Treatment of event-by-event fluctuations for comparison with experiment

Volodymyr Vovchenko (LBNL)

On-line seminar series III on "RHIC Beam Energy Scan: Theory and Experiment"

November 30, 2021

Unterstützt von / Supported by
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QCD phase diagram with heavy-ion collisions

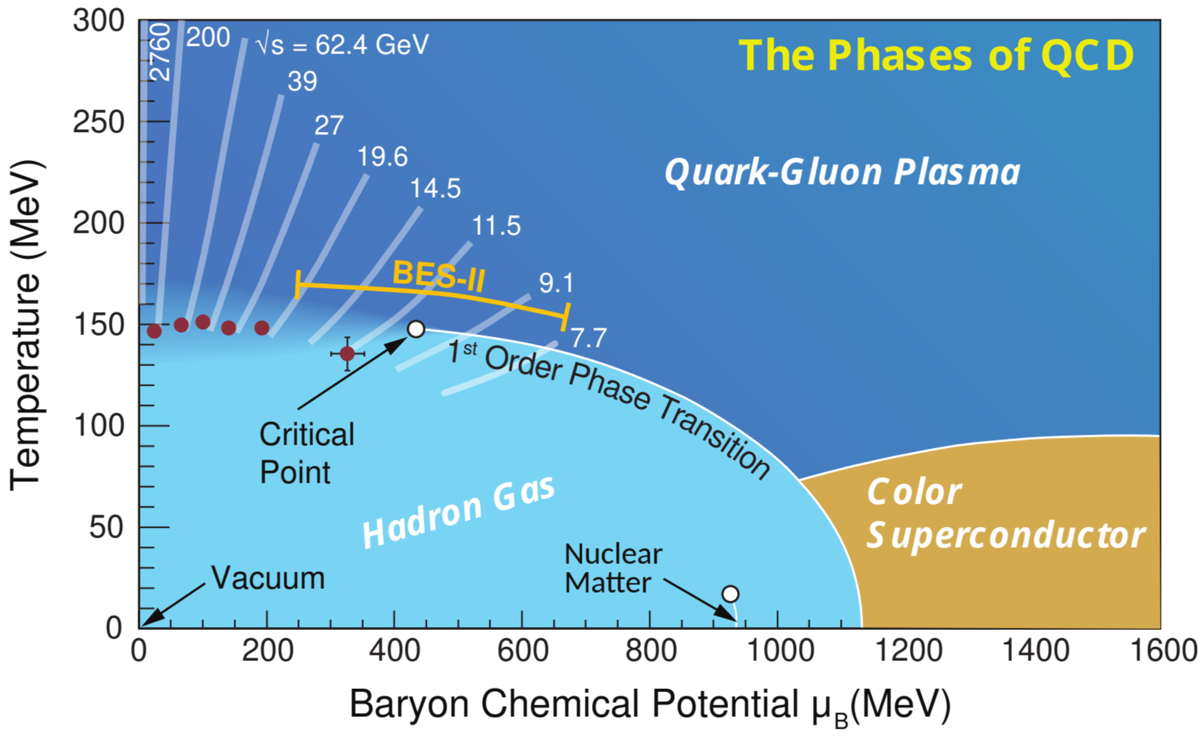
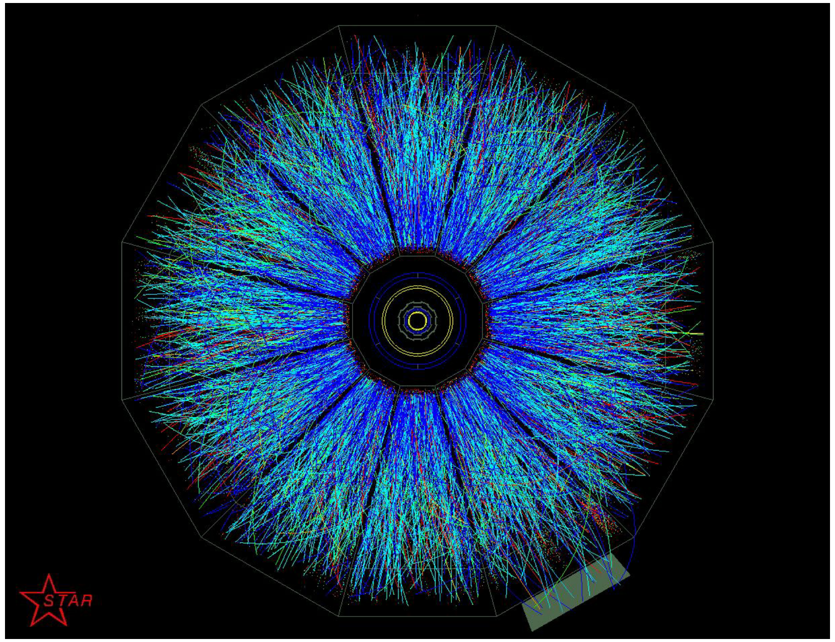


Figure from Bzdak et al., Phys. Rept. '20



STAR event display

Thousands of particles created in relativistic heavy-ion collisions

↓

Apply concepts of statistical mechanics

Event-by-event fluctuations and statistical mechanics

Cumulant generating function

$$K_N(t) = \ln\langle e^{tN}\rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

Grand partition function

$$\ln Z^{\rm gce}(T, V, \mu) = \ln\left[\sum_N e^{\mu N/T} Z^{\rm ce}(T, V, N)\right]$$

$$\kappa_n \propto \frac{\partial^n (\ln Z^{\rm gce})}{\partial \mu^n}$$

Cumulants measure chemical potential derivatives of the (QCD) equation of state

Event-by-event fluctuations and statistical mechanics

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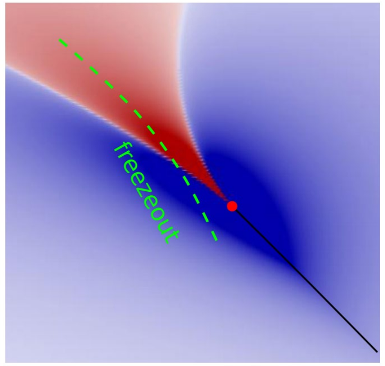
Cumulants measure chemical potential derivatives of the (QCD) equation of state

- (QCD) critical point



M. Stephanov, PRL '09
Energy scans at RHIC (STAR)
and CERN-SPS (NA61/SHINE)

- Test of (lattice) QCD at $\mu_B \approx 0$

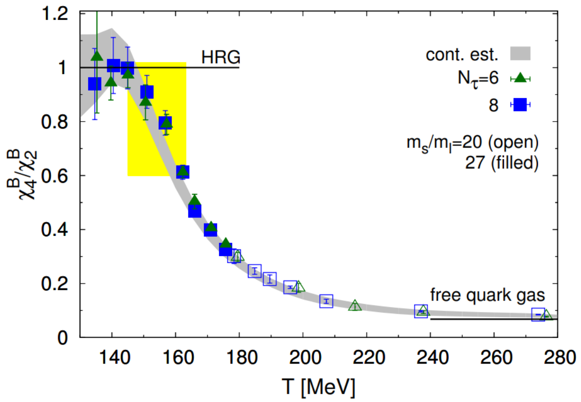
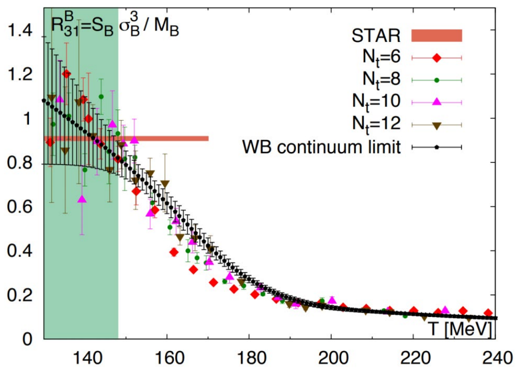


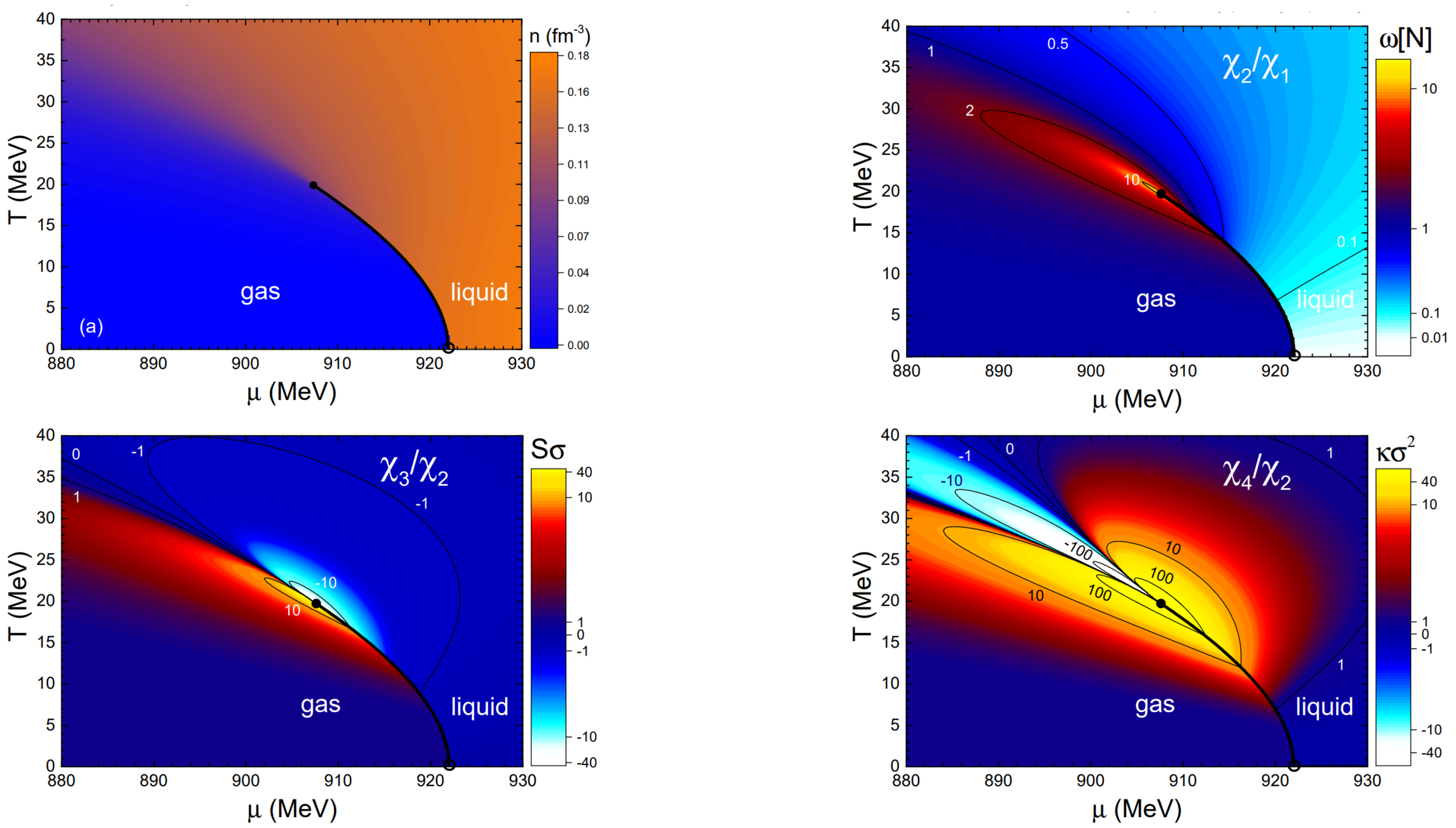
Figure from Bazavov et al. PRD 95, 054504 (2017)
Probed by LHC and top RHIC

- Freeze-out from fluctuations



Borsanyi et al. PRL 113, 052301 (2014)
Bazavov et al. PRL 109, 192302 (2012)
...

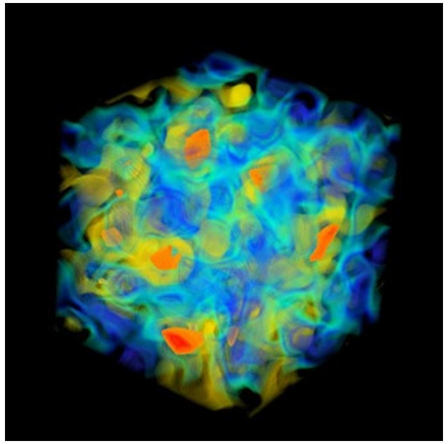
Example: Nuclear liquid-gas transition



VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)

Theory vs experiment

Theory



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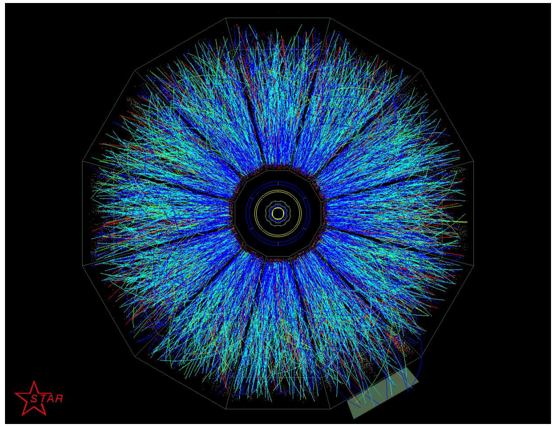
- Coordinate space
- In contact with the heat bath
- Conserved charges
- Uniform
- Fixed volume

Experiment



STAR event display

- Momentum space
- Expanding in vacuum
- Non-conserved particle numbers
- Inhomogenous
- Fluctuating volume

Theory vs experiment

- **accuracy of the grand-canonical ensemble (global conservation laws)**
  - **subensemble acceptance method (SAM)**

VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020); JHEP 089(2020); arXiv:2106.13775

- **coordinate vs momentum space**

Ling, Stephanov, PRC 93, 034915 (2016); Ohnishi, Kitazawa, Asakawa, PRC 94, 044905 (2016)

- **proxy observables in experiment (net-proton, net-kaon) vs conserved charges in QCD (net-baryon, net-strangeness)**

Kitazawa, Asakawa, PRC 85, 021901 (2012); VV, Jiang, Gorenstein, Stoecker, PRC 98, 024910 (2018)

- volume fluctuations

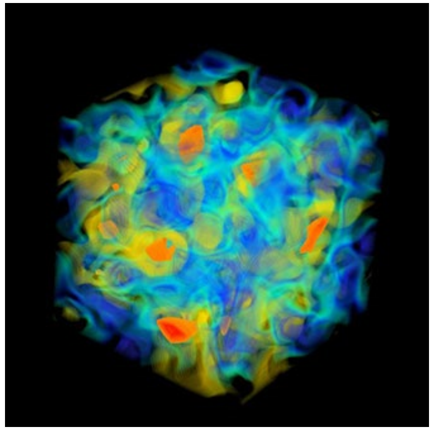
Gorenstein, Gazdzicki, PRC 84, 014904 (2011); Skokov, Friman, Redlich, PRC 88, 034911 (2013)
X. Luo, J. Xu, B. Mohanty, JPG 40, 105104 (2013); Braun-Munzinger, Rustamov, Stachel, NPA 960, 114 (2017)

- hadronic phase

Steinheimer, VV, Aichelin, Bleicher, Stoecker, PLB 776, 32 (2018)

- non-equilibrium (memory) effects

Mukherjee, Venugopalan, Yin, PRC 92, 034912 (2015)
Asakawa, Kitazawa, Müller, PRC 101, 034913 (2020)



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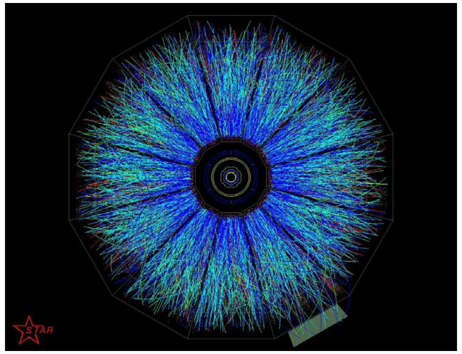


STAR event display

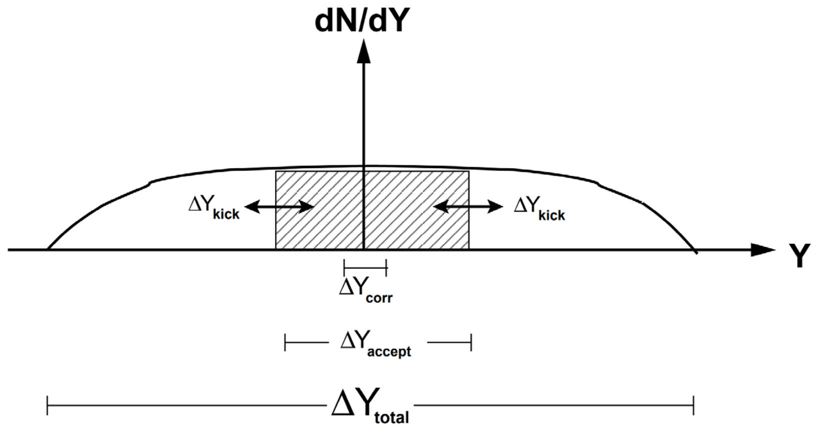
When are the measured fluctuations grand-canonical?

- Consider event-by-event fluctuations of particle number in acceptance ΔY_{accept} around midrapidity

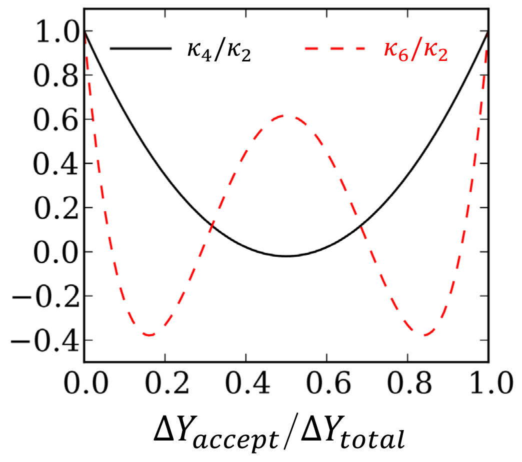
- Scales
  - ΔY_{accept} – acceptance
  - ΔY_{total} – full space
  - ΔY_{corr} – rapidity correlation length (thermal smearing)
  - ΔY_{kick} – diffusion in the hadronic phase

- **GCE applies if $\Delta Y_{total} \gg \Delta Y_{accept} \gg \Delta Y_{kick}, \Delta Y_{corr}$**

- In practice $\Delta Y_{total} \gg \Delta Y_{accept}$ and $\Delta Y_{accept} \gg \Delta Y_{corr}$ are not simultaneously satisfied
  - Corrections from global conservation are large [Bzdak et al., PRC '13]
  - $\Delta Y_{corr} \sim 1 \sim \Delta Y_{accept}$ [Ling, Stephanov, PRC '16]



V. Koch, arXiv:0810.2520



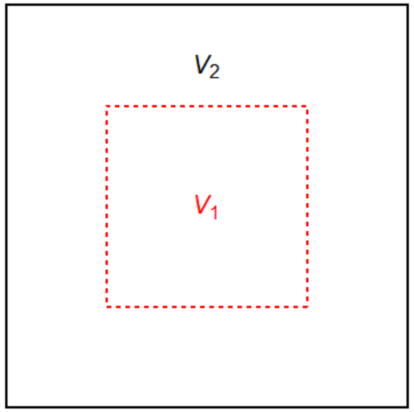
Subensemble acceptance method

VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

Subensemble acceptance method (SAM) – method to correct *any* EoS (e.g. *lattice QCD*) for **charge conservation** and extract the **susceptibilities**

Partition a thermal system with a globally conserved charge B *(canonical ensemble)* into two subsystems which can exchange the charge

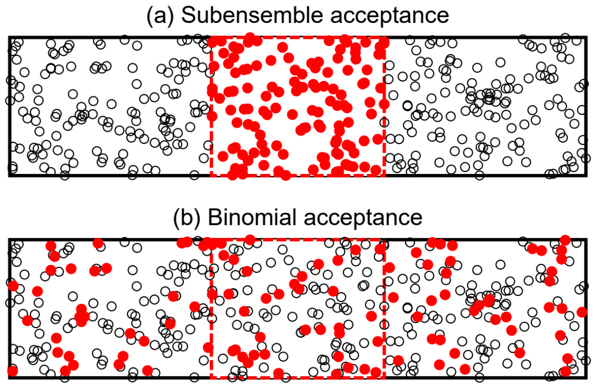
$$V_1 + V_2 = V$$



Assume thermodynamic limit:

$$V,\, V_1,\, V_2 \to \infty; \quad \frac{V_1}{V} = \alpha = const; \quad \frac{V_2}{V} = (1-\alpha) = const;$$

$$V_1,\, V_2 \gg \xi^3, \qquad \xi = \text{correlation length}$$



The canonical partition function then reads:

$$Z^{ce}(T,V,B) = \mathrm{Tr}\, e^{-\beta \hat{H}} \approx \sum_{B_1} Z^{ce}(T,V_1,B_1)\, Z^{ce}(T,V-V_1,B-B_1)$$

The probability to have charge B_1 is:

$$P(B_1) \propto Z^{ce}(T,\alpha V,B_1)\, Z^{ce}(T,(1-\alpha)V,B-B_1), \qquad \alpha \equiv V_1/V$$

SAM: Computing the cumulants

VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

Cumulant generating function for B_1:

$$G_{B_1}(t) \equiv \ln\langle e^{t\,B_1}\rangle = \ln\left\{\sum_{B_1} e^{t\,B_1}\,\exp\left[-\frac{\alpha V}{T}\,f(T,\rho_{B_1})\right]\exp\left[-\frac{\beta V}{T}\,f(T,\rho_{B_2})\right]\right\} + \tilde{C} \qquad \beta = 1-\alpha$$

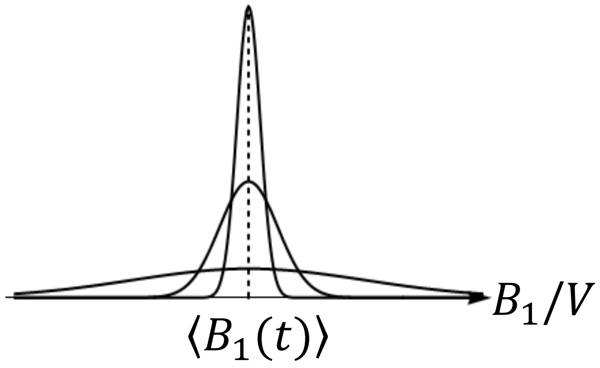
$$\tilde{\kappa}_1[B_1(t)] = \frac{\sum_{B_1} B_1\,\tilde{P}(B_1;t)}{\sum_{B_1}\tilde{P}(B_1;t)} \equiv \langle B_1(t)\rangle \qquad \text{with} \qquad \tilde{P}(B_1;t) = \exp\left\{tB_1 - V\,\frac{\alpha f(T,\rho_{B_1}) + \beta f(T,\rho_{B_2})}{T}\right\}.$$

Thermodynamic limit: $\tilde{P}(B_1;t)$ *highly peaked* at $\langle B_1(t)\rangle$

$\langle B_1(t)\rangle$ is a solution to equation $d\tilde{P}/\mathrm{d}B_1 = 0$:

$$t = \hat{\mu}_B[T,\rho_{B_1}(t)] - \hat{\mu}_B[T,\rho_{B_2}(t)]$$

$$\text{where} \quad \hat{\mu}_B \equiv \mu_B/T, \qquad \mu_B(T,\rho_B) = \partial f(T,\rho_B)/\partial\rho_B$$



$t = 0$: $\rho_{B_1} = \rho_{B_2} = B/V$, $B_1 = \alpha B$, i.e. charge uniformly distributed between the subsystems

SAM: Cumulant ratios in terms of GCE susceptibilities

$$\kappa_n[B_1] = \left.\frac{\partial^{n-1}\tilde{\kappa}_1[B_1(t)]}{\partial t^{n-1}}\right|_{t=0} \quad \longleftrightarrow \quad \frac{\partial^n}{\partial t^n}: \; t = \hat{\mu}_B[T, \rho_{B_1}(t)] - \hat{\mu}_B[T, \rho_{B_2}(t)]$$

$$\chi_n^B \equiv \partial^{n-1}(\rho_B/T^3)/\partial(\mu_B/T)^{n-1}$$

scaled variance $$\frac{\kappa_2[B_1]}{\kappa_1[B_1]} = (1-\alpha)\,\frac{\chi_2^B}{\chi_1^B},$$

skewness $$\frac{\kappa_3[B_1]}{\kappa_2[B_1]} = (1-2\alpha)\,\frac{\chi_3^B}{\chi_2^B},$$

kurtosis $$\frac{\kappa_4[B_1]}{\kappa_2[B_1]} = (1-3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B} - 3\alpha\beta\left(\frac{\chi_3^B}{\chi_2^B}\right)^2.$$

- Global conservation (α) and equation of state (χ_n^B) effects factorize in cumulants up to the 3rd order, starting from κ_4 not anymore
- $\alpha \to 0$ – GCE limit*, $\alpha \to 1$ – CE limit

*As long as $V_1 \gg \xi^3$ holds

For *multiple conserved charges* (joint B,Q,S cumulants up to 6th order) see VV, Poberezhnyuk, Koch, JHEP 10, 089 (2020)

Example: Lennard-Jones fluid

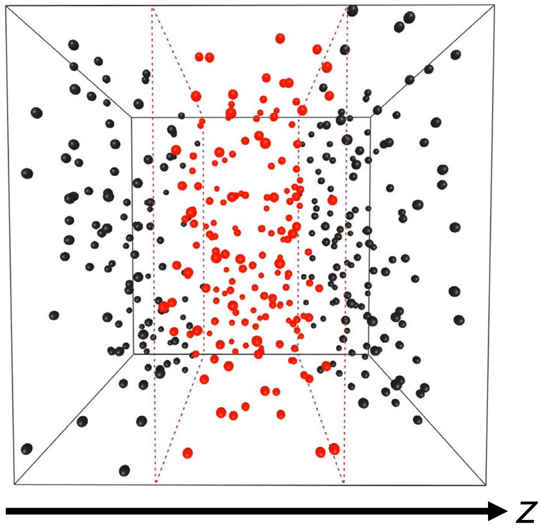
V. Kuznietsov, O. Savchuk, M.I. Gorenstein, V. Koch, VV, in preparation

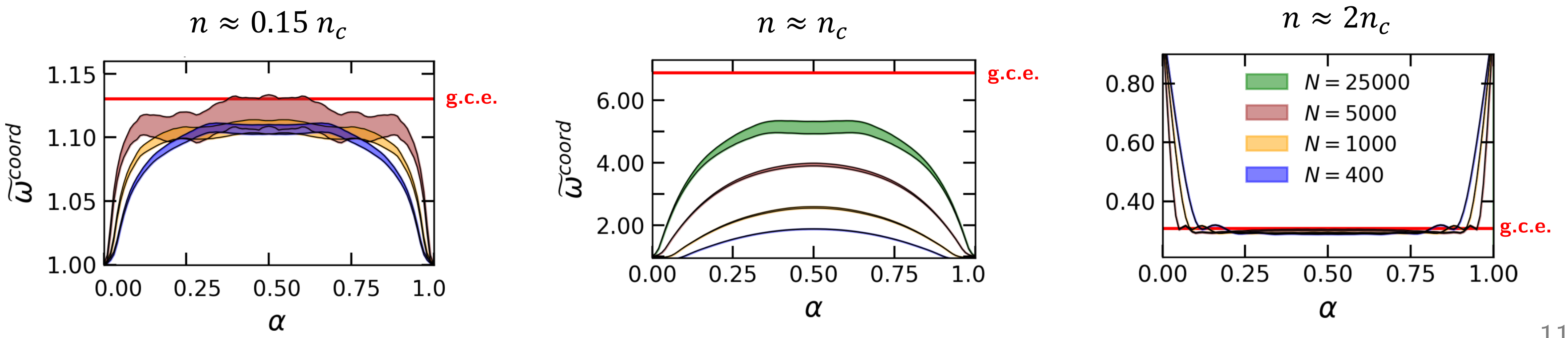
Classical molecular dynamics simulations* of a **Lennard-Jones fluid** along the (super)critical isotherm of the liquid-gas transition

Microcanonical (const. *EVN*) ensemble with periodic boundary conditions

Variance of conserved particle number distribution inside coordinate space subvolume $|z| < z^{max}$ as time average

$$\tilde{\omega}^{\text{coord}} = \frac{1}{1-\alpha} \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle}$$





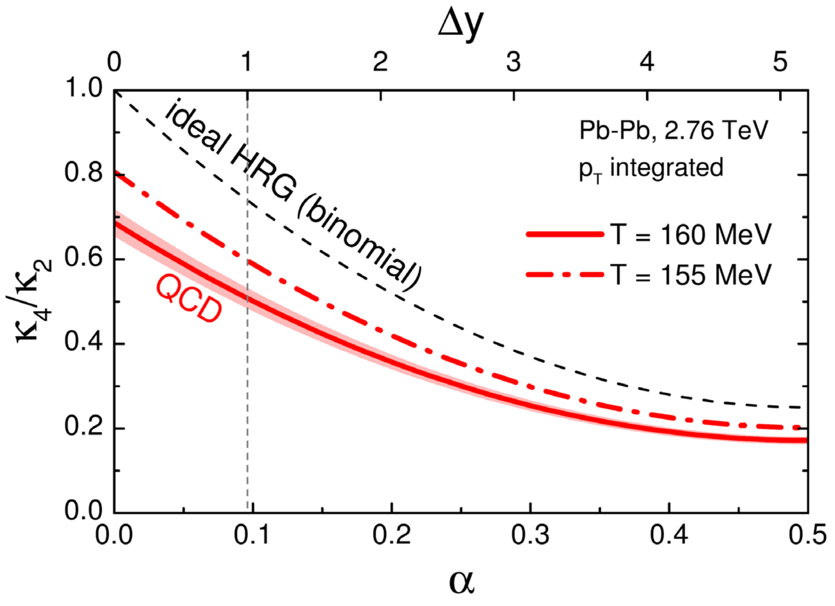
*Molecular dynamics code from https://github.com/vlvovch/lennard-jones-cuda

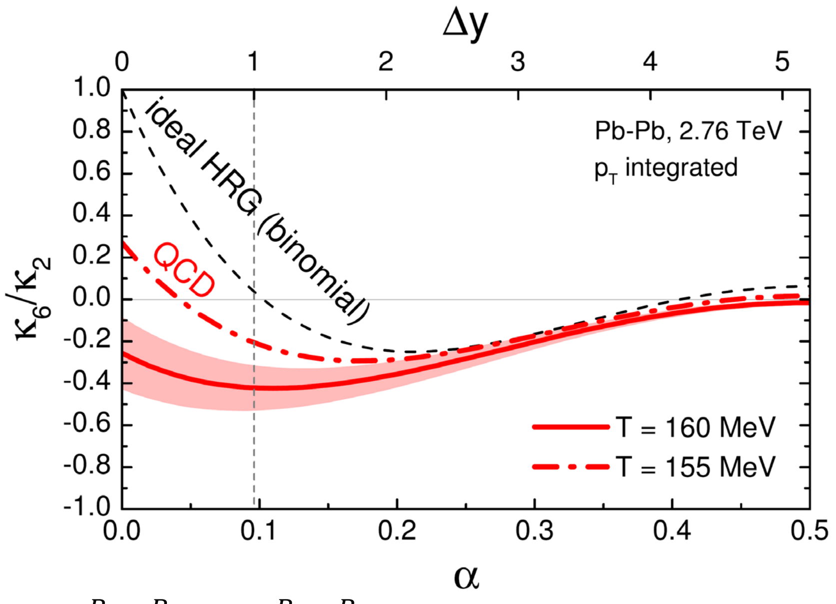
Net baryon fluctuations at LHC from lattice QCD ($\mu_B = 0$)

VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

Momentum rapidity $y \approx$ space-time rapidity η_s

$$\left(\frac{\kappa_4}{\kappa_2}\right)_{LHC} = (1 - 3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B} \qquad \left(\frac{\kappa_6}{\kappa_2}\right)_{LHC} = [1 - 5\alpha\beta(1-\alpha\beta)]\,\frac{\chi_6^B}{\chi_2^B} - 10\alpha(1-2\alpha)^2\beta\left(\frac{\chi_4^B}{\chi_2^B}\right)^2$$





Lattice data for χ_4^B/χ_2^B and χ_6^B/χ_2^B from Borsanyi et al., 1805.04445

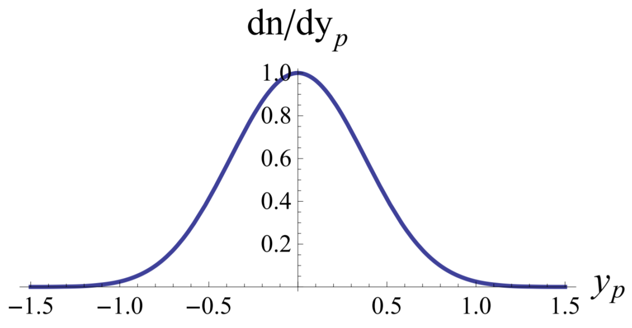
Theory: negative χ_6^B/χ_2^B is a possible signal of chiral criticality [Friman, Karsch, Redlich, Skokov, EPJC '11]

Experiment: $\alpha \approx \frac{N_{ch}(\Delta y)}{N_{ch}(\infty)} \approx \mathrm{erf}\left(\frac{\Delta y}{2\sqrt{2}\sigma_y}\right)$, for $\Delta y \approx 1$ the κ_6/κ_2 is mainly sensitive to the EoS

$N_{ch}(\Delta y)$ measurement: ALICE Collaboration, PLB 726 (2013) 610-622

SAM: Applicability and limitations

- Argument is based on partition in ***coordinate*** space but experiments measure in ***momentum*** space
  - OK at high energies where we have **Bjorken flow** $[Y \sim \eta_s = \text{atanh}(z/t)]$
    - For small $\Delta Y_{acc} < 1$: corrections due to thermal smearing and resonance decays
    - Limited applicability at lower energies
- **Thermodynamic limit** i.e. $V_1, V_2 \gg \xi^3$:
  - OK at LHC where $\frac{dV}{dy} \sim 4000 - 5000$ fm^3 vs. $V_{lattice} \sim 125$ fm^3
  - Applicability is more limited near the critical point
- Assumes $\boldsymbol{T, \mu_B = const}$ everywhere



[Ling, Stephanov, PRC '16]

To address these issues one needs a ***dynamical description***

Approaches to dynamical modeling of fluctuations

1. Dynamical model calculations of critical fluctuations
   - Fluctuating hydrodynamics
   - Equation of state with tunable critical point
   - Predict CP signatures dependent on its location

   Under development within the Beam Energy Scan Theory (BEST) Collaboration

   [X. An et al., Nucl. Phys. A 1017, 122343 (2022)]

2. **Study deviations from the non-critical baseline**
   - Include essential non-critical contributions to (net-)proton number cumulants
   - Exact baryon conservation + baryon excluded volume
   - Based on realistic hydrodynamic simulations

   [VV, C. Shen, V. Koch, arXiv:2107.00163]

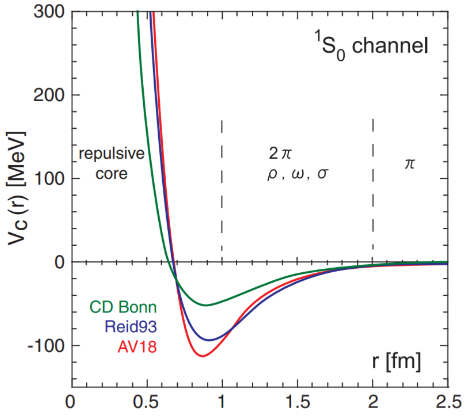


Figure from Ishii et al., PRL '07

Hydrodynamic description in a non-critical scenario

- Collision geometry based 3D initial state [Shen, Alzhrani, PRC '20]
  - Constrained to net proton distributions

- Viscous hydrodynamics evolution – MUSIC-3.0
  - Energy-momentum and baryon number conservation
  - NEOS-BQS crossover equation of state [Monnai, Schenke, Shen, PRC '19]
  - Shear viscosity via IS-type equation

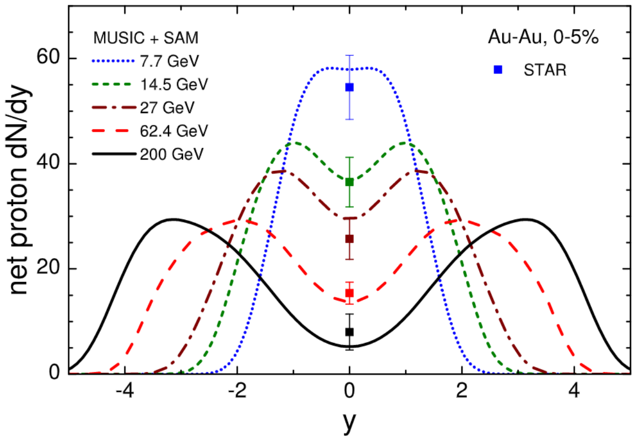
- Cooper-Frye particlization at $\epsilon_{sw} = 0.26$ GeV/fm^3

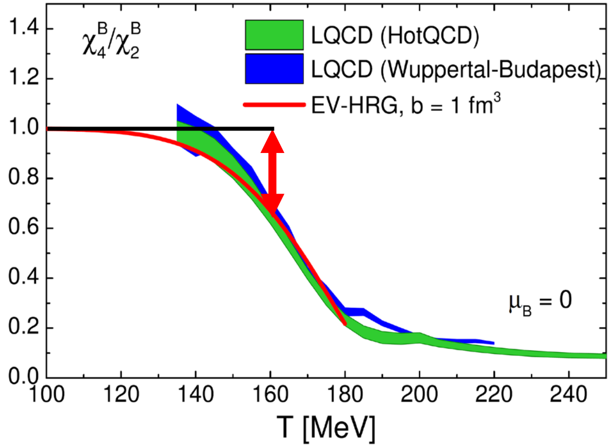
$$\omega_p \frac{dN_j}{d^3p} = \int_{\sigma(x)} d\sigma_\mu(x)\, p^\mu \, \frac{d_j\, \lambda_j^{\rm ev}(x)}{(2\pi)^3} \, \exp\left[\frac{\mu_j(x) - u^\mu(x) p_\mu}{T(x)}\right].$$

- Particlization includes QCD-based baryon number distribution
  - Here incorporated via baryon excluded volume

[VV, Pasztor, Fodor, Katz, Stoecker, PLB 775, 71 (2017)]

VV, C. Shen, V. Koch, arXiv:2107.00163





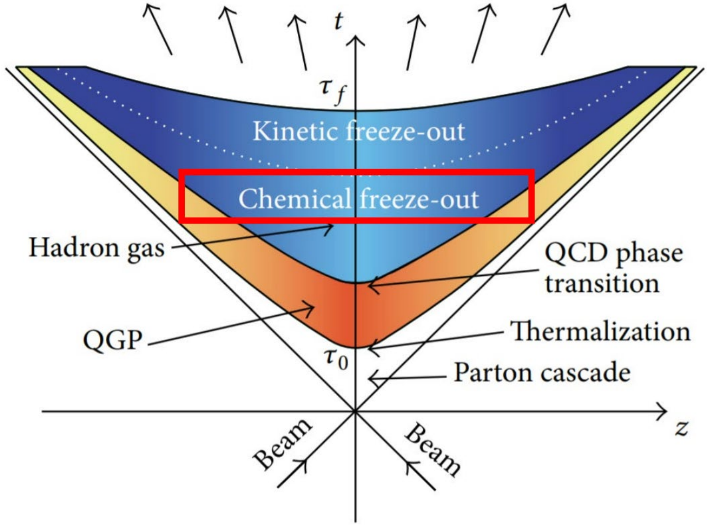
Calculating cumulants from hydrodynamics

- Strategy:
  1. Calculate proton cumulants in the experimental acceptance in the grand-canonical limit
  2. Apply correction for the exact global baryon number conservation

First step:

- Sum contributions from each hypersurface element x_i at freeze-out
  - Cumulants of joint (anti)proton/(anti)baryon distribution

$$\kappa_{n,m}^{B^\pm,p^\pm,\mathrm{gce}}(\Delta p_{\mathrm{acc}}) = \sum_{i\in\sigma} \delta\kappa_{n,m}^{B^\pm,p^\pm,\mathrm{gce}}(x_i;\Delta p_{\mathrm{acc}})$$



- To compute each contribution
  - GCE susceptibilities $\chi^{B^\pm}(x_i)$ define the distribution of the emitted (anti)baryons
  - Each baryon ends up in acceptance Δp_{acc} with binomial probability via the Cooper-Frye formula
  - Each baryon is a proton with probability $q(x_i) = \langle N_p(x_i)\rangle / \langle N_B(x_i)\rangle$

*For the Monte Carlo version of this procedure see [VV, Koch, Phys. Rev. C 103, 044903 (2021)]

Correcting for baryon number conservation with SAM-2.0

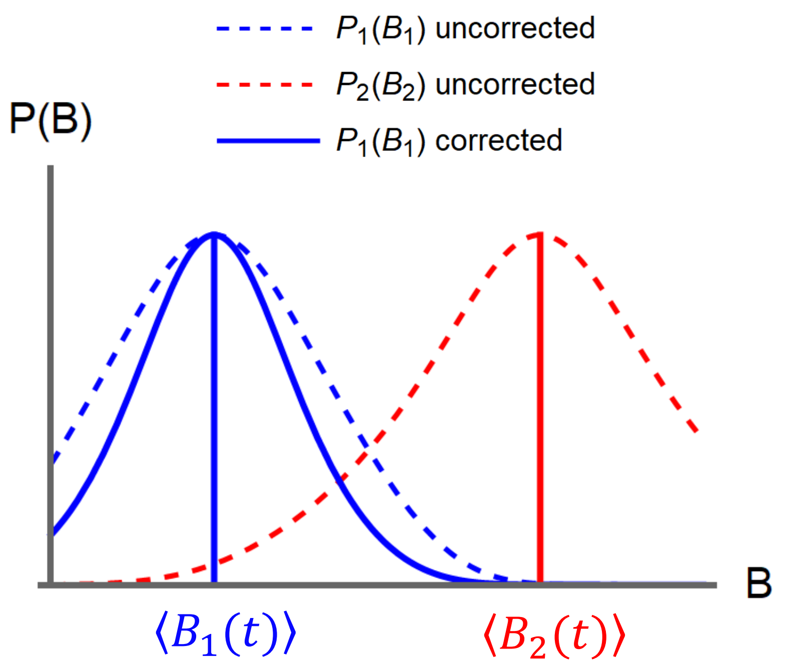
VV, arXiv:2107.00163 (to appear in PRC)

$$P_1^{\mathrm{ce}}(B_1) \propto \sum_{B_1,B_2} P_1^{\mathrm{gce}}(B_1) P_2^{\mathrm{gce}}(B_2) \times \delta_{B,B_1+B_2}$$

SAM-1.0: **uniform** thermal system and **coordinate** space

SAM-2.0: apply the correction for *arbitrary* distributions inside and outside the acceptance that are peaked at the mean

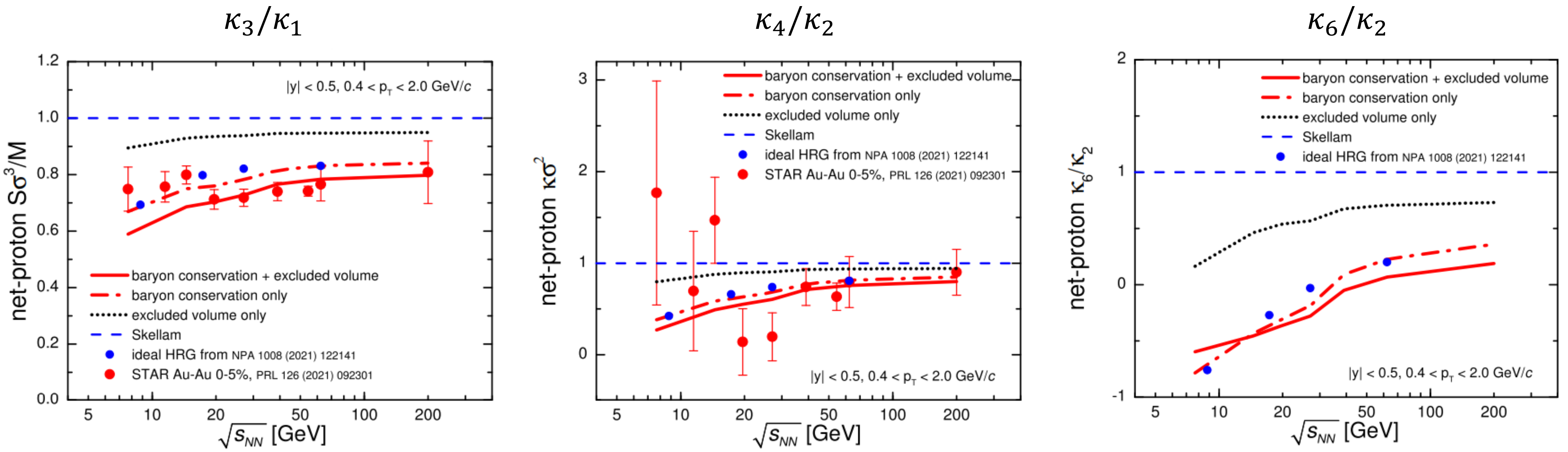
- Spatially inhomogeneous systems (e.g. RHIC)
- Momentum space
- Non-conserved quantities (e.g. proton number)
- Map "grand-canonical" cumulants inside and outside the acceptance to the "canonical" cumulants inside the acceptance



$$\kappa_{p,B}^{\mathrm{in,ce}} = \mathrm{SAM}\left[\kappa_{p,B}^{\mathrm{in,gce}}, \kappa_{p,B}^{\mathrm{out,gce}}\right]$$

*Explicit expressions for any cumulant available via a Mathematica notebook at https://github.com/vlvovch/SAM

Net proton cumulant ratios



- Both the baryon conservation and repulsion needed to describe data at $\sqrt{s_{NN}} \geq 20$ GeV quantitatively
- Effect from baryon conservation is larger than from repulsion
- Canonical ideal HRG limit is consistent with the data-driven study of [Braun-Munzinger et al., NPA 1008 (2021) 122141]
- κ_6/κ_2 turns negative at $\sqrt{s_{NN}}$~50 GeV

Cumulants vs Correlation Functions

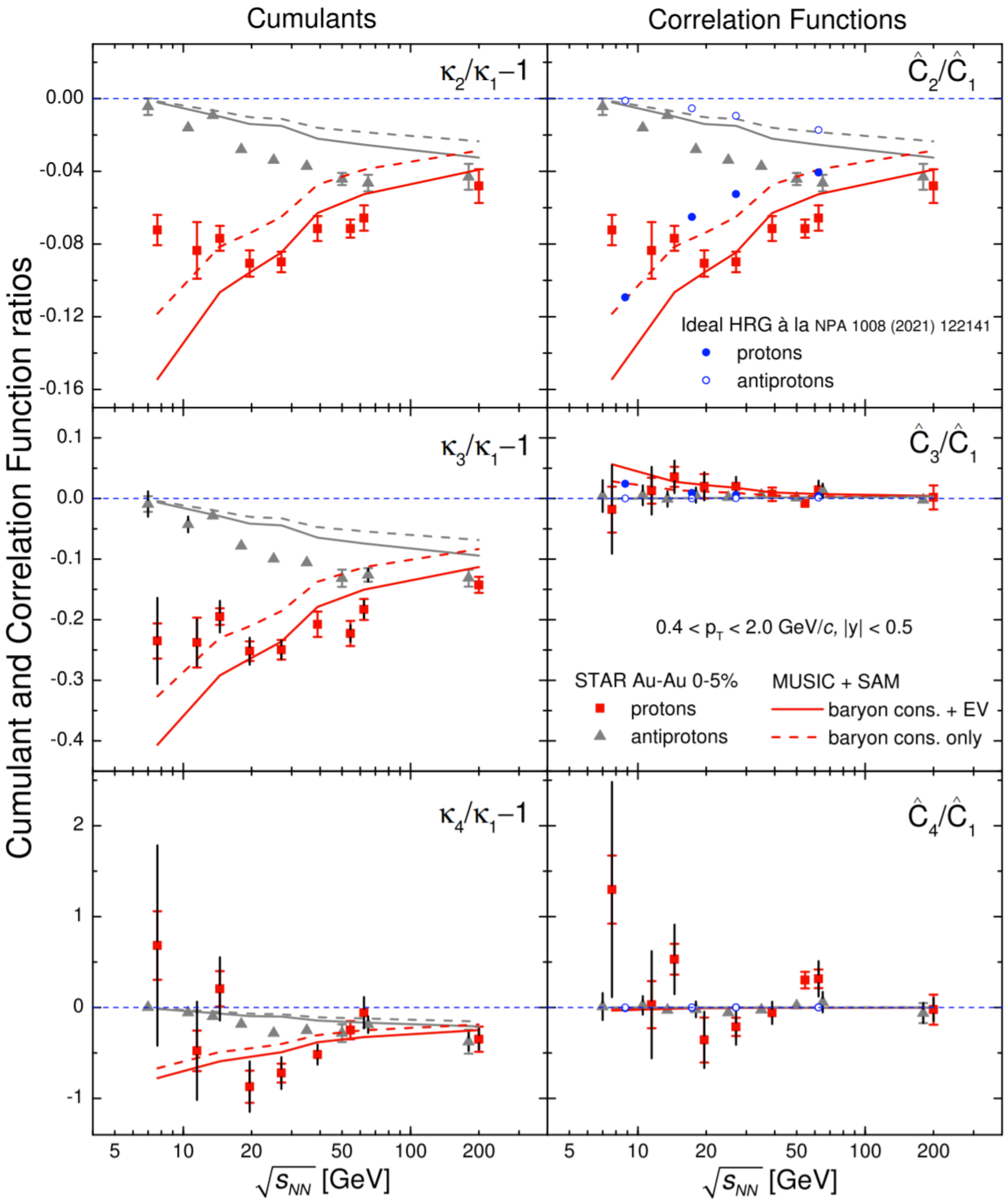
- Analyze genuine multi-particle correlations via **factorial cumulants** [Bzdak, Koch, Strodthoff, PRC '17]

$$\hat{C}_1 = \kappa_1, \qquad \hat{C}_3 = 2\kappa_1 - 3\kappa_2 + \kappa_3,$$
$$\hat{C}_2 = -\kappa_1 + \kappa_2, \qquad \hat{C}_4 = -6\kappa_1 + 11\kappa_2 - 6\kappa_3 + \kappa_4.$$

$$\hat{C}_n^{\text{cons}} \propto \alpha^n, \qquad \hat{C}_n^{\text{EV}} \propto b^n$$

[Bzdak, Koch, Skokov, EPJC '17] [VV et al, PLB '17]

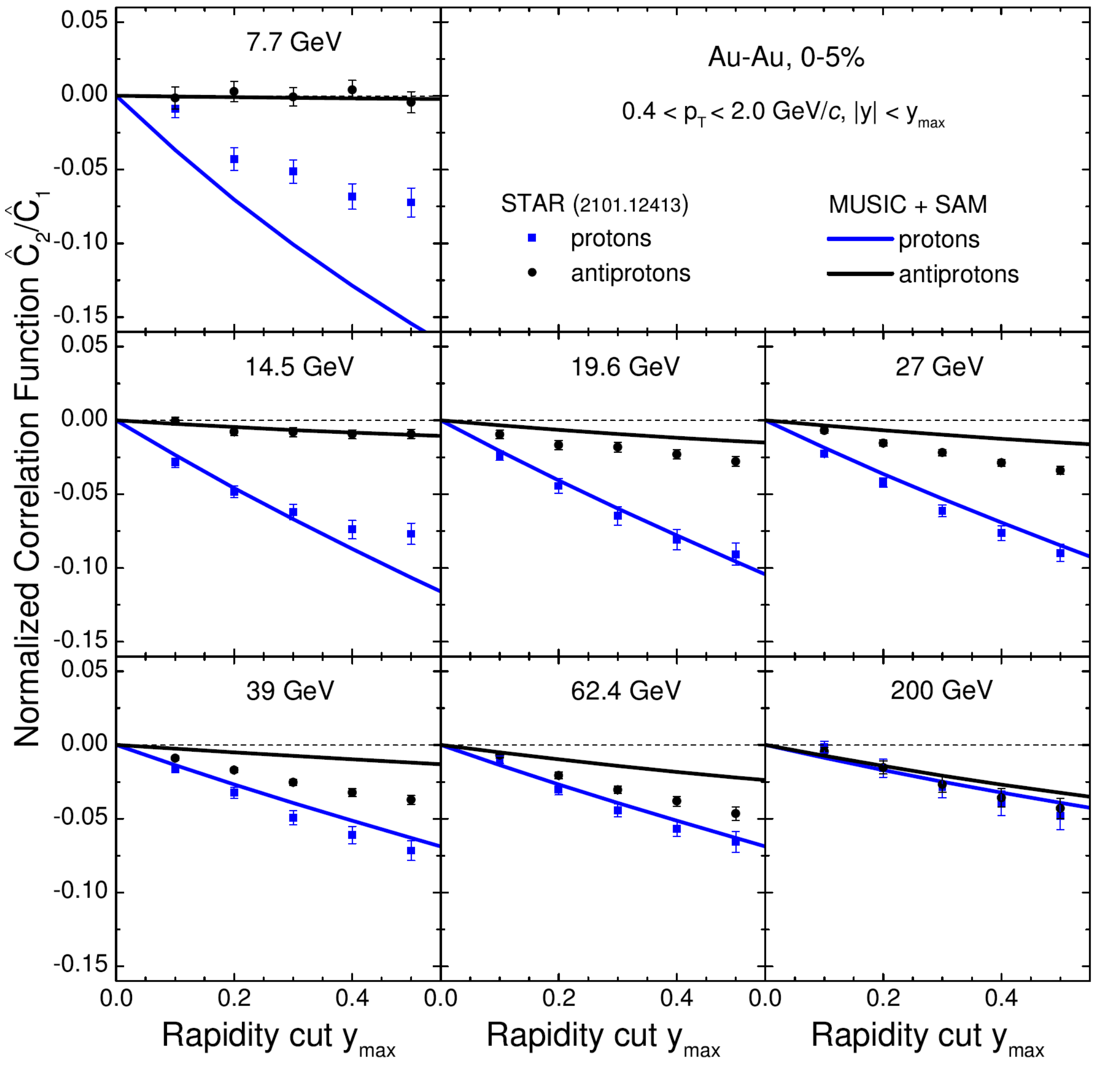
- Three- and four-particle correlations are small
  - Small positive $\hat{C}_3/\hat{C}_1$ in the data is explained by baryon conservation + excluded volume
  - Strong multi-particle correlations would be expected near the critical point [Ling, Stephanov, 1512.09125]
- Two-particle correlations are negative
  - Protons at $\sqrt{s_{NN}} \leq 14.5$ GeV overestimated
  - Antiprotons at $19.6 \leq \sqrt{s_{NN}} \leq 62.4$ GeV underestimated



*We use the notation for (factorial) cumulants from Bzdak et al., Phys. Rept. '20

Acceptance dependence of two-particle correlations

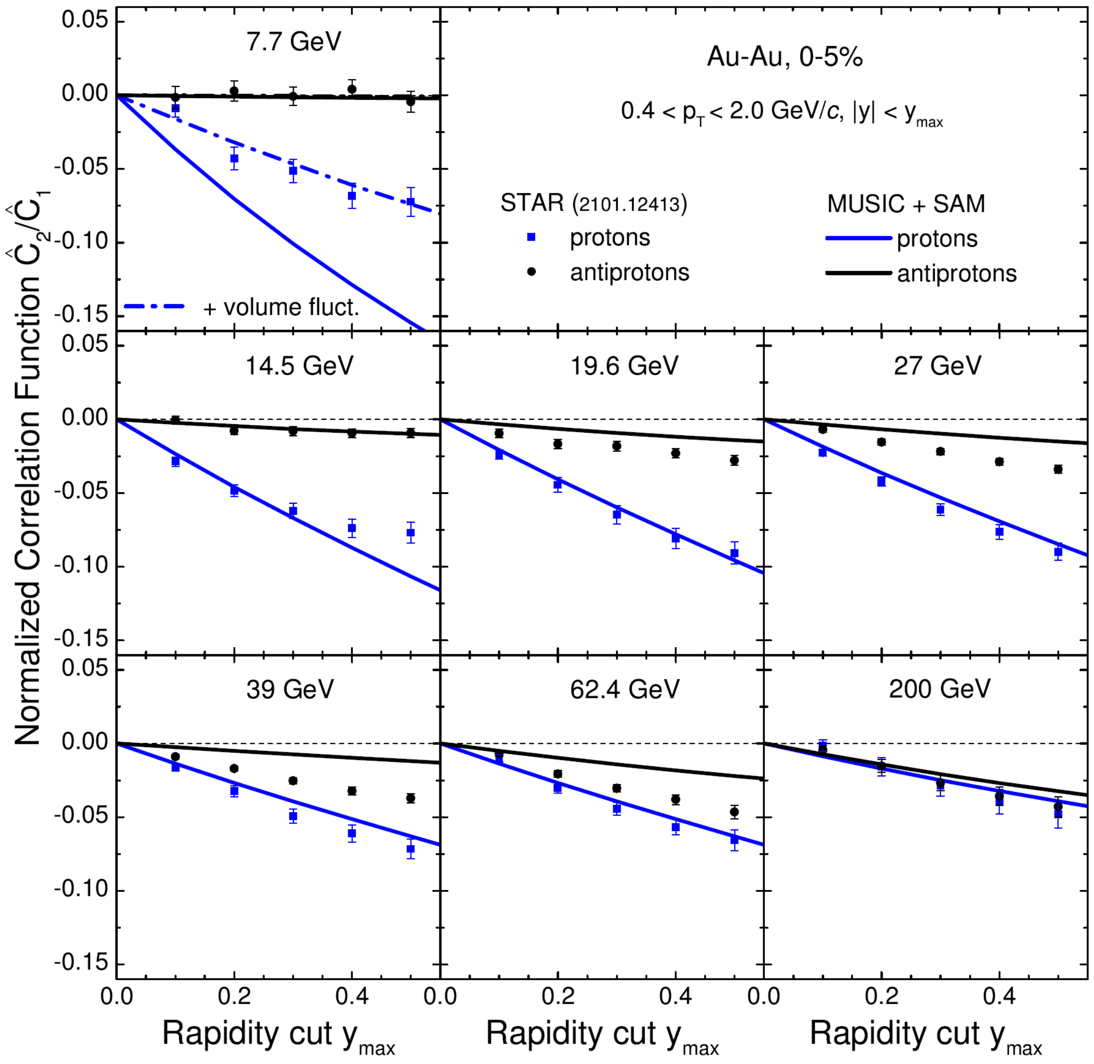
- Changing y_{max} slope at $\sqrt{s_{NN}} \leq 14.5$ GeV?



Acceptance dependence of two-particle correlations

- Changing y_{max} slope at $\sqrt{s_{NN}} \leq 14.5$ GeV?

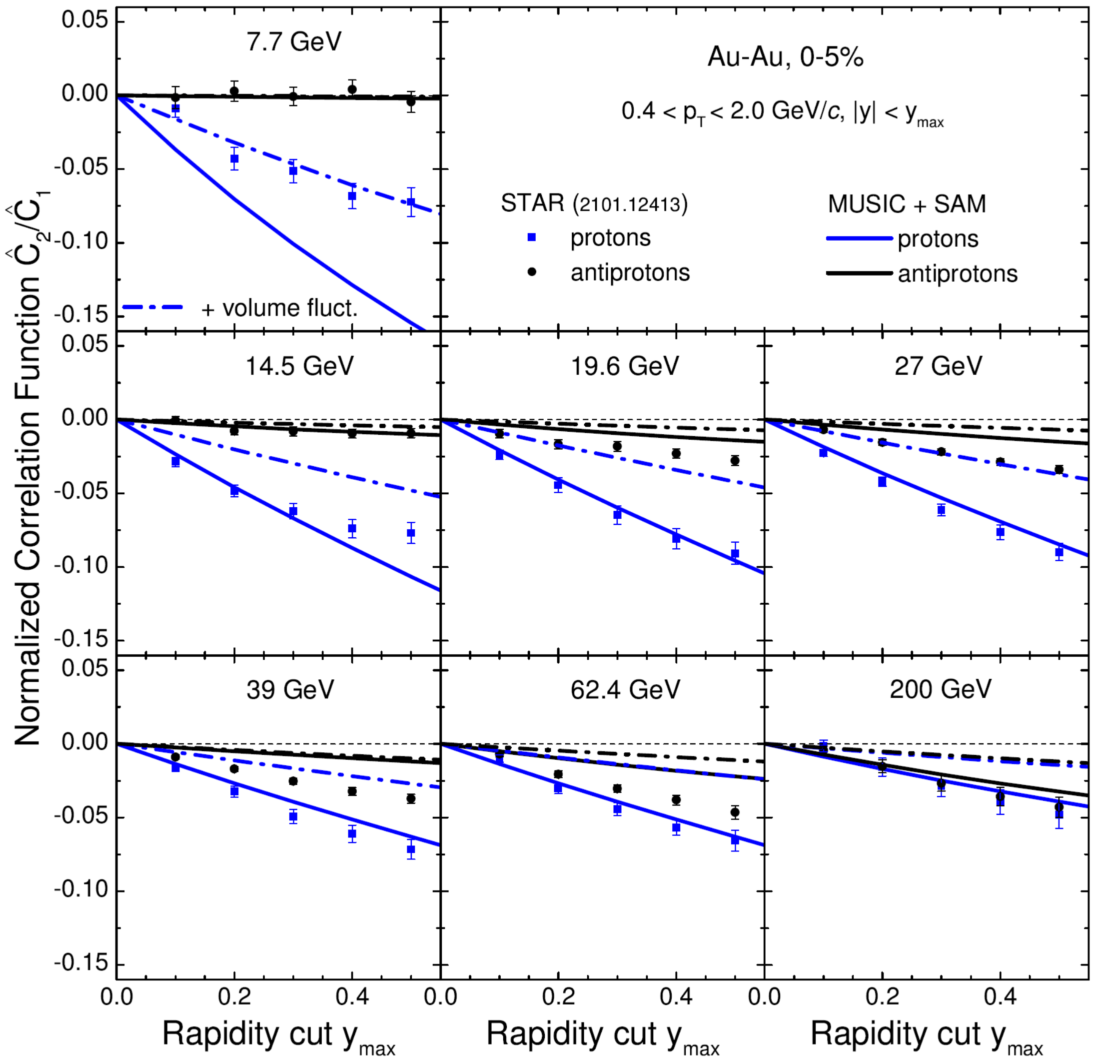
- **Volume fluctuations?** [Skokov, Friman, Redlich, PRC '13]
  - $C_2/C_1 \mathrel{+}= C_1 * v_2$



Acceptance dependence of two-particle correlations

- Changing y_{max} slope at $\sqrt{s_{NN}} \leq 14.5$ GeV?

- **Volume fluctuations?** [Skokov, Friman, Redlich, PRC ‘13]
  - $C_2/C_1 \mathrel{+}= C_1 * v_2$
  - Can improve low energies but spoil high energies?



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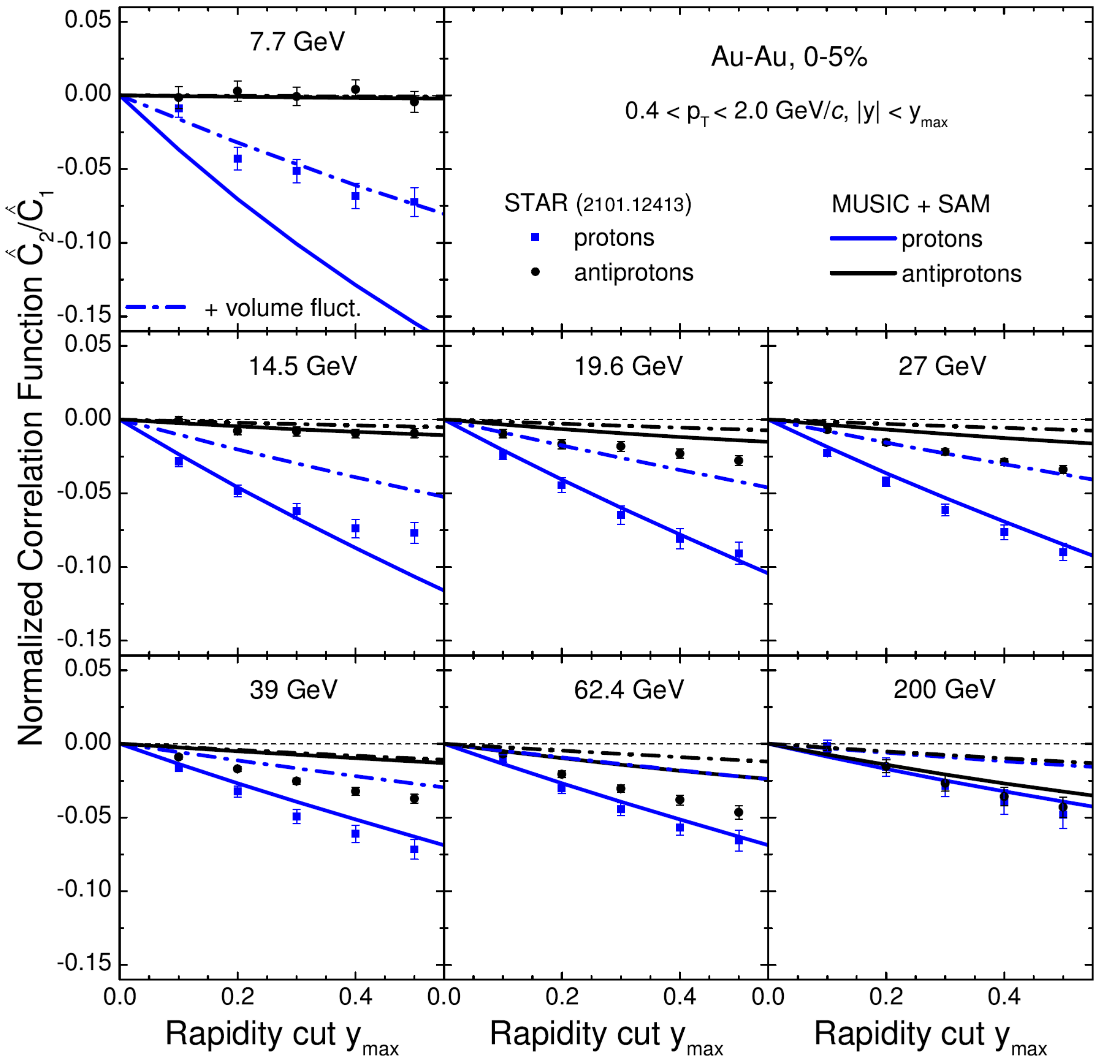

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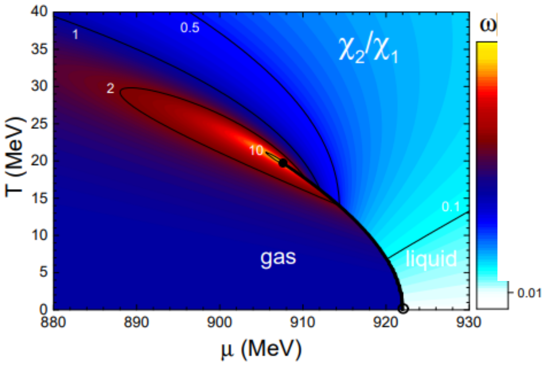
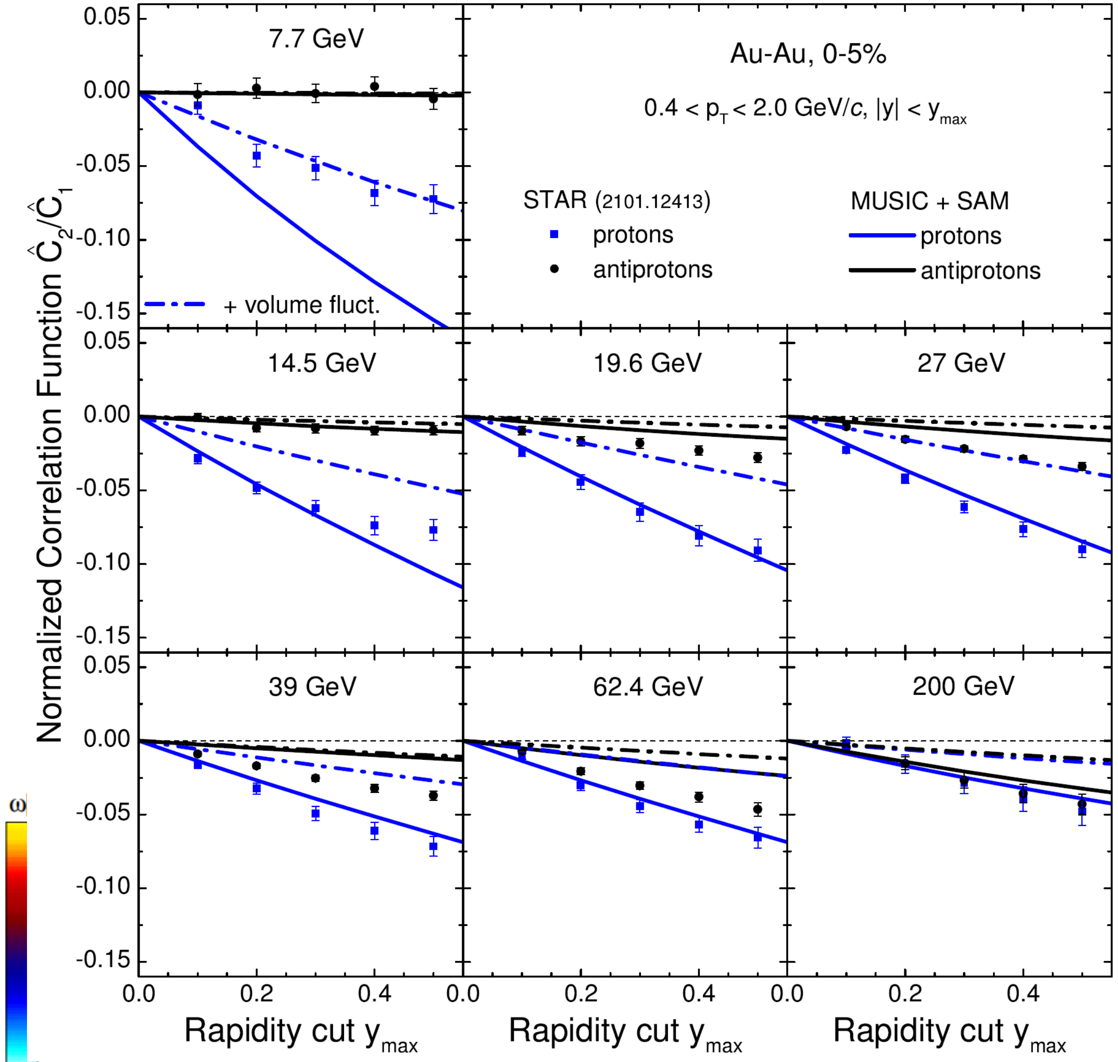
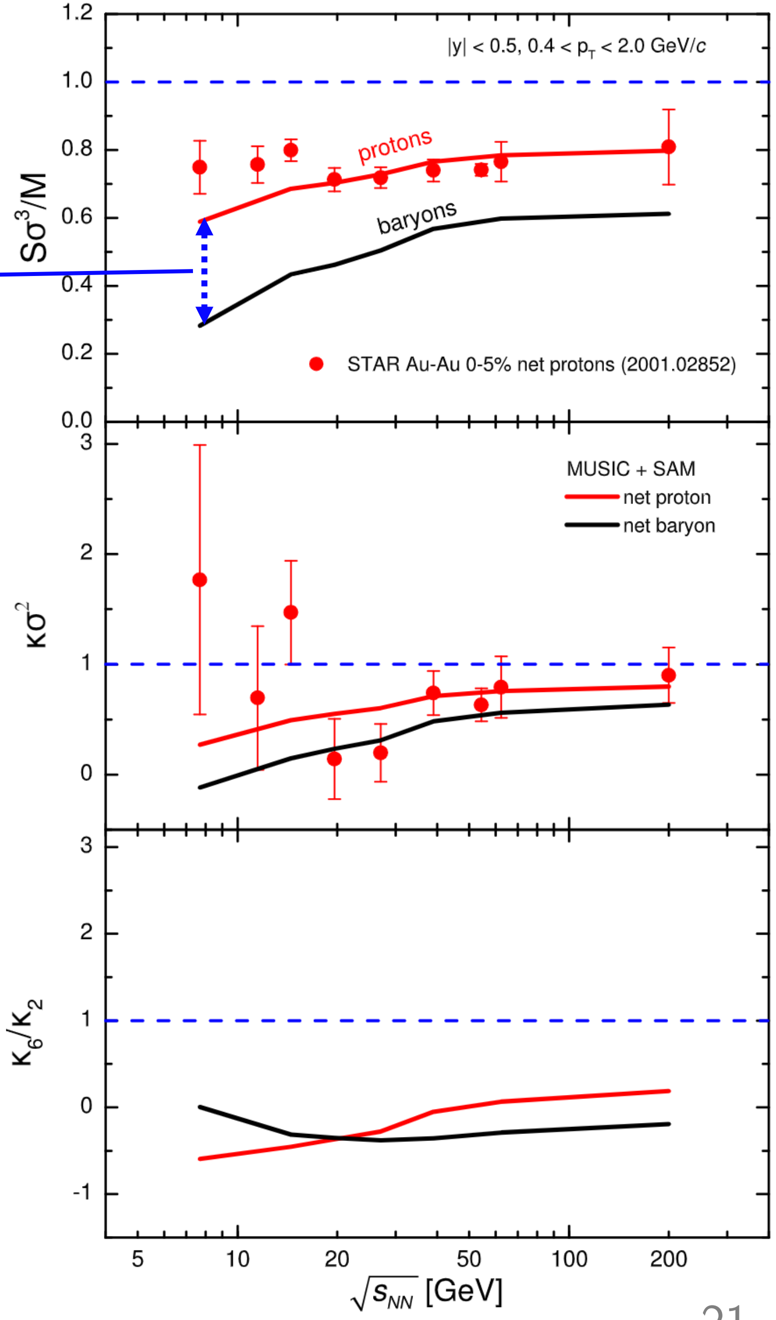
- **Attractive interactions?**
  - Could work if baryon repulsion turns into attraction in the high-μ_B regime
  - **Critical point?**

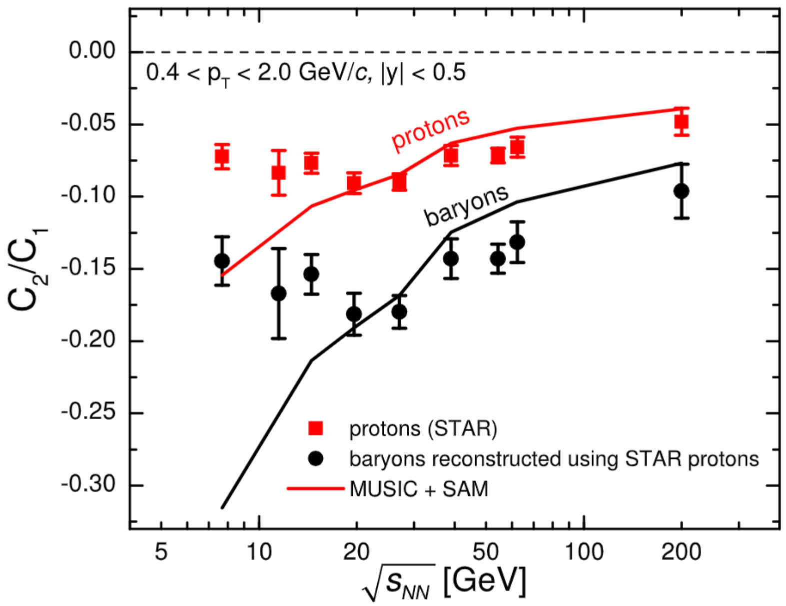




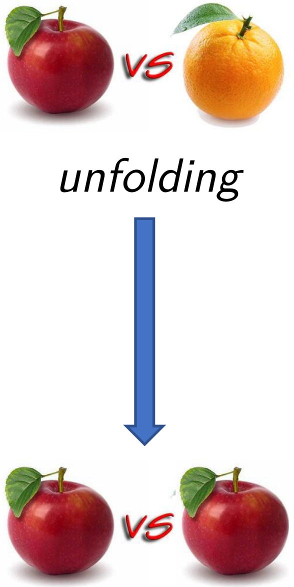
Outlook: baryon cumulants from protons

- **net baryon $\neq$ net proton**
- Baryon cumulants can be reconstructed from proton cumulants via binomial (un)folding based on isospin randomization [Kitazawa, Asakawa, Phys. Rev. C 85 (2012) 021901]
  - Amounts to an additional "efficiency correction" and requires the use of joint factorial moments, only experiment can do it model-independently





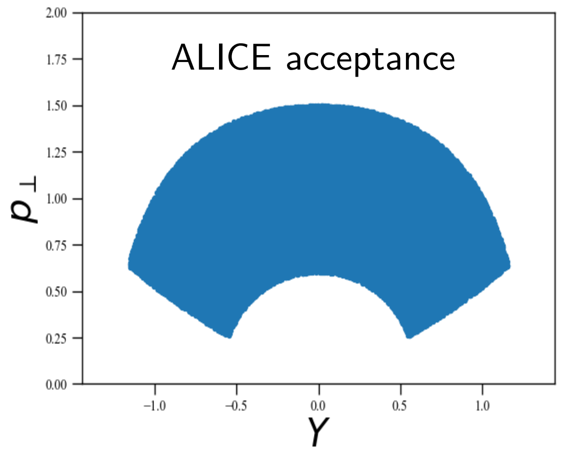
$$\frac{\hat{C}_2^B}{\hat{C}_1^B} \approx 2\,\frac{\hat{C}_2^p}{\hat{C}_1^p}$$



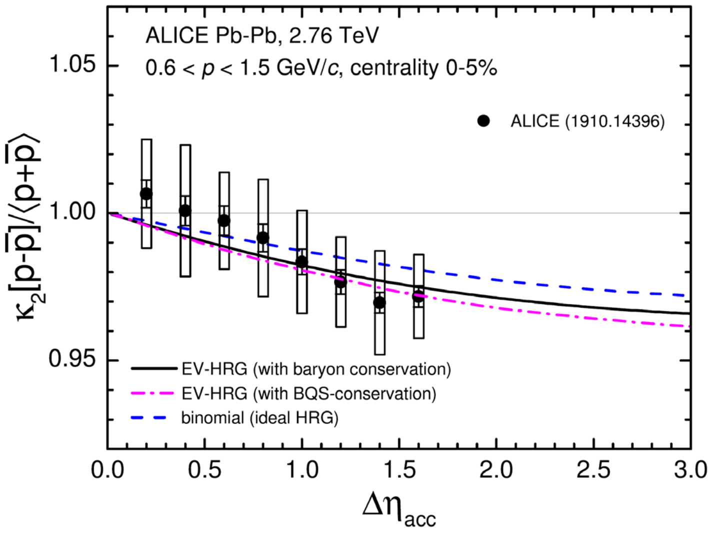
Net-particle fluctuations at the LHC

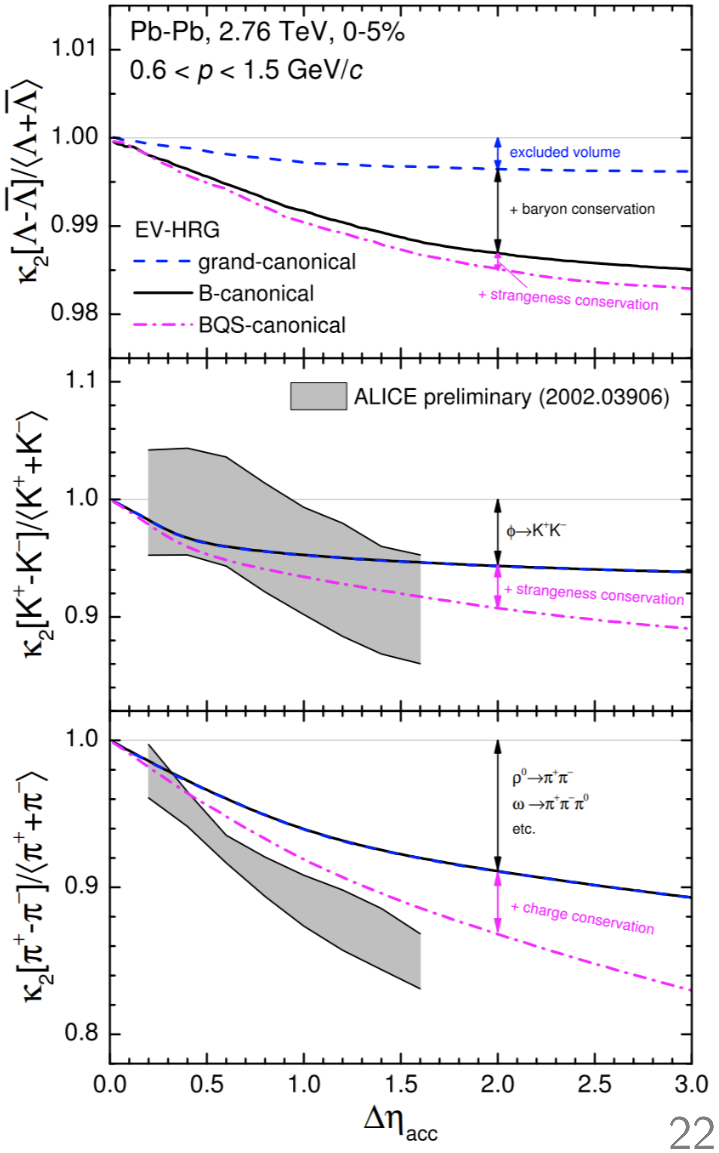
VV, Koch, Phys. Rev. C 103, 044903 (2021)

- Net protons described within errors but not sensitive to the equation of state for the present experimental acceptance
- Large effect from resonance decays for lighter particles + conservation of electric charge/strangeness
- Future measurements will require larger acceptance



$0.6 < p < 1.5$ GeV$/c$, $\Delta\eta_{acc} = 1.6$



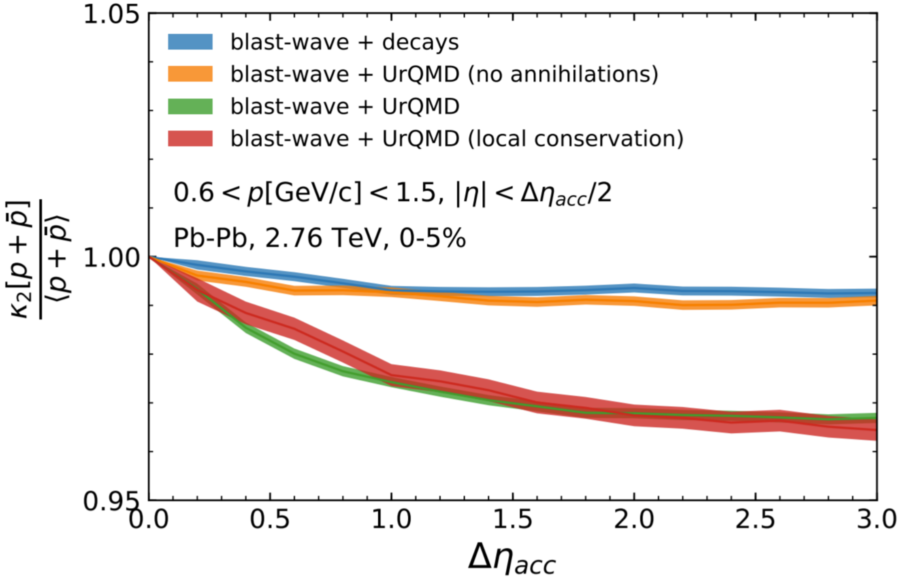


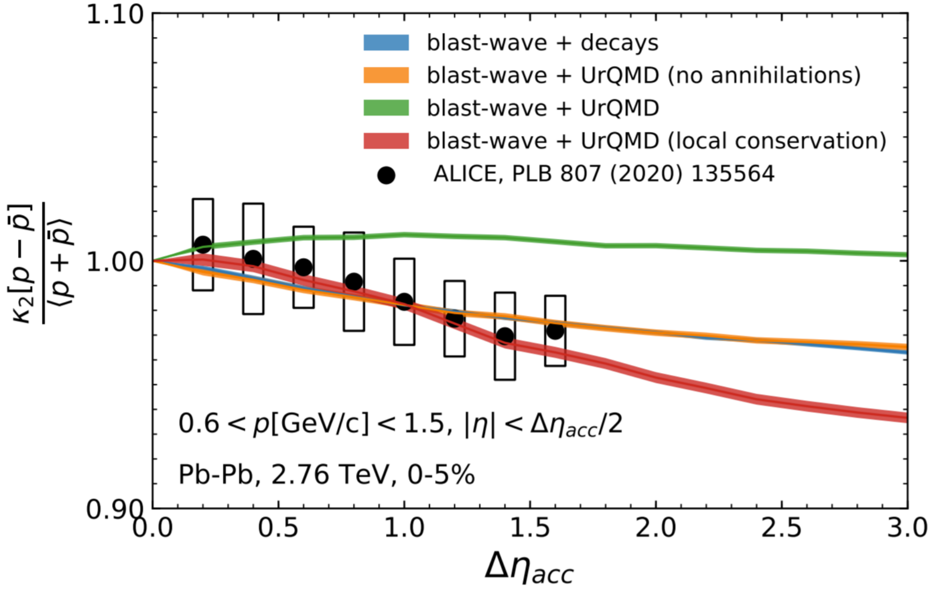
Effects of baryon annihilation and local conservation

O. Savchuk, V.V., V. Koch, J. Steinheimer, H. Stoecker, arXiv:2106.08239

Baryon annihilation $B\bar{B} \to n\pi$ in afterburners (UrQMD, SMASH) suppresses baryon yields

$\langle p + \bar{p} \rangle \searrow$ → $\frac{\kappa_2[p-\bar{p}]}{\langle p + \bar{p} \rangle} \nearrow$





- ALICE data requires local baryon conservation across $\Delta y \sim \pm 1.5$ with UrQMD annihilations (no regenerations) or global conservation ($\Delta y \sim \Delta y_{tot}$) without annihilations
- Local conservation and $B\bar{B}$ annihilation can be constrained from data through the combined analysis of $\kappa_2[p-\bar{p}]$ and $\kappa_2[p+\bar{p}]$

Lower energies $\sqrt{s_{NN}} \leq 7.7$ GeV

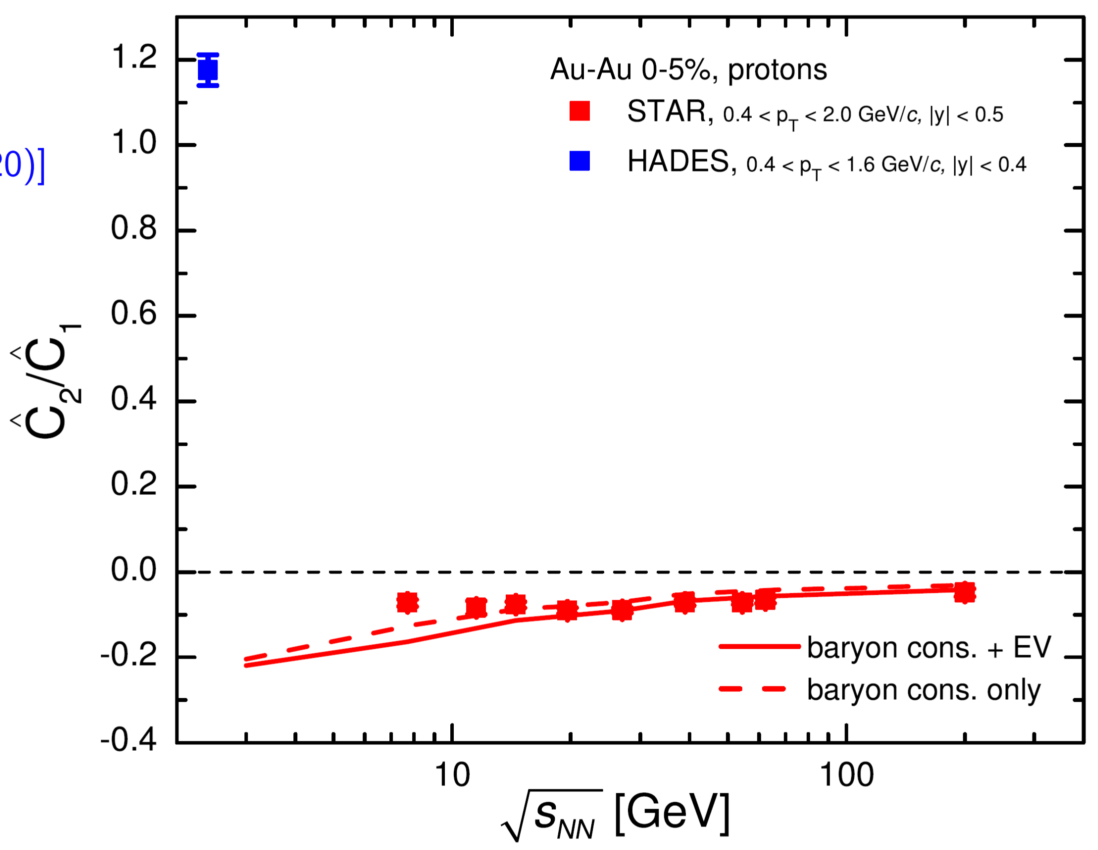
- Intriguing hint from HADES @ $\sqrt{s_{NN}} = 2.4$ GeV: huge two-particle correlations!

  [HADES Collaboration, PRC 102, 024914 (2020)]

- Extend the calculations down to $\sqrt{s_{NN}} = 3$ GeV by means of the blast-wave model

- No change of trend in the non-critical baseline

- Other important effects to consider
  - Light nuclei formation
  - Nuclear liquid-gas transition



Data from STAR-FXT eagerly awaited!

Thermodynamic analysis of HADES data

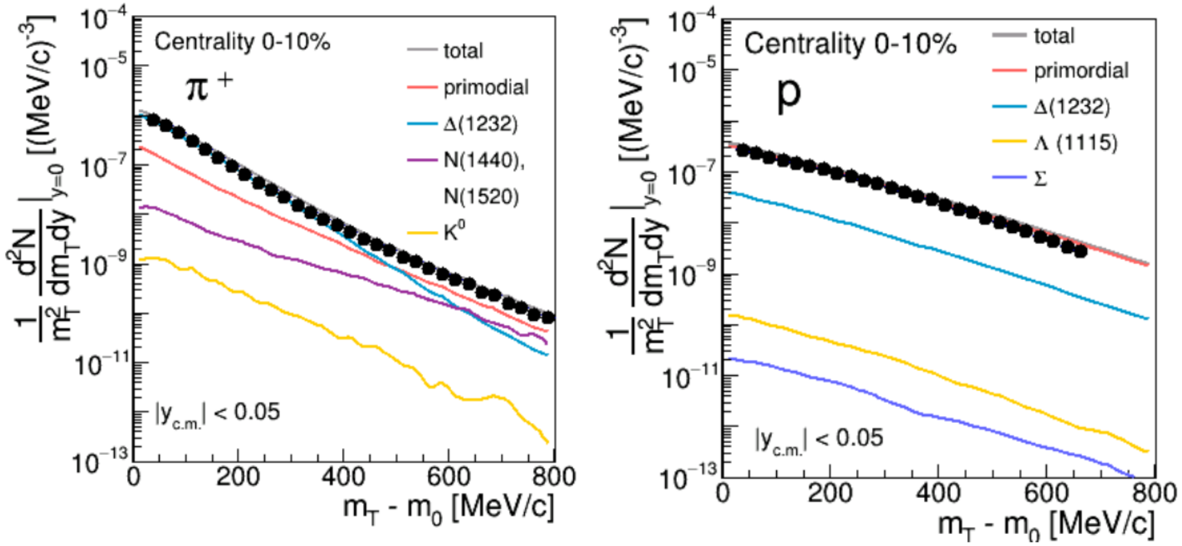
VV, Koch, in preparation

- Single freeze-out scenario: Emission from Siemens-Rasmussen hypersurface with Hubble-like flow

  $\rightarrow$ Pion and proton spectra o.k.

  [S. Harabasz et al., PRC 102, 054903 (2020)]

- Uniform $T \approx 70$ MeV, $\mu_B \approx 875$ MeV across the fireball [A. Motornenko et al., PLB 822, 136703 (2021)]



- Fluctuations:
  - Same as before but incorporate additional binomial filtering to account for protons bound in light nuclei
  - Uniform fireball $\rightarrow$ Final proton cumulants are linear combinations of baryon susceptibilities χ_n^B

Extract χ_n^B directly from experimental data

Thermodynamic analysis of HADES data

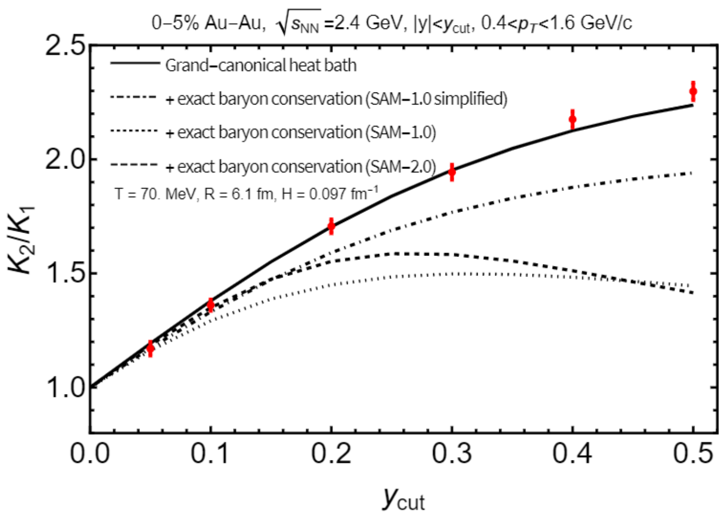
VV, Koch, in preparation

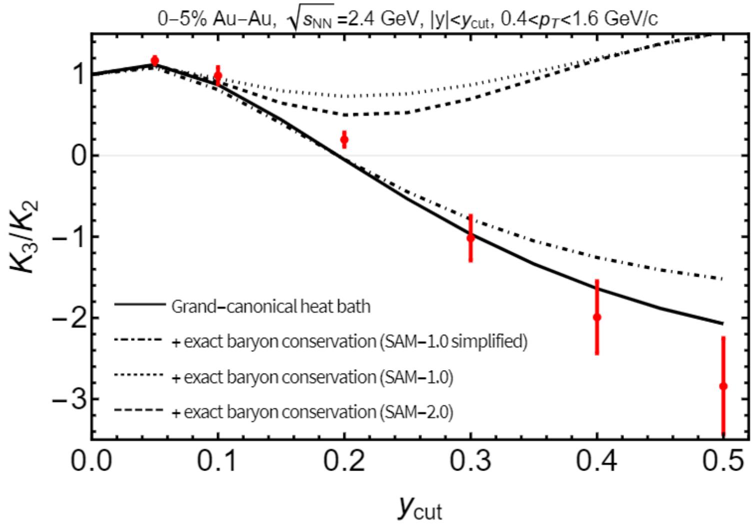
- In the grand-canonical limit (no baryon conservation) the data are described well with

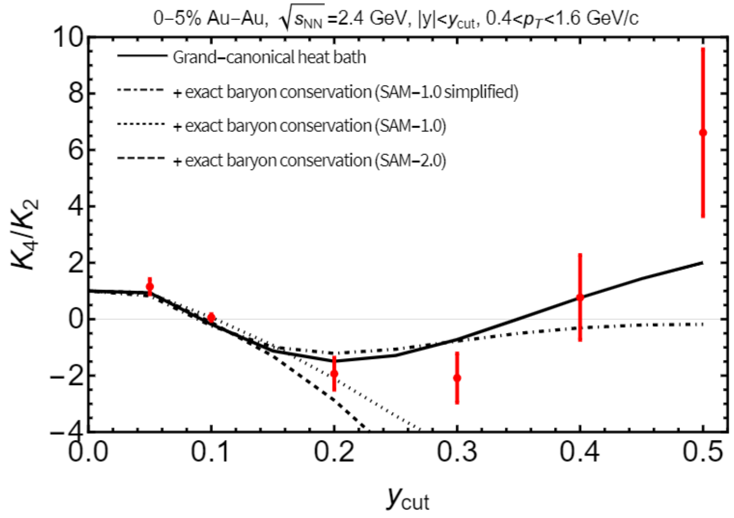
$$\frac{\chi_2^B}{\chi_1^B} = 9.35 \pm 0.40, \qquad \frac{\chi_3^B}{\chi_2^B} = -39.6 \pm 7.2, \qquad \frac{\chi_4^B}{\chi_2^B} = 1130 \pm 488$$

- Could be indicative of a critical point near the HADES freeze-out at $T \approx 70$ MeV, $\mu_B \approx 875$ MeV

- However, the results are challenging to describe with baryon conservation included

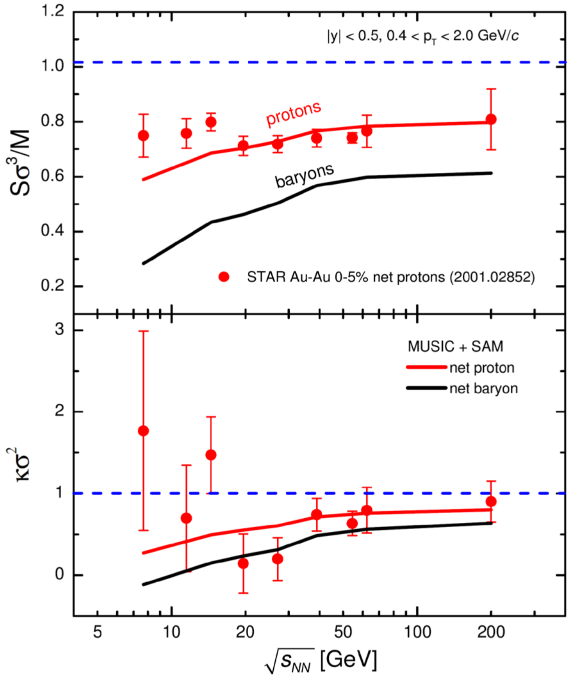


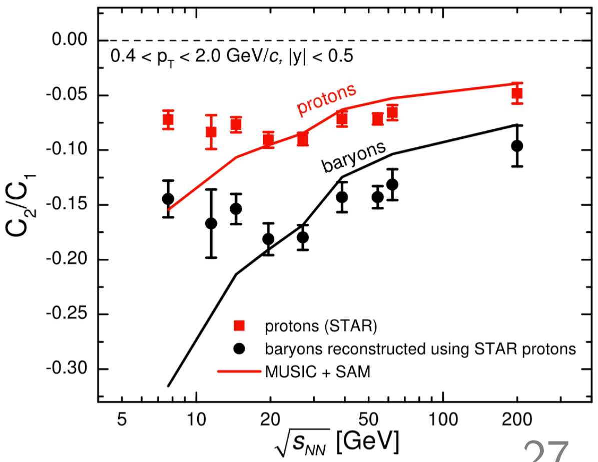




Summary

- Fluctuations are a powerful tool to explore the QCD phase diagram
  - test of lattice QCD and equilibration, probe the QCD critical point
- Quantitative analysis of central collisions at $\sqrt{s_{NN}}$=2.4-2760 GeV
  - Protons are described quantitatively at $\sqrt{s_{NN}} \geq 20$ GeV without critical point
  - Possible evidence for attractive proton interactions at $\sqrt{s_{NN}} \leq 14.5$ GeV
  - Significant quantitative difference between protons and baryons
- Factorial cumulants carry rich information
  - Small three- and four-particle correlations in absence of critical point effects
- HADES data point to potentially huge (multi-)proton correlations but at odds with baryon conservation





Thanks for your attention!