# *Flavor number schemes, evolution and PDF extraction*

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Heavy Flavor at the EIC Workshop, Zoom, Nov 04, 2020

## *Based on work done in collaboration with:*

- *Heavy-flavor PDF evolution and variable-flavor number scheme uncertainties in deep-inelastic scattering*
  S. Alekhin, J. Blümlein and S. M. arXiv:2006.07032
- *NLO PDFs from the ABMP16 fit*
  S. Alekhin, J. Blümlein and S. M. arXiv:1803.07537
- *Parton Distribution Functions, $\alpha_s$ and Heavy-Quark Masses for LHC Run II*
  S. Alekhin, J. Blümlein, S. M. and R. Plačakytė arXiv:1701.05838
- Many more papers of ABM and friends . . .
  2008 – ...

# *Treatment of heavy-quarks*

## *Light quarks*

- Neglect "light quark" masses $m_u, m_d \ll \Lambda_{QCD}$ and $m_s < \Lambda_{QCD}$ in hard scattering process
  - scale-dependent $u, d, s, g$ PDFs from mass singularities

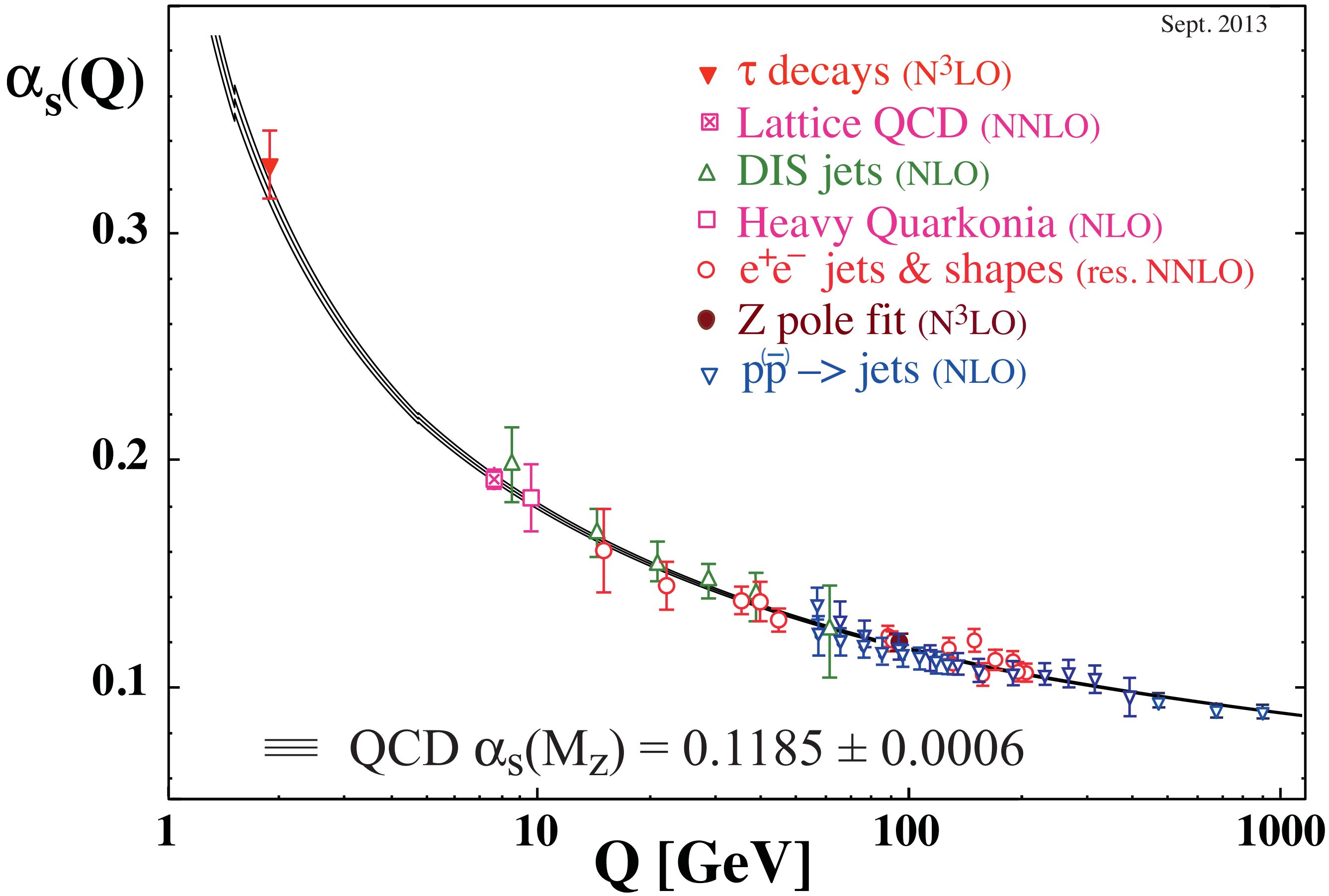
## *Heavy quarks*

- No mass singularities for $m_c, m_b, m_t \gg \Lambda_{QCD}$, no (evolving) PDFs
  - $c$ and $b$ PDFs for $Q \ggg m_c, m_b$ generated perturbatively
  - matching of two distinct theories
    $\longrightarrow n_f$ light flavors + heavy quark of mass $m$ at low scales
    $\longrightarrow n_f + 1$ light flavors at high scales

# *Strong coupling with flavor thresholds*

- Solution of QCD $\beta$-function for $\alpha_s^{n_l} \longrightarrow \alpha_s^{(n_l+n_h)}$
  - discontinuities for $n_f = 3 \to n_f = 4 \to n_f = 5$
- Big picture

Bethke for PDG 2014



# *Strong coupling with flavor thresholds*

- Solution of QCD $\beta$-function for $\alpha_s^{n_l} \longrightarrow \alpha_s^{(n_l+n_h)}$
  - discontinuities for $n_f = 3 \to n_f = 4 \to n_f = 5$
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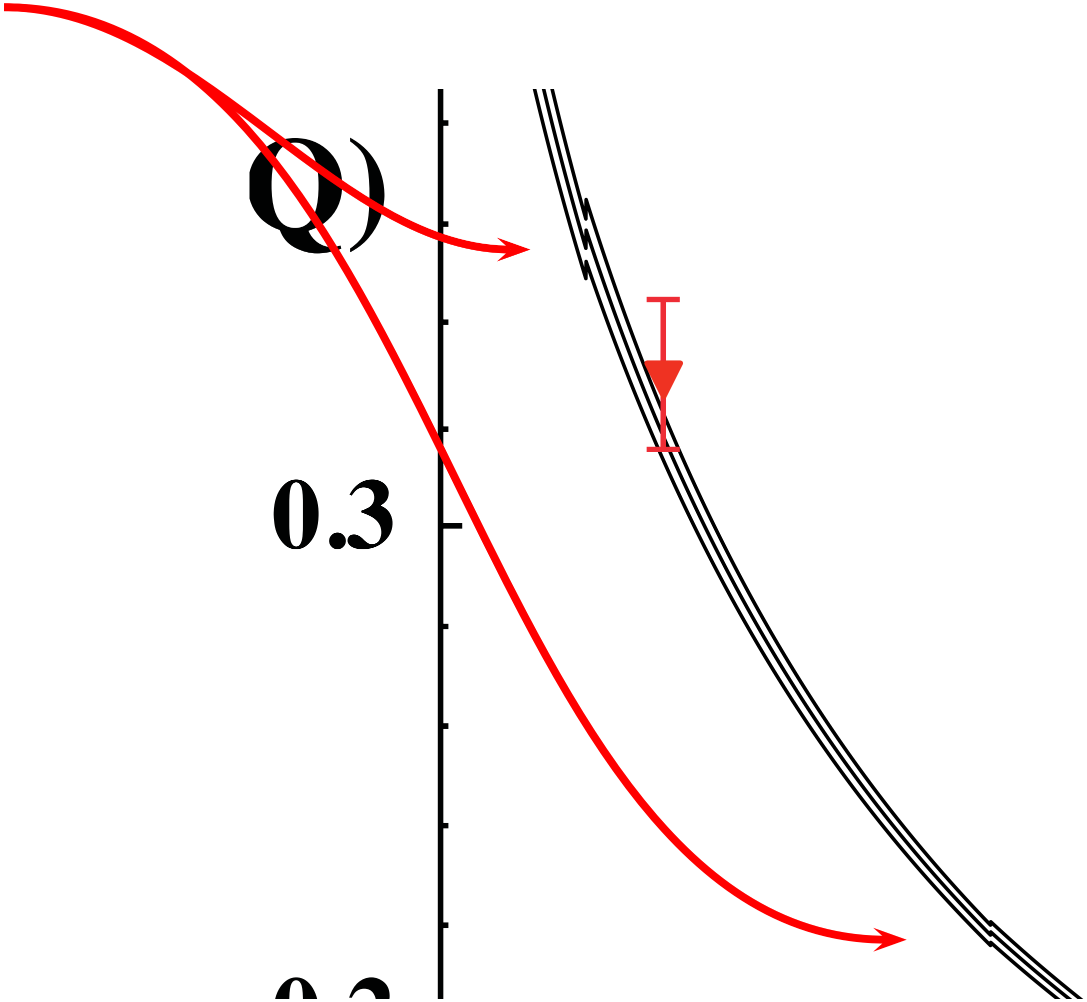


# *Cross sections with flavor thresholds*

## *Fixed flavor number scheme (FFNS) ("fixed order* $\ln(Q^2/m^2)$*")*

- Cross section with massive quarks at scales $Q \not\gg m_c$
  - top-quark hadro-production ($t\bar{t}$ pairs, single top in 4FS or 5FS, . . . ]
- $F_2^c$ at HERA with $u, d, s, g$ partons and massive charm coeff. fcts.
  - complete NLO predictions Laenen, Riemersma, Smith, van Neerven '92
  - approximations at NNLO Bierenbaum, Blümlein, Klein '09; Lo Presti, Kawamura, S.M., Vogt '12; Behring, Bierenbaum, Blümlein, De Freitas, Klein, Wissbrock '14

## *Variable flavor number scheme (VFNS) ("resum* $\ln(Q^2/m^2)$*")*

- (Smooth) matching of two distinct theories:
  $n_f$ light + heavy quark at low scales $\longrightarrow n_f + 1$ light flavors at high scales
  - Higgs boson production in $b\bar{b}$-annihilation ("Santander matching" Harlander, Krämer, Schumacher '11)
- $F_2^c$ at HERA with ACOT Aivazis, Collins, Olness, Tung '94, BMSN Buza, Matiounine, Smith, van Neerven '98, RT Thorne, Roberts '98, FONLL Forte, Laenen, Nason, Rojo '10
  - model assumptions in matching conditions
  - details of implementation matter in global fits

# GM-VFNS implementation (I)

- BSMN prescription for DIS structure function $F_2^h$ for heavy-quark $h$

$$F_2^{h,\mathrm{BMSN}}(N_f+1,x,Q^2) =$$
$$= \quad F_2^{h,\mathrm{exact}}(N_f,x,Q^2) + \left\{ F_2^{h,\mathrm{ZMVFN}}(N_f+1,x,Q^2) - F_2^{h,\mathrm{asymp}}(N_f,x,Q^2) \right\}$$

  - $F_2^{h,\mathrm{exact}}$: massive heavy-quark structure function ($m \neq 0$)
  - $F_2^{h,\mathrm{ZMVFN}}$: DIS structure function with zero mass ($m = 0$)
  - $F_2^{h,\mathrm{asymp}}$: asymptotic expansion of heavy-quark structure function (logarithms $\ln(Q^2/m^2)$)
- Difference $\{\ldots\}$ has to vanish at threshold $Q \simeq m$
- Generation of heavy-quark PDFs $h^{(n_f+1)}$ from light-flavor PDFs
  - heavy-quark operator matrix elements (OMEs) $A_{ji}$ at three loops
    Bierenbaum, Blümlein, Klein '09; Ablinger, Behring, Blümlein, De Freitas, von Manteuffel, Schneider '14

$$h^{(n_f+1)}(x,\mu) + \bar{h}^{(n_f+1)}(x,\mu) = A_{hq}(x) \otimes \Sigma^{(n_f)}(x,\mu) + A_{hg}(x) \otimes g^{(n_f)}(x,\mu)$$

# GM-VFNS implementation (II)

- Other variants of GM-VFNS implementations

$$F_2^{h,\mathrm{GM-VFNS}}(N_f+1,x,Q^2) =$$
$$= \quad F_2^{h,\mathrm{exact}}(N_f,x,Q^2) + \left\{F_2^{h,\mathrm{ZMVFN}}(N_f+1,x,Q^2) - F_2^{h,\mathrm{asymp}}(N_f,x,Q^2)\right\}$$

  - details of combining $F_2^{h,\mathrm{exact}}$, $F_2^{h,\mathrm{ZMVFN}}$ and $F_2^{h,\mathrm{asymp}}$ differ for other GM-VFNS implementations
- Different approaches to impose vanishing of $\{\ldots\}$ at threshold $Q \simeq m$
  - subject to model assumptions
  - ACOT: S-ACOT-$\chi$ for slow rescaling $x \to \chi(x) = x\left(1+\frac{4m^2}{Q^2}\right)$
  - FONLL: suppression of $\{\ldots\}$ with damping factor $\left(1+\frac{m^2}{Q^2}\right)^2$
  - RT: continuity of physical observables in threshold region
- Scale evolution of heavy-quark PDFs $h^{(n_f+1)}$

# PDFs with flavor thresholds (I)

- Generate heavy-quark PDFs $h^{(n_f+1)}$ from light-flavor PDFs
  - heavy-quark operator matrix elements (OMEs) $A_{ji}$ at three loops
    Bierenbaum, Blümlein, Klein '09; Ablinger, Behring, Blümlein, De Freitas, von Manteuffel, Schneider '14

$$h^{(n_f+1)}(x,\mu) + \bar{h}^{(n_f+1)}(x,\mu) = A_{hq}(x) \otimes \Sigma^{(n_f)}(x,\mu) + A_{hg}(x) \otimes g^{(n_f)}(x,\mu)$$

  - likewise light-quark PDFs $l_i^{(n_f)} \to l_i^{(n_f+1)}$ and gluon and the quark singlet PDFs $(\Sigma^{(n_f)}, g^{(n_f)}) \to (\Sigma^{(n_f+1)}, g^{(n_f+1)})$
- Perturbative expansion of OME $A_{hg}$

$$A_{hg}^{(1)}(x) = \underbrace{a_{hg}^{(10)}}_{=0} + \ln\left(\frac{\mu^2}{m^2}\right) P_{qg}^{(0)}$$

  - charm density at leading order with matching $c(x,\mu^2 = m_c^2) = 0$

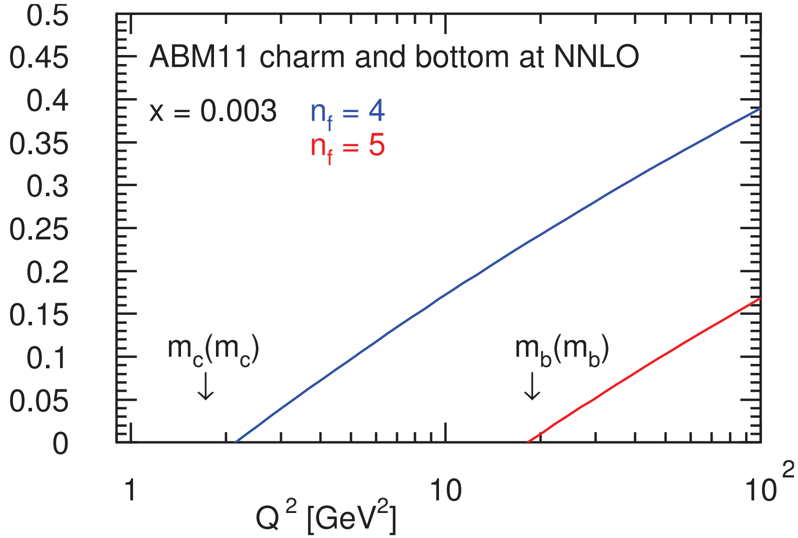
$$c^{(1)}(x,\mu^2) = a_s(\mu^2) \int_x^1 \frac{dz}{z} \ln\left(\frac{\mu^2}{m_c^2}\right) P_{qg}^{(0)}(z)\, g\left(\frac{x}{z},\mu^2\right)$$

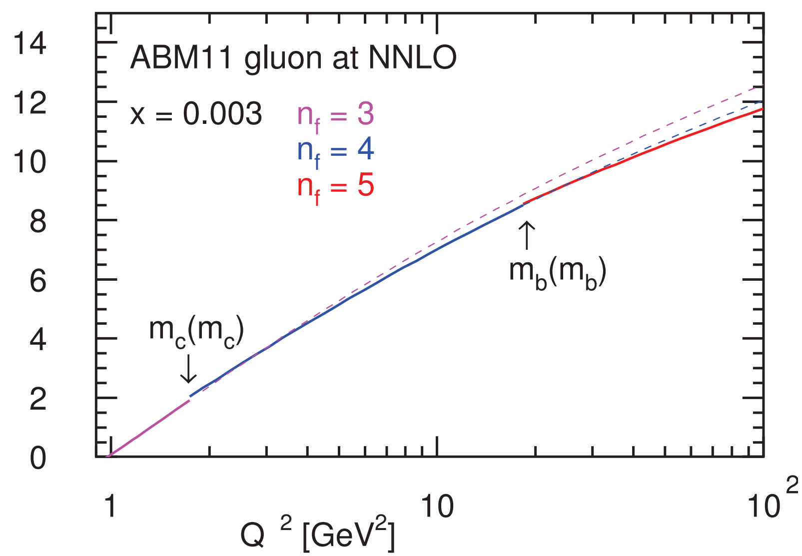
  - higher order matching $c(x,\mu^2 = m_c^2) \neq 0$

$$A_{hg}^{(2)}(x) = \underbrace{a_{hg}^{(20)}}_{\neq 0} + \ln\left(\frac{\mu^2}{m^2}\right) a_{hg}^{(21)} + \ln^2\left(\frac{\mu^2}{m^2}\right) a_{hg}^{(22)}$$

# *PDFs with flavor thresholds (II)*

- Solution of evolution equations between thresholds for $n_f \longrightarrow (n_f + 1)$ with fixed $n_f = 3 \to n_f = 4 \to n_f = 5$
  - discontinuities in PDFs across flavor thresholds
  - matching conditions known to NLO; $A_{hg}^{(3)}$ currently unknown



# PDFs with flavor thresholds (III)

- Scale dependence of charm quark PDF (here LO)

$$c^{(1)}(x,\mu^2) \quad = \quad a_s(\mu^2) \int_x^1 \frac{dz}{z} \ln\left(\frac{\mu^2}{m_c^2}\right) P_{qg}^{(0)}(z)\, g\left(\frac{x}{z},\mu^2\right)$$

## Fixed-order perturbation theory (FOPT)

- BSMN uses this and corresponding expression at higher orders to determine charm-quark PDF at all scales $\mu \geq m_c$

## Evolution at LO, NLO, ...

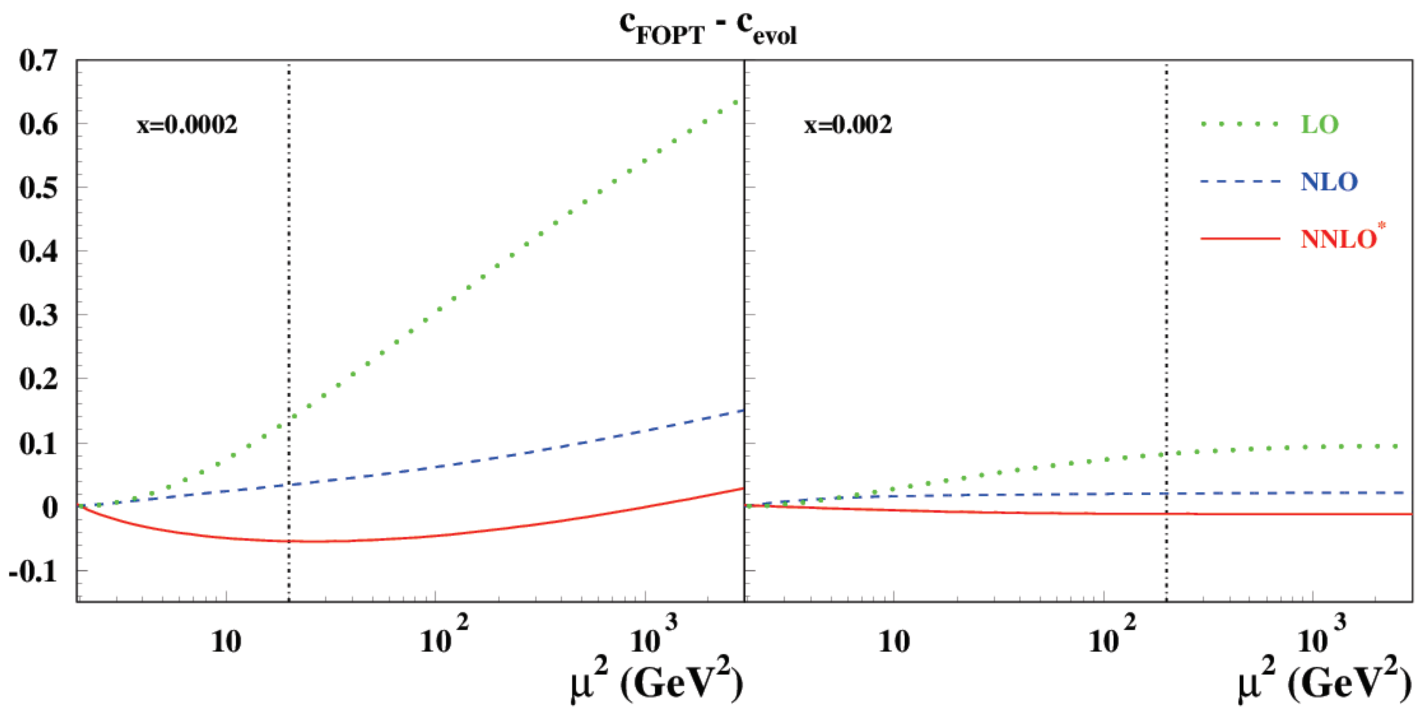
- Other GM-VFNS prescriptions (ACOT, FONLL, RT) use this as boundary condition for $c(x,\mu^2)$ at $\mu = m_c$ and derive scale dependence with standard QCD evolution equations

- Difference between FOPT and evolution illustrated with scale derivative

$$\frac{dc^{(1)}(x,\mu^2)}{d\ln\mu^2} \quad = \quad a_s(\mu^2) \int_x^1 \frac{dz}{z}\, a_{hg}^{(11)}(z)\; g\left(\frac{x}{z},\mu^2\right) + \left(\frac{da_s}{d\ln\mu^2}\right) \frac{c^{(1)}(x,\mu^2)}{a_s}$$
$$+ a_s(\mu^2) \ln\left(\frac{\mu^2}{m_c^2}\right) \int_x^1 \frac{dz}{z}\, a_{hg}^{(11)}(z)\, \frac{dg(x/z,\mu^2)}{d\ln\mu^2}$$

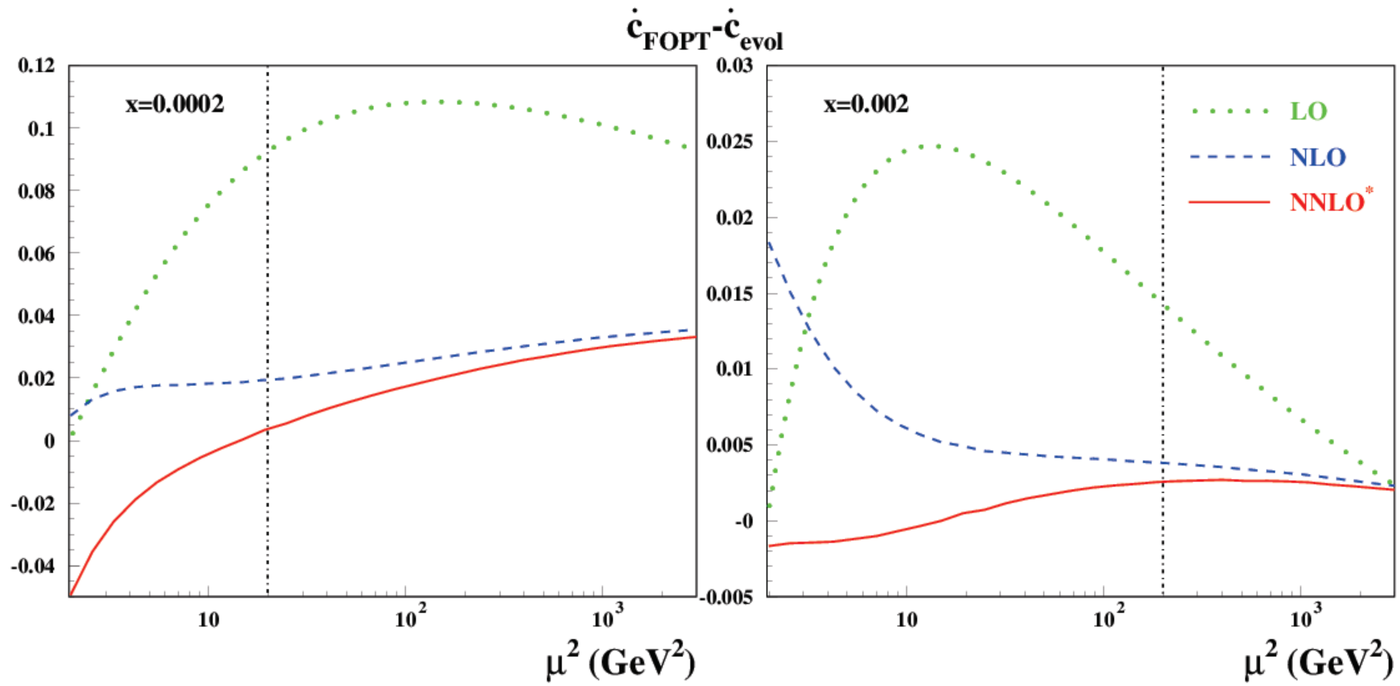
  - second and third term of higher order, but numerically important

# *Difference between FOPT and evolution (I)*



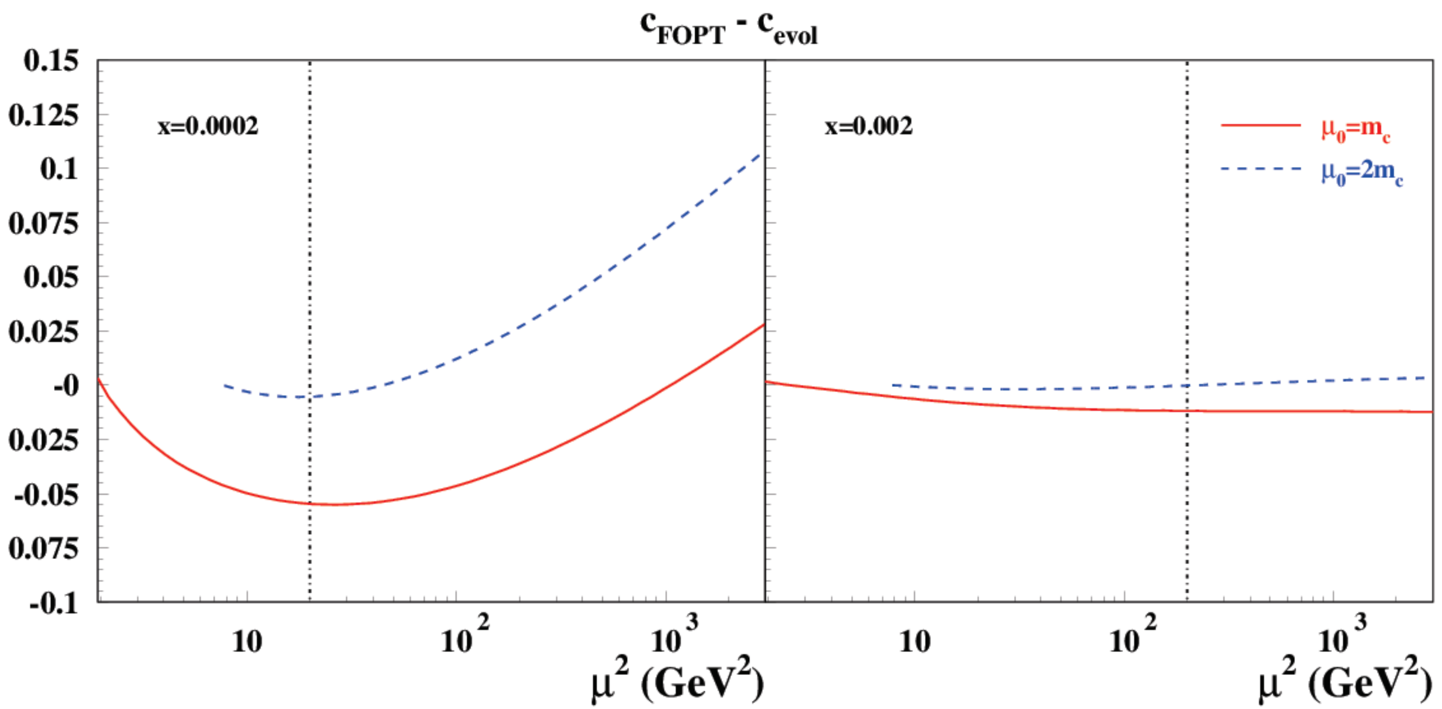

- Difference of charm-quark PDFs: $c_{\rm FOPT} - c_{\rm evol}$
  - LO (with one loop splitting functions and OMEs)
  - NLO (with two loop splitting functions and OMEs)
  - NNLO$^*$ (with three loop splitting functions; three loop OME $A_{hg}^{(3)}$ still missing)
- Matching scale $\mu_0 = m_c = 1.4\ {\rm GeV}$ with pole mass $m_c$
- Vertical dash-dotted lines display upper margin of kinematics at HERA

# Difference between FOPT and evolution (II)



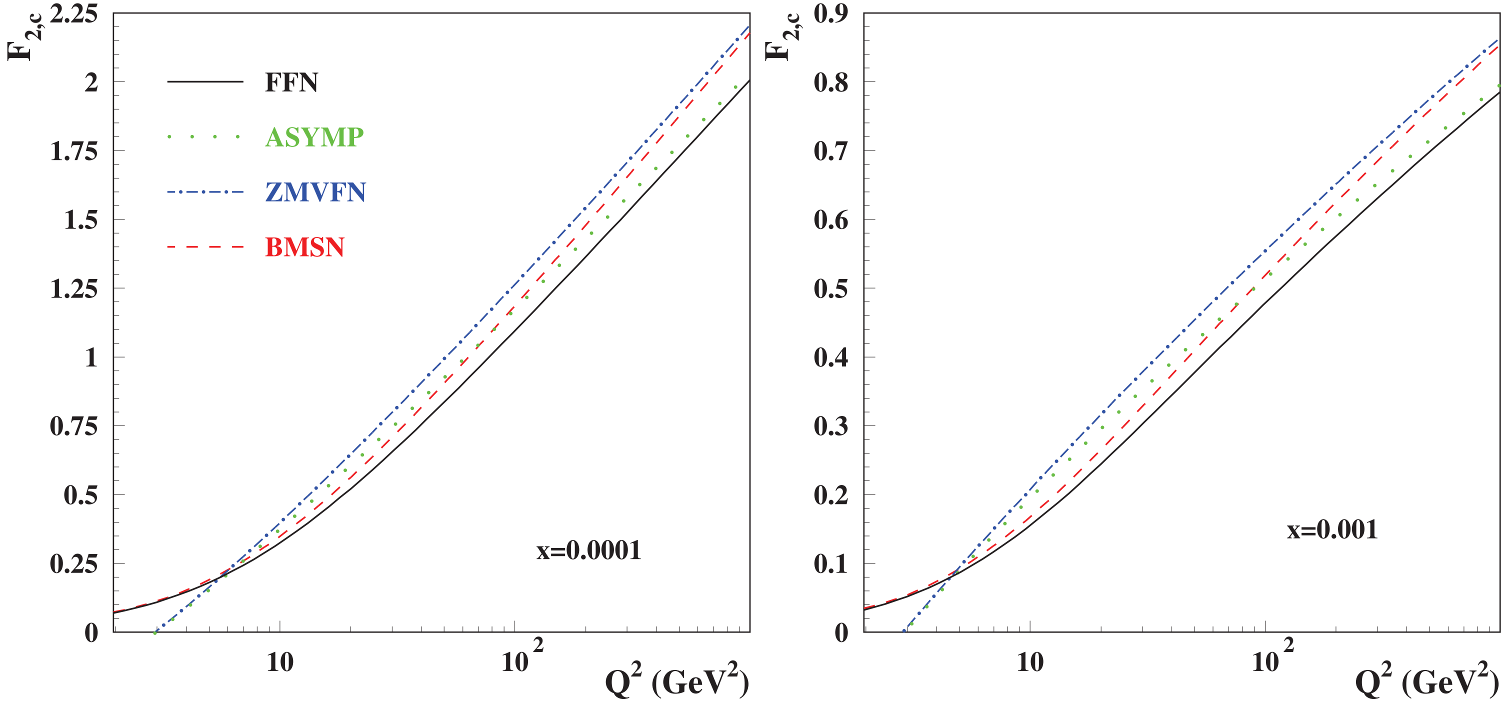

- Difference of scale derivatives of charm-quark PDFs: $\dot{c}_{\rm FOPT} - \dot{c}_{\rm evol}$

# Difference between FOPT and evolution (III)



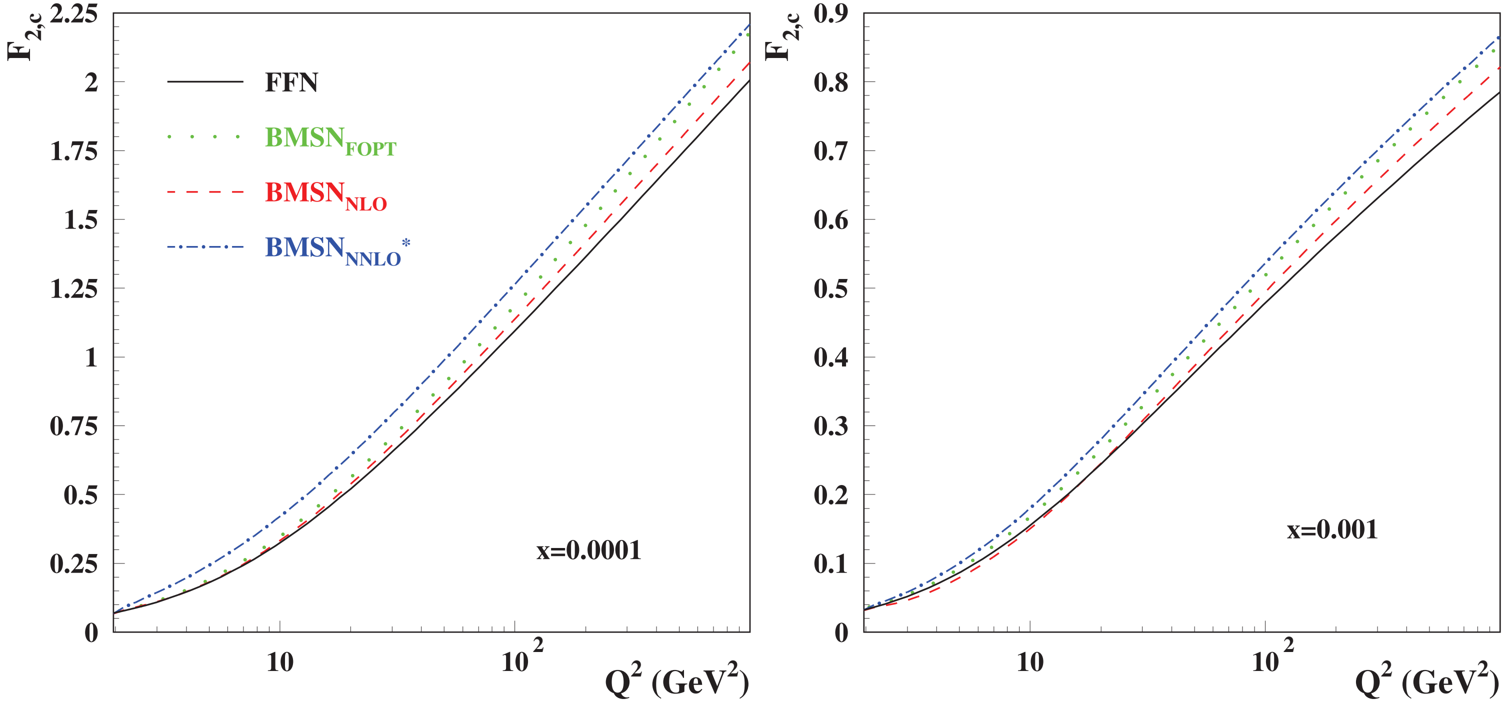

- Difference of charm-quark PDFs: $c_{\rm FOPT} - c_{\rm evol}$
  - NNLO$^*$ for FOPT and evolution
  - impact of different matching scales: $\mu_0 = m_c$ and $\mu_0 = 2m_c$

# $F_2^c$ structure function (I)



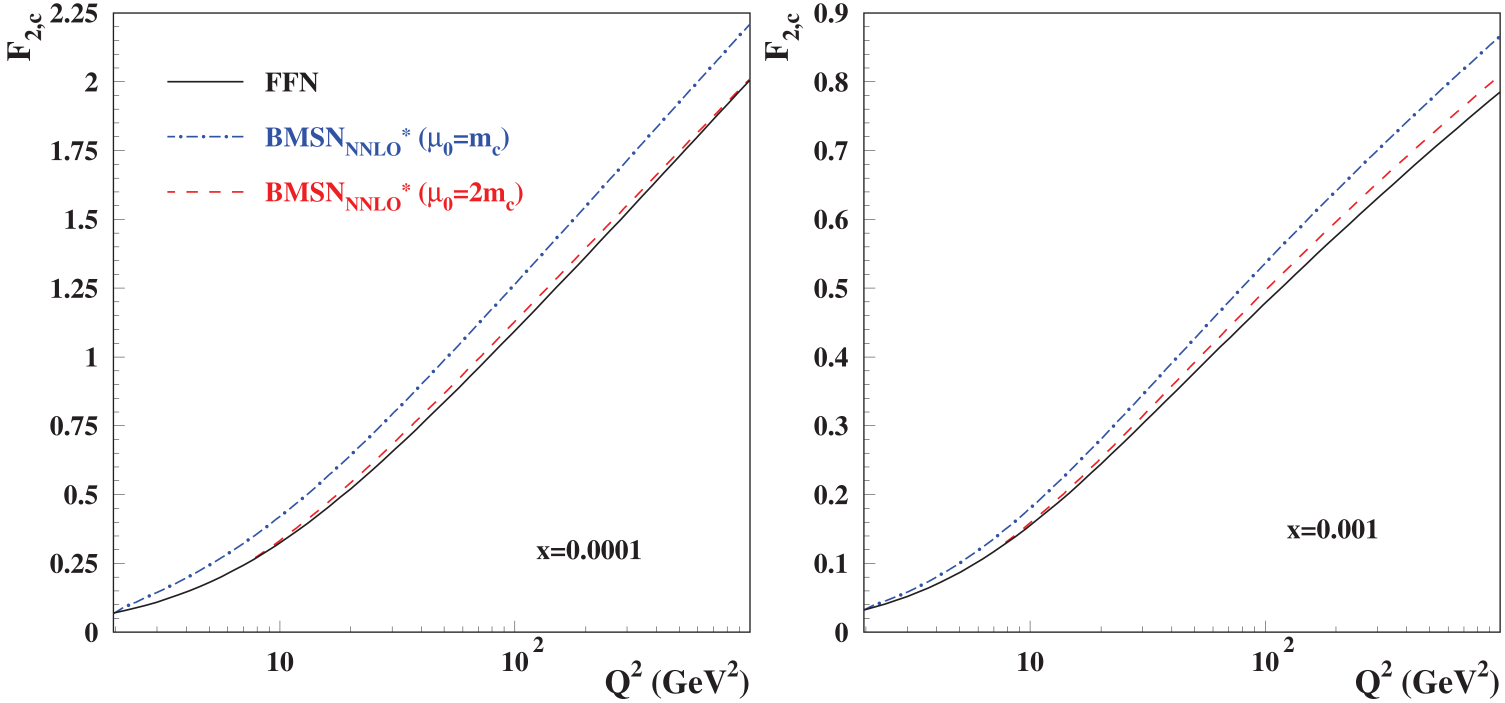

- DIS charm-quark structure function $F_2^c$ with comparison of
  - $F_2^{h,\mathrm{FFN}}$: exact massive heavy-quark structure function
  - $F_2^{h,\mathrm{asymp}}$: asymptotic expansion with logarithms $\ln(Q^2/m^2)$
  - $F_2^{h,\mathrm{ZMVFN}}$: DIS structure function with $m_c = 0$
  - $F_2^{h,\mathrm{BMSN}}$: DIS structure function with BMSN prescription of VFNS

# $F_2^c$ structure function (II)




- DIS charm-quark structure function $F_2^c$ with comparison of
  - $F_2^{h,\mathrm{FFN}}$: exact massive heavy-quark structure function
  - $F_2^{h,\mathrm{BMSN}}\big|_{\mathrm{FOPT}}$: BMSN prescription, FOPT $c$-quark PDFs
  - $F_2^{h,\mathrm{BMSN}}\big|_{\mathrm{NLO}}$: BMSN prescription, NLO evolved $c$-quark PDFs
  - $F_2^{h,\mathrm{BMSN}}\big|_{\mathrm{NNLO}^*}$: BMSN prescription, NNLO$^*$ evolved $c$-quark PDFs

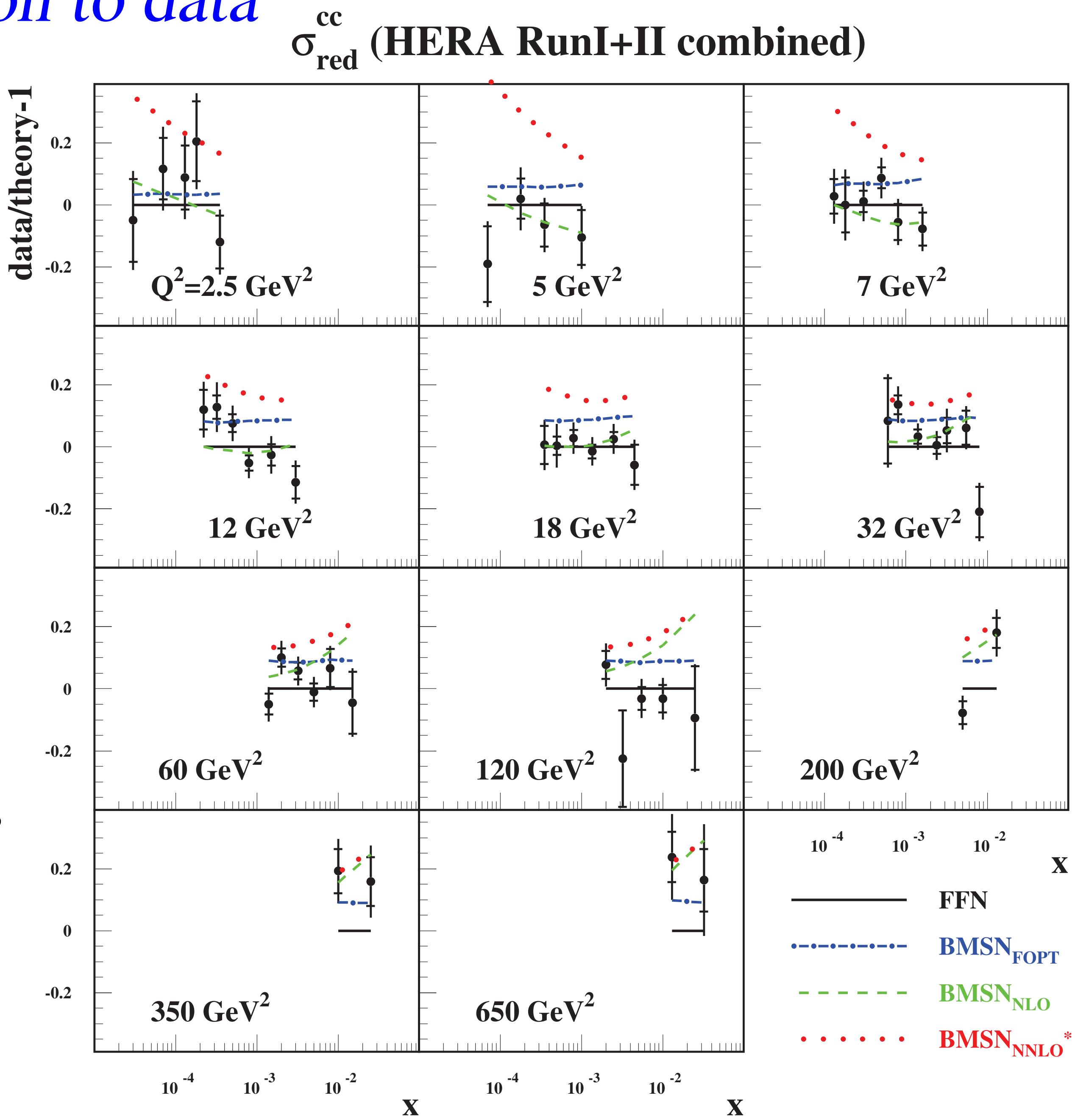
# $F_2^c$ structure function (III)



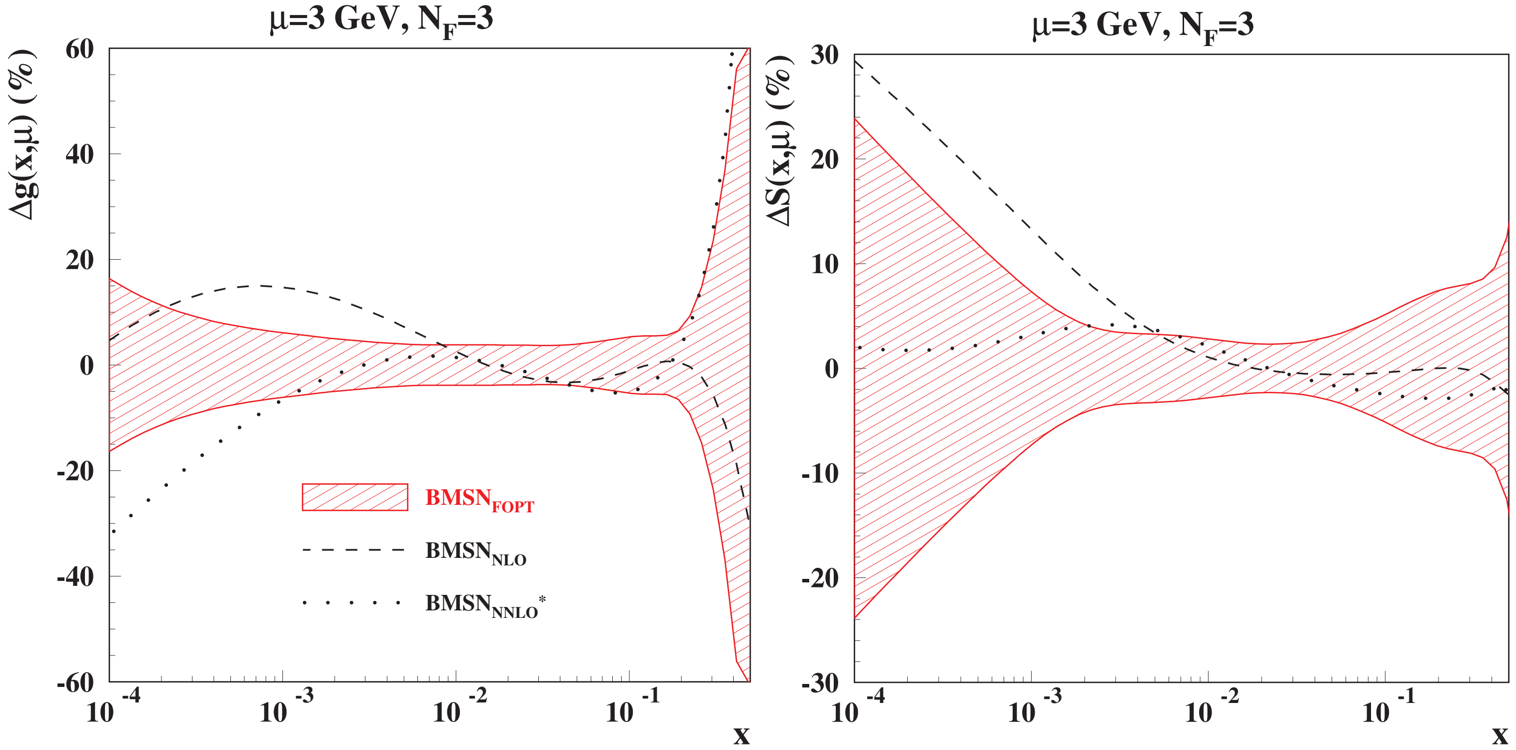

- DIS charm-quark structure function $F_2^c$ with comparison of
  - $F_2^{h,\mathrm{FFN}}$: exact massive heavy-quark structure function
  - $F_2^{h,\mathrm{BMSN}}\big|_{\mathrm{NNLO}^*}$: BMSN prescription, NNLO$^*$ evolved $c$-quark PDFs
- NNLO$^*$ evolution with impact of different matching scales
  - $\mu_0 = m_c$ and $\mu_0 = 2m_c$

# Comparision to data

- Pulls obtained for combined HERA DIS $c$-quark production data in FFNS version
- Comparison of
  - $F_2^{h,\mathrm{FFN}}$
  - $F_2^{h,\mathrm{BMSN}}\big|_{\mathrm{FOPT}}$
  - $F_2^{h,\mathrm{BMSN}}\big|_{\mathrm{NLO}}$
  - $F_2^{h,\mathrm{BMSN}}\big|_{\mathrm{NNLO*}}$
- Use in all cases PDFs from FFNS fit

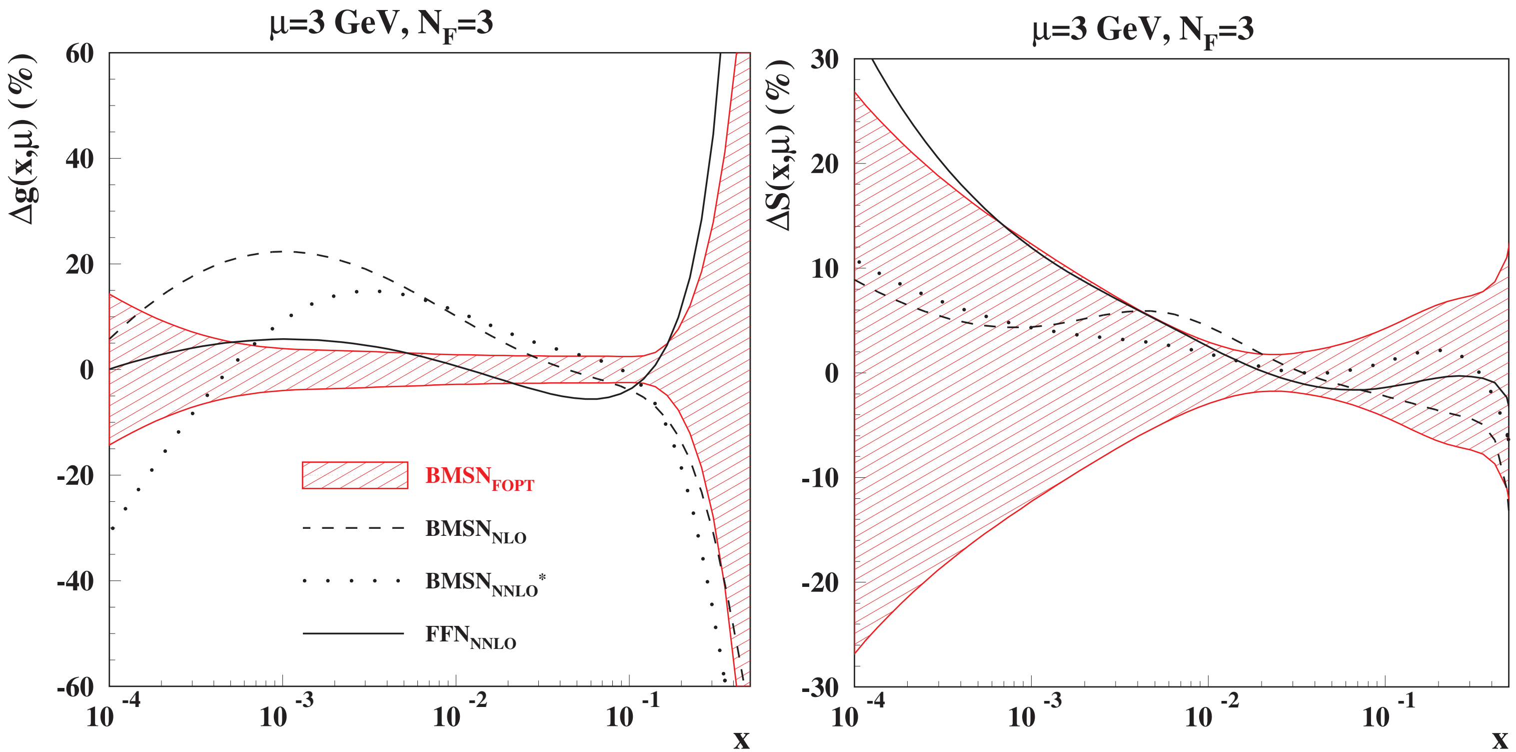


# Differences for PDFs (I)



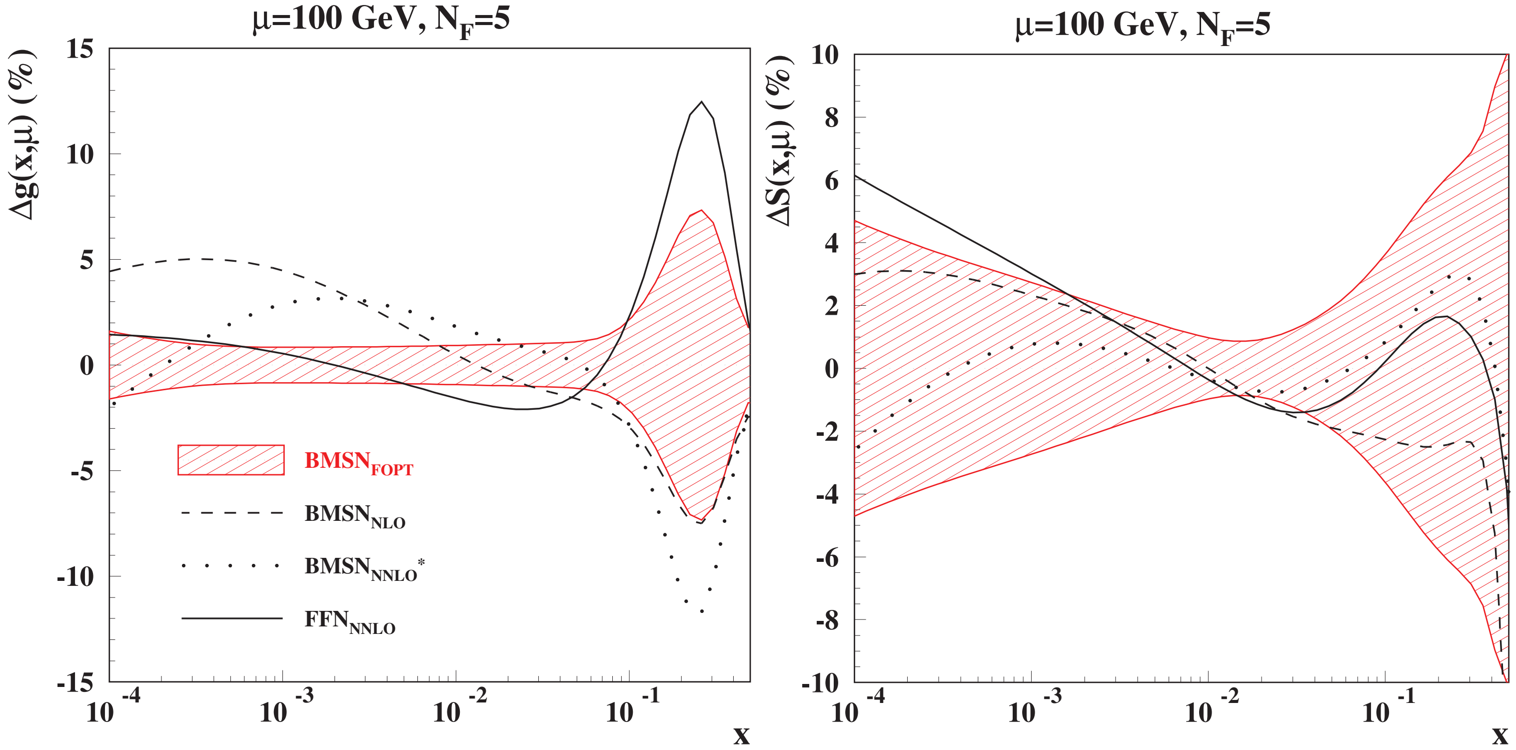

- Relative uncertainty in $n_f = 3$ flavor PDFs at scale $\mu = 3\ \mathrm{GeV}$
- Gluon $xg(x,\mu)$ and sea $xS(x,\mu)$ PDFs from fit with BMSN prescription and FOPT
  - $\mathrm{BMSN}\big|_{\mathrm{FOPT}}$: BMSN prescription, FOPT $c$-quark PDFs
  - $\mathrm{BMSN}\big|_{\mathrm{NLO}}$: BMSN prescription, NLO evolved $c$-quark PDFs
  - $\mathrm{BMSN}\big|_{\mathrm{NNLO}^*}$: BMSN prescription, NNLO$^*$ evolved $c$-quark PDFs

# *Differences for PDFs (II)*



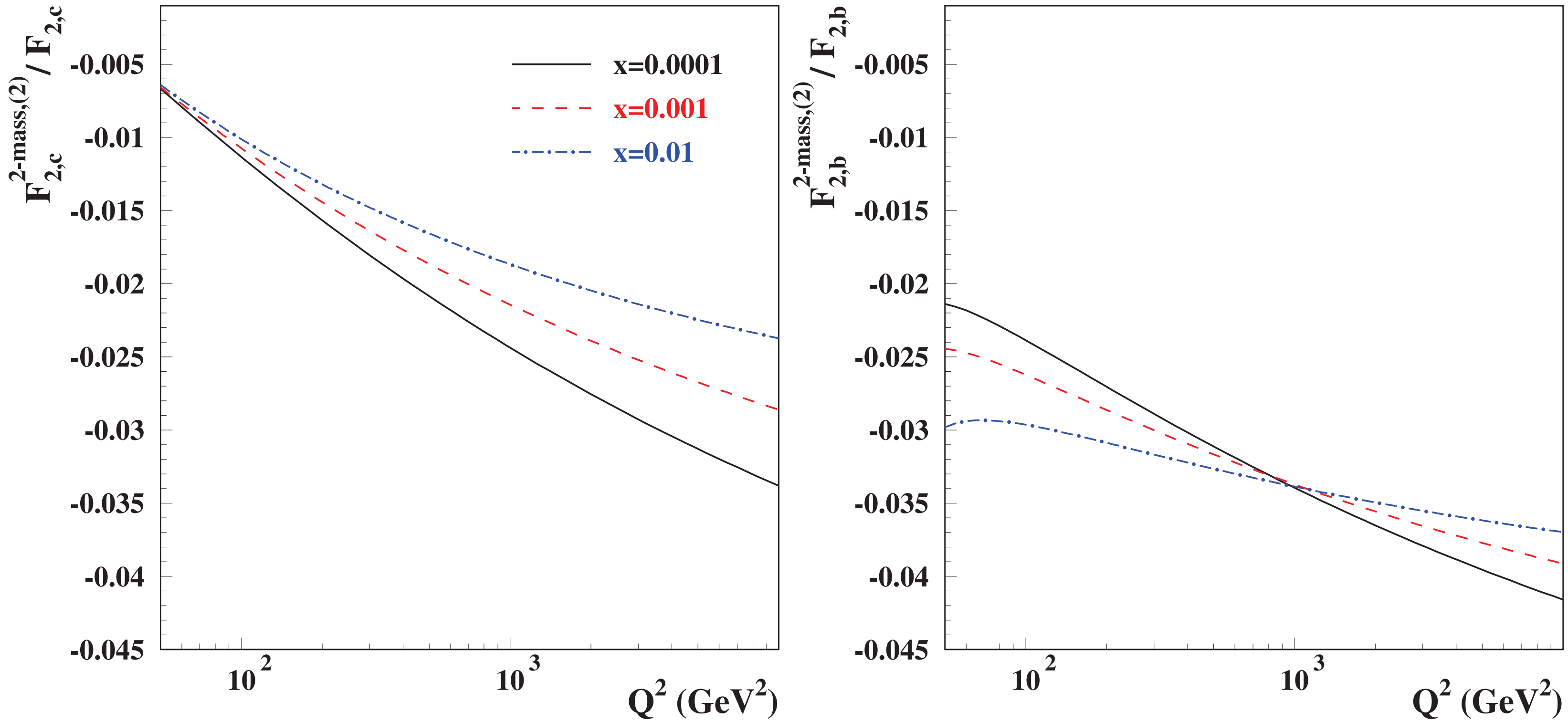

- Gluon $xg(x, \mu)$ and sea $xS(x, \mu)$ PDFs for $n_f = 3$ at scale $\mu = 3$ GeV
- Now from fit with HERA inclusive DIS data added
  - $\text{FFN}\big|_{\text{NNLO}}$: FFNS PDFs at NNLO
  - $\text{BMSN}\big|_{\text{FOPT}}$: BMSN prescription, FOPT $c$-quark PDFs
  - $\text{BMSN}\big|_{\text{NLO}}$: BMSN prescription, NLO evolved $c$-quark PDFs
  - $\text{BMSN}\big|_{\text{NNLO}^*}$: BMSN prescription, NNLO$^*$ evolved $c$-quark PDFs

# Differences for PDFs (III)




- Gluon $xg(x, \mu)$ and sea $xS(x, \mu)$ PDFs for $n_f = 5$ at scale $\mu = 100\ \mathrm{GeV}$
  - $\mathrm{FFN}\big|_{\mathrm{NNLO}}$: FFNS PDFs at NNLO
  - $\mathrm{BMSN}\big|_{\mathrm{FOPT}}$: BMSN prescription, FOPT $c$-quark PDFs
  - $\mathrm{BMSN}\big|_{\mathrm{NLO}}$: BMSN prescription, NLO evolved $c$-quark PDFs
  - $\mathrm{BMSN}\big|_{\mathrm{NNLO}^*}$: BMSN prescription, NNLO$^*$ evolved $c$-quark PDFs

# *Two-mass contributions*




- Two-mass contribution to DIS structure function $F_{2,c}$ (left) and $F_{2,b}$ (right)
  - Feynman diagrams with both heavy charm and bottom loops
  - VFNS with simultaneous transition for light flavors $n_f \to n_f + 2$

$$F_{2,h}^{\mathrm{cb},(2)}(x,Q^2) = -e_h^2\, a_s^2(Q^2)\, \frac{4}{3}\, x \ln\left(\frac{Q^2}{m_c^2}\right) \ln\left(\frac{Q^2}{m_b^2}\right) \int_x^1 \frac{dz}{z} \left(z^2 + (1-z)^2\right) g\left(\frac{x}{z}, Q^2\right)$$

# *Summary*

## *Big logs aren't always big*

- Study of VFNS in BMSN approach (FOPT, evolved at LO, NLO, NNLO)
- Study of charm-quark PDF in FFNS and VFNS (FOPT vs. subsequent evolution)
  - sizable impact of PDF evolution in BMSN prescription of VFNS $\longrightarrow$ $x$-dependent rather than $Q^2$-dependent
  - little impact on resummation of large logarithms on heavy-quark production for realistic kinematics
- Differences in charm-quark PDFs accuracy of evolution equations is intrinsic uncertainties VFNS
- FFNS gives very good description of HERA charm-quark data
  - no need for additional resummation of large logarithms in kinematic range covered by experiment