Hadronization into open heavy flavor

Ingo Schienbein
Université Grenoble Alpes/LPSC Grenoble

Opportunities with Heavy Flavor at the EIC
November 4-6 2020

Subject of this talk

- Extraction/Calculation of heavy flavor FFs and the related phenomenology
- How can this help understanding heavy flavor production at the EIC

Heavy flavor FFs depend on the heavy flavor scheme

Heavy Flavor Fragmentation Functions (FF)

- Need **factorization** to talk about FFs
- Here: **One-particle inclusive production** where we have **factorization theorems**
- FFs depend on:
  - Heavy flavor scheme:
    - FFNS: **scale-independent FF, D(z)**
    - VFNS: **scale-dependent, evolved FF, $D(z,\mu_F')$**
  - Perturbative order: **NLO** FF **harder** than **LO** FF
  - Factorization scheme: same way as PDFs, usually MSbar

$A+B\to H+X$: $\quad d\sigma = \sum_{i,j,k} f_i^A(x_1) \otimes f_j^B(x_2) \otimes d\sigma(ij\to kX) \otimes D_k^H(z)$

sum over all possible subprocesses $i+j\to k+X$

Parton distribution functions:
$f_i^A(x_1,\mu_F)$, $f_j^B(x_2,\mu_F)$
non-perturbative input
long distance
universal

Hard scattering cross section:
$d\sigma(\mu_F,\mu'_F,\alpha_s(\mu_R),[\frac{m_h}{p_T}])$
perturbatively computable
short distance
(coeffi cient functions)

Fragmentation functions:
$D_k^H(z,[\mu'_F])$
non-perturbative input
long distance
universal

Accuracy:

light hadrons: $\mathcal{O}((\Lambda/p_T)^p)$ with p_T hard scale, Λ hadronic scale, $p=1,2$
heavy hadrons: if m_h is neglected in $d\sigma$: $\mathcal{O}((m_h/p_T)^p)$

Details (subprocesses, PDFs, FFs; mass terms) depend on the Heavy Flavour Scheme

Theoretical approaches:

Fixed Flavor Number Scheme (FFNS)

FFNS/Fixed Order

Factorization formula for inclusive heavy quark (Q) production:

PDFs

$$d\sigma^Q \simeq \sum_{a,b} f_a^A \otimes f_b^B \otimes d\tilde{\sigma}_{ab \to Q+X}$$

sum over all possible partonic subprocesses
NO heavy quark PDF

Calculable short distance cross section;
log(pT/m) terms kept in **fixed order**

FFNS/Fixed Order

Factorization formula for inclusive heavy quark (Q) production:

PDFs

$$d\sigma^Q \simeq \sum_{a,b} f_a^A \otimes f_b^B \otimes d\tilde{\sigma}_{ab\to Q+X}$$

sum over all possible
partonic subprocesses
NO heavy quark PDF

Calculable short distance cross section;
log(pT/m) terms kept in **fixed order**

Inclusive heavy-flavored hadron (H) production:

$$d\sigma^H = d\sigma^Q \otimes D_Q^H(z)$$

Convolution with a
scale-independent FF

* non-perturbative
* describes hadronization
* not based on a fact. theorem

FFNS/Fixed Order

Factorization formula for inclusive heavy quark (Q) production:

PDFs

$$d\sigma^Q \simeq \sum_{a,b} f_a^A \otimes f_b^B \otimes d\tilde{\sigma}_{ab\to Q+X}$$

sum over all possible partonic subprocesses
NO heavy quark PDF

Calculable short distance cross section; log(pT/m) terms kept in **fixed order**

Inclusive heavy-flavored hadron (H) production:

$$d\sigma^H = d\sigma^Q \otimes D_Q^H(z)$$

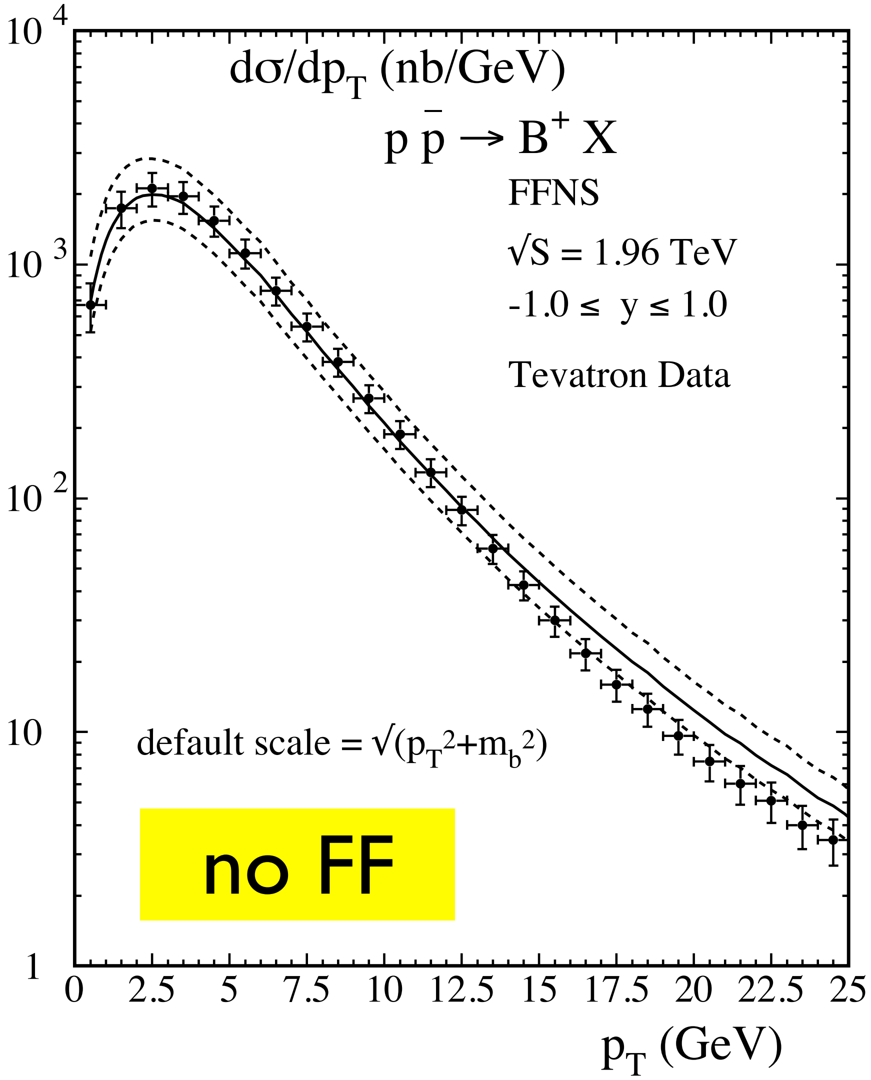
Convolution with a **scale-independent** FF

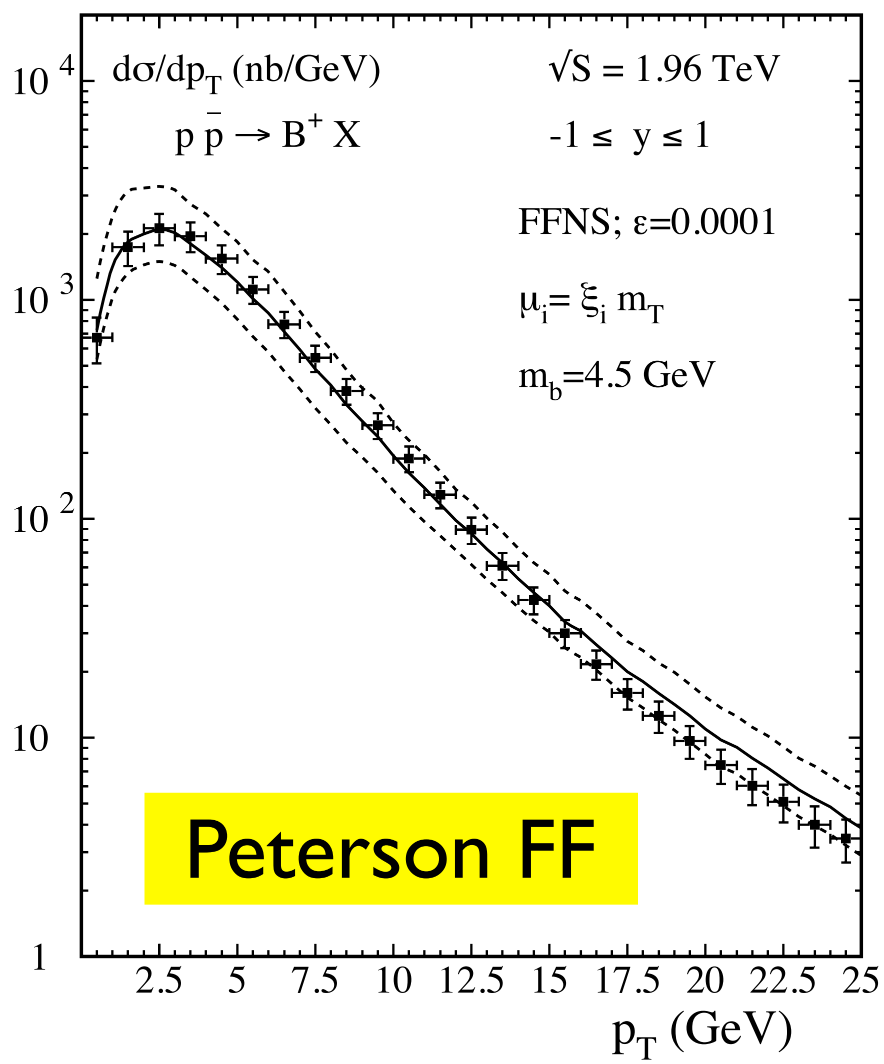
* non-perturbative
* describes hadronization
* not based on a fact. theorem

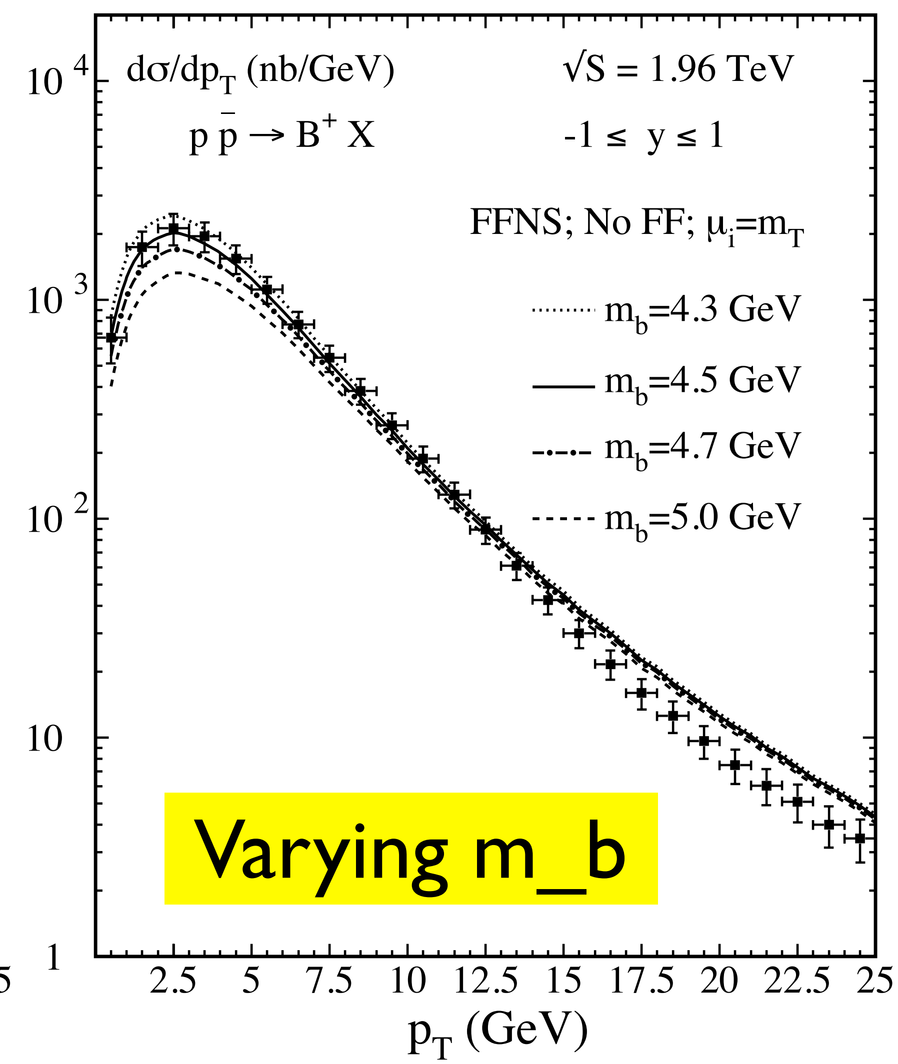
In the following, I will call $D_Q^H(z)$ the "hadronization function" (HF)

Some NLO results for B-meson production

Lesson from hadroproduction of heavy quarks:
NLO FFNS works very well for p_T up to roughly 5m

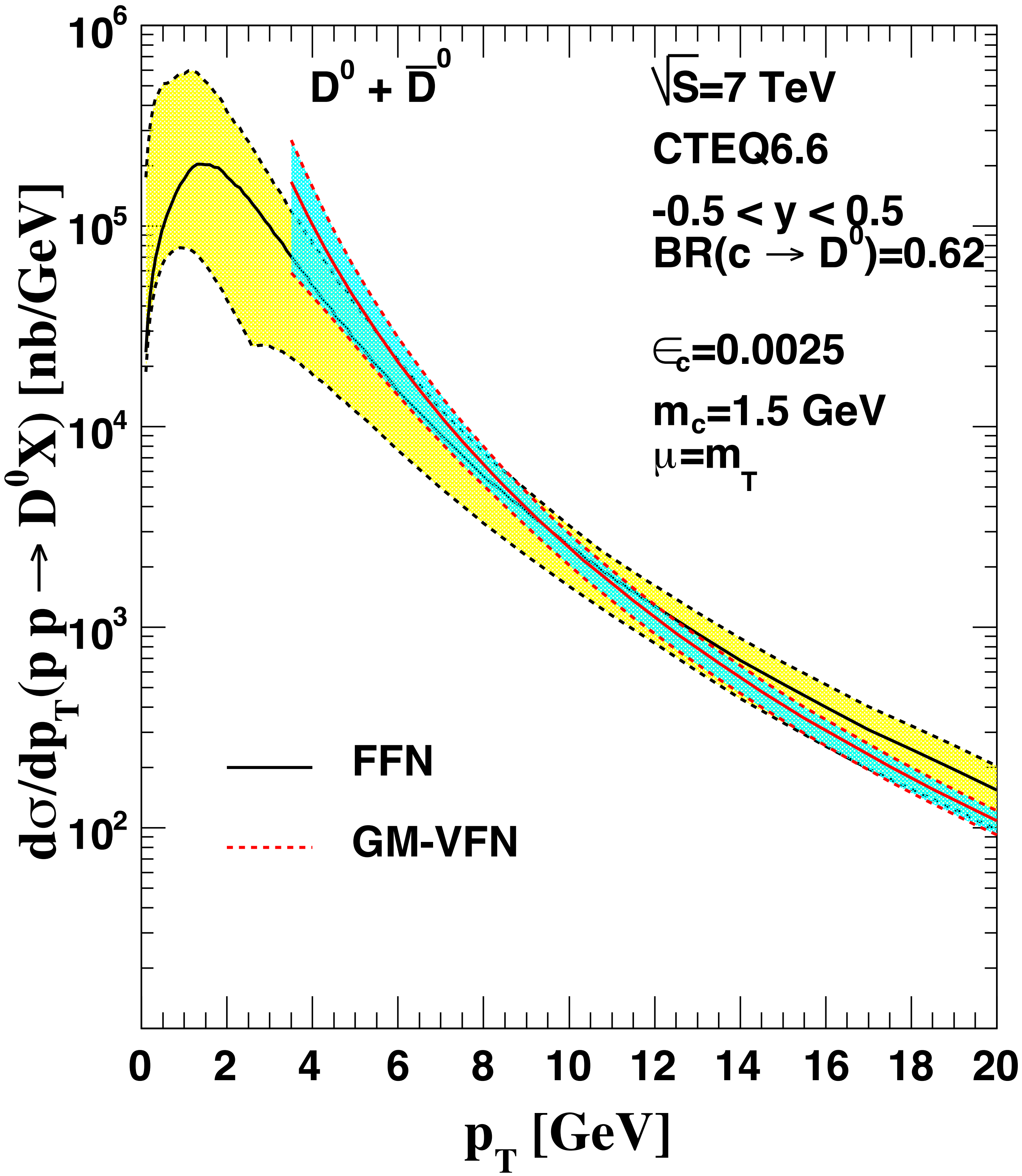






A Peterson HF with eps=0.0001 improves the agreement at larger p_T by lowering the cross section

LHC



Remarks:

The D-meson HF is softer than the B-meson HF ($\varepsilon_c > \varepsilon_b$)

At large p_T, the scale uncertainty in the GM-VFN is reduced

Theoretical approaches:

Zero Mass Variable Flavor Number Scheme (ZM-VFNS)

ZM-VFNS/RS

Factorization formula for inclusive heavy quark (Q) production:

$$d\sigma^{H+X} \simeq \sum_{a,b,c} \int_0^1 dx_a \int_0^1 dx_b \int_0^1 dz\ f_a^A(x_a,\mu_F) f_b^B(x_b,\mu_F) d\hat{\sigma}_{ab\to c+X} D_c^H(z,\mu'_F) + \mathcal{O}(m^2/p_T^2)$$

- Same factorization formula as for inclusive production of pions and kaons
- Quark mass neglected in kinematics and the short distance cross section
- Allows to compute p_T spectrum for $p_T >> m$
- Needs **scale-dependent** FFs of quarks and gluons into the observed heavy-flavored hadron (H)

List of subprocesses in the ZM-VFNS

Massless NLO calculation: [Aversa,Chiappetta,Greco,Guillet,NPB327(1989)105]

1. $gg \to qX$
2. $gg \to gX$
3. $qg \to gX$
4. $qg \to qX$
5. $q\bar{q} \to gX$
6. $q\bar{q} \to qX$
7. $qg \to \bar{q}X$
8. $qg \to \bar{q}'X$
9. $qg \to q'X$
10. $qq \to gX$
11. $qq \to qX$
12. $q\bar{q} \to q'X$
13. $q\bar{q}' \to gX$
14. $q\bar{q}' \to qX$
15. $qq' \to gX$
16. $qq' \to qX$

$\oplus$ charge conjugated processes

- In the **VFNS** we need FFs into the heavy meson/baryon for:
  - Light quarks
  - Heavy quarks
  - Gluon
  - The entire VFNS can be extended to the one-particle inclusive case: evolution equations for PDFs and FFs and α_s; the matching conditions across the heavy flavor thresholds for PDFs and FFs and α_s; calculation of the short distance cross sections
- In the **FFNS** we only had one scale-independent FF of the heavy quark into the heavy meson/baryon

Cacciari, Mitov, Moch, ...

Fragmentation functions

Approach I: Perturbative FFs (PFFs)

Caccciari, Greco, Nason, Oleari, ...

$$D_i^H(z, \mu'_F) = D_i^Q(z, \mu'_F) \otimes D_Q^H(z)$$

PFF evolved with DGLAP; short distance; boundary condition calculable

Non-pert., scale-independent HF describing hadronization of heavy quark Q into heavy hadron H

Mellin-moments of $D_Q^H(z)$ determined from e^+e^- data

Approach II: treat FFs into H in the same way as FFs into pions or kaons

Binnewies, Kniehl, Kramer, ...

Non-pert. boundary conditions $D_i^H(z,m)$ from fit to e^+e^- data;
Determine FFs directly in x-space; evolved with DGLAP

PFF approach

Cacciari, Nason, PRL89(2002)122003

Determine HF from N=2 moment in PFF approach; not from entire x-spectrum

$$D_N \equiv \int D(z) z^N \frac{dz}{z}$$

$$\frac{d\sigma}{dp_{\rm T}} = \int dz d\hat{p}_{\rm T}\, D(z) \frac{A}{\hat{p}_{\rm T}^n}\, \delta(p_{\rm T} - z\hat{p}_{\rm T}) = \frac{A}{p_{\rm T}^n} D_n$$

n~3,4,5

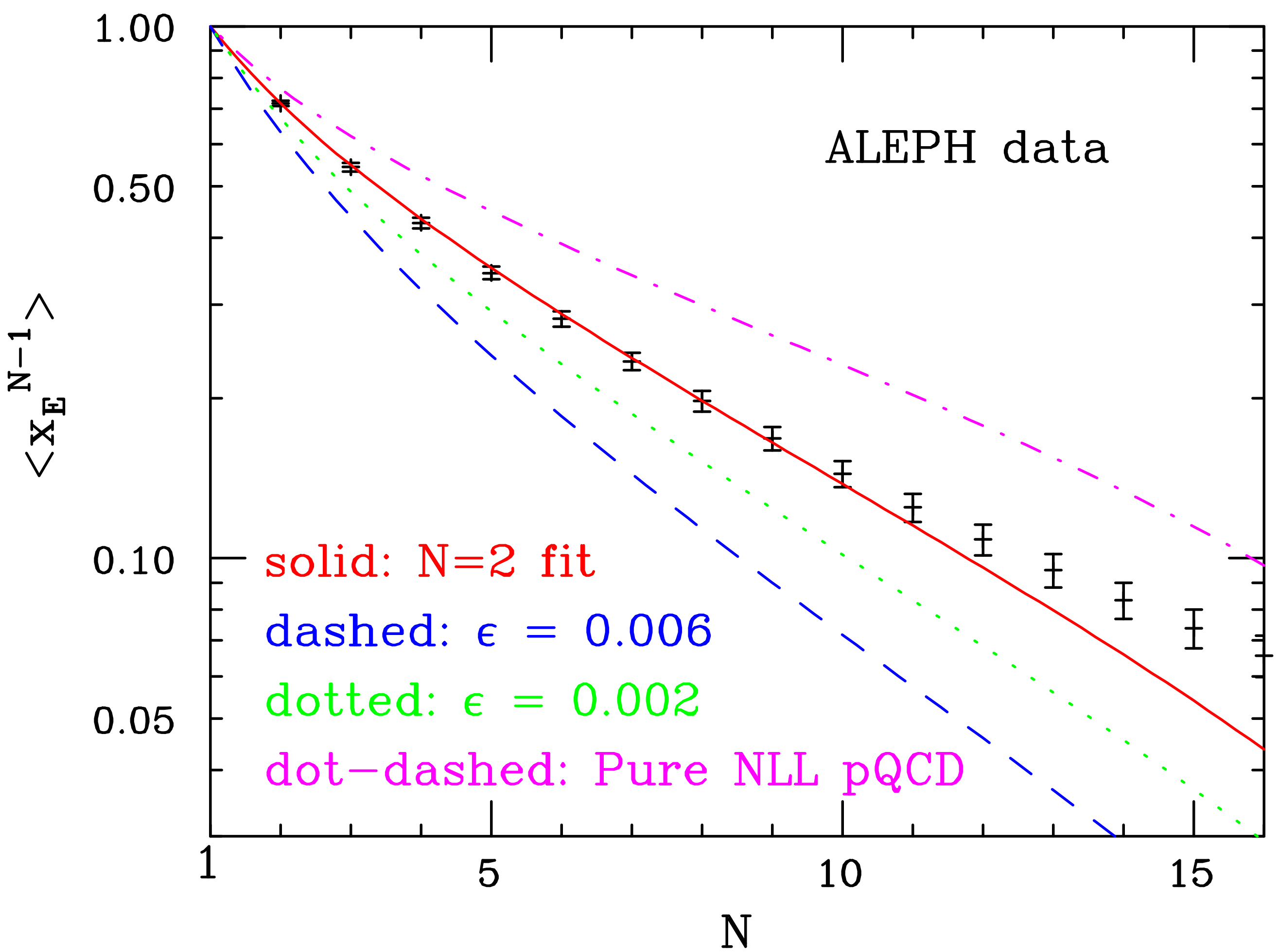
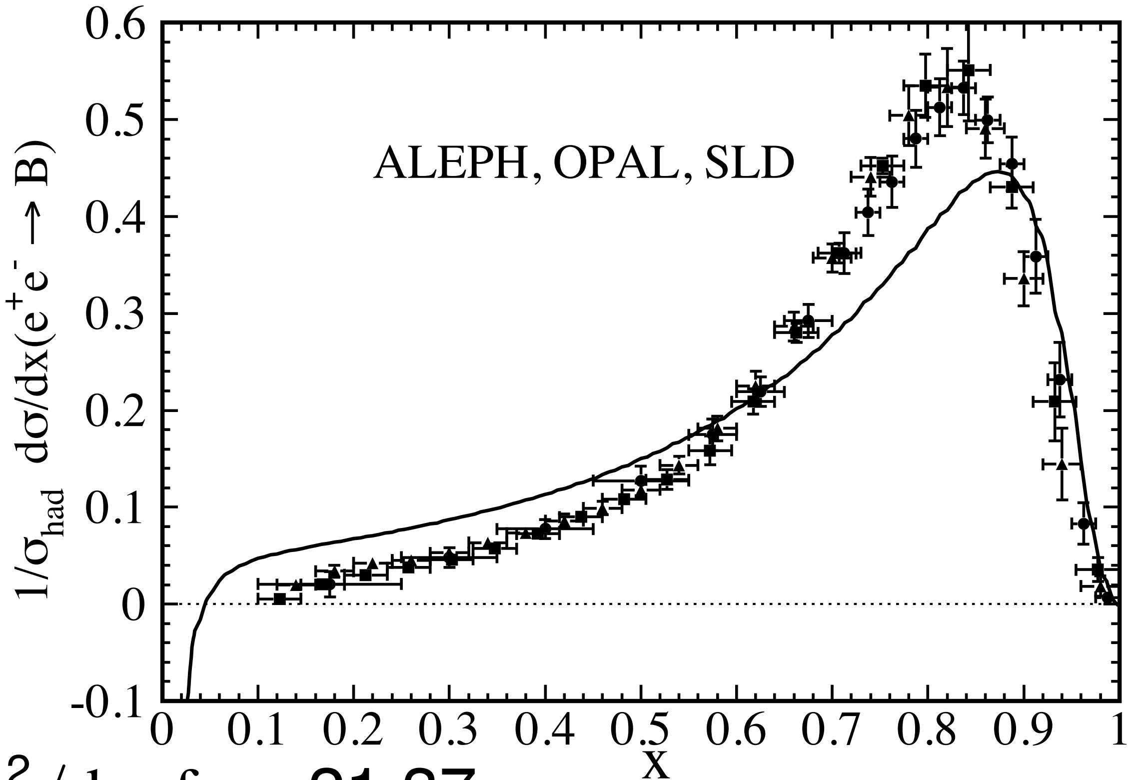


FIG. 1. Moments of the measured B meson fragmentation function, compared with the perturbative NLL calculation supplemented with different $D(z)$ non-perturbative fragmentation forms. The solid line is obtained using a one-parameter form fitted to the second moment.

FFs into B mesons [1] from LEP/SLC data [2]

Petersen

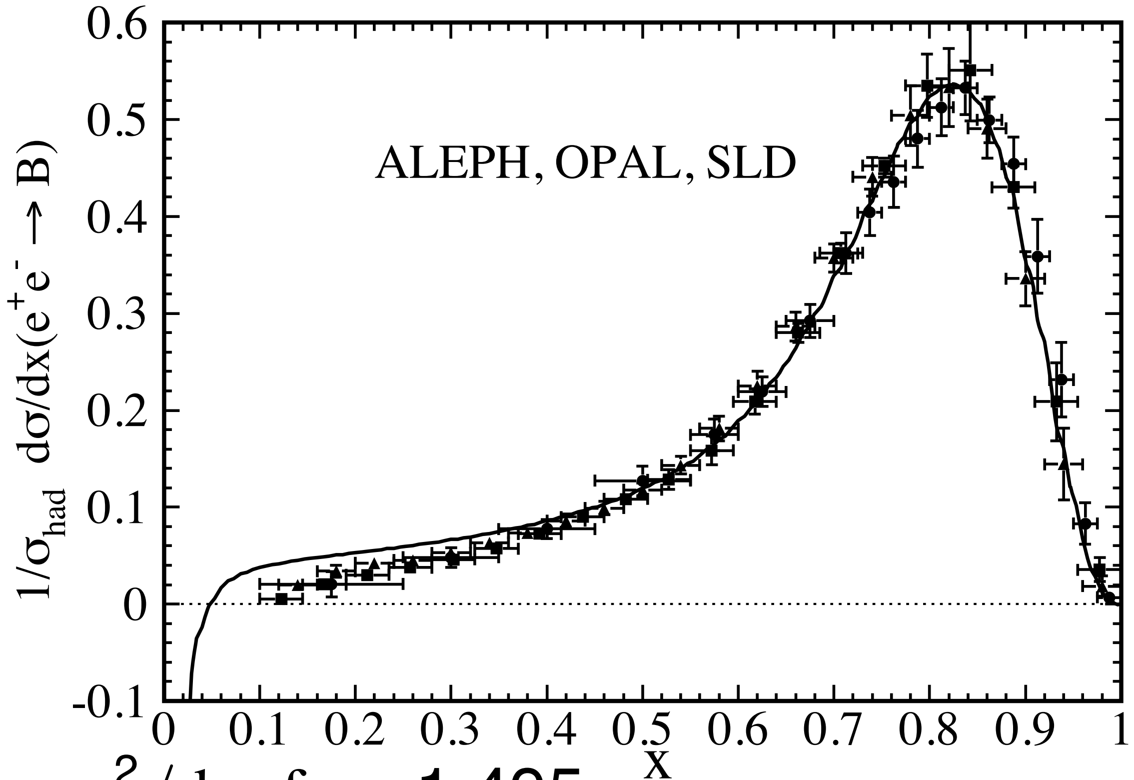
$$D(x,\mu_0^2) = N\frac{x(1-x)^2}{[(1-x)^2+\epsilon x]^2}$$



$\chi^2/\text{d.o.f.} = 21.37$

Kartvelishvili-Likhoded

$$D(x,\mu_0^2) = Nx^{\alpha}(1-x)^{\beta}$$



$\chi^2/\text{d.o.f.} = 1.495$

[1] Kniehl,Kramer,IS,Spiesberger,PRD77(2008)014011

[2] ALEPH, PLB512(2001)30; OPAL, EPJC29(2003)463; SLD, PRL84(2000)4300; PRD65(2002)092006

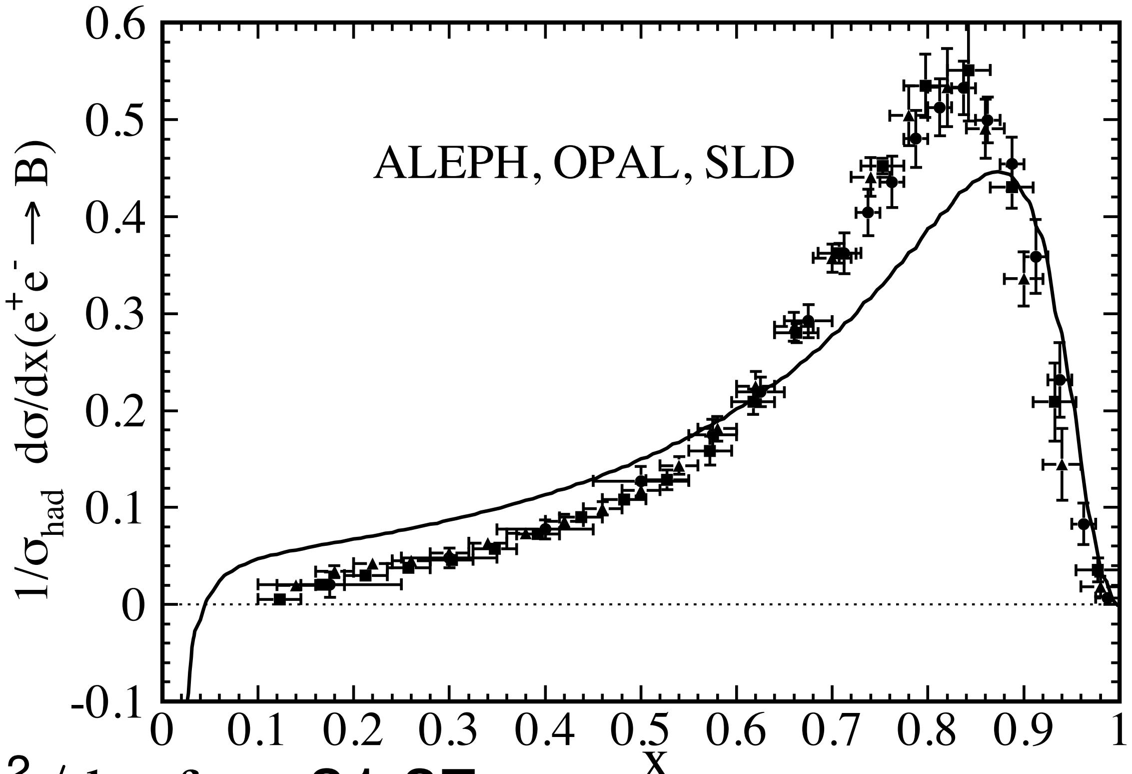
FFs into B mesons [1] from LEP/SLC data [2]

Petersen

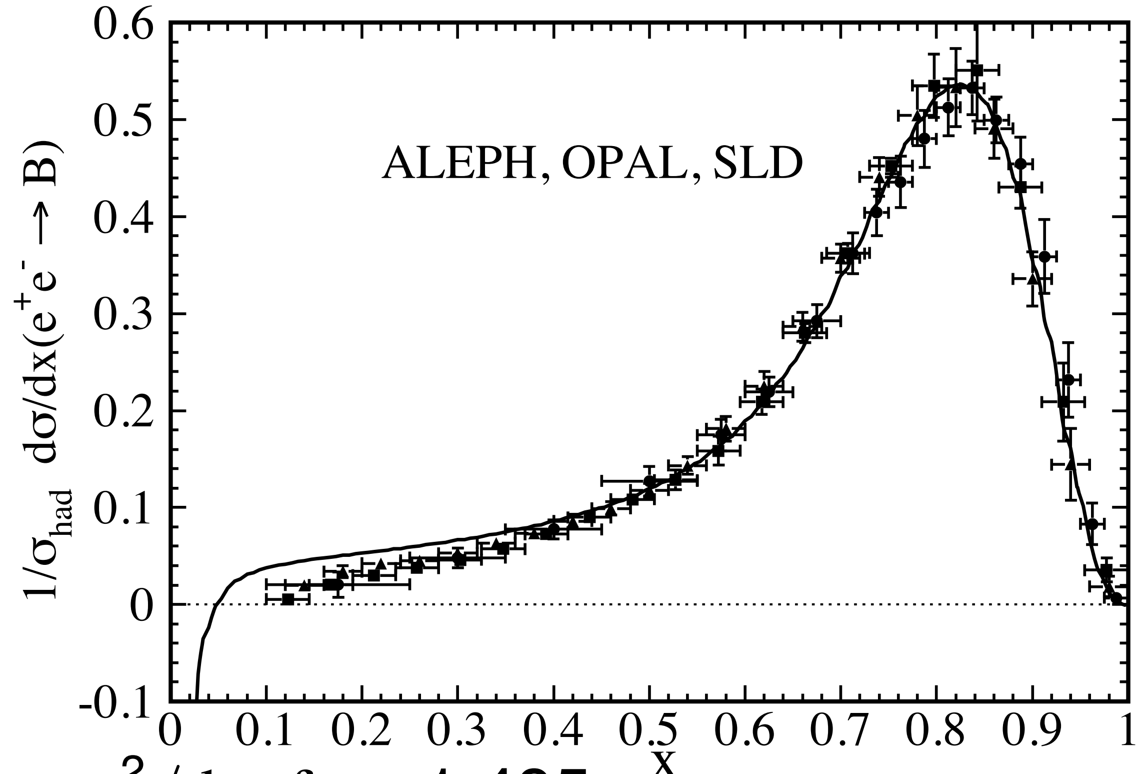
$$D(x,\mu_0^2) = N\frac{x(1-x)^2}{[(1-x)^2+\epsilon x]^2}$$

Kartvelishvili-Likhoded

$$D(x,\mu_0^2) = Nx^\alpha(1-x)^\beta$$



$\chi^2/\mathrm{d.o.f.} = 21.37$



$\chi^2/\mathrm{d.o.f.} = 1.495$

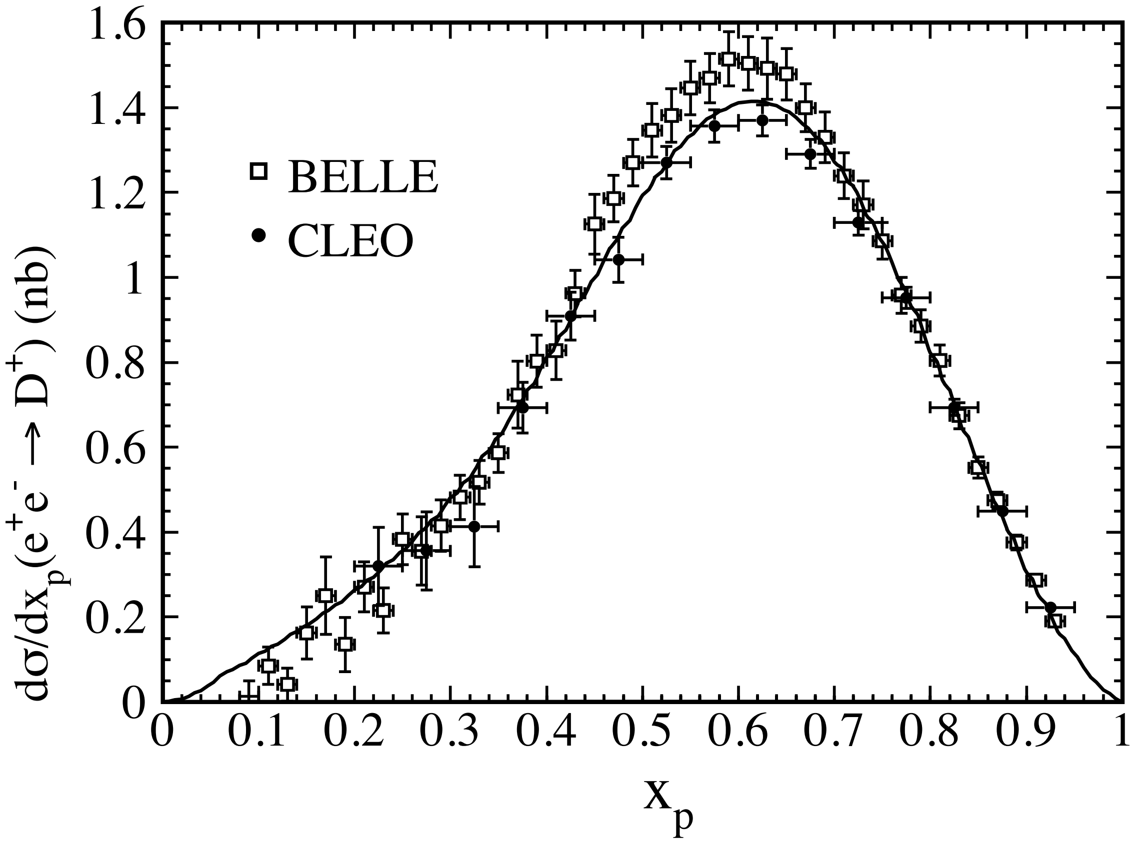
Note: The Petersen function or Kartvelishvili function is used here to parameterise the boundary condition for the heavy quark FF into the heavy meson which is then evolved.

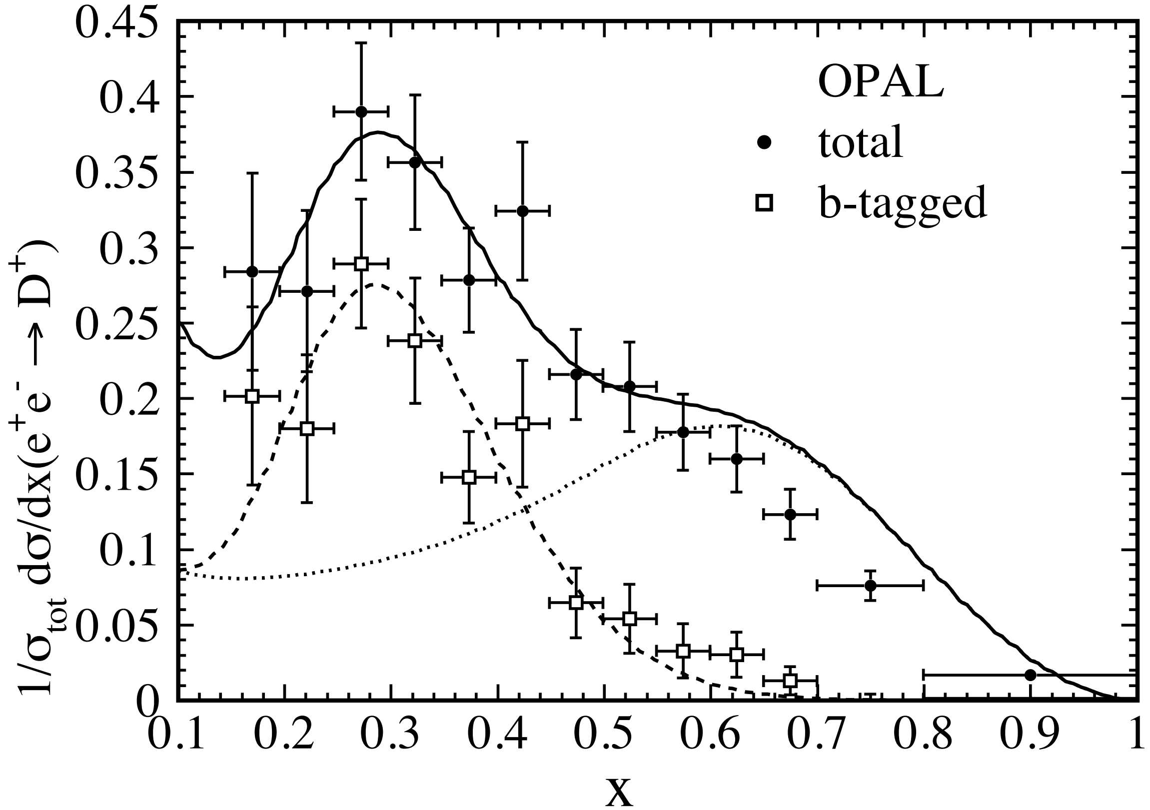
This is completely different from using a Petersen function as the scale independent "hadronization function".

[1] Kniehl,Kramer,IS,

[2] ALEPH, PLB512(... 00;

PRD65(2002)09200...





FF for $c \to D^*$
from fitting to e^+e^- data

2008 analysis based on GM-VFNS
$\mu_0 = m$

global fit: data from
ALEPH, OPAL, BELLE, CLEO

BELLE/CLEO fit

[KKKS: Kneesch, Kramer, Kniehl, IS NPB799 (2008)]

tension between low and high energy data sets $\to$ speculations about non-perturbative (power-suppressed) terms

Theoretical approaches:

General Mass Variable Flavor Number Scheme (GM-VFNS)

GM-VFNS

- Similar factorization formula as in the ZM-VFNS, BUT:
  - Quark mass retained in kinematics and the short distance cross section
  - Allows to compute p_T spectrum for $p_T >> m$ and $p_T \sim m$
- Uses the same **scale-dependent** PFFs of quarks and gluons (in the MSbar scheme)
- the **scale-independent** hadronization function might a priori differ in FFNS, ZM-VFNS and GM-VFNS determinations but to make **connection to the fixed order calculation** it is usually assumed to be the same in all cases

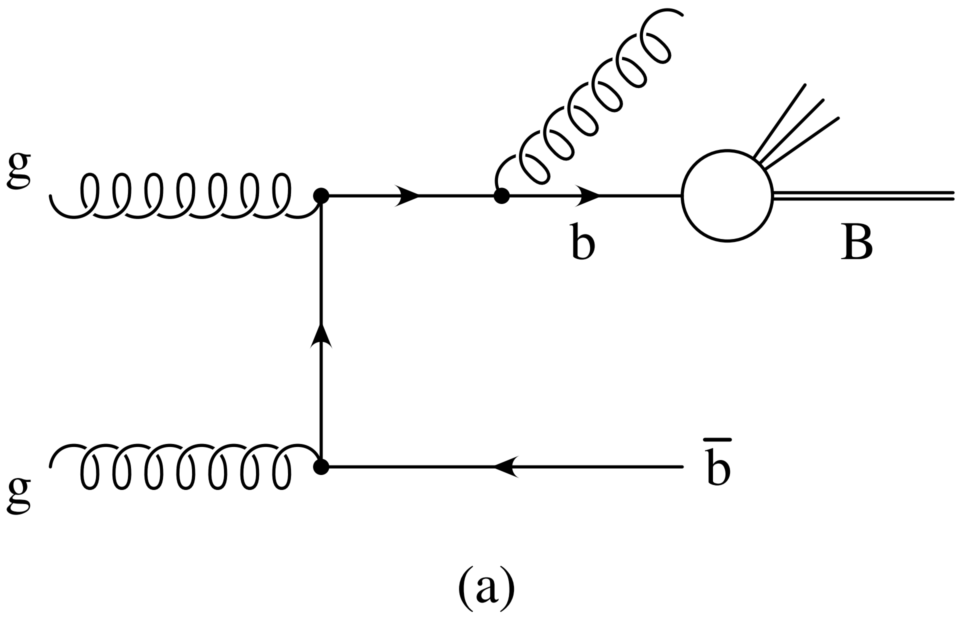
List of subprocesses in the GM-VFNS

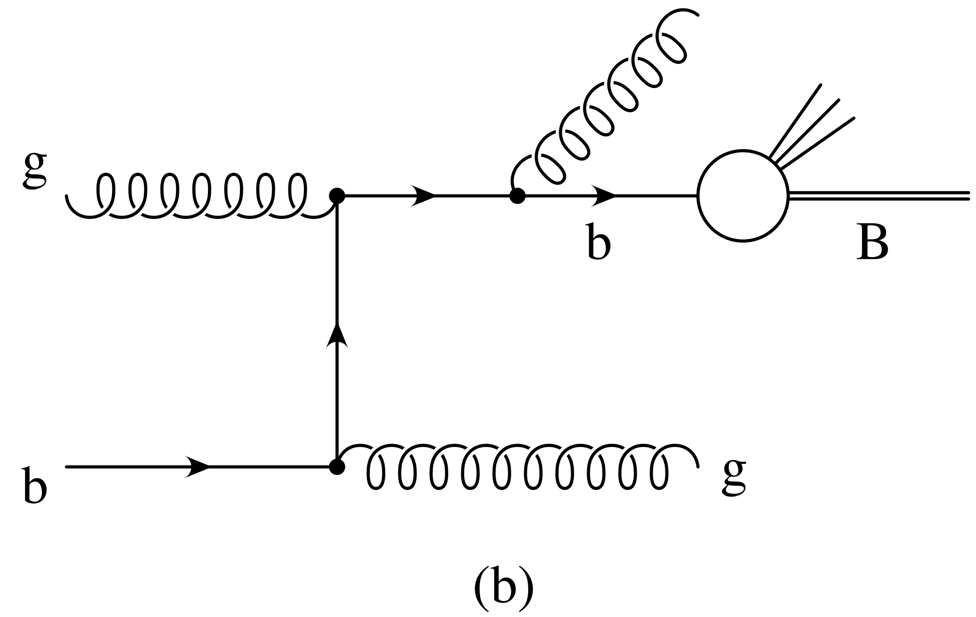
	Only light lines	Heavy quark initiated ($m_Q = 0$)	Mass effects: $m_Q \neq 0$
1	$gg \to qX$	-	$gg \to QX$
2	$gg \to gX$	-	-
3	$qg \to gX$	$Qg \to gX$	-
4	$qg \to qX$	$Qg \to QX$	-
5	$q\bar{q} \to gX$	$Q\bar{Q} \to gX$	-
6	$q\bar{q} \to qX$	$Q\bar{Q} \to QX$	-
7	$qg \to \bar{q}X$	$Qg \to \bar{Q}X$	-
8	$qg \to \bar{q}'X$	$Qg \to \bar{q}X$	$qg \to \bar{Q}X$
9	$qg \to q'X$	$Qg \to qX$	$qg \to QX$
10	$qq \to gX$	$QQ \to gX$	-
11	$qq \to qX$	$QQ \to QX$	-
12	$q\bar{q} \to q'X$	$Q\bar{Q} \to qX$	$q\bar{q} \to QX$
13	$q\bar{q}' \to gX$	$Q\bar{q} \to gX$, $q\bar{Q} \to gX$	-
14	$q\bar{q}' \to qX$	$Q\bar{q} \to QX$, $q\bar{Q} \to qX$	-
15	$qq' \to gX$	$Qq \to gX$, $qQ \to gX$	-
16	$qq' \to qX$	$Qq \to QX$, $qQ \to qX$	-

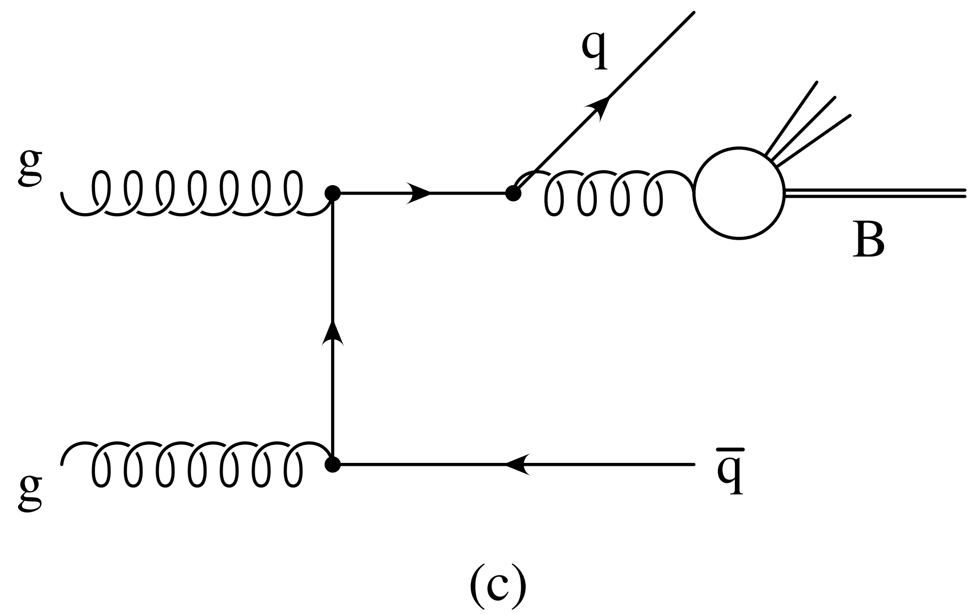
$\oplus$ charge conjugated processes

Example diagrams

m ≠ 0 — **m = 0 (S-ACOT)**







arXiv:0705.4392
Reaches 50% at Tevatron at small pT; decreases only mildly towards larger pT

FIG. 2: Examples of Feynman diagrams leading to contributions of (a) class (i), (b) class (ii), and (c) class (iii).

Mass terms contained in the hard scattering coeffi cients:

$$d\hat{\sigma}(\mu_F, \mu_{F'}, \alpha_s(\mu_R), \frac{m}{p_T})$$

Two ways to derive them:

(1) Compare massless limit of a massive fi xed-order calculation with a massless $\overline{\text{MS}}$ calculation to determine subtraction terms

[Kniehl,Kramer,IS,Spiesberger,PRD71(2005)014018]

OR

(2) Perform mass factorization using partonic PDFs and FFs

[Kniehl,Kramer,IS,Spiesberger,EPJC41(2005)199]

skip details

- Compare limit $m \to 0$ of the massive calculation (Merebashvili et al., Ellis, Nason; Smith, van Neerven; Bojak, Stratmann; ...) with massless $\overline{\mathrm{MS}}$ calculation (Aurenche et al., Aversa et al., ...)

$$\lim_{m\to 0} \mathrm{d}\tilde{\sigma}(m) = \mathrm{d}\hat{\sigma}_{\overline{\mathrm{MS}}} + \Delta \mathrm{d}\sigma$$

$\Rightarrow$ Subtraction terms

$$\mathrm{d}\sigma_{\mathrm{sub}} \equiv \Delta \mathrm{d}\sigma = \lim_{m\to 0} \mathrm{d}\tilde{\sigma}(m) - \mathrm{d}\hat{\sigma}_{\overline{\mathrm{MS}}}$$

- Subtract $\mathrm{d}\sigma_{\mathrm{sub}}$ from massive partonic cross section while keeping mass terms

$$\mathrm{d}\hat{\sigma}(m) = \mathrm{d}\tilde{\sigma}(m) - \mathrm{d}\sigma_{\mathrm{sub}}$$

$\to$ $\mathrm{d}\hat{\sigma}(m)$ short distance coeffi cient including m dependence

$\to$ allows to use PDFs and FFs with $\overline{\mathrm{MS}}$ factorization $\otimes$ massive short distance cross sections

- Treat contributions with charm in the initial state with $m = 0$
- Massless limit: technically non-trivial, map from phase-space slicing to subtraction method

Mass factorization

Subtraction terms are associated to mass singularities:
can be described by
partonic PDFs and FFs for collinear splittings $a \to b + X$

- initial state:

$$f^{(1)}_{g\to Q}(x,\mu^2) = \frac{\alpha_s(\mu)}{2\pi} P^{(0)}_{g\to q}(x) \ln \frac{\mu^2}{m^2}$$

$$f^{(1)}_{Q\to Q}(x,\mu^2) = \frac{\alpha_s(\mu)}{2\pi} C_F \left[\frac{1+z^2}{1-z} \left(\ln \frac{\mu^2}{m^2} - 2\ln(1-z) - 1 \right) \right]_+$$

$$f^{(1)}_{g\to g}(x,\mu^2) = -\frac{\alpha_s(\mu)}{2\pi} \frac{1}{3} \ln \frac{\mu^2}{m^2} \delta(1-x)$$

- final state:

$$d^{(1)}_{g\to Q}(z,\mu^2) = \frac{\alpha_s(\mu)}{2\pi} P^{(0)}_{g\to q}(z) \ln \frac{\mu^2}{m^2}$$

$$d^{(1)}_{Q\to Q}(z,\mu^2) = C_F \frac{\alpha_s(\mu)}{2\pi} \left[\frac{1+z^2}{1-z} \left(\ln \frac{\mu^2}{m^2} - 2\ln(1-z) - 1 \right) \right]_+$$

- Other partonic distribution functions are zero to order α_s

[Mele, Nason; Kretzer, Schienbein; Melnikov, Mitov]

Sketch of kinematics:

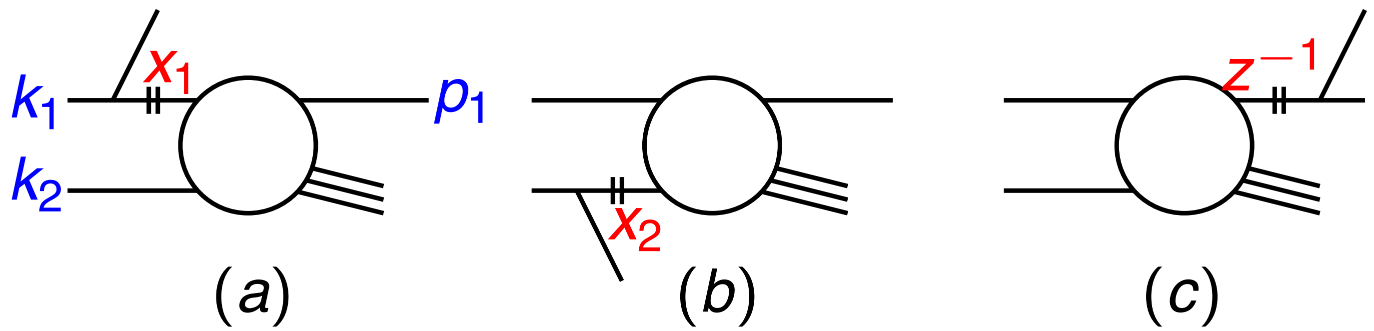


Fig. (a):

$$d\sigma^{\rm sub}(ab \to QX) = \int_0^1 dx_1\, f^{(1)}_{a\to i}(x_1, \mu_F^2)\, d\hat\sigma^{(0)}(ib \to QX)[x_1 k_1, k_2, p_1]$$
$$\equiv f^{(1)}_{a\to i}(x_1) \otimes d\hat\sigma^{(0)}(ib \to QX)$$

Fig. (b):

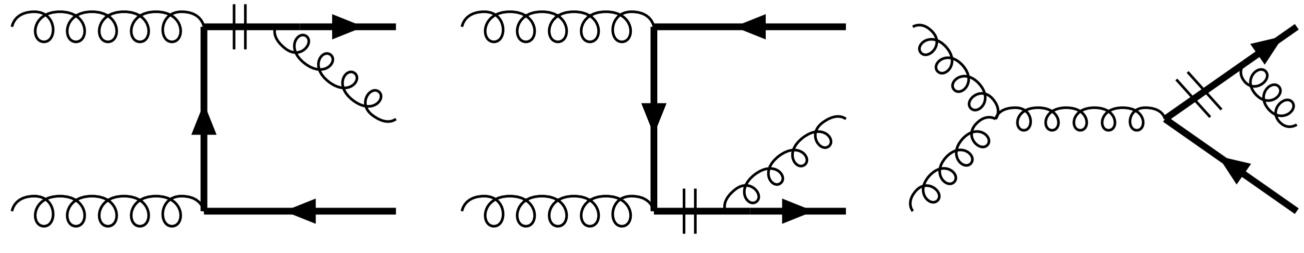
$$d\sigma^{\rm sub}(ab \to QX) = \int_0^1 dx_2\, f^{(1)}_{b\to j}(x_2, \mu_F^2)\, d\hat\sigma^{(0)}(aj \to QX)[k_1, x_2 k_2, p_1]$$
$$\equiv f^{(1)}_{b\to j}(x_2) \otimes d\hat\sigma^{(0)}(aj \to QX)$$

Fig. (c):

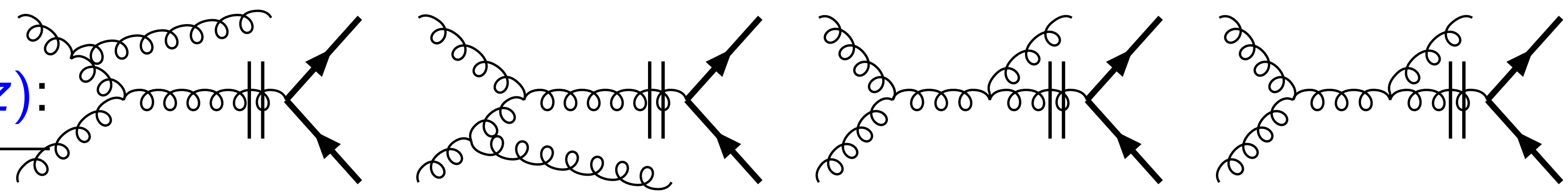
$$d\sigma^{\rm sub}(ab \to QX) = \int_0^1 dz\, d\hat\sigma^{(0)}(ab \to kX)[k_1, k_2, z^{-1} p_1]\, d^{(1)}_{k\to Q}(z, {\mu'_F}^2)$$
$$\equiv d\hat\sigma^{(0)}(ab \to kX) \otimes d^{(1)}_{k\to Q}(z)$$

[1] Kniehl, Kramer, I.S., Spiesberger, EPJC41(2005)199

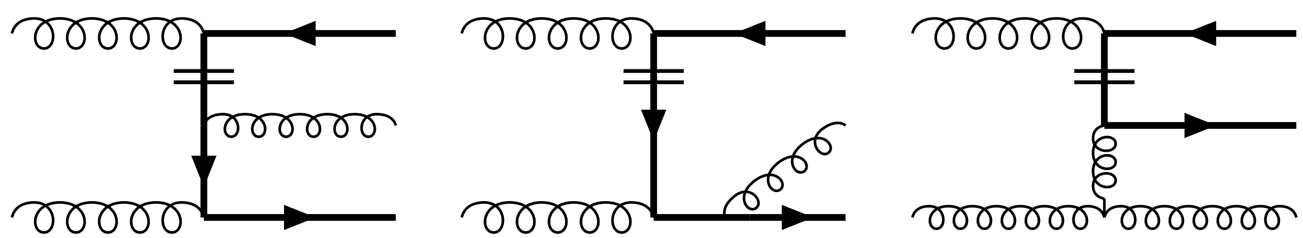
$\mathrm{d}\hat{\sigma}^{(0)}(gg \to Q\bar{Q}) \otimes d^{(1)}_{Q \to Q}(z)$:



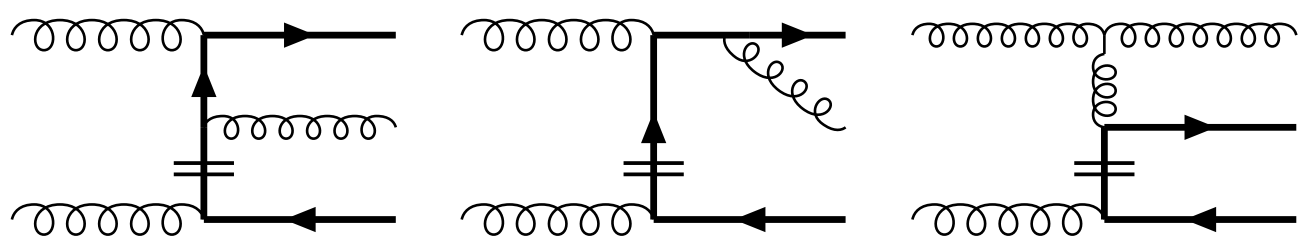
$\mathrm{d}\hat{\sigma}^{(0)}(gg \to gg) \otimes d^{(1)}_{g \to Q}(z)$:



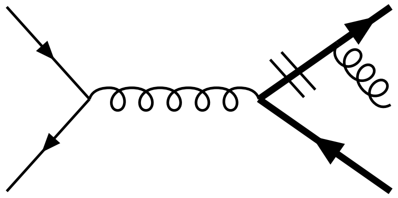
$f^{(1)}_{g \to Q}(x_1) \otimes \mathrm{d}\hat{\sigma}^{(0)}(Qg \to Qg)$:



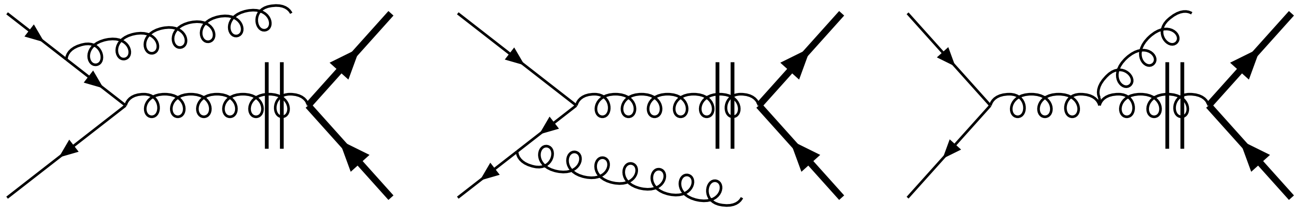
$f^{(1)}_{g \to Q}(x_2) \otimes \mathrm{d}\hat{\sigma}^{(0)}(gQ \to Qg)$:



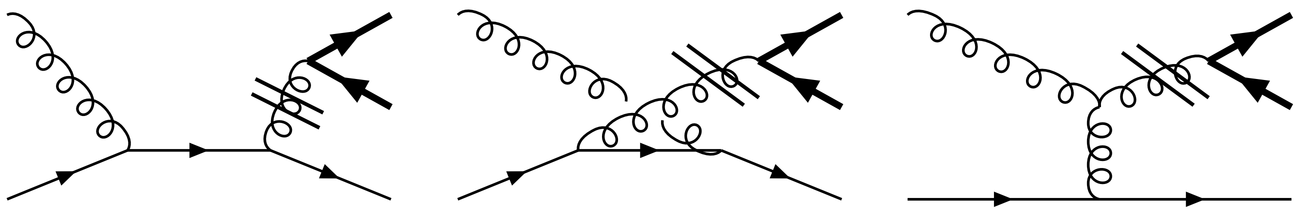
$\mathrm{d}\hat{\sigma}^{(0)}(q\bar{q} \to Q\bar{Q}) \otimes d^{(1)}_{Q\to Q}(z)$:



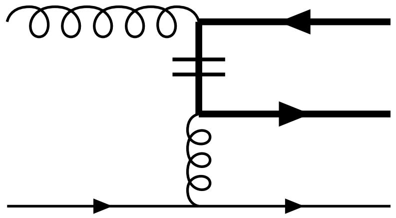
$\mathrm{d}\hat{\sigma}^{(0)}(q\bar{q} \to gg) \otimes d^{(1)}_{g\to Q}(z)$:



$\mathrm{d}\hat{\sigma}^{(0)}(gq \to gq) \otimes d^{(1)}_{g\to Q}(z)$:



$f^{(1)}_{g\to Q}(x_1) \otimes \mathrm{d}\hat{\sigma}^{(0)}(Qq \to Qq)$:



Are heavy flavor FFs universal?

Universality?

- Belle data prefer a harder D-meson FF than LEP data (a harder FF implies a bigger 4th moment and hence a bigger hadroproduction cross section)
- e^+e^- data do not constrain the gluon FF; perturbatively calculated boundary condition for the gluon (NLO, MSbar)
- The gluon FF is important in inclusive **hadroproduction** of heavy hadrons
- Problems with LHC data for Λ_c production

New fit of the Λ_c FF [arXiv:2004.04213]

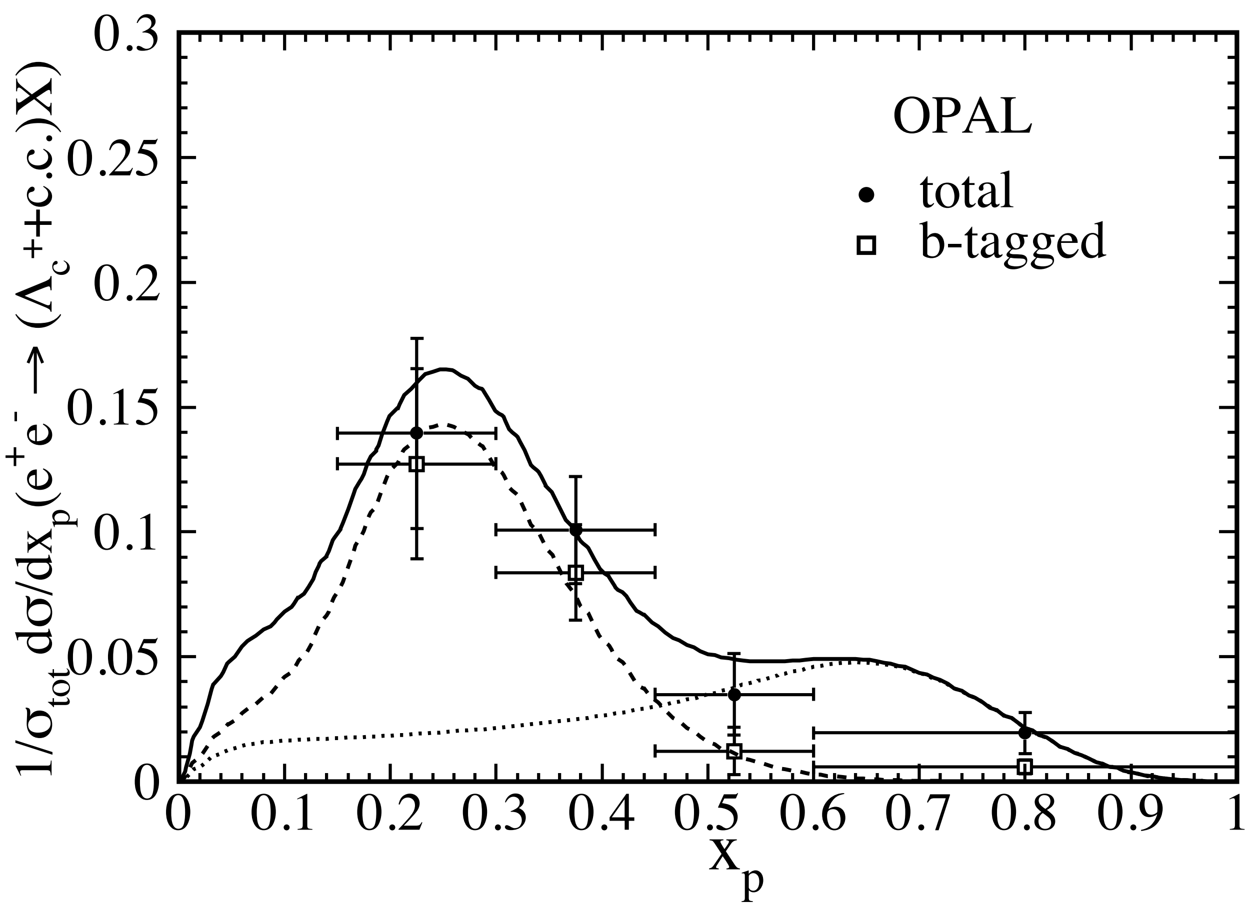
- OPAL data for $e^+e^- \to \gamma/Z \to \Lambda_c$: 4 points
- Belle data at Sqrt(S)=10.52 GeV: 35 points
- Use PDG2016 branching ratio
  $BR(\Lambda_c^+ \to \pi^+ K^- p)=0.0635$
  [correcting the OPAL data from 1996 which used $BR(\Lambda_c^+ \to \pi^+ K^- p)=0.044$]
- z-dependence of c- and b-quark FF at initial scale μ_0= 5 GeV parameterized by a Bowler function

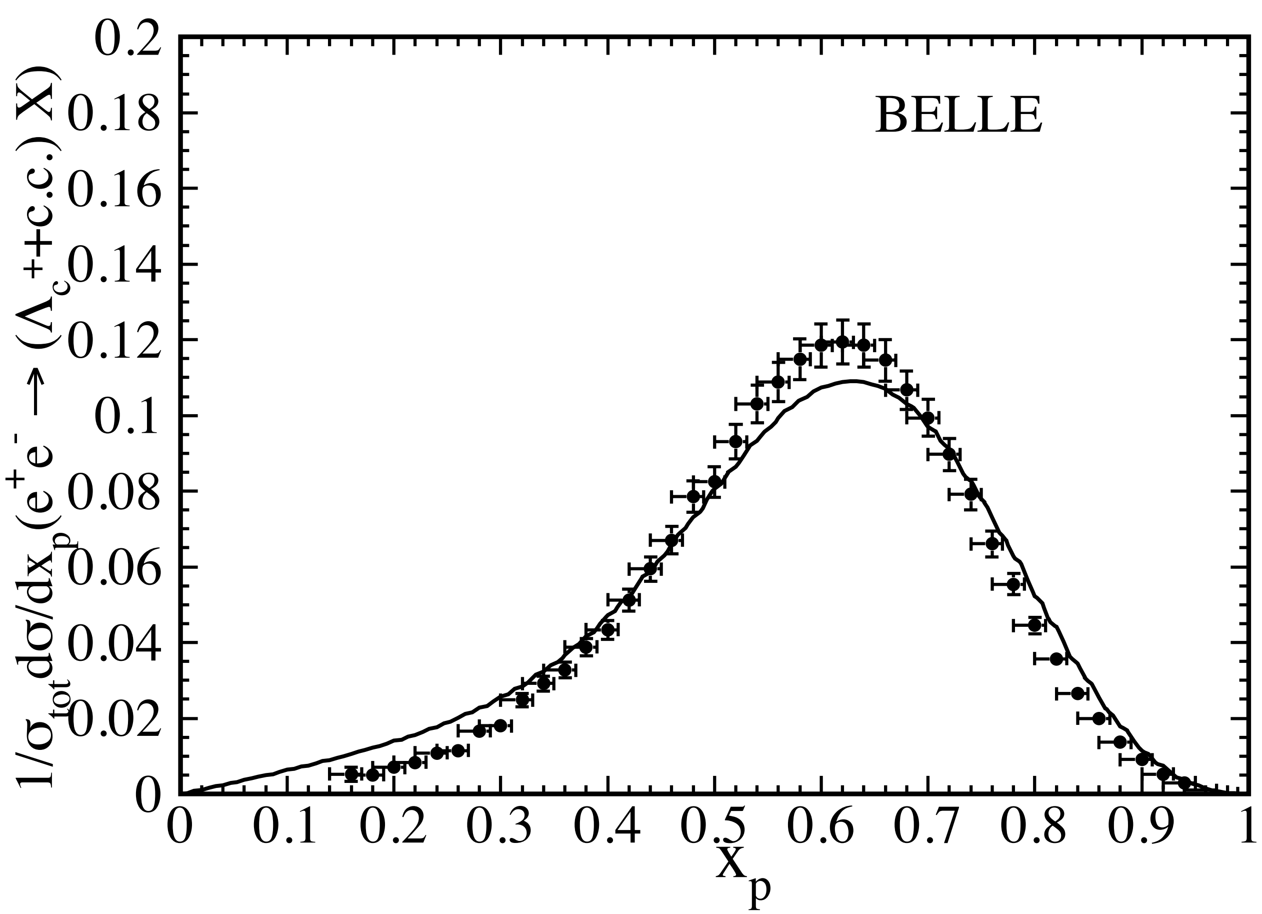
$$D_Q(x,\mu_0) = N x^{-(1+\gamma)^2}(1-x)^a e^{-\gamma^2/x}$$

New fit of the Λ_c FF [arXiv:2004.04213]

3 Fits to LEP and BELLE data: just OPAL, just BELLE, combined

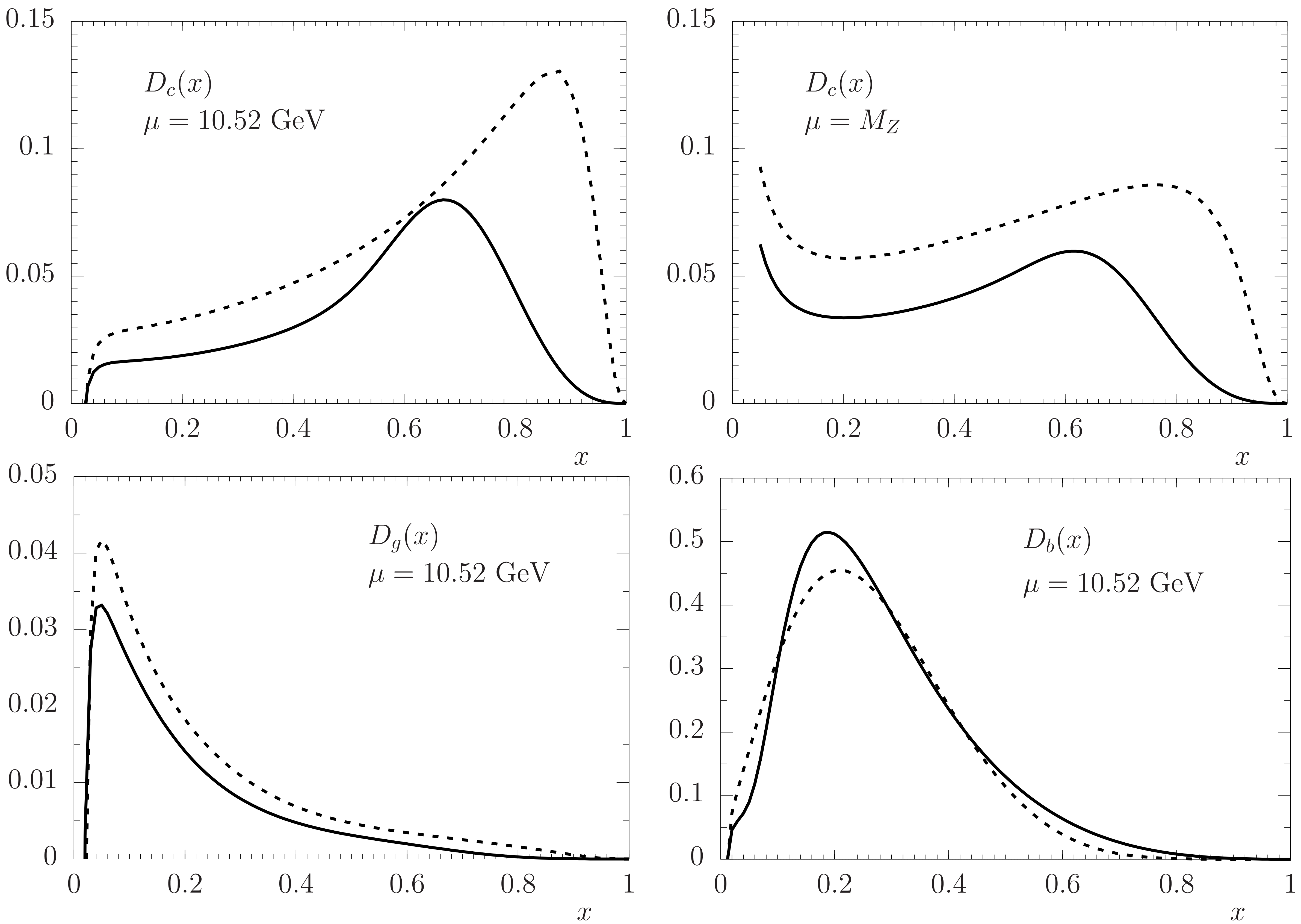
	OPAL	Belle	global
N_c	80345	1×10^{10}	1×10^{10}
a_c	0.35431×10^{-6}	2.1828	2.1821
γ_c	3.6432	4.5391	4.5393
N_b	19.953	19.953	41.973
a_b	6.3031	6.3031	7.4092
γ_b	1.1773	1.1773	1.2457
χ^2/d.o.f	0.4749	3.2928	2.8030





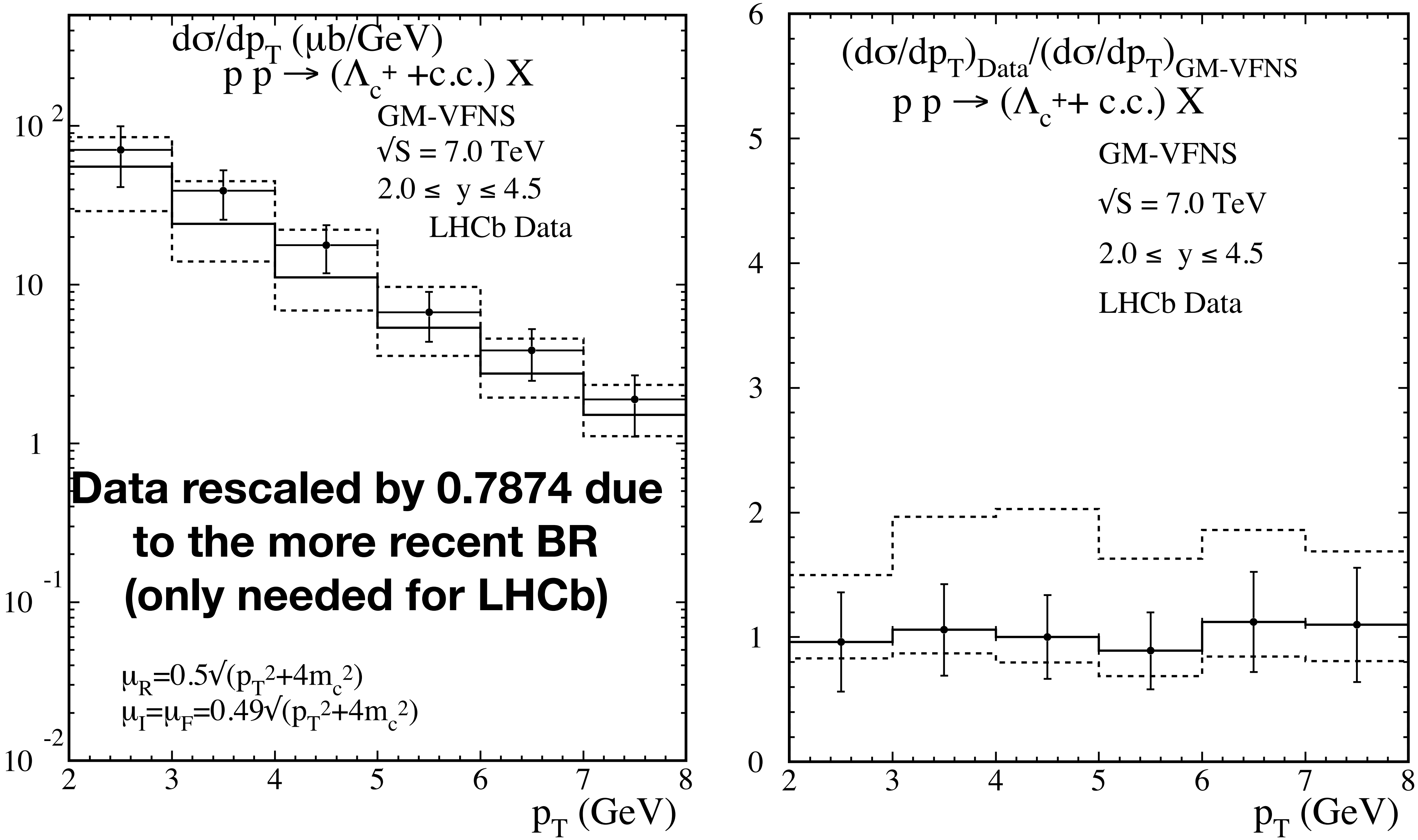
New fit of the Λ_c FF [arXiv:2004.04213]

Dashed lines: old fit (Kniehl, Kramer 2006), Solid lines: new fit



Comparison with LHC data [arXiv:2004.04213]

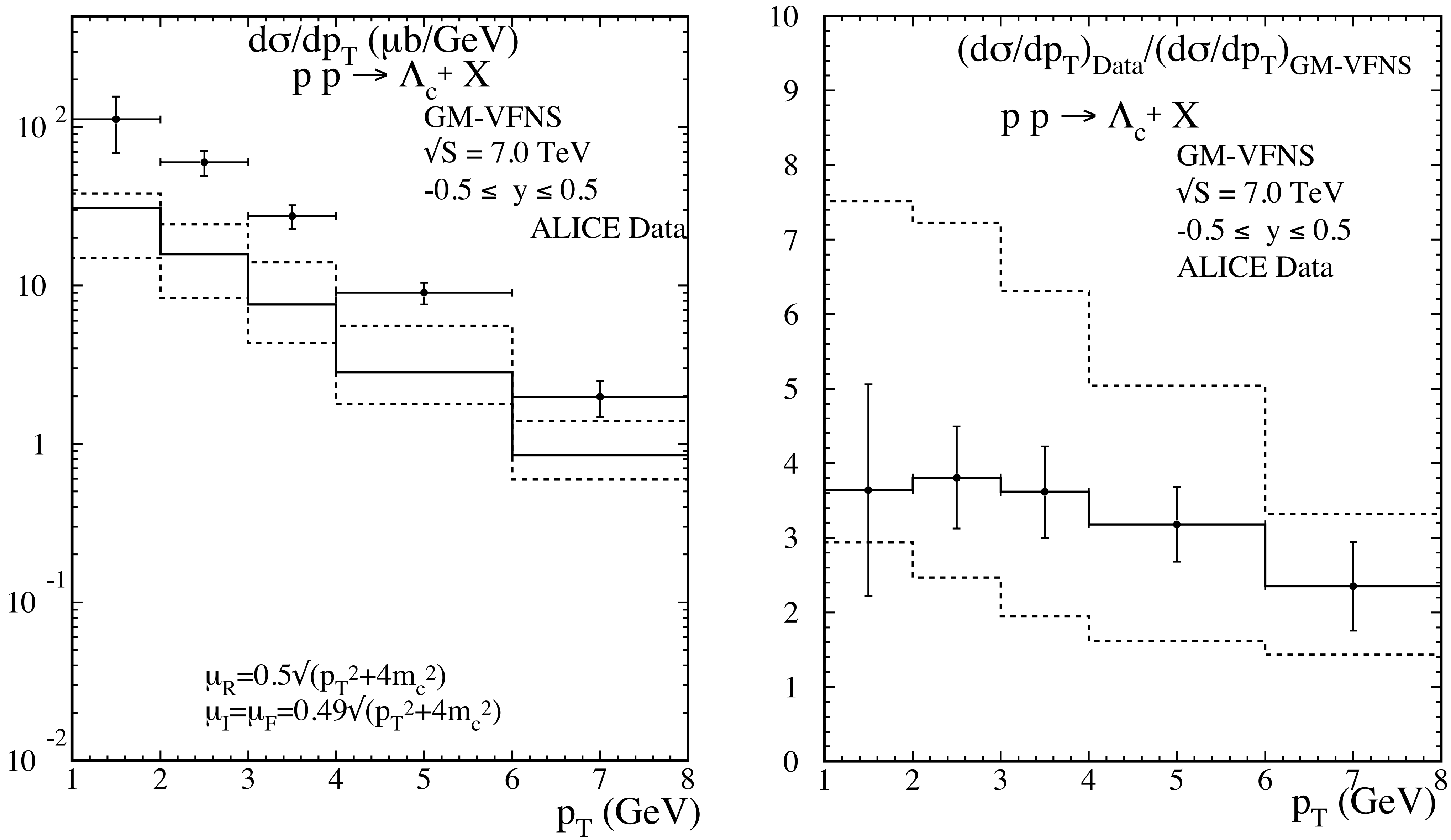
LHCb



Results are shown with the old Λ_c FFs from 2006.
With the new FFs the cross sections are slightly lower(!)
by 15% in the first p_T-bin to 35% in the last p_T-bin

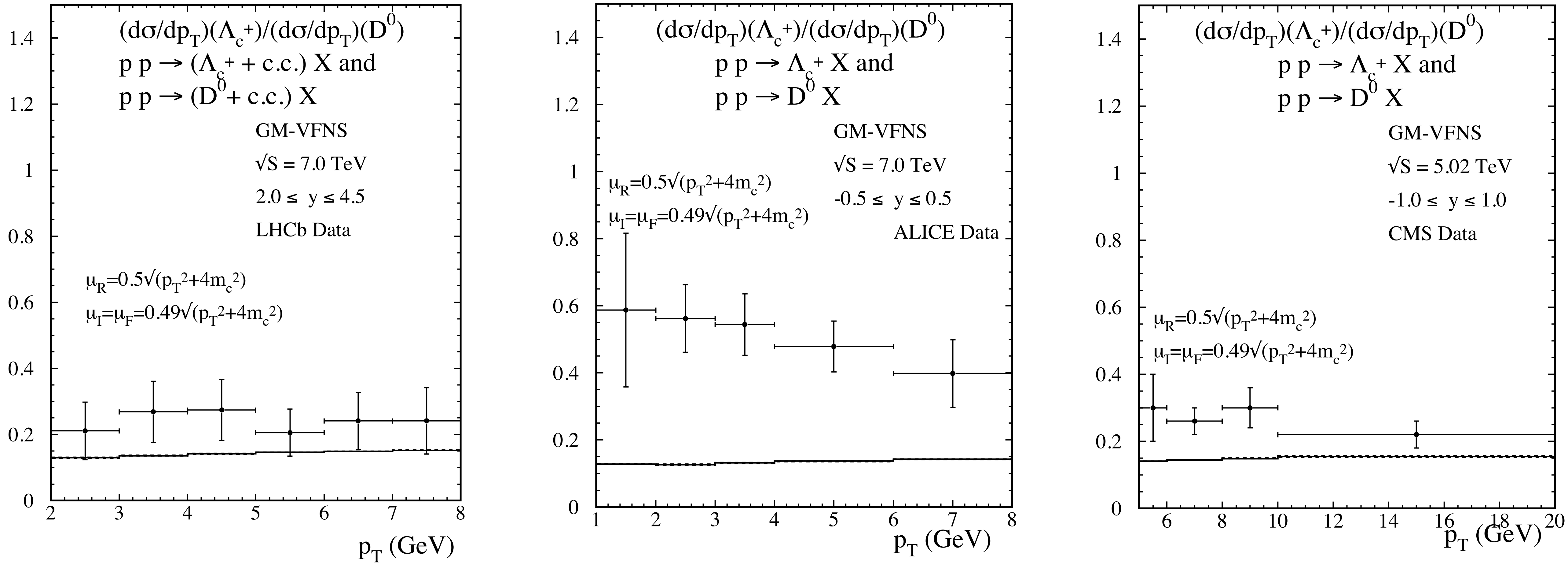
Comparison with LHC data [arXiv:2004.04213]

ALICE



Results are shown with the old Λ_c FFs from 2006.
With the new FFs the cross sections are slightly lower(!)
by 15% in the first p_T-bin to 35% in the last p_T-bin

Λ_c/D^0 ratio [arXiv:2004.04213]



- LHCb: Theory < Data by about 1 sigma (scale uncertainty largely cancels)
- ALICE: Theory ~ 0.15, Data ~ 0.6 … 0.4; **clear disagreement due to Λ_c cross section**
- CMS: Theory ~ 0.15, Data ~ 0.3; Are ALICE and CMS data compatible at pT~7 GeV?
- Note: pQCD predicts a flat p_T dependence for p_T > ~$2m_c$

Discussion [arXiv:2004.04213]

- Contribution from excited charm baryon states much bigger in pp at LHC than in e^+e^- at Belle?

- Higher twist effects beyond the pQCD factorisation present in pp and more important for Λ_c compared to D^0? Should fade away at larger p_T

- Could NNLO help? Unlikely for the Λ_c/D^0 ratio. Should affect all measurements in similar way

- More data differential in both p_T and y would be helpful. Overlapping kinematic regions: check compatibility

- More data at larger p_T would also be helpful. The higher the p_T the more reliable the twist-2 pQCD prediction.

Heavy flavours in DIS and the EIC

Lessons from HERA and neutrino DIS

Kretzer, Schienbein, hep-ph/9808375

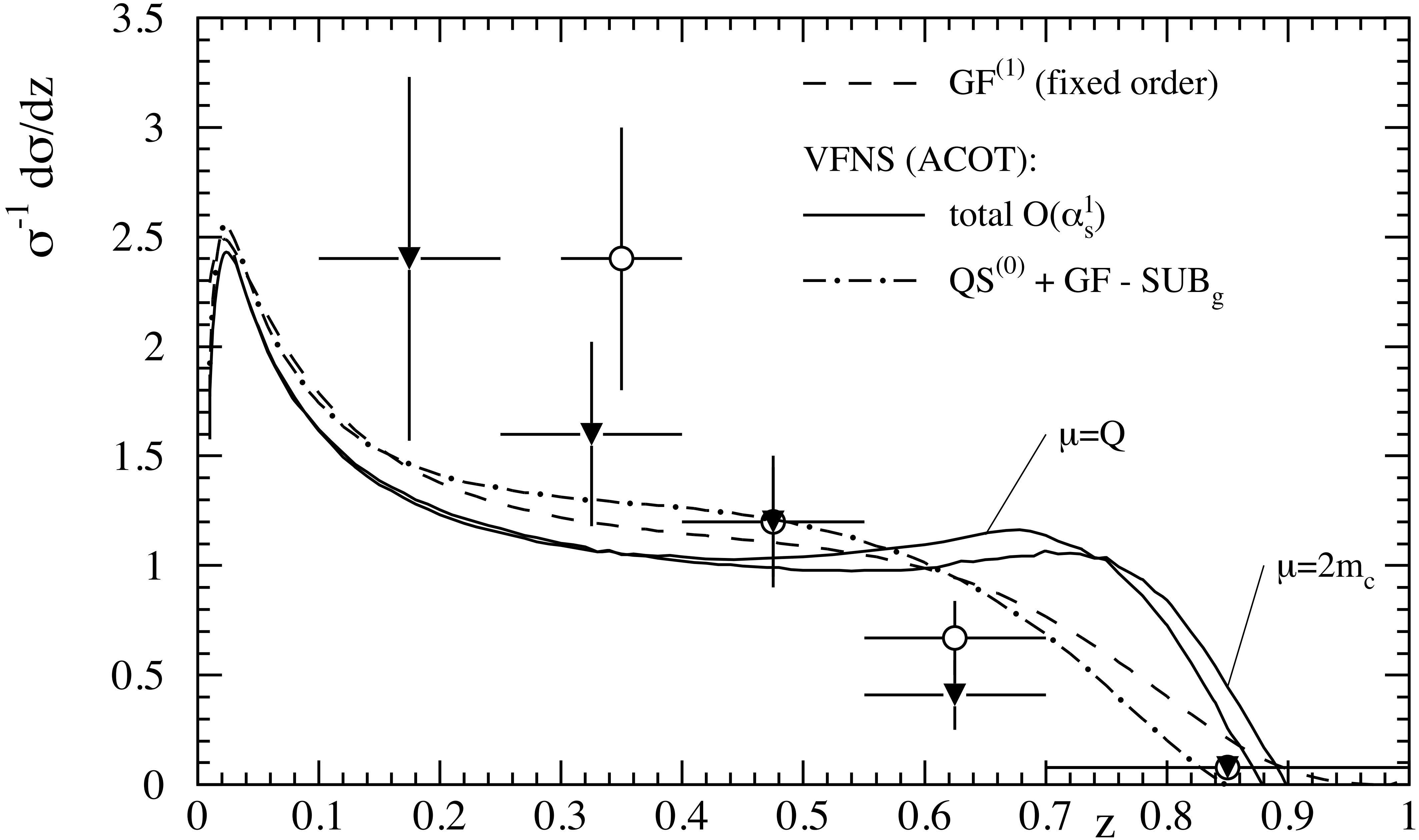
- Charm production contributes up to 30% to the cross section at small-x
- It has been often stated often that the main production mechanism is boson-gluon fusion. However, this statement doesn't make much sense.

  A better question would be: Is the FFNS more adequate than a GM-VFNS in semi-inclusive DIS?
- The energy distribution of the D meson in SIDIS is most sensitive to the charm fragmentation processes. The FFs play an important role in the **normalization** of the p_T and η distributions.

  Note that the p_T distribution in hadroproduction of heavy hadrons is only sensitive to the 4th or 5th moment of the FF (for $p_T \gtrsim 2m$) affecting again the **normalization**

z_D distribution at HERA in the ACOT scheme

Kretzer, Schienbein '98
Heavy quark fragmentation in DIS



$z = (p_D*p_N)/(q*p_N)$, the scaling variable for the D-meson energy distribution; in the nucleon rest frame $z = E_D/(E-E')$

Lessons from HERA and neutrino DIS

Kretzer, Schienbein, hep-ph/9808375

- It might be that the HF for D* mesons (LEP data, ε_c~0.02) is harder than the one for D mesons (CCFR, CDHSW, ARGUS, CLEO data, ε_c~0.06)

- Again the question: Is the D-meson HF universal? Going from low energies to LEP and comparing nu-A, e-p, e-A, e^+e^-

Conclusions for the EIC

- Fixed order calculations (NLO, NNLO) should work perfectly well at the EIC.
  Effects due to the resummation of collinear logs will be small at EIC kinematics.

- Still important to test GM-VFNS calculations against data for heavy quark production in SIDIS. Improve on how to account for the kinematics in the GM-VFNS in a more differential situation.

- Measurements of the E_D spectra in ep and eA will be interesting to compare.
  Best access to the charm fragmentation process. In the ep case without nuclear effects compared to the nuA scattering. (We don't have nu-p data for D-meson production.)

- Compare HF for D* and D mesons: is one harder than the other? What is <z>?

- Understand nuclear matter effects in eA collision **first in the FFNS**.
  Looks conceptually simpler: short-distance production of a heavy quark, then energy loss during propagation through the nucleus, then hadronization described by the scale-independent HF

  - How to constantly include energy loss effects in a VNFS?
    Medium-modified evolution? Avoid double-counting!

  - Other nuclear effects?

- Important to measure the production of different heavy flavoured mesons and baryons in ep and eA including Lc

Backup slides

Termes in the perturbation series

$L = \ln(m/p_T)$
$a = \alpha_s/(2\pi)$

Resummed ↓

Fixed Order →

	LL	NLL	NNLL	...
LO	1			
NLO	aL	a		
NNLO	$(aL)^2$	$a(aL)$	a^2	
...	...	...	...	...

FFNS/Fixed Order NLO

Resummed ↓

	LL	NLL	NNLL	...
LO $m \neq 0$	1			
NLO $m \neq 0$	aL	a		
NNLO	$(aL)^2$	$a(aL)$	a^2	
...	...	...	...	...

Fixed Order →

ZM-VFNS/Resummed NLO

Resummed ↓

Fixed Order →

	LL m=0	NLL m=0	NNLL	...
LO	1			
NLO	aL	a		
NNLO	$(aL)^2$	a(aL)	a^2	
...	...	...	...	...

GM-VFNS/FONLL (NLO+NLL)

Resummed ↓

Fixed Order →

	LL	NLL	NNLL	...
LO	1 $m \neq 0$			
NLO	aL $m \neq 0$	a $m \neq 0$		
NNLO	$(aL)^2$ $m=0$	$a(aL)$ $m=0$	a^2	
...	... $m=0$	... $m=0$	...	...

NLO Monte Carlo generators:
MC@NLO and POWHEG

NLO MC generators

- MC@NLO, POWHEG: hep-ph/0305252, arXiv:0707.3088
  consistent matching of NLO matrix elements with parton showers (PS)

- Flexible simulation of hadronic final state
  (PS, hadronization, detector effects)

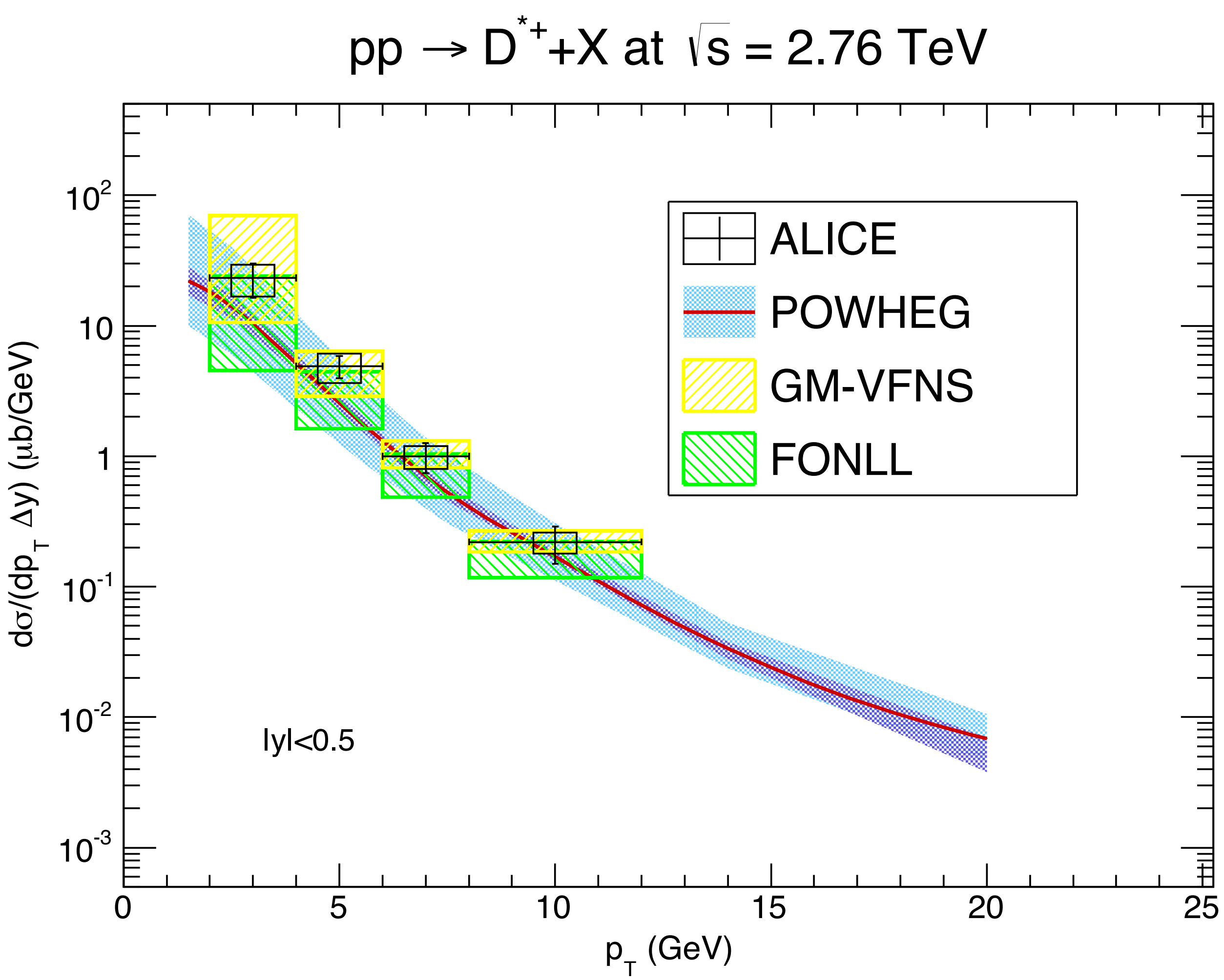
  Note: FONLL and GM-VFNS only one-particle inclusive observables

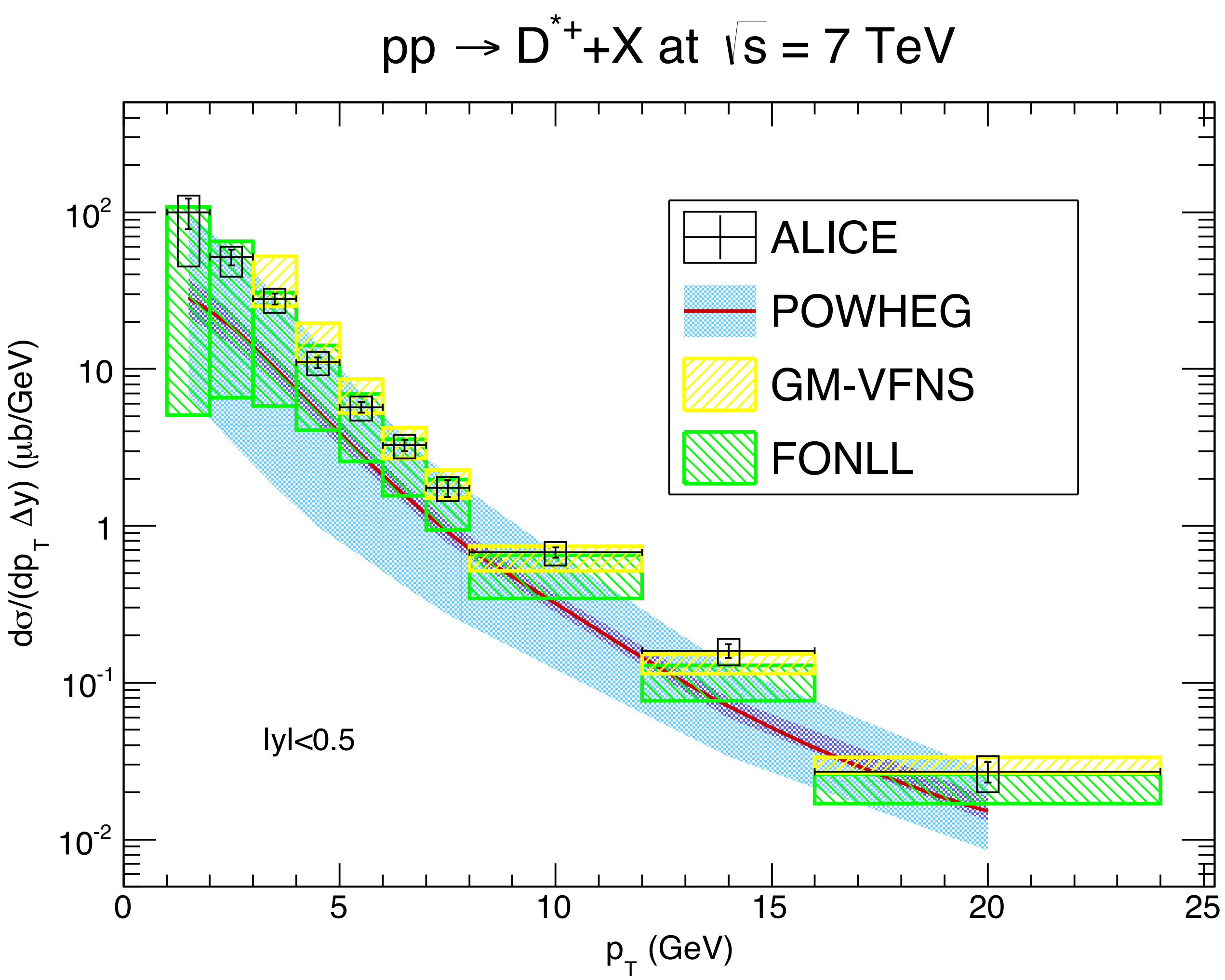
- High accuracy: NLO+LL*
  (FONLL and GM-VFNS have NLO+NLL accuracy)

- Simulation of hadronic final state involves tuning;
  NOT a pure theory prediction!

Comparison with ALICE data

arXiv:1405.3083





Comparison with ALICE data

arXiv:1405.3083