Stony Brook University | The State University of New York

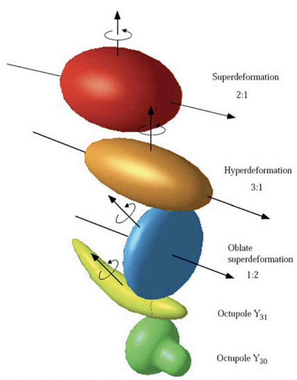
arXiv:2111.15559 arXiv:2105.05713
arXiv:2109.01631 arXiv:2105.01638
arXiv:2109.00604 arXiv:2102.08158
arXiv:2106.08768

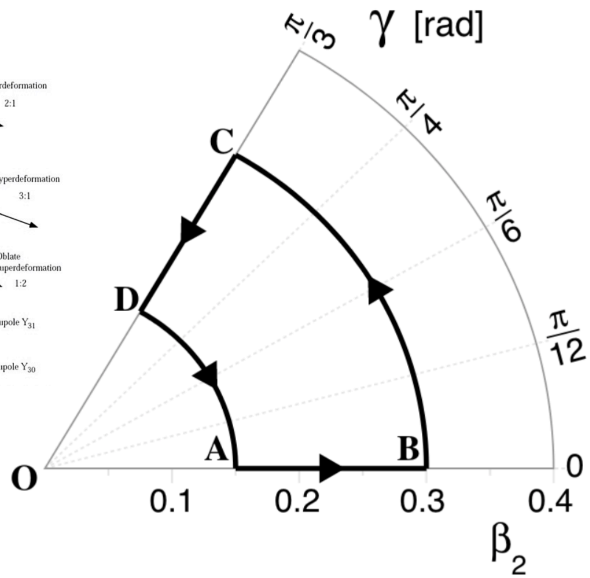
Imaging nuclear structure in heavy-ion collisions

Jiangyong Jia

Collaborators: Giuliano Giacalone and Chunjian Zhang

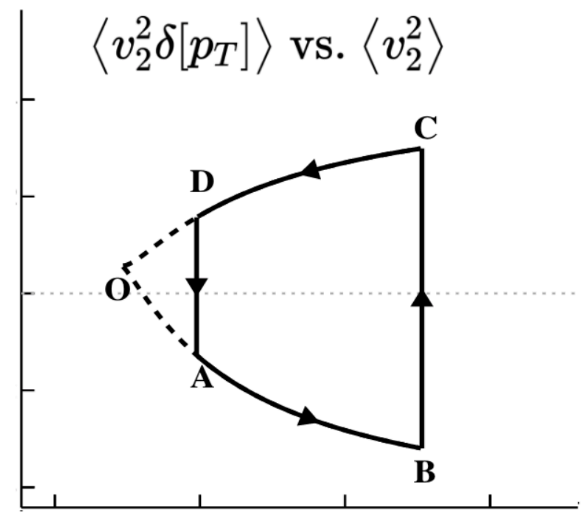
Nuclear structure





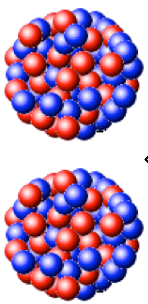
?

High-energy collisions

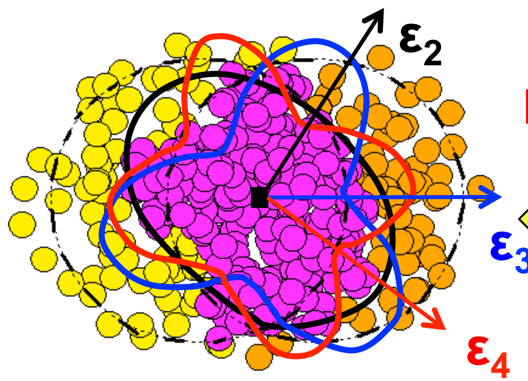


Hydrodynamic response to initial state

Nuclear Structure | **Initial State** | **Final Particle flow**

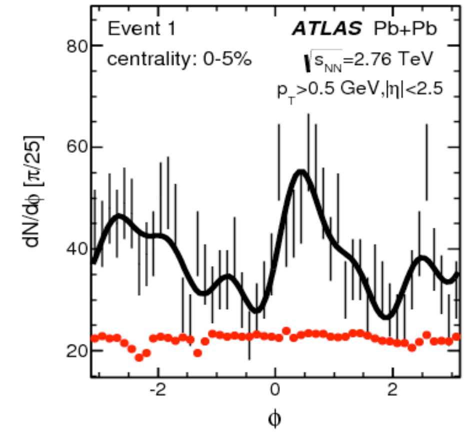


Imaging?



Hydro-response

Space-time dynamics



$$\frac{\rho_0}{1+e^{(r-R_0(1+\sum_n \beta_n Y_n^0(\theta,\phi))/a_0}}$$

- $\beta_2 \to$ Quadrupole deformation
- $\beta_3 \to$ Octupole deformation
- $a_0 \to$ Surface diffuseness
- $R_0 \to$ Nuclear size

Initial Size | Initial Shape

$$R_\perp \propto \langle r_\perp^2 \rangle, \quad \mathcal{E}_n \propto \langle r_\perp^n e^{in\phi} \rangle$$

??

$$R_0 \quad a_0 \quad \beta_n$$

Radial Flow | Harmonic Flow

$$\frac{d^2N}{d\phi dp_T} = N(p_T)\left(\sum_n V_n\, e^{-in\phi}\right)$$

arXiv:1206.1905

High energy: approx. linear response in each event:

$$\frac{\delta[p_T]}{[p_T]} \propto -\frac{\delta R_\perp}{R_\perp} \qquad V_n \propto \mathcal{E}_n$$

Previous works: PRC34.185, PRC61.021903, PRC61.034905, PRC.80.054903, nucl-th/0411054, 0712.0088, 1409.8375, 1507.03910, 1609.01949,1711.08499, 2007.00780.2103.05595. H. Stocker, W. Geiner, BA Li, E. Shuryak, PBM, U. Heinz, P. Philip, N.Xu, Q. Shou, P. Sorensen, F. Videbaek, A. Tang, P. Dasgupta, R. Chatterjee, D. Krivastava, F.Wang, H. Xu, Jaki. M. Luzum, P.Carzon...

Influence of shape fluctuations in relativistic heavy ion collisions

A. Rosenhauer, H. Stöcker, J. A. Maruhn, and W. Greiner
Phys. Rev. C **34**, 185 – Published 1 July 1986

Article | References | No Citing Articles | PDF | Export Citation

Focus particle production in UCC tip-tip vs body-body, start consider effect of collective flow

Uranium on uranium collisions at relativistic energies

Bao-An Li
Phys. Rev. C **61**, 021903(R) – Published 12 January 2000

Article | References | Citing Articles (25) | PDF | Export Citation

High energy collisions of strongly deformed nuclei: An old idea with a new twist

E. V. Shuryak
Phys. Rev. C **61**, 034905 – Published 22 February 2000

Article | References | Citing Articles (26) | PDF | Export Citation

Choice of colliding beams to study deformation effects in relativistic heavy ion collisions

S. Das Gupta and C. Gale
Phys. Rev. C **62**, 031901(R) – Published 23 August 2000

Article | References | Citing Articles (2) | PDF | Export Citation

Modern concept of elliptic flow and eccentricies, still focus on UCC region

Elliptic flow in central collisions of deformed nuclei

P. Filip

Physics of Atomic Nuclei **71**, 1609–1618 (2008) | Cite this article

Anisotropic Flow and Jet Quenching in Ultrarelativistic $\mathrm{U}+\mathrm{U}$ Collisions

Ulrich Heinz and Anthony Kuhlman
Phys. Rev. Lett. **94**, 132301 – Published 6 April 2005

Initial eccentricity in deformed $^{197}\mathrm{Au}+{}^{197}\mathrm{Au}$ and $^{238}\mathrm{U}+{}^{238}\mathrm{U}$ collisions at $\sqrt{s_{NN}}=200$ GeV at the BNL Relativistic Heavy Ion Collider

Peter Filip, Richard Lednicky, Hiroshi Masui, and Nu Xu
Phys. Rev. C **80**, 054903 – Published 5 November 2009

Parameterization of deformed nuclei for Glauber modeling in relativistic heavy ion collisions

Q.Y. Shou [a, b], Y.G. Ma [a], P. Sorensen [c], A.H. Tang [c], F. Videbæk [c], H. Wang [c]

Collision geometry and flow in uranium + uranium collisions

Andy Goldschmidt, Zhi Qiu, Chun Shen, and Ulrich Heinz
Phys. Rev. C **92**, 044903 – Published 7 October 2015

Article | References | Citing Articles (20) | PDF | HTML | Export Citation

Spectra and elliptic flow of thermal photons from full-overlap U+U collisions at energies available at the BNL Relativistic Heavy Ion Collider

Pingal Dasgupta, Rupa Chatterjee, and Dinesh K. Srivastava
Phys. Rev. C **95**, 064907 – Published 15 June 2017

Article | References | Citing Articles (7) | PDF | HTML | Export Citation

Hydrodynamic predictions for 5.44 TeV Xe+Xe collisions

Giuliano Giacalone, Jacquelyn Noronha-Hostler, Matthew Luzum, and Jean-Yves Ollitrault
Phys. Rev. C **97**, 034904 – Published 6 March 2018

Article | References | Citing Articles (50) | PDF | HTML | Export Citation

Possible octupole deformation of $^{208}\mathrm{Pb}$ and the ultracentral v_2 to v_3 puzzle

P. Carzon, S. Rao, M. Luzum, M. Sievert, and J. Noronha-Hostler
Phys. Rev. C **102**, 054905 – Published 9 November 2020

Article | References | Citing Articles (1) | PDF | HTML | Export Citation

Sophisticated e-by-e hydrodynamic model comparison including deformations.

Observing the Deformation of Nuclei with Relativistic Nuclear Collisions

Giuliano Giacalone
Phys. Rev. Lett. **124**, 202301 – Published 19 May 2020

v_2^2-p_T correlation

Article | References | Citing Articles (8) | PDF | HTML | Export Citation

Constraining the quadrupole deformation of atomic nuclei with relativistic nuclear collisions

Giuliano Giacalone
Phys. Rev. C **102**, 024901 – Published 3 August 2020

Article | References | Citing Articles (9) | PDF | HTML | Export Citation

Multiphase transport model predictions of isobaric collisions with nuclear structure from density functional theory

Radial structure

Hanlin Li, Hao-jie Xu, Jie Zhao, Zi-Wei Lin, Hanzhong Zhang, Xiaobao Wang, Caiwan Shen, and Fuqiang Wang
Phys. Rev. C **98**, 054907 – Published 26 November 2018

Probing the Neutron Skin with Ultrarelativistic Isobaric Collisions

Hanlin Li, Hao-jie Xu, Ying Zhou, Xiaobao Wang, Jie Zhao, Lie-Wen Chen, and Fuqiang Wang
Phys. Rev. Lett. **125**, 222301 – Published 23 November 2020

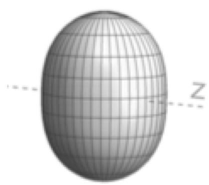
Article | References | Citing Articles (5) | PDF | HTML | Export Citation

Shape of nuclei

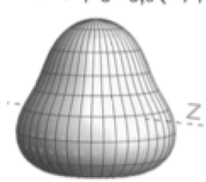
$$\rho(r,\theta,\phi) = \frac{\rho_0}{1 + e^{(r - R(\theta,\phi))/a_0}}$$

$$R(\theta,\phi) = R_0\left(1 + \beta_2[\cos\gamma Y_{2,0} + \sin\gamma Y_{2,2}] + \beta_3 \sum_{m=-3}^{3} \alpha_{3,m} Y_{3,m} + \beta_4 \sum_{m=-4}^{4} \alpha_{4,m} Y_{4,m}\right)$$

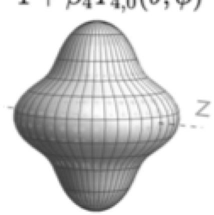
Quadrupole: $1 + \beta_2 Y_{2,0}(\theta,\phi)$



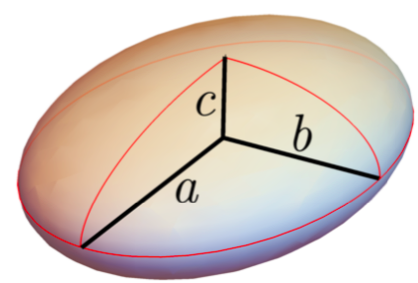
Octupole: $1 + \beta_3 Y_{3,0}(\theta,\phi)$



Hexadecapole: $1 + \beta_4 Y_{4,0}(\theta,\phi)$



Triaxial spheroid: $a \neq b \neq c$.



$0 \leq \gamma \leq \pi/3$

Prolate: a=b<c → β_2, $\gamma=0$

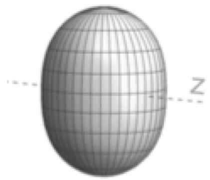
Oblate: a<b=c → β_2, $\gamma=\pi/3$ or $-\beta_2$, $\gamma=0$

Shape of nuclei

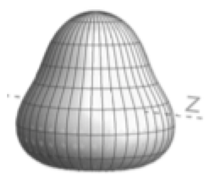
$$\rho(r,\theta,\phi) = \frac{\rho_0}{1 + e^{(r-R(\theta,\phi))/a_0}}$$

$$R(\theta,\phi) = R_0\left(1 + \beta_2[\cos\gamma Y_{2,0} + \sin\gamma Y_{2,2}] + \beta_3 \sum_{m=-3}^{3} \alpha_{3,m} Y_{3,m} + \beta_4 \sum_{m=-4}^{4} \alpha_{4,m} Y_{4,m}\right)$$

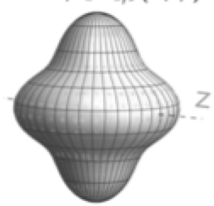
Quadrupole: $1+\beta_2 Y_{2,0}(\theta,\phi)$

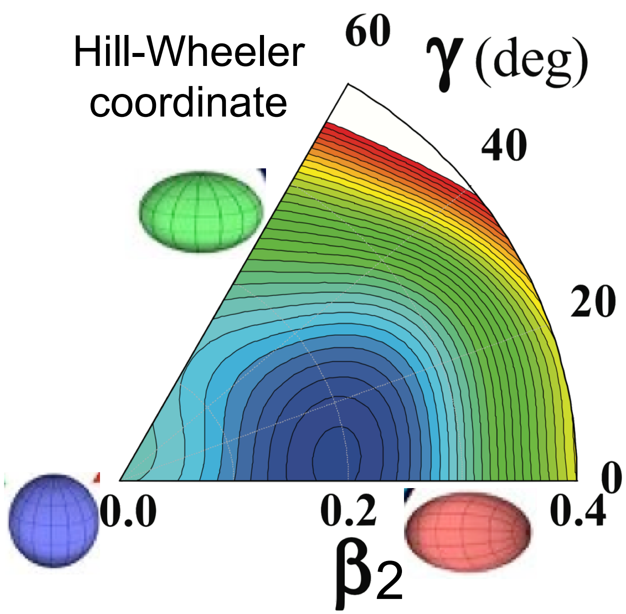


Octupole: $1+\beta_3 Y_{3,0}(\theta,\phi)$



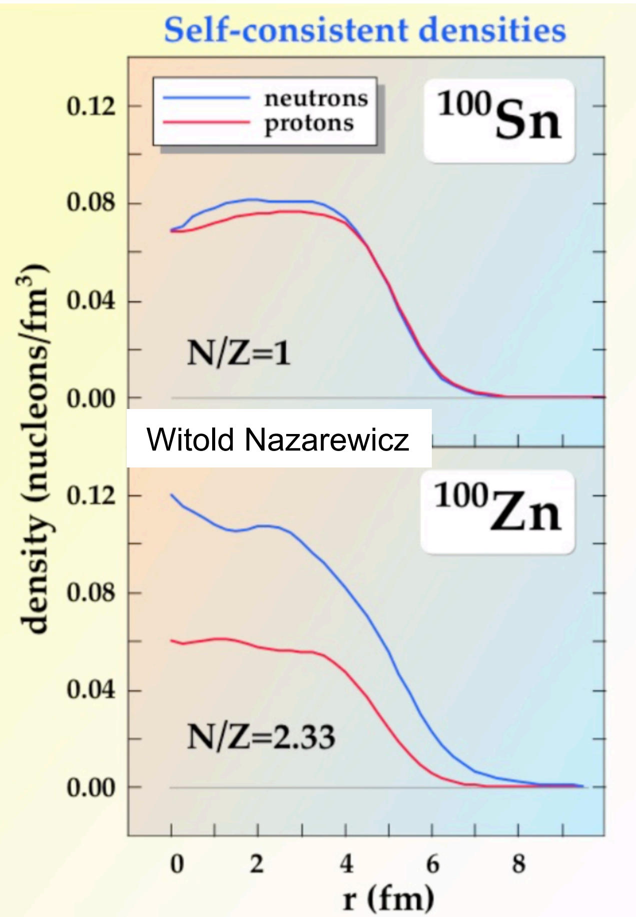
Hexadecapole: $1+\beta_4 Y_{4,0}(\theta,\phi)$





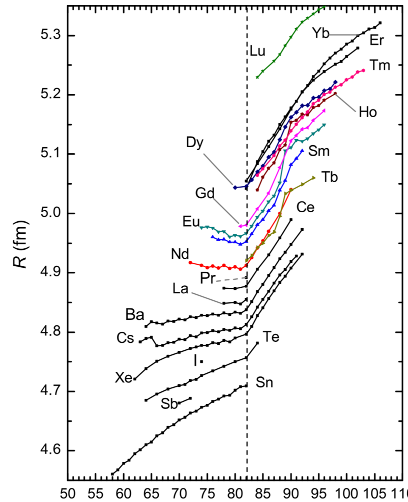
Radial structure of nuclei

$$\rho = \frac{\rho_0}{1 + e^{(r - R_0(1+\sum_n \beta_n Y_n^0(\theta,\phi))/a_0}}$$

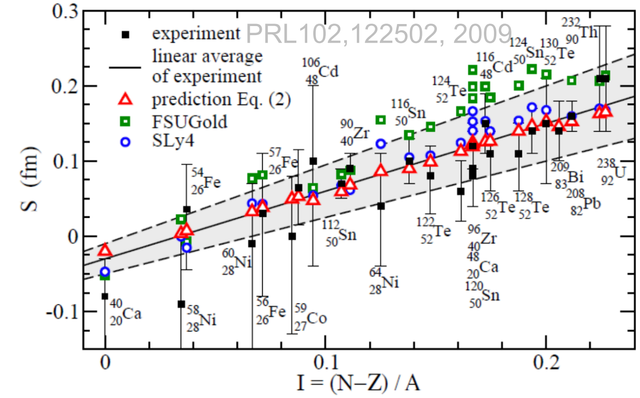


Witold Nazarewicz

R_0

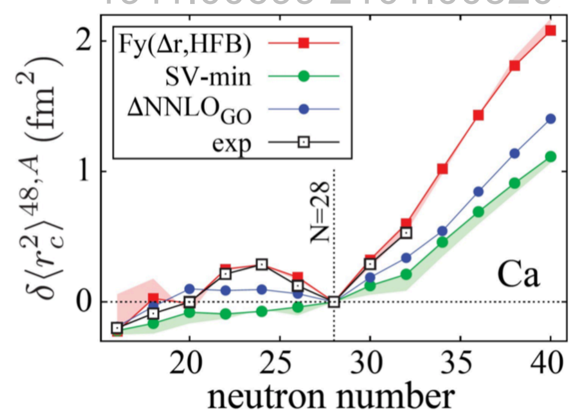


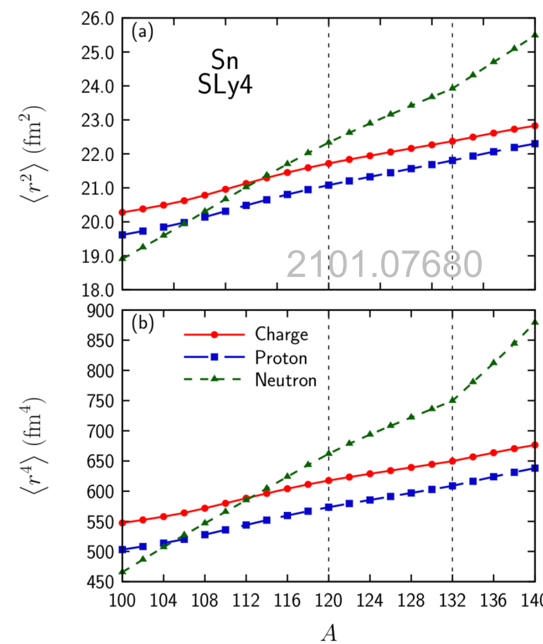
$\Delta r_{np} = R_n - R_p$.



Higher radial moments

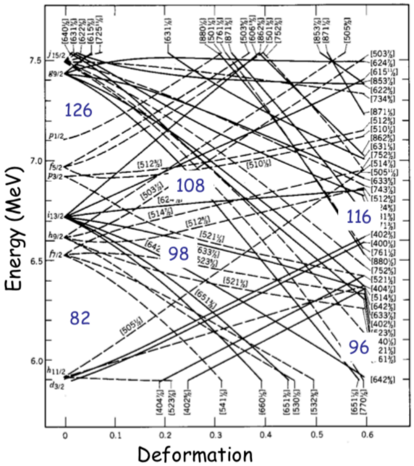
1911.00699 2101.00320



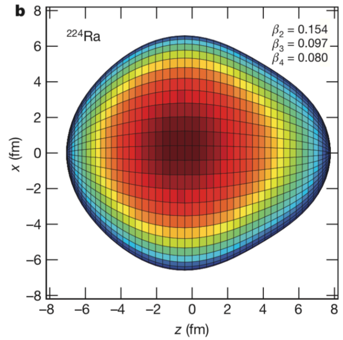


Some topics in nuclear structure

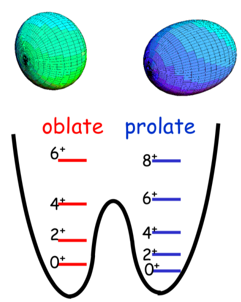
- How the shape/size/skin evolves along isotopic chain
  - Strong test on nuclear structure model

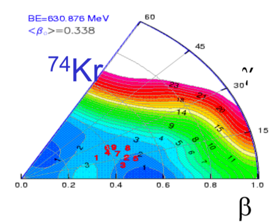


- Octuple (pear-shaped) deformation
  - Octupole correlation or static deformation
  - Strong test on EDM effects



- Triaxility : infers from γ-band, Chiral and Wobbling bands. Have large uncertainties.
  - shape coexistance



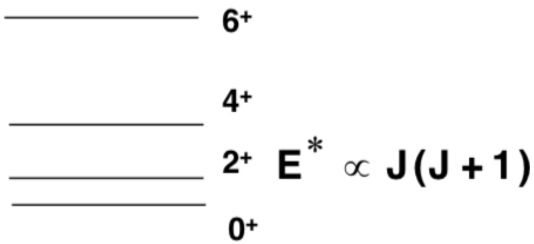


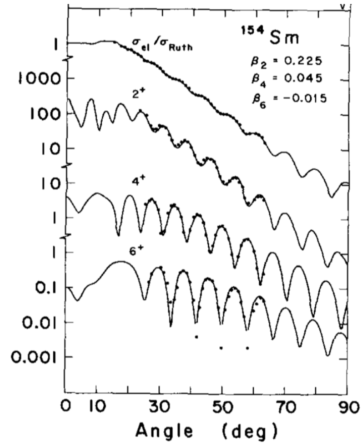
Use shape imaging in heavy-ion collision to help?

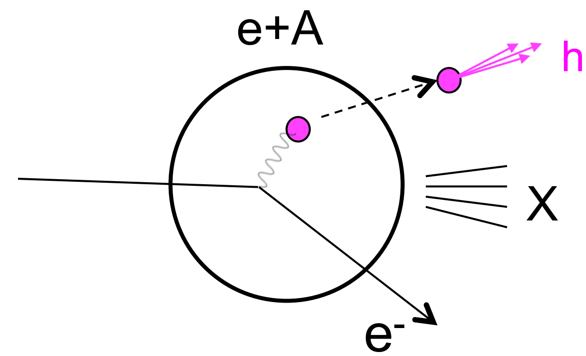
Nuclear structure vs HI method

- Shape from B(En), radial profile from e+A or ion-A scattering

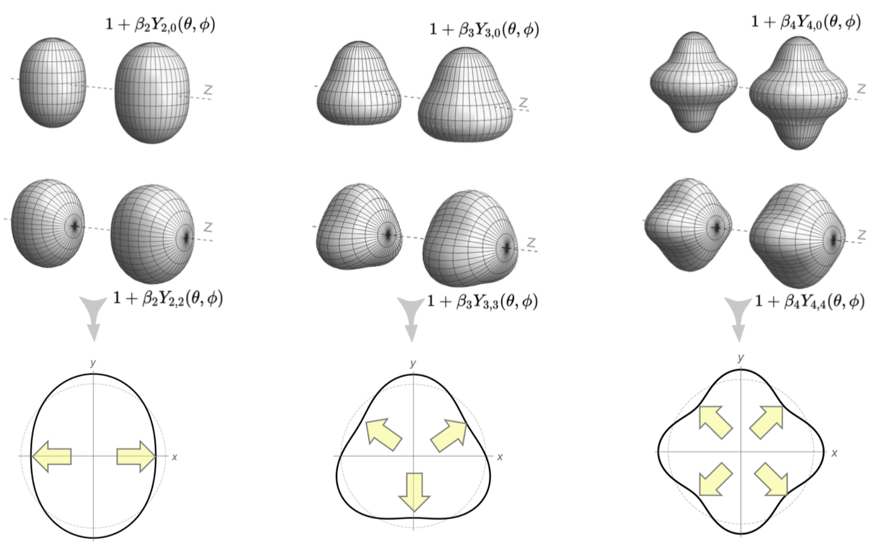
«rotational» spectrum

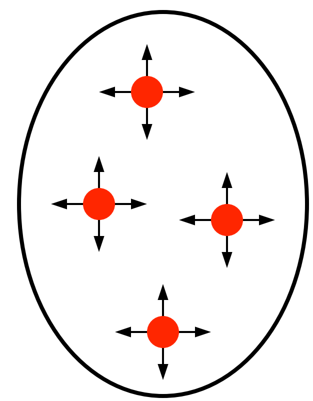






- Probe entire mass distribution: multi-point correlations





$$S(\mathbf{s}_1,\mathbf{s}_2) \equiv \langle \delta\rho(\mathbf{s}_1)\delta\rho(\mathbf{s}_2)\rangle = \langle \rho(\mathbf{s}_1)\rho(\mathbf{s}_2)\rangle - \langle \rho(\mathbf{s}_1)\rangle\langle \rho(\mathbf{s}_2)\rangle.$$

Flow response to probe the nuclear structure

Evidence of deformation in U+U vs Au+Au

https://indico.cern.ch/event/854124/contributions/4135480/

Collisions at $\sqrt{s_{NN}}$=193-200 GeV

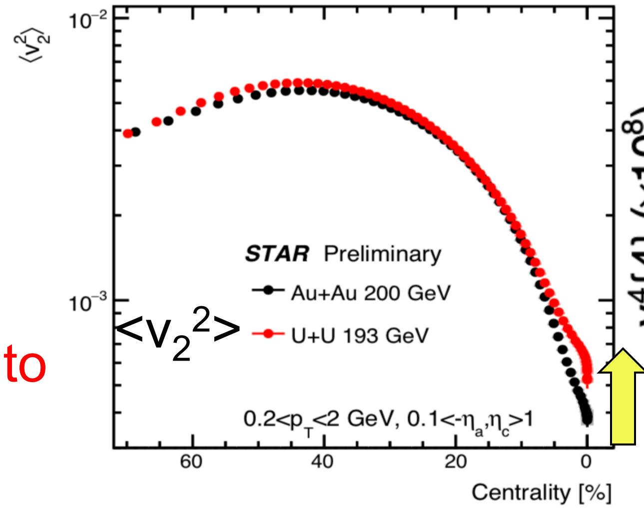
Large deformation in ^{238}U relative to ^{197}Au strongly influence flow signals

β_{2U} ~ 0.28

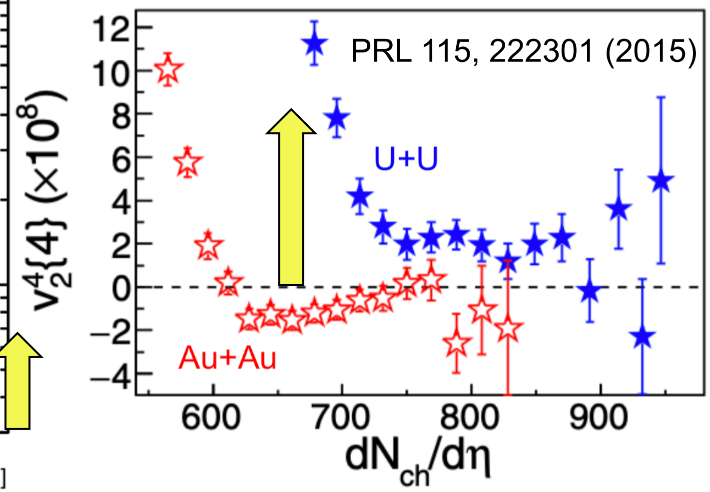
β_{2Au}~ -0.13?

Working to turn these into a quantitative tool !

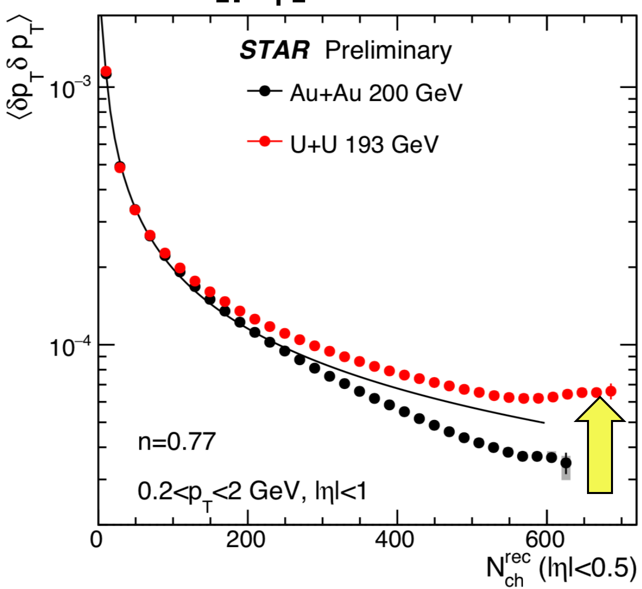
v_2 variance



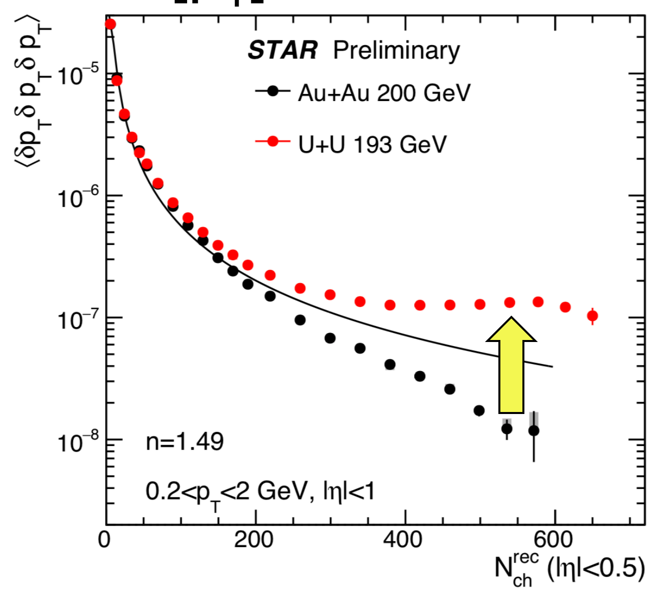
v_2 kurtosis



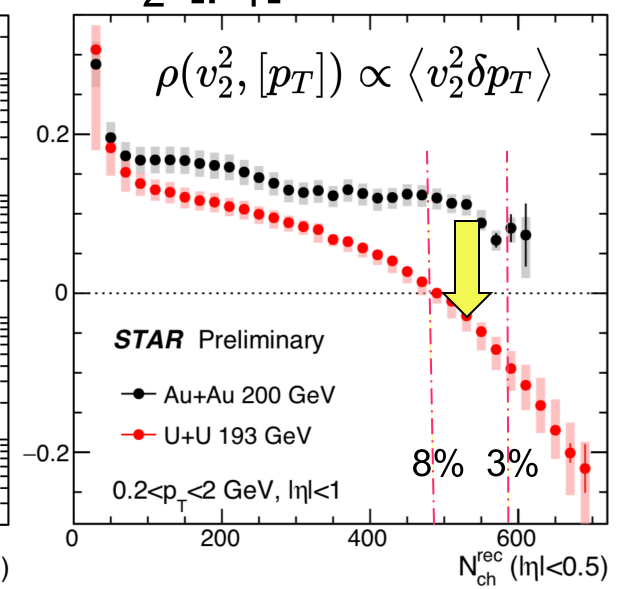
$[p_T]$ variance



$[p_T]$ skewness



v_2-$[p_T]$ covariance

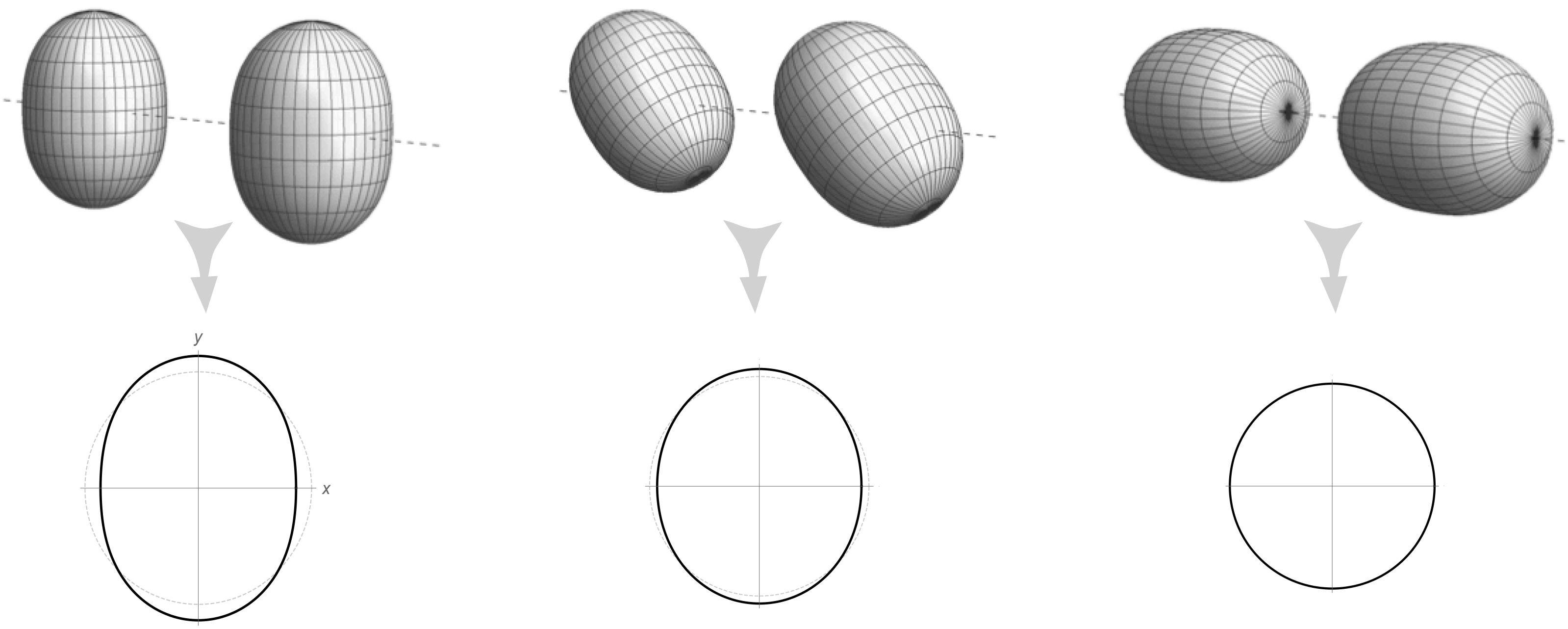


Shape of the initial state in HI

Body-Body

$$\mathcal{E}_n = \epsilon_n e^{in\Phi} \propto \left\langle r_\perp^n e^{in\phi} \right\rangle$$

Tip-Tip



$\varepsilon_2 \sim 0.95\beta_2$ $\varepsilon_2 \sim 0.48\beta_2$ $\varepsilon_2 \sim 0$

shape of overlap = shape of nucleon dist. projected along Euler angle $\Omega = \varphi\theta\psi$

Parametric dependence

- ε_n has the form $\epsilon_n = \underbrace{\epsilon_{n;0}}_{\text{undeformed}} + \sum_{m=2}^{4} \underbrace{\boldsymbol{p}_{n;m}(\Omega_1,\Omega_2)}_{\text{phase factor}}\beta_m + \mathcal{O}(\beta^2)$

  γ only appear here, in the form of cos3γ, cos6γ,…

- $R_\perp^2 = \langle x^2\rangle + \langle y^2\rangle$ has the form $\delta d_\perp/d_\perp = \delta_d + \sum_{m=2}^{4} p_{0;m}(\Omega_1,\Omega_2)\beta_m + \mathcal{O}(\beta^2)$

  $d_\perp \equiv 1/R_\perp$

- Two particle correlation

$$\left\langle \varepsilon_n^2 \right\rangle \approx \left\langle \varepsilon_{n;0}^2 \right\rangle + \sum_m \left\langle \boldsymbol{p}_{n;m}\boldsymbol{p}_{n;m}^* \right\rangle \beta_m^2 \qquad \left\langle \left(\frac{\delta d_\perp}{d_\perp}\right)^2 \right\rangle \approx \left\langle \delta_d^2 \right\rangle + \sum_m \left\langle p_{0;m}^2 \right\rangle \beta_m^2$$

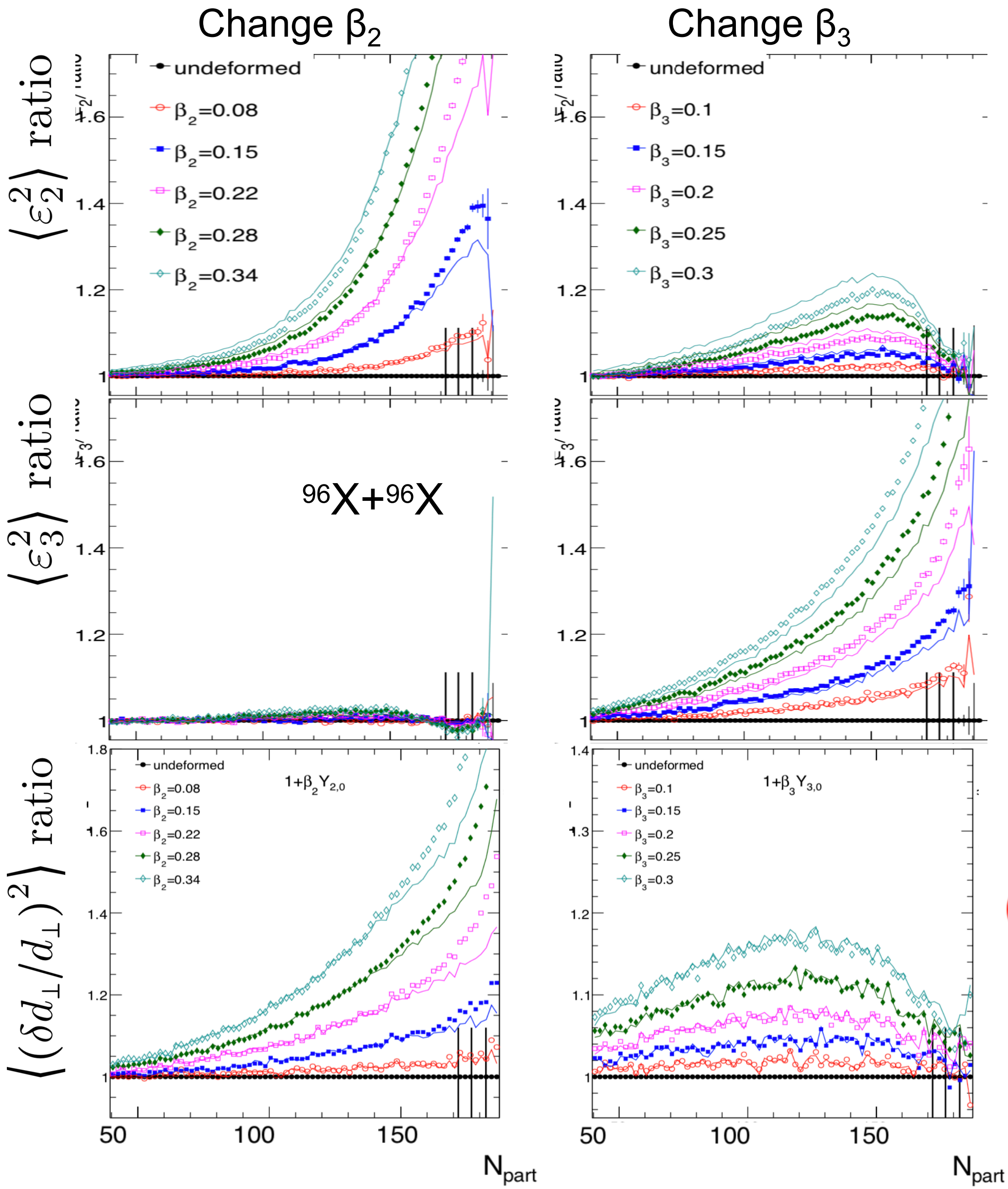
- Consider also the influence of R_0 and a $\dfrac{\rho_0}{1+e^{(r-R_0(1+\sum_n \beta_n Y_n^0(\theta,\phi))/a_0}}$

$$\left\langle \varepsilon_n^2 \right\rangle \approx b_0 + b_1\beta_2^2 + b_2\beta_3^2 + b_3(R_0 - R_{0,\,\mathrm{ref}}) + b_4(a - a_{\mathrm{ref}})$$

- Again linear response to relate to final state: $v_n \propto \varepsilon_n \quad \dfrac{\delta[p_T]}{[p_T]} \propto \dfrac{\delta d_\perp}{d_\perp}$

Parametric dependence

See 2106.08768



medium size system:

$$\varepsilon_2^2 = a_2' + b_2'\beta_2^2 + b_{2,3}'\beta_3^2$$

$$\varepsilon_3^2 = a_3' + b_3'\beta_3^2$$

$$(\delta d_\perp/d_\perp)^2 = a_0' + b_0'\beta_2^2 + b_{0,3}'\beta_3^2$$

$$v_2^2 = a_2 + b_2\beta_2^2 + b_{2,3}\beta_3^2$$

$$v_3^2 = a_3 + b_3\beta_3^2$$

$$(\delta p_{\mathrm{T}}/p_{\mathrm{T}})^2 = a_0 + b_0\beta_2^2 + b_{0,3}\beta_3^2$$

Isobar collisions as precision tool

- Unique running mode of RHIC and STAR to minimize the detector systematics
  - 0.4% precision is achieved in ratio of many observables between two isobar systems→ precision imaging tool

A key question for any HI observable O

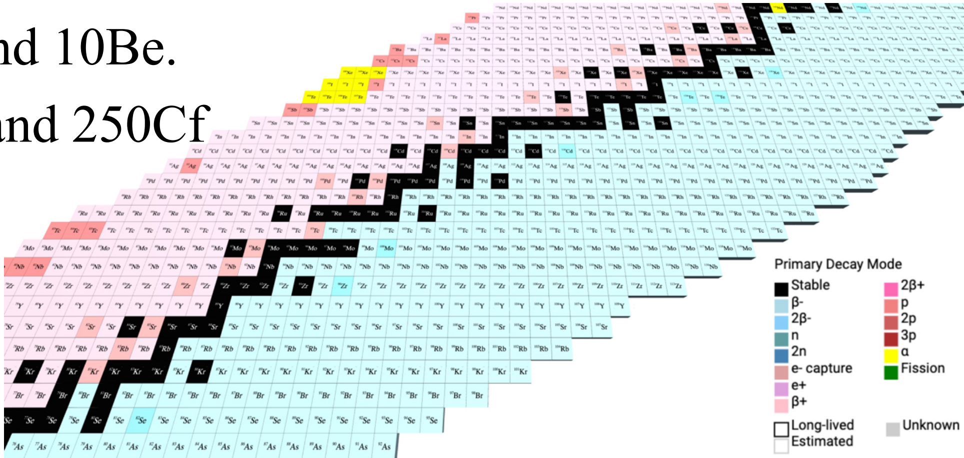
$$\frac{O_{\mathrm{X+X}}}{O_{\mathrm{Y+Y}}} \stackrel{?}{=} 1$$

$$^{A}\mathrm{X} + {}^{A}\mathrm{X} \; vs \; {}^{A}\mathrm{Y} + {}^{A}\mathrm{Y}$$

Deviation from 1 must has its origin in the nuclear structure, which is reflected by the initial state and then survives the final state. A precision tool to study initial state and final state responses

- Many such pairs of isobars in the nuclear chart.
  - Small system isobar such as 10B and 10Be.
  - Large system isobar up to 250Cm and 250Cf

A new handle to probe heavy ion physics



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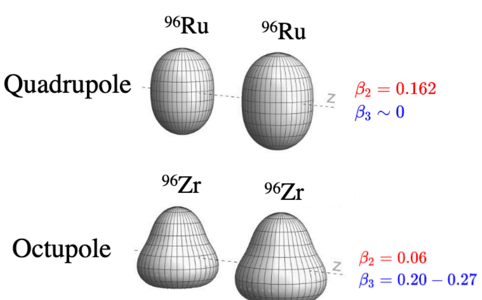
A key question for any HI observable O

$$\frac{O_{\mathrm{X+X}}}{O_{\mathrm{Y+Y}}} \stackrel{?}{=} 1 \qquad {}^{A}\mathrm{X} + {}^{A}\mathrm{X}\ vs\ {}^{A}\mathrm{Y} + {}^{A}\mathrm{Y}$$

Deviation from 1 must has its origin in the nuclear structure, which is reflected by the initial state and then survives the final state. A precision tool to study initial state and final state responses

- Expectation



J. Jia, aXiv:2106.08768

$$\mathcal{O} \approx b_0 + b_1\beta_2^2 + b_2\beta_3^2 + b_3(R_0 - R_{0,\mathrm{ref}}) + b_4(a - a_{\mathrm{ref}})$$

$$R_{\mathcal{O}} \equiv \frac{\mathcal{O}_{\mathrm{Ru}}}{\mathcal{O}_{\mathrm{Zr}}} \approx 1 + c_1\Delta\beta_2^2 + c_2\Delta\beta_3^2 + c_3\Delta R_0 + c_4\Delta a$$

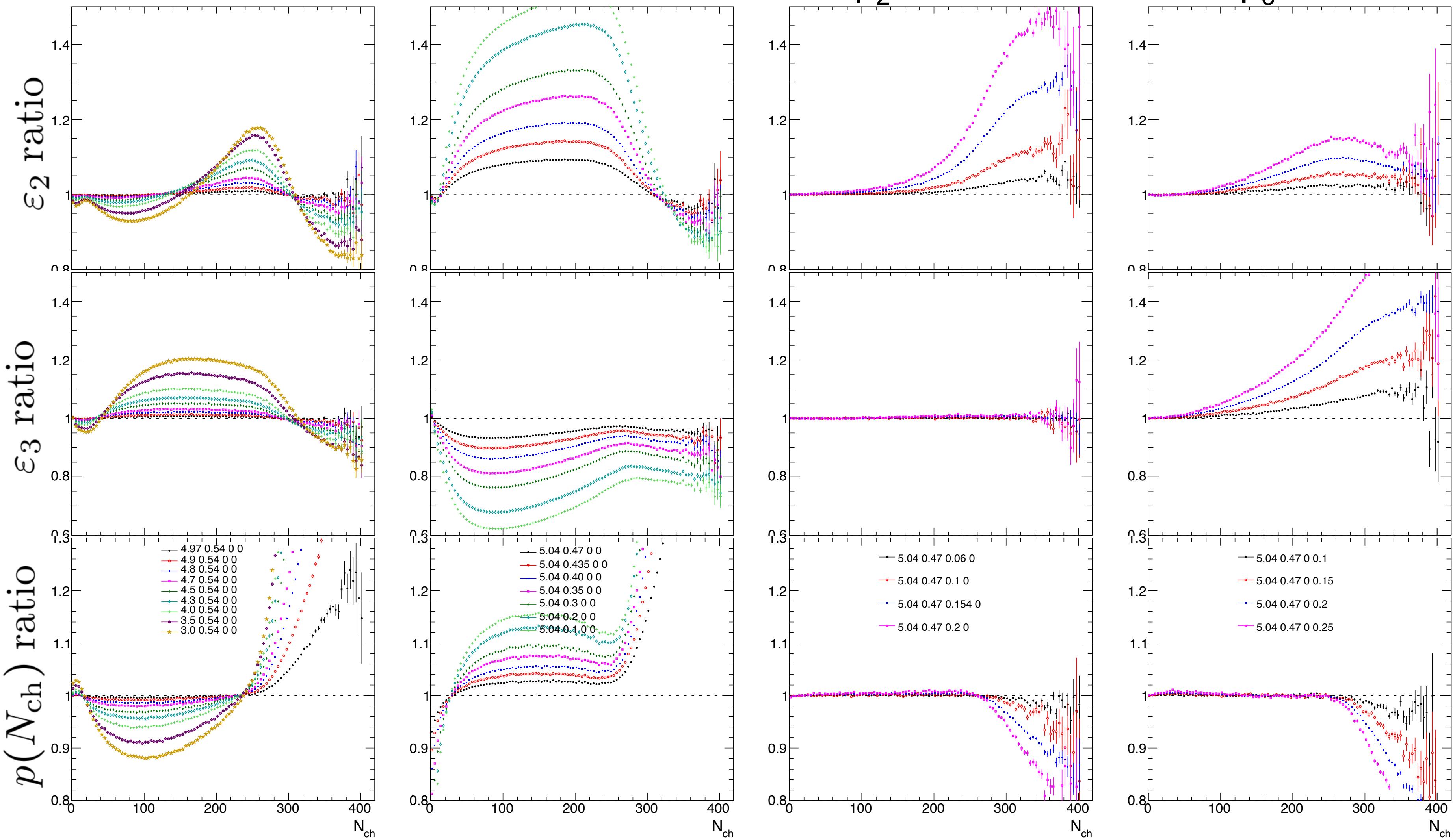
Species	β_2	β_3	a_0	R_0
Ru	0.162	0	0.46 fm	5.09 fm
Zr	0.06	0.20	0.52 fm	5.02 fm

difference	$\Delta\beta_2^2$	$\Delta\beta_3^2$	Δa_0	ΔR_0
	0.0226	-0.04	-0.06 fm	0.07 fm

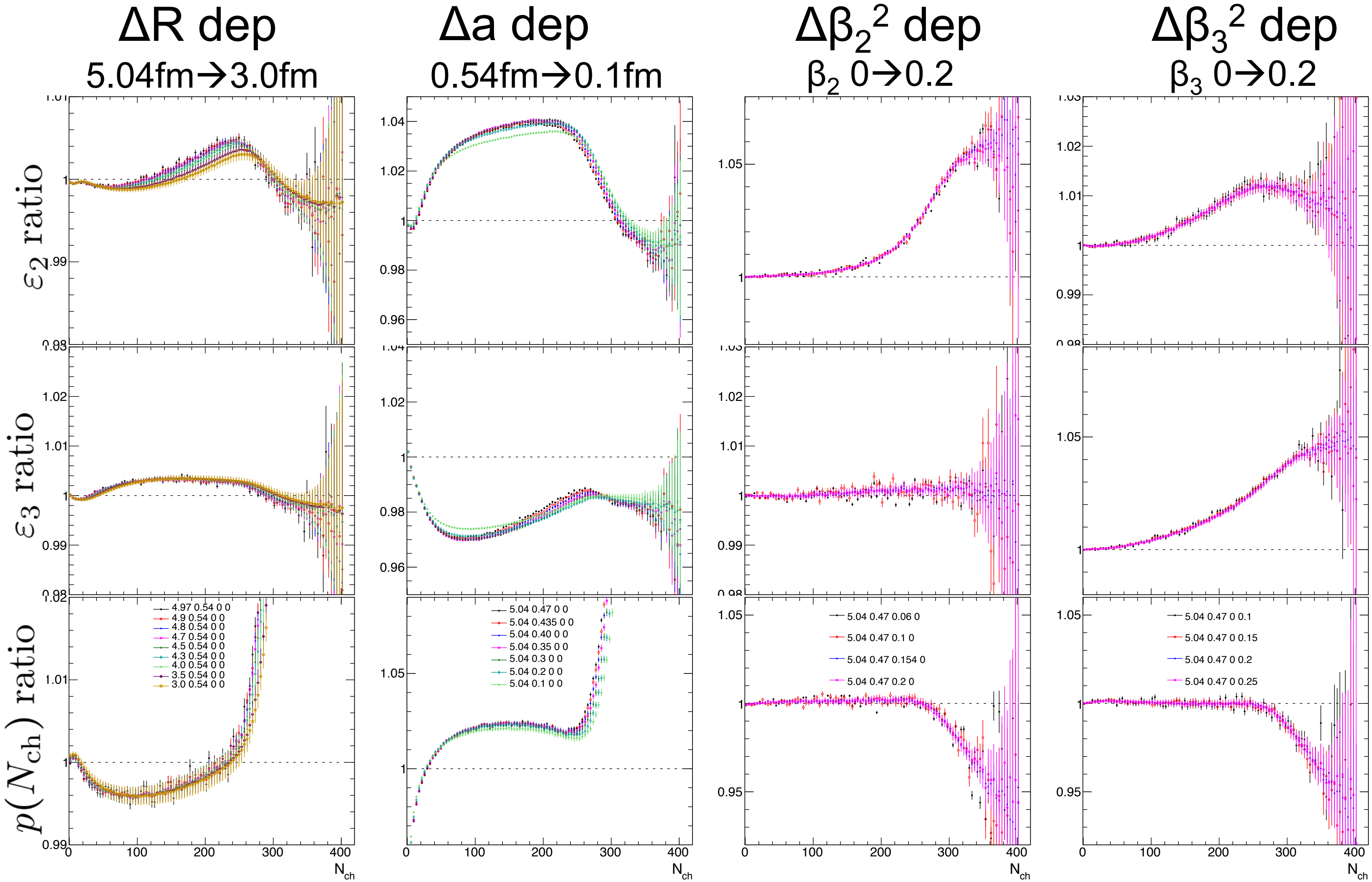
Valid for most single- and two-particle observable: v2,v3, p(N),$\langle p_T \rangle$,$\langle \delta p_T^2 \rangle$..

Glauber results: N_{ch} dep

ΔR dep	Δa dep	$\Delta\beta_2^2$ dep	$\Delta\beta_3^2$ dep
5.04fm→3.0fm	0.54fm→0.1fm	β_2 0→0.2	β_3 0→0.2

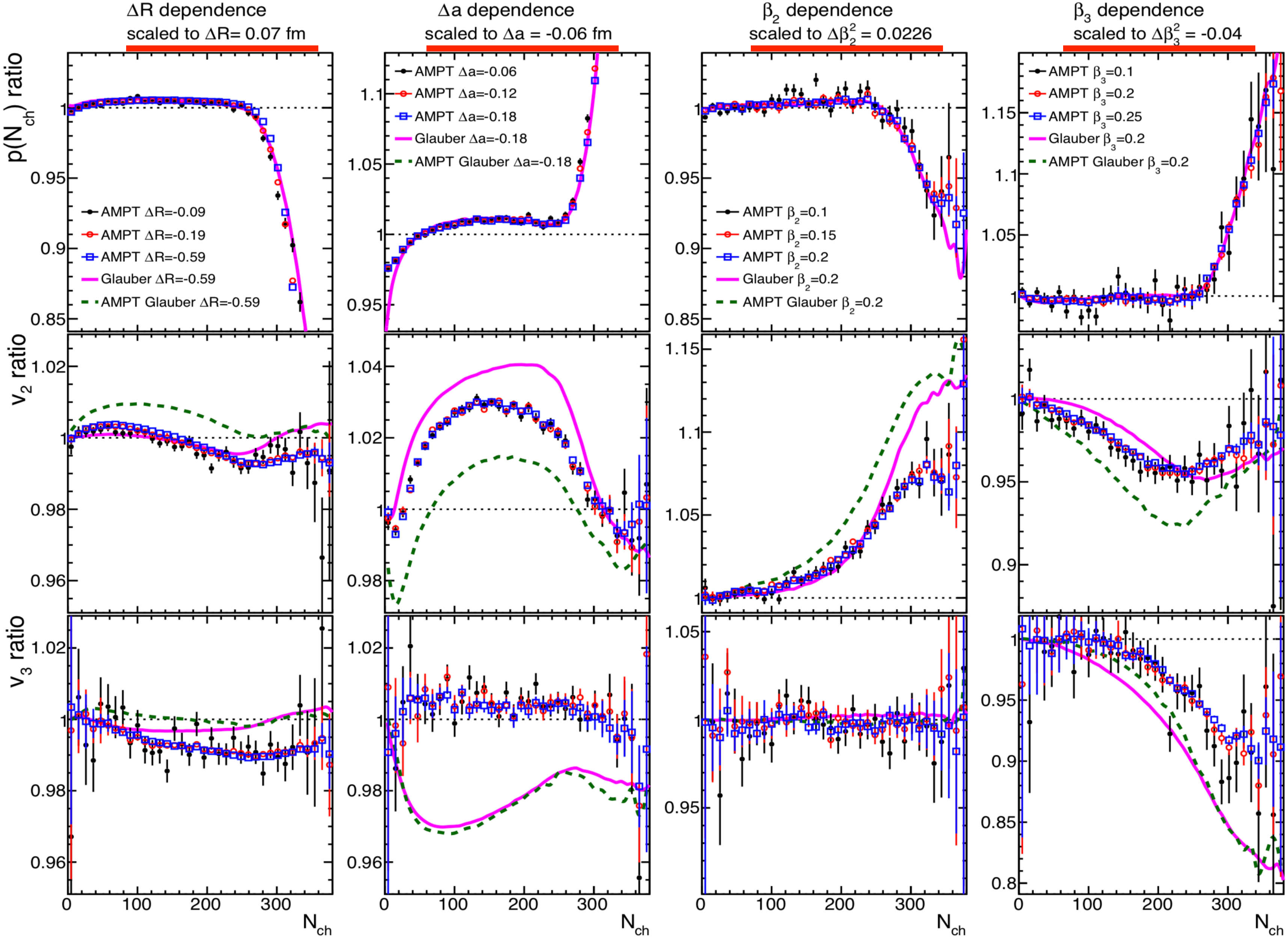


Glauber results: scaled



Verifies the relation: $1 + c_1\Delta\beta_2^2 + c_2\Delta\beta_3^2 + c_3\Delta a + c_4\Delta R$

AMPT results: scaled



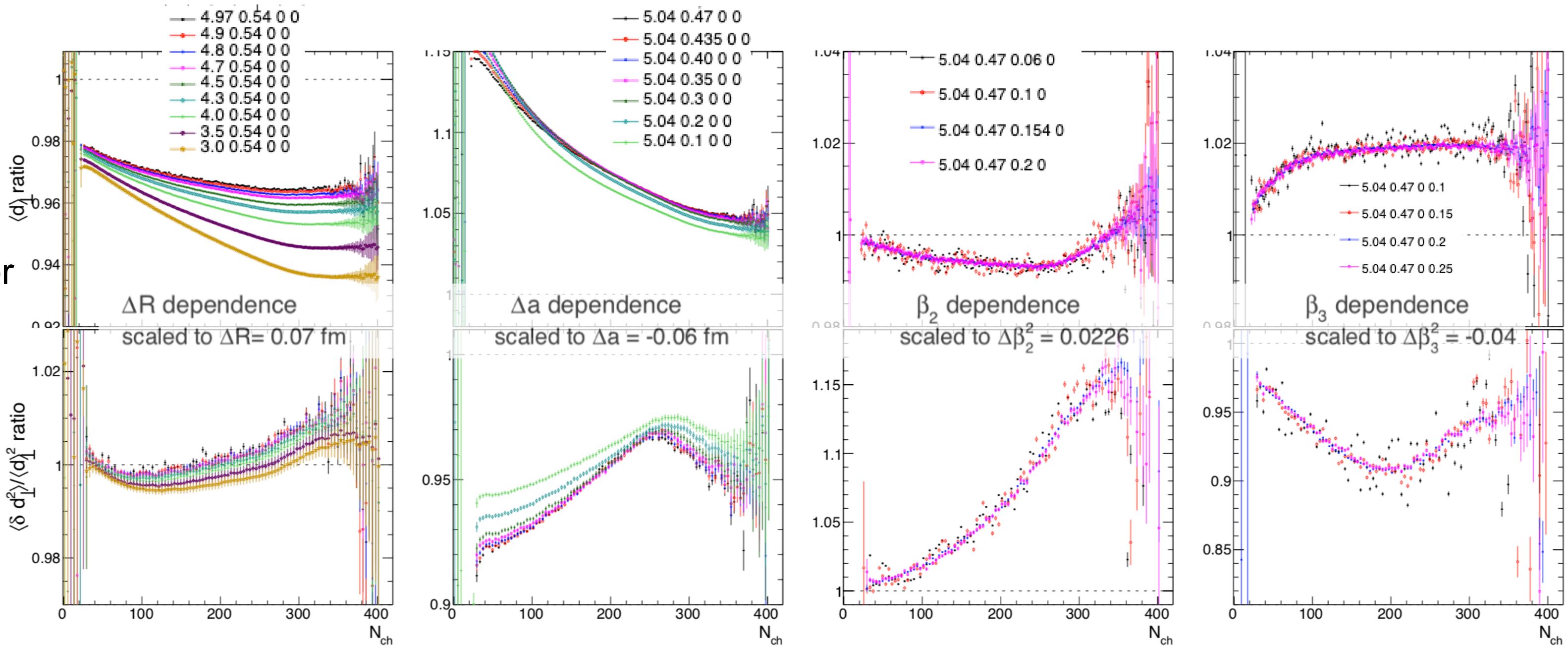
Verifies the relation: $1 + c_1 \Delta\beta_2^2 + c_2 \Delta\beta_3^2 + c_3 \Delta a + c_4 \Delta R$

Scaling approach to nuclear structure

$$\frac{\mathcal{O}_{\rm Ru}}{\mathcal{O}_{\rm Zr}} \approx 1 + c_1\Delta\beta_2^2 + c_2\Delta\beta_3^2 + c_3\Delta R_0 + c_4\Delta a$$

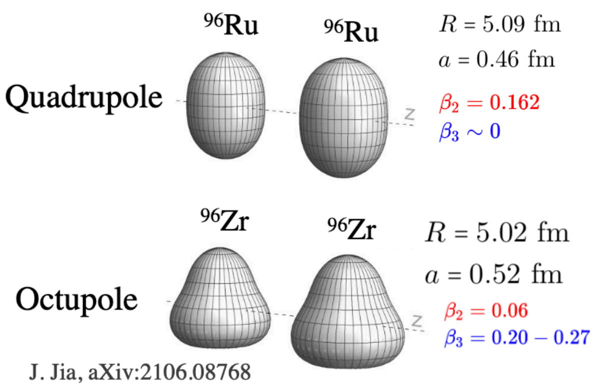
Works well for most single- and two-particle observables:
v2,v3, p(N), but also $\langle p_T \rangle$, $\langle \delta p_T^2 \rangle$…

Departure seen for large ΔR and Δa

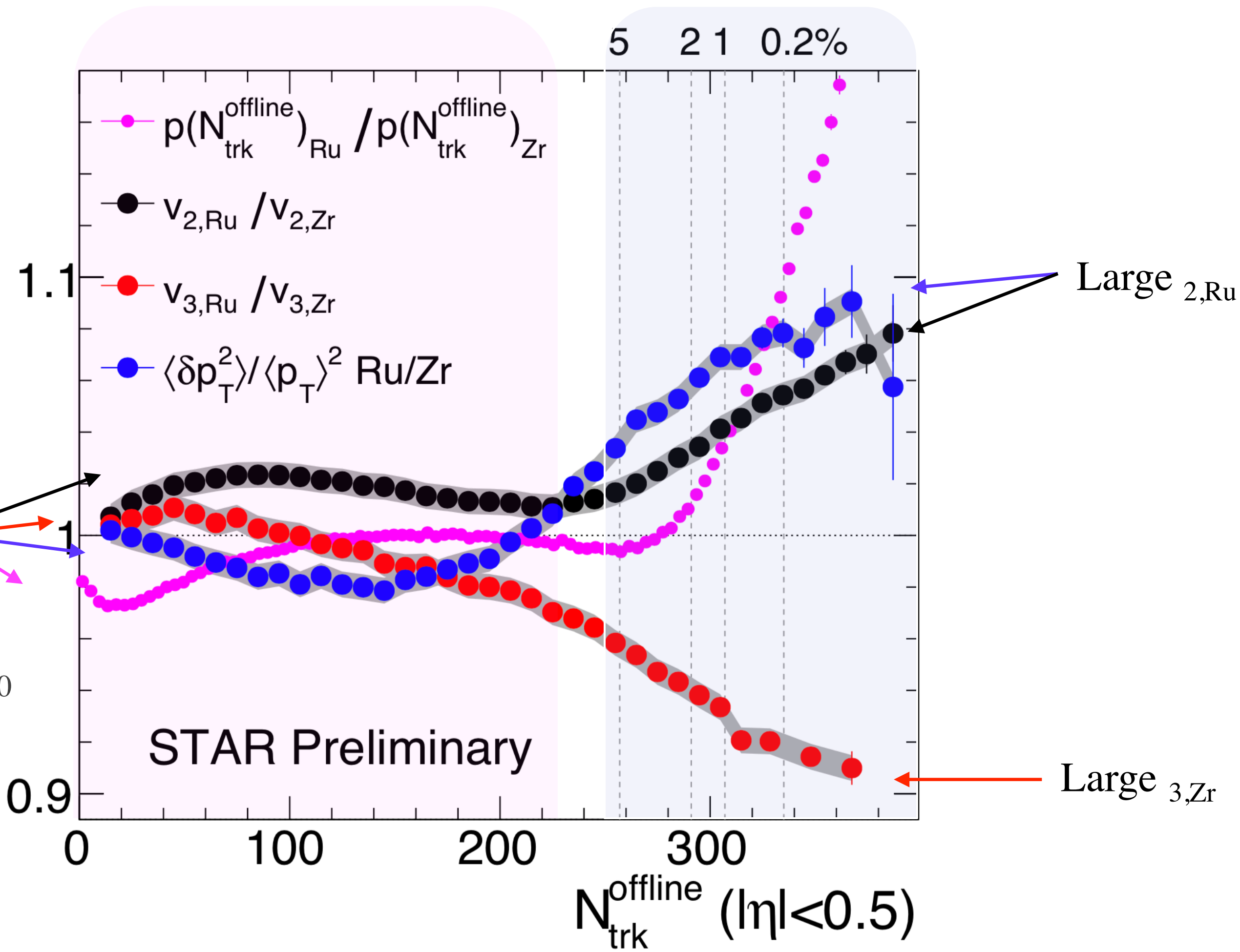


- Determine c_n once, and predict ratios for other parameter values.
- Constrain parameters via χ^2 analysis or Bayesian inference.
- Generalize to multi-particle observables…

Compare with isobar data



from Chunjian Zhang DNP STAR talk



Nuclear deformation
+ Radial structures

STAR, arXiv:2109.00131

H. Xu et al., arXiv:1808.06711,1910.06170
2103.05595, 2111.14812

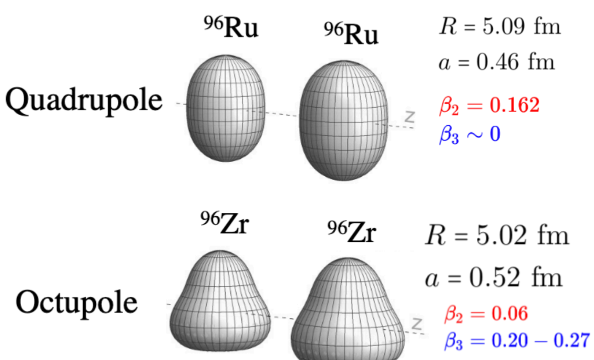
Q. Shou et al., arXiv:1409.8375

C. Zhang et al., arXiv:2109.01631

G. Giacalone et al., 2105.01638

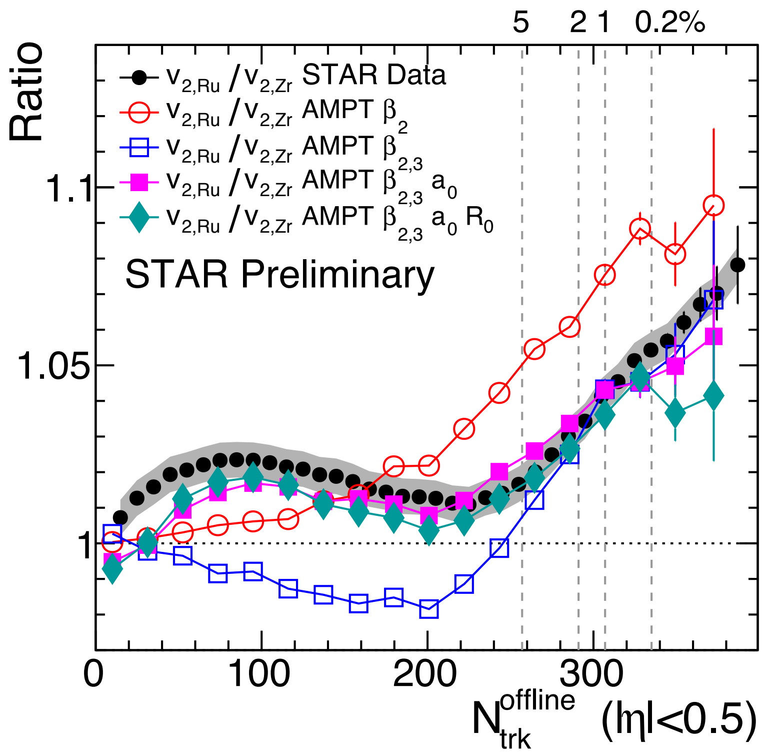
Use these ratios to probe shape and radial structure of nuclei.

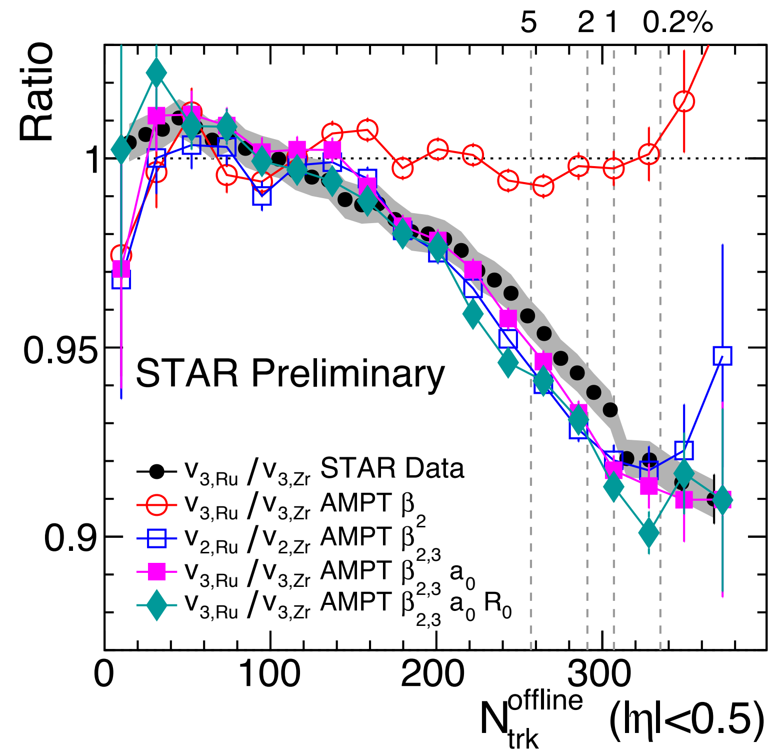
Nuclear structure via v_n-ratio



J. Jia, aXiv:2106.08768

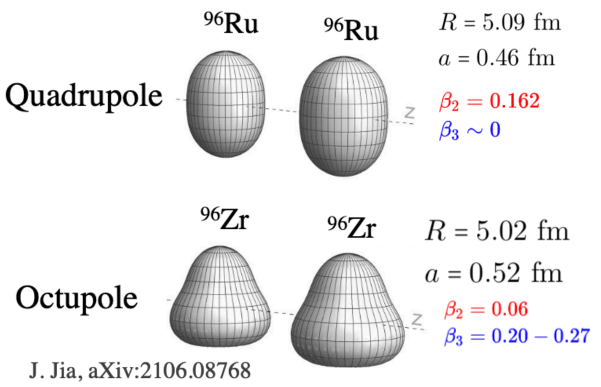
- β_{2Ru}~0.16 increase v_2, no influence on v_3 ratio
- β_{3Zr}~0.2 decrease v_2 in mid-central, decrease v_3 ratio
- diffu. Δa_0=-0.06fm increase v_2 mid-central, no influe. on v_3.
  - Similar study by Haojie et.al.
- Radius ΔR_0=0.07fm only slightly affects v_2 and v_3 ratio.



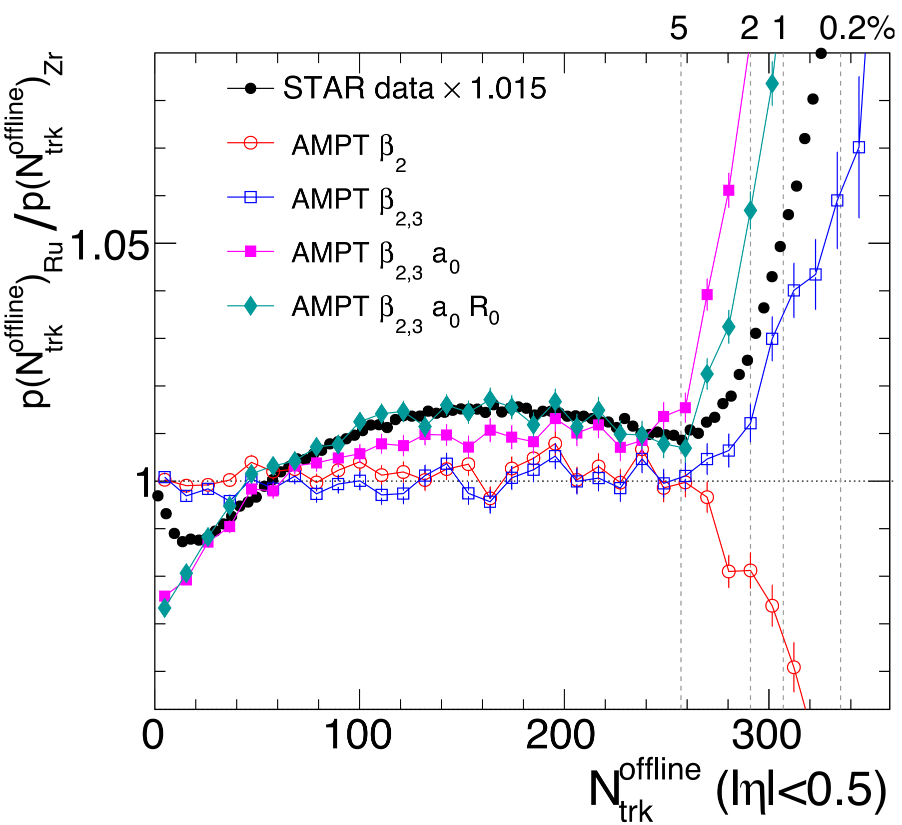


Simultaneously constrain these parameters using different N_{ch} regions

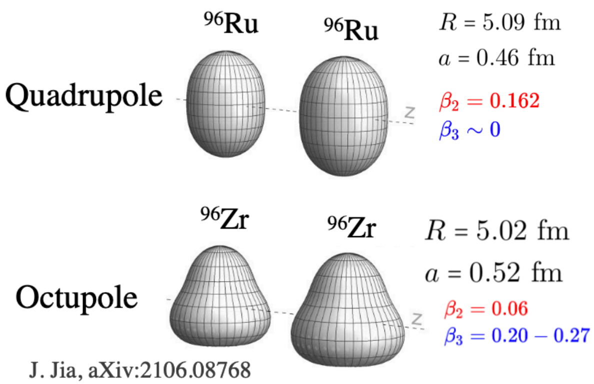
Nuclear structure via p(N_{ch})-ratio



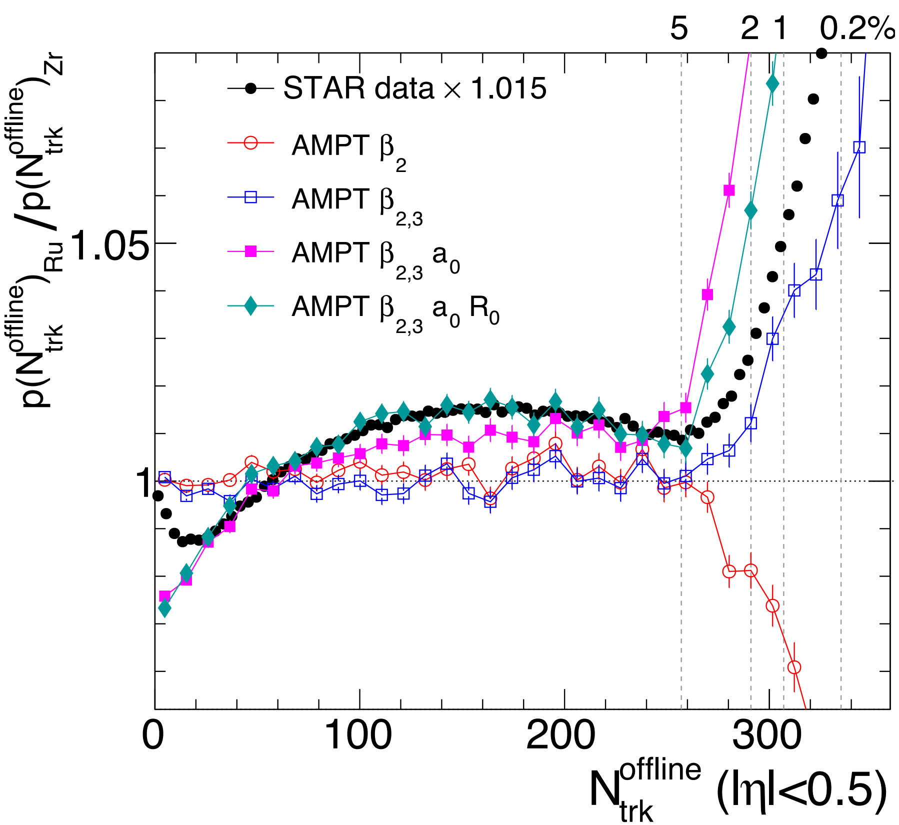
- β_{2Ru}~0.16 decrease ratio, increase after considering β_{3Zr}~0.2
- The bump structure in non-central region mostly sensitive to differences in surface diffuseness Δa_0 and radius ΔR_0

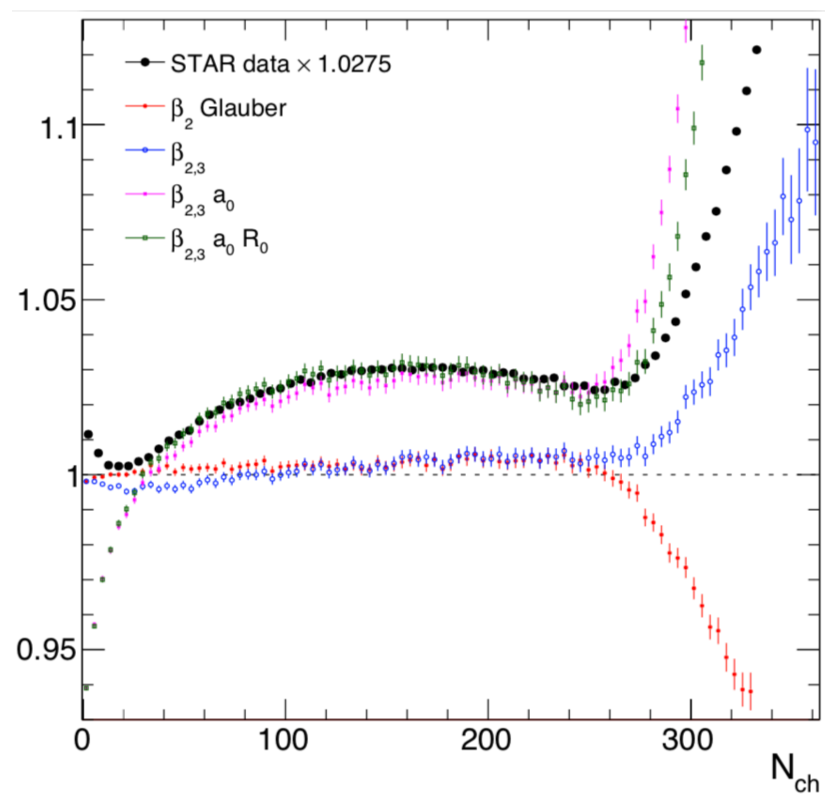


Nuclear structure via p(N_{ch})-ratio

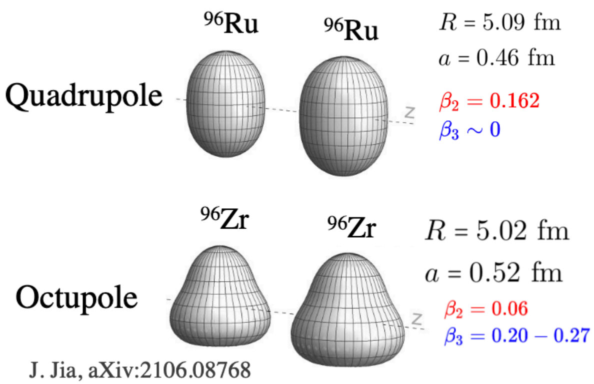


- β_{2Ru}~0.16 decrease ratio, increase after considering β_{3Zr}~0.2
- The bump structure in non-central region mostly sensitive to differences in surface diffuseness Δa_0 and radius ΔR_0
- All these trends are quantitatively reproduced by Glauber



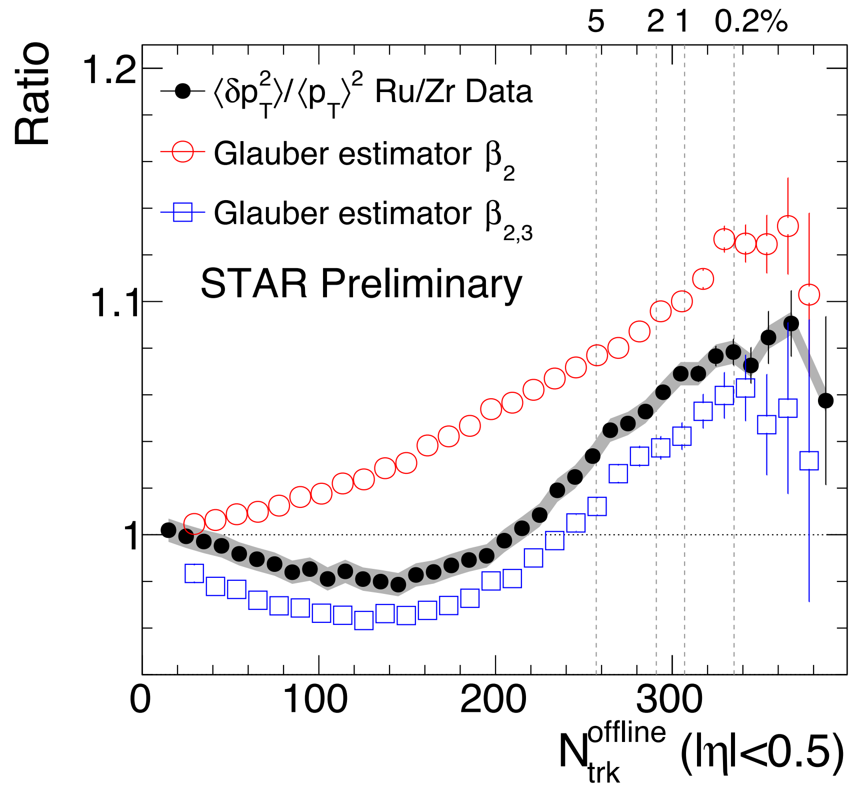


Structure via radial flow: <p_T> & its flucs.

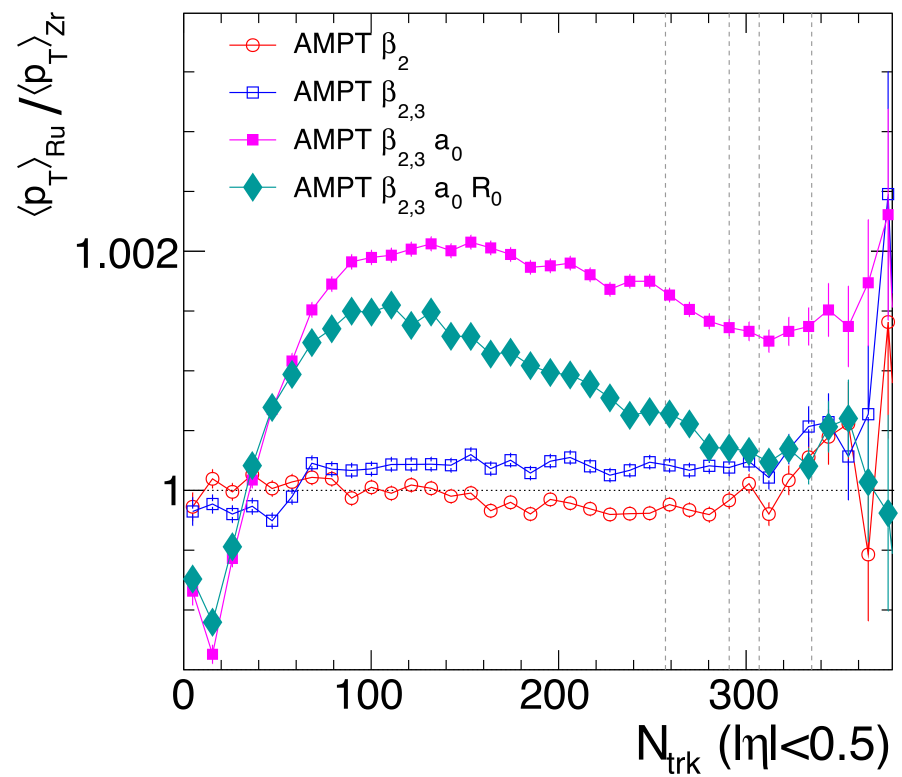


- Glauber model is used by assuming $\frac{\delta[p_T]}{[p_T]} \propto \frac{\delta d_\perp}{d_\perp}$
  - Variance: β_{2Ru}~0.16 increase ratio, β_{3Zr}~0.2 decrease it
- AMPT has wrong <pT> responses, only for qualitative info
  - R_0 and a plays most important role

<pT> variance



<pT> mean

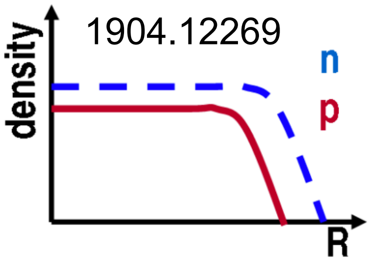


Probing the Neutron Skin

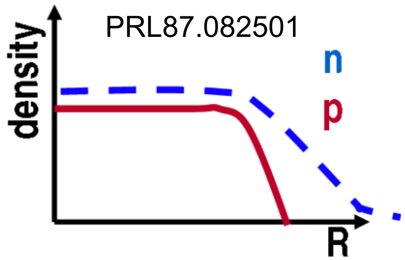
$$\rho = \frac{\rho_0}{1 + e^{-(r-R_0)/a}}$$

Neutron skin:

$$\Delta r_{np} = \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}$$

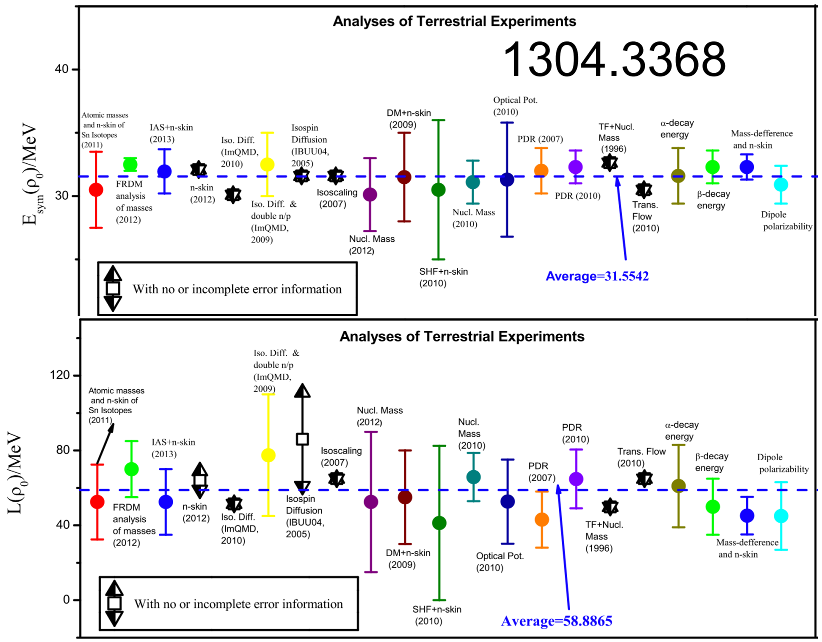


skin-type



halo-type

Constraints from structure and low-energy heavy-ion experiments



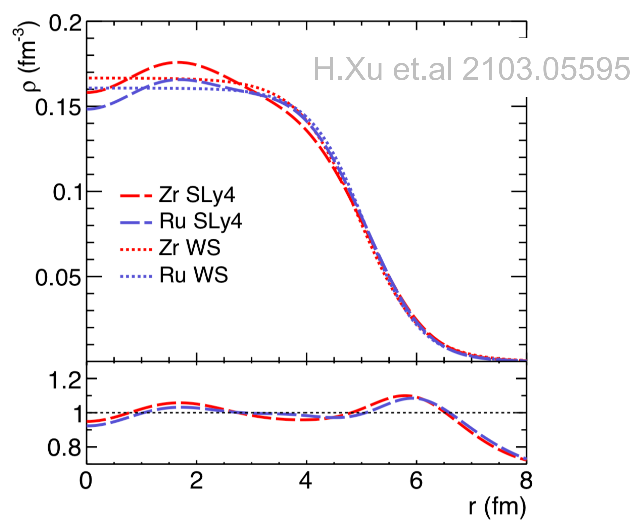
Related to the EOS of symmetry energy,
in particular the slope parameter "**L**"

$$E(\rho,\alpha) \approx E_{\mathrm{SNM}}(\rho) + E_{\mathrm{sym}}(\rho)\cdot\alpha^2 \quad \alpha = (\rho_{\mathrm{n}} - \rho_{\mathrm{p}})/\rho$$

$$E_{\mathrm{sym}}(\rho) \approx J + Lx + \frac{1}{2}K_{\mathrm{sym}}x^2 \qquad x = (\rho - \rho_{\mathrm{sat}})/3\rho_{\mathrm{sat}}$$

Fundamental importance in nuclear and astro-physics

Distribution has more sophisticated form,
but typically parameterized with WS



Hydro-response to Neutron skin

Sensitive to L parameter via hydro response: $\langle p_\perp \rangle \sim d_\perp \equiv \sqrt{N_{\rm part} / S_\perp}$

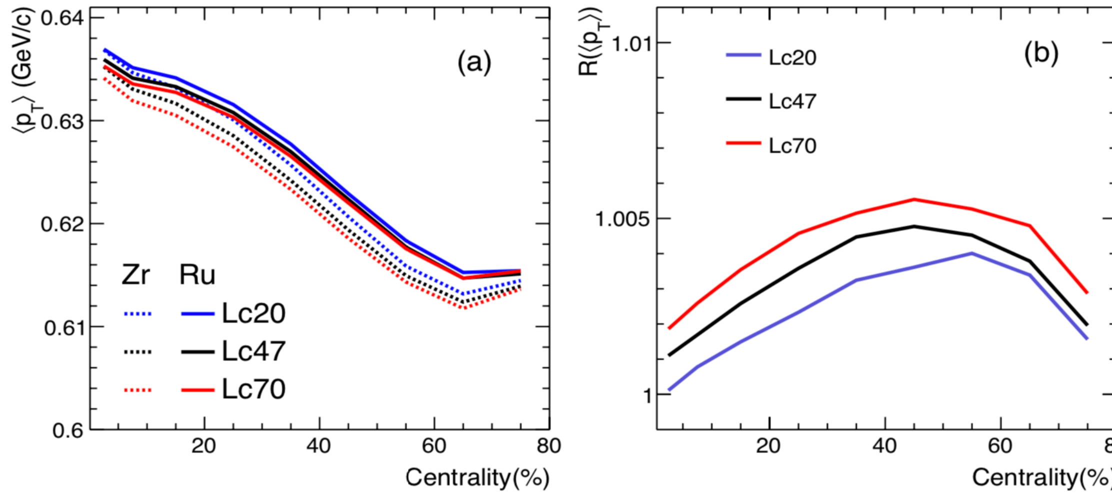
H. Xu et.al.2111.14812

$$E(\rho, \alpha) \approx E_{\rm SNM}(\rho) + E_{\rm sym}(\rho) \cdot \alpha^2$$

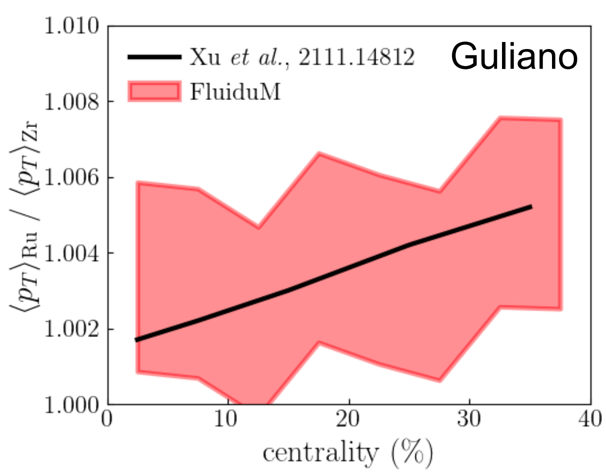
$$E_{\rm sym}(\rho) \approx J + Lx + \frac{1}{2} K_{\rm sym} x^2$$

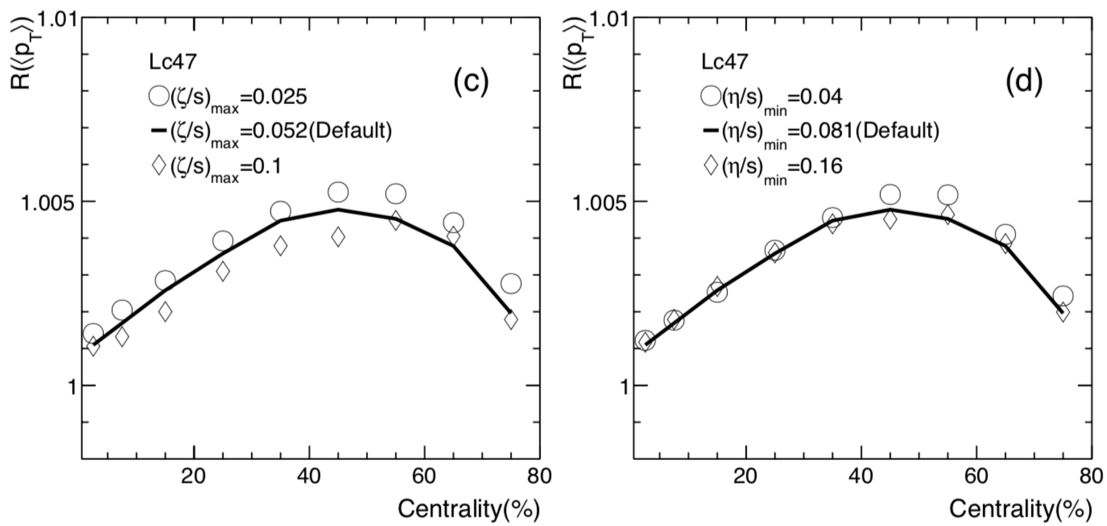
$$\alpha = (\rho_{\rm n} - \rho_{\rm p})/\rho$$

$$x = (\rho - \rho_{\rm sat})/3\rho_{\rm sat}$$



Insensitive to final state effects, direct probe of the initial state:

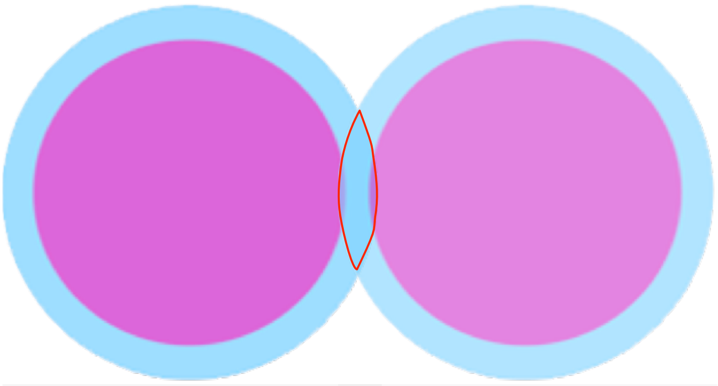




Skin by peripheral isobaric collision

Enhanced skin contribution in the collisions reduces net charge

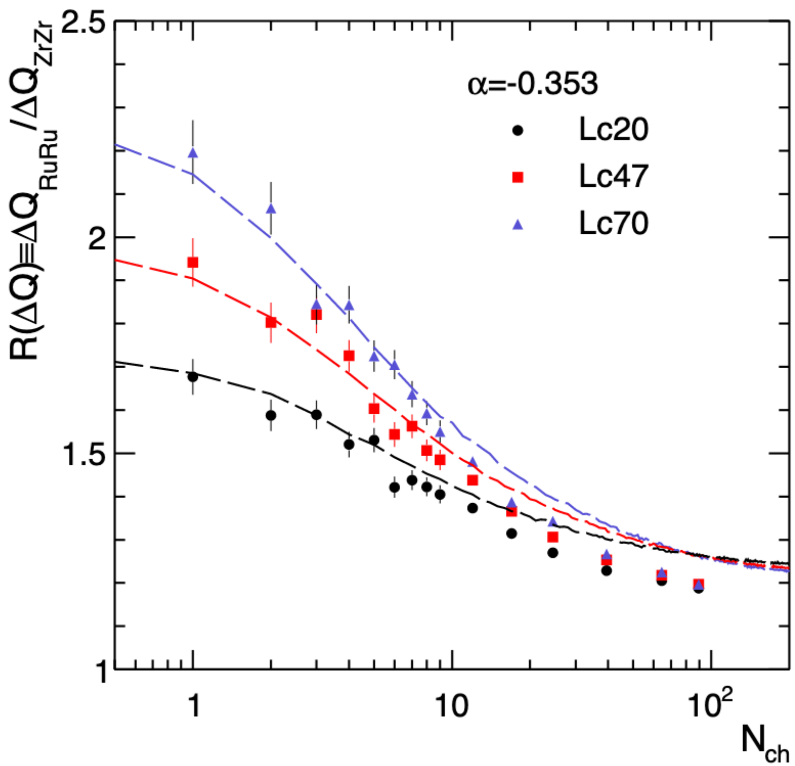
H. Xu et.al. 2105.04052



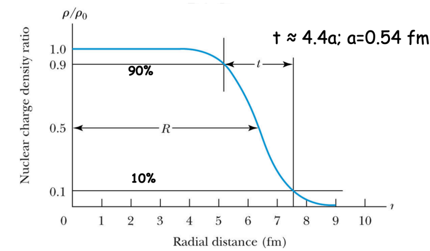
$$R_{\Delta Q} \equiv \frac{\Delta Q_{\mathrm{RuRu}}}{\Delta Q_{\mathrm{ZrZr}}} = \frac{q_{\mathrm{RuRu}} + \alpha/(1-\alpha)}{q_{\mathrm{ZrZr}} + \alpha/(1-\alpha)}$$

$\alpha \simeq -0.352$ is net-Q ratio in nn vs pp collisions

Large neutron skin of 96Zr is sensitive to L parameter



A direct link to Neutron Skin



Using relation for WS: $R^2 \equiv \langle r^2 \rangle \approx \left(\frac{3}{5}R_0^2 + \frac{7}{5}\pi^2 a^2\right) / \left(1 + \frac{5}{4\pi^2}\sum_n \beta_n^2\right)$

Neutron skin expressed by R and a parameters for nucleons and protons:

$$\Delta r_{np} \approx \frac{R^2 - R_p^2}{R(\delta+1)} \approx \frac{3(R_0^2 - R_{0,p}^2) + 7\pi^2(a^2 - a_p^2)}{\sqrt{15}R_0\sqrt{1 + \frac{7\pi^2}{3}\frac{a^2}{R_0^2}}(1 + \delta + \frac{5}{8\pi^2}\sum_n \beta_n^2)} \qquad \delta = (N - Z)/A$$

The difference between two isobar can be expressed as

$$\Delta(\Delta r_{np}) = \Delta r_{np,1} - \Delta r_{np,2} \approx \frac{\Delta Y - \frac{7\pi^2}{3}\frac{\bar{a}^2}{\bar{R}_0^2}\left(\frac{\Delta Y}{2} + \bar{Y}\left(\frac{\Delta a}{\bar{a}} - \frac{\Delta R_0}{\bar{R}_0}\right)\right)}{\sqrt{15}\bar{R}_0(1 + \bar{\delta} + \frac{5}{8\pi^2}\sum_n \overline{\beta_n^2})}$$

where $Y \equiv 3(R_0^2 - R_{0,p}^2) + 7\pi^2(a^2 - a_p^2) \qquad \Delta x = x_1 - x_2 \quad \bar{x} = (x_1 + x_2)/2$

Can obtain skin diff. from ΔR_0 Δa for nucleons and known ΔR_0 Δa for protons

Example: 2103.05595

	^{96}Ru		^{96}Zr	
	R	a	R	a
p	5.060	0.493	4.915	0.521
n	5.075	0.505	5.015	0.574
p+n	5.067	0.500	4.965	0.556

Direct calc.: $\Delta(\Delta r_{np})$=0.0296 fm – 0.1606fm = -0.1310fm

Formula: $\Delta(\Delta r_{np})$= -0.1319 fm

How to do system (isobar) scan

arXiv:2106.08768

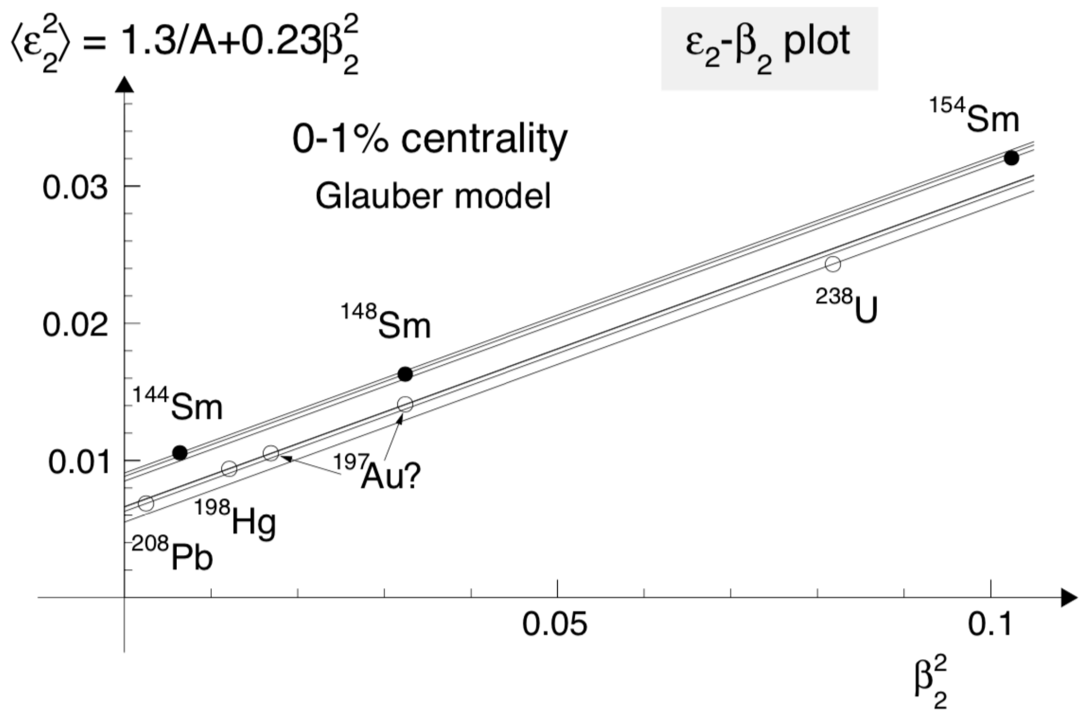
$$\left\langle \epsilon_2^2 \right\rangle = a' + b' \beta_2^2$$

$$\left\langle v_2^2 \right\rangle = a + b\beta_2^2$$

In central collisions

$$a' = \langle \varepsilon_2^2 \rangle_{|\beta_2=0} \propto 1/A$$

$$a = \langle v_2^2 \rangle_{|\beta_2=0} \propto 1/A$$



b', b are ~ independent of system

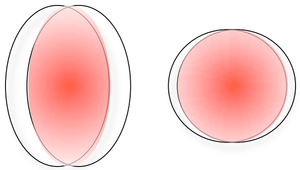
Systems with similar A fall on the same curve.

Fix a and b with two isobar systems with known β_n, then make predictions for third one

Similar approach also works for R_0 and diffusivity.

Application in ^{197}Au+^{197}Au vs ^{238}U+^{238}U

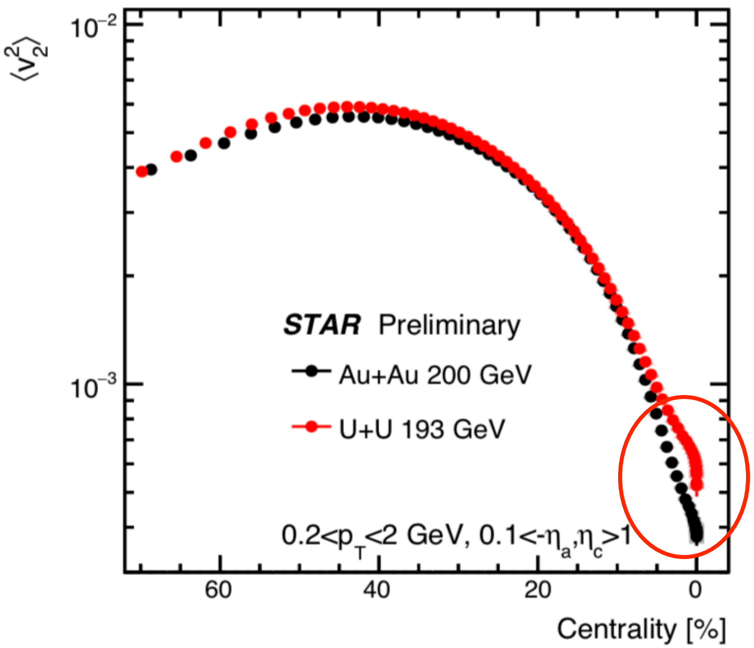
Collisions at $\sqrt{s_{NN}}$=193-200 GeV See:arXiv:2105.01638

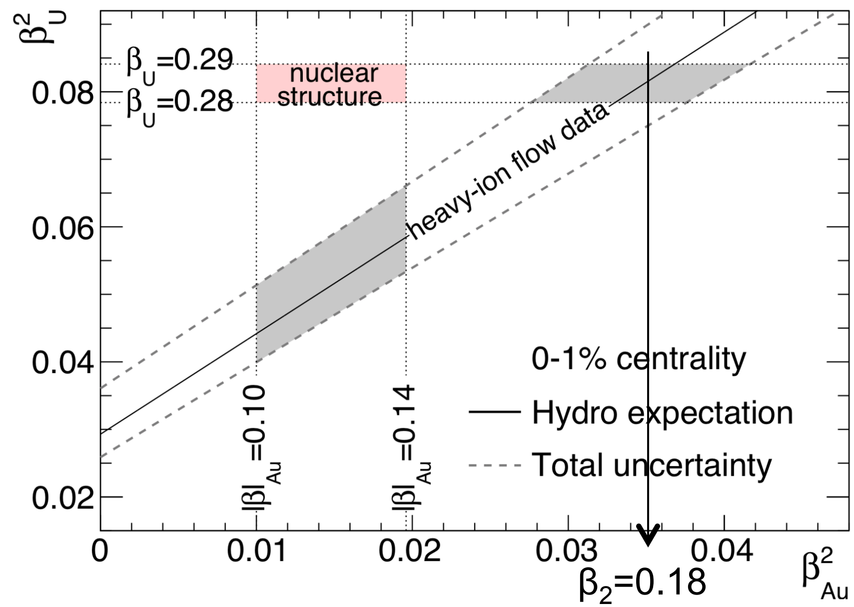


$$v_{2,\mathrm{Au}}^2 = a_{\mathrm{Au}} + b\beta_2^2 \qquad v_{2,\mathrm{U}}^2 = a_{\mathrm{U}} + b\beta_2^2$$

Need to correct for slightly different size: $a \propto 1/A\,,\quad r_a = \frac{a_{\mathrm{Au}}}{a_{\mathrm{U}}} = \frac{238}{197} = 1.21$

A simple relation for β_{2U} and β_{2Au}: $\beta_{\mathrm{U}}^2 = \frac{r_{v_2^2} r_a - 1}{b/a_{\mathrm{U}}} + r_{v_2^2}\beta_{\mathrm{Au}}^2 \quad r_{v_2^2} = \frac{v_{2,\mathrm{U}}^2}{v_{2,\mathrm{Au}}^2}$





Suggests $|\beta_2|_{Au}$~0.18+-0.02, larger than NS model of 0.13+-0.02

Note: 197Au is a odd-mass nuclei, β_2 not measured.

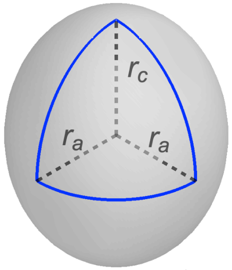
Higher-order correlations

A wide range of possibilities, focus only one topic…

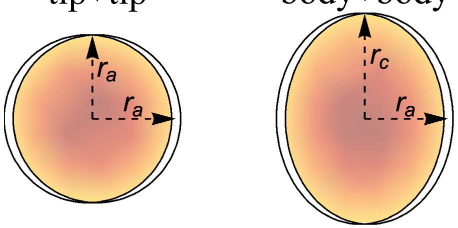
Triaxiality γ: $R(\theta,\phi) = R_0\left(1 + \beta_2[\cos\gamma Y_{2,0} + \sin\gamma Y_{2,2}]\right.$

Prolate

$\beta_2 = 0.25, \cos(3\gamma) = 1$

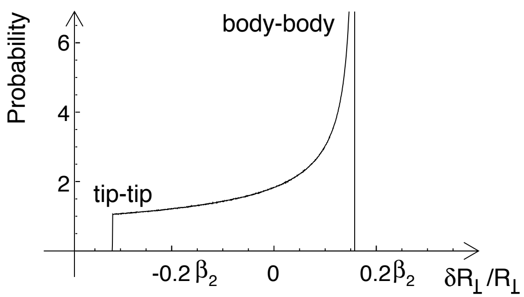


tip+tip body+body



$\epsilon_2 \downarrow, R_\perp \downarrow$ $\epsilon_2 \uparrow, R_\perp \uparrow$

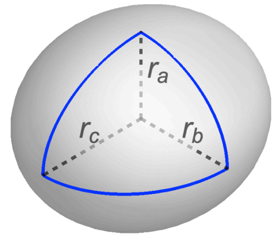
$\mathrm{cov}\left(\epsilon_2^2, R_\perp\right) > 0$

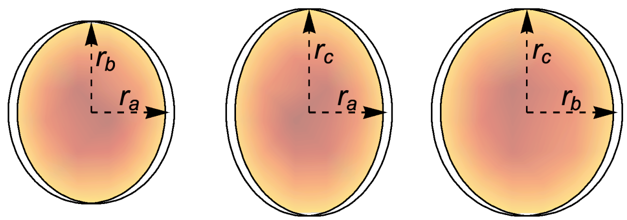


$\left\langle (\delta R_\perp)^3 \right\rangle < 0$

Triaxial

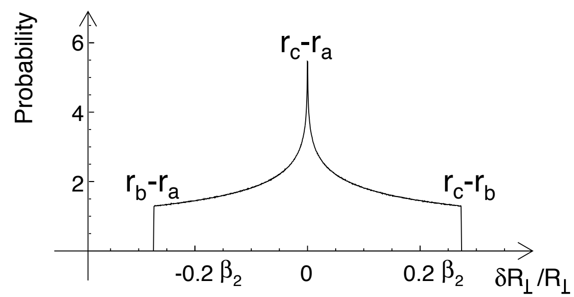
$\beta_2 = 0.25, \cos(3\gamma) = 0$





$\epsilon_2, R_\perp$ no linear correlation

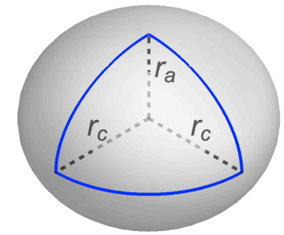
$\mathrm{cov}\left(\epsilon_2^2, R_\perp\right) = 0$



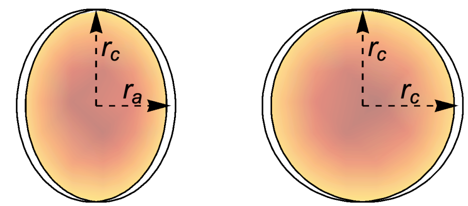
$\left\langle (\delta R_\perp)^3 \right\rangle = 0$

Oblate

$\beta_2 = 0.25, \cos(3\gamma) = -1$

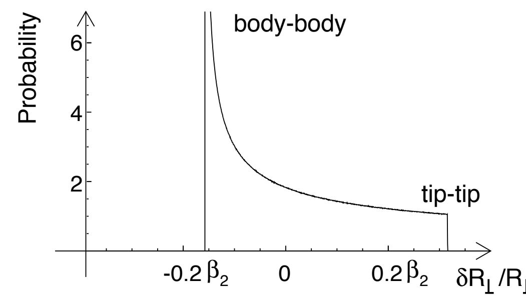


body+body tip+tip



$\epsilon_2 \uparrow, R_\perp \downarrow$ $\epsilon_2 \downarrow, R_\perp \uparrow$

$\mathrm{cov}\left(\epsilon_2^2, R_\perp\right) < 0$



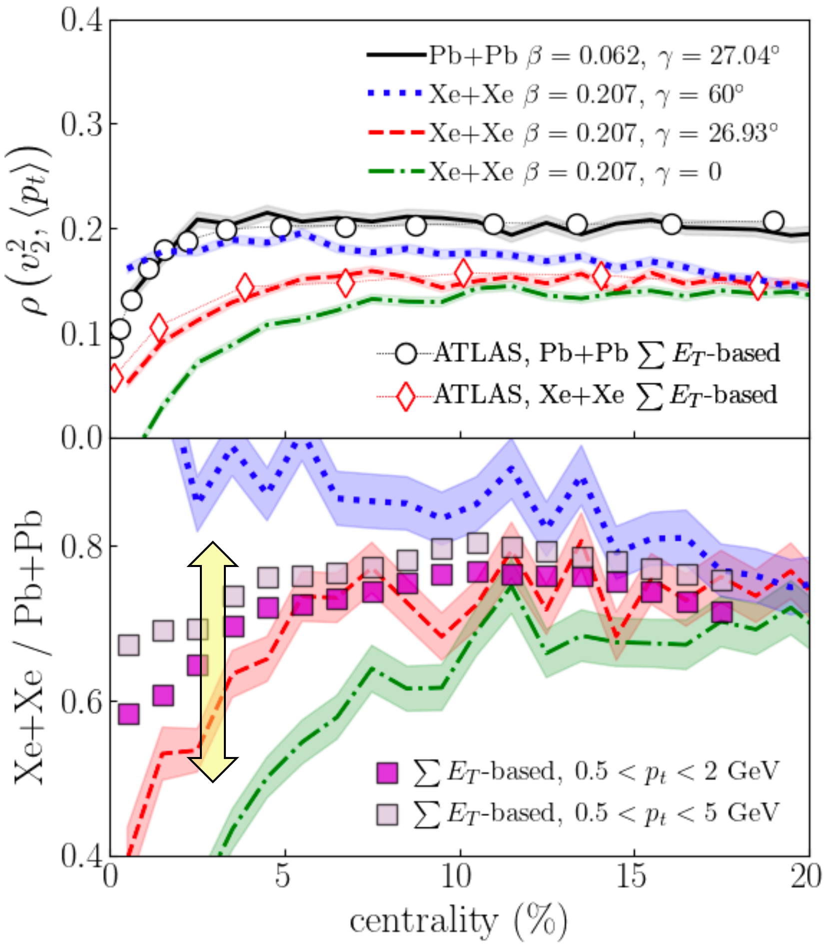
$\left\langle (\delta R_\perp)^3 \right\rangle > 0$

$\mathrm{cov}\left(v_2^2, p_\mathrm{T}\right) < 0$ $\quad [p_T] \sim 1/R_\perp = d_\perp \quad$ $\mathrm{cov}\left(v_2^2, p_\mathrm{T}\right) = 0$ $\quad$ $\mathrm{cov}\left(v_2^2, p_\mathrm{T}\right) > 0$

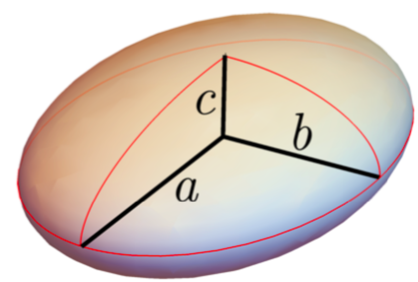
$\left\langle (\delta p_\mathrm{T})^3 \right\rangle > 0$ $\quad$ $\left\langle (\delta p_\mathrm{T})^3 \right\rangle = 0$ $\quad$ $\left\langle (\delta p_\mathrm{T})^3 \right\rangle < 0$

v_2^2-p_T correlation at LHC

B Bally, M Bender, G Giacalone, V Somà 2108.09578



$$R(\theta,\phi) = R_0\left(1 + \beta_2[\cos\gamma Y_{2,0} + \sin\gamma Y_{2,2}]\right.$$



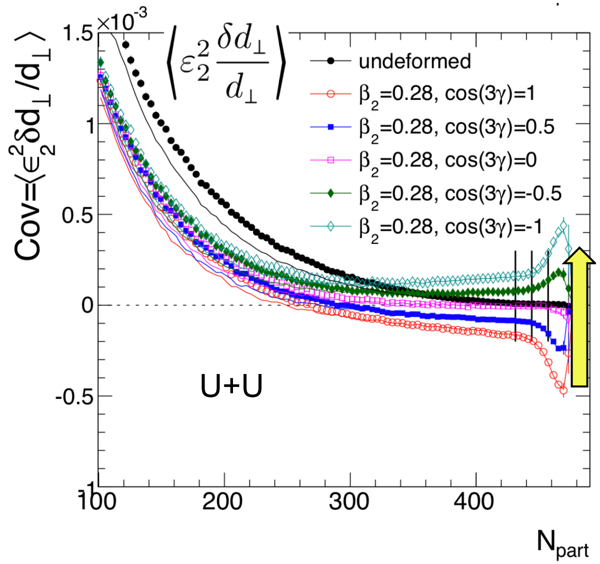
- Clear sensitivity to the triaxiality of 129Xe.

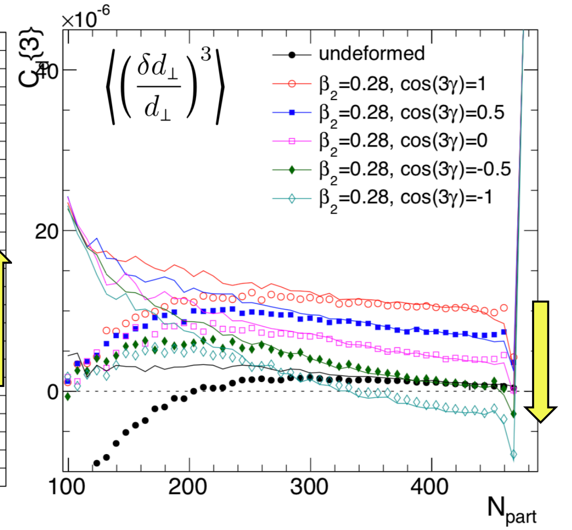
Influence of triaxiality on initial state

Skewness super sensitive ☺

Described by

$$a' + (b' + c' \cos(3\gamma))\beta_2^3$$

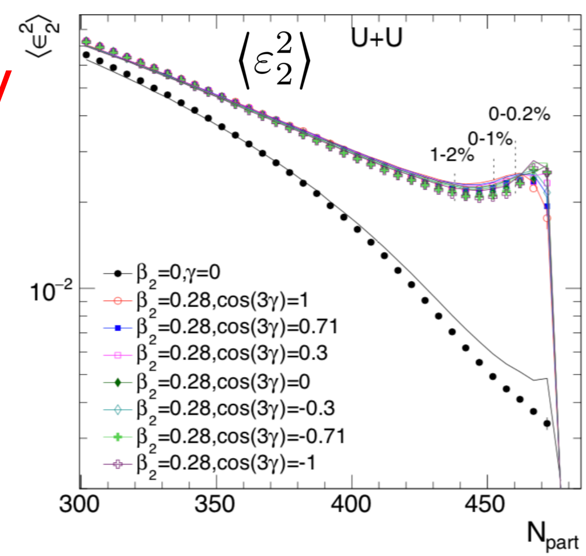


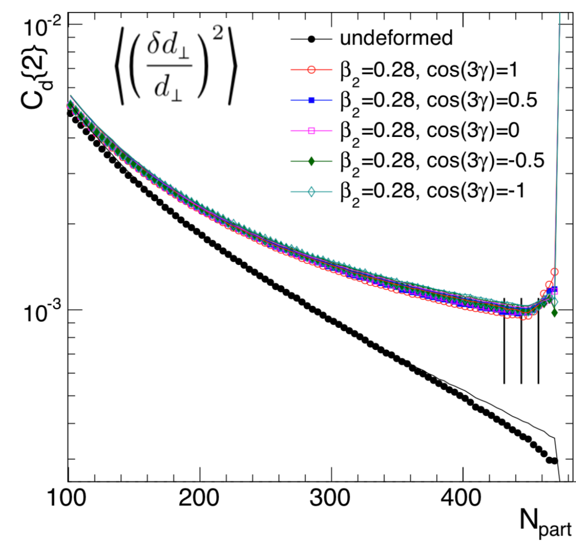


variances insensitive to γ

Only a function of β_2 in central

$$a' + b'\beta_2^2$$





Use variance to constrain β_2, use skewness to constrain γ

(β_2,γ) diagram in heavy-ion collisions

The (β_2,γ) dependence in 0-1% U+U Glauber model can be approximated by:

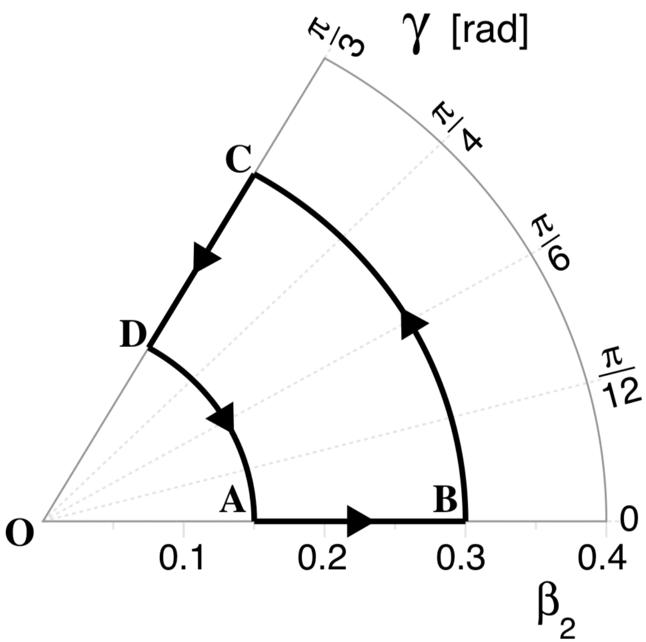
$$\langle \varepsilon_2^2 \rangle \approx [0.02 + \beta_2^2] \times 0.235$$

$$\langle (\delta d_\perp / d_\perp)^2 \rangle \approx [0.035 + \beta_2^2] \times 0.0093$$

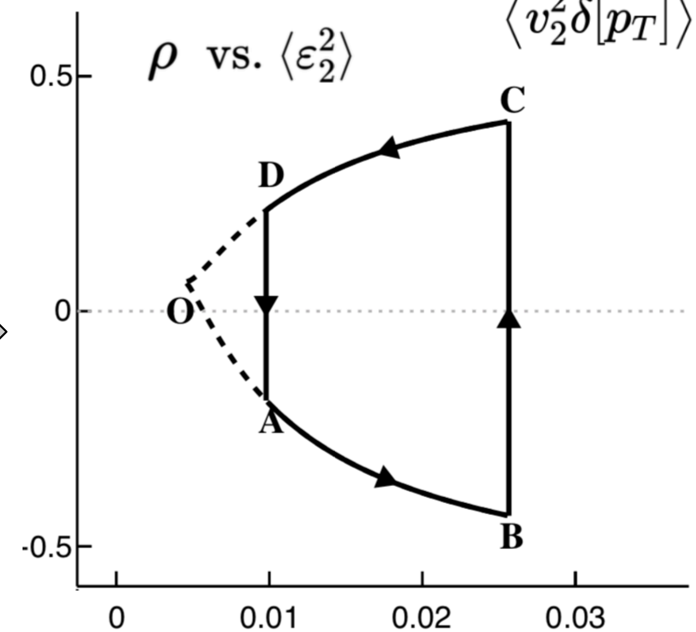
$$\langle \varepsilon_2^2 \delta d_\perp / d_\perp \rangle \approx [0.0005 - (0.07 + 1.36\cos(3\gamma))\beta_2^3] \times 10^{-2}$$

$$\rho = \frac{\langle \varepsilon_2^2 \delta d_\perp \rangle}{\langle \varepsilon_2^2 \rangle \sqrt{\langle (\delta d_\perp)^2 \rangle}}$$

Map from (β_2,γ) plane to HI observables



?

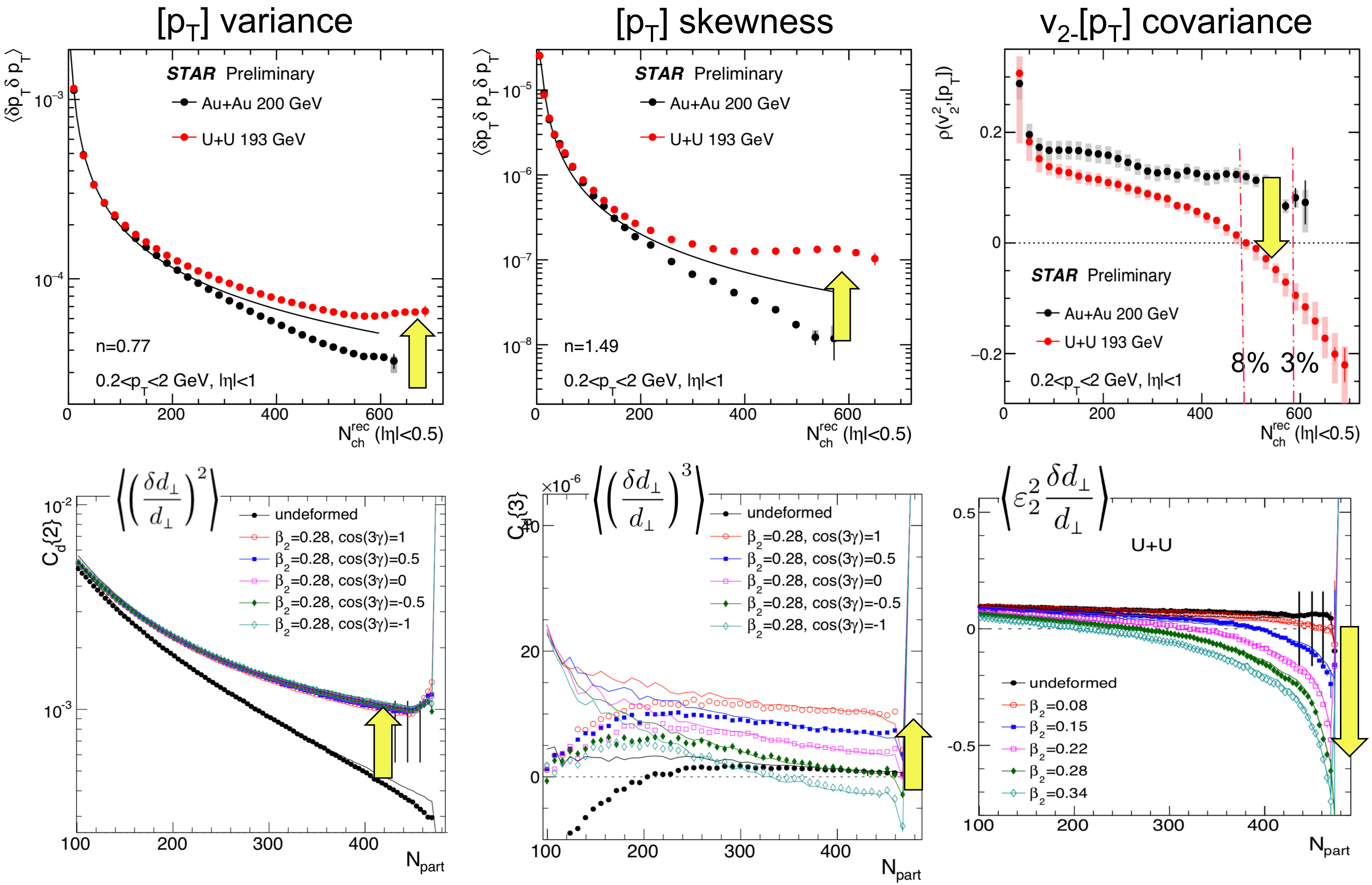


How about

$\langle v_2^2 \delta[p_T] \rangle$ vs. $\langle v_2^2 \rangle$

Collision system scan to map out this trajectory: calib. coefficients with species with known β,γ, then predict for species of interest.

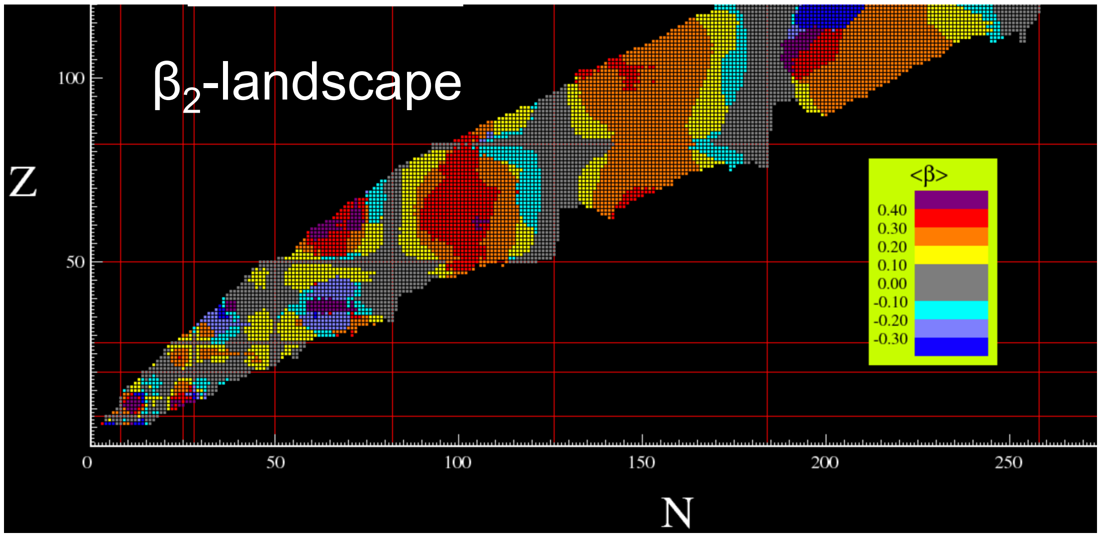
Contrast Glauber model with STAR data

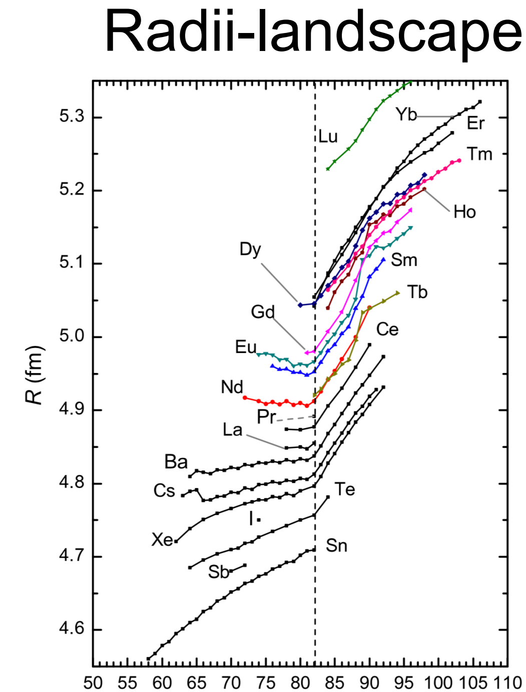


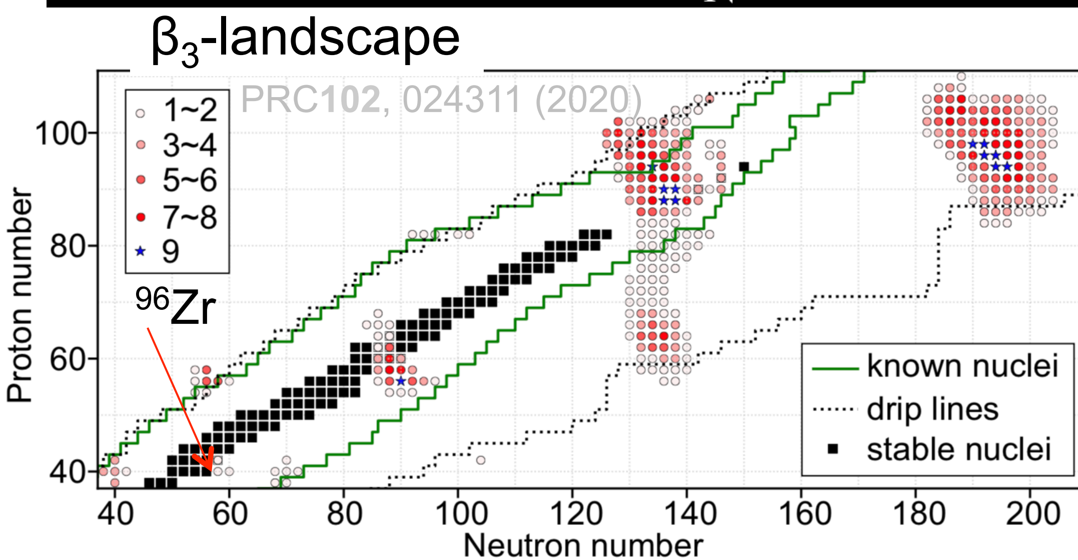
Require high-stat. hydro simulation to quantify the response!

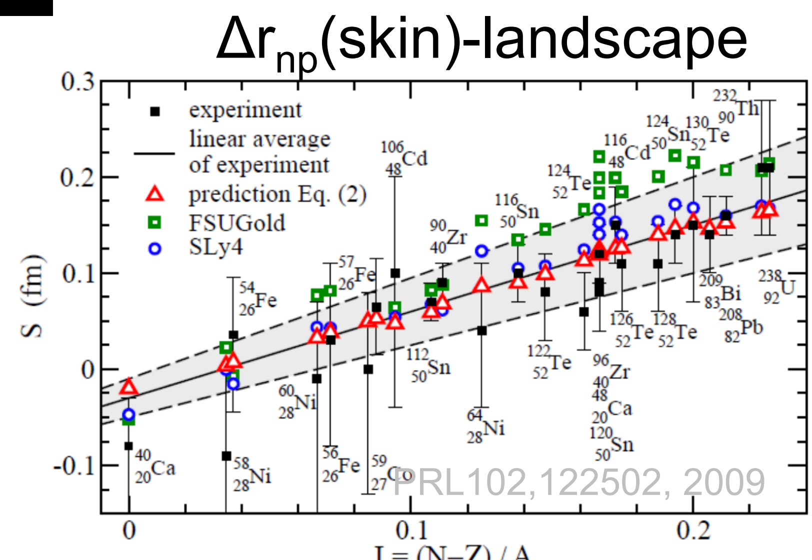
nuclear structure shape/size landscapes

A lot of possibilities, need to identify most impactful ones!









Thoughts on scan strategy

- Even-even nuclei (0^+ ground state) can be compared with Nuclear Structure exp.
  - We collided several odd-mass ones ☹

A list of large systems from RHIC and LHC

	β_2	β_3	β_4		β_2	β_3	β_4
^{238}U	0.286 [9]	0.078 [10]	0.094 [10]	^{208}Pb	0.06 [9]	0.04[11]	?
^{197}Au	-(0.13-0.16) [12, 13]	?	-0.03 [12]	^{129}Xe	0.16 [12]	?	?
^{96}Ru	0.16 [14]	?	?	^{96}Zr	0.06 [14]	0.20-0.27	0.06 [12]

- A partial list of isobar pairs
  - Mapping the nuclear structure within each isobar pair.
  - Control system-size dependence hydro-response by stepping between different isobar pairs
  - How to optimize the choices?

2102.08158

A	isobars	A	isobars	A	isobars
36	Ar, S	106	Pd, Cd	148	Nd, Sm
40	Ca, Ar	108	Pd, Cd	150	Nd, Sm
46	Ca, Ti	110	Pd, Cd	152	Sm, Gd
48	Ca, Ti	112	Cd, Sn	154	Sm, Gd
50	Ti, V, Cr	113	Cd, In	156	Gd, Dy
54	Cr, Fe	114	Cd, Sn	158	Gd, Dy
64	Ni, Zn	115	In, Sn	160	Gd, Dy
70	Zn, Ge	116	Cd, Sn	162	Dy, Er
74	Ge, Se	120	Sn, Te	164	Dy, Er
76	Ge, Se	122	Sn, Te	168	Er, Yb
78	Se, Kr	123	Sb, Te	170	Er, Yb
80	Se, Kr	124	Sn, Te, Xe	174	Yb, Hf
84	Kr, Sr, Mo	126	Te, Xe	176	Yb, Lu, Hf
86	Kr, Sr	128	Te, Xe	180	Hf, W
87	Rb, Sr	130	Te, Xe, Ba	184	W, Os
92	Zr, Nb, Mo	132	Xe, Ba	186	W, Os
94	Zr, Mo	134	Xe, Ba	187	Re, Os
96	Zr, Mo, Ru	136	Xe, Ba, Ce	190	Os, Pt
98	Mo, Ru	138	Ba, La, Ce	192	Os, Pt
100	Mo, Ru	142	Ce, Nd	198	Pt, Hg
102	Ru, Pd	144	Nd, Sm	204	Hg, Pb
104	Ru, Pd	146	Nd, Sm		

Future Opportunities

- STAR proposed to explore such opportunities in the next few years
  - Part1: calibrate systematics with two species around ^{197}Au: ^{208}Pb & ^{198}Hg
    - ^{208}Pb $\sqrt{s}$=0.2 RHIC vs 5 TeV @LHC: Precision on IS and pre-equlibrium dynamics
    - ^{208}Pb $\sqrt{s}$=0.2 vs ^{197}Au $\sqrt{s}$=0.2 TeV: Quantify effects of Au deformation
    - ^{198}Hg $\sqrt{s}$=0.2 TeV: with known β_2 cross-check the consistency of β_{2Au}, γ in ^{197}Au.

5 days each opportunistically

  - Part2: explore more exotic regions for triaxial and octupole deformations
    - Scan a isotopic chain: ^{144}Sm (β_2=0.08),^{148}Sm (β_2=0.14,triaxial),^{154}Sm (β_2=0.34)
    - Compare a pair with equal mass: ^{154}Sm (β_2 = 0.34) and ^{154}Gd (β_2 = 0.31)

potentially for 2026 if running

But priority is sPHENIX, very limited possibility at RHIC.

- Maybe at LHC beyond 2030 (RUN5)? See CERN yellow report

Table 4: Parameters and performance for a range of light nuclei with a moderately optimistic value of the scaling parameter $p = 1.5$ in (5).

	$^{16}O^{8+}$	$^{40}Ar^{18+}$	$^{40}Ca^{20+}$	$^{78}Kr^{36+}$	$^{129}Xe^{54+}$	$^{208}Pb^{82+}$
γ	3760.	3390.	3760.	3470.	3150.	2960.
$\sqrt{s_{NN}}$/TeV	7.	6.3	7.	6.46	5.86	5.52

- Other heavy ion facilities such as NICA? $\sqrt{s_{NN}}$ up to 11 GeV
  - Measure the same isobar pair at different energies.

Manifestation of nuclear structure are $\sqrt{s}$ and rapidity dependent!

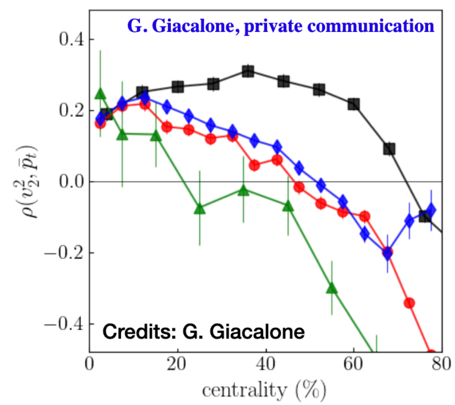
Energy dependence of nucleon size

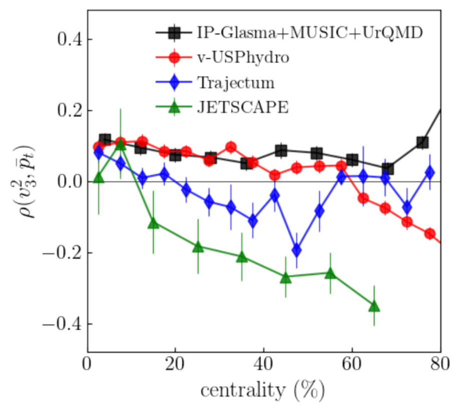
❖ Sensitive to the nucleon width parameter (size of nucleon)

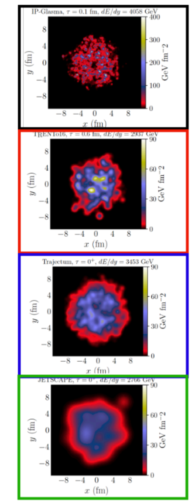
- IP-Glasma ~ 0.3; v-USPhydro ~ 0.5; Trajectum~0.7; JETSCAPE (TRENTo) ~ 1.1
- w(IP-Glasma) < w(v-USPhydro)<w(Trajectum) < w(JETSCAPE)

Slide from You Zhou

v_n^2-p_T has strong sensitivity to nucleon width

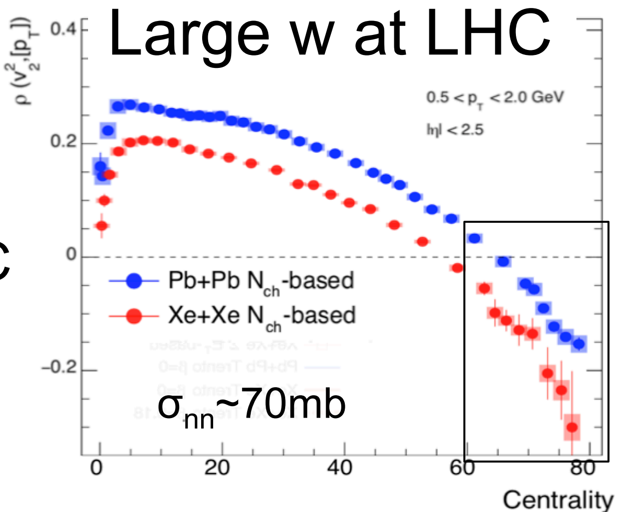


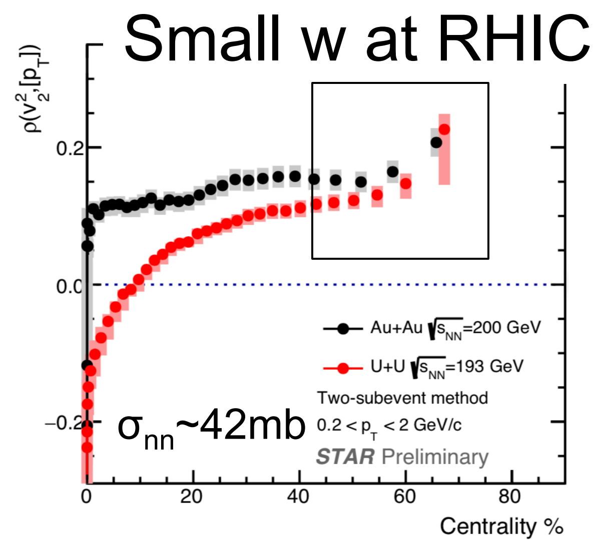




Nucleon size have strong $\sqrt{s_{nn}}$ dependence

Very different peripheral behavior at RHIC vs LHC





Questions

- How are nuclear shape and radial profile inferred from hydrodynamic response related to properties measured in nuclear structure experiments?
- How does the uncertainty brought by nuclear structure impact the initial state of heavy-ion collisions and extraction of QGP transport properties?
- What is the energy and longitudinal dependence of nuclear structure?

INSTITUTE for NUCLEAR THEORY

“Intersection of nuclear structure and high-energy nuclear collisions”

January 23 – February 24, 2023

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Matthew Luzum, Jaki Noronha-Hostler , Fuqiang Wang