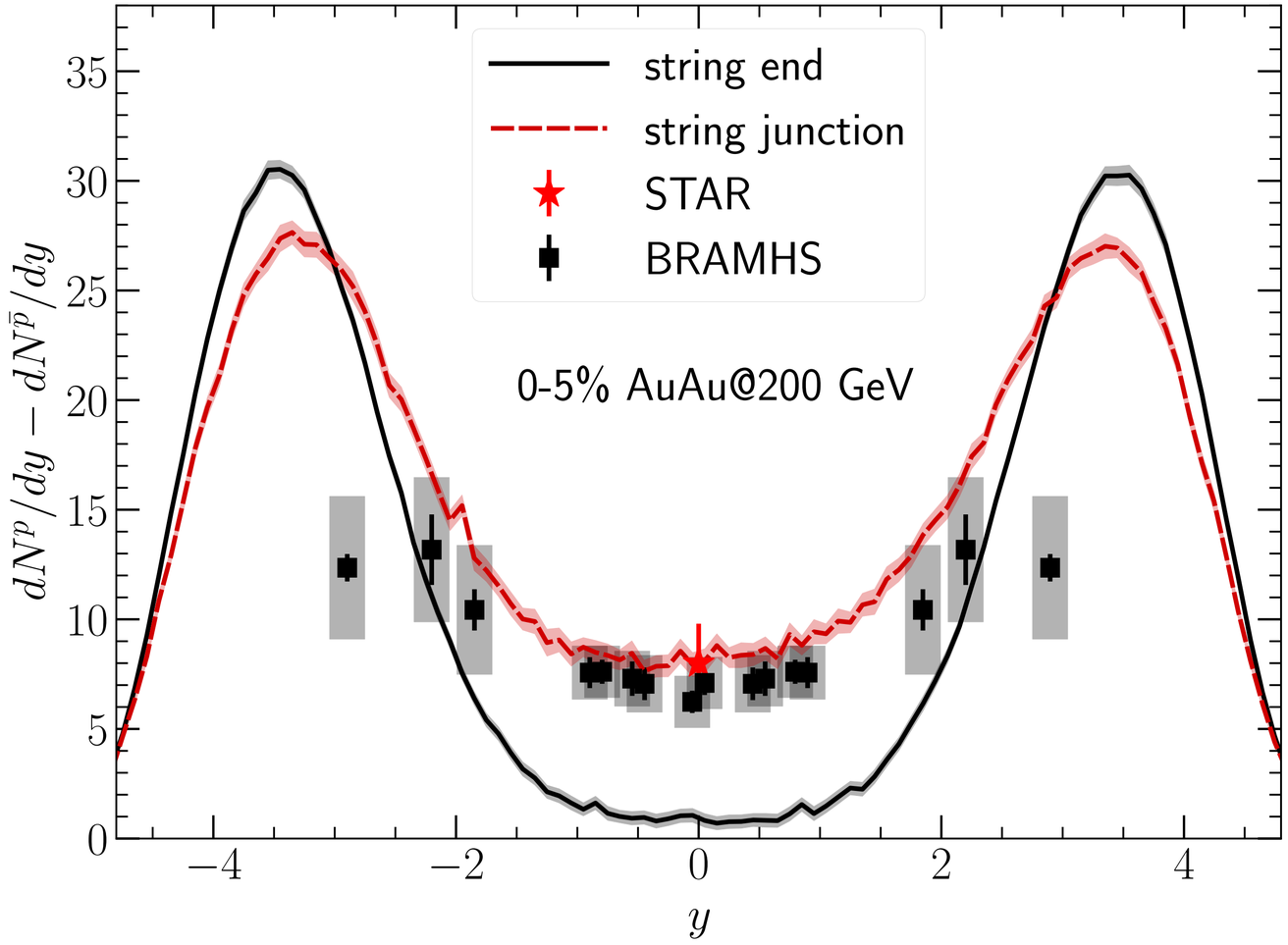


## Effect of baryon junctions