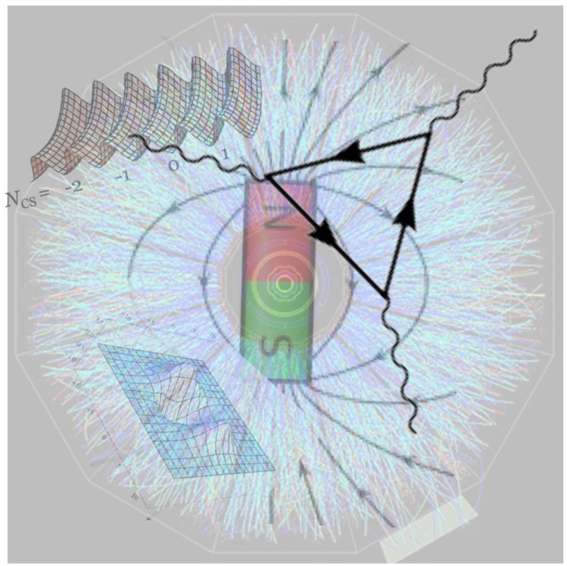
Fudan Open Seminar Series

Mar. 30, 2022

Novel Phenomena in Quantum Chromo Matter
II. Rotational Polarization in the Subatomic Swirls



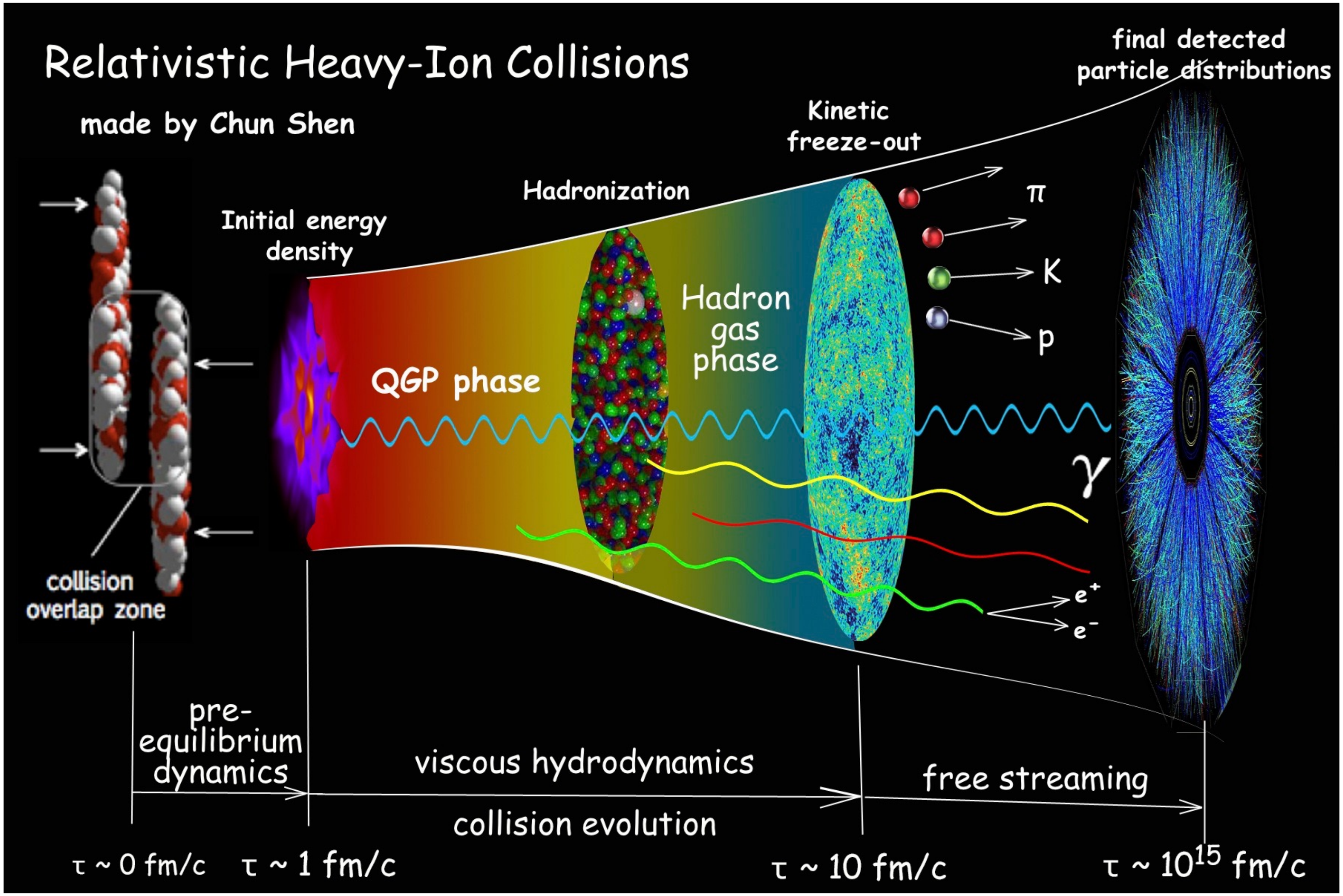
Jinfeng Liao

Indiana University, Physics Dept. & CEEM

Research Supported by NSF & DOE

Introduction

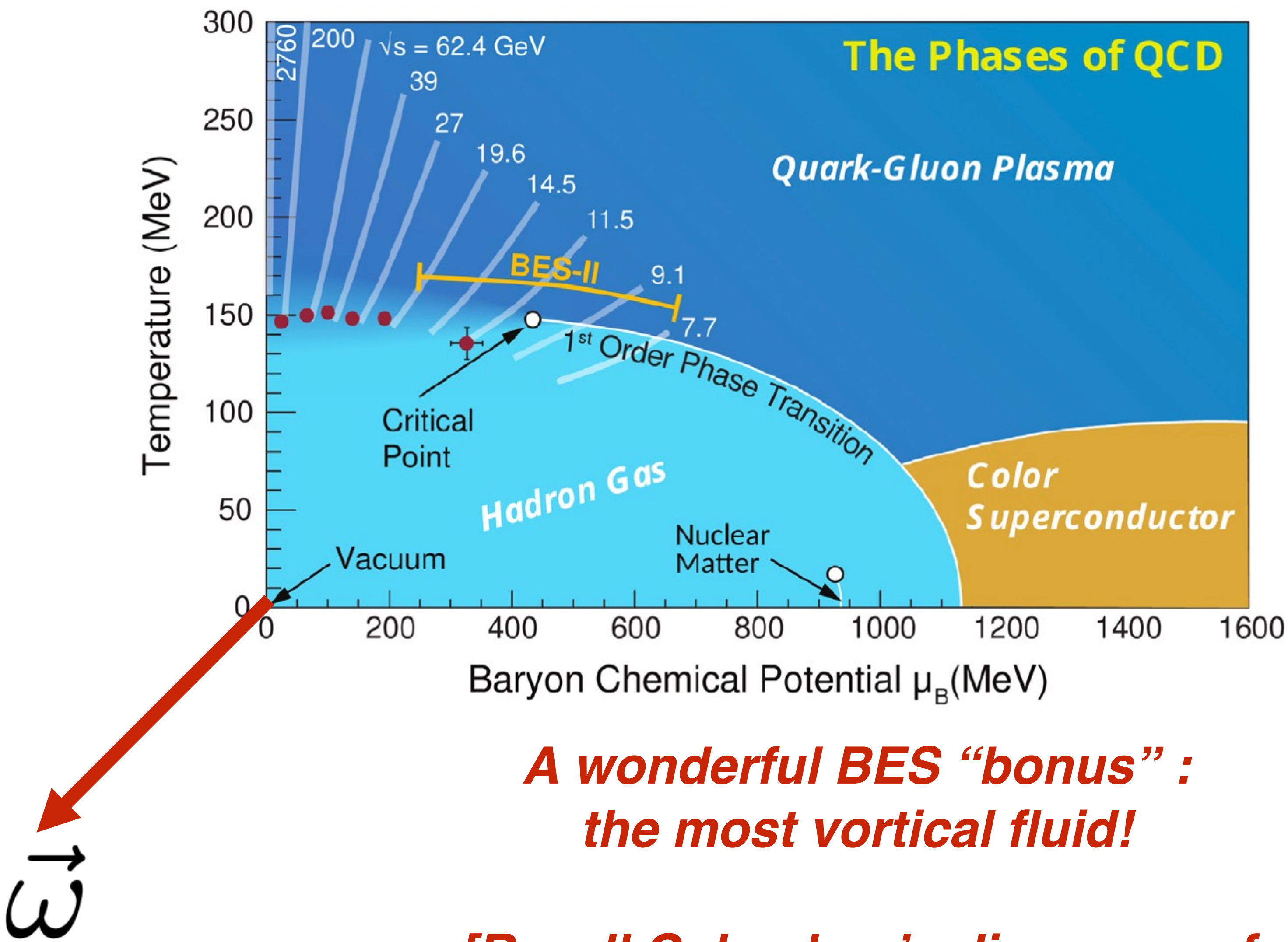
The Most Perfect Fluid in High Energy Nuclear Collisions



relativistic hydrodynamics @ 1~ 10 fm scale.

Image credit: C. Shen & U. Heinz

Beam Energy Scan & High Baryon Density Region

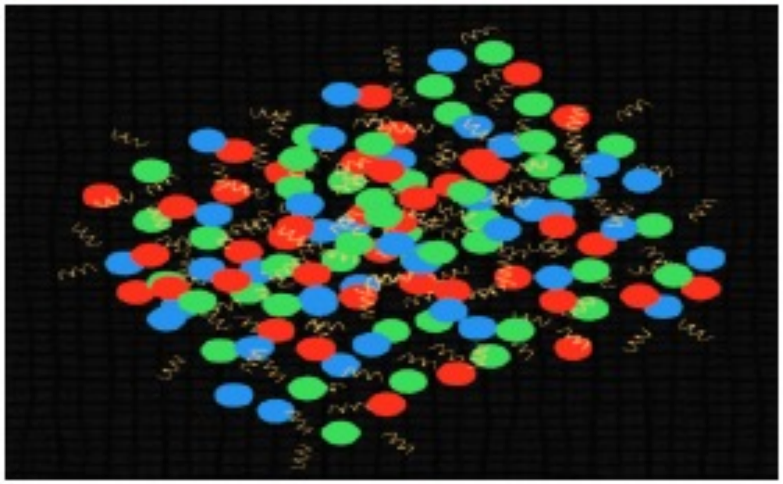


A wonderful BES "bonus" :
the most vortical fluid!

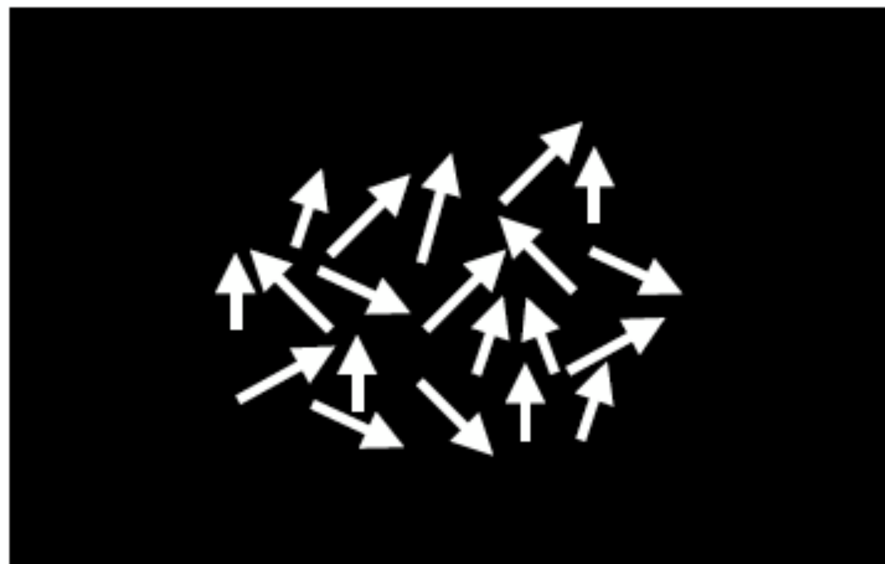
[Recall Columbus's discovery of
the American continents]

A New “Handle”: Rotation

A nearly perfect fluid (of energy-momentum)



What happens to the spin DoF in the fluid???



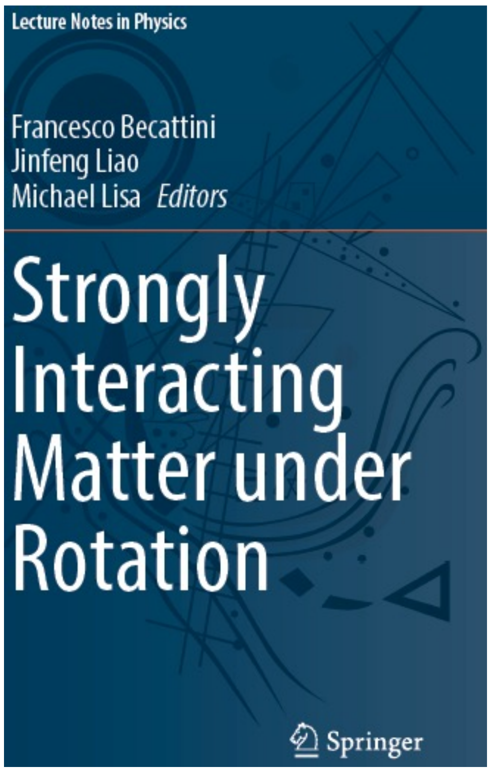
We now have a new external control on strong interaction matter: angular momentum!

Angular momentum transport, to be imaged by spin polarization.

Strongly Interacting Matter under Rotation

Opening doors for a whole new array of interesting studies:

– ***Phase structure change? Equation of state change?***
– ***Global and local polarization? Vector mesons?***
– ***Spin transport theory? Spin hydrodynamics?***
– ***Novel transport processes?***
– ***……***



A recent volume in Springer Lecture Notes in Physics!

Strongly Interacting Matter Under Rotation: An Introduction **1**

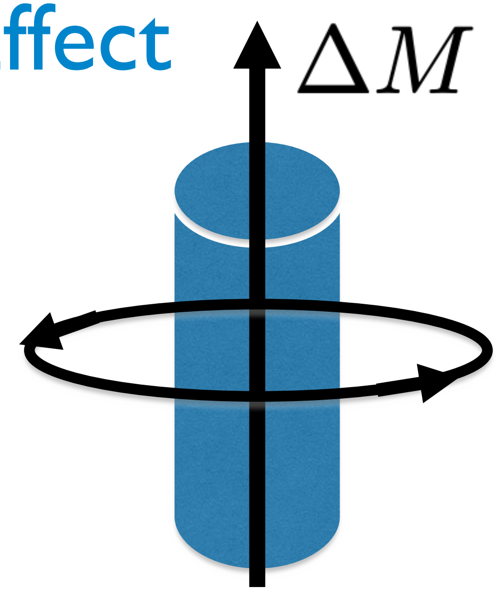
Francesco Becattini, Jinfeng Liao and Michael Lisa

Abstract

Ultrarelativistic collisions between heavy nuclei briefly generate the Quark–Gluon Plasma (QGP), a new state of matter characterized by deconfined partons last seen microseconds after the Big Bang. The properties of the QGP are of intense interest, and a large community has developed over several decades, to produce, measure, and understand this primordial plasma. The plasma is now recognized to be a strongly coupled fluid with remarkable properties, and hydrodynamics is commonly used to quantify and model the system. An important feature of any fluid is its vorticity, related to the local angular momentum density; however, this degree of freedom has received relatively little attention because no experimental signals of vorticity had been detected. Thanks to recent high-statistics datasets from experiments with precision tracking and complete kinetic coverage at collider energies, hyperon spin polarization measurements have begun to uncover the vorticity of the QGP created at the Relativistic Heavy Ion Collider. The injection of this new degree of freedom into a relatively mature field of research represents an enormous opportunity to generate new insights into the physics of the QGP. The community has responded with enthusiasm, and this book represents some of the diverse lines of inquiry into aspects of strongly interacting matter under rotation.

[arXiv:2102.00933; 2010.08937; 2009.04803; 2101.04963; 2004.04050; 2011.09974; 1908.10244; 2007.04029; 2001.00359; 2108.00586; …]

A Long Story: Barnett Effect



Lehrbuch der Kristalloptik, by E. B. Wilson; "Notes"; "New Publications."

SPECIAL ARTICLES

ON MAGNETIZATION BY ANGULAR ACCELERATION

Some time ago, while thinking about the origin of the earth's magnetism, it occurred to me that any magnetic substance must, according to current theory, become magnetized by receiving an angular velocity.

Thus consider a cylinder of iron or other substance constituted of atomic or molecular systems whose individual magnetic moments

fectly definite and unquestionable, but exceedingly difficult to account for, viz., a magnetization along the rod in a definite direction independent of the direction of rotation and of the direction of the original residual magnetism of the rod. It was not due to the jarring of the cylinder as it was rotated in the earth's field, nor to a possible minute change in the direction of its axis produced by the pull of the motor. In magnitude this effect was several times as great as the other, which became manifest only at the higher of the two speeds used.

Second Series. *October, 1915* *Vol. VI., No. 4*

Rotating solid sample
—> magnetization

$$\Delta J \Rightarrow \Delta M$$

THE

PHYSICAL REVIEW.

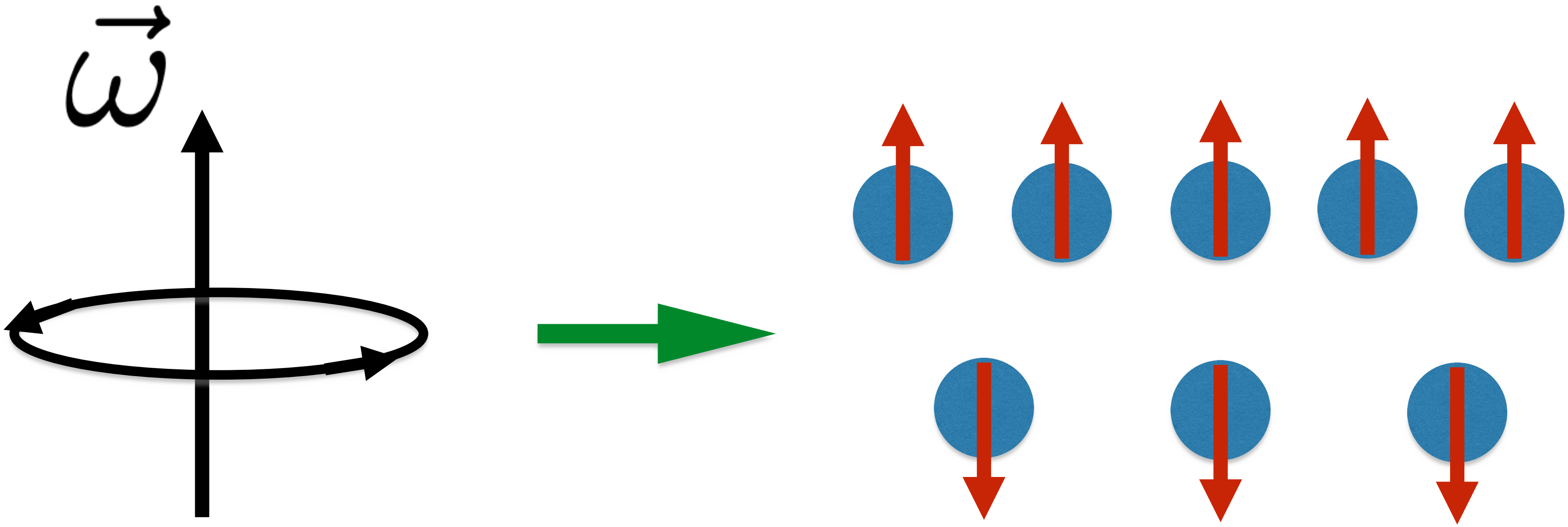
MAGNETIZATION BY ROTATION.[1]

By S. J. Barnett.

§1. In 1909 it occurred to me, while thinking about the origin of terrestrial magnetism, that a substance which is magnetic (and therefore, according to the ideas of Langevin and others, constituted of atomic or molecular orbital systems with individual magnetic moments fixed in magnitude and differing in this from zero) must become magnetized by a sort of molecular gyroscopic action on receiving an angular velocity.

In Short: Rotational Polarization

Essential assumption underlying the Barnett effect: rotational polarization



Macroscopic rotation; Global angular momentum

Microscopic spin alignment

It however is tricky to be directly observed for a flowing fluid.

Spin & Rotational Polarization

Dirac Lagrangian in rotating frame:

$$g_{\mu\nu} = \begin{pmatrix} 1-\vec{v}^2 & -v_1 & -v_2 & -v_3 \\ -v_1 & -1 & 0 & 0 \\ -v_2 & 0 & -1 & 0 \\ -v_3 & 0 & 0 & -1 \end{pmatrix}$$

$$\vec{v} = \vec{\omega} \times \vec{x}.$$

$$\bar{\gamma}^\mu = e_a{}^\mu \gamma^a$$

$$\Gamma_\mu = \frac{1}{4} \times \frac{1}{2}[\gamma^a, \gamma^b]\Gamma_{ab\mu}$$

$$\longrightarrow \quad \mathcal{L} = \bar{\psi}\left[i\bar{\gamma}^\mu(\partial_\mu + \Gamma_\mu) - m\right]\psi$$

Under slow rotation:

$$\mathcal{L} = \psi^\dagger \left[i\partial_0 + i\gamma^0\vec{\gamma}\cdot\vec{\partial} + (\vec{\omega}\times\vec{x})\cdot(-i\vec{\partial}) + \vec{\omega}\cdot\vec{S}_{4\times4}\right]\psi$$

$$\hat{H} = \gamma^0(\vec{\gamma}\cdot\vec{p} + m) - \vec{\omega}\cdot(\vec{x}\times\vec{p} + \vec{S}_{4\times4}) = \hat{H}_0 - \vec{\omega}\cdot\hat{\vec{J}}$$

Rotational polarization effect!

[Yin Jiang, JL, PRL2016]

Spin & Rotational Polarization

Eigenstates of Dirac Hamiltonian in rotating frame:

$$\hat{H} = \gamma^0(\vec{\gamma}\cdot\vec{p} + m) - \vec{\omega}\cdot(\vec{x}\times\vec{p} + \vec{S}_{4\times4}) = \hat{H}_0 - \vec{\omega}\cdot\hat{\vec{J}}$$

$$\longrightarrow \quad \hat{H},\ \hat{p}_z,\ \hat{\vec{p}}_t^{\,2},\ \hat{J}_z, \text{ and } \hat{h}_t \equiv \gamma^5\gamma^3\vec{p}_t\cdot\vec{S}_{4\times4}$$

$$u_{k_z,k_t,n,s} = \sqrt{\frac{E_k+m}{4E_k}}e^{ik_z z}e^{in\theta}\begin{pmatrix} J_n(k_t r) \\ s e^{i\theta}J_{n+1}(k_t r) \\ \frac{k_z - isk_t}{E_k+m}J_n(k_t r) \\ \frac{-sk_z+ik_t}{E_k+m}e^{i\theta}J_{n+1}(k_t r)\end{pmatrix},$$

$$v_{k_z,k_t,n,s} = \sqrt{\frac{E_k+m}{4E_k}}e^{-ik_z z}e^{in\theta}\begin{pmatrix} \frac{k_z - isk_t}{E_k+m}J_n(k_t r) \\ \frac{sk_z-ik_t}{E_k+m}e^{i\theta}J_{n+1}(k_t r) \\ J_n(k_t r) \\ -s e^{i\theta}J_{n+1}(k_t r)\end{pmatrix},$$

$$E_k \equiv \sqrt{k_z^2 + k_t^2 + m^2}$$

$$E = \pm E_k - (n+1/2)\omega$$

Rotational polarization energy

[Yin Jiang, JL, PRL2016]

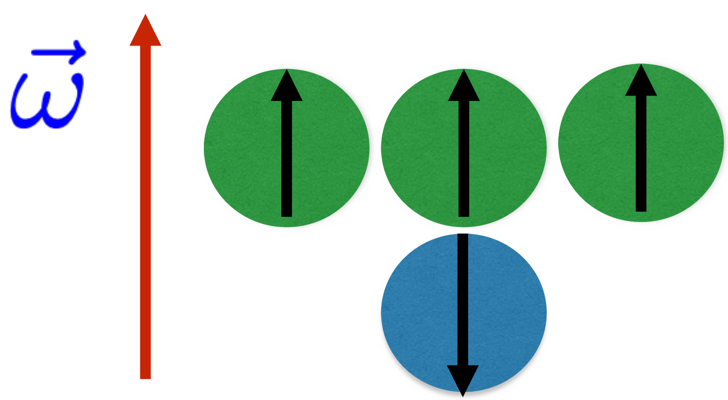
Rotational Polarization in Thermal Source

$$\hat{H} = \gamma^0(\vec{\gamma}\cdot\vec{p} + m) - \vec{\omega}\cdot(\vec{x}\times\vec{p} + \vec{S}_{4\times4}) = \hat{H}_0 - \vec{\omega}\cdot\hat{\vec{J}}$$

Rotational polarization effect!

For thermally produced particles:
"equal-partition" of angular momentum

$$dN \propto e^{\frac{\vec{\omega}\cdot\vec{J}}{T}}$$



Rotational Polarization in Condensed Matter

Spin hydrodynamic generation

R. Takahashi, M. Matsuo, M. Ono, K. Harii, H. Chudo, S. Okayasu, J. Ieda, S. Takahashi, S. Maekawa & E. Saitoh

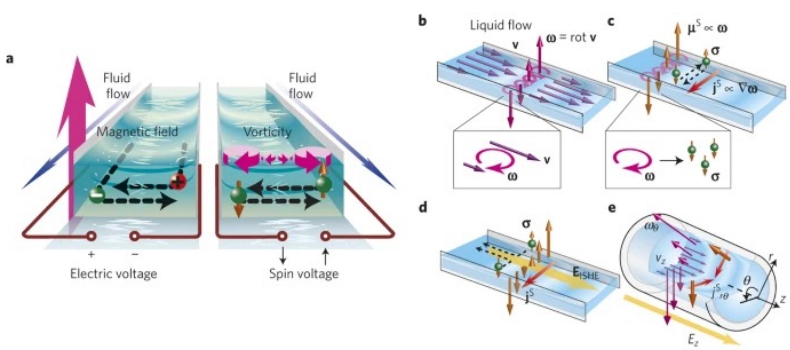
Nature Physics **12**, 52–56(2016) | Cite this article

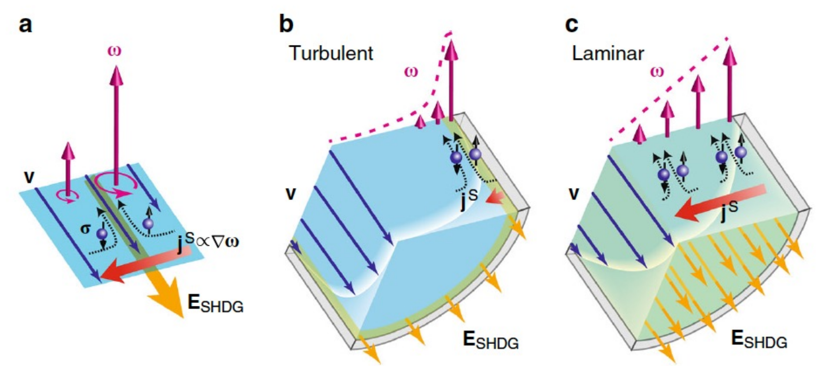
Viscous fluid flow —> vorticity —> spin polarization

Giant spin hydrodynamic generation in laminar flow

R. Takahashi, H. Chudo, M. Matsuo, K. Harii, Y. Ohnuma, S. Maekawa & E. Saitoh

Nature Communications **11**, Article number: 3009 (2020) | Cite this article





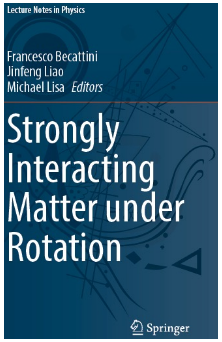
"Fluid Spintronics": Based on spin-fluid-vorticity coupling

Plan for the Rest of the Talk

- *Phenomenology of rotational polarization in heavy ion collisions: global polarization of hyperons*
- *Theoretical understanding of angular momentum in hydrodynamic framework*
- *Nontrivial effects of rotational polarization on the phase structures of matter*

Rotational Polarization in Heavy Ion Collisions

Strongly Interacting Matter under Rotation



7 Global Polarization Effect and Spin-Orbit Coupling in Strong Interaction

Jian-Hua Gao, Zuo-Tang Liang, Qun Wang and Xin-Nian Wang

Abstract

In non-central high energy heavy ion collisions, the colliding system posses a huge orbital angular momentum in the direction opposite to the normal of the reaction plane. Due to the spin-orbit coupling in strong interaction, such huge orbital angular momentum leads to the polarization of quarks and antiquarks in the same direction. This effect, known as the global polarization effect, has been recently observed by STAR Collaboration at RHIC that confirms the theoretical prediction made more than ten years ago. The discovery has attracted much attention to the study of spin effects in heavy ion collision. It opens a new window to study properties of QGP and a new direction in high energy heavy ion physics—Spin Physics in Heavy Ion Collisions. In this chapter, we review the original ideas

8 Vorticity and Polarization in Heavy-Ion Collisions: Hydrodynamic Models

Iurii Karpenko

Abstract

Fluid dynamic approach is a workhorse for modelling collective dynamics in relativistic heavy-ion collisions. The approach has been successful in describing various features of the momentum distributions of hadrons produced in the heavy-ion collisions, such as p_T spectra and flow coefficients v_n. As such, the description of the phenomenon of polarization of Λ hyperons in heavy-ion collisions has to be incorporated into the hydrodynamic approach. We start this chapter by introducing different definitions of vorticity in relativistic fluid dynamics. Then we present a derivation of the polarization of spin 1/2 fermions in the relativistic fluid. The latter is directly applied to compute the spin polarization of the Λ hyperons, which are produced from the hot and dense medium, described with fluid dynamics. It is followed by a review of the existing calculations of global or local polarization of Λ hyperons in different hydrodynamic models of relativistic heavy-ion collisions. We particularly focus on the explanations of the collision energy dependence of the global Λ polarization from the different hydrodynamic models, the polarization component in the beam direction as well as on the origins of the global and local Λ polarization.

9 Vorticity and Spin Polarization in Heavy Ion Collisions: Transport Models

Xu-Guang Huang, Jinfeng Liao, Qun Wang and Xiao-Liang Xia

Abstract

Heavy ion collisions generate strong fluid vorticity in the produced hot quark–gluon matter which could in turn induce measurable spin polarization of hadrons. We review recent progress on the vorticity formation and spin polarization in heavy ion collisions with transport models. We present an introduction to the fluid vorticity in non-relativistic and relativistic hydrodynamics and address various properties of the vorticity formed in heavy ion collisions. We discuss the spin polarization in a vortical fluid using the Wigner function formalism in which we derive the freeze-out formula for the spin polarization. Finally, we give a brief overview of recent theoretical results for both the global and local spin polarization of Λ and $\bar{\Lambda}$ hyperons.

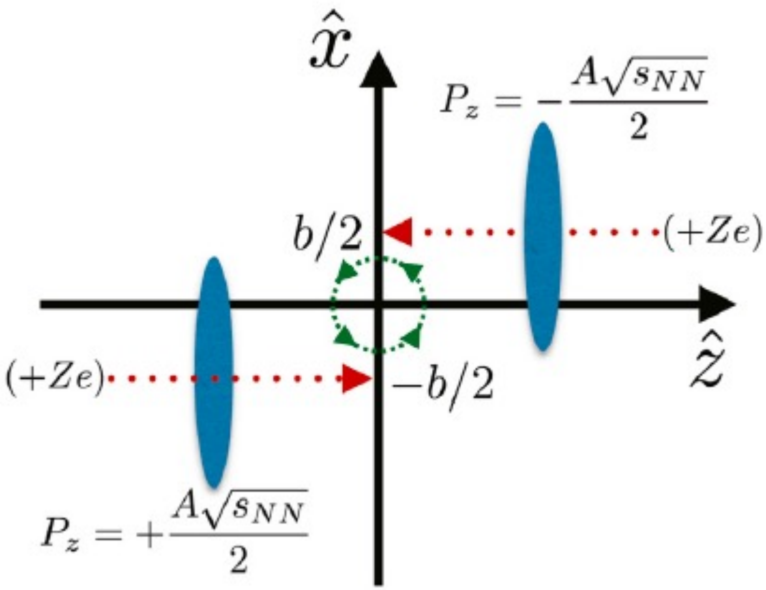
10 Connecting Theory to Heavy Ion Experiment

Gaoqing Cao and Iurii Karpenko

Abstract

Only a fraction of all Λ and $\bar{\Lambda}$ hyperons detected in heavy ion collisions are produced from the hot and dense matter directly at the hadronization. These hyperons are called the *primary* hyperons. The rest of the hyperons are products of the decays of heavier hyperon states, which in turn are produced at the hadronization. As such, the polarization of only primary hyperons can be described with the formulae introduced in Sect. 8. For the rest of the hyperons, the polarization transfer in the decays has to be computed, and convoluted with the polarization of the mother hyperon. In this chapter, a derivation of the polarization transfer coefficients, as well as the computation of the mean polarization of all Λ hyperons detected in the experiment, is presented. The chapter is concluded with the calculation of the resonance contributions to the global and local Λ polarizations.

Angular Momentum in Heavy Ion Collisions



Huge angular momentum for the system in non-central collisions at high energy

$$L_y = \frac{Ab\sqrt{s}}{2} \sim 10^{4\sim5}\hbar$$

Liang & Wang ~ 2005: spin-orbital coupling
orbital L —> spin polarization via partonic collision processes

Becattini, et al ~ 2008, 2013: A fluid dynamical scenario

$$S^\mu(p) = -\frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_\sigma\frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\varpi_{\nu\rho}n_F(1-n_F)}{\int_\Sigma \mathrm{d}\Sigma \cdot pn_F} \qquad \varpi_{\mu\nu} = \frac{1}{2}\left[\partial_\nu\left(\frac{1}{T}u_\mu\right) - \partial_\mu\left(\frac{1}{T}u_\nu\right)\right]$$

“Rotating” Quark-Gluon Plasma

$$L_y = \frac{Ab\sqrt{s}}{2} \sim 10^{4\sim5}\hbar$$

What fraction stays in QGP?
— up to ~20%, depending on collision energy.

Is this portion conserved?
—YES!

How QGP accommodates this angular momentum?
— Fluid vorticity!

PHYSICAL REVIEW C **94**, 044910 (2016)

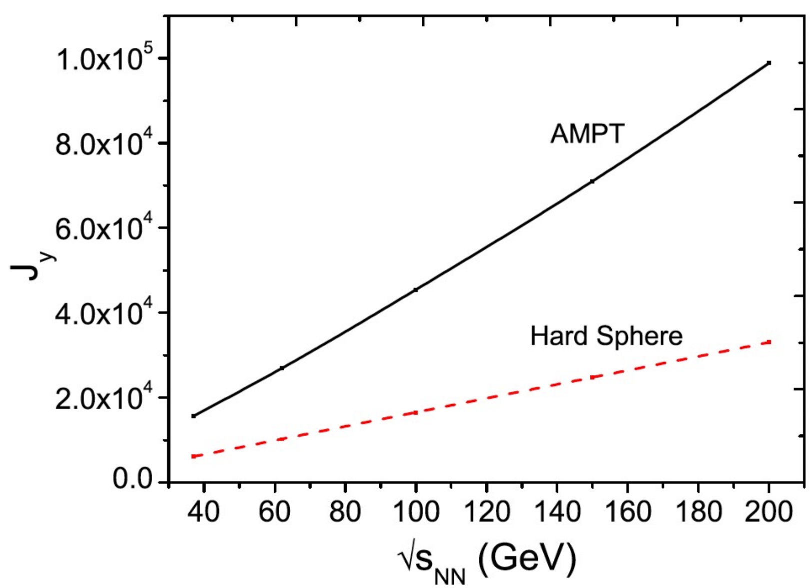
Rotating quark-gluon plasma in relativistic heavy-ion collisions

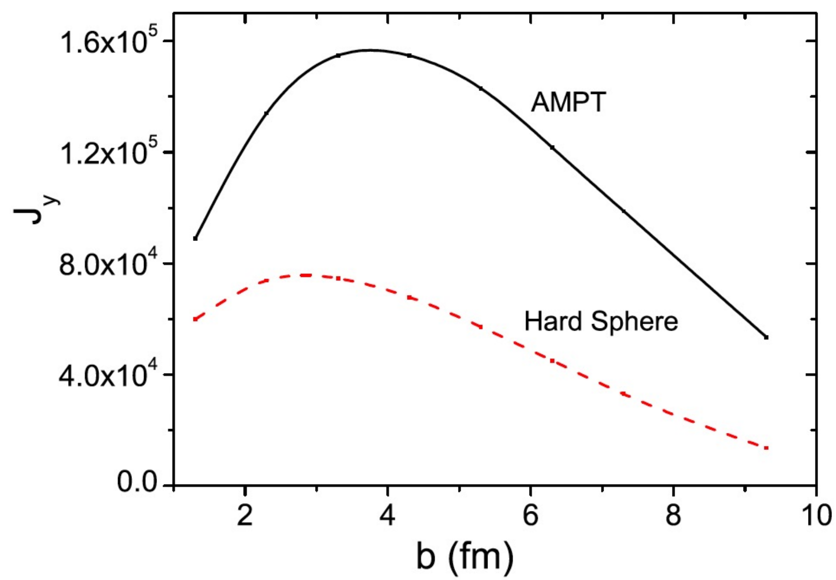
Yin Jiang,[1] Zi-Wei Lin,[2] and Jinfeng Liao[1,3]

[1] *Physics Department and Center for Exploration of Energy and Matter, Indiana University, 2401 North Milo B. Sampson Lane, Bloomington, Indiana 47408, USA*

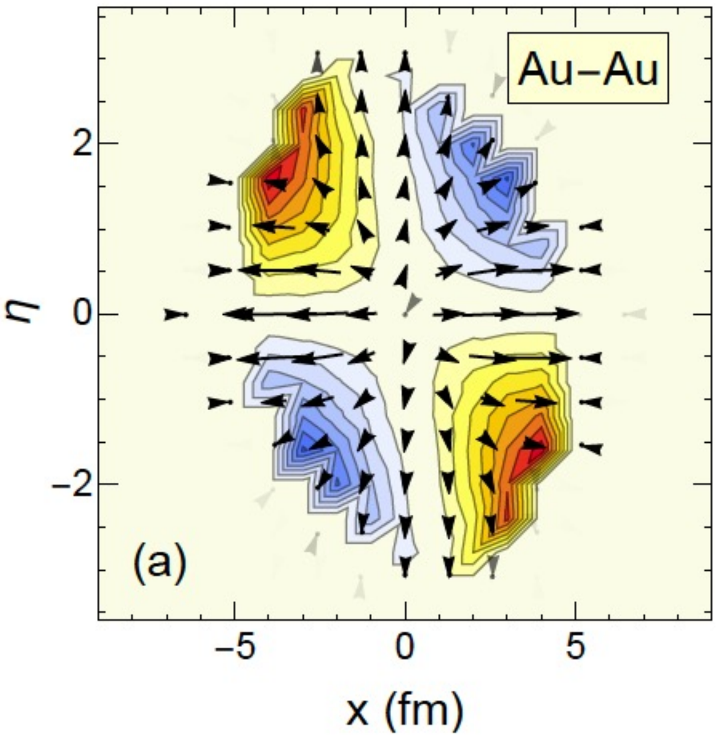
[2] *Department of Physics, East Carolina University, Greenville, North Carolina 27858, USA*

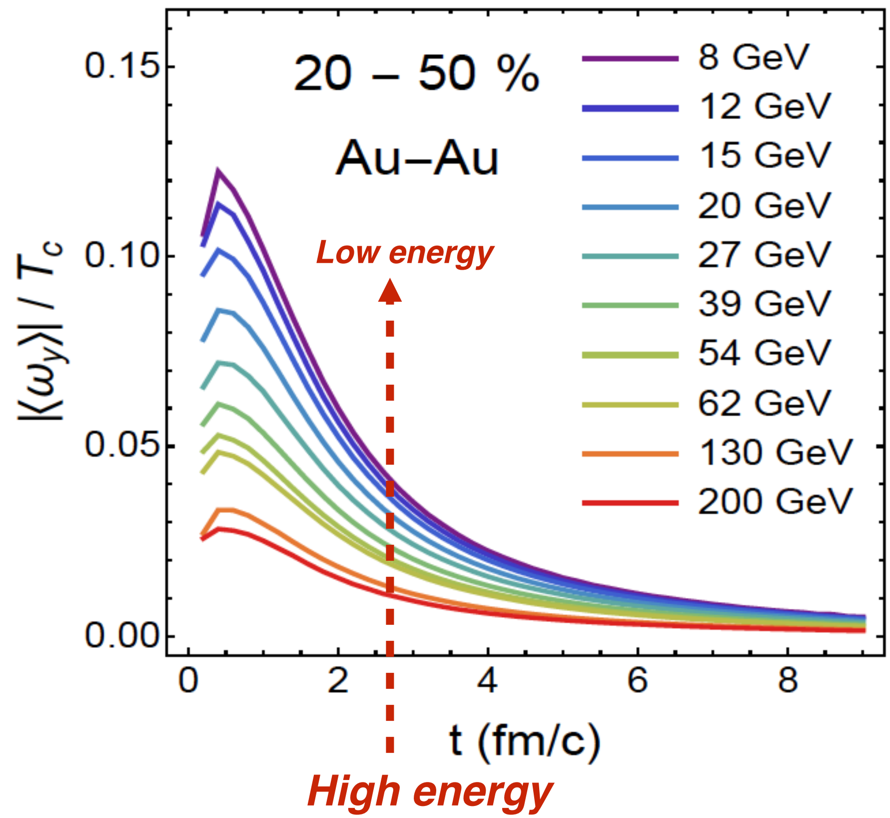
[3] *RIKEN BNL Research Center, Building 510A, Brookhaven National Laboratory, Upton, New York 11973, USA*





Nontrivial Vorticity Structures





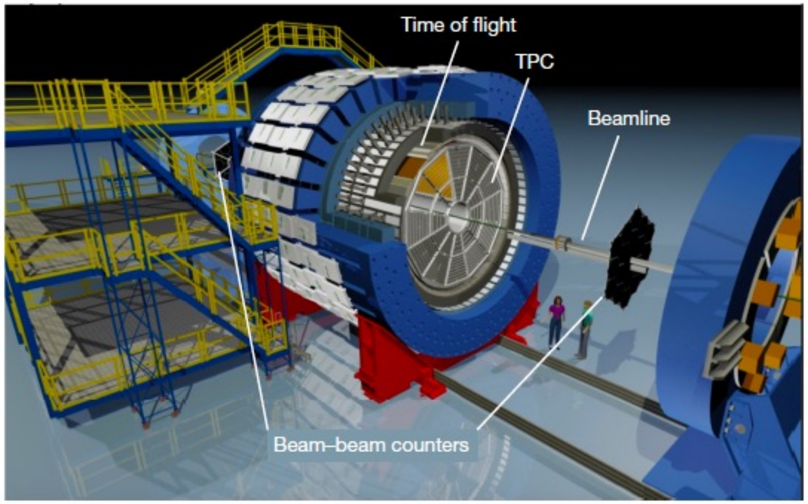
Sizable vorticity in QGP, due to the global orbital angular momentum.

It could be manifested via spin-fluid coupling.

Jiang, Lin, JL, PRC2016;
Deng, Huang, PRC2016; ……

Vorticity @ O(10) GeV >> Vorticity @ O(100) GeV

The Most Vortical Fluid



An exciting discovery from STAR Collaboration at RHIC: The most vortical fluid!

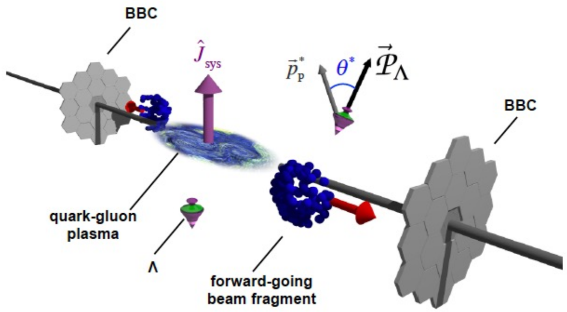
LETTER

doi:10.1038/nature23004

Global Λ hyperon polarization in nuclear collisions

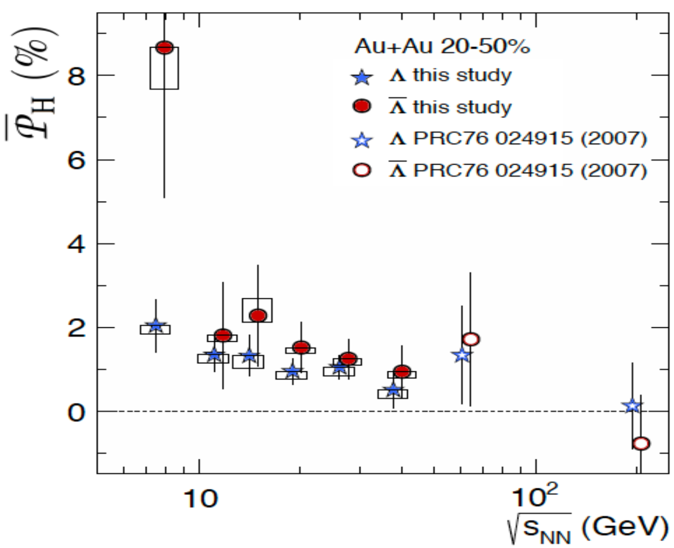
The STAR Collaboration*

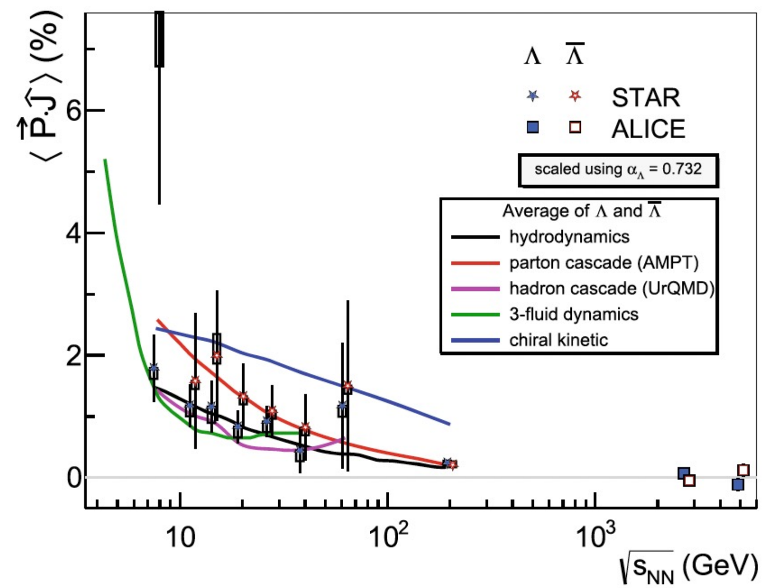
Spin Polarization in the Subatomic Swirls



STAR Collaboration, Nature 2017

The most vortical fluid!





$$\omega \approx (9 \pm 1) \times 10^{21} s^{-1}$$

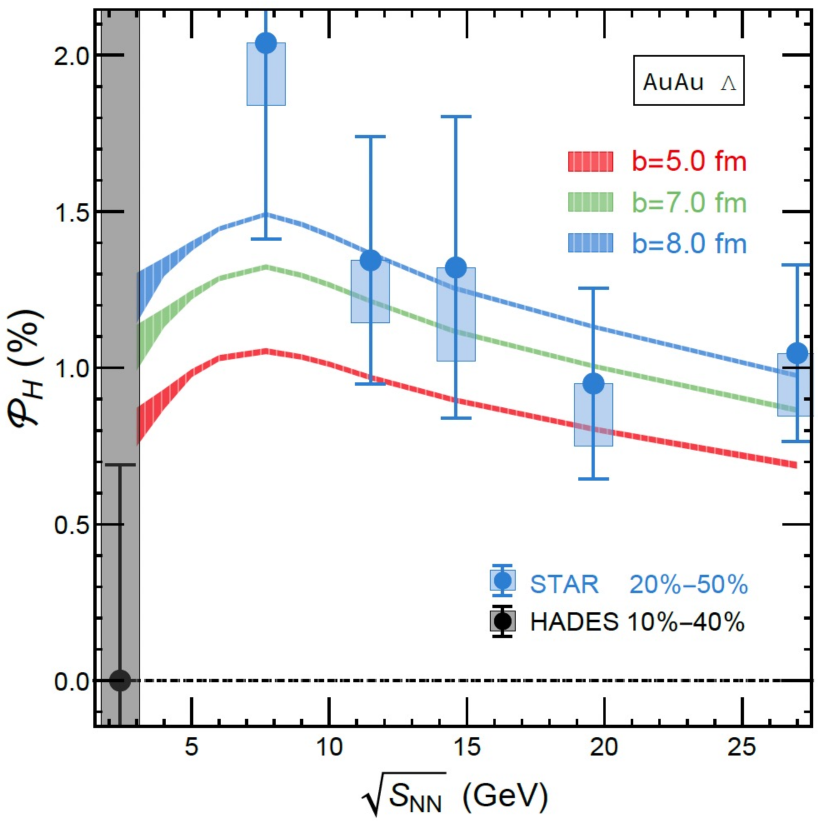
Many calculations based on hydro or transport models

Recent Developments after the Discovery

- ***Various local polarization patterns, leading to interesting puzzles and new ideas***
- ***Vector meson alignment measurements and their interpretations?***
- ***Trend toward sub-10 GeV collisions? Where is THE most vortical fluid?***
- ***Possible splitting between polarization signals of particle/anti-particle?***
- ***……***

Trend of Global Polarization toward O(1) GeV

The Question: Trend for global hyperon polarization @ O(1~10) GeV ???



Yu Guo, et al, PRC2021 arXiv:2105.13481

AMPT calculations predict non-monotonic behavior in the dependence of global polarization on beam energy —> maximum around 7.7 GeV

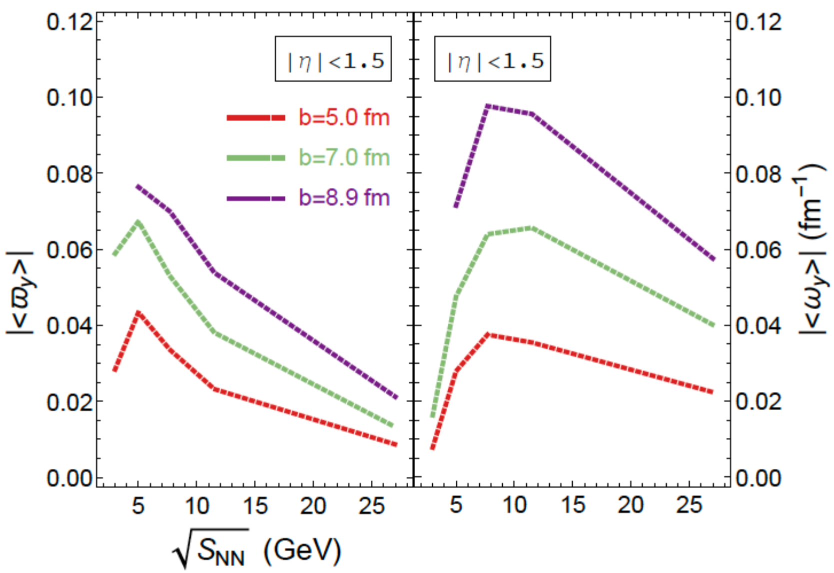
See also results for differential dependence and local polarization in the paper.

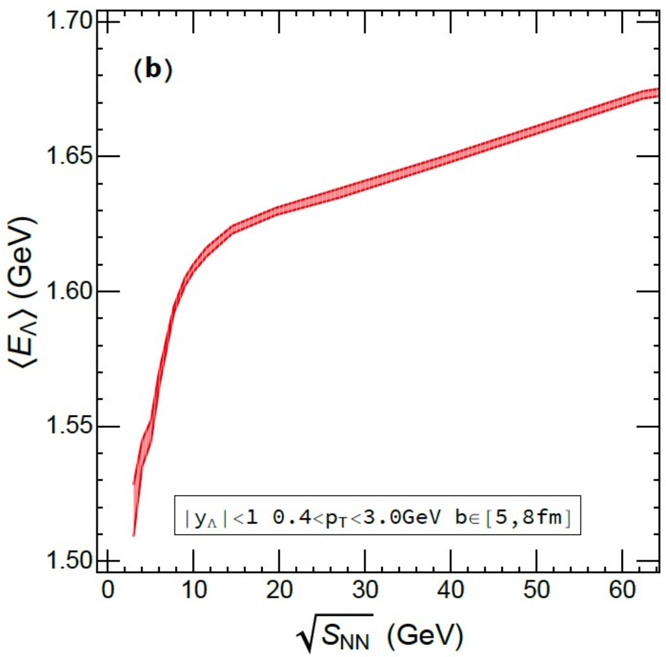
Understanding the Trend

"Thermal model" formula for spin d.o.f.

$$S^\mu(p) = -\frac{1}{8m}\epsilon^{\mu\nu\rho\sigma} p_\sigma \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p \varpi_{\nu\rho} n_F(1-n_F)}{\int_\Sigma \mathrm{d}\Sigma \cdot p n_F}$$

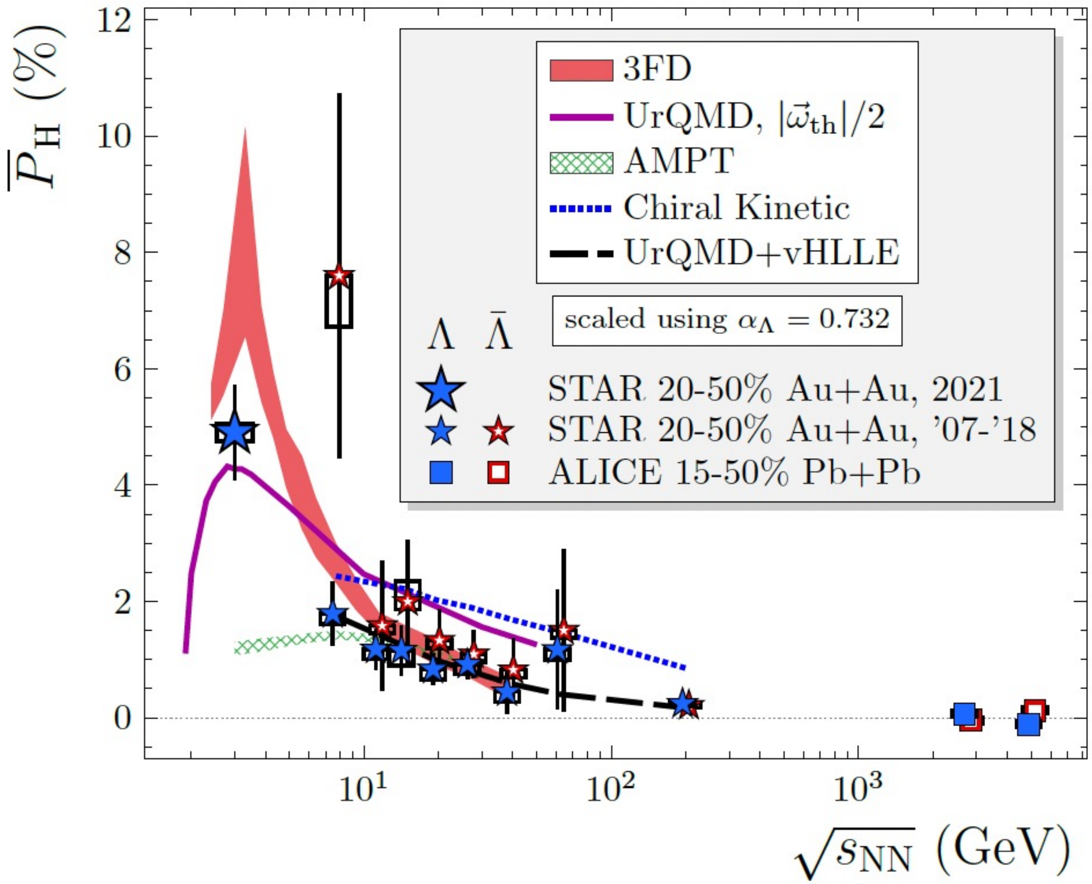
$$\varpi_{\mu\nu} = \frac{1}{2}\left[\partial_\nu\left(\frac{1}{T}u_\mu\right) - \partial_\mu\left(\frac{1}{T}u_\nu\right)\right]$$





The decrease of polarization toward O(1) GeV region is due to the decrease in both vorticity and the produced hyperon energy.

STAR Results



STAR, arXiv:2108.00044

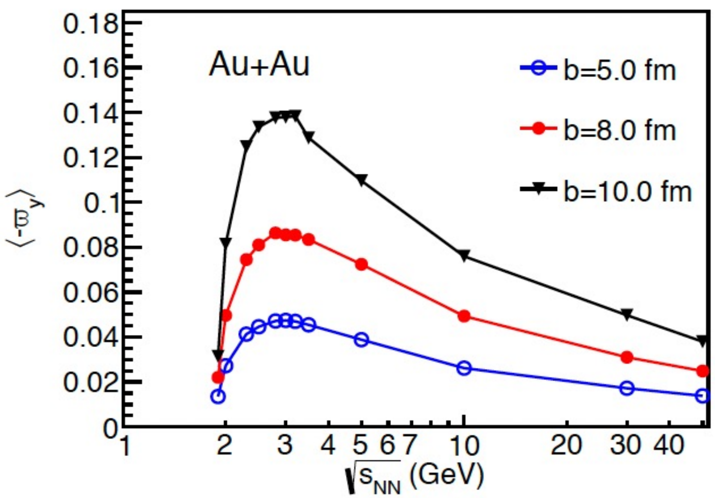
Somewhat surprisingly large signal at 3GeV …

$$L_y = \frac{1}{2} A b \sqrt{s} \sqrt{1 - (2M/\sqrt{s})^2}$$

Digesting the STAR Results

URQMD

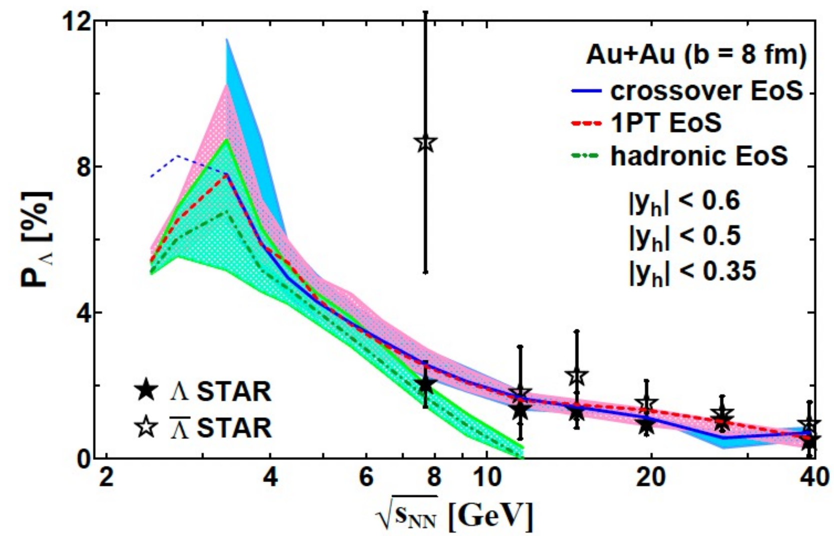
Deng, Huang, Ma, Zhang, arXiv:2001.01371



— computed the vorticity
— URQMD & AMPT results for vorticity are similar.

3FD

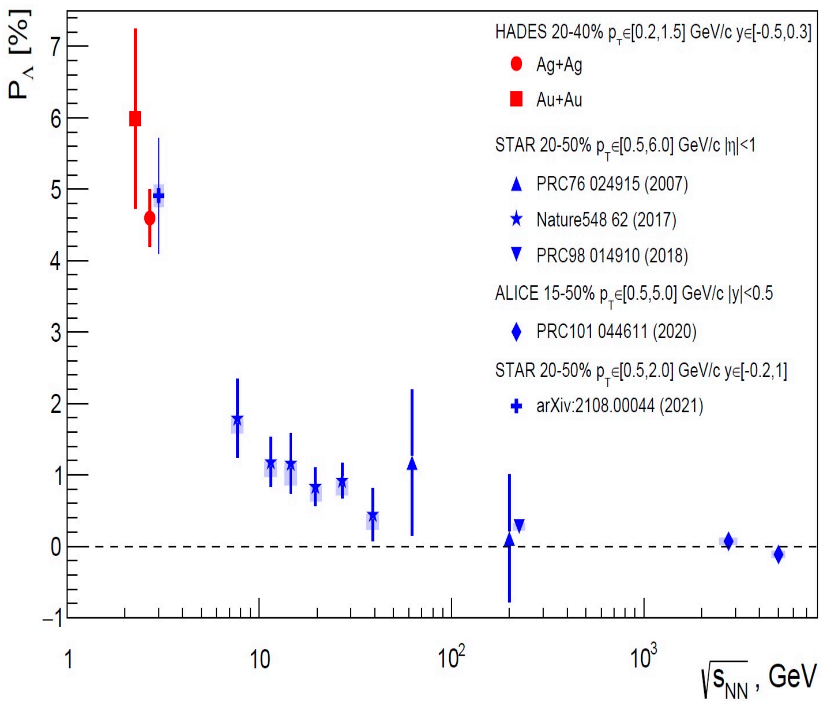
Ivanov, arXiv:2012.07597



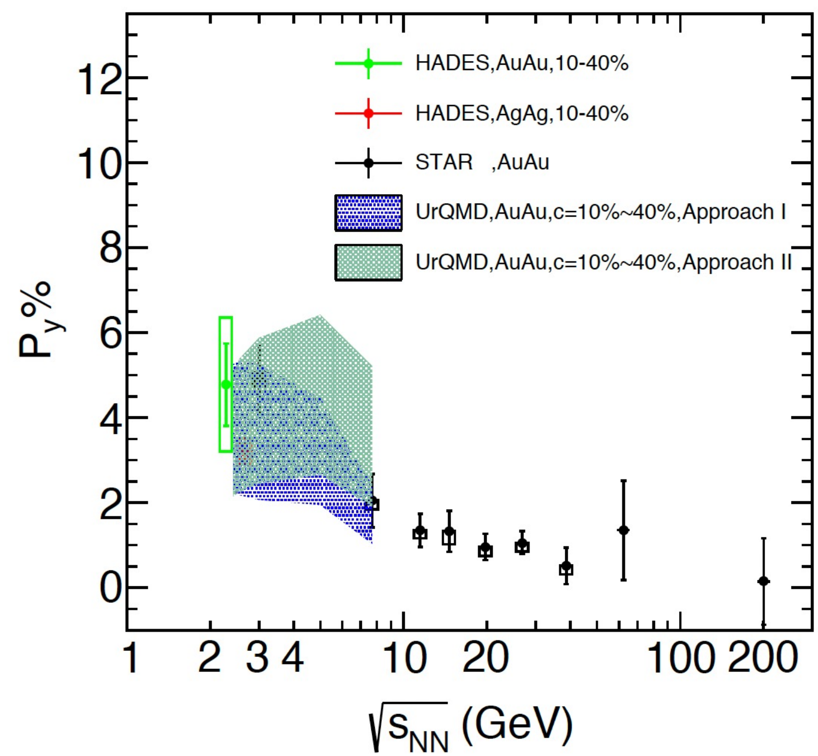
— Many model details could be quite different and need to be understood
— Likely having more spectator-participant interactions and angular momentum transport

STAR + HADES Results

Kornas @ Chirality 2021



Deng, Huang, Ma, arXiv:2109.09956



Even more surprisingly large signal at ~2.4 GeV …

$$L_y = \frac{1}{2} A b \sqrt{s} \sqrt{1 - (2M/\sqrt{s})^2}$$

Digesting the New Results

What can go "wrong" in calculating the hyperon global polarization?

$$S^{\mu}(p) = -\frac{1}{8m}\epsilon^{\mu\nu\rho\sigma} p_{\sigma} \frac{\int_{\Sigma} \mathrm{d}\Sigma \cdot p \varpi_{\nu\rho} n_F (1 - n_F)}{\int_{\Sigma} \mathrm{d}\Sigma \cdot p n_F}$$

— complete failure of fluid-vorticity-polarization scenario

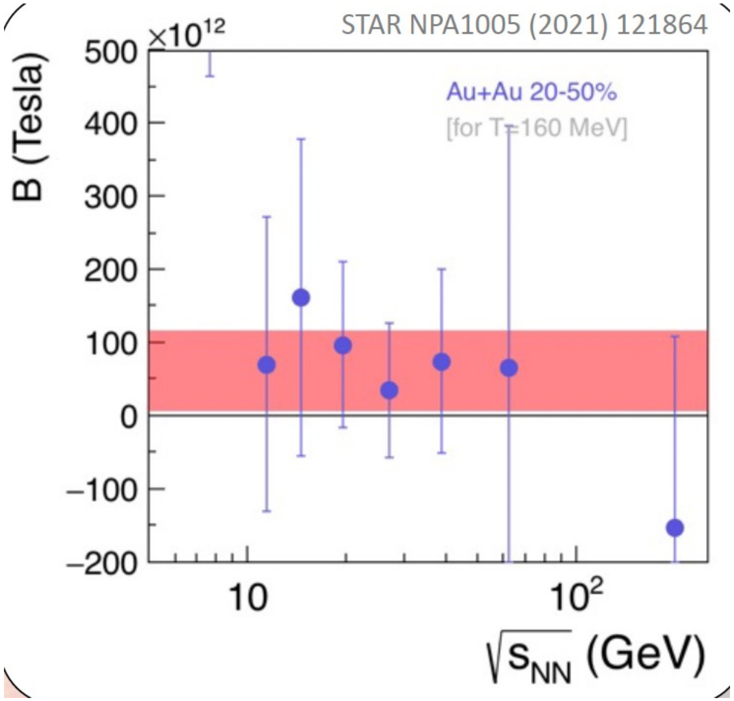
— substantial out-of-equilibrium correction

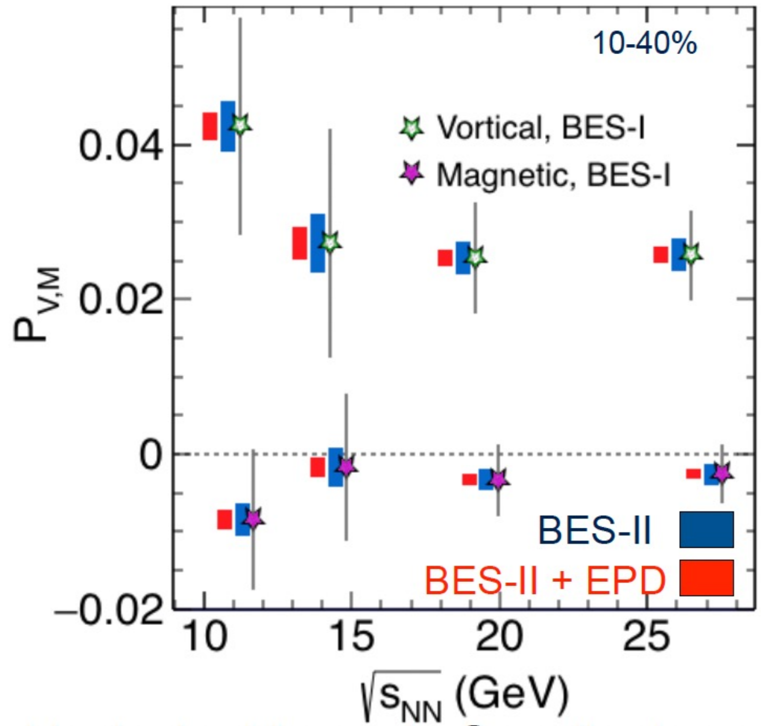
— inaccurate bulk fluid property (e.g. thermal vorticity)

— particle production mechanism (e.g. hadronic versus partonic)

The low AMPT signal could indicate:
A shift of dominance in hyperon production from partonic coalescence to direct hadronic reaction.

“Magnetic Polarization” at Low Energy?

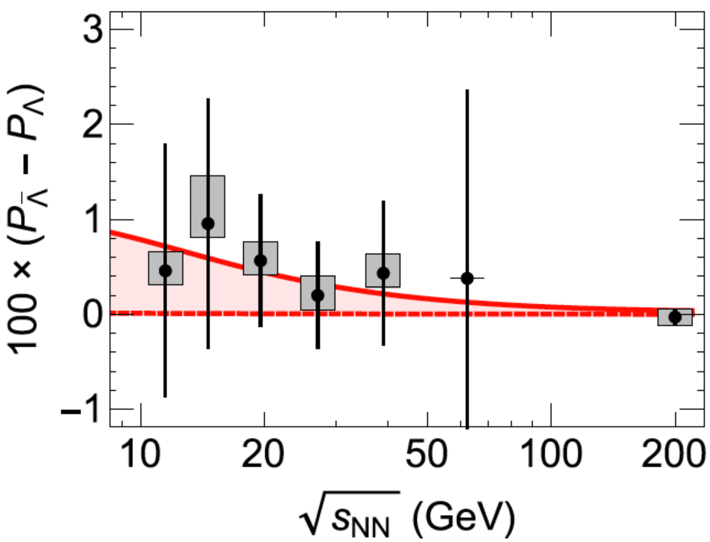


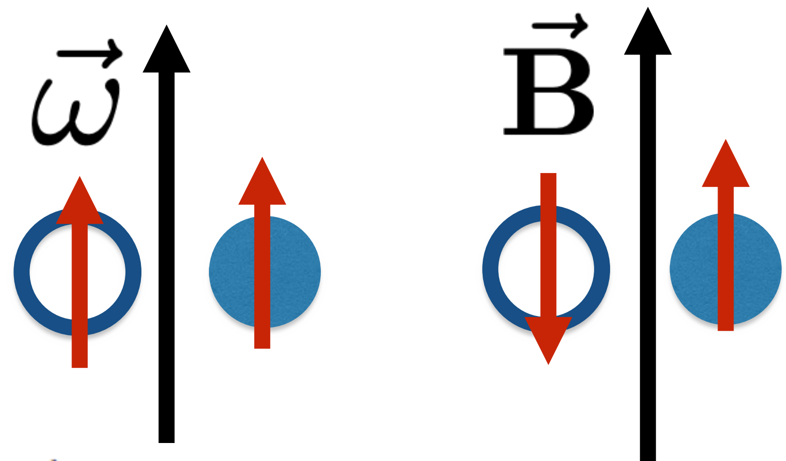


Access to possible dynamical in-medium B-field and magnetic polarization phenomenon

A Subatomic Version of Barnett Effect

A possible solution to a puzzle in STAR data at low energy:
polarization difference between particle/anti-particle





$$\tilde{S}^\mu = -\frac{1}{8m}\epsilon^{\mu\nu\rho\sigma} p_\nu \left[\varpi_{\rho\sigma} \mp 2(eF_{\rho\sigma})\mu_\Lambda/T_f\right]$$

Late-time magnetic field could explain the polarization difference;

$$\tau_B \simeq \frac{115\mathrm{GeV}\cdot\mathrm{fm/c}}{\sqrt{s_{NN}}}$$

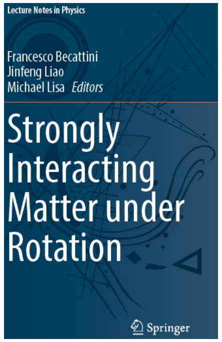
[Guo, Shi, Feng, JL, arXiv:1905.12613, PLB2019; Mueller, Schaefer, 1806.10907]

Charged rotating fluid contributes to late-time B field via Barnett-like mechanism.

[Guo, JL, Wang, arXiv:1904.04704, Scientific Reports 2020]

Hydrodynamics with Angular Momentum

Strongly Interacting Matter under Rotation



2 Polarization in Relativistic Fluids: A Quantum Field Theoretical Derivation

Francesco Becattini

Abstract

We review the calculation of polarization in a relativistic fluid within the framework of statistical quantum field theory. We derive the expressions of the spin density matrix and the mean spin vector both for a single quantum relativistic particle and for a quantum-free field. After introducing the formalism of the covariant Wigner function for the scalar and the Dirac field, the relation between the spin density matrix and the covariant Wigner function is obtained. The formula is applied to the fluid produced in relativistic nuclear collisions by using the local thermodynamic equilibrium density operator and recovering previously known formulae. The dependence of these results on the spin tensor and pseudo-gauge transformations of the stress-energy tensor is addressed.

3 Thermodynamic Equilibrium of Massless Fermions with Vorticity, Chirality and Electromagnetic Field

Matteo Buzzegoli

Abstract

We present a study of the thermodynamics of the massless free Dirac field at equilibrium with axial charge, angular momentum and external electromagnetic field to assess the interplay between chirality, vorticity and electromagnetic field in relativistic fluids. After discussing the general features of global thermodynamic equilibrium in quantum relativistic statistical mechanics, we calculate the thermal expectation values. Axial imbalance and electromagnetic field are included non-perturbatively by using the exact solutions of the Dirac equation, while a perturbative expansion is carried out in thermal vorticity. It is shown that the chiral vortical effect and the axial vortical effect are not affected by a constant homogeneous electromagnetic field.

12 Relativistic Decomposition of the Orbital and the Spin Angular Momentum in Chiral Physics and Feynman's Angular Momentum Paradox

Kenji Fukushima and Shi Pu

Abstract

Over recent years we have witnessed tremendous progress in our understanding of the angular momentum decomposition. In the context of the proton spin problem in high-energy processes, the angular momentum decomposition by Jaffe and Manohar, which is based on the canonical definition, and the alternative by Ji, which is based on the Belinfante improved one, have been revisited under light shed by Chen et al. leading to seminal works by Hatta, Wakamatsu, Leader, etc. In chiral physics as exemplified by the chiral vortical effect and applications to the relativistic nucleus–nucleus collisions, sometimes referred to as a relativistic extension of the Barnett and the Einstein–de Haas effects, such arguments of the angular momentum decomposition would be of crucial importance. We pay our special attention to the fermionic part in the canonical and the Belinfante conventions and discuss a difference between them, which is reminiscent of a classical example of Feynman's angular momentum paradox. We point out its possible relevance to early-time dynamics in the nucleus–nucleus collisions, resulting in excess by the electromagnetic angular momentum.

4 Exact Solutions in Quantum Field Theory Under Rotation

Victor E. Ambruş and Elizabeth Winstanley

Abstract

We discuss the construction and properties of rigidly rotating states for free scalar and fermion fields in quantum field theory. On unbounded Minkowski space-time, we explain why such states do not exist for scalars. For the Dirac field, we are able to construct rotating vacuum and thermal states, for which expectation values can be computed exactly in the massless case. We compare these quantum expectation values with the corresponding quantities derived in relativistic kinetic theory.

5 Particle Polarization, Spin Tensor, and the Wigner Distribution in Relativistic Systems

Leonardo Tinti and Wojciech Florkowski

Abstract

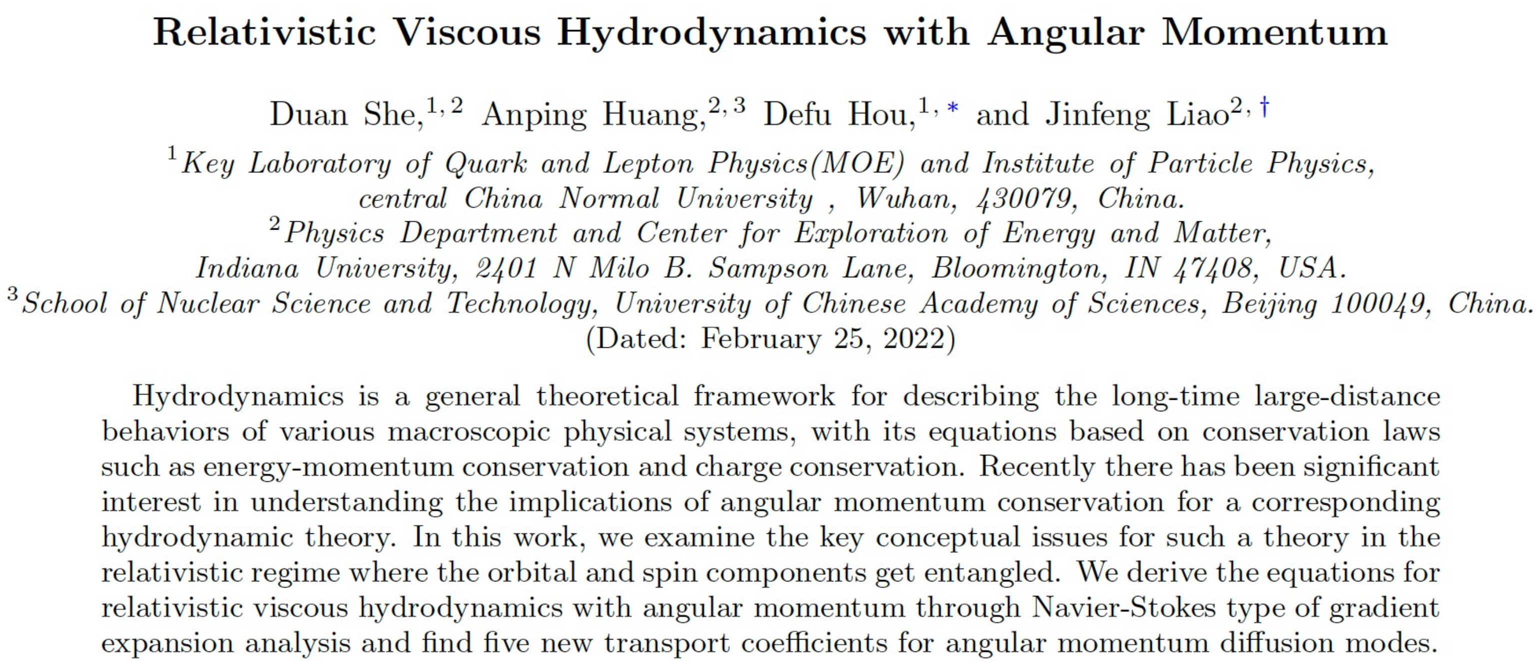
Particle spin polarization is known to be linked both to rotation (angular momentum) and magnetization of many particle systems. However, in the most common formulation of relativistic kinetic theory, the spin degrees of freedom appear only as degeneracy factors multiplying phase-space distributions. Thus, it is important to develop theoretical tools that allow to make predictions regarding the spin polarization of particles, which can be directly confronted with experimental data. Herein, we discuss a link between the relativistic spin tensor and particle spin polarization, and elucidate the connections between the Wigner function and average polarization. Our results may be useful for the theoretical interpretation of heavy-ion data on spin polarization of the produced hadrons.

Hydrodynamics with Angular Momentum

Phenomenological issues?
How to incorporate the angular momentum into the hydrodynamic framework?
In particular, how to include spin degrees of freedom?

Florkowski, Ryblewski, Kumar, …;
Becattini, Tinti, Buzzegoli, …;
Hattori, Hongo, Huang, …;
Fukushima, Pu;
Shi, Gale, Jeon;
Weickgenannt, Speranza, Sheng, Wang, Rischke;
Liu, Yin, …;
Gallegos, Gursoy, Yarom;
Li, Stephanov, Yee;
……

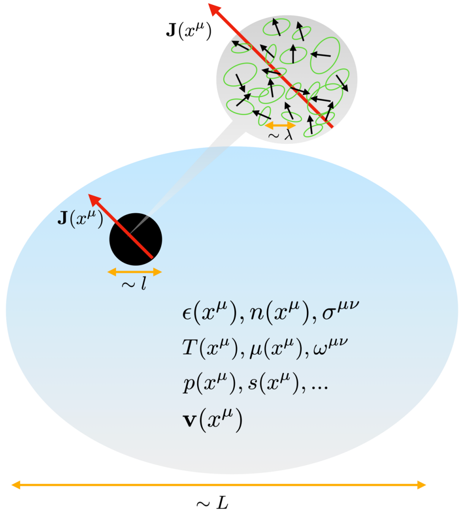
Relativistic Viscous Hydro with Ang. Mom.


Relativistic Viscous Hydrodynamics with Angular Momentum

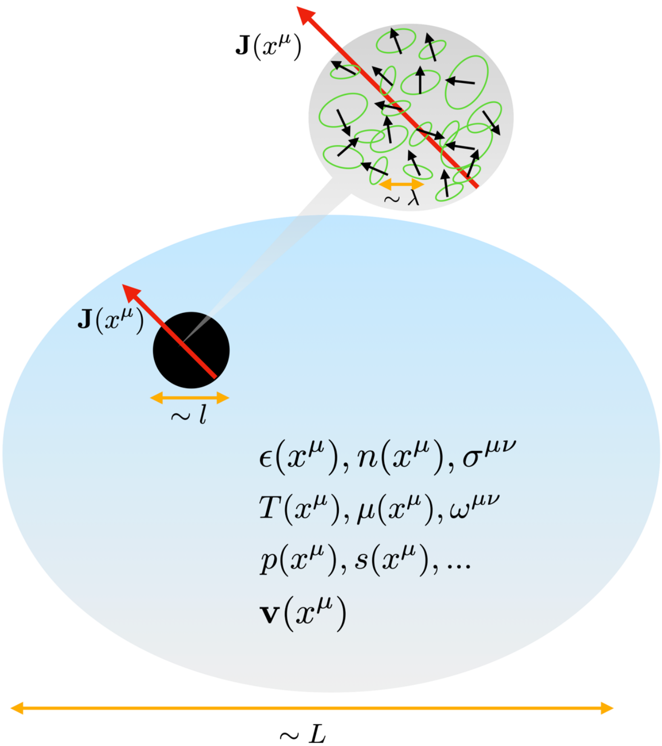
Duan She,[1,2] Anping Huang,[2,3] Defu Hou,[1,*] and Jinfeng Liao[2,†]
[1]Key Laboratory of Quark and Lepton Physics(MOE) and Institute of Particle Physics, central China Normal University , Wuhan, 430079, China.
[2]Physics Department and Center for Exploration of Energy and Matter, Indiana University, 2401 N Milo B. Sampson Lane, Bloomington, IN 47408, USA.
[3]School of Nuclear Science and Technology, University of Chinese Academy of Sciences, Beijing 100049, China.
(Dated: February 25, 2022)

Hydrodynamics is a general theoretical framework for describing the long-time large-distance behaviors of various macroscopic physical systems, with its equations based on conservation laws such as energy-momentum conservation and charge conservation. Recently there has been significant interest in understanding the implications of angular momentum conservation for a corresponding hydrodynamic theory. In this work, we examine the key conceptual issues for such a theory in the relativistic regime where the orbital and spin components get entangled. We derive the equations for relativistic viscous hydrodynamics with angular momentum through Navier-Stokes type of gradient expansion analysis and find five new transport coefficients for angular momentum diffusion modes.

[arXiv:2105.04060]



Ideal Hydrodynamics



$\lambda \ll l \ll L$, a coarse-graining process

$$\partial_\mu T^{\mu\nu} = 0,$$
$$\partial_\mu N^\mu = 0,$$

$$T^{\mu\nu}_{(0)} = \epsilon u^\mu u^\nu - p\Delta^{\mu\nu},\ N^\mu_{(0)} = n u^\mu.$$

$$\epsilon = -p + Ts + \mu n$$

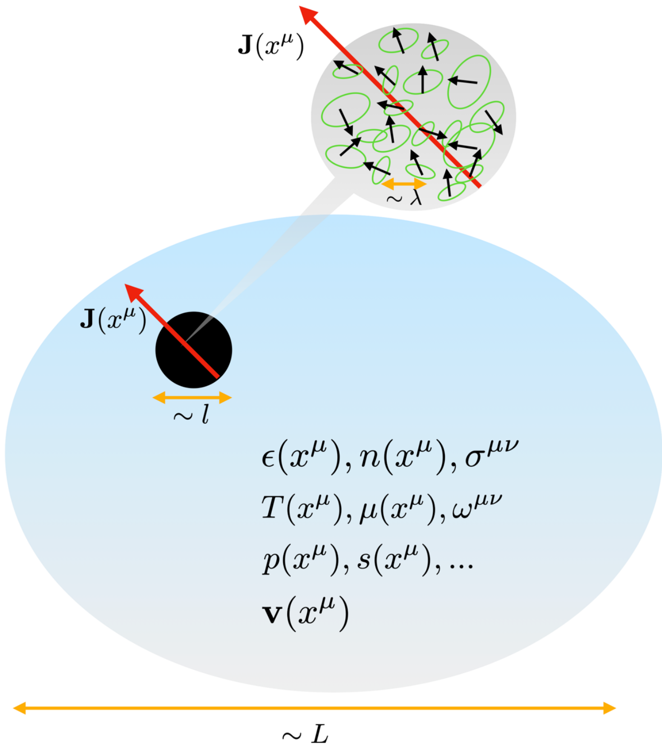
$$S^\mu_{(0)} = s u^\mu$$

$$\partial_\mu S^\mu_{(0)} = \partial_\mu \left(s u^\mu \right) = 0$$

Microscopic physics enters via thermodynamic relations (i.e. EOS).

See e.g. Landau and Lifshitz

Navier-Stokes Viscous Hydrodynamics



$$\partial_\mu T^{\mu\nu} = 0,$$
$$\partial_\mu N^\mu = 0,$$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - p\Delta^{\mu\nu} + \widetilde{T}^{\mu\nu},$$
$$N^\mu = n u^\mu + \widetilde{N}^\mu,$$
$$S^\mu = s u^\mu + \widetilde{S}^\mu.$$

$$\partial_\mu S^\mu \geq 0.$$

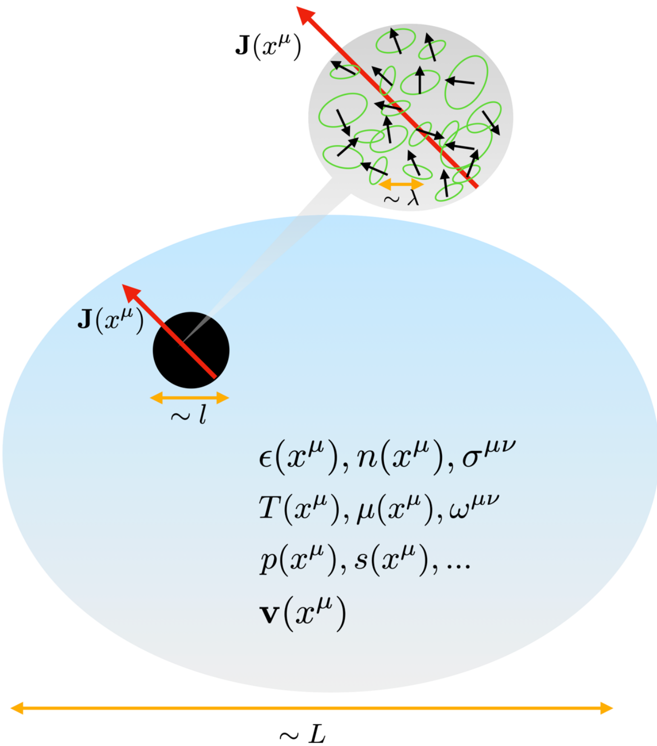
$\lambda \ll l \ll L$, a coarse-graining process

See e.g. Landau and Lifshitz

$$\Pi = -\zeta\theta,$$
$$\pi^{\mu\nu} = 2\eta\nabla^{\langle\mu}u^{\nu\rangle},$$
$$q^\mu = \lambda T\left(\frac{\nabla^\mu T}{T} - Du^\mu\right)$$

Microscopic physics also enters via transport coefficients in viscous terms.

Goal: Navier-Stokes Program for Ang. Mom.



$\lambda \ll l \ll L$, a coarse-graining process

$$\partial_\mu T^{\mu\nu} = 0,$$

$$\partial_\mu N^\mu = 0,$$

$$\partial_\mu J^{\mu\alpha\beta} = 0,$$

$$J^{\mu\alpha\beta} = \left(x^\alpha T^{\mu\beta} - x^\beta T^{\mu\alpha}\right) + \Sigma^{\mu\alpha\beta}.$$

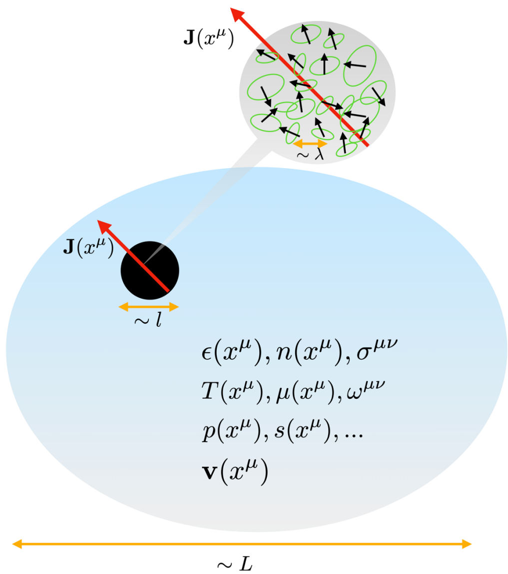
We choose to deal with only the conserved quantities, i.e. angular momentum.

Local angular momentum current $\Sigma^{\mu\alpha\beta}$

Local angular momentum density $\sigma^{\alpha\beta}(x^\mu)$

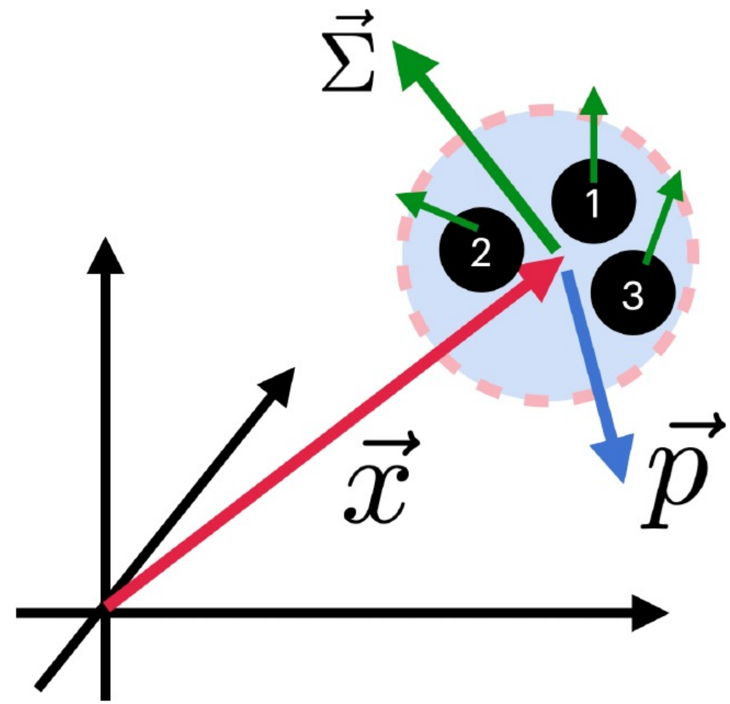
Local angular momentum chemical potential $\omega_{\alpha\beta}(x^\mu)$

Decomposition of Fluid Cell Angular Momentum



$\lambda \ll l \ll L$, a coarse-graining process

$$J^{\mu\alpha\beta} = \left(x^\alpha T^{\mu\beta} - x^\beta T^{\mu\alpha}\right) + \Sigma^{\mu\alpha\beta}\,.$$



Conceptually, it may NOT be feasible to further separate the spin part out of the local angular momentum of a coarse-grained fluid cell.

$$\vec{\mathbf{J}} = \vec{\mathbf{x}}\times\vec{\mathbf{p}}+\sum_{i=1,2,3}\left(\vec{\mathbf{x}'}_i \times \vec{\mathbf{p}'}_i + \vec{\mathbf{s}}_i\right).$$

$$\vec{\mathbf{\Sigma}} = \sum_{i=1,2,3}\left(\vec{\mathbf{x}'}_i \times \vec{\mathbf{p}'}_i + \vec{\mathbf{s}}_i\right)$$

Ideal Hydro with Ang. Mom.

$$T^{\mu\nu}_{(0)} = \epsilon u^\mu u^\nu - p\Delta^{\mu\nu},\ N^\mu_{(0)} = n u^\mu.$$

$$\Sigma^{\mu\alpha\beta}_{(0)} = \sigma^{\alpha\beta} u^\mu.$$

$$\partial_\mu J^{\mu\alpha\beta}_{(0)} = \sigma^{\alpha\beta}\theta + D\sigma^{\alpha\beta} = 0.$$

Generalized thermodynamics:

$$\epsilon = -p + Ts + \mu n + \omega_{\alpha\beta}\sigma^{\alpha\beta}$$

It is straightforward to verify: no entropy generation

$$\partial_\mu S^\mu_{(0)} = \partial_\mu \left(s u^\mu \right) = 0.$$

Viscous Hydro with Ang. Mom.

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - p\Delta^{\mu\nu} + \widetilde{T}^{\mu\nu} \,,$$
$$N^\mu = n u^\mu + \widetilde{N}^\mu \,,$$
$$\Sigma^{\mu\alpha\beta} = u^\mu \sigma^{\alpha\beta} + \widetilde{\Sigma}^{\mu\alpha\beta} \,,$$
$$S^\mu = s u^\mu + \widetilde{S}^\mu .$$

Let us first focus on entropy current:

$$S^\mu = p\beta^\mu + \beta_\nu T^{\mu\nu} - \alpha N^\mu - \beta\omega_{\alpha\beta}\Sigma^{\mu\alpha\beta}$$

Leading order:

$$\partial_\mu S^\mu_{(0)} = 2\beta\omega_{\alpha\beta}\widetilde{T}^{\alpha\beta}_{(a)} . \quad \Rightarrow \quad \widetilde{T}^{\alpha\beta}_{(a)} = 0.$$

Next order (2nd-gradient):

$$\begin{aligned}
\partial_\mu S^\mu &= \partial_\mu \left(p\beta^\mu + \beta_\nu T^{\mu\nu} - \alpha N^\mu - \beta\omega_{\alpha\beta}\Sigma^{\mu\alpha\beta} \right) \\
&= \widetilde{T}^{\mu\nu}\partial_\mu\beta_\nu - \widetilde{N}^\mu\partial_\mu\alpha - \widetilde{\Sigma}^{\mu\alpha\beta}\partial_\mu\left(\beta\omega_{\alpha\beta}\right),
\end{aligned}$$

$$\partial_\mu S^\mu \geq 0.$$

Viscous Hydro with Ang. Mom.: Eckart Frame

$$u_E^\mu = \frac{N^\mu}{\sqrt{N^\nu N_\nu}},$$

Write down all allowed Lorentz structures to the correct order of gradient expansion

$$\begin{aligned}
T^{\mu\nu} &= \epsilon u^\mu u^\nu - (p + \Pi)\,\Delta^{\mu\nu} + 2u^{(\mu} q^{\nu)} + \pi^{\mu\nu}, \\
N^\mu &= n u^\mu, \\
\Sigma^{\mu\alpha\beta} &= u^\mu \sigma^{\alpha\beta} + 2u^{[\alpha}\Delta^{\mu\beta]}\Phi \\
&\qquad + 2u^{[\alpha}\tau_{(s)}^{\mu\beta]} + 2u^{[\alpha}\tau_{(a)}^{\mu\beta]} + \Theta^{\mu\alpha\beta}.
\end{aligned}$$

Plug these into the entropy current divergence and look for conditions of positivity:

$$\begin{aligned}
\partial_\mu S^\mu &= \partial_\mu \left(p\beta^\mu + \beta_\nu T^{\mu\nu} - \alpha N^\mu - \beta\omega_{\alpha\beta}\Sigma^{\mu\alpha\beta} \right) \\
&= \widetilde{T}^{\mu\nu}\partial_\mu \beta_\nu - \widetilde{N}^\mu \partial_\mu \alpha - \widetilde{\Sigma}^{\mu\alpha\beta}\partial_\mu \left(\beta\omega_{\alpha\beta}\right),
\end{aligned}$$

$$\partial_\mu S^\mu \geq 0.$$

Viscous Hydro with Ang. Mom.: Eckart Frame

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (p+\Pi)\,\Delta^{\mu\nu} + 2u^{(\mu}q^{\nu)} + \pi^{\mu\nu},$$
$$N^\mu = n u^\mu,$$
$$\Sigma^{\mu\alpha\beta} = u^\mu \sigma^{\alpha\beta} + 2u^{[\alpha}\Delta^{\mu\beta]}\Phi$$
$$+2u^{[\alpha}\tau_{(s)}^{\mu\beta]} + 2u^{[\alpha}\tau_{(a)}^{\mu\beta]} + \Theta^{\mu\alpha\beta}.$$

$$\partial_\mu S^\mu \geq 0.$$

$$\Pi = -\zeta\theta,$$
$$\pi^{\mu\nu} = 2\eta\nabla^{\langle\mu}u^{\nu\rangle},$$
$$q^\mu = \lambda T\left(\frac{\nabla^\mu T}{T} - Du^\mu\right)$$
$$= -\frac{\lambda n T^2}{\epsilon+p}\left[\nabla^\mu\left(\frac{\mu}{T}\right) + \frac{\sigma^{\alpha\beta}}{n}\nabla^\mu\left(\frac{\omega_{\alpha\beta}}{T}\right)\right]$$

$$\Phi = -\chi_1 u^\alpha \nabla^\beta\left(\frac{\omega_{\alpha\beta}}{T}\right), \tag{25}$$
$$\tau_{(s)}^{\mu\beta} = -\chi_2 u^\alpha \Big[\left(\Delta^{\beta\rho}\Delta^{\mu\gamma} + \Delta^{\mu\rho}\Delta^{\beta\gamma}\right) - \frac{2}{3}\Delta^{\mu\beta}g^{\rho\gamma}\Big]\nabla_\gamma\left(\frac{\omega_{\alpha\rho}}{T}\right) \tag{26}$$
$$\tau_{(a)}^{\mu\beta} = -\chi_3 u^\alpha \left(\Delta^{\beta\rho}\Delta^{\mu\gamma} - \Delta^{\mu\rho}\Delta^{\beta\gamma}\right)\nabla_\gamma\left(\frac{\omega_{\alpha\rho}}{T}\right), \tag{27}$$
$$\Theta^{\mu\alpha\beta} = -\chi_4\left(u^\beta u^\rho \Delta^{\alpha\delta} - u^\alpha u^\rho \Delta^{\beta\delta}\right)\Delta^{\mu\gamma}\nabla_\gamma\left(\frac{\omega_{\delta\rho}}{T}\right) + \chi_5 \Delta^{\alpha\delta}\Delta^{\beta\rho}\Delta^{\mu\gamma}\nabla_\gamma\left(\frac{\omega_{\delta\rho}}{T}\right). \tag{28}$$

Five new positive angular momentum transport coefficients:

$\chi_1, \chi_2, \chi_3, \chi_4$ and χ_5

Viscous Hydro with Ang. Mom.: Landau Frame

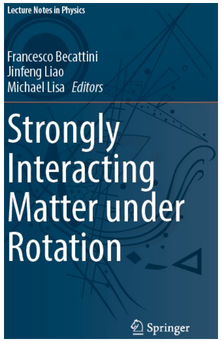
$$u_L^\mu = \frac{T^\mu_\nu u_L^\nu}{\sqrt{u_L^\alpha T_\alpha^\beta T_{\beta\gamma} u_L^\gamma}}$$

$$\begin{aligned}
T^{\mu\nu} &= \epsilon u_L^\mu u_L^\nu - (p+\Pi)\,\Delta_L^{\mu\nu} + \pi^{\mu\nu}, \\
N^\mu &= n u_L^\mu - n\frac{q^\mu}{\epsilon+p}, \\
\Sigma^{\mu\alpha\beta} &= u_L^\mu \sigma^{\alpha\beta} - \frac{q^\mu}{\epsilon+p}\sigma^{\alpha\beta} + 2u_L^{[\alpha}\Delta_L^{\mu\beta]}\Phi \\
&\quad + 2u_L^{[\alpha}\tau_{(s)}^{\mu\beta]} + 2u_L^{[\alpha}\tau_{(a)}^{\mu\beta]} + \Theta^{\mu\alpha\beta}.
\end{aligned}$$

Following the same procedure, one obtains essentially the same consistent results.

Phase Structures under Rotation

Strongly Interacting Matter under Rotation

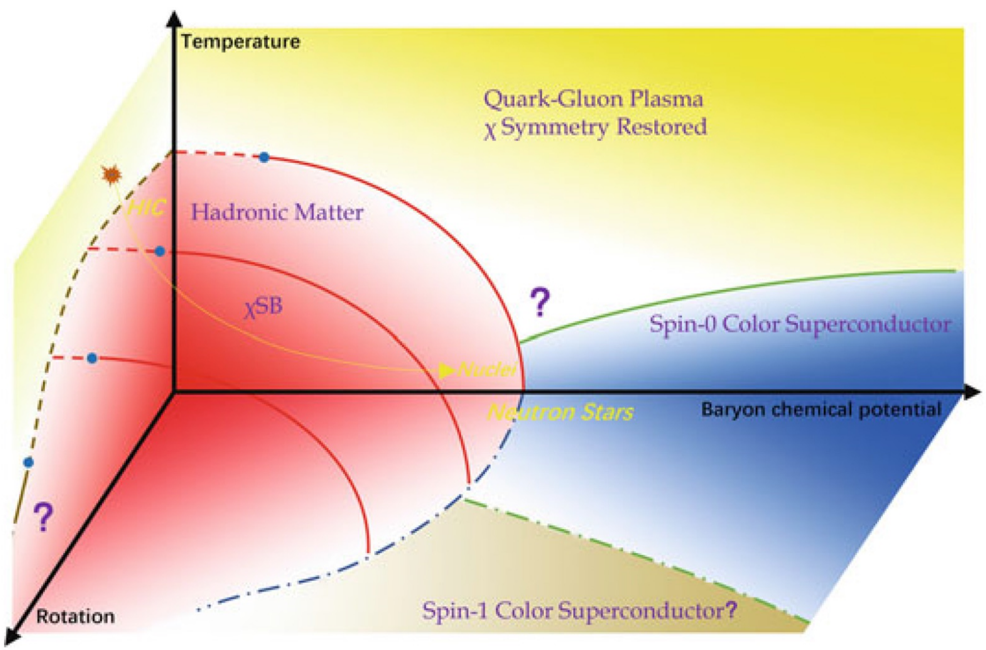


11 QCD Phase Structure Under Rotation

Hao-Lei Chen, Xu-Guang Huang and Jinfeng Liao

Abstract

We give an introduction to the phase structure of QCD matter under rotation based on effective four-fermion models. The effects of the magnetic field on the rotating QCD matter are also explored. Recent developments along these directions are overviewed, with special emphasis on the chiral phase transition. The rotational effects on pion condensation and color superconductivity are also discussed.

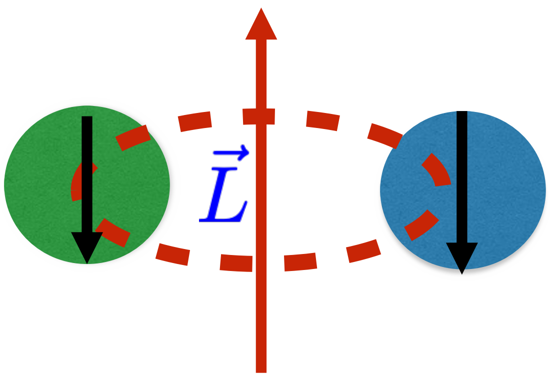


Fermion Pairing under Rotation

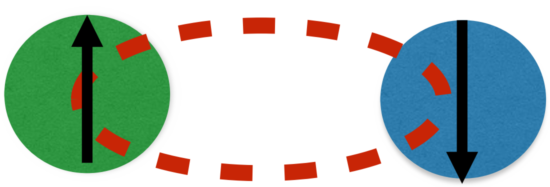
Let us consider pairing phenomenon in fermion systems.
There are many examples:
superconductivity, superfluidity, chiral condensate, diquark, …

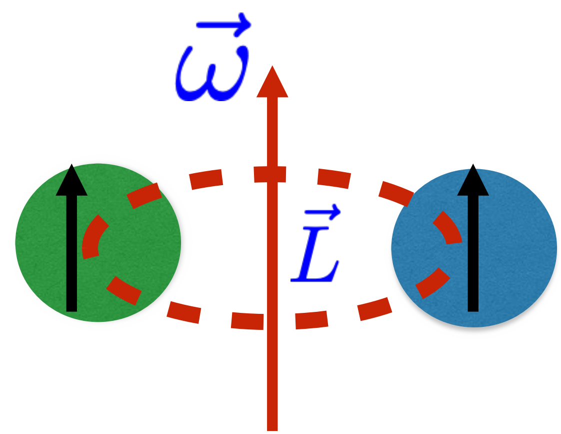
We consider scalar pairing state, with J=0.

$$\vec{S} = \vec{s}_1 + \vec{s}_2 \qquad \vec{J} = \vec{L} + \vec{S}$$



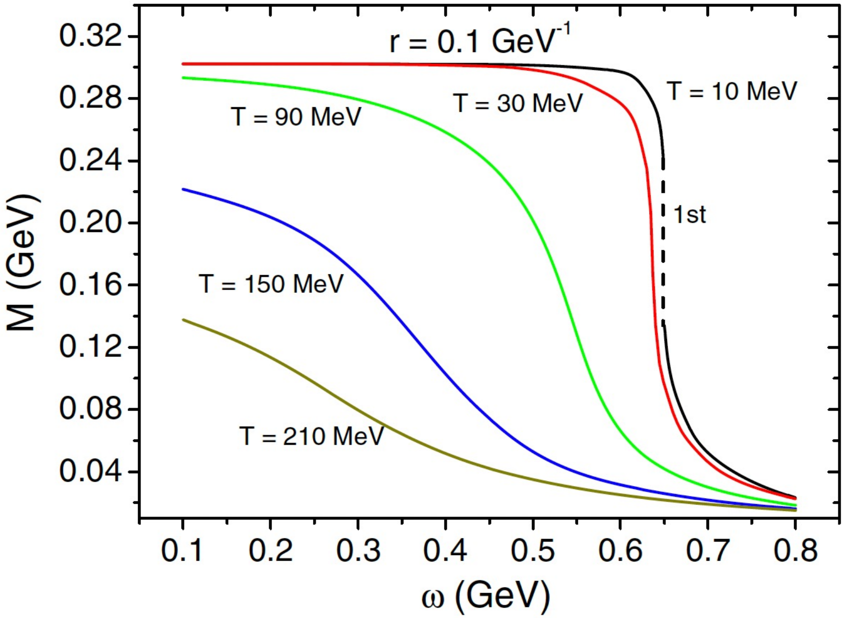
Rotation tends to polarize ALL angular momentum, both L and S, thus suppressing scalar pairing.

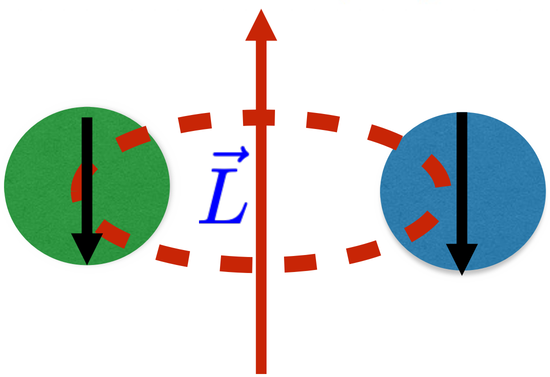


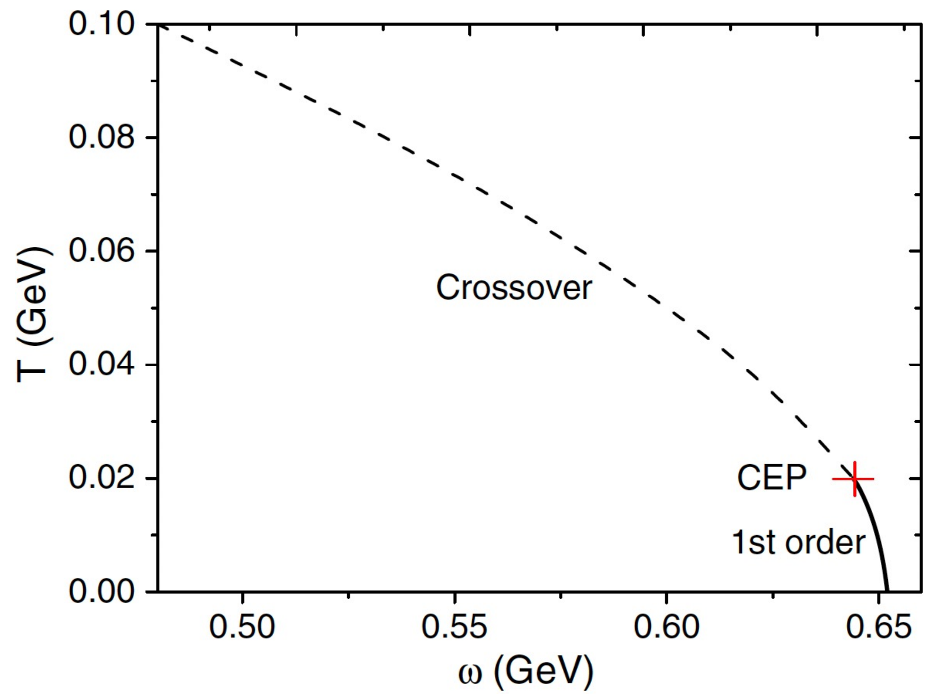


[Yin Jiang, JL, PRL2016]

Chiral Condensate under Rotation

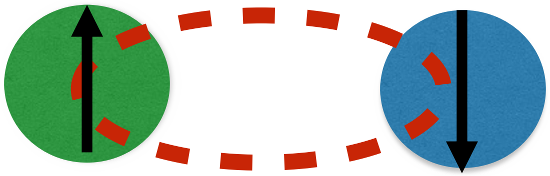


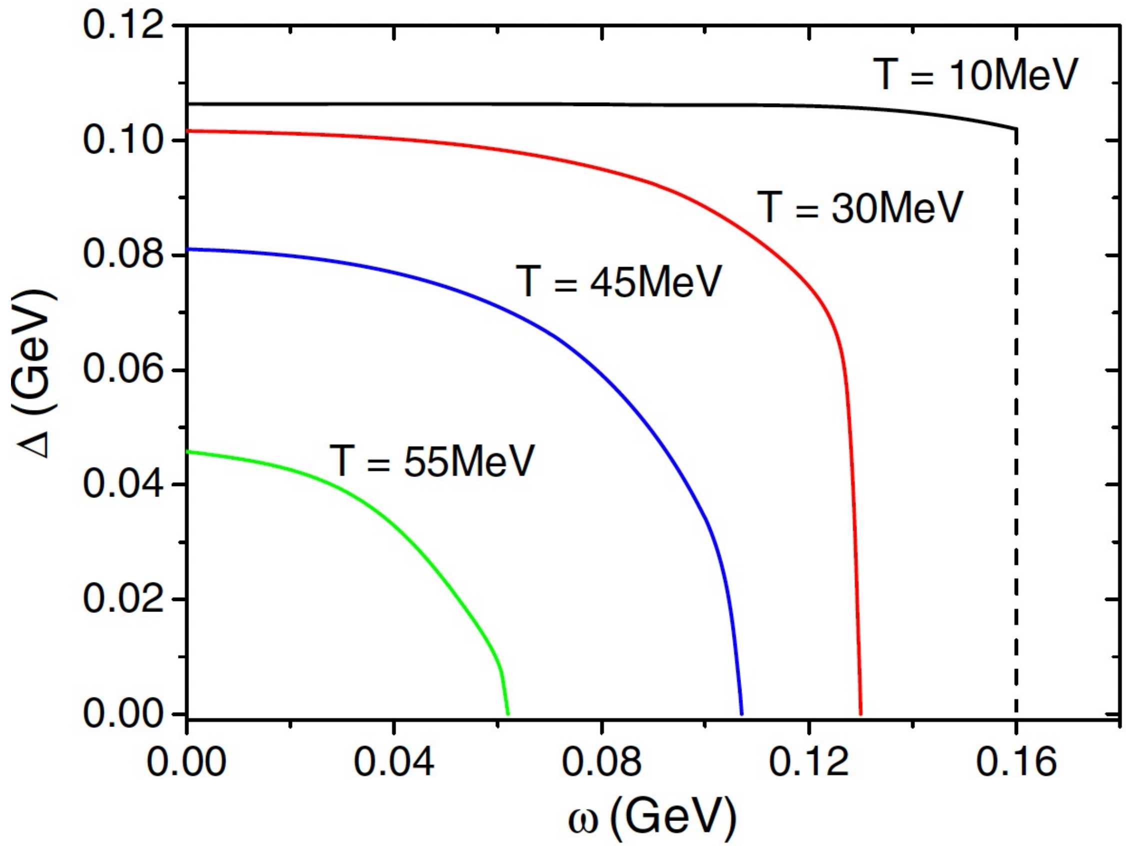




[Yin Jiang, JL, PRL2016]

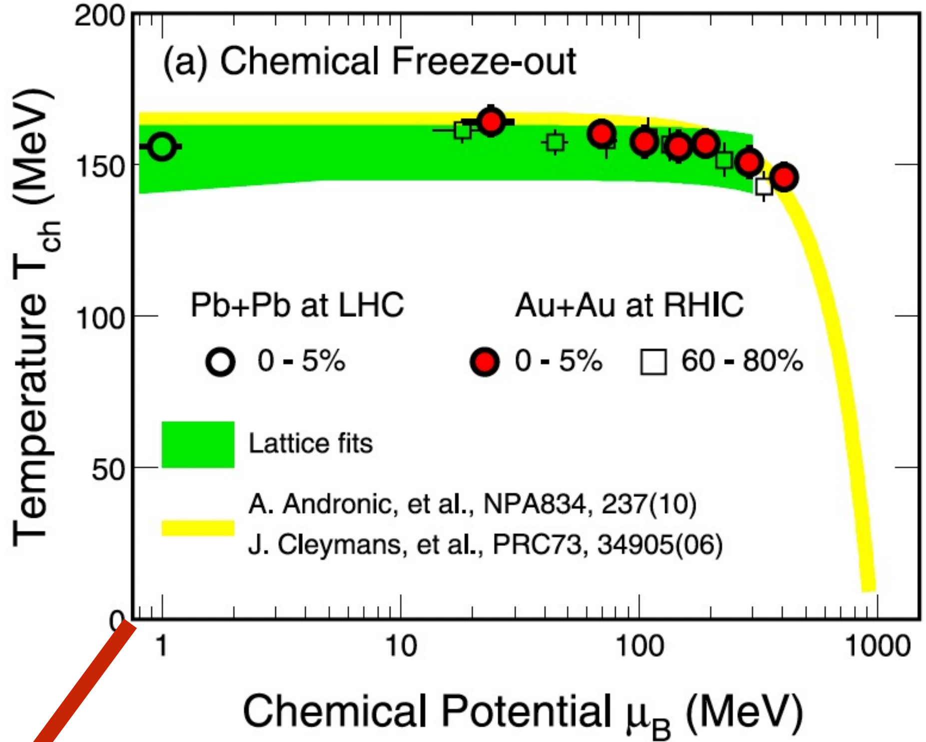
Color Superconductor under Rotation





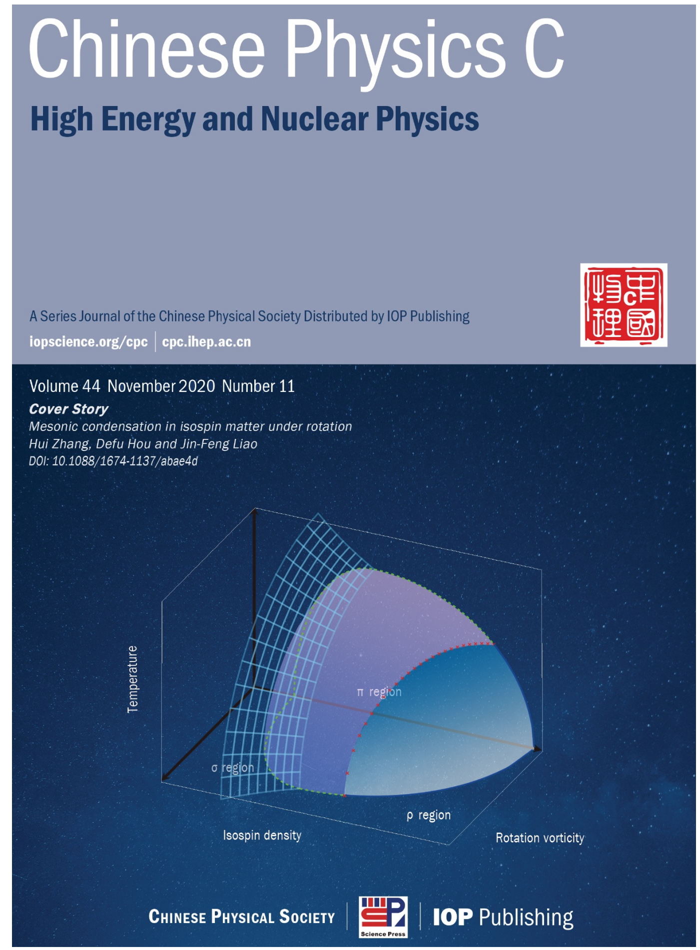
[Yin Jiang, JL, PRL2016]

Isospin Matter under Rotation



$\vec{\omega}$

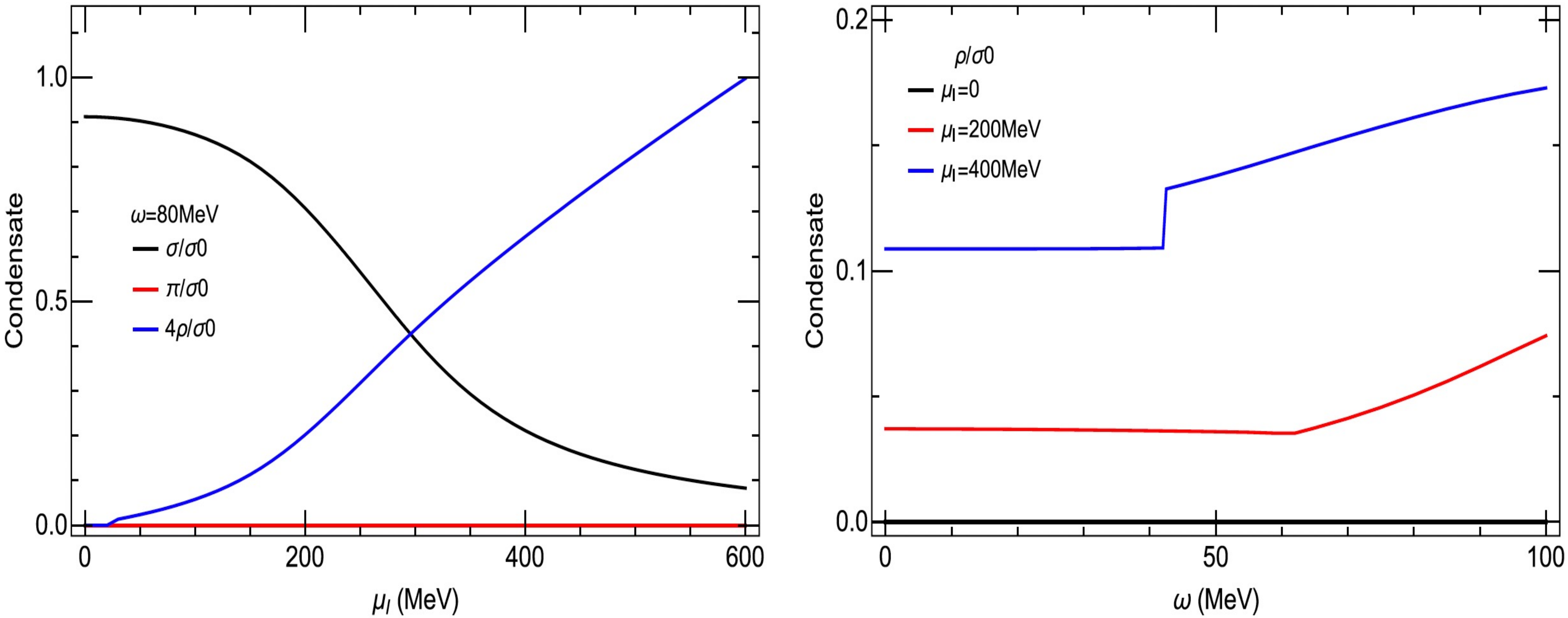
Finite isospin density for both low energy collisions and neutron stars



[Hui Zhang, Defu Hou, JL, CPC44(2020)11,111001]

Isospin Matter under Rotation

Vacuum: sigma condensate;
Static isospin matter: pion superfluidity;
Isospin matter under rotation: emergence of rho condensate!
This effect is more significant at high baryon density.



[Hui Zhang, Defu Hou, JL, CPC44(2020)11,111001]

Isospin Matter under Rotation

Vacuum: sigma condensate;
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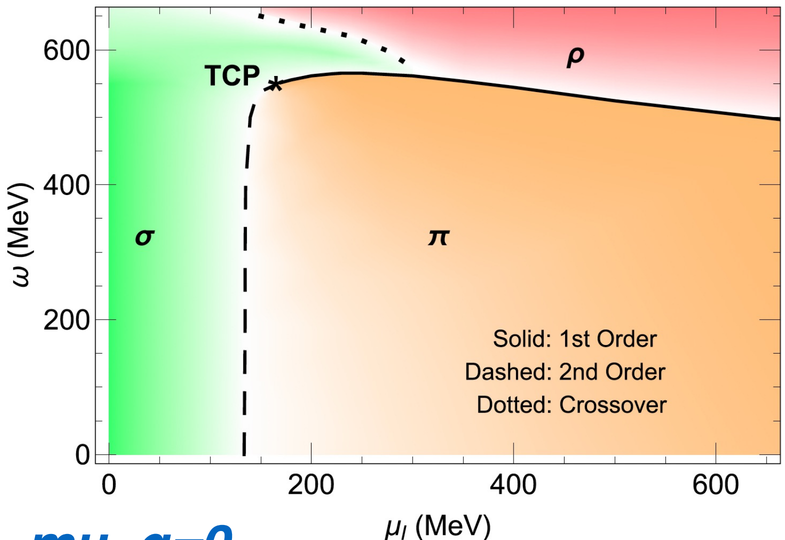
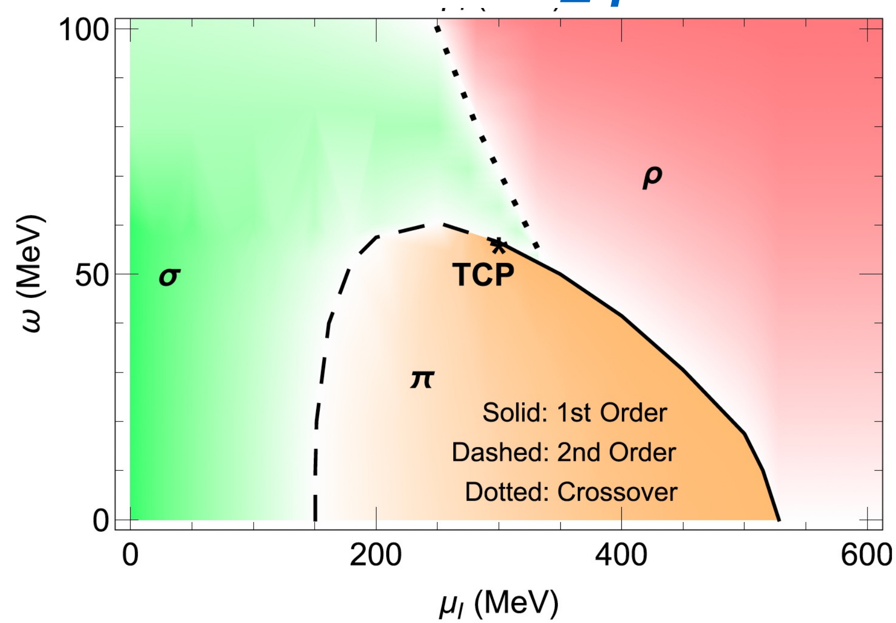


mu_q=0

mu_q=250MeV



Rich phase structures of isospin matter under rotation;
Relevant to low energy HIC or neutron star matter;
Implications for particle polarization / dileptons?!

[Hui Zhang, Defu Hou, JL, CPC44(2020)11,111001]

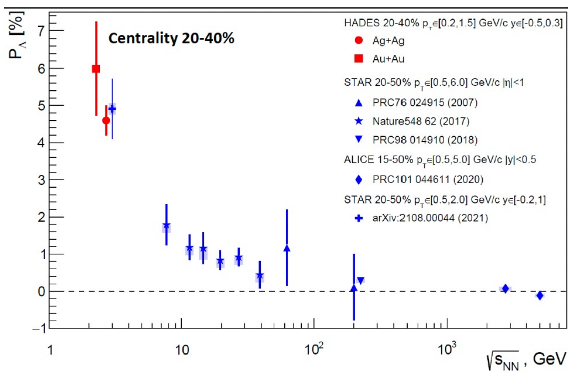
SUMMARY

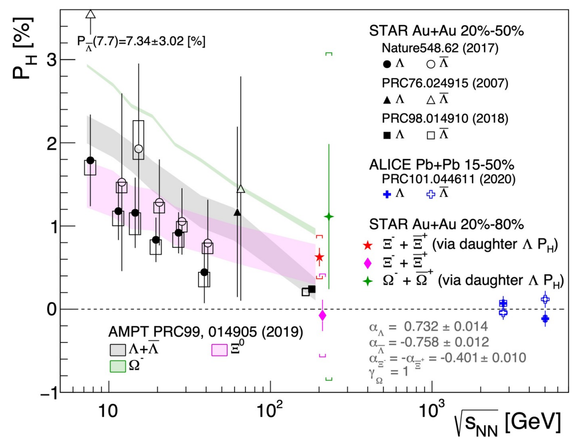
Summary

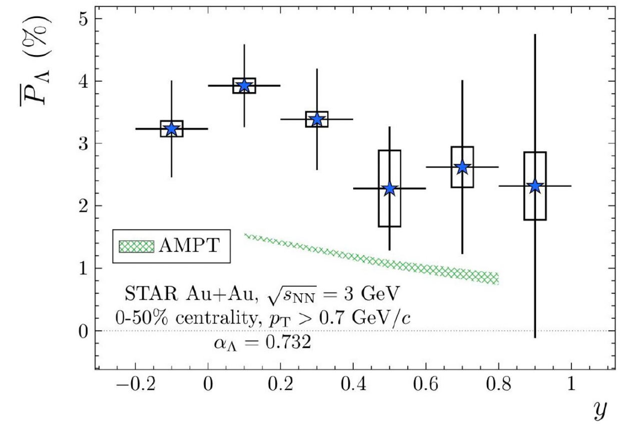
- *A new regime of study for strong interaction matter:*

  Large angular momentum as external control;
- *Nontrivial fluid vorticity structures and induced global and local polarization phenomena, especially at low energy;*
- *Development of hydrodynamic theoretical framework with angular momentum;*
- *New and rich phase structures under rotation;*
- *Many more interesting questions to be fully explored!*

Exciting New Regime @ Low Energy: Highly Polarized Strong Interaction Matter







Also a similar trend for vector meson spin alignment

Going from O(100)GeV down to O(1)GeV:

— the disappearance of partonic collectivity in a nearly perfect QGP fluid

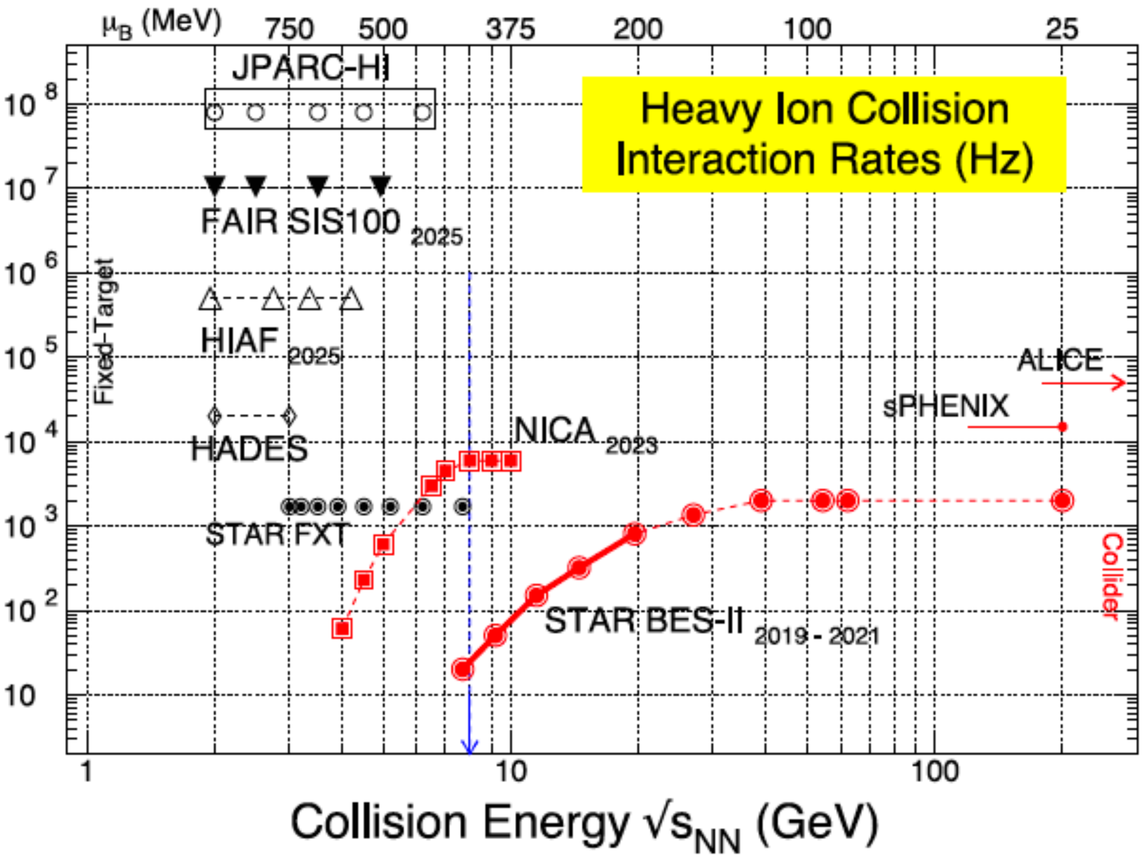
— maybe the appearance of spin collectivity in a high-polarization strong interaction fluid

Note: fireball OAM is a global quantity!

High-Polarization Matter in Low Energy Collisions

Relativistic nuclear collisions have been and will continue to be done from O(1) GeV to O(1000) GeV beam energy!

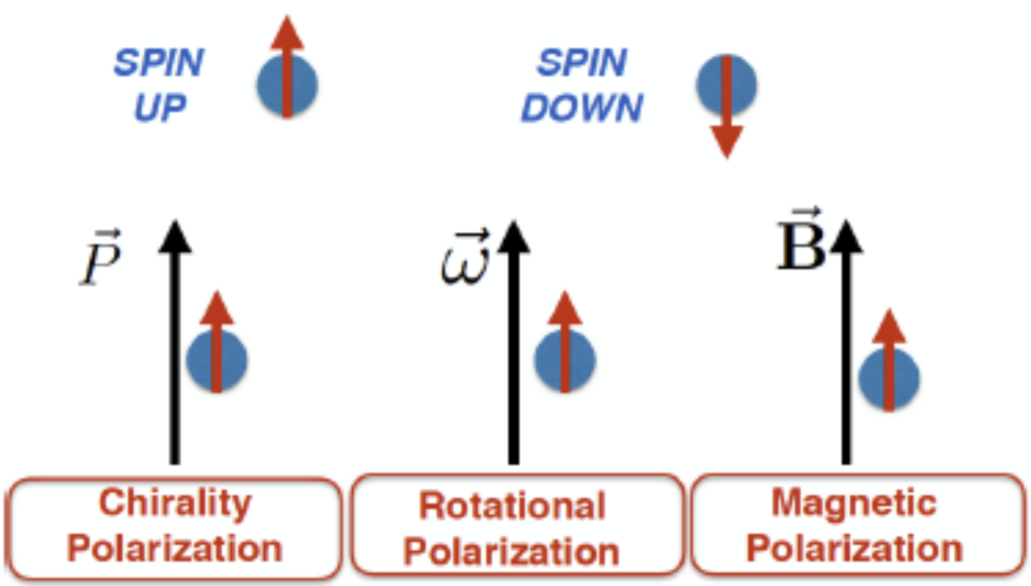
"Mapping the Phases of Quantum Chromodynamics with Beam Energy Scan", Bzdak, Esumi, Koch, JL, Stephanov, Xu, Phys. Rep. 853(2020)1-87.

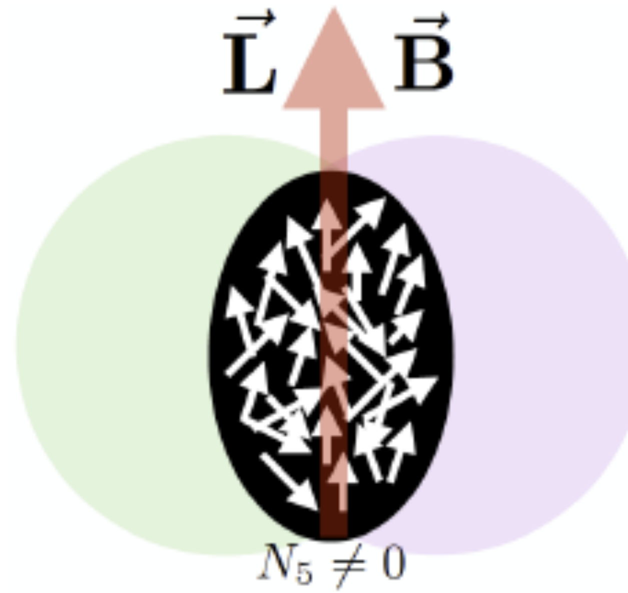


High Polarization Matter: Exciting new regime!

Potentially a new, interesting and integral component of a rich and diverse low energy nuclear collision physics program.

Spin @ Chirality, Vorticity and Magnetic Field





[arXiv:2004.00569]

The interplay of microscopic spin with
Macroscopic chirality/vorticity/magnetic field
<—>
novel phenomena in strong interaction matter