QED Corrections to Hadronic Processes in Lattice QCD

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Multi Hadron and Nonlocal Matrix Elements in Lattice QCD

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(arXiv:1502.00257)

- Electromagnetic corrections to hadronic masses are now being calculated. For a review see A.Portelli at Lattice 2014.
- The results of (some) weak matrix elements obtained from lattice QCD are now being quoted with $O(1\%)$ precision e.g. FLAG Collaboration, arXiv:1310.8555

f_π	f_K	f_D	f_{D_s}	f_B	f_{B_s}
130.2(1.4)	156.3(0.8)	209.2(3.3)	248.6(2.7)	190.5(4.2)	227.7(4.5)

(results given in MeV)

- We therefore need to start considering electromagnetic (and other isospin breaking) effects if we are to use these results to extract CKM matrix elements at a similar precision.
- For illustration, we consider f_π but the discussion is general. we do not use ChPT. For a ChPT based discussion of f_π, see J.Gasser & G.R.S.Zarnauskas, arXiv:1008.3479
- At $O(\alpha^0)$

$$\Gamma(\pi^+ \to \ell^+\nu_\ell) = \frac{G_F^2\,|V_{ud}|^2 f_\pi^2}{8\pi}\, m_\pi\, m_\ell^2 \left(1-\frac{m_\ell^2}{m_\pi^2}\right)^2 .$$

Infrared Divergences

- At $O(\alpha)$ infrared divergences are present and we have to consider

$$\begin{aligned}\Gamma(\pi^+ \to \ell^+\nu_\ell(\gamma)) &= \Gamma(\pi^+ \to \ell^+\nu_\ell) + \Gamma(\pi^+ \to \ell^+\nu_\ell\gamma) \\ &\equiv \Gamma_0 + \Gamma_1\,,\end{aligned}$$

  where the suffix denotes the number of photons in the final state.
- Each of the two terms on the rhs is infrared divergent, the divergences cancel in the sum.
- The cancelation of infrared divergences between contributions with virtual and real photons is an old and well understood issue.

F.Bloch and A.Nordsieck, PR 52 (1937) 54

- The question for our community is how best to combine this understanding with lattice calculations of non-perturbative hadronic effects.
- This is a generic problem if em corrections are to be included in the evaluation of a decay process.

- In principle, particularly as techniques and resources improve in the future, it may be better to compute Γ_1 nonperturbatively over a larger range of photon energies.
- At present we do not propose to compute Γ_1 nonperturbatively. Rather we consider only photons which are sufficiently soft for the point-like (pt) approximation to be valid.
  - A cut-off ΔE of $O(10\,\text{-}\,20\,\mathrm{MeV})$ appears to be appropriate both experimentally and theoretically.

F.Ambrosino et al., KLOE collaboration, hep-ex/0509045; arXiv:0907.3594

- We now write

$$\boxed{\Gamma_0 + \Gamma_1(\Delta E) = \lim_{V\to\infty}(\Gamma_0 - \Gamma_0^{\rm pt}) + \lim_{V\to\infty}(\Gamma_0^{\rm pt} + \Gamma_1(\Delta E))\,.}$$

  - The second term on the rhs can be calculated in perturbation theory. It is infrared convergent, but does contain a term proportional to $\log\Delta E$.
  - The first term is also free of infrared divergences.
  - Γ_0 is calculated nonperturbatively and $\Gamma_0^{\rm pt}$ in perturbation theory. The subtraction in the first term is performed for each momentum and then the sum over momenta is performed (see below).

Outline of Talk

$$\boxed{\Gamma_0 + \Gamma_1(\Delta E) = \lim_{V\to\infty}(\Gamma_0 - \Gamma_0^{\mathrm{pt}}) + \lim_{V\to\infty}(\Gamma_0^{\mathrm{pt}} + \Gamma_1(\Delta E))\,.}$$

1. Introduction
2. What is G_F at $O(\alpha)$?
3. Proposed calculation of $\Gamma_0 - \Gamma_0^{\mathrm{pt}}$
4. Calculation of $\Gamma_0^{\mathrm{pt}} + \Gamma_1(\Delta E)$
5. Estimates of structure dependent contributions to $\Gamma_1(\Delta E)$
6. Summary and Conclusions

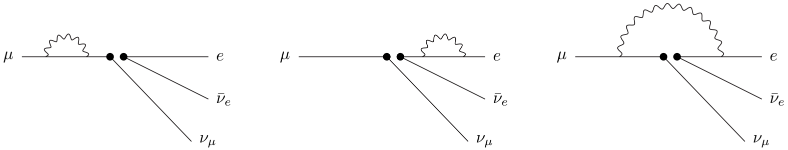
2. What is G_F at $O(\alpha)$?

1. The results for the widths are expressed in terms of G_F, the Fermi constant ($G_F = 1.16632(2)\times 10^{-5}\,\mathrm{GeV}^{-2}$). This is obtained from the muon lifetime:

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}\left[1-\frac{8m_e^2}{m_\mu^2}\right]\left[1+\frac{\alpha}{2\pi}\left(\frac{25}{4}-\pi^2\right)\right].$$

S.M.Berman, PR 112 (1958) 267; T.Kinoshita and A.Sirlin, PR 113 (1959) 1652

- This expression can be viewed as the definition of G_F. Many EW corrections are absorbed into the definition of G_F; the explicit $O(\alpha)$ corrections come from the following diagrams in the effective theory:

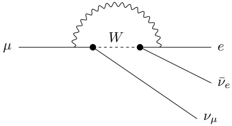


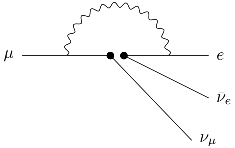
together with the diagrams with a real photon.

- These diagrams are evaluated in the W-regularisation in which the photon propagator is modified by: A.Sirlin, PRD 22 (1980) 971

$$\frac{1}{k^2}\to\frac{M_W^2}{M_W^2-k^2}\,\frac{1}{k^2}\,. \qquad \left(\frac{1}{k^2}=\frac{1}{k^2-M_W^2}+\frac{M_W^2}{M_W^2-k^2}\,\frac{1}{k^2}\right)$$

- The $\gamma - W$ box diagram:





As an example providing some evidence & intuition that the W-regularization is useful consider the $\gamma - W$ box diagram.

- In the standard model (left-hand diagram) it contains both the γ and W propagators.
- In the effective theory this is preserved with the W-regularization where the photon propagator is proportional to

$$\frac{1}{k^2}\,\frac{1}{k^2 - M_W^2}$$

and the two diagrams are equal up to terms of $O(q^2/M_W^2)$, where q is the momentum of the e and ν_e.

- Most (but not all) of the EW corrections which are absorbed in G_F are common to other processes (including pion decay) $\Rightarrow$ factor in the amplitude of $(1 + 3\alpha/4\pi(1+2\bar{Q})\log M_Z/M_W)$, where $\bar{Q} = \frac{1}{2}(Q_u + Q_d) = 1/6$.

  A.Sirlin, NP B196 (1982) 83; E.Braaten & C.S.Li, PRD 42 (1990) 3888

- We therefore need to calculate the pion-decay diagrams in the effective theory with

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}}\,V_{ud}^*\left(1 + \frac{\alpha}{\pi}\log\frac{M_Z}{M_W}\right)(\bar{d}_L\gamma^\mu u_L)(\bar{\nu}_{\ell,L}\,\gamma_\mu \ell_L)$$

  in the W-regularization.

- Thus for example, with the Wilson action for both the gluons and fermions:

$$O_1^{\rm W-reg} = \left(1 + \frac{\alpha}{4\pi}\left(2\log a^2M_W^2 - 15.539\right)\right)O_1^{\rm bare} + \frac{\alpha}{4\pi}\left(0.536\,O_2^{\rm bare} + 1.607\,O_3^{\rm bare} - 3.214\,O_4^{\rm bare} - 0.804\,O_5^{\rm bare}\right),$$

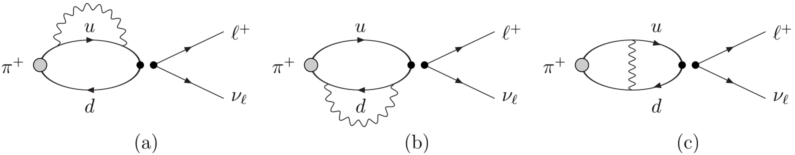
where

$$O_1 = (\bar{d}\gamma^\mu(1-\gamma^5)u)\,(\bar{\nu}_\ell\gamma_\mu(1-\gamma^5)\ell) \qquad O_2 = (\bar{d}\gamma^\mu(1+\gamma^5)u)\,(\bar{\nu}_\ell\gamma_\mu(1-\gamma^5)\ell)$$

$$O_3 = (\bar{d}(1-\gamma^5)u)\,(\bar{\nu}_\ell(1+\gamma^5)\ell) \qquad O_4 = (\bar{d}(1+\gamma^5)u)\,(\bar{\nu}_\ell(1+\gamma^5)\ell)$$

$$O_5 = (\bar{d}\sigma^{\mu\nu}(1+\gamma^5)u)\,(\bar{\nu}_\ell\sigma_{\mu\nu}(1+\gamma^5)\ell)\,.$$

Consider now the evaluation of Γ_0.



- The correlation function for this set of diagrams is of the form:

$$C_1(t) = -\frac{1}{2}\int d^3\vec{x}\; d^4x_1\; d^4x_2\; \langle 0|T\left\{J_W^\nu(0)\, j_\mu(x_1) j_\mu(x_2)\phi^\dagger(\vec{x},-t)\right\}|0\rangle\; \Delta(x_1,x_2)\,,$$

  where $j_\mu(x) = \sum_f Q_f \bar{f}(x)\gamma_\mu f(x)$, J_W is the weak current, ϕ is an interpolating operator for the pion and Δ is the photon propagator.

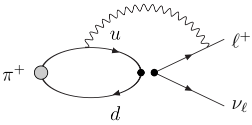
- Combining C_1 with the lowest order correlator:

$$C_0(t) + C_1(t) \simeq \frac{e^{-m_\pi t}}{2m_\pi} Z^\phi \,\langle 0\,|J_W^\nu(0)\,|\,\pi^+\rangle\,,$$

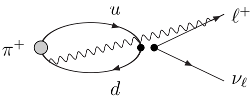
  where now $O(\alpha)$ terms are included.

- $e^{-m_\pi t} \simeq e^{-m_\pi^0 t}\,(1 - \delta m_\pi\, t)$ and Z^ϕ is obtained from the two-point function.

Proposed calculation of $\Gamma_0 - \Gamma_0^{\rm pt}$ (Cont)



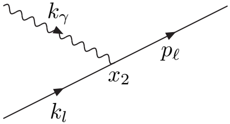
(e)



(f)

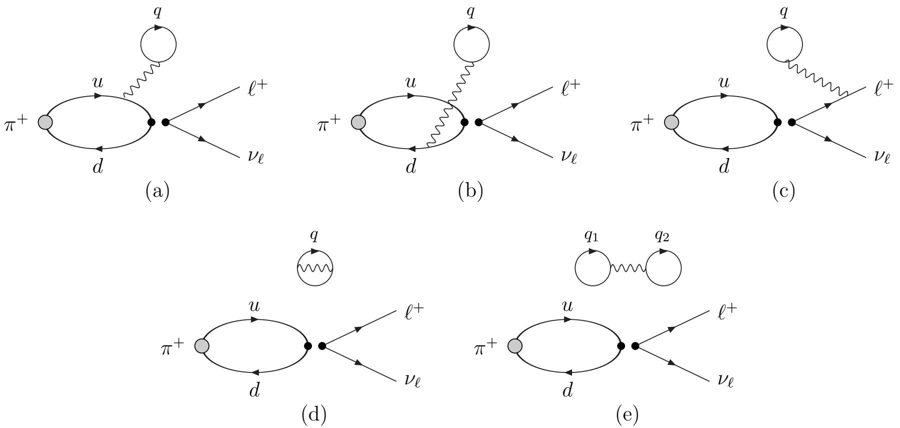
$$\begin{aligned}\bar{C}_1(t)_{\alpha\beta} &= -\int d^3\vec{x}\,d^4x_1\,d^4x_2\,\langle 0|T\{J_W^\nu(0)\,j_\mu(x_1)\phi^\dagger(\vec{x},-t)\}|\,0\rangle\,\Delta(x_1,x_2)\\ &\qquad\qquad \times\left(\gamma_\nu(1-\gamma^5)S(0,x_2)\gamma_\mu\right)_{\alpha\beta}e^{E_\ell\,t_2}e^{-i\vec{p}_\ell\cdot\vec{x}_2}\\ &\simeq Z_0^\phi\,\frac{e^{-m_\pi^0 t}}{2m_\pi^0}\,(\bar{M}_1)_{\alpha\beta}\end{aligned}$$

- Corresponding contribution to the amplitude is $\bar{u}_\alpha(p_{\nu_\ell})(\bar{M}_1)_{\alpha\beta}v_\beta(p_\ell)$.
- Diagrams (e) and (f) are not simply generalisations of the evaluation of f_π.
- The lepton's wave function renormalisation cancels in the difference $\Gamma_0 - \Gamma_0^{\rm pt}$.
- We have to be able to isolate the finite-volume ground state (pion).
- The Minkowski $\leftrightarrow$ Euclidean continuation can be performed (the time integrations are convergent).



- For every term in the $\vec{k}_\gamma$ integration, $\omega_\gamma + \omega_l > E_l$ so the t_2 behaviour, $\exp[-(\omega_k + \omega_l - E_l)t_2]$ is convergent.

There are also disconnected diagrams to be evaluated.



- The total width, $\Gamma^{\rm pt}$ was calculated in 1958/9 using a Pauli-Villars regulator for the UV divergences and m_γ for the infrared divergences.

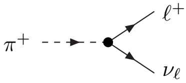
  S.Berman, PR **112** (1958) 267, T.Kinoshita, PRL **2** (1959) 477

  - This is a useful check on our perturbative calculation.

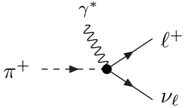
- In the perturbative calculation we use the following Lagrangian for the interaction of a point-like pion with the leptons:

$$\mathcal{L}_{\pi\text{-}\ell\text{-}\nu_\ell} \;=\; i\,G_F f_\pi V_{ud}^* \,\{(\partial_\mu - ieA_\mu)\pi\}\left\{\bar\psi_{\nu_\ell}\frac{1+\gamma_5}{2}\gamma^\mu\psi_\ell\right\} + \text{H.C.}\,.$$

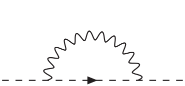
The corresponding Feynman rules are:



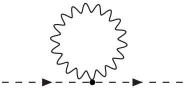
$$= -iG_F f_\pi V_{ud}^*\, p_\pi^\mu\, \tfrac{1+\gamma^5}{2}\gamma_\mu$$

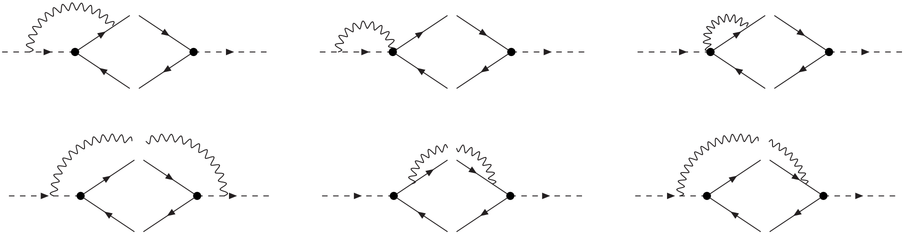


$$= ie\, G_F f_\pi V_{ud}^*\, g^{\mu\nu}\, \tfrac{1+\gamma^5}{2}\gamma_\mu$$

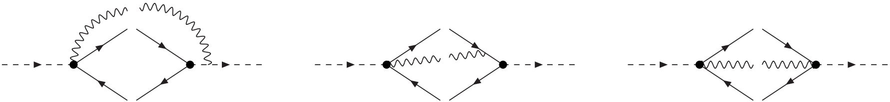


and





(a) (b) (c)



(d) (e) (f)

- We find, for $E_\gamma < \Delta E$

$$\begin{aligned}
\Gamma^{\rm pt}(\Delta E) \;=\;& \Gamma_0^{\rm tree} \times \Bigg(1 + \frac{\alpha}{4\pi}\Bigg\{3\log\left(\frac{m_\pi^2}{M_W^2}\right) + \log\left(r_\ell^2\right) - 4\log(r_E^2) + \frac{2-10r_\ell^2}{1-r_\ell^2}\log(r_\ell^2) \\
& -2\frac{1+r_\ell^2}{1-r_\ell^2}\log(r_E^2)\log(r_\ell^2) - 4\frac{1+r_\ell^2}{1-r_\ell^2}\,\mathrm{Li}_2(1-r_\ell^2) - 3 \\
& + \Bigg[\frac{3+r_E^2-6r_\ell^2+4r_E(-1+r_\ell^2)}{(1-r_\ell^2)^2}\log(1-r_E) + \frac{r_E(4-r_E-4r_\ell^2)}{(1-r_\ell^2)^2}\log(r_\ell^2) \\
& - \frac{r_E(-22+3r_E+28r_\ell^2)}{2(1-r_\ell^2)^2} - 4\frac{1+r_\ell^2}{1-r_\ell^2}\,\mathrm{Li}_2(r_E)\Bigg]\Bigg\}\Bigg)\,,
\end{aligned}$$

where $r_E = 2\Delta E/m_\pi$ and $r_\ell = m_\ell/m_\pi$.

- We believe that this is a new result.

- The total rate is readily computed by setting r_E to its maximum value, namely $r_E=1-r_\ell^2$, giving

$$\Gamma^{\rm pt} \;=\; \Gamma_0^{\rm tree}\times\Bigg\{1+\frac{\alpha}{4\pi}\left(3\log\left(\frac{m_\pi^2}{M_W^2}\right)-8\log(1-r_\ell^2)-\frac{3r_\ell^4}{(1-r_\ell^2)^2}\log(r_\ell^2)\right.$$
$$\left.-8\frac{1+r_\ell^2}{1-r_\ell^2}\,\mathrm{Li}_2(1-r_\ell^2)+\frac{13-19r_\ell^2}{2(1-r_\ell^2)}+\frac{6-14r_\ell^2-4(1+r_\ell^2)\log(1-r_\ell^2)}{1-r_\ell^2}\,\log(r_\ell^2)\right)\Bigg\}\,.$$

- This result agrees with the well known results in literature providing an important check of our calculation.

4. Calculation of $\Gamma^{\rm pt} = \Gamma_0^{\rm pt} + \Gamma_1^{\rm pt}$ (cont)

- It is of course possible instead to impose a cut-off on the energy of the final-state lepton, requiring it to be close to its maximum value $E_\ell^{\rm max} = \frac{m_\pi}{2}(1+r_\ell^2)$.
- We also give, up to $O(\Delta E_\ell)$, the distribution for $\Gamma^{\rm pt}(\Delta E_\ell)$ defined as

$$\Gamma^{\rm pt}(\Delta E_\ell) = \int_{E_\ell^{max}-\Delta E_\ell}^{E_\ell^{max}} dE'_\ell \, \frac{d\Gamma^{\rm pt}}{dE'_\ell}\,,$$

where $0 \leq \Delta E_\ell \leq (m_\pi - m_\ell)^2/(2m_\pi)$;

$$\begin{aligned}\Gamma^{\rm pt}(\Delta E_\ell) = \Gamma_0^{\rm tree} \times \Bigg\{ 1 + \frac{\alpha}{4\pi}\Bigg[& 3\log\left(\frac{m_\pi^2}{M_W^2}\right) + 8\log\left(1-r_\ell^2\right) - 7 \\ & + \log\left(r_\ell^2\right)\frac{3 - 7r_\ell^2 + 8\Delta E_\ell + 4\left(1+r_\ell^2\right)\log\left(1-r_\ell^2\right)}{1-r_\ell^2} \\ & + \log\left(2\Delta E_\ell\right)\left(-8 - 4\frac{1+r_\ell^2}{1-r_\ell^2}\log\left(r_\ell^2\right)\right)\Bigg]\Bigg\}.\end{aligned}$$

- Summary: The perturbative calculation of $\Gamma_0^{\rm pt} + \Gamma_1(\Delta E)$ is done.

- For sufficiently small ΔE the structure dependent contributions to Γ_1 can be neglected.
- How big might they be for experimentally accessible values of ΔE?
  To estimate this for f_π and f_K we use Chiral Perturbation Theory.

  J.Bijnens, G.Ecker and J.Gasser, hep-ph/9209261,
  J.Bijnens, G.Colangelo, G.Ecker and J.Gasser, hep-ph/9411311.
  V. Cirigliano and I. Rosell, arXiv:0707.3439 [hep-ph]],
  L. Ametller, J. Bijnens, A. Bramon and F. Cornet, hep-ph/9302219.
- We define

$$R_1^A(\Delta E) = \frac{\Gamma_1^A(\Delta E)}{\Gamma_0^{\alpha,\mathrm{pt}} + \Gamma_1^{\mathrm{pt}}(\Delta E)}\,, \qquad A = \{\mathrm{SD,INT}\}\,,$$

  where SD and INT refer to the structure dependent and interference (between SD and pt) contributions respectively.
  - Note that the notation I am using here differs from that in the paper.

- Start with a decomposition in terms of Lorenz invariant form factors of the hadronic matrix element

$$H^{\mu\nu}(k,p_\pi) \quad = \quad \int d^4x\, e^{ikx}\, T\langle 0|j^\mu(x) J_W^\nu(0)|\pi(p_\pi)\rangle$$

and separate the contribution corresponding to the approximation of a point-like pion $H_{\rm pt}^{\mu\nu}$, from the structure dependent part $H_{\rm SD}^{\mu\nu}$,

$$H^{\mu\nu} = H_{\rm SD}^{\mu\nu} + H_{\rm pt}^{\mu\nu}\ .$$

- $H_{\rm pt}^{\mu\nu}$ is simply given by

$$H_{\rm pt}^{\mu\nu} = f_\pi \left[g^{\mu\nu} - \frac{(2p_\pi - k)^\mu (p_\pi - k)^\nu}{(p_\pi - k)^2 - m_\pi^2}\right]\ .$$

- The structure dependent component can be parametrised by four independent invariant form factors which we define as

$$H_{\rm SD}^{\mu\nu} \quad = \quad H_1\left[k^2 g^{\mu\nu} - k^\mu k^\nu\right] + H_2\left\{\left[(k\cdot p_\pi - k^2)k^\mu - k^2(p_\pi - k)^\mu\right](p_\pi - k)^\nu\right\}$$
$$-i\frac{F_V}{m_\pi}\,\epsilon^{\mu\nu\alpha\beta}k_\alpha p_{\pi\,\beta} + \frac{F_A}{m_\pi}\left[(k\cdot p_\pi - k^2)g^{\mu\nu} - (p_\pi - k)^\mu k^\nu\right]\ .$$

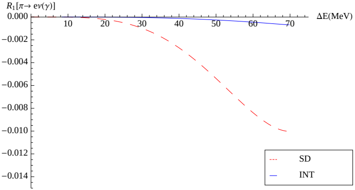
- For the decay into a real photon, only F_V and F_A contribute.
- At $O(p^4)$ in chiral perturbation theory,

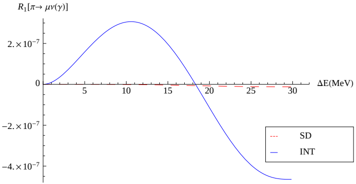
$$F_V = \frac{m_P}{4\pi^2 f_\pi} \quad \text{and} \quad F_A = \frac{8m_P}{f_\pi}\,(L_9^r + L_{10}^r)\,,$$

  where $P = \pi$ or K and $L_9^r,\, L_{10}^r$ are Gasser-Leutwyler coefficients.
- The numerical values of these constants have been taken from the review by M.Bychkov and G.D'Ambrosio in the PDG. F_V and F_A are 0.0254 and 0.0119 for the pion and 0.096 and 0.042 for the Kaon (for the pion these values of the form factors, obtained from direct measurements, can be found in the supplement to the PDG.)

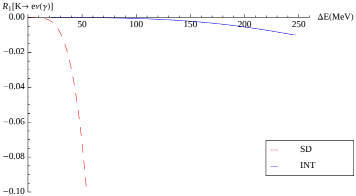
5. Estimates of structure dependent contributions to $\Gamma_1(\Delta E)$ (cont)

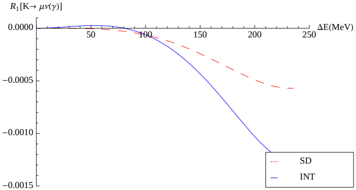
Pion





Kaon





- For heavy-light mesons we don't have such ChPT calculations.
- For the B-meson in particular we have another small scale $< \Lambda_{\text{QCD}}$, $m_{B^*} - m_B \simeq 45$ MeV so that we may expect that we will have to go to smaller ΔE in order to be able to neglect SD effects.
- Calculations based on the extreme approximation of single pole dominance suggest that this is likely to be the case.

  D. Becirevic, B. Haas and E. Kou, arXiv:0907.1845 [hep-ph]
- To be investigated further!

6. Summary and Conclusions

- Lattice calculations of some physical quantities are approaching $O(1\%)$ precision $\Rightarrow$ we need to include isospin-breaking effects, including electromagnetic effects, to make the tests of the SM even more stringent.
- For decay widths and scattering cross sections including em effects introduces infrared divergences.
- In this work we propose a method for dealing with these divergences, illustrating the procedure by a detailed study of the leptonic (and semileptonic) decays of pseudoscalar mesons.
- Although challenging, the method is within reach of present simulations and we will now implement the procedure in an actual numerical computation.
  - Power-like FV corrections, $O(1/(L\Lambda_{\rm QCD})^n)$, to be evaluated.
  - $O(\alpha\alpha_s)$ matching factors to be studied.
- In the future one can envisage relaxing the condition $\Delta E \ll \Lambda_{\rm QCD}$, including the emission of real photons with energies which do resolve the structure of the initial hadron. Such calculations can be performed in Euclidean space under the same conditions as above, i.e. providing that there is a mass gap.
  - The natural extension of the present proposal is to subtract and add $\Gamma_1^{\rm pt}(\Delta E)$ to determine $\Gamma_1(\Delta E) - \Gamma_1^{\rm pt}(\Delta E)$, so that our calculation of $\Gamma_0^{\rm pt} + \Gamma_1^{\rm pt}(\Delta E)$ will still be useful.