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Recent developments of spin physics in relativistic nuclear collisions

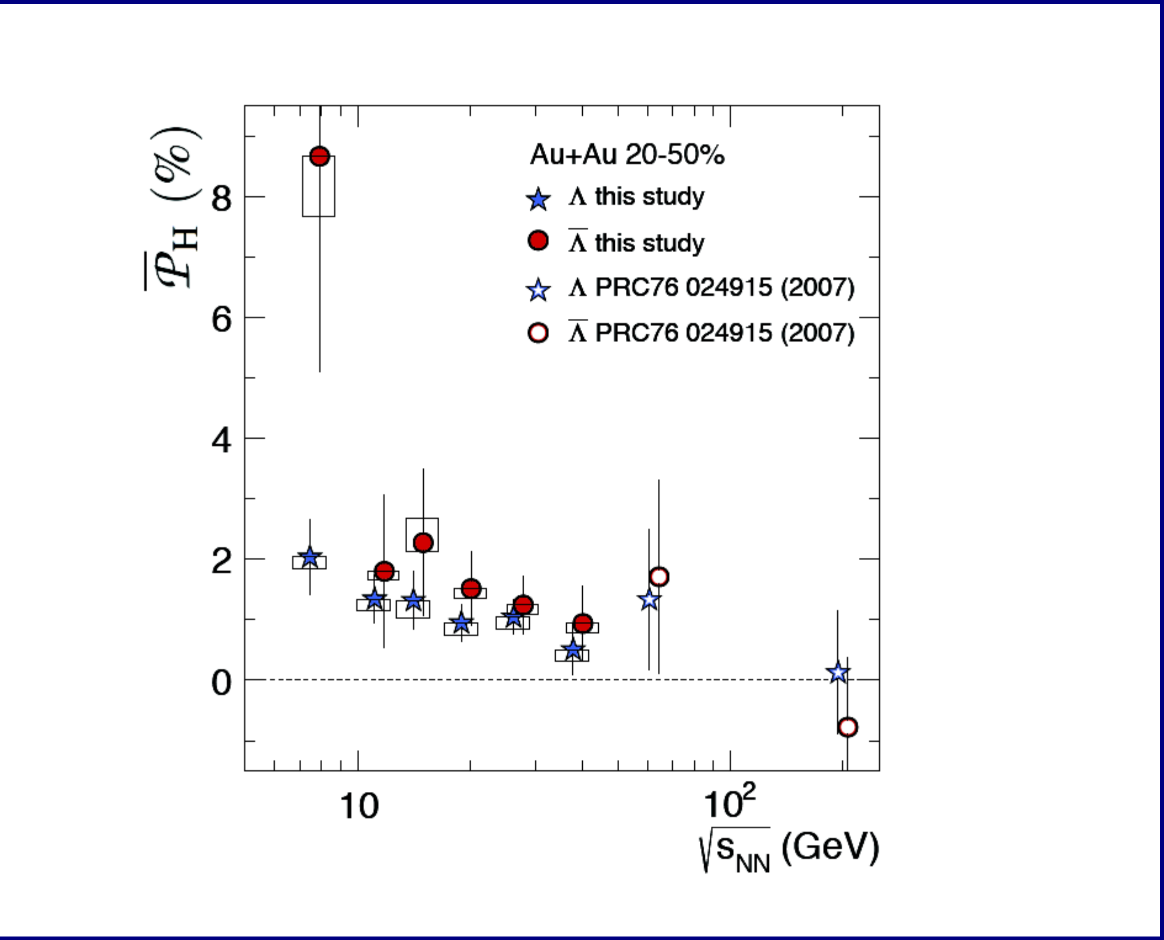
OUTLINE

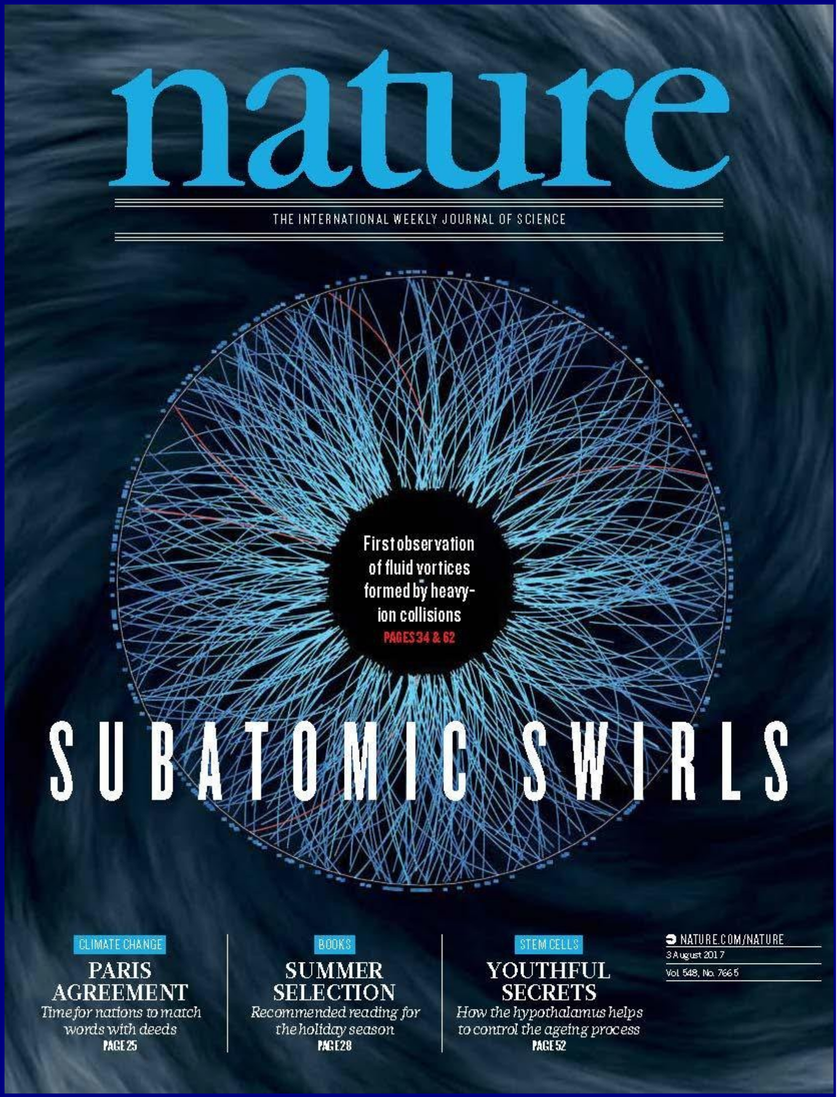
- Introduction and local polarization puzzles
- Spin in relativistic fluids: quantum-relativistic theory
- Local thermodynamic equilibrium and its expansion
- Spin-thermal shear coupling and application to heavy ion physics
- Isothermal local equilibrium

RHIC-BES seminar series III Nov 23 2021

Introduction

STAR Collaboration, *Global Lambda hyperon polarization in nuclear collisions,* Nature 548 62-65, 2017

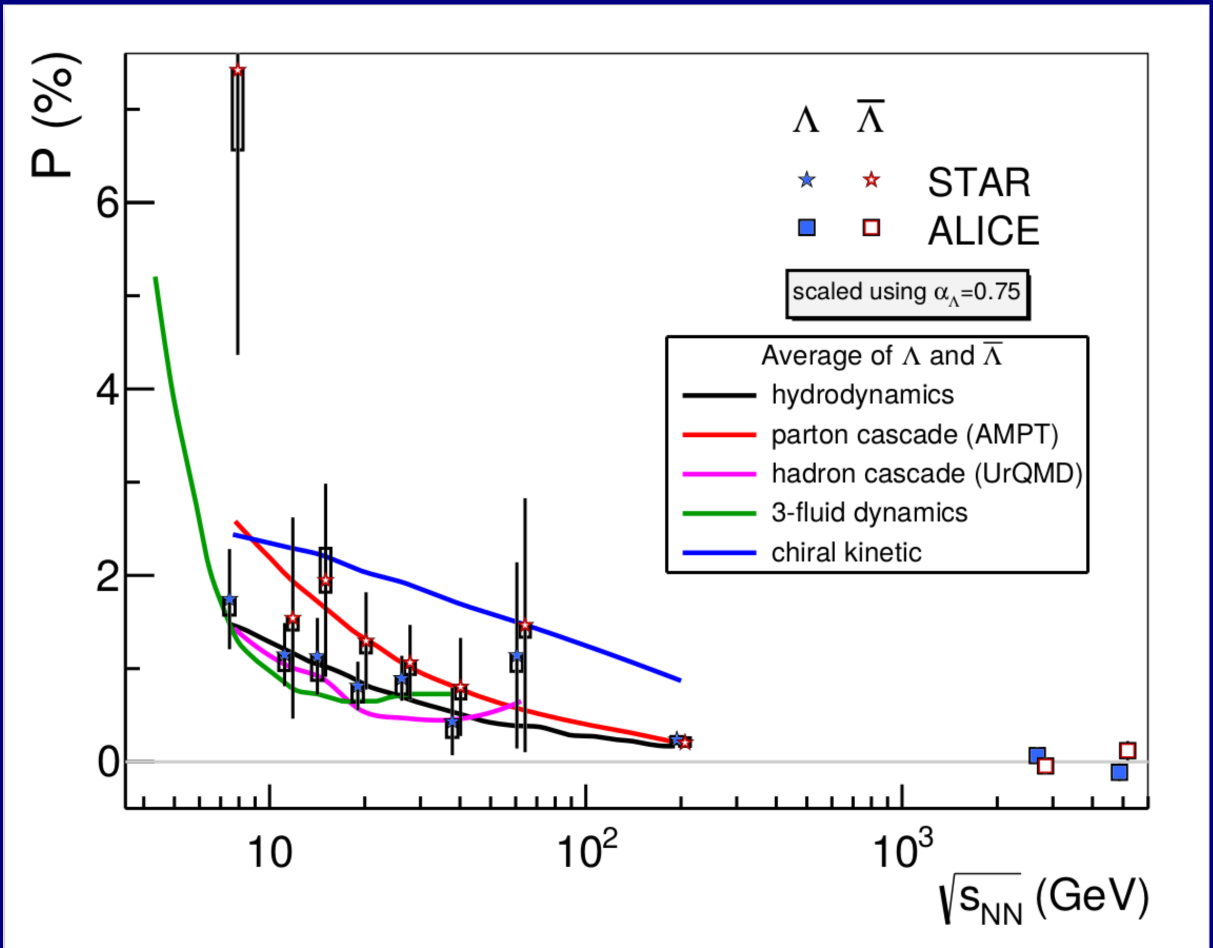




Particle and antiparticle have the same polarization sign.
This shows that the phenomenon cannot be driven
by a mean field (such as EM) whose coupling is *C-odd.*
Definitely favours the thermodynamic (equipartition) interpretation

Comparison with the data (date Jan 2020)

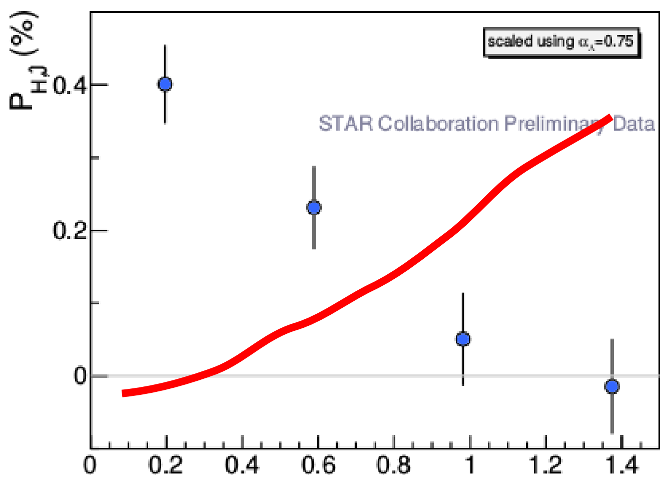
F. B., M. Lisa, Polarization and vorticity in the QGP, Ann. Rev. Part, Nucl. Sc. 70, 395 (2020)



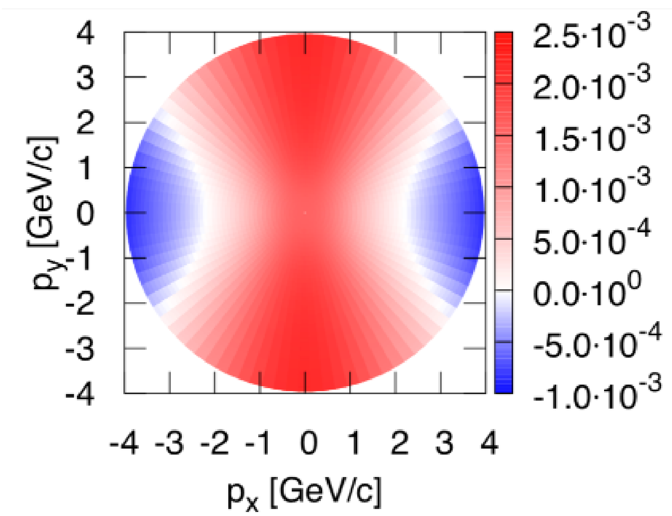
$$S^\mu(p) = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma} p_\sigma \frac{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau n_F (1 - n_F) \partial_\nu \beta_\rho}{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau \ n_F}$$

Different models of the collision, same formula for polarization

Puzzles: momentum dependence of polarization

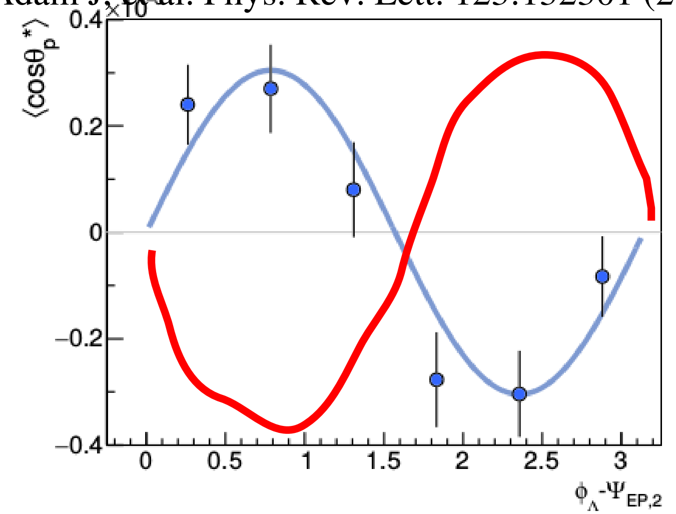


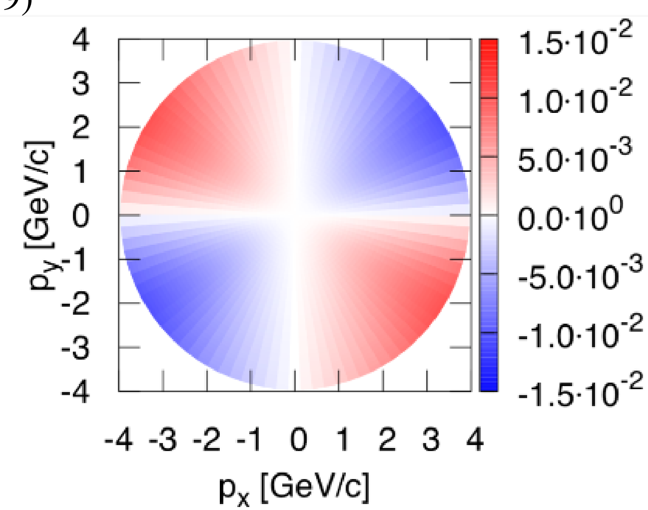
Niida T. Nucl. Phys. A982:511 (2019)



Theory prediction

Adam J, et al. Phys. Rev. Lett. 123:132301 (2019)





Not the effect of decays:

X. L. Xia, H. Li, X.G. Huang and H. Z. Huang, Phys. Rev. C 100 (2019), 014913

F. B., G. Cao and E. Speranza, Eur. Phys. J. C 79 (2019) 741

How to solve the problem?

- *Final hadronic interactions/rescattering*

L. Csernai, J. Kapusta. Y. Xie, C. Barros, ...

- *Dissipative corrections*

K. Hattori, M. Hongo, S. Bhadury, W. Florkowski, A. Jaiswal, S. Shi, A. Kumar, R. Sing, D. Hou, J. Liao, …..

- *Lack of local thermodynamic equilibrium in the spin sector: kinetics*

Q. Wang, X. L. Sheng, X. N. Wang. Z. T. Liao, N. Weickgennant, D. Rischke, J. H. Gao, E. Speranza, X. G. Huang, P. Zhuang, C. M. Ko, Y. Sun, J. Kapusta, ….

- *Role of the spin tensor: additional spin potential required and spin-hydrodynamics*

W. Florkowski, R. Ryblewski, E. Speranza, M. Hongo, M. Stephanov, H. U. Yee,…..

Polarization in a relativistic fluid: theory

F. Becattini, Lecture Notes in Physics 987, 15 (2021) arXiv:2004.04050

The covariant Wigner function of the free Dirac field:

$$W(x,k)_{AB} = -\frac{1}{(2\pi)^4}\int \mathrm{d}^4y\, \mathrm{e}^{-ik\cdot y}\langle :\Psi_A(x-y/2)\overline{\Psi}_B(x+y/2):\rangle$$
$$= \frac{1}{(2\pi)^4}\int \mathrm{d}^4y\, \mathrm{e}^{-ik\cdot y}\langle :\overline{\Psi}_B(x+y/2)\Psi_A(x-y/2):\rangle$$

$$\langle\widehat{X}\rangle = \mathrm{tr}(\widehat{\rho}\widehat{X})$$

It allows to calculate the spin density matrix for spin ½:

$$\Theta(p)_{rs} = \frac{\int \mathrm{d}\Sigma_\mu p^\mu \bar{u}_r(p) W_+(x,p) u_s(p)}{\sum_t \int \mathrm{d}\Sigma_\mu p^\mu \bar{u}_t(p) W_+(x,p) u_t(p)}$$

And the mean spin vector in these three equivalent forms:

$$S^\mu(p) = -\frac{1}{2m}\epsilon^{\mu\beta\gamma\delta}p_\delta \frac{\int \mathrm{d}\Sigma_\lambda p^\lambda \mathrm{tr}_4(\Sigma_{\beta\gamma}W_+(x,p))}{\int \mathrm{d}\Sigma_\lambda p^\lambda \mathrm{tr}_4 W_+(x,p)}$$

$$S^\mu(p) = \frac{1}{2}\frac{\int \mathrm{d}\Sigma\cdot p\ \mathrm{tr}_4(\gamma^\mu\gamma^5 W_+(x,p))}{\int \mathrm{d}\Sigma\cdot p\ \mathrm{tr}_4 W_+(x,p)}$$

$$S^\mu(p) = -\frac{1}{4}\epsilon^{\mu\beta\gamma\delta}p_\delta \frac{\int \mathrm{d}\Sigma_\lambda \mathrm{tr}_4(\{\gamma^\lambda,\Sigma_{\beta\gamma}\}W_+(x,p))}{\int \mathrm{d}\Sigma_\lambda p^\lambda \mathrm{tr}_4 W_+(x,p)}$$

Density operator of quantum relativistic fluid

Needed to calculate the Wigner function!

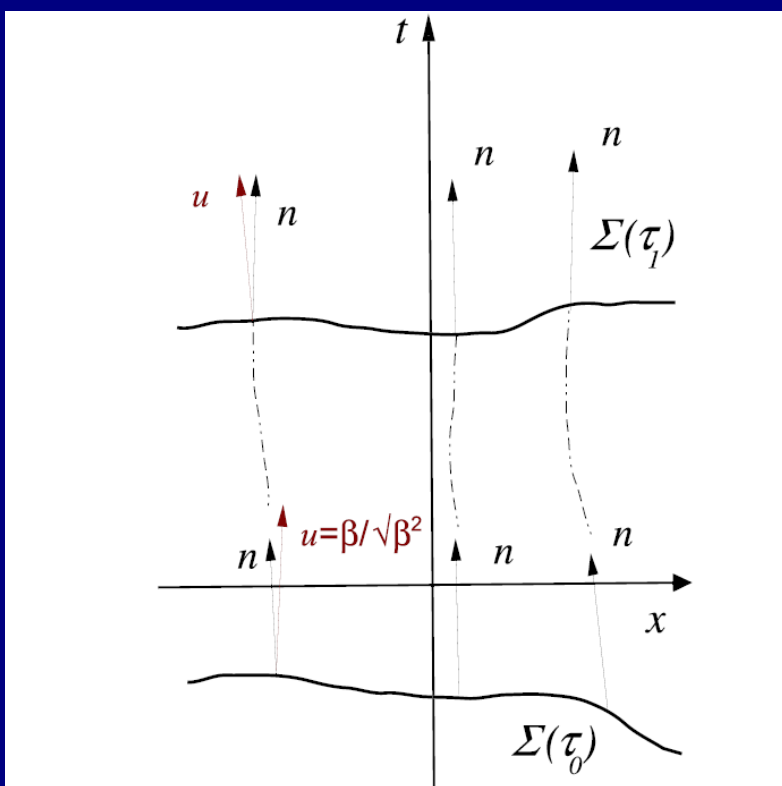
$$W(x,k) = \mathrm{Tr}(\widehat{\rho}\widehat{W}(x,k))$$

$$\beta = \frac{1}{T}u$$

$$\zeta = \frac{\mu}{T}$$

General covariant
Local thermodynamic
Equilibrium density operator

$$\widehat{\rho} = \frac{1}{Z}\exp\left[-\int_\Sigma \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta\widehat{j}^\mu\right)\right]$$



The operator is obtained by maximizing the entropy

$$S = -\mathrm{tr}(\widehat{\rho}\log\widehat{\rho})$$

with the constraints of fixed energy-momentum density

Zubarev, 1979, Ch, Van Weert 1982

See also:
F. B., L. Bucciantini, E. Grossi, L. Tinti,
Eur. Phys. J. C 75 (2015) 191

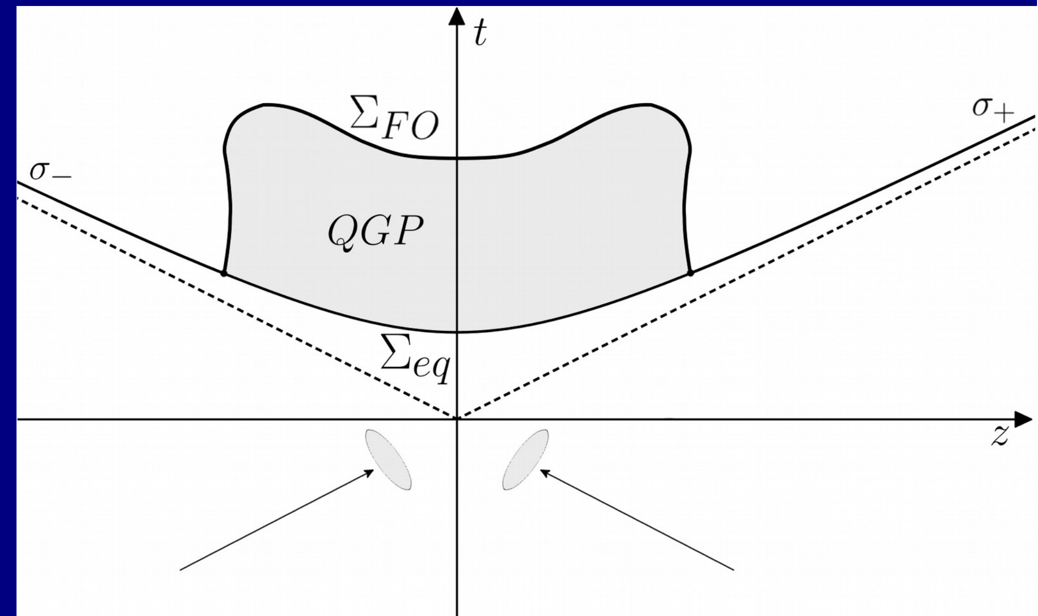
T. Hayata, Y. Hidaka, T. Noumi, M. Hongo,
Phys. Rev. D 92 (2015) 065008

The actual statistical operator

The above density operatot is “time” dependent, cannot be the actual one!

In the Zubarev’s theory, this is the LTE at some initial “time”:

$$\widehat{\rho} = \frac{1}{Z} \exp \left[- \int_{\Sigma(\tau_0)} \mathrm{d}\Sigma_\mu \left(\widehat{T}_B^{\mu\nu} \beta_\nu - \zeta \widehat{j}^\mu \right) \right] .$$



NOTE: T_B stands for the symmetrized Belinfante stress-energy tensor

with the Gauss theorem

$$\widehat{\rho} = \frac{1}{Z} \exp \left[- \int_{\Sigma(\tau)} \mathrm{d}\Sigma_\mu \left(\widehat{T}_B^{\mu\nu} \beta_\nu - \zeta \widehat{j}^\mu \right) + \int_\Theta \mathrm{d}\Theta \; \left(\widehat{T}_B^{\mu\nu} \nabla_\mu \beta_\nu - \widehat{j}^\mu \nabla_\mu \zeta \right) \right] ,$$

Local equilibrium, non-dissipative terms

Dissipative terms

Incidentally: global thermodynamic equilibrium

$$\widehat{\rho} = \frac{1}{Z} \exp\left[- \int_\Sigma \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu} \beta_\nu - \zeta \widehat{j}^\mu \right) \right]$$

Independent of the 3D hypersurface Σ if

$$\partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0 \qquad \partial_\mu \zeta = 0$$

Killing equation

$$\beta_\mu = b_\mu + \varpi_{\mu\nu} x^\nu$$

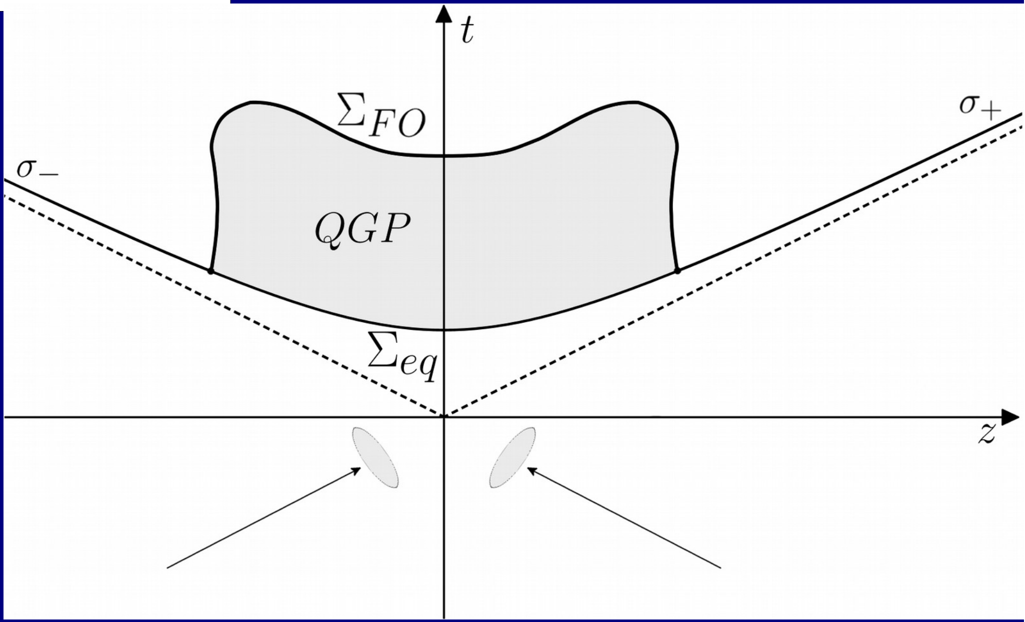
The density operator becomes

$$\widehat{\rho} = \frac{1}{Z} \exp\left[-b_\mu \widehat{P}^\mu + \frac{1}{2} \varpi_{\mu\nu} \widehat{J}^{\mu\nu} + \zeta \widehat{Q} \right]$$

Local thermodynamic equilibrium approximation

$$\widehat{\rho} \simeq \widehat{\rho}_{\rm LE} = \frac{1}{Z}\exp\left[-\int_{\Sigma(\tau)} {\rm d}\Sigma_\mu \, \widehat{T}^{\mu\nu}_B \beta_\nu - \zeta \widehat{j}^\mu\right]$$
$$= \frac{1}{Z}\exp\left[-\int_{\Sigma_{FO}} {\rm d}\Sigma_\mu \, \widehat{T}^{\mu\nu}_B \beta_\nu - \zeta \widehat{j}^\mu\right]$$

Corresponding to the ideal fluid: Neglecting dissipative term in the exponent of the density operator



$$W(x,k) \simeq W(x,k)_{\rm LE} = {\rm Tr}(\widehat{\rho}_{\rm LE}\widehat{W}(x,k))$$

Mean value of a local operator: Taylor expansion

$$W(x,k)_{\rm LE} = \frac{1}{Z}{\rm Tr}\left(\exp\left[-\int_{\Sigma_{FO}} {\rm d}\Sigma_\mu(y)\widehat{T}_B^{\mu\nu}(y)\beta_\nu(y) - \zeta(y)\widehat{j}^\mu(y)\right]\widehat{W}(x,k)\right)$$

Expand the β and ζ fields from the point *x* where the Wigner operator is to be evaluated

$$\beta_\nu(y) = \beta_\nu(x) + \partial_\lambda\beta_\nu(x)(y-x)^\lambda + \ldots$$

$$\int_\Sigma {\rm d}\Sigma_\mu T_B^{\mu\nu}(y)\beta_\nu(x) = \beta_\nu(x)\int_\Sigma {\rm d}\Sigma_\mu T_B^{\mu\nu}(y) = \beta_\nu(x)\widehat{P}^\nu$$

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z}\exp\left[-\beta_\mu(x)\widehat{P}^\mu - \frac{1}{2}(\partial_\mu\beta_\nu(x) - \partial_\nu\beta_\mu(x))\widehat{J}_x^{\mu\nu} - \frac{1}{2}(\partial_\mu\beta_\nu(x) + \partial_\nu\beta_\mu(x))\widehat{Q}_x^{\mu\nu} + \ldots\right]$$

$$\widehat{J}_x^{\mu\nu} = \int {\rm d}\Sigma_\lambda (y-x)^\mu \widehat{T}_B^{\lambda\nu}(y) - (y-x)^\nu \widehat{T}_B^{\lambda\mu}(y)$$

$$\widehat{Q}_x^{\mu\nu} = \int {\rm d}\Sigma_\lambda (y-x)^\mu \widehat{T}_B^{\lambda\nu}(y) + (y-x)^\nu \widehat{T}_B^{\lambda\mu}(y)$$

neglected in < 2021

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z} \exp[-\beta_\mu(x)\widehat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}(x)\widehat{J}^{\mu\nu}_x - \frac{1}{2}\xi_{\mu\nu}(x)\widehat{Q}^{\mu\nu}_x + \ldots]$$

$$\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu)$$

Thermal vorticity
Adimensional in natural units

$$\xi_{\mu\nu} = \frac{1}{2}(\partial_\mu\beta_\nu + \partial_\nu\beta_\mu)$$

Thermal shear
Adimensional in natural units

At global equilibrium the thermal shear vanishes because of the Killing equation

Surprise: thermal shear does contribute!

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z} \exp[-\beta_\mu(x)\widehat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}(x)\widehat{J}^{\mu\nu}_x - \frac{1}{2}\xi_{\mu\nu}(x)\widehat{Q}^{\mu\nu}_x + \ldots]$$

$$S^\mu(p) = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma} p_\sigma \frac{\int_\Sigma {\rm d}\Sigma_\tau p^\tau n_F(1-n_F)\partial_\nu\beta_\rho}{\int_\Sigma {\rm d}\Sigma_\tau p^\tau \, n_F}$$

$$n_F = ({\rm e}^{\beta\cdot p - \xi} + 1)^{-1}$$

NON-dissipative effect

$$S^\mu_\xi(p) = -\frac{1}{4m}\epsilon^{\mu\nu\sigma\tau}\frac{p_\tau p^\rho}{\varepsilon}\frac{\int_\Sigma {\rm d}\Sigma\cdot p\, n_F(1-n_F)\hat{t}_\nu\xi_{\sigma\rho}}{\int_\Sigma {\rm d}\Sigma\cdot p\, n_F},$$

F. B., M. Buzzegoli, A. Palermo, Phys. Lett. B 820 (2021) 136519

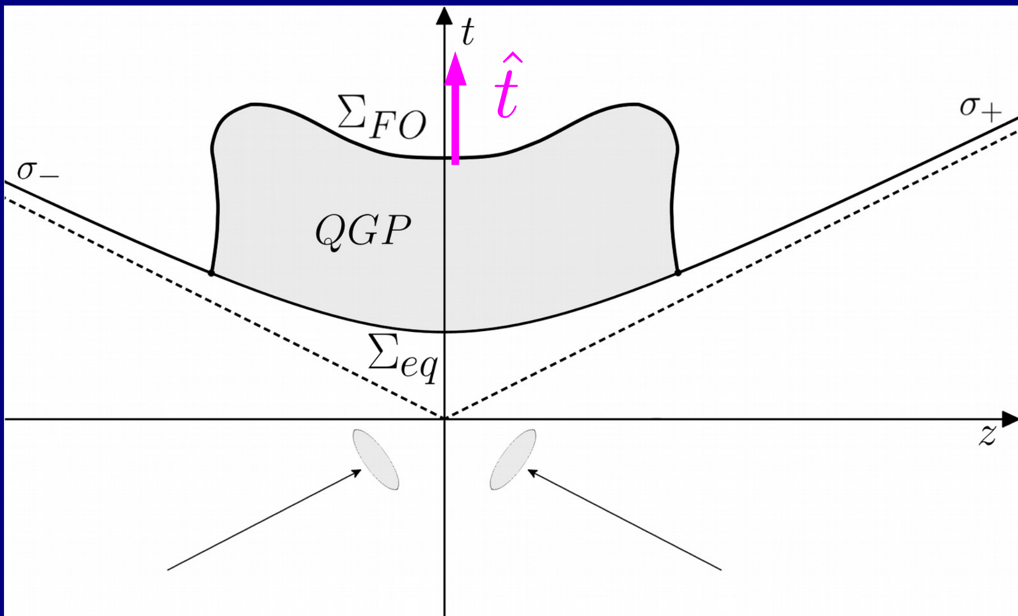
Same (though not precisely the same) formula obtained by Liu and Yin with a different method:

S. Liu, Y. Yin, JHEP 07 (2021) 188 *See next talk!*

The additional local equilibrium term has been confirmed in more analyses:

C. Yi, S. Pu, D. L. Yang, 2106.00238
Y. C. Liu, X. G. Huang, arXiv 2109.15034

Why a dependence on Σ ?



The thermal shear term depends on the correlator:

$$\langle \widehat{Q}_x^{\mu\nu} \widehat{W}(x,p) \rangle$$

$$\widehat{J}_x^{\mu\nu} = \int \mathrm{d}\Sigma_\lambda (y-x)^\mu \widehat{T}_B^{\lambda\nu}(y) - (y-x)^\nu \widehat{T}_B^{\lambda\mu}(y)$$

$$\widehat{Q}_x^{\mu\nu} = \int_{\Sigma_{FO}} \mathrm{d}\Sigma_\lambda (y-x)^\mu \widehat{T}_B^{\lambda\nu}(y) + (y-x)^\nu \widehat{T}_B^{\lambda\mu}(y)$$

The divergence of the integrand of $J^{\mu\nu}$ vanishes, therefore it does not depend on the integration hypersurface (it is a constant of motion) and J is thus a tensor operator:

$$\widehat{\Lambda} \widehat{J}_x^{\mu\nu} \widehat{\Lambda}^{-1} = \Lambda^{-1\mu}_{\alpha} \Lambda^{-1\nu}_{\beta} \widehat{J}_x^{\alpha\beta}$$

The divergence of the integrand of $Q^{\mu\nu}$ does not vanish, therefore it does depend on the integration hypersurface and Q is NOT a tensor operator

$$\widehat{\Lambda} \widehat{Q}^{\mu\nu} \widehat{\Lambda}^{-1} \neq \Lambda^{-1\mu}_{\alpha} \Lambda^{-1\nu}_{\beta} \widehat{Q}^{\alpha\beta}$$

What is this new term?

It is a quantum, *non-dissipative*, correction to local equilibrium

$$S^\mu_\xi(p) = -\frac{1}{4m}\epsilon^{\mu\nu\sigma\tau}\frac{p_\tau p^\rho}{\varepsilon}\frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\; n_F(1-n_F)\hat{t}_\nu \xi_{\sigma\rho}}{\int_\Sigma \mathrm{d}\Sigma\cdot p\; n_F},$$

Does it have a non-relativistic limit?

$$\xi_{\sigma\rho} = \frac{1}{2}\partial_\sigma\left(\frac{1}{T}\right)u_\rho + \frac{1}{2}\partial_\rho\left(\frac{1}{T}\right)u_\sigma + \frac{1}{2T}\left(A_\rho u_\sigma + A_\sigma u_\rho\right) + \frac{1}{T}\sigma_{\rho\sigma} + \frac{1}{3T}\theta\Delta_{\rho\sigma}$$

A is the acceleration field

$$\sigma_{\mu\nu} = \frac{1}{2}(\nabla_\mu u_\nu + \nabla_\nu u_\mu) - \frac{1}{3}\Delta_{\mu\nu}\theta$$

All terms are relativistic (they vanish in the infinite *c* limit) EXCEPT grad T terms, which give rise to:

$$\mathbf{S}_\xi = \frac{1}{8}\mathbf{v}\times\frac{\int \mathrm{d}^3\mathrm{x}\; n_F(1-n_F)\nabla\left(\frac{1}{T}\right)}{\int \mathrm{d}^3\mathrm{x}\; n_F}$$

There is an equal contribution in the NR limit from thermal vorticity

Application to relativistic heavy ion collisions

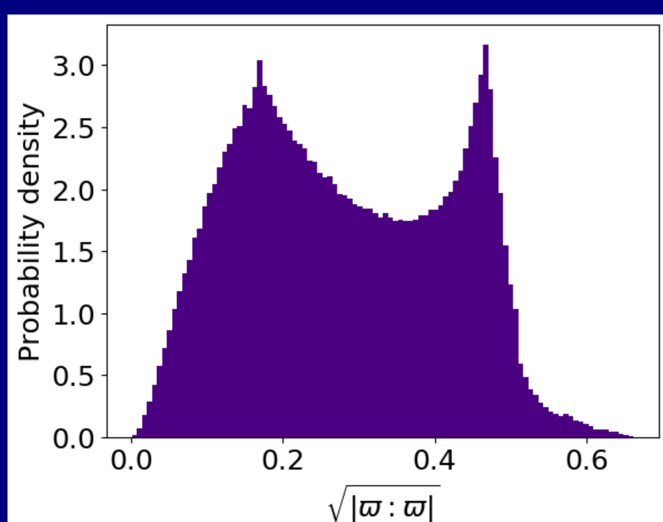
F. B., M. Buzzegoli, A. Palermo, G. Inghirami and I. Karpenko, arXiv:2103.14621, maybe appearing in PRL soon

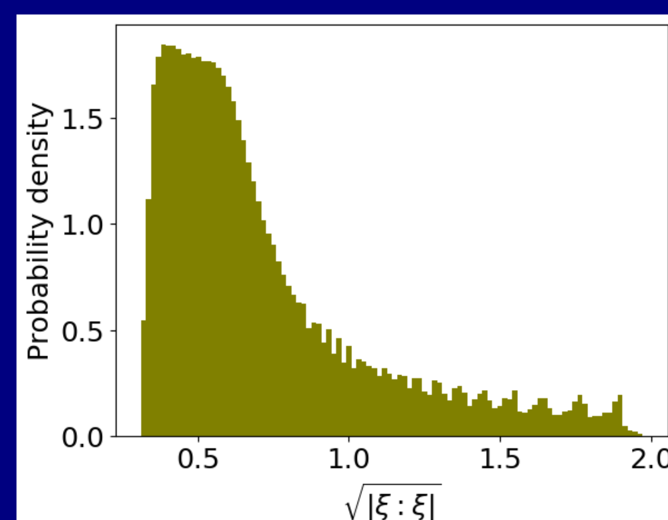
$$S^\mu = S^\mu_\varpi + S^\mu_\xi$$

$$S^\mu_\varpi(p) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\varpi_{\rho\sigma}}{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F}$$

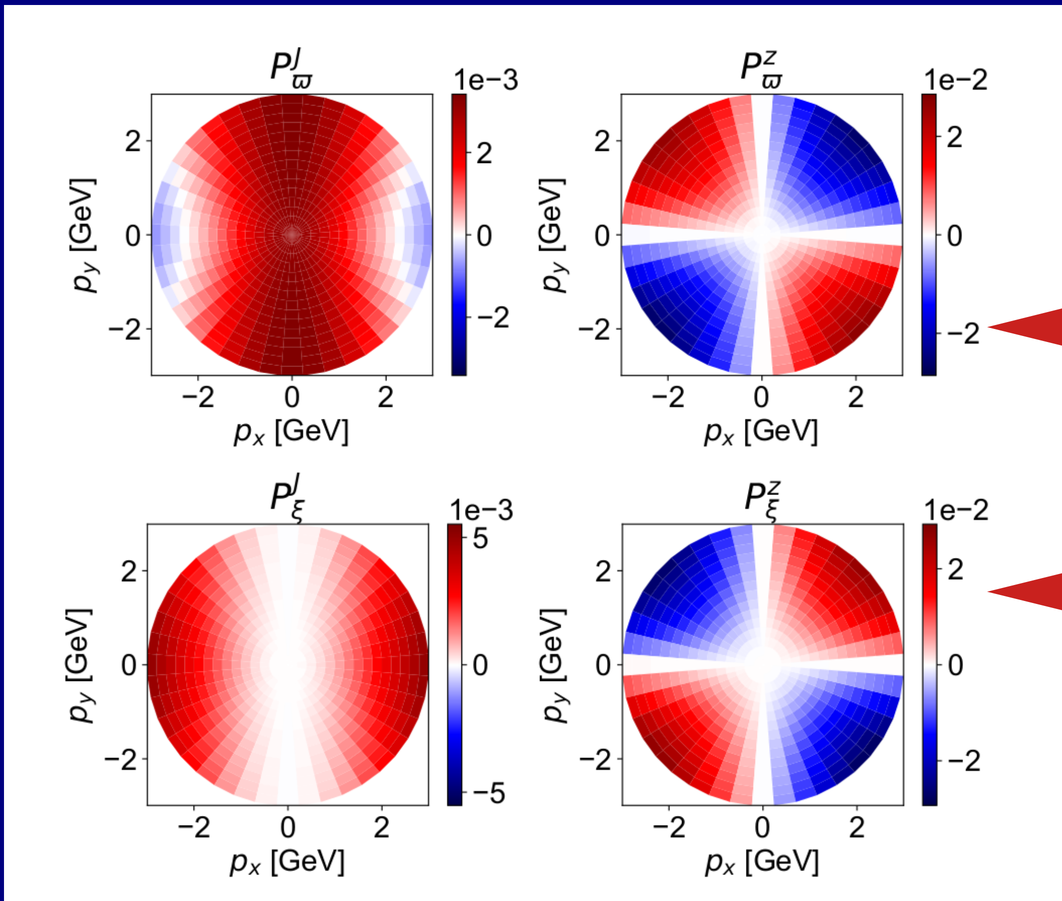
$$S^\mu_\xi(p) = -\frac{1}{4m}\epsilon^{\mu\rho\sigma\tau}\frac{p_\tau p^\lambda}{\varepsilon}\frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\hat{t}_\rho\xi_{\sigma\lambda}}{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F}$$

Is linear response theory adequate?





New calculations

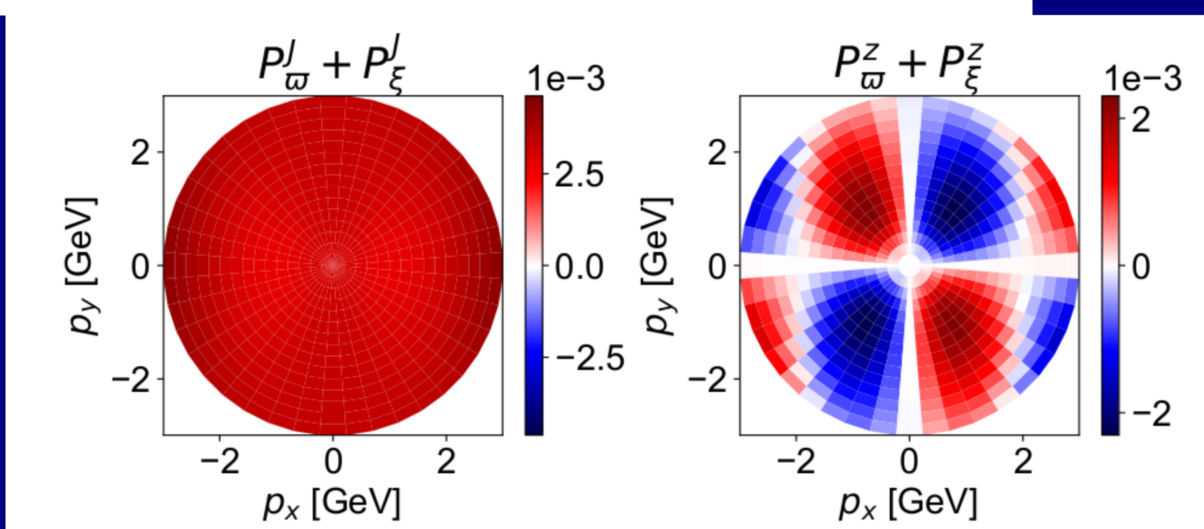


Based on the hydrodynamic code VHLLE (author I. Karpenko) tuned to reproduce Au-Au momentum spectra at RHIC top energy.
Similar output with ECHO-QGP (main author G. Inghirami).

Thermal vorticity

Thermal shear

Right pattern!



SUMMING UP

Not sufficient to restore the agreement between data and model

Calculations fully consistent with:
B. Fu, S. Liu, L. Pang, H. Song and Y. Yin, Phys. Rev. Lett. 127 (2021) 14, 142301

Isothermal local equilibrium

The most appropriate setting for relativistic heavy ion collisions at very high energy!

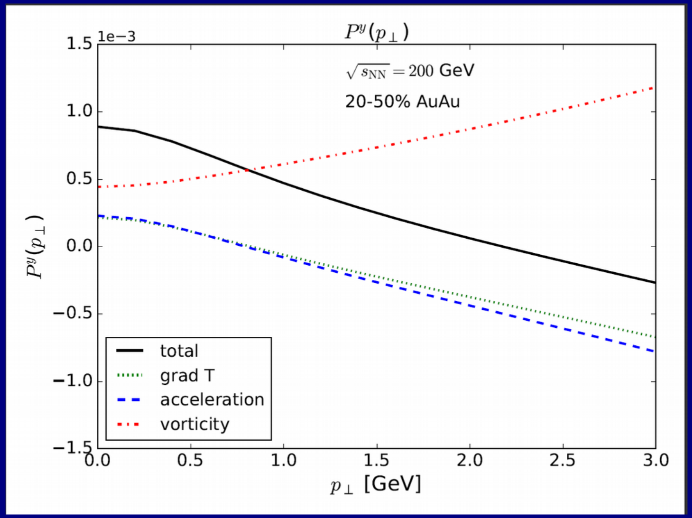
Both thermal shear and thermal vorticity include temperature gradients

$$\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu)$$

$$\xi_{\mu\nu} = \frac{1}{2}(\partial_\mu \beta_\nu + \partial_\nu \beta_\mu)$$

$$\beta^\mu = (1/T) u^\mu$$

Thermal gradients do contribute to the polarization



$$\mathbf{S}^* \propto \frac{\hbar}{KT^2}\mathbf{u} \times \nabla T + \frac{\hbar}{KT}(\boldsymbol{\omega} - \boldsymbol{\omega} \cdot \mathbf{v}\mathbf{u}/c^2) + \frac{\hbar}{KT}\mathbf{A} \times \mathbf{u}/c^2$$

I. Karpenko, Nucl. Phys. A 982 (2019), 519-522, talk given at QM 2018

Is it the best thing to do?

The formulae of the spin vector are based on a Taylor expansion of the density operator

$$W(x,k)_{\rm LE} = \frac{1}{Z} {\rm Tr} \left(\exp \left[- \int_{\Sigma_{FO}} {\rm d}\Sigma_\mu(y) \widehat{T}^{\mu\nu}(y) \beta_\nu(y) - \zeta(y) \widehat{j}^\mu(y) \right] \widehat{W}(x,k) \right)$$

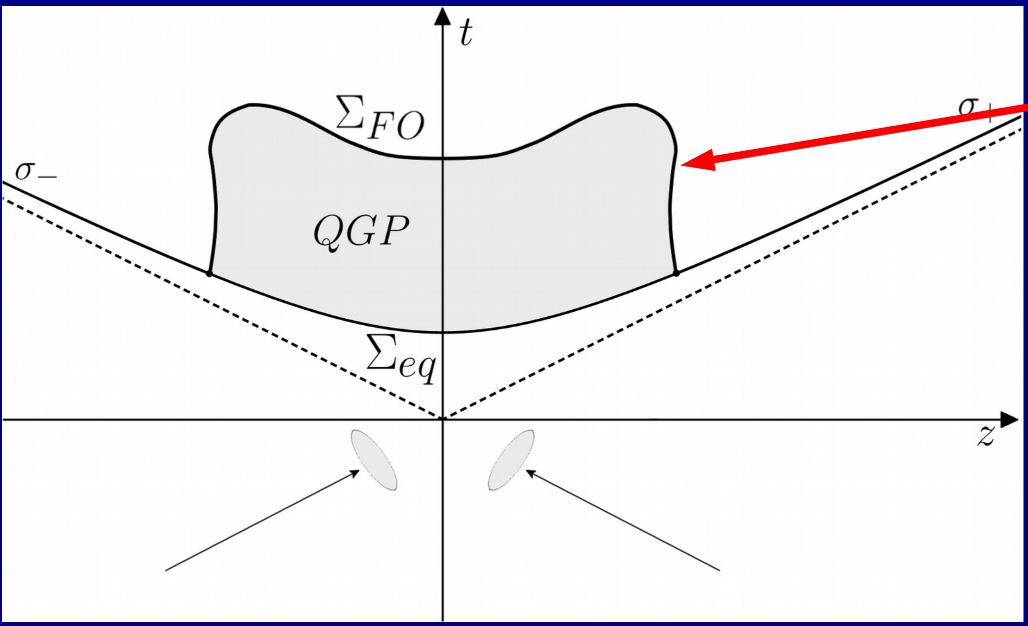
$$\beta_\nu(y) = \beta_\nu(x) + \partial_\lambda \beta_\nu(x) (y-x)^\lambda + \ldots$$

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z} \exp[-\beta_\mu(x) \widehat{P}^\mu - \frac{1}{2}(\partial_\mu \beta_\nu(x) - \partial_\nu \beta_\mu(x)) \widehat{J}^{\mu\nu}_x - \frac{1}{2}(\partial_\mu \beta_\nu(x) + \partial_\nu \beta_\mu(x)) \widehat{Q}^{\mu\nu}_x + \ldots]$$

This is generally correct, but it is an approximation after all.

Can we find a better approximation for a special case?

Isothermal hadronization at very high energy



At high energy, Σ_{FO} expected to be T= constant!

$$\beta^\mu = (1/T) u^\mu$$

$$\widehat{\rho}_{\rm LE} = \frac{1}{Z} \exp\left[- \int_{\Sigma_{FO}} {\rm d}\Sigma_\mu \; \widehat{T}^{\mu\nu} \beta_\nu \right] = \frac{1}{Z} \exp\left[- \frac{1}{T} \int_{\Sigma_{FO}} {\rm d}\Sigma_\mu \; \widehat{T}^{\mu\nu} u_\nu \right]$$

NOW u (and just u) can be expanded!

$$u_\nu(y) = u_\nu(x) + \partial_\lambda u_\nu(x) (y-x)^\lambda + \ldots$$

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z} \exp[-\beta_\mu(x) \widehat{P}^\mu - \frac{1}{2T} (\partial_\mu u_\nu(x) - \partial_\nu u_\mu(x)) \widehat{J}^{\mu\nu}_x - \frac{1}{2T} (\partial_\mu u_\nu(x) + \partial_\nu u_\mu(x)) \widehat{Q}^{\mu\nu}_x + \ldots]$$

Spin mean vector at leading order with isothermal local equilibrium (ILE)

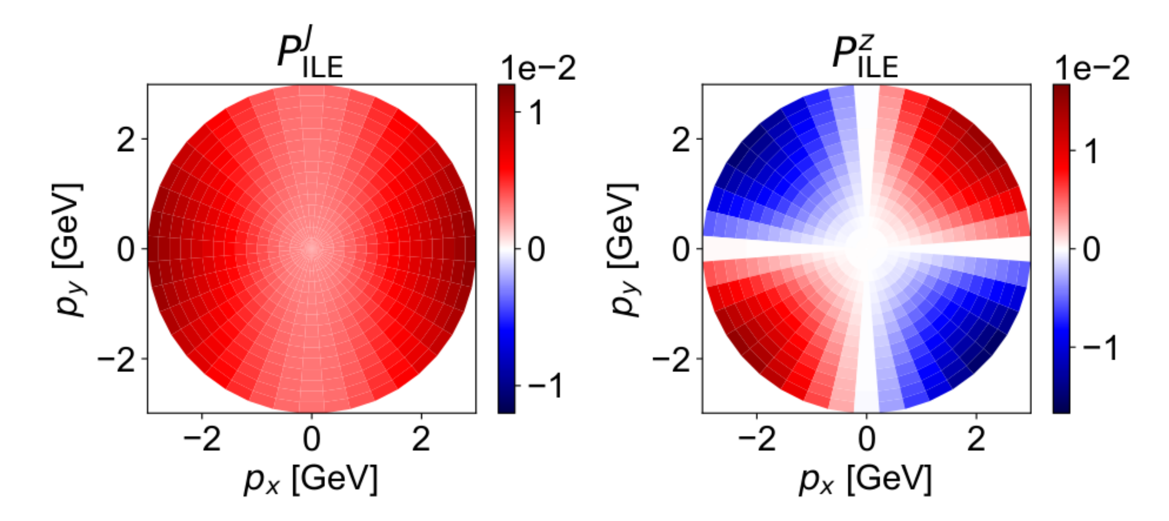
Readily found by replacing the gradients of β with those of *u*

$$S^{\mu}_{\rm ILE}(p) = - \epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma {\rm d}\Sigma \cdot p \, n_F (1-n_F) \left[\omega_{\rho\sigma} + 2\, \hat{t}_\rho \frac{p^\lambda}{\varepsilon} \Xi_{\lambda\sigma} \right]}{8 m T_{\rm dec} \int_\Sigma {\rm d}\Sigma \cdot p \, n_F}$$

$$\omega_{\rho\sigma} = \frac{1}{2} \left(\partial_\sigma u_\rho - \partial_\rho u_\sigma \right)$$

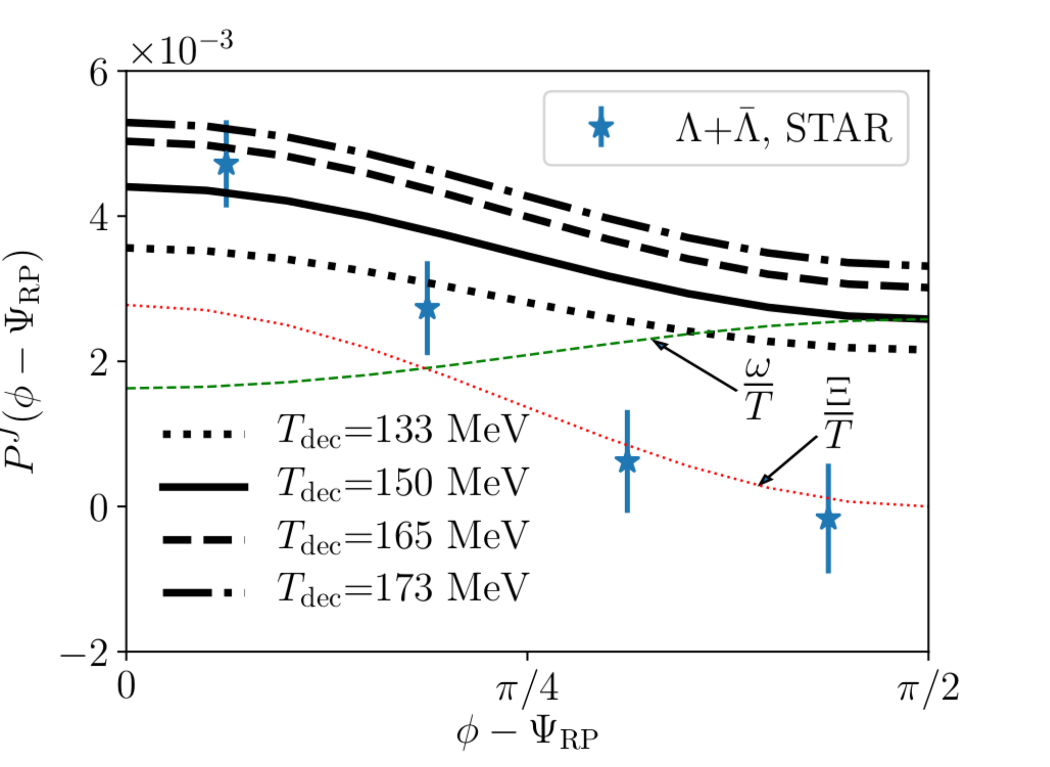
$$\Xi_{\rho\sigma} = \frac{1}{2} \left(\partial_\sigma u_\rho + \partial_\rho u_\sigma \right)$$

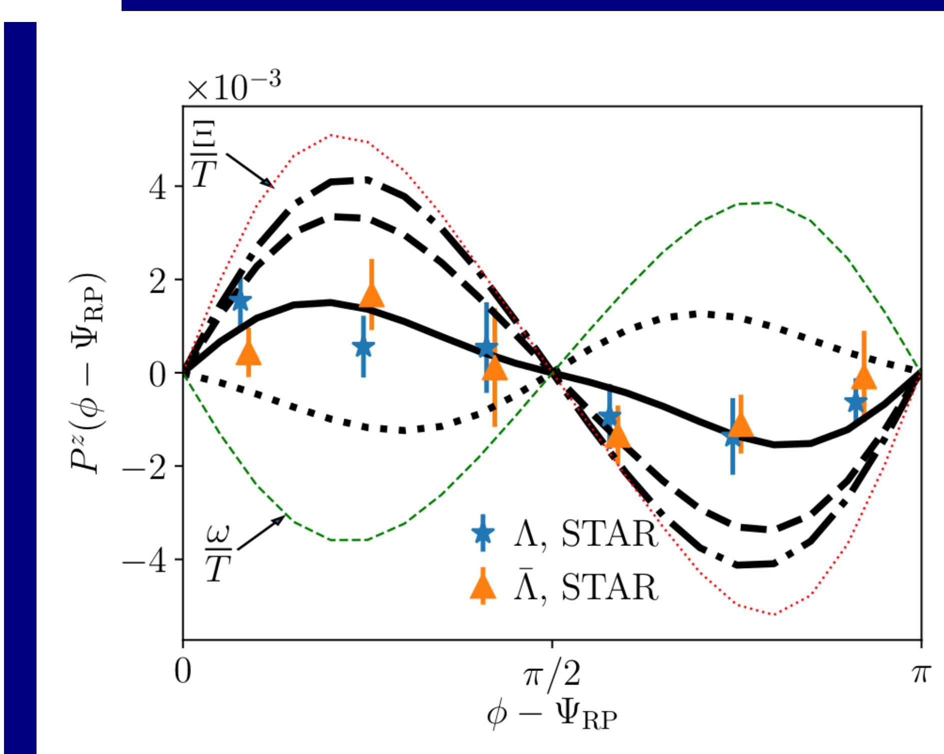
Isothermal local equilibrium: results



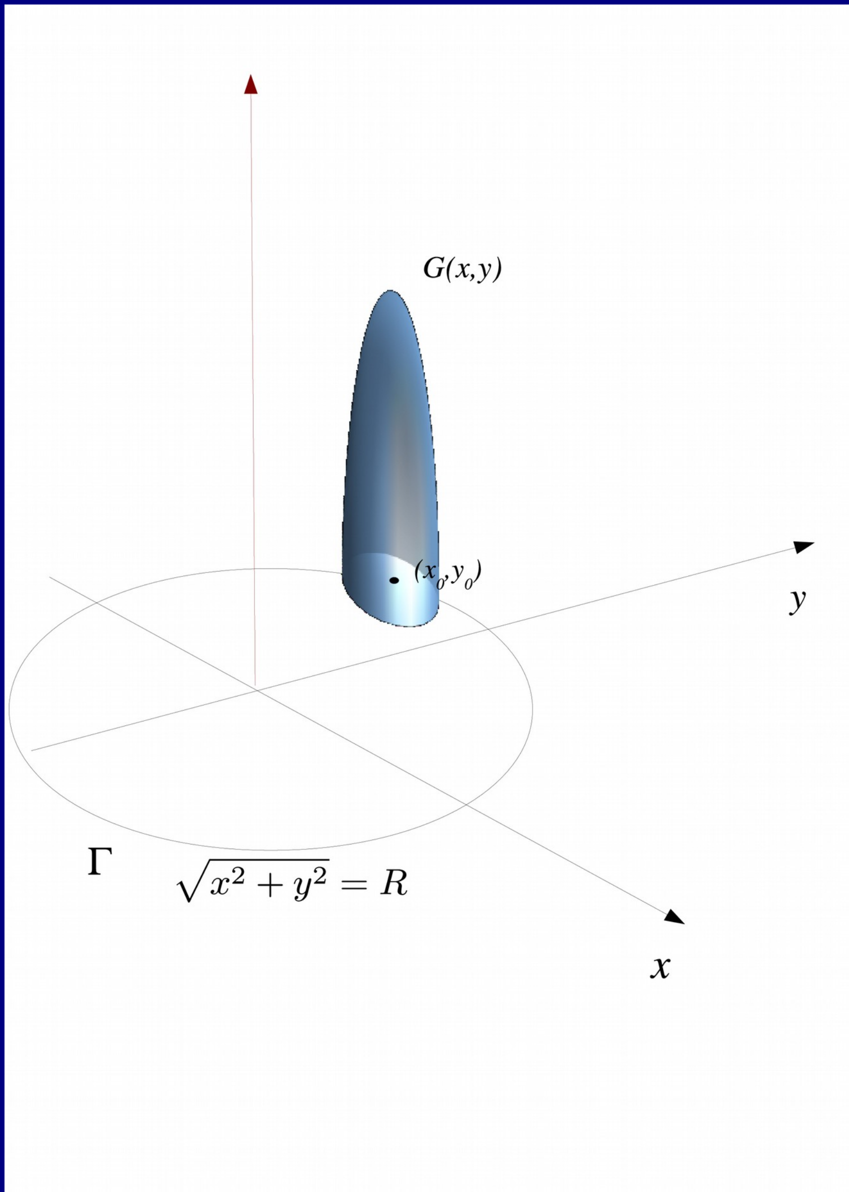
Apply the new formula (for primary hadrons)

$$S^\mu_{\rm ILE}(p) = - \epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma {\rm d}\Sigma \cdot p \, n_F (1-n_F) \left[\omega_{\rho\sigma} + 2 \, \hat{t}_\rho \frac{p^\lambda}{\varepsilon} \Xi_{\lambda\sigma} \right]}{8 m T_{\rm dec} \int_\Sigma {\rm d}\Sigma \cdot p \, n_F}$$





Why the thermal gradients matter: a simple example



Task: approximate the integral

$$W = \int_\Gamma \mathrm{e}^{\sqrt{x^2+y^2}} G(x,y)ds$$

where *G(x,y)* is a peaked function around the point (x_0,y_0) on the circle.

Since G is peaked, one can Taylor expand the exponent about (x_0,y_0)

$$\begin{aligned} W &\simeq \mathrm{e}^{\sqrt{x_0^2+y_0^2}} \int_\Gamma \mathrm{e}^{x_0(x-x_0)/R+y_0(y-y_0)/R} G(x,y)\mathrm{d}s \\ &= \mathrm{e}^R \int_\Gamma \mathrm{e}^{x_0(x-x_0)/R+y_0(y-y_0)/R} G(x,y)\mathrm{d}s \\ &= \mathrm{e}^R \int_\Gamma \mathrm{e}^{\nabla r|_{(x_0,y_0)}\cdot(\mathbf{x}-\mathbf{x}_0)} G(x,y)\mathrm{d}s \end{aligned}$$

But it is just pointless if we integrate over the circle!

$$W = \mathrm{e}^R \int_\Gamma G(x,y)\mathrm{d}s$$

In the previous example, the Taylor expansion at first order introduces an undesired term:

$$\mathrm{W} = \mathrm{e}^{R} \int_{\Gamma} G(x,y)\mathrm{d}s \qquad W \simeq \mathrm{e}^{R} \int_{\Gamma} \mathrm{e}^{\nabla r|_{(x_0,y_0)} \cdot (\mathbf{x}-\mathbf{x}_0)} G(x,y)\mathrm{d}s$$

exact — With gradient of r expansion

which is proportional to the gradient of the constant quantity on the circle, perpendicular to the integration line. This term does not vanish in the integration!

Summary and outlook

- Local thermodynamic equilibrium: new unexpected term relating spin polarization with the symmetric gradient of four-temperature. Under further investigation.

- Local polarization puzzles at very high energy can be solved by including this term in the numerical analysis along with the isothermal hadronization setting:

- If confirmed, an important phenomenological consequence: local equilibrium ("ideal fluid" picture) seems to hold in the spin sector too!

- Sensitivity to hadronization temperature: spin as a probe of the plasma formation?