Extract collinear parton distributions from lattice QCD calculations

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Based on work done with

Tomomi Ishikawa, Yan-Qing Ma, Shinsuke Yoshida, …

arXiv:1404.6860, 1412.2688, …

RBRC workshop on "Multi-hadron and nonlocal matrix elements in lattice QCD"

Brookhaven National Lab, Upton, NY, February 5-6, 2015

Outline

- ❑ Why Parton distribution functions (PDFs)?

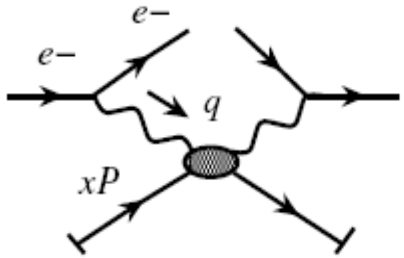
  No PDFs, no predictions for Higgs production x-sections, …

- ❑ PDFs from lattice QCD calculations
- ❑ Our proposal – QCD collinear factorization
- ❑ Case study – Extract PDFs from quasi PDFs
- ❑ Summary and outlook

QCD factorization: PDFs

- **One hadron:** $\ell + h(p) \to \ell' + X$

$\sigma_{\mathrm{tot}}^{\mathrm{DIS}}$:



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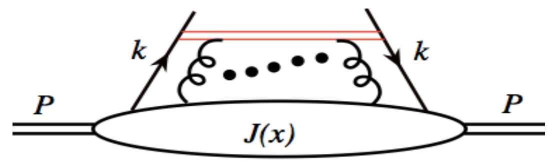

$+ \, O\left(\frac{1}{QR}\right)$

Hard-part	Parton-distribution	Power corrections
Probe	Structure	Approximation

QCD factorization: PDFs

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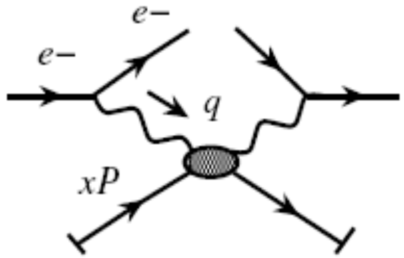

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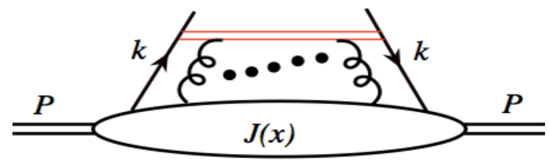


$+\,O\left(\dfrac{1}{QR}\right)$

Hard-part
Probe

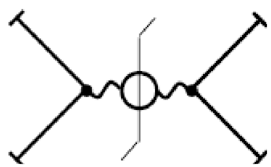
Parton-distribution
Structure

Power corrections
Approximation

- **Two hadrons:**

$h(p) + h'(p') \to V(\gamma^*, Z^0, \ldots) + X$

$\sigma_{\text{tot}}^{\text{DY}}$:



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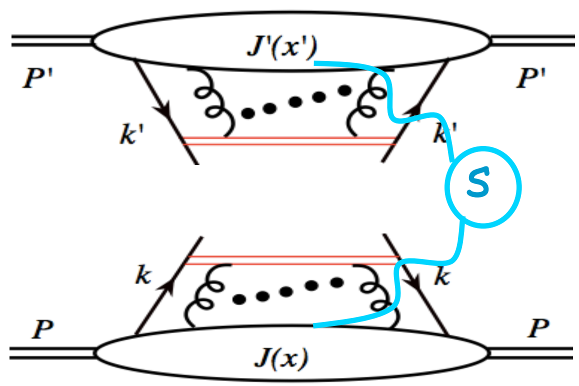

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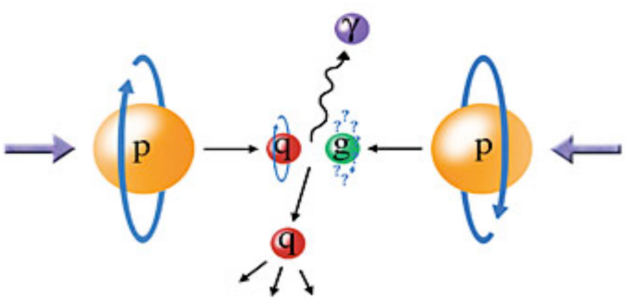


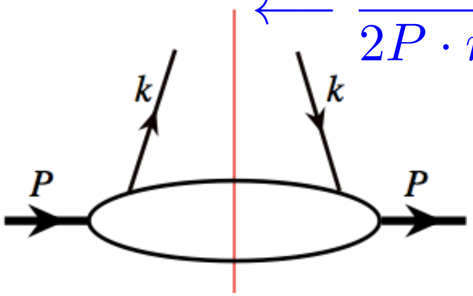
Predictive power:
Universal Parton Distributions

Operator definition of PDFs

- **Quark distribution (spin-averaged):**

$$q(x,\mu^2) = \int \frac{d\xi_-}{4\pi} e^{-ix\xi_- P_+} \langle P|\overline{\psi}(\xi_-)\gamma_+ \exp\left\{-ig\int_0^{\xi_-} d\eta_- A_+(\eta_-)\right\}\psi(0)|P\rangle \; + \text{UVCT}$$

- **Cut-vertex notation:**



$$\longleftarrow \frac{\gamma\cdot n}{2P\cdot n}\,\delta\left(x - \frac{k\cdot n}{P\cdot n}\right)\frac{d^4k}{(2\pi)^4}$$

$$+\,\text{UVCT}$$

PDFs are not direct physical observables, such as cross sections!
But, well-defined in QCD and process independent!

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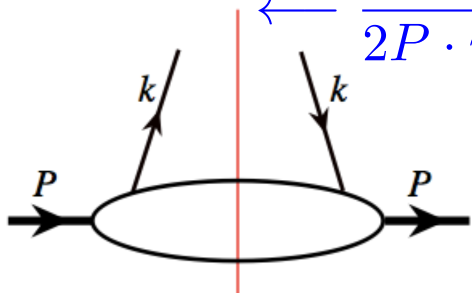


$$+\,\mathrm{UVCT}$$

PDFs are not direct physical observables, such as cross sections!
But, well-defined in QCD and process independent!

- Parton interpretation emerges in *n.A = 0* gauge
- Independent of hadron momentum *P*
- Simplest of all parton correlation functions of the hadron

Global QCD analyses – a successful story

- World data with "Q" > 2 GeV

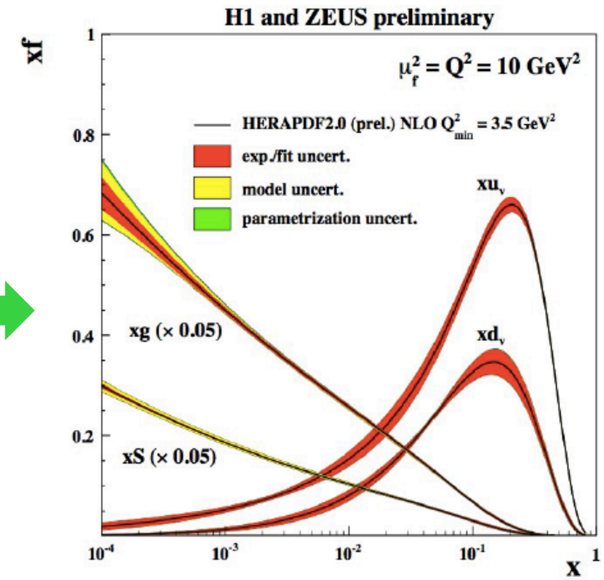
  + Factorization:

DIS: $$F_2(x_B,Q^2) = \Sigma_f C_f(x_B/x,\mu^2/Q^2)\otimes f(x,\mu^2)$$

H-H: $$\frac{d\sigma}{dydp_T^2} = \Sigma_{ff'} f(x)\otimes\frac{d\hat{\sigma}_{ff'}}{dydp_T^2}\otimes f'(x')$$

+ DGLAP Evolution:

$$\frac{\partial f(x,\mu^2)}{\partial\ln\mu^2} = \Sigma_{f'} P_{ff'}(x/x')\otimes f'(x',\mu^2)$$



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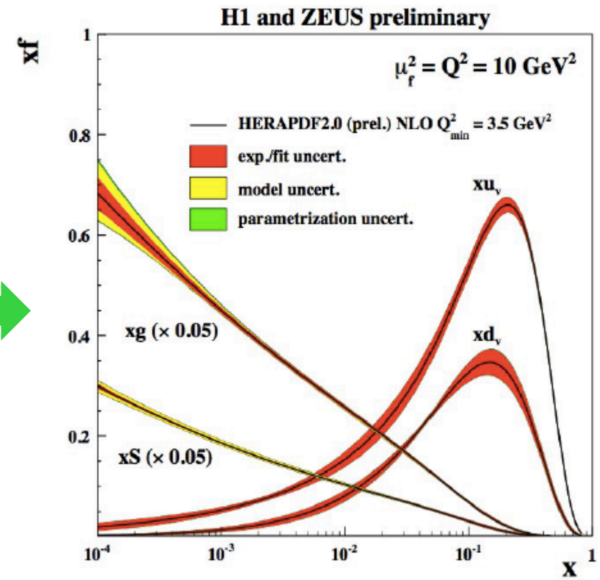
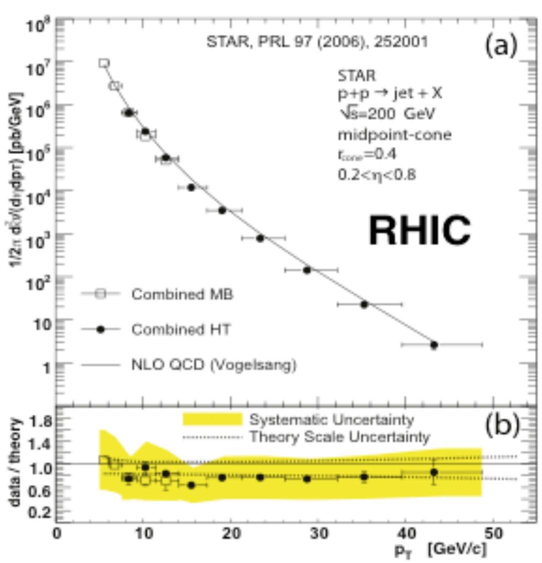
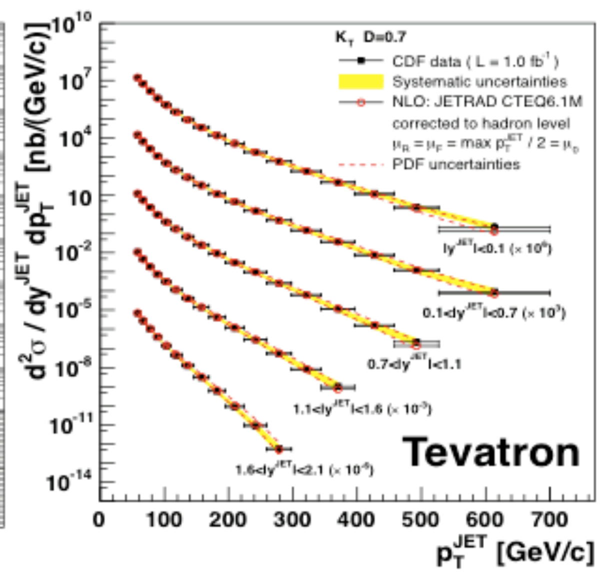
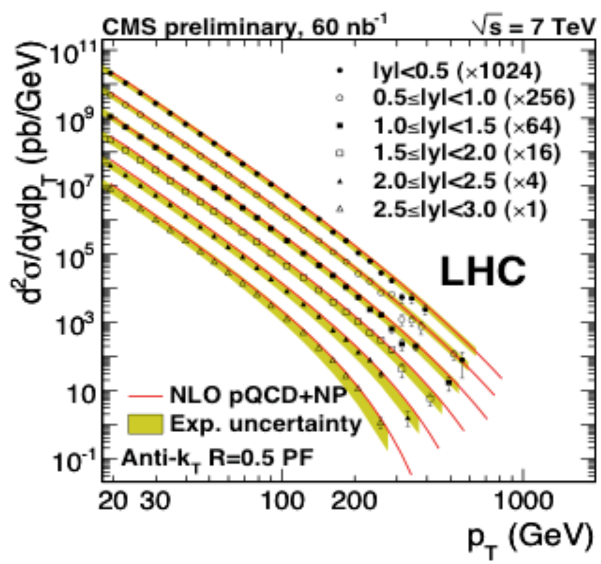
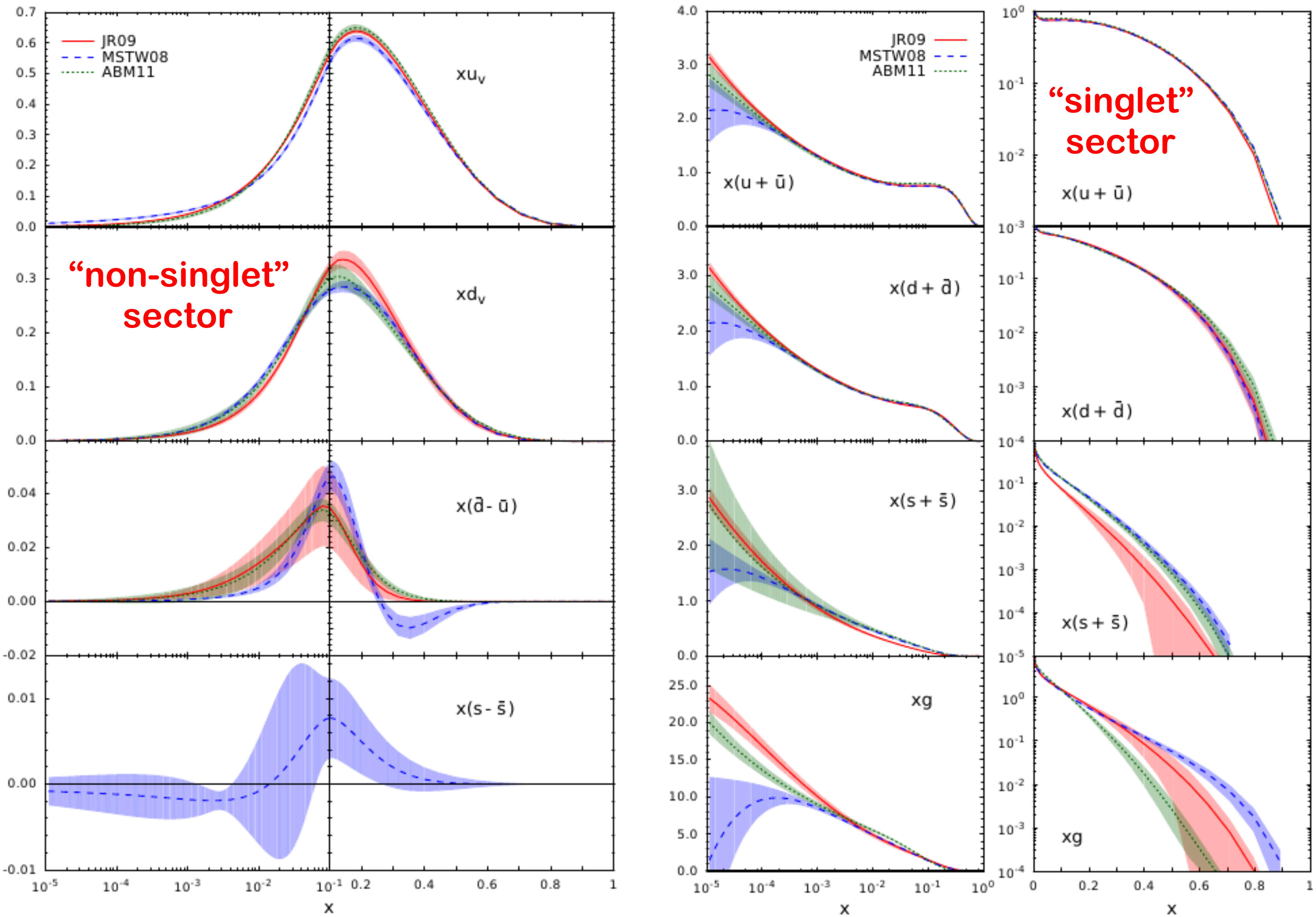






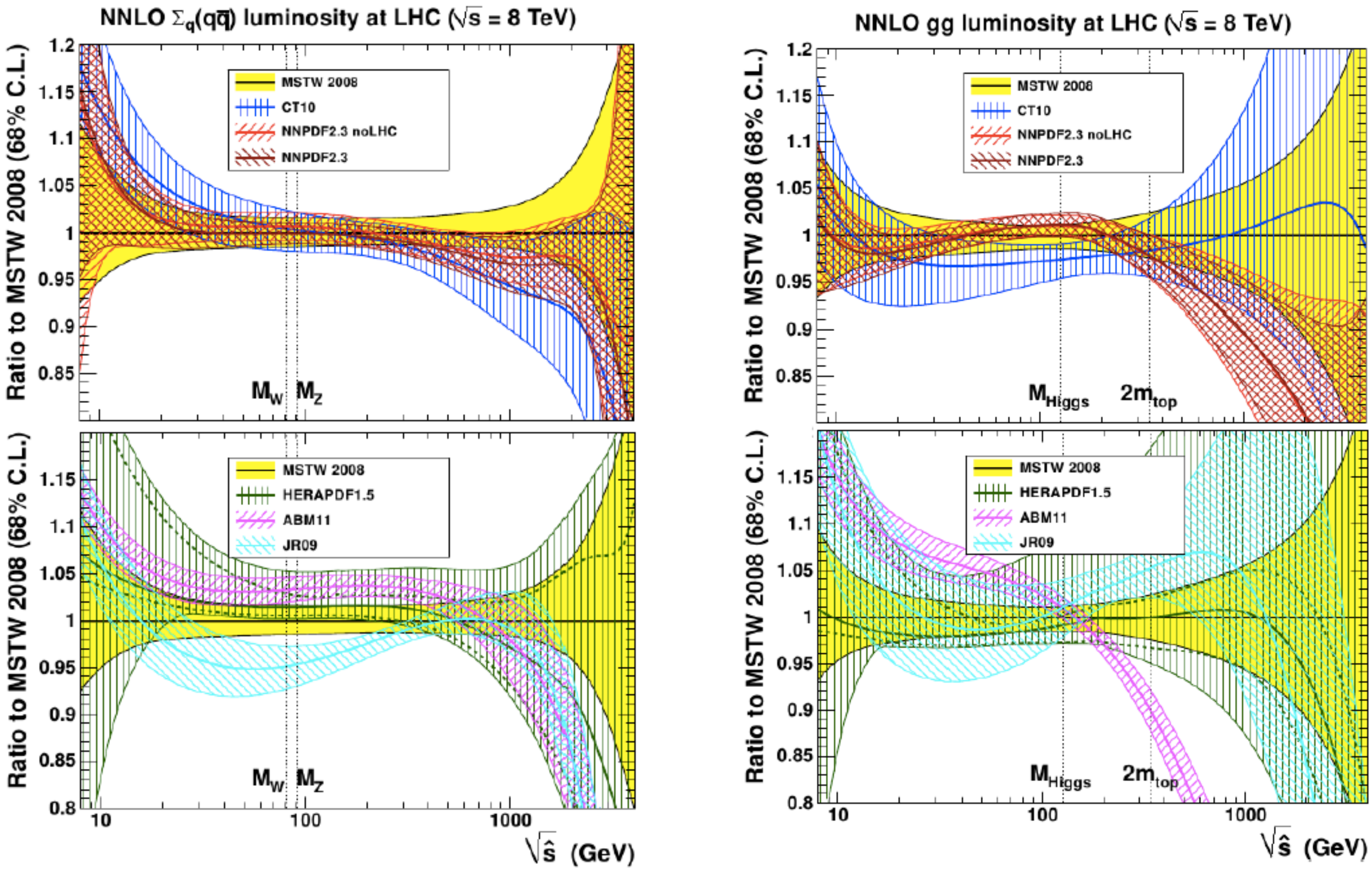


Uncertainties of PDFs



Partonic luminosities

q - qbar | **g - g**



PDFs at large x

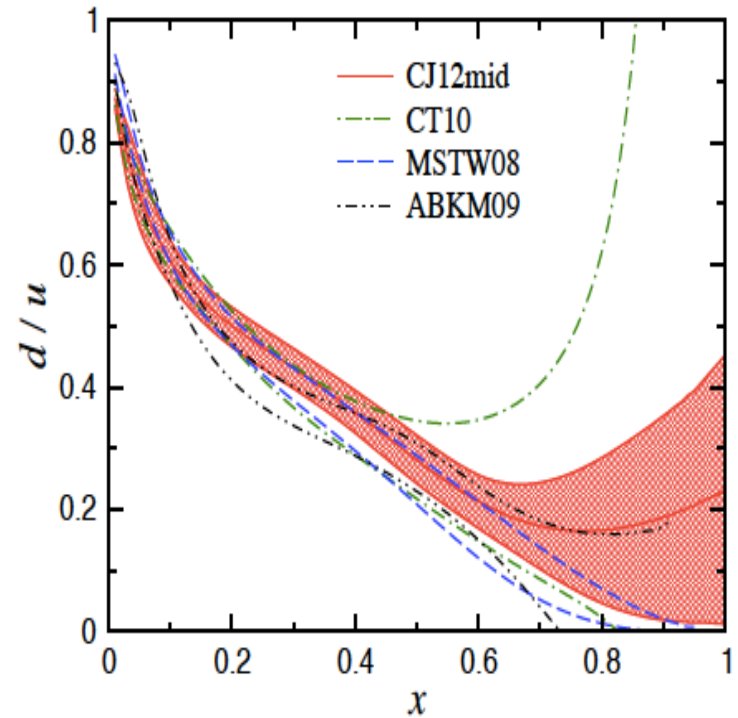
❑ **Testing ground for hadron structure at x → 1:**

✧ $d/u \to 1/2$ — **SU(6) Spin-flavor symmetry**

✧ $d/u \to 0$ — **Scalar diquark dominance**

✧ $d/u \to 1/5$ — **pQCD power counting**

✧ $d/u \to \dfrac{4\mu_n^2/\mu_p^2 - 1}{4 - \mu_n^2/\mu_p^2} \approx 0.42$ — **Local quark-hadron duality**



PDFs at large x

❑ **Testing ground for hadron structure at x → 1:**

$\diamond\ d/u \to 1/2$

SU(6) Spin-flavor symmetry

$\diamond\ \Delta u/u \to 2/3$

$\Delta d/d \to -1/3$

$\diamond\ d/u \to 0$

Scalar diquark dominance

$\diamond\ \Delta u/u \to 1$

$\Delta d/d \to -1/3$

$\diamond\ d/u \to 1/5$

pQCD power counting

$\diamond\ \Delta u/u \to 1$

$\Delta d/d \to 1$

$$\diamond\ d/u \to \frac{4\mu_n^2/\mu_p^2 - 1}{4 - \mu_n^2/\mu_p^2} \approx 0.42$$

Local quark-hadron duality

$\diamond\ \Delta u/u \to 1$

$\Delta d/d \to 1$

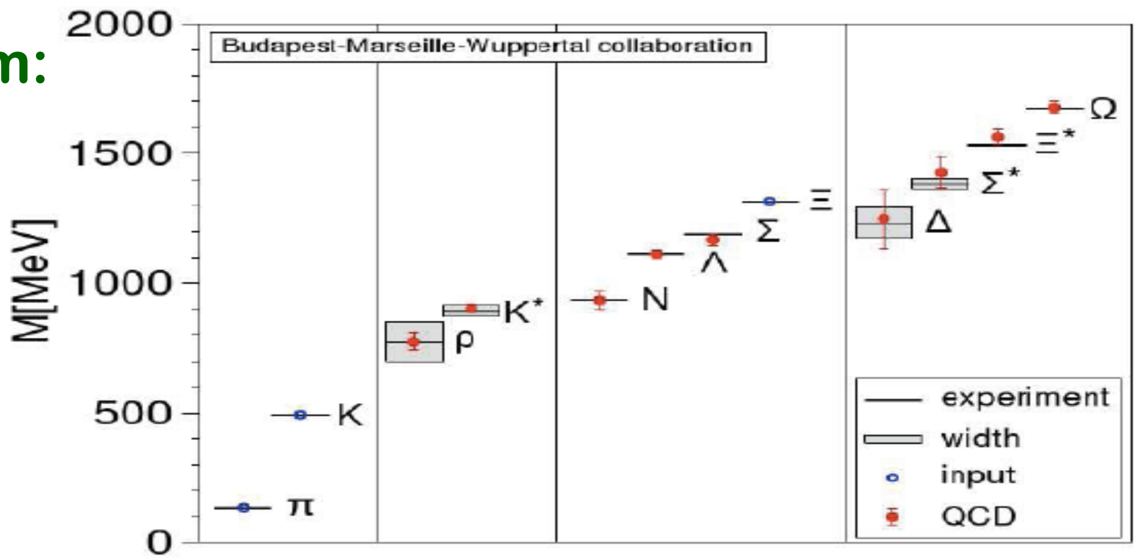
Can lattice QCD help?

Lattice QCD

- The main non-perturbative approach to solve QCD

- Hadron mass spectrum:

Predict the spectrum
with limited inputs



Lattice QCD

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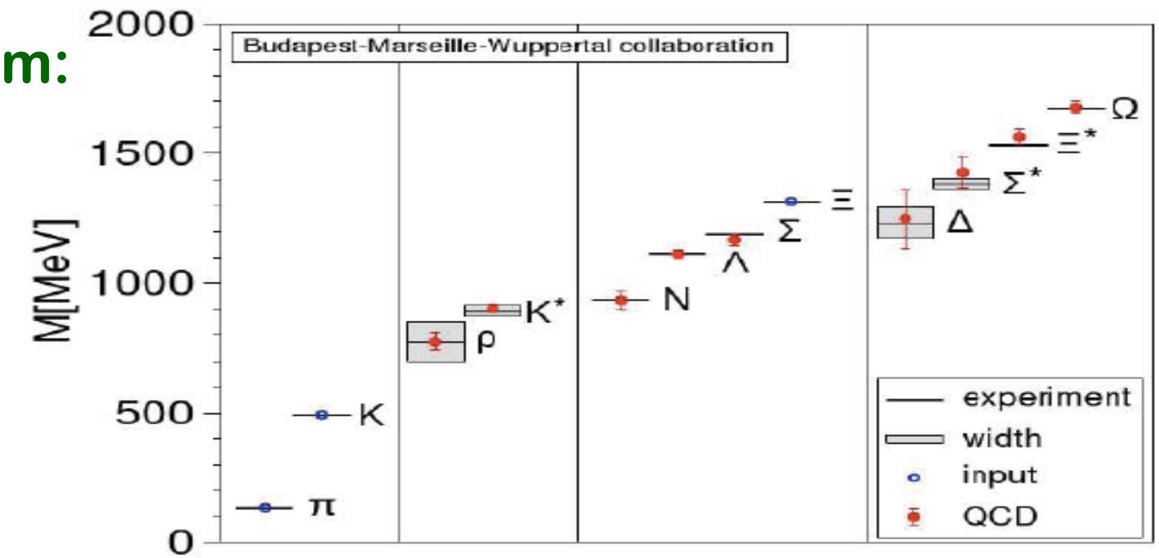


- An intrinsically Euclidean approach:
  - ✧ Lattice "time" is Euclidean: $\tau = i\,t$
  - ✧ No direct implementation of physical time

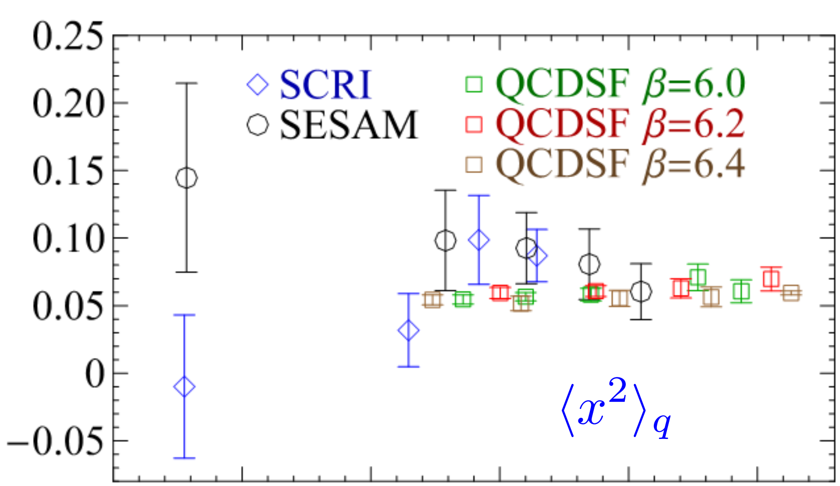
Cannot calculate PDFs directly, whose operators are time-dependent

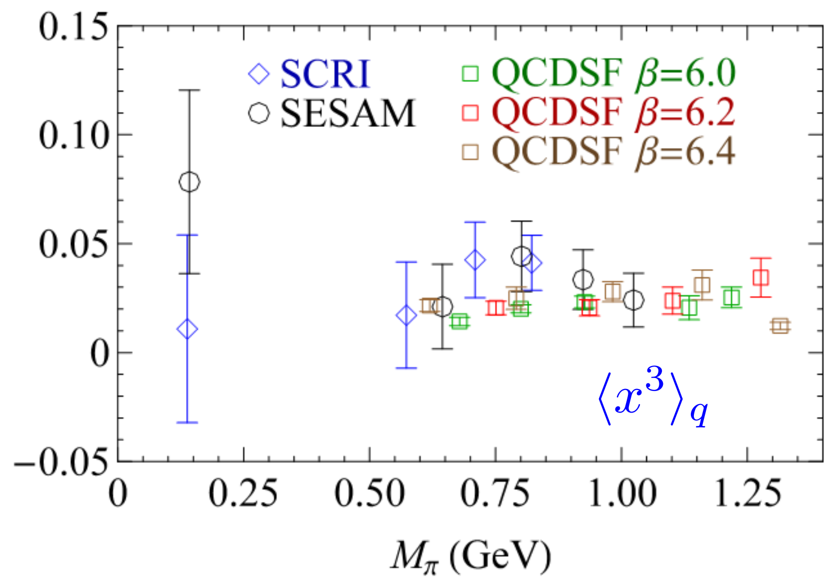
PDFs from lattice QCD

- Moments of PDFs – matrix elements of local operators

$$\langle x^n(\mu^2)\rangle_q \equiv \int_0^1 dx\, x^n\, q(x,\mu^2)$$

- Works, but, hard and limited moments:





Dolgov et al., hep-lat/0201021

Gockeler et al., hep-ph/0410187

Limited moments – hard to get the full x-dependent distributions!

From quasi-PDFs to PDFs (Ji's idea)

Ji, arXiv:1305.1539

❑ "Quasi" quark distribution (spin-averaged):

$$\tilde{q}(x,\mu^2,P_z) \equiv \int \frac{d\xi_z}{4\pi} e^{-ixP_z\xi_z} \langle P|\overline{\psi}(\xi_z)\gamma_z \exp\left\{-ig\int_0^{\xi_z} d\eta_z A_z(\eta_z)\right\}\psi(0)|P\rangle + \mathrm{UVCT}(\mu^2)$$

❑ Features:

- Quark fields separated along the z-direction – not boost invariant!
- Perturbatively UV power divergent: $\propto (\mu/P_z)^n$ with $n > 0$ - renormalizable?
- Quasi-PDFs ➔ *Normal PDFs* *when P_z ➔ ∞*
- Quasi-PDFs could be calculated using standard lattice method

❑ Proposed matching:

Ji, arXiv:1305. 1539

$$\tilde{q}(x,\mu^2,P_z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y},\frac{\mu}{P_z}\right) q(y,\mu^2) + \mathcal{O}\left(\frac{\Lambda^2}{P_z^2},\frac{M^2}{P_z^2}\right)$$

- Size of $O(1/P_z^2)$ terms
- UV renormalization of power divergence, and potential operator mixing, …

Our observation

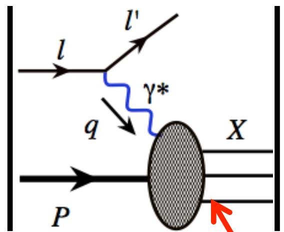
❑ **QCD factorization of single-hadron cross section:**

Momentum transfer

CO Factorization

PDFs

$$\sigma^{\mathrm{DIS}}(x, Q^2; \sqrt{s}) \propto$$



$$\left|\ \cdots\ \right|^2 \approx \sum_f \mathcal{C}_f(x, \frac{Q^2}{\mu^2}, \sqrt{s}) \otimes f_i(x, \mu^2) + \mathcal{O}\left[\frac{1}{Q^2}\right]$$

Collision energy

Single hadron Matrix elements

Perturbative coefficients

Power corrections

✧ **PDFs are UV and IR finite, but, absorb perturbative CO divergence!**

✧ **With a large momentum transfer, PDFs completely cover all leading power CO divergence of single hadron matrix elements**

Our observation

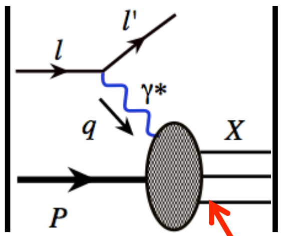
❑ QCD factorization of single-hadron cross section:

Momentum transfer

CO Factorization

PDFs

$$\sigma^{\text{DIS}}(x, Q^2; \sqrt{s}) \propto \left| \quad \right|^2 \approx \sum_f \mathcal{C}_f(x, \frac{Q^2}{\mu^2}, \sqrt{s}) \otimes f_i(x, \mu^2) + \mathcal{O}\left[\frac{1}{Q^2}\right]$$



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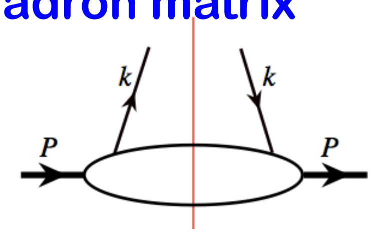
❑ Collinear divergences are from the region when $k_T \rightarrow 0$:

Leading power perturbative CO divergences of single hadron matrix elements are logarithmic, $\propto \int dk_T^2/k_T^2$, and

are the same for both Minkowski and Euclidean time



Our ideas

❑ **Lattice QCD can calculate "single" hadron matrix elements:**

$$\langle 0|\, \mathcal{O}(\overline{\psi},\psi,A)\, |0\rangle = \frac{1}{Z}\int \mathcal{D}A\mathcal{D}\overline{\psi}\mathcal{D}\psi\; e^{iS(\overline{\psi},\psi,A)}\mathcal{O}(\overline{\psi},\psi,A)$$

$$\sum_{P'}|P'\rangle\langle P'| \quad \sum_{P}|P\rangle\langle P| \quad \Longrightarrow \quad \langle P_z|\mathcal{O}(\overline{\psi},\psi,A)|P_z\rangle$$

With an Euclidean time

Ma and Qiu,
arXiv:1404.6860
1412.2688

✧ **Operators made of conserved currents – Physical – No need for UVCT**

Need a large scale, μ^2, e.g., the offshellness of the current(s)

✧ **Operators lead to perturbative UV divergence – Renormalizable!**

$$\widetilde{\sigma}(\tilde{x},P_z;\mu^2)_{\mathrm{E}} \equiv \;\mathrm{F.T.\,of}\;\langle P_z|\mathcal{O}(\overline{\psi},\psi,A)(\delta_z)|P_z\rangle + \mathrm{UVCT}(\mu^2)$$

quasi-PDFs

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❑ **Collinear factorization:**

Normal PDFs

$$\widetilde{\sigma}(\tilde{x},P_z;\mu^2)_{\mathrm{E}} = \Sigma_f \int_0^1 \frac{dx}{x}\, \mathcal{C}_f\left(\frac{\tilde{x}}{x},\frac{\bar{\mu}^2}{\mu^2},\alpha_s;P_z\right) f(x,\bar{\mu}^2) + \mathcal{O}\left[\frac{1}{\mu^\alpha}\right]$$

✧ Perturbatively, $\widetilde{\sigma}(\tilde{x},P_z;\mu^2)_{\mathrm{E}}$ and $f(x,\bar{\mu}^2)$ have the same CO divergence

✧ Matching coefficients, $\mathcal{C}_f$, are IR safe and perturbatively calculable

Extract PDFs from lattice "cross sections"

❑ Lattice "cross section":

$$\widetilde{\sigma}_{\mathrm{E}}^{\mathrm{Lat}}(\tilde{x}, 1/a, P_z) \propto \text{F.T. of } \langle P_z|\mathcal{O}(\overline{\psi}, \psi, A)|P_z\rangle + \mathrm{UVCT}(1/a)$$

- ✧ Its continuum limit is UV renormalizable
- ✧ It is calculable in lattice QCD with an Euclidean time, "E"
- ✧ It is infrared (IR) safe, calculated in lattice perturbation theory
- ✧ All CO divergences of its continuum limit ($a \to 0$) can be factorized into the normal PDFs with perturbatively calculable hard coefficients

"Collision energy" $P_z \sim$ "$\sqrt{s}$" *"rapidity"* $\tilde{x} \sim$ "y"

"Hard momentum transfer" $1/a \sim \tilde{\mu} \sim$ "Q"

❑ UV renormalization:

- ✧ No UVCT needed if $\mathcal{O}(\overline{\psi}, \psi, A)$ is made of conserved currents
- ✧ The quasi-PDFs are not made of conserved currents – UVCT needed

❑ CO Factorization – IR safe matching coefficients:

$$\widetilde{\sigma}_{\mathrm{E}}^{\mathrm{Lat}}(\tilde{x}, \frac{1}{a}, P_z) \approx \sum_i \int_0^1 \frac{dx}{x} f_{i/h}(x, \mu^2)\, \widetilde{\mathcal{C}}_i(\frac{\tilde{x}}{x}, \frac{1}{a}, \mu^2, P_z)$$

QCD Global analysis of lattice data

Differences between Ji's approach and ours

❑ For the quasi-PDFs:

✧ **Ji's approach – high P_z effective field theory:**

Ji, arXiv:1305.1539
1404.6680

$$\tilde{q}(x,\mu^2,P_z) = \int_x^1 \frac{dy}{y}\, Z\left(\frac{x}{y},\frac{\mu}{P_z}\right) q(y,\mu^2) + \mathcal{O}\left(\frac{\Lambda^2}{P_z^2},\frac{M^2}{P_z^2}\right)$$

✧ **Our approach – QCD collinear factorization:**

Ma and Qiu,
arXiv:1404.6860
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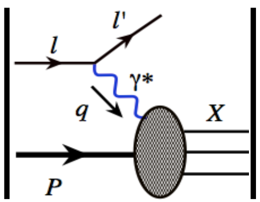
$$\tilde{q}(x,\mu^2,P_z) = \sum_f \int_0^1 \frac{dy}{y}\, \mathcal{C}_f\left(\frac{x}{y},\frac{\mu^2}{\bar{\mu}^2},P_z\right) f(y,\bar{\mu}^2) + \mathcal{O}\left(\frac{1}{\mu^2}\right)$$

Parameter like $\sqrt{s}$

Factorization scale

High twist
Power corrections

$\sigma^{\rm DIS}(x,Q^2;\sqrt{s}) \propto$



^{2} $$\approx \sum_f \mathcal{C}_f\left(x,\frac{Q^2}{\mu^2},\sqrt{s}\right) \otimes f_i(x,\mu^2) + \mathcal{O}\left[\frac{1}{Q^2}\right]$$

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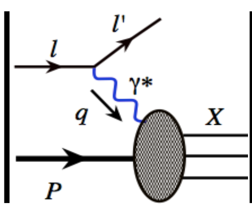
High twist Power corrections

$$\sigma^{\mathrm{DIS}}(x,Q^2;\sqrt{s}) \propto \Big|$$



$$\Big|^2 \approx \sum_f \mathcal{C}_f(x,\frac{Q^2}{\mu^2},\sqrt{s}) \otimes f_i(x,\mu^2) + \mathcal{O}\left[\frac{1}{Q^2}\right]$$

❑ Beyond quasi-PDFs:

Lattice "cross-sections" – *lattice calculable single hadron matrix elements*

UV and IR safe with a large momentum transfer, CO factorized into PDFs

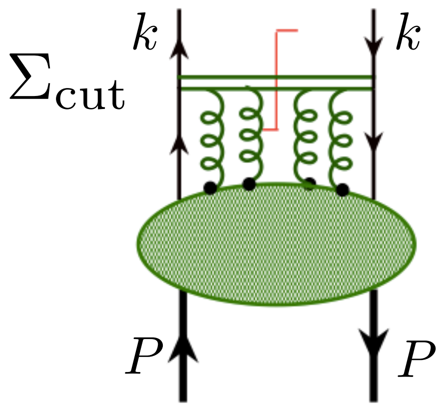
Case study – factorization of quasi-PDFs

Ma and Qiu,
arXiv:1404.6860
1412.2688

- **The "Quasi-quark" distribution, as an example:**

$$\tilde{q}(\tilde{x}, \tilde{\mu}^2, P_z) = \int \frac{dy_z}{4\pi} e^{i\tilde{x}P_z y_z} \langle P|\overline{\psi}(y_z)\gamma_z \exp\left\{-ig\int_0^{y_z} dy'_z A_z(y'_z)\right\}\psi(0)|P\rangle$$

- ✧ **Feynman diagram representation:** $\Phi_{n_z}^{(f,a)}(\{\xi_z, 0\}) = \Phi_{n_z}^{\dagger(f,a)}(\{\infty, \xi_z\})\,\Phi_{n_z}^{(f,a)}(\{\infty, 0\})$



$$\longleftarrow \frac{\gamma_z}{2P_z}\,\delta\left(\tilde{x} - \frac{k_z}{P_z}\right)$$

$P_z \sim \text{“}\sqrt{s}\text{”} \sim \tilde{\mu}/\tilde{x}$ ***Sufficiently large***

$P^2 \sim 0$

Case study – factorization of quasi-PDFs

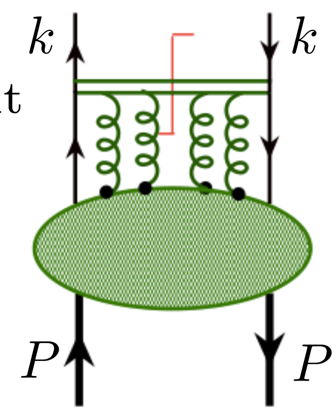
Ma and Qiu, arXiv:1404.6860
1412.2688

- **The "Quasi-quark" distribution, as an example:**

$$\tilde{q}(\tilde{x},\tilde{\mu}^2,P_z) = \int \frac{dy_z}{4\pi} e^{i\tilde{x}P_z y_z} \langle P|\overline{\psi}(y_z)\gamma_z \exp\left\{-ig\int_0^{y_z} dy_z' A_z(y_z')\right\}\psi(0)|P\rangle$$

✧ **Feynman diagram representation:** $\Phi_{n_z}^{(f,a)}(\{\xi_z,0\}) = \Phi_{n_z}^{\dagger(f,a)}(\{\infty,\xi_z\})\,\Phi_{n_z}^{(f,a)}(\{\infty,0\})$

$\Sigma_{\rm cut}$



$$\longleftarrow \frac{\gamma_z}{2P_z}\,\delta\left(\tilde{x}-\frac{k_z}{P_z}\right)$$

$$P_z \sim \text{“}\sqrt{s}\text{”} \sim \tilde{\mu}/\tilde{x}$$ ***Sufficiently large***

$$P^2 \sim 0$$

✧ **Like PDFs, it is IR finite**

✧ **Like PDFs, it is UV divergent, but, worse (linear UV divergence)**

Potential trouble! - mixing with the Log UV of PDFs?

✧ **Like PDFs, it is CO divergent – factorizes CO divergence into PDFs**

Show to all orders in perturbation theory

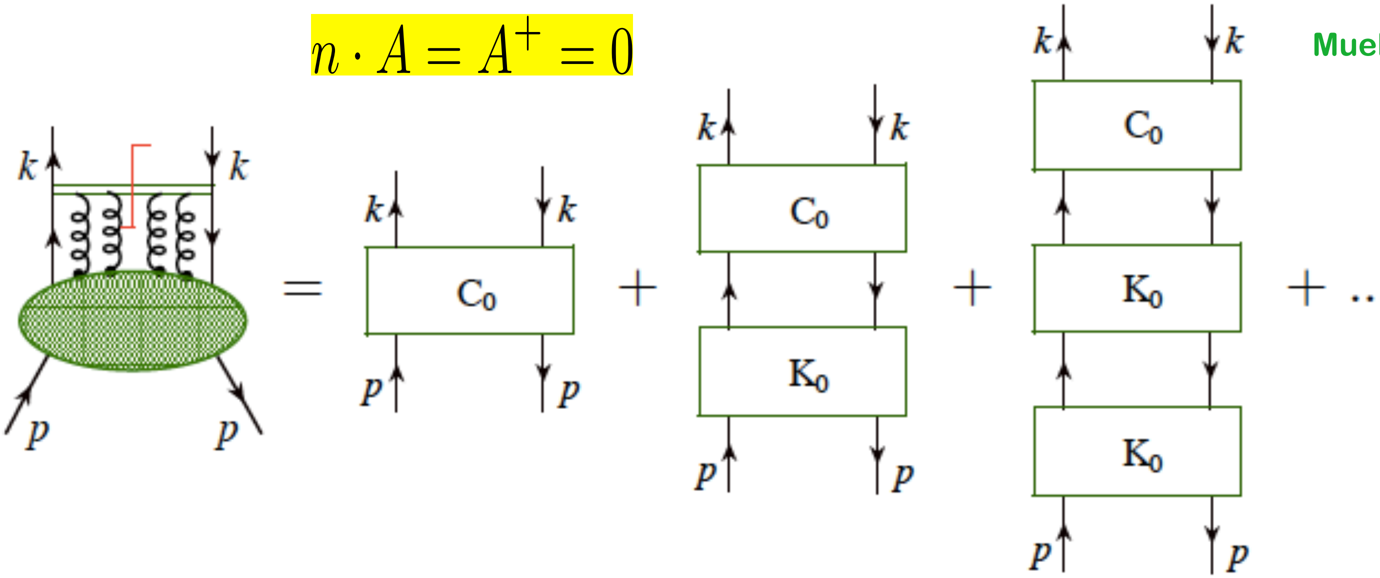
All order QCD factorization of CO divergence

Ma and Qiu, arXiv:1404.6860

- Generalized ladder decomposition in a physical gauge

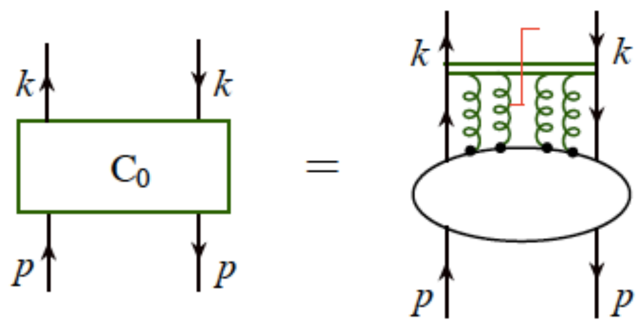
$n \cdot A = A^{+} = 0$

Mueller, PRD 1974



- $C_0, \;\; K_0$:2PI kernels

  ✧ Only process dependence:

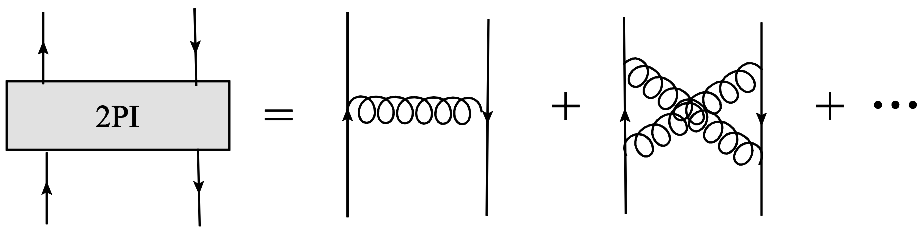


  ✧ 2PI are finite in a physical gauge for fixed *k* and *p*:

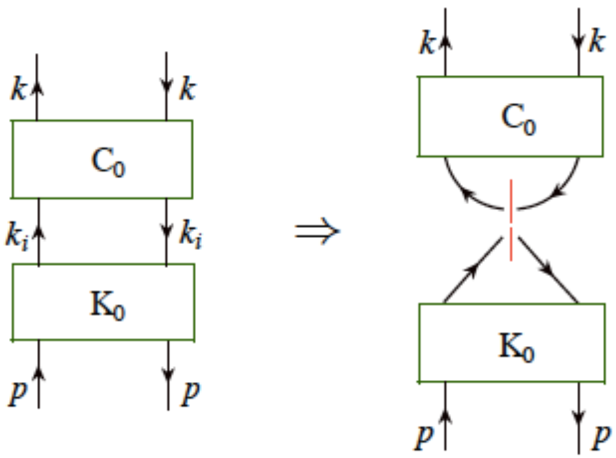
Ellis, Georgi, Machacek, Politzer, Ross, 1978, 1979

All order QCD factorization of CO divergence

- **2PI kernels – Diagrams:**



- **Ordering in virtuality:** $P^2 \ll k^2 \lesssim \tilde{\mu}^2$ **– Leading power in** $\dfrac{1}{\tilde{\mu}}$



$\leftarrow \frac{1}{2}\gamma \cdot p$

$\leftarrow \frac{\gamma \cdot n}{2p \cdot n}\, \delta\left(x_i - \frac{k_i \cdot n}{p \cdot n}\right)$ **+ power suppressed**

Cut-vertex for normal quark distribution
Logarithmic UV and CO divergence

- **Renormalized kernel - parton PDF:**

$$K \equiv \int d^4 k_i\, \delta\left(x_i - \frac{k^+}{p^+}\right) \mathrm{Tr}\left[\frac{\gamma \cdot n}{2p \cdot n}\, K_0\, \frac{\gamma \cdot p}{2}\right] + \mathrm{UVCT}_{\mathrm{Logarithmic}}$$

All order QCD factorization of CO divergence

- Projection operator for CO divergence:

$\widehat{\mathcal{P}}\, K$ Pick up the logarithmic CO divergence of K

- Factorization of CO divergence:

$$\tilde{f}_{q/p} = \lim_{m\to\infty} C_0 \sum_{i=0}^{m} K^i + \mathrm{UVCTs}$$

$$= \lim_{m\to\infty} C_0 \left[1 + \sum_{i=0}^{m-1} K^i (1-\widehat{\mathcal{P}}) K \right]_{\mathrm{ren}} + \tilde{f}_{q/p}\, \widehat{\mathcal{P}}\, K$$

$$= \lim_{m\to\infty} C_0 \left[1 + \sum_{i=1}^{m} \left[(1-\widehat{\mathcal{P}}) K \right]^i \right]_{\mathrm{ren}} + \tilde{f}_{q/p}\, \widehat{\mathcal{P}}\, K$$

$$\Rightarrow \quad \tilde{f}_{q/P} = \left[C_0 \frac{1}{1-(1-\widehat{\mathcal{P}})K} \right]_{\mathrm{ren}} \left[\frac{1}{1-\widehat{\mathcal{P}} K} \right]$$

Normal Quark distribution

CO divergence free

All CO divergence of quasi-quark distribution

All order QCD factorization of CO divergence

- Projection operator for CO divergence:

$\widehat{\mathcal{P}}\,K$ Pick up the logarithmic CO divergence of *K*

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Normal Quark distribution

CO divergence free

All CO divergence of quasi-quark distribution

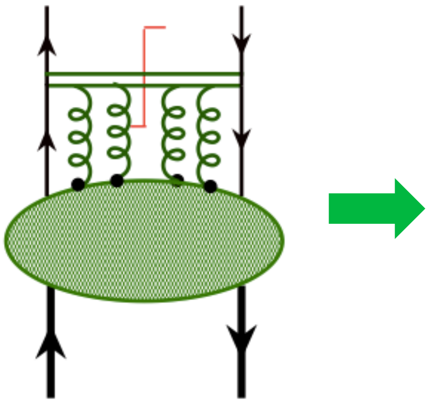
$$\Rightarrow \quad \tilde{\sigma}_{\mathrm{M}}(\tilde{x},\tilde{\mu}^2,P_z) \approx \sum_i \int_0^1 \frac{dx}{x}\, f_{i/h}(x,\mu^2)\, \mathcal{C}_i\!\left(\frac{\tilde{x}}{x},\tilde{\mu}^2,\mu^2,P_z\right)$$

UV finite?

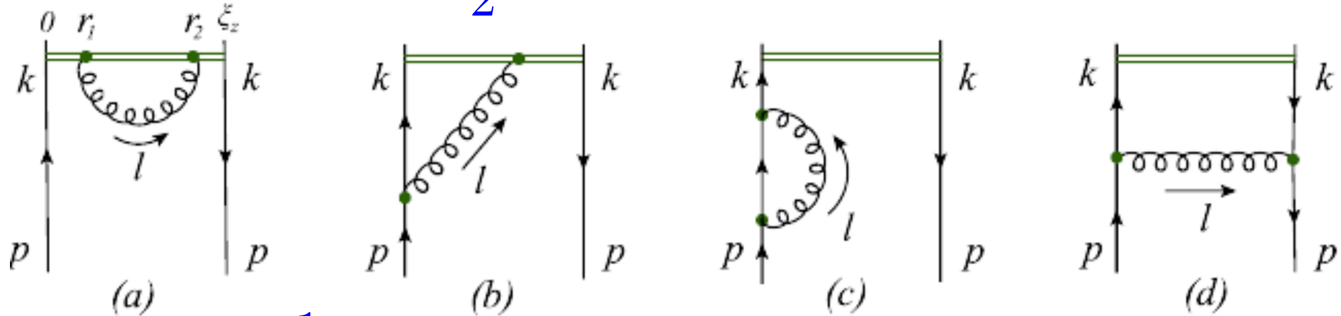
UV renormalization

Ma and Qiu, arXiv:1404.6860, …

- **UV divergences (difference in gauge link):**



$$n_z^2 = -1 \qquad \frac{1}{2}\gamma^z$$



$$\frac{1}{2}\gamma\cdot p \propto \left[\gamma^0 - \gamma^z\right]$$

- **Renormalization:**

$$\left[C_0 \frac{1}{1-(1-\hat{\mathcal{P}})K}\right]_{\rm ren} \equiv C_0 \frac{1}{1-(1-\hat{\mathcal{P}})K} + \text{UVCTs}$$

In coordinate space:
ξ_z
Independence!

✧ **Power divergence:** **Diagram (a) – independent of ξ_z**

Removed by "mass" renormalization of a test particle – the gauge link

✧ **Left-over log divergence:** Dotsenko and Vergeles NPB, (1980)

Dimensional regularization – ξ_z independence of $1/\varepsilon$ – finite CTs

✧ **Log(ξ_z) – term:** **Artifact of dimensional regularization**

One-loop example: quark →quark

Ma and Qiu, arXiv:1404.6860

❑ Expand the factorization formula:

$$\tilde{f}^{(1)}_{q/q}(\tilde{x}) = f^{(0)}_{q/q}(x) \otimes \mathcal{C}^{(1)}_{q/q}(\tilde{x}/x) + f^{(1)}_{q/q}(x) \otimes \mathcal{C}^{(0)}_{q/q}(\tilde{x}/x)$$

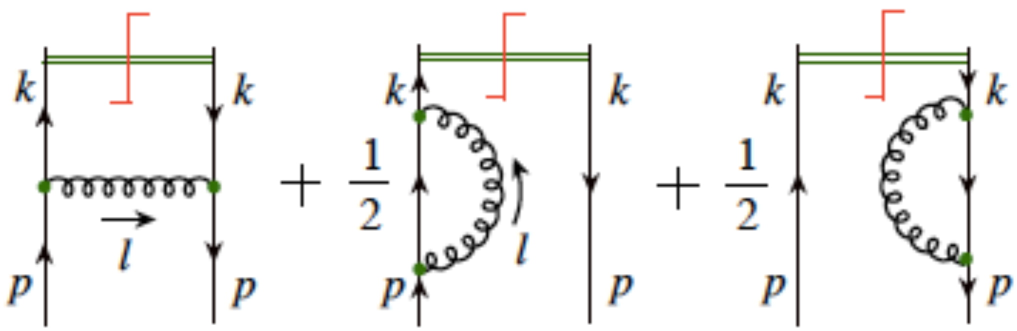
$$\Rightarrow \quad \mathcal{C}^{(1)}_{q/q}(t, \tilde{\mu}^2, \mu^2, P_z) = \tilde{f}^{(1)}_{q/q}(t, \tilde{\mu}^2, P_z) - f^{(1)}_{q/q}(t, \mu^2)$$

❑ Feynman diagrams:

Same diagrams for both

$\tilde{f}_{q/q}$ **and** $f_{q/q}$

But, in different gauge



❑ Gauge choice:

$n_z \cdot A = 0$ for $\tilde{f}_{q/q}$ $\qquad$ $n \cdot A = 0$ for $f_{q/q}$

Gluon propagator:

$$\tilde{d}^{\alpha\beta}(l) = -g^{\alpha\beta} + \frac{l^\alpha n_z^\beta + n_z^\alpha l^\beta}{l_z} - \frac{n_z^2\, l^\alpha l^\beta}{l_z^2} \qquad \textbf{with} \quad n_z^2 = -1$$

One-loop "quasi-quark" distribution in a quark

Ma and Qiu, arXiv:1404.6860

❑ Real + virtual contribution:

$$\tilde{f}^{(1)}_{q/q}(\tilde{x},\tilde{\mu}^2,P_z) = C_F \frac{\alpha_s}{2\pi}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\int_0^{\tilde{\mu}^2}\frac{dl_\perp^2}{l_\perp^{2+2\epsilon}}\int_{-\infty}^{+\infty}\frac{dl_z}{P_z}\left[\delta(1-\tilde{x}-y)-\delta(1-\tilde{x})\right]\left\{\frac{1}{y}\left(1-y+\frac{1-\epsilon}{2}y^2\right)\right.$$

$$\left.\times\left[\frac{y}{\sqrt{\lambda^2+y^2}}+\frac{1-y}{\sqrt{\lambda^2+(1-y)^2}}\right]+\frac{(1-y)\lambda^2}{2y^2\sqrt{\lambda^2+y^2}}+\frac{\lambda^2}{2y\sqrt{\lambda^2+(1-y)^2}}+\frac{1-\epsilon}{2}\frac{(1-y)\lambda^2}{[\lambda^2+(1-y)^2]^{3/2}}\right\}$$

where $\quad y = l_z/P_z,\ \lambda^2 = l_\perp^2/P_z^2,\ C_F = (N_c^2-1)/(2N_c)$

❑ Cancelation of CO divergence:

$$\frac{y}{\sqrt{\lambda^2+y^2}}+\frac{1-y}{\sqrt{\lambda^2+(1-y)^2}} = 2\theta(0<y<1)-\left[\mathrm{Sgn}(y)\frac{\sqrt{\lambda^2+y^2}-|y|}{\sqrt{\lambda^2+y^2}}+\mathrm{Sgn}(1-y)\frac{\sqrt{\lambda^2+(1-y)^2}-|1-y|}{\sqrt{\lambda^2+(1-y)^2}}\right]$$

Only the first term is CO divergent for $0 < y < 1$, which is the same as the divergence of the normal quark distribution – necessary!

One-loop "quasi-quark" distribution in a quark

Ma and Qiu, arXiv:1404.6860

❑ Real + virtual contribution:

$$\tilde{f}^{(1)}_{q/q}(\tilde{x},\tilde{\mu}^2,P_z) = C_F\frac{\alpha_s}{2\pi}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\int_0^{\tilde{\mu}^2}\frac{dl_\perp^2}{l_\perp^{2+2\epsilon}}\int_{-\infty}^{+\infty}\frac{dl_z}{P_z}\left[\delta(1-\tilde{x}-y)-\delta(1-\tilde{x})\right]\left\{\frac{1}{y}\left(1-y+\frac{1-\epsilon}{2}y^2\right)\right.$$
$$\left.\times\left[\frac{y}{\sqrt{\lambda^2+y^2}}+\frac{1-y}{\sqrt{\lambda^2+(1-y)^2}}\right]+\frac{(1-y)\lambda^2}{2y^2\sqrt{\lambda^2+y^2}}+\frac{\lambda^2}{2y\sqrt{\lambda^2+(1-y)^2}}+\frac{1-\epsilon}{2}\frac{(1-y)\lambda^2}{[\lambda^2+(1-y)^2]^{3/2}}\right\}$$

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Only the first term is CO divergent for $0 < y < 1$, which is the same as the divergence of the normal quark distribution – necessary!

❑ UV renormalization:

Different treatment for the upper limit of $l_\perp^2$ integration - "scheme"

Here, a UV cutoff is used – other scheme is discussed in the paper

One-loop coefficient functions

Ma and Qiu, arXiv:1404.6860

- $\overline{\text{MS}}$ scheme for $f_{q/q}(x,\mu^2)$:

$$\mathcal{C}^{(1)}_{q/q}(t,\tilde{\mu}^2,\mu^2,P_z) = \tilde{f}^{(1)}_{q/q}(t,\tilde{\mu}^2,P_z) - f^{(1)}_{q/q}(t,\mu^2)$$

$$\frac{\mathcal{C}^{(1)}_{q/q}(t)}{C_F\frac{\alpha_s}{2\pi}} = \left[\frac{1+t^2}{1-t}\ln\frac{\tilde{\mu}^2}{\mu^2}+1-t\right]_+ + \left[\frac{t\Lambda_{1-t}}{(1-t)^2}+\frac{\Lambda_t}{1-t}+\frac{\text{Sgn}(t)\Lambda_t}{\Lambda_t+|t|} - \frac{1+t^2}{1-t}\left[\text{Sgn}(t)\ln\left(1+\frac{\Lambda_t}{2|t|}\right)+\text{Sgn}(1-t)\ln\left(1+\frac{\Lambda_{1-t}}{2|1-t|}\right)\right]\right]_N$$

where $\Lambda_t = \sqrt{\tilde{\mu}^2/P_z^2+t^2}-|t|$, $\text{Sgn}(t)=1$ if $t\geq 0$, and -1 otherwise.

- Generalized "+" description: $t=\tilde{x}/x$

$$\int_{-\infty}^{+\infty} dt\left[g(t)\right]_N h(t) = \int_{-\infty}^{+\infty} dt\, g(t)\,[h(t)-h(1)]$$

For a testing function $h(t)$

- Explicit verification of the factorization at one-loop:

Coefficient functions for all partonic channels are IR safe and finite!

$$\mathcal{C}^{(1)}_{i/j}(t,\tilde{\mu}^2,\mu,P_z) \quad \text{with } i,j=q,\bar{q},g$$

From Lattice quasi-PDFs to PDFs

- **BNL – RBRC efforts:**

One-loop Perturbative Matching

$$\tilde{f}_{i/h}^{\text{Latt.}}(\tilde{x}, \frac{1}{a}, P_z)_{\text{E}} \longleftrightarrow \tilde{f}_{j/h}^{\text{Cont.}}(\tilde{x}, \mu^2, P_z)_{\text{E}}$$

Tomomi's talk

Same

One-loop Perturbative Matching

$$f_{i/h}(x, \bar{\mu}^2)_{\text{M}} \longleftrightarrow \tilde{f}_{j/h}^{\text{Cont.}}(\tilde{x}, \mu^2, P_z)_{\text{M}}$$

This talk

From Lattice quasi-PDFs to PDFs

❑ BNL – RBRC efforts:

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
\tilde{f}^{\,\mathrm{Latt.}}_{i/h}\left(\tilde{x}, \frac{1}{a}, P_z\right)_{\mathrm{E}} \arrow[r, leftrightarrow, "\text{One-loop Perturbative Matching}", "\text{Tomomi's talk}"'] \arrow[d, leftrightarrow, "\text{Direct Matching in progress}"'] & \tilde{f}^{\,\mathrm{Cont.}}_{j/h}(\tilde{x}, \mu^2, P_z)_{\mathrm{E}} \arrow[d, leftrightarrow, "\text{Same}"] \\
f_{i/h}(x, \bar{\mu}^2)_{\mathrm{M}} \arrow[r, leftrightarrow, "\text{One-loop Perturbative Matching}", "\text{This talk}"'] & \tilde{f}^{\,\mathrm{Cont.}}_{j/h}(\tilde{x}, \mu^2, P_z)_{\mathrm{M}}
\end{tikzcd}
$$

❑ To do list:

- ✧ Matching with more realistic lattice fermion (no principle difficulty)
- ✧ Lattice numerical simulations of the quasi-PDFs
- ✧ First physics project: d(x)/u(x) at large x

Summary and outlook

- "lattice cross sections" = single hadron matrix elements calculable in Lattice QCD and factorizable in QCD

  Key difference from Ji's idea:

  Expansion in $1/\mu$ instead of that in $1/P_z$

- Extract PDFs by global analysis of data on "Lattice cross sections". Same should work for other distributions

$$\widetilde{\sigma}_{\mathrm{E}}^{\mathrm{Lat}}(\tilde{x}, \frac{1}{a}, P_z) \approx \sum_i \int_0^1 \frac{dx}{x} f_{i/h}(x, \mu^2)\, \widetilde{C}_i(\frac{\tilde{x}}{x}, \frac{1}{a}, \mu^2, P_z)$$

- Conservation of difficulties – complementarity:

  High energy scattering experiments

  – less sensitive to large x parton distribution/correlation

  "Lattice factorizable cross sections"

  – more suited for large x PDFs, and more: PDFs of meson?

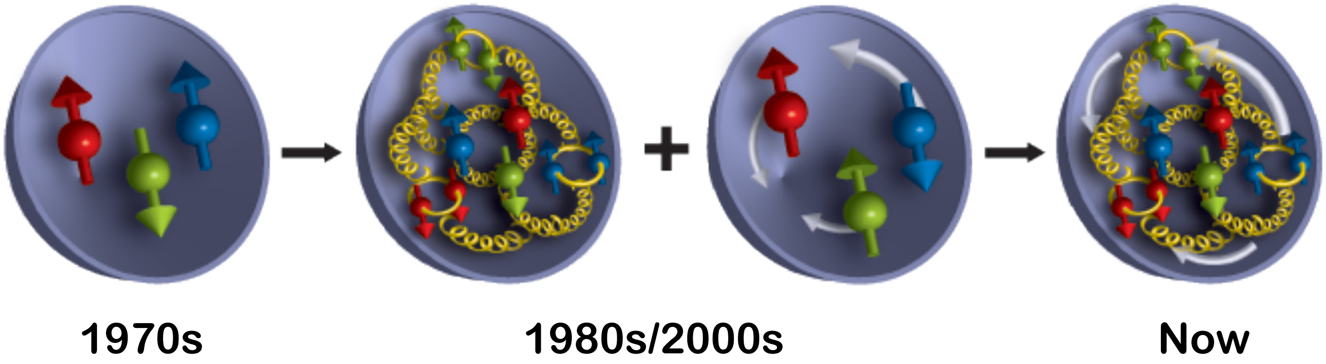
- Lattice QCD can calculate PDFs, but, more works are needed!

Thank you!

BACKUP SLIDES

Nucleon's internal structure

- ❑ Our understanding of the nucleon evolves



Nucleon is a strongly interacting, relativistic bound state of quarks and gluons

- ❑ QCD bound states:
  - ✧ Neither quarks nor gluons appear in isolation!
  - ✧ Understanding such systems completely is still beyond the capability of the best minds in the world

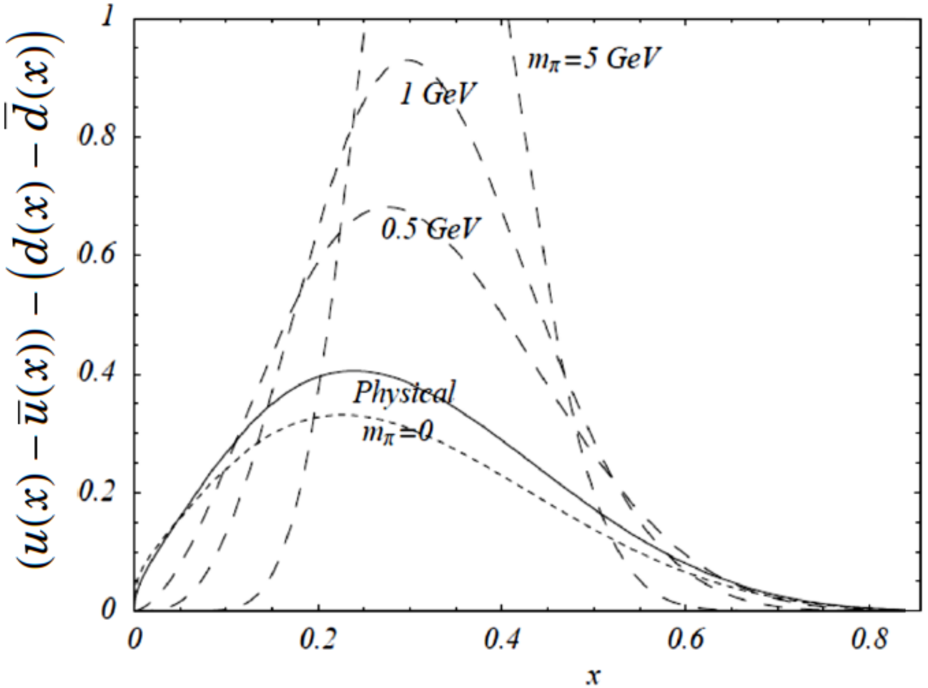
- ❑ The great intellectual challenge:

  Probe nucleon structure without "seeing" quarks and gluons?

PDFs from lattice QCD

- **How to get x-dependent PDFs with a limited moments?**
  - ✧ **Assume a smooth functional form with some parameters**
  - ✧ **Fix the parameters with the lattice calculated moments**

$$xq(x) = a\,x^b(1-x)^c(1+\epsilon\sqrt{x}+\gamma\,x)$$



W. Dermold et al., Eur.Phys.J.direct C3 (2001) 1-15

Cannot distinguish valence quark contribution from sea quarks

"Quasi-PDFs" have no parton interpretation

❑ **Normal PDFs conserve parton momentum:**

$$M = \sum_q \left[\int_0^1 dx\, x f_q(x) + \int_0^1 dx\, x f_{\bar{q}}(x) \right] + \int_0^1 dx\, x f_g(x)$$
$$= \sum_q \int_{-\infty}^{\infty} dx\, x f_q(x) + \frac{1}{2} \int_{-\infty}^{\infty} dx\, x f_g(x)$$
$$= \frac{1}{2(P^+)^2} \langle P | T^{++}(0) | P \rangle = \text{constant}$$

$T^{\mu\nu}$

Energy-momentum tensor

❑ **"Quasi-PDFs" do not conserve "parton" momentum:**

$$\widetilde{\mathcal{M}} = \sum_q \left[\int_0^\infty d\tilde{x}\, \tilde{x} \tilde{f}_q(\tilde{x}) + \int_0^\infty d\tilde{x}\, \tilde{x} \tilde{f}_{\bar{q}}(\tilde{x}) \right] + \int_0^\infty d\tilde{x}\, \tilde{x} \tilde{f}_g(\tilde{x})$$
$$= \sum_q \int_{-\infty}^{\infty} d\tilde{x}\, \tilde{x} \tilde{f}_q(\tilde{x}) + \frac{1}{2} \int_{-\infty}^{\infty} d\tilde{x}\, \tilde{x} \tilde{f}_g(\tilde{x})$$
$$= \frac{1}{2(P_z)^2} \langle P | \left[T^{zz}(0) - g^{zz}(\ldots) \right] | P \rangle \neq \text{constant}$$

Note: "Quasi-PDFs" are not boost invariant

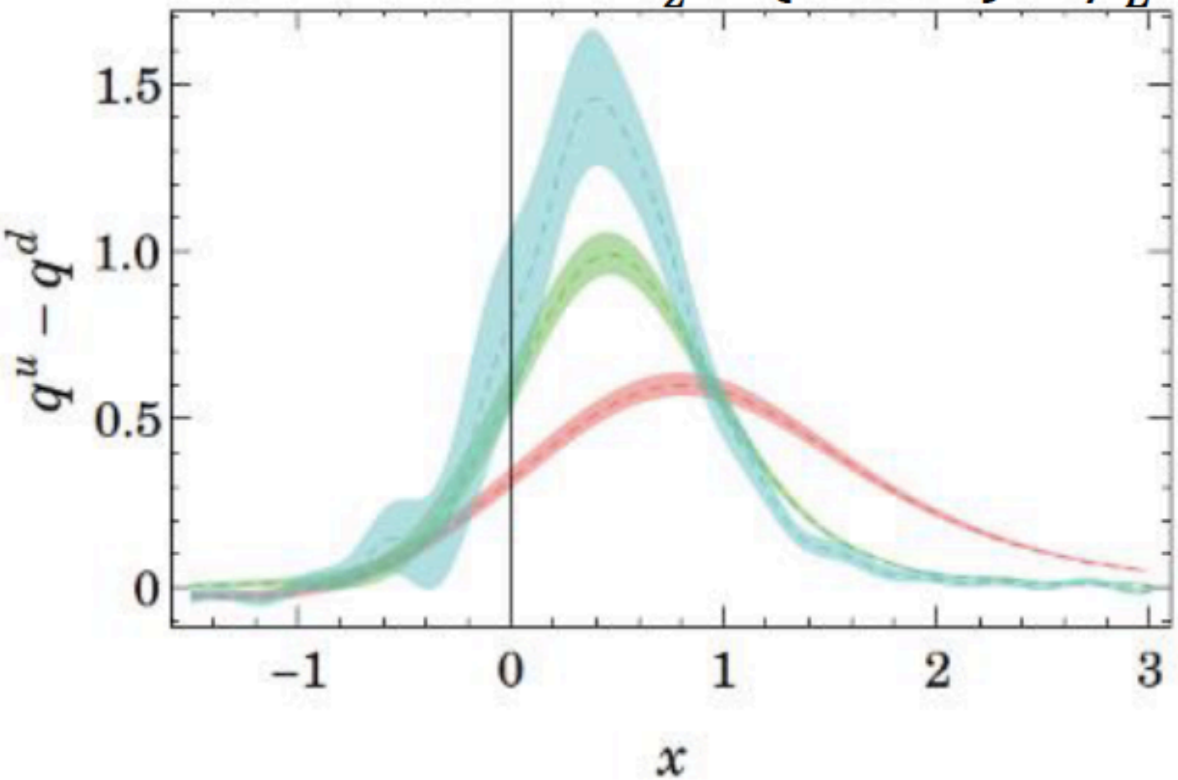
The first try

Lin *et al.*, arXiv:1402.1462

§ Exploratory study

$$\int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \middle| \bar{\psi}(z)\, \gamma_z \exp\!\left(-ig \int_0^z dz'\, A_z(z')\right) \psi(0) \middle| P \right\rangle$$

$$P_z \in \{1, 2, 3\}\, {}^{2\pi}/_{L}$$



Distribution gets sharper as P_z increases

Artifacts due to finite P_z on the lattice

Improvement?

Work out leading-P_z corrections