The Curious Case of Life on the Lattice

Dorota M Grabowska

CERN

Standard Model

Classification of Elementary Particles

<u>Force Carriers</u>

Photon

$W^{\pm}$ and Z^0

Gluon

<u>Quarks</u>

Up	Down
Charm	Strange
Top	Bottom

Higgs

<u>Leptons</u>

Electron	Electron Neutrino
Muon	Muon Neutrino
Tau	Tua Neutrino

Quantum Field Theory

Mathematical Formulation of Particle Physics

Quantum field theory = special relativity + quantum mechanics

Path Integral Formulation of Quantum Field Theory

Sum over all possible trajectories (field configurations)

$$\mathcal{Z} = \int \mathcal{D}\Phi \exp\left[i \int d^4x \, \mathcal{L}(\Phi[x]) \right]$$

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But how do we actually calculate anything?

The Lattice

Extremely Powerful Toolkit

Definition of "The Lattice"

Numerical Monte-Carlo Finite Volume Discretized Field Theory

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Numerical Monte-Carlo

Importance sampling of path integral, using large scale computing resources

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Numerical Monte-Carlo Finite Volume

Boundary effects cannot be ignored and are used to extract infinite volume scattering

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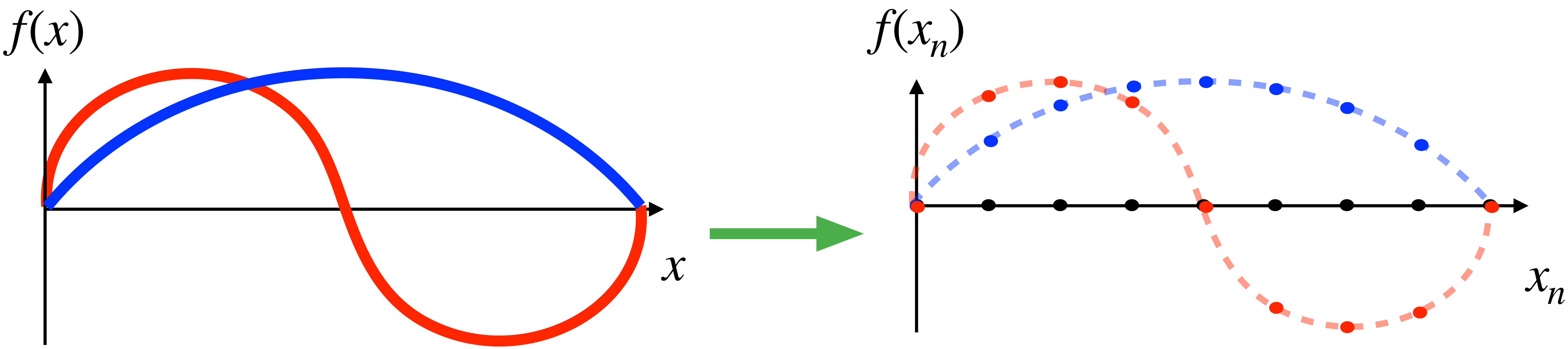
QFT defined on discretized Euclidean spacetime instead continuous Minkowski

Discretized Field Theory

Euclidean Spacetime

Idea: Map continuous functions into discrete ones



Discretized Field Theory

Euclidean Spacetime

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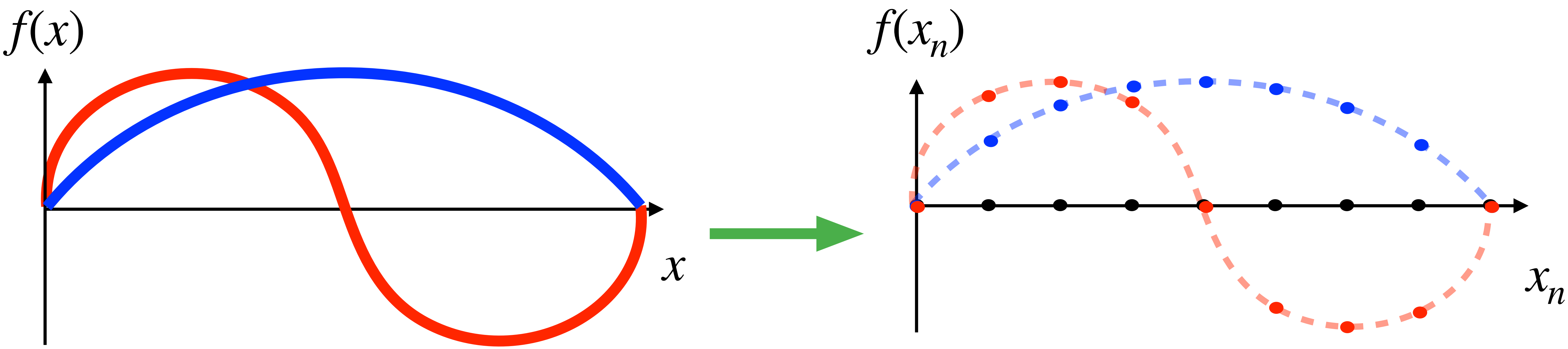

Path Integral Formulation of (Euclidean) QFT

Sum over all possible discretized trajectories

$$\mathcal{Z} = \int \prod_x d\Phi(x) \exp\left[-a^4 \sum_x \mathcal{L}(\Phi[x])\right]$$

Partition Function   Sum over all paths   Accumulated weight

Numerical Monte Carlo

Importance Sampling

Monte Carlo: Computational method using random sampling to numerically approximate an observable

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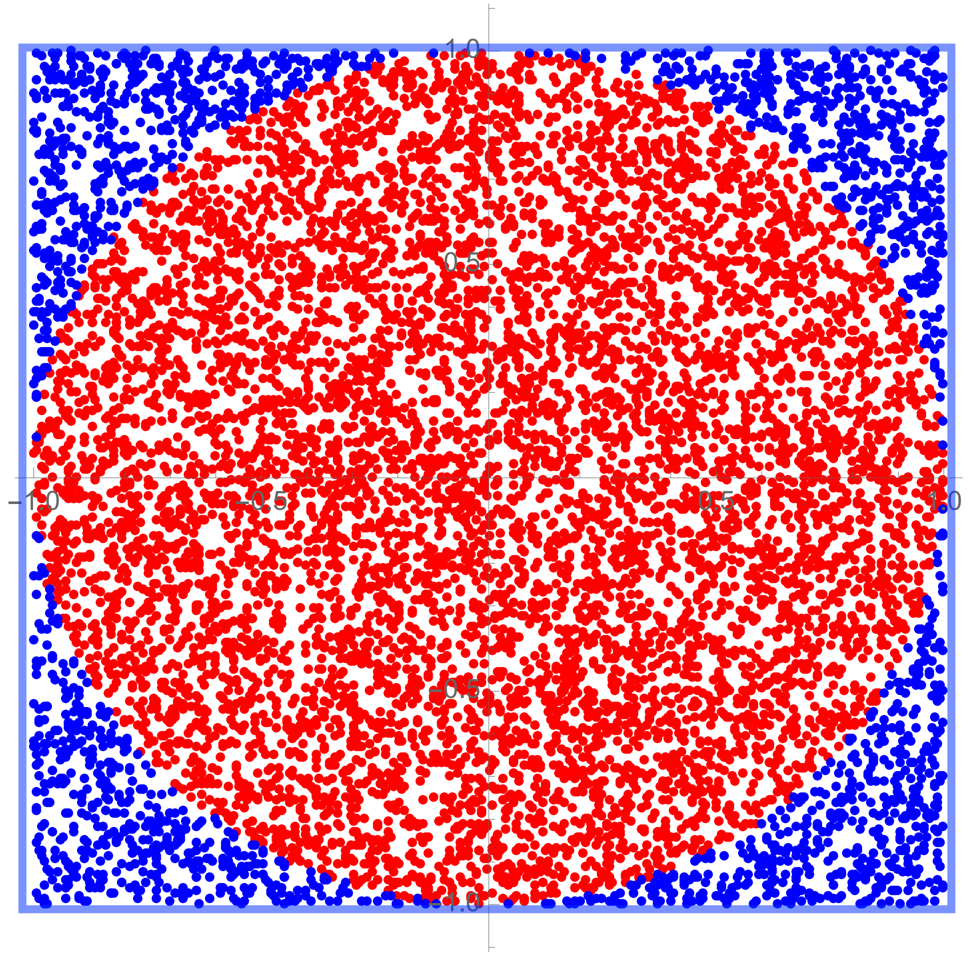


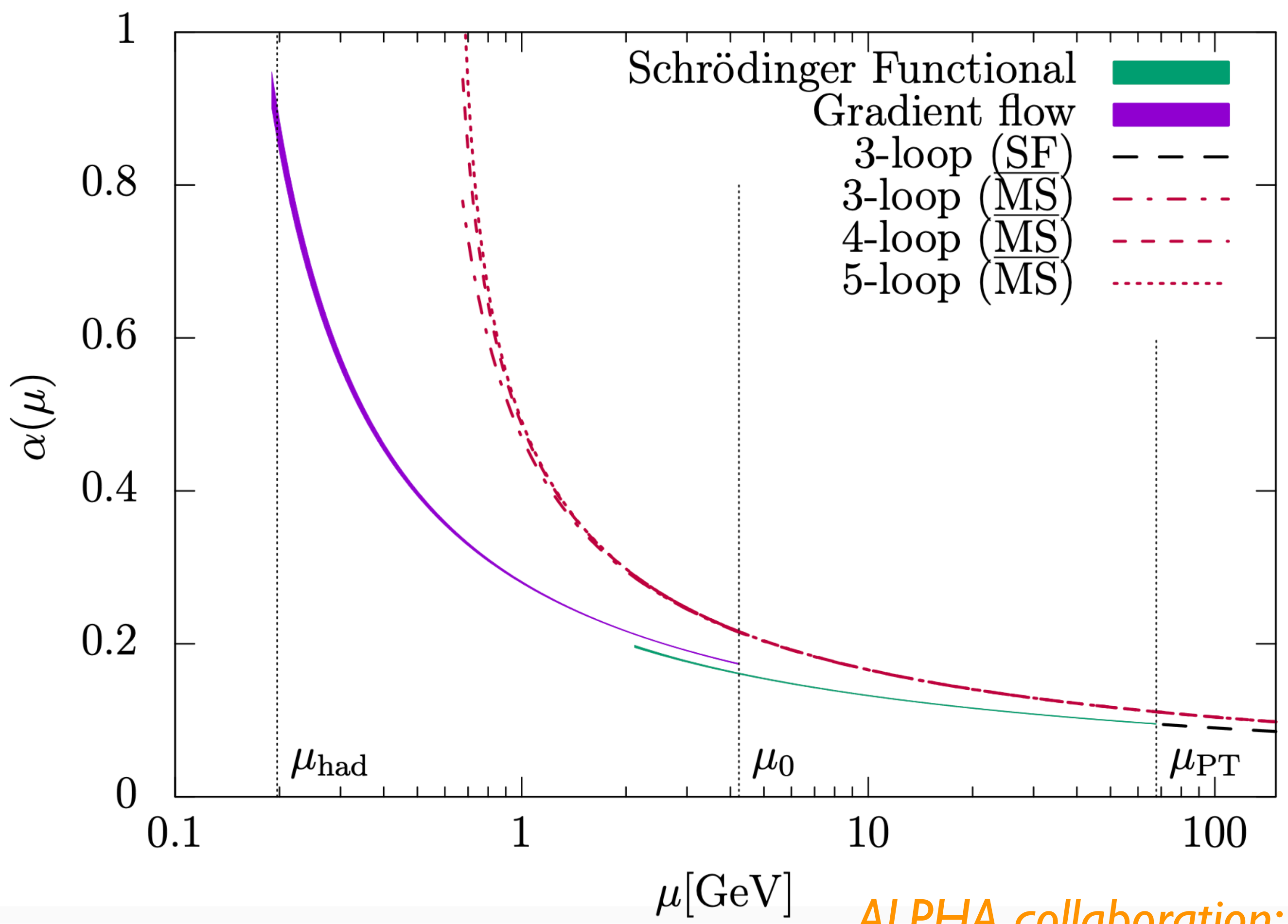
Ex: Find approximate value of π by randomly firing at a square

Numerical Monte Carlo

Evaluating Correlation Functions

Ex: Running of the strong couplings constant, α_s, with energy

Notice: QCD becomes strongly coupled at low energies



ALPHA collaboration; arXiv:1706.03821

Lattice is only method for calculating low energy QCD observables from first principles

Numerical Monte Carlo

Computationally Intense

Ex: Cost of simulating QCD with Wilson fermions in 2001*

$$\text{cost} \approx 2.8 \left[\frac{\#\text{of conf.}}{1000}\right] \left[\frac{M_\pi/M_\rho}{0.6}\right]^{-6} \left[\frac{L}{3\text{fm}}\right] \left[\frac{a^{-1}}{2\text{GeV}}\right]^{7} \text{Tflops} \cdot \text{year}$$

*Ukawa, Nucl. Phys. B Proc. Suppl. 2002

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Example Simulation

Physical point, 100 configs, $196\text{x}96^3$ lattice points with a=0.064fm

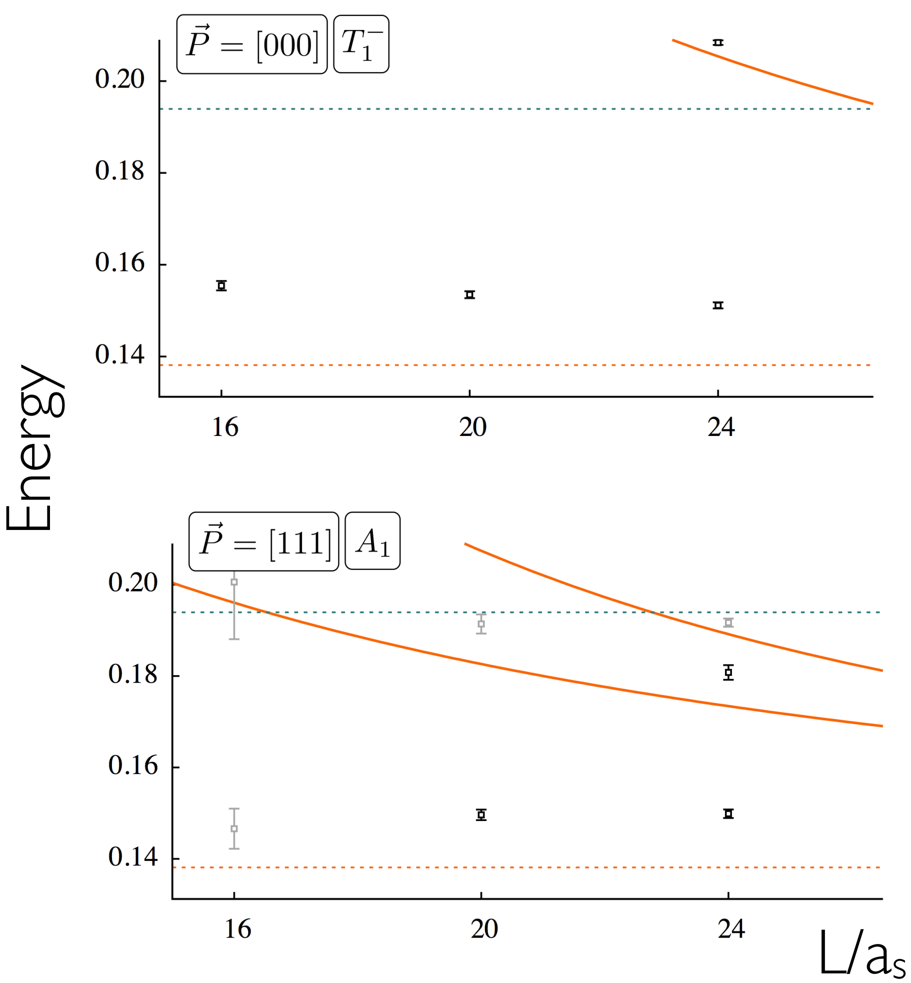
Cost in 2001: 640 billion core hours
Cost in 2020 : 20 million core hours

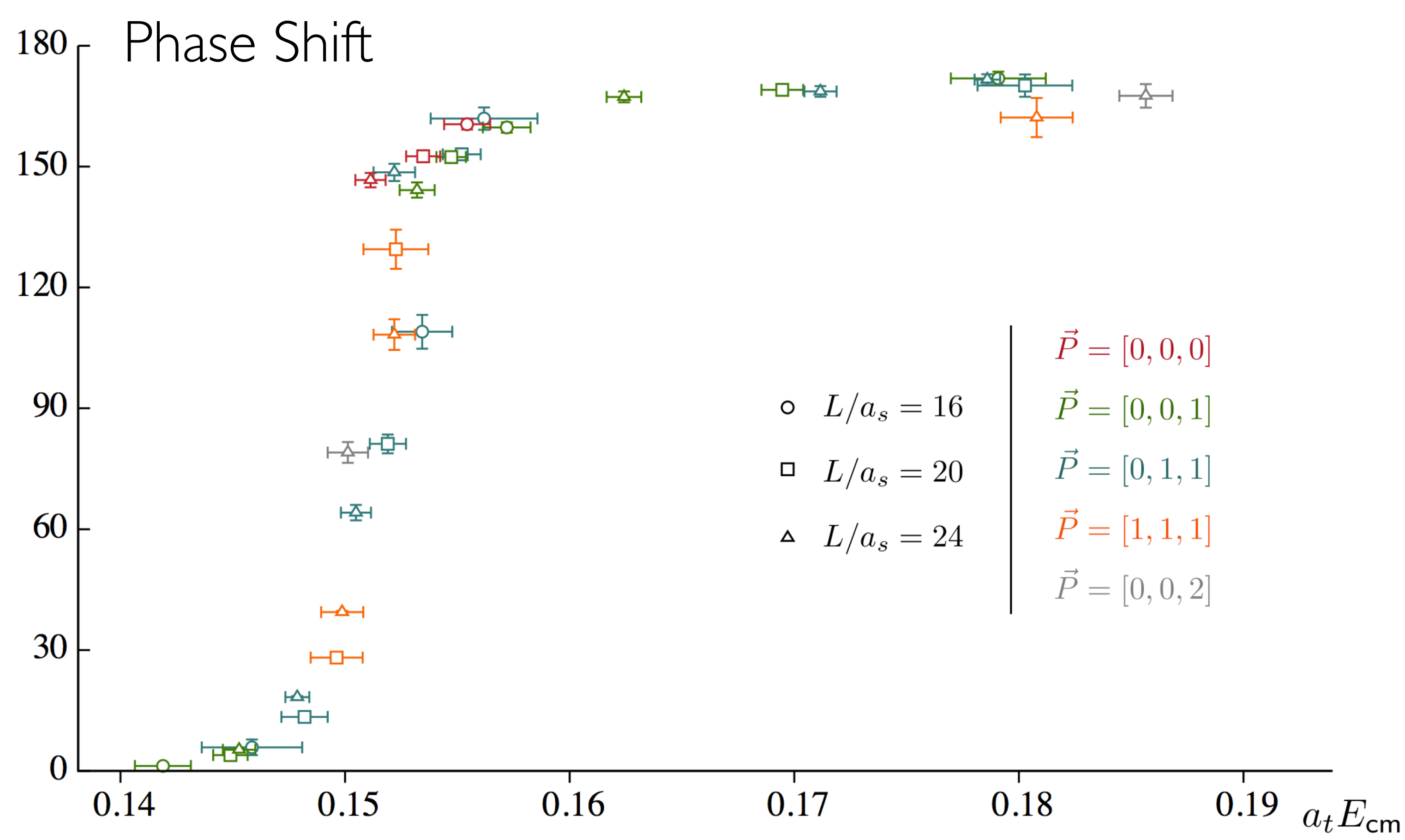
This dramatic decrease is mainly due to algorithmic advances, not Moore's Law!

Finite Volume

Necessary for Simulations

Idea: Dependence of energies at finite volume can be used to extract infinite volume scattering





Dudek, Edwards, Thomas; arXiv:1212.0830

Ex: Extraction of infinite volume ππ scattering phase shift from finite volume energy shifts

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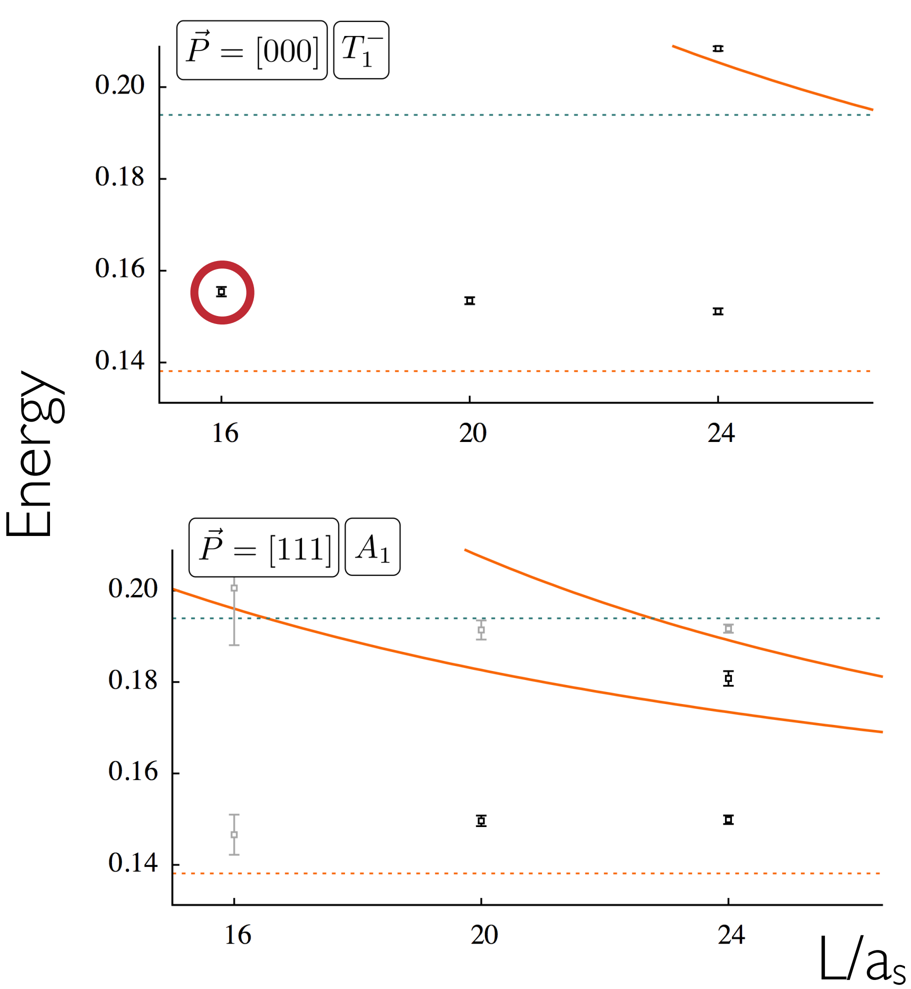

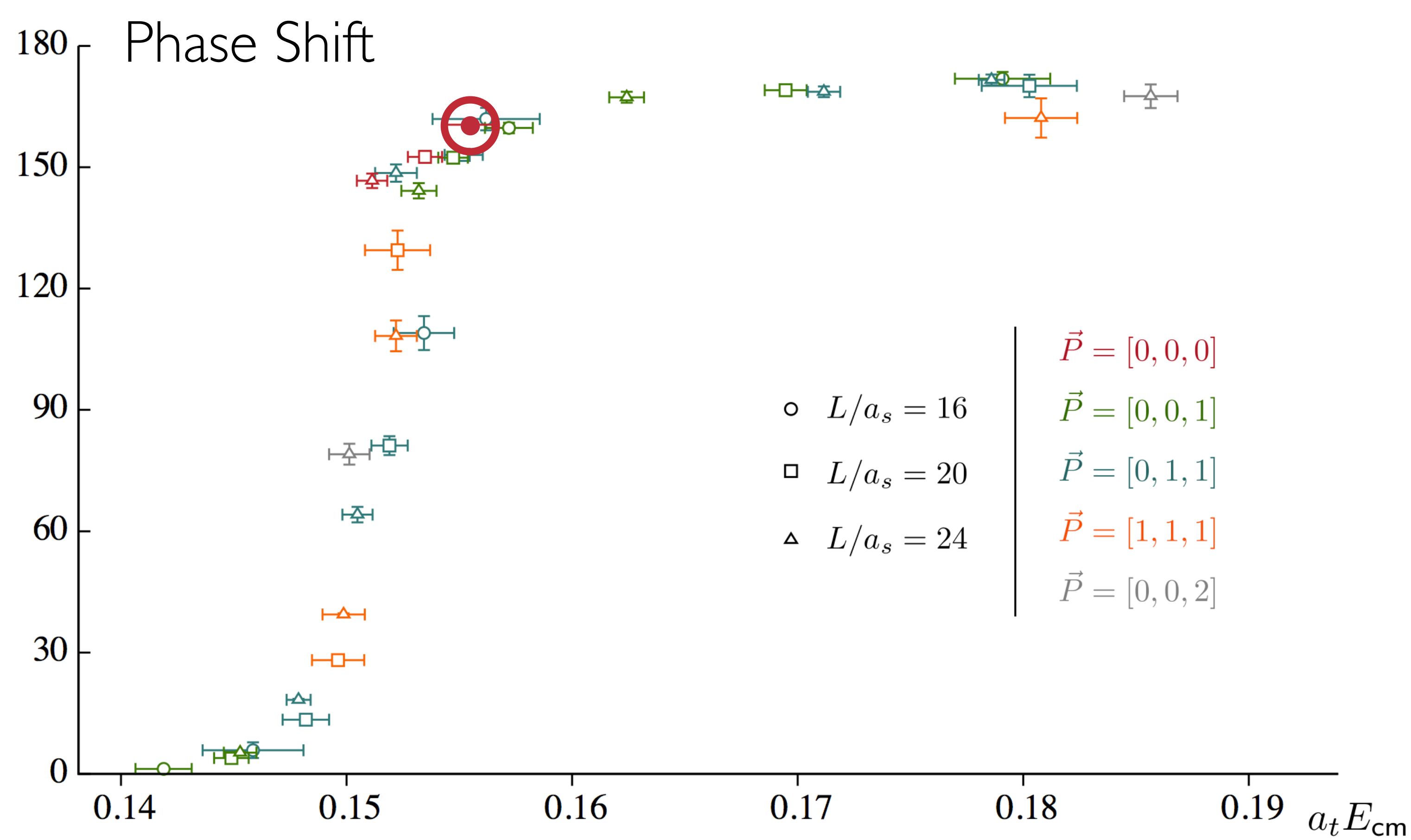

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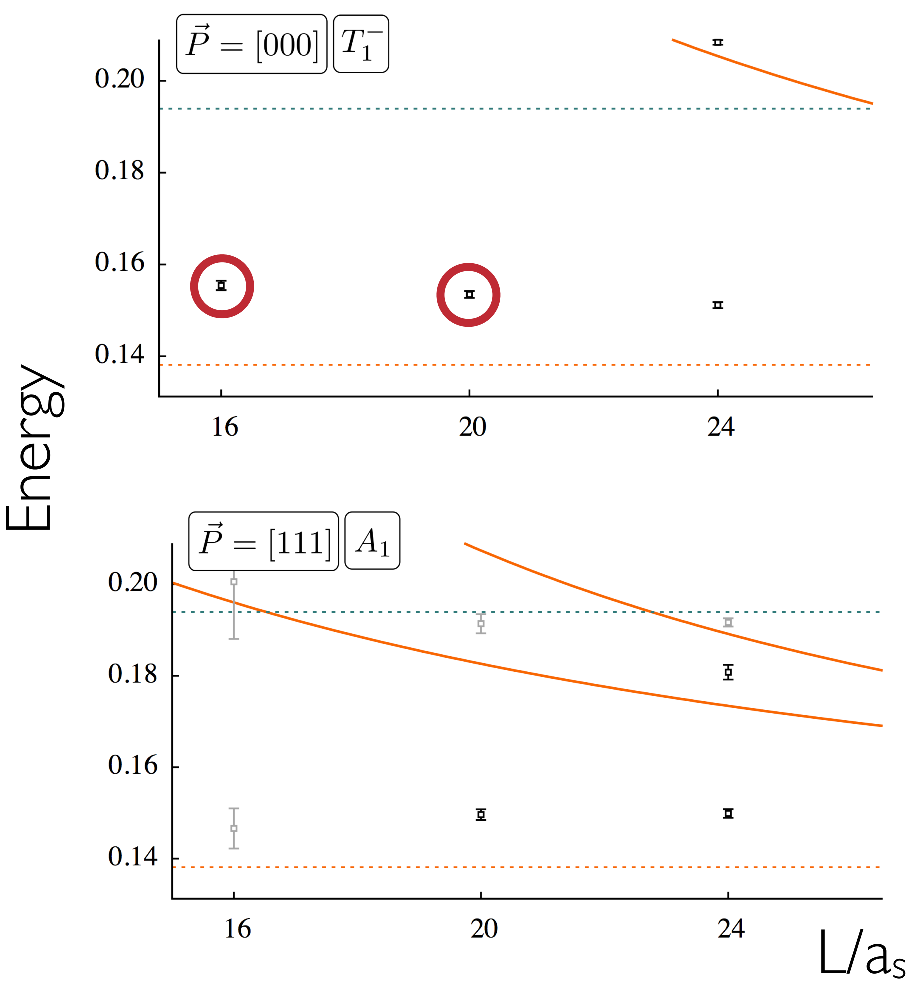

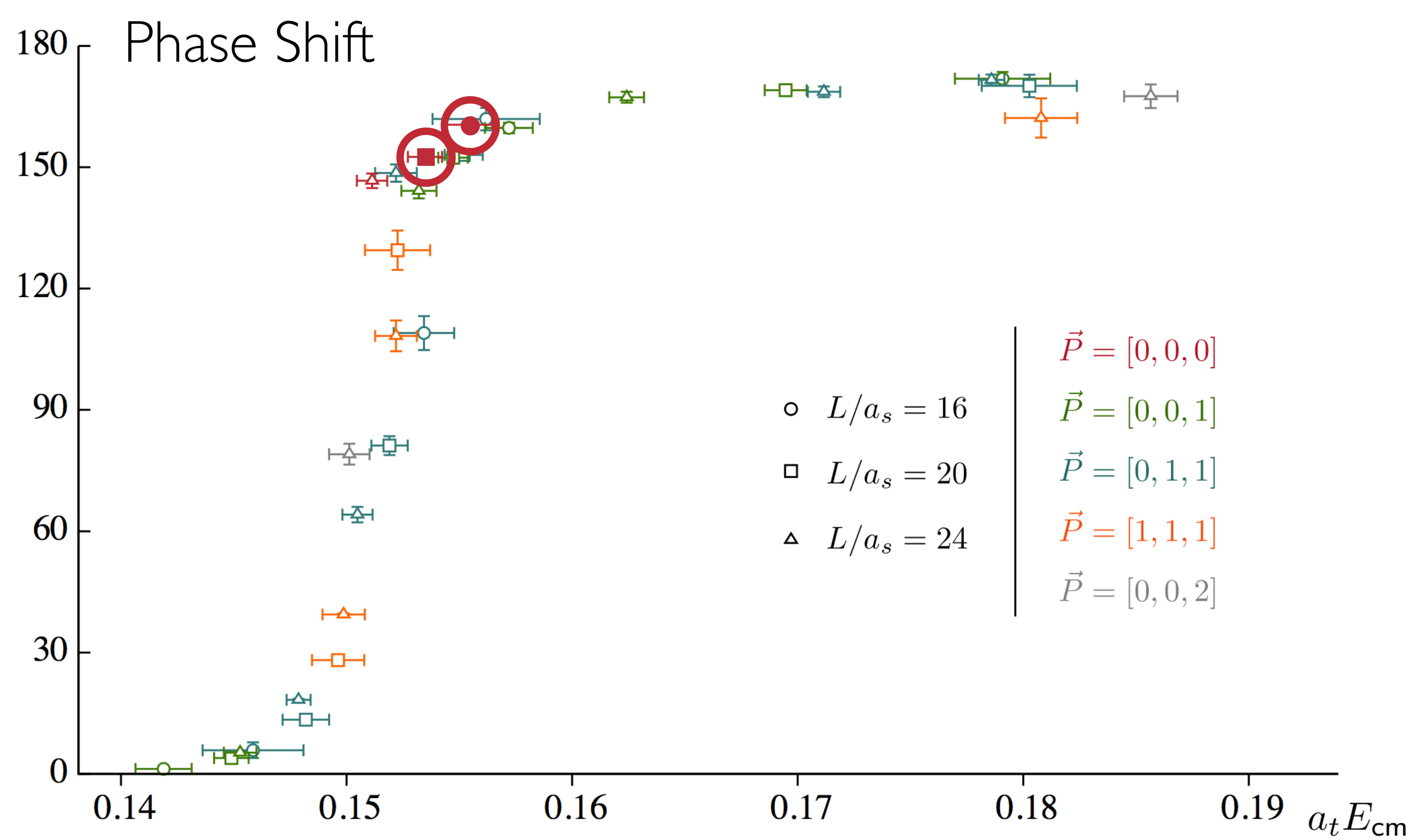

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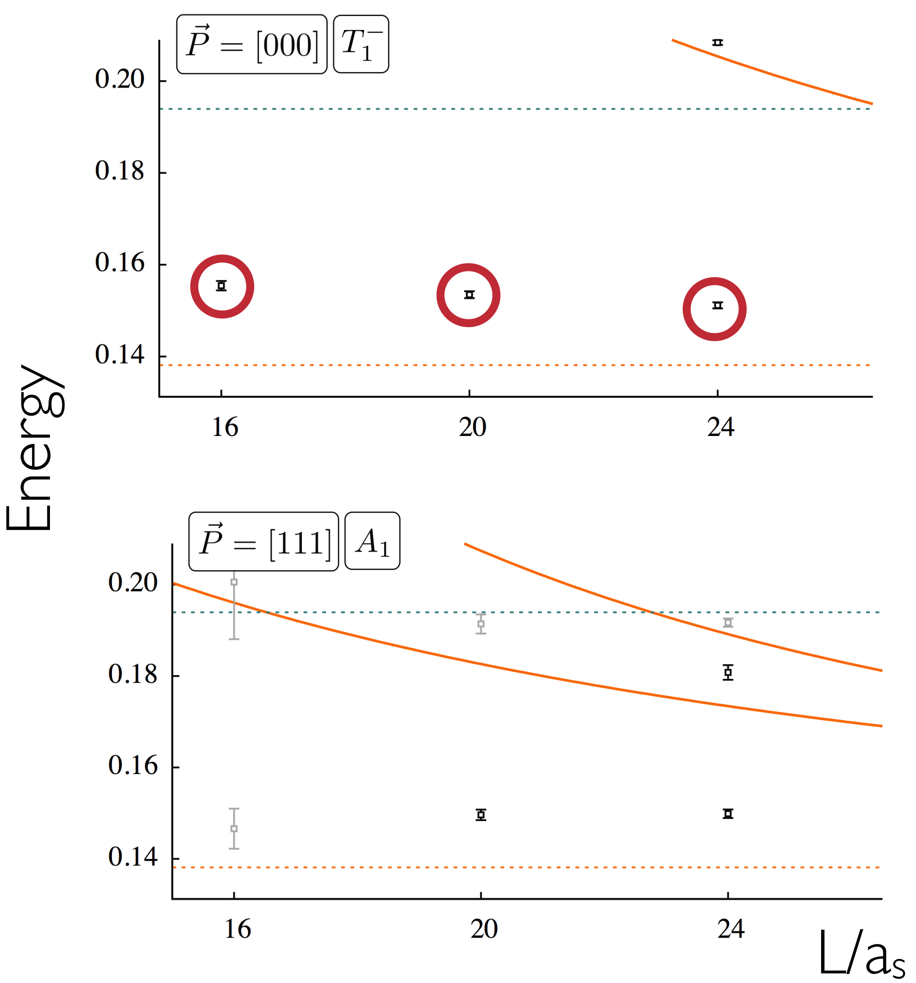

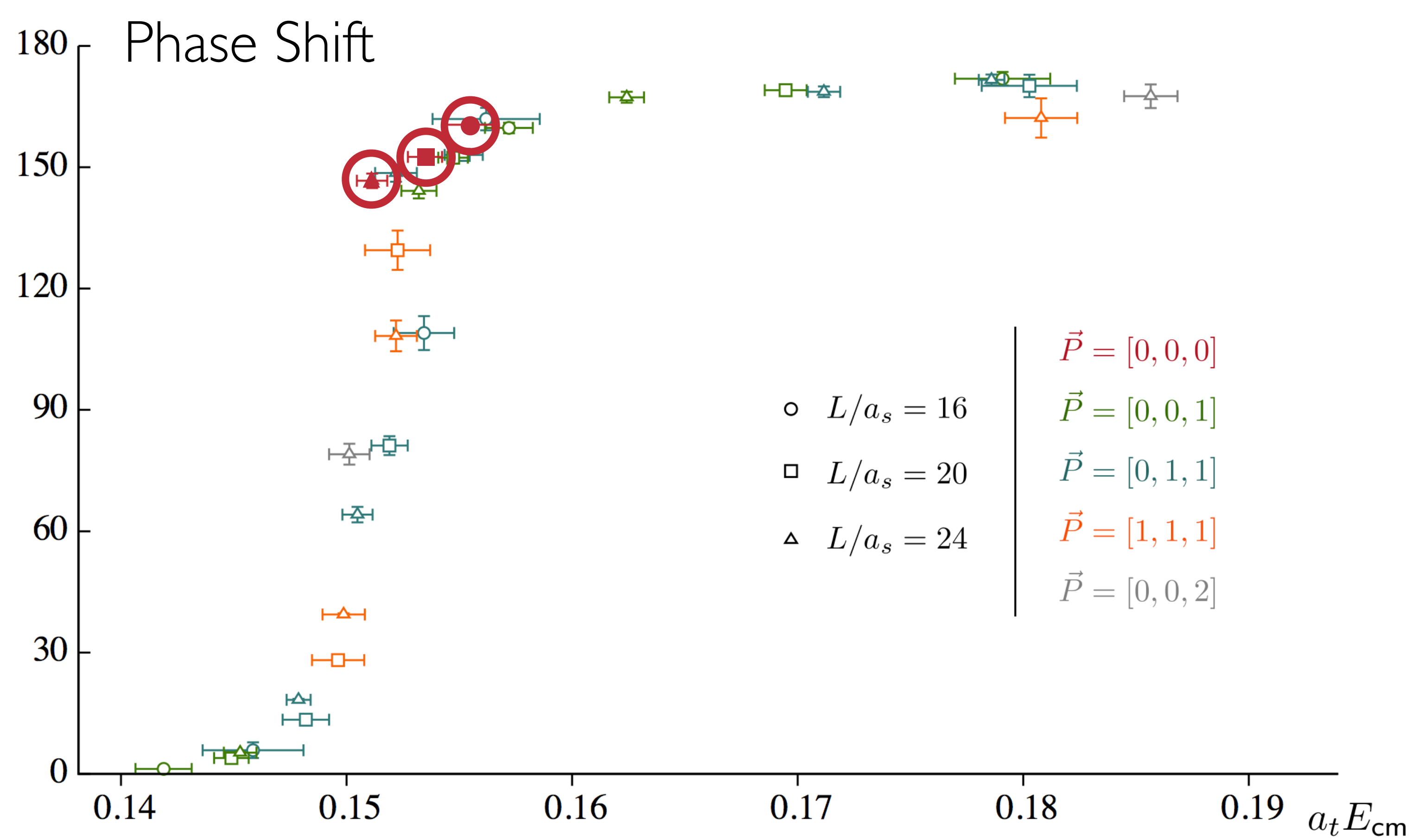

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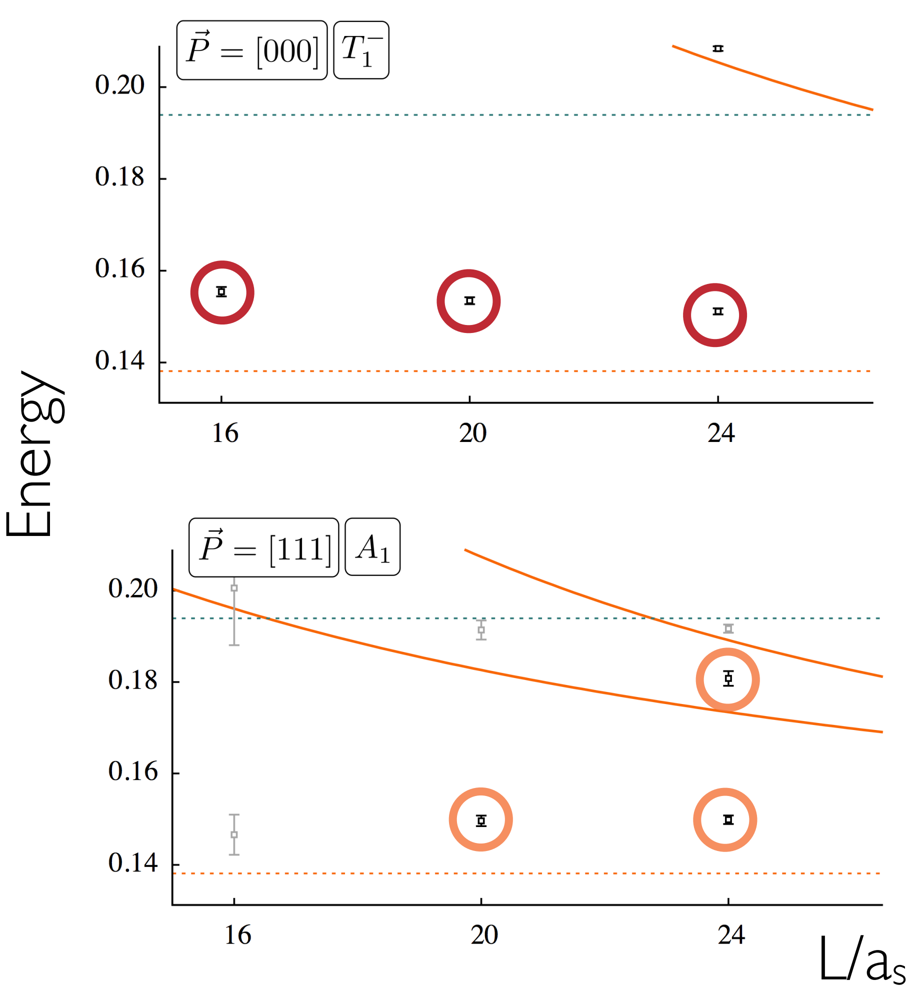

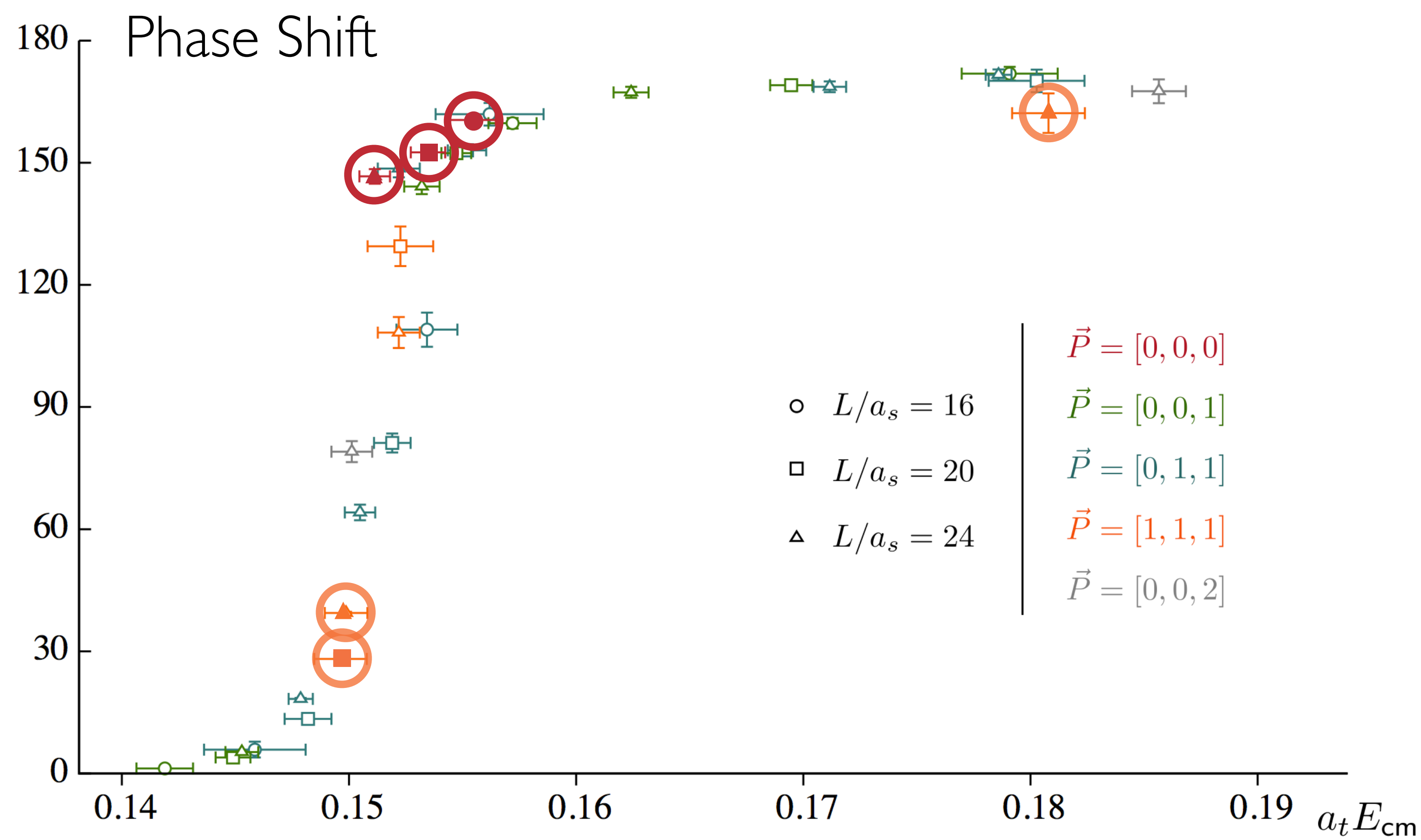

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Ex: Extraction of infinite volume ππ scattering phase shift from finite volume energy shifts

Outline

1. Introduction: What is “the Lattice”
2. Chiral fermions on the lattice
3. Finite volume effects

Main Messages

Dual Sides of the Lattice

First Main Message

The lattice provides both a deeper theoretical understanding of quantum field theories and the only way to extract many numbers of experimental relevance

Second Main Message

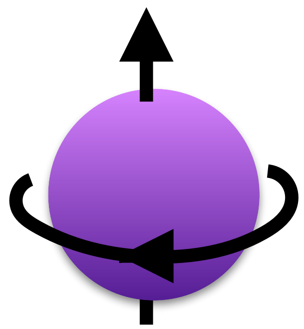
Life on the lattice is full of complications that must be overcome, but it is incredibly worthwhile to do so

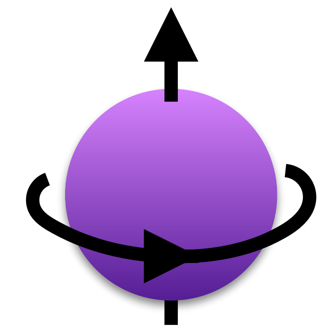
Fermions on the Lattice

Chiral Symmetry in Standard Model

Property of Fermions

Chiral Symmetry: is a mirror image identical to the original?





Left Handed: Spin and momentum are anti-parallel

Right Handed: Spin and momentum are parallel

Handiness is frame independent only for massless fermions

Chiral symmetry plays in incredibly important role in the Standard Model

Anomaly

Mystery of the Heavy η'

Anomaly: Classical symmetry broken at the quantum level

Ex: Symmetries of the QCD Lagrangian

Two symmetries of the Lagrangian

- $U(1)_V$ counts sum of left handed and right handed fermions
- $U(1)_A$ counts difference between number of left handed and right handed fermions

These two symmetries, plus flavor symmetries, predict 9 light mesons. But we only have 8, as $m_{\eta'} \sim 960$ MeV

Solution: $U(1)_A$ is not a symmetry of the quantum theory

Chiral Symmetry in Standard Model

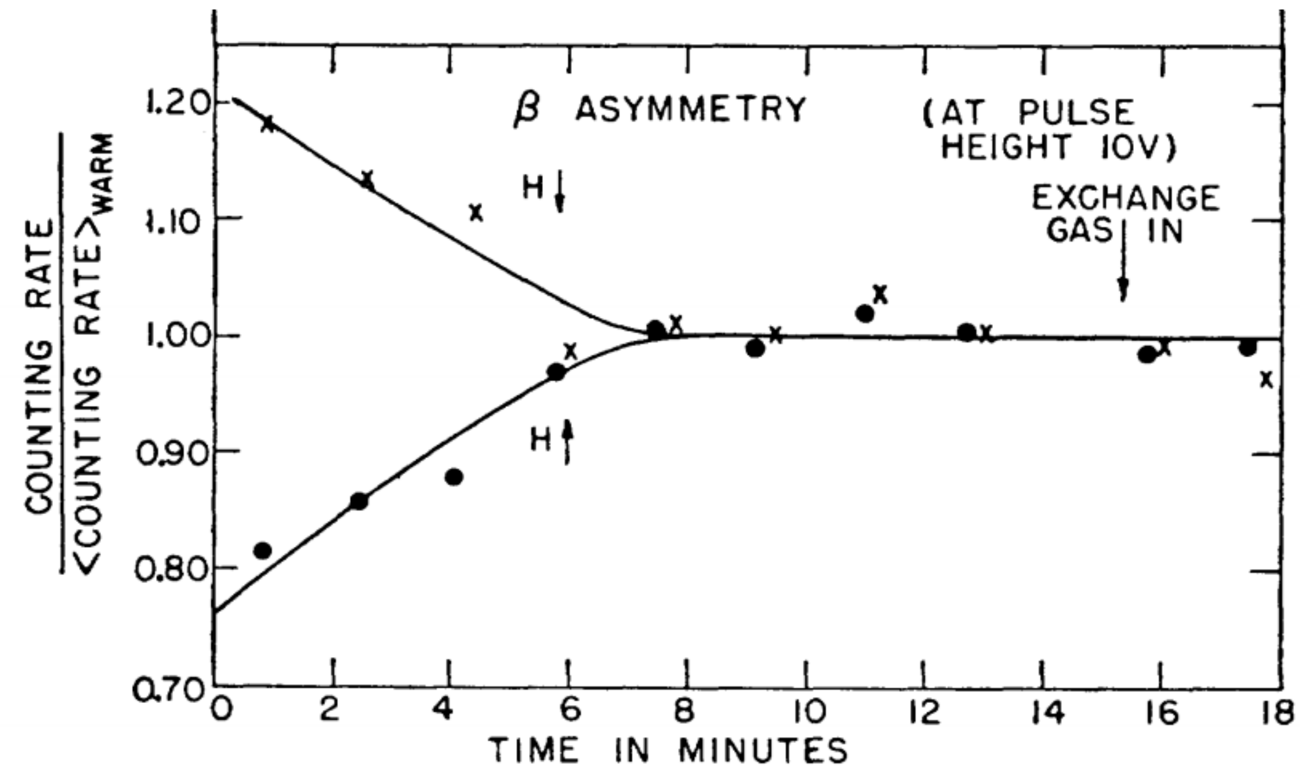
Gauge Theories

Electroweak interactions distinguish left from right

Ex: decay of ^{60}Co

Set-Up

1) Align spins of cobalt atoms using strong magnetic field
2) Unstable cobalt decays
3) Measure distribution of emitted electrons and gamma rays
4) Compare two distributions



$$^{60}_{27}\mathrm{Co} \rightarrow ^{60}_{28}\mathrm{Ni} + e^- + \bar{\nu}_e + 2\gamma$$

Wu et al.; Phys. Rev. 105, 1413

Goal: construct a discretized theory that has the necessary symmetries to faithfully reflect the continuum theory

No-Go Theorem

Nielson-Ninomiya Theorem

No-Go Theorem: No lattice fermion operator can satisfy all four conditions simultaneously

1. Fourier transform of operator is local

2. At least one Dirac fermion in the continuum limit

3. No more than one Dirac fermion in the continuum limit

4. Full chiral symmetry from the continuum is preserved

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Fermions on the lattice must violate at least one condition

Fermion Doubling Problem

Nielson-Ninomiya Theorem

Recall 3rd condition: no more than one fermion in the continuum

Continuum Theory

$$\mathcal{L} = \int d^4x \, \bar{\psi} \partial\!\!\!/ \psi$$

Spectrum has single massless fermion

Discrete Theory

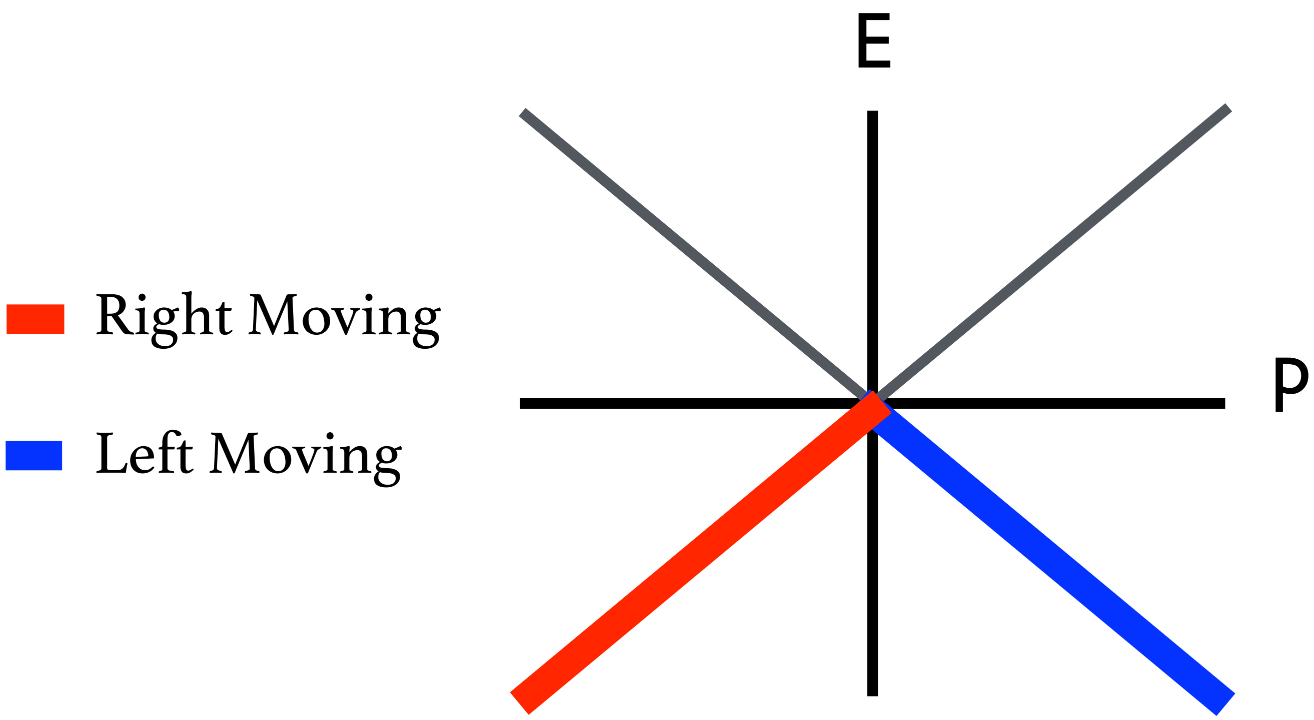
$$\mathcal{L} = \sum_n \bar{\psi}_n \gamma_\mu \left(\frac{\psi_{n+\mu} - \psi_{n-\mu}}{2} \right)$$

Spectrum has ***16*** massless fermion!

Number of Doublers grows like 2^d

Anomalies and the Dirac Sea

Ex: Massless electrons in two dimensions



Initial Ground State

Anomalies and the Dirac Sea

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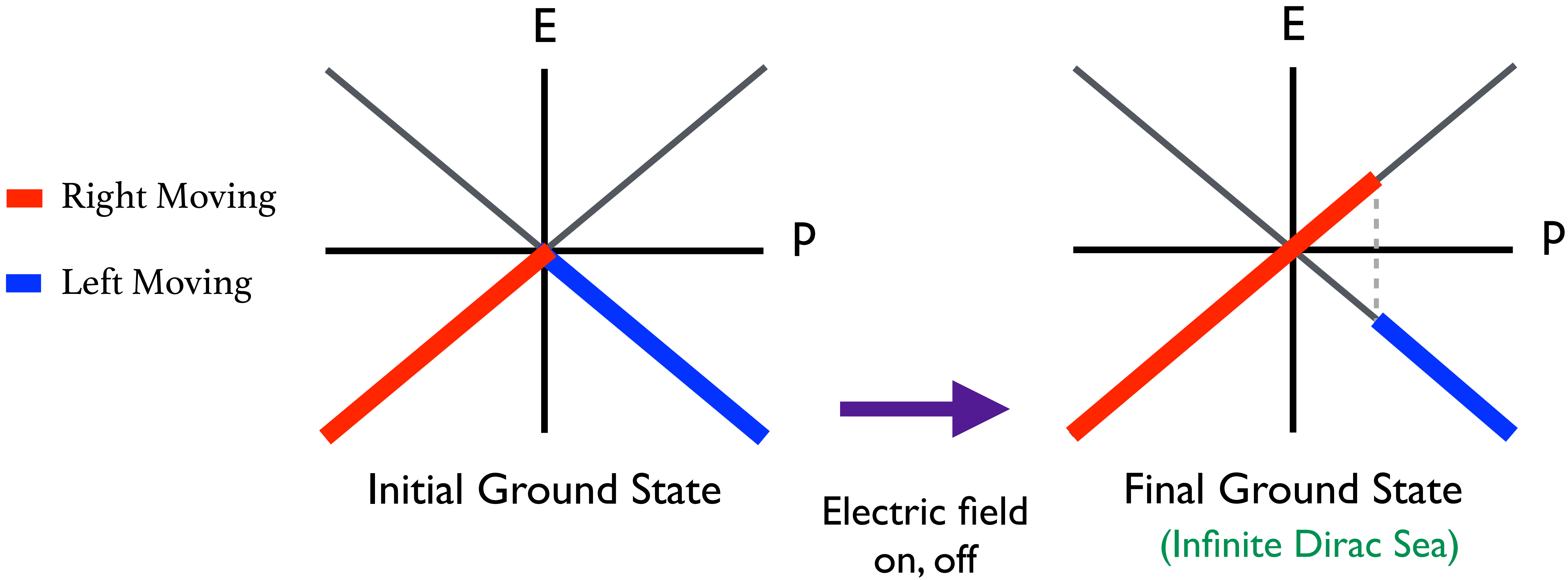

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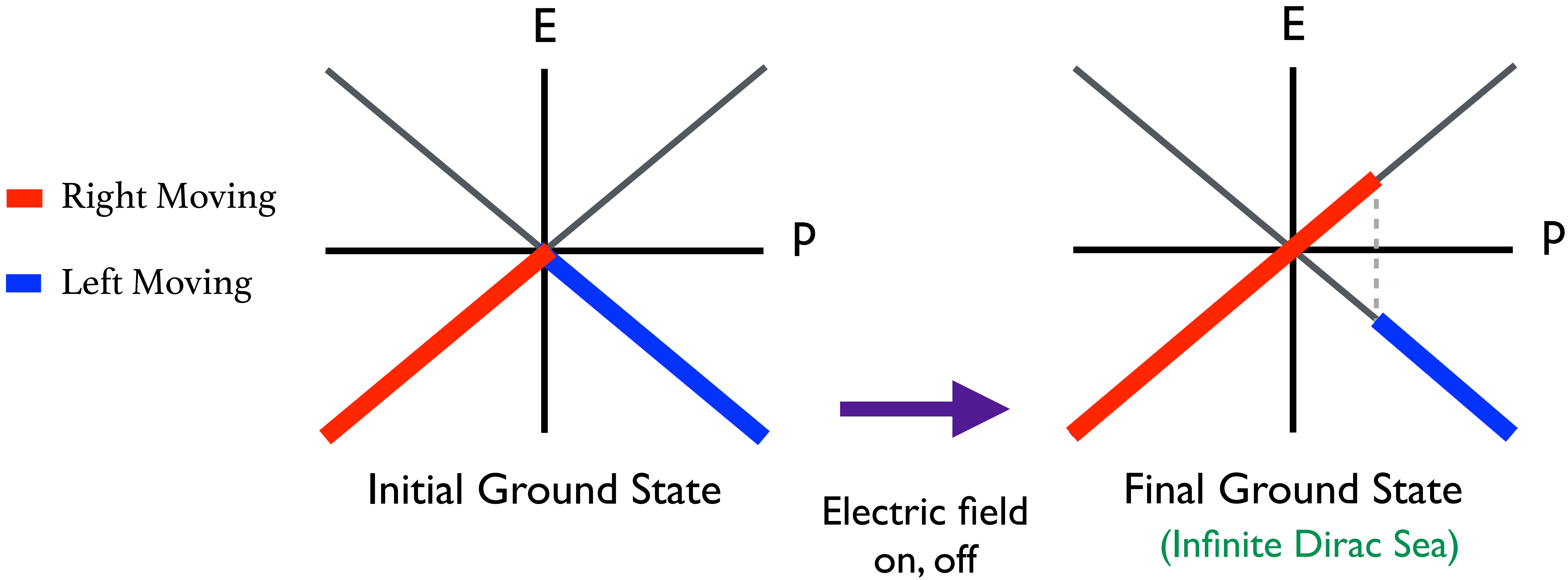

- $U(1)_V$ preserved
- $U(1)_A$ violated

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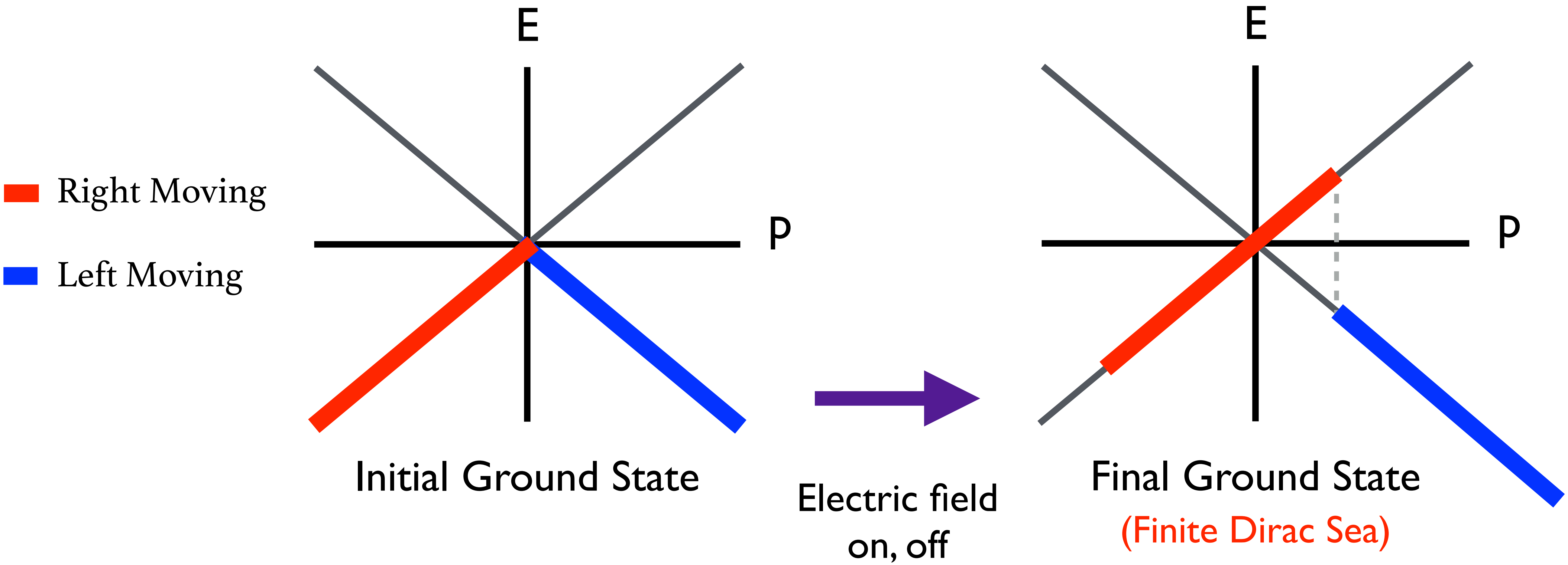

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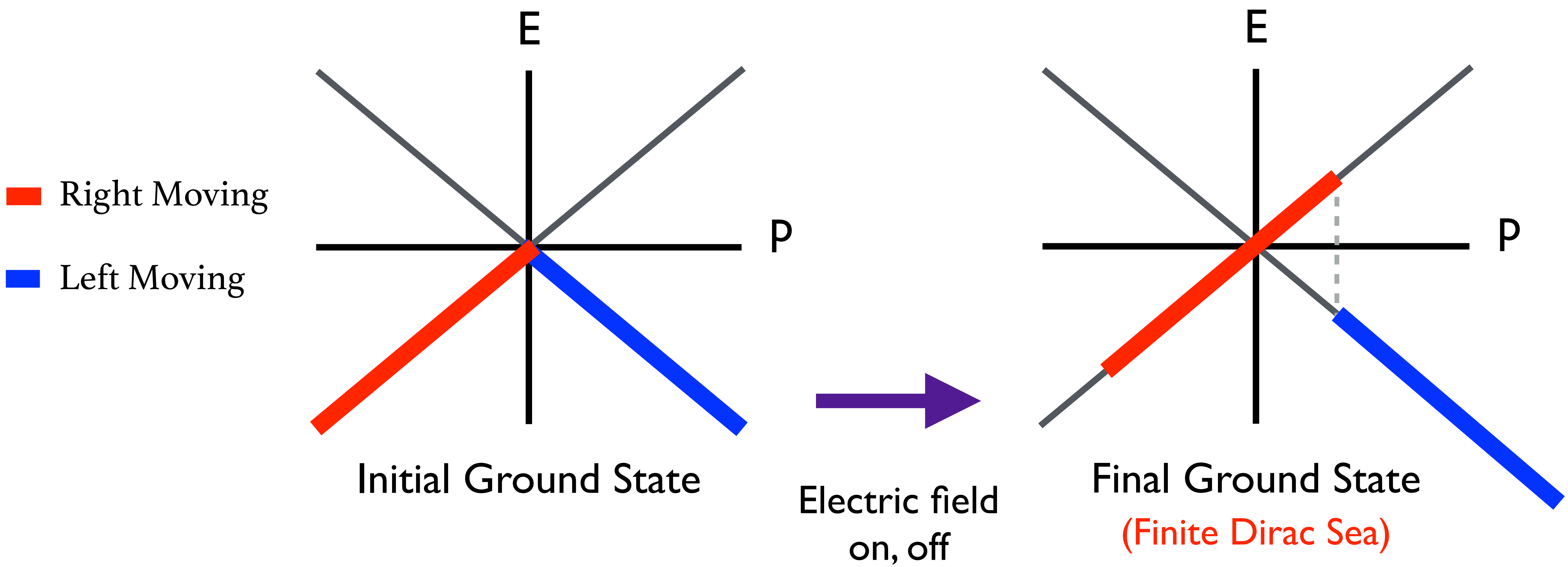

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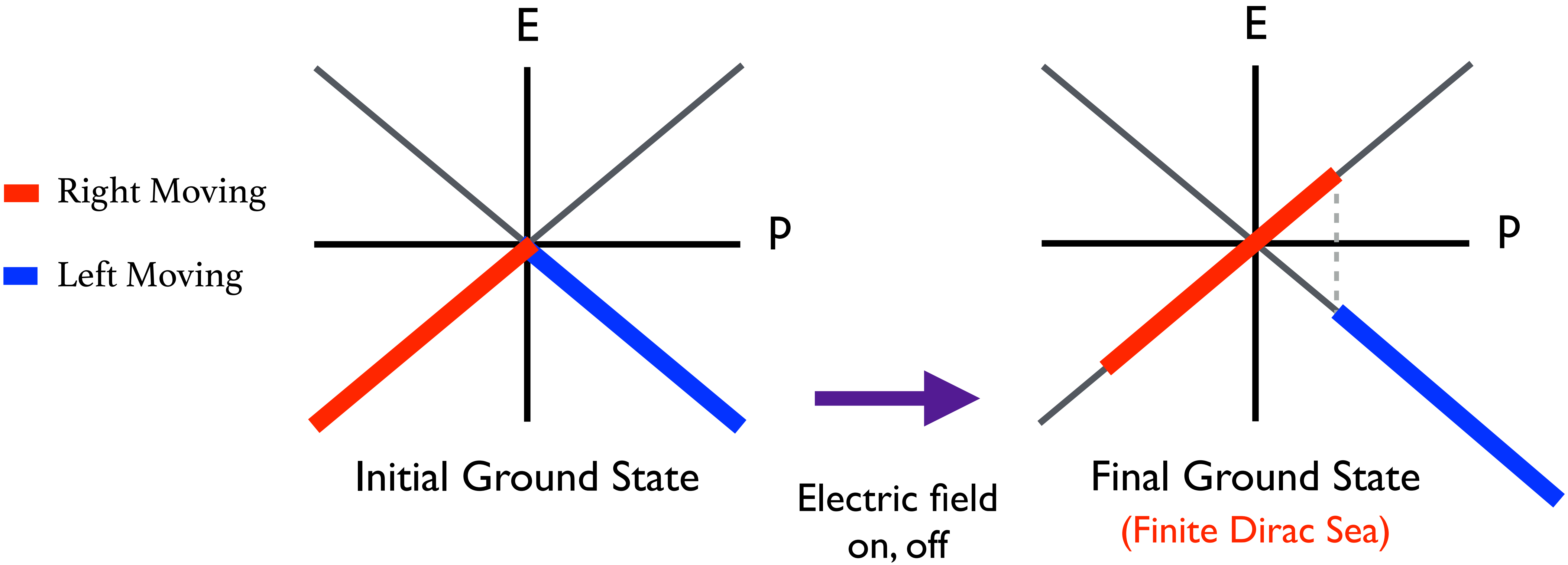

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Anomalies require infinite number of degrees of freedom

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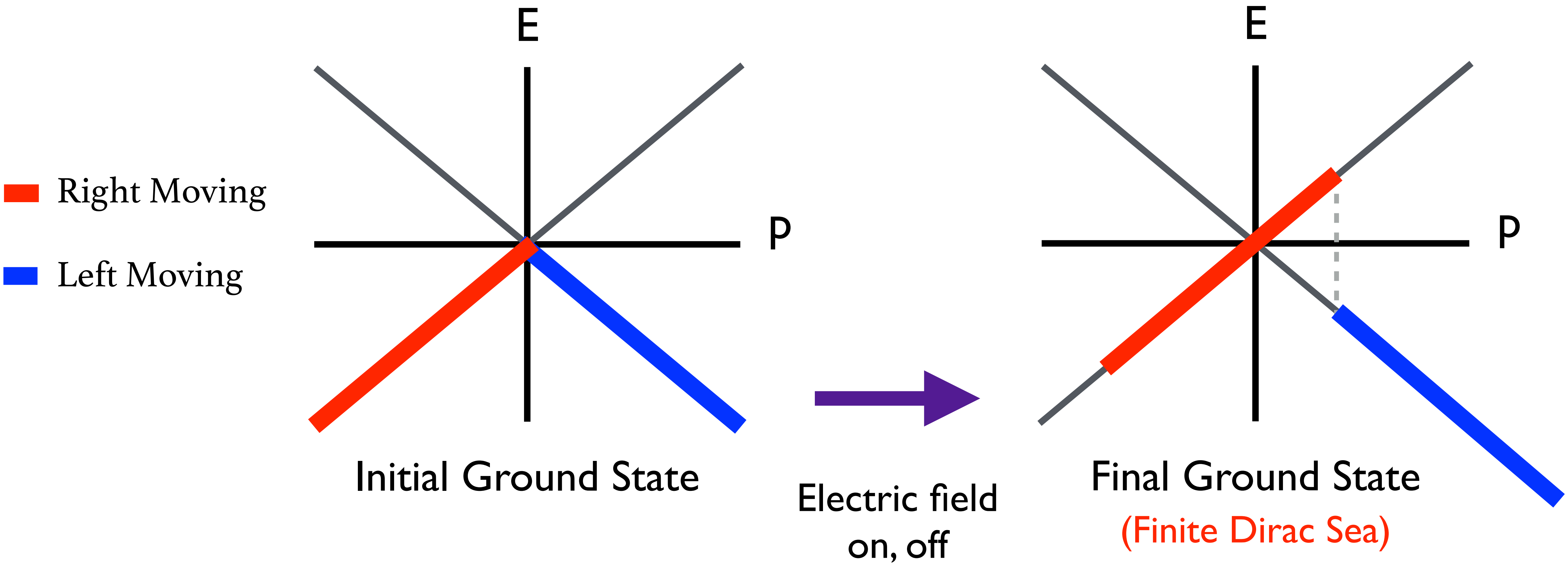

- $U(1)_V$ preserved
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Anomalies require infinite number of degrees of freedom

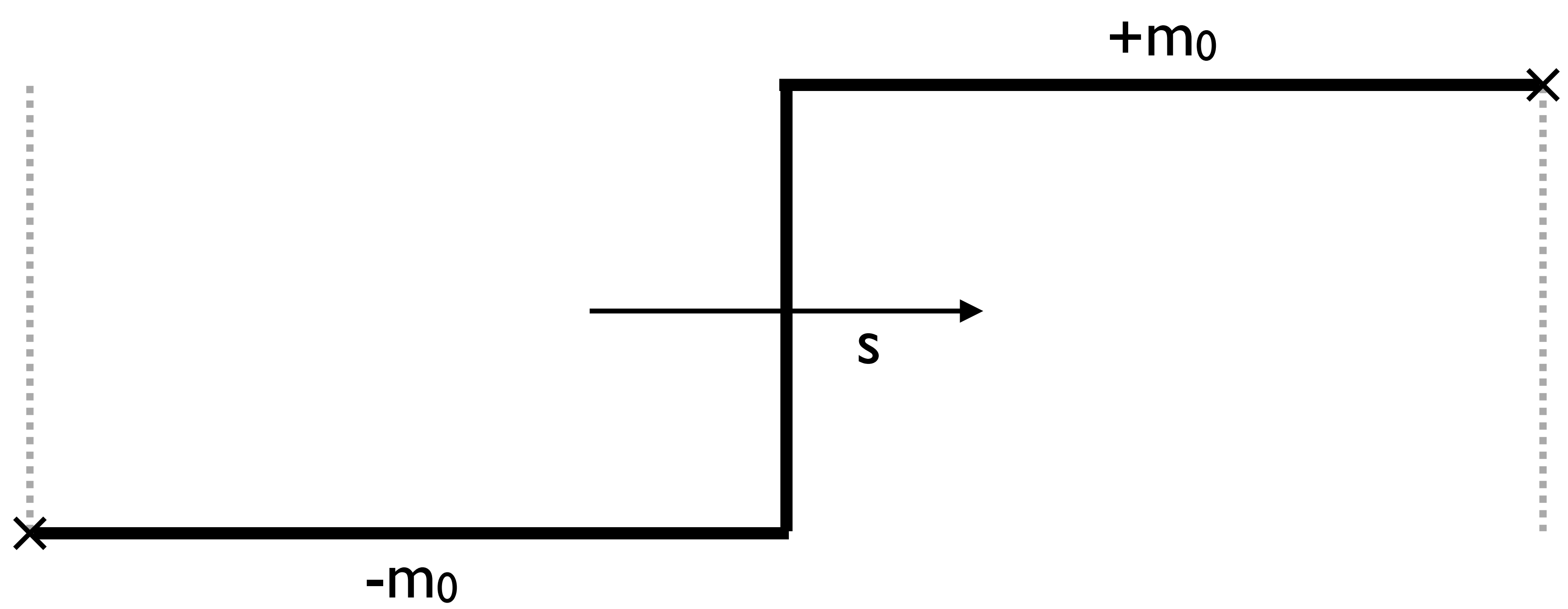
The lattice always has a finite number of degrees of freedom

Domain Wall Fermions

Extra Dimension

Idea: Utilize an extra dimension to localize light fermion modes on the boundaries

$$\mathcal{L}_F = \bar{\Psi}(\mathbf{x}, s)\left[\gamma^\mu D_\mu(\mathbf{x}) + \Gamma\partial_s + m(s)\right]\Psi(\mathbf{x}, s)$$



Kaplan, arXiv: 0912.2560

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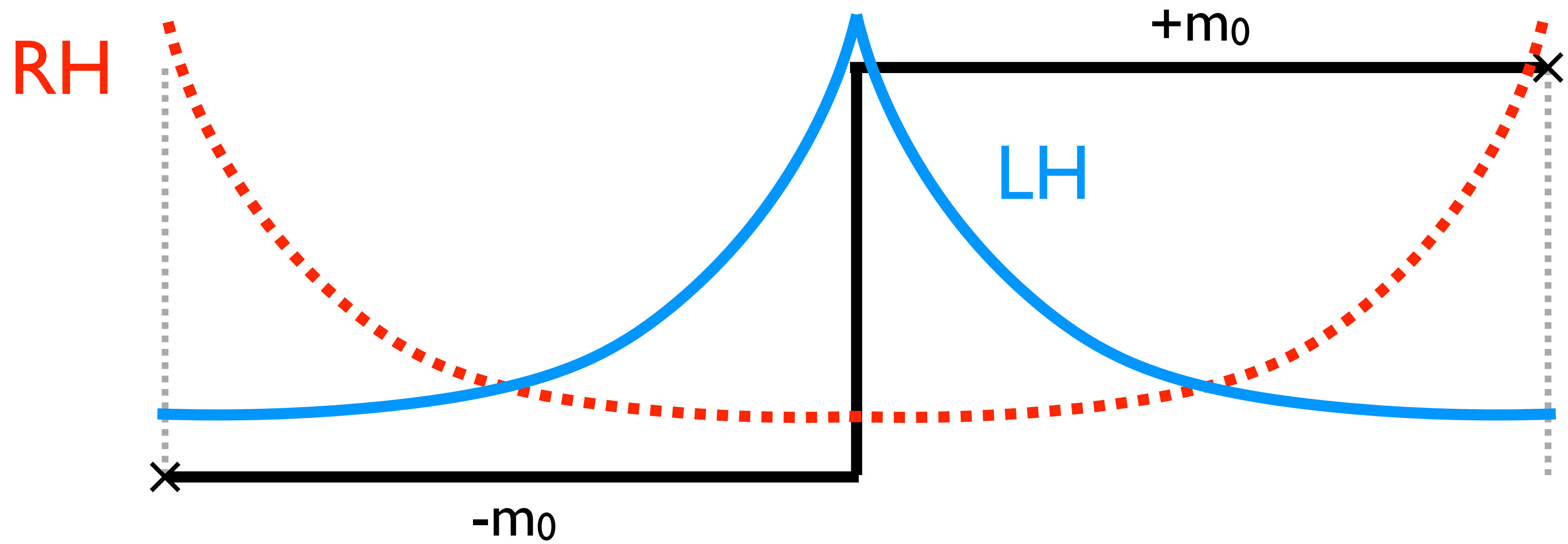

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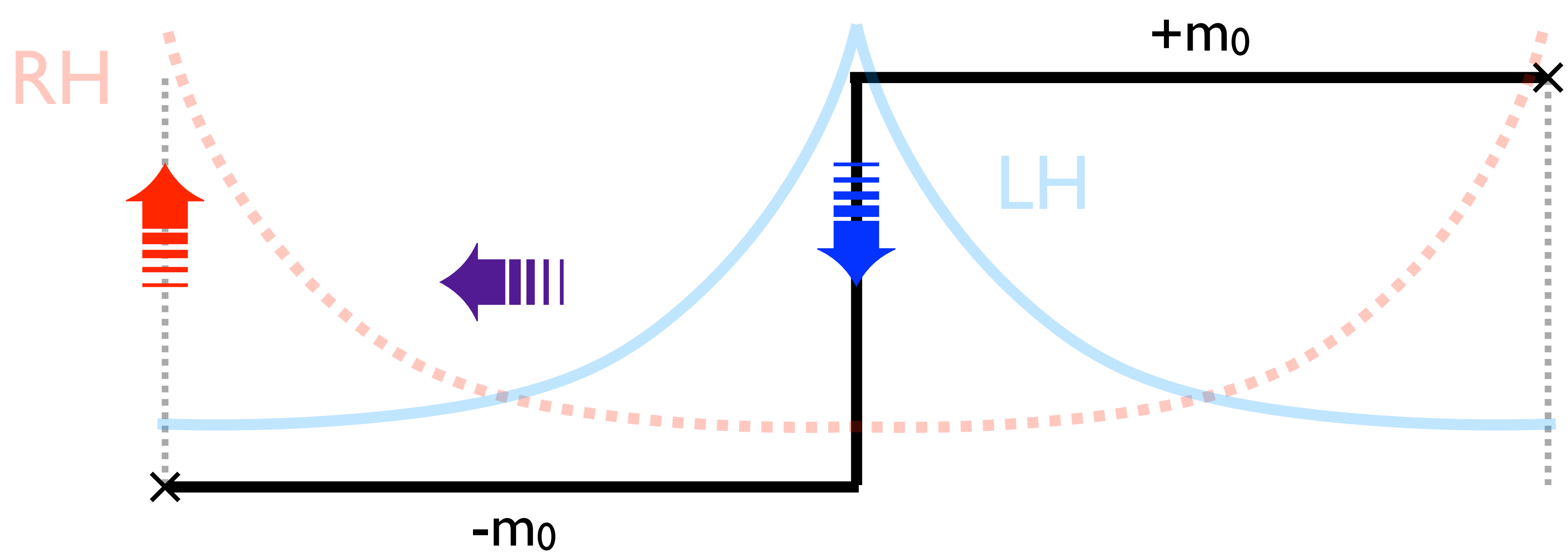


$U(1)_A$ is broken ***explicitly*** by particles flowing between walls

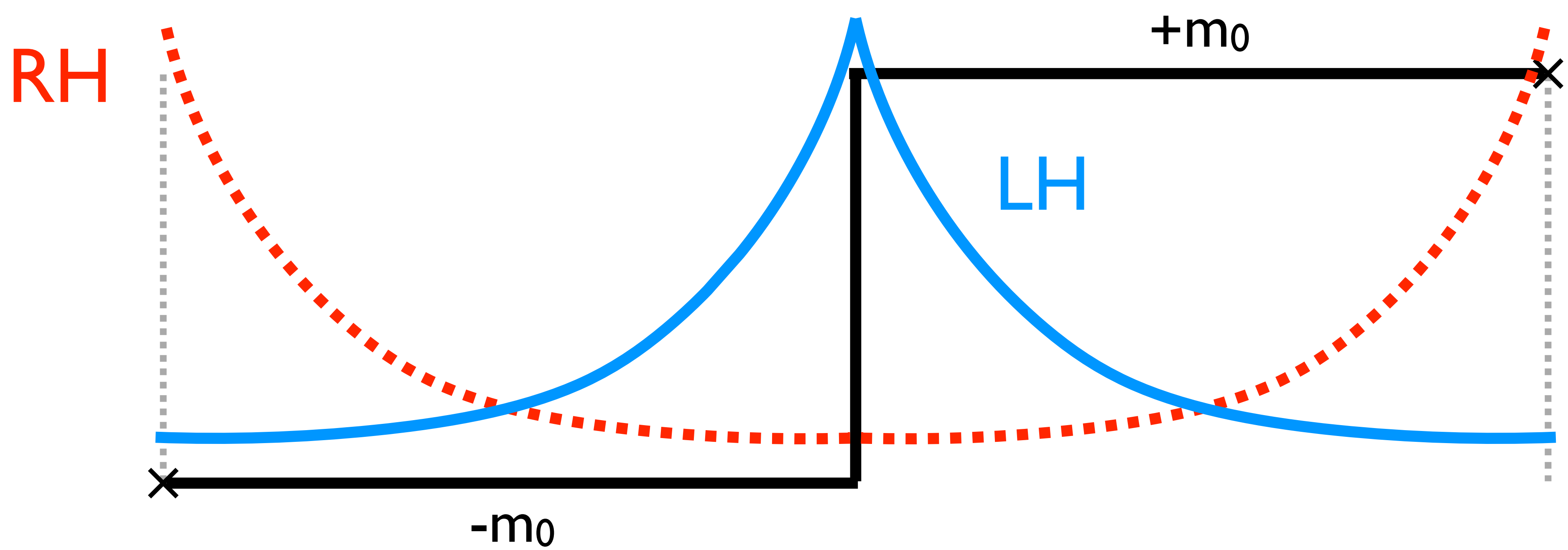
Kaplan, arXiv: 0912.2560

Flowed Domain Wall Fermions

Chiral Gauge Theories on the Lattice

Idea: Decouple unwanted chiral modes by turning off gauge field in gauge invariant way

$$\partial_s \mathcal{A}_\mu(\mathbf{x}, s) = \tau \mathrm{sgn}(s) \mathcal{D}_\nu \mathcal{F}_{\mu\nu}$$



*DMG + Kaplan arXiv: 1610.02151, 1511.03649

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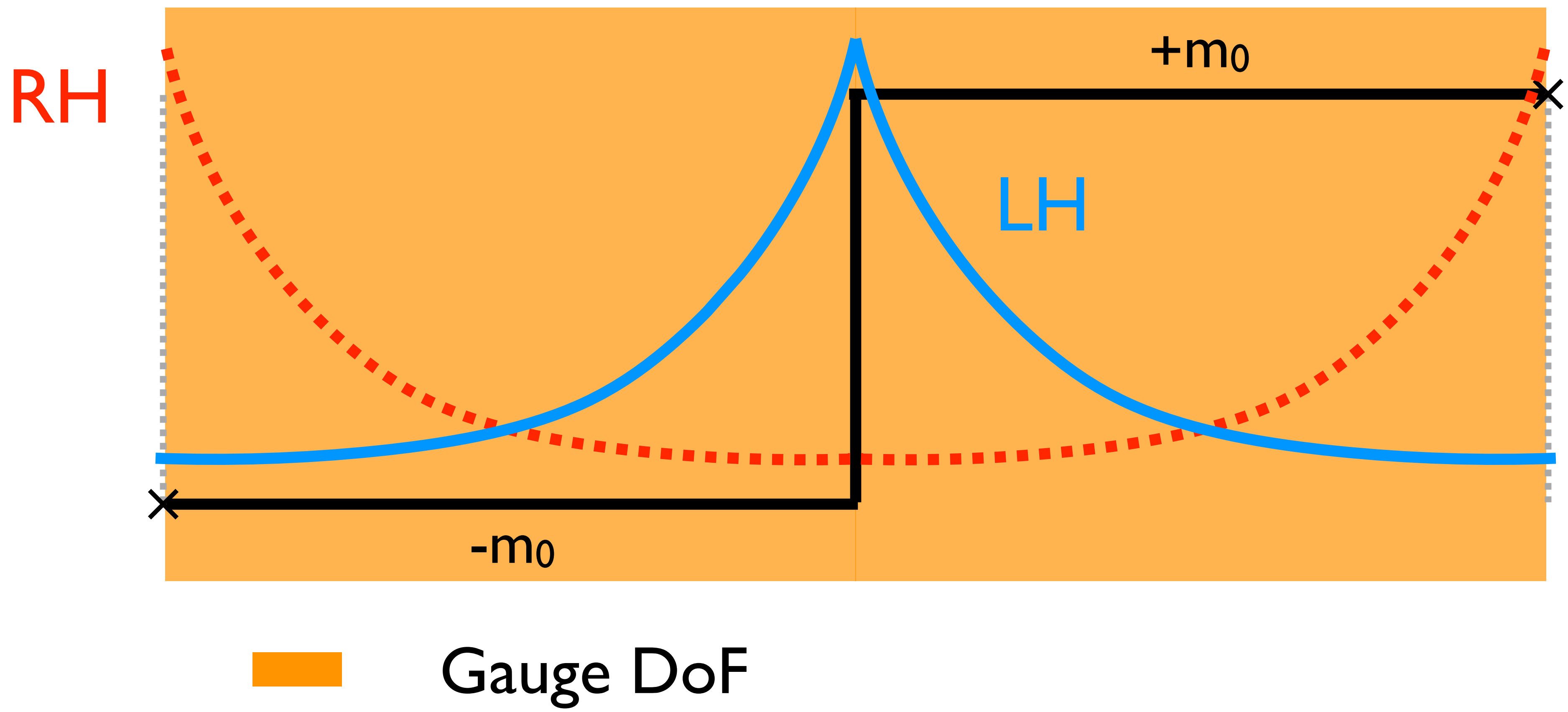

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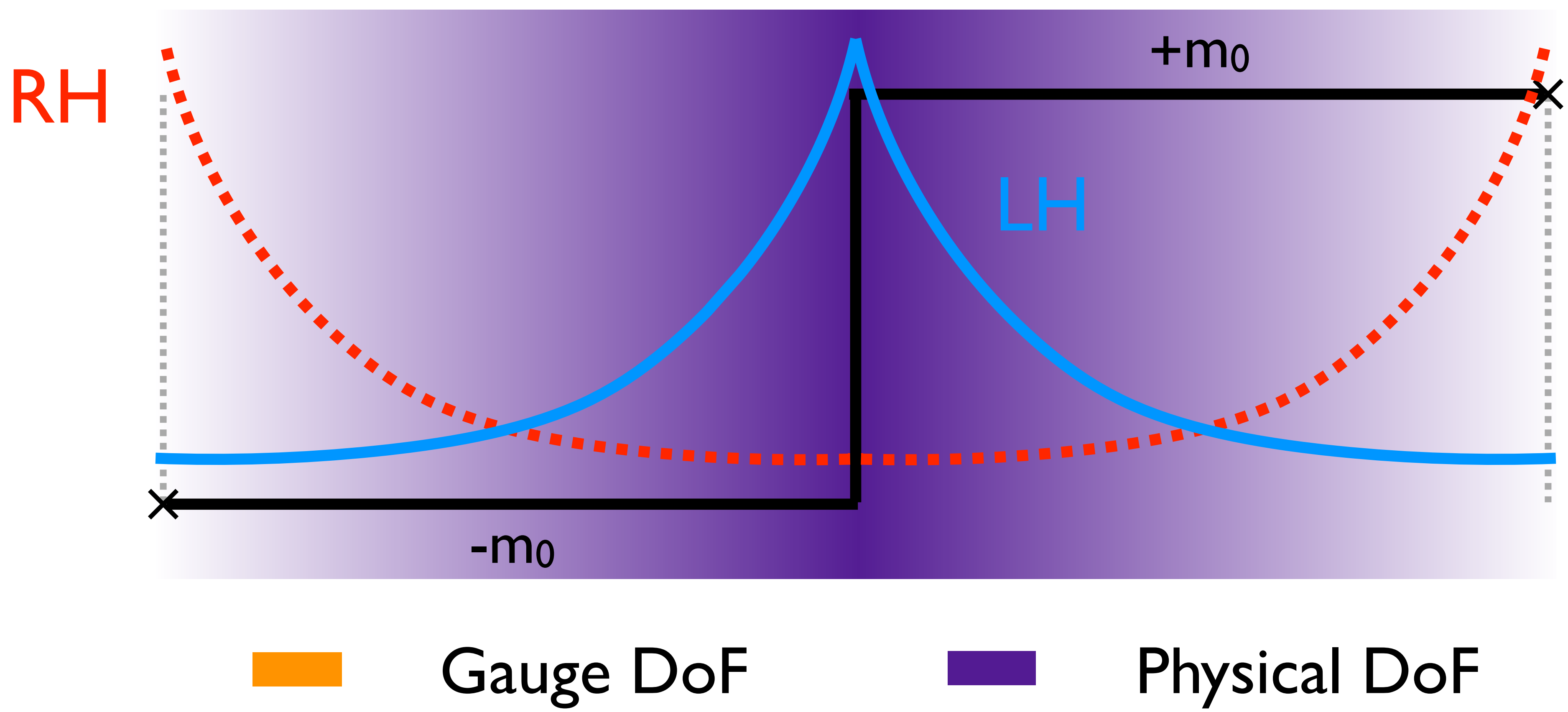

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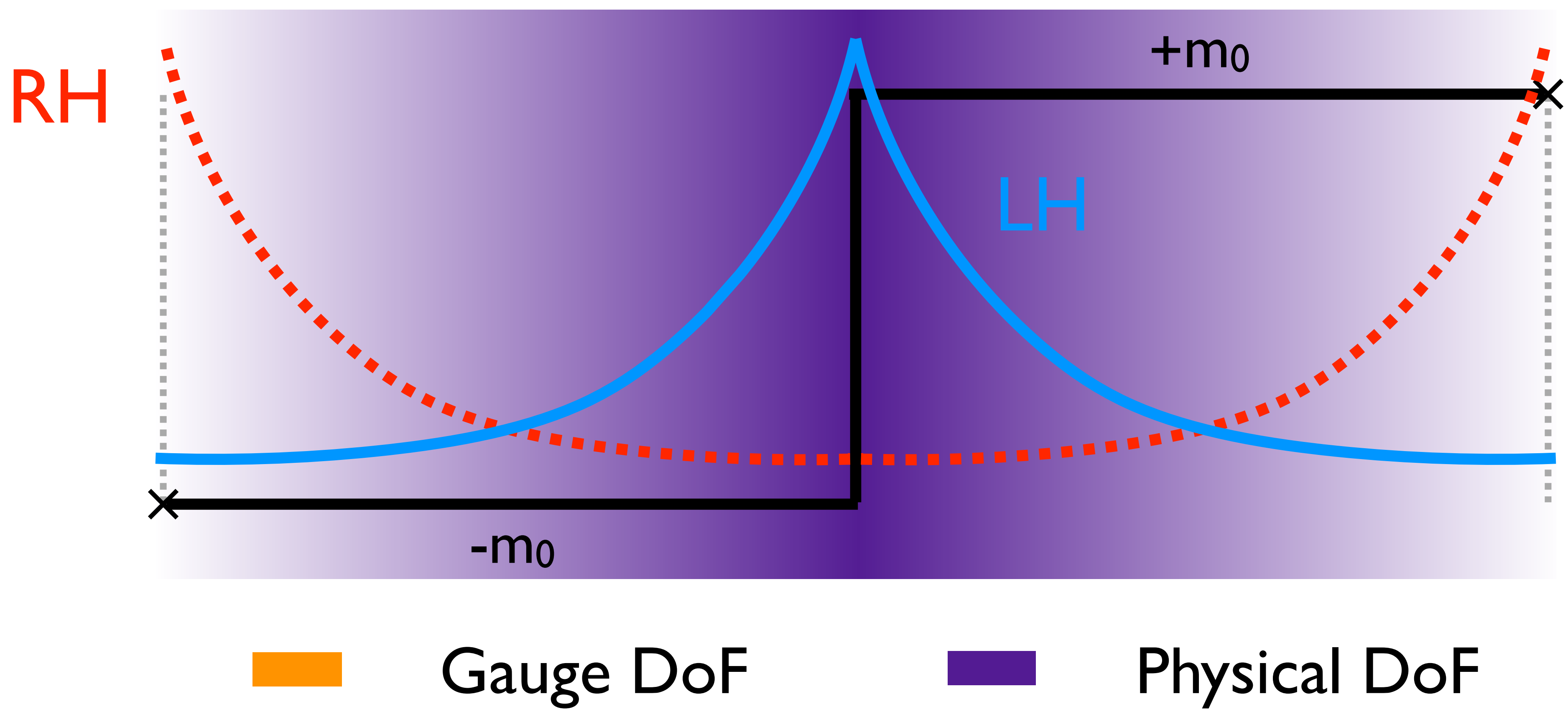


Important open questions remain

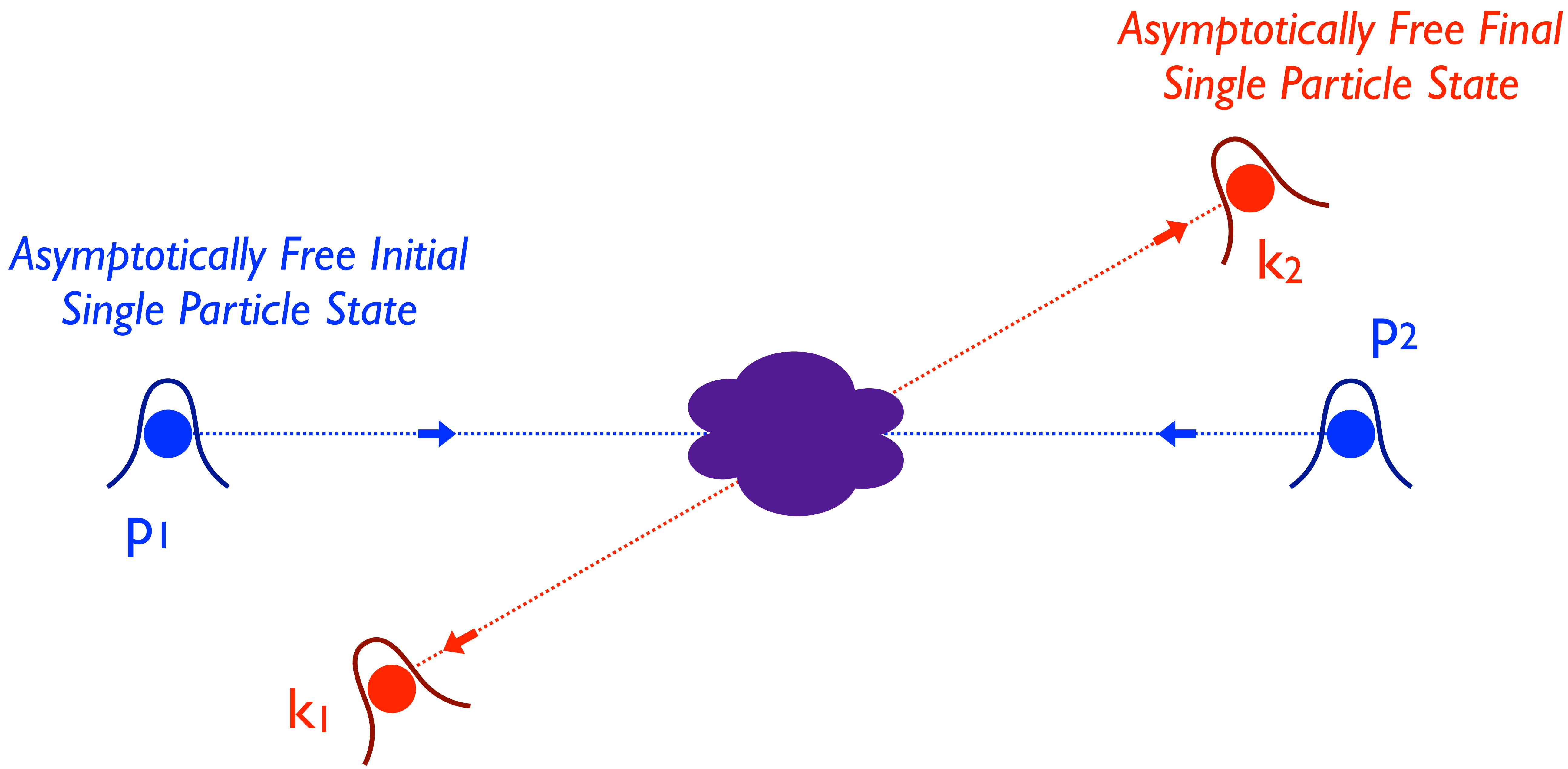
**DMG + Kaplan arXiv: 1610.02151, 1511.03649*

Numerical Limitations

Finite Volume Effects

QFT Picture for Scattering

Infinite Volume

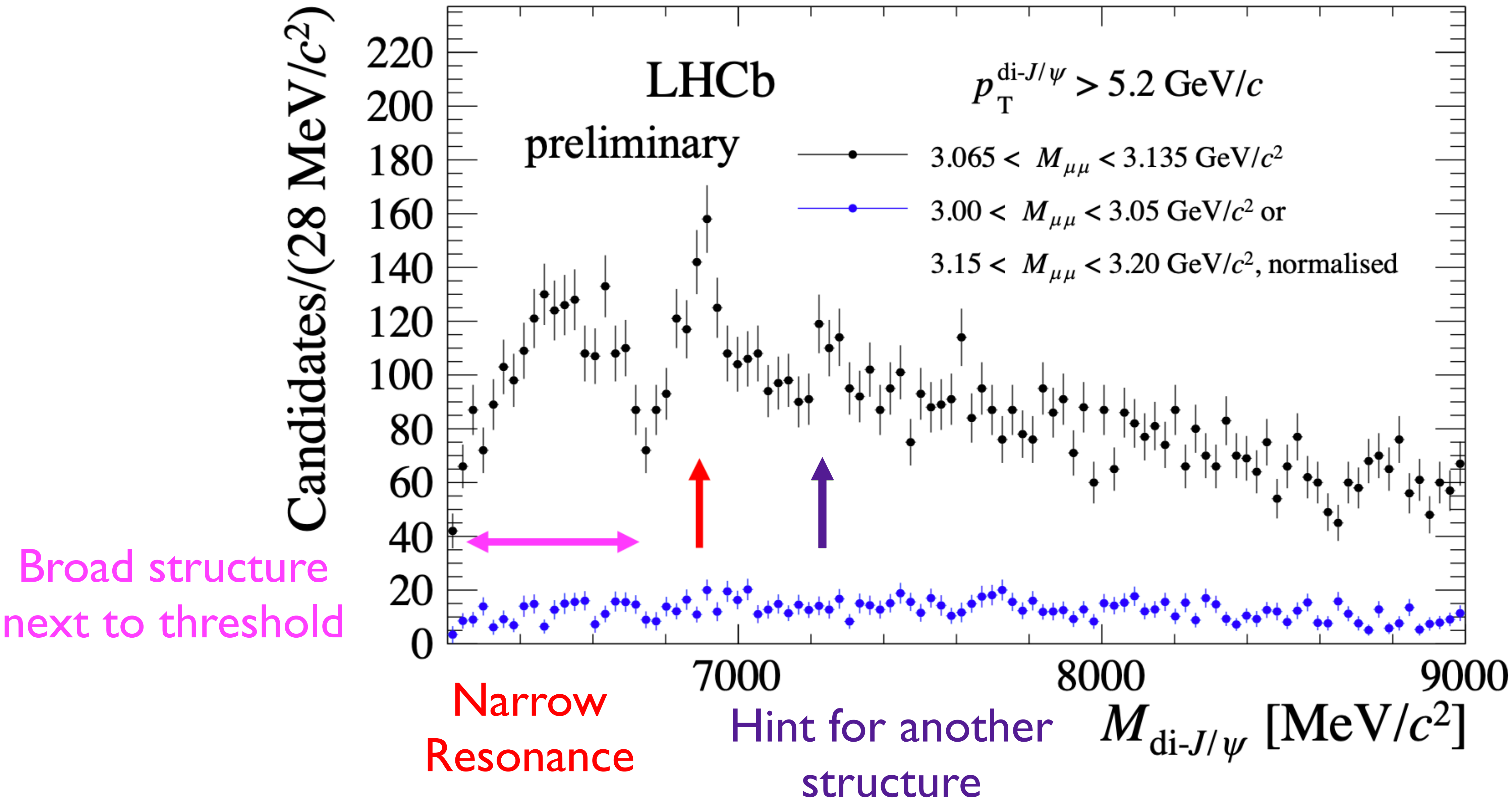


Time Dependent Phenomena

Scattering Experiments

J/ψ-pair mass spectrum

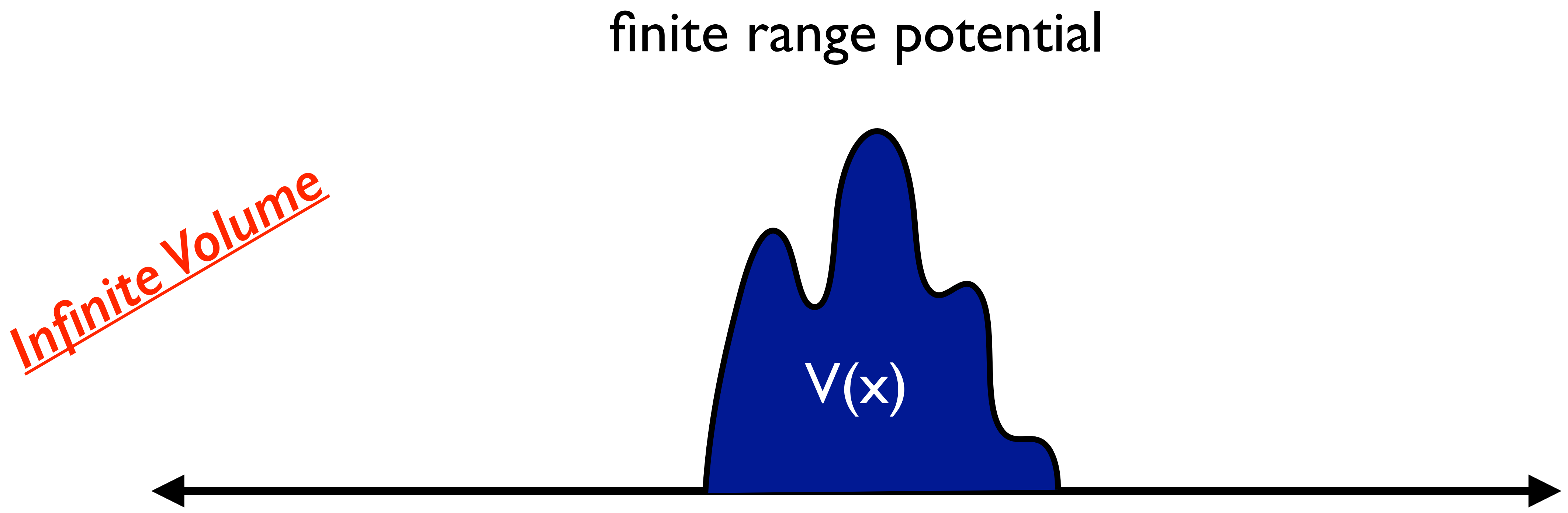
LHCb, arχiv: 2006.16957



Extraction of scattering information from experimental data requires an assumption about the spectral function form

Scattering With Boundary Conditions

Toy Model: elastic scattering of two identical spinless bosons in 1D in non-relativistic quantum mechanics



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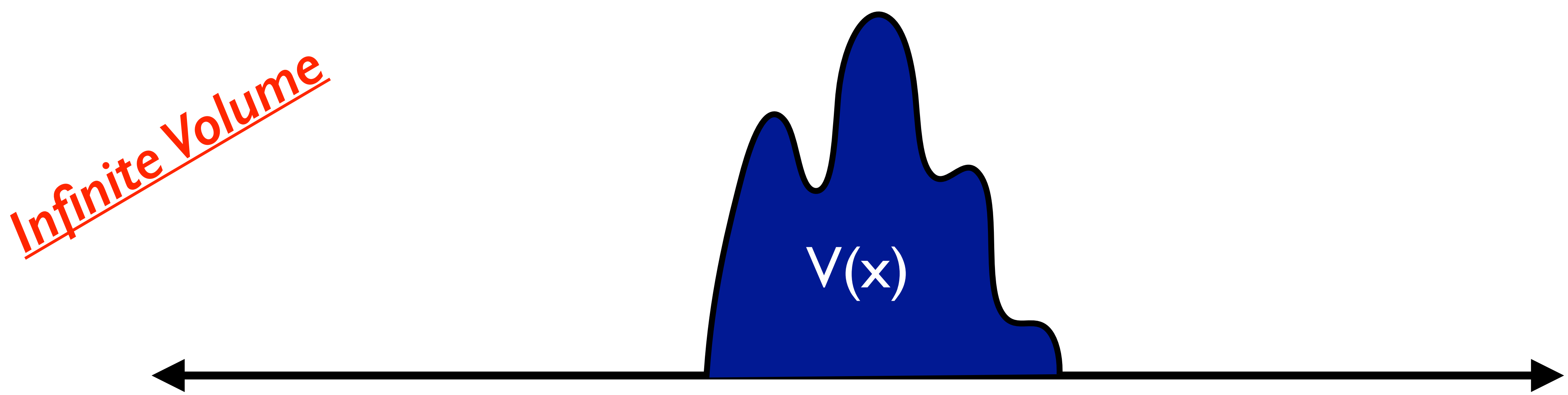
finite range potential



$$\psi(x) \sim \cos\left[p|x| + \boxed{\delta(p)}\right]$$

Scattering Phase Shift

Infinite Volume: No constraint on p

$$p \in \mathbb{R}$$

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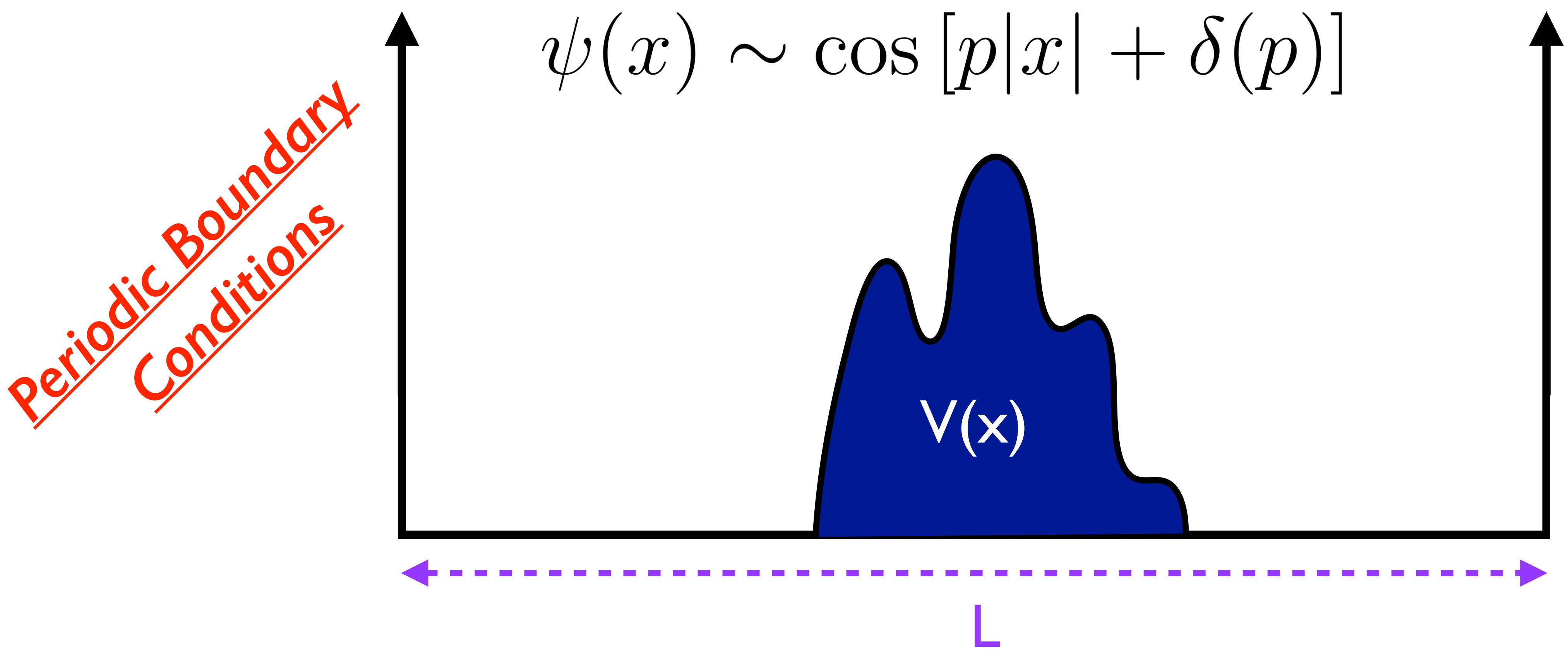

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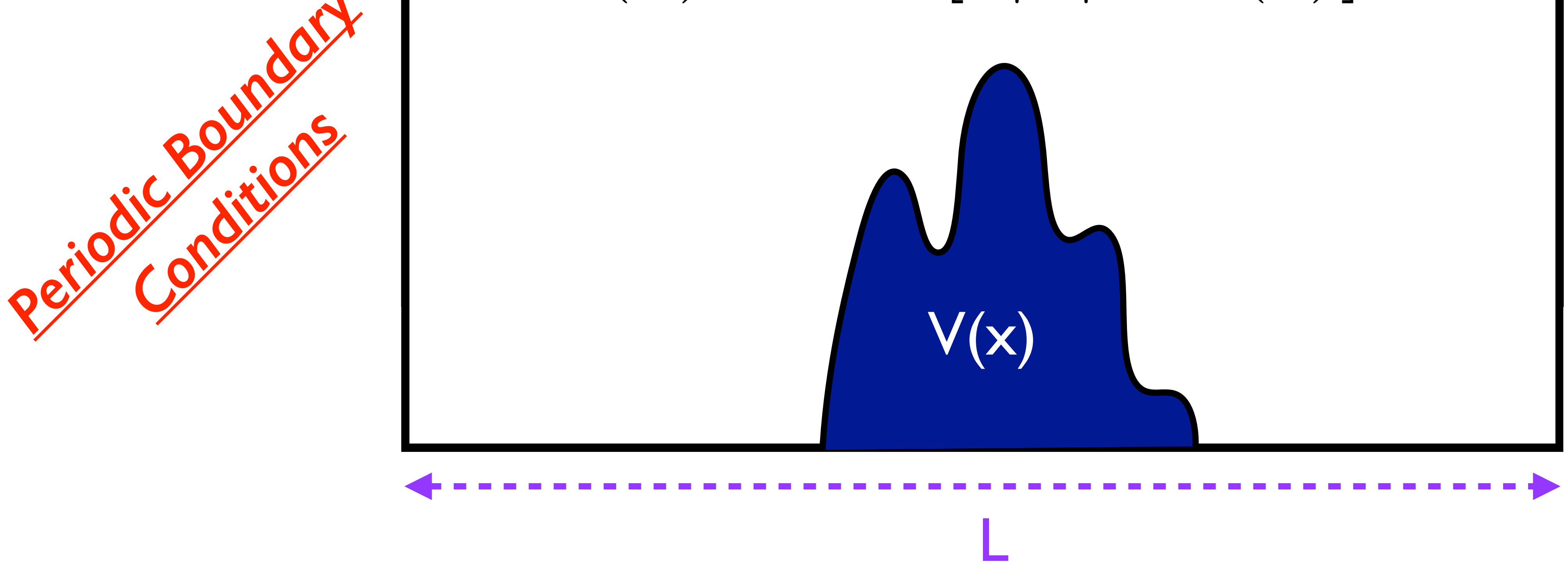


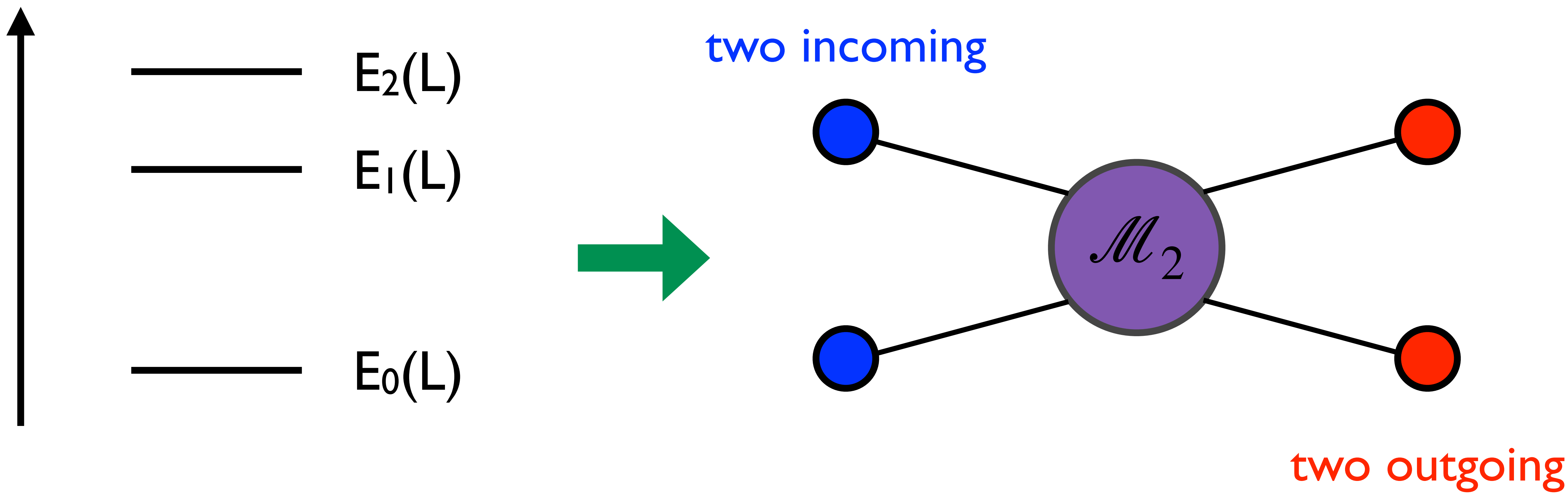
Finite Volume: Allowed momenta depend on both infinite volume scattering phase shift and finite volume size

$$\psi(x) = \psi(x+L) \longrightarrow p = \frac{2\pi}{L}n + \frac{2\delta(p)}{L}$$

Quantization Condition

Two to Two

Idea: Energy levels of particles in a box map to infinite volume scattering amplitude



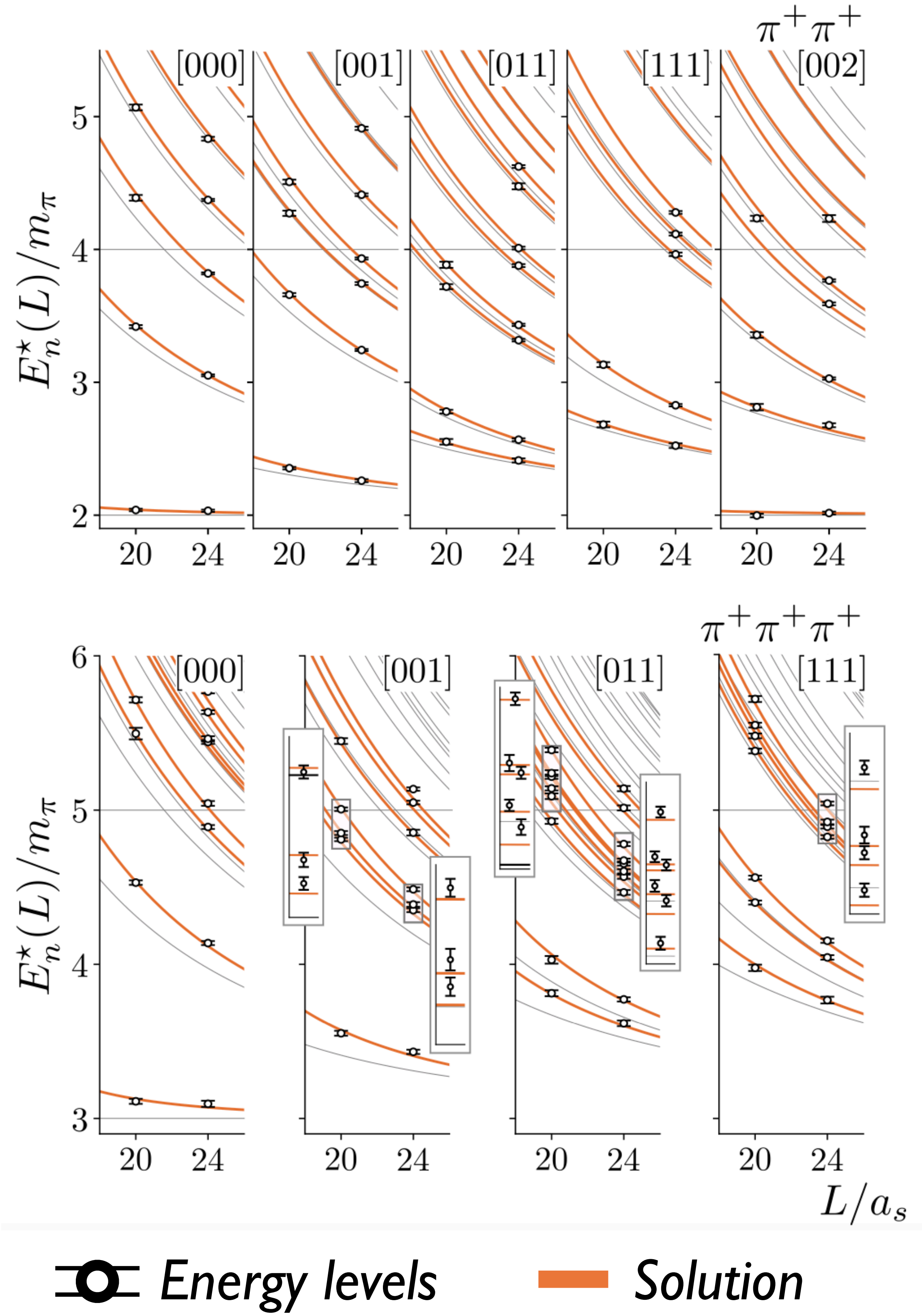
Solution for Two to Two: Discrete energy levels satisfy the Luscher quantization condition

$$\det\left[1 + \mathcal{M}_2 F_2\right] = 0$$

Luscher, Commun. Math.Phys. 105 (1986), Nucl.Phys.B 364 (1991)

Lattice Energy Levels

HadSpec arXiv: 2009.04931



Must solve quantization condition to extract scattering information

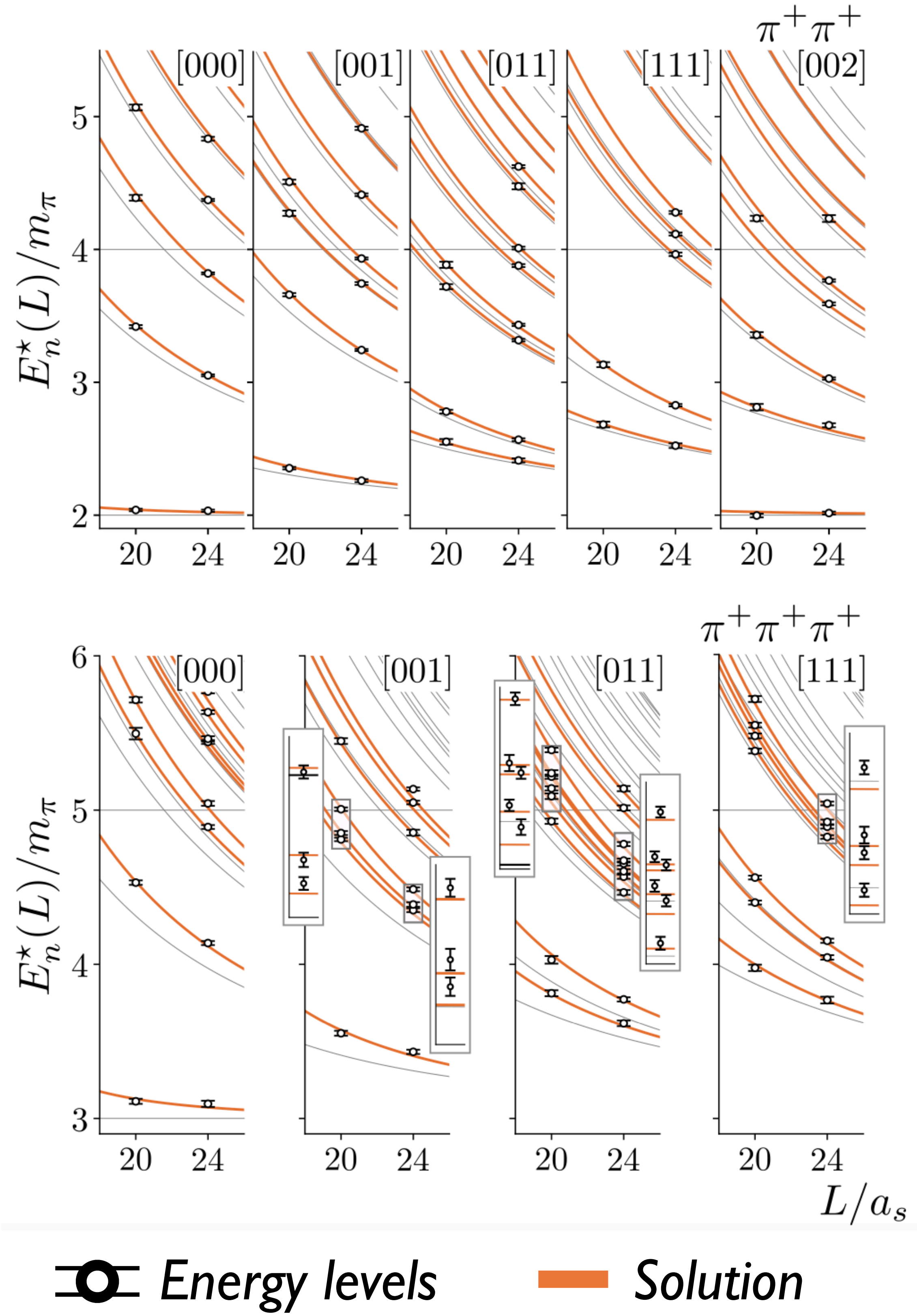
(Usually numerically)

Note that $\mathscr{M}_n$ and F_n are infinite dimensional matrices

**DMG + Hansen, to appear*

Lattice Energy Levels

HadSpec arXiv: 2009.04931



Must solve quantization condition to extract scattering information

(Usually numerically)

Note that $\mathscr{M}_n$ and F_n are infinite dimensional matrices

Current work

Solve quantization condition to have analytic control over energy splitting

- Two to Two and Three to Three
- Center of Mass and Moving Frame

**DMG + Hansen, to appear*

Conclusions

Dual Sides of the Lattice

First Main Message

The lattice provides both a deeper theoretical understanding of quantum field theories and the only way to extract many numbers of experimental relevance

Second Main Message

Life on the lattice is full of complications that must be overcome, but it is incredibly worthwhile to do so