

CLUSTERING AND MICROSCOPICALLY SEPARATED STATES FORMATION IN FISSION POTENTIAL ENERGY CALCULATIONS

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Outline

- Introduction
- Experimental evidence of clustering in fission
- Theoretical background and models of clustering
- Cluster formation model
- Conclusions

Section 1

INTRODUCTION

Overview

V. V. Pashkevich, Yu. V. Pyatkov, A. V. Unzhakova, *et al.*:

- **Manifestation of clustering in the $^{252}\text{Cf}(\text{sf})$ and $^{249}\text{Cf}(\text{n}_{\text{th}},\text{f})$ reactions,** Nuclear Physics A624, 1997
- **Collinear cluster tripartition of actinides: mass-energy correlations of fragments,** Conf. Clustering phenomena in nuclear physics, St. Petersburg, 2000
- **Nontrivial manifestation of clustering in fission of heavy nuclei at low and middle excitations,** Physics of Atomic Nuclei 67, 2004
- **Structure of Fission Potential-Energy Surfaces in Ten-Dimensional Deformation Spaces,** 4th Int. Conf. on Fission and Properties of Neutron-Rich Nuclei, Sanibel Island, 2007

Section 2

EXPERIMENTAL EVIDENCE OF CLUSTERING IN FISSION

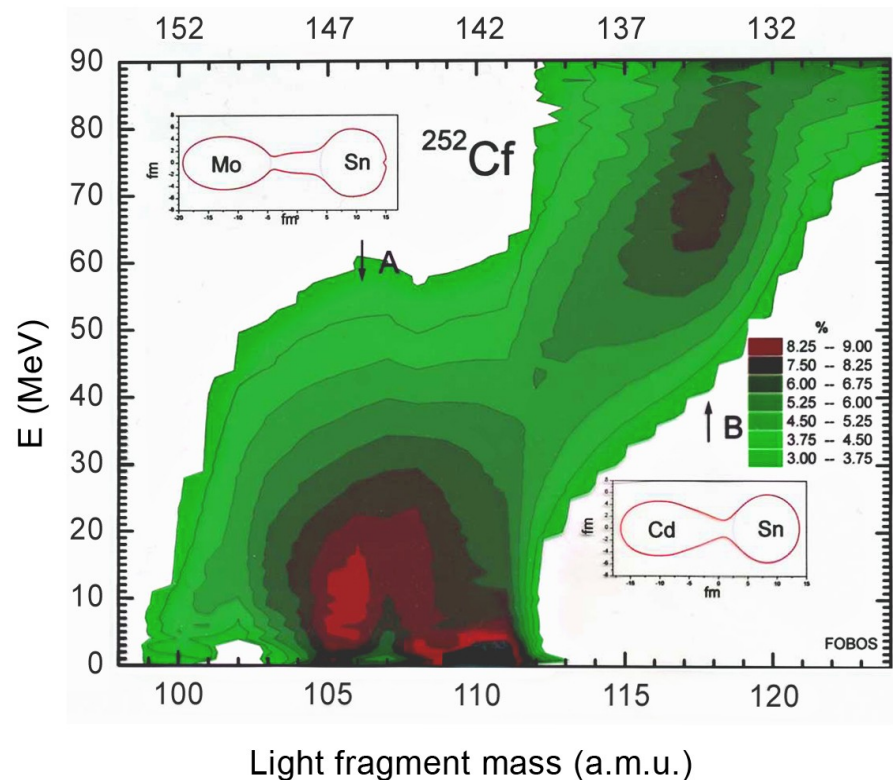
Clustering in Binary Fission

The contour map of the conditional distribution $P(M | E^*)$.

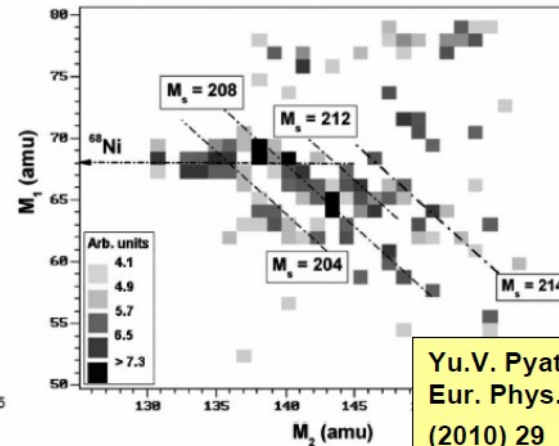
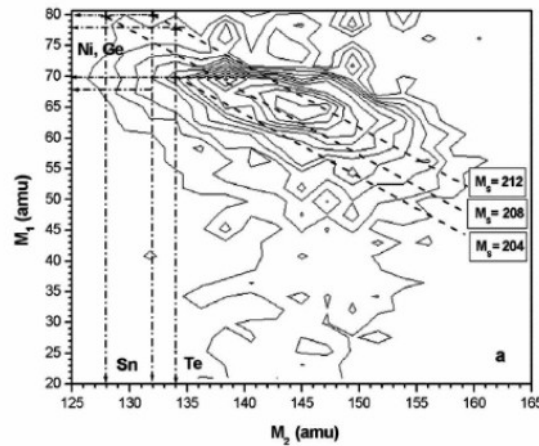
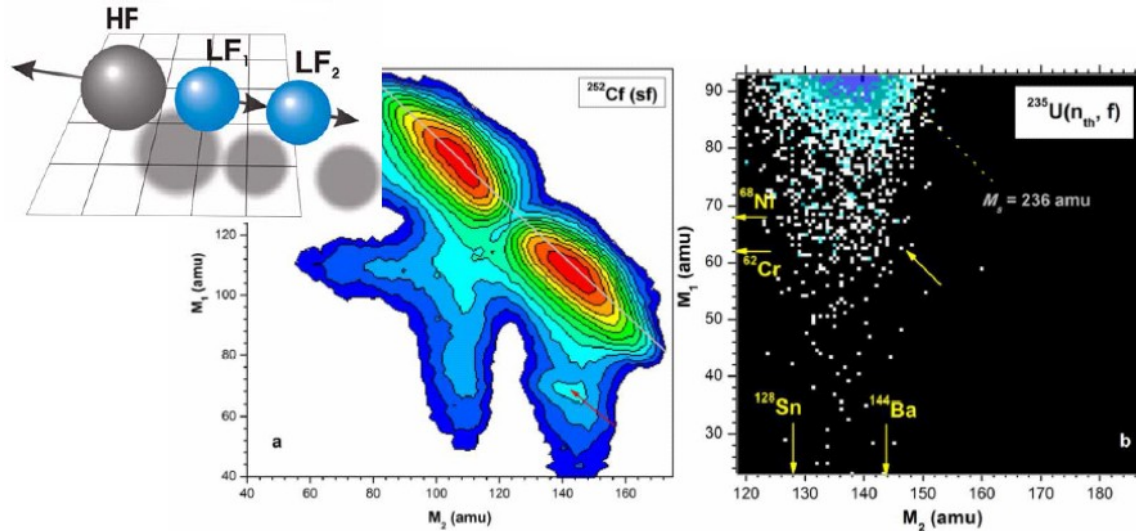
The panels depict the shapes of the fissioning system in the ten-dimensional Potential Energy Surface calculations ascribed to the two dominant cluster structures.

Manifestation of clustering in the $^{252}\text{Cf}(sf)$ and $^{249}\text{Cf}(n_{th},f)$ reactions.

Nuclear Physics A624, 1997



Clustering in Ternary Fission

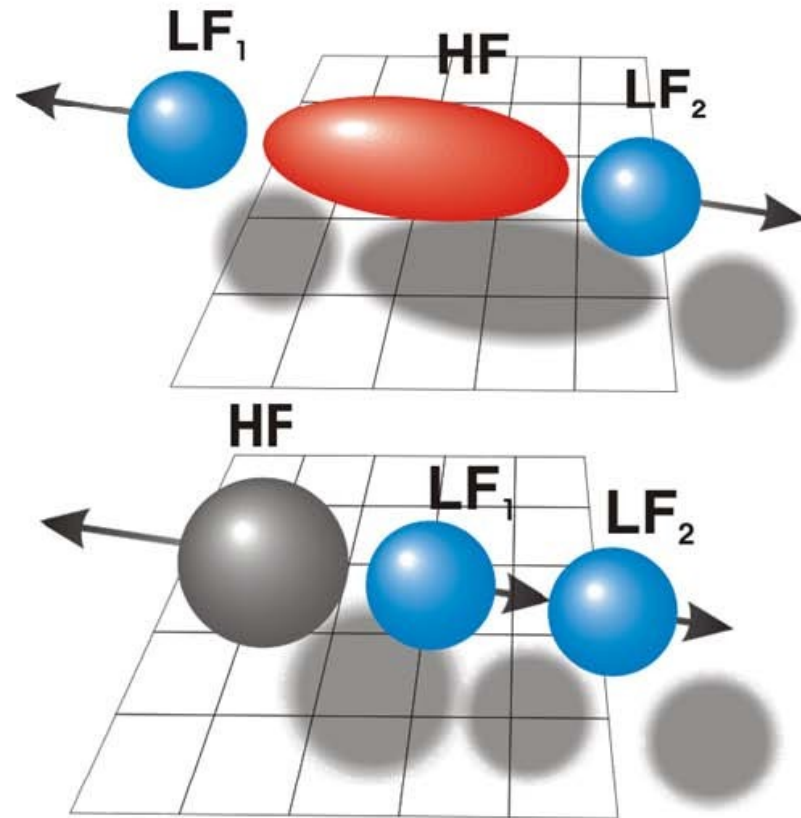
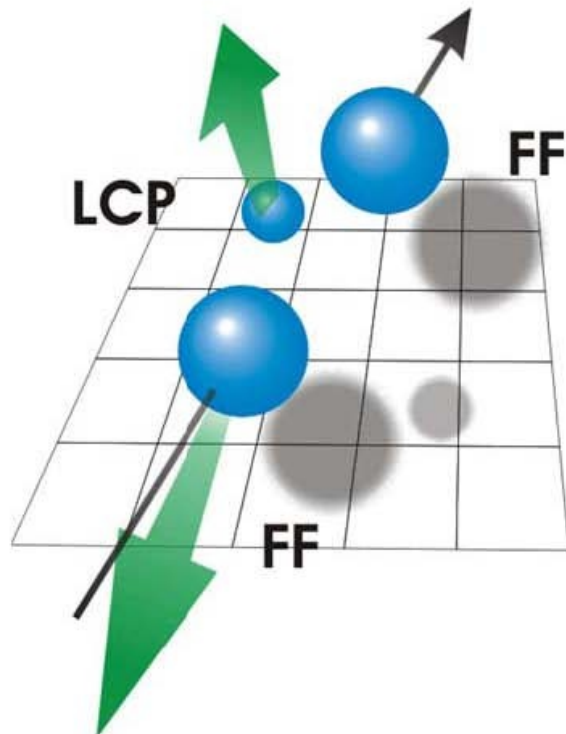


Yu.V. Pyatkov et al.,
Eur. Phys. J. A 45
(2010) 29

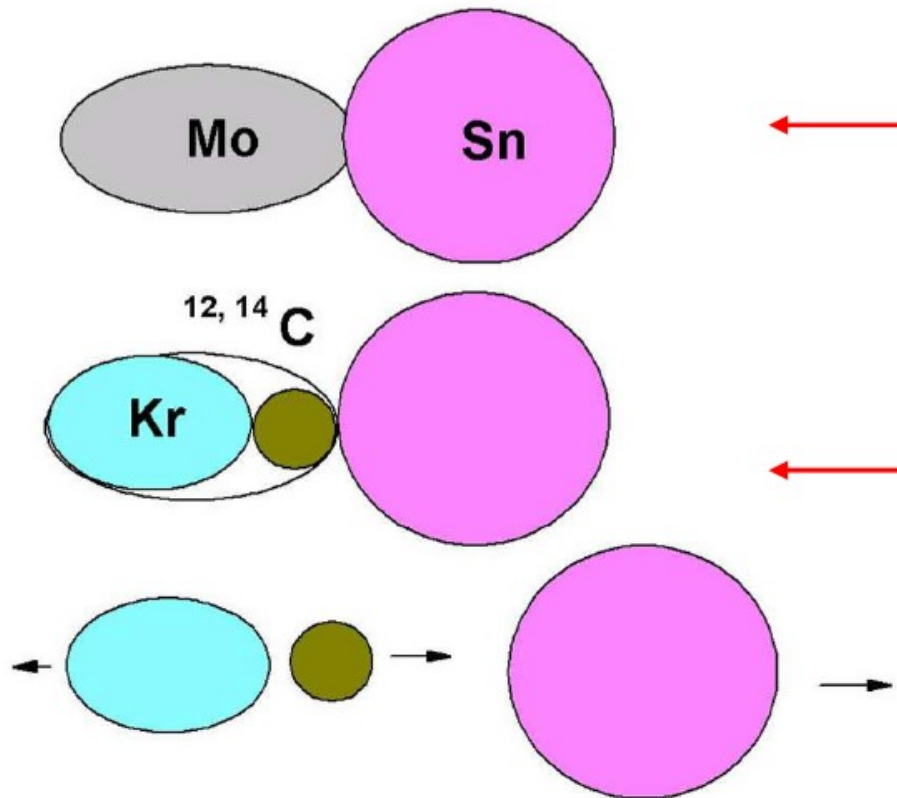
CCT Channels

Collinear Cluster Tripartition

Conventional ternary fission



Secondary Clusterization



- At some elongation stage the fissioning system *clusterizes* into two magic constituents such as Sn/Mo, Cd/Ru etc.
- Each initial constituent *reclusterizes* with elongation (secondary clusterization), forming lighter magic cluster and at least one light particle.

D.V. Kamanin, Y.V. Pyatkov,

Clusterization in ternary fission

Clusters in Nuclei, Volume 3, 2014 - Springer

Yu. V. Pyatkov, D. V. Kamanin, A. A. Alexandrov, I. A. Alexandrova, Z. I. Goryainova, V. Malaza, N. Mkaza, E. A. Kuznetsova, A. O. Strelakovsky, O. V. Strelakovsky, and V. E. Zhuchko,

Examination of evidence for collinear cluster tri-partition

Physical Review C 96, 064606, 2017

Yu.V. Pyatkov, D. V. Kamanin, Yu. M. Tchuvil'sky, A. A. Alexandrov, I. A. Alexandrova, Z. I. Goryainova, V. Malaza, E. A. Kuznetsova, A. O. Strelakovsky, O. V. Strelakovsky, Sh. Wyngaardt, V. E. Zhuchko,

Refined scenario of the collinear cluster tri-partition mode with the greatest yield

Nuclear Experiment, 2020 <https://arxiv.org/abs/2003.08591v2>

Yu.V. Pyatkov, D.V. Kamanin, A.A. Alexandrov, I.A. Alexandrova, Z.I. Goryainova, E.A. Kuznetsova, A.O. Strelakovsky, O.V. Strelakovsky, V.E. Zhuchko, V. Malaza,

Manifestations of Pear-Shaped Clusters in Collinear Cluster Tri-Partition of ^{252}Cf

Exotic Nuclei, pp. 338-344, 2019

Section 3

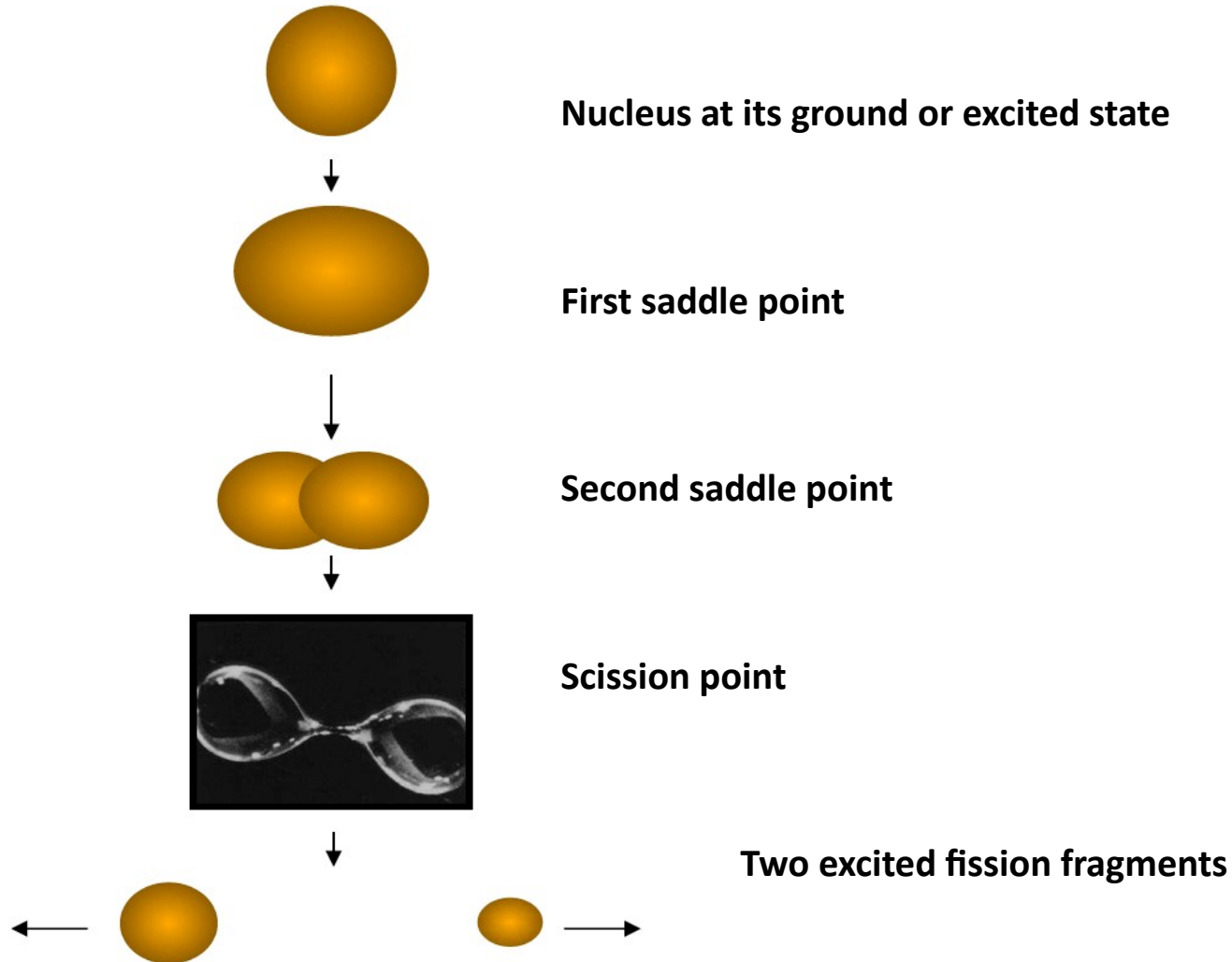
THEORETICAL BACKGROUND AND MODELS OF CLUSTERING

Macroscopic-Microscopic Approach

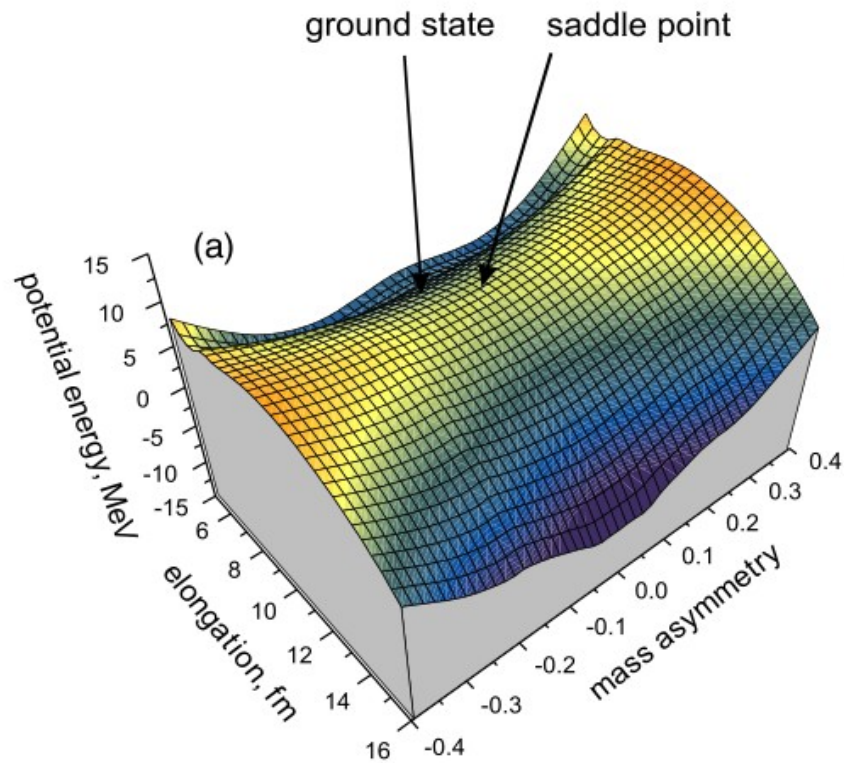
- The macroscopic-microscopic method presents a relatively easy alternative calculational approach that takes advantage of a characteristic feature of nuclear properties that is well brought out for nuclear masses: There is an overall smooth trend overlaid by relatively small deviations from the average.
- The central idea is, therefore, to write the potential energy of a nuclear system as a sum of a smooth term, $E_{\text{macro}}(Z, N, \text{shape})$, and a fluctuating correction term that reflects the shell and pairing effects associated with the particular level scheme at the specified shape:

$$E(Z, N, \text{shape}) = E_{\text{macro}}(Z, N, \text{shape}) + E_{\text{micro}}(Z, N, \text{shape})$$

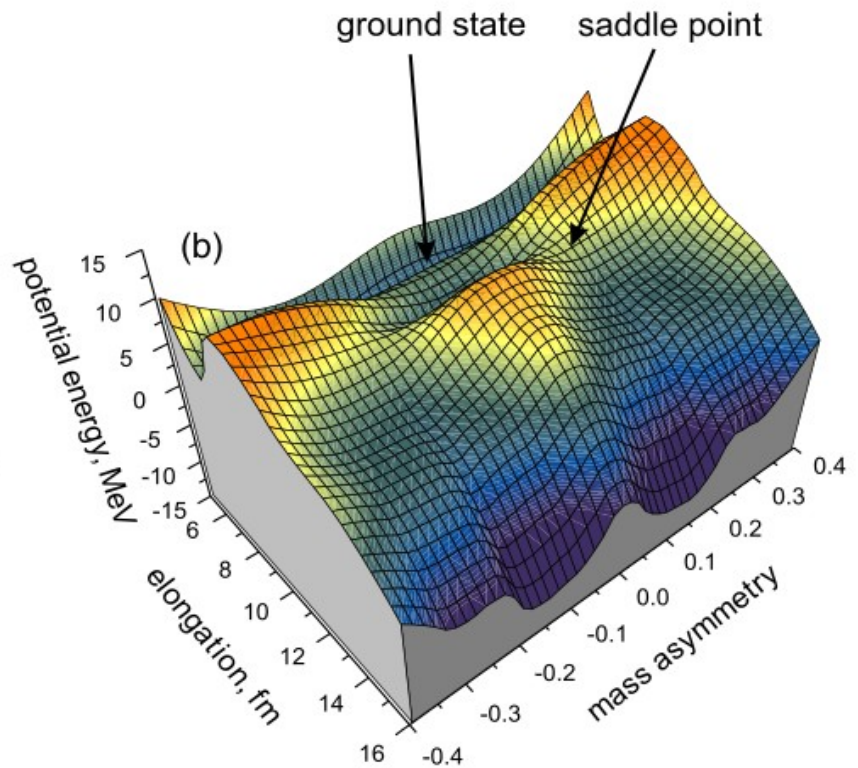
Fission in the Liquid Drop Model



Macroscopic-microscopic model

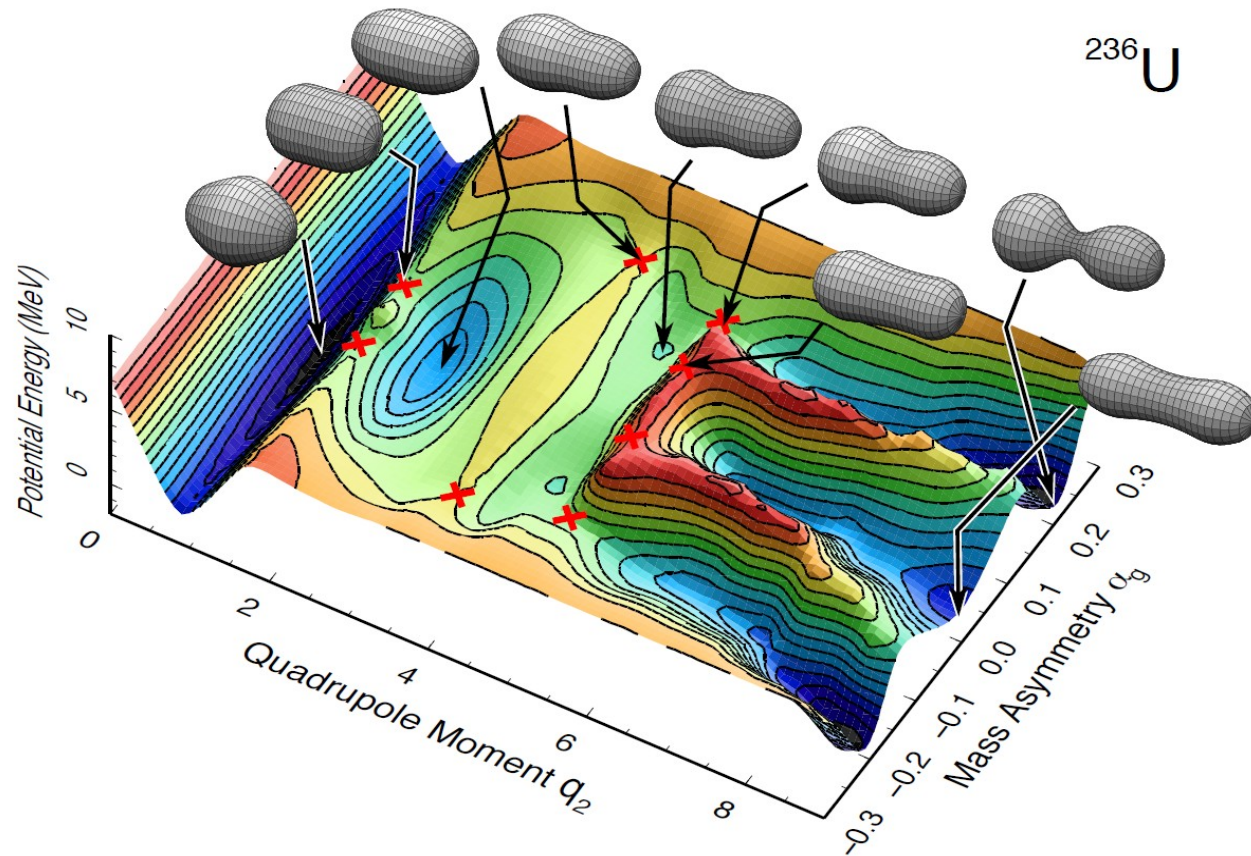


liquid-drop potential



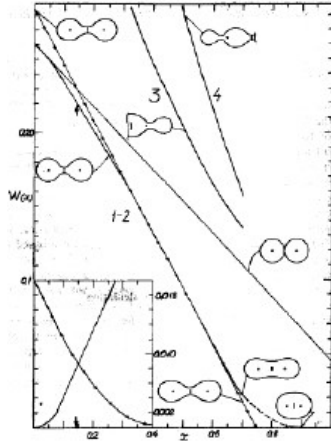
plus shell effects

Potential Energy Surface



A Predictive Theory for Fission. A. J. Sierk, Peter Moller, John Lestone

Theoretical background

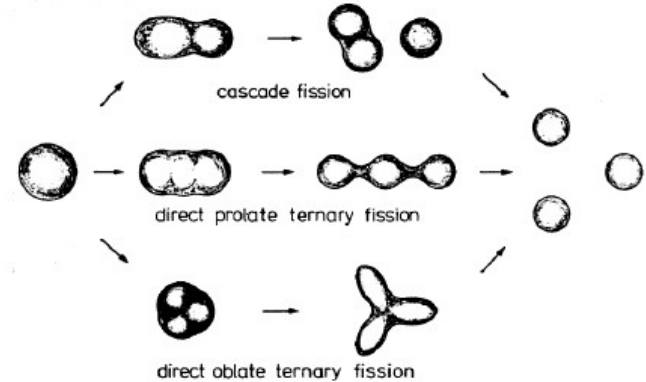


SYMMETRICAL SHAPES OF EQUILIBRIUM FOR A LIQUID DROP MODEL

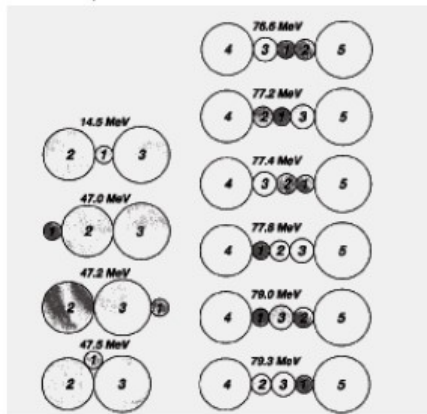
V.M. STRUTINSKY, N.Ya. LYASHCHENKO and N.A. POPOV

Nucl. Phys. **46** (1963) 639

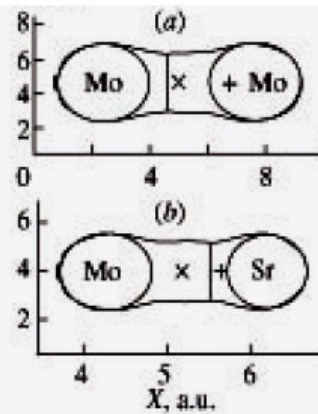
two-neck and three-neck shapes



H. Diehl & W. Greiner,
Nuclear Physics **A229** (1974)



Aligned and compact configurations for α -accompanied and $\alpha+{}^6\text{He}+{}^{10}\text{Be}$ accompanied cold fission of ${}^{252}\text{Cf}$
D.N. Poenaru et al.,
Phys. Rev. C **59** (1999) 3457



Yu.V. Pyatkov, V.V. Pashkevich, A.V. Unzhakova et al.,
Physics of Atomic Nuclei **66** (2003) 1631

Three-clustering
In quasi-fission
V. Zagrebaev,
2007

Section 4

CLUSTER FORMATION MODEL

Basic Notations of the Model

Assume vector $x_i \in \mathbb{R}^m$ denotes the characteristics of particle $i \in \mathcal{N}$. Neighboring particles in a considered volume interact with each other. The connections among the neighboring particles could be represented by graph $\mathcal{G} = \{\mathcal{N}, \mathcal{E}, C\}$

- with set of vertices \mathcal{N} denoting particles $i, i \in N$,
- set of edges $\mathcal{E} = \{(i, j) : i, j \in \mathcal{N}\}$ comprised of pairs (i, j) denoting interactions between particles i and j , corresponding to connections in graph \mathcal{G} ;
- and connectivity matrix C with elements $c_{ij} = 1$ in case the particles i and j are connected and $c_{ij} = 0$ otherwise.

System Dynamics Model

Let us introduce an external disturbance z_i affecting particle x_i . The dynamics of the system of particles is given by the following set of \mathcal{N} equations in discrete time:

$$x_i^{t+1} = x_i^t + z_i^t + \gamma \sum_{j \in \mathcal{N}_i^t} c_{ij}^t (x_j^t - x_i^t) =$$

$$x_i^t + z_i^t + \gamma \left(\sum_{j \in \mathcal{N}_i^t} c_{ij}^t x_j^t \right) - \gamma d_i(C^t) x_i^t, \quad i \in \mathcal{N}, \quad (1)$$

where $d_i(C) = \sum_{j=1}^n c_{ij}$ is the weighted sum of i -th row of matrix C . Denote $D(C) = \text{diag}\{d_i(C)\}$ the corresponding diagonal matrix.

Graph Laplacian Potential

The disagreement in the characteristics among the particles in the fissioning system could be defined as the value of the *Laplacian potential* of the graph \mathcal{G} :

$$\Phi_{\mathcal{G}}(x) = \frac{1}{2} \sum_{j \in \mathcal{N}^i} c_{ij} (x_j - x_i)^2$$

where \mathcal{N}^i is the set of neighboring particles of particle i .

System Dynamics Model in Matrix Form

Define the \mathbb{R}^{nm} -valued vectors $X_t = \{x_1, \dots, x_m\}$ and $Z_t = \{z_1, \dots, z_m\}$ composed of corresponding vectors x_t^i and z_t^i .
The system dynamics in matrix form:

$$X_{t+1} = X_t + \gamma(C_t \otimes I_k)X_t - \gamma(D(C_t) \otimes I_m)X_t + Z_t, \quad (2)$$

where $C_t \otimes I_m$ is the Kronecker product the $nm \times nm$ which is the block matrix:

$$C_t \otimes I_m = \begin{bmatrix} c_t^{1,1} I_m & \cdots & c_t^{1,n} I_m \\ \vdots & \ddots & \vdots \\ c_t^{n,1} I_m & \cdots & c_t^{n,n} I_m \end{bmatrix}.$$

The proposed model describes the evolution of the nuclear system consisting of the interconnected interacting elements corresponding to particles.

In the considered system connections exist only among the neighboring particles.

The system with the given dynamics demonstrates the clusterization and reclusterization during the evolution of its state.

Section 5

CONCLUSIONS

In the process of fission, one centered nuclear system transforms into two or three new nuclear systems of future fragments. This transition is the result of the new nuclear shell structures emerging in several regions of the multidimensional potential energy surface.

It is the formation, change and development of the multi-cluster system with main elongation of the nuclear system that determines fission multimodality.

It was shown that nonequilibrium fluctuations in the collective motion of nucleons are responsible for clusterization process and microscopically separated states formation .

Theoretical description of reclusterization dynamics is beyond the frame of realistic mean field approaches to fission and requires specific dynamical modeling. A new type of microscopical model aimed to describe the cluster formation inside the fissioning system is suggested.

T. Vicsek, A. Czirok, E. Ben-Jacob, I. Cohen, O. Shochet,
Novel Type of Phase Transition in a Systems of Self-Driven Particles
Phys. Rev. Lett., Vol. 75, #6, p.1226-1229, 1995

T. Vicsek, A. Zafeiris,
Collective Motion
Physics Reports 517. D, Vol. 517, #3-4, p.71, 2012

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Self-organization and adaptation in smart coating modeling
Smart NanoMaterials 2018: Advances, Innovation and Applications, 10-13 December, Paris, p.
78, 2018

A.V. Unzhakova , T.A. Khantuleva, and O.N. Granichin,
Cluster degrees of freedom in fission of actinides
Fission and Properties of Neutron-Rich Nuclei: Proceedings of the Sixth International Conference
on ICFN6 pp. 582-589, 2018

K. Amelin, N. Amelina, O. Granichin, O. Granichina, and Y. Ivanskiy,
**Synchronization of Multi-Agent System of “Feathers” on the Surface of the Wing in a Turbulent
Airflow**
2019 IEEE Conference on Control Technology and Applications, pp. 355-359