QCD物理研讨会(山东青岛)

Propagation of spin waves

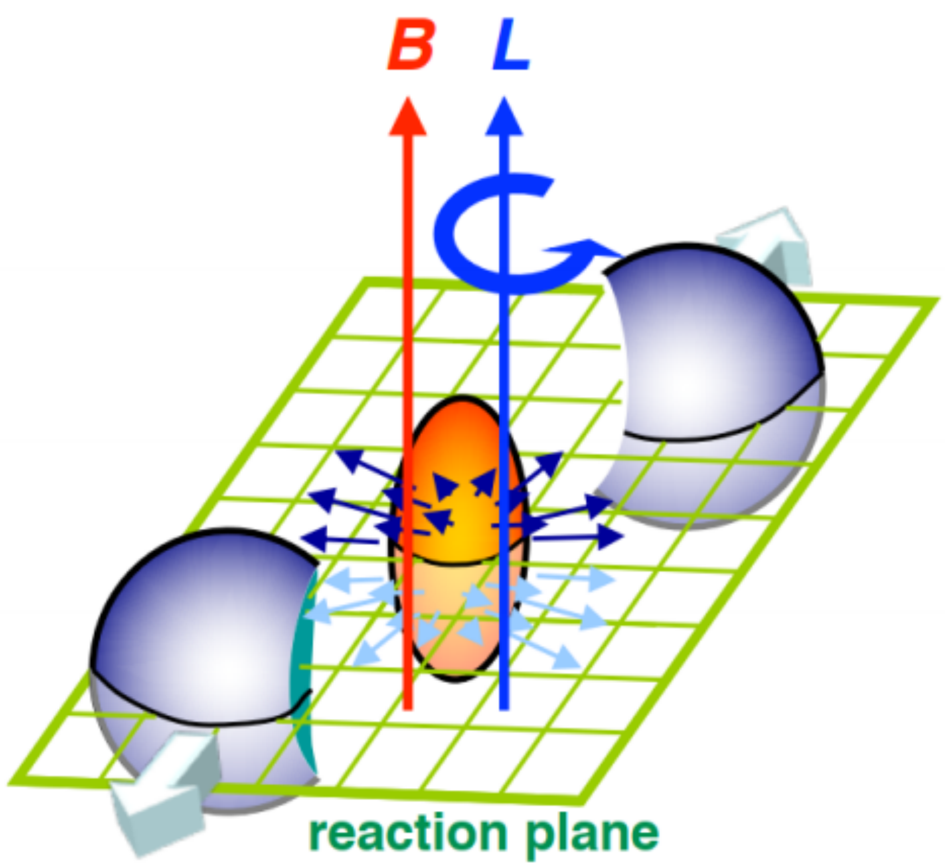
Speaker： Jin Hu(胡进)
Supervisor： Zhe Xu
Tsinghua University

Outline

- Introduction
- Spin Boltzmann equation
- Results and discussion
- Summary

Introduction

Liang, Wang, PRL (2005)

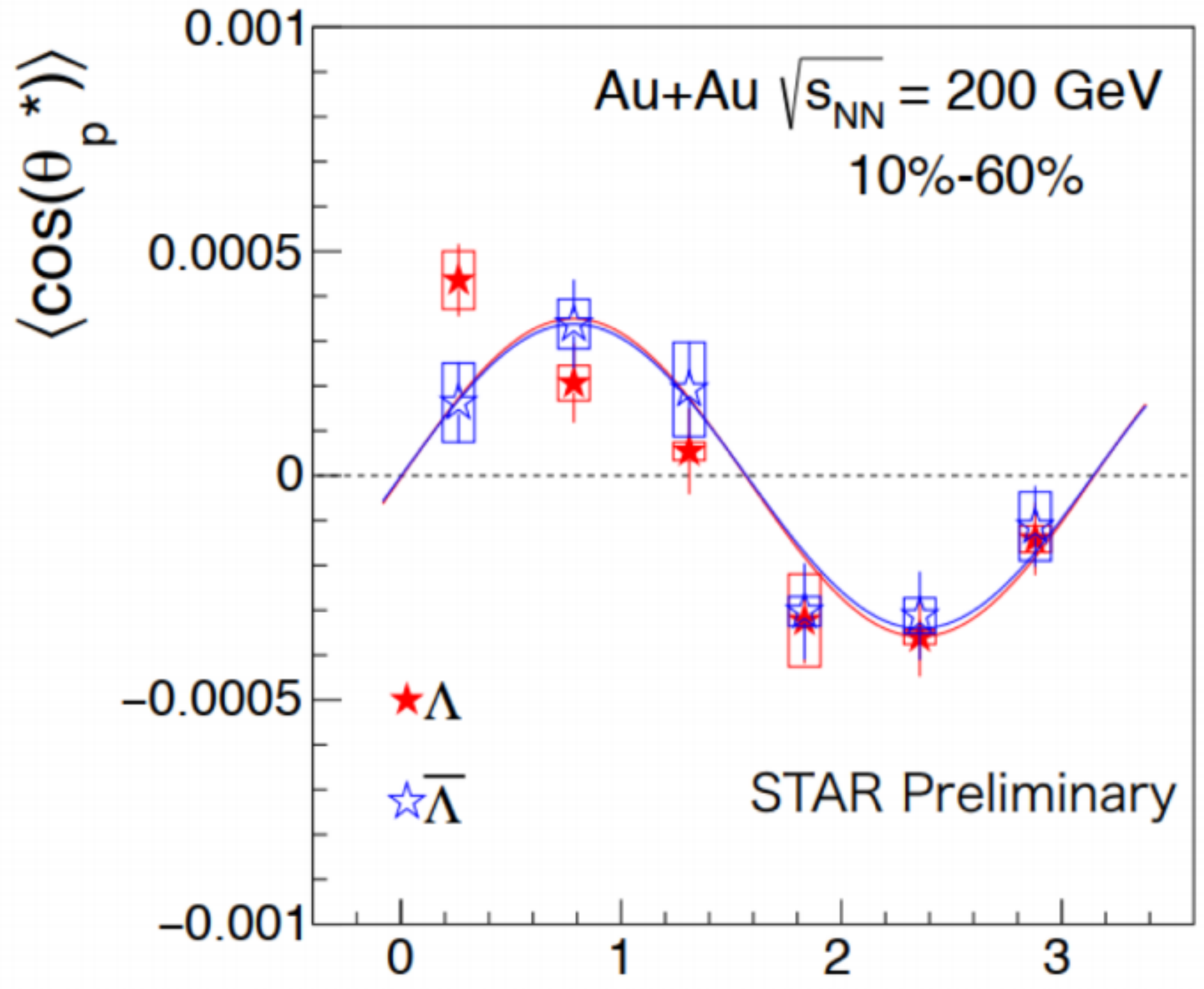


Produce huge angular momentum L

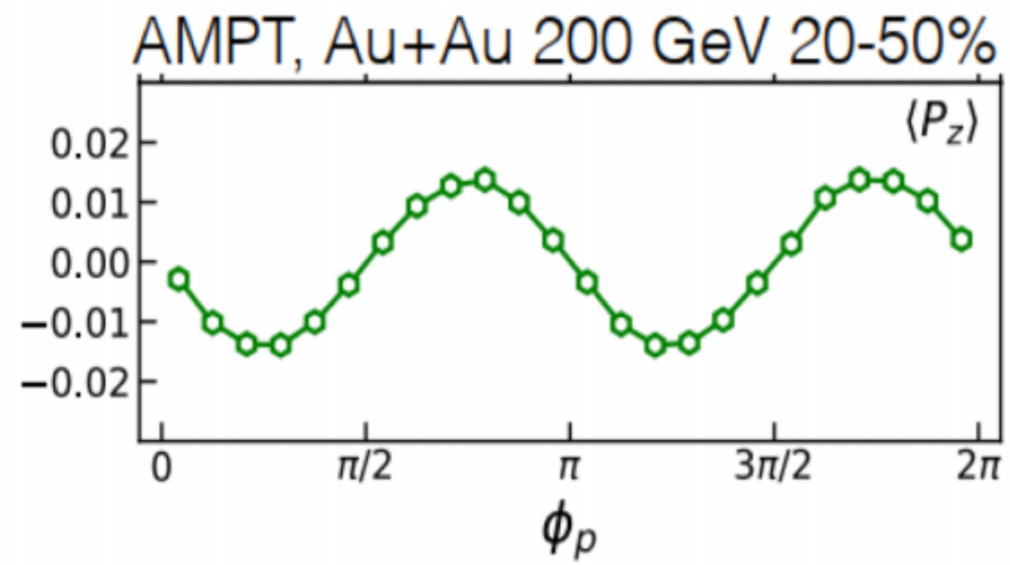
Global polarization :
theoretical results fit the data well (**equilibrium** picture)

Local polarization : even predicts the **opposite** azimuthal angular dependance.

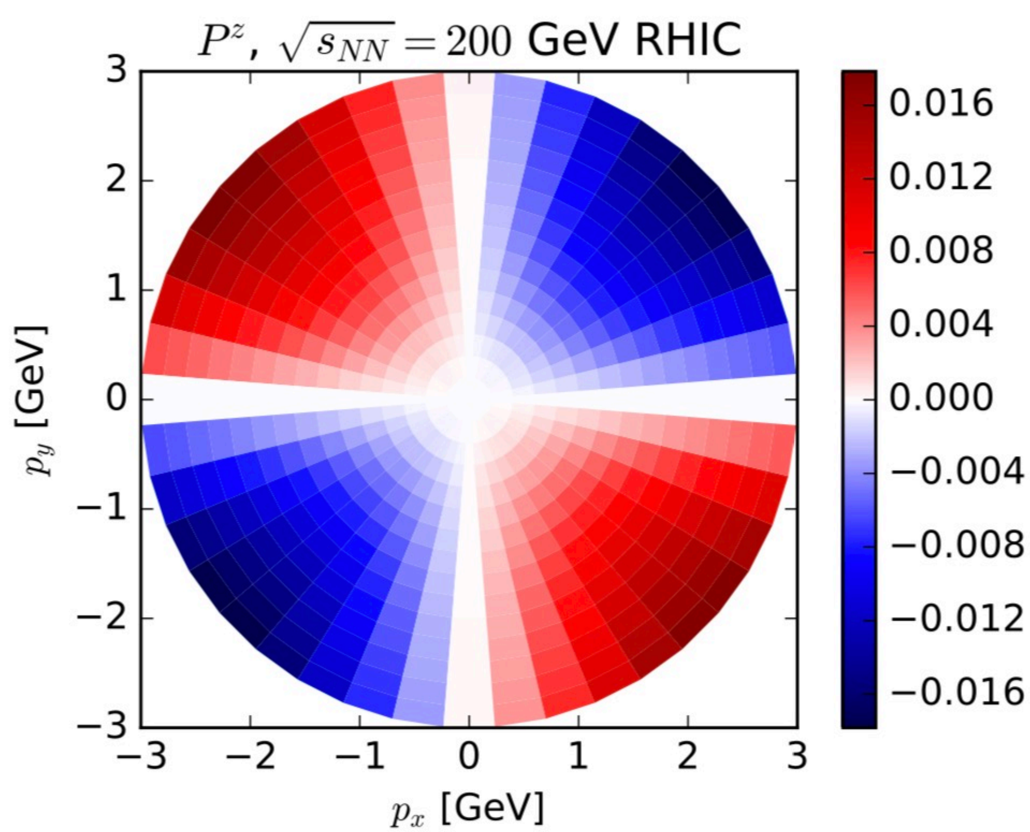
Opposite sign !



Xia, Li, Tang, Wang PRC(2018)



F.Becattini, I.Karpenko PRL (2018)



Introduction

- Stimulate the development of spin transport and spin hydrodynamics.
- Spin relaxation rate becomes essential for modeling the evolution of spinful fluids.
- Similar to propagation of sound waves, spin waves come into play when taking about spin dynamic evolution.

Spin Boltzmann equation

$$p \cdot \partial f(x, p, \mathbf{s}) = C[f]$$

N. Weickgenannt, E. Speranza, X.-l. Sheng, Q. Wang, and D. H. Rischke (2021)

$$C[f] \equiv \int d\Gamma_1 d\Gamma_2 d\Gamma' \mathscr{W} \, [f(x+\Delta_1, p_1, \mathbf{s}_1) f(x+\Delta_2, p_2, \mathbf{s}_2) - f(x+\Delta, p, \mathbf{s}) f(x+\Delta', p', \mathbf{s}')],$$

Spatial shift $\Delta \equiv -\dfrac{1}{2m(p^0+m)} \epsilon^{\mu\nu\beta} p_\nu \mathbf{s}_\beta$ ⇒ Spin-orbit coupling

Introduce **"classical" spin s** to extend phase space :

$$\int dS(p) = 2,$$

$$\int dS(p)\, \mathbf{s}^\mu = 0,$$

$$\int dS(p)\, \mathbf{s}^\mu \mathbf{s}^\nu = -2 \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right)$$

$$\int dS(p) \equiv \sqrt{\frac{p^2}{3\pi^2}} \int d^4\mathbf{s}\, \delta(\mathbf{s}\cdot\mathbf{s}+3)\delta(p\cdot\mathbf{s})$$

When no spin, return to Boltzmann equation.

Equilibrium state

N. Weickgenannt, E. Speranza, X.-l. Sheng,
Q. Wang, and D. H. Rischke (2021)

$$C[f_{\mathbf{leq}}] = -\frac{1}{(2\pi)^6}\int d\Gamma' d\Gamma_1 d\Gamma_2 \mathscr{W} \exp(2\xi - \beta\cdot(p+p'))$$

$$\times\Big[-\partial_\mu\xi\left(\Delta_1^\mu + \Delta_2^\mu - \Delta^\mu - \Delta'^\mu\right) + \partial_\mu\beta_\nu\left(\Delta_1^\mu p_1^\nu + \Delta_2^\mu p_2^\nu - \Delta^\mu p^\nu - \Delta'^\mu p'^\nu\right)$$

$$-\frac{1}{4}\Omega_{\mu\nu}\left(\Sigma_{\mathbf{s}_1}^{\mu\nu} + \Sigma_{\mathbf{s}_2}^{\mu\nu} - \Sigma_{\mathbf{s}}^{\mu\nu} - \Sigma_{\mathbf{s}'}^{\mu\nu}\right)\Big]$$

with $f_{\mathbf{leq}}(x,p,\mathbf{s}) = \frac{1}{(2\pi)^3}\exp[\xi - \beta\cdot p + \frac{\Omega_{\mu\nu}\Sigma_{\mathbf{s}}^{\mu\nu}}{4}]$

$\xi \equiv \frac{\mu}{T}, \beta^\alpha \equiv \frac{u^\alpha}{T}, \Omega$ **are Lagranian mutipliers**

total angular momentum

$$J^{\mu\nu} = \Delta^\mu p^\nu - \Delta^\nu p^\mu + \frac{1}{2}\Sigma_{\mathbf{s}}^{\mu\nu}$$

orbit **spin**

→

Conditions for global eq

$$\partial_{(\mu}\beta_{\nu)} = 0,$$

$$\xi = \textbf{const},$$

$$\Omega_{\mu\nu} = -\partial_{[\mu}\beta_{\nu]} = \textbf{const}\,.$$

Linear mode analysis

arxiv: 2202.07373

Choose one background profile in global eq,

$$\Omega = 0, \quad u^\alpha = (1,0,0,0)$$

Linearized transport equation in k space $k = (\omega, \vec{\kappa})$

$$p^0 \omega \tilde{\chi} + p^i \kappa_i \tilde{\chi} + L_2[\tilde{\chi}] = -iL_1[\tilde{\chi}] \qquad \tilde{\chi} = \int d^4x \exp(ik\cdot x)(f - f_{eq})$$

$$L_1[\phi] \sim \int d\Gamma' d\Gamma_1 d\Gamma_2 \mathscr{W} \exp(-\beta\cdot p')\Big[\phi(k,p,\mathbf{s}) + \phi(k,p',\mathbf{s}') - \phi(k,p_1,\mathbf{s}_1) - \phi(k,p_2,\mathbf{s}_2)\Big],$$

$$L_2[\phi] \sim \int d\Gamma' d\Gamma_1 d\Gamma_2 \mathscr{W} \exp(-\beta\cdot p')\Big[\Delta\cdot\kappa\phi(k,p,\mathbf{s}) + \Delta'\cdot\kappa\phi(k,p',\mathbf{s}') - \Delta_1\cdot\kappa\phi(k,p_1,\mathbf{s}_1) - \Delta_2\cdot\kappa\phi(k,p_2,\mathbf{s}_2)\Big].$$

Focus on hydro mode of $L_1 : \chi(t) \sim \exp(-\frac{L_1}{u\cdot p}t)\chi(0),$

$L_1\psi = 0, \qquad \psi = 1, \quad p^\alpha, \quad J^{\alpha\beta}$ **collisional invariants**

Linear mode analysis

arxiv : 2202.07373

Linearized transport equation in k space $k = (\omega, \vec{\kappa})$

$$p_0\omega\tilde{\chi} + p^i\kappa_i\tilde{\chi} + L_2[\tilde{\chi}] = -iL_1[\tilde{\chi}]$$

the LHS all start with $O(\kappa)$ **perturbation to** L_1

Solve the dispersion relations via degenerate perturbation

$$\tilde{\chi} = \tilde{\chi}^{(0)} + \tilde{\chi}^{(1)} + \cdots, \quad \omega = \omega^{(1)} + \omega^{(2)} + \cdots.$$

The spinless sector $(1, p^\alpha)$ **remain unchanged.**

Linear mode analysis

arxiv : 2202.07373

The spin sector $(J^{\alpha\beta})$ **:**

Two longitudinal modes, purely decaying $\omega_{L,i} = i\Gamma_{L,i}\kappa^2 \quad i = 1,2$

Four propagating transverse modes (double degeneracy)

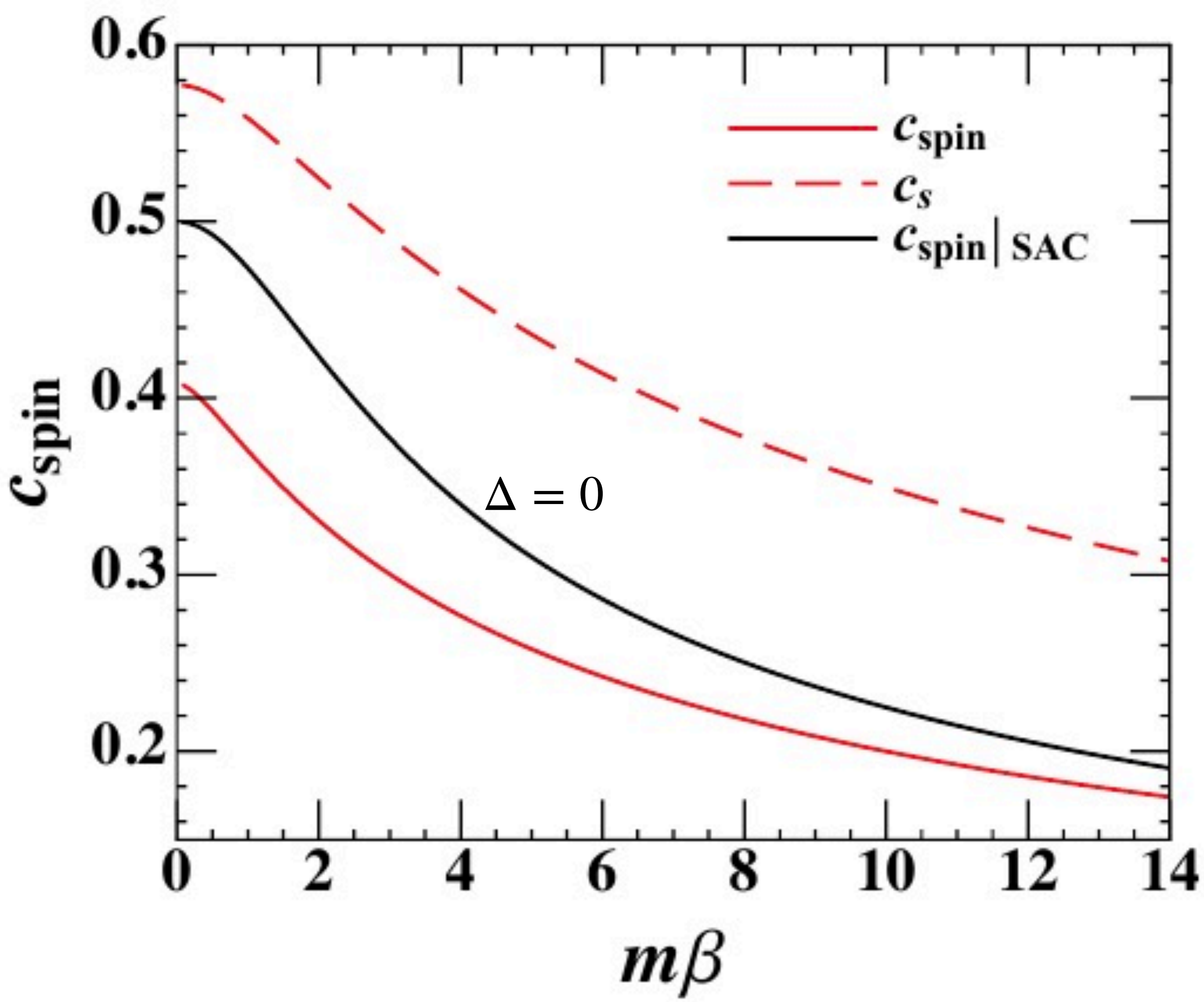
$$\omega_{T,1} = c_{spin}\kappa + i\Gamma_{T,1}\kappa^2, \quad \omega_{T,2} = -c_{spin}\kappa + i\Gamma_{T,2}\kappa^2$$

No gapped non-hydro modes (closely related to spin relaxation)

M.Hongo, X.G.Huang, M.Kaminski, M.Stephanov, H.U.Yee, JHEP (2021)

Results and discussion

arxiv : 2205.15755



Black line taken from V.E. Ambrus, R.Ryblewski, and R.Singh, PRD(2022)

sound propagation

EOS unchanged

(1, p still collision invariants)

Mutilated model

arxiv : 2205.15755

Approximate full linearized operator with a mutilated one

$$-L_1 \sim \left(-\gamma + \gamma \sum_{n=1}^{11} |\lambda_n\rangle\langle\lambda_n|\right)$$

γ **reciprocal of relaxation time** τ^{-1}

$|\lambda_n\rangle$ **is zero eigenfunction**

without the 2nd counter term, $L_1 \sim L_{AW}$

L_1 **can be parameterized scale-dependent**

while for Anderson-Witting RTA

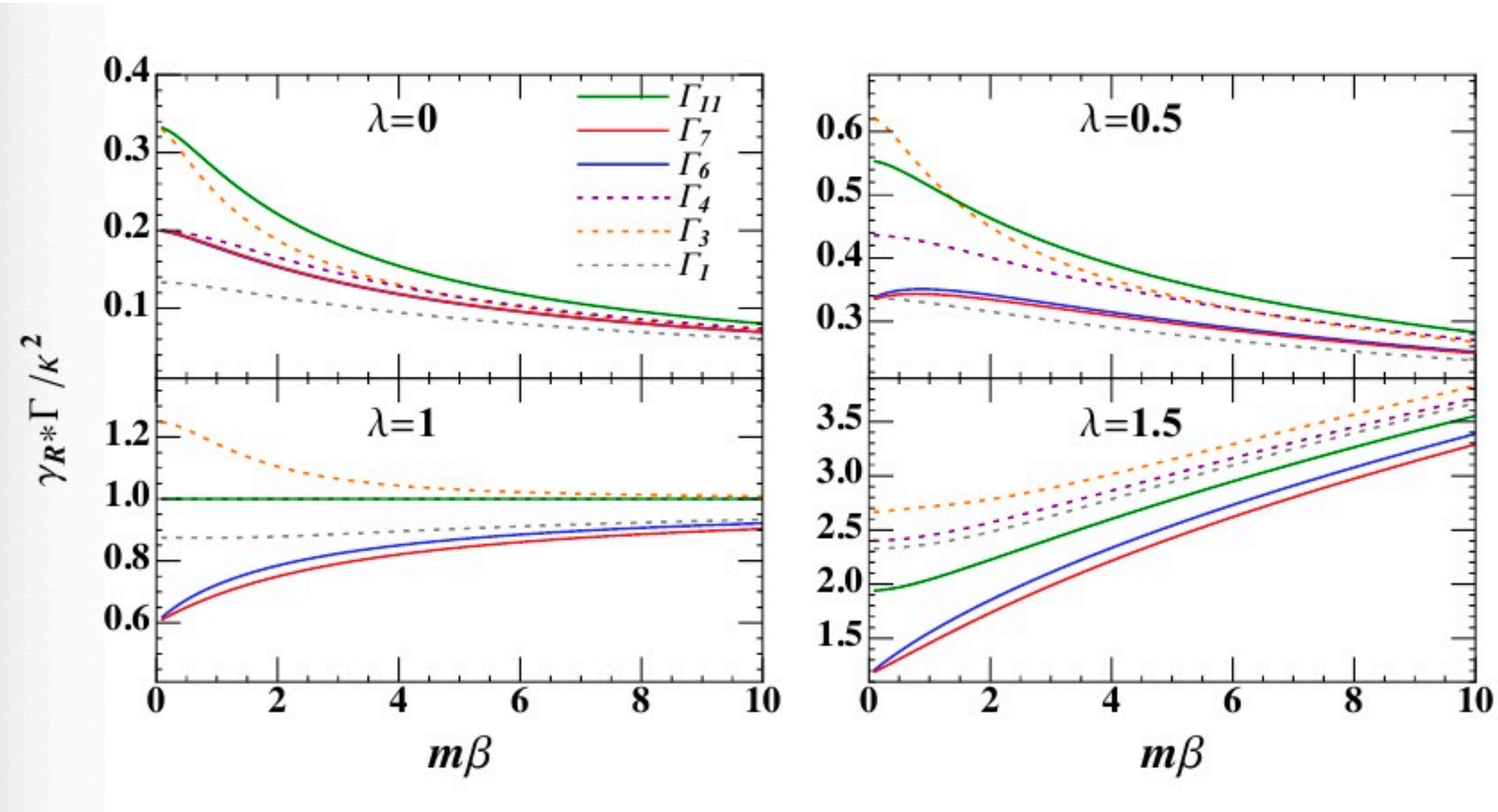
Landau matching

$$\partial_\alpha T^{\alpha\beta} = -\int dP\, p^\beta \frac{E_p}{\tau} f_0 \phi_p = 0 \quad \overset{?}{\longleftrightarrow} \quad \int dP\, p^\beta E_p f_0 \phi_p = 0$$

see also G.S.Rocha, G.S.Denicol, J.Noronha PRL (2021)

Results and discussion

arxiv : 2205.15755



$$\gamma = \gamma_R\left(\frac{E_p}{T}\right)^{-\lambda}$$

Most theories lie between $\lambda \in [0,1]$

K. Dusling, G.D. Moore, D.Teaney PRC(2010)

1, 3, 4 sound, heat, shear
6,11 longitudinal spin
7 transverse spin (4 deg)

Over a wide value range, decay rates are comparable

Related to spin dissipation

In HW gauge, only when global eq or local collisions, $S_{HW}^{\lambda,\mu\nu}$ is conserved

$$T_{HW}^{\mu\nu} \equiv \int d\Gamma\, p^\mu p^\nu f(x,p,\mathbf{s}) + O(\partial^2),$$

Hilgevoord Wouthuysen gauge

$$S_{HW}^{\lambda,\mu\nu} \equiv \int d\Gamma\, p^\lambda (\frac{1}{2}\Sigma_{\mathbf{s}}^{\mu\nu} - \frac{1}{2m^2} p^{[\mu}\partial^{\nu]}) f(x,p,\mathbf{s}) + O(\partial^2)$$

$$\partial_\lambda S_{HW}^{\lambda,\mu\nu} = \int d\Gamma\, \frac{1}{2}\Sigma_{\mathbf{s}}^{\mu\nu} C[f] = T_{HW}^{[\mu\nu]} = O(\partial^2)$$

E.Speranza, N.Weickgenannt EPJA(2021)

Spin is not conserved, but happens at $O(\hbar^2\partial^2)$

$T_{HW}^{[\mu\nu]}$ **cannot be solely expressed by** f, $C[f]$

$\delta S^{\mu\nu}$ **related to the attenuation of spin modes** arxiv : 2202.07373

Results and discussion

arxiv : 2202.07373

Define fluctuation amplitudes

$$\rho_n \equiv (\lambda_n, \tilde{\chi}) = \int d\Gamma e^{-\beta\cdot p} \lambda_n \tilde{\chi}$$

$$\tilde{\chi} \sim \frac{\frac{\gamma}{|\kappa|} \sum_{n=1}^{11} \rho_n \lambda_n}{\frac{-i\omega + \gamma}{-i|\kappa|} + \frac{p_z}{p_0}}$$

Equation set for ρ_n

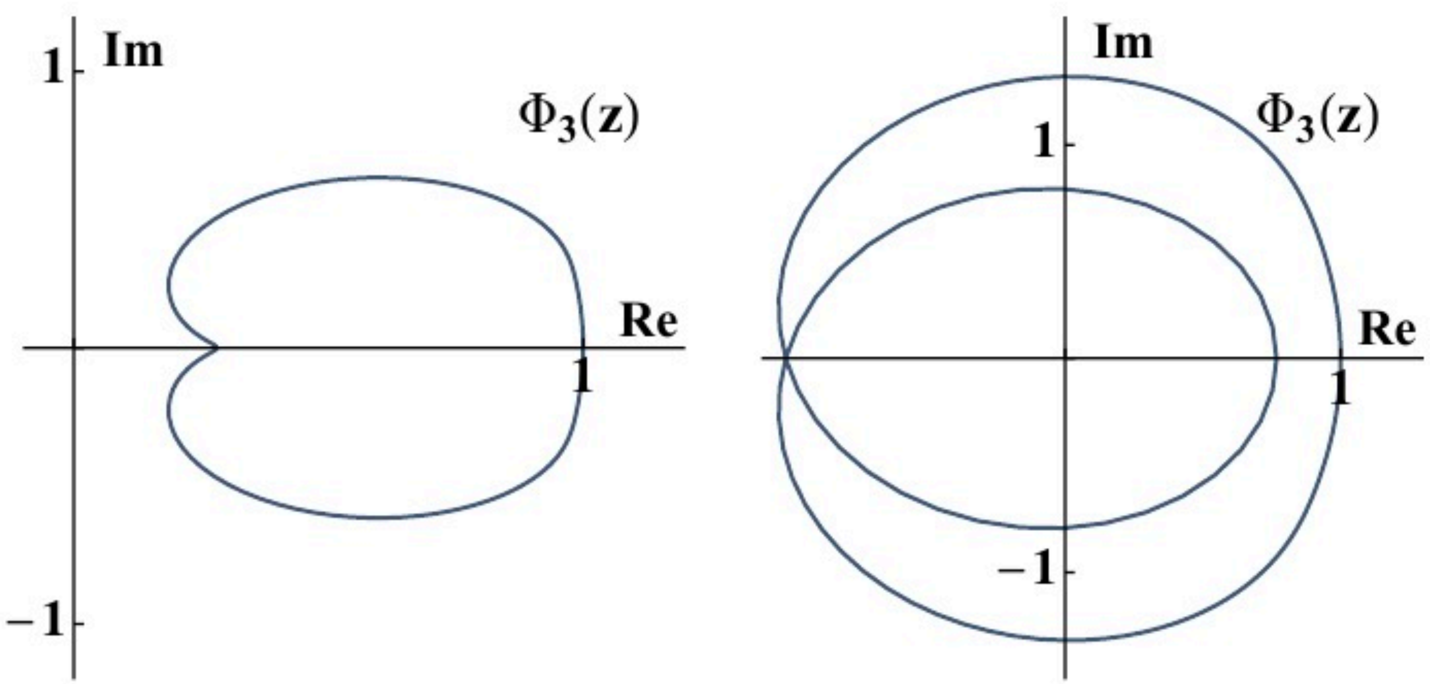


FIG. 3. The typical trajectories of $\Phi_3(z)$ with $\kappa > \kappa_o$ (left) and $\kappa < \kappa_o$(right)

varying $\frac{\gamma}{|\kappa|}$

Φ_3 determinant

Results and discussion

arxiv : 2202.07373

critical wave vector in non-relativistic limit

	L(6th)	L(11th)	T	sound	shear	heat
$\tau\kappa_o/n\sigma$	1.772	1.762	1.754	1.853	1.772	1.918

existence conditions for hydro modes

$\kappa \uparrow$ **collision-dominated to Knudsen region**

Onset of hydrodynamics ? see P.Romatschke EPJC(2016)

Sharp transition due to oversimplified collision kernel

Summary

- We present a linear mode analysis , find two longitudinal spin modes and four transverse propagating modes.
- Based on spin Boltzmann eq, these spin modes are responsible for spin dissipation. Over a wide parameter value range, spin relaxation is as slow as momentum relaxation.
- Existence conditions for hydro modes. A sharp transition.

Outlook

- How to find non-hydro modes in kinetic theory ?
- Calculate retarded correlators from kinetic theory, then see whether gapped poles exist in physical (principal) Riemann sheet.

Back Up

For (spin) Boltzmann eq

Relativistic kinetic theory, S.R.Degroot

$$\mathcal{F}[\psi] = \tfrac{1}{2} \sum_{i,j,k,l} \int \frac{d^3p_i}{p_i^0} \frac{d^3p_j}{p_j^0} \frac{d^3p_k}{p_k^0} \frac{d^3p_l}{p_l^0} \psi_k (f_i f_j W_{ij|kl} - f_k f_l W_{kl|ij}) . \quad (6)$$

Next we interchange the initial and final integration variables in the last term giving

$$\mathcal{F}[\psi] = \tfrac{1}{2} \sum_{i,j,k,l} \int \frac{d^3p_i}{p_i^0} \frac{d^3p_j}{p_j^0} \frac{d^3p_k}{p_k^0} \frac{d^3p_l}{p_l^0} (\psi_k - \psi_i) f_k f_l W_{ij|kl} . \quad (7)$$

For Enskog eq

R.Malfliet , Nuclear physics A (1984)

$$\left(\frac{\partial f}{\partial t}\right)^{\mathrm{E}}_{\mathrm{coll}} \equiv \iiint d\boldsymbol{v}_1\, d\boldsymbol{v}'\, d\boldsymbol{v}'_1\; W(\boldsymbol{v}\boldsymbol{v}_1|\boldsymbol{v}'\boldsymbol{v}'_1)\{\cdots\} ,$$

$$\{\cdots\} = Y_{\mathrm{E}}(n(\boldsymbol{r}+\tfrac{1}{2}\boldsymbol{\varepsilon} d, t)) f(\boldsymbol{r}, \boldsymbol{v}', t) f(\boldsymbol{r}+\boldsymbol{\varepsilon} d, \boldsymbol{v}'_1, t) - Y_{\mathrm{E}}(n(\boldsymbol{r}-\tfrac{1}{2}\boldsymbol{\varepsilon} d, t)) f(\boldsymbol{r}, \boldsymbol{v}, t) f(\boldsymbol{r}-\boldsymbol{\varepsilon} d, \boldsymbol{v}_1, t) ,$$

Back Up

N.Weickgenannt, D.Wagner, and E.Speranza PRD(2022)

By construction, not all information are well retained by $f, C[f]$

$$\mathfrak{f}(x,p,\mathfrak{s}) \equiv \frac{1}{2}\left[\mathcal{F}(x,p) - \hbar\delta V(x,p) - \mathfrak{s}\cdot\mathcal{A}(x,p)\right] \ ,$$

$$T^{\mu\nu}_{D,HW} = \frac{1}{m}\int d^4p \left[p^\nu\left(p^\mu\mathcal{F} - \hbar D^\mu_\mathcal{V}\right) + \frac{\hbar^2}{4}(\partial^\nu\partial^\mu - g^{\mu\nu}\partial^2)\mathcal{F} + \frac{\hbar^2}{4}\epsilon^{\lambda\mu\nu\alpha}\partial_\lambda D_{\mathcal{A}\alpha}\right] + \mathcal{O}(\hbar^3) \ .$$

$$-\frac{\hbar}{2}\partial^\mu\mathcal{P} + \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}p_\nu S_{\alpha\beta} + m\mathcal{A}^\mu \ = \ -\hbar D^\mu_\mathcal{A} \ ,$$

Back Up

$$\hat{L}_{\mathrm{RTA}}\phi_{\mathbf{k}} = -\frac{E_{\mathbf{k}}}{\tau_R} f_{0\mathbf{k}} \left[\phi_{\mathbf{k}} - \frac{\langle (E_{\mathbf{k}}/\tau_R)\, \phi_{\mathbf{k}} \rangle_0}{\langle E_{\mathbf{k}}/\tau_R \rangle_0} - P_1 \frac{\left\langle (E_{\mathbf{k}}/\tau_R)\, P_1^{(0)} \phi_{\mathbf{k}} \right\rangle_0}{\left\langle (E_{\mathbf{k}}/\tau_R)\, P_1^{(0)} P_1^{(0)} \right\rangle_0} - k^{\langle \mu \rangle} \frac{\left\langle (E_{\mathbf{k}}/\tau_R)\, k_{\langle \mu \rangle} \phi_{\mathbf{k}} \right\rangle_0}{(1/3) \left\langle (E_{\mathbf{k}}/\tau_R)\, k_{\langle \nu \rangle} k^{\langle \nu \rangle} \right\rangle_0} \right].$$

$$\int dK\, L_{RTA}\, \phi_k = 0$$

consistent with Landau matching

Results and discussion

arxiv : 2205.15755

Compare with other results

M.Hongo, X.G.Huang, M. Kaminski, M.Stephanov, H.U. Yee JHEP (2021)

Linear response theory + spin hydro with torsion,
non-propagating and gapped dof

V.E. Ambrus, R.Ryblewski, R.Singh, PRD (2022)

hydro from AW RTA approximation
conserved spin angular momentum, propagating transverse dof

arxiv : 2205.15755

Spin Boltzmann eq + mutilated model

conserved total angular momentum, propagating transverse dof