Double Beta Decay I: Why 0νββ?

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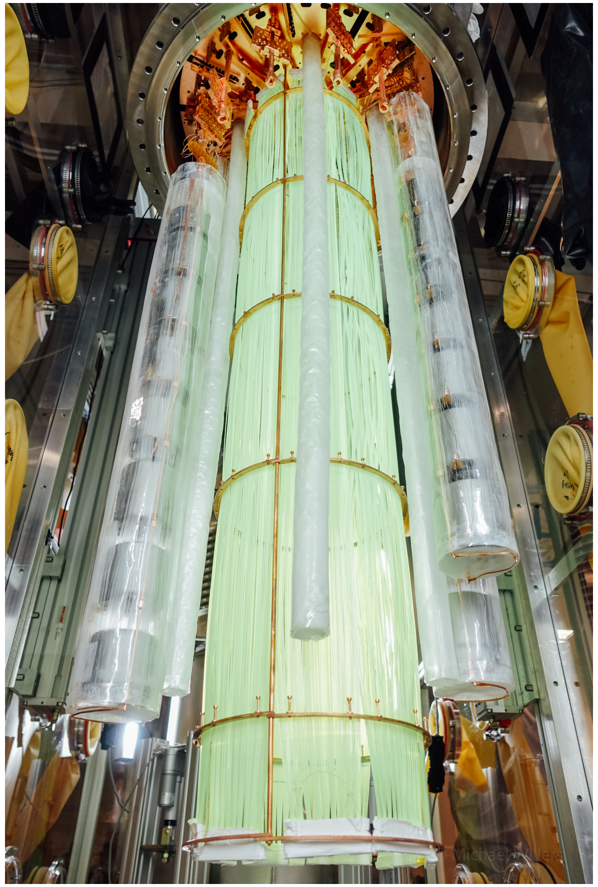
THE UNIVERSITY
of NORTH CAROLINA
at CHAPEL HILL

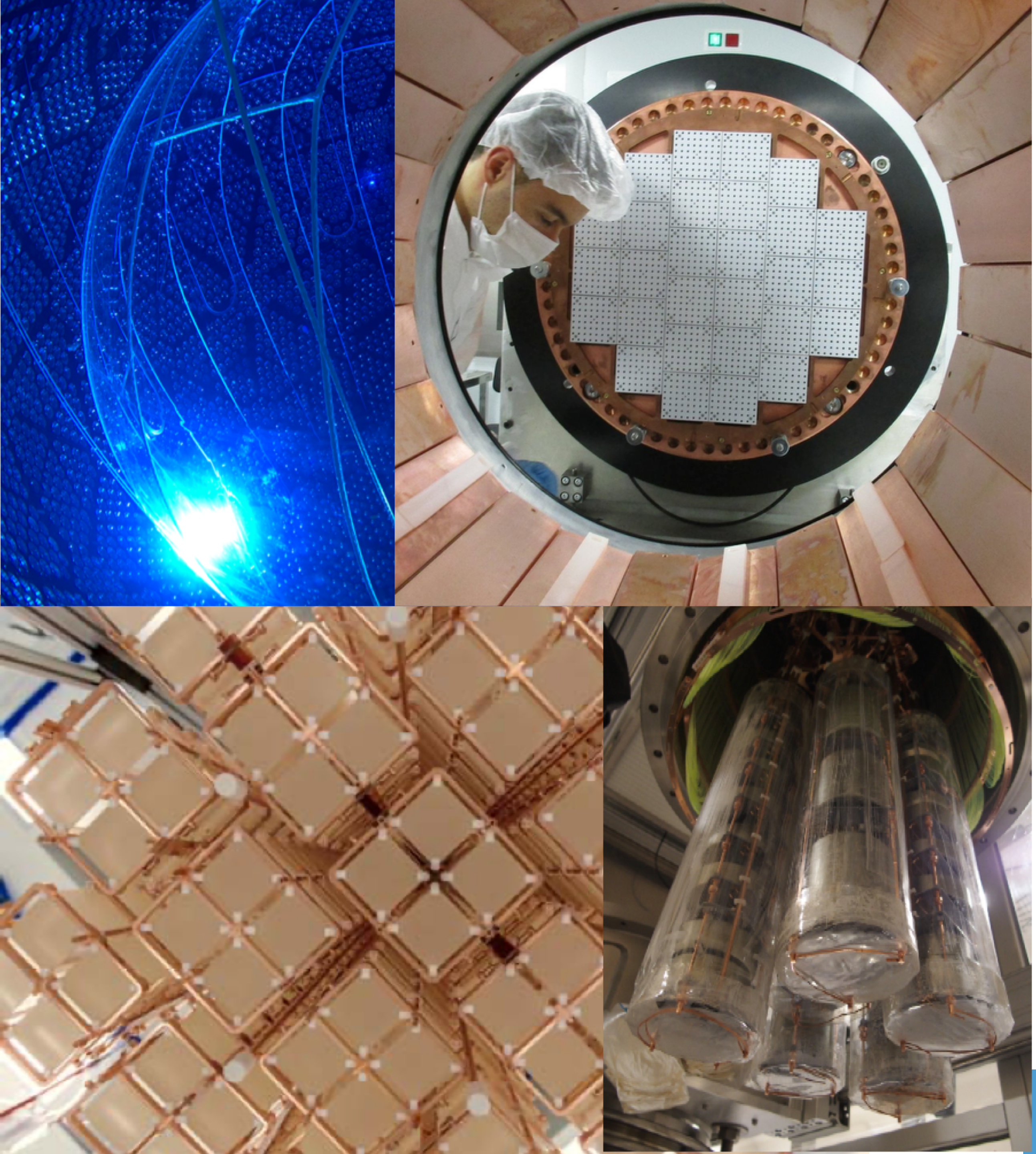
TUNL
TRIANGLE UNIVERSITIES NUCLEAR LABORATORY

Introduction

- My research focuses on two approaches to searching for 0νββ:
  - Current and next-generation experiments in 76Ge: The Majorana Demonstrator, LEGEND-200, and LEGEND-1000
  - R&D for next-next-generation experiments in liquid scintillator: NuDot
- Primary interest: improving understanding of the physics in our detectors, and using that information to improve discovery potential.



LEGEND-200 Commissioning



Outline

- Wednesday: Why look for 0νββ?
  - Intro to Neutrinos and Weak Interactions
  - Neutrino Masses: Dirac and Majorana
  - Intro to Double-Beta Decay
  - The See-Saw Mechanism(s)
  - The Rate of Double-Beta Decay
  - Leptogenesis and Baryogenesis
- Thursday: How to look for 0νββ?
- Friday: The State of the Field

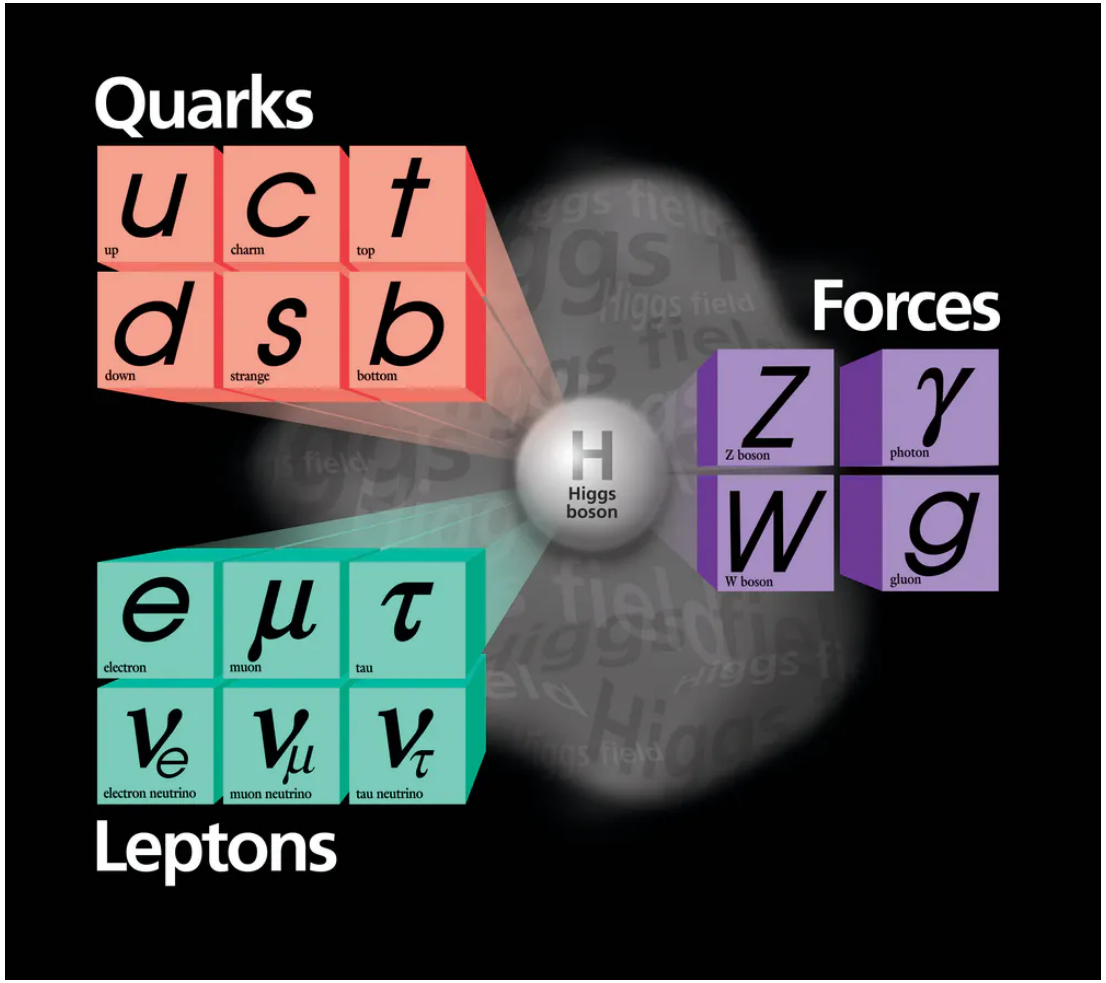
Motivating BSM Physics

- Need answers to:
  - What is dark matter?
  - What is dark energy/the mechanism behind the cosmological constant?
  - What generates neutrino mass?
  - What created the matter/antimatter asymmetry?
- Would like answers to:
  - Naturalness problems
  - CP conservation in QCD
  - Unification, flavor, etc…

Intro to Neutrinos and Weak Interactions

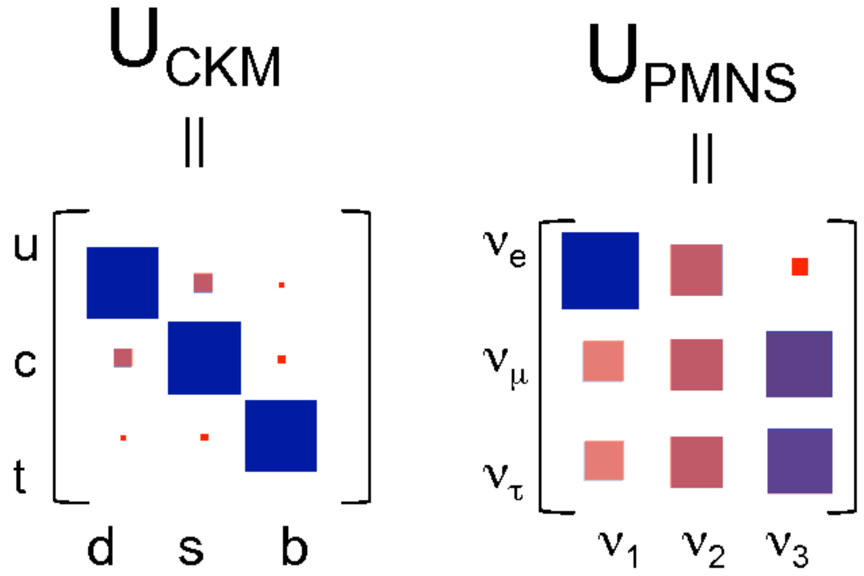
Neutrinos: The Basics

- Neutrinos: neutral fermions that only interact via the weak force
- Interactions are very rare! 10^{11} per cm^2 per second on earth from the sun, but you don't notice them.
- 3 mass states, and 3 flavor states, but these don't match up. They oscillate between flavors as they travel.
- Surprise: neutrinos have non-zero mass!
- We need new physics to explain the mass of neutrinos



Neutrino Mixing

- 3 flavors, 3 mass states
  - Mixing is large!
  - Described by 3 mixing angles and a phase angle



Neutrino Mixing

- Mixing angles are measured with neutrino oscillation experiments
- Two of the three mixing angles, $\sin^2(\theta_{12})$ and $\sin^2(\theta_{13})$, have been measured to better than 4% precision
- The octant of $\sin^2(\theta_{23})$ is degenerate with δ_{CP} and has not been determined yet

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\mu 2} & U_{\mu 3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

$$c_{ij} = \cos\theta_{ij} \text{ and } s_{ij} = \sin\theta_{ij}$$

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta_{CP}} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Atmospheric Reactor Solar

Neutrino Mass Hierarchy

- Neutrino oscillations depend on $(\Delta m_{ij})^2$ and the mixing angles
- We know the magnitudes of the $(\Delta m_{ij})^2$, but matter effects are needed to measure their signs
- From ν_e oscillations in the sun, we know $m_2 > m_1$
- We don't know where m_3 lies relative to those: need to measure matter effects on ν_μ oscillations
- This is one of the main goals of DUNE and other long-baseline neutrino experiments

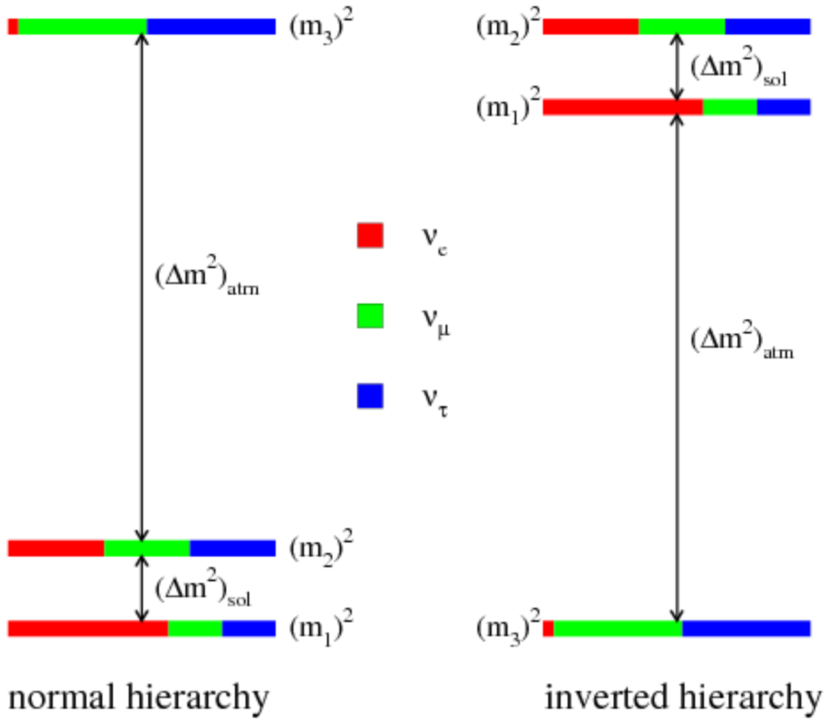
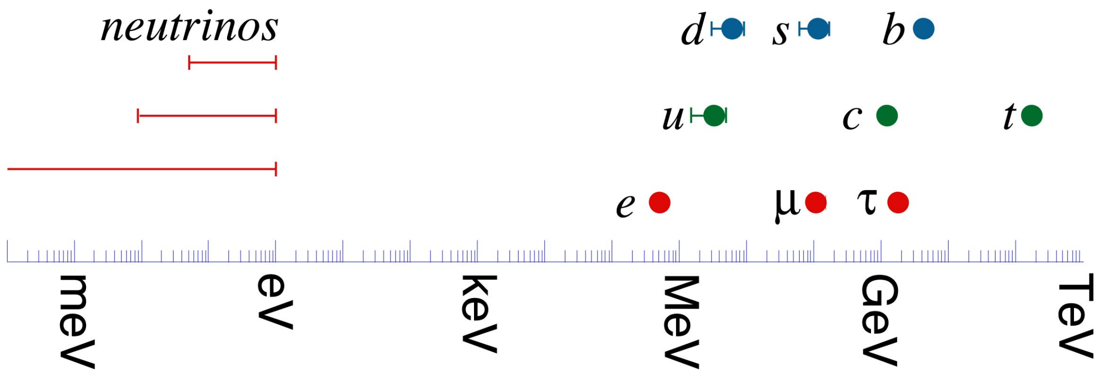


Fig. courtesy of H. Murayama

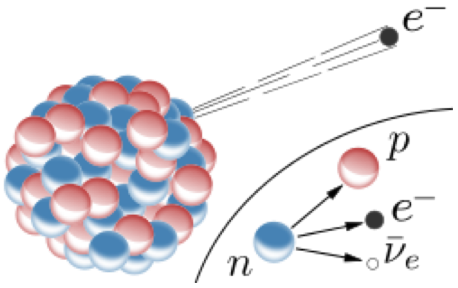
Neutrino Mass

- Because they oscillate, neutrinos must have non-zero mass
- We know neutrino masses are small, but not what they are



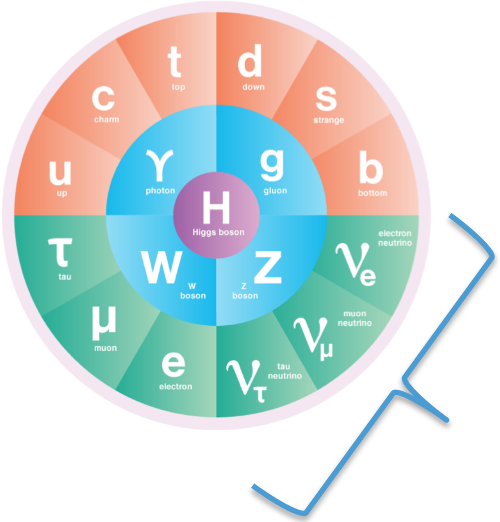
Neutrino Mass

m_β:



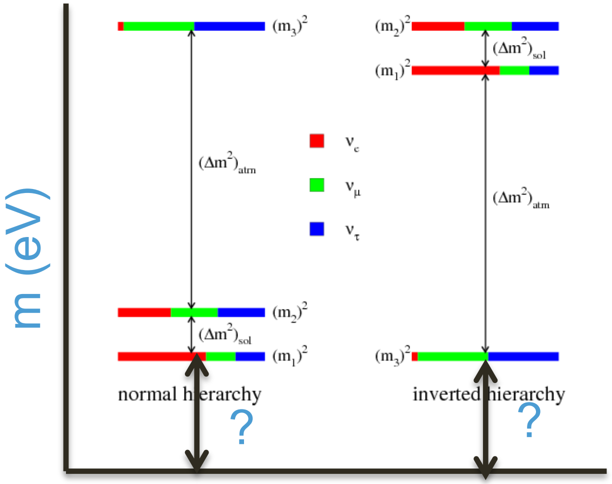
Kinematics experiments

Σm:



Cosmology experiments

$m_{lightest}$:



Neutrino Mass Measurements

Technique	Sensitivity	Current limit
Neutrino Oscillations	$\Delta m_{ij}^2 = m_i^2 - m_j^2$	IH: $\Sigma m > 98$ meV NH: $\Sigma m > 59$ meV
Cosmological modeling of Astrophysical Observations	$\sum m_i + \ light\ dof$	$\Sigma m < 120 - 230$ meV, depending on data sets used
Beta Decay Kinematics	$\sum \lvert U_{ei} \rvert^2 m_i^2$	$m_\beta < 0.8$ eV $\Sigma m <$ ~2400 meV

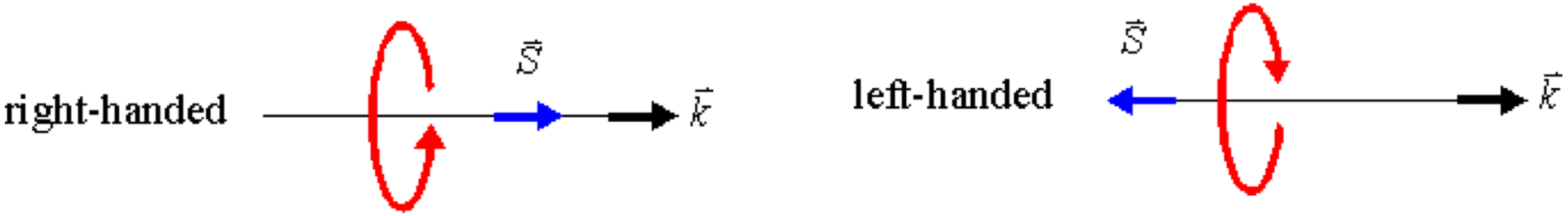
Neutrinos and the Weak Interactions

- To go a bit deeper, let’s quickly review some concepts related to fields and weak interactions

Helicity

- Helicity describes the alignment of spin and momentum:

$$\hat{h} = \frac{\vec{S}\cdot\vec{P}}{s\left|\vec{P}\right|}$$



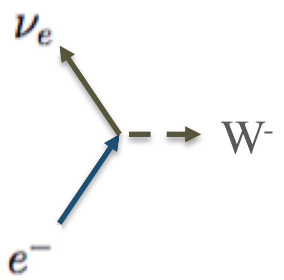
  - For massless particles, eigenvalues are ±1
  - For massive particles, reference-frame dependent: if you boost to a reference frame moving faster than the particle, p reverses but s does not

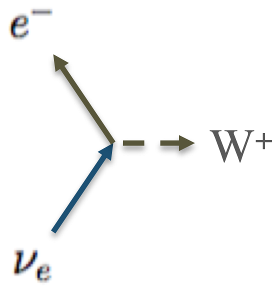
Chirality

- Chirality describes the field's transformation under γ^5:
  - Left-chiral: eigenstate of γ^5 with eigenvalue = -1
  - Right-chiral: eigenstate of γ^5 with eigenvalue = 1
  - $\frac{1}{2}(1- \gamma^5)$ is a projection operator, choosing the left-chiral state
- Mass couples the left- and right-chiral fields together
  - If $E \gg mc^2$, helicity~chirality

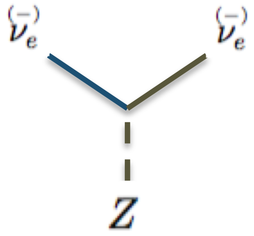
Weak Interactions

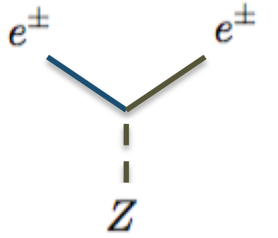
- Charged weak interactions:
  - V-A form: $\mathcal{L}_{I,L}^{(CC)} = -\frac{g}{2\sqrt{2}}\overline{\nu_e}\gamma^\mu(1-\gamma^5)eW_\mu + H.c.$
  - Notice: only couples to left-chiral states!

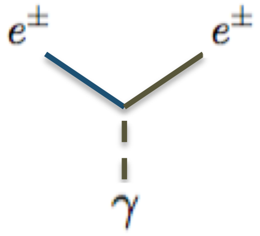




- Neutral weak interactions:
  - Z coupling: $\frac{-ig_z}{2}\gamma^\mu({c_V}^f - {c_A}^f\gamma^5)$
  - No longer pure V-A, couples to right-handed particles







Weak Interactions and Neutrinos

- Full neutral current Lagrangian:

$$\mathcal{L}_{I,L}^{(NC)} = -\frac{g}{2\cos\vartheta_W}(\overline{\nu_e}\gamma^\mu(g_V^\nu - g_A^\nu\gamma^5)\nu_e + \bar{e}\gamma^\mu(g_V^l - g_A^l\gamma^5)e)Z_\mu + -e(-\bar{e}\gamma^\mu e)A_\mu$$

$$\cos(\theta_W) = \frac{m_W}{m_Z}$$

- For the neutrinos:

$$({g_V}^\nu - {g_A}^\nu\gamma^5) = \frac{1}{2}(1-\gamma^5)$$

so right-handed neutrinos don't participate!

	g_V	g_A
v	½	½
l	-½ + 2 $\sin(\theta_W)^2$	-½

Take-away: right handed neutrinos and left-handed antineutrinos don't have **any** interactions

In the SM, it is assumed that neutrino fields have only left-handed components – their observed interactions don't require right-handed components (and we don't have a way to produce them)

Majorana and Dirac Mass Terms

*This section freely borrows from Ben Jones's "The Physics of Neutrinoless Double Beta Decay: A Primer": 2108.09364

Spinors and the Dirac Equation

- Neutrinos are massive, relativistic spin ½ particles, so they obey the Dirac equation.
- Solutions to the Dirac equation are 4-component spinors, ψ:

$$\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi = 0$$

- You can interpret this as a Schrodinger-like equation for a 4-component wave function: probability of finding four distinct kinds of particle at each point in space
- You can also interpret this as an equation of motion for a 4-component field
- Either way, this equation carries **up to** 4 pieces of information about what exists at each point in space, but the 4 components may or may not be independent, as we'll see

Weyl Basis

- Let’s choose a basis that decouples left- and right-chiral components, called the Weyl basis

$\psi = \begin{pmatrix} \xi \\ \eta \end{pmatrix}$, where the two upper components are a left-chiral field ξ and the two lower components are a right-chiral field η

- Then the γ matrices are:

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \qquad \gamma^5 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}$$

- We can confirm the chirality of ξ and η by applying the chirality operator γ^5:

$$\gamma^5 \begin{pmatrix} \xi \\ 0 \end{pmatrix} = - \begin{pmatrix} \xi \\ 0 \end{pmatrix} \qquad \gamma^5 \begin{pmatrix} 0 \\ \eta \end{pmatrix} = \begin{pmatrix} 0 \\ \eta \end{pmatrix}$$

Left-chiral

Right-chiral

Dirac Equation in the Weyl Basis

- Writing it out the Dirac equation in terms of the chiral components:

$$i(\partial_0 + \vec{\sigma} \cdot \nabla)\eta = m\,\xi$$
$$i(\partial_0 - \vec{\sigma} \cdot \nabla)\xi = m\,\eta$$

- The two sub-fields are coupled! Chirality is not conserved unless m = 0

Dirac vs. Majorana

- The 4-component spinor representation and Dirac equation are always a valid description of neutrinos, whether or not they are Majorana
- The question is whether ξ and η are independent fields that can have independent values everywhere in space
- Dirac: 4 degrees of freedom at every point in space; we can interpret them as independent left- and right-chiral fermions and anti-fermions
- Majorana: Only 2 degrees of freedom at every point in space; if you know 2 components (say, both components of η), you immediately know the other 2

The Majorana Condition

- We need to construct a spinor such that $\eta = f(\xi)$ at some initial time, and this relationship continues to hold as the field evolves according to the Dirac equation
- Ettore Majorana came up with a solution:

$$\psi = \begin{pmatrix} \xi \\ -i\sigma_2\xi^* \end{pmatrix}$$

- This satisfies the Dirac equation and the Majorana condition. Whatever the top two components are at a given moment in time, the bottom two will stay related to them in the same way.

Majorana Fermions

- The 4-component Dirac equation is no longer needed. Instead, we can write the two-component Majorana equation of motion:

$$\sigma^{\mu}\partial_{\mu}\xi + m\sigma_2\xi^* = 0$$

- Looking at the full 4-component spinor, we see that the Majorana spinor is invariant under charge conjugation:

$$\psi^c \equiv i\gamma^2\psi^* = \psi$$

- This means Majorana particles are their own antiparticles
- Take-away: Majorana spinors have 2 independent components, which correspond to amounts of left- and right-handed stuff at each point in space. Each of those is both particle and anti-particle at the same time.

Traits of Majorana Particles

A Majorana particle needs to…

- have mass (or the Dirac equations decouple and the distinction is irrelevant)
- be spin ½ (so it obeys the Dirac eqn)
- not interact with any gauge fields

An example of why gauge charges aren't allowed:

Consider a particle that interacts electromagnetically. The equation of motion would be:

$$\left(i\gamma^{\mu}\left(\partial_{\mu}+iqA_{\mu}\right)-m\right)\psi=0$$

Taking the complex conjugate and multiplying by $i\gamma^{2}$ to get an equation of motion for ψ^{c}:

$$\left(i\gamma^{\mu}\left(\partial_{\mu}-iqA_{\mu}\right)-m\right)\psi^{c}=0$$

If $\psi=\psi^{c}$, these two equations would be inconsistent unless q = 0

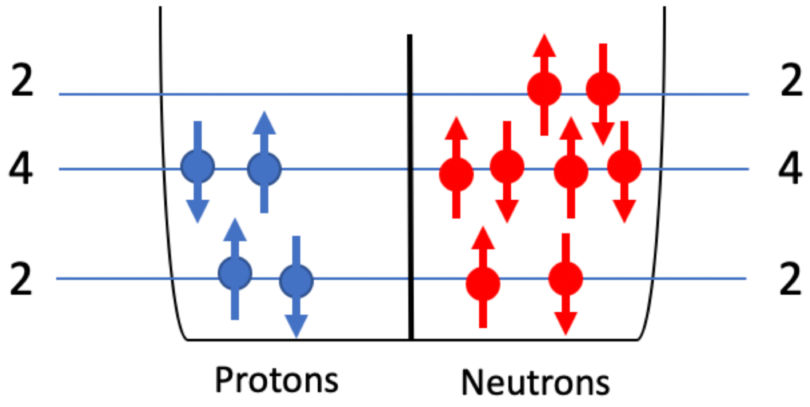
Neutrinos as Majorana Fermions

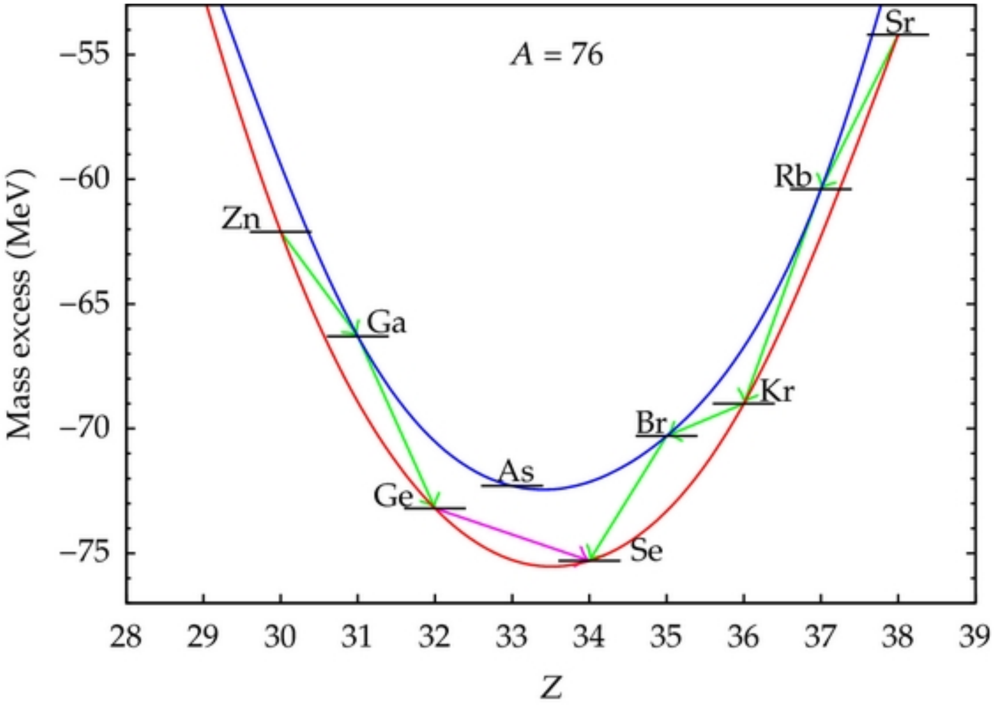
- The neutrino is the only Standard Model fermion that meets all the conditions, and could therefore be a Majorana particle
- Beyond that, there are good reasons to think neutrinos might be Majorana particles:
  - No need for non-interacting fields to explain non-zero neutrino mass; can naturally get small neutrino masses
  - Lowest-order new physics we can add to the standard model
  - Could resolve the matter asymmetry problem

Intro to Double-Beta Decay

Double-Beta Decay

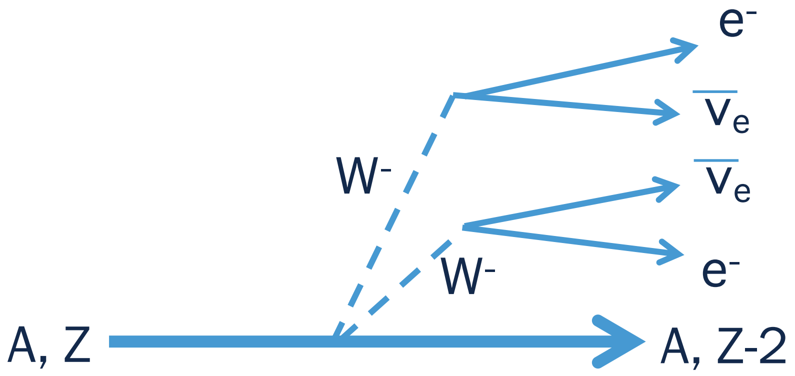
- Because of Pauli exclusion, nuclei are lower in energy if they have even numbers of protons and neutrons– they prefer to have paired spins
- For certain even-even nuclei, single beta decay is disallowed b/c of energy or momentum
- Instead, they double-beta decay, as predicted in 1935



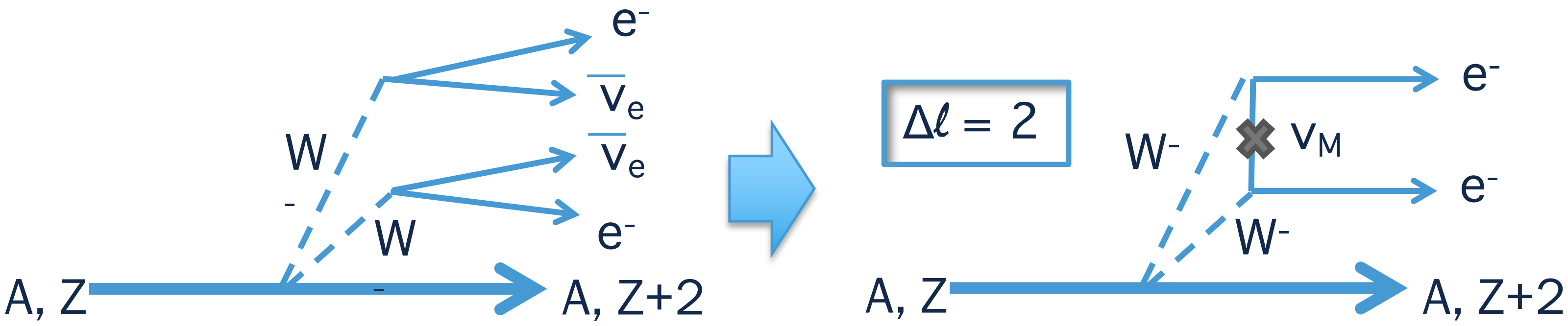


Two-Neutrino Double-Beta Decay

- Second-order weak process, $t_{1/2} \sim 10^{19}$ to 10^{21} years
- One of the longest-lifetime process we've ever observed. Not seen until 1987!
- In the SM, two electron antineutrinos are emitted

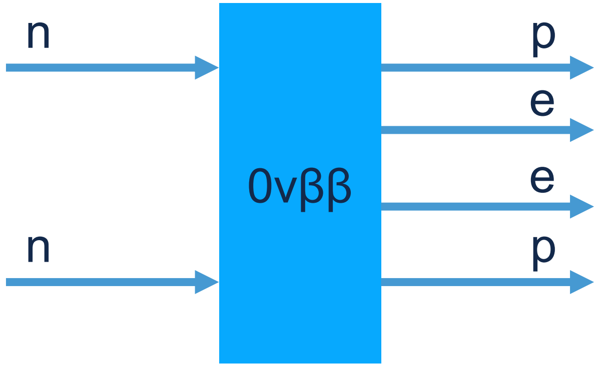


Neutrinoless Double-Beta Decay

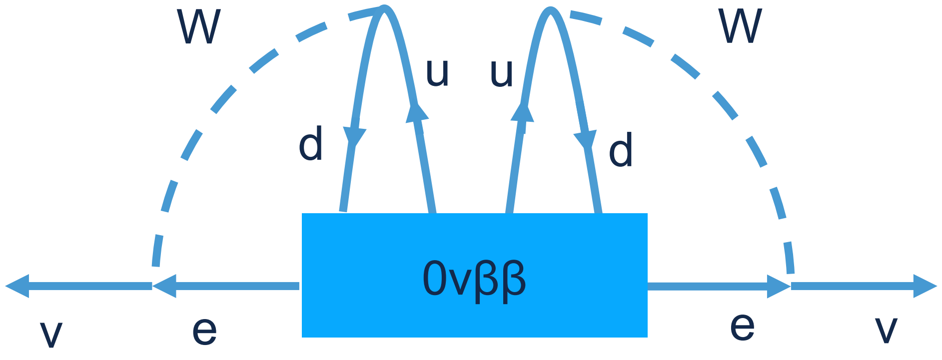


- If neutrinos are Majorana, 0vββ could occur
- Lepton number conservation is violated by 2 units
- In this case, I've drawn the exchange of a light neutrino, but you can think of that "x" as a contracted diagram of any sort (with new physics in it)

Majorana Neutrinos and 0νββ



(A, Z) → (A, Z+2) +2e-



(A, Z) → (A, Z+2) +2e-

Model-independent implications of 0νββ:

- Lepton number violation
- Neutrino-antineutrino oscillation, implying a non-zero Majorana mass term

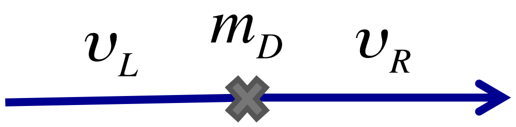
The mechanism of 0νββ determines the rate along with the parameters of the model

See-Saw Mechanisms

Dirac Mass Term

- The neutrino could get its mass the same way other leptons do
- Add a non-interacting right-chiral neutrino field to the SM

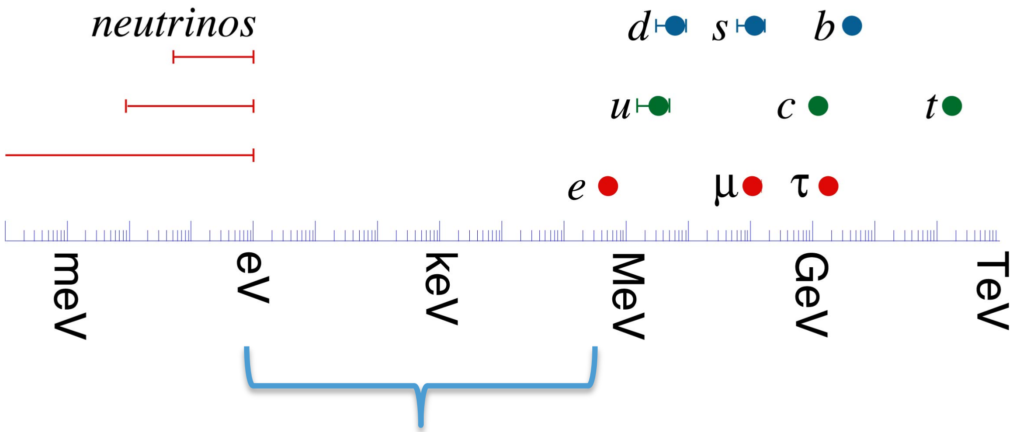
$$L^{D}_{mass} = -m_{D}\left(\overline{\upsilon_{L}}\upsilon_{R} + \overline{\upsilon_{R}}\upsilon_{L}\right)$$



υ_L ↑ makes ℓ^+

$\overline{\upsilon_R}$ ↑ makes ℓ^-

$\underbrace{\upsilon_R \; \overline{\upsilon_L}}$ non-interacting



Leads to the "hierarchy problem"

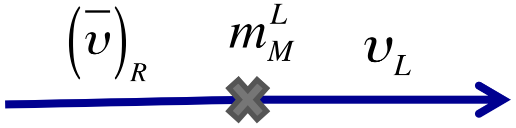
Over 6 orders of magnitude difference in Yukawa couplings: Why?

Majorana Mass Term

- If the neutrino is a Majorana fermion, then we can write a non-zero left-handed Majorana mass term:

$$L^L_{mass} = -\frac{1}{2} m^L_M \left(\overline{\nu_L} \nu_L^C + \overline{\nu_L^C} \nu_L \right)$$

- We can identify ν_L^C with the particle we observe as the anti-neutrino:



Lepton number not conserved!

2 mass-degenerate states:

makes ℓ^+ makes ℓ^-

More on Majorana Mass Terms

- One problem: left-handed term isn't renormalizable in the SM. It's not invariant under SU(2)xU(1):

$$L_{mass}^{L} = -\frac{1}{2} m_M^L \left(\overline{\upsilon_L} \upsilon_L^C + \overline{\upsilon_L^C} \upsilon_L \right)$$

$I_3 = 1, Y = -2$

- This term is allowed if you introduce new physics at high energy to cut off the infinities
- The right-handed Majorana mass term is allowed:

$$L_{mass}^{R} = -\frac{1}{2} m_M^R \left(\overline{\upsilon_R} \upsilon_R^C + \overline{\upsilon_R^C} \upsilon_R \right)$$

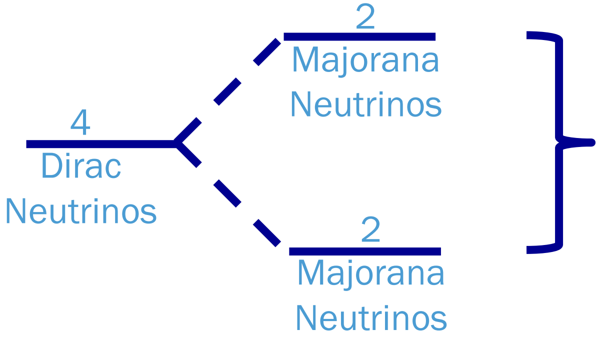
$I_3 = 0, Y = 0$

The Type I See-Saw Mechanism

- If we include all the terms (Dirac, left-handed Majorana, right-handed Majorana):

$$L_{mass} = \frac{1}{2}\left(\overline{\nu_L}\;\overline{\nu_R}^{C}\right)\begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}\begin{pmatrix} \nu_L^C \\ \nu_R \end{pmatrix} + h.c.$$

- Setting m_L to 0, mass eigenvalues are $\lambda = \frac{m_R}{2} \pm \frac{m_R}{2}\sqrt{1+\frac{4m_D^2}{m_R^2}}$
- If $m_R >> m_D$, $\lambda_1 = m_R \quad \lambda_2 = \frac{2m_D^2}{m_R}$
- Called the "see-saw mechanism":



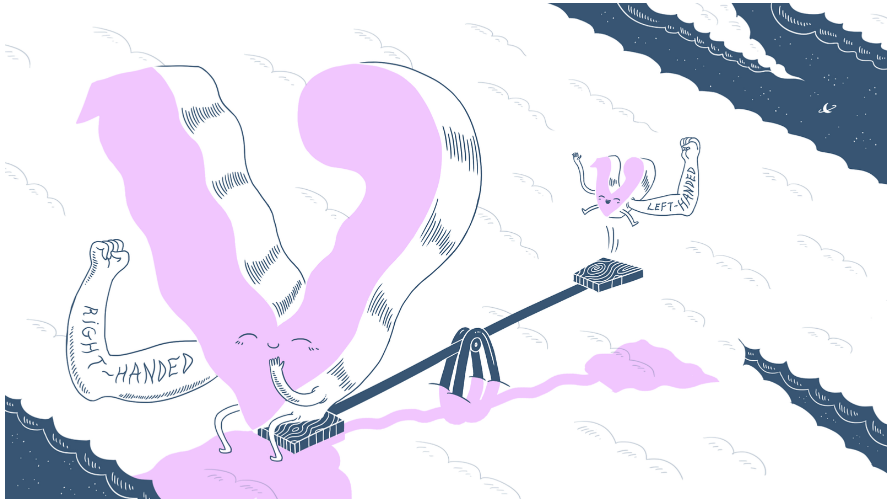
Splitting due to Majorana mass

The Type I See-Saw Mechanism

- If m_R is of GUT scale (about 10^{15} GeV) and m_D is EW scale (about 100 GeV), mass eigenvalues are:

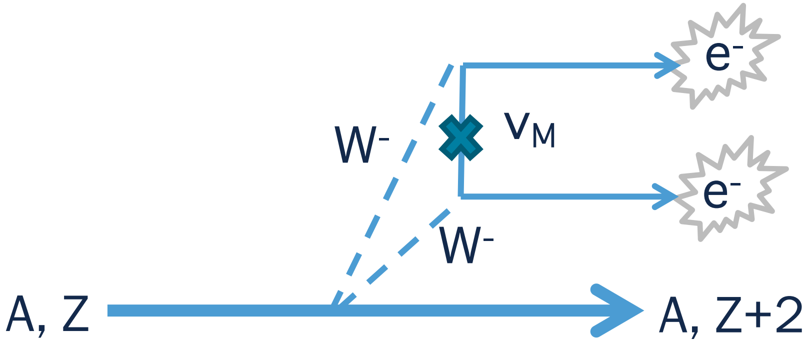
$$M_\upsilon \sim \frac{m_D^2}{m_R} \sim .01\,eV \quad M_N \sim m_R \sim 10^{15}\,GeV$$

- So you get a “natural” neutrino of the correct mass by introducing a new GUT-scale particle



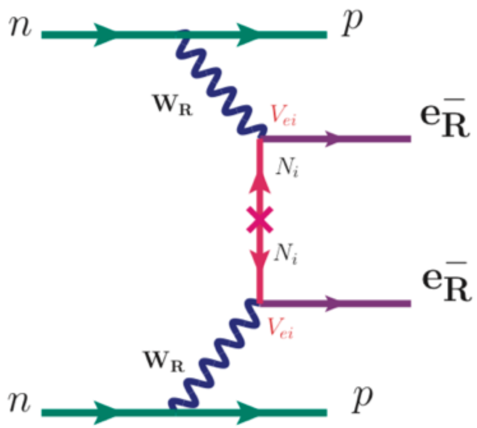
Type I See-Saw and 0νββ

- In the Type-I see-saw, the right-handed neutrino is heavy, so the neutrino exchanged in 0νββ propagates as a light Majorana neutrino
- This is considered the “simplest” model for Majorana neutrinos because it requires only one new particle, at energies where we expect new physics
- Any number of heavy N’s can be added, but you need at least 2



Other See-Saw Mechanisms

- Type II See-Saw: add a complex scalar triplet, $\Delta_L = (1, 3, 2)$
  - No right-handed neutrinos needed to explain neutrino masses and mixing
  - Example: Left-Right Symmetric Model



Left-Right Symmetric Model,
$M_{W_R} \sim 2$ TeV, $M_N = 1$ TeV,
$g_R \sim 2/3\, g_L$

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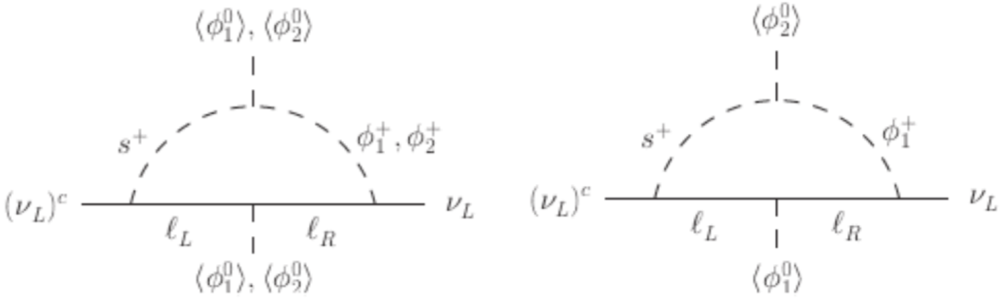
Other See-Saw Mechanisms

- Type III See-Saw: add a “fermionic triplet,” $\Delta_L = (1, 3, 0)$
  - Leads to same mass matrix as Type I, but adds heavy charged leptons
  - Example: GUTs, see P. Fileviez Perez, 1501.01886
- Radiative Majorana masses: add new scalars and Majorana fermions
  - See H. Sugiyama, 1505.01738

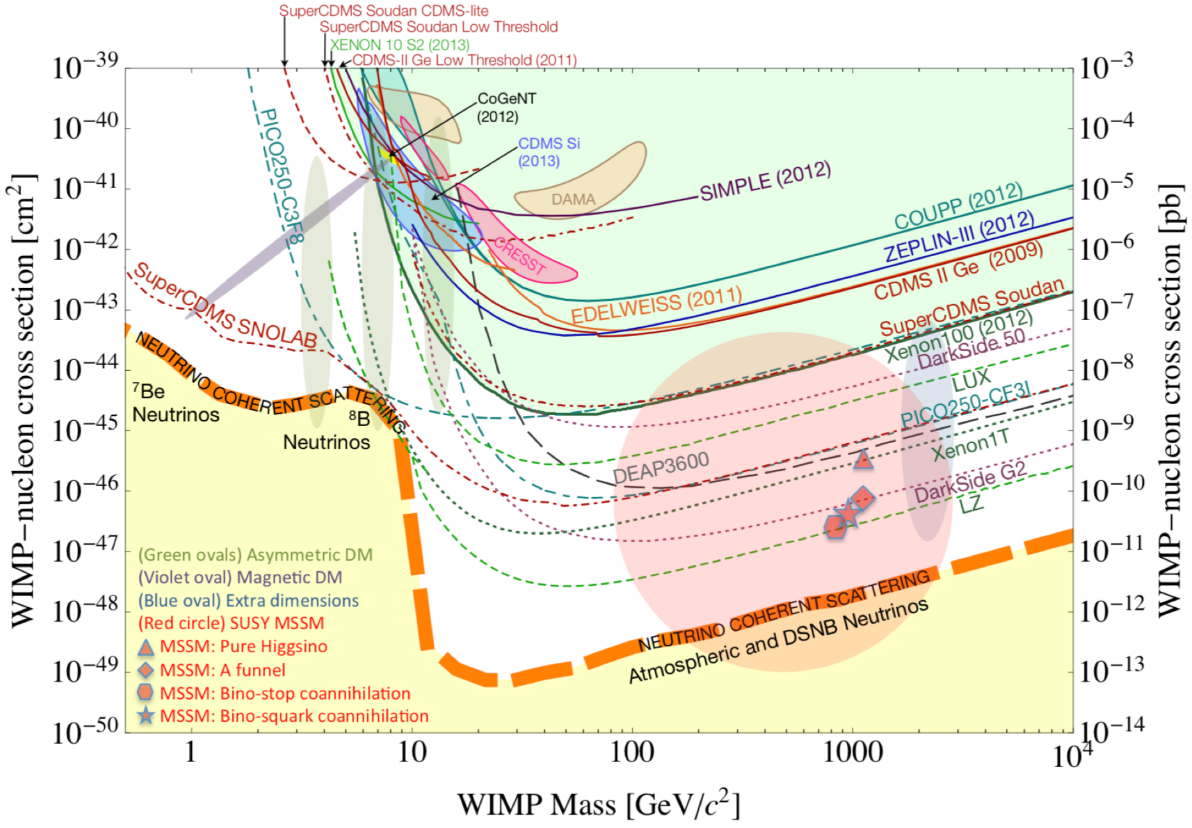


Distinguishing Between Mechanisms

- Different mechanisms make predictions about $m_{\beta\beta}$. If we measure $m_{\beta\beta}$, neutrino mass hierarchy, and neutrino mass, we can see which "theory island" we're in
- Many mechanisms predict relatively low-energy (TeV-scale) new particles. Collider experiments could see these.
- $0\nu\beta\beta$-related measurements could also help us distinguish mechanisms: energy and momentum of each outgoing electron, $0\nu\beta\beta$ decays to excited states, and ratios between different $0\nu\beta\beta$ isotopes

”Theory Islands” for 0νββ

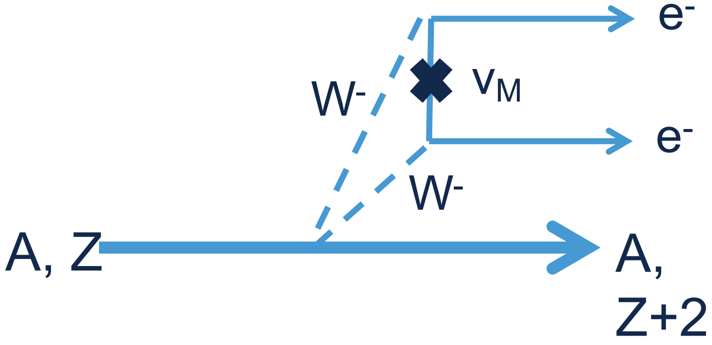
- There’s still a lot we don’t know!
- Simplest Type 1 seesaw model gives us a set of “theory islands” but isn’t the whole story
- Understanding the other properties of neutrinos can give us hints of where to look, but they can’t really rule out parameter space for us



WIMP Searches: Snowmass 2013 Status

The Rate of Double-Beta Decay

The 0νββ Rate for Light Majorana Neutrino Exchange



$$(T^{0\nu}_{1/2})^{-1} = G^{0\nu}|M_{0\nu}|^2\left(\frac{\langle m_{\beta\beta}\rangle}{m_e}\right)^2$$

Effective Majorana mass for light neutrino exchange:

$$\langle m_{\beta\beta}\rangle = |\sum_{i=1}^{3} U_{ei}^2 m_i|$$

$$U = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix}$$

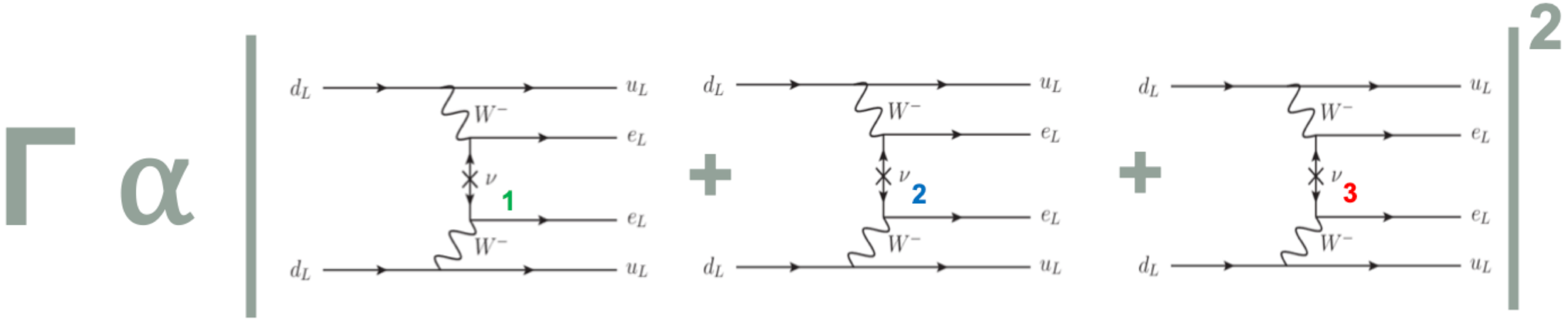
$$= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \begin{bmatrix} e^{i\alpha 1/2} & 0 & 0 \\ 0 & e^{i\alpha 2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

c_{ij} = cos θ_{ij} , s_{ij} = sin θ_{ij} , δ = Dirac CP violation, α_i = Majorana CP violation

Even under simple assumptions, the 0νββ rate depends on:

- ν mixing angles
- ν masses
- mass hierarchy
- 2 totally unknown phases

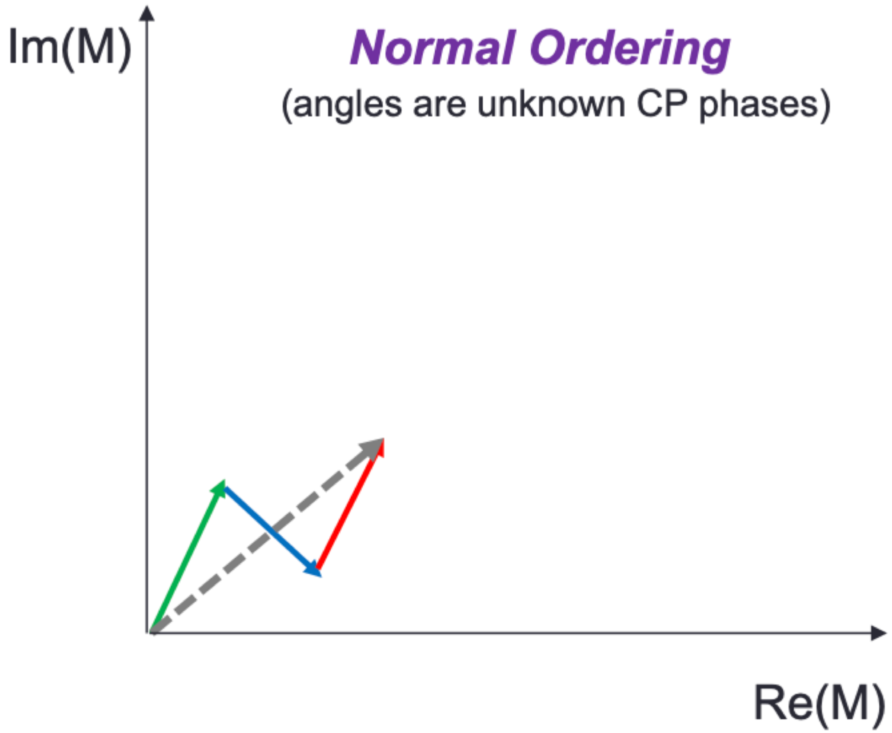
Understanding $m_{\beta\beta}$

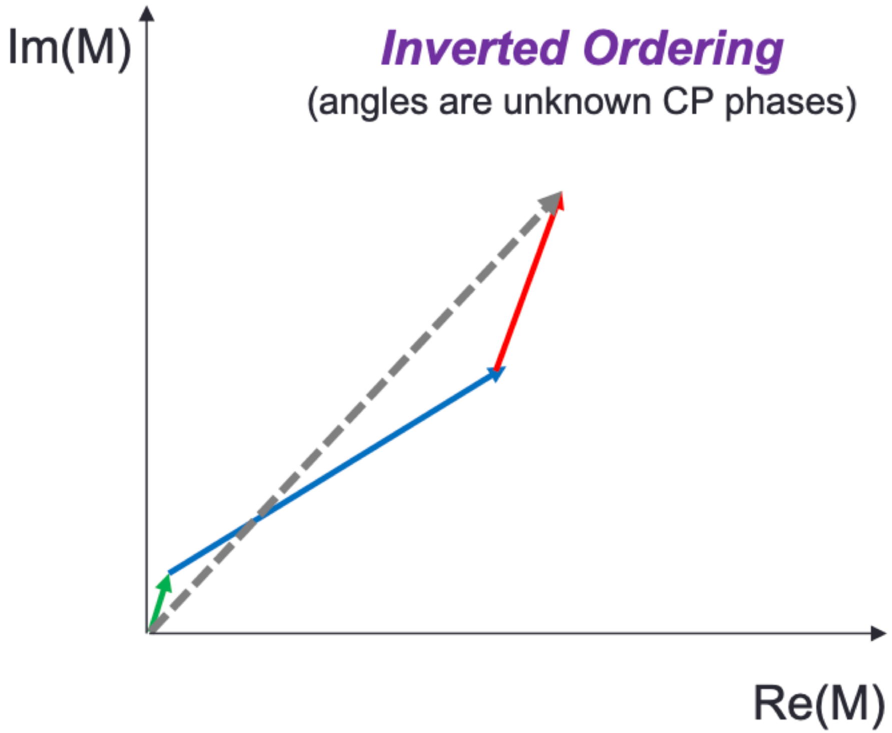


$$\langle m_{\beta\beta}\rangle = |\sum_{i=1}^{3} U_{ei}^2 m_i|$$

$$\langle m_{\beta\beta}\rangle = \cos\theta_{12}^2 \cos\theta_{13}^2 e^{2i\alpha} m_1 + \cos\theta_{12}^2 \sin\theta_{12}^2 e^{2i\beta} m_2 + \sin\theta_{13}^2 m_3$$

Understanding $m_{\beta\beta}$





Figures by B. Jones

Interpretation of Half-Life Sensitivity

$$(T_{1/2}^{0\nu})^{-1} = G^{0\nu}|M_{0\nu}|^2 \left(\frac{\langle m_{\beta\beta}\rangle}{m_e}\right)^2$$

$$\langle m_{\beta\beta}\rangle = |\sum_{i=1}^{3} U_{ei}^2 m_i|$$

- Light Majorana neutrino exchange: assumes new physics is at GUT scale, 0νββ mediated by dim. 5 operator
- Used to compare and set goals for future experiments

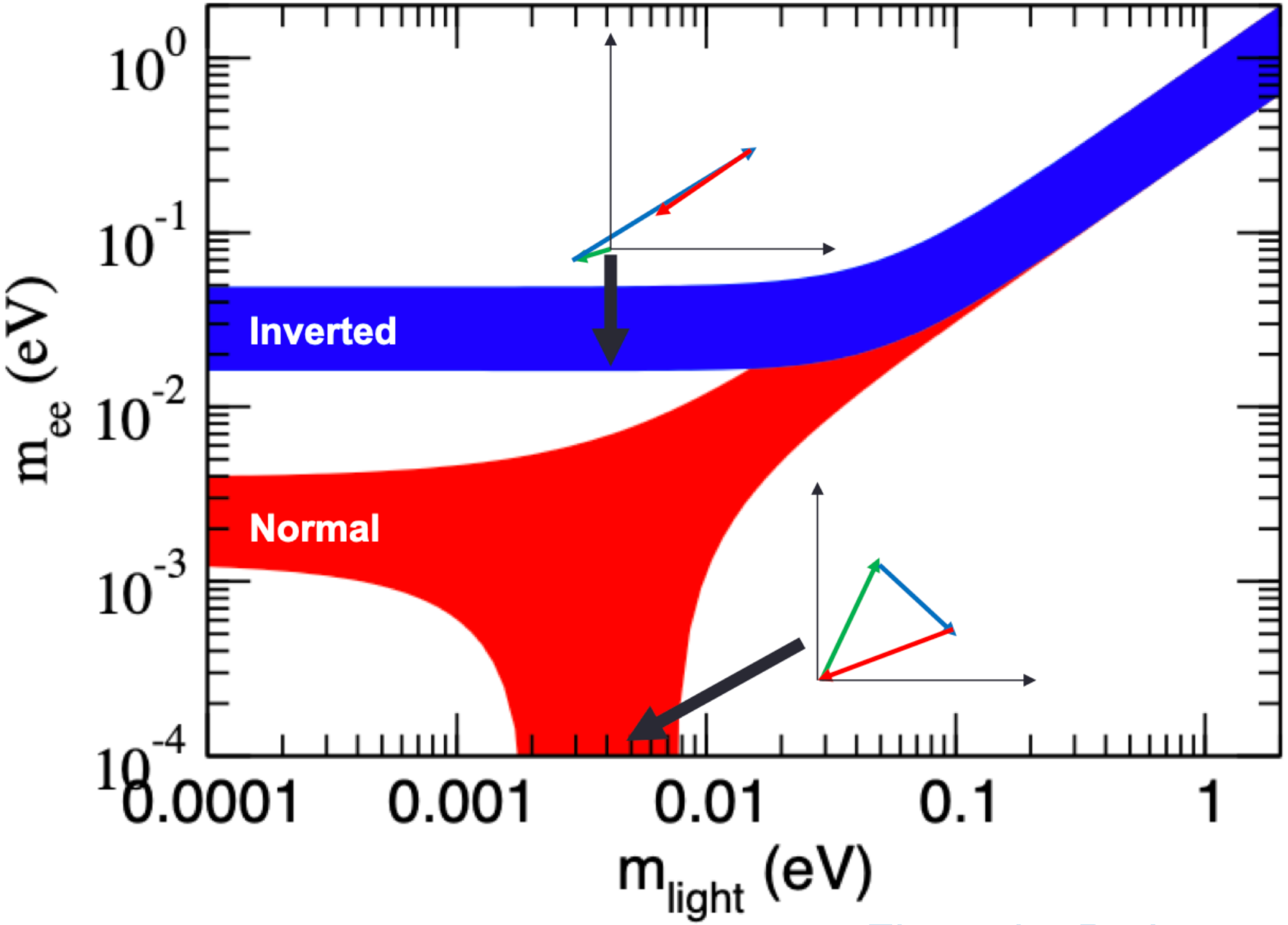
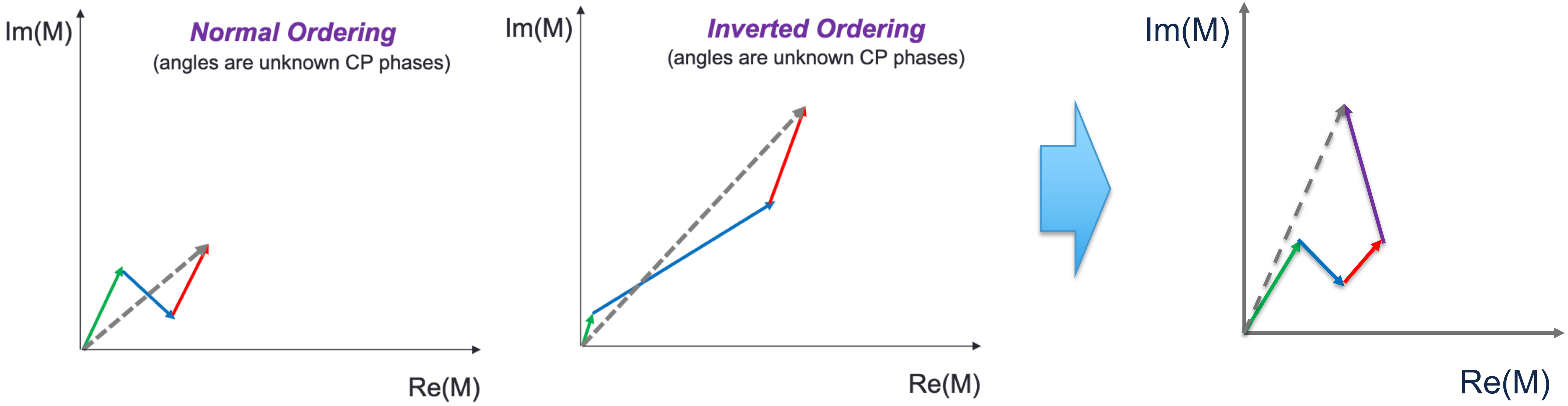


Figure by B. Jones

Adding Sterile Neutrino(s)

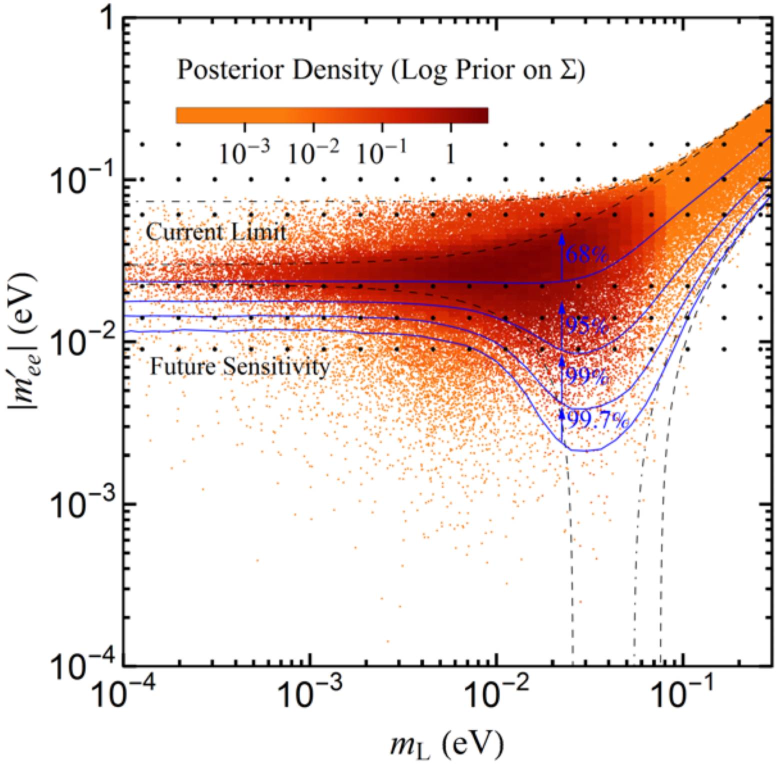


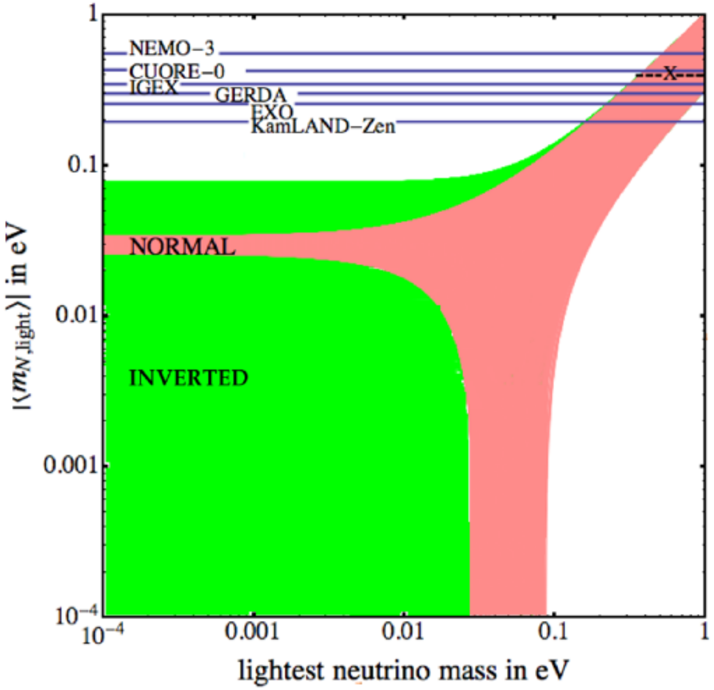
Figures by B. Jones

Sterile Neutrinos and the 0νββ Rate

The addition of sterile neutrinos would modify the rate of 0νββ and can switch IO/NO allowed regions

(3+1)ν mixing, flat prior on Σm
$\Delta m^2_{41} \equiv 1.7$ eV^2 and $\sin^2\theta = 0.019$
Nuc. Phys. B 945, 114691 (2019)

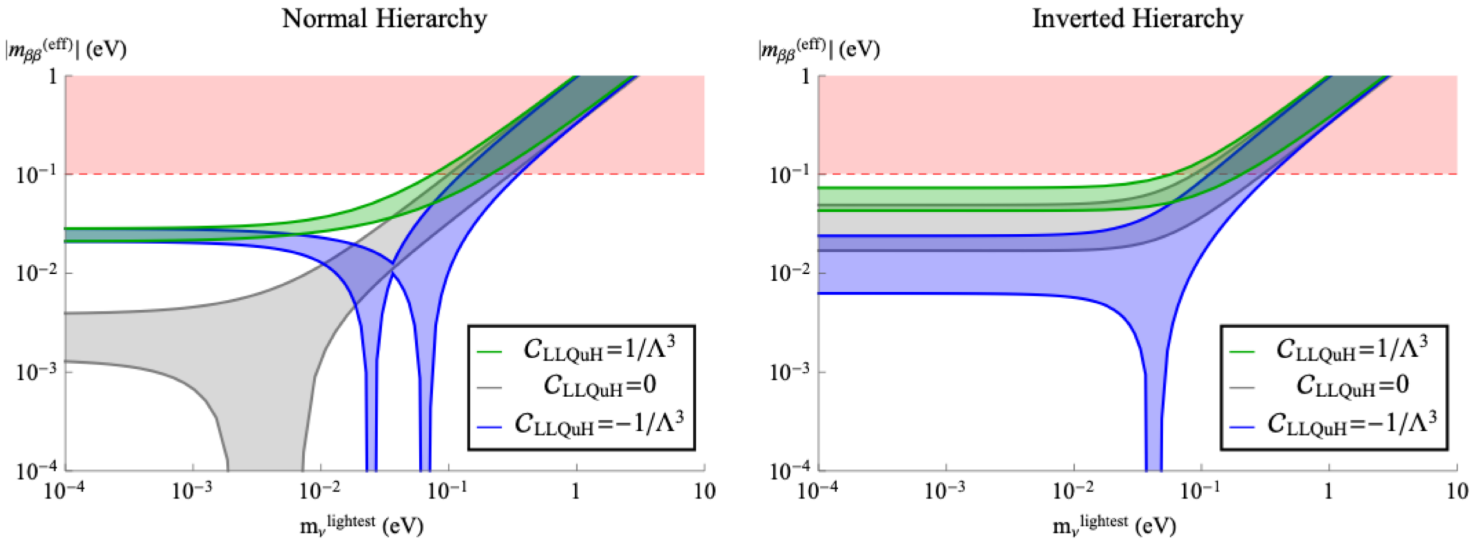




(3+1)ν mixing
$m_4 = 1$ eV and
$|U_{e4}|^2 = 0.03$
PRD 92, 093001 (2015)

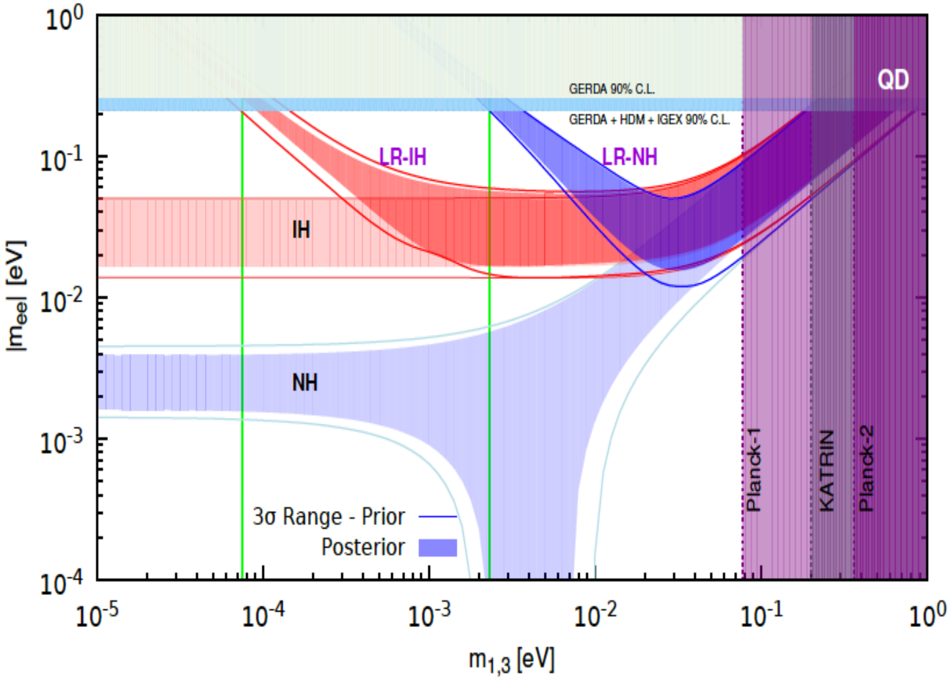
The Rate In Alternative Mechanisms

- The situation changes significantly if new physics is at lower scales
- EFT methods are being used to describe the effects of generic operators, which can then be matched to specific particle physics scenarios



Role of additional dimension-7 operators, Λ = 600 TeV
JHEP 2017, 82 (2017)

Left-Right Symmetric Model
JHEP 10 (2015) 077



A Preview…

- Tomorrow we'll go into a lot more detail about calculating the rate of 0νββ, including:
  - Effective field theory methods
  - The role of lattice QCD calculations
  - Nuclear matrix element calculation techniques

Leptogenesis and Baryogenesis

*This section freely borrows from S. Davidson, E. Nardi, and Y. Nir’s “Leptogenesis”: 0802.2962

The Matter Asymmetry Problem

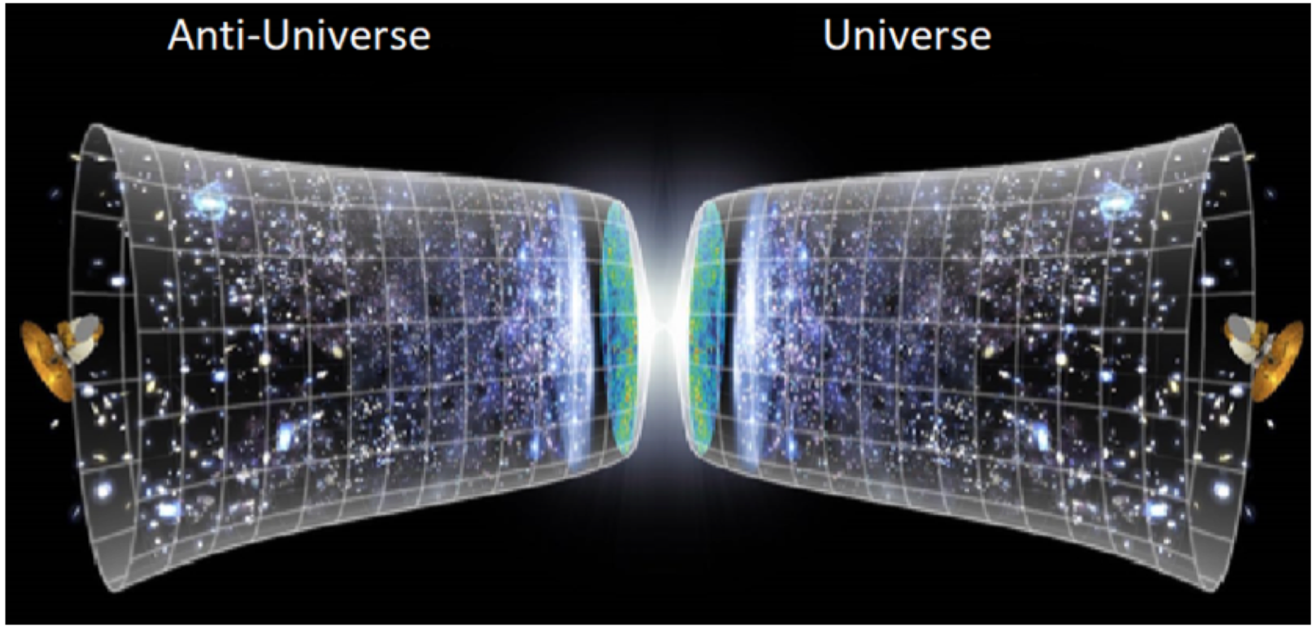
- Today, all the structure we see in the universe is made up of matter, with no significant quantity of antimatter
  - We don’t observe annihilation
  - There isn’t much antimatter in cosmic rays
  - There is no electric dipole moment of the universe
- The two main ways to measure baryon asymmetry agree
  - Big Bang Nucleosynthesis: abundances of D and ^{3}H are highly sensitive to baron density
  - CMB anisotropy: higher baryon density enhances compression in potential wells, leads to higher odd power spectrum peaks
  - Both measurements give $\eta \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} \sim 6\times10^{-10}$

Solving Matter Asymmetry: Initial Conditions?

- Could the asymmetry just be an initial condition of the universe?
  - Initial condition would require fine-tuning: 6,000,001 quarks for every 6,000,000 antiquarks
  - Based on CMB, we think the universe underwent inflation: this would have diluted away any primordial baryon asymmetry

Solving Matter Asymmetry: The “Big Separation”?

- Could there have been a “big separation,” with our observable universe being the matter one?
  - CMB measurements suggest that matter and anti-matter were created homogenously almost in equal amounts, with active annihilation happening in the first seconds
  - Known laws of physics don’t explain why the matter and anti-matter would go in different directions/separate



Solving Matter Asymmetry: Dynamic Baryogenesis

- We believe this asymmetry has to have been **generated dynamically**, not as an initial condition
- Two options:
  - A process made more matter than anti-matter
  - Matter and anti-matter were created in equal amounts, and an annihilation process destroyed more anti-matter

- Note: you'll hear this described as "baryogenesis," since baryons make up almost all the mass. We'll get to the leptons soon.

Making an Asymmetry: The Sakharov Conditions

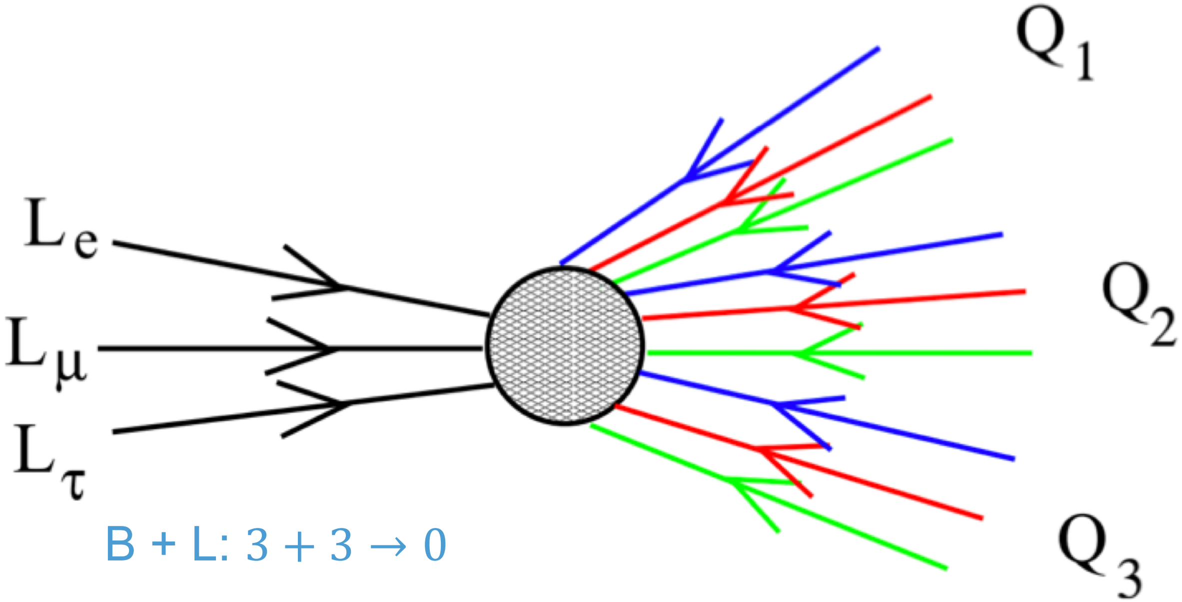
In 1967, Sakharov proposed 3 conditions required for baryon-generating interactions that would generate an asymmetry:

1. Baryon number violation: need to remove anti-baryons without removing all the baryons
2. Interactions out of thermal equilibrium: in equilibrium, backwards and forwards directions of the baryon-creating process would occur at equal rate (i.e. CPT is conserved, and there would be no way to define a time direction)
3. C and CP violation: if either C or CP is conserved, then processes generating baryons would proceed at the same rate as processes generating anti-baryons

Baryon Number Violation in the Standard Model

- At tree level in the SM (i.e. in the renormalizable Lagrangian), B and L are conserved
- At 1-loop level, there are non-perturbative gauge field configurations that violate B and L
- Called the "chiral anomaly" or the "triangle anomaly": left-handed quarks annihilate with leptons
- Leads to B+L violation
- B-L is still conserved

$$\partial_\mu J^\mu_{B_L+L_L} = \frac{3g}{32\pi}\epsilon_{\mu\nu\rho\sigma} W_a^{\mu\nu} W_a^{\rho\sigma}$$



B + L: $3 + 3 \to 0$

B - L: $3 - 3 = 0 \to 0$

The Chiral Anomaly

- Consider massless fermions in d =1 + 1 dimensions
- In this case, the Pauli matrices are:

$$\gamma^0 = \sigma^1 \qquad \gamma^1 = i\sigma^2 \qquad \gamma^5 = -\gamma^0\gamma^1 = -i\sigma^1\sigma^2 = \sigma^3$$

- And the Dirac spinors are 2-component objects ψ
- In this case, the action becomes:

$$S = \int d^2x \, i\bar{\psi}\,\partial\psi = \int d^2x i\psi^\dagger(\partial_t - \gamma^5\partial_x)\psi$$

- We can decompose the massless Dirac fermion into chiral components:

$$\psi_\pm = \frac{1}{2}(1 \pm \gamma^5)\psi \qquad \psi_+ = \begin{pmatrix} \chi_+ \\ 0 \end{pmatrix} \qquad \psi_- = \begin{pmatrix} 0 \\ \chi_- \end{pmatrix}$$

Note: This explanation comes from Ch. 3 of David Tong's "Lectures on Gauge Theory" (https://www.damtp.cam.ac.uk/user/tong/gaugetheory.html)

The Chiral Anomaly

- Then in terms of chiral fermions:

$$S = \int d^2x \, i\chi_+^\dagger \partial_- \chi_+ + i\chi_-^\dagger \partial_+ \chi_- \qquad \partial_\pm = \partial_t \pm \partial_x$$

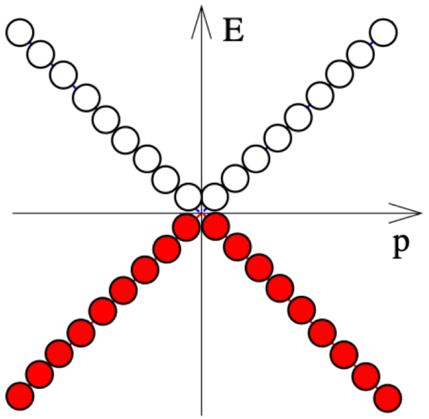
- So we have 2 equations of motion:
  - For left-chiral fermions χ_+,: $\partial_- \chi_+ = 0$, solution is $\chi_+ = \chi_+(t + x)$
  - For right-chiral fermions χ_-,: $\partial_+ \chi_- = 0$, solution is $\chi_- = \chi_-(t - x)$
  - Left-chiral particles move to the left, right-chiral move to the right
  - To make a particle stand still, we need to add a mass term that couples the left-moving and right-moving particles

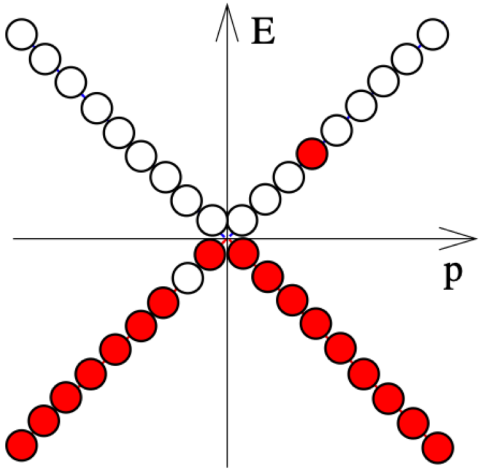
The Chiral Anomaly

- In this theory, there are particles and anti-particles, and all particles have $E = |p|$; right-moving have p>0, left-moving have p<0
- Two global symmetries: $\psi \rightarrow e^{i\alpha}\psi, \psi \rightarrow e^{i\alpha\gamma^5}\psi$; number of left-moving and right-moving fermions are each separately conserved. This is called “chiral symmetry”

Dirac sea vacuum configuration:

- all negative energy states filled
- all positive energy states vacant





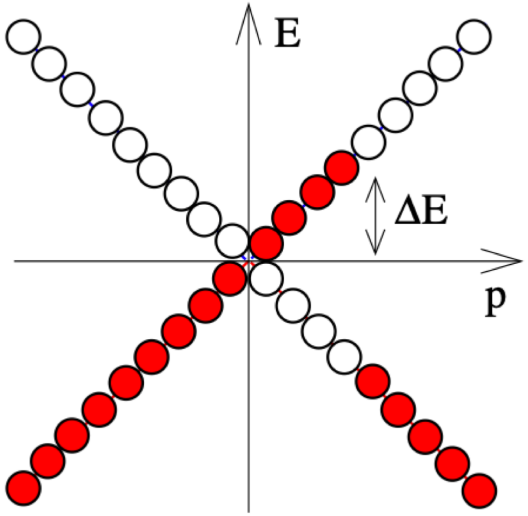
Under operations that preserve chiral symmetry:

- E.g.: right-moving particle/anti-particle pair created
- Left-moving hole = right-moving particle

The Chiral Anomaly

- Adding a constant background electric field $\mathcal{E}$ for some time t, pointing to the right, increases the momentum and energy of all the particles: $\Delta p = e\mathcal{E}t$

Despite the symmetry, we've created left-moving anti-particles and right-moving particles!

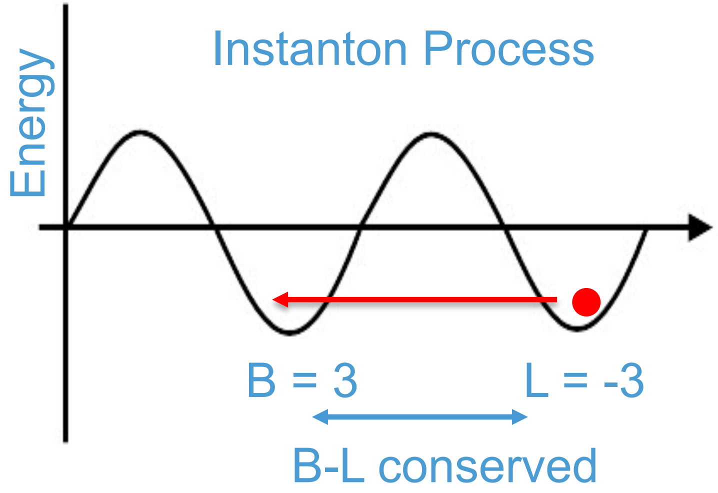


This is called a chiral anomaly: it is enabled by the infinite Dirac sea

The anomaly arises because we're dealing with a continuum quantum theory that has an infinite number of states, rather than a finite quantum system

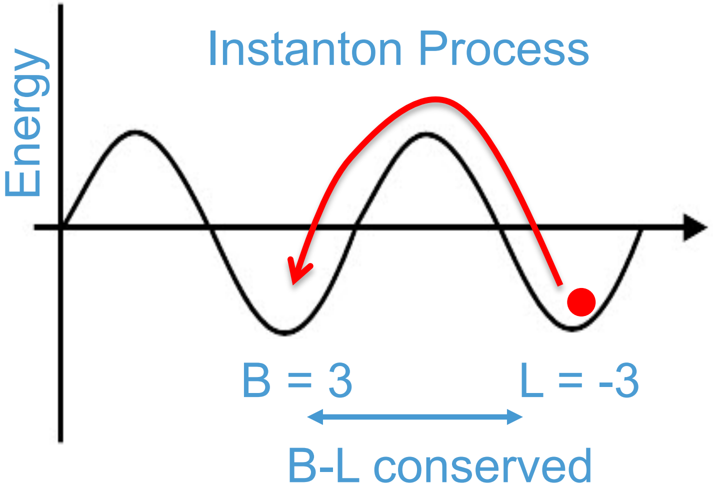
Chiral Anomalies in the Standard Model

- In the SM, SU(2) gauge interactions cause a similar "level crossing": $\partial^\mu j_\mu^i = \frac{1}{64\pi^2} F_{\mu\nu}^A \tilde{F}^{\mu\nu A}$
- Ground state of the gauge fields is like a period potential: minima are different values of B+L
- At T = 0, driven by tunneling, called "instantons": rate is highly suppressed, B+L violation is unobservably small



Chiral Anomalies in the Standard Model

- At finite T, thermal fluctuations of the field can climb over the barrier; called the "sphaleron process"
- Rate of B+L violation depends strongly on temperature: $\Gamma_{sph} \propto e^{\frac{2Bm_W}{\alpha_W T}}$, B~2
- From lattice computations: $\frac{\Gamma}{V} \sim 25\,\alpha_W^5 T^4$
- **B+L violation happens at high temperature in the SM**



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From SM at high T

Out-of-Equilibrium Baryogenesis

- If the expansion rate of the universe is larger than the interaction rate, products of a decay can't "find each other" to undergo inverse decay and the particle is "frozen out"
- How this happens depends on the baryogenesis theory: depends on when the asymmetry is generated and the strength of the interactions
- Leptogenesis scenarios offer a few ways to do this: e.g. in Type I see-saw, high N mass and low interaction rates mean the N decay goes out of equilibrium

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From SM at high T

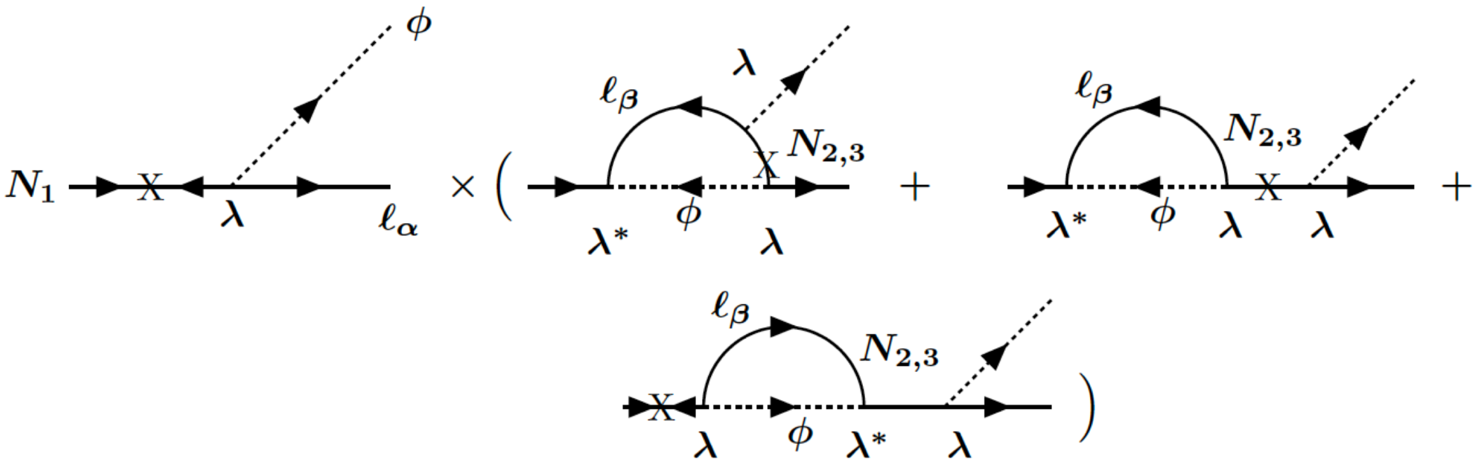
( ) MJ neutrinos can do this, depending on model

C and CP Violation in the Standard Model

- CP violation occurs in the Standard Model:
  - In the CKM matrix: leads to CP violation in Kaon, B, and D^0 decays
  - Potentially in the PMNS matrix: hints of differences in neutrino and anti-neutrino oscillations from long-baseline experiments
- The amount of CP violation is far too small to produce the observed baryon asymmetry (10 orders of magnitude too small!)
- A new source of CP violation is needed to explain the matter asymmetry

CP Violation and Majorana Neutrinos

- In the Type 1 see-saw, the heavy right-handed neutrino N decays to Higgs (φ) + v
- Interference of tree and loop-level diagrams creates additional CP violation
- Required CP violation gives lower limit on M_N:
  - Assuming strongly hierarchical N's, $M_N \geq \sim 10^9$ GeV
  - Limit can be loosened if there are multiple degenerate N's or other new particles (4th lepton generation, multi-Higgs models, etc)



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   From SM at high T

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   ( ) MJ neutrinos can do this, depending on model

Leptogenesis Summary

- Leptogenesis via Majorana neutrinos, along with the SM, can satisfy all 3 Sakharov conditions!
- Leptogenesis via Majorana neutrinos can “freeze in” matter asymmetry at the right point in time and in the right amounts to lead to the universe we see today

Summary

- Neutrinos are the only SM particles that could be Majorana fermions
- Majorana neutrinos could explain why the neutrino mass is small but non-zero, and the origin of the matter/anti-matter asymmetry
- There are many models that predict Majorana neutrinos
- If neutrinos are Majorana, 0νββ **may** occur; if 0νββ is observed, the neutrino **must** have a non-zero Majorana mass component
- Tomorrow we'll discuss how to calculate the rate of 0νββ and how to look for it in experiments