Spin alignments of vector mesons - new frontier of spin dynamics

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Outline

- Introduction
- Non-relativistic coalescence model for spin alignments of vector mesons
- Relativistic spin Boltzmann equations from Closed-Time-Path formalism (CTP) or Kadanoff-Baym equation (KBE)
- Questions for discussions

Global OAM and Magnetic field in HIC

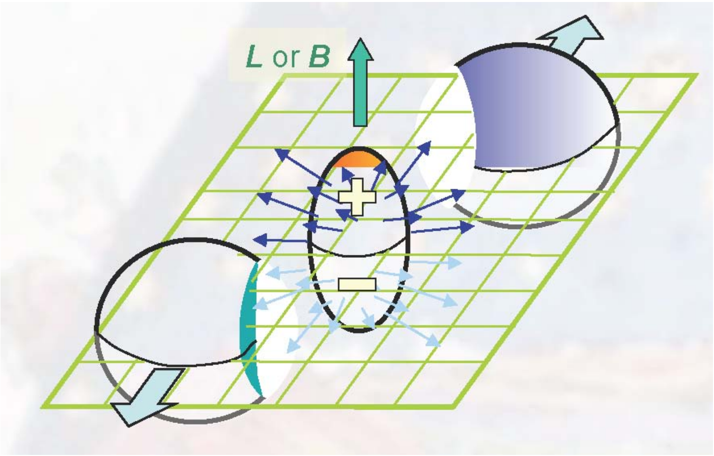
- Huge global orbital angular momenta are produced

$$L \sim 10^5 \hbar$$

- Very strong magnetic fields are produced

$$B \sim m_\pi^2 \sim 10^{18}\ \mathrm{Gauss}$$

- How do orbital angular momenta be transferred to the matter created?
- Any way to measure angular momentum? How is spin coupled to local vorticity in a fluid?



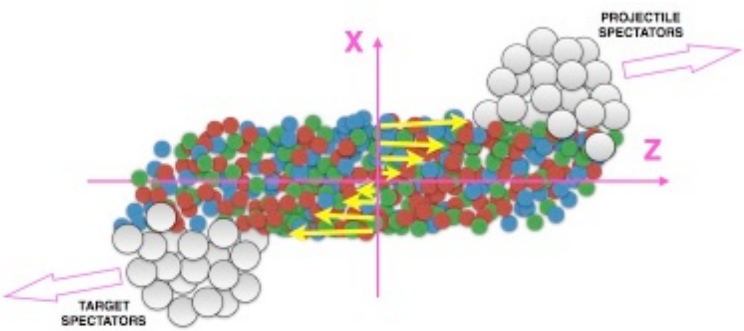


Figure taken from
Becattini et al, 1610.02506

Theoretical models and proposals: early works on global polarization in HIC

With such correlation between rotation and polarization in materials, we expect the same phenomena in heavy ion collisions. Some early works along this line:

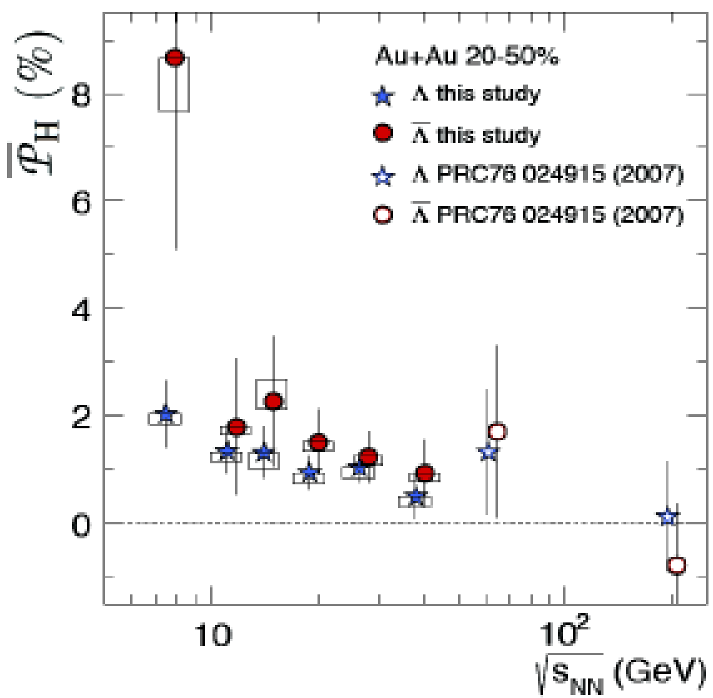
- **Polarizations of Λ hyperons and spin alignments of vector mesons through spin-orbital coupling in HIC from global OAM**
- **-- Liang and Wang, PRL 94,102301(2005), PRL 96, 039901(E) (2006) [nucl-th/0410079]**
- **-- Liang and Wang, PLB 629, 20(2005) [nucl-th/0411101]**

- **Polarized secondary particles in un-polarized high energy hadron-hadron collisions**
- **-- Voloshin, nucl-th/0410089**

- **Polarization as probe to vorticity in HIC**
- **-- Betz, Gyulassy, Torrieri, PRC 76, 044901(2007) [0708.0035]**

- **Statistical model for relativistic spinning particles**
- **-- Becattini, Piccinini, Annals Phys. 323, 2452 (2008) [0710.5694]**

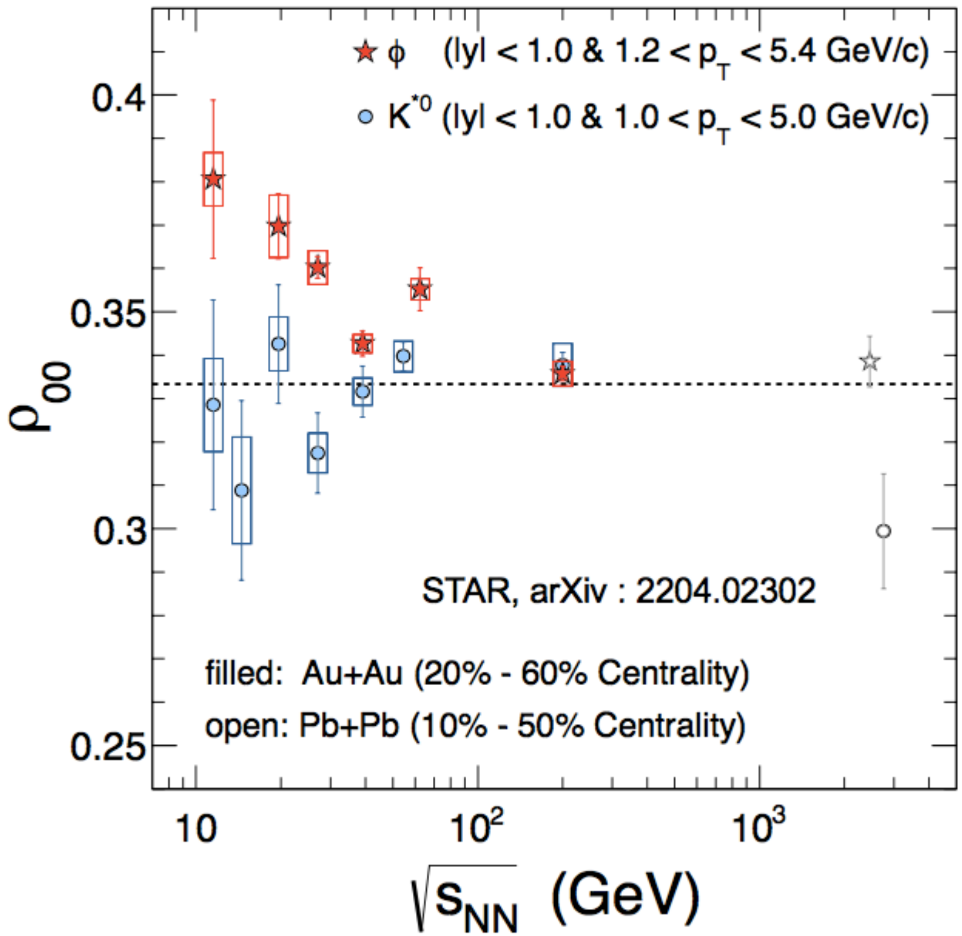
STAR results: Hyperon Polarization



STAR Collab., Nature 548 (2017) 62

Theoretical works (China):
THU, FDU, CCNU, SYU, SDU, PKU-IMP, USTC, ……
(international): Florence, Frankfurt, Riken, Berkeley, Chicago, ……

STAR results: spin alignments of vector mesons



STAR Collab., 2204.02302

"the global spin alignment for phi unexpectedly large, while that for K^{*0} is consistent with zero. The observed spin-alignment pattern and magnitude for the phi cannot be explained by conventional mechanisms, while a model with strong force fields [2,3] accommodates the current data. "

[2] Sheng, Oliva, QW (2019, 2020, Erratum 2022)
[3] Sheng, QW, Wang (2020)

Possible contributions to ρ_{00}^{ϕ}

$$\rho_{00}^{\phi} = \frac{1}{3} + c_{\varepsilon} + c_{\omega} + c_{E} + c_{B} + c_{F} + c_{A} + c_{L} + c_{\phi}$$

E-part of vorticity tensor [1,2]

B-part of vorticity tensor [1,2]

Electric field [1]

Magnetic field [1,2]

Frag. [4]

Turbulent color field [5]

Local+ Helicity [6,7]

ϕ field [1]

cannot explain large positive deviation from 1/3

[1] Sheng, Luica, QW (2019);
[2] Becattini, Csernai, Wang (2013);
[3] Yang, Fang, QW, Wang (2018);
[4] Liang, Wang (2005);
[5] Muller, Yang (2022);
[6] Xia, Li, Huang, Huang (2021);
[7] Gao (2021);

Polarization of strange quarks by ϕ vector fields (non-relativistic model)

- Like electric charges in motion can generate an EM field, s and $\bar{s}$ quarks in motion can generate an effective ϕ vector field.
- The ϕ vector field can polarize s and $\bar{s}$ with a large magnitude due to strong interaction, in analogy to how EM field polarize (anti)quarks.

$$
\begin{aligned}
\vec{\mathcal{P}}_{s/\bar{s}} = & \frac{1}{2}\omega + \frac{1}{2m_s}\varepsilon \times \mathbf{p}_{s/\bar{s}} \\
& \pm \frac{Q_s}{2m_s T}\mathbf{B} \pm \frac{Q_s}{2m_s^2 T}\mathbf{E} \times \mathbf{p}_{s/\bar{s}} \\
& \pm \frac{g_\phi}{2m_s T}\mathbf{B}_\phi \pm \frac{g_\phi}{2m_s^2 T}\mathbf{E}_\phi \times \mathbf{p}_{s/\bar{s}}
\end{aligned}
$$

Electric part corresponds to spin-orbit couplings (spin-Hall effects) not accessible via Λ polarization:

$$
\mathbf{E} \times \mathbf{p} \sim -\frac{1}{r}\frac{d\Phi}{dr}(\mathbf{r} \times \mathbf{p})
$$

Sheng, Oliva, QW (2019)

ρ_{00}^{ϕ} from ϕ fields in non-relativistic coalescence model

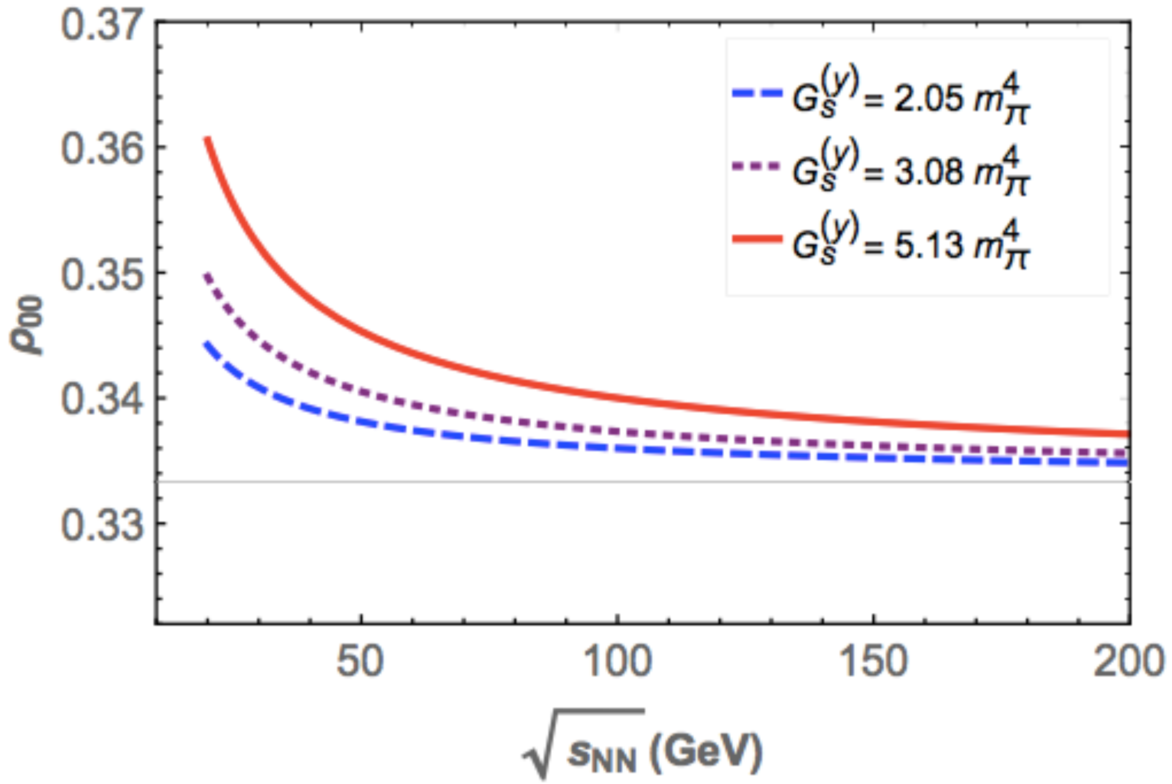
- **The spin density matrix has off-diagonal elements in spin. Assuming the spin quantization direction is y-direction (OAM), we have**

ϕ **meson's non-relativistic wave function**

$$
\begin{aligned}
\rho_{00}^{\phi}(t,\mathbf{x}) &\approx \frac{1}{3}-\frac{4}{9}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\left|\psi_{\phi}(\mathbf{p})\right|^2 \\
&\quad\times\left\{P_s^y(\mathbf{p})P_{\bar{s}}^y(-\mathbf{p})-\frac{1}{2}\left[P_s^x(\mathbf{p})P_{\bar{s}}^x(-\mathbf{p})+P_s^z(\mathbf{p})P_{\bar{s}}^z(-\mathbf{p})\right]\right\} \\
&\approx \left.\begin{aligned}
&\frac{1}{3}+\frac{g_\phi^2}{9m_s^2T_{\mathrm{eff}}^2}\left[\left\langle B_{\phi,y}^2\right\rangle-\frac{1}{2}\left\langle B_{\phi,x}^2+B_{\phi,z}^2\right\rangle\right] \\
&+\frac{g_\phi^2\left\langle\mathbf{p}^2\right\rangle_\phi}{27m_s^4T_{\mathrm{eff}}^2}\left[\left\langle E_{\phi,y}^2\right\rangle-\frac{1}{2}\left\langle E_{\phi,x}^2+E_{\phi,z}^2\right\rangle\right]
\end{aligned}\right\}\equiv\frac{1}{27m_s^2T_{\mathrm{eff}}^2}C_s^{(y)}
\end{aligned}
$$

- **Coalescence model in HIC: cf. Lie-Wen Chen's talk**

Prediction for ρ_{00} from ϕ field (non-relativistic coalescence model)



Sheng, Oliva, QW (2019)

Shortcomings of non-relativistic coalescence model for ρ_{00}^{ϕ}

- Spins are decoupled from momenta in spin density matrix: too simple to account for spin dynamics. The sign of anti-quark's momentum is not easy to determine (easy to make a mistake)
- Without Lorentz covariance, only valid for quasi-static ϕ mesons, cannot be applied to ϕ mesons with non-vanishing momenta with confidence
- It is not a model based on relativistic quantum field theory
- The deeper implication of ϕ field has not been explored

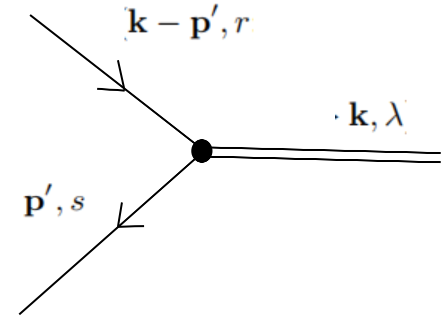
$\Rightarrow$

- To solve above problems, it is necessary to develop a relativistic spin transport theory for ϕ mesons, which can describe the relativistic fusion process $s\bar{s} \rightarrow \phi$ with spin dof

Relativistic spin Boltzmann equation for fusion process

- A phenomenological Relativistic Spin Boltzmann Equation (RSBE) for fusion process

$$
\begin{aligned}
k\cdot\partial_x f_\lambda^V(x,\mathbf{k}) \sim & \sum_{r,s=\pm 1/2}\int \frac{d^3\mathbf{p}'}{E_{\mathbf{p}'}^{\bar{q}} E_{\mathbf{k}-\mathbf{p}'}^{q}}\delta\left(E_{\mathbf{k}}^V - E_{\mathbf{k}-\mathbf{p}'}^{q} - E_{\mathbf{p}'}^{\bar{q}}\right) \\
& \times \left|M(\mathbf{k}-\mathbf{p}',r;\mathbf{p}',s\to\mathbf{k},\lambda)\right| \\
& \times \Big\{ \underbrace{f_r^q(\mathbf{k}-\mathbf{p}') f_s^{\bar{q}}(\mathbf{p}')\left[1+f_\lambda^V(\mathbf{k})\right]}_{\text{Gain term}} - \underbrace{f_\lambda^V(\mathbf{k})\left[1-f_r^q(\mathbf{k}-\mathbf{p}')\right]\left[1-f_s^{\bar{q}}(\mathbf{p}')\right]}_{\text{Loss term}} \Big\}
\end{aligned}
$$



- It is more rigorous to derive RSBE from CTP or KBE in terms of Matrix Valued Spin Dependent Distributions (MVSD) for quarks and vector mesons

$$
\begin{aligned}
f_r^q &\to f_{r_1 r_2}^q \\
f_s^{\bar{q}} &\to f_{s_1 s_2}^{\bar{q}}
\end{aligned}
\qquad
f_\lambda^V \to f_{\lambda_1\lambda_2}^V
$$

MVSD:
Sheng, Weickgenannt, Speranza, Rischke, QW (2021); Weickgenannt, Speranza, Sheng, QW, Rischke (2021)

RSBE from CTP or KBE

- **A general RSBE based on relativistic quantum field theory (CTP or KBE) for fusion process**

$$k\cdot\partial_x f^V_{\lambda_1\lambda_2}(x,\mathbf{k}) = \frac{1}{16}\sum_{\lambda_1',\lambda_2'}\left[\epsilon^*_\mu(\lambda_1,\mathbf{k})\epsilon_\nu(\lambda_1',\mathbf{k})\delta_{\lambda_2\lambda_2'} + \delta_{\lambda_1\lambda_1'}\epsilon^*_\mu(\lambda_2',\mathbf{k})\epsilon_\nu(\lambda_2,\mathbf{k})\right]\underbrace{\mathcal{C}^{\mu\nu}_{\lambda_1'\lambda_2'}(x,\mathbf{k})}_{\text{collision kernel}},$$

collision kernel

$$\begin{aligned}
&\mathcal{C}^{\mu\nu}_{\lambda_1'\lambda_2'}(x,\mathbf{k})\\
&= \sum_{r_1,s_1,r_2,s_2}\int\frac{d^3\mathbf{p}'}{(2\pi\hbar)^2}\frac{1}{E^{\bar{q}}_{\mathbf{p}'}E^{q}_{\mathbf{k}-\mathbf{p}'}}\delta\left(E^V_{\mathbf{k}} - E^{\bar{q}}_{\mathbf{p}'} - E^{q}_{\mathbf{p}-\mathbf{p}'}\right)\\
&\times\left\{f^{\bar{q}}_{r_1s_1}(\mathbf{p}')f^{q}_{r_2s_2}(\mathbf{k}-\mathbf{p}')\left[\delta_{\lambda_1'\lambda_2'} + f^V_{\lambda_1'\lambda_2'}(\mathbf{k})\right]\right.\\
&\left.-\left[\delta_{r_1s_1} - f^{\bar{q}}_{r_1s_1}(\mathbf{p}')\right]\left[\delta_{r_2s_2} - f^{q}_{r_2s_2}(\mathbf{k}-\mathbf{p}')\right]f^V_{\lambda_1'\lambda_2'}(\mathbf{k})\right\}\\
&\times \mathrm{Tr}\left[\Gamma^\nu v_{s_1}(\mathbf{p}')\bar{v}_{r_1}(\mathbf{p}')\Gamma^\mu u_{r_2}(\mathbf{k}-\mathbf{p}')\bar{u}_{s_2}(\mathbf{k}-\mathbf{p}')\right], \qquad (2)
\end{aligned}$$

Sheng, Lucia, Liang, QW, Wang, 2205.15689, 2206.05868

$$\boxed{\Gamma^\alpha \approx g_V B(\mathbf{p}-\mathbf{p}',\mathbf{p}')\gamma^\alpha}$$

Bethe-Salpeter wave function for vector mesons [Roberts et al (2019, 2021)]

Fusion and dissociation process

- **In the dilute gas limit**

Sheng, Lucia, Liang, QW, Wang, 2205.15689, 2206.05868

$$f^V_{\lambda_1\lambda_2} \sim f^q_{rs} \sim f^{\bar{q}}_{rs} \ll 1.$$

- **RSBE for fusion (coalescence) and dissociation process $q\bar{q} \leftrightarrow V$ can be simplied as**

$$k\cdot\partial_x f^V_{\lambda_1\lambda_2}(x,\mathbf{k}) = \frac{1}{8}\left[\underline{\epsilon^*_\mu(\lambda_1,\mathbf{k})\epsilon_\nu(\lambda_2,\mathbf{k})}\mathcal{C}^{\mu\nu}_{\mathrm{coal}}(x,\mathbf{k}) - \mathcal{C}_{\mathrm{diss}}(\mathbf{k})f^V_{\lambda_1\lambda_2}(x,\mathbf{k})\right],$$

$$\epsilon^\mu(\lambda,\mathbf{k}) = \left(\frac{\mathbf{k}\cdot\boldsymbol{\epsilon}_\lambda}{m_V}, \boldsymbol{\epsilon}_\lambda + \frac{\mathbf{k}\cdot\boldsymbol{\epsilon}_\lambda}{m_V(E^V_{\mathbf{k}}+m_V)}\mathbf{k}\right) \Longrightarrow k_\mu\epsilon^\mu(\lambda,\mathbf{k}) = 0$$

$$\begin{aligned}\boldsymbol{\epsilon}_0 &= \mathbf{n}_y\\ \boldsymbol{\epsilon}_{+1} &= -\frac{1}{\sqrt{2}}(\mathbf{n}_z + i\mathbf{n}_x)\\ \boldsymbol{\epsilon}_{-1} &= \frac{1}{\sqrt{2}}(\mathbf{n}_z - i\mathbf{n}_x)\end{aligned}$$

- **The fusion part depends on MVSDs of q and $\bar{q}$, while the dissociation part is independent of MVSDs.**

MVSD or spin density matrix element for vector mesons

- **Forml solution to MVSD (spin density matrix elements) for vector mesons**

$$f^V_{\lambda_1\lambda_2}(x,\mathbf{k}) \sim \frac{1}{\mathcal{C}_{\mathrm{diss}}(\mathbf{k})}\left[1-e^{-\mathcal{C}_{\mathrm{diss}}(\mathbf{k})\Delta t}\right] \times \epsilon^*_\mu(\lambda_1,\mathbf{k})\epsilon_\nu(\lambda_2,\mathbf{k})\mathcal{C}^{\mu\nu}_{\mathrm{coal}}(x,\mathbf{k})$$

Sheng, Lucia, Liang, QW, Wang, 2205.15689, 2206.05868

- **where the coalescence collision kernel $C^{\mu\nu}_{coal}$ is given by**

$$\begin{aligned}\mathcal{C}^{\mu\nu}_{\mathrm{coal}}(x,\mathbf{k}) = \int \frac{d^3\mathbf{p}'}{(2\pi\hbar)^2}\frac{1}{E^{\bar{q}}_{\mathbf{p}'}E^{q}_{\mathbf{k}-\mathbf{p}'}}\delta\left(E^V_{\mathbf{k}}-E^{\bar{q}}_{\mathbf{p}'}-E^{q}_{\mathbf{k}-\mathbf{p}'}\right)\\ \times \mathrm{Tr}\left\{\underline{\Gamma^\nu}\left(p'\cdot\gamma-m_{\bar{q}}\right)\left[1+\gamma_5\gamma\cdot \underline{P^{\bar{q}}(x,\mathbf{p}')}\right]\right.\\ \left.\times\underline{\Gamma^\mu}\left[(k-p')\cdot\gamma+m_q\right]\left[1+\gamma_5\gamma\cdot \underline{P^q(x,\mathbf{k}-\mathbf{p}')}\right]\right\}\\ \times \underline{f_{\bar{q}}(x,\mathbf{p}')f_q(x,\mathbf{k}-\mathbf{p}')},\end{aligned}$$

Polarization vector of quark and antiquark

BS wave function of vector meson

Un-polarized quark distribution functions

Spin density matrix element for vector mesons

- **Spin density matrix elements (normalized MVSD) for vector mesons**

$$f^V_{\lambda_1\lambda_2} \propto \rho^V_{\lambda_1\lambda_2} = \frac{\epsilon^*_\mu(\lambda_1,\mathbf{k})\epsilon_\nu(\lambda_2,\mathbf{k})\mathcal{C}^{\mu\nu}_{\mathrm{coal}}}{\sum_{\lambda=0,\pm1}\epsilon^*_\mu(\lambda,\mathbf{k})\epsilon_\nu(\lambda,\mathbf{k})\mathcal{C}^{\mu\nu}_{\mathrm{coal}}}$$

- **Focus on ϕ meson, the polarization vector for s and $\bar{s}$ appear in the collision kernel**

$$P^\mu_s(x,\mathbf{p}) \approx \frac{1}{4m_s}\epsilon^{\mu\nu\rho\sigma}\left(\omega_{\rho\sigma} + \frac{g_\phi}{(u\cdot p)T_{\mathrm{eff}}}F^\phi_{\rho\sigma}\right)p_\nu$$

$$P^\mu_{\bar{s}}(x,\mathbf{p}) \approx \frac{1}{4m_s}\epsilon^{\mu\nu\rho\sigma}\left(\omega_{\rho\sigma} - \frac{g_\phi}{(u\cdot p)T_{\mathrm{eff}}}F^\phi_{\rho\sigma}\right)p_\nu$$

Field strength tensor for ϕ field

Sheng, Lucia, Liang, QW, Wang, 2205.15689, 2206.05868

Spin density matrix element for vector mesons

- **The collision kernel can be evaluated in the rest frame of ϕ meson. We then obtain ρ_{00}^{ϕ}**

$$
\begin{aligned}
\rho_{00}(x,\mathbf{0}) \approx & \frac{1}{3} + C_1\left[\frac{1}{3}\boldsymbol{\omega}'\cdot\boldsymbol{\omega}' - (\boldsymbol{\epsilon}_0\cdot\boldsymbol{\omega}')^2\right] \\
& + C_2\left[\frac{1}{3}\boldsymbol{\varepsilon}'\cdot\boldsymbol{\varepsilon}' - (\boldsymbol{\epsilon}_0\cdot\boldsymbol{\varepsilon}')^2\right] \\
& - \frac{4g_\phi^2}{m_\phi^2 T_{\mathrm{eff}}^2}C_1\left[\frac{1}{3}\mathbf{B}'_\phi\cdot\mathbf{B}'_\phi - (\underline{\boldsymbol{\epsilon}_0}\cdot\mathbf{B}'_\phi)^2\right] \\
& - \frac{4g_\phi^2}{m_\phi^2 T_{\mathrm{eff}}^2}C_2\left[\frac{1}{3}\mathbf{E}'_\phi\cdot\mathbf{E}'_\phi - (\underline{\boldsymbol{\epsilon}_0}\cdot\mathbf{E}'_\phi)^2\right],
\end{aligned}
$$

$$
C_1 = \frac{8m_s^4 + 16m_s^2 m_\phi^2 + 3m_\phi^4}{120m_s^2(m_\phi^2 + 2m_s^2)},
$$

$$
C_2 = \frac{8m_s^4 - 14m_s^2 m_\phi^2 + 3m_\phi^4}{120m_s^2(m_\phi^2 + 2m_s^2)}.
$$

All fields with prime are defined in the rest frame of ϕ meson

Spin quantization direction

- **Perfect cancellation occurs: all ϕ fields appear as local correlation between strong force fields of the same kinds and same components. Suprising results!**

Lorentz transformation for ϕ fields

- We can express ρ_{00}^{ϕ} in terms of ϕ fields in the lab frame and obtain the dependence on momenta of ϕ mesons

$$\mathbf{B}'_{\phi} = \gamma \mathbf{B}_{\phi} - \gamma \mathbf{v} \times \mathbf{E}_{\phi} + (1-\gamma)\frac{\mathbf{v}\cdot\mathbf{B}_{\phi}}{v^2}\mathbf{v},$$

$$\mathbf{E}'_{\phi} = \gamma \mathbf{E}_{\phi} + \gamma \mathbf{v} \times \mathbf{B}_{\phi} + (1-\gamma)\frac{\mathbf{v}\cdot\mathbf{E}_{\phi}}{v^2}\mathbf{v},$$

- where $\gamma = E_{\mathbf{p}}^{\phi}/m_{\phi}$ and $\mathbf{v} = \mathbf{p}/E_{\mathbf{p}}^{\phi}$

- In terms of lab-frame fields we obtain

$$\overline{\rho}_{00}^{\phi}(x,\mathbf{p}) \approx \frac{1}{3} + \frac{1}{3}\sum_{i=1,2,3} I_{B,i}(\mathbf{p})\left[\boldsymbol{\omega}_i^2 - \frac{4g_{\phi}^2}{m_{\phi}^2 T_{\text{eff}}^2}(\mathbf{B}_i^{\phi})^2\right]$$
$$+\frac{1}{3}\sum_{i=1,2,3} I_{E,i}(\mathbf{p})\left[\boldsymbol{\varepsilon}_i^2 - \frac{4g_{\phi}^2}{m_{\phi}^2 T_{\text{eff}}^2}(\mathbf{E}_i^{\phi})^2\right],$$

Parameters and comparison with data

- **Two parameters (transverse and longitudinal field squares)**

$$\left\langle (g_\phi \mathbf{B}^\phi_{x(y)})^2 \right\rangle = \left\langle (g_\phi \mathbf{E}^\phi_{x(y)})^2 \right\rangle = F^2$$

$$\left\langle (g_\phi \mathbf{B}^\phi_z)^2 \right\rangle = \left\langle (g_\phi \mathbf{E}^\phi_z)^2 \right\rangle = r_z F^2 < F^2$$

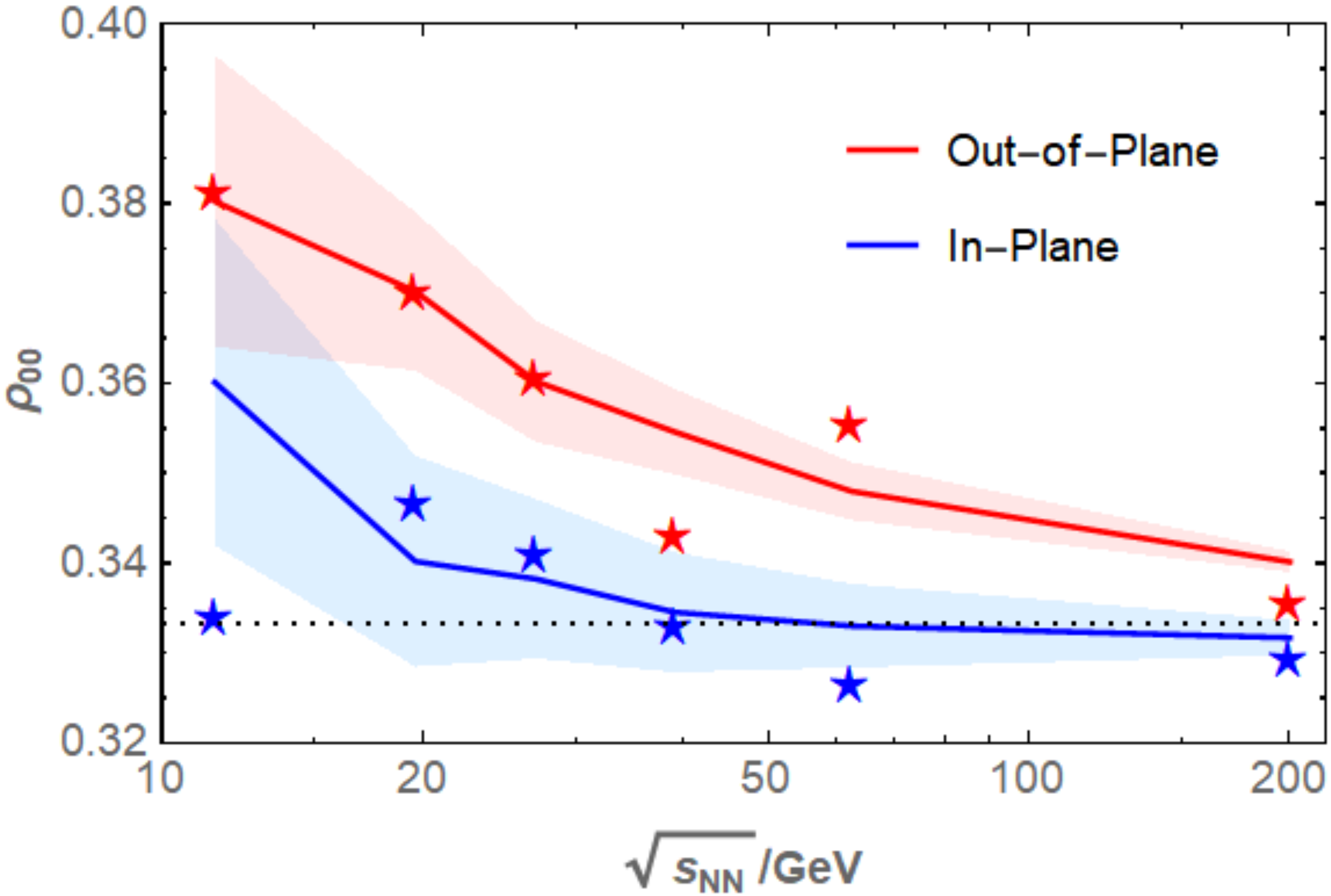
- **Two sets of parameter values give the same result**

$$F^2 = 0.45 m_\pi^4,\ m_s = 170\,\mathrm{MeV}$$
$$F^2 = 5.02 m_\pi^4,\ m_s = 530\,\mathrm{MeV}$$

$$r_z = 0.79$$

- **The magnitude of electric field's contribution decreases with increasing m_s**

Collision energy dependence



Transverse momentum dependence

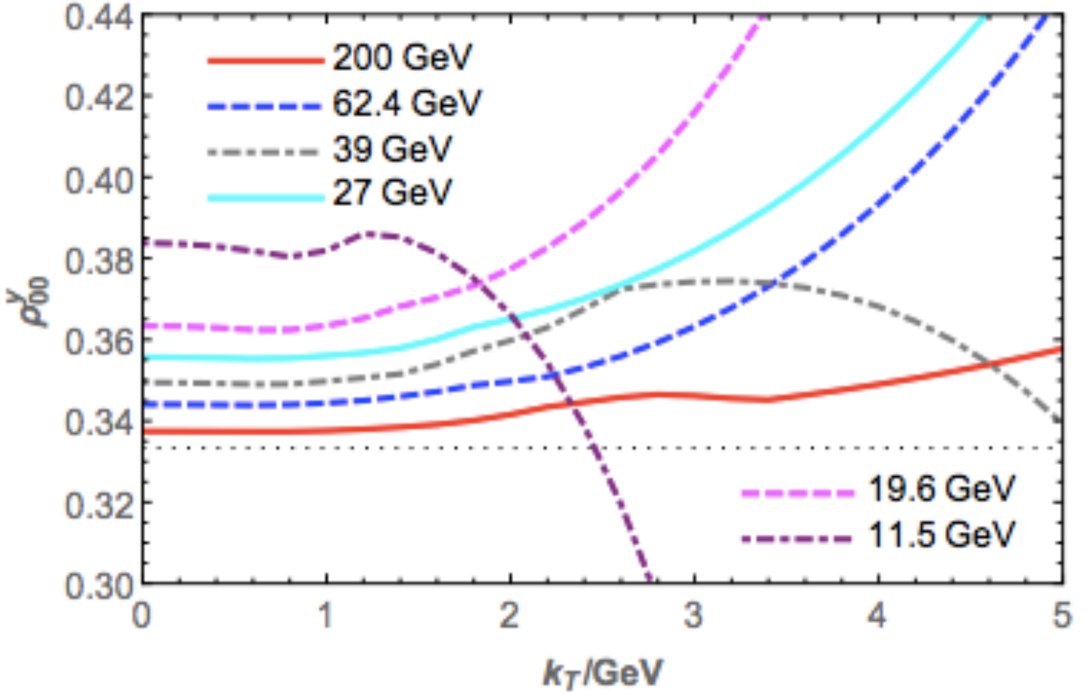


Figure 2. The ϕ meson's ρ_{00}^{y} as functions of transverse momenta at different collision energies.

Centrality dependence

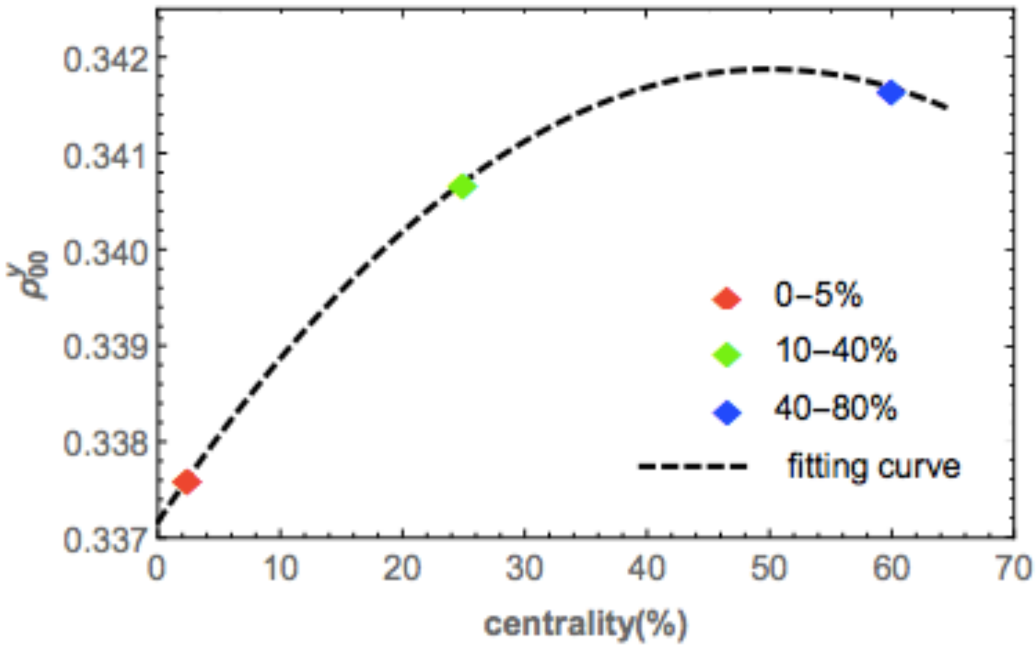


Figure 4. The ϕ meson's ρ_{00}^{y} as a function of centrality at 200 GeV. Red, green, and blue diamond points are our results for centrality ranges 0-5%, 10-40%, and 40-80%, respectively. The dashed line is the fitting curve using a second order polynomial.

Take-home messages and Questions for discussions

- **Take-home messages: P_Λ measures the fields (mean field), ρ_{00}^{ϕ} measures field squares (field fluctuation).**

- **Questions for dicussions**
- **What are particles? What are fields? Particle-field duality?**
- **What is the nature of vector meson fields? Are they real entities? Can we calculate field squares on Lattice?**
- **Any connection with QCD sum rules and QCD vacuum properties? Any connection with quark or gluon condensates (trace anomaly)?**
- **Any implication for J/Psi polarization (gluon fields)?**

- **Welcome to HENPIC seminar on August 4 (Thursday) and join the discussion !**