Perturbative matching for quasi-PDFs between continuum and lattice

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normal-PDFs v.s. quasi-PDFs

normal-PDFs

$$q(x,\mu) = \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle \mathcal{N}(P)|O(\xi^-)|\mathcal{N}(P)\rangle,$$
$$O(\xi^-) = \overline{\psi}(\xi^-)\gamma^+ U_+(\xi^-,0)\psi(0)$$

- $\xi^\pm = (t \pm z)/\sqrt{2}$: light-cone coordinate
- Time-dependent. $\Longrightarrow$ It cannot be calculated on the lattice directly.

quasi-PDFs [Ji (2013)]

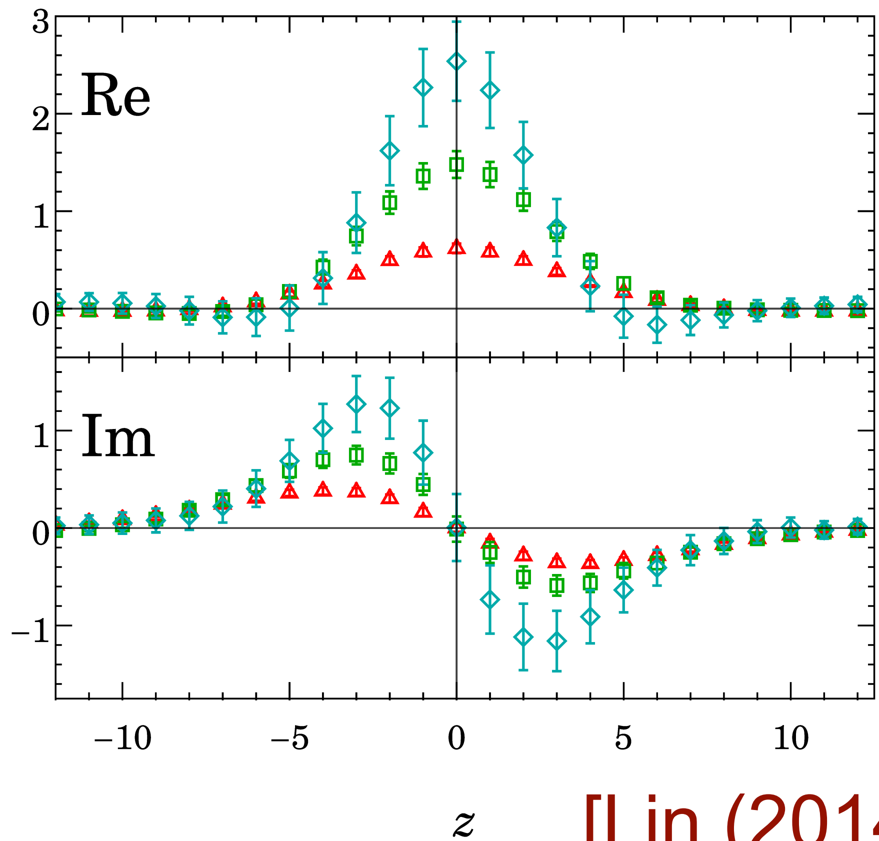
$$\widetilde{q}(\tilde{x},\mu,P_z) = \int \frac{d\delta z}{2\pi} e^{-i\tilde{x}P_z\delta z} \langle \mathcal{N}(P_z)|\widetilde{O}(\delta z)|\mathcal{N}(P_z)\rangle,$$
$$\widetilde{O}(\delta z) = \overline{\psi}(\delta z)\gamma^z U_z(\delta z,0)\psi(0)$$

- P_z may not be infinite.
- Time-independent. It is computable on the lattice.

Lattice quasi-PDFs, so far

- Non-local matrix element

$$\langle \mathcal{N}(P_z)|O(\delta z)|\mathcal{N}(P_z)\rangle$$



[Lin (2014)]

FIG. 1. The real (top) and imaginary (bottom) parts of the nonlocal isovector matrix element h of Eq. 3 computed on a lattice with the nucleon momentum P_z (in units of $2\pi/L$) = 1 (red triangles), 2 (green squares), 3 (cyan diamonds).

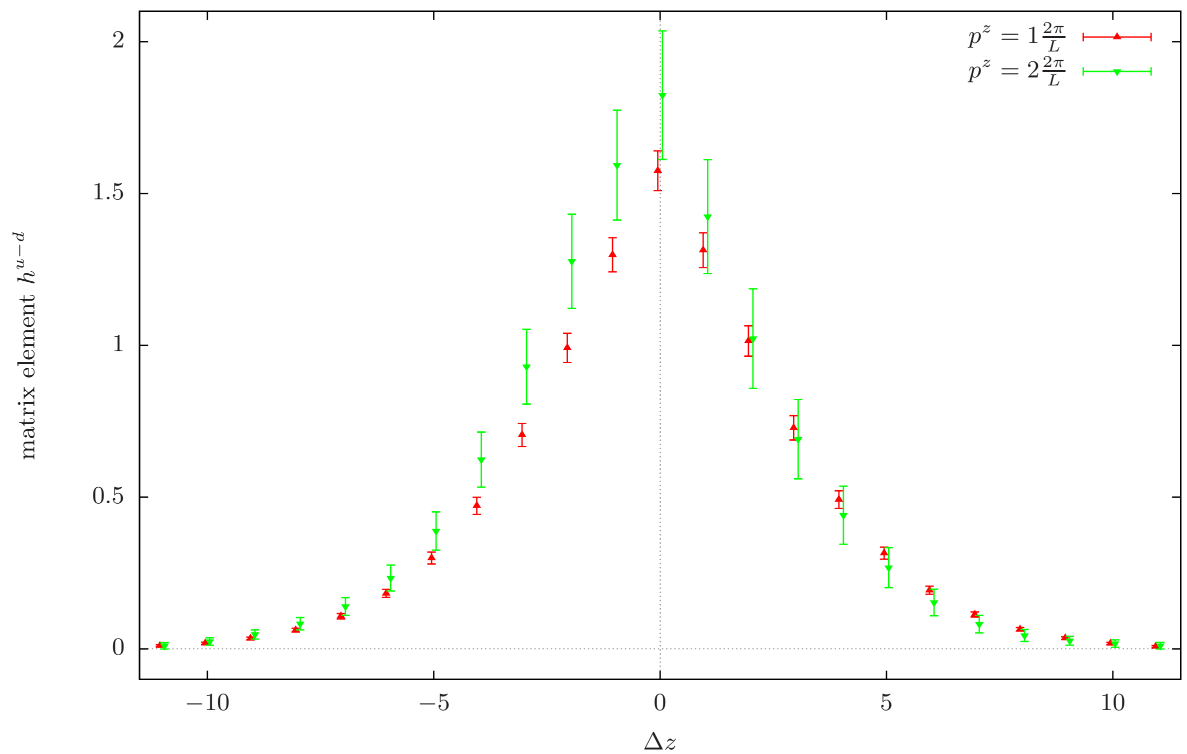
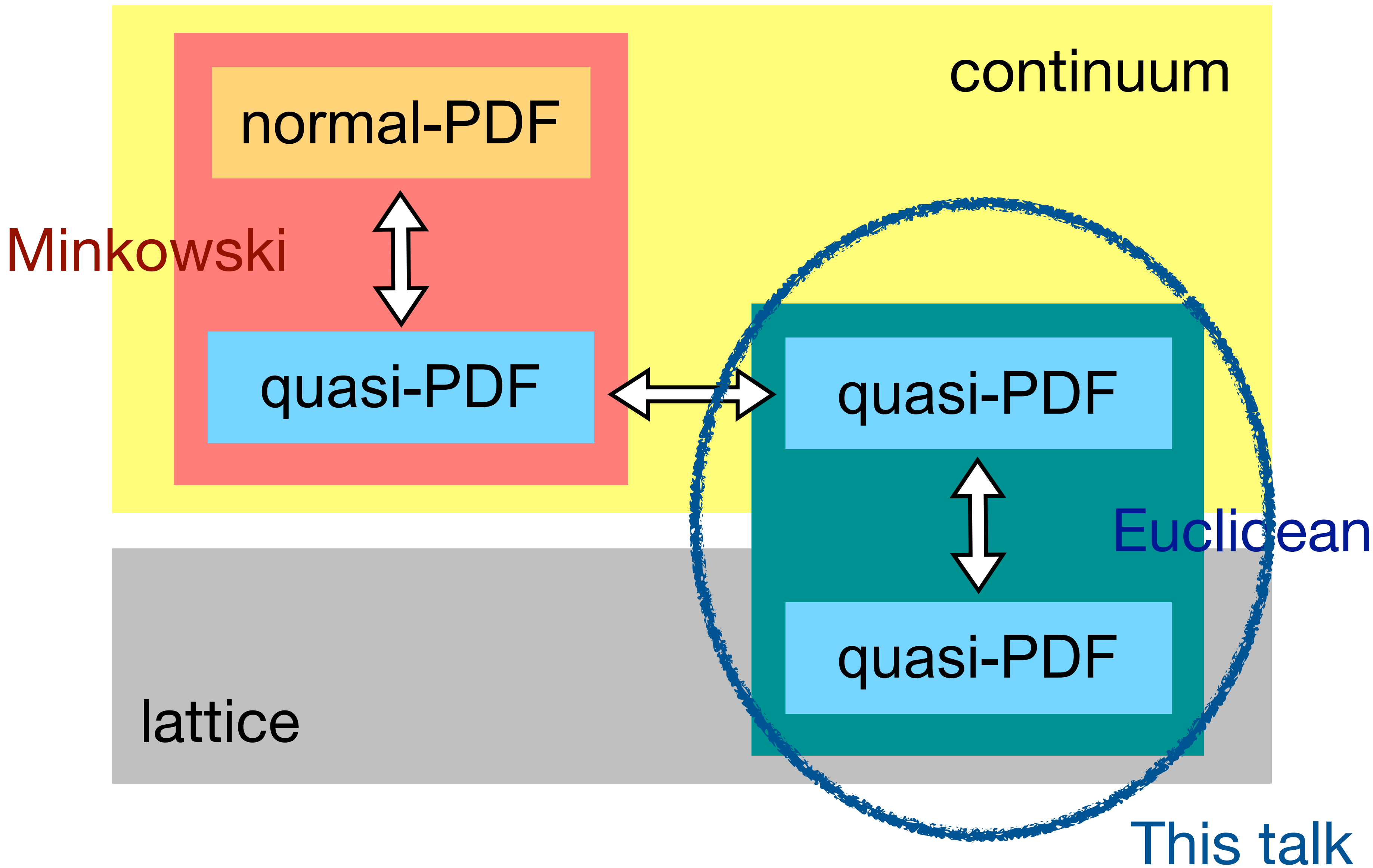


Figure 4: Real part of the matrix element for the first two momenta with 1000 measurements.

[Wiese (2014)]

Matching between continuum and lattice has not been implemented.

Matching overview



- Matching in continuum Minkowski space has been done.

[Ji (2013), Xiong et. al. (2013), Ma and Qiu (2014)]

- Minkowski and Euclidean space should be equivalent in quasi-PDF.

Momentum space v.s. Coordinate space

$$\tilde{q}(\tilde{x},\mu,P_z) = \int \frac{d\delta z}{2\pi} e^{-i\tilde{x}P_z\delta z} \langle \mathcal{N}(P_z)|\tilde{O}(\delta z)|\mathcal{N}(P_z)\rangle,$$
$$\tilde{O}(\delta z) = \overline{\psi}(\delta z)\gamma^z U_z(\delta z,0)\psi(0)$$

- ## Matching in momentum space

$$\boxed{\tilde{q}_{\rm cont}(\tilde{x},\mu,P_z) \quad \Longleftrightarrow \quad \tilde{q}_{\rm latt}(\tilde{x},a^{-1},P_z)}$$

- z-component of the momentum is restricted to be P_z .
- Loop-momentum becomes 3-dimensional.

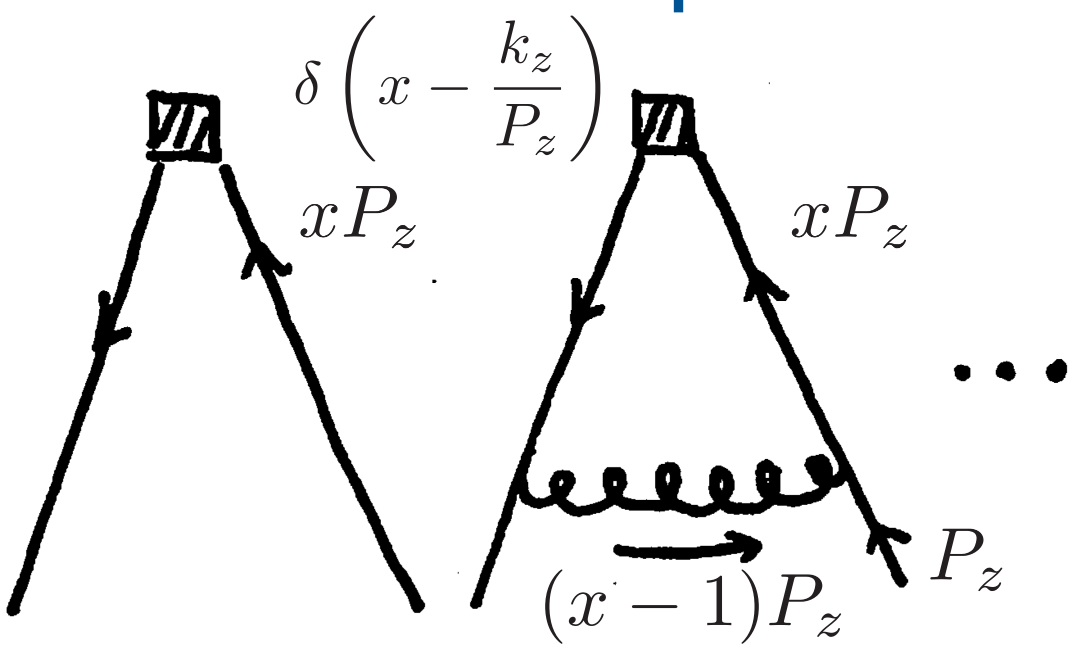
- ## Matching in coordinate space

$$\boxed{\tilde{O}_{\rm cont}(\delta z) \quad \Longleftrightarrow \quad \tilde{O}_{\rm latt}(\delta z)}$$

- There is no restriction on momenta.

Momentum space v.s. Coordinate space

▸ momentum space

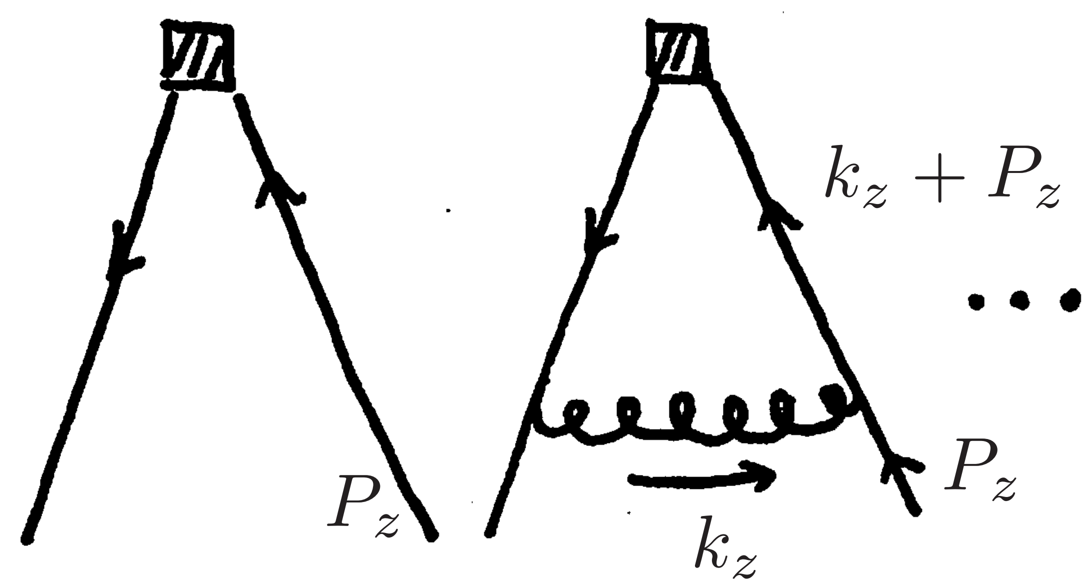


- z-component of the momentum is restricted to be xP_z .

- Loop-momentum becomes 3-dimensional.

Shinsuke Yoshida is working on this.

▸ coordinate space



- No restriction on momentum.

- Loop-momentum is 4-dimensional.

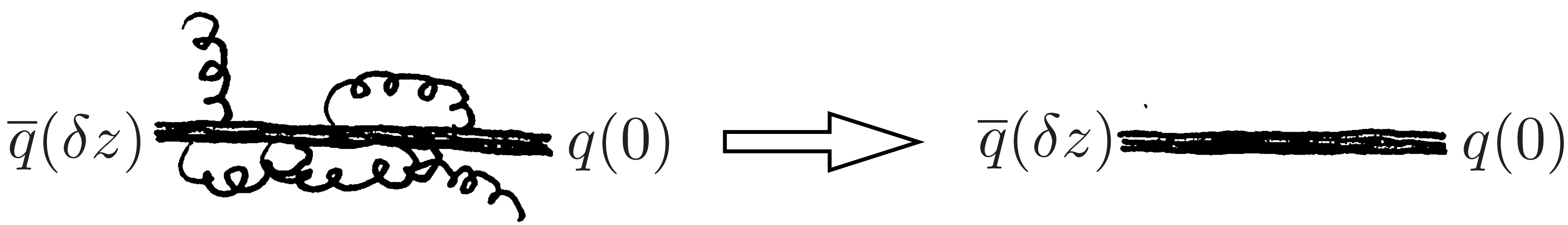
This talk

Covariant gauge v.s. Axial gauge

$$\widetilde{O}(\delta z) = \overline{\psi}(\delta z)\gamma^z U_z(\delta z, 0)\psi(0)$$

▸ Axial gauge $A_z(x) = 0$

- It looks convenient, because $U_z(\delta z, 0) = 1$



- No free lunch, because gluon propagators introduce complication.

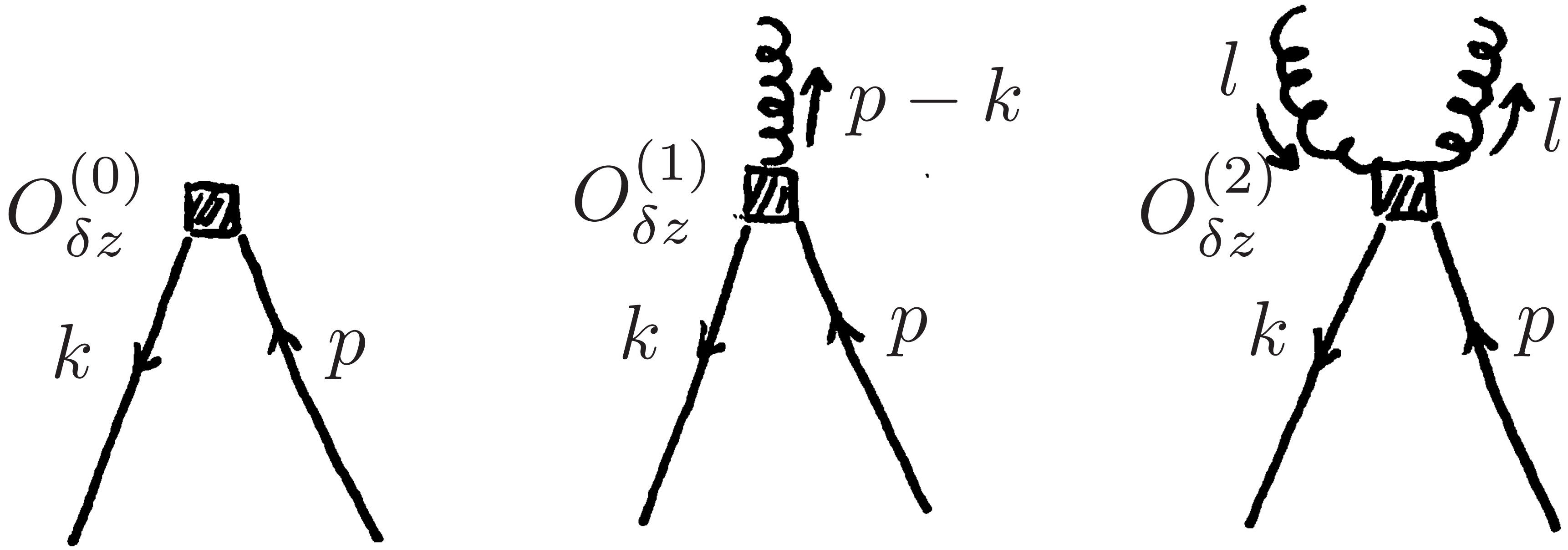
$$G_{\mu\nu}(k) = \frac{1}{k^2}\left(\delta_{\mu\nu} - \frac{\delta_{\mu,z}k_\nu + k_\mu\delta_{\nu,z}}{k_z} + \frac{k_\mu k_\nu}{k_z^2}\right).$$

- Spurious pole exists. Pole prescription is required in many cases. $k_z \longrightarrow 0$

Feynman rules in covariant gauge

$$\widetilde{O}(\delta z) = \overline{\psi}(\delta z)\gamma^z U_z(\delta z, 0)\psi(0)$$

- Tree, one-gluon, two-gluon (at one-loop level)



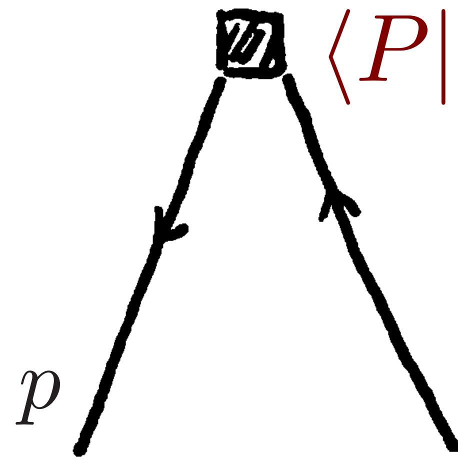
$$O^{(0)}_{\delta z}(p,k) = \gamma_z \delta(p-k) e^{-ip_z\delta z}$$

$$O^{(1)}_{\delta z}(p,k) = ig\gamma_z \frac{e^{-ip_z\delta z} - e^{-ik_z\delta z}}{i(p-k)_z}$$

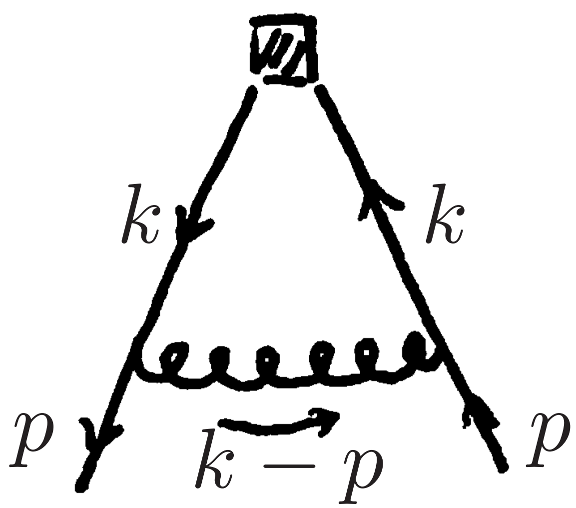
$$O^{(2)}_{\delta z}(p,k,l) = -g^2\gamma_z \delta(p-k) e^{-ip_z\delta z}\left(\frac{1-e^{il_z\delta z}}{l_z^2} - \frac{\delta z}{il_z}\right)$$

There is no pole.

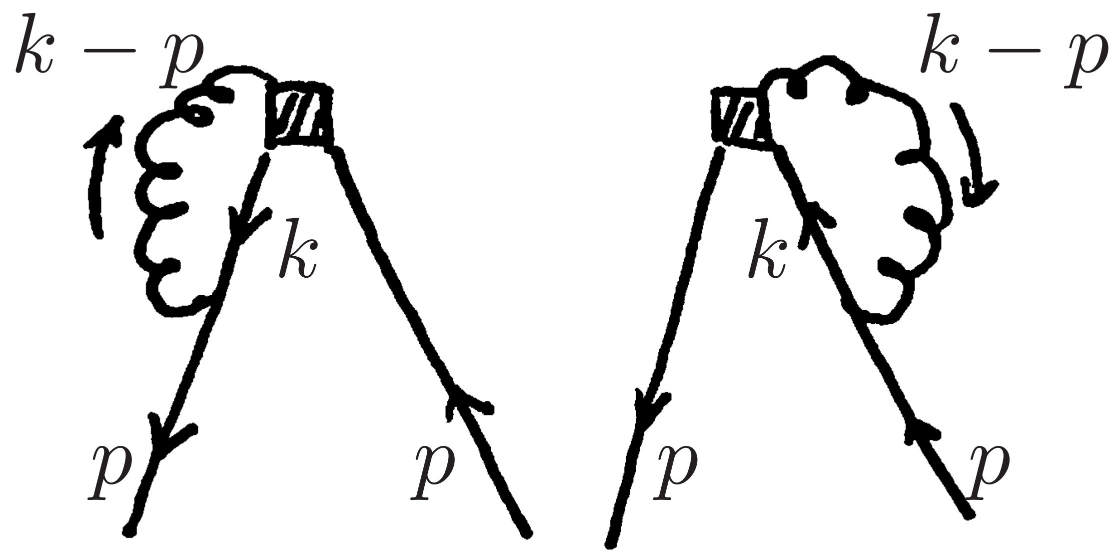
Diagrams at 1-loop



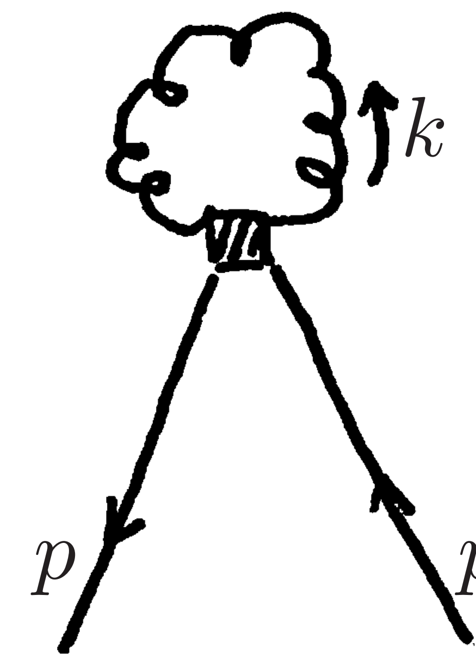
$\langle P|O(\delta z)|P\rangle_0 = e^{-iP_z\delta z}$



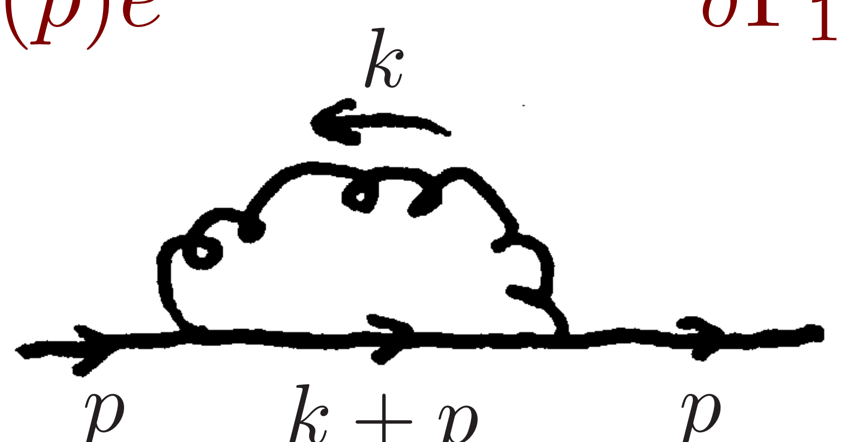
$\delta\Gamma_0(p)e^{-iP_z\delta z}$



$\delta\Gamma_1(p)e^{-iP_z\delta z}$



$\delta\Gamma_2 e^{-iP_z\delta z}$



$\Sigma_{\mathrm{RS}}(p)$

$\Sigma_{\mathrm{TP}}(p)$ (only in lattice)

Momentum dependent v.s. independent

$$\langle P|\widetilde{O}(\delta z)|P\rangle_{\rm cont} = Z(\delta z, P)\langle P|\widetilde{O}(\delta z)|P\rangle_{\rm latt}$$

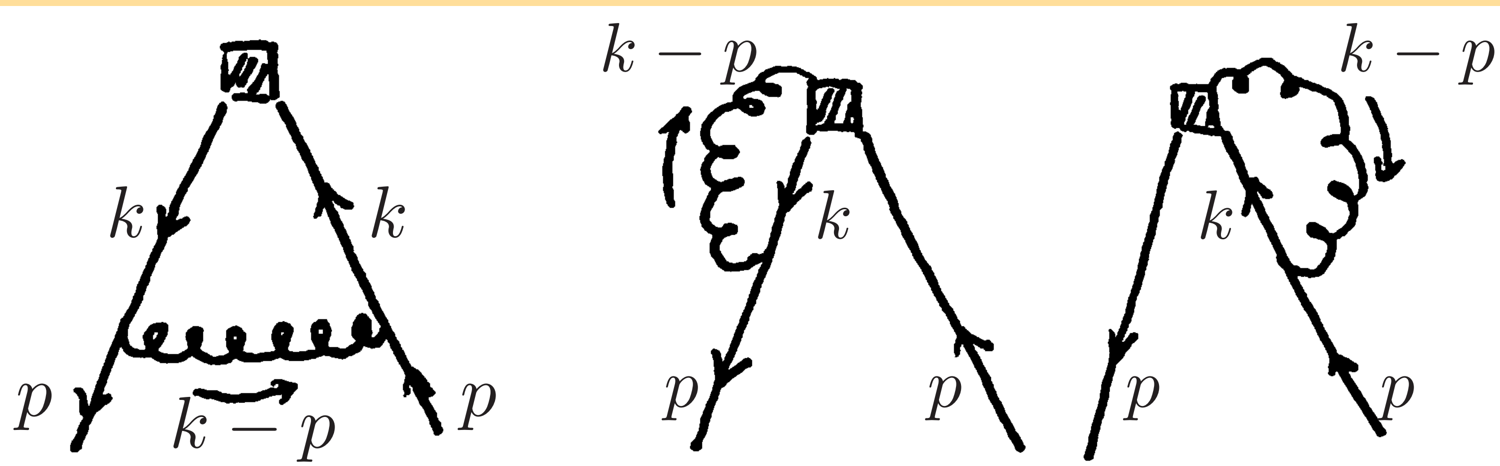
?

Momentum dependence

- The difference of momentum dependence between continuum and lattice is related to UV-divergences in loop integral. $\int dk f(k,p)$

When the loop-integral involves UV-log divergence at most,

$$\int_{-\pi/a}^{\pi/a} dk \underbrace{\left[f^{\rm latt}(k,p+\Delta p) - f^{\rm latt}(k,p)\right]}_{\rm no\ UV-divergence} \xrightarrow[a\to 0]{} \int_{-\infty}^{\infty} dk \left[f^{\rm cont}(k,p+\Delta p) - f^{\rm cont}(k,p)\right]$$



Common momentum dependence between continuum and lattice.

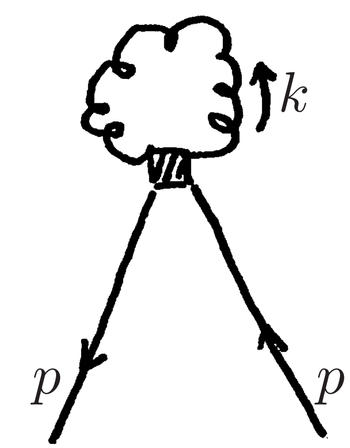
Momentum dependent v.s. independent

Momentum dependence

- $\delta\Gamma_2$ has UV-linear divergence, but external momentum is not involved in the loop integral.

continuum

$$\delta\Gamma_2 = -g^2 C_F \int_k \frac{1}{k^2}\left(\frac{1-e^{ik_z\delta z}}{k_z^2} - \frac{\delta z}{ik_z}\right)$$



same as on the lattice

Common momentum dependence between continuum and lattice.

$\delta\Gamma_0(p=0)$ $\delta\Gamma_1(p=0)$

tree-level

$e^{-iP_z\delta z}$

$$\frac{\langle P|\widetilde{O}(\delta z)|P\rangle_{\mathrm{cont}}}{\langle P|\widetilde{O}(\delta z)|P\rangle_0} = 1 + g^2\mathcal{A}_{\mathrm{cont}}(\delta z) + g^2\mathcal{B}(\delta z, P)$$

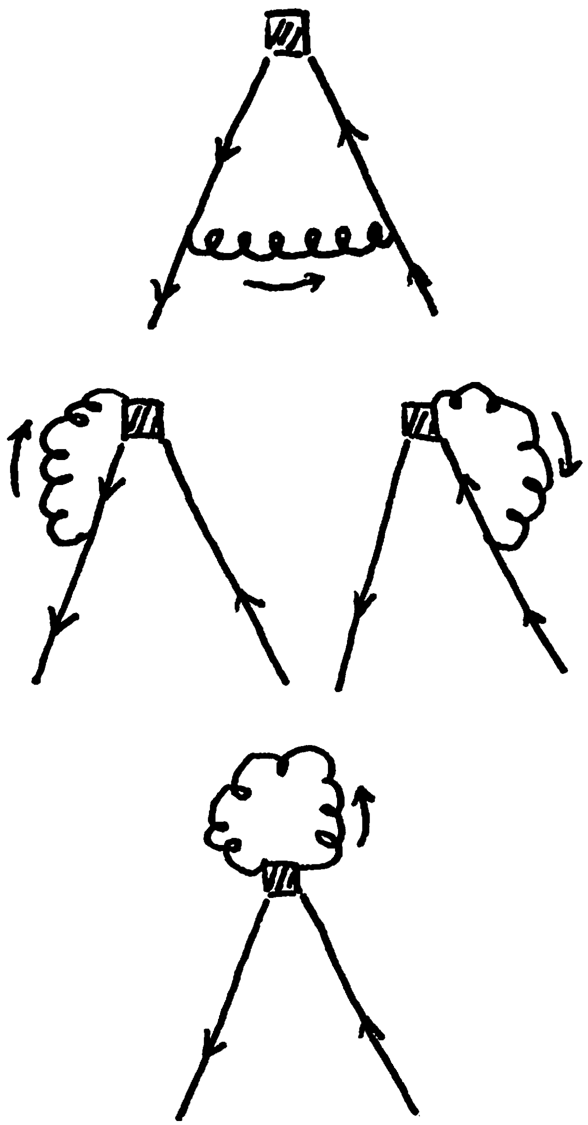
$$\frac{\langle P|\widetilde{O}(\delta z)|P\rangle_{\mathrm{latt}}}{\langle P|\widetilde{O}(\delta z)|P\rangle_0} = 1 + g^2\mathcal{A}_{\mathrm{latt}}(\delta z) + g^2\mathcal{B}(\delta z, P)$$

common.
vanished
in the matching.

$$\boxed{\langle P|\widetilde{O}(\delta z)|P\rangle_{\mathrm{cont}} = Z(\delta z, \cancel{P})\langle P|\widetilde{O}(\delta z)|P\rangle_{\mathrm{latt}}}$$

1-loop in continuum

Divergence structure (P=0)

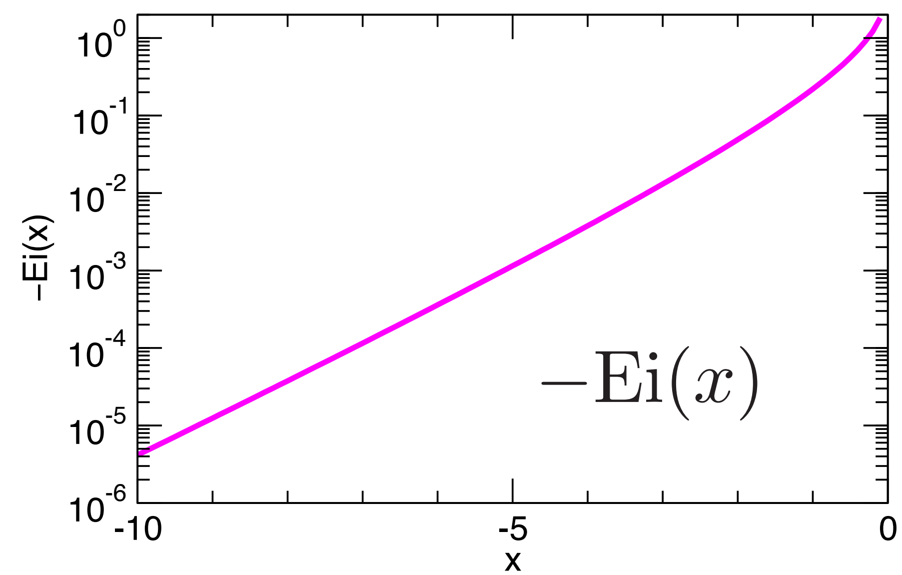


$$\delta\Gamma_0 = \frac{1}{8\pi^2}\left(\mathrm{Ei}(-k_\perp) - (2+k_\perp)e^{-k_\perp}\right)\Big|_{\lambda|\delta z|}^{\mu|\delta z|} \xrightarrow[\delta z \to 0]{} \frac{1}{8\pi^2}\ln\frac{\mu}{\lambda}$$

$$\delta\Gamma_1 = \frac{1}{4\pi^2}\left(\ln(k_\perp) - \mathrm{Ei}(-k_\perp) + e^{-k_\perp}\right)\Big|_{\lambda|\delta z|}^{\mu|\delta z|} \xrightarrow[\delta z \to 0]{} 0$$

$$\delta\Gamma_2 = \frac{1}{4\pi^2}\left(\ln(k_\perp) - \mathrm{Ei}(-k_\perp) - k_\perp\right)\Big|_{\lambda|\delta z|}^{\mu|\delta z|} \xrightarrow[\delta z \to 0]{} 0$$

Linear divergence



$$\mathrm{Ei}(x) = -\int_{-x}^{\infty} dt\,\frac{e^{-t}}{t}$$

: exponential integral

- Local case ($\delta z \to 0$) can be safely reproduced.
- Linear divergence from the tad-pole like diagram.
- UV(μ) and IR(λ) regulators are introduced in $\perp = (t, x, y)$ direction.

Back to the Axial gauge

- 1-loop correction

$$\delta\Gamma + \left.\frac{\partial\Sigma(p)}{\partial \not{p}}\right|_{p=0} = +g^2 C_F \int_k \frac{1}{k^4}\left(1 - \frac{4k_z^2}{k^2}e^{-ik_z\delta z}\right) - g^2 C_F \int_k \frac{1}{k^2}\left(\frac{1-e^{ik_z\delta z}}{k_z^2} - \frac{\delta z}{ik_z}\right) + g^2 C_F \int_k \frac{1}{k^2}\frac{\delta z}{ik_z}$$

same as Feynman gauge　　　　extra part

- The extra term includes a spurious pole.
- The spurious pole needs a prescription to be dealt with:

$$\int_k \frac{1}{k^2}\frac{1}{k_z} = \int_{k_\perp}\frac{1}{k_\perp^2}\int_{k_z}\frac{1}{k_z}.$$

$$\boxed{\frac{1}{k_z} \longrightarrow \frac{1}{2}\left(\frac{1}{k_z - i\epsilon} + \frac{1}{k_z + i\epsilon}\right)}$$

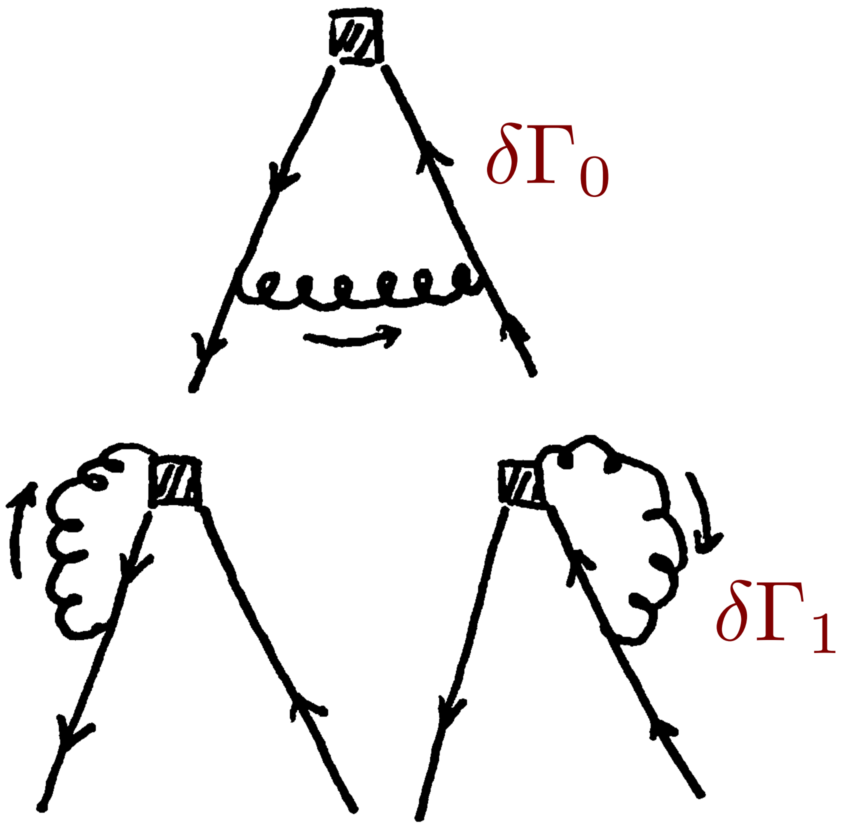
- Do not use axial gauge to avoid the pole prescription ambiguity.

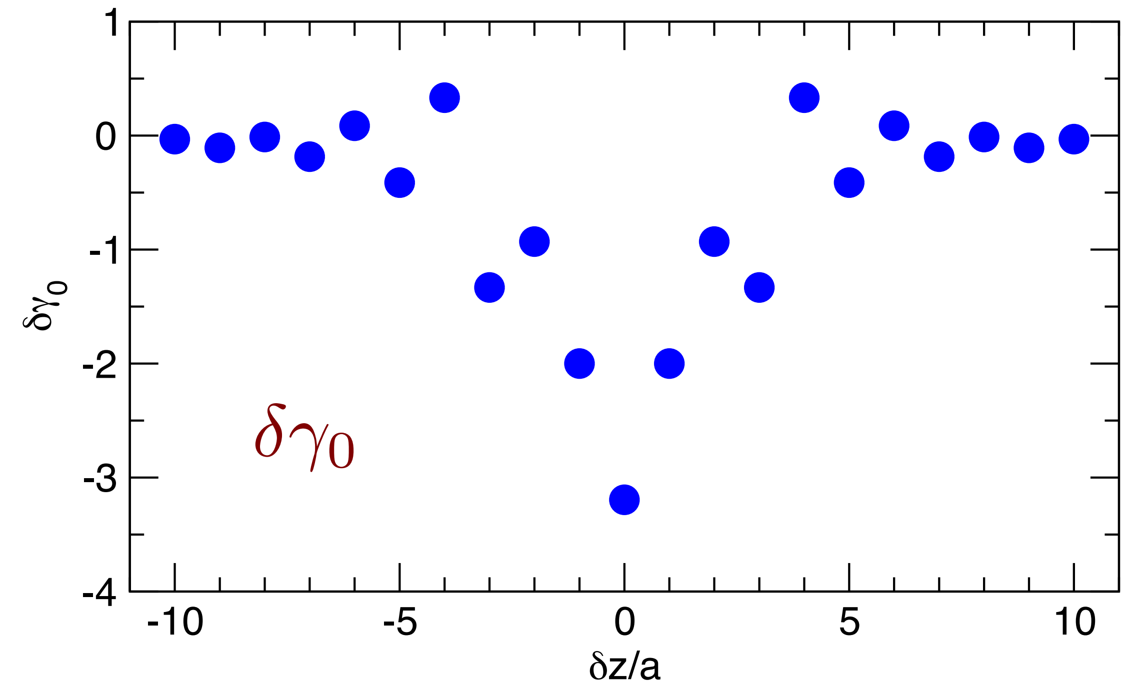
1-loop matching

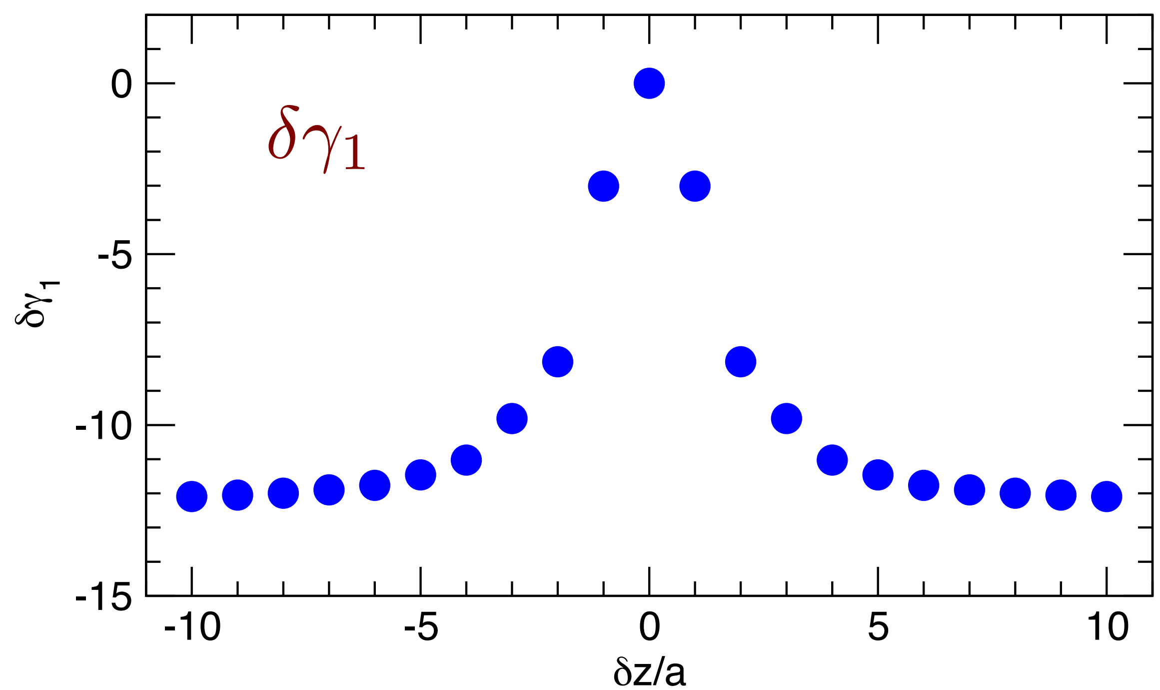
1-loop matching coefficients

- UV cut-off is set to be $\mu = a^{-1}$.
- Naive fermion is used.
(not practical, but OK.)

$$\delta\Gamma_{\rm cont} - \delta\Gamma_{\rm latt} \equiv \frac{g^2}{16\pi^2} C_F \gamma_z \delta\gamma$$



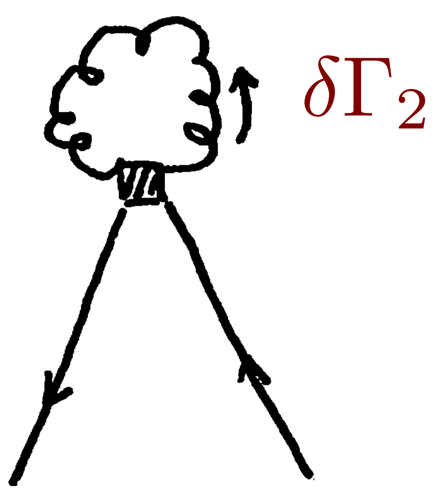


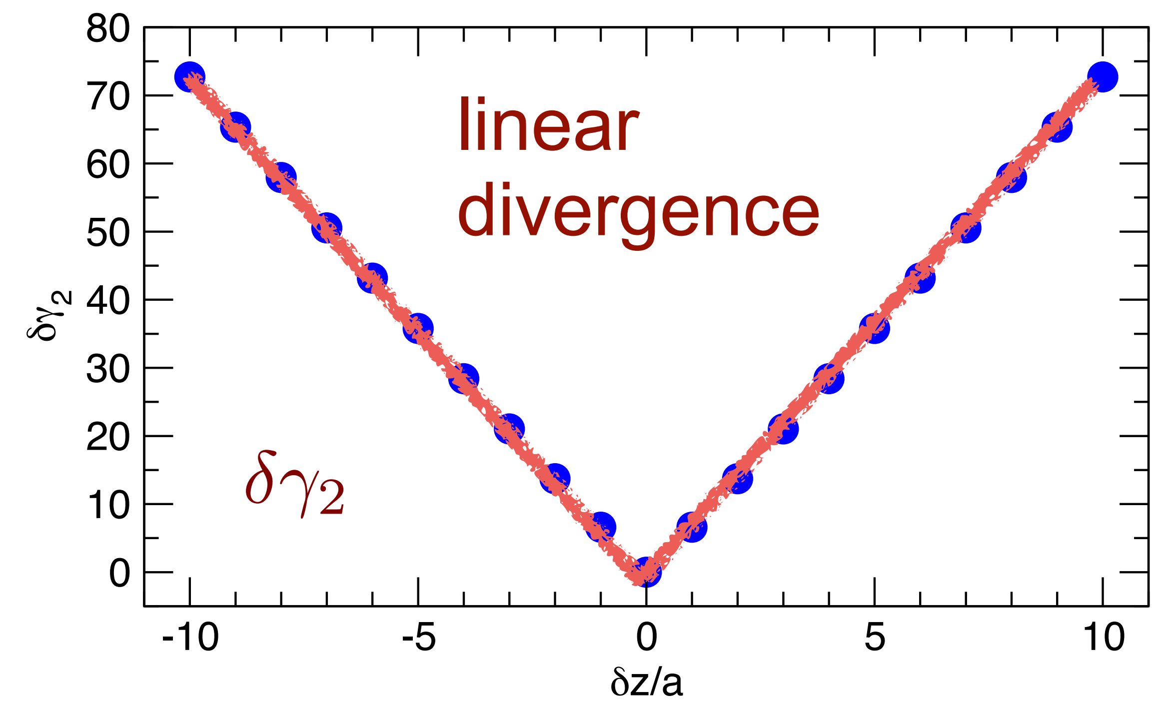


1-loop matching

- 1-loop matching coefficients

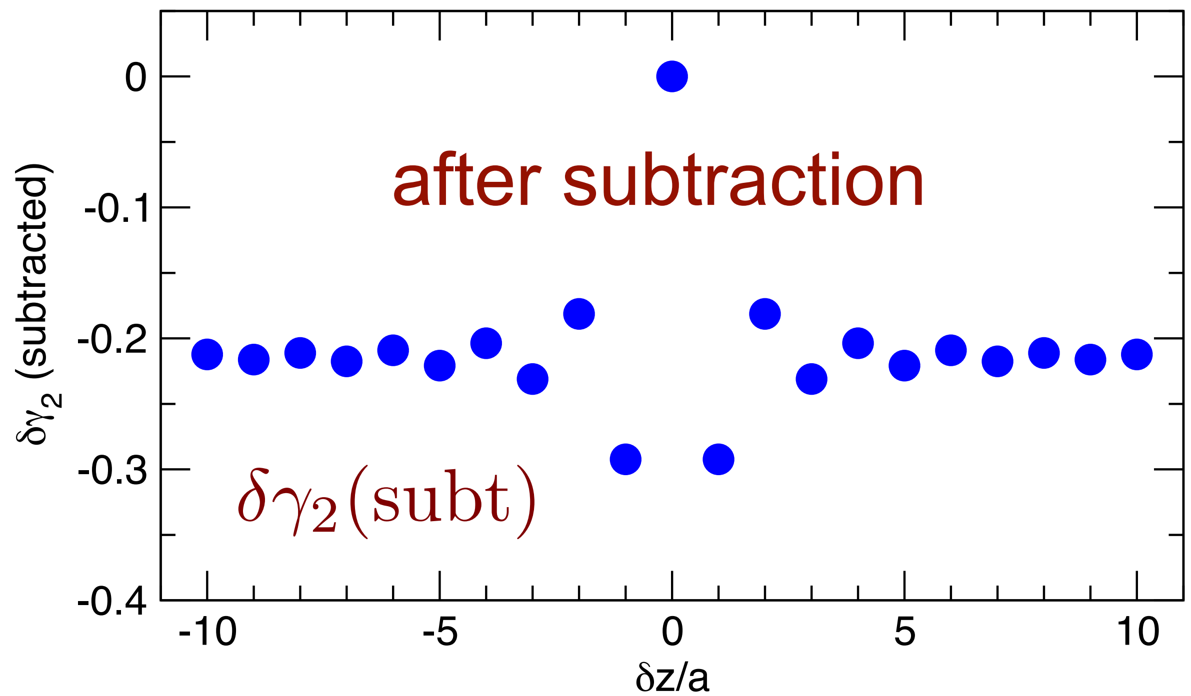
$$\delta\Gamma_{\rm cont} - \delta\Gamma_{\rm latt} \equiv \frac{g^2}{16\pi^2} C_F \gamma_z \delta\gamma$$





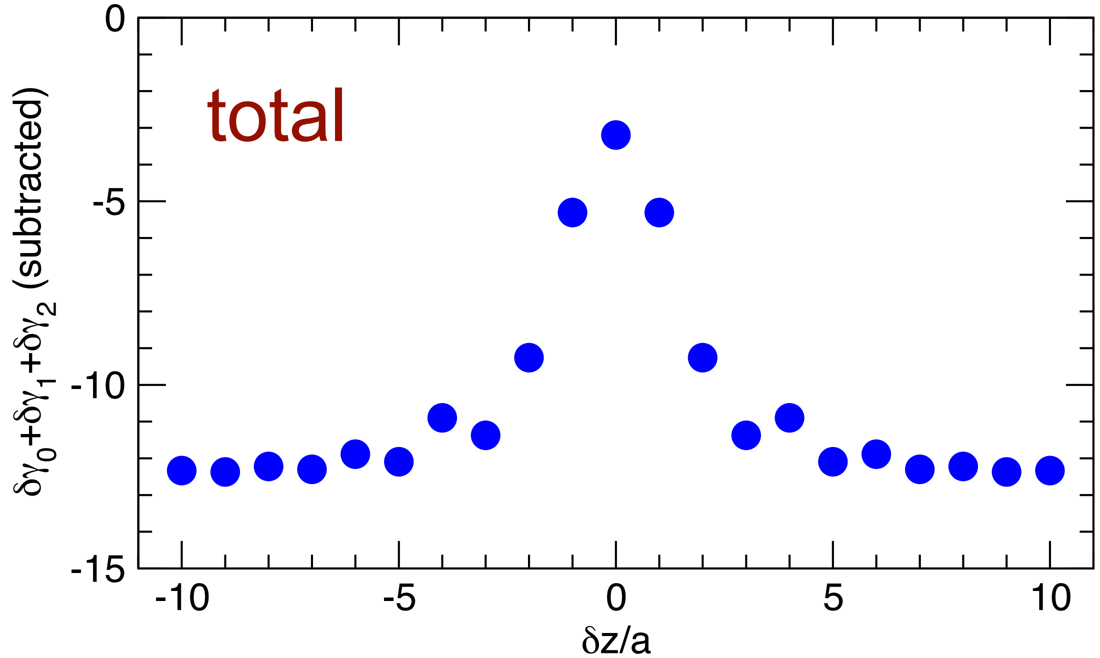
- There is a mismatch in linear divergence between continuum and lattice.

- The linear divergence should be subtracted, otherwise the continuum limit cannot be taken.



1-loop matching

1-loop matching coefficients



$$\delta\Gamma_{\rm cont} - \delta\Gamma_{\rm latt} \equiv \frac{g^2}{16\pi^2} C_F \gamma_z \delta\gamma$$

Wave function part is not included.
(It is the same as usual local operator case.)

Comments

- MF-improvement should be used in the actual matching factor.
- Other lattice actions and link smearings can be easily implemented.
- In the Large Momentum Effective Theory (Ji's context), non-perturbative subtraction of the linear divergence would be required, once $O(1/P_z^2)$ correction is included.
(Mixing with lower dimensional operators cannot be treated perturbatively.)

Summary and outlook

- 1-loop perturbative matching factor of quasi-PDFs between continuum and lattice is discussed.
- Matching method in coordinate space is applied in this talk.
- When axial gauge is used, there is a prescription ambiguity to deal with a spurious pole.
- External momentum dependence is common between continuum and lattice, which results in momentum independent matching factor.
- Linear divergent behavior can be seen. This linear divergent should be subtracted, otherwise continuum limit cannot be taken.
- We are preparing numerical simulations of the quasi-PDFs.