New effects in the Monte Carlo model of pp, pA and AA collisions with string fusion

Vladimir Kovalenko

Saint Petersburg State University

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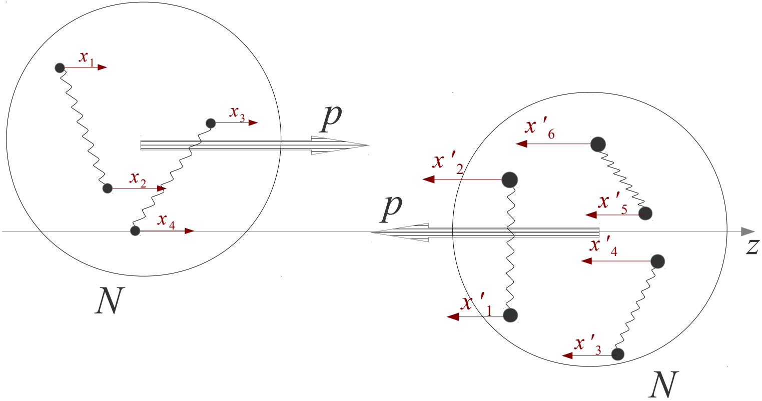
Outline

- Formulation of the main ideas of the dipole-based the Monte Carlo model of pp, pA and AA collisions with string fusion
- Energy dependence of number of partons
- Effects of the running coupling on the high-pt observables
- Conclusions and Outlook

Model description: Color dipoles inside a nucleon



$$\sum_i x_i p = p \qquad \sum_i x_i' p = p$$

$$\sum_i x_i = 1 \qquad \sum_i x_i' = 1$$

p-p interaction: parton distributions

- Inclusive momentum distributions are taken from [1,2]:

$$
\begin{aligned}
f_u(x) &= f_{\bar{u}}(x) = C_{u,n}\, x^{-\frac{1}{2}}(1-x)^{\frac{1}{2}+n}, \\
f_d(x) &= f_{\bar{d}}(x) = C_{d,n}\, x^{-\frac{1}{2}}(1-x)^{\frac{3}{2}+n}, \\
f_{ud}(x) &= C_{ud,n}\, x^{\frac{3}{2}}(1-x)^{-\frac{3}{2}+n}, \\
f_{uu}(x) &= C_{uu,n}\, x^{\frac{5}{2}}(1-x)^{-\frac{3}{2}+n}.
\end{aligned}
$$

- At n>1 the sea quarks and antiquarks have the same distribution as the valence quarks.
- Poisson distribution for the number of quark-antiquark (diquark) pairs (n) is assumed with some parameter λ

[1] A.B. Kaidalov, O.I.Piskunova. Zeitschrift fur Physik C 30(1):145-150, 1986
[2] G.H. Arakelyan, A.Capella, A.B.Kaidalov, and Yu.M.Shabelski. Eur.Phys.J (C), 26(1):81-90, 2002

p-p interaction: parton distributions

- Corresponding exlusive distribution of the momentum fractions:

$$\rho(x_1, ...x_N) = c \cdot \prod_{j=1}^{N-1} x_j^{-\frac{1}{2}} \cdot x_N^{\alpha_N} \cdot \delta(\sum_{i=1}^{N} x_i - 1)$$

- Valence quark is labelled by N-1, the diquark by N, and the other refers to sea quarks and antiquarks.
- N=2*n

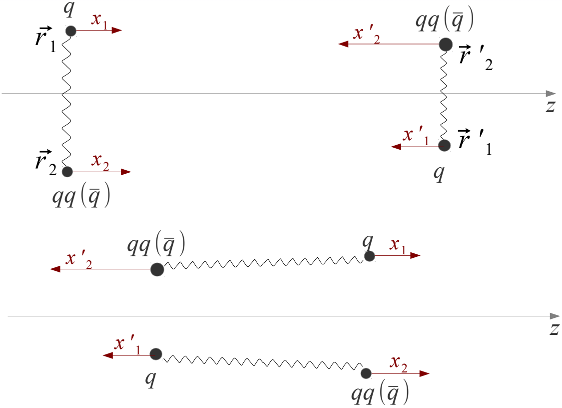
V. N. Kovalenko. Phys. Atom. Nucl. 76, 1189 (2013), arXiv:1211.6209 [hep-ph]
V. Kovalenko, V. Vechernin., PoS (Baldin ISHEPP XXI) 077, arXiv:1212.2590 [nucl-th], 2012

6 Distribution in the impact parameter plane

- Exclusive distribution in the impact parameter plane is constructed from the following suppositions:
  1. Centre of mass is fixed: $\sum_{j=1}^{N} \vec{r}_j \cdot x_j = 0$.
  2. Inclusive distribution of each parton is the 2-dimentional Gaussian distribution.
  3. Normalization condition $< r^2 >=< \frac{1}{N} \sum_{j=1}^{N} r_j{}^2 >= r_0{}^2$.
- The parameter $r_0{}^2$ is connected with the mean square radius of the proton by the formula: $< r_N^2 >= \frac{3}{2} r_0^2$.

V. N. Kovalenko. Phys. Atom. Nucl. 76, 1189 (2013), arXiv:1211.6209 [hep-ph]
V. Kovalenko, V. Vechernin., PoS (Baldin ISHEPP XXI) 077, arXiv:1212.2590 [nucl-th], 2012

Monte Carlo model: Color dipoles



Interaction probability amplitude [4, 5]:

$$(1)\quad f = \frac{\alpha_s^2}{2} \ln^2 \frac{|\vec{r}_1 - \vec{r}_1{}'||\vec{r}_2 - \vec{r}_2{}'|}{|\vec{r}_1 - \vec{r}_2{}'||\vec{r}_2 - \vec{r}_1{}'|}$$

Two dipoles interact more probably, if the ends are close to each other, and (others equal) if they are wide.

[4] G. Gustafson, Acta Phys. Polon. B40, 1981 (2009)
[5] C. Flensburg, G. Gustafson, and L. Lonnblad, Eur. Phys. J. (C) 60, 233 (2009)

Confinement radius

- With confinement taken into we obtain [4, 5]:

$$f=\frac{\alpha_s^2}{2}\left[K_0\left(\frac{|\vec{r}_1-\vec{r}_1{}'|}{r_{max}}\right)+K_0\left(\frac{|\vec{r}_2-\vec{r}_2{}'|}{r_{max}}\right)-K_0\left(\frac{|\vec{r}_1-\vec{r}_2{}'|}{r_{max}}\right)-K_0\left(\frac{|\vec{r}_2-\vec{r}_1{}'|}{r_{max}}\right)\right]^2 \quad (2)$$

  where K_0 is modified Bessel function.

- At $r \to 0$ $K_0(r/r_{max}) \approx -\ln(r/(2\mathrm{r}_{max}))$ and we return back to the formula (1).
- At $r \to \infty$: $K_0(r/r_{max}) \approx \sqrt{\frac{\pi r_{max}}{2\mathrm{r}}}e^{-r/r_{max}}$

and amplitude decrease exponentially.

- The total probability of the inelastic interaction of two protons in the eikonal approximation:

$$p = 1 - e^{-\sum_{i,j} f_{ij}}$$

[4] G. Gustafson, Acta Phys. Polon. B40, 1981 (2009)
[5] C. Flensburg, G. Gustafson, and L. Lonnblad, Eur. Phys. J. (C) 60, 233 (2009)

Calculation of multiplicity

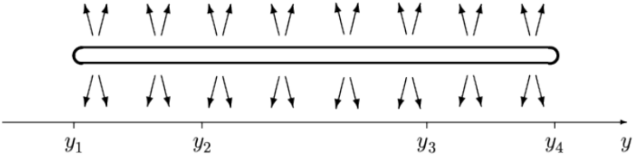
- Multiplicity is calculated in the framework of colour strings, stretched between colliding partons; x_i determine rapidity ends of strings.

y_{min} and y_{min} are calculated supposing that a string fragments into only two particle with masses 0.15 GeV (for pion) and 0.94 GeV for proton and transverse momentum of 0.3 GeV (and higher at LHC)

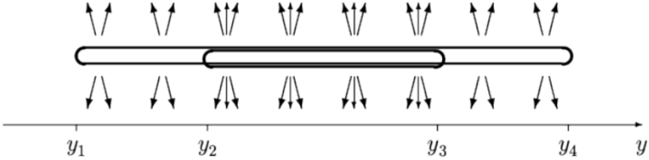
- dN/dy from one string is supposed to be constant μ_0.
- String fusion effects considered

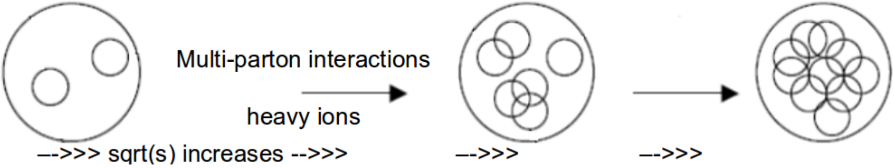
string fusion

- Uniform and independent distribution of particles on rapidity from y_{min} to y_{max}



- Can study string overlaps:





string fusion

$$Q^2(n) = \left(\sum_{i=1}^{n} \vec{Q}_i(1) \right)^2 = \sum_{i=1}^{n} Q_i^2(1) + \sum_{i \neq j} \vec{Q}_i(1) \cdot \vec{Q}_i(1)$$

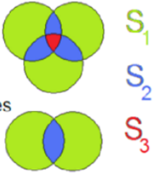
$$\langle Q^2(n) \rangle = n Q^2(1)$$

overlaps

SFM

$C = \{S_1, S_2, \ldots\}$

S_k – area covered k-times



$$\langle \mu \rangle_k = \mu_0 \sqrt{k} \frac{S_k}{\sigma_0} \qquad \langle p_t^2 \rangle_k = p_0^2 \sqrt{k} \qquad \langle p_t \rangle_k = p_0 \sqrt[4]{k}$$

S_k – area, where k strings are overlapping, σ_0 single string transverse area, μ_0 and p_0 – mean multiplicity and transverse momentum from one string

String fusion mechanism predicts (agrees with experiment):

– decrease of multiplicity
– increase of p_T
– growth of p_T with multiplicity in pp, pA and AA collisions
– growth of strange particle yields

Key parameter – transverse radius of the string r_{str}: lager string area – bigger overlapping
r_{str} =0 - no fusion;

M. A. Braun, C. Pajares, Nucl. Phys. B 390 (1993) 542.
M. A. Braun, R. S. Kolevatov, C. Pajares, V. V. Vechernin, Eur. Phys. J. C 32 (2004) 535.
N.S. Amelin, N. Armesto, C. Pajares, D. Sousa, Eur.Phys.J.C22:149-163 (2001), arXiv:hep-ph/0103060
G. Ferreiro and C Pajares J. Phys. G: Nucl. Part. Phys. 23 1961 (1997)

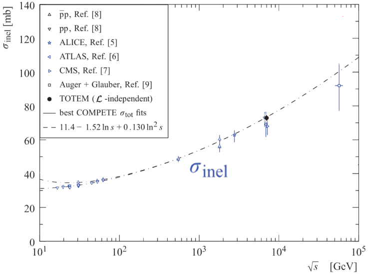
p-p interaction: parameter fixing

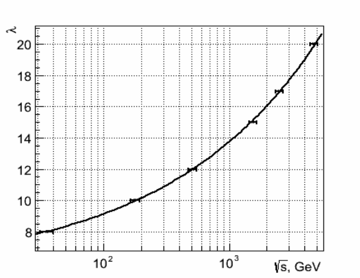
Strategy for parameters fixing:

- Correspondence of mean number of dipoles λ and energy is obtained using data on total inelastic cross section

- Performed for each parameters combination and tabulated

σ_{inel}, mb





Parameter range

- r_0: 0.4 – 0.7 fm
- r_{max}/r_0: 0.3 – 0.6
- α_S: 0.2 – 2.8
- r_{str}: 0 (no fusion); 0.2-0.6 fm
- Energy range: 53 – 13000 GeV

How number of partons depends on the energy?

Glauber on the parton level

- Let's take Glauber option first:
 assume that target and projectile protons are ensambles of *A* and *B* partons, colliding at some transverse distance

- The collision probability and cross-section:

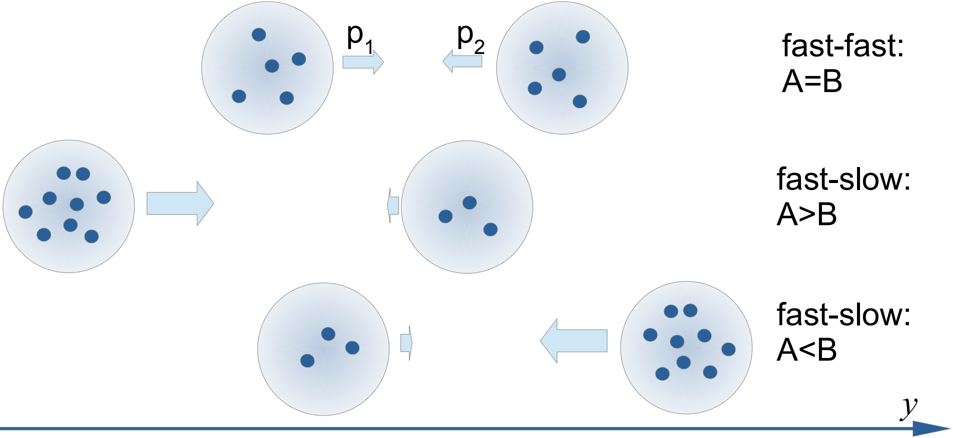
$$P(\mathbf{b}) = \sum_{n=1}^{AB} P(n, \mathbf{b}) = 1 - (1 - T(\mathbf{b})\sigma_{inel}^{pp})^{AB}.$$

$$\frac{d\sigma_{inel}^{AB}}{d\mathbf{b}} = P(\mathbf{b}) => \sigma_{inel}^{AB} = \int d\mathbf{b}\,(1 - (1 - T(\mathbf{b})\sigma_{inel}^{pp})^{AB})$$

- Cross section depends on the product *AB*

Glauber on the parton level

- A depends on the proton momentum: A=f(p_1),
  B on the other proton momentum: B=f(p_2).
- Cross-section must be Lorentz-invariant



Glauber on the parton level

- Replace momentum by the rapidity:

$A = g(y_1)$; $B = g(y_2)$. $y_1 + y_2 = Y = const$

$\sigma_{inel}(A, B) = \sigma_{inel}(g(y), g(Y-y)) = const(y)$

$g(y) \cdot g(Y-y) = const$

$g'(y) \cdot g(Y-y) — g(y) \cdot g'(Y-y) = 0$

$g(y) = b \cdot e^{ay}$

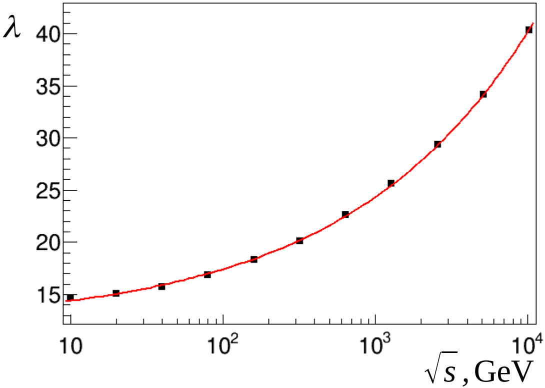
- Power-low energy dependence of the number of partons:

$A = f(E) = E^{a} \cdot b$

V. P. Mikhailovsky, V. N. Kovalenko, Nucleus-2020. Phys. Part. Nucl. (in press), 2021

Energy dependence of the number of partons in dipole-based model

- r_0: =0.6 fm
- r_{max}/r_0 =0.5
- α_S: = 0.4



$$\lambda = (\sqrt{s})^a \cdot b + c$$

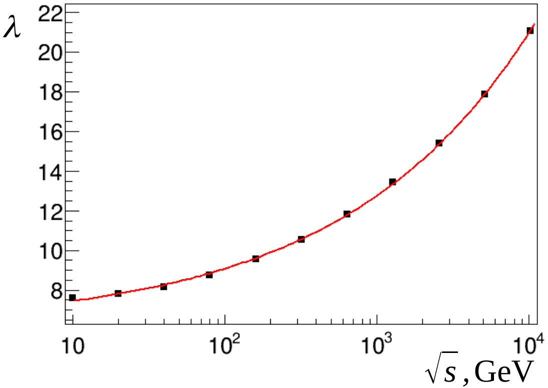
$$a = 0.36 \pm 0.01$$

$$b = 1.1 \pm 0.1$$

$$c = 12.1 \pm 0.2$$

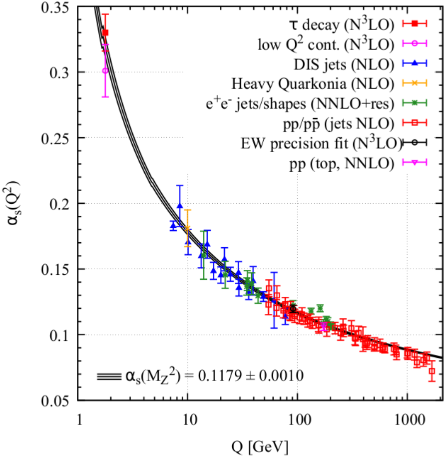
18 Energy dependence of the number of partons in dipole-based model

- r_0: =0.6 fm
- r_{max}/r_0 =0.5
- α_S: = 0.9



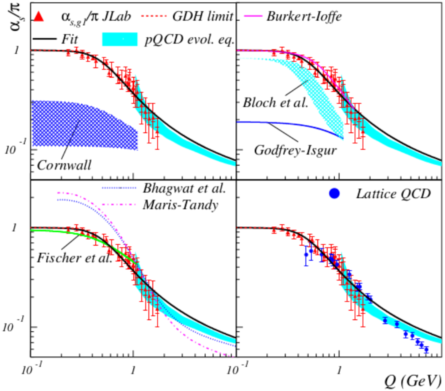
$$\lambda = (\sqrt{s})^a \cdot b + c$$
$$a = 0.35 \pm 0.01$$
$$b = 0.6 \pm 0.1$$
$$c = 6.1 \pm 0.1$$

More definite parameters: running coupling constant

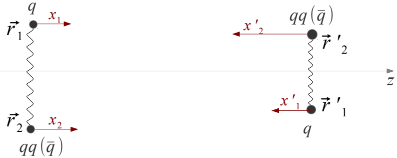


P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

More definite parameters: running coupling constant



A. Deur, AIP Conf.Proc.1149:281-284, 2009, arXiv:0901.2190 [hep-ph]
Effective strong coupling constant at large distances



$$f=\frac{\alpha_s^2}{2}\left[K_0\left(\frac{|\vec{r}_1-\vec{r}_1{}'|}{r_{max}}\right)+K_0\left(\frac{|\vec{r}_2-\vec{r}_2{}'|}{r_{max}}\right)-K_0\left(\frac{|\vec{r}_1-\vec{r}_2{}'|}{r_{max}}\right)-K_0\left(\frac{|\vec{r}_2-\vec{r}_1{}'|}{r_{max}}\right)\right]^2$$

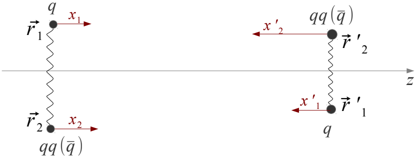
More definite parameters: running coupling constant

- The **hardness** of the elementary collisions is defined by transverse size of dipoles:

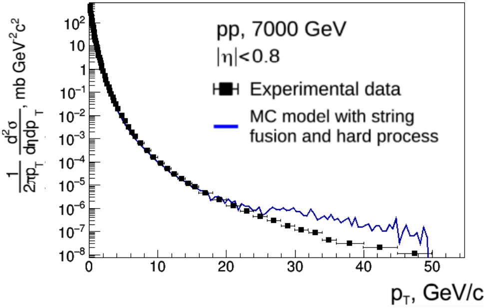
$$d_{1i} = |\vec{r}_1 - \vec{r}_2|, \; d_i' = |\vec{r}_1' - \vec{r}_2'|$$

- Mean transverse momentum of a cluster of k strings:

$$p_{T\,str\,i}^{\;2} = \frac{1}{d_i^2} + \frac{1}{d_i'^2} + p_0^{\;2}$$



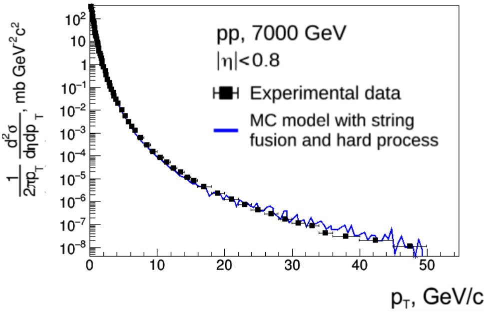
More definite parameters: running coupling constant



Fixed effective strong coupling

Experimental data:
B. Abelev, et. al. (ALICE Collaboration), Eur. Phys. J. C 73 (2013) 2662, arXiv:1307.1093 [nucl-ex]

More definite parameters: running coupling constant



Running effective strong coupling

Experimental data:
B. Abelev, et. al. (ALICE Collaboration), Eur. Phys. J. C 73 (2013) 2662, arXiv:1307.1093 [nucl-ex]

Conclusions and Outlook

Conclusions

The collision energy dependence of the number of partons can be described by power-law formula, allowing making predictions at higher energies

At large transverse momentum the spectra is better described with taking into account the Q- running of the strong coupling

Outlook

The fully theoretical explanation of the energy dependence of the number of dipoles λ.

Update in the model parameter tuning with reduced freedom in the parameters

The end

Thank you!

Backup

References

- V.Kovalenko. Modelling of exclusive parton distributions and long-range rapidity correlations for pp collisions at the LHC energy
  accepted at Phys. Atom. Nucl. Vol. 93, N 10 (2013)
  arXiv:1211.6209 [hep-ph]

- V.Kovalenko, V.Vechernin. Model of pp and AA collisions for the description of long-range correlations
  PoS (Baldin ISHEPP XXI) 077
  arXiv:1212.2590 [nucl-th]

Confinement

- We have to introduce a new parameter – r_{max}
- Confinement effects can be taken into account by the replacement of the Coulomb propagator $\Delta(\vec{r}) = \int \frac{d^2\vec{k}}{(2\pi)^2}\frac{e^{i\vec{k}\cdot\vec{r}}}{k^2}$, by the Yukawa one: $\frac{1}{k^2+M^2}$, where $M = 1/r_{max}$ is the confinement specific scale.
- As a result, we get for the probability amplitude the following:

$$f = \frac{\alpha_s^2}{2}\Big[K_0\left(\frac{|\vec{r}_1-\vec{r}_3|}{r_{max}}\right) + K_0\left(\frac{|\vec{r}_2-\vec{r}_4|}{r_{max}}\right) - \\ -K_0\left(\frac{|\vec{r}_1-\vec{r}_4|}{r_{max}}\right) - K_0\left(\frac{|\vec{r}_2-\vec{r}_3|}{r_{max}}\right)\Big]^2 \qquad (4)$$

- Squared ratio of the quark and hadron radiuses should be about $\frac{1}{10}$. It leads $r_{max} \simeq 0.2 - 0.3 fm$.

p-p interaction: color dipoles

- The probability amplitude for the collision of two dipoles with coordinates (r_1,r_2), (r_3,r_4) [3,4]:

$$f = \frac{\alpha_S^2}{2} \ln^2 \frac{|\vec{r_1} - \vec{r_3}| \cdot |\vec{r_2} - \vec{r_4}|}{|\vec{r_1} - \vec{r_4}| \cdot |\vec{r_2} - \vec{r_3}|}$$

- Confienment is taken into account by introduction of some cut off at $r_{max} \simeq 0.2 - 0.3$fm. It leads:

$$f = \frac{\alpha_s^2}{2}\Big[K_0\left(\frac{|\vec{r}_1 - \vec{r}_3|}{r_{max}}\right) + K_0\left(\frac{|\vec{r}_2 - \vec{r}_4|}{r_{max}}\right) - \\ -K_0\left(\frac{|\vec{r}_1 - \vec{r}_4|}{r_{max}}\right) - K_0\left(\frac{|\vec{r}_2 - \vec{r}_3|}{r_{max}}\right)\Big]^2$$

- The total probability of the inelastic interaction of two protons in the eikonal approximation:

$$p = 1 - e^{-\sum\limits_{i,j} f_{ij}}$$

[3] G. Gustafson, Acta Phys. Polon. B40, 1981 (2009)
[4] C. Flensburg, G. Gustafson, and L. Lonnblad, Eur. Phys. J. (C) 60, 233 (2009)

p-p interaction: string fusion

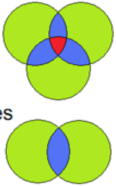
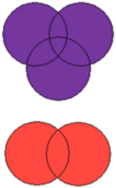
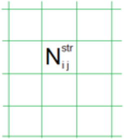
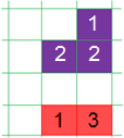
The interaction of colour strings in transverse plane is carried out in the framework of local string fusion model [5]
with the introduction of the lattice in the impact parameter plane.
The finite rapidity length of strings is taken into account [6-8].

$$\langle \mu \rangle_k = \mu_0 \sqrt{k} \frac{S_k}{\sigma_0} \qquad \langle p_t^2 \rangle_k = p_0^2 \sqrt{k} \qquad \langle p_t \rangle_k = p_0 \sqrt[4]{k}$$

Sk – area, where k strings are overlapping, σ_0 single string transverse area, μ_0 and p_0 – mean multiplicity and transverse momentum from one string

[5] Braun, M.A. and Pajares, C. Eur. Phys. J. (C),16,349,2000
[6] V. Vechernin and R. Kolevatov, Physics of Atomic Nuclei 70, 1797 (2007)
[7] V. Vechernin and R. Kolevatov, Physics of Atomic Nuclei 70, 1809 (2007)
[8] Vechernin, V. V. and Kolevatov, R. S., Simple cellular model of long-range multiplicity and pt correlations in high-energy nuclear collisions 2003 http://arxiv.org/abs/hep-ph/0304295v1

string fusion mechanism versions

	"overlaps" (local fusion)	"clusters" (global fusion)
SFM	○ $C=\{S_1,S_2,\dots\}$ S_k – area covered k-times  S_1 S_2 S_3	● $C=\left\{\begin{matrix} S_1^{cl}, S_2^{cl},\dots \\ N_1^{str}, N_2^{str},\dots \end{matrix}\right\}$ $k_i^{cl}=\frac{N_i^{str}\cdot\sigma_0}{S_i^{cl}}$  $N_1^{str}=3$ S_1^{cl} $N_2^{str}=2$ S_2^{cl}
cellular analog of SFM	□ $C=\{N_{ij}^{str}\}$  $k_{ij}=N_{ij}^{str}$ – "occupation" numbers	■ $C=\left\{\begin{matrix} S_1^{cl}, S_2^{cl},\dots \\ N_1^{str}, N_2^{str},\dots \end{matrix}\right\}$ $k_i^{cl}=\frac{N_i^{str}\cdot\sigma_0}{S_i^{cl}}$  $S_1^{cl}=3\sigma_0$; $N_1^{str}=5$; $k_1^{cl}=5/3$ $S_2^{cl}=2\sigma_0$; $N_2^{str}=4$; $k_2^{cl}=2$

Glauber Model [2, 3]

- Nucleus-Nucleus collision is a sequence of nucleons collisions
- Nucleons are distributed according to Woods-Saxon:

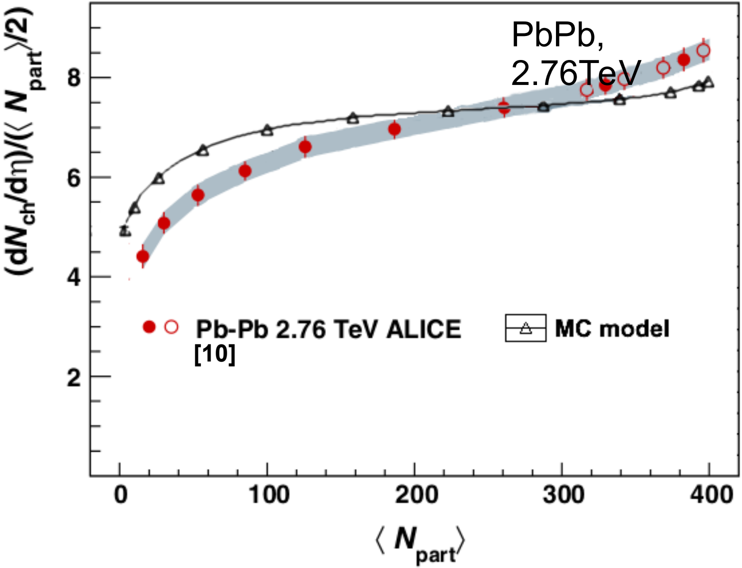
$$\rho(r) = \rho_0 \frac{1}{1 + exp\left(\frac{r-R}{\alpha}\right)}$$

- Trajectories of nucleons are linear
- Each nucleus can collide several times with the same inelastic cross section: ${\sigma_{inel}}^{nn}$=const

  corresponding to proton-proton inelastic cross section
- Energy loss due to particle production is not considered

[2] Bialas A, Bleszynski M, Czy W. Nucl.Phys.B 111:461,1976; Glauber RJ. Nucl. Phys.A 774:3, 2006
[3] M. L. Miller, K. Reygers, S. J. Sanders, P. Steinberg. Ann. Rev. Nucl. Part. Sci., 57:205–243, 2007

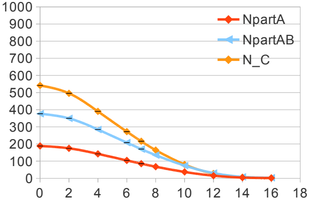
AA interaction: charged multiplicity

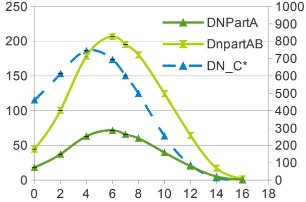


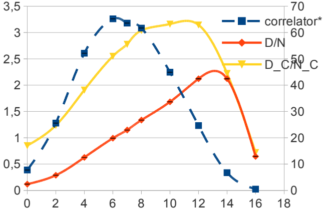
[10] K. Aamodt et al (ALICE Collaboration).Phys. Rev. Lett.,106, 032301, 2011.

AA interactions
Compare with Glauber's model

Number of participant, number of binary collisions, their variations and scaled variatons and correlator for σ^{inel}_{NN} =34mb, calculated in the *model of this work*:







The same for the *Glauber's model* (σ_{NN} =34mb) :