Systematics-hardened foreground subtraction

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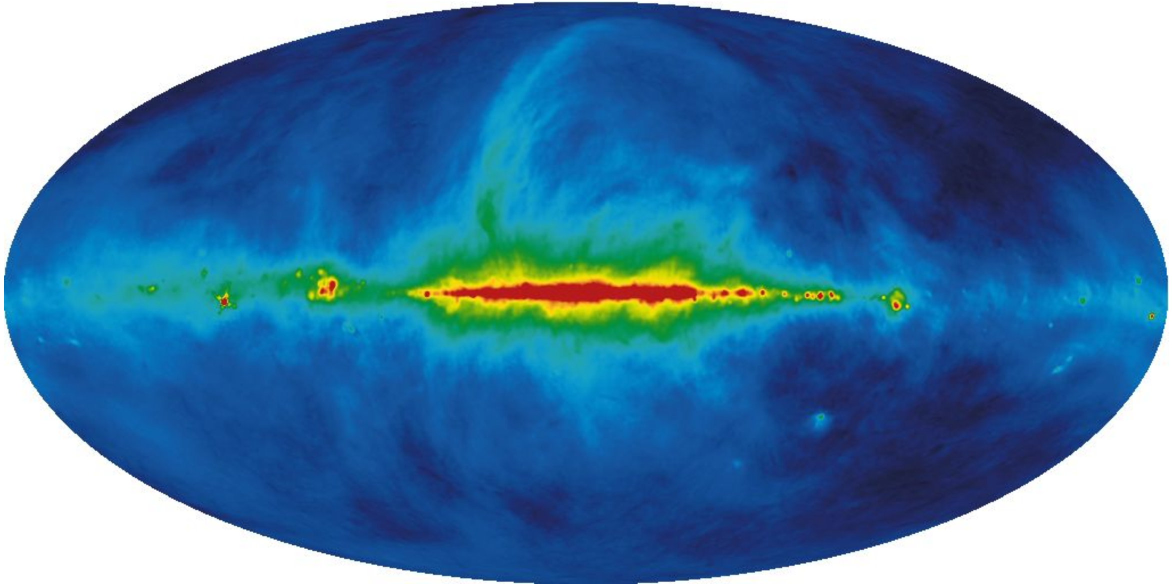
Observations with 21 cm line are hard

Need to separate 21 cm signal from astrophysical foregrounds that are ~3-5 orders of magnitude brighter



- Reprocessed Haslam map (Remazeilles et al. 2014)
- Galactic radio emission at 408 MHz
- ~10-100 K

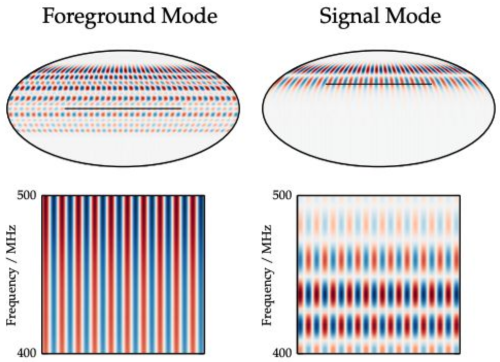
Foreground cleaning strategies

Linear:

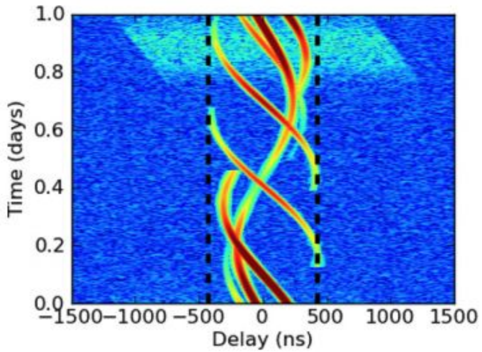
- Karhunen-Loève transform (Shaw et al. 2014)
- Delay transform (Parsons et al. 2012)
- Polynomial fitting (Liu & Tegmark 2012)

Non-linear:

- Principal Component Analysis (Masui et al. 2013)
- Independent Component Analysis (Wolz et al. 2014)



Shaw et al. 2014

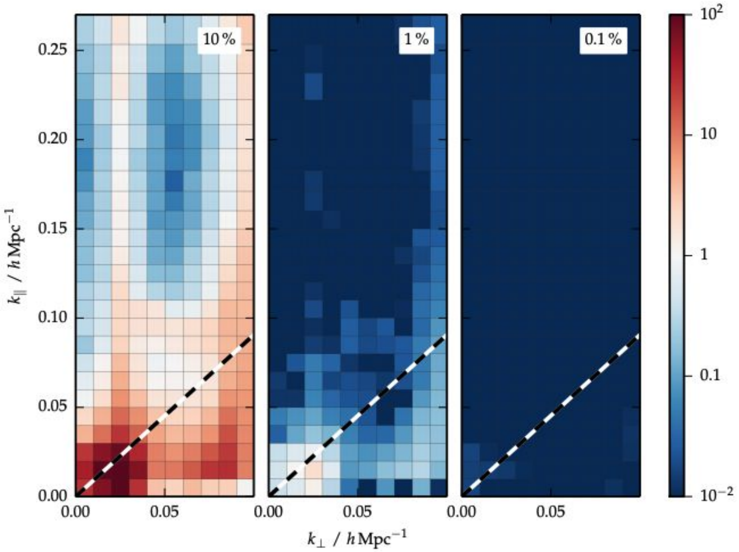


Parsons et al. 2012

Even with that, 21 cm observations are still hard

Instrumental systematics

- Polarization leakage
- Beam errors
- Bandpass errors
- Stability
- Noise



Biasing of the power spectrum from gain perturbations (Shaw et al. 2014)

Our strategy

- Goal:
  - Develop foreground removal algorithms that are more robust to known systematic errors
- Assumption:
  - Filtered data is dominated by (has information about) systematics
- Strategy:
  - We assume there is a solution to the “no-systematics” foreground filtering problem (eg KLT filter, delay filter)
  - Perturbatively estimate the effect of known systematics from the data residuals

Model

Data Instrument Signal Foregrounds

$$\mathbf{d} = \mathbf{B}(\mathbf{s} + \mathbf{f}) = \overline{\mathbf{s}} + \overline{\mathbf{f}} \quad \longrightarrow \quad \mathbf{C} = \langle \mathbf{d}\mathbf{d}^{\dagger} \rangle = \overline{\mathbf{S}} + \overline{\mathbf{F}}$$

- $\mathbf{L}$ some linear filter that projects data onto a subspace where there is significantly more signal than foregrounds

$$\mathbf{L}\mathbf{d} = \mathbf{s}' + \beta \mathbf{f}' \quad \longrightarrow \quad \mathbf{C}' = \mathbf{S}' + \beta^2 \mathbf{F}'$$

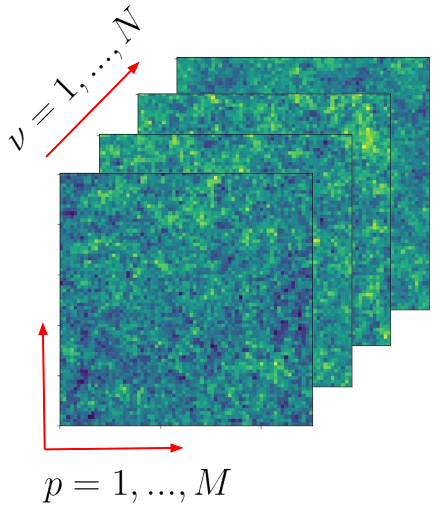
- Imperfect instrument knowledge causes bright foregrounds to leak into signal dominated subspace

Toy example: Achromatic foregrounds

$$d_{\nu p} = (s_{\nu p} + f_p)$$

$$\mathbf{S}_{(\nu p)(\nu' p')} = \delta_{\nu\nu'}\delta_{pp'}\sigma_s^2, \quad \mathbf{F}_{(\nu p)(\nu' p')} = \delta_{pp'}\sigma_f^2, \quad \frac{\sigma_s^2}{\sigma_f^2} \ll 1$$

- Foreground filter = subtract average over frequencies



$$\hat{f}_p = \frac{1}{N}\sum_{\nu'} d_{\nu' p}$$

$$\hat{s}_{\nu p} = d_{\nu p} - \hat{f}_p = s_{\nu p} - \frac{1}{N}\sum_{\nu'} s_{\nu' p}$$

✔

- “No-systematics” foreground filtering problem solved

Toy example: Achromatic foregrounds + bandpass perturbations

$$d_{\nu p} = (s_{\nu p} + f_p)\,(1 + g_\nu)$$

- Repeat procedure

Foregrounds leaking into signal-dominated subspace

$$\hat{s}_{\nu p} = s_{\nu p}\,(1 + g_\nu) - \frac{1}{N}\sum_{\nu'} s_{\nu' p}\,(1 + g_{\nu'}) + \underbrace{f_p\left(g_\nu - \frac{1}{N}\sum_{\nu'} g_{\nu'}\right)}$$

- Estimated signal power dominated by residual term $\sigma_f^2 g_\nu^2$

Estimate and subtract residuals

- Estimate g_ν by correlating ν-th frequency of estimated signal with estimated foregrounds

$$\hat{g}_\nu = \frac{\sum_p \hat{s}_{\nu p} \hat{f}_p}{\sum_p \hat{f}_p^2}$$

- Subtract foreground residuals

$$\tilde{s}_{\nu p} = \hat{s}_{\nu p} - \hat{f}_p \hat{g}_\nu \qquad \longrightarrow \langle \tilde{s}_{\nu p} \rangle = 0$$

$$\longrightarrow \langle \tilde{s}_{\nu p}^2 \rangle = \sigma_s^2 \left(1 + 2g_\nu + g_\nu^2 - \frac{1}{M} - \frac{1}{N} \right)$$

- Cleaned signal power unaffected by foregrounds

General model

$$\mathbf{d} = \mathbf{s} + \mathbf{f} + \sum_k g_k \mathbf{G}_k \mathbf{f}$$

- $\mathbf{L}$ foreground filter: $\hat{\mathbf{s}} = \mathbf{L}\mathbf{d}$
- $\mathbf{A}$ signal filter: $\hat{\mathbf{f}} = \mathbf{A}\mathbf{d}$
- What we did in the achromatic foreground example was to form quantities that look like

$$\hat{y}_k = \hat{\mathbf{f}}^\dagger \mathbf{E}_k \hat{\mathbf{s}} - b_k$$

- Reminiscent of quadratic estimators (Tegmark 1997)

General model

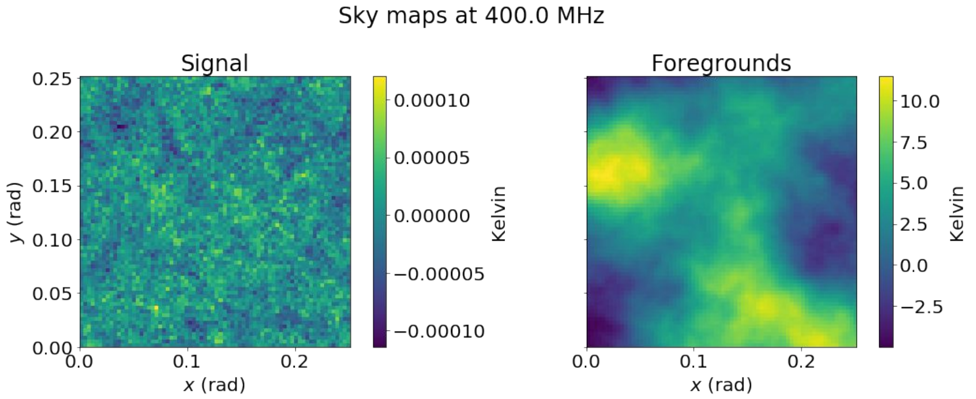
$$\hat{y}_k = \hat{\mathbf{f}}^\dagger \mathbf{E}_k \hat{\mathbf{s}} - b_k \quad \longrightarrow \quad \langle \hat{y}_k \rangle = \sum_{k'} W_{kk'} g_{k'}$$

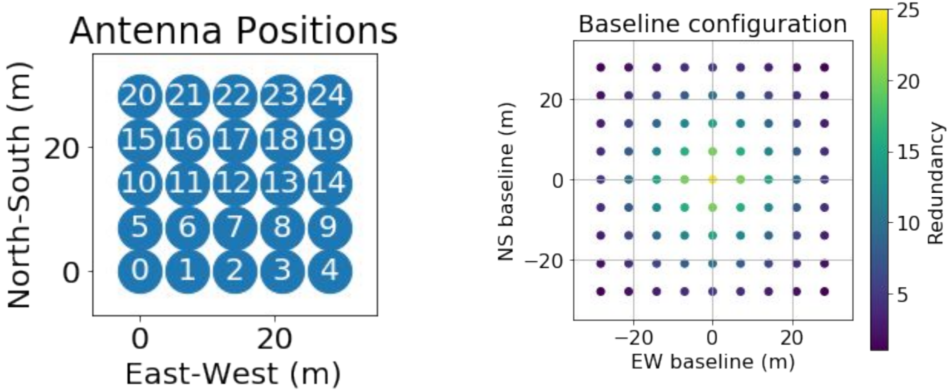
$$W_{kk'} = \mathrm{Tr}\left(\mathbf{E}_k \, \frac{\partial \mathbf{C}^{\hat{s}\hat{f}}}{\partial g_{k'}} \right)$$

- $\mathbf{E}_k$ chosen by analyst based on desired properties of estimator
- b_k chosen to remove bias (if present)
- $\partial \mathbf{C}^{\hat{s}\hat{f}} / \partial g_{k'}$ approximately independent of the g_k

Example: interferometer + bandpass errors

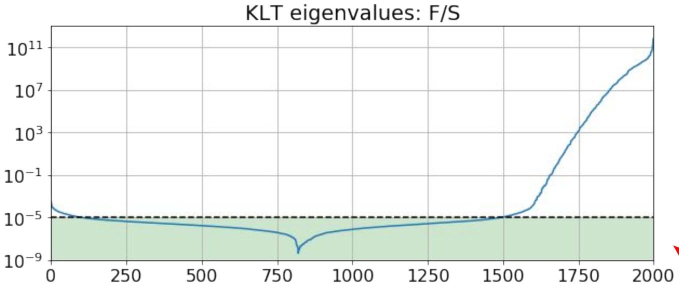
- 5x5 dish array
- 6m dish diameter, 7m spacing
- 50 frequencies in 400-500 MHz range
- 72 x 72 pixels per map
- 0.2x0.2 deg pixel resolution
- FWHM=λ/D gaussian primary beam



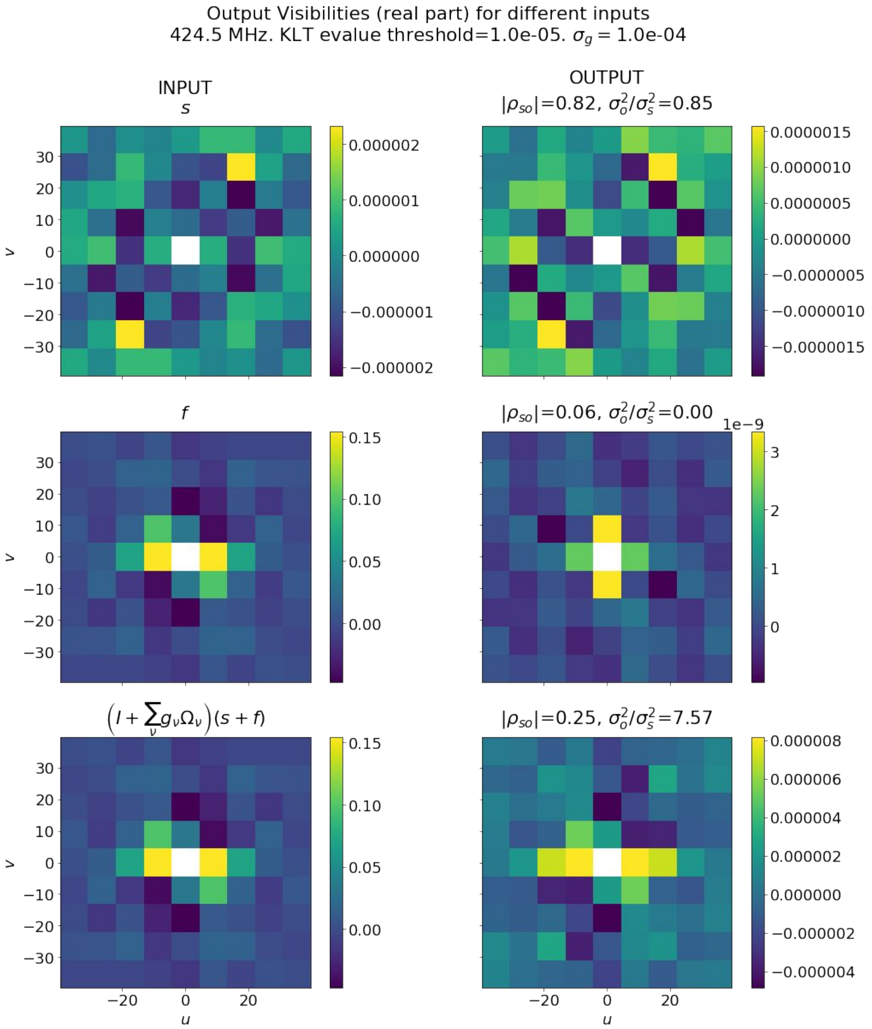


Foreground filter

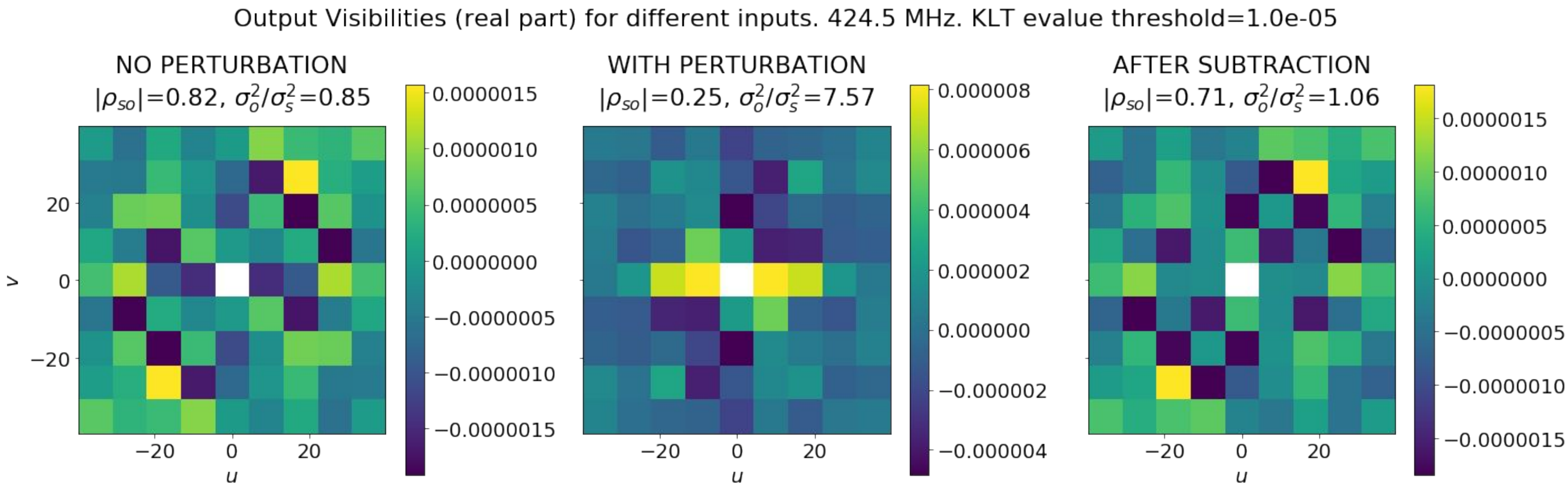
- Based on the Karhunen-Loève transform
- Requires: models for signal and foreground statistics
- Provides: Set of modes ordered by F/S ratio
- Procedure: Discard modes with high F/S ratio



Foreground filter ***L*** constructed from these modes



Subtracting foreground residuals



Using information in foreground residuals and cross correlating with estimated foregrounds we obtain ~2 orders of magnitude of additional foreground suppression

Status and next steps

- Status
  - Early stages of algorithm development
  - Pipeline for small-scale simulations
- Next steps
  - Scale up simulations
  - Model and simulate realistic instrument and systematics
  - How well can we constrain the perturbations? Is it good enough?

Conclusions

- Residuals of foreground cleaning are typically dominated by instrumental systematics
- Developed quadratic estimator formalism to extract information about known systematics and remove them from the data
- Demonstrated procedure with toy model of achromatic foregrounds and bandpass errors
- In more realistic sky simulations with a small-scale array obtain ~1-2 orders of magnitude of additional foreground suppression