

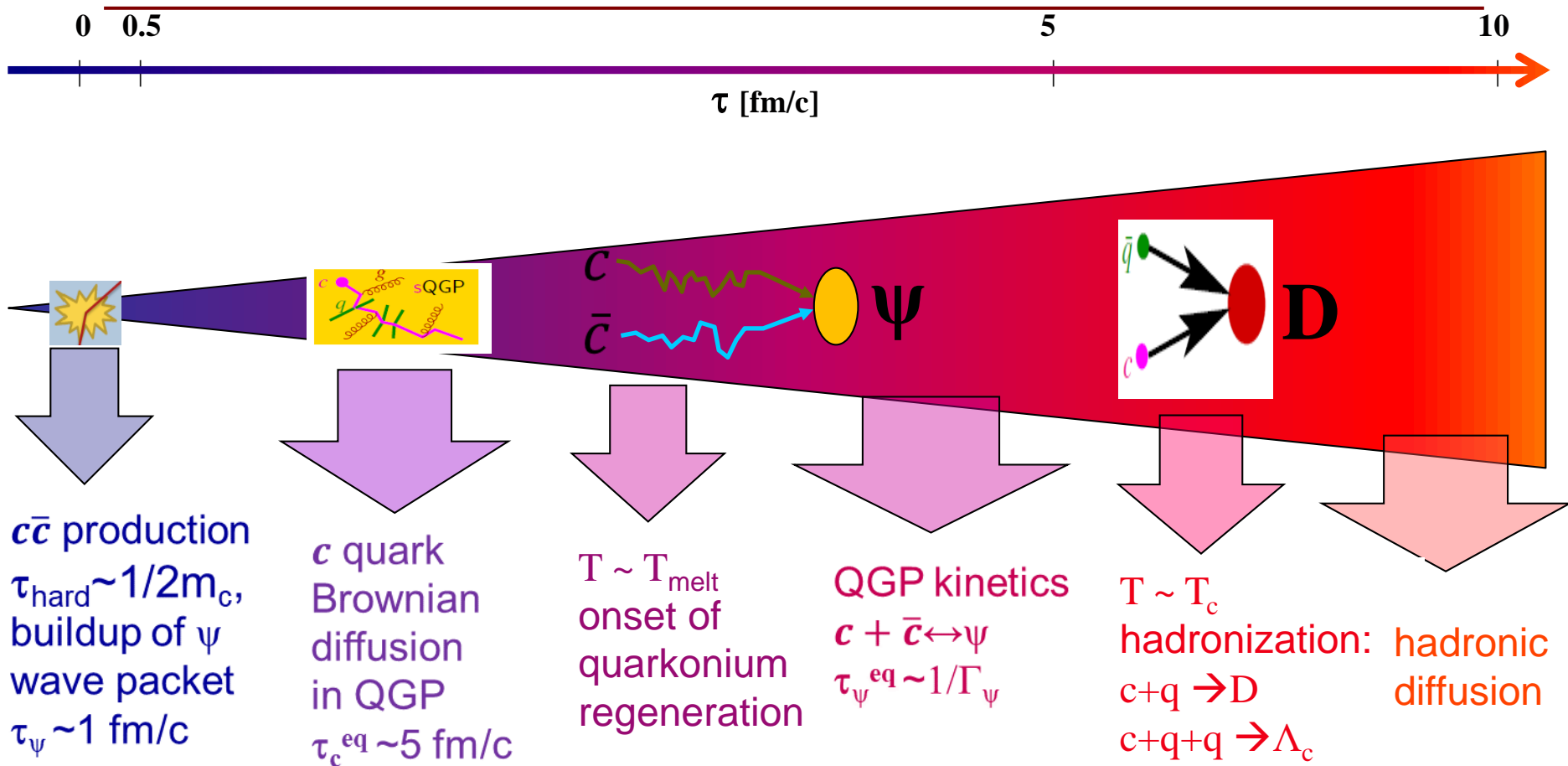
29-31 July 2022  
山东大学青岛校区

# Selected Overview of Heavy-flavor Production

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# Heavy flavor transport as probes of QGP



# Part I: Charm-hadron production in pp

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- pQCD factorization & fragmentation
- $\Lambda_c/D^0$  enhancement vs hadronization models
- $\Sigma_c$  &  $\Xi_c$  production

# pQCD factorization & fragmentation

$$\frac{d\sigma^{H_c}}{dp_T^{H_c}}(p_T; \mu_F, \mu_R) = \text{PDF}(x_1, \mu_F) \cdot \text{PDF}(x_2, \mu_F) \otimes \frac{d\sigma^c}{dp_T^c}(x_1, x_2, \mu_R, \mu_F) \otimes D_{c \rightarrow H_c}(z = p_{H_c}/p_c, \mu_F)$$

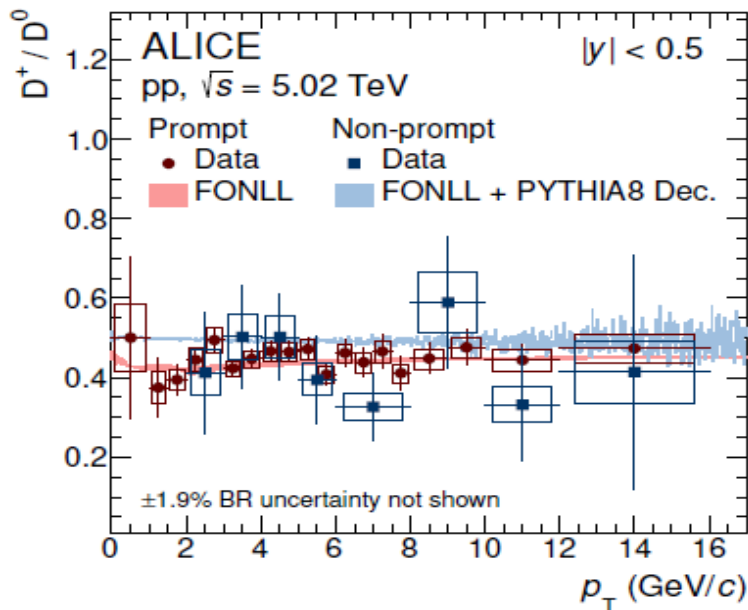
Parton distribution functions (PDFs)

Hard scattering cross section (pQCD)

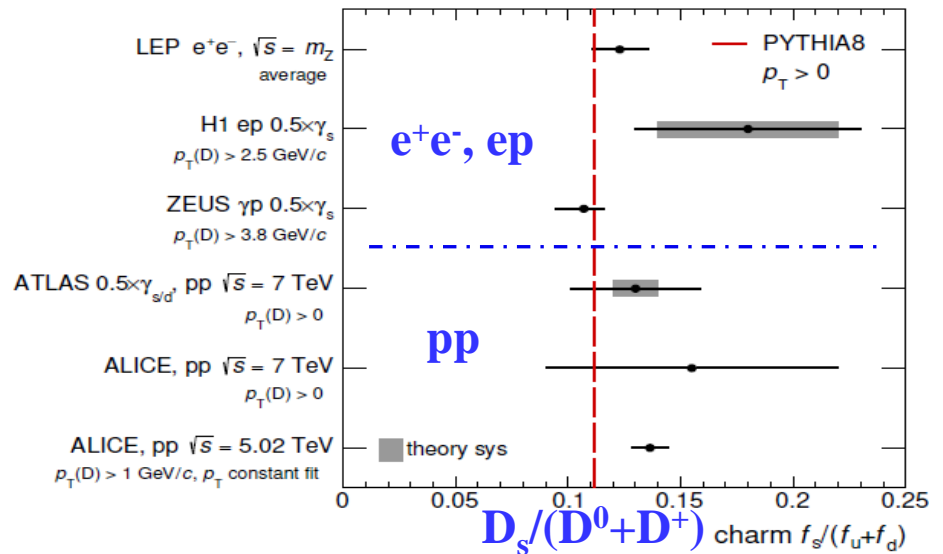
Fragmentation function (hadronization)

→ from e<sup>+</sup>e<sup>-</sup>, universal?

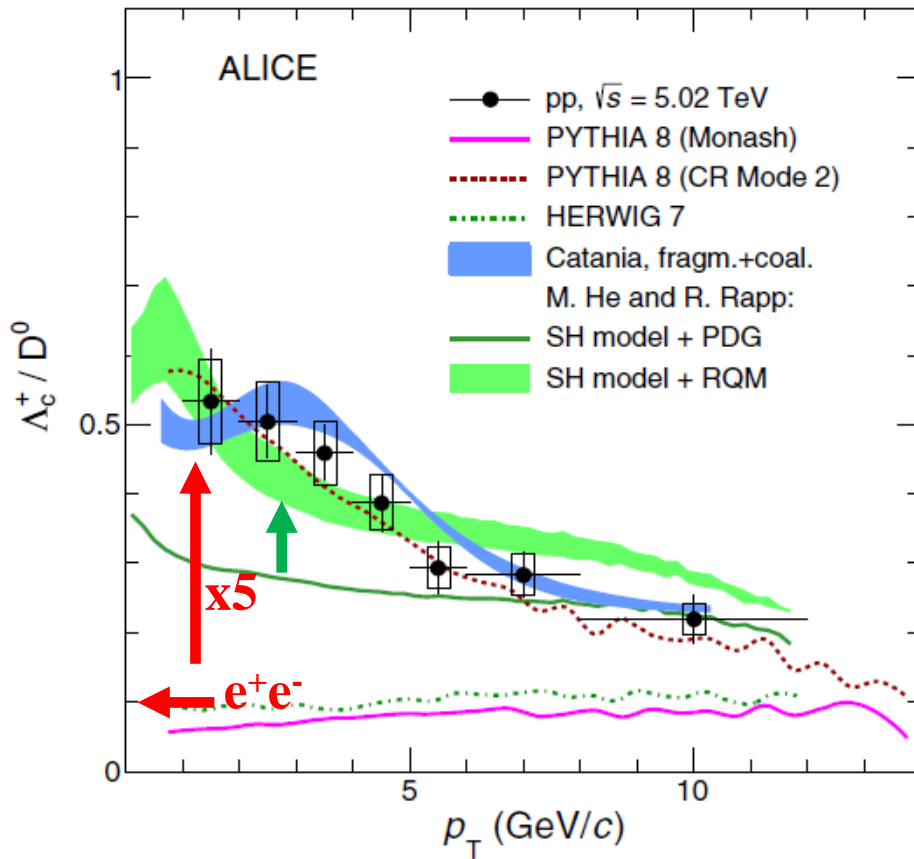
❖ phenomenological FF: assumed universal & constrained by e<sup>+</sup>e<sup>-</sup>



charm-meson ratios in pp reproduced

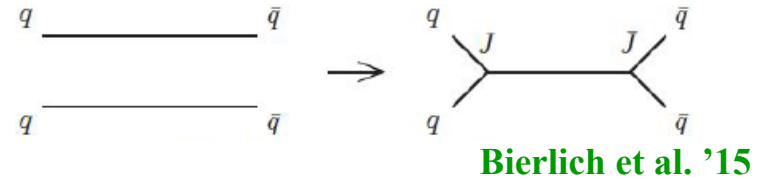


# $\Lambda_c^+ / D^0$ @ 5 TeV pp collisions



ALICE, PRL 127, 202301(2021)  
ALICE, PRC 104, 054905(2021)

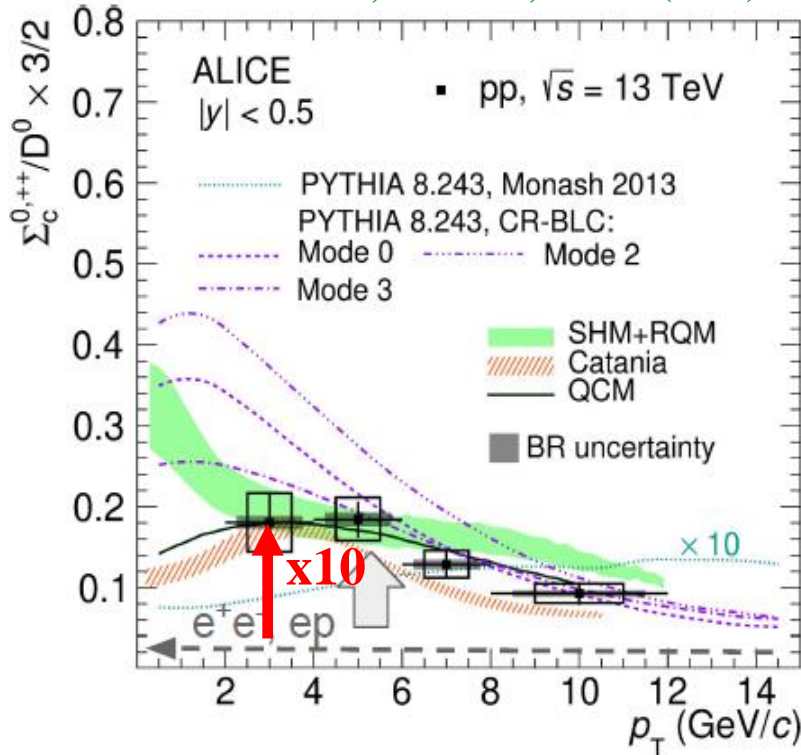
- ❖ PYTHIA8: Color-reconnection with junctions frag. into baryons



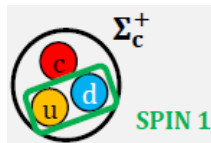
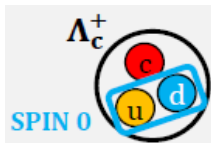
- ❖ Catania: c-q(-q) coalescence in a small QGP fireball [Minissale et al. '21](#)
  - ❖ Statistical hadronization in q-rich environment (unlike  $e^+e^-$ )
    - augmented by “missing” charm-baryons assuming *relative* chemical equilibrium
    - $\Lambda_c^+$  enhanced at low  $p_T$  by feeddowns
- MH & Rapp '19

# $\Sigma_c/D^0$ & $\Xi_c/D^0$

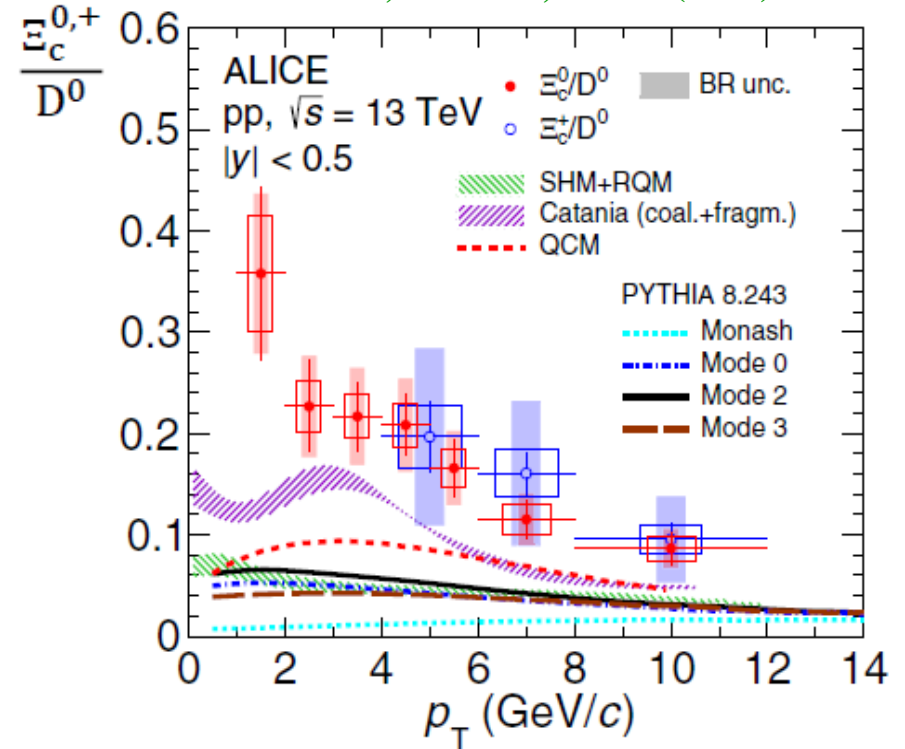
ALICE, PRL 128, 012001(2022)



❖  $\Sigma_c/D^0$  x10 enhanced despite more massive spin-1 diquark



ALICE, PRL 127, 272001(2021)



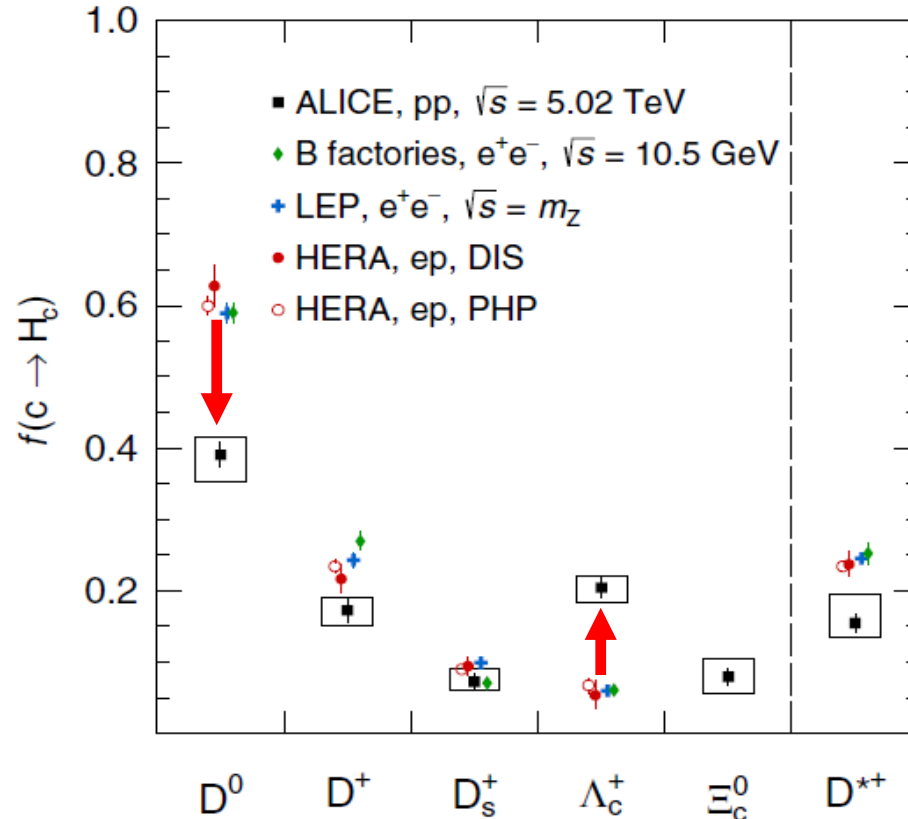
❖  $\Xi_c/D^0$  ratio underestimated by all models



# Take-aways from Part I

- ❖ charm quark fragmentation is **non-universal** from  $e^+e^-$  to pp

ALICE, PRD 105, L011103 (2022)



- $c \rightarrow \Lambda_c$  much enhanced vs  $c \rightarrow D^0$  reduced
- full charm-hadrons measured  $d\sigma^{c\bar{c}}/dy \sim 1.16$  mb at mid-y
- ➔ significant impact on charmonia production

# Part II: Charm-hadron production in AA

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- Microscopic interactions & diffusion coefficient
- Hadronization: Coalescence, Resonance Reco., SHMc
- D-meson  $R_{AA}$  &  $v_2$ : Diffusion coefficient
- Charm hadro-chemistry:  $\Lambda_c/D^0$

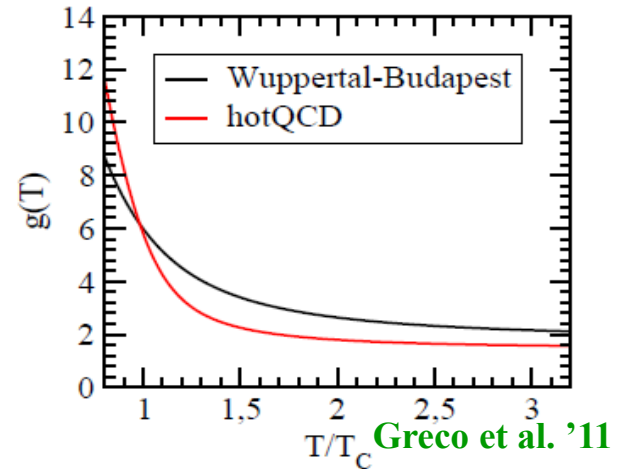
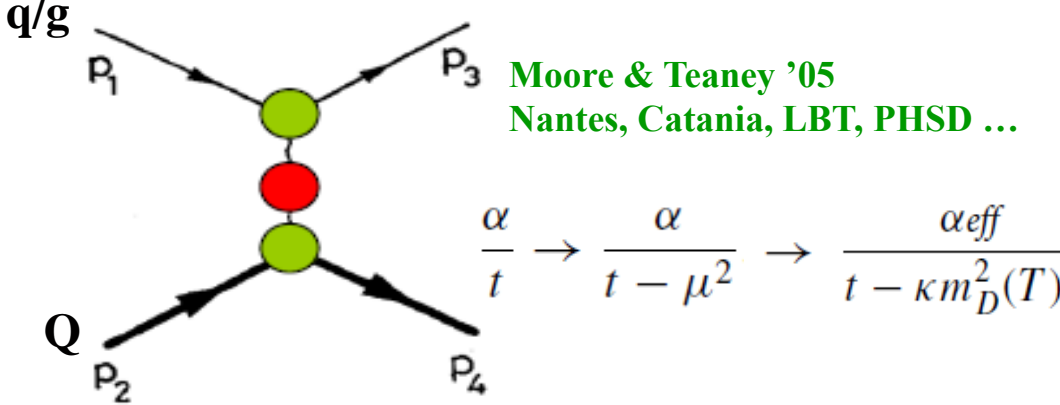


# Transport coefficient: pQCD vs T-matrix

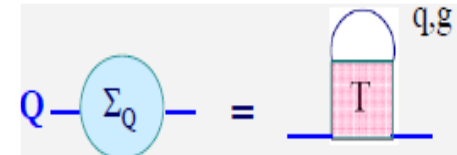
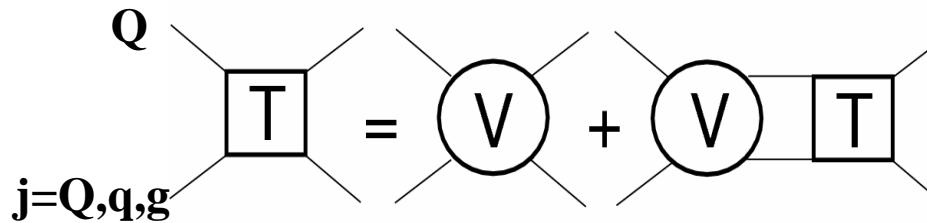
- ❖ HQ Brownian diffusion via Fokker-Planck

$$\frac{\partial}{\partial t} f_Q(t, p) = \gamma \frac{\partial}{\partial p_i} [p_i f_Q(t, p)] + D_p \Delta_{\vec{p}} f_Q(t, p) \quad \gamma = A \sim \int |T_{Qj}|^2 (1 - \cos\theta) f^j$$

- ❖ effective Born diagram w/ large  $\alpha_s$



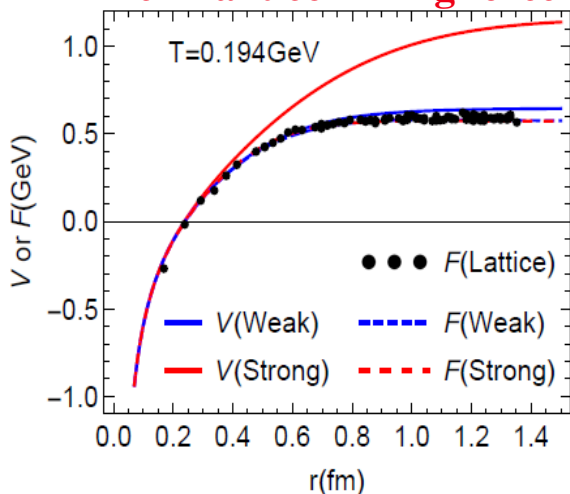
- ❖ T-matrix: coupled two- and one-body integral equations



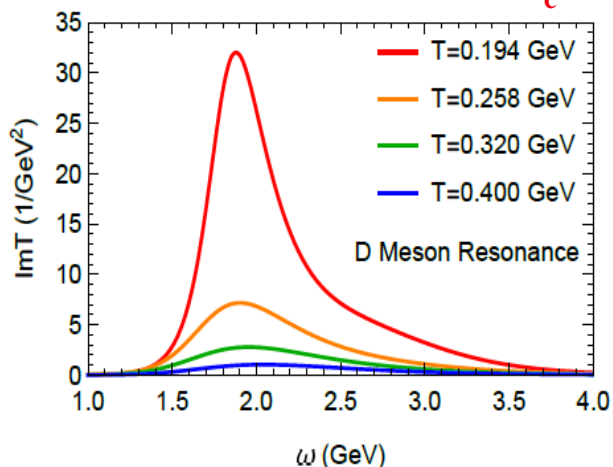
TAMU '05-'19

# T-matrix approach: Spectral + Transport Properties

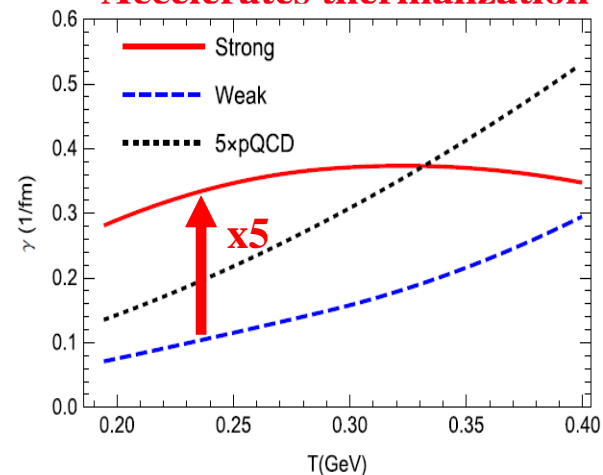
**Remnant confining force**



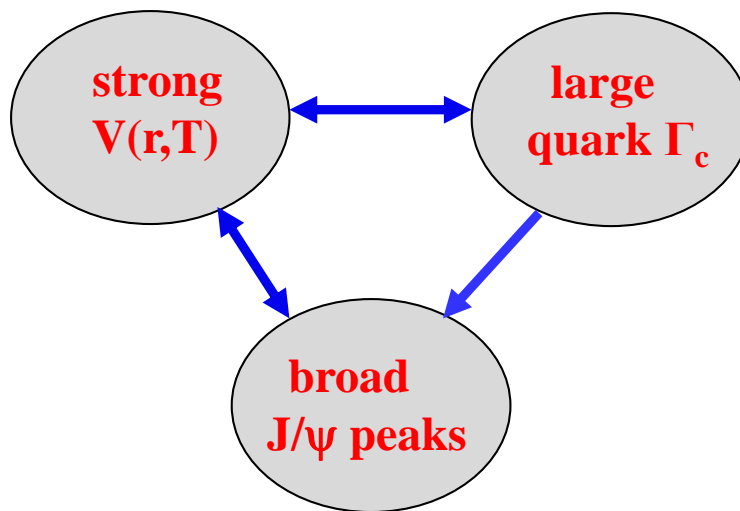
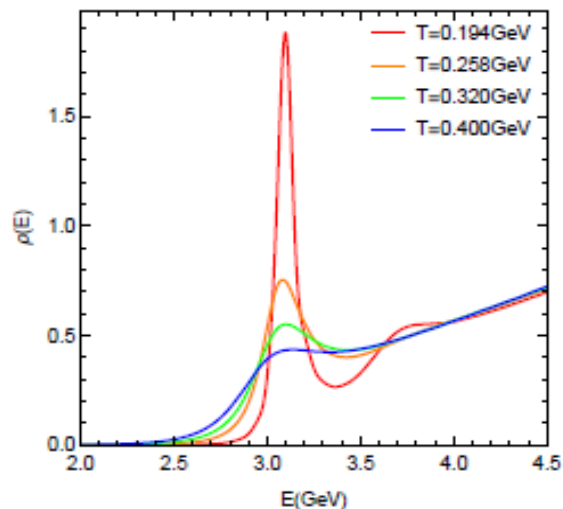
**Resonances form near  $T_c$**



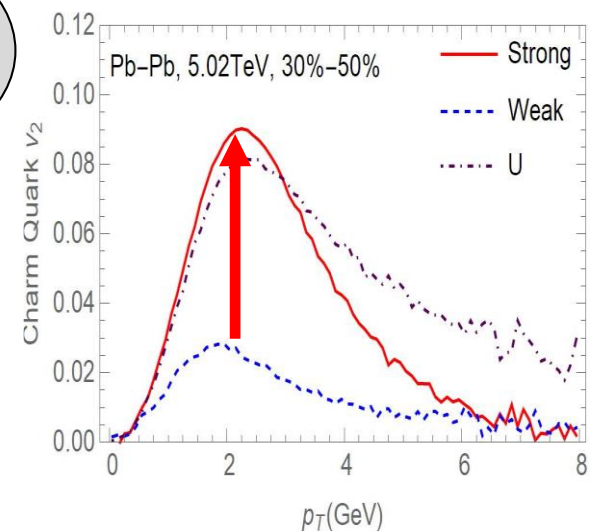
**Accelerates thermalization**



**$J/\psi$  survives in QGP**



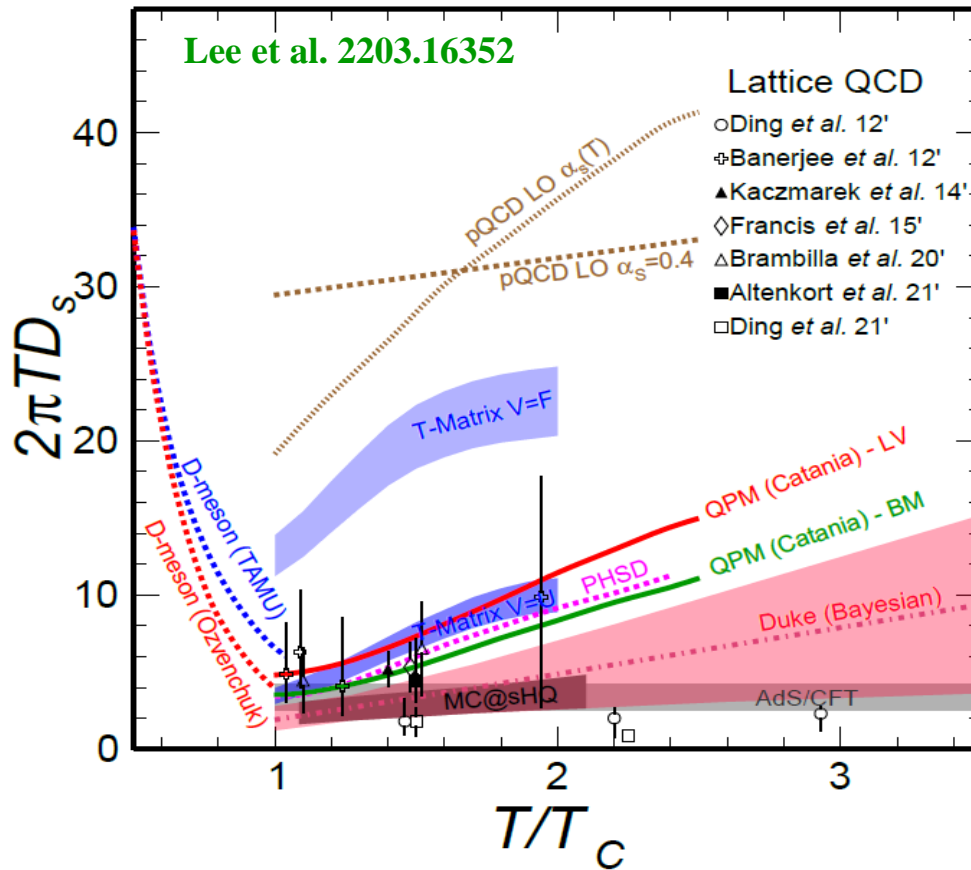
**Coupling strength  $\leftrightarrow$  HQ  $v_2$**



S.Y.F Liu et al '18, '19

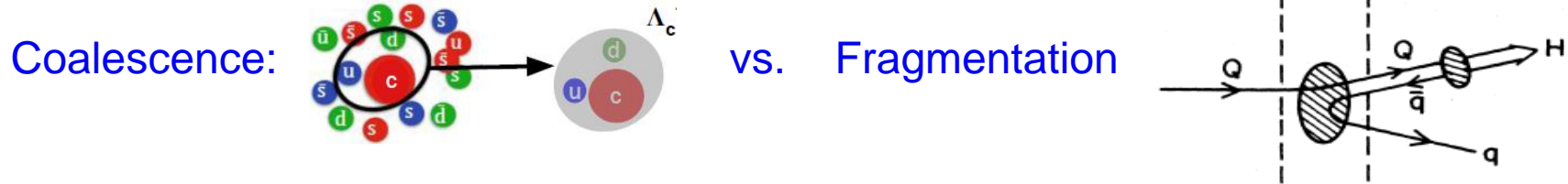
# Transport coefficient: $\mathcal{D}_s(2\pi T)$

❖ HQ spatial diffusion coefficient:  $\mathcal{D}_s = T/m_Q A(p=0) = T/m_Q \gamma$



- models & lattice  $\mathcal{D}_s(2\pi T) \sim 2-4$  near  $T_c$ , x10 smaller than pQCD
- maximum coupling strength near  $T_c$ , remnant of confining force?

# HQ hadronization in QGP



## ❖ Instantaneous coalescence models (ICM)

$$f_h(\mathbf{p}'_h) = \int \left[ \prod_i d\mathbf{p}_i f_i(\mathbf{p}_i) \right] W(\{\mathbf{p}_i\}) \delta(\mathbf{p}'_h - \sum_i \mathbf{p}_i) \quad \text{Fries et al., Greco et al., Voloshin '03}$$

- static Wigner distribution w/ hadron radius
- equilibrium limit challenging at low  $p_T$
- improvement: c-q(-q) form excited cluster + decay

Greco et al. '04, Oh et al.'09, Plumari et al.'18,  
Cao et al. '16, '20, Katz '20, Li+Liao '20

Beraudo et al. '15, '22  
Cao et al.'20

## ❖ Statistical hadronization with charm (SHMc) Andronic et al. '21

- *thermalized* c-quarks hadronize at  $T_c$

$$\frac{dN(h_{oc,\alpha}^i)}{dy} = g_c^\alpha V n_i^{\text{th}} \frac{I_\alpha(N_c^{\text{tot}})}{I_0(N_c^{\text{tot}})} \propto g_c^\alpha \leftarrow d\sigma^{c\bar{c}}/dy$$

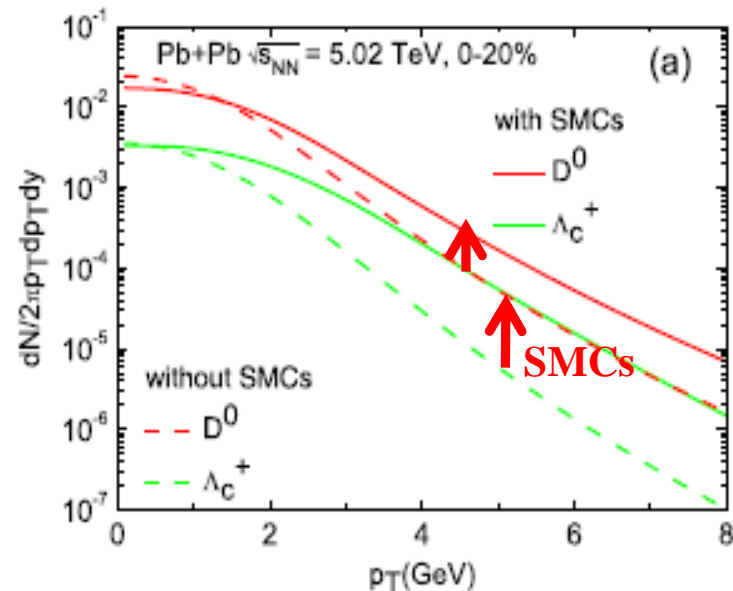
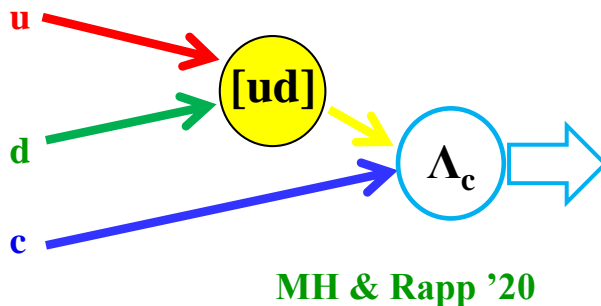
- $p_T$ -spectrum by hydrodynamic blast wave at  $T_c$

# HQ hadronization II

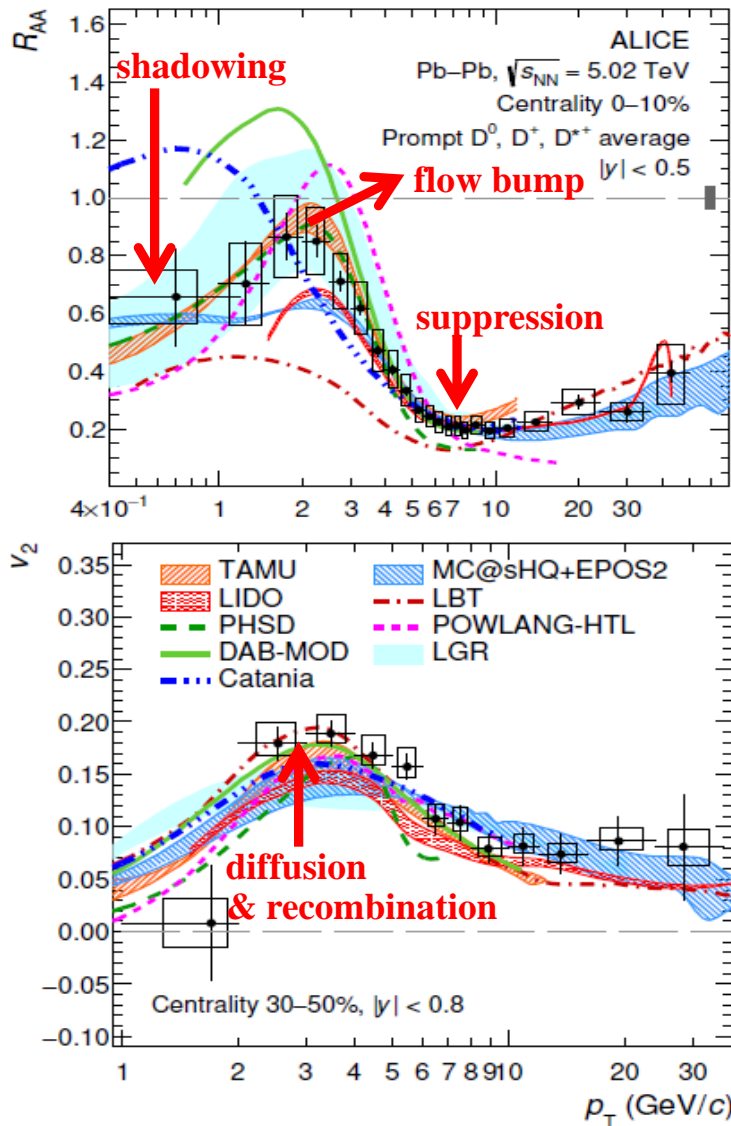
- ❖ Resonance recombination model (RRM) Ravagli et al.'07, MH et al.'12

$$f_M(\vec{x}, \vec{p}) = \frac{\gamma_M(p)}{\Gamma_M} \int \frac{d^3 \vec{p}_1 d^3 \vec{p}_2}{(2\pi)^3} f_q(\vec{x}, \vec{p}_1) f_{\bar{q}}(\vec{x}, \vec{p}_2) \underline{\sigma_M(s)} v_{\text{rel}}(\vec{p}_1, \vec{p}_2) \delta^3(\vec{p} - \vec{p}_1 - \vec{p}_2)$$

- $\sigma_M(s)$  resonant cross section: energy conservation & equilibrium limit
- 3-body RRM & space-momentum correlations (SMCs)  $\rightarrow$  enhancing  $\Lambda_c/D^0$



# D-meson $R_{AA}$ & $v_2$ : extracting $\mathcal{D}_s(2\pi T)$

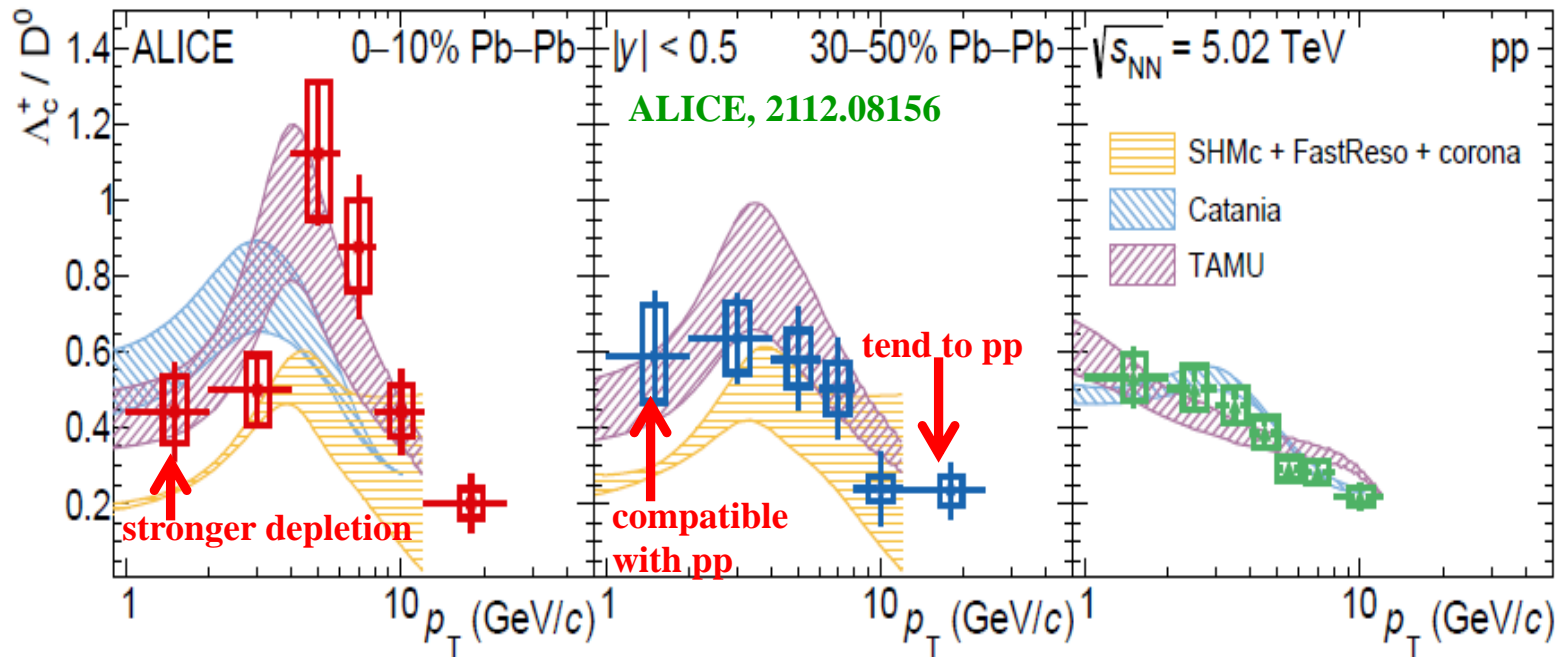


ALICE, JHEP01(2022)174; PLB 813(2021)136054

Model	$\chi^2/\text{ndf}$	
	$R_{AA}$	$v_2$
Catania [6, 7]	143.8/30	14.0/8
DAB-MOD [9]	234.1/30	9.8/6
LBT [10, 11]	411.8/30	15.8/12
LIDO [13]	46.4/26	62.0/11
LGR [12]	9.2/30	15.5/11
MC@sHQ+EPOS2 [8]	56.6/30	5.7/12
PHSD [5]	294.7/30	19.6/11
POWLANG-HTL [3, 4]	468.6/30	13.5/8
TAMU [2]	30.2/30	8.15/9

- models with  $\chi^2/\text{ndf} < 5$  (2) for  $R_{AA}$  ( $v_2$ )  
 $\rightarrow \mathcal{D}_s(2\pi T) = 1.5-4.5$  near  $T_c$
- caveat: also affected by hadronization, hydro, hadronic phase

# Charm hadro-chemistry: $\Lambda_c/D^0$

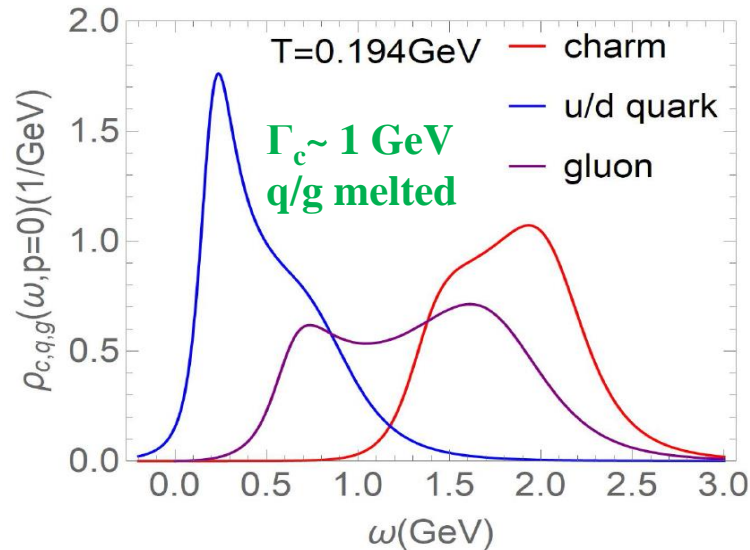


- Catania: instantaneous coalescence + fragmentation Plumari et al. '18
- SHMc: hydrodynamic blast wave spectrum on PDG-only baryons + corona pp Andronic et al. '21
- TAMU: RQM charm-baryons + RRM w/ SMCs  
integrated ratio compatible with pp MH & Rapp '20

# Take-aways from Part II

## ❖ Heavy-quark diffusion

- $\mathcal{D}_s(2\pi T)=1.5-4.5$  near  $T_c \rightarrow$  scattering rate  $\Gamma_{\text{coll}} \sim 3/\mathcal{D}_s \sim 1 \text{ GeV} > M_{q,g}$   
 $\rightarrow$  thermal partons melt, Brownian markers survive
- strong coupling via remnants of confining force



## ❖ Heavy-quark hadronization

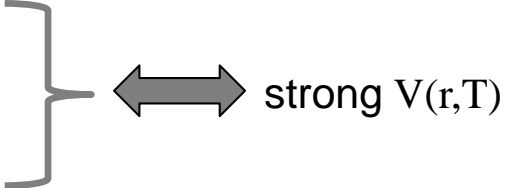
- recombination  $\rightarrow$   $p_T$ -dependent charm hadro-chemistry
- $p_T$ -integrated  $\Lambda_c/D^0$  compatible with  $pp \rightarrow$  kinematic redistribution in  $p_T$  in AA



# Summary & outlook

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❖ HFs: excellent probes of sQGP structure, transport properties, in-medium force & hadronization

- a small open HF diffusion coefficient  $\mathcal{D}_s$
  - recombination/color neutralization important
  - quarkonia melting by large reaction rates
- 
- strong  $V(r,T)$

➔ connection between open- & hidden-HF, e.g. via  $J/\psi$  regeneration (not discussed here)

Thanks for attention!

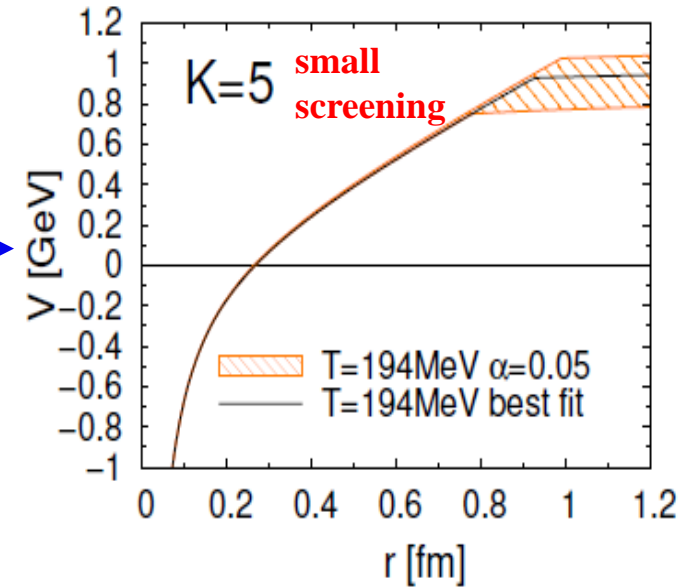
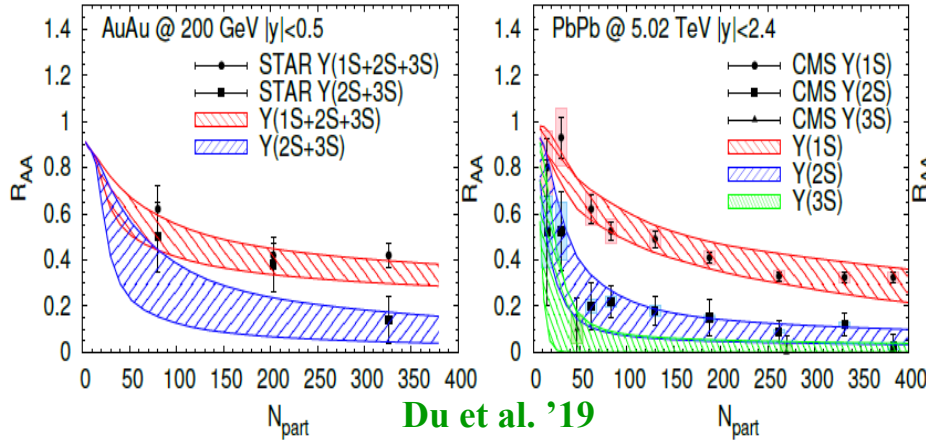
# Part III: Heavy quarkonium production in AA

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- Strong HQ potential
- Semi-classical approach: suppression vs regeneration
- $J/\psi$   $v_2$  puzzle
- Open quantum system approach to  $Y$  states

# Extraction of HQ potential

- statistical transport analysis of Y data with trial input potential

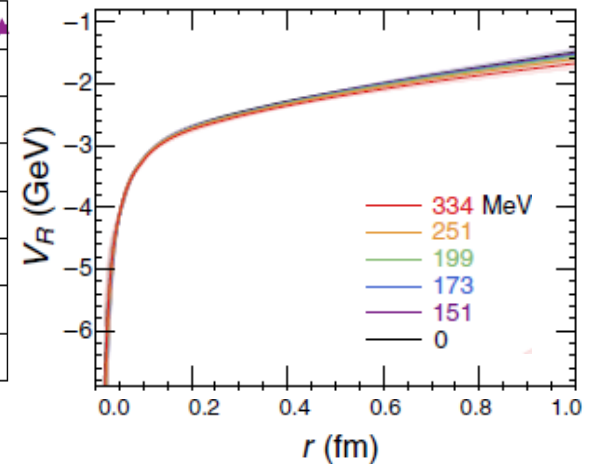
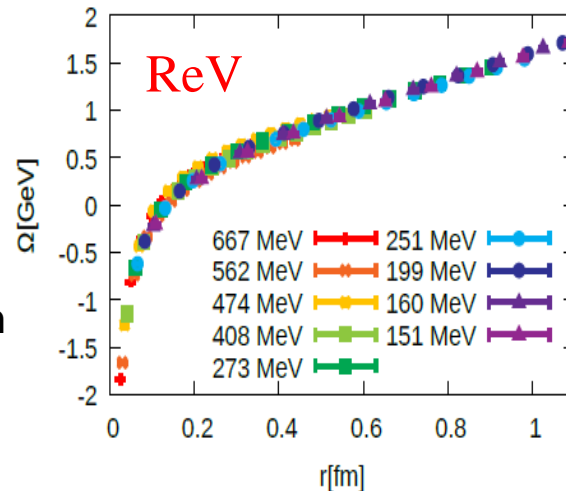


- lattice QCD extraction of static potential

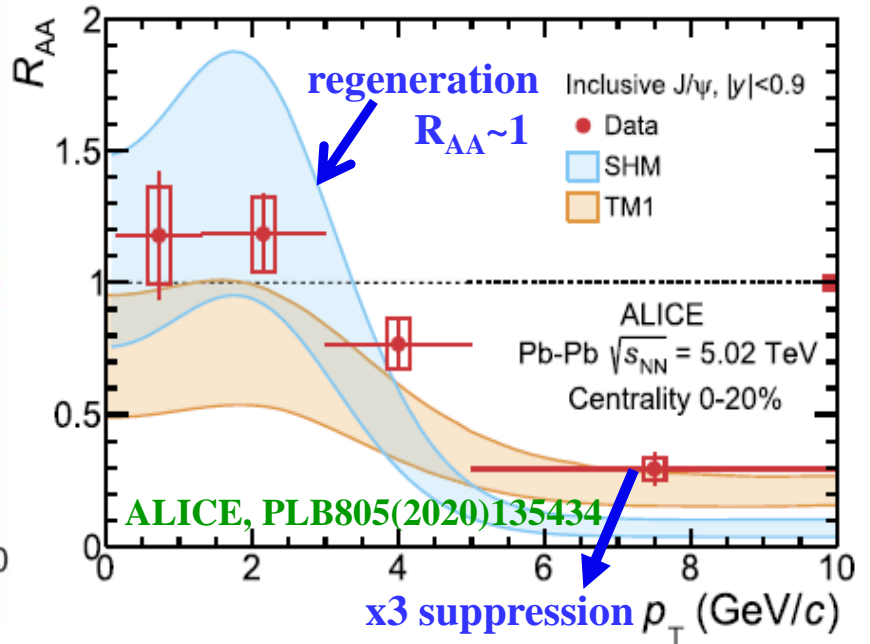
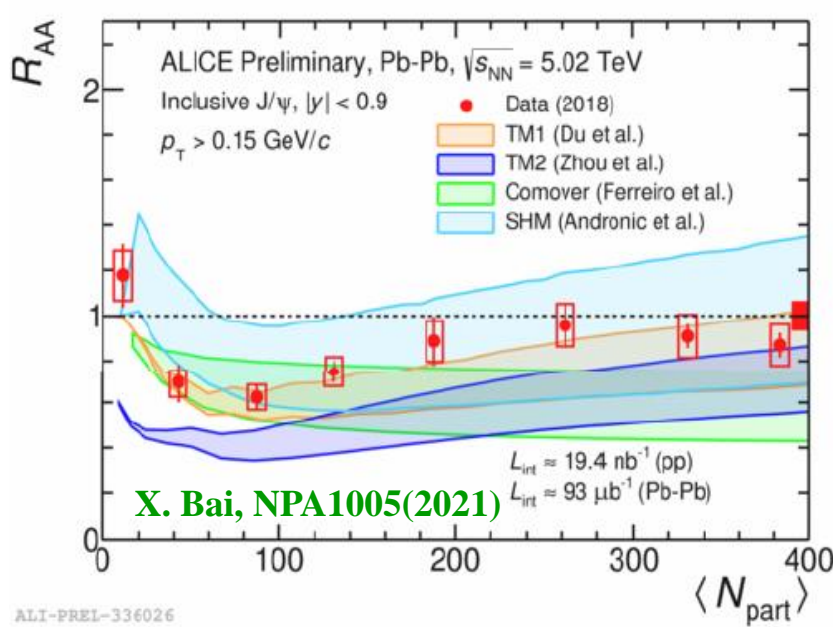
Bala et al., 2110.11659

- neural network approach

Shi et al. '22



# J/ψ: suppression vs regeneration



- semi-classical transport **Du et al. '15, Zhou et al. '14, Ferreiro et al. '14**

$$\frac{dN_{\Psi}(\tau(T))}{d\tau} = -\Gamma_{\Psi}(T)[N_{\Psi}(\tau(T)) - N_{\Psi}^{\text{eq}}(T)]$$

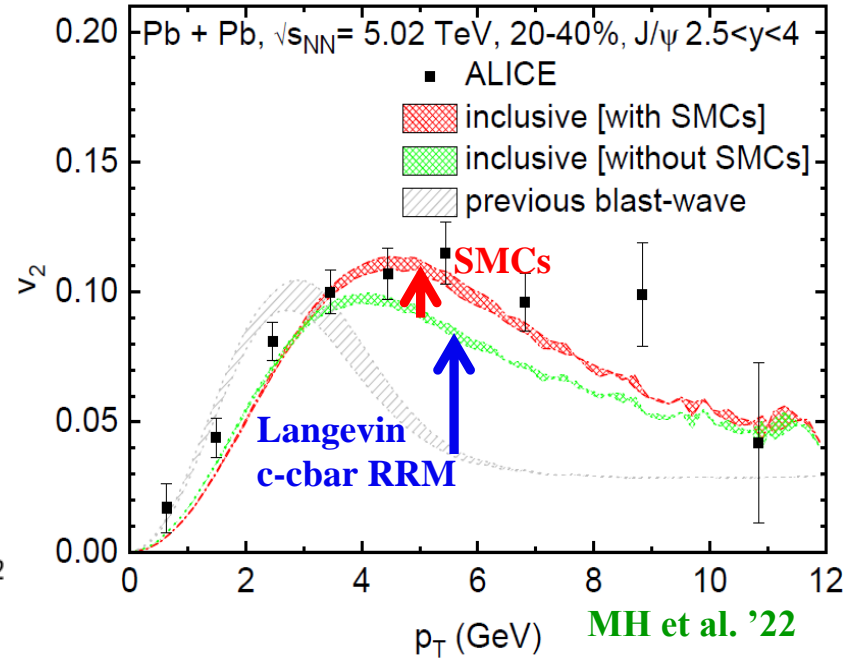
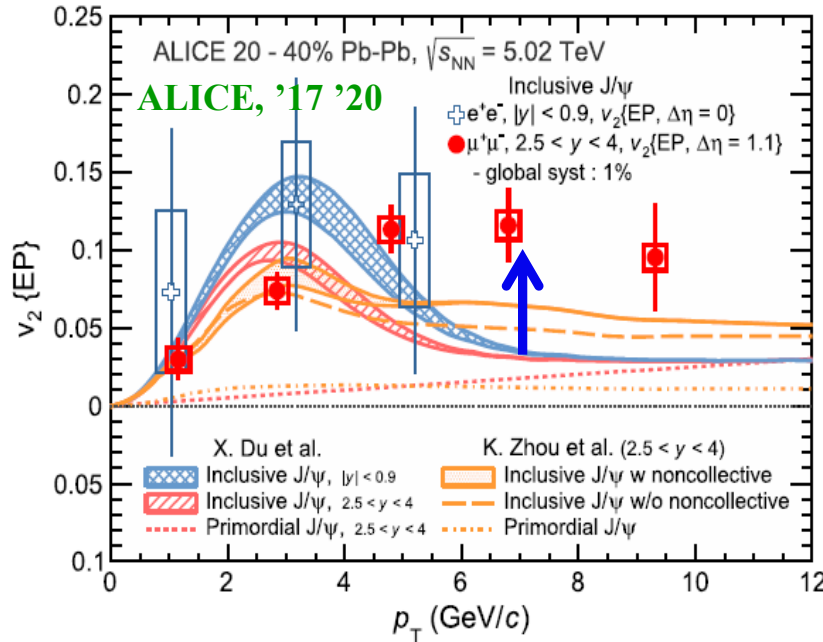
reaction rate  $\Gamma_{\Psi}$

regeneration toward equilibrium

- SHMc: hydrodynamic blastwave spectrum + pp corona **Andronic et al. '19**

$$\frac{dN(h_{hc}^j)}{dy} = g_c^2 V n_j^{\text{th}} \propto g_c^2 \leftarrow d\sigma^{\text{ccbar}}/dy$$

# J/ψ “v<sub>2</sub> puzzle”



- regeneration via RRM

$$f_{\Psi}(\vec{x}, \vec{p}) = C_{\Psi} \frac{E_{\Psi}(\vec{p})}{m_{\Psi} \Gamma_{\Psi}} \int \frac{d^3 \vec{p}_1 d^3 \vec{p}_2}{(2\pi)^3} \underbrace{f_c(\vec{x}, \vec{p}_1) f_{\bar{c}}(\vec{x}, \vec{p}_2)}_{\text{transported } c \text{ \& } \bar{c} \text{ quark spectra}} \times \sigma_{\Psi}(s) v_{\text{rel}}(\vec{p}_1, \vec{p}_2) \delta^3(\vec{p} - \vec{p}_1 - \vec{p}_2)$$

transported  $c$  &  $\bar{c}$  quark spectra  
 constrained by D-meson observables

- off-equilibrium  $c/\bar{c}$  spectra + space-momentum correlations (SMCs)  
 $\rightarrow$  regeneration up to  $p_T \sim 8$  GeV  $\rightarrow v_2$  enhanced

# Open quantum system approach to Y states

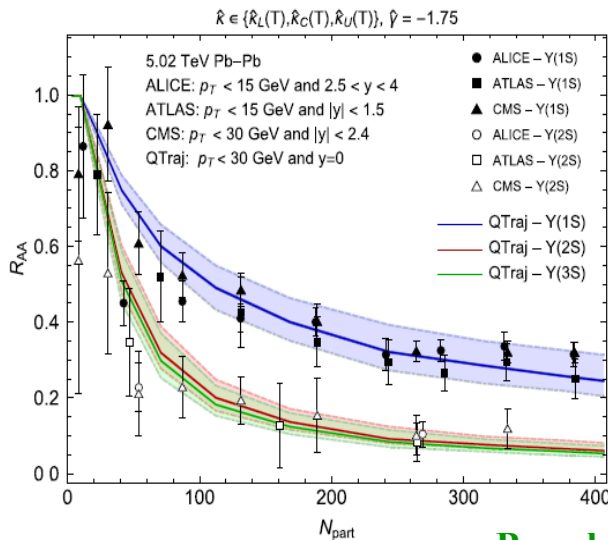
❖ OQS + pNRQCD → Lindblad equation

Brambilla et al. '17-21, Yao et al., '21, Blaizot '18  
Akamatsu '21, Rothkopf '20, Gossiaux et al. '21

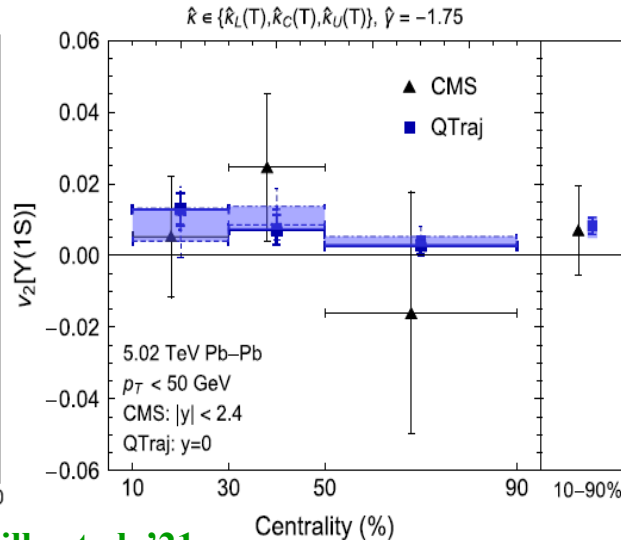
$$\frac{d\rho(t)}{dt} = -i[H, \rho(t)] + \sum_n \left( C_n \rho(t) C_n^\dagger - \frac{1}{2} \{ C_n^\dagger C_n, \rho(t) \} \right)$$

$$M \gtrsim 1/a_0 \gg \pi T \sim m_D \gg E.$$

- quantum transition between different states included, lacking in semi-classical
- regeneration currently limited to diagonal  $b\bar{b}$
- Coulomb potential + **transport coefficient  $\kappa$**  encoded in  $C_n$



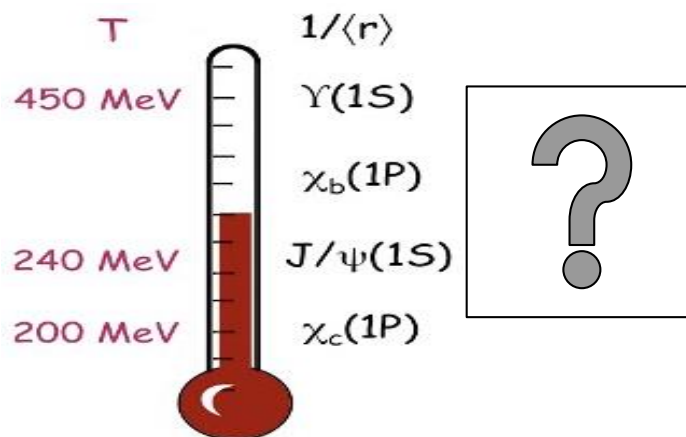
Brambilla et al. '21



- **Y(1S) in-medium width**  
 $\Gamma_{Y(1S)} = 3a_0^2 \kappa \sim 50 \text{ MeV}$  at  $T \sim 250 \text{ MeV}$
- values & results comparable to semi-classical approach  
Strickland et al. '15

# Take-aways from Part III

- ❖ strong HQ potential with little screening close to  $T_c$   
quarkonia melt through large reaction rates ( $\leftrightarrow$  small HQ  $\mathcal{D}_s$ )
  - probe of in-medium force via in-medium “spectroscopy”, not “thermometer”

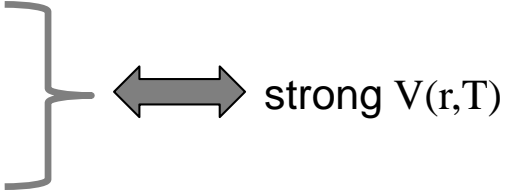


- ❖ Quantitative connections open-  $\leftrightarrow$  hidden-charm transport
  - transported  $c/\bar{c}$  distributions &  $d\sigma^{c\bar{c}}/dy$

# Summary & outlook

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- a small open HF diffusion coefficient  $\mathcal{D}_s$
  - recombination/color neutralization important
  - quarkonia melting by large reaction rates
- 
- strong  $V(r,T)$

➔ connection between open- & hidden-HF, e.g. via  $J/\psi$  regeneration

## ❖ outlook

- $\Xi_c$  production in pp
- p-dependence of  $\mathcal{D}_s$ :  
nonperturbative diffusion → perturbative radiative e-loss?
- tension:  $\Lambda_c/D^0$  in pp/pA by LHCb at forward-y vs ALICE at mid-y
- more ...

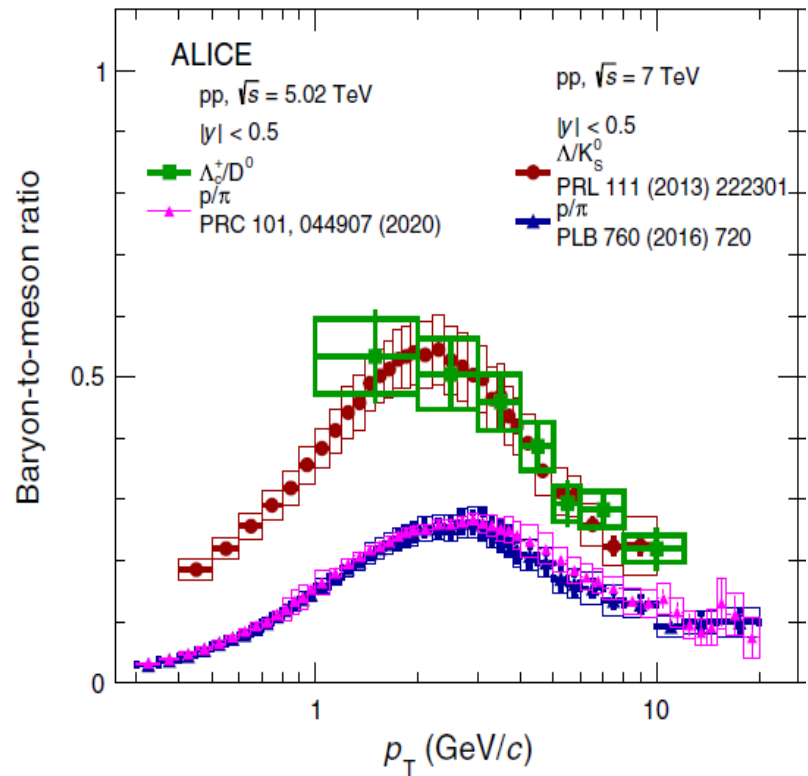
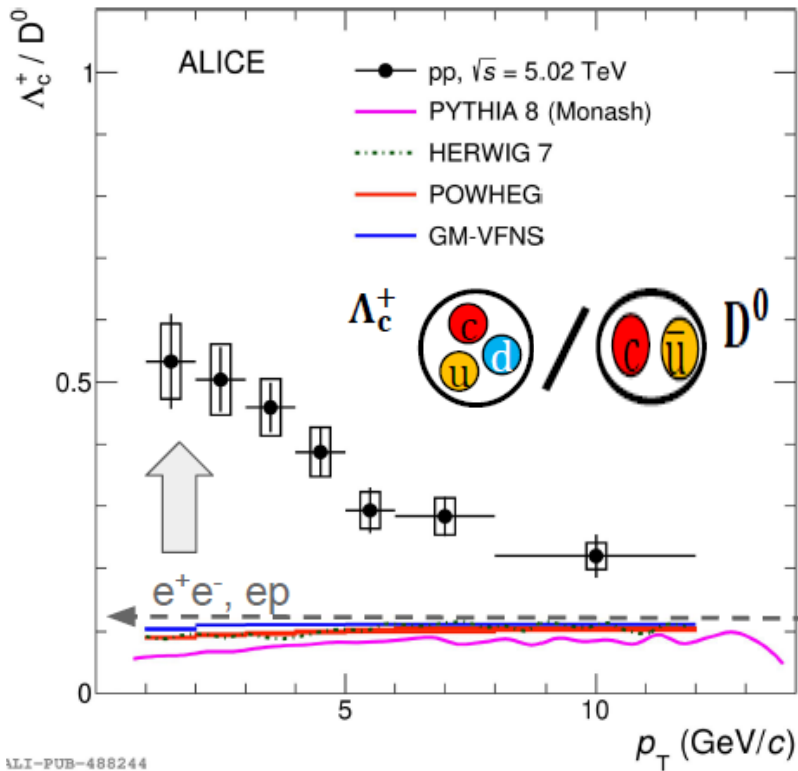


# Back-up

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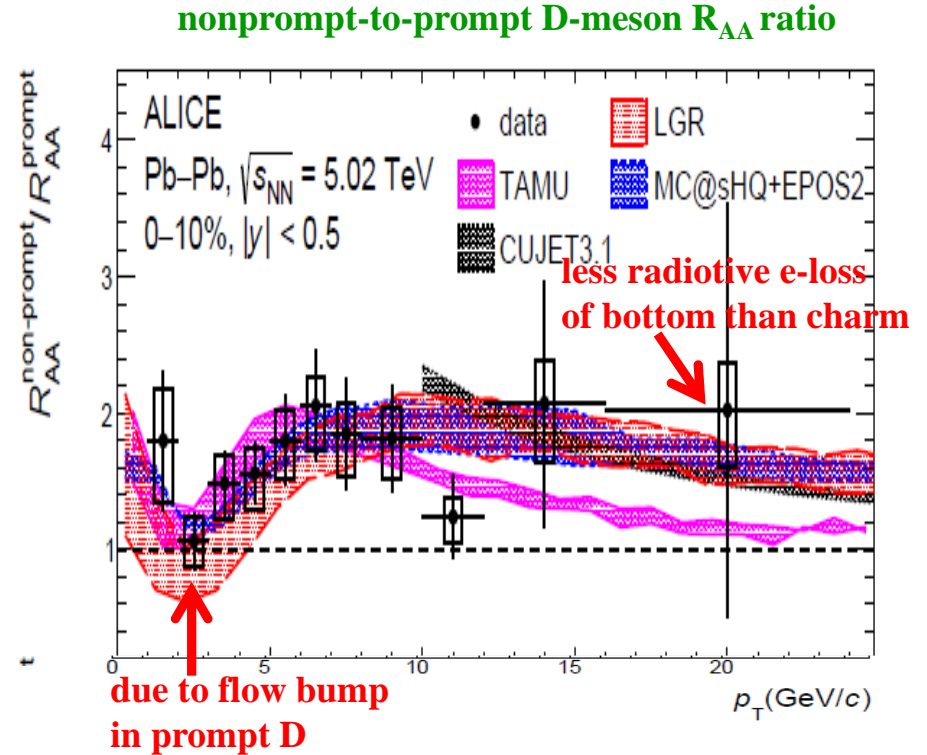
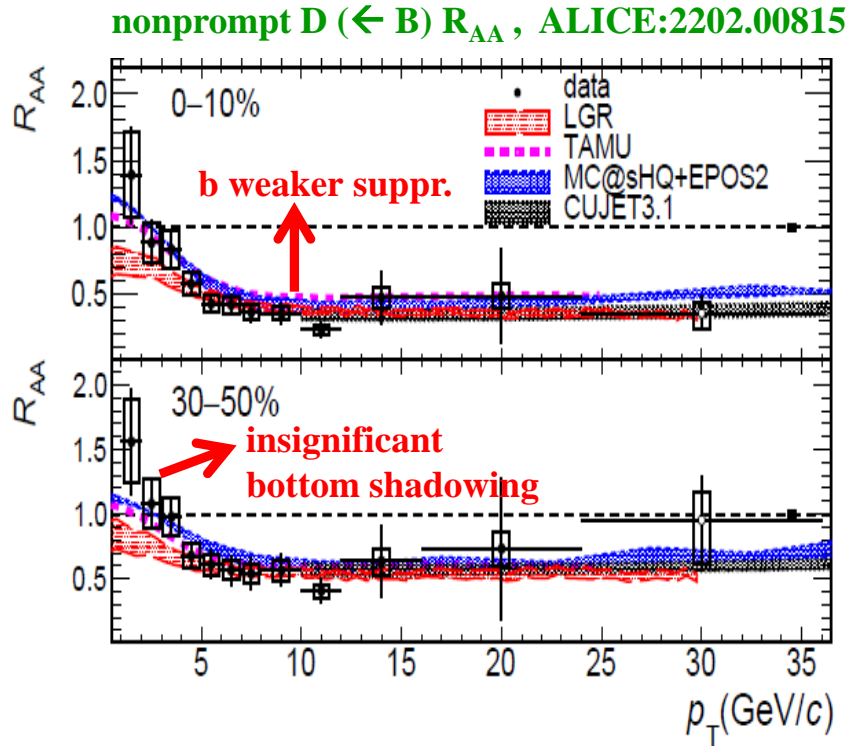
**The following are back-up pages**

# $\Lambda_c^+/\text{D}^0$ enhancement surprise



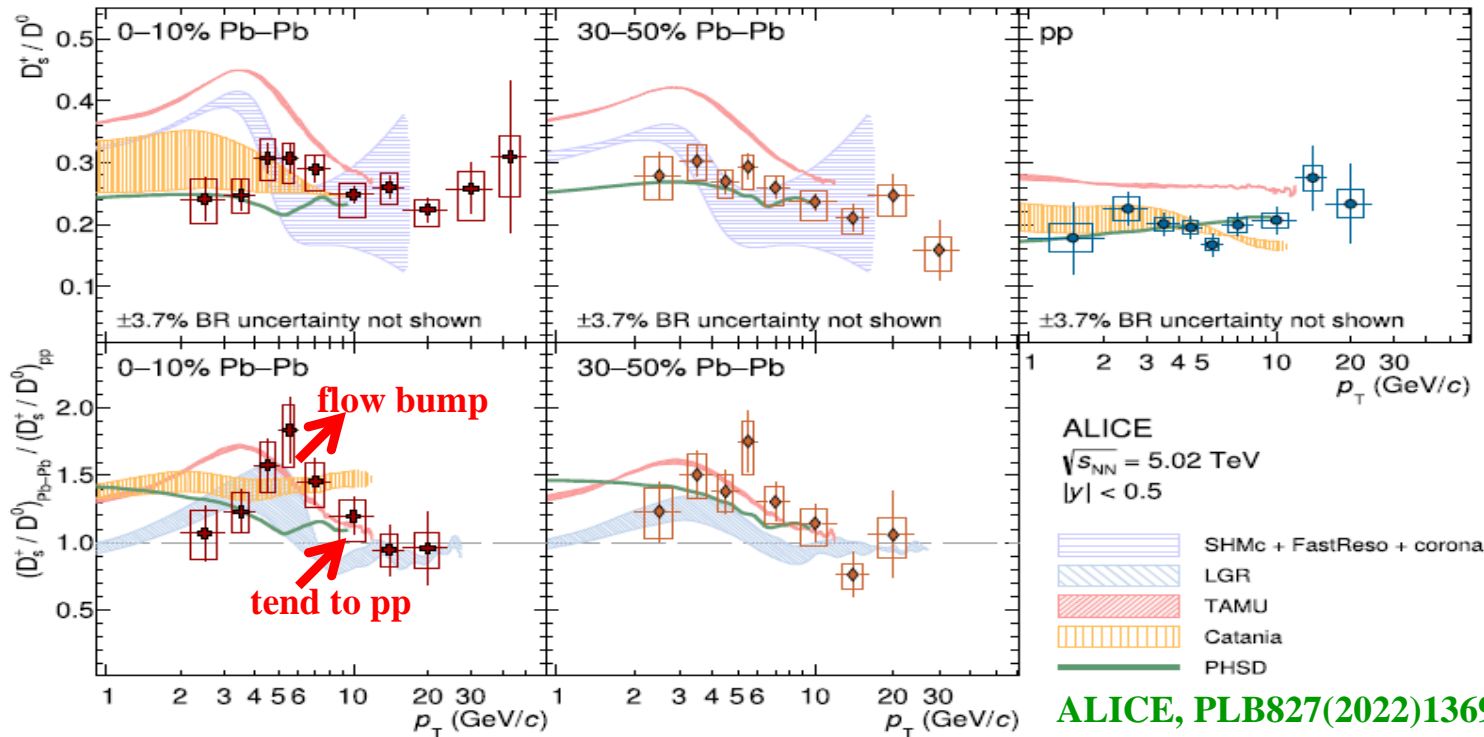
- ❖ a factor  $\sim 5$  enhancement w.r.t.  $e^+e^-$  at low  $p_T$ , much underestimated by FFs tuned to  $e^+e^-$
- ❖ decreasing toward high  $p_T$ , trend similar to  $\Lambda/K$  and  $p/\pi$

# Flavor dependence: charm vs bottom



- ❖ x3 mass: b-quark longer thermalization time at low  $p_T$  than charm  
less flow added to b from recombination with u/d/s
- ❖ high  $p_T > 15$  GeV: b-quark less radiative e-loss  $\leftarrow$  stronger “dead cone”

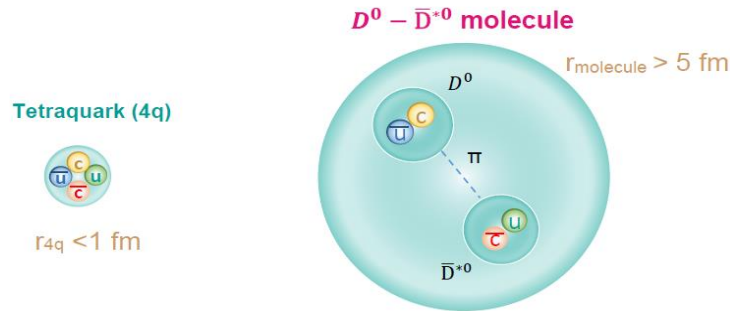
# Charm hadro-chemistry: $D_s^*/D^0$



- low  $p_T$ : enhancement due to charm recombination in a strangeness-equilibrated QGP reproduced by Catania & PHSD; overestimated by TAMU in both pp and PbPb
- high  $p_T$ : tending to pp value as fragmentation takes over
- flow bump due to recombination with flowing s-quark heavier than u/d, predicted by TAMU (RRM w/ SMCs) & SHMc (hydro blastwave spectrum)

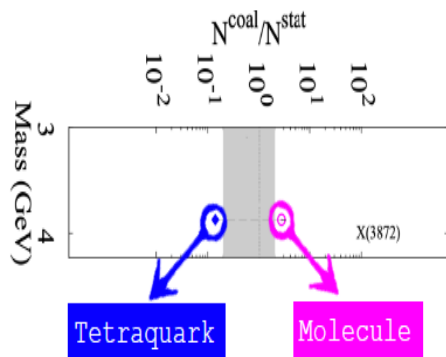
# X(3872) production in HIC

❖ inner structure: compact tetraquark vs loosely bound molecule

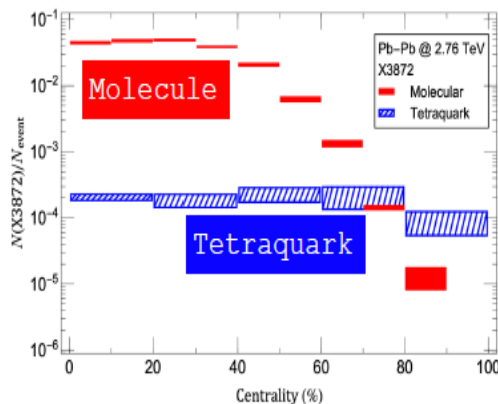


coalescence model

Cho et al. '11

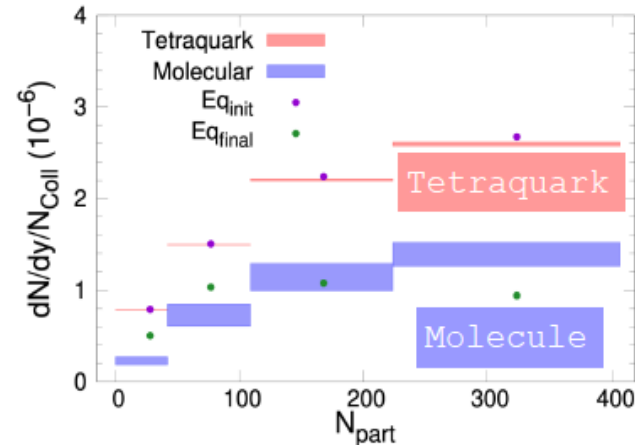


coalescence within AMPT zhang et al. '21



transport model

Wu et al. '21



- $N_{\text{molecule}} > N_{\text{tetraquark}}$  by x10 or 100, yet no account of hadron phase reactions  $\pi X \leftrightarrow DD^*$   
 → to be better constrained

- $N_{\text{tetraquark}} > N_{\text{molecule}}$  by x2, molecule regenerated in late hadronic phase, tetraquark chem. freezeout at  $T_c$

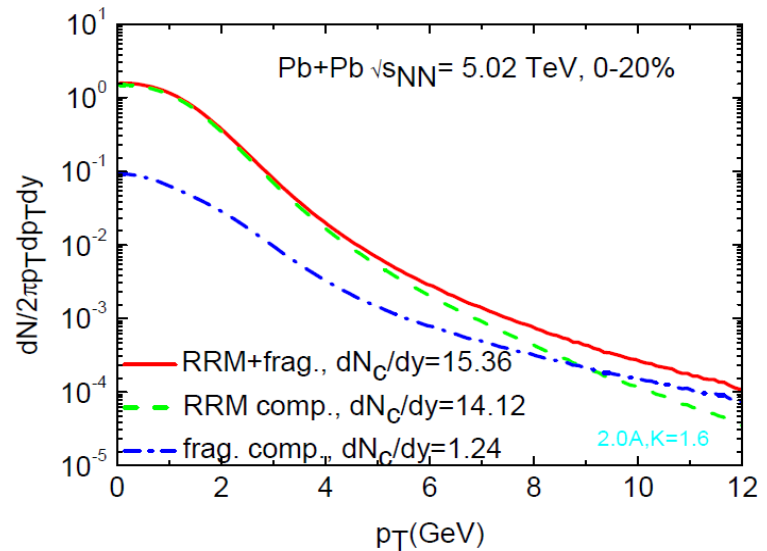
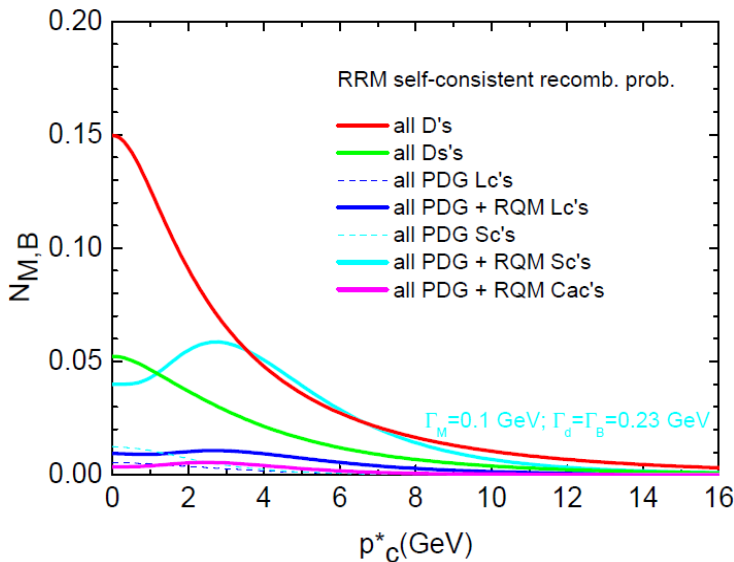
# Charm quark recombination probability

- No. of mesons/baryons formed from a single c-quark of rest frame  $p_c^*$

$$N_M(p_c^*) = \int \frac{d^3 \vec{p}_1^*}{(2\pi)^3} g_q e^{-E(\vec{p}_1^*)/T_{pc}} \frac{E_M(\vec{p}^*)}{m_M \Gamma_M} \sigma(s) v_{rel},$$

$$N_B(p_c^*) = \int \frac{d^3 p_1 d^3 p_2}{(2\pi)^6} g_1 e^{-E(\vec{p}_1)/T_c} g_2 e^{-E(\vec{p}_2)/T_c} \frac{E_d(\vec{p}_{12})}{m_d \Gamma_d} \sigma(s_{12}) v_{rel}^{12}(\vec{p}_1, \vec{p}_2) \frac{E_B(\vec{p})}{m_B \Gamma_B} \sigma(s_{d3}) v_{rel}^{d3}(\vec{p}_{12}, \vec{p}_{30}),$$

- Renormalizing  $N_M(p_c^*)$  and  $N_B(p_c^*)$  by a **common** factor  $\sim 4$  for all charmed mesons/baryons such that  $\sum_M P_{coal,M}(p_c^* = 0) + \sum_B P_{coal,B}(p_c^* = 0) = 1$



➔ **charm conservation** consistently built in, in an (e-by-e) way without spoiling the relative chemical equilibrium realized by RRM

# Space-momentum correlations: light-q

- hydro: a manifestation of SMCs

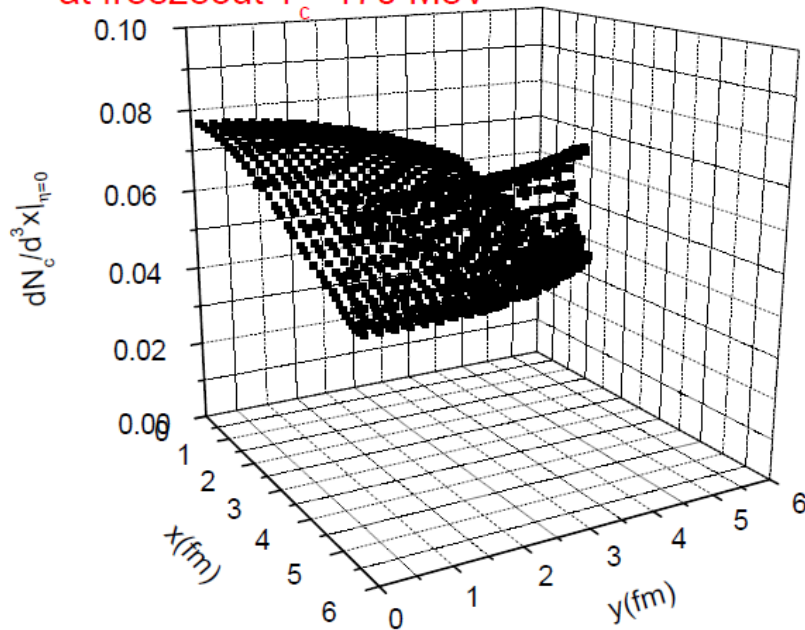
$$f_q^{eq}(\vec{x}, \vec{p}) = g_q e^{-p \cdot u(x)/T(x)} = g_q e^{-\gamma_T(x)[m_T \cosh(y-\eta) - \vec{p}_T \cdot \vec{v}_T(x)]/T(x)}$$

longitudinal boost invariance:  $y - \eta$

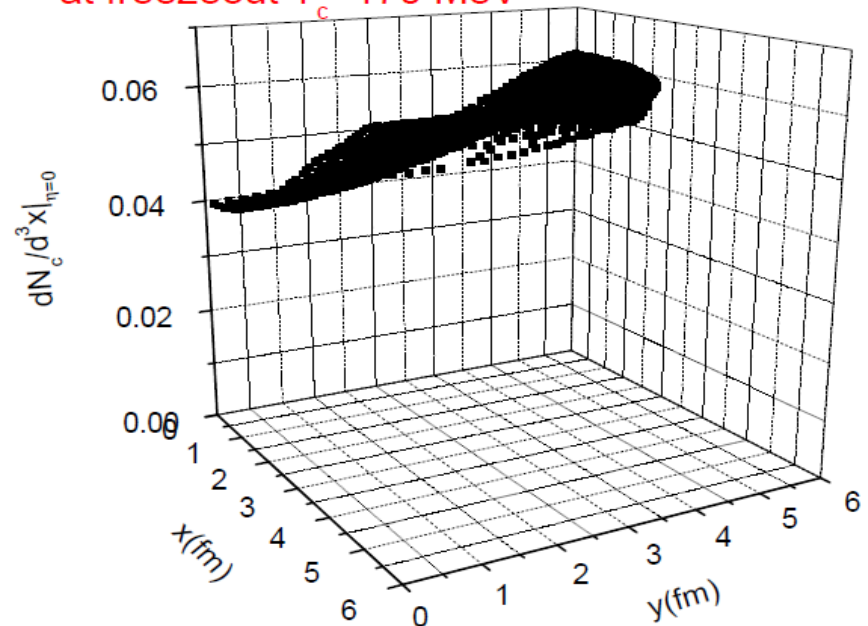
transverse SMCs  $p_T \cdot v_T$

- hydro-q: low (high)  $p_T$  more concentrated in center (boundary)

hydro light quarks  $p_T = 0.0 - 0.3$  GeV,  
at freezeout  $T_c = 170$  MeV



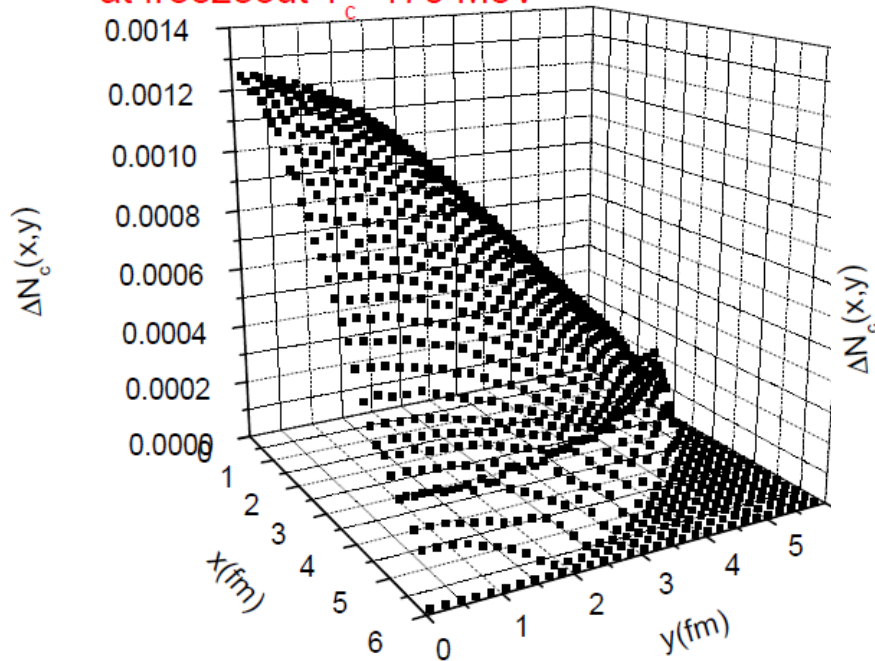
hydro light quarks  $p_T = 0.6 - 0.9$  GeV,  
at freezeout  $T_c = 170$  MeV



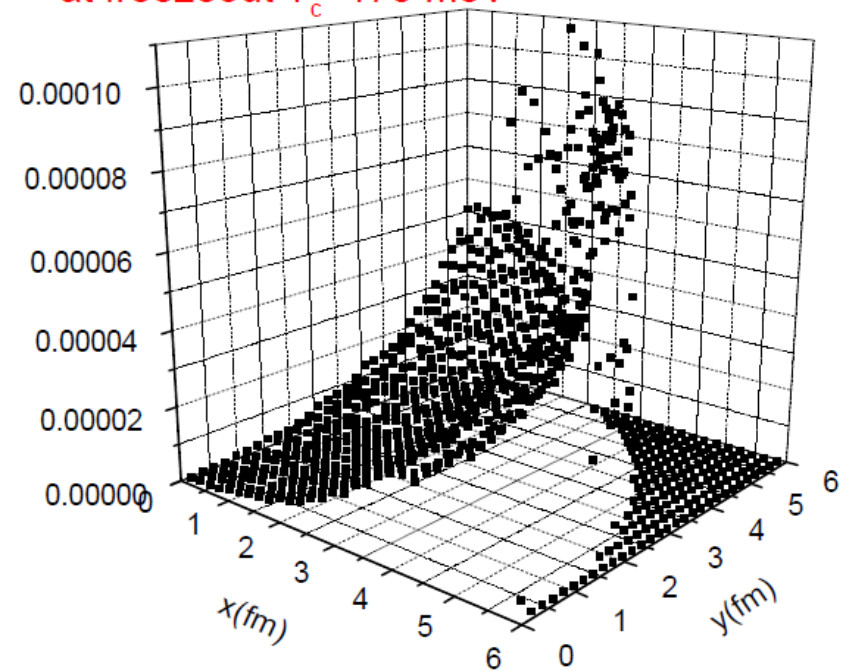
# SMCs: Langevin charm quarks

- Langevin-c: low (high)  $p_T$  more populated in central (outer)

Langevin charm quarks  $p_T=0.0-1.0$  GeV,  
at freezeout  $T_c=170$  MeV



Langevin charm quarks  $p_T=3.0-4.0$  GeV,  
at freezeout  $T_c=170$  MeV



- SMCs usually neglected in ICMs: uniformly distributed independent of  $p_T$

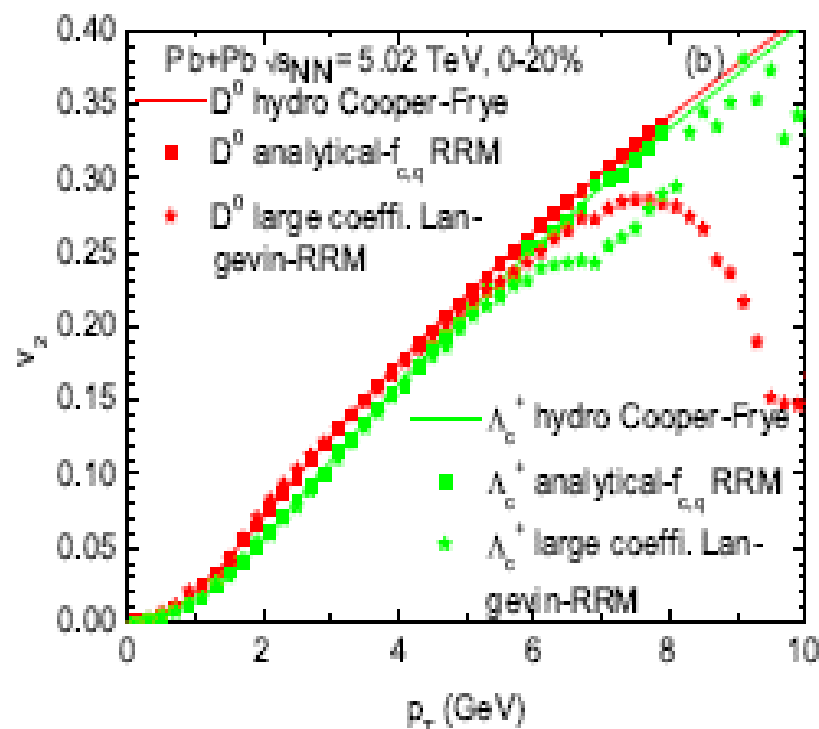
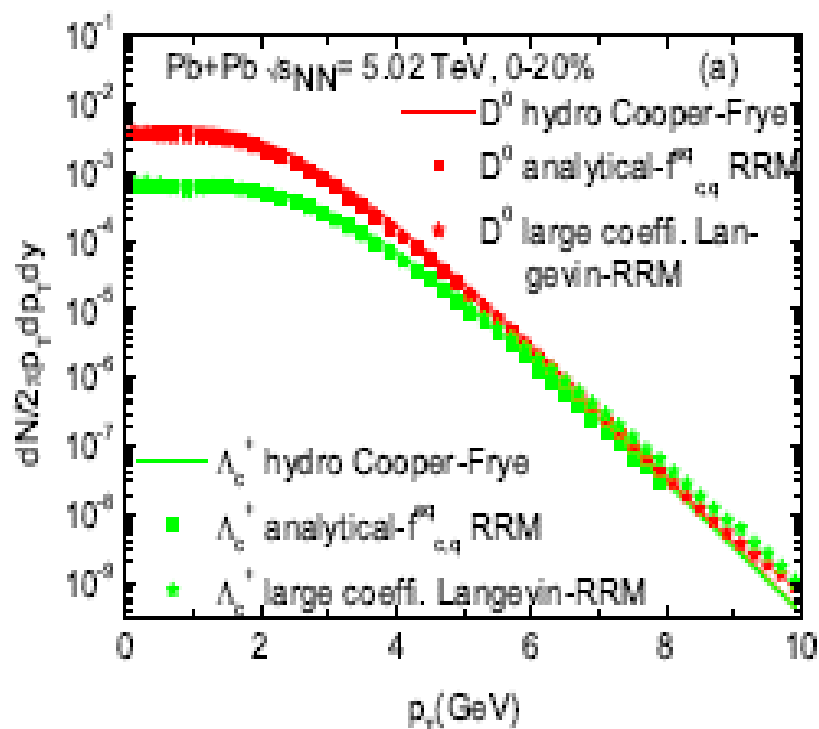
$$f_{c,q}(\vec{x}, \vec{p}) = (2\pi)^3 \frac{dN_{c,q}}{d^3\vec{x}d^3\vec{p}} = \frac{(2\pi)^3}{V E(\vec{p})} \frac{dN_{c,q}}{p_T dp_T d\phi_q dy}$$



# RRM equilibrium mapping

- Event-by-event Langevin-RRM simulation with **very large trans. coeffi.** & with **SMCs** properly incorporated

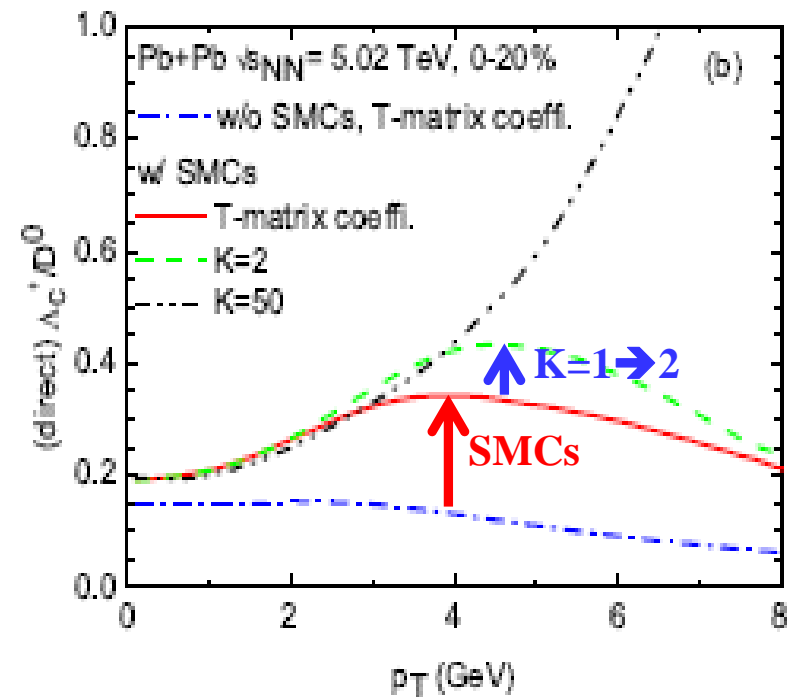
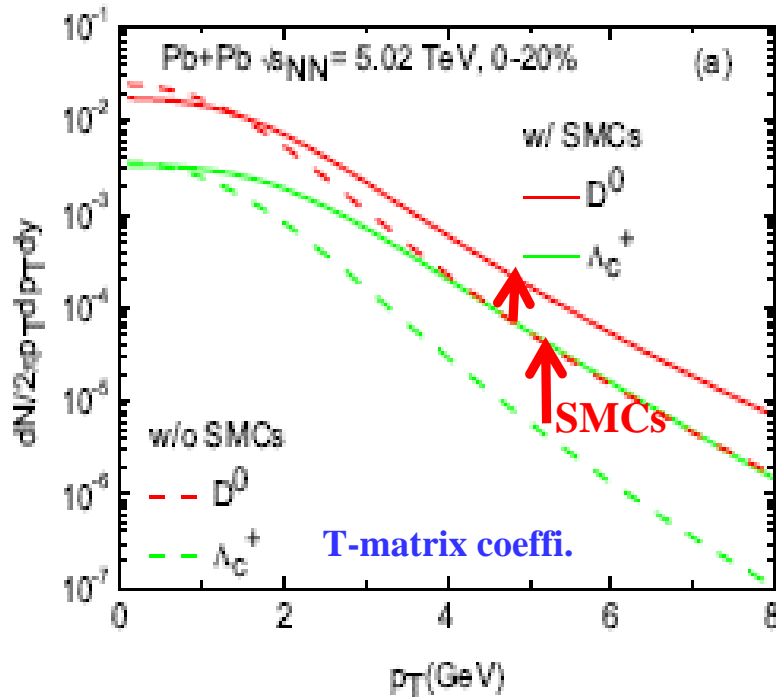
➔ kinetic & chemical equil. mapping



➔ Observables come out as RRM predictions with realistic T-matrix coeffi.

# Direct $D^0$ & $\Lambda_c^+$ production via RRM

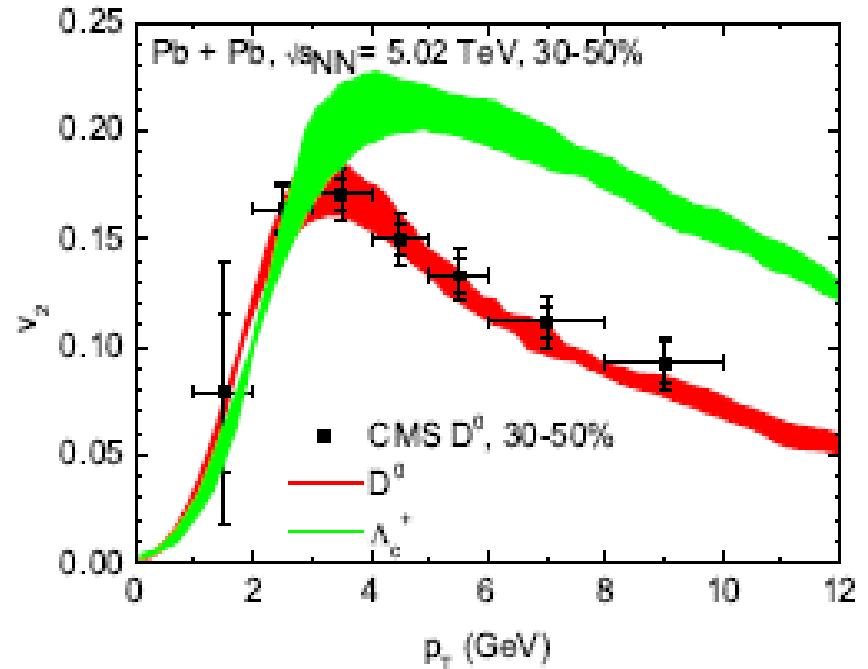
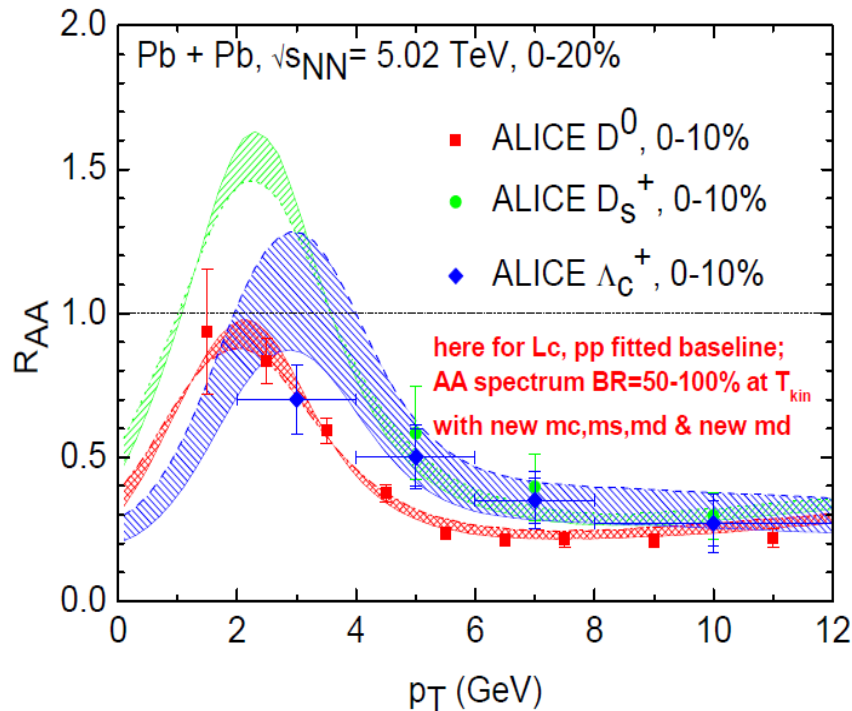
- Including SMCs makes spectra **harder** & **enhances** the  $\Lambda_c^+/D^0$



- Fast-moving c-quarks [ $p_T \sim 3-4$  GeV] moving to outer part of fireball find higher-density of harder [ $p_T \sim 0.6-0.9$  GeV] light quarks for recombination
- An effect entering **squared** for the recombination production of  $\Lambda_c^+$   
**➔ larger enhancement for  $\Lambda_c^+ \rightarrow \Lambda_c^+/D^0$  ratio enhanced!**

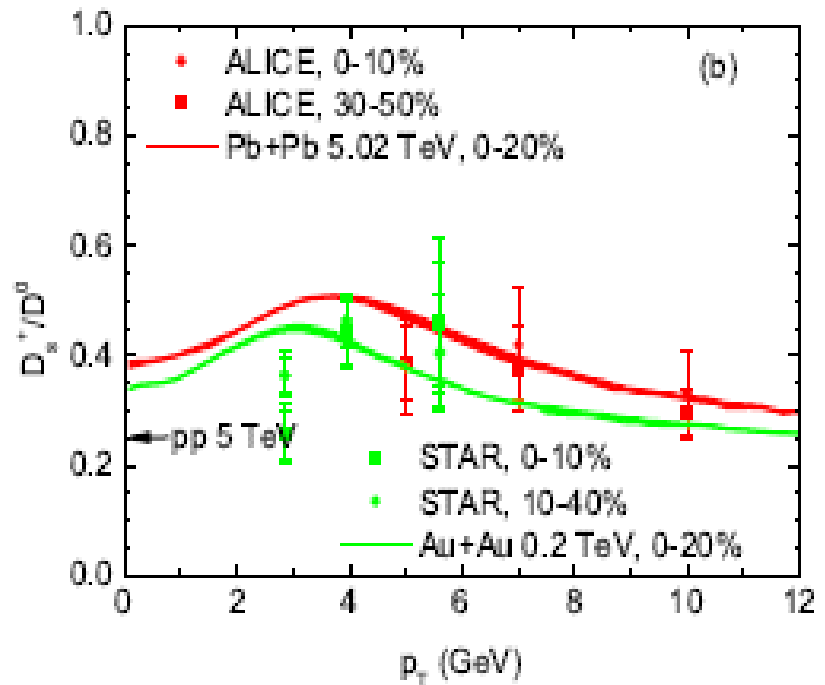
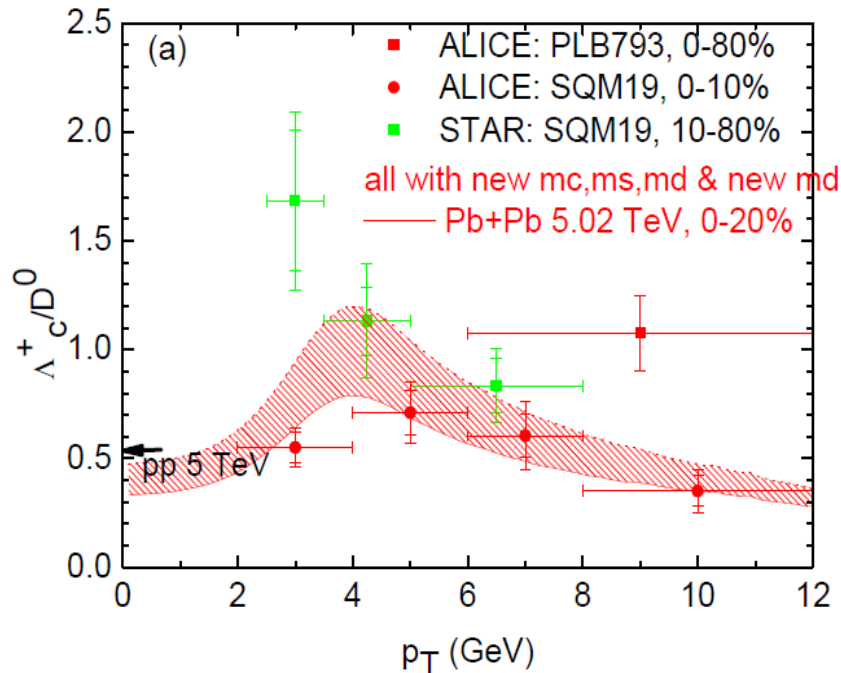
# $D^0$ , $D_s^+$ & $\Lambda_c^+$ suppression & elliptic flow

➤ Final  $D^0$ ,  $D_s^+$  &  $\Lambda_c^+$ , including feeddowns from all RQM baryons



- T-matrix coefficient\*K-factor(=1.6), to compensate for radiative e-loss; uncertainty: BR=50-100% to  $\Lambda_c^+$  for  $\Lambda_c^+$ 's &  $\Sigma_c^+$ 's above DN (2805 MeV)
- Hadronic phase diffusion also included: seamlessly connected to hadronization (RRM+frag), increasing D-meson  $v_2$  by  $\sim 15\%$

# Charm-hadron ratios: $\Lambda_c^+/D^0$ & $D_s^+/D^0$

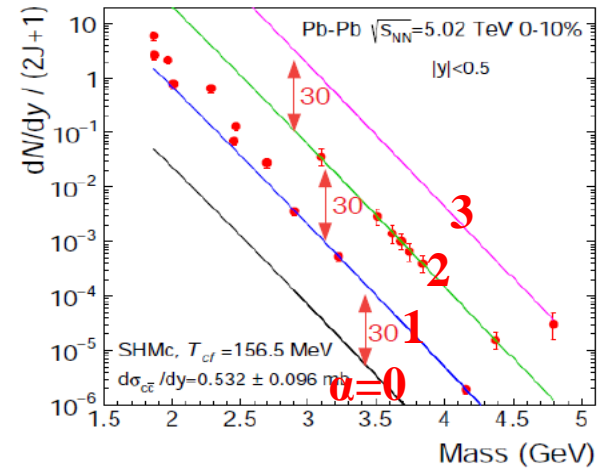


- $\Lambda_c^+/D^0$ : low  $p_T$  approaching RRM equil. limit = SHM pp; intermediate  $p_T$  enhancement from RRM with SMCs; high  $p_T$  fragmentation tending to pp value
- $D_s^+/D^0$  enhancement: recombination of charm in a strangeness-equilibrated QGP

# Hadronization: SHMc Andronic, PBM et al. 2104.12754

- SHMc: open-charm statistical hadronization at  $T_c$ 

$$\frac{dN(h_{oc,\alpha}^i)}{dy} = g_c^\alpha V n_i^{\text{th}} \frac{I_\alpha(N_c^{\text{tot}})}{I_0(N_c^{\text{tot}})}$$
  - ❑ multicharm baryons  $\alpha=1,2,3$  emerging pattern
  - ❑ yields enhanced by  $g_c^\alpha \sim 30^\alpha$  than pure thermal  $\rightarrow$  strong signal of deconfinement



- SHMc yields + blast wave  $\rightarrow p_T$  spectra

