Strongly intense observables as a tool for studying clusters of quark-gluon strings in relativistic hadronic interactions

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The string model

A. Capella, U. Sukhatme, Chung-I Tan, J. Tran Thanh Van, Phys. Lett. B 81, 68 (1979).
A.B. Kaidalov, Phys. Lett. B 116, 459 (1982).
A.B. Kaidalov, K.A.Ter-Martirosyan, Phys. Lett., 117B (1982) 247.
A. Capella, U. Sukhatme, Chung-I Tan, J. Tran Thanh Van, Phys. Rep. 236 (1994) 225.

First stage: colour quark-gluon strings (colour flux tubes) are formed
Second stage: hadronization of these strings produces the observed hadrons

A. Capella and A. Krzywicki, Phys.Rev.D**18**, 4120 (1978)

Motivation. String fusion effects.

pp $\rightarrow$ pA $\rightarrow$ AA - the increase of the string density in transverse plain leads to the string fusion (color ropes or string cluster formation)

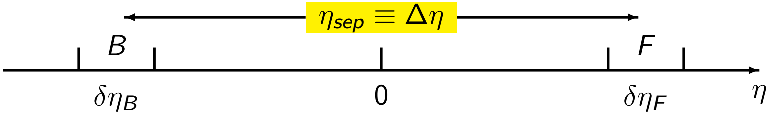
T.S. Biro, H.B. Nielsen, J. Knoll, Nucl. Phys. B **245**, 449 (1984)
A. Bialas, W. Czyz, Nucl. Phys. B **267**, 242 (1986)
M.A. Braun,C. Pajares, Phys.Lett. **B287**, 154 (1992);
Nucl. Phys. **B390**, 542 (1993)

The same in pp collisions with increasing energy and centrality

LHC !

$\Rightarrow$ Reduction of multiplicity, increase of transverse momenta.
$\Rightarrow$ The influence on the Long-Range FB Correlations (LRC).
N.S. Amelin, N. Armesto, M.A. Braun, E.G. Ferreiro, C. Pajares, Phys.Rev.Lett. **73**, 2813 (1994).

Forward-Backward (FB) Rapidity Correlations



Forward-Backward (FB) Rapidity Correlations: $(k_z, \mathrm{k}_\perp) \Rightarrow (\eta, \mathrm{k}_\perp)$

$$\eta \equiv \frac{1}{2} \ln \frac{k_0 + k_z}{k_0 - k_z} \,, \qquad \eta' \equiv \frac{1}{2} \ln \frac{|\mathrm{k}| + k_z}{|\mathrm{k}| - k_z} = - \ln \mathrm{tg} \left(\frac{\theta^*}{2} \right)$$

The correlation coefficient:

$$b_{BF} = \frac{\langle FB \rangle - \langle F \rangle \langle B \rangle}{\langle F^2 \rangle - \langle F \rangle^2} = \frac{cov(F, B)}{D_F} \tag{1}$$

Short- and long-range rapidity correlations

Traditional Observables

Traditional FB correlation:

B, $F \Rightarrow n_B$, n_F - the extensive variables $\Rightarrow b_{nn}$

A. Capella and A. Krzywicki, Phys.Rev.D**18**, 4120 (1978)

Unfortunately for traditional observables
the n_F-n_B correlation is strongly influenced by the "volume" fluctuations.

We'll look for observables, which is not sensitive to the fluctuation in the number of sources (strings), but is sensitive to the fluctuation in the quality of sources (e.g. to the formation of string clusters by string fusion).

The strongly intensive observable $\Sigma(n_F, n_B)$

The strongly intensive quantities [*M.I.Gorenstein,M.Gazdzicki,Phys.Rev.C84(2011)014904*].
We define the strongly intensive observable $\Sigma(n_F, n_B)$ between multiplicities in forward (n_F) and backward (n_B) windows [*E.V.Andronov,Theor.Math.Phys.185(2015)1383*] as

$$\Sigma(n_F, n_B) \equiv \frac{1}{\langle n_F \rangle + \langle n_B \rangle}[\langle n_F \rangle \omega_{n_B} + \langle n_B \rangle \omega_{n_F} - 2\,\mathrm{cov}(n_F\, n_B)]\ , \qquad (2)$$

where

$$\mathrm{cov}(n_F, n_B) \equiv \langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle\ , \qquad (3)$$

and ω_{n_F} and ω_{n_B} are the corresponding scaled variances of the multiplicities:

$$\omega_n \equiv \frac{D_n}{\langle n \rangle} = \frac{\langle n^2 \rangle - \langle n \rangle^2}{\langle n \rangle}\ . \qquad (4)$$

Σ in the model with independent identical strings

The fundamental characteristics of a string:
one- and two-particle rapidity distributions from a single string decay:

$$\lambda(\eta) = \mu_0, \qquad \lambda_2(\eta_1, \eta_2) = \lambda_2(\eta_1 - \eta_2) = \lambda_2(\Delta\eta)$$

$\Lambda(\Delta\eta)$ - two-particle correlation function of a string:

$$\Lambda(\eta_1, \eta_2) \equiv \frac{\lambda_2(\eta_1, \eta_2)}{\lambda(\eta_1)\lambda(\eta_2)} - 1 = \frac{\lambda_2(\Delta\eta)}{\mu_0^2} - 1 = \Lambda(\Delta\eta) \ .$$

$\delta\eta$ - the width of the observation windows (below we suppose $\delta\eta \ll \eta_{corr}$),
$\Delta\eta = \eta_{sep}$ - the distance between the observation windows.

$$\boxed{\Sigma(\Delta\eta) = 1 + \mu_0 \delta\eta [\Lambda(0) - \Lambda(\Delta\eta)]}$$

Vechernin V 2018 Eur.Phys.J.: Web of Conf. 191 04011
Andronov E, Vechernin V 2019 Eur.Phys.J. A 55 14
Vechernin V, Andronov E 2019 Universe 5 15

Properties of Σ in model with independent identical strings

$$\Sigma(\Delta\eta) = 1 + \mu_0 \delta\eta [\Lambda(0) - \Lambda(\Delta\eta)]$$

- We see that in the model with identical strings the $\Sigma(\Delta\eta)$ is a really strongly intensive quantity. It does not depend nor on the mean number of strings $\langle N \rangle$, nor on their event-by-event fluctuations $\omega_N \equiv D_N / \langle N \rangle$.
It depends ONLY on string parameters: μ_0 and $\Lambda(\Delta\eta)$.

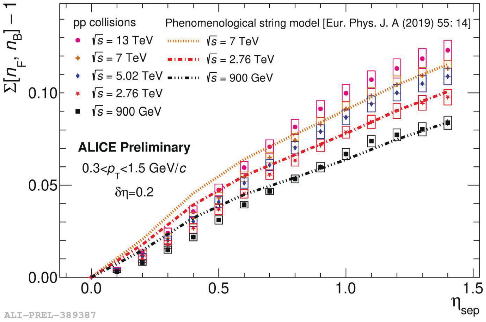
- The $\Sigma(0) = 1$ and increases with the gap between windows, $\Delta\eta$, as the $\Lambda(\Delta\eta)$ decrease to 0 with $\Delta\eta$, since the correlations in a string go off with increase of $\Delta\eta$.
- The rate of the $\Sigma(\Delta\eta)$ growth with $\Delta\eta$ is proportional to the width of the observation window $\delta\eta$ and μ_0 - the multiplicity produced from one string.
- The model predicts saturation of the $\Sigma(\Delta\eta)$ on the level

$$\Sigma(\Delta\eta) = 1 + \mu_0 \delta\eta \, \Lambda(0) = \omega_\mu = D_\mu / \langle \mu \rangle$$

at large $\Delta\eta$, since $\Lambda(\Delta\eta) \to 0$ at the $\Delta\eta \gg \eta_{corr}$, where the η_{corr} is a string correlation length.

Comparing the $\Sigma(n_F, n_B)$ with preliminary ALICE data

The comparison of the string model predictions with preliminary ALICE data for the $\Sigma(n_F, n_B)$ in pp collisions at energies 0.9 - 7 TeV [Andrey Erokhin (for the ALICE Collaboration) "Forward-backward multiplicity correlations with strongly intensive observables in pp collisions", The VI-th International Conference on the Initial Stages of High-Energy Nuclear Collisions (IS2021), 10-15 January 2021]:



"Phenomenological string model from [Eur.Phys.J.A55.1(2019),p.14] reproduces the quantitative behavior better than PYTHIA"

$\Sigma(n_F, n_B)$ in the model with string fusion

In the model with string fusion on transverse grid we find [*S.N. Belokurova, V.V.Vechernin, Theor.Math.Phys. 200(2019)1094*]:

$$\Sigma(n_F, n_B) = \sum_{k=1}^{\infty} \alpha_k \, \Sigma_k(\mu_F, \mu_B) \; , \qquad \alpha_k = \frac{\langle n^{(k)} \rangle}{\langle n \rangle} \; , \qquad (5)$$

where k is a degree of string overlapping and $\langle n^{(k)} \rangle$ is a mean number of particles produced from areas with such overlapping. $\sum \alpha_k = 1$.
Here $\Sigma_k(\mu_F, \mu_B)$ is the variable Σ for the cluster formed by k strings:

$$\Sigma_k(\mu_F, \mu_B) = \Sigma_k(\Delta\eta) = 1 + \mu_0^{(k)} \delta\eta [\Lambda_k(0) - \Lambda_k(\Delta\eta)] \; ,$$

where $\mu_0^{(k)}$ and $\Lambda_k(\Delta\eta)$ are the corresponding parameters of the string cluster.

$$\Lambda_k(\Delta\eta) = \Lambda_0^{(k)} exp[-|\Delta\eta| / \eta_{corr}^{(k)}]$$

$\Sigma(n_F, n_B)$ in the model with string fusion

For such string cluster, formed by k fused strings, we expect, basing on the string decay picture [*V.Vechernin*, Baldin ISHEPP XIX v.1(2008)276; arXiv:0812.0604]:

1) larger multiplicity from one string, $\mu_0^{(k)} > \mu_0$,
2) smaller correlation length, $\eta_{corr}^{(k)} < \eta_{corr}$.

This corresponds to the analysis of the net-charge fluctuations in the framework of the string model for pp and AA collisions *[A.Titov, V.Vechernin, PoS(Baldin ISHEPP XXI)047(2012)]*.

Both factors lead to the steeper increase of $\Sigma_k(\Delta\eta)$ with $\Delta\eta$ and its saturation at a higher level

That is in accordance with the energy dependence obtained above for $\Sigma(n_F, n_B)$ from the ALICE pp data.

$\Sigma(n_F, n_B)$ in the model with string fusion

[*M.A.Braun,C.Pajares* Nucl.Phys.B 390 (1993) 542]

$$\mu_0^{(k)} = \mu_0^{(1)}\sqrt{k} \ , \qquad \Lambda_0^{(k)} = \Lambda_0^{(1)} = const \ , \qquad \eta_{corr}^{(k)} = \eta_{corr}^{(1)}/\sqrt{k} \ ,$$

which is instructive to compare with

$$\mu_0^{(k)} = \mu_0^{(1)}\, k \ , \qquad \Lambda_0^{(k)} = \Lambda_0^{(1)}/k \ , \qquad \eta_{corr}^{(k)} = \eta_{corr}^{(1)} = const \ .$$

for the case without string fusion in a given transverse cell.

(In last case $\Sigma(n_F, n_B) = \Sigma_1(\mu_F, \mu_B)$ and does not depends on α_k.)

The values of the parameters $\Lambda_0^{(1)} = 0.8$ and $\eta_{corr}^{(1)} = 2.7$ were chosen so that to obtain a correspondence with the values of the $\Sigma(n_F,\ n_B)$ obtained in [Vechernin V 2018 Eur.Phys.J.:Web of Conf. 191 04011].

Note that in that paper the $\Sigma(n_F,\ n_B)$ was calculated on the base of the string pair correlation function, $\Lambda(\Delta\eta)$, extracted in [V.Vechernin, Nucl.Phys.A939(2015)21] from the ALICE data on the FB correlations [ALICE collab.,JHEP05(2015)097] in the approx. of IDENTICAL strings.

MC calculations of $\Sigma(n_F, n_B)$ in the model with string clusters formation

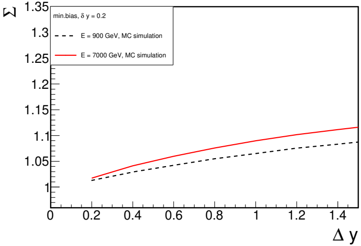
[V.V. Vechernin, S.N. Belokurova, J.Phys.:Conf.Ser. 1690(2020)012088, arXiv:2012.07682, S. Belokurova, Phys.Part.Nucl. (in press), arXiv:2011.10434]

- Modelling the initial string distribution in the impact parameter plane of pp collisions for different initial energies to take into account string fusion processes. Like in [*V. Vechernin, I. Lakomov. Proceedings of Science (Baldin ISHEPP XXI) (2013) 072.*].
- Monte Carlo simulations of string configurations and calculation of weighting factors α_k as a function of centrality and initial energy of pp collision.

$$\alpha_k = \frac{\langle n^{(k)}\rangle}{\sum_{k=1}^{\infty}\langle n^{(k)}\rangle} = \frac{\langle m^{(k)}\rangle \mu_0^{(k)} \delta\eta}{\sum_{k=1}^{\infty}\langle m^{(k)}\rangle \mu_0^{(k)} \delta\eta} = \frac{\langle m^{(k)}\rangle \sqrt{k}}{\sum_{k=1}^{\infty}\langle m^{(k)}\rangle \sqrt{k}} ,$$

where the $\langle m^{(k)}\rangle$ is the mean number of clusters with k fused strings, which we take from our MC simulations of the string configurations.
- Calculation the $\Sigma(n_F, n_B)$ for different centralities of pp collision at few LHC energies using the relation (5).

Energy dependence



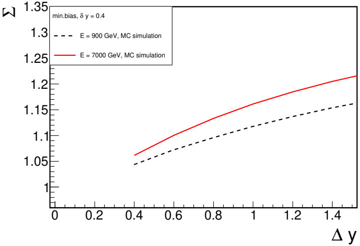


Figure: The strongly intensive observable $\Sigma(n_F, n_B)$ for pp collisions as a function of the rapidity distance $\Delta\eta = \Delta y$ between the centers of the FB observation windows, for two widths of windows: $\delta\eta$=0.2 (left panel) and $\delta\eta$=0.4 (right panel), and for two initial energies: 0.9 TeV (dashed lines) and 7 TeV (solid lines), calculated for particles with transverse momenta in the interval 0.3-1.5 GeV/c, as in the experimental analysis in [ALICE collab.,JHEP05(2015)097].

The increase of the $\Sigma(n_F, n_B)$ in pp collisions with energy is caused by the increasing contribution of string fusion processes and the formation of string clusters with new properties.

Centrality (multiplicity) dependence

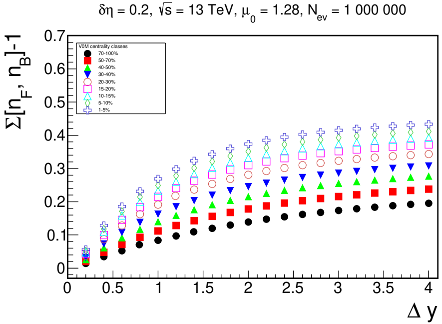


Figure: The strongly intensive observable $\Sigma(n_F, n_B) - 1$ for pp collisions as a function of the rapidity distance $\Delta\eta = \Delta y$ between the centers of the FB observation windows, for the widths of windows $\delta\eta$=0.2 and for initial energy: 13 TeV for different centrality classes.

The increase of the $\Sigma(n_F, n_B)$ in pp collisions with the collision centrality is also caused by the increasing contribution of string fusion processes and the formation of string clusters with new properties.

The model with independent identical strings

- In this version of the model the variable $\Sigma(n_F, n_B)$ depends only on the individual characteristics of a string and is independent of both the mean number of strings and its fluctuation, which reflects its strongly intensive character.
- So the studies of this observable enable to extract from the experimental data these fundamental characteristics of an individual string - a mean number of particles per unit of rapidity, μ_0, and the pair correlation function, $\Lambda(\Delta\eta, \Delta\phi)$, for particles produced from a fragmentation of a single string.
- However in this version of the model the string parameters occur dependent on collision energy. This fact can be considered as a signal that with increasing of the initial energy of a pp collision due to the string fusion the formation of the sources with new properties - the string clusters - takes place

The model with string fusion and string clusters formation

- In this case the observable $\Sigma(n_F, n_B)$ is equal to a weighted average of its values for different string clusters, $\Sigma_k(\mu_F, \mu_B)$, with weight factors, α_k, which are proportional to the mean number of the particles, produced from all clusters formed by the k fused strings.
- The $\Sigma(n_F, n_B)$, through these weight factors, α_k becomes dependent on collision conditions - its energy and centrality.
- Analyzing these dependencies of the $\Sigma(n_F, n_B)$ we can extract from the experimental data the information on the individual characteristics of the string clusters - the multiplicity density, $\mu_0^{(k)}$, and the pair correlation function, $\Lambda_k(\Delta\eta)$, for particles, produced from a decay of a given cluster.
- In the framework of this approach it was shown that the overall increase of the $\Sigma(n_F, n_B)$ in pp collisions with collision energy and centrality can be explained by the formation of string clusters with new properties.

The research was supported by the Russian Foundation for Basic Research, grant 18-02-40075 and the SPbSU grant ID:75252518.

Backup

Backup slides

Fitting the parameters of the initial string distribution in the impact parameter plane of pp collisions

Table: The non-diffractive cross section, the multiplicity density at mid-rapidity and the mean number of initial strings in *pp* collisions at different initial energies.

$\sqrt{s}(GeV)$	$\sigma_{th}^{ND}(mb)$	$\sigma_{MC}^{ND}(mb)$	dN^{ND}/dy	$\langle N_{str} \rangle$
60	24.9	24.9	2.44	4.2
900	39.9	39.9	3.76	7.8
7000	52.5	52.4	5.44	13.4
13000	56.5	56.6	6.03	16.0

$$\sigma_{MC\ \ simulations}^{ND} = \frac{n_{sim}(N=0)}{n_{sim}(N \geq 0)} S_b$$

$$\mu_0^{(k)} = \mu_0^{(1)} \sqrt{k} \text{ with } \mu_0^1 = 0.7$$

The parametrization of the single correlation function

The parametrization for the pair correlation function $\Lambda(\eta,\phi)$ of a single string (reflecting the Schwinger mechanism of a string decay, was suggested in [V.Vechernin,Nucl.Phys.A939(2015)21]:

$$\Lambda(\eta,\phi) = \Lambda_1 e^{-\frac{|\eta|}{\eta_1}} e^{-\frac{\varphi^2}{\varphi_1^2}} + \Lambda_2 \left(e^{-\frac{|\eta-\eta_0|}{\eta_2}} + e^{-\frac{|\eta+\eta_0|}{\eta_2}} \right) e^{-\frac{(|\varphi|-\pi)^2}{\varphi_2^2}} \; . \qquad (6)$$

This formula has the nearside peak, characterizing by parameters Λ_1, η_1 and φ_1, and the awayside ridge-like structure, characterizing by parameters Λ_2, η_2, η_0 and φ_2 (two wide overlapping hills shifted by $\pm\eta_0$ in rapidity, η_0 - the mean length of a string decay segment). We imply that in formula (6)

$$|\varphi| \leq \pi \; . \qquad (7)$$

If $|\varphi| > \pi$, then we use the replacement $\varphi \to \varphi + 2\pi k$, so that (7) was fulfilled. With such completions the $\Lambda(\eta,\phi)$ meets the following properties

$$\Lambda(-\eta,\phi) = \Lambda(\eta,\phi) \; , \quad \Lambda(\eta;-\phi) = \Lambda(\eta,\phi) \; , \quad \Lambda(\eta,\phi+2\pi k) = \Lambda(\eta,\phi) \qquad (8)$$

Fitting the model parameters by FBC in small windows

$\Lambda(\eta_{sep}, \phi_{sep})$ was fitted by the ALICE b_{nn} pp data with FB windows of small acceptance, $\delta\eta = 0.2, \delta\phi = \pi/4$, separated in azimuth and rapidity *[ALICE collab.,JHEP 05(2015)097]*. It gives for the parameters:

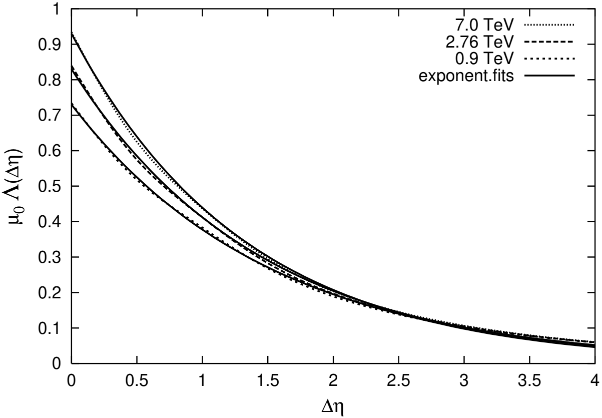
$\sqrt{s}$, TeV		0.9	2.76	7.0
LRC	$\mu_0\omega_N$	0.7	1.4	2.1
SRC	$\mu_0\Lambda_1$	1.5	1.9	2.3
	η_1	0.75	0.75	0.75
	ϕ_1	1.2	1.15	1.1
	$\mu_0\Lambda_2$	0.4	0.4	0.4
	η_2	2.0	2.0	2.0
	ϕ_2	1.7	1.7	1.7
	η_0	0.9	0.9	0.9

$\omega_N = \frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle}$ is the e-by-e scaled variance of the number of strings,
μ_0 is the average rapidity density of the charged particles from one string,
$i=1$ corresponds to the nearside and $i=2$ to the awayside contributions,
η_0 is the mean length of a string decay segment.
[V.Vechernin,Nucl.Phys.A939(2015)21]

The string correlation function $\Lambda(\Delta\eta)$

Then we find $\Lambda(\Delta\eta)$ integrating over azimuth:

$$\Lambda(\eta_{sep}) = \frac{1}{\pi}\int_0^{\pi} \Lambda(\eta_{sep}, \phi_{sep})\, d\phi_{sep} \ .$$



The string correlation function $\Lambda(\Delta\eta)$

The obtained dependencies in this fugure for three initial energies are well approximated by the exponent:

$$\Lambda(\Delta\eta) = \Lambda_0 e^{-\frac{|\Delta\eta|}{\eta_{corr}}}\ , \tag{9}$$

with the parameters presented in the table:

$\sqrt{s}$, TeV	0.9	2.76	7.0
$\mu_0\Lambda_0$	0.73	0.83	0.93
η_{corr}	1.52	1.43	1.33

[V.Vechernin, EPJ Web Conf. 191(2018)04011]

We see that the correlation length, η_{corr}, decreases with the increase of collision energy.

This can be interpreted as a signal of an increase with energy of the admixture of strings of a new type - the fused strings in pp collisions.

$\Sigma(n_F, n_B)$ in windows separated in azimuth and rapidity

For small observation windows:

$$\Sigma(\Delta\eta, \Delta\phi) = 1 + \frac{\delta\eta\,\delta\phi}{2\pi}\mu_0\,\left[\Lambda(0,0) - \Lambda(\Delta\eta, \Delta\phi)\right]$$

$$\Delta\eta \equiv \eta_{sep}, \qquad \Delta\phi \equiv \phi_{sep}$$

For observation windows of an arbitrary width $\delta\eta_F\delta\phi_F$ and $\delta\eta_B\delta\phi_B$:

$$\Lambda(\Delta\eta, \Delta\phi) \to J_{FB}(\Delta\eta, \Delta\phi) = \frac{1}{\delta\eta_F\delta\phi_F\delta\eta_B\delta\phi_B} \times$$

$$\times \int_{\delta\eta_F\delta\phi_F} d\eta_1 d\phi_1 \int_{\delta\eta_B\delta\phi_B} d\eta_2 d\phi_2\, \Lambda(\eta_1 - \eta_2; \phi_1 - \phi_2)\ ,$$

$$\Lambda(0,0) \to J_{FF} = \frac{1}{(\delta\eta_F\delta\phi_F)^2} \int_{\delta\eta_F\delta\phi_F} d\eta_1 d\phi_1 \int_{\delta\eta_F\delta\phi_F} d\eta_2 d\phi_2\, \Lambda(\eta_1 - \eta_2; \phi_1 - \phi_2)\ .$$

V.Vechernin, Nucl.Phys.A 939 (2015) 21

Various versions of string fusion

local fusion (overlaps)

M.A. Braun, C. Pajares Eur.Phys.J. **C16**, 349, (2000)

$\langle n\rangle_k = \mu_0\sqrt{k}\ S_k/\sigma_0\ , \qquad \langle p_t^2\rangle_k = p_0^2\sqrt{k}\ , \qquad k = 1, 2, 3, ...$

global fusion (clusters)

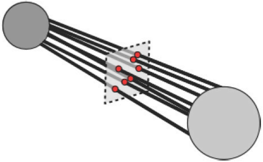
M.A. Braun, F. del Moral, C. Pajares, Phys.Rev. **C65**, 024907, (2002)

$\langle p_t^2\rangle_{cl} = p_0^2\sqrt{k_{cl}}\ , \qquad \langle n\rangle_{cl} = \mu_0\sqrt{k_{cl}}S_{cl}/\sigma_0\ , \qquad k_{cl} = k\sigma_0/S_{cl}$

the version of SFM with the finite lattice (grid) in transverse plane

Vechernin V.V., Kolevatov R.S., hep-ph/0304295; hep-ph/0305136

Braun M.A., Kolevatov R.S., Pajares C., Vechernin V.V., Eur.Phys.J. C32 (2004) 535



Domains in transverse area

The approach with string fusion on a transverse lattice (grid) was exploited later for a description of various phenomena (correlations, anisotropic azimuthal flows, the ridge) in high energy hadronic collisions in

ALICE collaboration et al., J. Phys. G **32** 1295 (2006), [Sect. 6.5.15]
Vechernin V.V., Kolevatov R.S. Phys.of Atom.Nucl. **70** (2007) 1797; 1858
M.A. Braun, C. Pajares, Eur. Phys. J. C **71**, 1558 (2011)
M.A. Braun, C. Pajares, V.V. Vechernin, Nucl. Phys. A **906**, 14 (2013)
V.N. Kovalenko, Phys. Atom. Nucl. **76**, 1189 (2013)
M.A. Braun, C. Pajares, V.V.V., Eur. Phys. J. A **51**, 44 (2015)
V.V.Vechernin, Theor. Math. Phys. 184 (2015) 1271
V.V.Vechernin, Theor. Math. Phys. 190 (2017) 251

It leads to the splitting of the transverse area into domains with different, fluctuating values of color field within them.

What was also considered in the CGC approach
A.Kovner., M. Lublinsky, Phys.Rev. D **83**, 034017 (2011)