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Collins asymmetry in EIC and EicC

——phenomenology prediction within TMD factorization

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EIC YR: semi-inclusive working group meeting

Remote meeting

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Collaborated with Shi-Chen Xue, De-Min Li(Zhengzhou U.) and Zhun Lu(Southeast U., China)

Based on S. C. Xue, X. Wang, D. M. Li and Z. Lu, arXiv: 2003.05679, accepted by EPJC

OUTLINE

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Why Collins asymmetry?

Differential Cross Section in SIDIS Process

Bacchetta et al., JHEP0702, 093 (2007)

$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \\
&\quad \frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} \\
&\qquad +\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} \\
&\qquad +S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&\qquad +S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&\qquad +|\boldsymbol{S}_\perp|\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
&\qquad\quad +\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} \\
&\qquad\quad +\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] \\
&\qquad +|\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} \\
&\qquad\quad +\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\},
\end{aligned}
$$

- Assuming single photon exchange
- 18 structure functions $F_{XY,Z}$
- in terms of TMD PDFs and FFs

Why Collins asymmetry?

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TMD PDFs at leading-twist

Unpolarized

Longitudinally polarized

Transversely polarized

Nucleon \ Quark, Gluon	U	L	T
U	number density $f_1^{q,g}(x,k_T^2)$		Boer-Mulders $h_1^{\perp q,g}(x,k_T^2)$
L		Helicity $g_{1L}^{q,g}(x,k_T^2)$	worm-gear L $h_{1L}^{\perp q,g}(x,k_T^2)$
T	Sivers $f_{1T}^{\perp q,g}(x,k_T^2)$	Kotzinian-Mulders worm-gear T $g_{1T}^{\perp q,g}(x,k_T^2)$	Transversity $h_1^{q,g}(x,k_T^2)$ Pretzelosity $h_{1T}^{\perp q,g}(x,k_T^2)$

TMD PDFs give the number density of partons, with their intrinsic motion and spin, inside a fast moving proton, with its spin.

Unpolarized distribution function, helicity, transversity: Survive at collinear limit

Boer-Mulders,Sivers: Time reversal odd

Why Collins asymmetry?

Transversity distribution function

- Fundamental distribution to encode the nucleon structure
- Chiral-odd
- Hard to access compared to helicity and unpolarized distribution function

How to access transversity

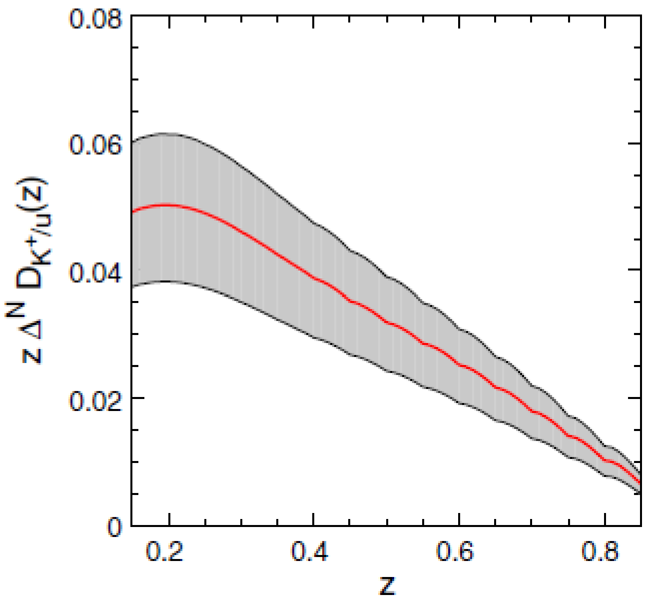
- (TMD) factorization frame in SIDIS ——Collins function
  J. C. Collins, NPB 396, 161 (1993)
- Collinear factorization in SIDIS——twist-3 fragmentation function
  X.Wang, Z. Lu PRD 93, 074009 (2016)
- Collinear factorization in SIDIS——dihadron fragmentation function
  A. Bacchetta et al., PRL107, 012001(2011)
- Drell-Yan process——the antiquark transversity
  V. Barone et al., Phys.Rept. 359 (2002) 1-168

Although progress has been made, sea quark transversity is almost unknown.

Why Collins asymmetry?

Collins function

- Correlation between fragmenting quark transverse spin and unpolarized hadron transverse momentum
- Counterpart of Boer-Mulders function
- Probe of quark spin in nucleon
- Kaon Collins function has been extracted from the e^+e^- annihilation data from BaBar



M. Anselmino et al., PRD 93, 034025 (2016)

The Collins asymmetry in Kaon production SIDIS process makes an ideal tool to access sea quark transversity due to its strange constituent.

What have we done?

Collins asymmetry

- apply the TMD factorization formalism
- adopt the parametrization of the non-perturbative Sudakov form factor
- NLL accuracy

Approach-TMD evolution formalism

Approach-TMD evolution formalism

TMD factorization and TMD evolutions

- TMD factorization frame:

  valid in the region $q_\perp \ll Q$

  observables : convolutions of hard factor and well-defined TMD PDFs/FFs

- TMD evolution :

  convenient to perform in b-space (conjugate to $k_\perp$ via Fourier Transformation)

  Collins-Soper equation + renormalization group equation

  Sudakov form factor plays an important role

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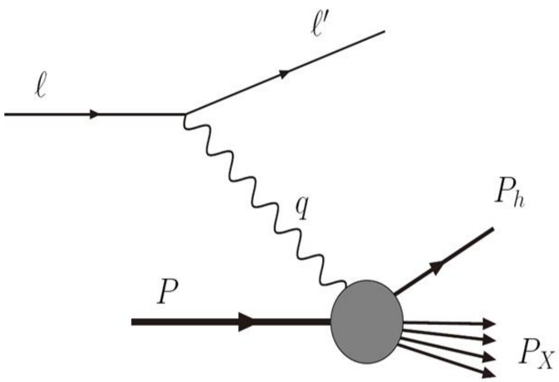
Reference frame and invariants

- SIDIS with unpolarized electron beam and transversely polarized target

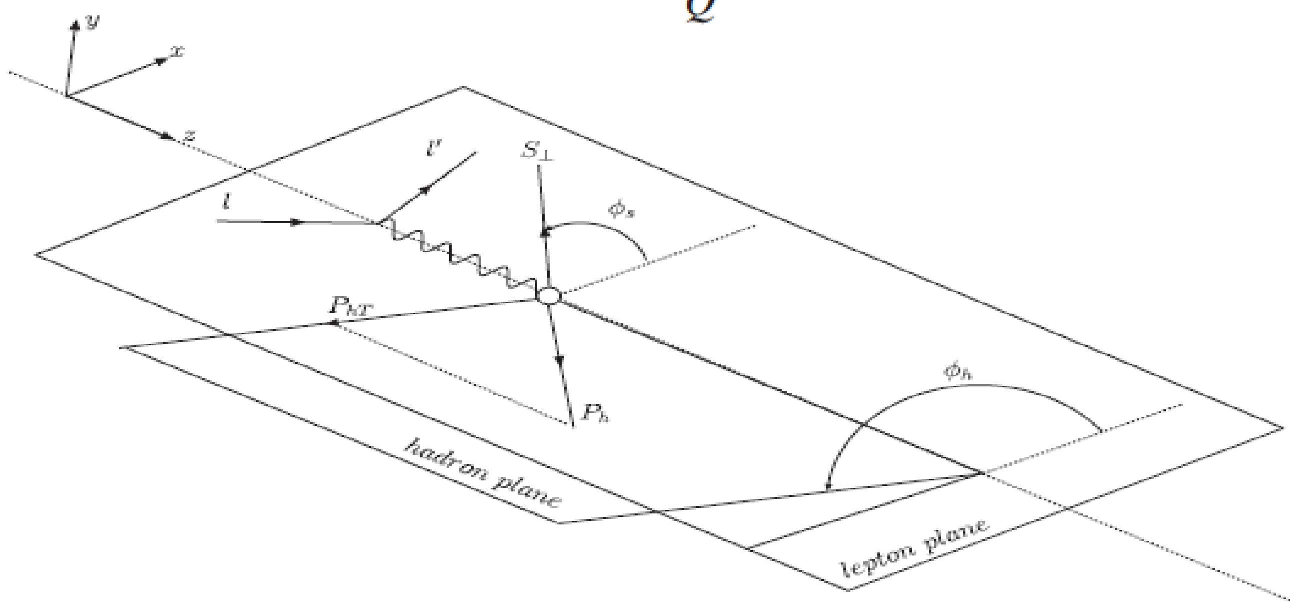
$$e(\ell) + p^{\uparrow}(P) \longrightarrow e(\ell') + K(P_h) + X(P_X),$$



- The invariants defined as

$$x = \frac{Q^2}{2P\cdot q}, \qquad y = \frac{P\cdot q}{P\cdot l}, \qquad z = \frac{P\cdot P_h}{P\cdot q},$$

$$\gamma = \frac{2Mx}{Q}, \qquad Q^2 = -q^2, \qquad s = (P+l)^2.$$



Definition of Collins asymmetry

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- The 5-fold differential cross section with a transversely polarized target has the following general form

$$\frac{d^5\sigma(S_T)}{dx_B dy dz_h d^2\boldsymbol{P}_{hT}} = \sigma_0(x_B, y, Q^2)\left[F_{UU} + \sin(\phi_h+\phi_s)\frac{2(1-y)}{1+(1-y)^2}F_{UT}^{\sin(\phi_h+\phi_s)} + \dots\right],$$

- The Collins asymmetry can be written in terms of the structure functions

$$A_{UT}^{\sin(\phi_h+\phi_s)} = \frac{\sigma_0(x_B, y, Q^2)}{\sigma_0(x_B, y, Q^2)}\frac{2(1-y)}{1+(1-y)^2}\frac{F_{UT}^{\sin(\phi_h+\phi_s)}}{F_{UU}},$$

$$F_{UU}(Q; P_{hT}) = \mathcal{C}[f_1 D_1],$$

$$F_{UT}^{\sin(\phi_h+\phi_s)}(Q; P_{hT}) = \mathcal{C}\left[\frac{-\hat{h}\cdot k_T}{M_h}h_1 H_1^{\perp}\right].$$

$$\mathcal{C}[\omega f D] = \sum_q e_q^2 \int d^2p_T d^2k_T \delta^{(2)}(p_T - k_T - P_{hT}/z_h)\omega(p_T, k_T) f^q(x_B, p_T^2)\, D^q(z_h, k_T^2).$$

The structure functions

- Performing the Fourier Transformation and introducing the TMDs definition

$$
\begin{aligned}
F_{UU}(Q;P_{hT}) &= \mathcal{C}[f_1 D_1] \\
&= \sum_q e_q^2 \int d^2 p_T d^2 k_T \delta^{(2)}\left(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{hT}/z_h\right) f_1^q(x_B, p_T^2)\, D_1^q(z_h, k_T^2) \\
&= \frac{1}{z_h^2} \sum_q e_q^2 \int d^2 p_T d^2 K_T \delta^{(2)}\left(\boldsymbol{p}_T + \boldsymbol{K}_T/z_h - \boldsymbol{P}_{hT}/z_h\right) f_1^q(x_B, p_T^2)\, D_1^q(z_h, K_T^2) \\
&= \frac{1}{z_h^2} \sum_q e_q^2 \int d^2 p_T d^2 K_T \int \frac{d^2 b}{(2\pi)^2} e^{-i\left(\boldsymbol{p}_T + \boldsymbol{K}_T/z_h - \boldsymbol{P}_{hT}/z_h\right)\cdot \boldsymbol{b}} f_1^q(x_B, p_T^2)\, D_1^q(z_h, K_T^2) \\
&= \frac{1}{z_h^2} \sum_q e_q^2 \int \frac{d^2 b}{(2\pi)^2} e^{i\boldsymbol{P}_{hT}/z_h \cdot \boldsymbol{b}} \tilde{f}_1^q(x_B, b)\, \tilde{D}_1^q(z_h, b).
\end{aligned}
$$

$$
\int d^2 p_T e^{-i\boldsymbol{p}_T\cdot \boldsymbol{b}} f_1^q(x_B, p_T^2) = \tilde{f}_1^q(x_B, b)
$$

$$
\int d^2 K_T e^{-i\boldsymbol{K}_T/z_h \cdot \boldsymbol{b}} D_1^q(z_h, K_T^2) = \tilde{D}_1^q(z_h, b)
$$

The structure functions

➢ The Collins structure function

$$
\begin{aligned}
F_{UT}^{\sin(\phi_h+\phi_s)}(Q;P_{hT}) &= \mathcal{C}[\frac{-\hat{h}\cdot k_T}{M_h}h_1 H_1^{\perp}] \\
&= \sum_q e_q^2 \int d^2p_T d^2k_T \delta^{(2)}(p_T - k_T - P_{hT}/z_h)\frac{-\hat{h}\cdot k_T}{M_h} h_1^q(x_B,p_T^2) H_1^{\perp,q}(z_h,k_T^2) \\
&= \sum_q e_q^2 \frac{1}{z_h^2}\int d^2p_T d^2K_T \delta^{(2)}(p_T + K_T/z_h - P_{hT}/z_h)\frac{\hat{h}\cdot K_T}{z_h M_h} h_1^q(x_B,p_T^2) H_1^{\perp,q}(z_h,K_T^2) \\
&= \sum_q e_q^2 \frac{1}{z_h^2}\int d^2p_T d^2K_T \int \frac{d^2b}{(2\pi)^2} e^{-i(p_T + K_T/z_h - P_{hT}/z_h)\cdot b}\frac{\hat{h}\cdot K_T}{z_h M_h} h_1^q(x_B,p_T^2) H_1^{\perp,q}(z_h,K_T^2) \\
&= \sum_q e_q^2 \frac{1}{z_h^2}\frac{1}{z_h}\int \frac{d^2b}{(2\pi)^2} e^{iP_{hT}/z_h\cdot b}\hat{h}_\alpha \tilde{h}_1^q(x_B,b)\tilde{H}_1^{\perp\alpha,q}(z_h,b).
\end{aligned}
$$

$$
\int d^2p_T e^{-ip_T\cdot b} h_1^q(x_B,p_T^2) = \tilde{h}_1^q(x_B,b);
$$

$$
\int d^2K_T e^{-iK_T/z_h\cdot b}\frac{K_T^\alpha}{M_h} H_1^{\perp,q}(z_h,K_T^2) = \tilde{H}_1^{\perp\alpha,q}(z_h,b).
$$

TMD evolution formalism

- The TMD evolution equation for the ζ_F dependence is encoded in a Collins-Soper (CS) equation through

$$\frac{\partial\ \ln\tilde{F}(x_B,b;\mu,\zeta_F)}{\partial\ \ln\sqrt{\zeta_F}}=\frac{\partial\ \ln\tilde{D}(z_h,b;\mu,\zeta_D)}{\partial\ \ln\sqrt{\zeta_D}}=\tilde{K}(b;\mu),$$

- μ dependence is encoded in a RG equation through

$$\frac{d\ \tilde{K}}{d\ \ln\mu}=-\gamma_K(\alpha_s(\mu)),$$
$$\frac{d\ \ln\tilde{F}(x_B,b;\mu,\zeta_F)}{d\ \ln\mu}=\gamma_F(\alpha_s(\mu);\frac{\zeta_F^2}{\mu^2}),$$
$$\frac{d\ \ln\tilde{D}(z_h,b;\mu,\zeta_D)}{d\ \ln\mu}=\gamma_D(\alpha_s(\mu);\frac{\zeta_D^2}{\mu^2}),$$

- The solution of the energy dependence for TMDs has the general form as

$$\tilde{F}(x_B,b;Q)=\mathcal{F}\times e^{-S}\times\tilde{F}(x_B,b;\mu),$$

$$\tilde{D}(z_h,b;Q)=\mathcal{D}\times e^{-S}\times\tilde{D}(z_h,b;\mu),$$

TMD evolution formalism

- When Fourier transform back to the momentum space, one needs perform in the whole b region, which requires the region analysis
- To combine the information at small b (perturbative region) with that at large b(non-perturbative region), a matching procedure must be introduced, with $b_{\max}$ serving the boundary between the two regions and define

$$b_* = b/\sqrt{1+b^2/b_{\max}^2} \qquad \begin{array}{l} b_* \approx b \text{ at low values of } b \\ b_* \approx b_{\max} \text{ at large } b \text{ values.} \end{array}$$

- At small b region, PDFs/FFs can be written as convolutions of the perturbatively calculable hard coefficients and the corresponding collinear counterparts at fixed energy μ

$$\tilde{F}_{q/H}(x,b;\mu) = \sum_i C_{q\leftarrow i} \otimes F_{i/H}(x,\mu),$$

- The Sudakov-like form factor can be separated into a perturbatively calculable part and a nonperturbative part

$$S(Q;b) = S_{\text{pert}}(Q;b_*) + S_{\text{NP}}(Q;b).$$

TMD evolution formalism

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- The perturbative part of S being

$$S_{\rm pert}(Q,b)=\int_{\mu_b^2}^{Q^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[A(\alpha_s(\bar{\mu}))\ln\frac{Q^2}{\bar{\mu}^2}+B(\alpha_s(\bar{\mu}))\right].$$

- Assume the universal Gaussian form for the S_{NP} of the PDF and FF

$$S_{\rm NP}^{\rm pdf}(Q;b)=\frac{g_2}{2}\ln(\frac{Q}{Q_0})b^2+g_1^{\rm pdf}b^2,$$
$$S_{\rm NP}^{\rm ff}(Q;b)=\frac{g_2}{2}\ln(\frac{Q}{Q_0})b^2+g_1^{\rm ff}b^2.$$

$$g_1^{\rm pdf}=\frac{\langle k_\perp^2\rangle_{Q_0}}{4},\qquad g_1^{\rm ff}=\frac{\langle p_\perp^2\rangle_{Q_0}}{4z_h^2},$$

- The general form of the energy dependent TMDs

$$\tilde{F}_{q/H}(x_B,b;Q)=e^{-\frac{1}{2}S_{\rm Pert}(Q;b_*)-S_{\rm NP}^{\rm pdf}(Q;b)}\mathcal{F}(\alpha_s(Q))\sum_i C_{q\leftarrow i}\otimes F^{i/H}(x_B,\mu_b),$$
$$\tilde{D}_{H/q}(z_h,b;Q)=e^{-\frac{1}{2}S_{\rm Pert}(Q;b_*)-S_{\rm NP}^{\rm ff}(Q;b)}\mathcal{D}(\alpha_s(Q))\sum_j \hat{C}_{j\leftarrow q}\otimes D^{H/j}(z_h,\mu_b).$$

Structure functions

- Unpolarized structure function

$$F_{UU}(Q;P_{hT}) = \frac{1}{z_h^2}\int_0^\infty \frac{db\ b}{(2\pi)} J_0(P_{hT}/z_h\ b)\widetilde{F}_{UU}(Q;b)$$

$$= \frac{1}{z_h^2}\sum_q e_q^2 \int_0^\infty \frac{db\ b}{(2\pi)} J_0(P_{hT}/z_h\ b) e^{-S_{\rm pert}(Q;b_*)-S_{\rm NP}^{\rm SIDIS}(Q;b)}$$

$$\left(\sum_i C_{q\leftarrow i}^{(\rm SIDIS)} \otimes f_1^{i/p}(x_B,\mu_b)\right) \times \left(\sum_j \hat{C}_{j\leftarrow q}^{(\rm SIDIS)} \otimes D_1^{K/j}(z_h,\mu_b)\right) .$$

- Collins structure function

$$F_{UT}(Q;P_{hT}) = \frac{-1}{2z_h^3}\sum_q e_q^2 \int_0^\infty \frac{db\ b^2}{(2\pi)} J_1(P_{hT}/z_h\ b) e^{-S_{\rm pert}(Q;b_*)-S_{\rm NP\ Collins}^{\rm SIDIS}(Q;b)}$$

$$\left(\sum_i \delta C_{q\leftarrow i}^{(\rm SIDIS)} \otimes h_1^{i/p}(x_B,\mu_b)\right)\left(\sum_j \delta\hat{C}_{j\leftarrow q}^{(\rm SIDIS)} \otimes \hat{H}_{K/j}^{(3)}(z_h,\mu_b)\right) .$$

$$\hat{H}_{h/j}^{(3)}(z_h) = \int d^2p_\perp \frac{|p_\perp^2|}{M_h} H_{1\,h/j}^\perp(z_h,p_\perp) .$$

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Parametrizations

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- Parametrization for the collinear transversity distribution function

Z. B. Kang et al., PRD 93, 014009 (2016)

$$h_1^q(x,Q_0) = N_q^h x^{a_q}(1-x)^{b_q}\frac{(a_q+b_q)^{a_q+b_q}}{a_q^{a_q}b_q^{b_q}}\frac{1}{2}(f_1^q(x,Q_0)+g_1^q(x,Q_0)) ,$$ Valence quark

$$h_1^q(x,Q_0) = N_s\frac{1}{2}(f_1^q(x,Q_0)+g_1^q(x,Q_0)) ,$$ Sea quark

$N_s \leq 1$ to satisfy positivity bound

- Parametrization for Collins function

M. Anselmino et al., PRD 93, 034025 (2016)

$$\Delta^N D_{h/q^\uparrow}(z_h,p_\perp) = \tilde{\Delta}^N D_{h/q^\uparrow}(z_h)\, h(p_\perp)\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle} .$$

$$\hat{H}_{h/j}^{(3)}(z_h) = \frac{\sqrt{2e}}{M_C}N_q^C(z_h)\,D_{h/q}(z_h)\left(\frac{M_C^2}{M_C^2+\langle p_\perp^2\rangle}\right)^2\langle p_\perp^2\rangle .$$

Kinematical region

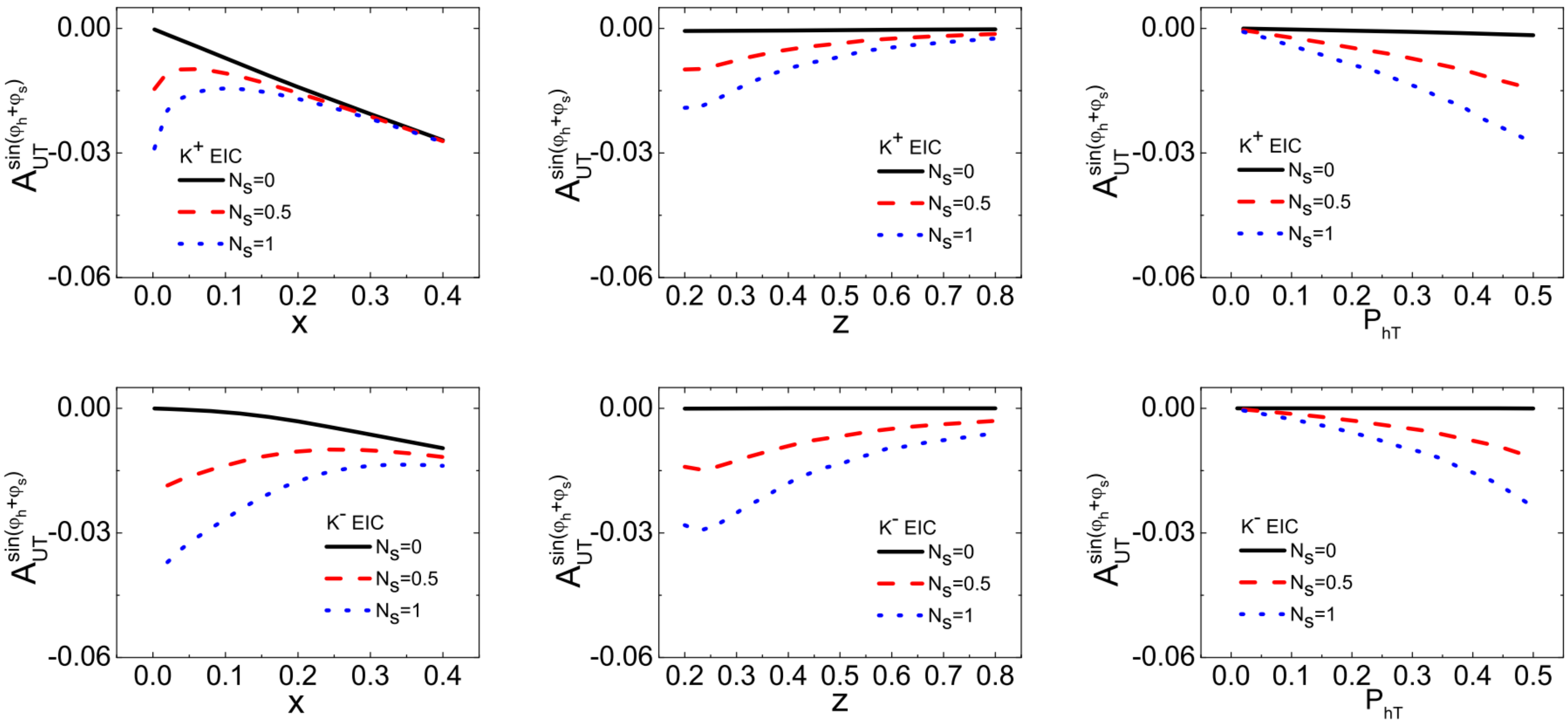
- For EIC

$$0.001 < x < 0.4, \quad 0.07 < y < 0.9, \quad 0.2 < z < 0.8,$$
$$1\ \mathrm{GeV}^2 < Q^2\ , \quad W > 5\ \mathrm{GeV}, \quad \sqrt{s} = 100\ \mathrm{GeV}, \quad P_{hT} < 0.5\ \mathrm{GeV}.$$

- For EicC

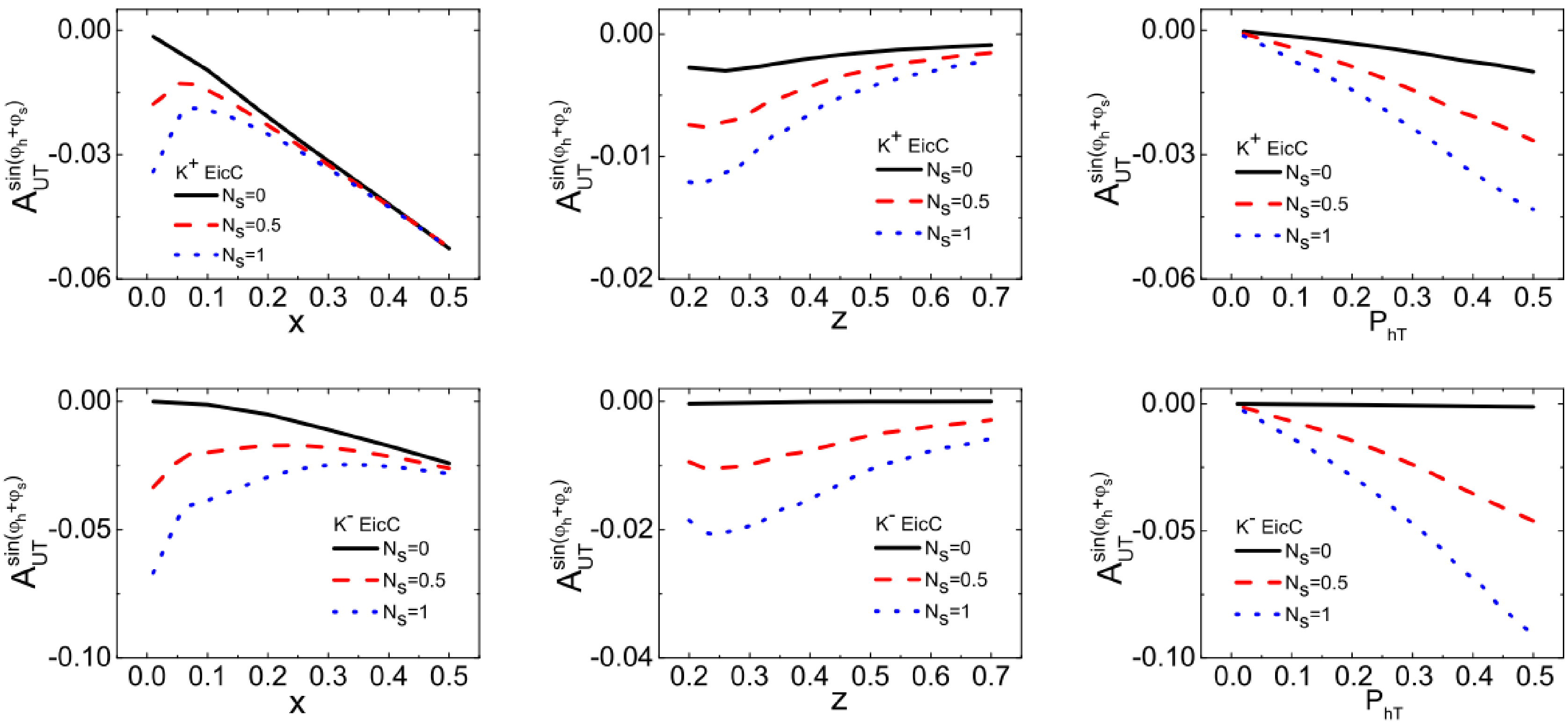
$$0.005 < x < 0.5, \quad 0.07 < y < 0.9, \quad 0.2 < z < 0.7,$$
$$1\mathrm{GeV}^2 < Q^2 < 200\ \mathrm{GeV}^2\ , \quad W > 2\ \mathrm{GeV}, \quad \sqrt{s} = 16.7\ \mathrm{GeV}, \quad P_{hT} < 0.5\ \mathrm{GeV},$$

EIC

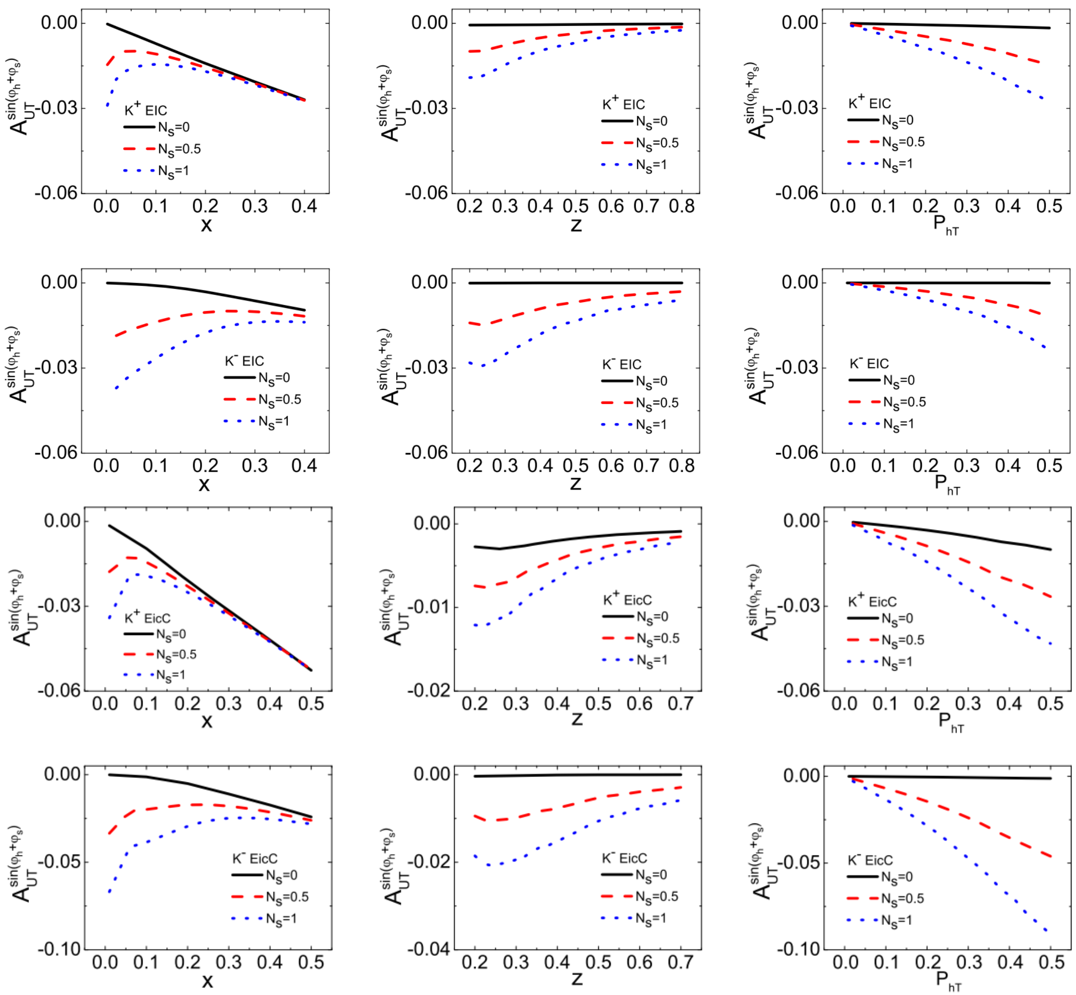


K+ EIC
K- EIC
Ns=0
Ns=0.5
Ns=1
$A_{UT}^{\sin(\varphi_h+\varphi_s)}$
0.00
-0.03
-0.06
x
z
P_{hT}
0.0 0.1 0.2 0.3 0.4
0.2 0.3 0.4 0.5 0.6 0.7 0.8
0.0 0.1 0.2 0.3 0.4 0.5

EicC

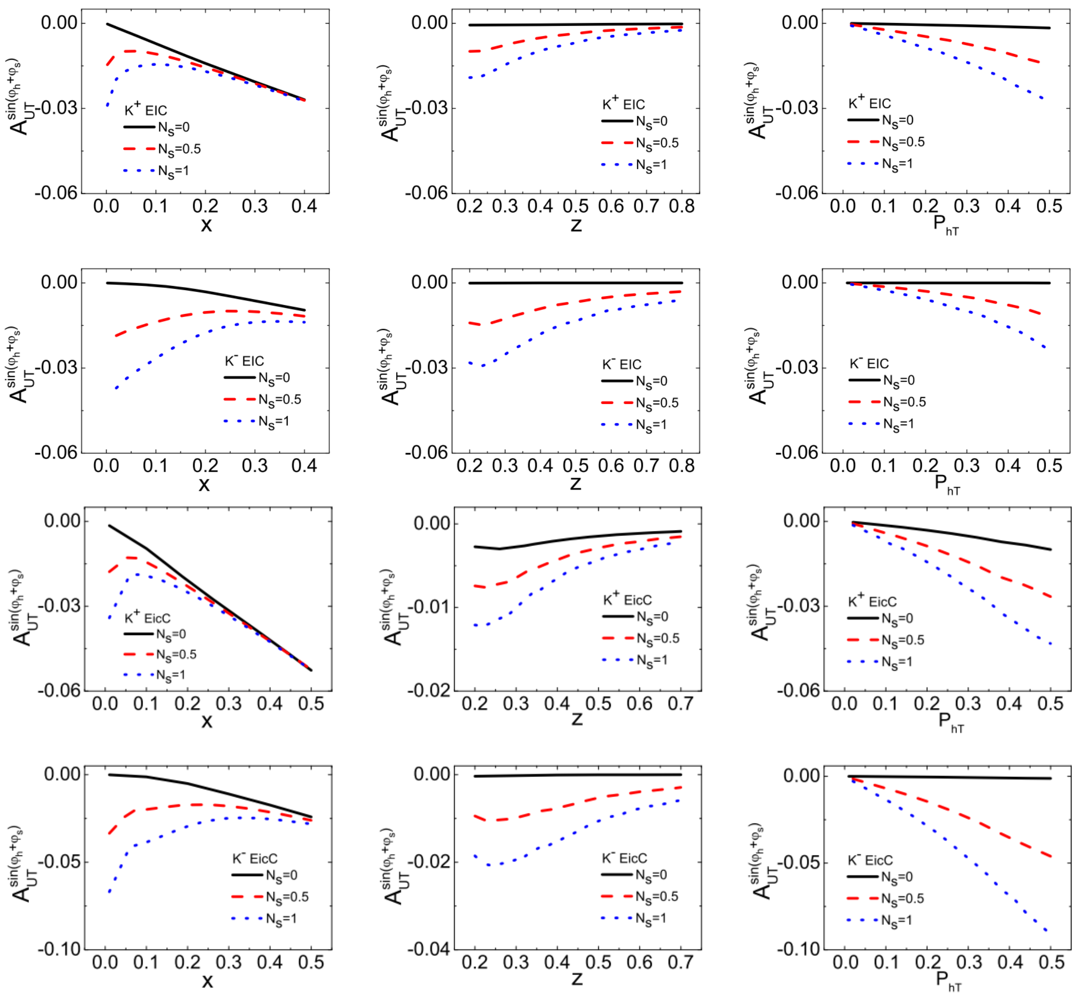


Discussion



- Both sizable at EIC and EicC
- The magnitude of the asymmetry increases with increasing N_s of the collinear sea quark transversity
- The effect of the transversity of the sea quarks turns out to be smaller in the K^+ production process than that in the K^- process

Discussion



- For the asymmetry as the function of x, there is a clear peak at $x \approx 0.05$ at EicC when considering the non-zero sea quark transversity, while the peak vanishes with zero sea contribution of transversity. Although the peak turns to be vague at EIC, the tendency still remains.

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Conclusion

- The measurement on the Collins asymmetry of semi-inclusive Kaon production at future electron ion colliders can provide useful constraints on the sea quark transversity
- The access to the sea quark transversity distribution function depends on the Kaon Collins function, which brings the combination between the SIDIS data and e^+e^- data to constrain the Kaon Collins function as well as the transversity distribution function

What will we do in the future?

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- Once we have the Collins function with high accuracy, we can perform the prediction on the $\cos 2\phi$ asymmetry contributed by convolution of the Boer-Mulders function and Collins function
- The study of Boer-Mulders function can be useful to verify the sign change of the T-odd PDFs
- Cahn effect vs $\cos 2\phi$ effect
- With the transversity, the information of the dihadron fragmentation process can be access

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Thank you!