Combinant analysis of multiplicity distributions in p+p interactions in multipomeron exchange model

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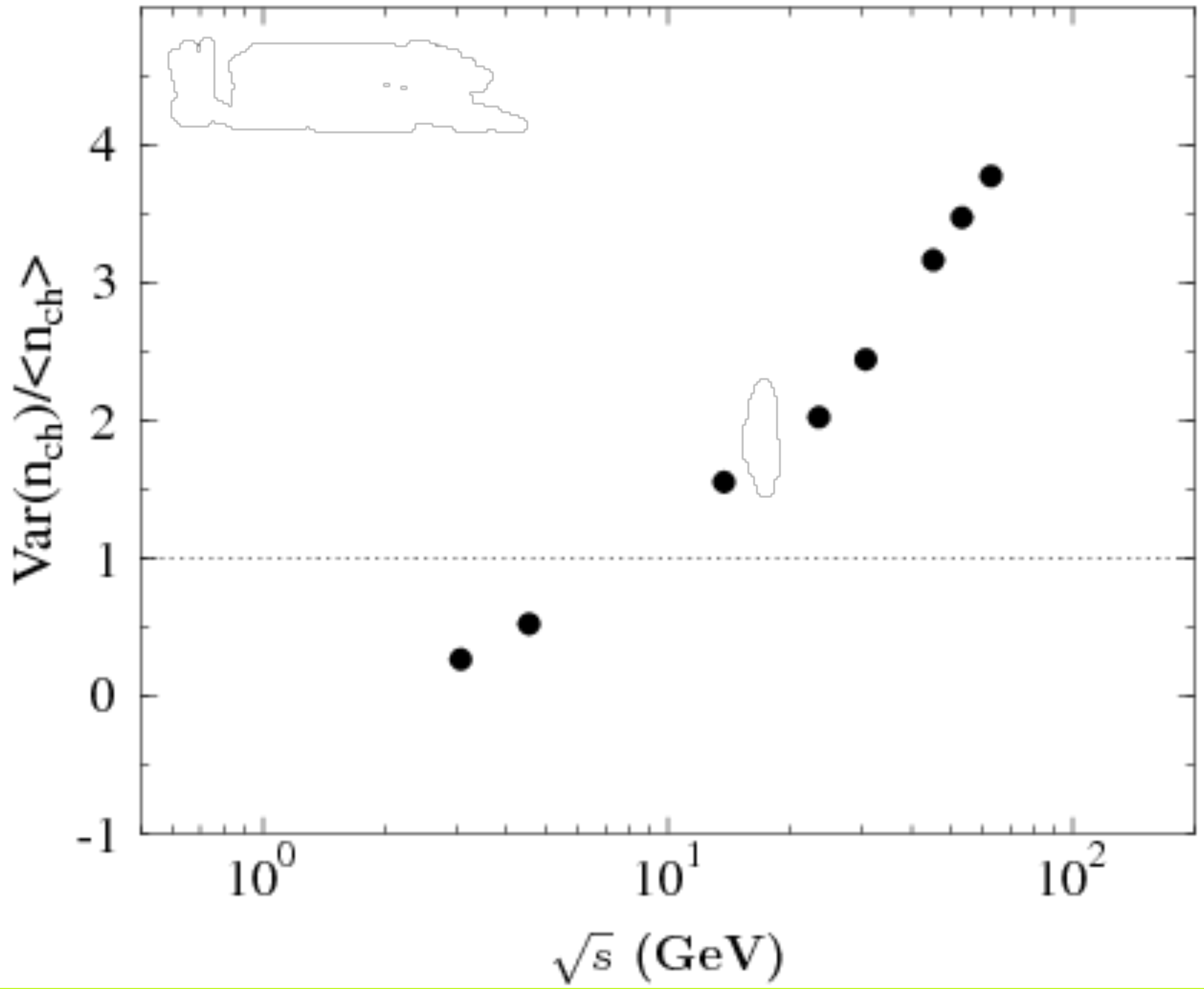
Multiplicity

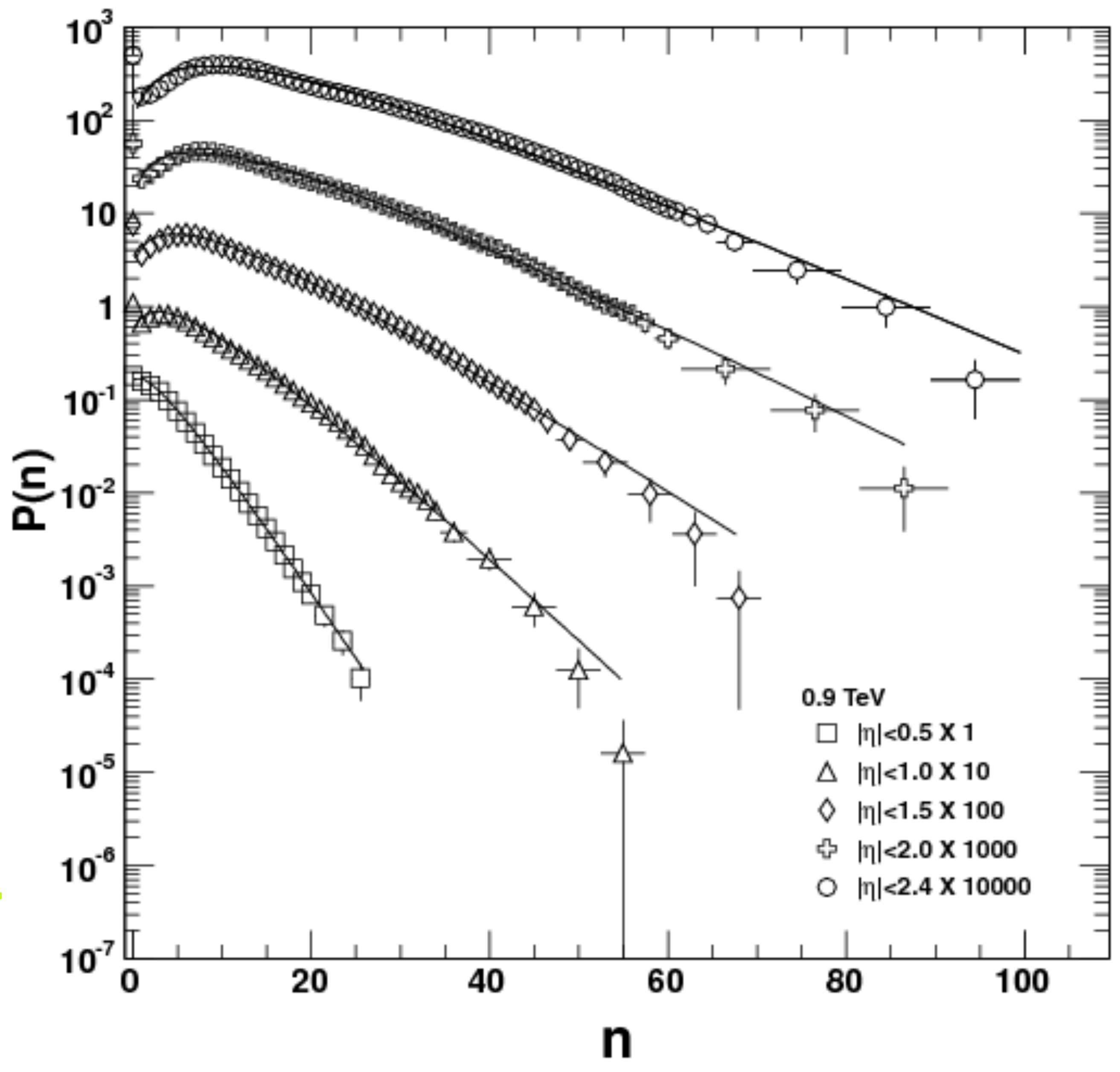
- Number of hadrons produced in an interaction - N
- Event-by-event multiplicity fluctuations are sensitive to critical phenomena and formation of quark-gluon plasma
- Scaled variance $\omega[N] = \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle} > 1$ for p+p collisions starting from $\sqrt{s} = 10$ GeV - important reference for A+A collisions



NA49, *Phys.Rev.C* 75 (2007) 064904

Multiplicity

- Number of hadrons produced in an interaction - N
- Event-by-event multiplicity fluctuations are sensitive to critical phenomena and formation of quark-gluon plasma
- Remarkably good description of multiplicity distribution in p+p interactions is provided by negative binomial distribution (NBD)) $p(N) = \frac{\Gamma(N+q)}{\Gamma(N+1)\Gamma(q)} \cdot \left[\frac{\langle N \rangle}{q+\langle N \rangle}\right]^N \cdot \left[\frac{q}{q+\langle N \rangle}\right]^q$



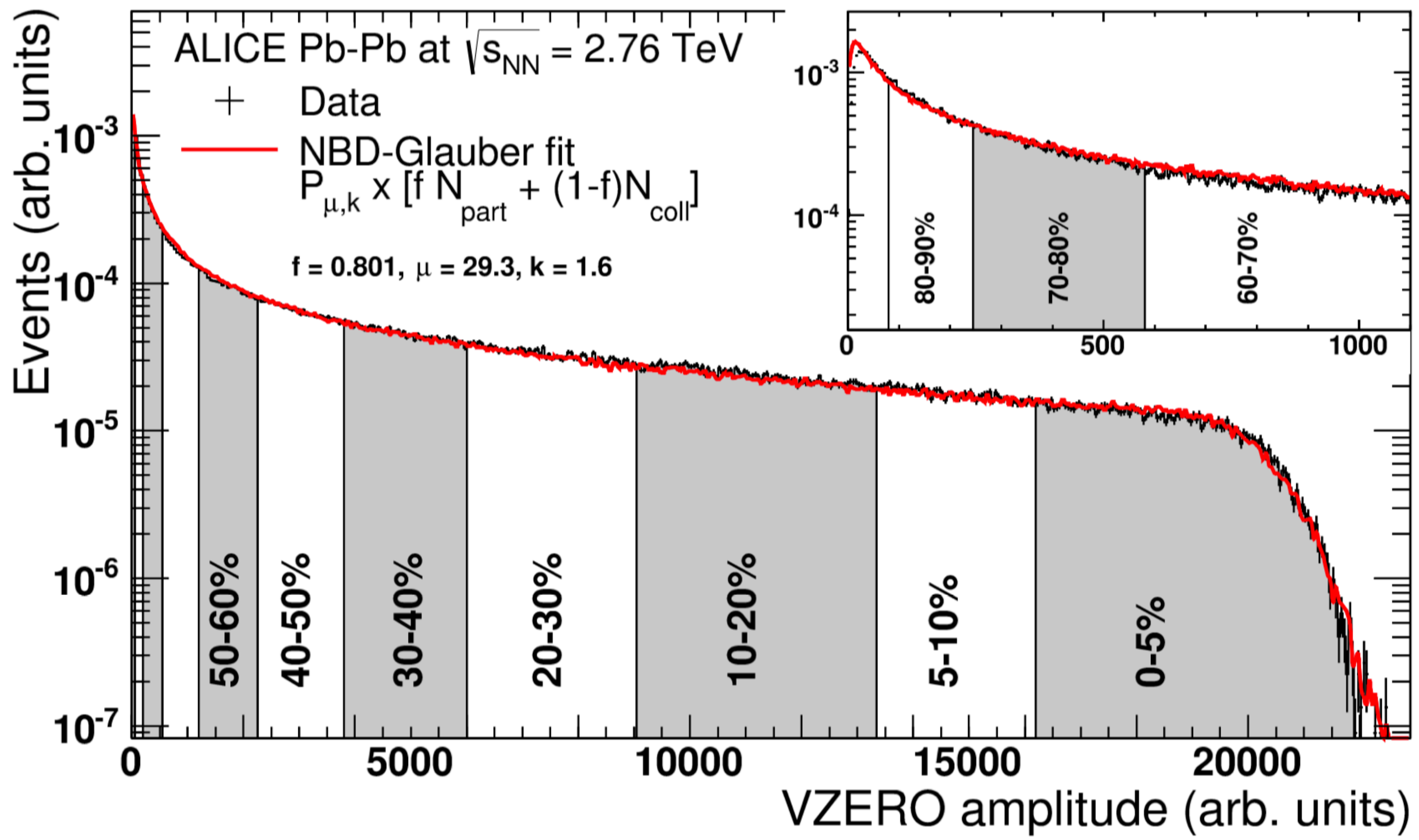
Scaled variance $\omega[N] = \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle} = 1 + \frac{\langle N \rangle}{q} > 1$

2-NBD fits works reasonably good too

Fits of ALICE measurements at 900 GeV by NBD
Premomoy Ghosh *Phys.Rev.D* 85 (2012) 054017

Multiplicity

- Centrality determination using fitting of multiplicity in Glauber Monte Carlo+NBD approach
- Ancestor model: $f \cdot N_{part} + (1 - f) \cdot N_{coll}$ is treated as number of particle producing sources (ancestors), each of them is producing particles according to NBD



ALICE *Phys.Rev.C* 88 (2013) 4, 044909

Another look at distribution

- For a given discrete distribution $P(N)$ one can define:
- Moments: $\nu_k = E[N^k] = \sum_N N^k \cdot P(N)$ $(\omega[N] = \frac{\nu_2}{\nu_1})$
  - Generating function $E[e^{N \cdot t}]$ (derivatives at $t = 0$)
- Central moments: $\mu_k = E[(N - E[N])^k]$
  - Generating function $E[e^{(N - E[N]) \cdot t}]$ (derivatives $t = 0$)
- Factorial moments: $\mu'_k = E[N * (N-1) * \ldots * (N-k+1)]$
  - Generating function $E[s^N]$ (derivatives $s = 1$)
- Genereting function of cumulants: $lnE[e^{N \cdot t}]$
- Genereting function of factorial cumulants: $lnE[s^N]$ (derivatives $s = 1$)
- Generating function of combinants: $lnE[t^N]$ (derivatives $t = 0$)

First appearance

Multiplicity Distributions of Created Bosons: The Combinants Tool

- S.K. Kauffmann, M. Gyulassy *J.Phys.A* 11 (1978) 1715-1727

Combinants

- $F(t) = lnE[t^N] = \sum_{i=0}^{\infty} t^i P(i); F(1) = 1; F(0) = P(0)$
- $ln(F(t)) = ln(F(0)) + \sum_{i=1}^{\infty} t^i C*(i) = ln(P(0)) + \sum_{i=1}^{\infty} t^i C*(i) = \sum_{i=1}^{\infty} \left(t^i - 1\right) C*(i)$
- $C*(1) = \frac{P(1)}{P(0)}$
- $C*(2) = \frac{P(2)}{P(0)} - \frac{1}{2}\left(\frac{P(1)}{P(0)}\right)^2$
- …

Why combinants?

How to retrieve additional information from the multiplicity distributions

- Grzegorz Wilk, Zbigniew Włodarczyk *J.Phys.G* 44 (2017) 1, 015002

- Main idea:
  - for a lot of distribution the following recurrence relation holds:
    - $(N+1) \cdot P(N+1) = g(N) \cdot P(N)$, where g(N) - linear function
    - or more generally: $(N+1) \cdot P(N+1) = \langle N \rangle \sum_{j=0}^{N} C(j) \cdot P(N-j)$
    - this relation holds in clans and cascade models
    - C(j) - «modified» combinants: $C(j) = \frac{j+1}{\langle N \rangle} C^*(j+1)$
    - $\langle N \rangle C(j) = (j+1) \cdot \frac{P(j+1)}{P(0)} - \langle N \rangle \sum_{i=0}^{j-1} C(i) \cdot \frac{P(j-i)}{P(0)}$

Problems of NBD

$$C(j) = \frac{\langle N \rangle}{\langle N^2 \rangle - \langle N \rangle^2}\left(1 - \frac{\langle N \rangle}{\langle N^2 \rangle - \langle N \rangle^2}\right)^j$$

NBD has a monotonic
rank dependence of combinants

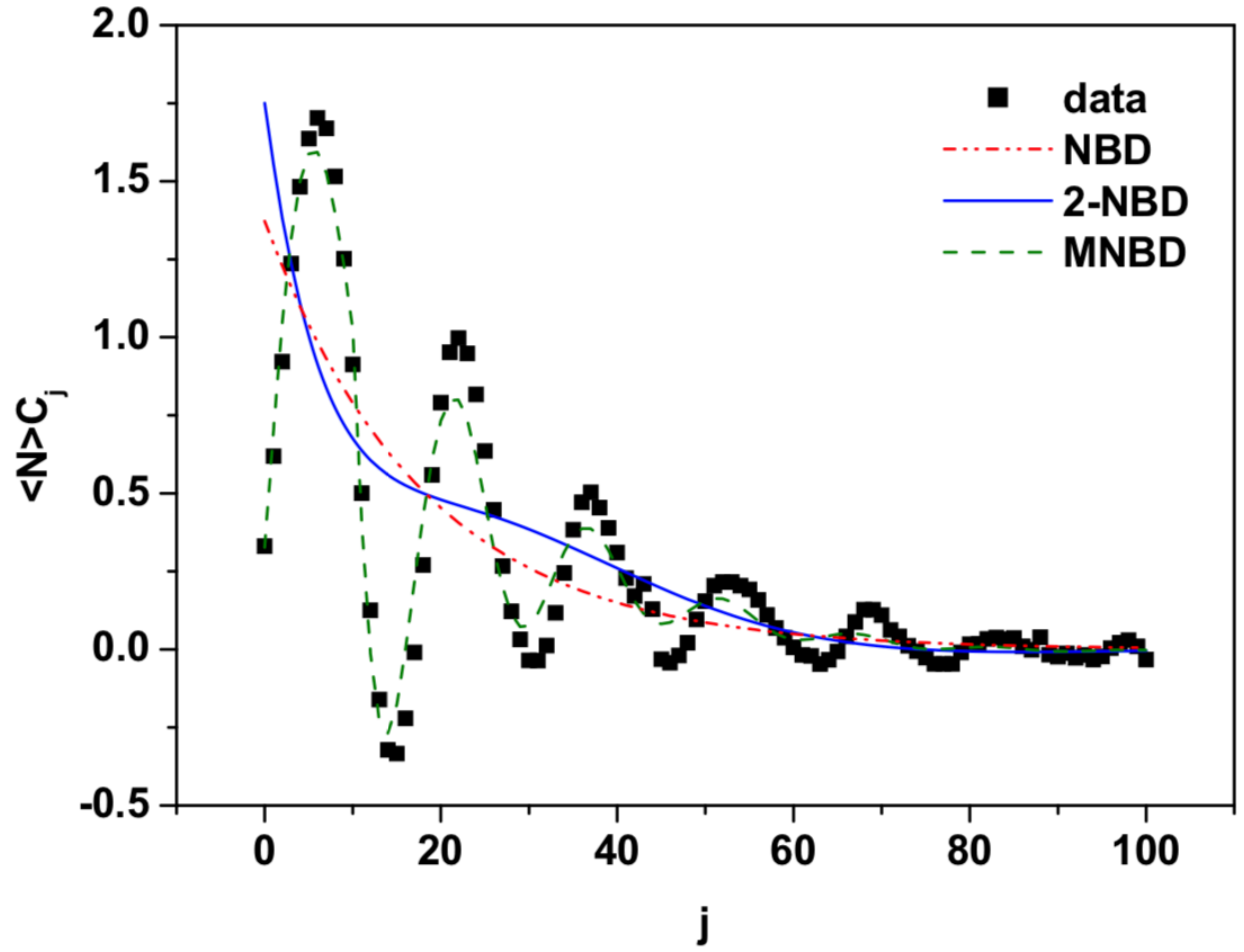


Figure 9. (Color online) Coefficients C_j emerging from the MNBD fit to the CMS data [40] taken for $\sqrt{s} = 7$ TeV and pseudorapidity window $|\eta| < 2$ compared with the C_j obtained from the single NBD and from the 2-component NBD (2-NBD) fits to the CMS data with parameters from [42].

Grzegorz Wilk, Zbigniew Włodarczyk *Entropy* 19 (2017) 12, 670
Grzegorz Wilk, Zbigniew Włodarczyk *Int.J.Mod.Phys.A* 33 (2018) 10, 1830008
Maciej Rybczynski, Grzegorz Wilk, Zbigniew Włodarczyk *Phys.Rev.D* 99 (2019) 9, 094045
Han Wei Ang et al. *Eur.Phys.J.A* 56 (2020) 4, 117
Grzegorz Wilk, Zbigniew Włodarczyk *Int.J.Mod.Phys.A* 36 (2021) 13, 2150072
H.W. Ang et al. Mod.Phys.Lett.A 34 (2019) 39, 1950324
I. Zborovský Eur.Phys.J.C 78 (2018) 10, 816
R. Aggarwal, M. Kaur Adv.High Energy Phys. 2020 (2020) 5464682
Aayushi Singla, M. Kaur Adv.High Energy Phys. 2020 (2020) 5192193

Multipomeron exchange model

- N. Armesto, D.A. Derkach, G.A. Feofilov *Phys.Atom.Nucl.* 71 (2008) 2087-2095
- E.O. Bodnya, V.N. Kovalenko, A.M. Puchkov, G.A. Feofilov *AIP Conf.Proc.* 1606 (2015) 1, 273-282
- E.V. Andronov, V.N. Kovalenko *Theor.Math.Phys.* 200 (2019) 3, 1282-1293, *Teor.Mat.Fiz.* 200 3, 415-428

$$P(N) = C(z) \sum_{N_{pom}} \frac{1}{z \cdot N_{pom}} \left(1 - e^{-z} \sum_{l=0}^{N_{pom}-1} \frac{z^l}{l!} \right) \cdot P_{N_{pom}}(N)$$

$$z = \frac{2C\gamma s^{\Delta}}{R^2 + \alpha' \log(s)}, \ \Delta = 0.139, \ \alpha' = 0.21 \text{ GeV}^{-2}, \ \gamma = 1.77 \text{ GeV}^{-2}, \ R_0^2 = 3.18 \text{ GeV}^{-2}, \ C = 1.5$$

$P_{N_{pom}}(N)$ was taken to be Poisson distribution with mean $2 \cdot N_{pom} \cdot \delta\eta \cdot k(\sqrt{s})$ where $k = 0.255 + 0.0653 \cdot ln\sqrt{s}$

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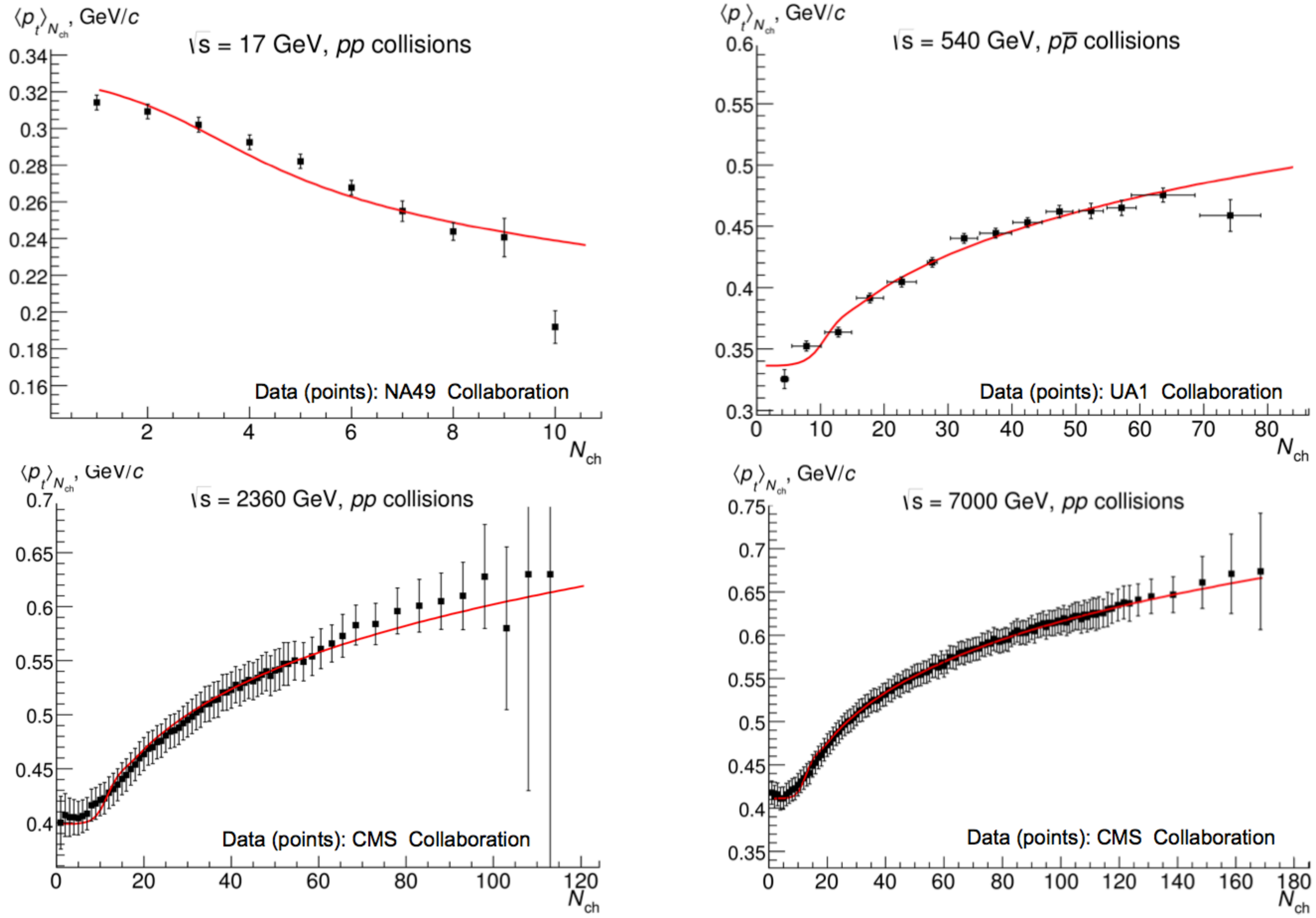
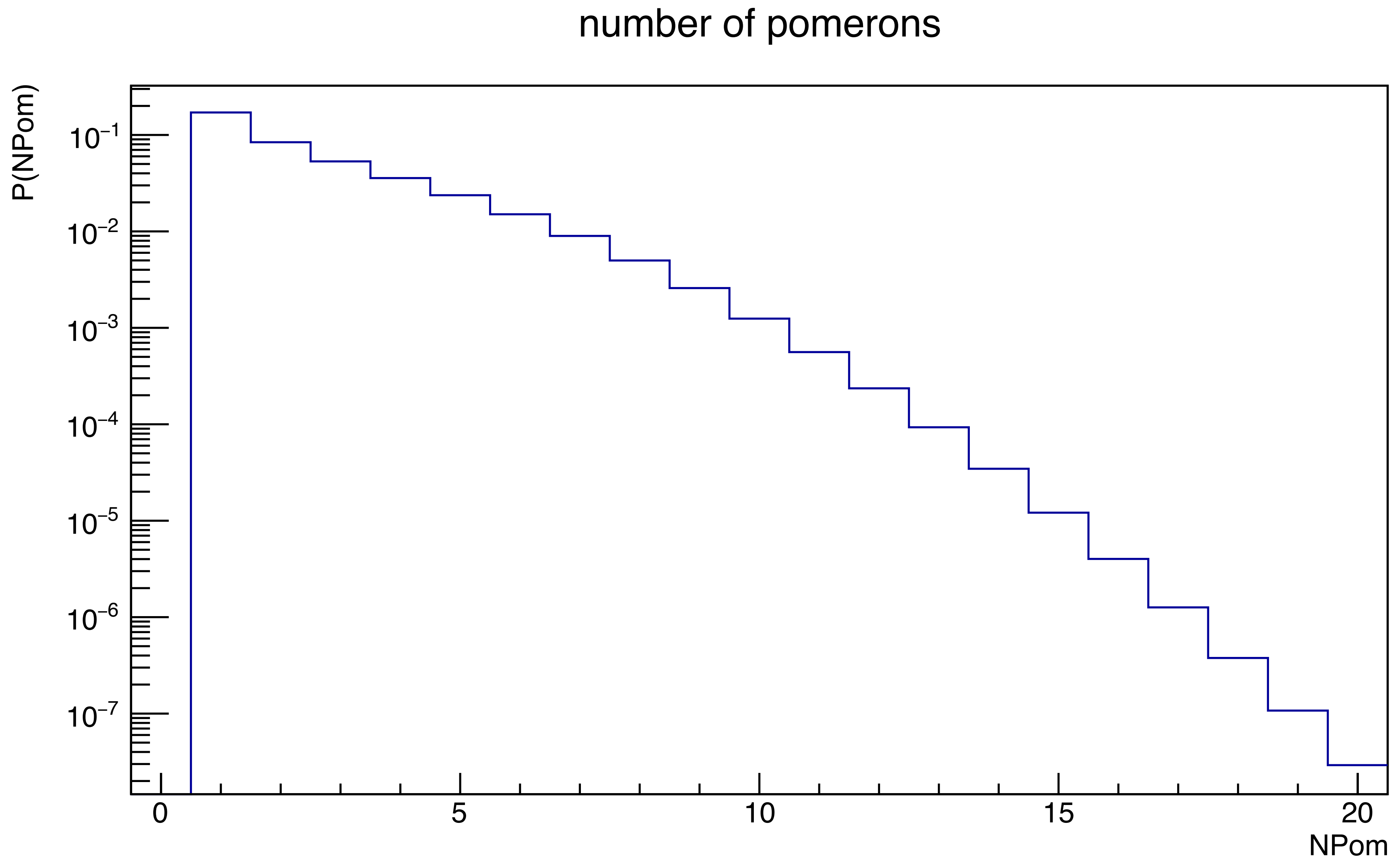
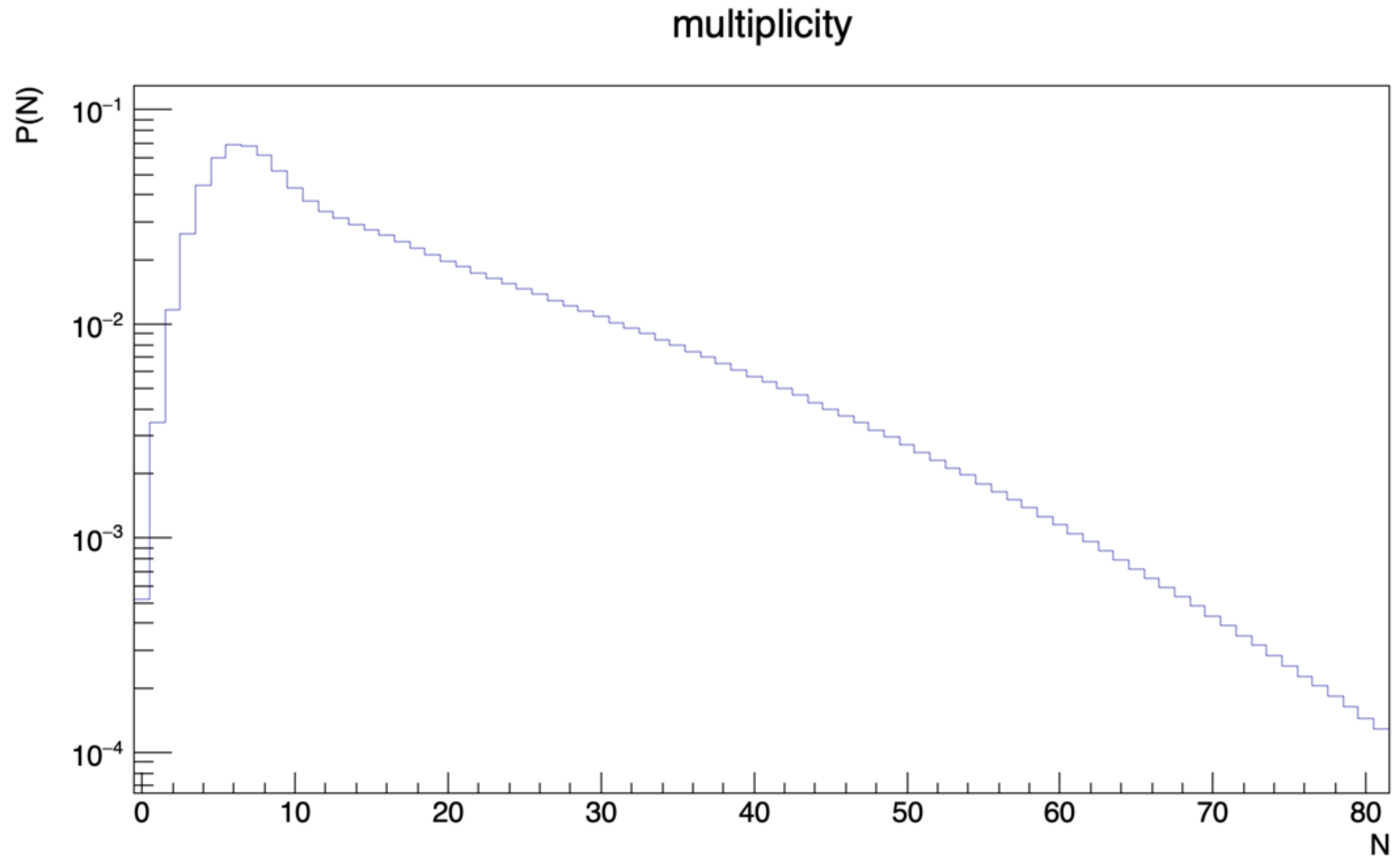


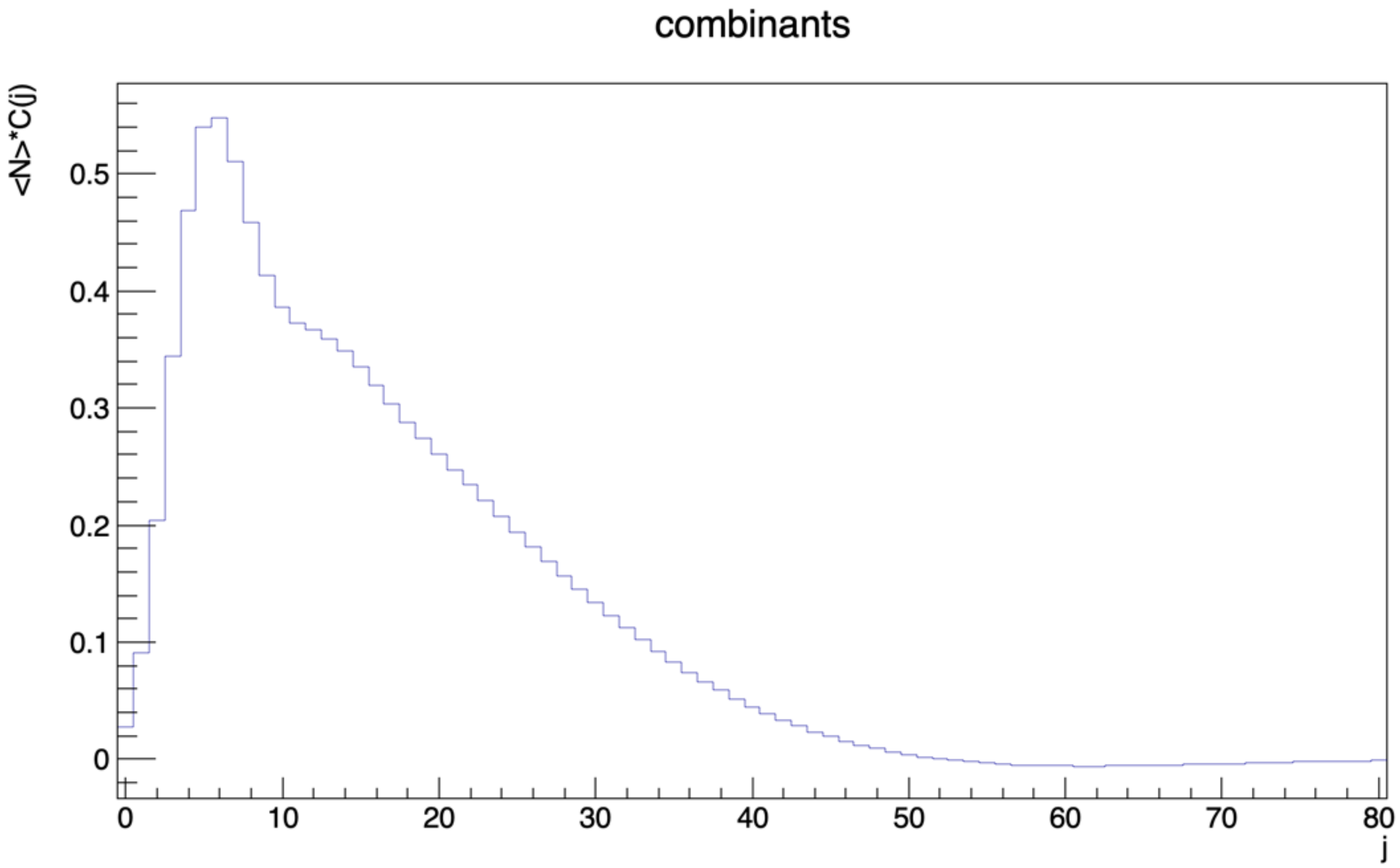
Multipomeron exchange model results





$\sqrt{s} = 900$ GeV; $|\Delta\eta| < 2.4$

no oscillations of combinants



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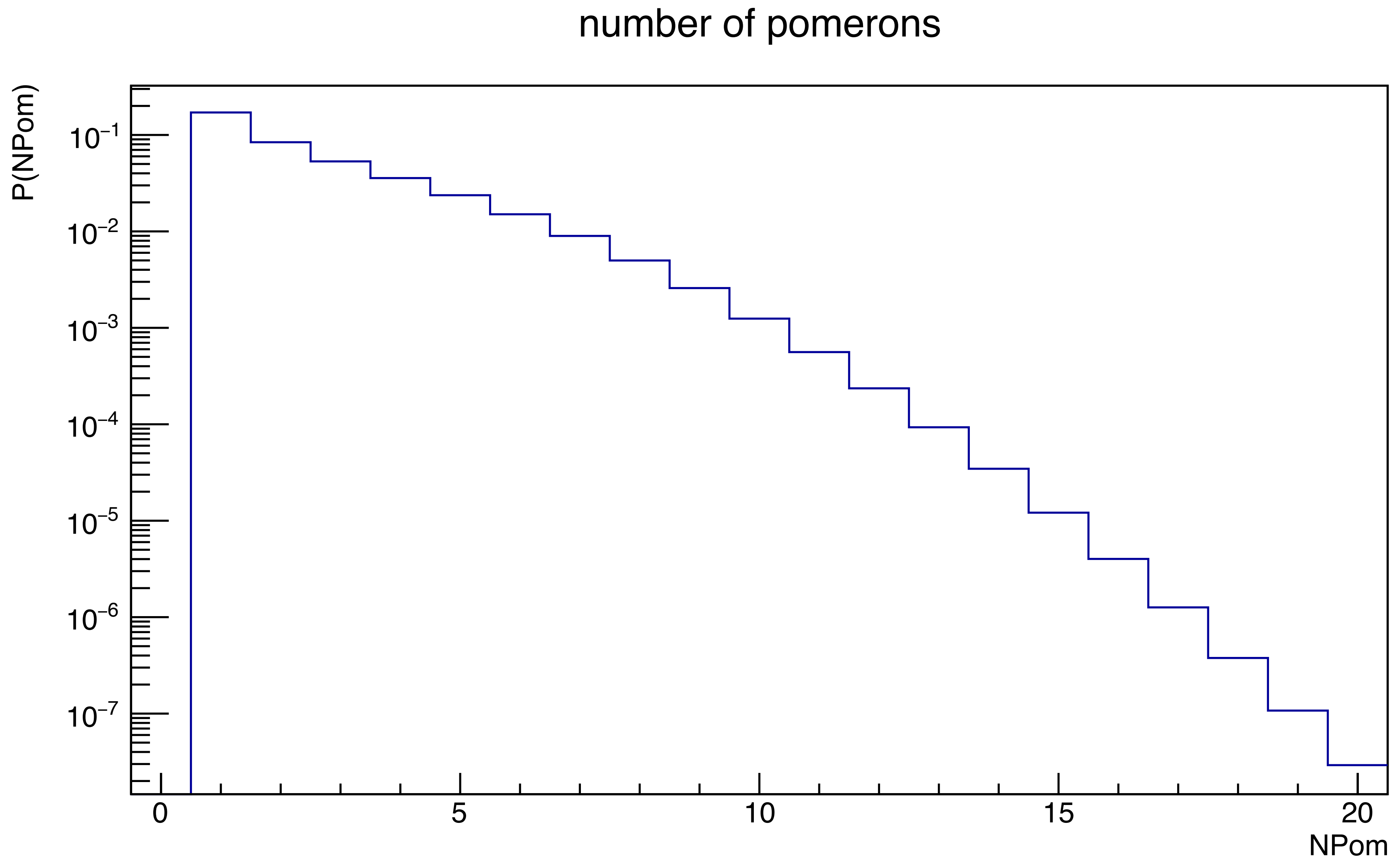
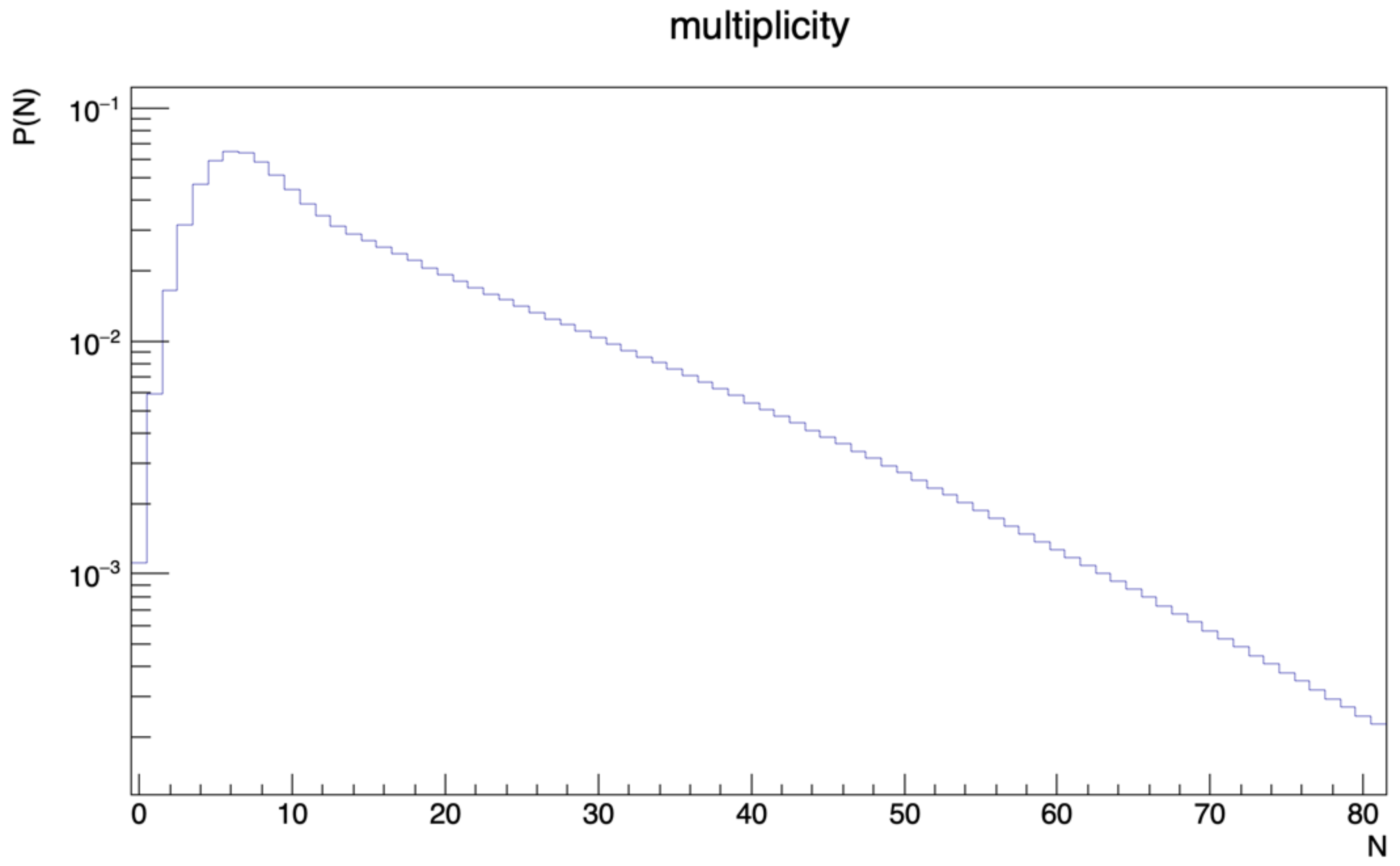
$$P(N) = C(z) \sum_{N_{pom}} \frac{1}{z \cdot N_{pom}} \left(1 - e^{-z} \sum_{l=0}^{N_{pom}-1} \frac{z^l}{l!} \right) \cdot P_{N_{pom}}(N)$$

$$z = \frac{2C\gamma s^{\Delta}}{R^2 + \alpha' \log(s)}, \ \Delta = 0.139, \ \alpha' = 0.21 \text{ GeV}^{-2}, \ \gamma = 1.77 \text{ GeV}^{-2}, \ R_0^2 = 3.18 \text{ GeV}^{-2}, \ C = 1.5$$

Let us consider $P_{N_{pom}}(N)$ to be NBD such that its expectation value is again $2 \cdot N_{pom} \cdot \delta\eta \cdot k(\sqrt{s})$ where $k = 0.255 + 0.0653 \cdot ln\sqrt{s}$

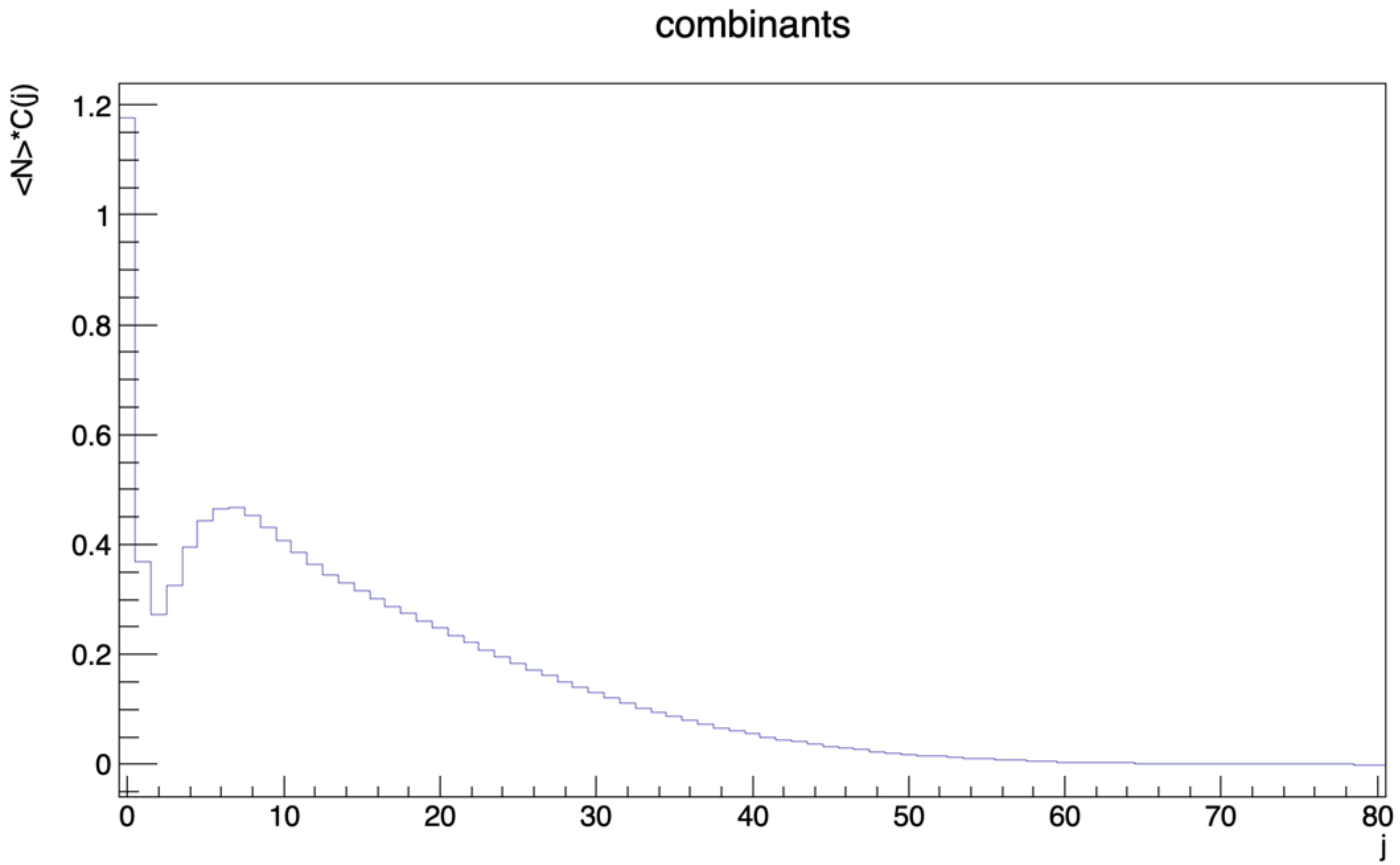
Multipomeron exchange model





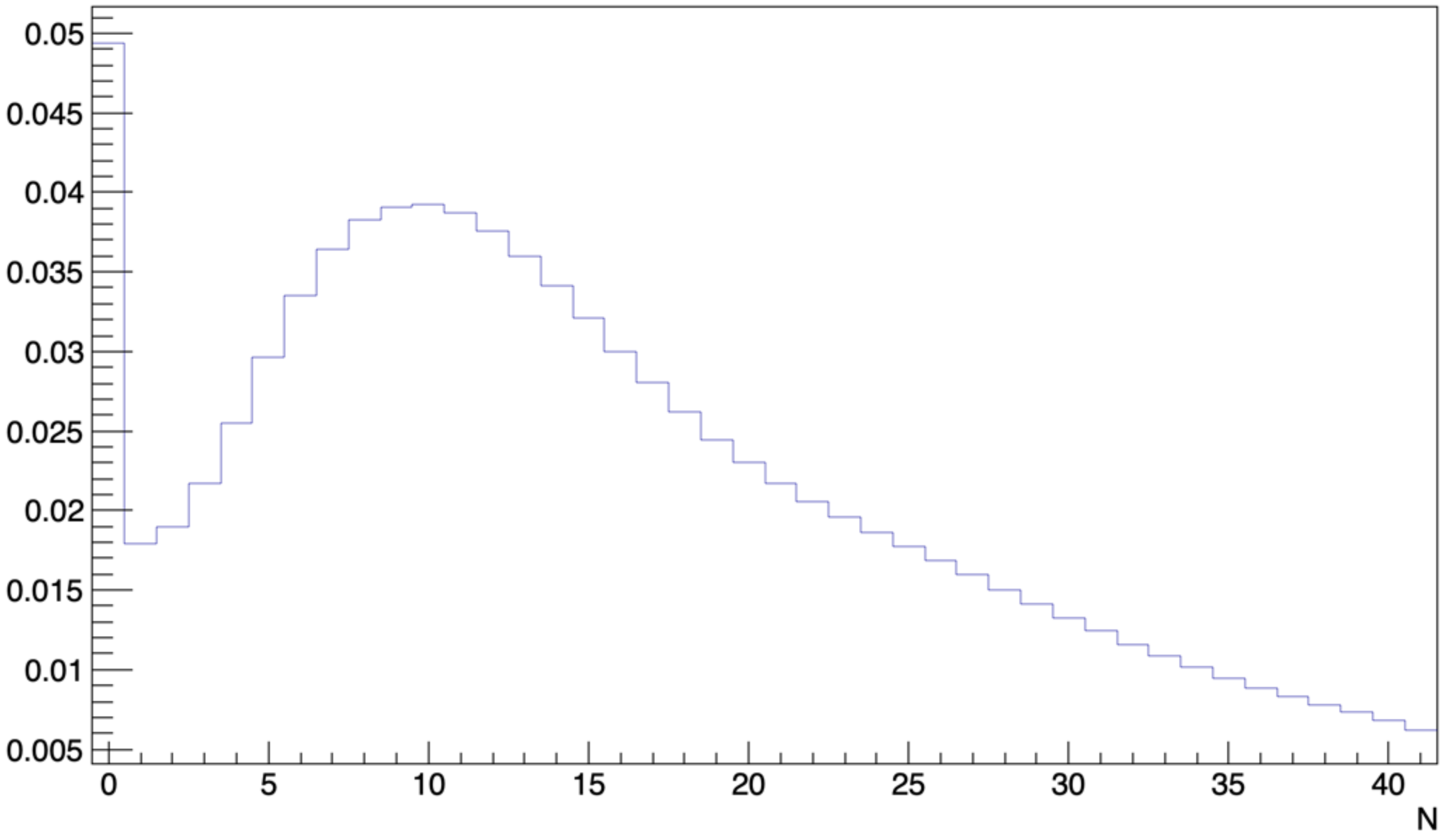
$\sqrt{s} = 900$ GeV; $|\Delta\eta| < 2.4$

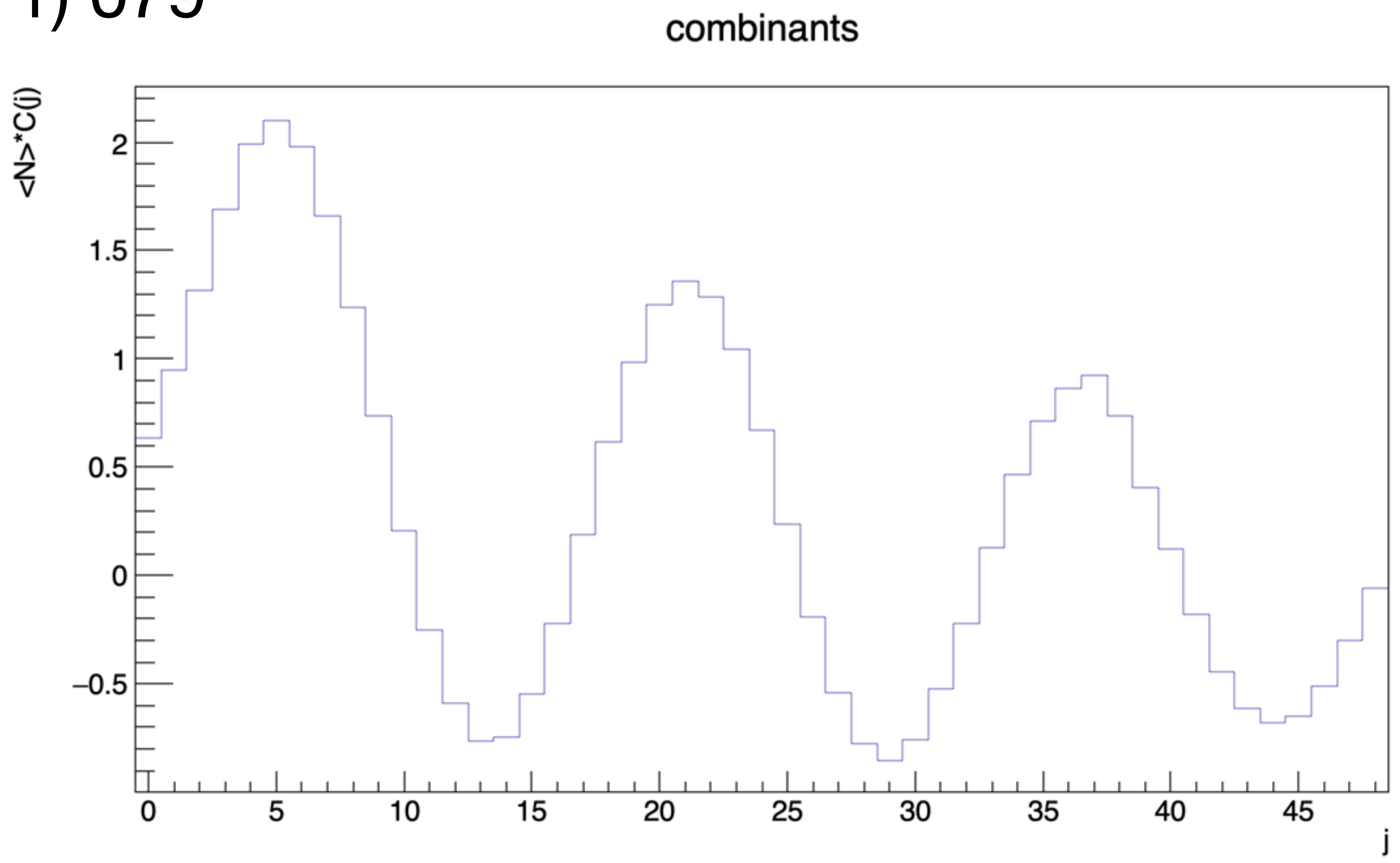
no pronounced periodic structure



Sensitivity of combinants to 0-th bin

$\sqrt{s} = 900$ GeV; $|\Delta\eta| < 2.4$ CMS, *JHEP* 01 (2011) 079

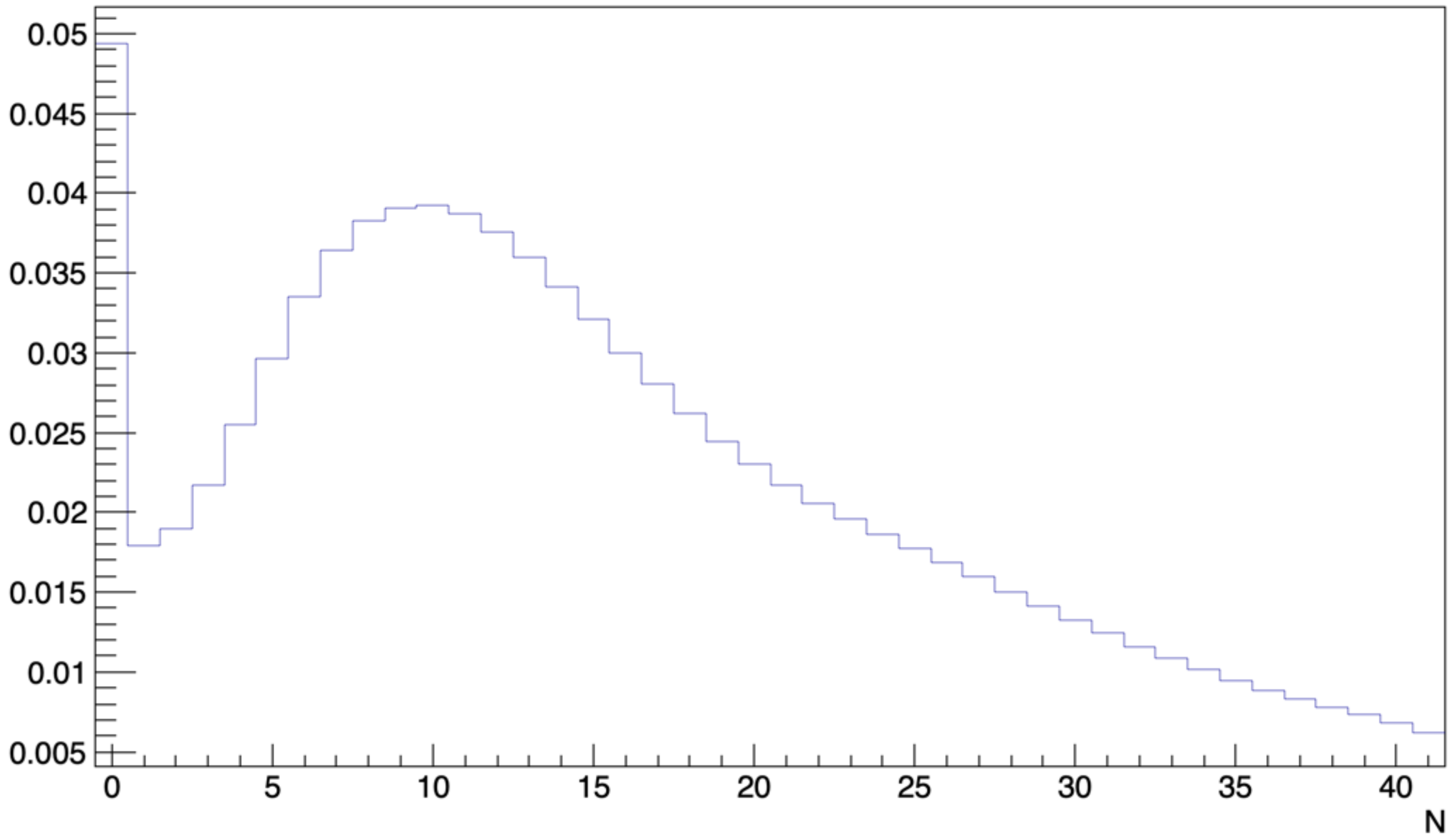


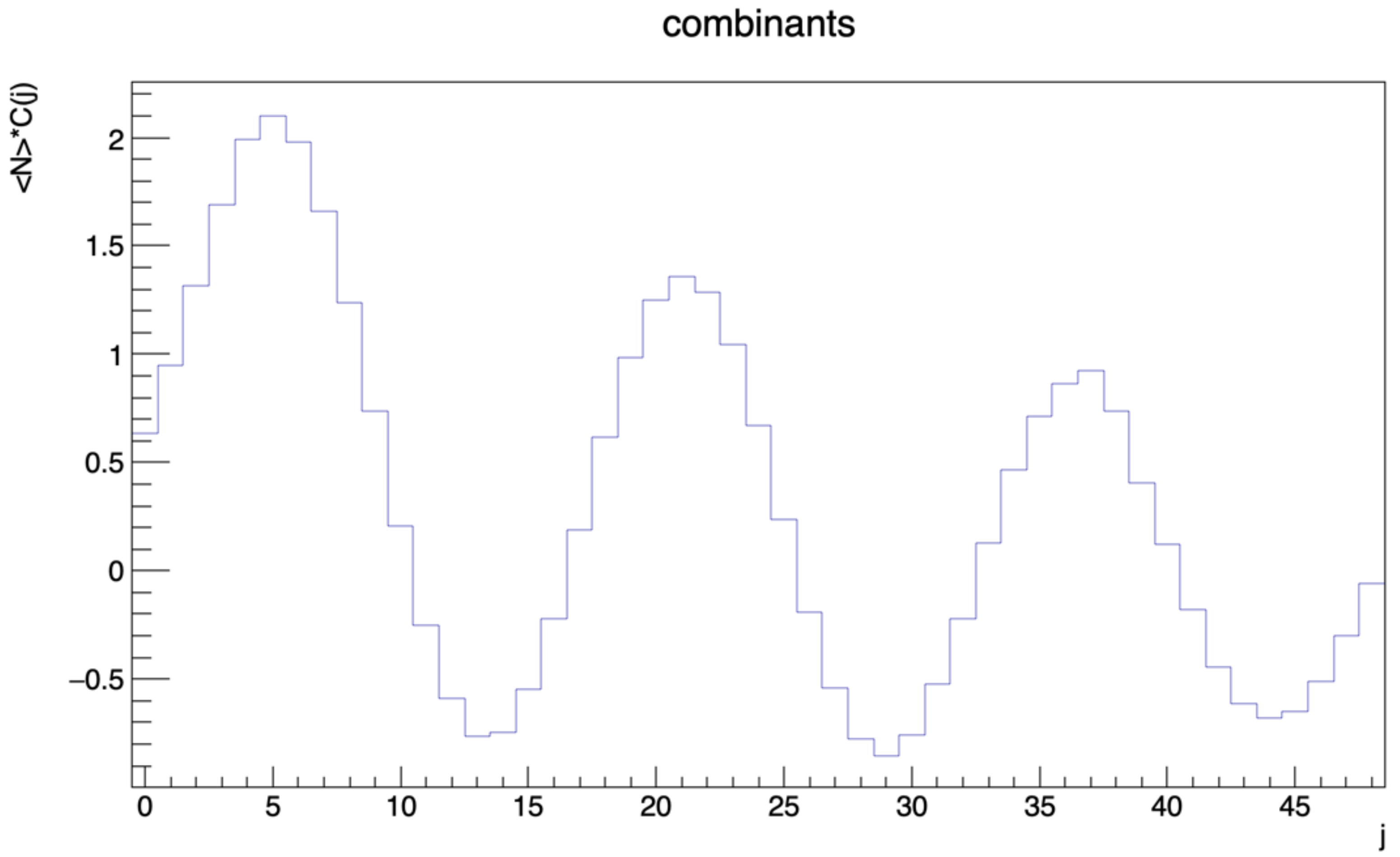


All combinants strongly depend on P(0) by definition
Let us check how they would behave for truncated distribution

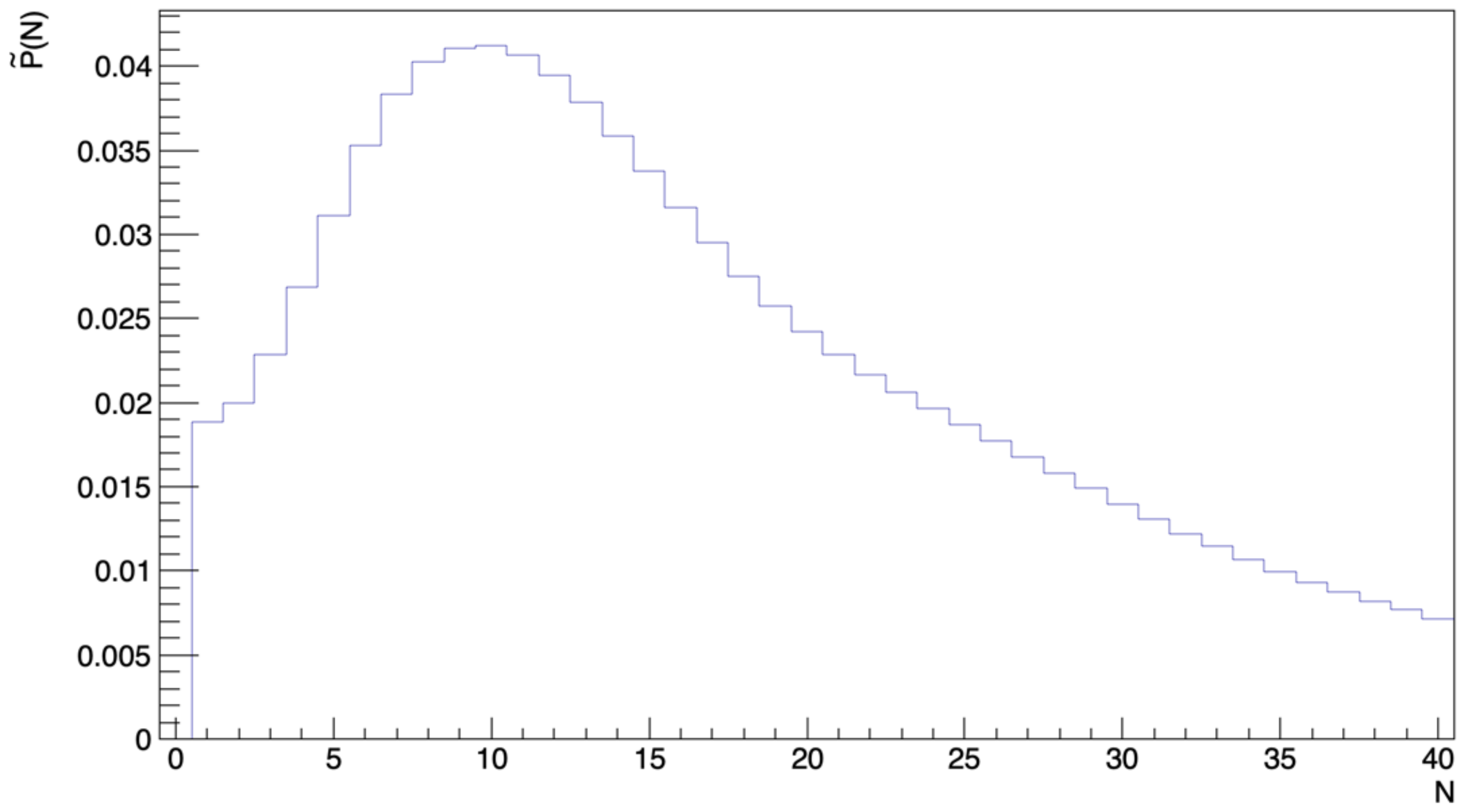
Sensitivity of combinants to 0-th bin

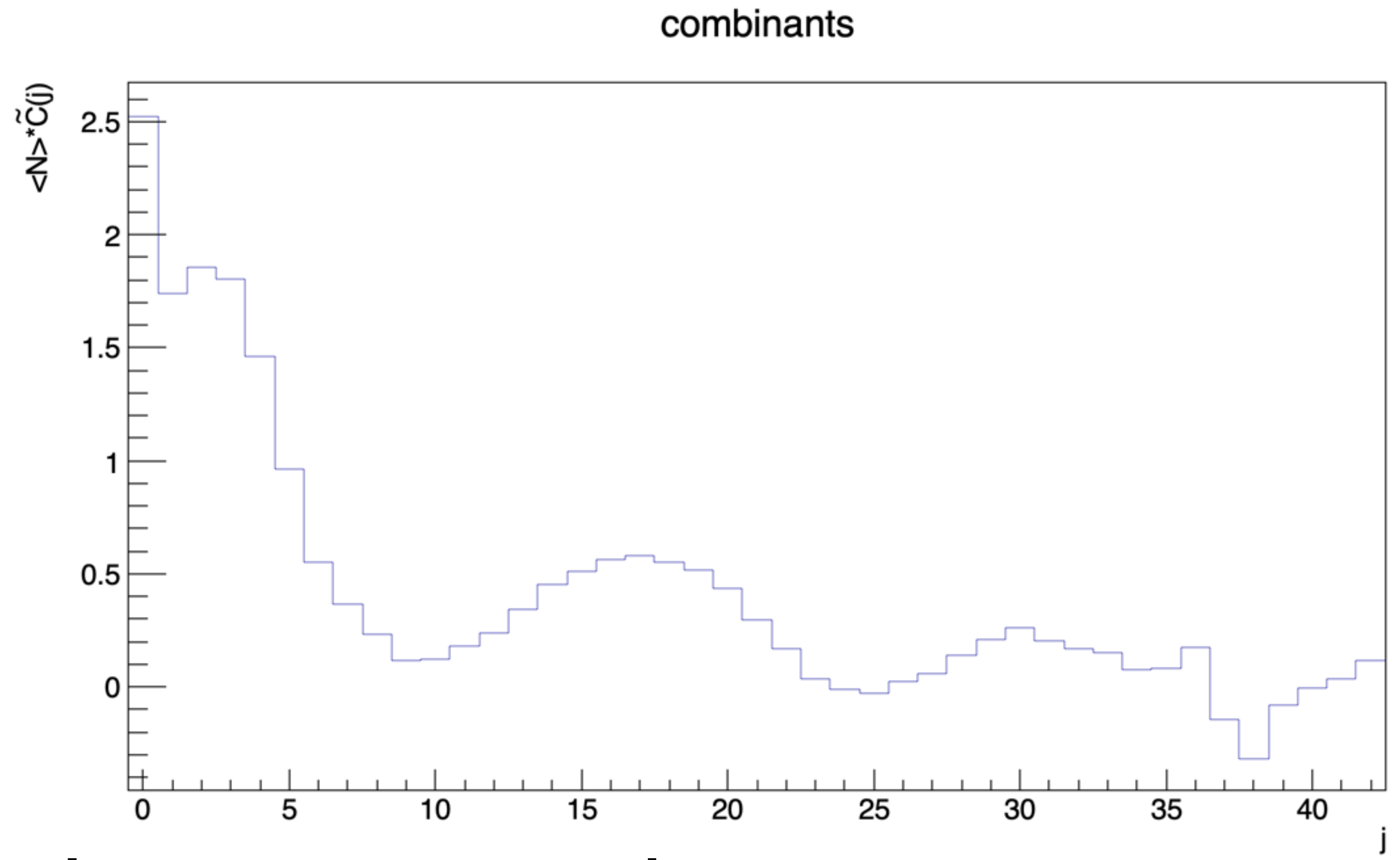
$\sqrt{s} = 900$ GeV; $|\Delta\eta| < 2.4$ CMS, *JHEP* 01 (2011) 079





$$P(N) \to \tilde{P}(N) = \frac{P(N)}{1 - P(0)}$$





After redefinition amplitude significantly suppressed

Conclusions

- Modified combinants exhibit oscillating behaviour for p+p interactions at LHC energies - not described by negative binomial distribution fits to data
- Default multipomeron exchange model does not produce oscillations
- NBD as distribution of particle from a single string leads to rapidly fading wave
- Redefinition of combinants omitting ‘problematic’ 0-th bin results in change of amplitude and does not affect periodicity

Thank you for your attention!

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Combinants and cumulants

As in the case of the combinants, $C_j^\star$, the set of modified combinants, C_j, provides a similar measure of fluctuations as the set of cumulant factorial moments, K_q, which are very sensitive to the details of the multiplicity distribution and were frequently used in phenomenological analyses of data (cf., [1, 19]),

$$K_q = F_q - \sum_{i=1}^{q-1} \binom{q-1}{i-1} K_{q-i} F_i, \qquad (13)$$

where

$$F_q = \sum_{N=q}^{\infty} N(N-1)(N-2)\dots(N-q+1)P(N), \quad (14)$$

are the factorial moments. The K_q can be expressed as an infinite series of the C_j,

$$K_q = \sum_{j=q}^{\infty} \frac{(j-1)!}{(j-q)!} \langle N \rangle C_{j-1}, \qquad (15)$$

and, conversely, the C_j can be expressed in terms of the K_q [1, 19],

$$C_j = \frac{1}{\langle N \rangle} \frac{1}{(j-1)!} \sum_{p=0}^{\infty} \frac{(-1)^p}{p!} K_{p+j}. \qquad (16)$$

Sensitivity to statistics

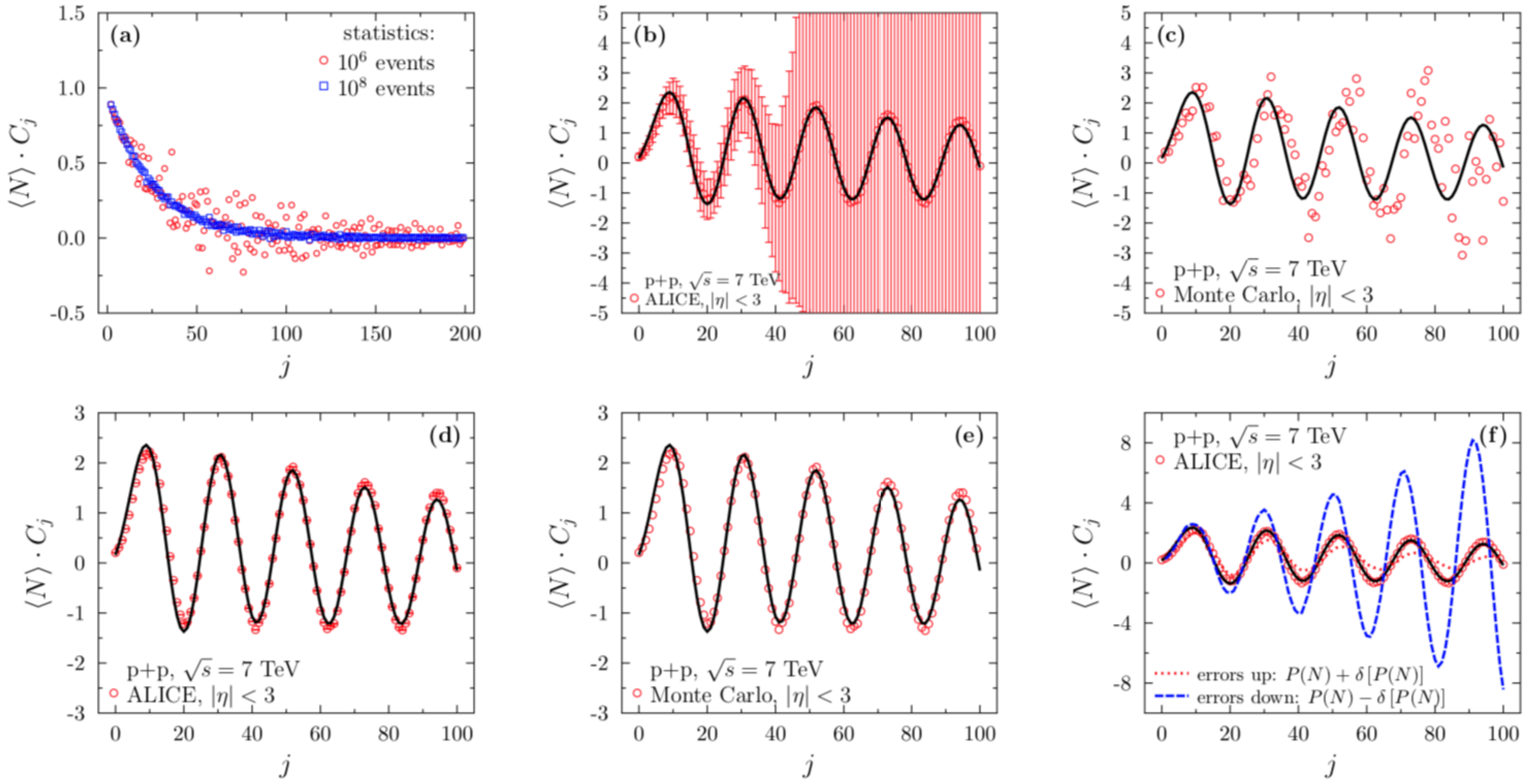

FIG. 7. (Color online) (a) Monte Carlo evaluated coefficients C_j emerging from NBD with parameters: $\langle N \rangle = 25.5$ and $k = 1.45$. With increasing statistics points are merging to a continuous line. (b) Errors of $\langle N \rangle C_j$ evaluated using the systematic and statistical uncertainties of $P(N)$ given by ALICE [30]. (c) Monte Carlo evaluated coefficients $\langle N \rangle C_j$ emerging from the systematic and statistical errors of $P(N)$. The curve presented here denotes the fit to the original coefficients C_j obtained from the measured $P(N)$, it is not the fit to the points shown. (d) For the same data as before the errors were evaluated assuming only statistical uncertainties of the measured $P(N)$ with a poissonian distribution of events in each bin, i.e., $Var[P(N)] = P(N)/N_{stat}$. Note that in this case statistical errors do not give any noticeable errors of C_j. (e) Monte Carlo evaluated coefficients $\langle N \rangle C_j$ with only statistical errors of $P(N)$ accounted for. The continuous curve represents the fit to the original coefficients C_j obtained from the measured $P(N)$. (f) The modified combinants C_j emerging from the ALICE data on $P(N)$ [30] (continuous curve) in envelope corresponding to the systematic uncertainties of data, $P(N) \pm \delta[P(N)]$.

Combinants and independent sources

Physics Letters B 266 (1991) 231–235
North-Holland

PHYSICS LETTERS B

Description of pion multiplicities using combinants

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Received 28 May 1991

Combinants, certain linear combinations of ratios of probabilities, were introduced earlier by Kauffmann and Gyulassy to study boson multiplicities. It is shown that combinants can be a useful tool to distinguish between bosons coming from the secondary decay of other particles such as deltas and bosons emitted from thermally equilibrated sources.