SOMMERFELD EFFECT AT THE NEXT-TO-LEADING ORDER FOR WINO DARK MATTER

Robert Szafron

CERN Theory Department

18 November 2020

OVERVIEW

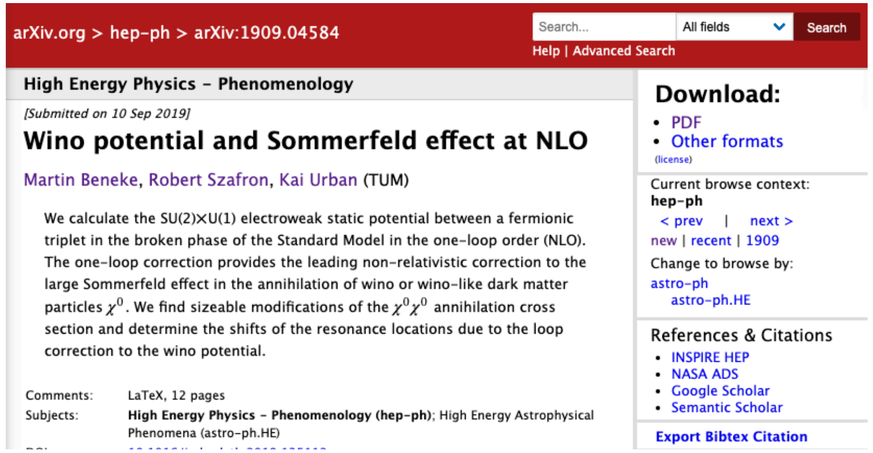
- Introduction
- NREFT approach
- Corrections to the potential
- Sommerfeld effect at NLO
- Relic density

Based on:

1909.04584 and 2009.00640

with

Martin Beneke and Kai Urban



arXiv.org > hep-ph > arXiv:1909.04584

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High Energy Physics – Phenomenology

[Submitted on 10 Sep 2019]

Wino potential and Sommerfeld effect at NLO

Martin Beneke, Robert Szafron, Kai Urban (TUM)

We calculate the SU(2)×U(1) electroweak static potential between a fermionic triplet in the broken phase of the Standard Model in the one-loop order (NLO). The one-loop correction provides the leading non-relativistic correction to the large Sommerfeld effect in the annihilation of wino or wino-like dark matter particles χ^0. We find sizeable modifications of the $\chi^0\chi^0$ annihilation cross section and determine the shifts of the resonance locations due to the loop correction to the wino potential.

Comments: LaTeX, 12 pages

Subjects: **High Energy Physics – Phenomenology (hep-ph)**; High Energy Astrophysical Phenomena (astro-ph.HE)

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High Energy Physics – Phenomenology

[Submitted on 1 Sep 2020]

Sommerfeld-corrected relic abundance of wino dark matter with NLO electroweak potentials

Martin Beneke, Robert Szafron, Kai Urban

Extending previous work, we calculate the electroweak potentials for all co-annihilation channels of wino dark matter at the one-loop order and obtain the wino relic abundance including the Sommerfeld effect at the next-to-leading order (NLO).

Comments: 48 pages, LaTeX

Subjects: **High Energy Physics – Phenomenology (hep-ph)**

Report number: TUM-HEP-1281/20, CERN-TH-2020-144

Cite as: arXiv:2009.00640 **[hep-ph]**

(or arXiv:2009.00640v1 **[hep-ph]** for this version)

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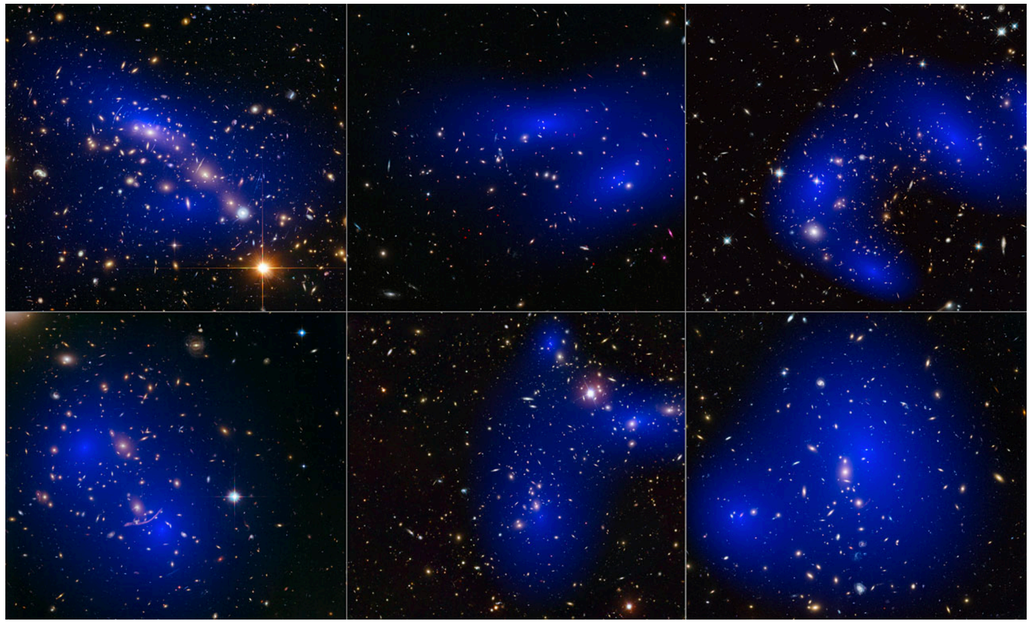
DARK MATTER?

- We know it constitutes about 80% of the matter in the observable universe
- We don’t see it (doesn't emit/absorb light)
- It interacts gravitationally
- We don't have a candidate for DM in the Standard Model



[NASA/ESA]

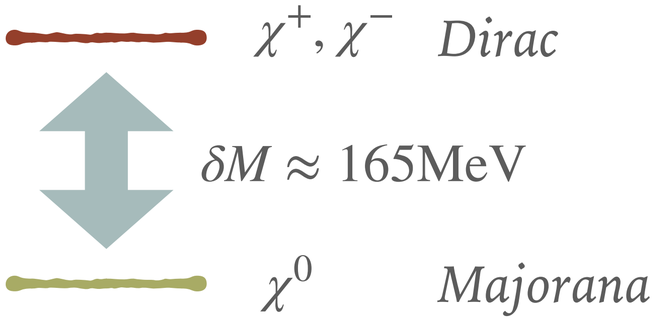
THE WINO-LIKE DM

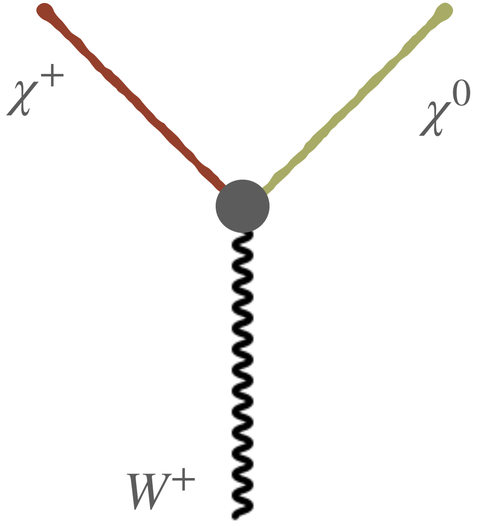
Minimal models: fermionic electroweak doublet ("pure Higgsino"), triplet ("pure Wino"), quintuplet, ...

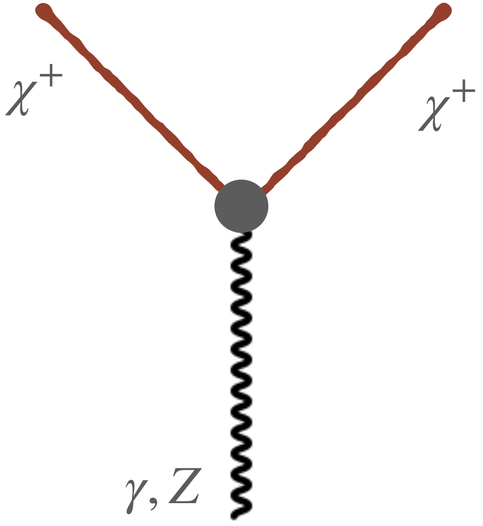
We consider a fermionic Majorana triplet of SM $SU(2)$ (Wino-like DM)

$$\mathscr{L}_{\mathrm{DM}} = \frac{1}{2}\bar{\chi}\left(i\gamma^{\mu}D_{\mu} - M_{\chi}\right)\chi$$

After EW symmetry breaking:







WHY WINO?

- Simple and nice model which doesn't require new interactions
- Mass is the only new parameter (TeV range)
- Motivated by MSSM
- Great playground to understand physics of WIMP before going to more complicated models with complete dark sector having its own force carriers …

Only weak coupling, but the annihilation cross section at LO (Born level) is unreliable — needs EFT approach

See also

Beneke et al. (2016)

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What can we learn from wino?

See also

Beneke et al. (2016)

INDIRECT DETECTION AND RELIC DENSITY

We are going to look at two observables

Relic Density

$$\frac{dn}{dt} + 3Hn = -\langle\sigma v\rangle_{\text{eff}}(n^2 - n_{\text{eq}}^2)$$

Thermally averaged, fully inclusive cross section

Freeze-out time, $T \sim M_\chi/25$

Hisano et al. (2006)
Beneke et al. (2016)

Cosmic Rays

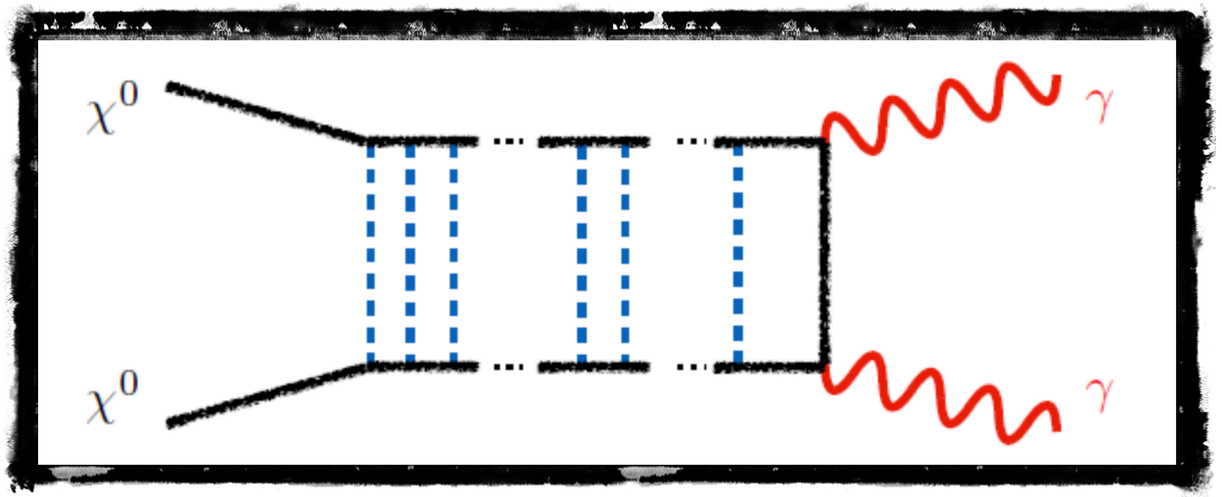
$$\Phi(E_\gamma) = \frac{1}{8\pi M_\chi^2}\int_{\Delta\Omega} d\Omega \int ds \rho_{DM}^2(\mathbf{r}(s)) \frac{d}{dE_\gamma}[\sigma v]_{\chi\chi\to\gamma X}$$

Exclusive cross section

Non-relativistic, $v \sim 10^{-3}$

Hryczuk et al. (2019)
Rinchiuso et al. (2020)

EW SOMMERFELD EFFECT



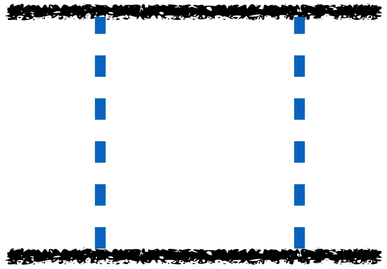
Hisano et al. (2004,2006)

For massless gauge bosons

$$S(v) = \frac{\pi\alpha}{v}\frac{1}{1-e^{-\pi\alpha/v}}$$

For massive gauge bosons enhancement when $v \lesssim \frac{m_W}{M_\chi}$

Each loop gives



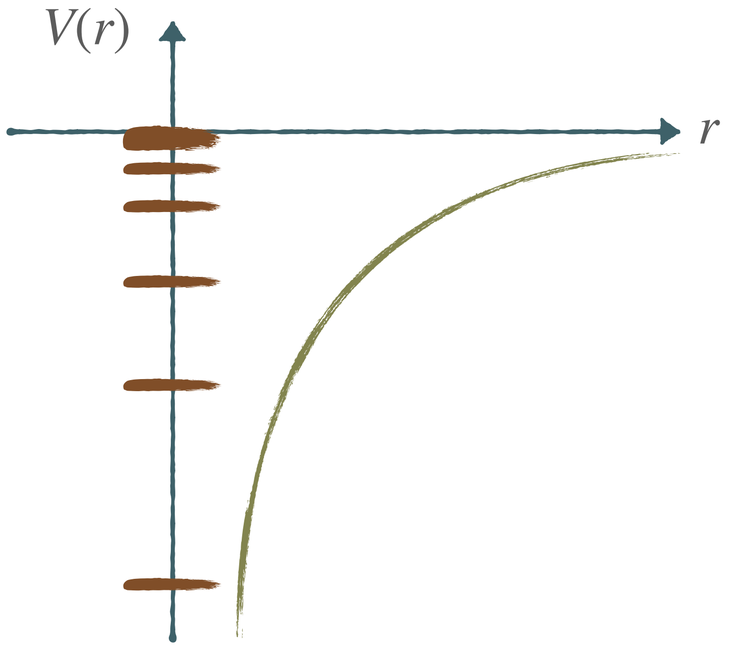
$$\alpha_2 \frac{M_\chi}{m_W} \sim 1$$

Need to sum to all orders!

Contrary to the massless case, the enhancement saturates at small velocities $\sim \min\left(\frac{1}{v}, \frac{M_\chi}{m_W}\right)$

Except near the resonance

RESONANCE FOR THE YUKAWA POTENTIAL



RESONANCE FOR THE YUKAWA POTENTIAL

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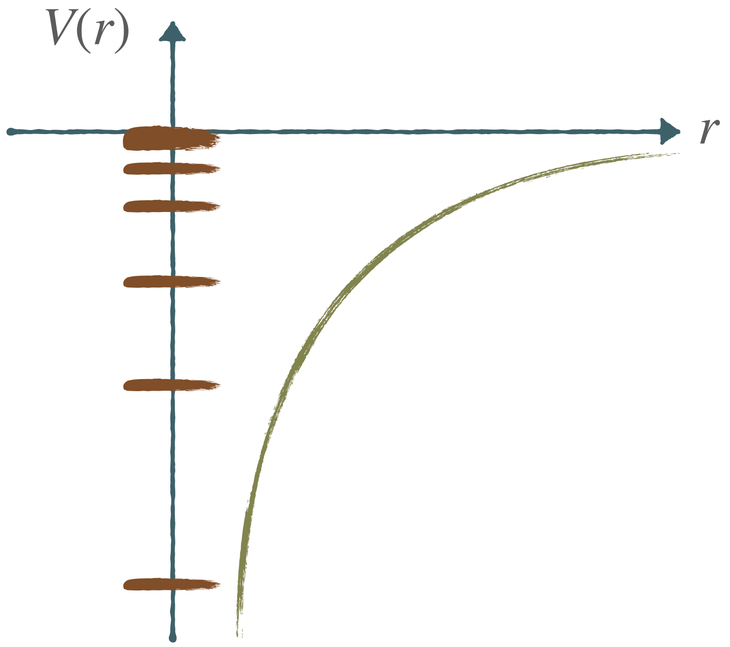
  $E_n \sim \frac{1}{4} M_\chi \alpha_2^2/n^2$



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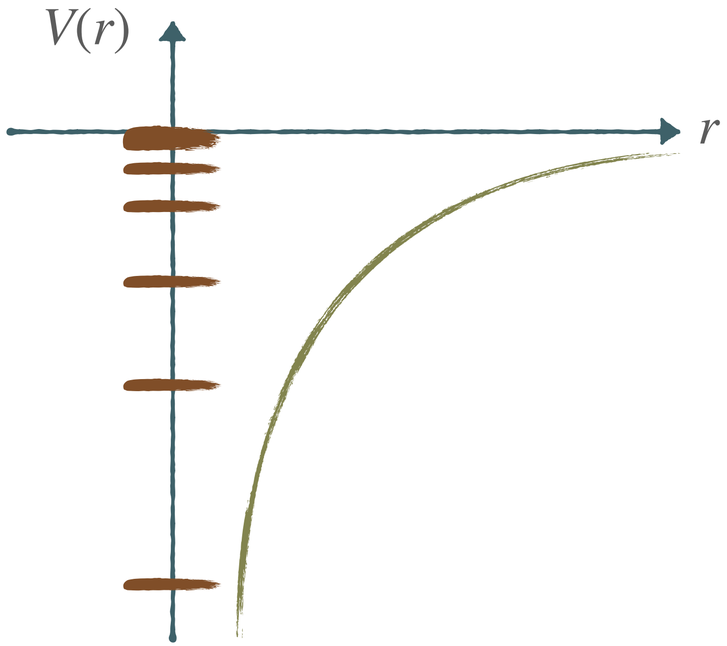

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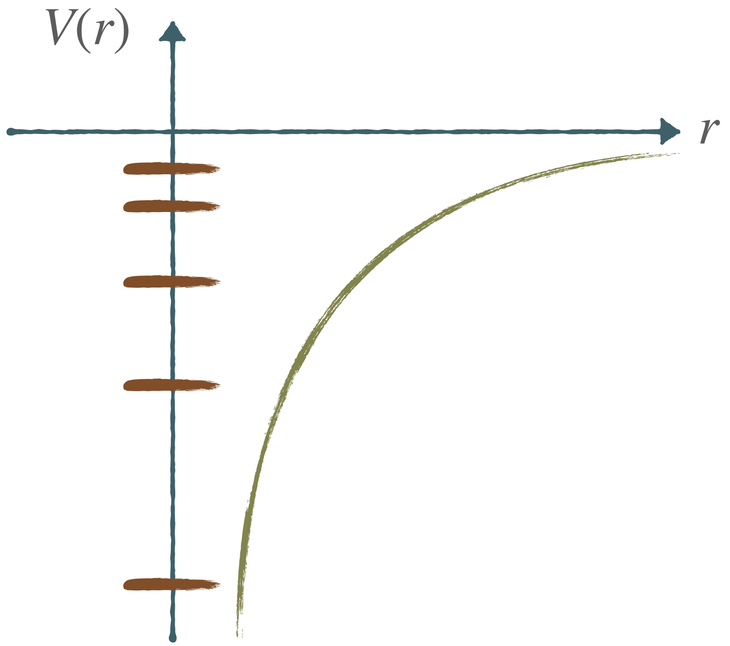

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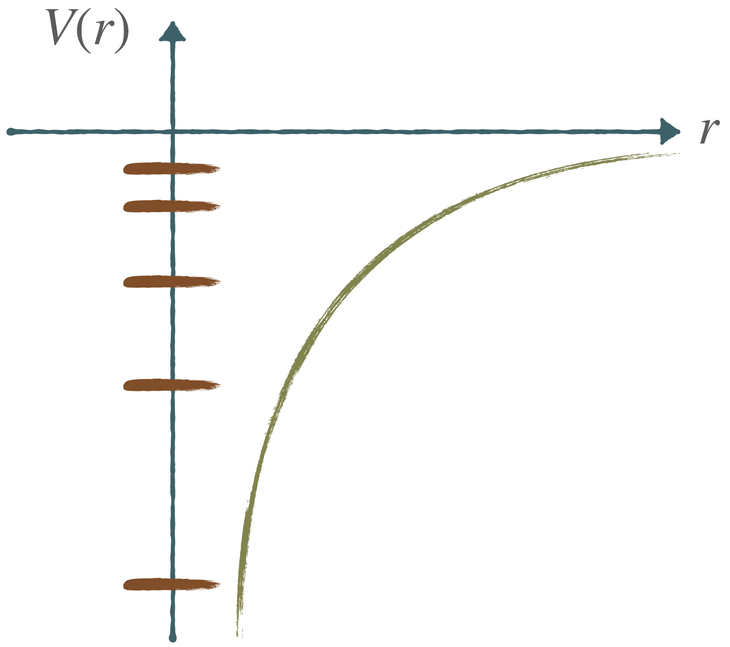

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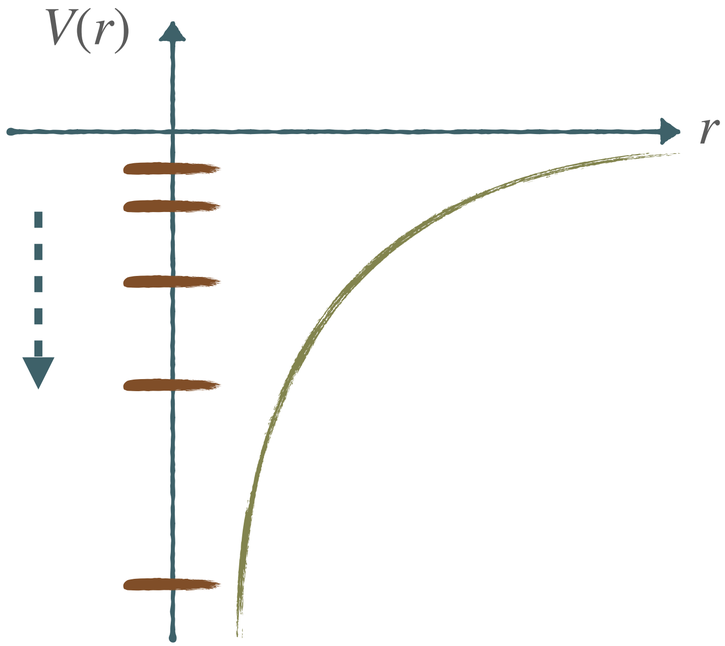

$$S \sim \frac{1}{E - E_b} \to \frac{1}{E} \sim \frac{1}{v^2}$$

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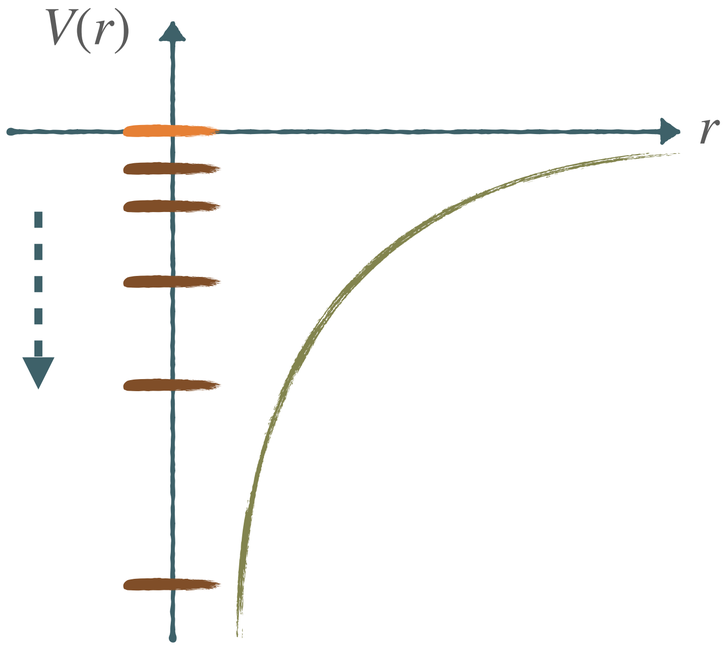
- As M_χ increases, new states enter the spectrum
- Zero-energy bound states for certain values of M_χ



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Resonant enhancement

EFT FRAMEWORK

- We focus on the non-relativistic part
- Expansion in velocity $v \ll 1$

Modes

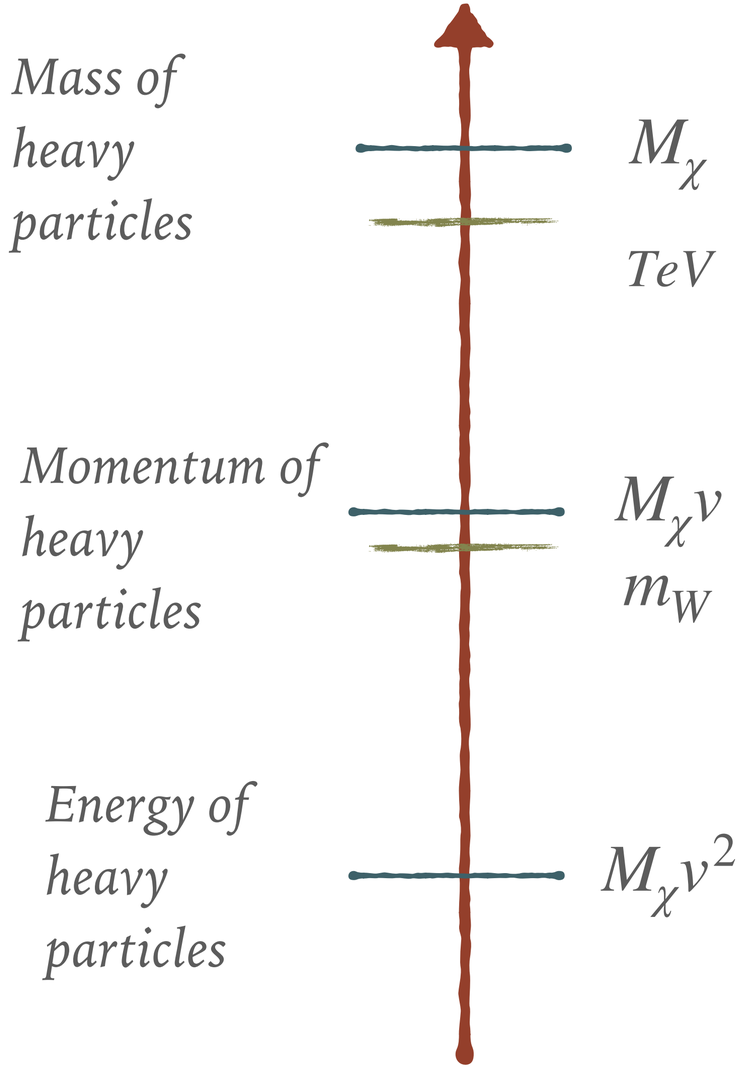
Hard $E \sim M_\chi, \;\; p \sim M_\chi$

Soft $E \sim vM_\chi, \;\; p \sim vM_\chi$

Potential $E \sim v^2 M_\chi, \;\; p \sim vM_\chi$

Ultra-soft $E \sim v^2 M_\chi, \;\; p \sim v^2 M_\chi$

Full Theory



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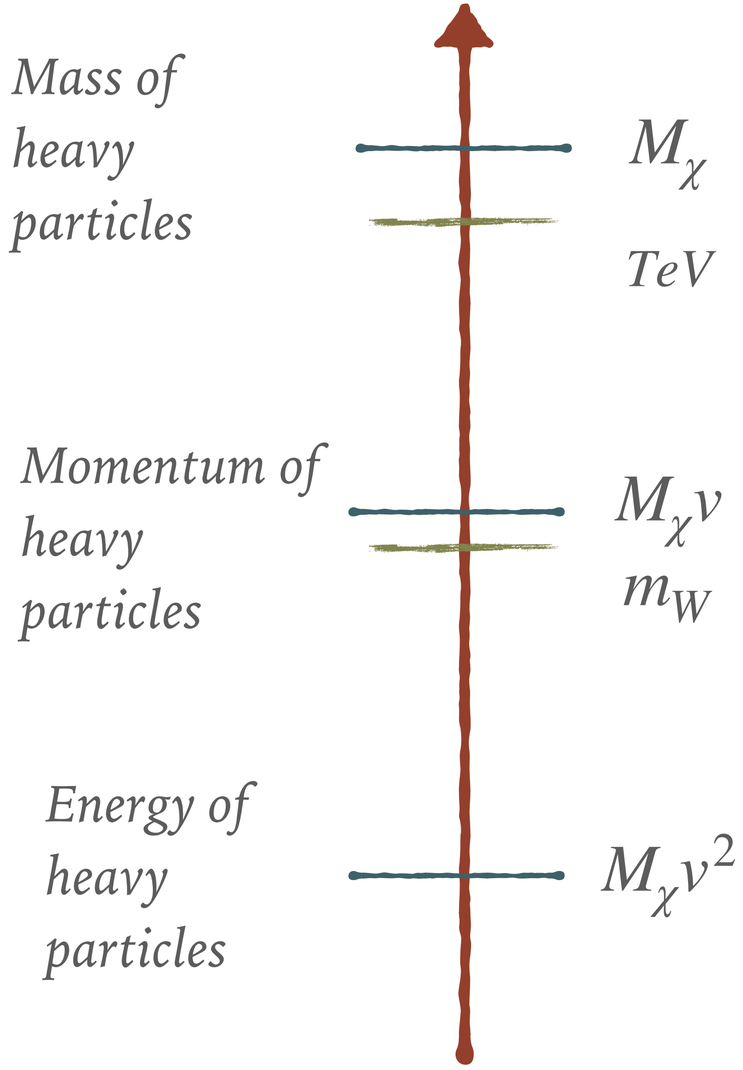
Soft $E \sim vM_\chi, \;\; p \sim vM_\chi$

Potential $E \sim v^2 M_\chi, \;\; p \sim vM_\chi$

Ultra-soft $E \sim v^2 M_\chi, \;\; p \sim v^2 M_\chi$

Full Theory ⇨ Non-relativistic EFT



EFT FRAMEWORK

- We focus on the non-relativistic part
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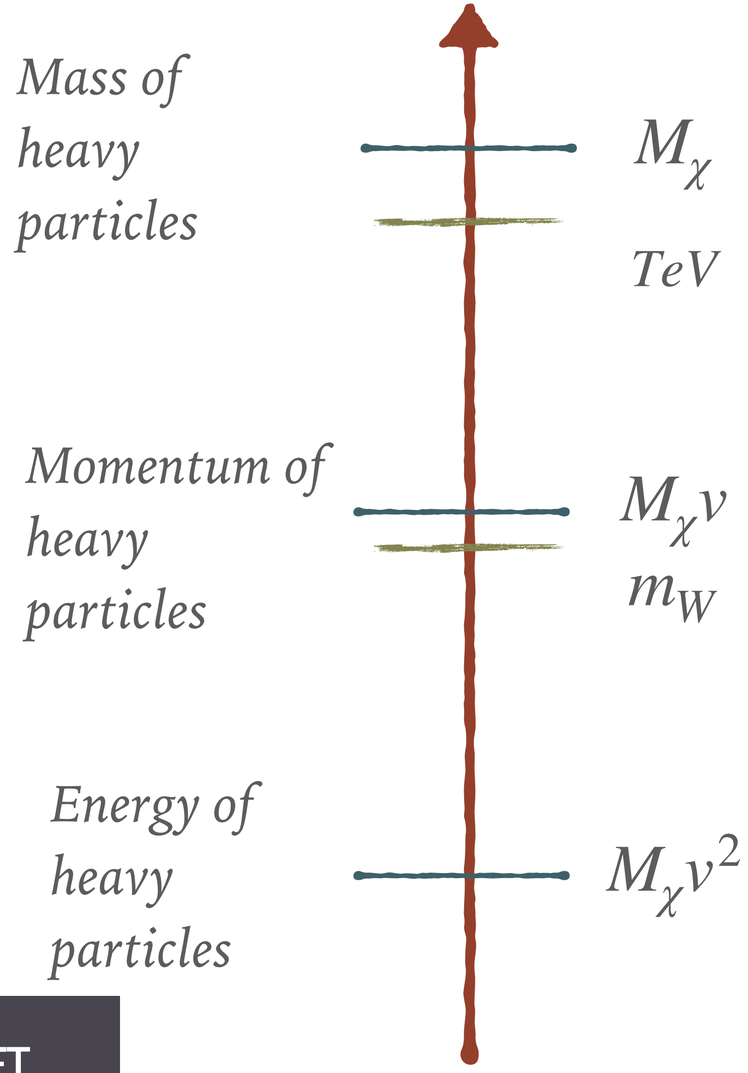
Modes

Potential $E \sim v^2 M_\chi, \;\; p \sim v M_\chi$ — *Only heavy d.o.f.*

Ultra-soft $E \sim v^2 M_\chi, \;\; p \sim v^2 M_\chi$ — *Only light d.o.f.*



Full Theory ⇨ Non-relativistic EFT ➡ Potential NREFT

NREFT

Obtained from the full theorem after integrating out hard modes $k \sim M_\chi$

For energies below M_χ *and above* m_W *we can use non relativistic EFT*

$SU(2)$ covariant derivatives

$$\mathscr{L}_{\text{NREFT}} = \chi^\dagger \left(iD^0 + \frac{\mathbf{D}^2}{2M_\chi} + \frac{\mathbf{D}^4}{8M_\chi^3} \right) \chi - \frac{d_1 g_2}{2M_\chi} \chi^\dagger \sigma \cdot \mathbf{B} \chi + \ldots$$

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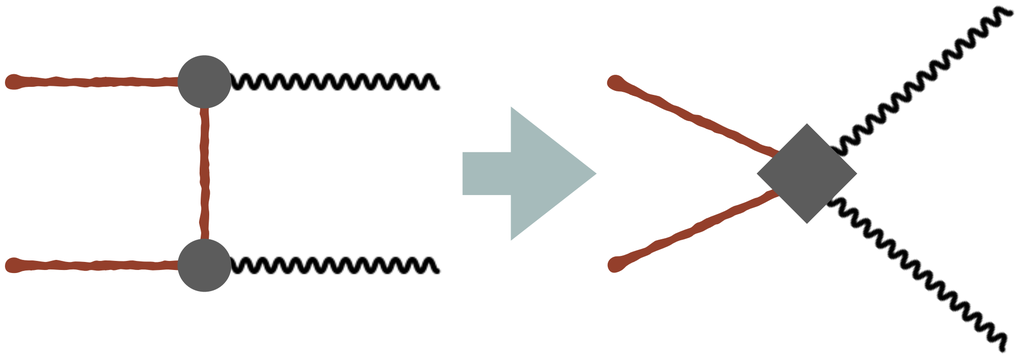
DM annihilation is described by an operator $\mathcal{O} = \chi^{c\dagger} \Gamma^A \chi \, O_A$



Operator O_A *describes finial state*

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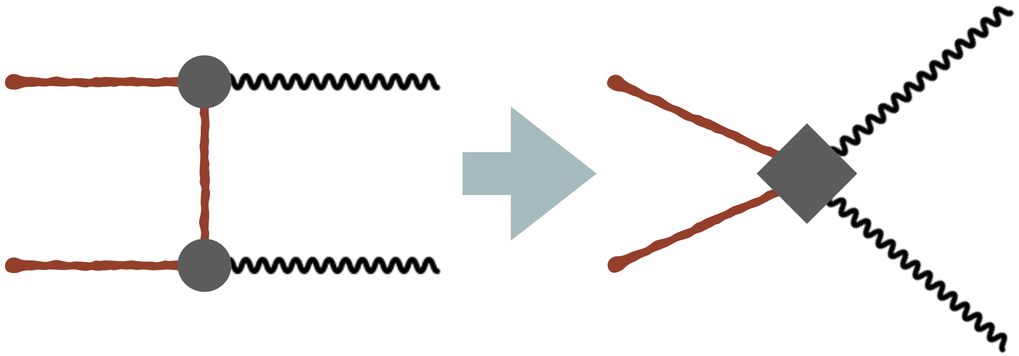
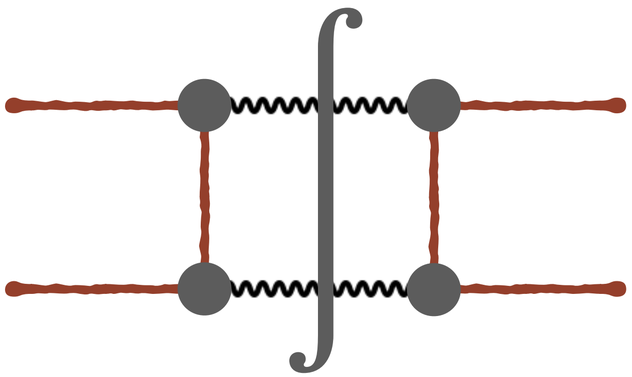
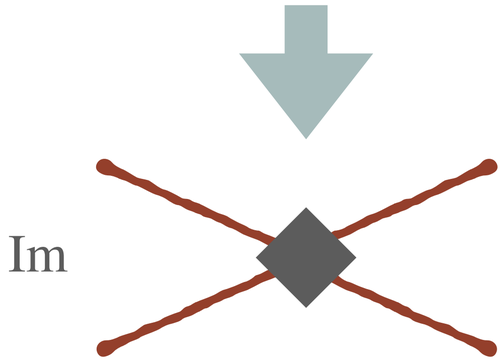


Operator O_A *describes finial state*

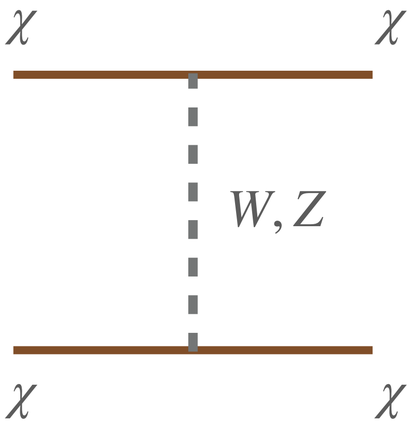
We can also be inclusive





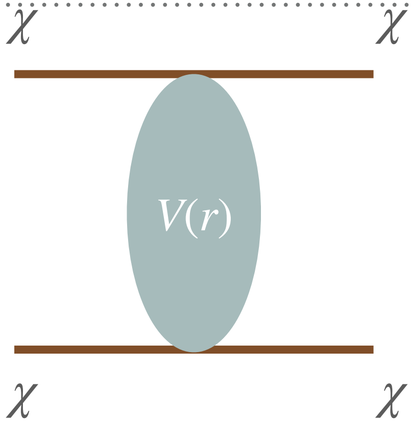
$$\mathcal{O} = \chi^{c\dagger} \Gamma^A \chi \, \chi^{c\dagger} \Gamma_A \chi$$

LO AND NLO POTENTIALS



$$\frac{4\pi\alpha_2}{(\mathbf{q}^2 + m_W^2)}$$

$$\frac{\alpha_2\, e^{-m_W r}}{r}$$



$U(1)_{\rm em}$ covariant derivative

$$\mathscr{L}_{\rm PNRDM} = \sum_{i=\pm,0} \chi_{vi}^\dagger(x)\left(iD_i^0(t,\mathbf{0}) - \delta m_i + \frac{\partial^2}{2M_\chi} - e_i\, e\, \mathbf{x}\cdot\mathbf{E}(t,\mathbf{0}) \right)\chi_{vi}(x)$$

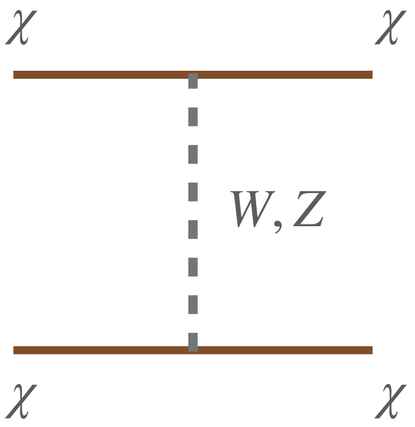
$$- \sum_{\{i,j\},\{k,l\}} \int d^3\mathbf{r}\, V_{\{ij\}\{kl\}}(r)\, \chi_{vk}^\dagger(t,\mathbf{x})\chi_{vl}^\dagger(t,\mathbf{x}+\mathbf{r})\chi_{vi}(t,\mathbf{x})\chi_{vj}(t,\mathbf{x}+\mathbf{r})$$

In the EFT, potential is just a matching coefficient of non-local four fermion operator

Local in time, non-local in space

Beneke (1998)
Pineda et al. (1998)

LO AND NLO POTENTIALS



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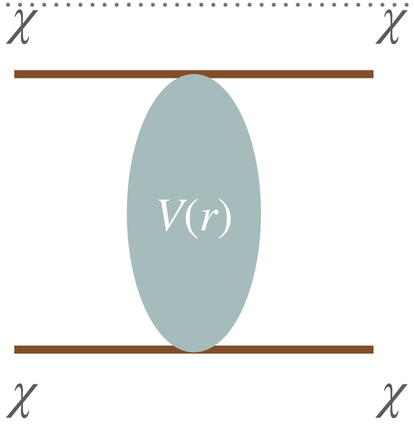

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Leading terms in velocity expansion

EOM is just Schrödinger equation

Local in time, non-local in space

Beneke (1998)
Pineda et al. (1998)

LO POTENTIALS

$$V_{\{ij\}\{kl\}}(r) = \int \frac{d^3\mathbf{q}}{(2\pi)^3}\, e^{i\mathbf{q}\cdot\mathbf{x}}\, i\, T^{\chi\chi\to\chi\chi}_{ijkl}(\mathbf{q}^2)$$

$\chi^0\chi^0 \to \chi^0\chi^0$

$$V_{\rm LO}(r) = \begin{pmatrix} 0 & -\sqrt{2}\alpha_2 \frac{e^{-m_W r}}{r} \\ -\sqrt{2}\alpha_2 \frac{e^{-m_W r}}{r} & 2\delta m - \frac{\alpha}{r} - \alpha_2 c_W^2 \frac{e^{-m_Z r}}{r} \end{pmatrix}$$

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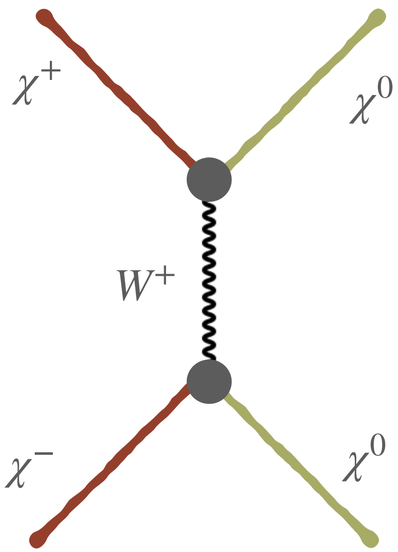

$\chi^0\chi^0 \to \chi^+\chi^-$

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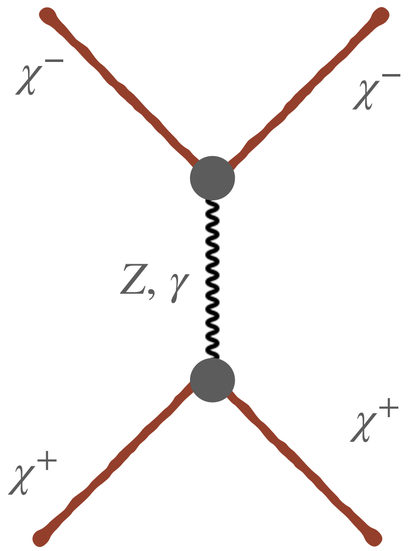
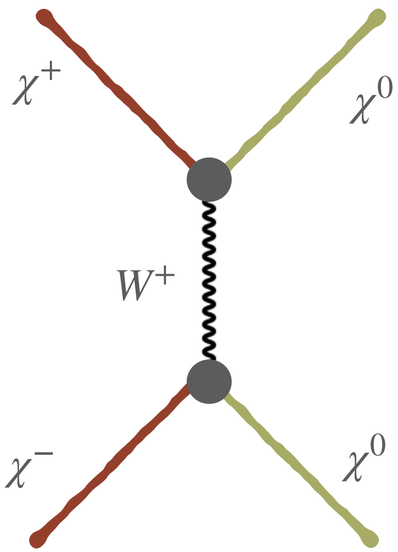


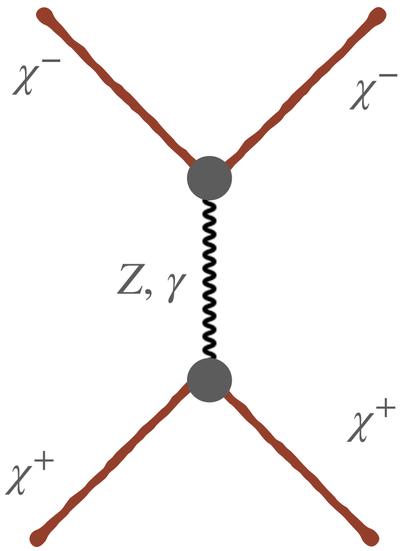
$\chi^+\chi^- \to \chi^+\chi^+$



$\chi^0\chi^0 \to \chi^+\chi^-$

LO POTENTIALS

$$V_{\{ij\}\{kl\}}(r) = \int \frac{d^3\mathbf{q}}{(2\pi)^3}\, e^{i\mathbf{q}\cdot\mathbf{x}}\, i\, T^{\chi\chi\to\chi\chi}_{ijkl}(\mathbf{q}^2)$$



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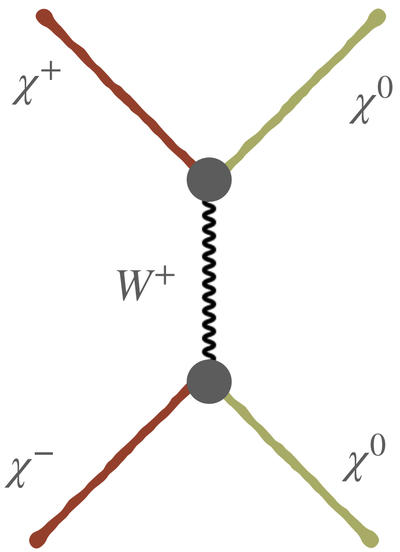


$\chi^0\chi^0 \to \chi^+\chi^-$

Mass splitting

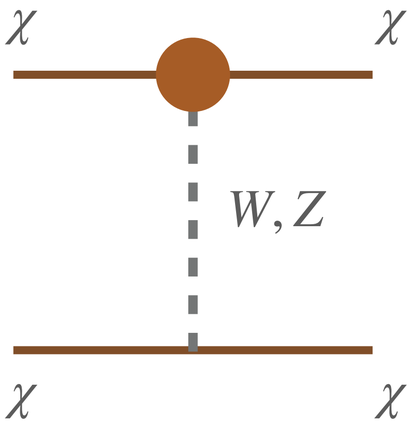
$$\delta m \sim \alpha_2 m_W \sim \frac{m_W^2}{M_\chi}$$

$$\partial_0 \sim E \sim M_\chi v^2 \sim \delta m$$

For our NLO computation we need 2-loop splitting

Yamada (2010)

LO AND NLO POTENTIALS



$$\mathscr{L}_{\text{NREFT}} = \chi^\dagger \left(iD^0 + \frac{\mathbf{D}^2}{2M_\chi} + \frac{\mathbf{D}^4}{8M_\chi^3} \right) \chi - \frac{d_1 g_2}{2M_\chi} \chi^\dagger \sigma \cdot \mathbf{B} \chi + \ldots$$

Leading interaction

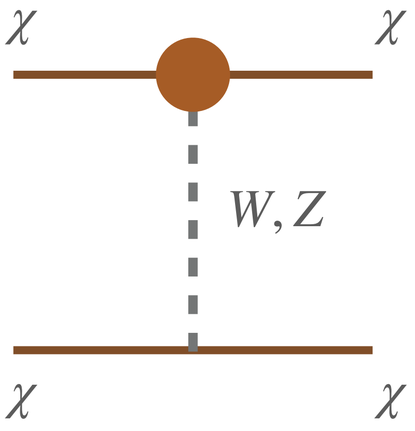
Subleading interactions

One can in principle have velocity suppressed interactions in the vertices …

…but there are no $\frac{1}{M_\chi}$ *corrections*

Power-counting $v \sim \alpha_2$ *means that we only get these terms at NNLO.*

LO AND NLO POTENTIALS



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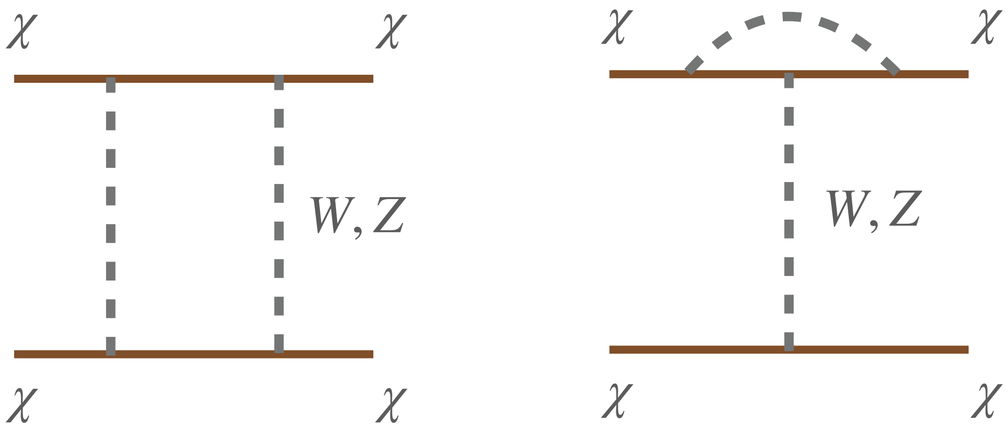
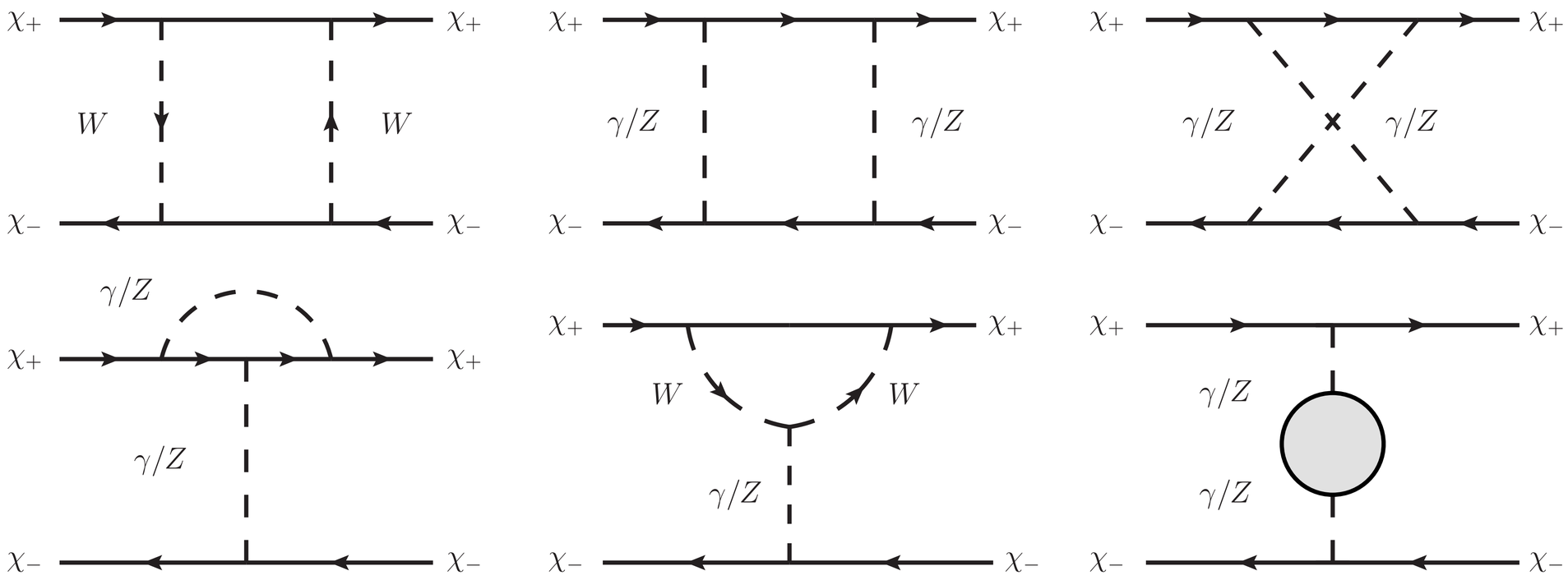
At NLO, we need to consider only soft loops



QUANTUM CORRECTIONS TO THE YUKAWA POTENTIAL IN SM

Boxes



Vertex-corrections

Self-energy

We set all the masses to zero except m_t, m_H, m_Z, m_W

NLO POTENTIALS

Basic checks of our computations:

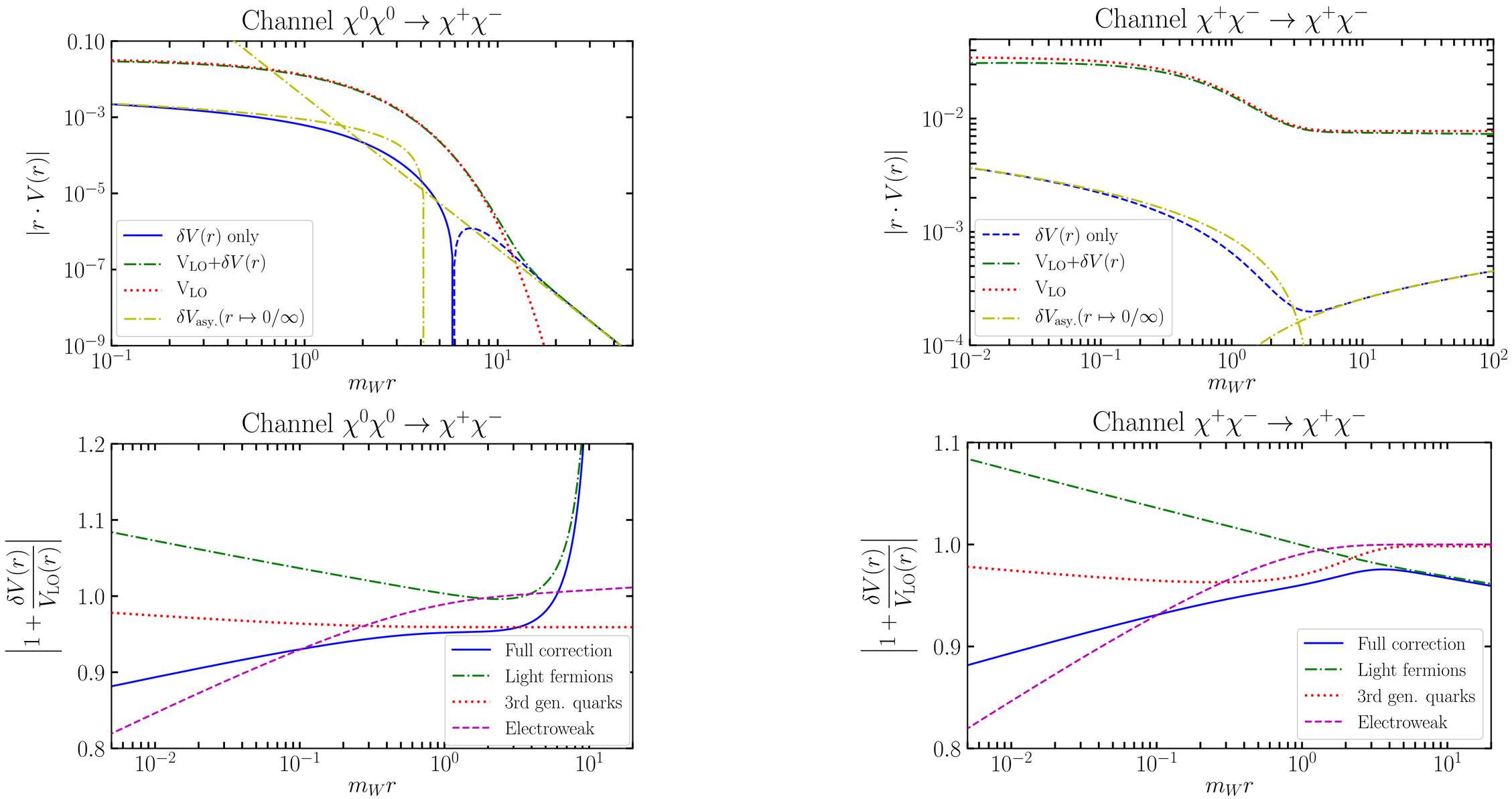
- Pole cancellation in on-shell scheme with $\alpha_{\rm os}(M_z)$, m_W, m_Z, m_t, m_H as input parameters
- Gauge invariance — computations done in Feynman and general R_ξ gauge, including tadpoles

We will focus on $\delta V_{\chi^+\chi^-\to\chi^+\chi^-}$ and $\delta V_{\chi^0\chi^0\to\chi^+\chi^-}$

We also found that

- $\delta V_{\chi^0\chi^0\to\chi^0\chi^0} = 0$
- $\delta V_{\chi^0\chi^\pm\to\chi^0\chi^\pm} = -\,\delta V_{\chi^0\chi^0\to\chi^+\chi^-}$
- $\delta V_{\chi^\pm\chi^\pm\to\chi^\pm\chi^\pm} = -\,\delta V_{\chi^+\chi^-\to\chi^+\chi^-}$

NLO YUKAWA POTENTIALS



ASYMPTOTIC CORRECTIONS

- Short-distance limit $r \to 0$ / $\mathbf{k}^2 \to \infty$

$$\delta V_{light\ ferm.}(\mathbf{k}^2 \to \infty) = -\frac{3\alpha_2^2}{\mathbf{k}^2} \ln \frac{\mathbf{k}^2}{m_Z^2}$$

$$\delta V_{\text{3rd gen. quarks}}(\mathbf{k}^2 \to \infty) = -\frac{\alpha_2^2}{\mathbf{k}^2} \ln \frac{\mathbf{k}^2}{m_Z^2}$$

$$\delta V_{\text{electroweak}}(\mathbf{k}^2 \to \infty) = \frac{\alpha_2^2}{\mathbf{k}^2} \frac{43}{6} \ln \frac{\mathbf{k}^2}{m_Z^2}$$

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Large logarithmic corrections as $r \to 0$

Breakdown of perturbation theory

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Large logarithmic corrections as $r \to 0$

Breakdown of perturbation theory

$$\delta V^{r\to 0}_{\chi^+\chi^- \to \chi^+\chi^-}(r) = \delta V^{r\to 0}_{\chi^0\chi^0 \to \chi^+\chi^-}(r) = \frac{\alpha_2^2}{2\pi r}\left(-\beta_{0,\mathrm{SU}(2)}(\ln(m_Z r) + \gamma_E) + \frac{1}{2}\Delta\right)$$

The same limit for both channels — $SU(2)$ symmetry is restored at high energy

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Screening theorem — only logarithmic dependence on m_H

$$\delta V^{r\to 0}_{\chi^+\chi^- \to \chi^+\chi^-}(r) = \delta V^{r\to 0}_{\chi^0\chi^0 \to \chi^+\chi^-}(r) = \frac{\alpha_2^2}{2\pi r}\left(-\beta_{0,\mathrm{SU}(2)}(\ln(m_Z r) + \gamma_E) + \frac{1}{2}\Delta\right)$$

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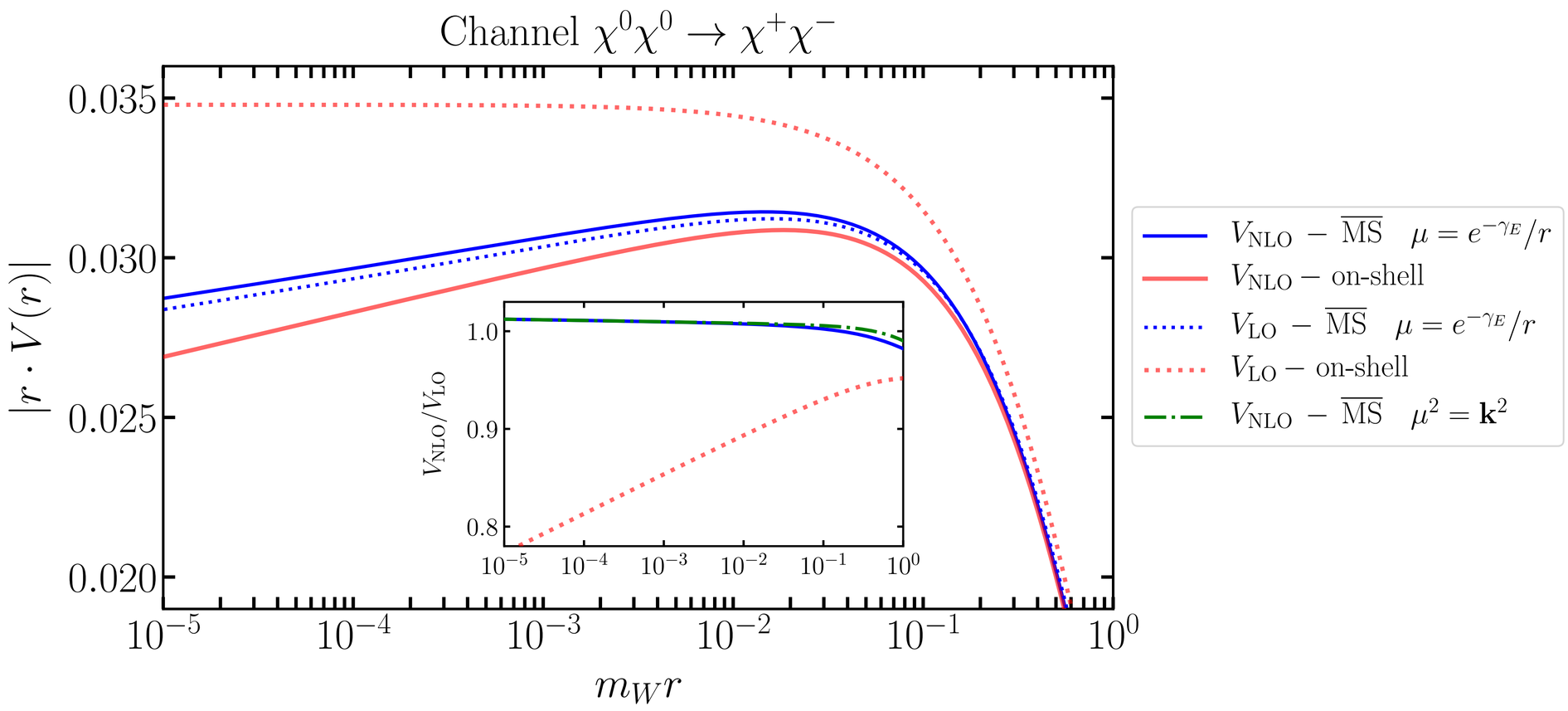
NLO POTENTIAL IN $\overline{\mathrm{MS}}$ SCHEME



$\overline{\mathrm{MS}}$ scheme is suitable for short distance corrections

Large logarithmic corrections can be removed by choosing dynamical running scale $\mu \sim 1/r$

ASYMPTOTIC CORRECTIONS

- Long distance limit $r \to \infty$ / $\mathbf{k}^2 \to 0$
- Let us start with $\chi^+\chi^- \to \chi^+\chi^-$ channel
- At LO we have diagrams due to γ and Z exchange: $V(r) = -\frac{\alpha}{r} - \alpha_2 c_W^2 \frac{e^{-m_Z r}}{r}$
- At NLO, for $r \to \infty$ photon dominates

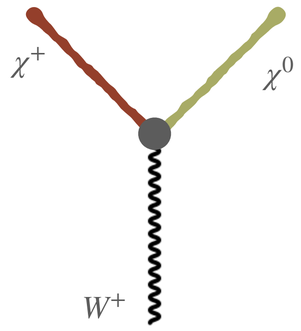
$$\delta V^{r\to\infty}_{\chi^+\chi^-\to\chi^+\chi^-}(r) = \frac{\alpha^2}{2\pi r}(-\beta_{0,\mathrm{em}})(\ln(m_Z r) + \gamma_E)$$

$\beta_{0,\mathrm{em}} = -80/9$ *electromagnetic beta function for all SM fermions except top quark*

— massless degrees of freedom

ASYMPTOTIC CORRECTIONS

- Long distance limit $r \to \infty$ / $\mathbf{k}^2 \to 0$
- Now we focus on $\chi^0\chi^0 \to \chi^+\chi^-$ channel
- At LO we have a diagram due W exchange: $V(r) = -\alpha_2 \dfrac{e^{-m_W r}}{r}$



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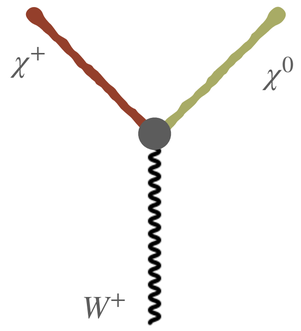

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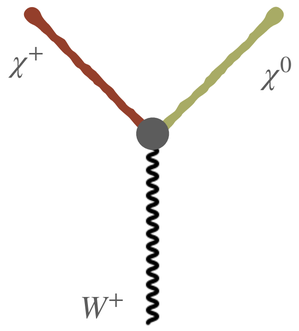

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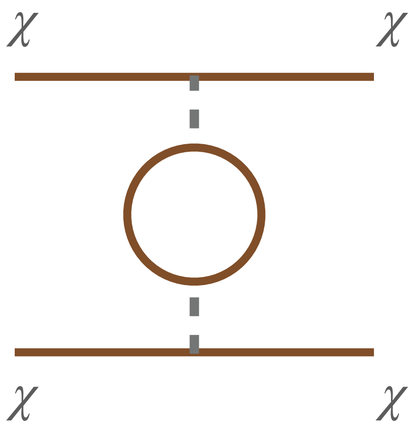

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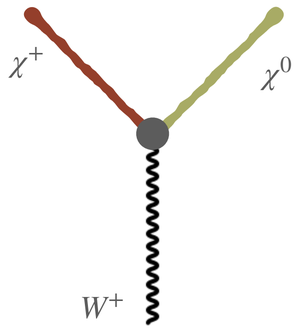

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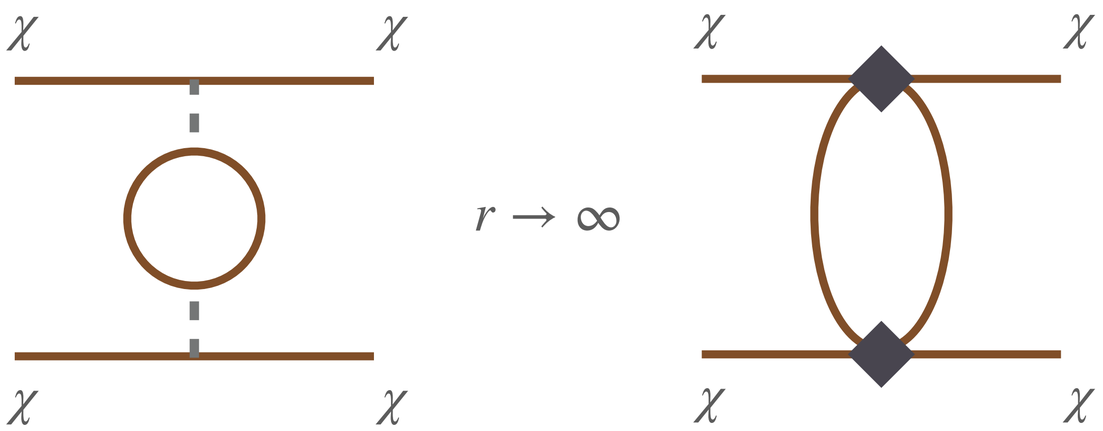

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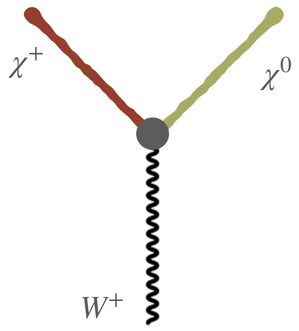

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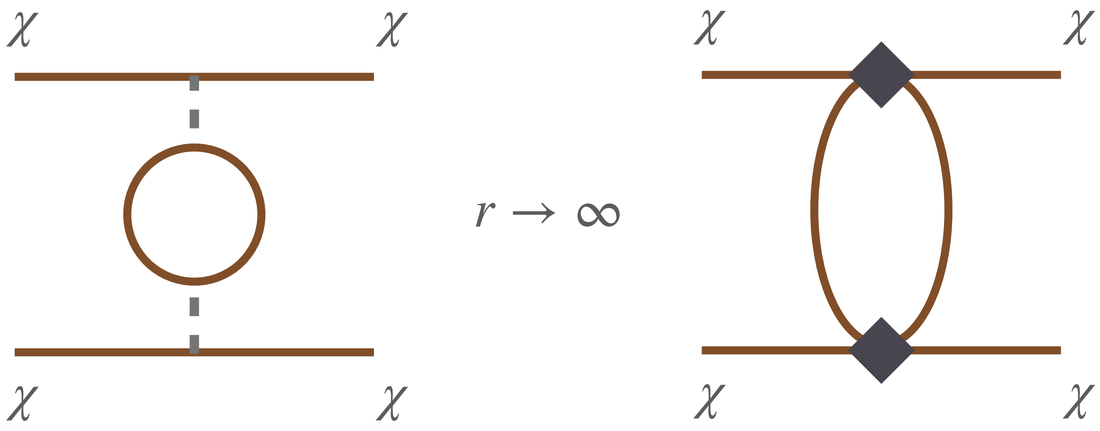

Integral measures $\int d^3q d^4k \sim 1/r^7$

Each fermionic propagator $\dfrac{\gamma^\mu k_\mu}{k^2} \sim r$

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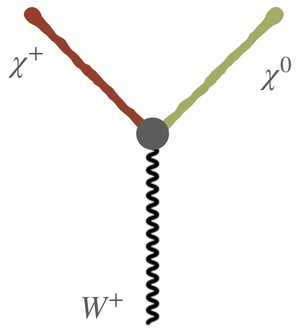

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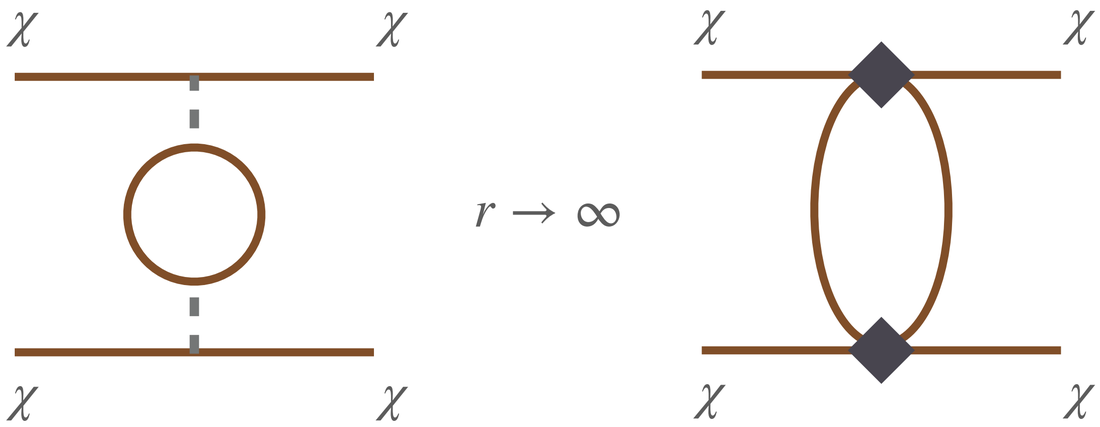

Integral measures $\int d^3q d^4k \sim 1/r^7$

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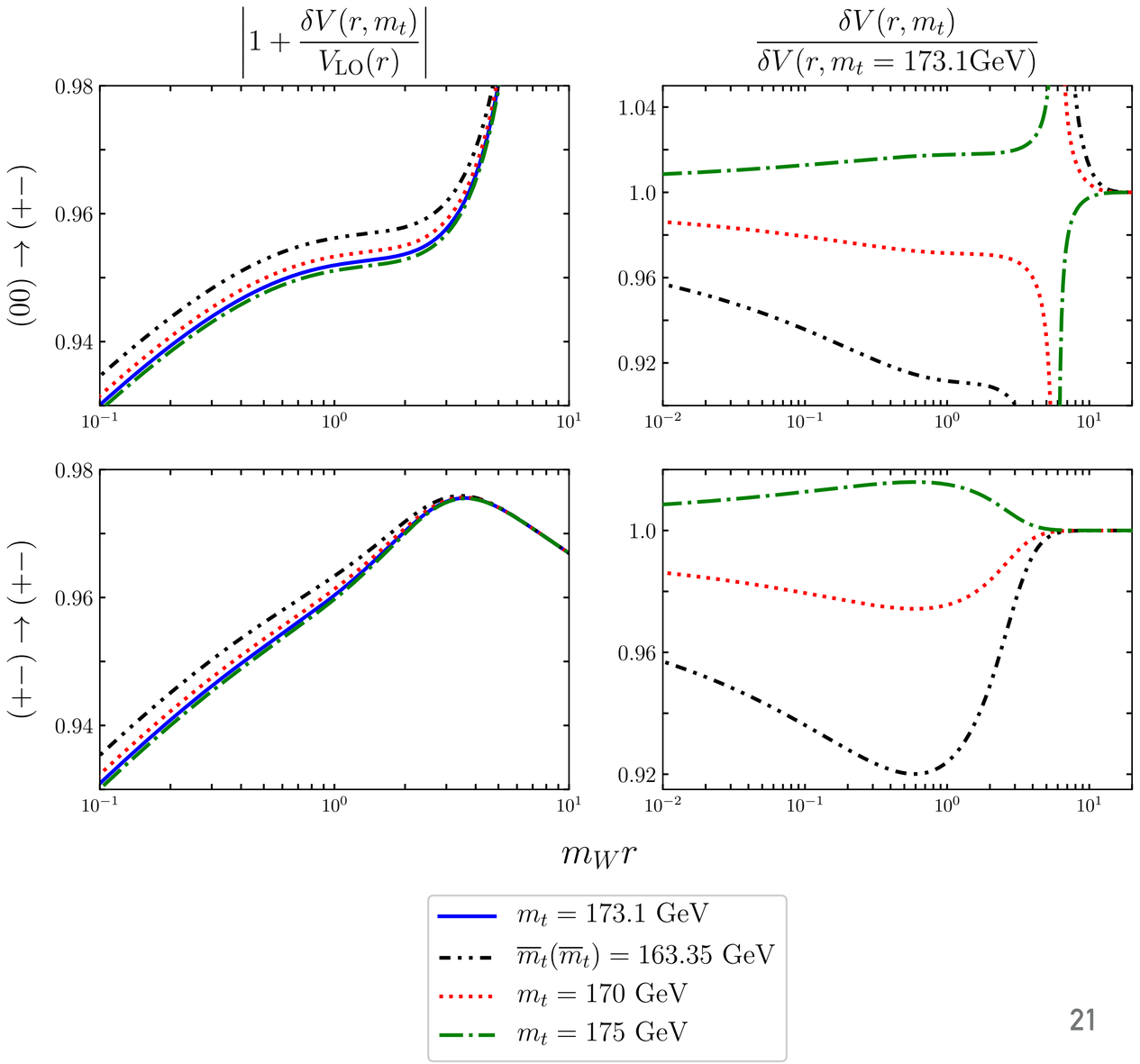
$1/r^5$

For massive fermions we will get extra suppression factor

$\sim e^{-m_f r}$

TOP-MASS EFFECTS

- At NLO, the potential depends on the top mass
- Dependence on m_t is quadratic
- Top mass is the only parameter with large uncertainty
- It is not clear which scheme for top mass should be used
- In computations, we use on-shell value $m_t = 173.1\text{GeV}$



ANNIHILATION CROSS-SECTION

$$\chi^0\chi^0 \to \gamma + X$$

Most important effect is the shift of the resonance position by about 6%

$$2.283 \to 2.419 \ \text{TeV}$$
$$8.773 \to 9.335 \ \text{TeV}$$

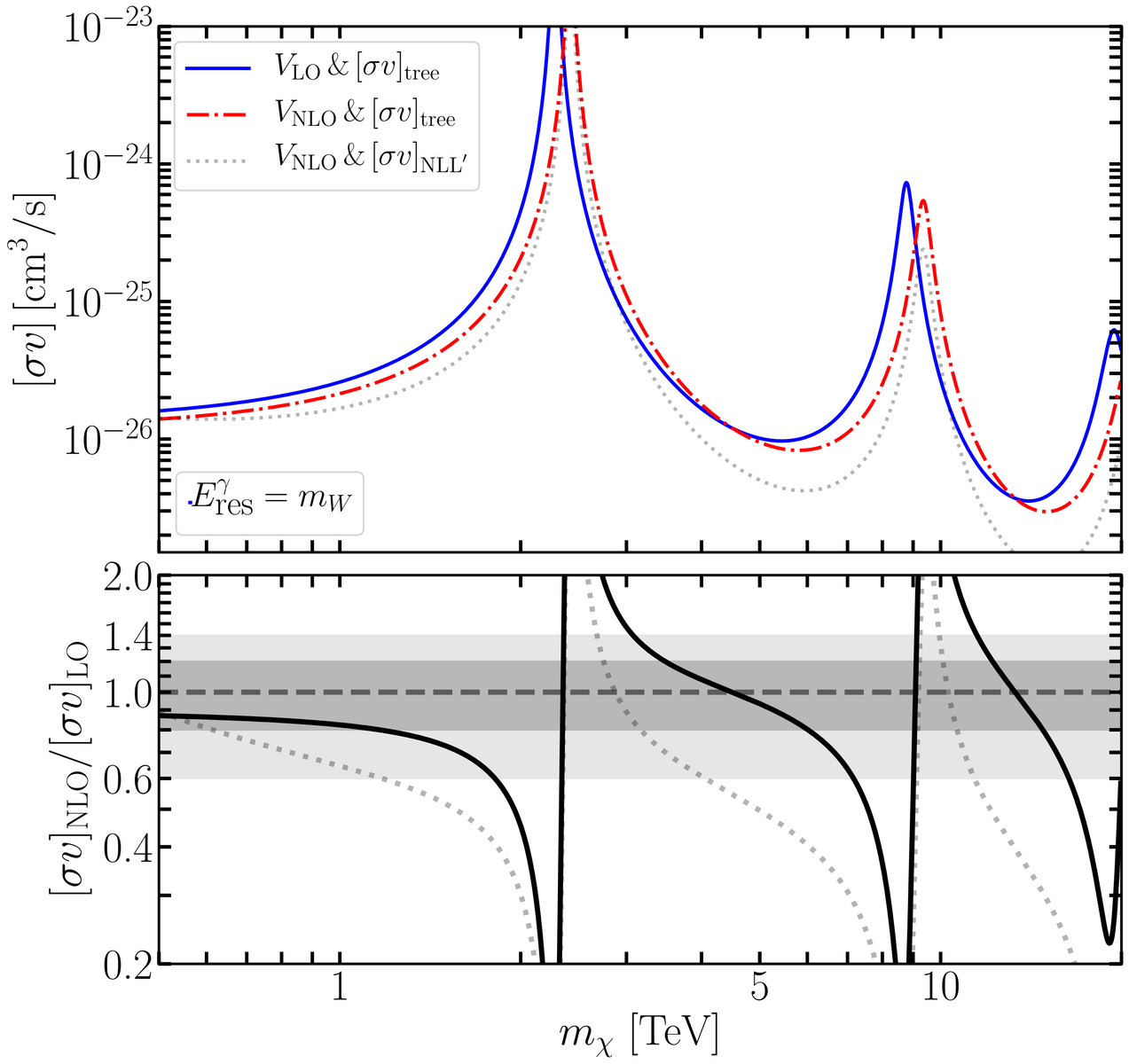
Effect is larger than the typical 3% for an electroweak loop correction

For $\overline{\text{MS}}$ *top mass* $\overline{m}_t(\overline{m}_t) = 163.35$ GeV

$$2.283 \to 2.408 \ \text{TeV}$$
$$8.773 \to 9.311 \ \text{TeV}$$

For SCET part see:

Beneke et al. (2018,2019)
Baumgart et al. (2018,2019)



20%

40%

SOMMERFELD FACTORS FOR INCLUSIVE CROSS-SECTION

$$S_{ij}[\hat{f}(^{2S+1}L_J)] = \frac{\left[\psi^{(L,S)}_{e_4e_3,\,ij}\right]^* \hat{f}^{\chi\chi\to\chi\chi}_{\{e_1e_2\}\{e_4e_3\}}(^{2S+1}L_J)\,\psi^{(L,S)}_{e_1e_2,\,ij}}{\hat{f}^{\chi\chi\to\chi\chi}_{\{ij\}\{ij\}}(^{2S+1}L_J)\,|_{\mathrm{LO}}}$$

$\psi^{(L,S)}_{e_1e_2,\,ij}$ *is the wave-function at* $\mathbf{r} = 0$

$\hat{f}^{\chi\chi\to\chi\chi}_{\{ij\}\{ij\}}(^{2S+1}L_J)$ *absorptive part of Wilson coefficient — short distance annihilation*

Hisano et al. (2004,2006)
Slatyer (2010)
Hryczuk et al. (2010)
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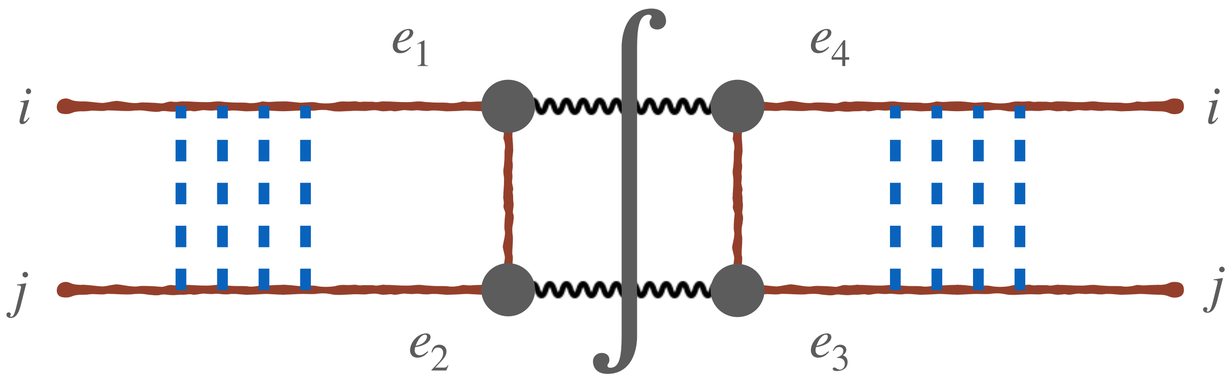
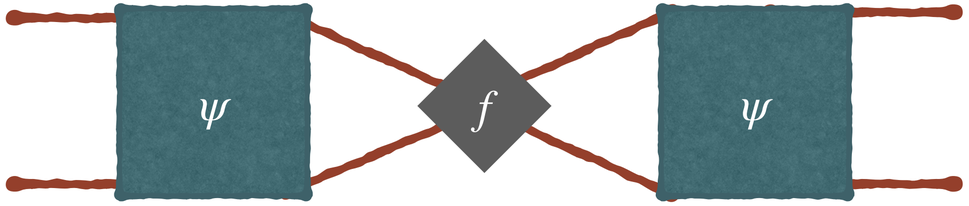
Hellmann et al. (2013)





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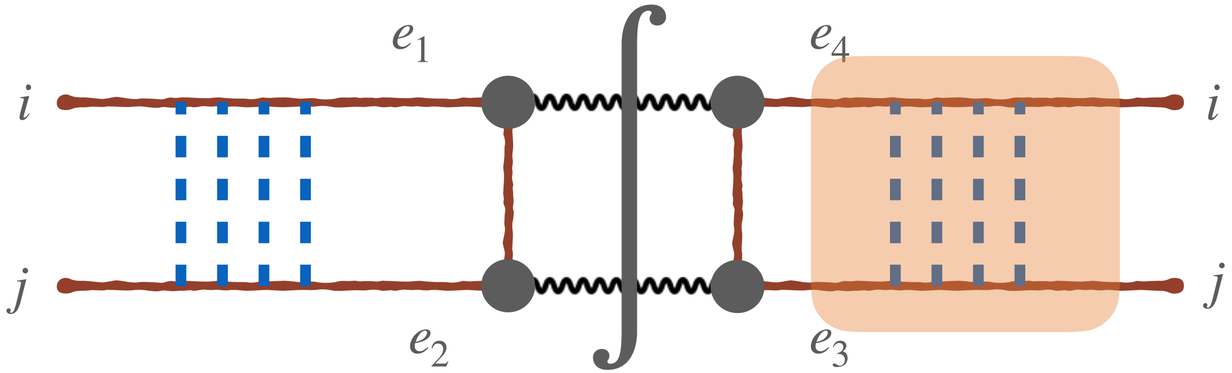

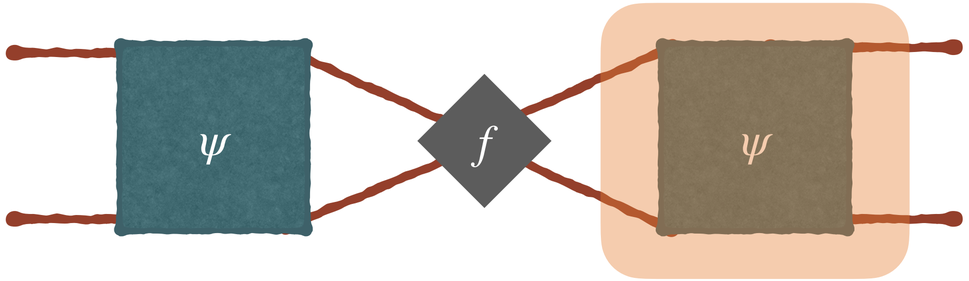

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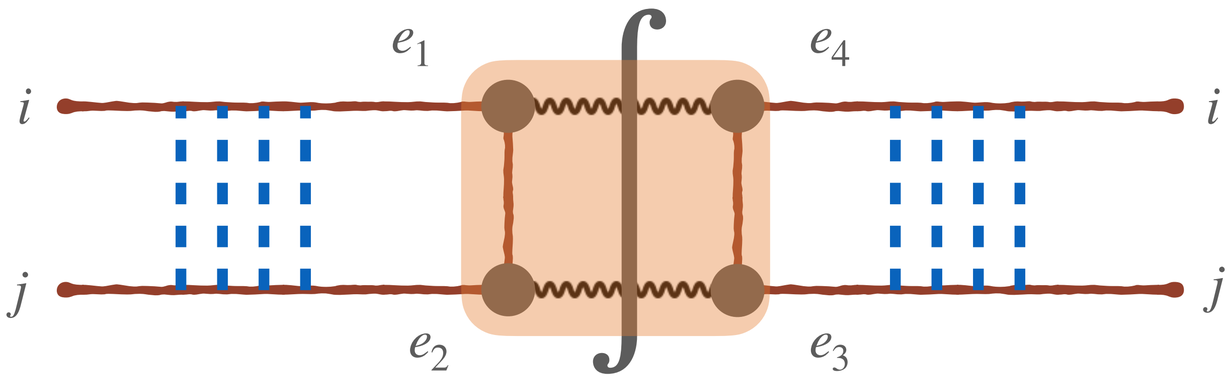
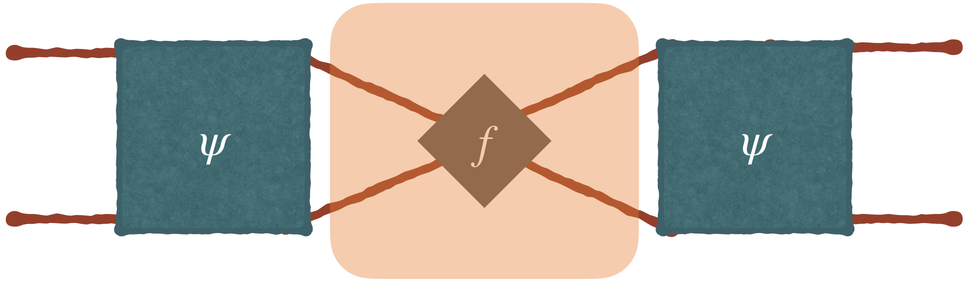




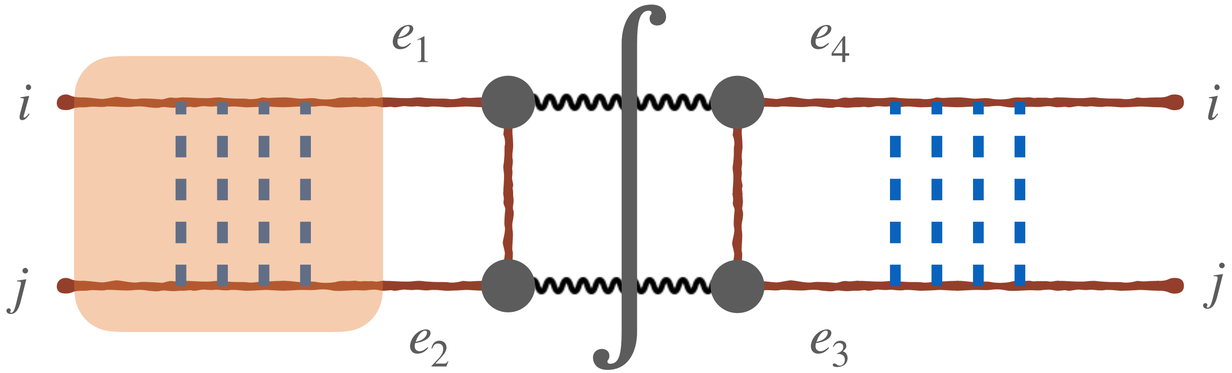
SOMMERFELD FACTORS FOR INCLUSIVE CROSS-SECTION

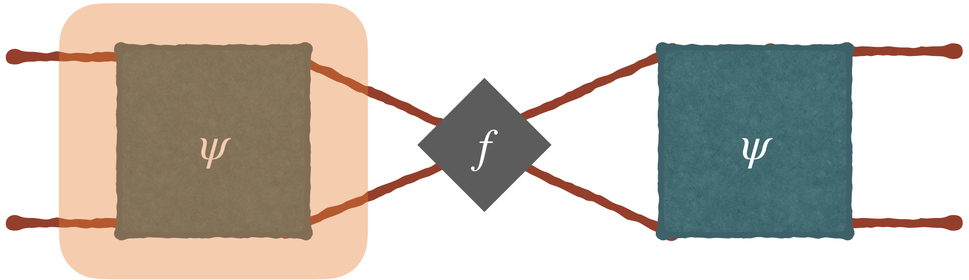
$$S_{ij}[\hat{f}(^{2S+1}L_J)] = \frac{\left[\psi^{(L,S)}_{e_4e_3,\,ij}\right]^* \hat{f}^{\chi\chi\to\chi\chi}_{\{e_1e_2\}\{e_4e_3\}}(^{2S+1}L_J)\,\psi^{(L,S)}_{e_1e_2,\,ij}}{\hat{f}^{\chi\chi\to\chi\chi}_{\{ij\}\{ij\}}(^{2S+1}L_J)\,|_{\mathrm{LO}}}$$

$\psi^{(L,S)}_{e_1e_2,\,ij}$ *is the wave-function at* $\mathbf{r} = 0$

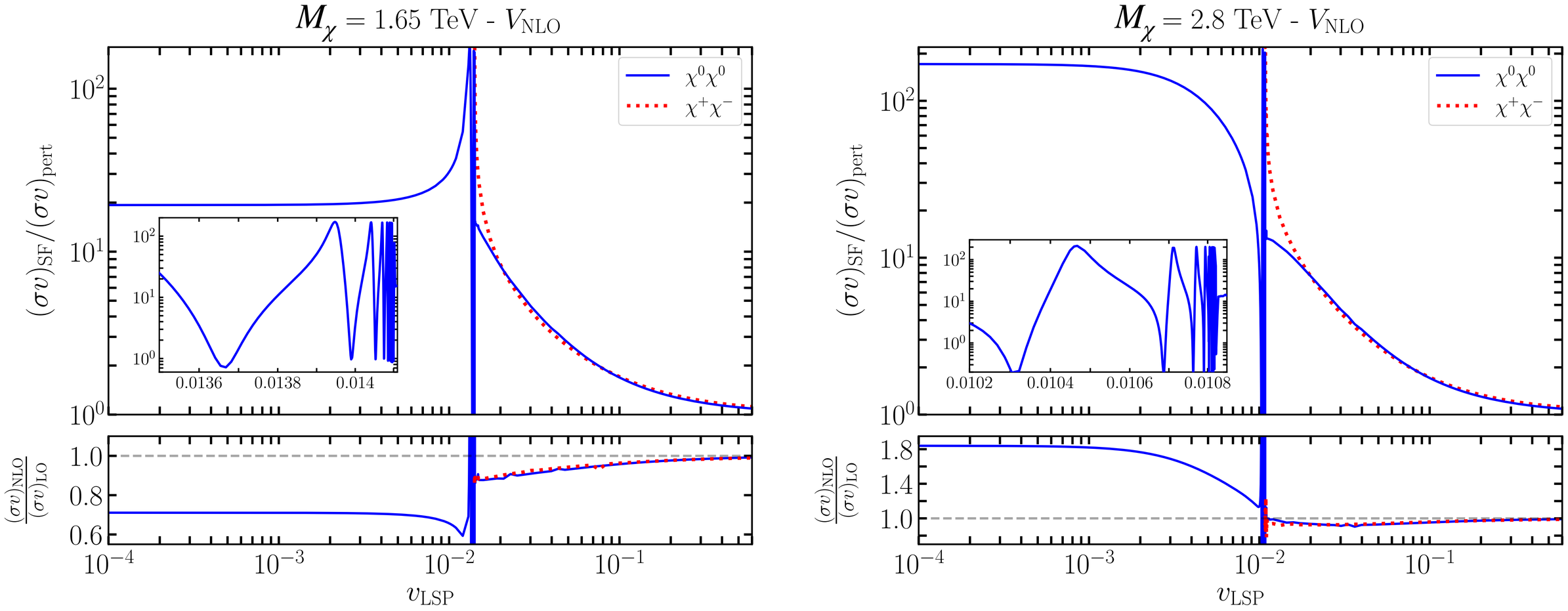
$\hat{f}^{\chi\chi\to\chi\chi}_{\{ij\}\{ij\}}(^{2S+1}L_J)$ *absorptive part of Wilson coefficient — short distance annihilation*

Hisano et al. (2004,2006)
Slatyer (2010)
Hryczuk et al. (2010)
Beneke et al. (2012)
Hellmann et al. (2013)





SOMMERFELD FACTORS AS FUNCTIONS OF VELOCITY



NLO potential gives large corrections for small velocities

COMPUTING RELIC ABUNDANCE

Boltzmann equation

$$\frac{dn}{dt} + 3Hn = -\langle \sigma v \rangle_{\text{eff}} (n^2 - n_{\text{eq}}^2)$$

$$n = \sum_{i=1}^{N} n_i \quad \textit{with } n_i \textit{ being particle number density of } \chi_i$$

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We need to include all channels

$\langle \sigma_{ij} v_{ij} \rangle$ *Thermal average computed with* $f_i \sim e^{-E_i/T}$ *distribution*

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Introduce Yield $Y = n/s$

Replace time t for temperature T

Use variable $x = \frac{M_\chi}{T}$

$$\frac{dY}{dx} = \frac{1}{3H} \frac{ds}{dx} \langle \sigma v \rangle_{\text{eff}} (Y^2 - Y_{\text{eq}}^2)$$

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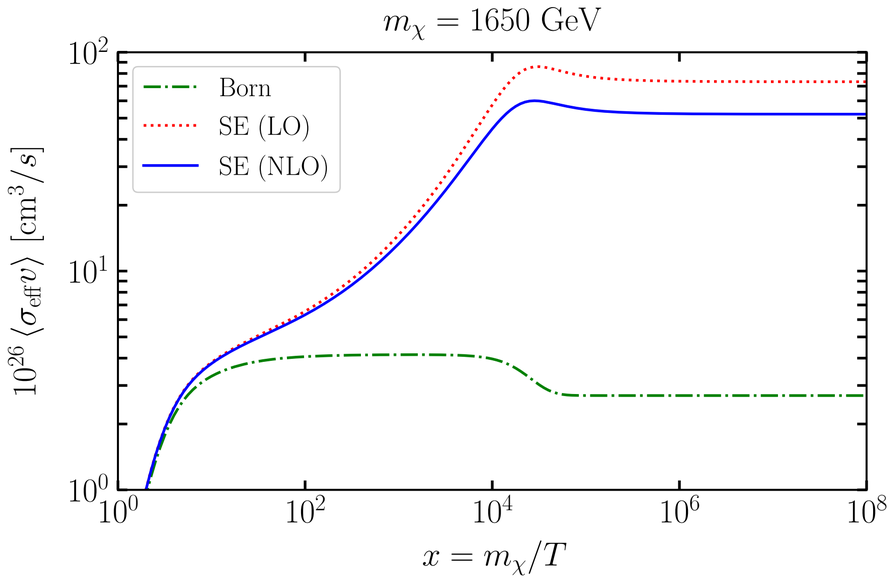
Relic density

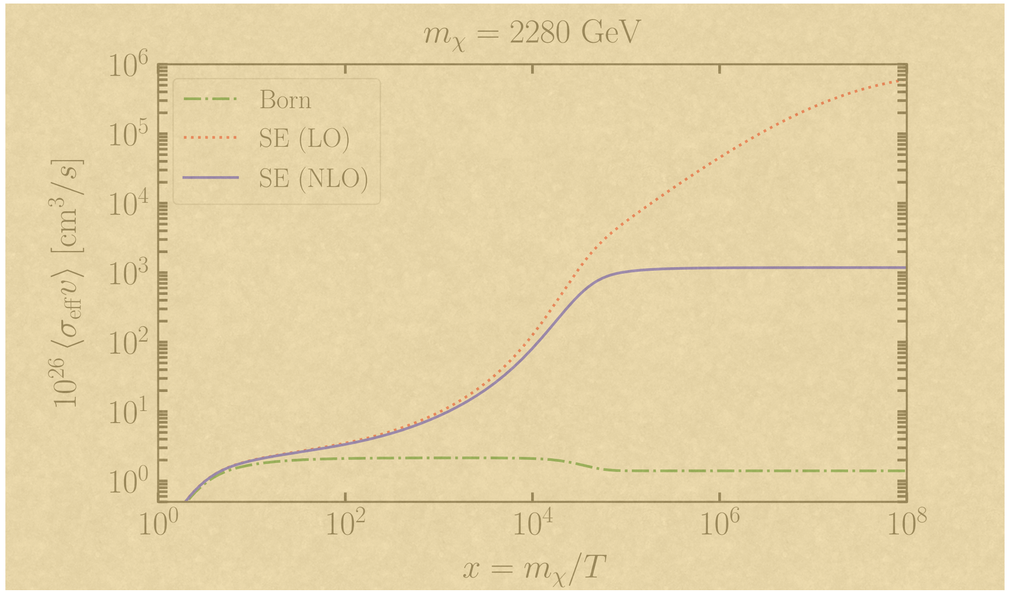
$$\Omega = M_\chi s_0 Y_0 / \rho_{\text{crit}}$$

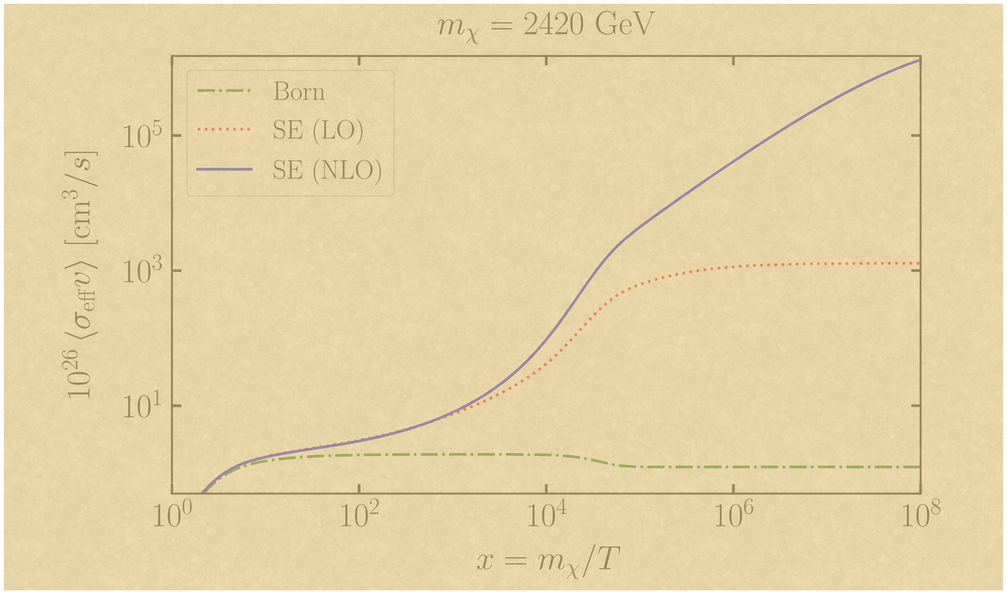
Present day value

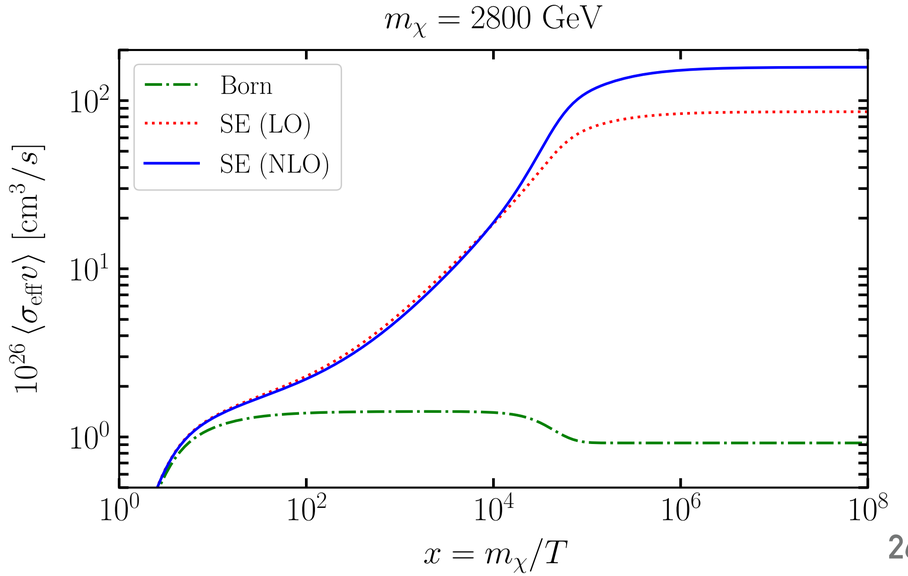
THE THERMALLY-AVERAGED CROSS SECTION



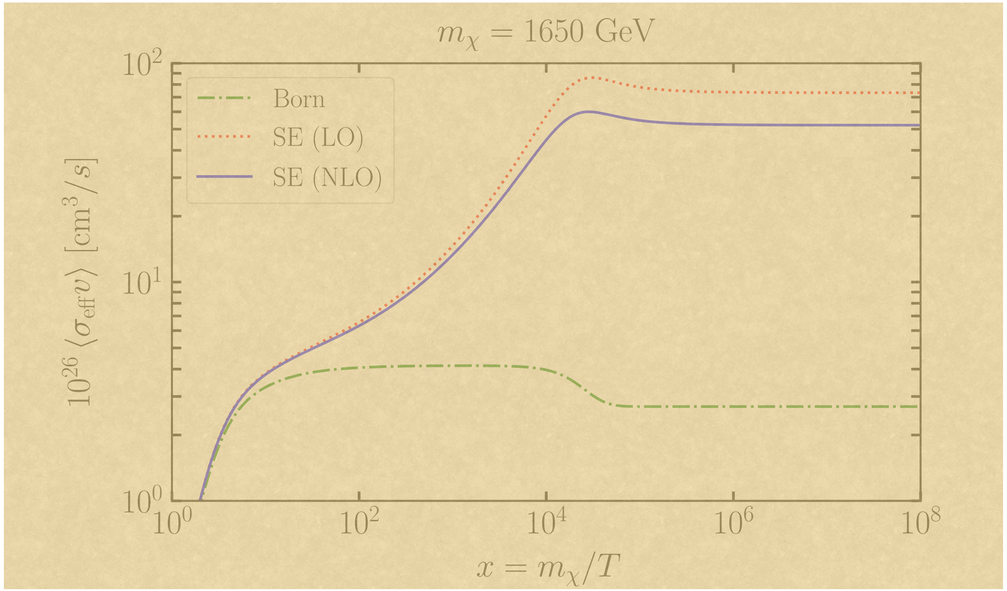
Few % corrections away from resonance

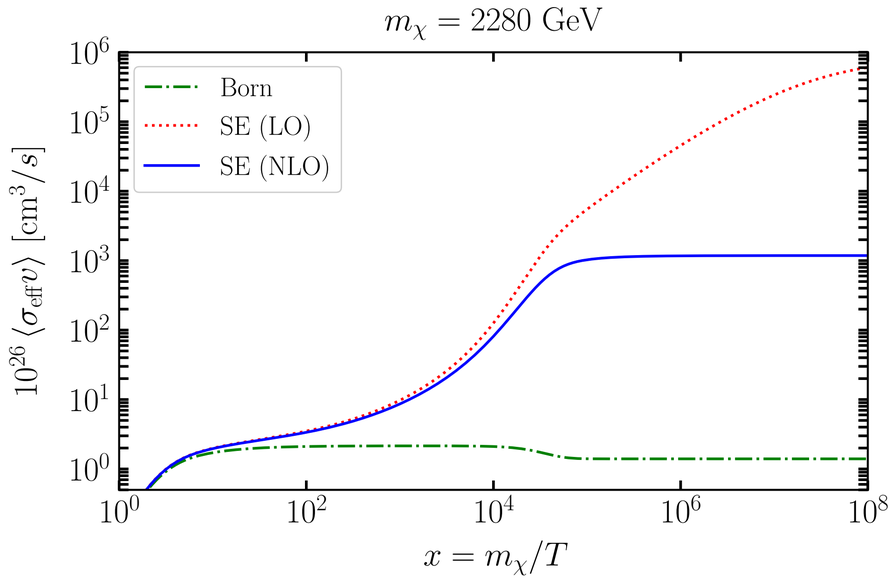


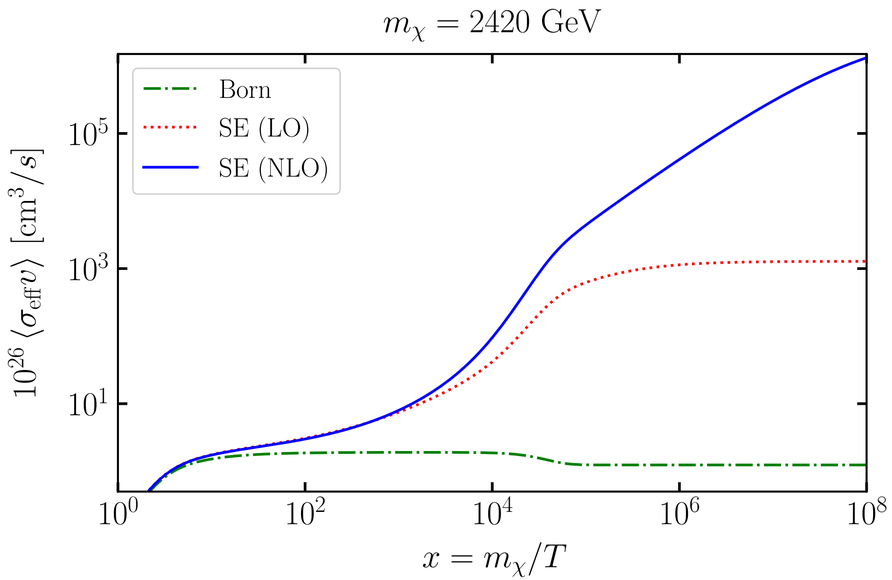




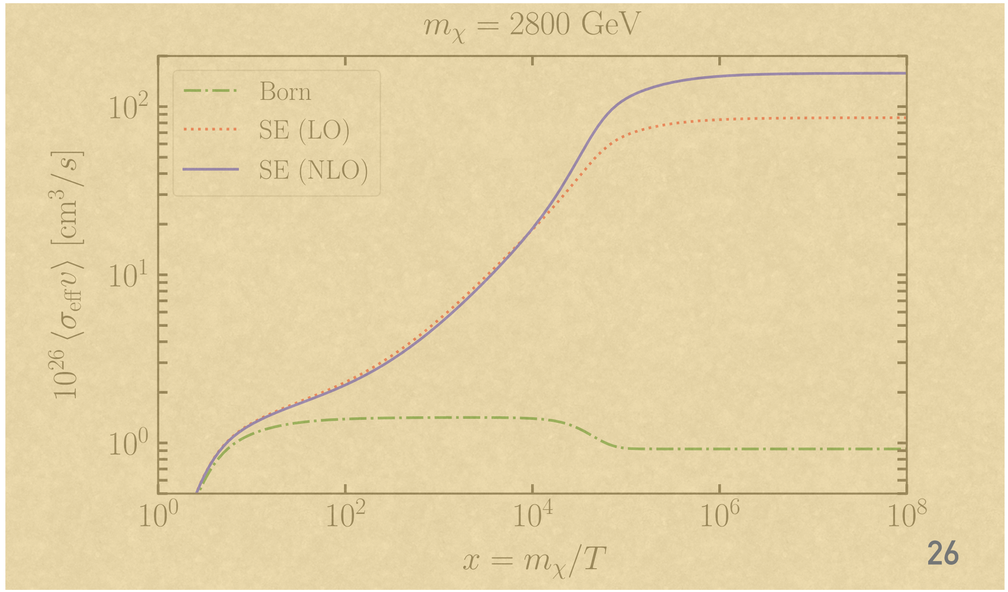
THE THERMALLY-AVERAGED CROSS SECTION



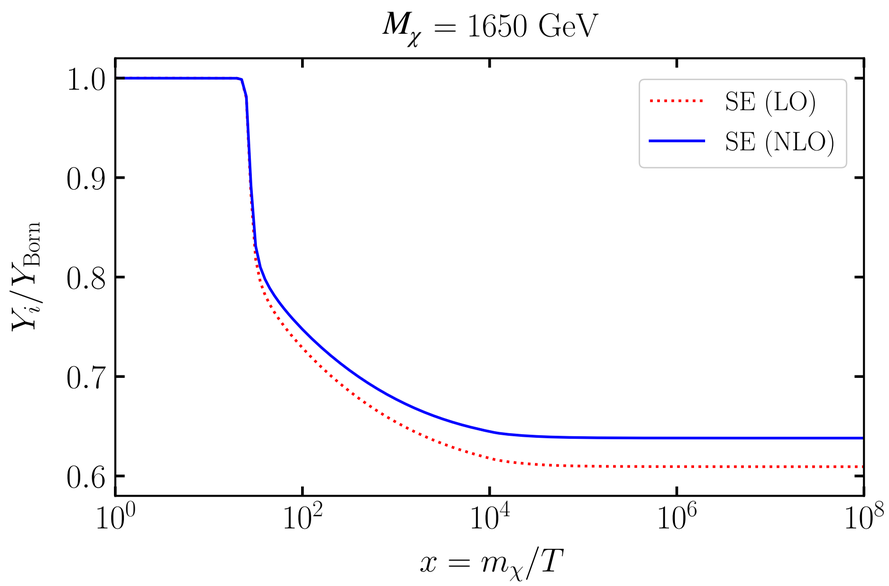


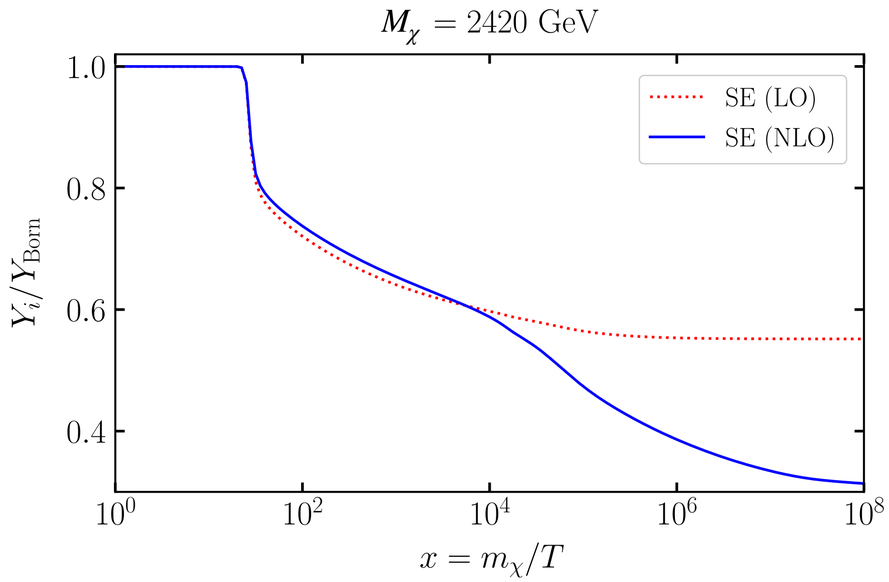


Large effects near the resonance — NLO cannot be neglected



YIELDS

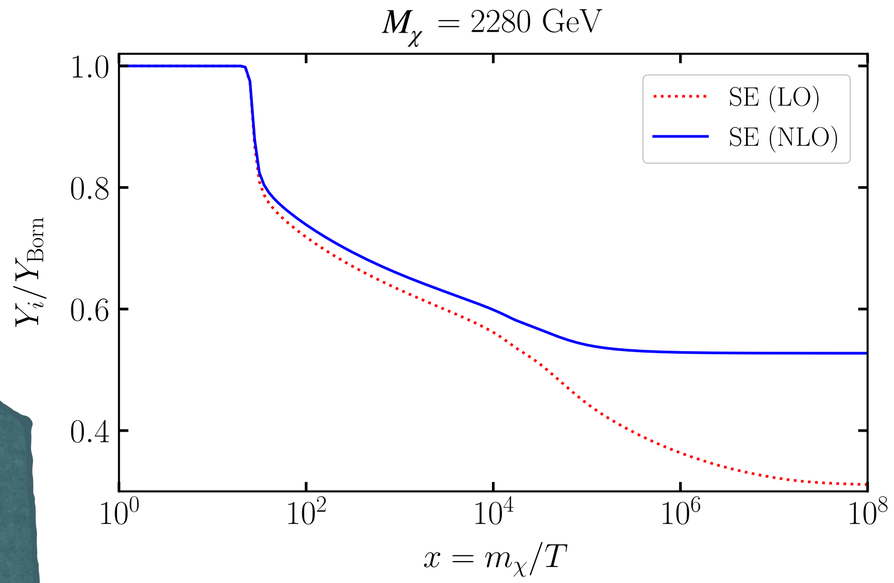


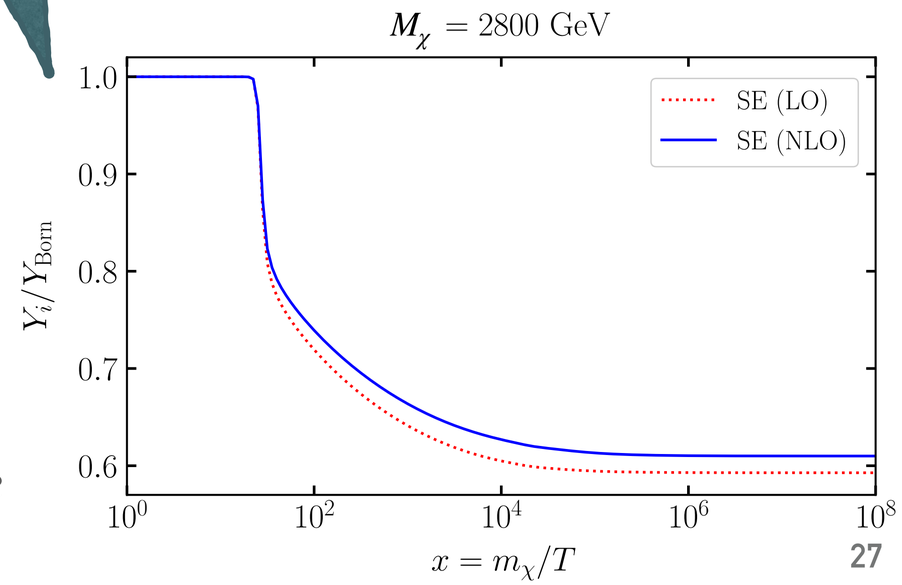


Until $x \sim 20$, freeze-out has not started yet

Near the resonance

Near $x \sim 10^4$, late-time annihilations ceases, except near the resonance





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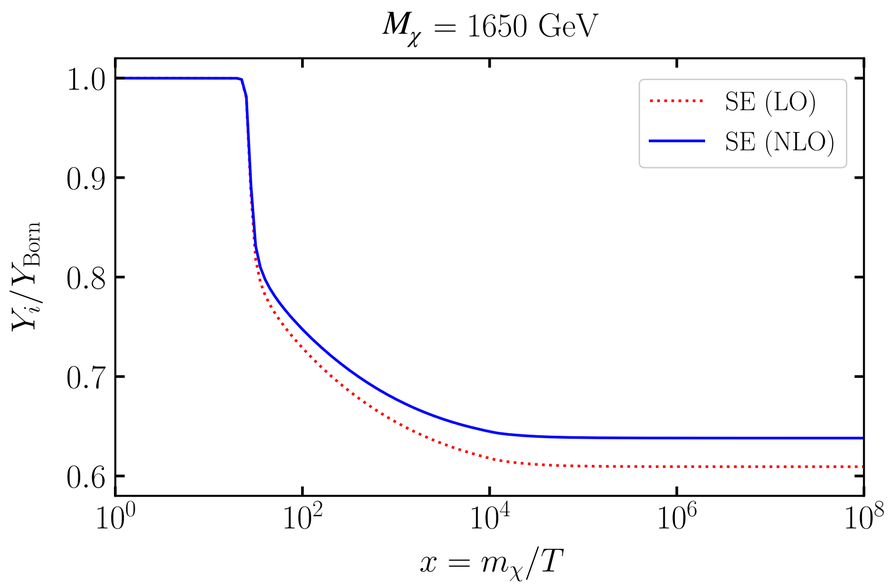
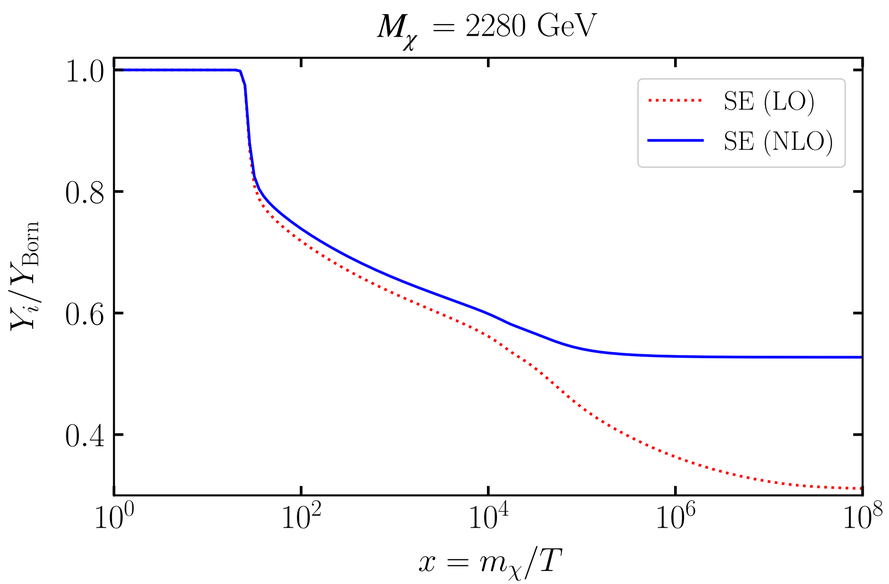
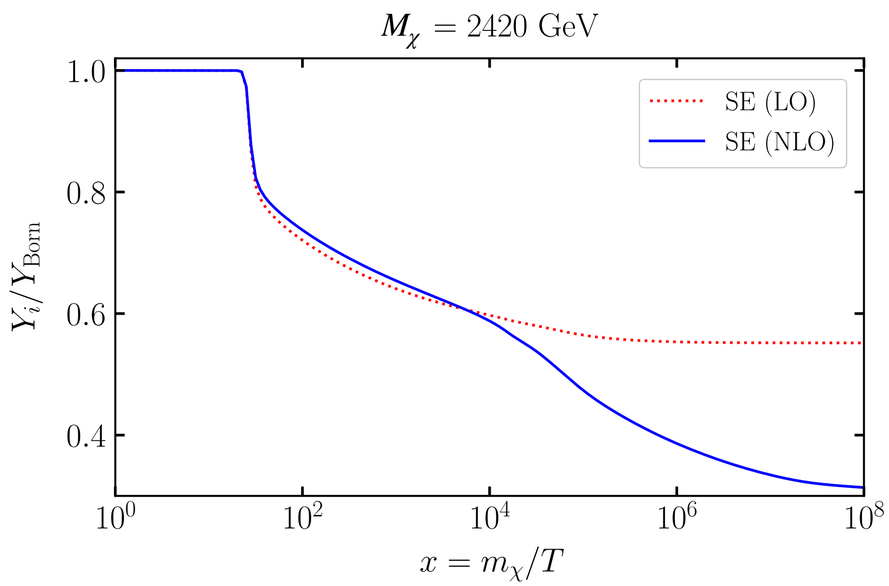
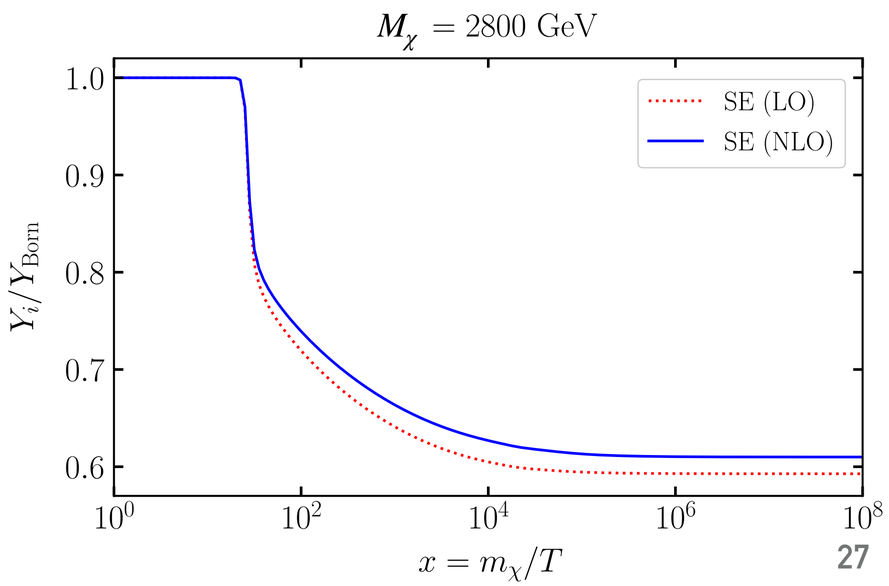


At early times (low x), the NLO yield is always larger than the LO one.





Near $x \sim 10^4$, late-time annihilations ceases, except near the resonance



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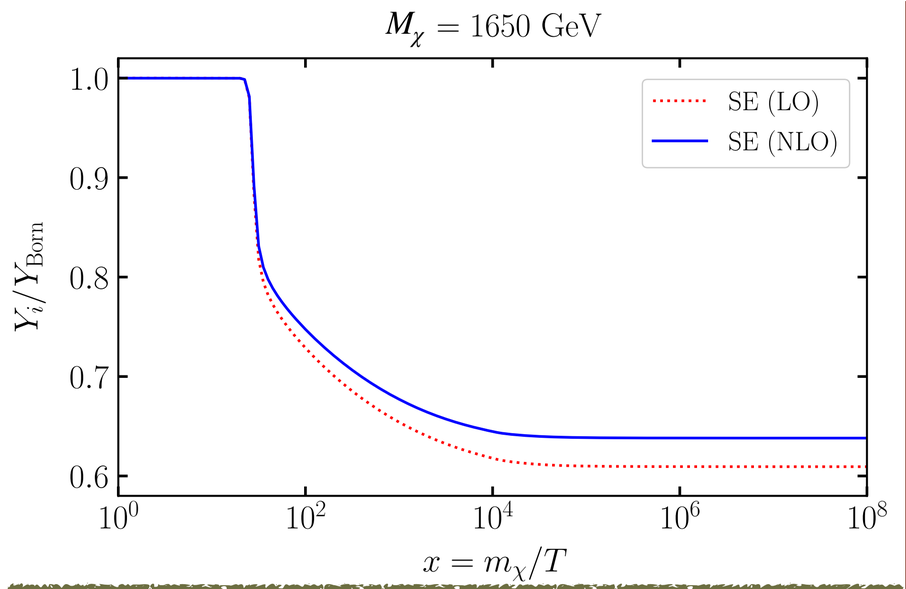

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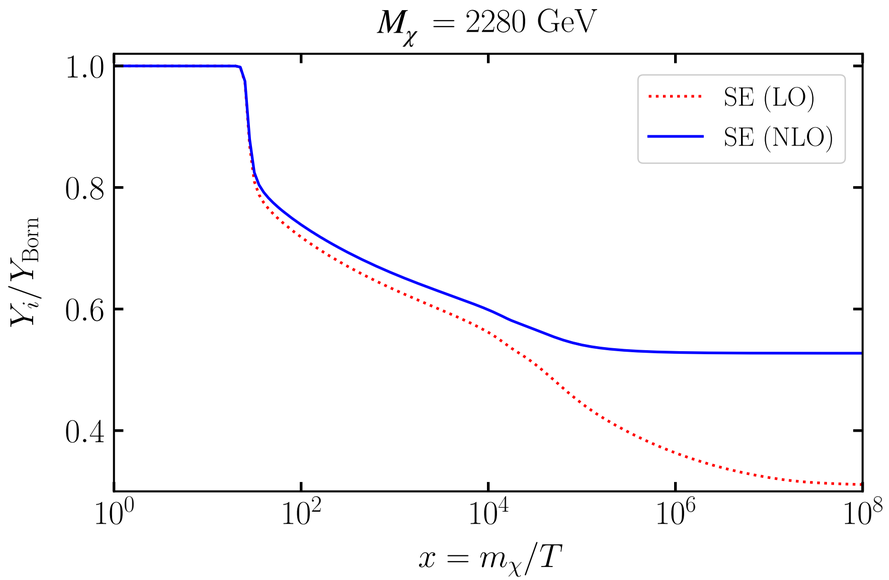
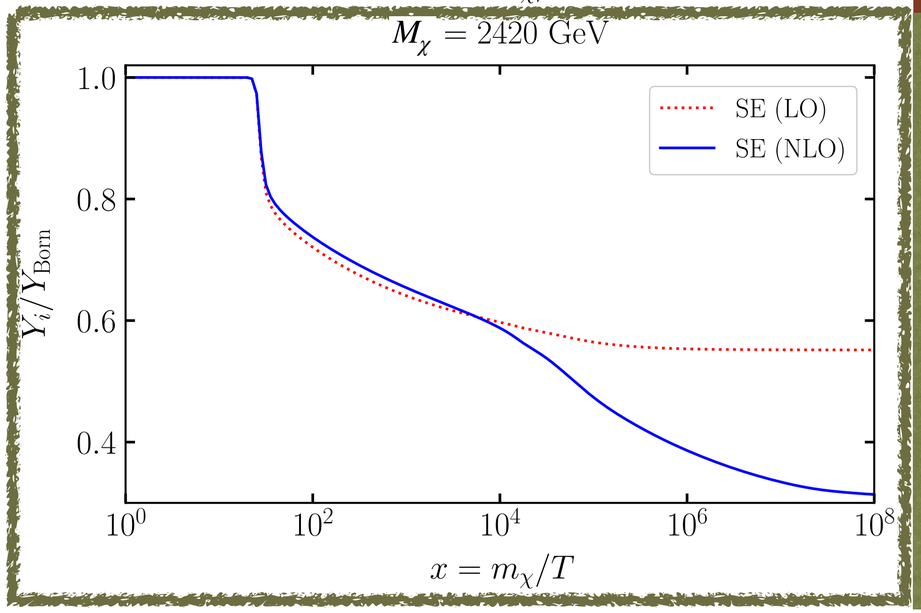
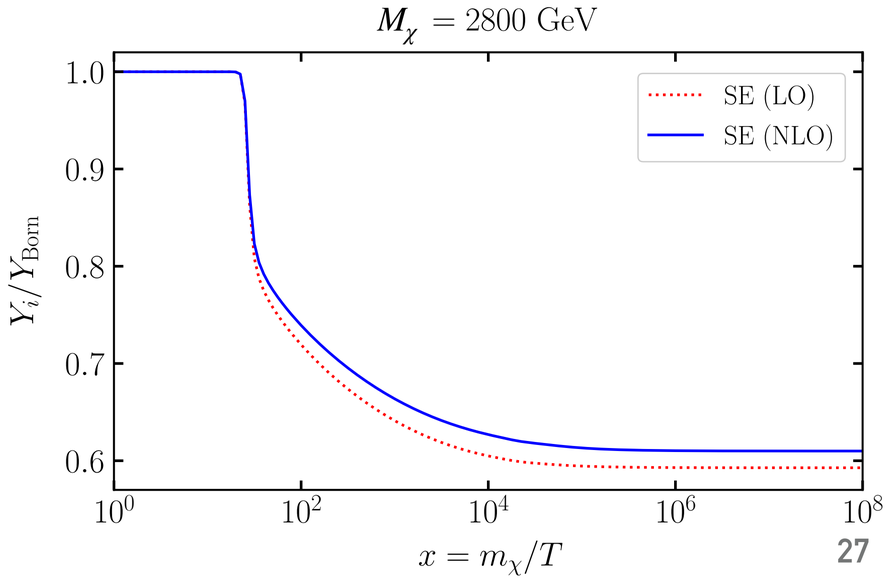


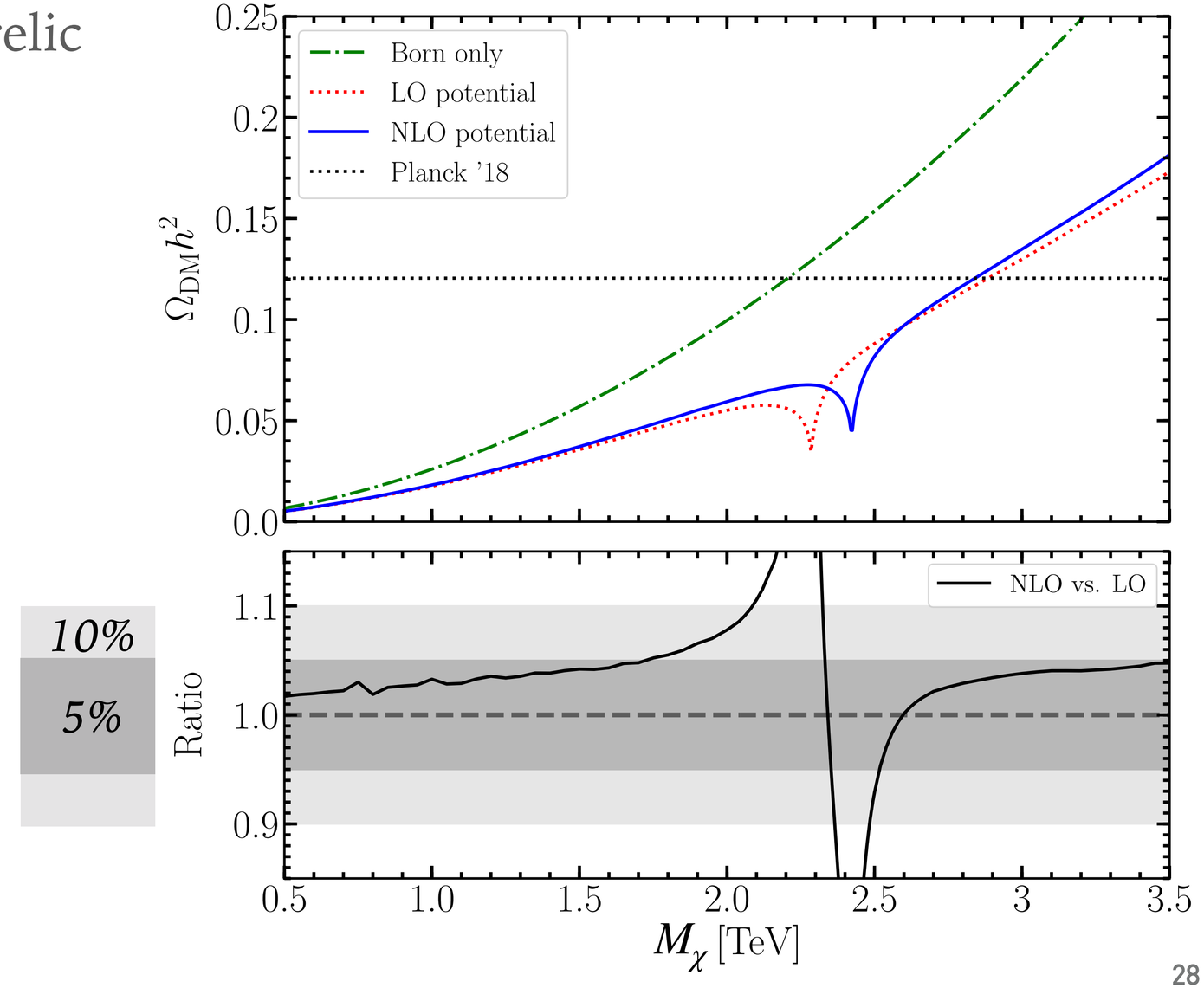


At late times, the $1/v^2$ $(1/T)$ enhancement for masses around the resonance can lead to substantial late-time annihilations.



WINO DM RELIC ABUNDANCE

- Away from the resonance, NLO relic abundance is larger by about $2-5\,\%$ than the LO one
- Effect is larger than typically included $\mathcal{O}(v^2)$ terms which give $1-3\,\%$ correction
- Observed value $\Omega_{\rm DM}h^2 = 0.1205$ corresponds to $M_\chi = 2.842$TeV (at LO: $M_\chi = 2.886$TeV)



SUMMARY AND OUTLOOK

- Computation of WIMP annihilation is a highly non-trivial problem
- If your physics is close to a resonance, better include higher order potentials
- If you are away from a resonance, then the correction are still larger than typical EW effects
- NLO corrections weaken the potential and shift resonance towards larger values
- Higher order effects can change the picture both quantitatively and qualitatively — new phenomena emerge
- Pure wino will be excluded by CTA, we need to look towards more complicated scenarios (Higgsino, quintuplet …)