Structure of Charmed Tetraquarks from LQCD

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HAL QCD (Hadrons to Atomic nuclei from Lattice QCD)

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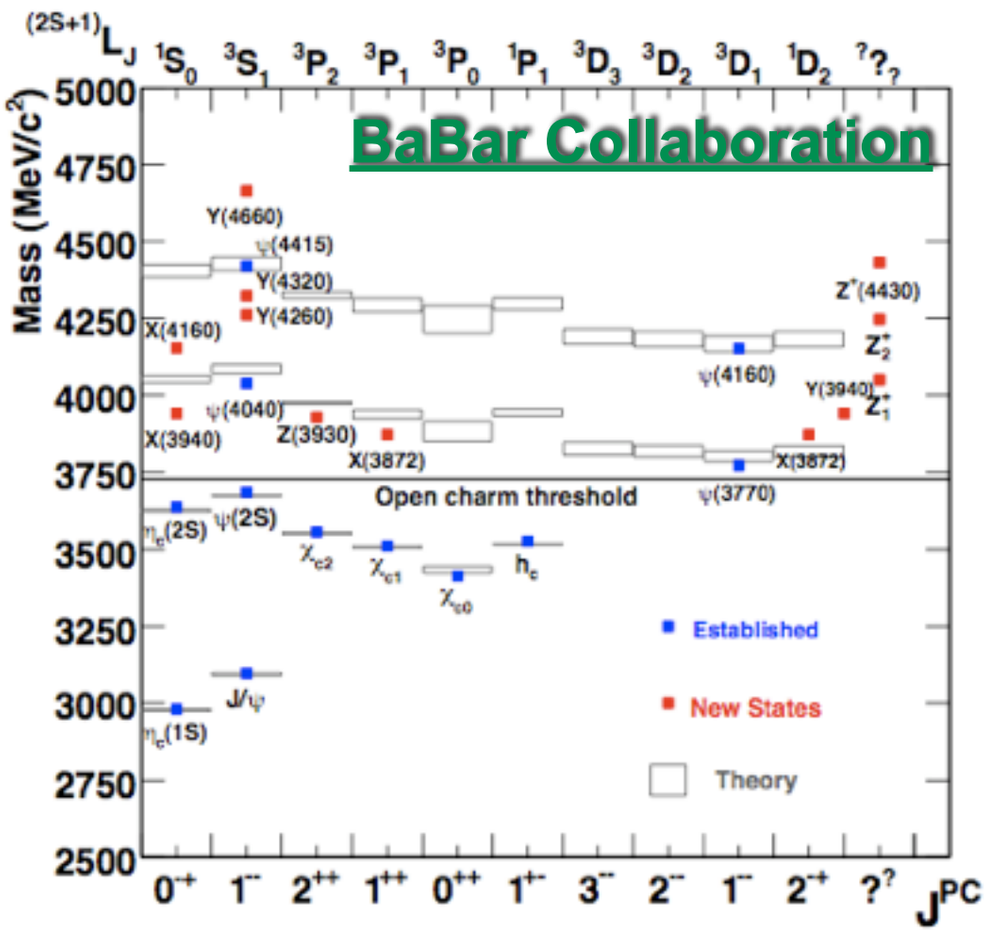
Takumi Doi, Tetsuo Hatsuda, Yoichi Ikeda, Vojtech Krejcirik (RIKEN)
Takashi Inoue (Nihon Univ.)
Noriyoshi Ishii, Keiko Murano (RCNP, Osaka Univ.)
Hidekatsu Nemura, Kenji Sasaki,
Masanori Yamada, Takaya Miyamoto (Univ. Tsukuba)

Hadrons to Atomic nuclei HAL from Lattice QCD

"Multi-Hadron and Nonlocal Matrix Elements in Lattice QCD"
@BNL, 5-6Feb. 2015.

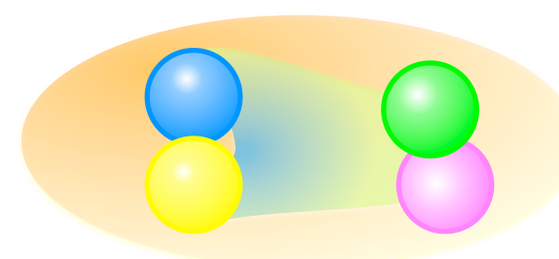
Spectrum of charmonium(-like) system



- Quark potential models well describe mass spectra below open charm threshold

  Godfrey, Isgur, PRD 32 (1985).
  Barnes, Godfrey, Swanson, PRD 72 (2005).

- "NEW" charmonium-like (X, Y, Z) states:
  ➡ not within quark model spectrum
  ➡ candidates of exotic hadrons

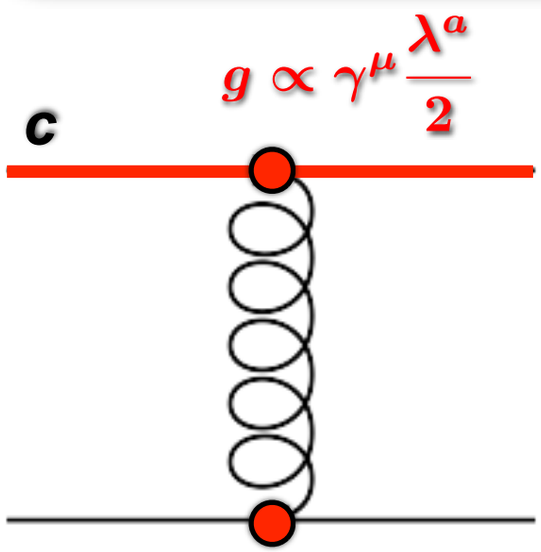


- "Other" exotic candidates (expected from quark models):
  ➡ doubly charmed tetra-quark, but experimentally not observed so far

Our target: tetra-quark channels

- "Tetraquark" Tcc ($ccu^{bar}d^{bar}$) is manifest 4-quark channel
- "Charged" charmonium-like states ($cc^{bar} + \pi^{+/-}$) require at least 4 quarks

Key dynamics involving heavy quarks



$$\sim \bar{c}\gamma^\mu \frac{\lambda^a}{2} c A^a$$

➡ color magnetic + color electric forces

magnetic interactions << electric interactions

➡ magnetic gluon coupling is suppressed by $O(1/m_c)$

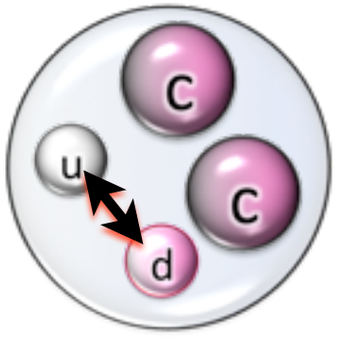
✓ Heavy quark spin symmetry

- **Color magnetic interaction : mass splitting**

$$V_{ij}^{\mathrm{CMI}} \propto -\frac{(\vec{\lambda}(i)\cdot\vec{\lambda}(j))(\vec{\sigma}(i)\cdot\vec{\sigma}(j))}{M_i M_j}$$

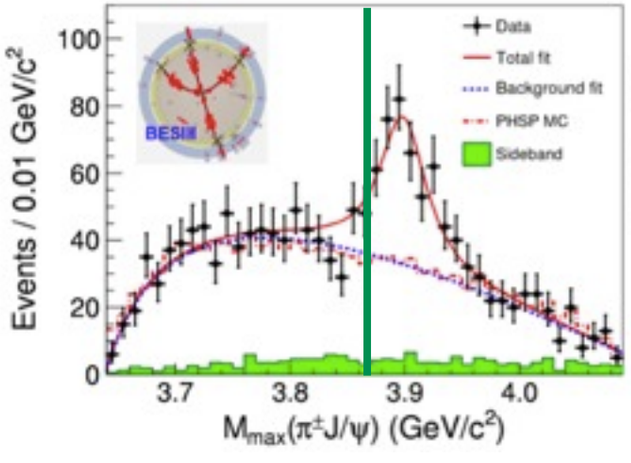
H. J. Lipkin, PLB172, 242 (1986).

➡ I=0 [ud]-diquark correlation (good diquark) --> Tcc bound state?

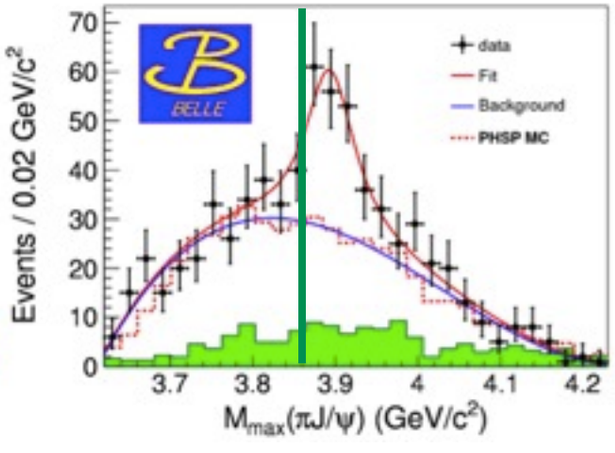


- **Color electric interaction : threshold**

- Zc(3900) is near threshold resonance?
- $J^P = 1^+$ seems most probable
- $D^{bar}D^*$ molecule? cc^{bar} + meson cloud?



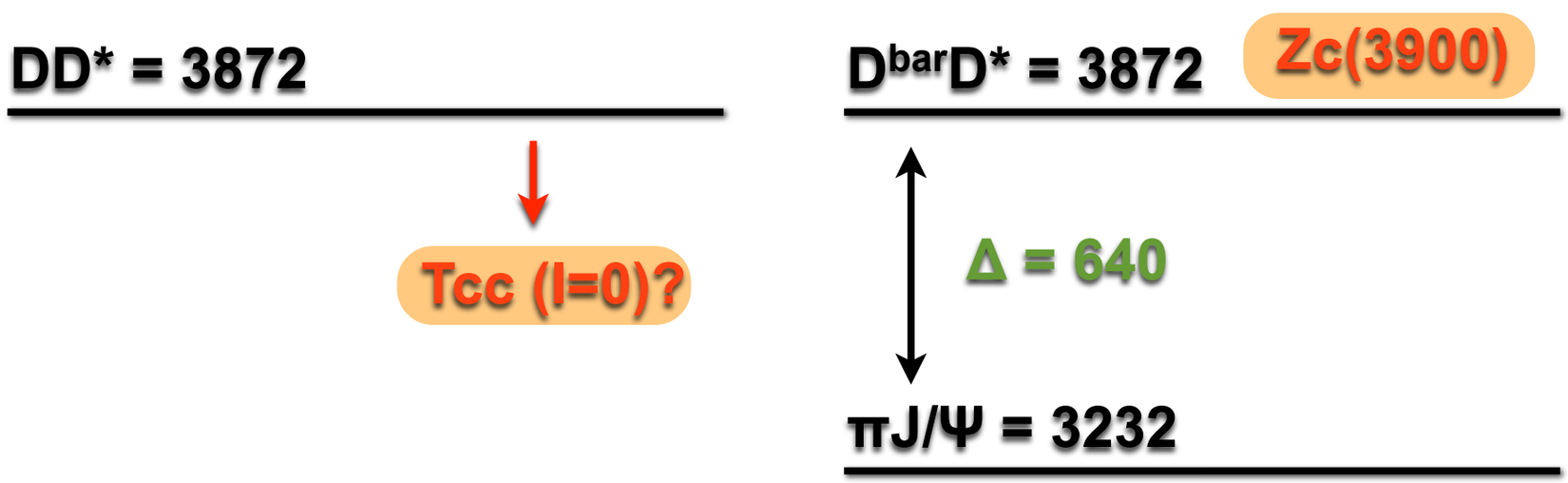
BESIII Coll., PRL110 (2013).



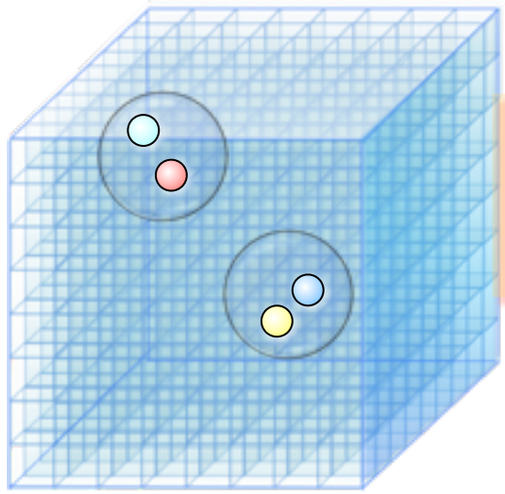
Belle Coll., PRL110 (2013).

Contents

- Introduction
- HAL QCD method to define (coupled-channel) potentials
- Tcc in $I(J^P)=0,1(1^+)$ channels [DD* single-channel]
- Zc(3900) in $I(J^P)=1(1^+)$ [$\pi J/\Psi$-$\rho\eta_c$-$D^{bar}D^*$ coupled-channel]
- Summary



Two identical methods for scattering



- ***Lüscher's finite size formula***

  interaction energy --> phase shift

  Lüscher, NPB354, 531 (1991).

- ***Scattering parameters***

$$k\cot\delta(k) = \frac{1}{a} - \frac{1}{2}r_e k^2 + \cdots$$

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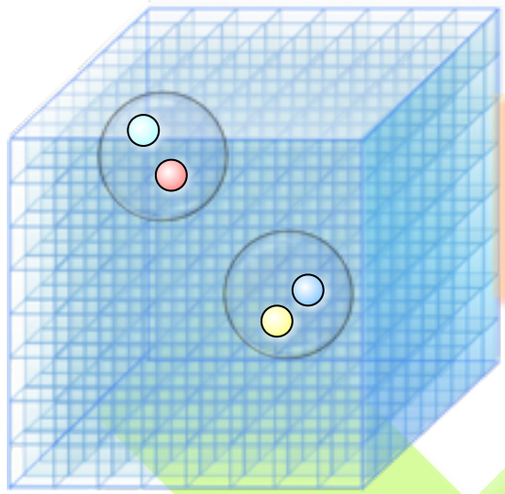

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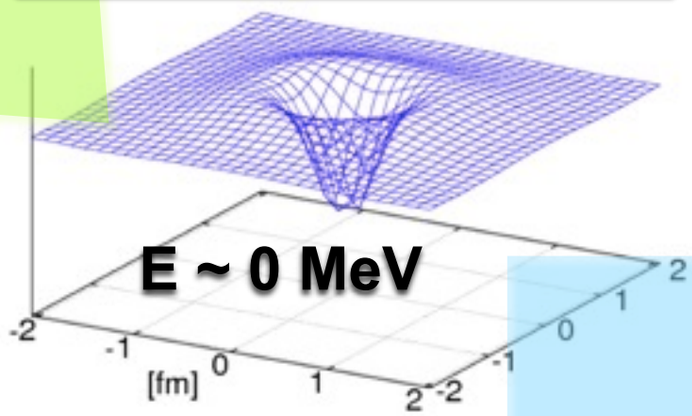
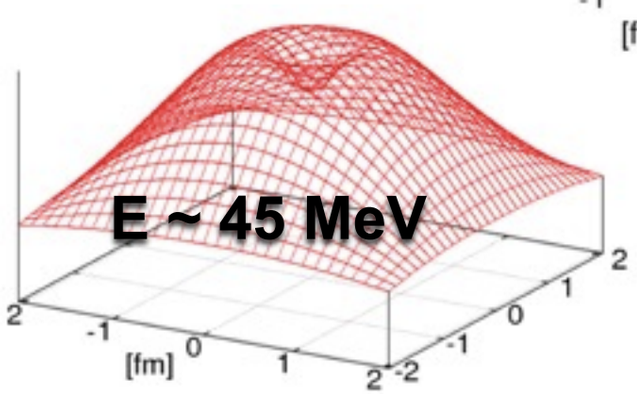
- **Guaranteed to be the same**

  Kurth et al., JHEP 1312 (2013) 015.

- ***NBS wave function***

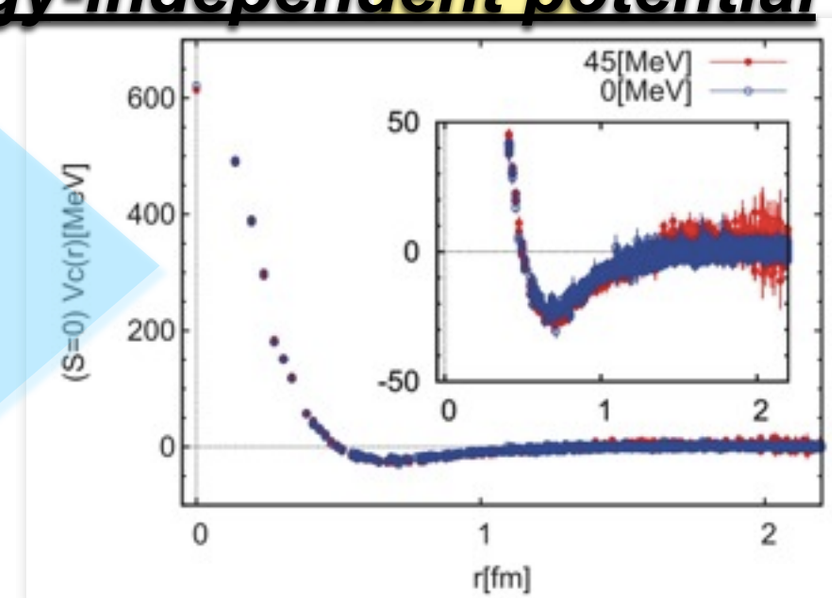




Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).

Aoki, Hatsuda, Ishii, PPTP123, 89 (2010).

- ***Energy-independent potential***



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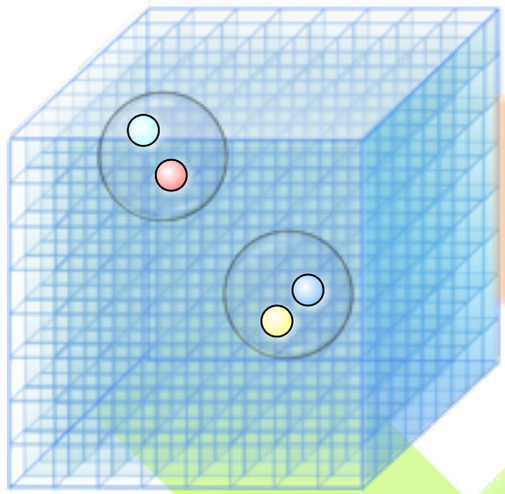

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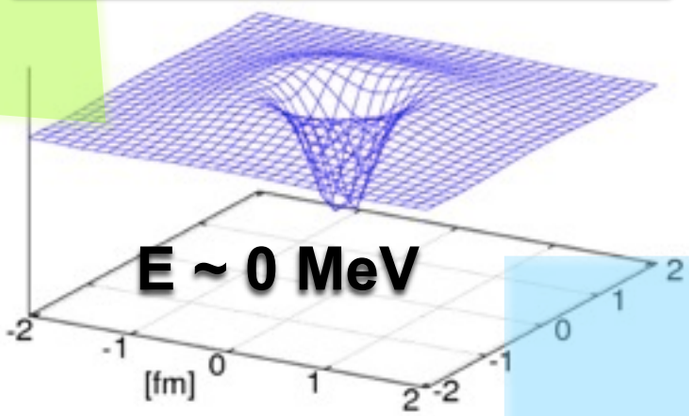

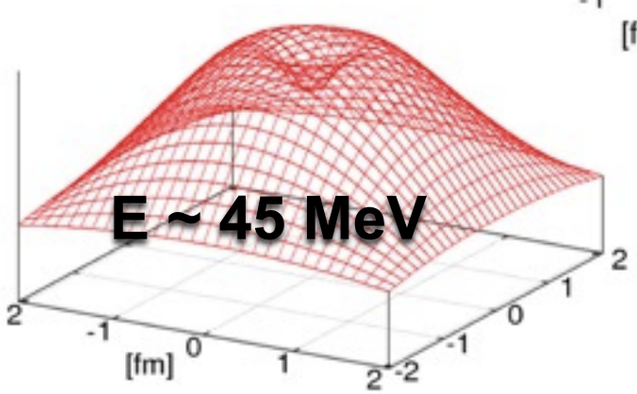

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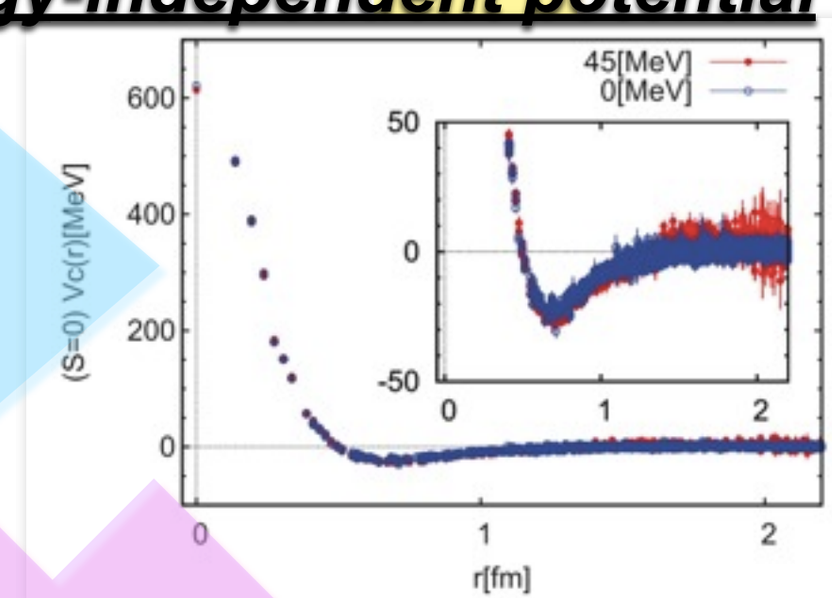

✓ **LQCD potentials can be applied to...**

properties of hadrons & nuclei, construction of EOS, etc.

Resonance from LQCD

T-matrix in formal scattering theory (N/D method)

$$T^{-1}(\sqrt{s}) = V^{-1} + \frac{1}{2\pi}\int_{s_+}^{\infty} ds' \frac{\rho(s')}{s'-s}$$

Interaction part is not determined within scattering theory

➡ **interactions faithful to phase shift from LQCD**

Resonance from LQCD

T-matrix in formal scattering theory (N/D method)

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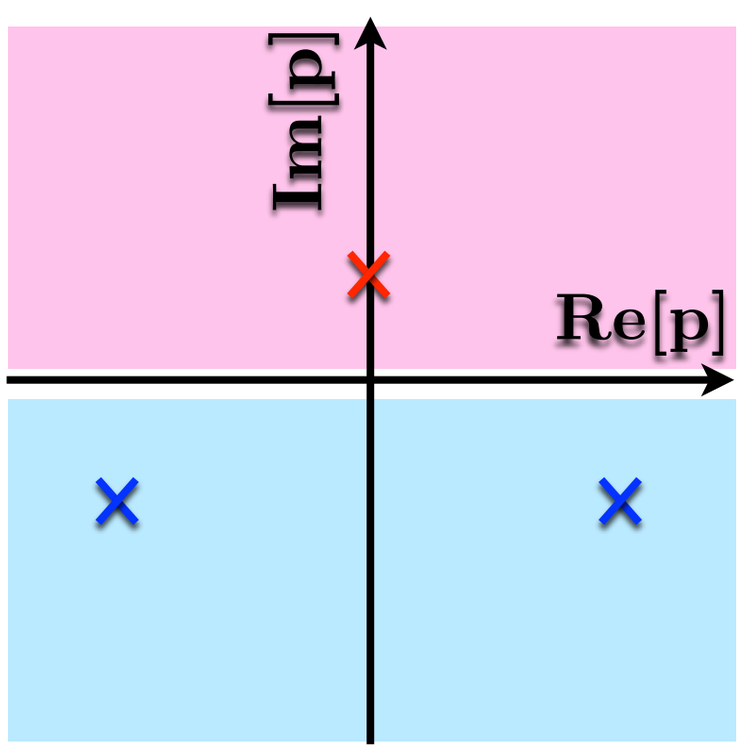
Interaction part is not determined within scattering theory

➡ **interactions faithful to phase shift from LQCD**

Analyticity of T-matrix is uniquely determined



Bound states (physical sheet, 1st)

- binding energy --> T-matrix pole position
- coupling --> residue of pole

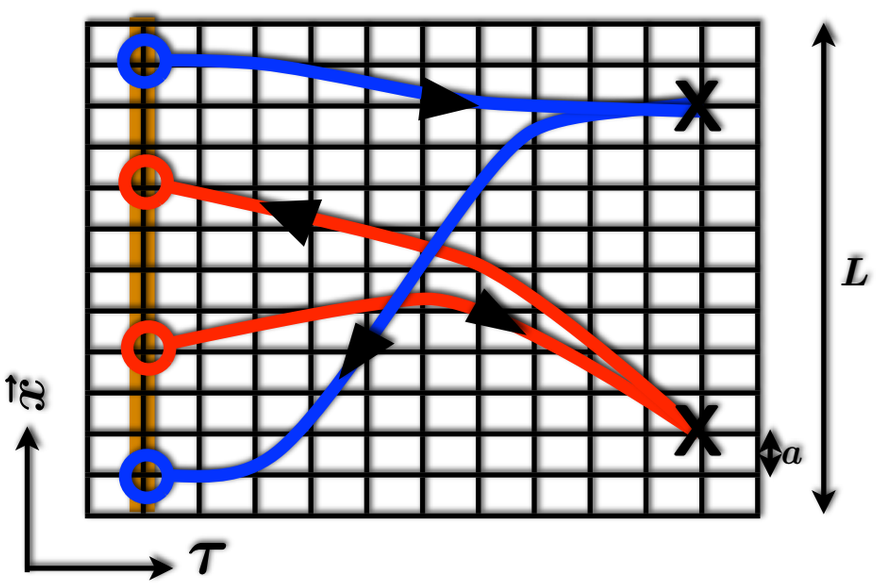
Resonance/Virtual states (unphysical sheet, 2nd)

- Analytic continuation of T-matrix
- resonance energy --> T-matrix pole position
- coupling --> (complex) residue of pole?

“Potentials” in QCD

Hadron 4pt functions & Nambu-Bethe-Salpeter (NBS) wave function

$$\psi^{ab}(\vec{r},\tau) = \sum_{\vec{x}} \langle 0|\phi_1^a(\vec{x}+\vec{r},\tau)\phi_2^a(\vec{x},\tau)\mathcal{J}^{b\dagger}(\tau=0)|0\rangle$$
$$= \sum_n A_n^b \exp\left[-W_n\tau\right]\sqrt{Z_1^a}\sqrt{Z_2^a}\psi_n^a(\vec{r})$$



- Helmholtz eq. of NBS wave func.

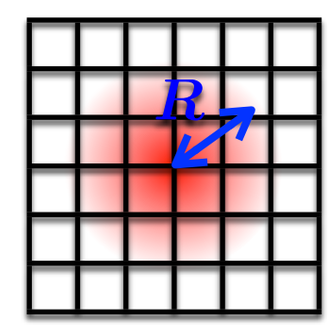
$$(\nabla^2 + (\vec{k}^a)^2)\psi^a_{W(\vec{k})}(\vec{r}) = 0 \quad (|\vec{r}| > R)$$

$$\psi^{(l)}_{W(\vec{k})}(r) \sim \frac{e^{i\delta_l(k)}}{kr}\sin\left(kr + \delta_l(k) - l\pi/2\right)$$

- NBS wave func. in QFT ~ wave func. in Q.M.

- Coupled-channel potential matrix (faithful to phase shifts)

$$\left(\nabla^2 + (\vec{k}^a)^2\right)\psi^a_n(\vec{r}) = 2\mu^a \sum_b \int d\vec{r}' U^{ab}(\vec{r},\vec{r}')\psi^b_n(\vec{r}')$$



Aoki et al. [HAL QCD Coll.], Proc. Jpn. Acad., Ser. B, 87 (2011); PTEP 2012, 01A105 (2012).

- Coupled-channel potentials are energy-independent (non-local in general)

HAL QCD method

✓ **Definition of energy-independent coupled-channel potentials :**

Aoki, Hatsuda, Ishii, PTP123, 89 (2010).

$$\psi_n(\vec{r}) = \langle 0|\phi_1^a(\vec{r}+\vec{x})\phi_2^a(\vec{x})|W_n; J^P\rangle$$

$$\left(\nabla^2 + (\vec{k}^a)^2\right)\psi_n^a(\vec{r}) = 2\mu \sum_b \int d\vec{r}' U^{ab}(\vec{r},\vec{r}')\psi_n^b(\vec{r}')$$

✓ **Extract energy-independent potential from time-dependent Schrödinger-type eq.**

Ishii et al.(HAL QCD), PLB712, 437(2012).

$$R^{ab}(\vec{r},\tau) \equiv \psi^{ab}(\vec{r},\tau)\frac{e^{(m_1^a+m_2^a)\tau}}{\sqrt{Z_1^a}\sqrt{Z_2^a}} \qquad \delta = \frac{m_1^a - m_2^a}{m_1^a + m_2^a} \qquad \Delta^{ac} = \frac{e^{(m_1^a+m_2^a)t}}{e^{(m_1^c+m_2^c)t}}$$

$$\left[-\partial_\tau + \nabla^2/2\mu^a + \partial_\tau^2/8\mu^a + \mathcal{O}(\delta^2)\right]R^{ab}(\vec{r},\tau) = \sum_c \int d\vec{r}' \Delta^{ac} U^{ac}(\vec{r},\vec{r}')R^{cb}(\vec{r}',\tau)$$

✤ **Since energy-independent potential can produce all scattering states, single-state saturations in simulations is not required**

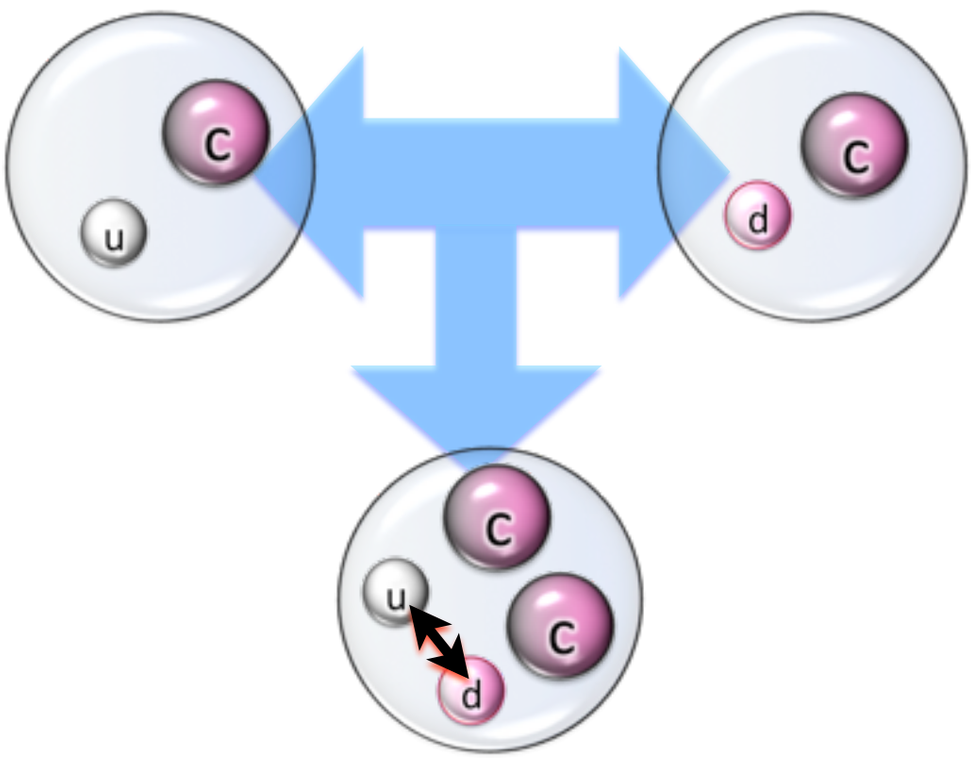
✓ **Velocity expansion:**

$$U(\vec{r},\vec{r}') = V(\vec{r},\nabla)\delta(\vec{r}-\vec{r}')$$

$$\Rightarrow V(\vec{r},\nabla) = \underbrace{V_C(\vec{r})}_{\text{(LO)}} + \underbrace{\vec{L}\cdot\vec{S}V_{LS}(\vec{r})}_{\text{(NLO)}} + \mathcal{O}(\nabla^2)$$

✓ **Calculate observable: phase shift, binding energy, pole position, ...**

Tcc in $I(J^P)=0,1(1^+)$



Asymptotic states : DD* (s-wave)

Lattice QCD Setup

N_f=2+1 full QCD configurations generated by PACS-CS Coll.

PACS-CS Coll., S. Aoki et al., PRD79, 034503, (2009).

- Iwasaki gauge & O(a)-improved Wilson quark actions
- a=0.0907(13) fm --> L~2.9 fm (32^3 x 64)

Light meson mass [conf.1, conf.2, conf.3] (MeV)

M_π=699(1), 572(2), 411(2) [PDG:135 (π^0)]

M_K=787(1), 714(1), 635(2) [PDG:498 (K^0)]

Tsukuba-type Relativistic Heavy Quark (RHQ) action for charm quark

S. Aoki et al., PTP109, 383 (2003)

➡ remove leading cutoff errors $O(m_c a)$, $O(\Lambda_{QCD} a)$, ...

- We are left with $O((a\Lambda_{QCD})^2)$ error (~ a few %)
- We employ RHQ parameters tuned by Namekawa et al.

Y. Namekawa et al., PRD84, 074505 (2011)

Charmed meson mass [conf.1, conf.2, conf.3] (MeV)

$M_{\eta c}$=3024(1), 3005(1), 2988(2) [PDG:2981]

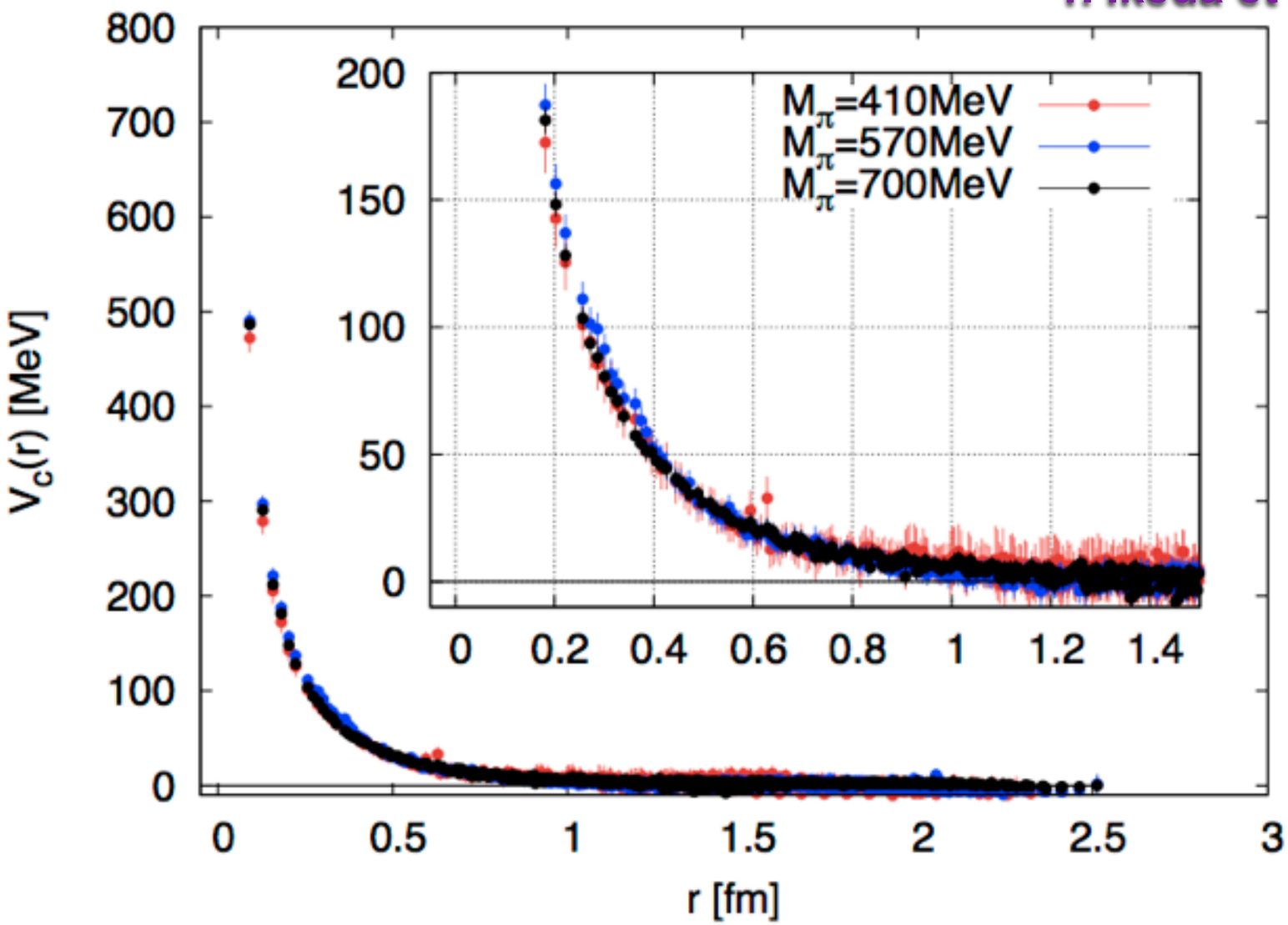
$M_{J/\psi}$=3142(1), 3118(1), 3097(2) [PDG:3097]

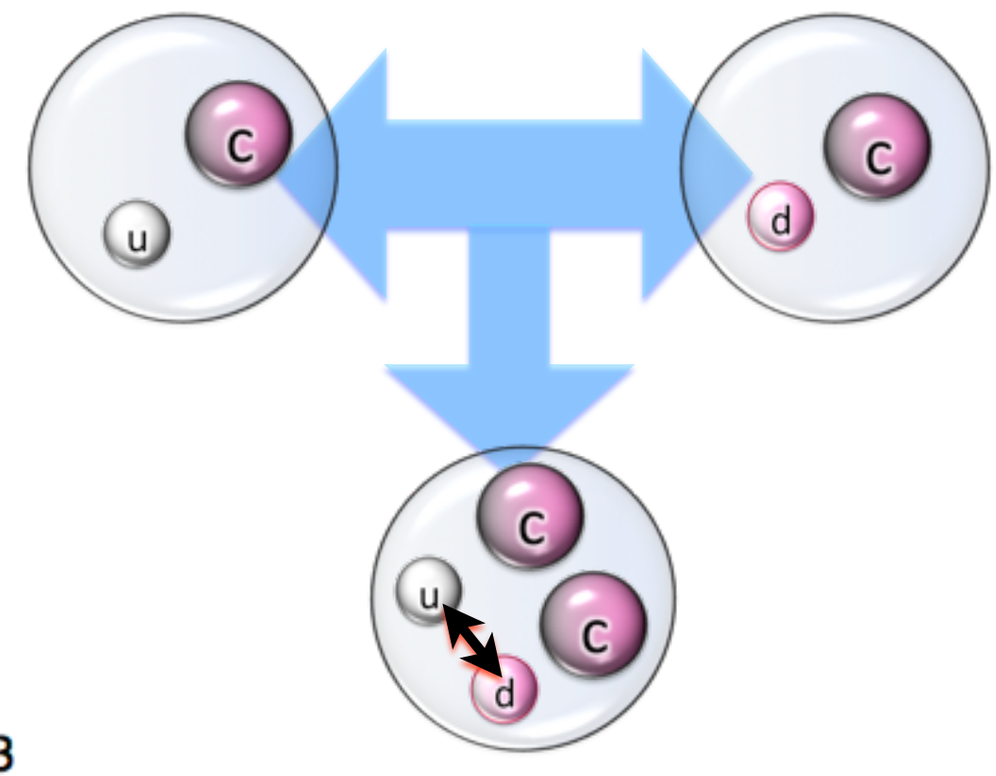
M_D=1999(1), 1946(1), 1902(3) [PDG:1865 (D^0)]

M_{D^*}=2159(4), 2099(6), 2048(12) [PDG:2007 (D^{*0})]

S-wave DD* in I=1 : "bad" diquark

Y. Ikeda et al. (HAL QCD), PLB729, 85 (2014).

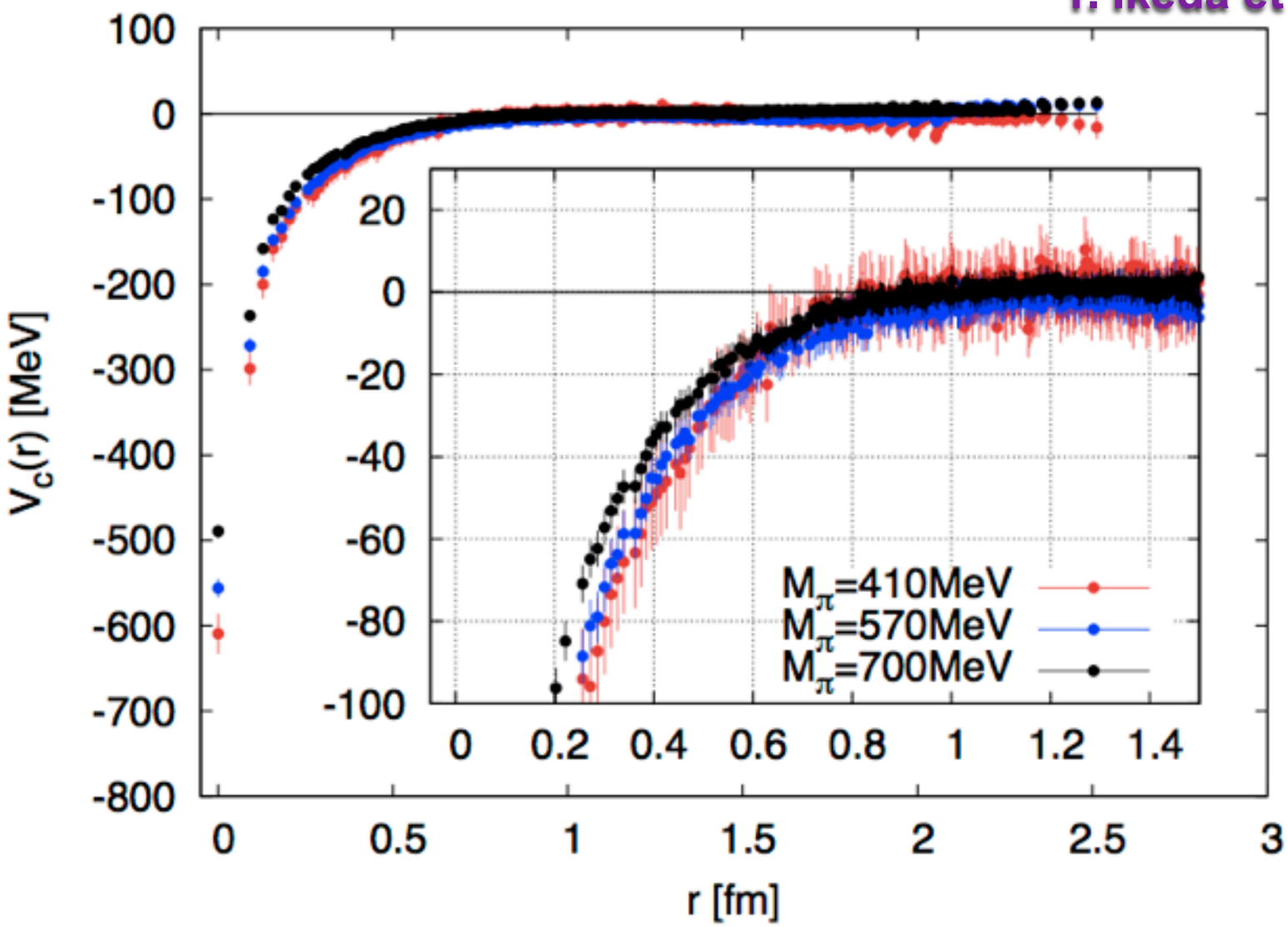


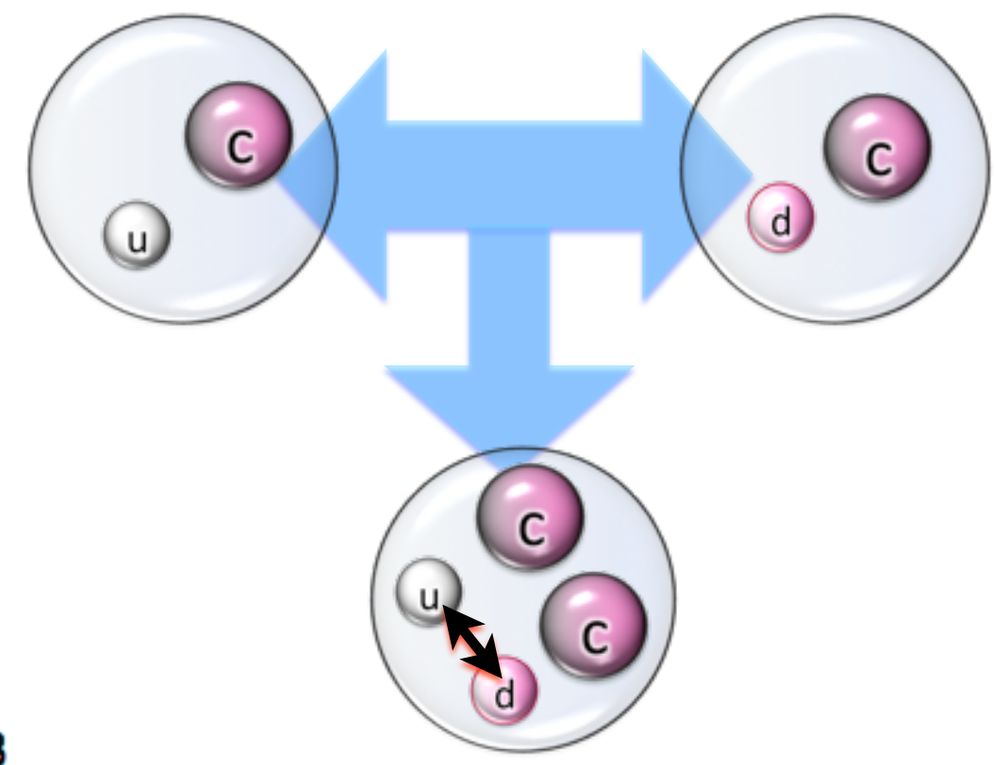


- **Repulsive s-wave potentials of DD***
- **Weak quark mass dependence**
- **It is unlikely to form bound state even at physical point**

S-wave DD* in I=0 : "good" diquark

Y. Ikeda et al. (HAL QCD), PLB729, 85 (2014).



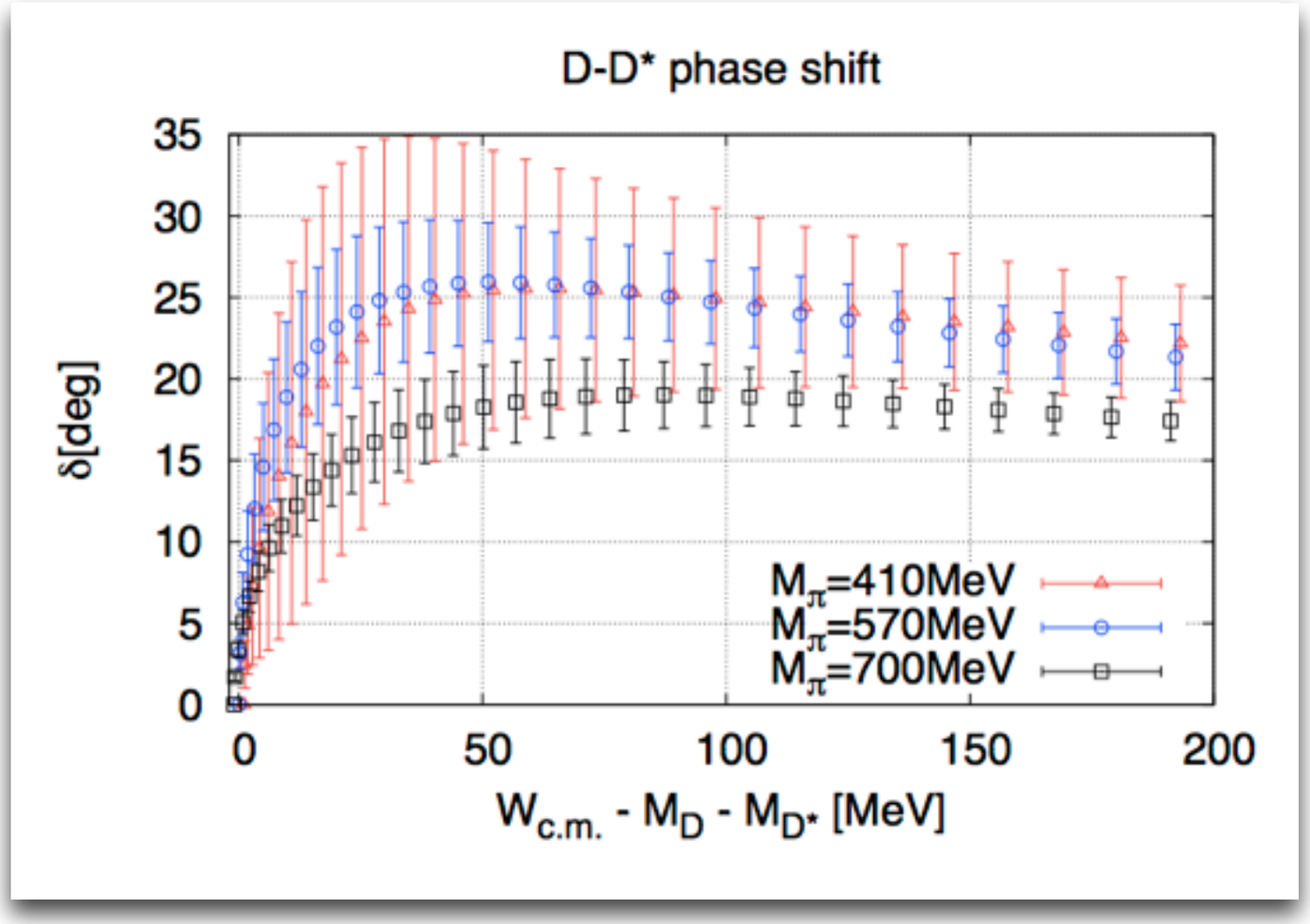


- **Attractive S-wave potentials**
- **Attraction increases, as m_q decreases**
- **Check whether bound T_{cc} exist or not --> phase shift analysis**

S-wave phase shifts : Tcc in I=0

Y. Ikeda et al. (HAL QCD), PLB729, 85 (2014).

- solve Schrödinger equation --> phase shifts

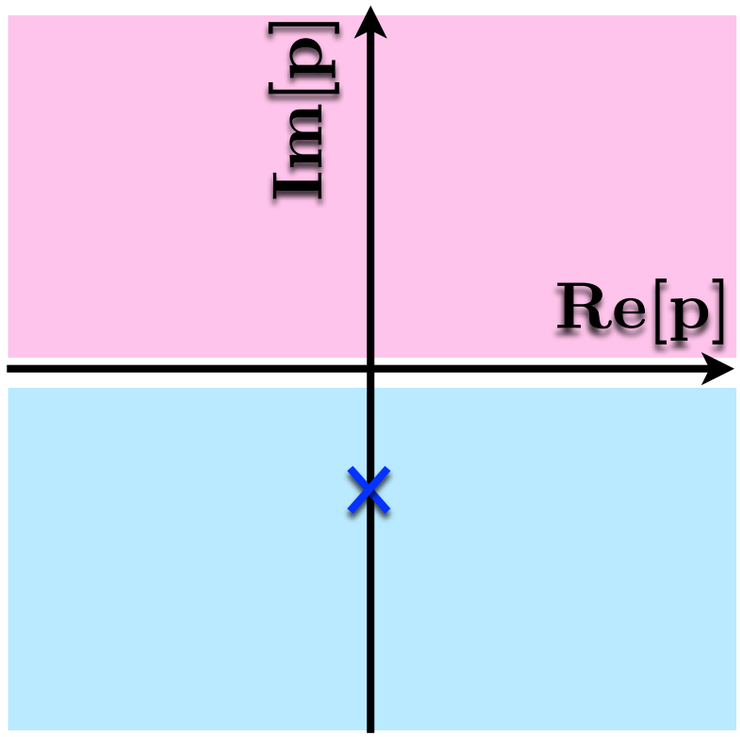


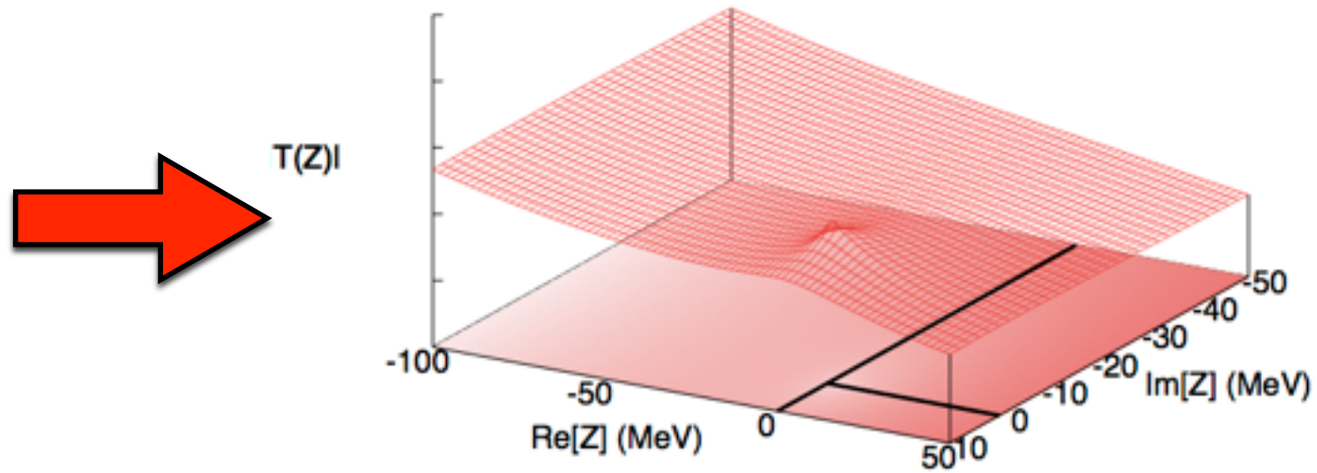
- Attraction is not sufficiently strong to generate bound state
- Rapid increase at threshold of DD* phase shift --> effect of virtual state?

➡ examine pole position

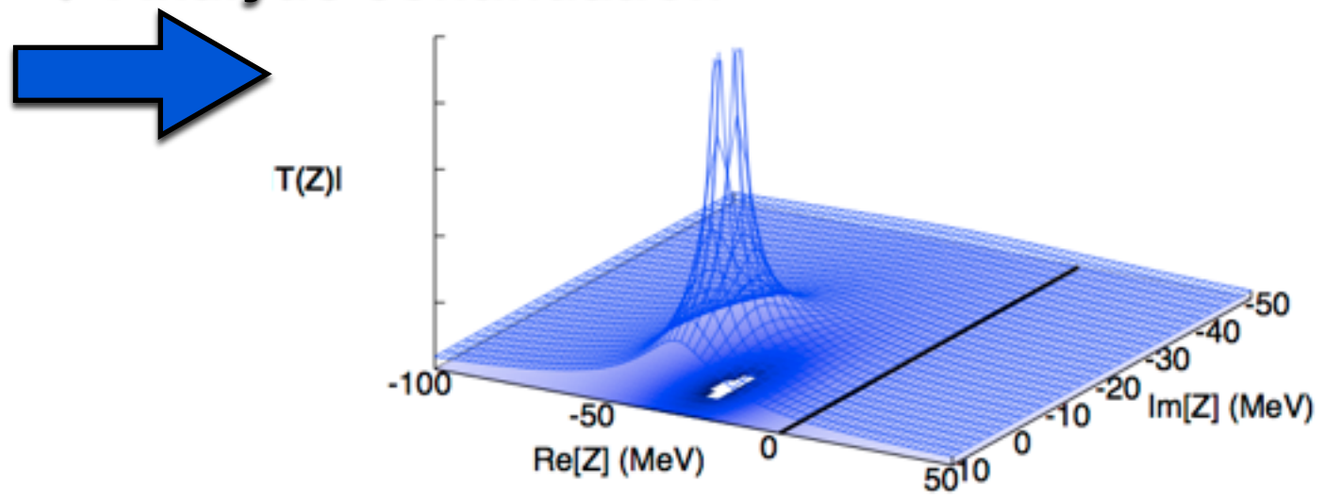
I=0 DD* T-matrix on complex energy plane

- **Pole search w/ LQCD potential@m_π=410MeV**





❖ **Analytic continuation**



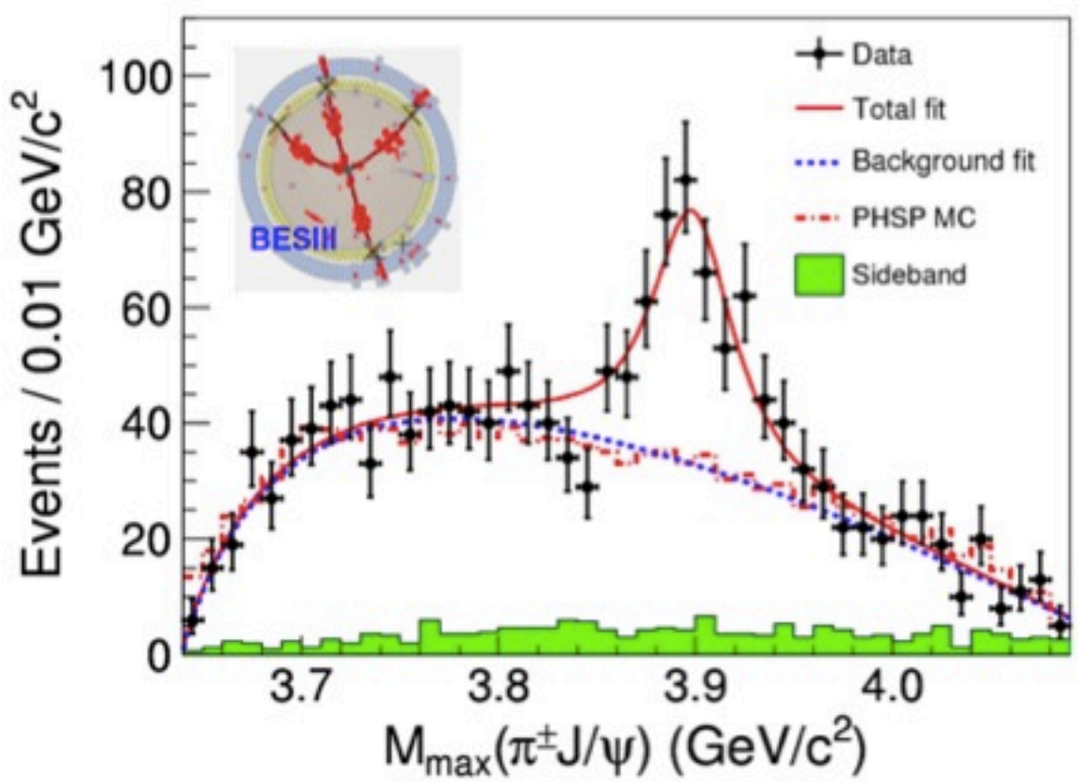
- **Virtual pole on the DD* unphysical energy plane**

➡ **threshold cusp of the amplitude**

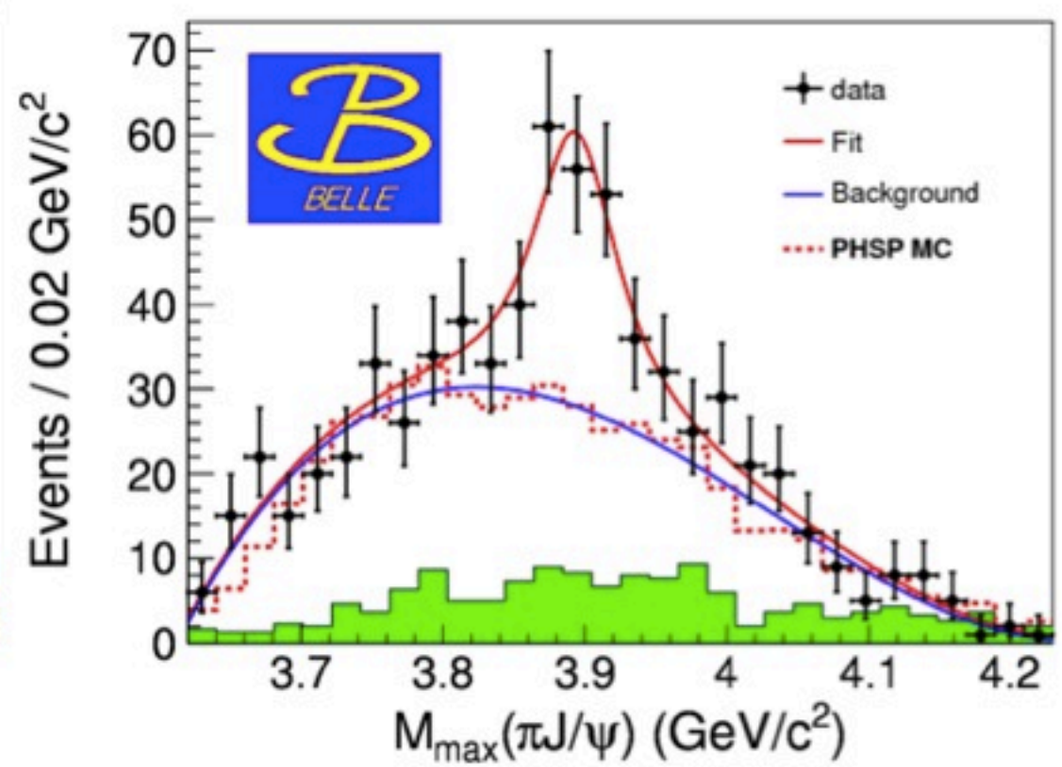
➡ **rapid increase of scattering phase shift**

Zc(3900) in $I^G(J^P)=1^+(1^+)$

BESIII Coll., PRL110, 252001, (2013).



Belle Coll., PRL110, 252002, (2013).



Lattice QCD setup

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- Tsukuba-type RHQ action for charm quark

  S. Aoki et al., PTP109, 383 (2003)

  Y. Namekawa et al., PRD84, 074505 (2011)

Light meson mass (MeV)

M_π= 411(2) [PDG:135 (π^0)]

M_ρ= 895(14) [PDG:775]

Charmed meson mass (MeV)

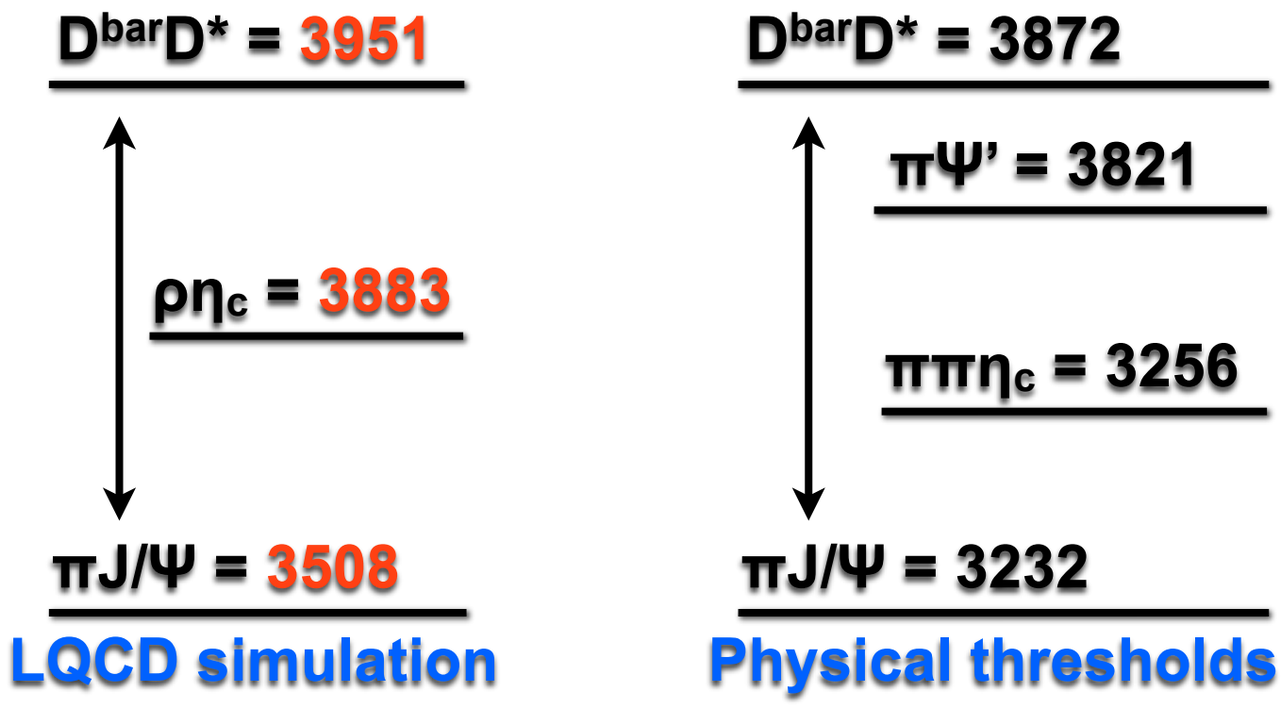
$M_{\eta c}$= 2988(2) [PDG:2981]

$M_{J/\Psi}$= 3097(2) [PDG:3097]

M_D= 1902(3) [PDG:1865 (D^0)]

M_{D^*}= 2048(12) [PDG:2007 (D^{*0})]

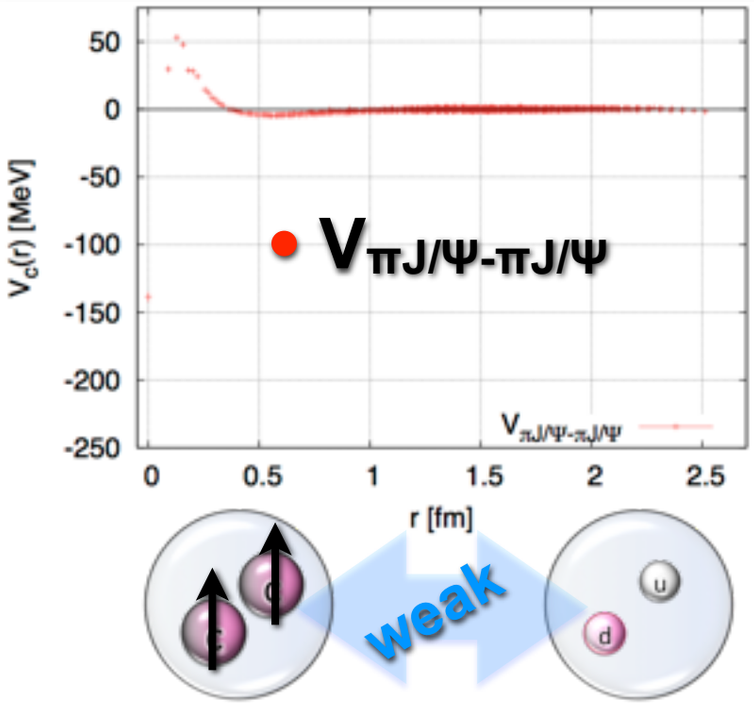
Thresholds in $I^G J^P$=1^+1^+ channel

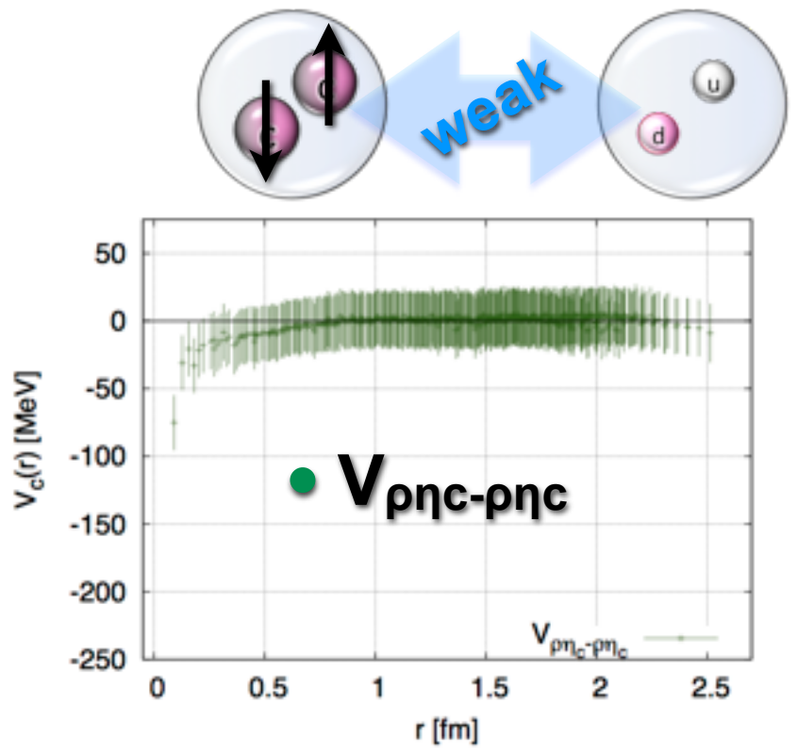


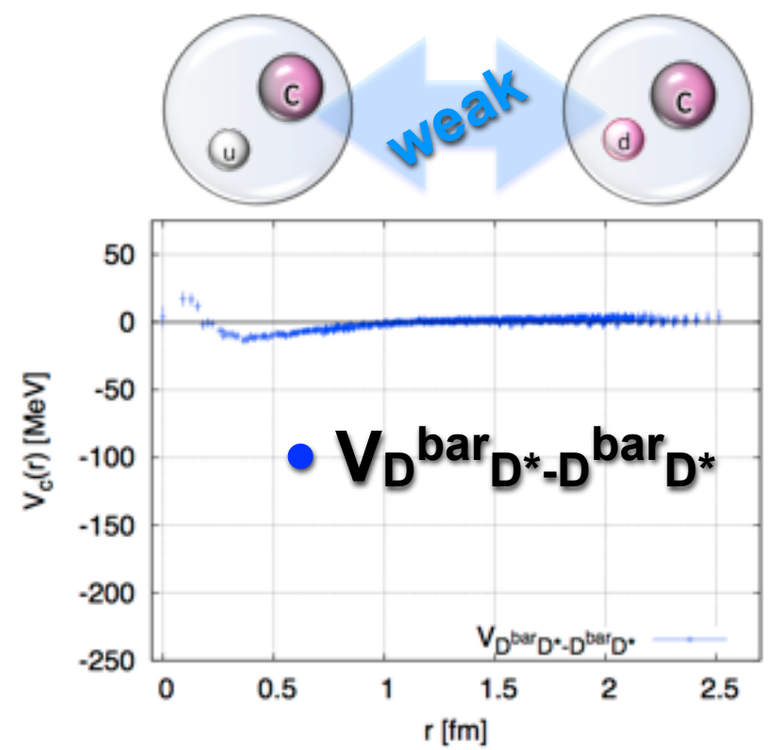
- $M_{\pi\Psi'}$ > $M_{D^{bar}D^*}$ due to heavy pion mass
- $\rho \to \pi\pi$ decay not allowed in our setup

- S-wave $\pi J/\Psi$ - $\rho\eta_c$ - $D^{bar}D^*$ coupled-channel analysis is performed

Potential matrix ($\pi J/\Psi$ - $\rho\eta_c$ - $D^{bar}D^*$)



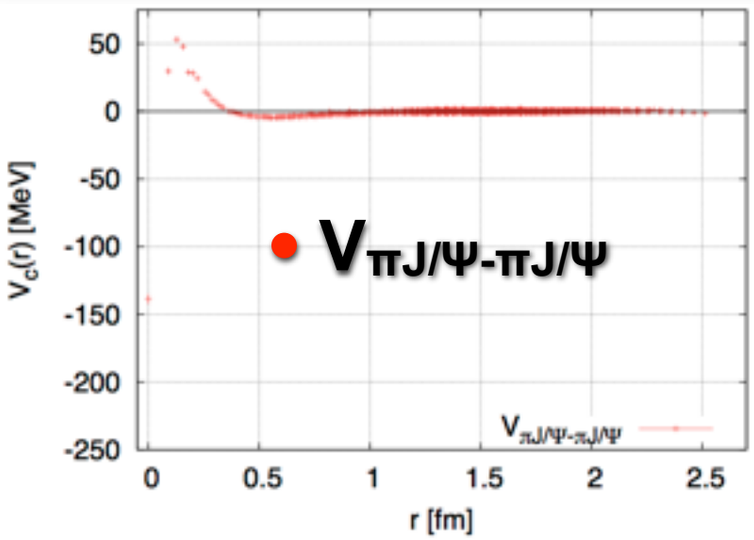


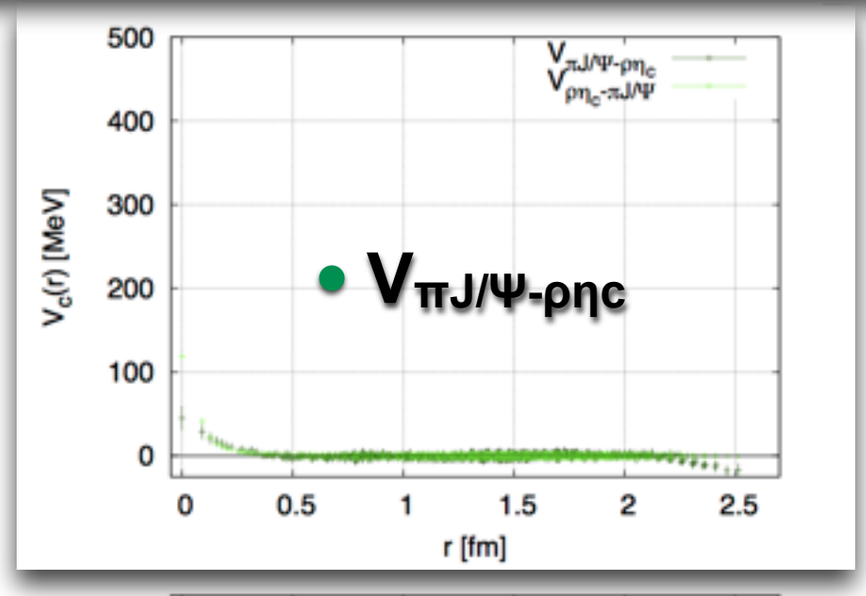


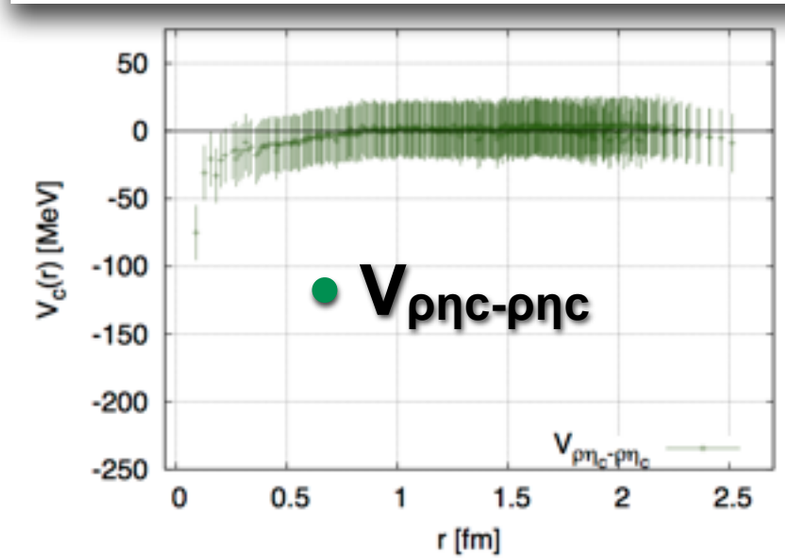
- **All diagonal potentials are weak**

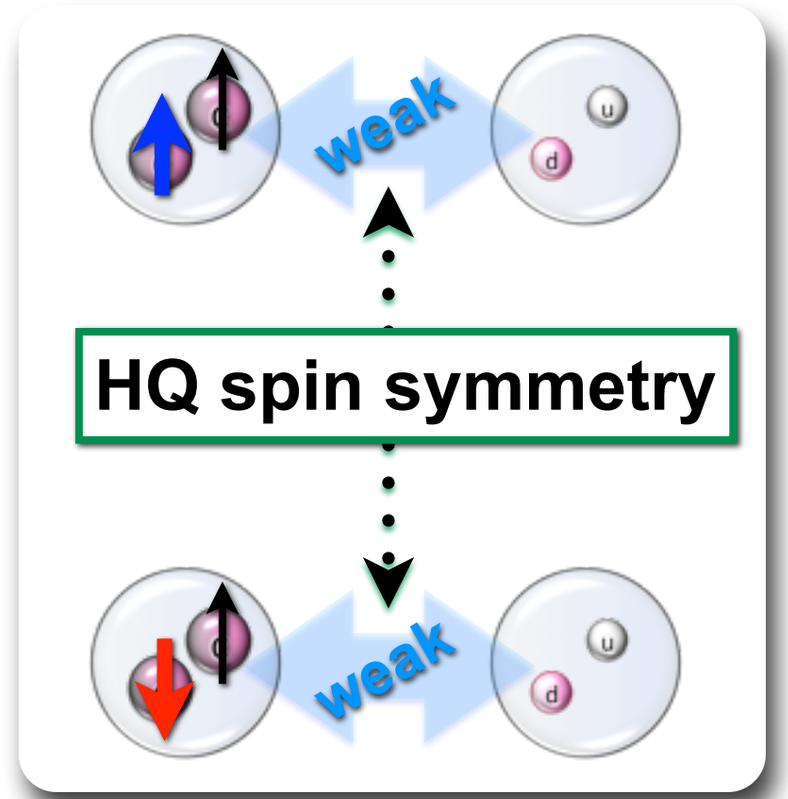
  ➡ **no bound $D^{bar}D^*$**

Potential matrix ($\pi J/\Psi$ - $\rho\eta_c$ - $D^{bar}D^*$)





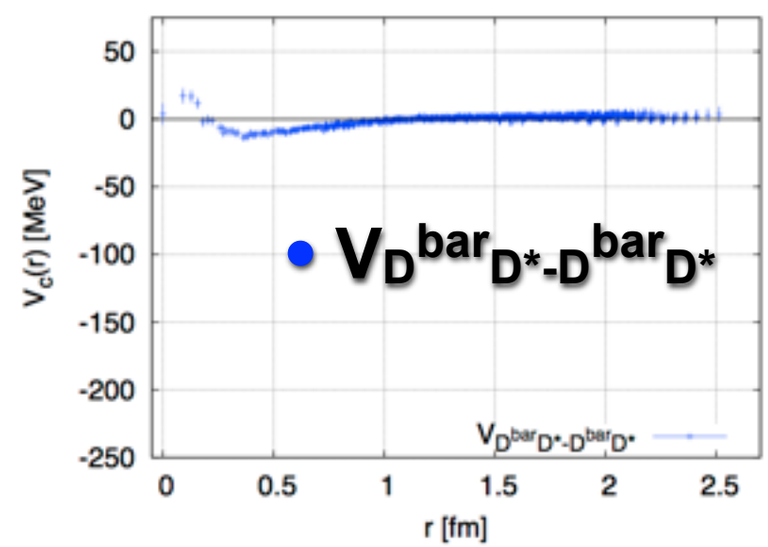




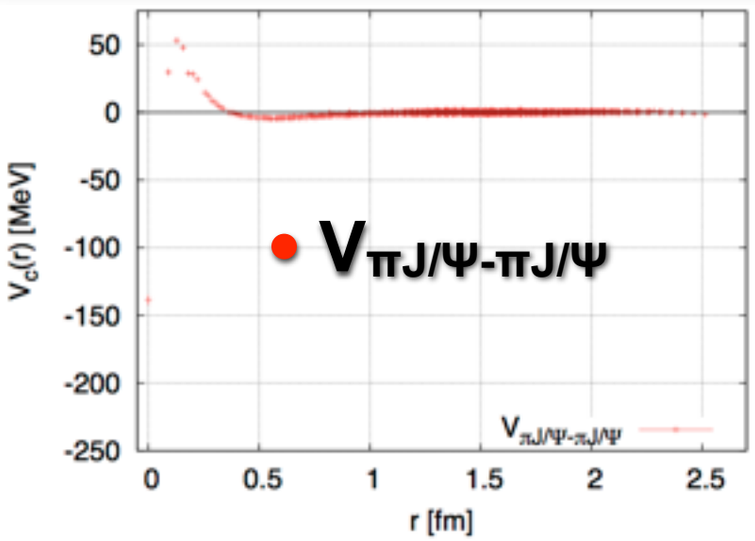
- **Weak charm spin-flip potential**

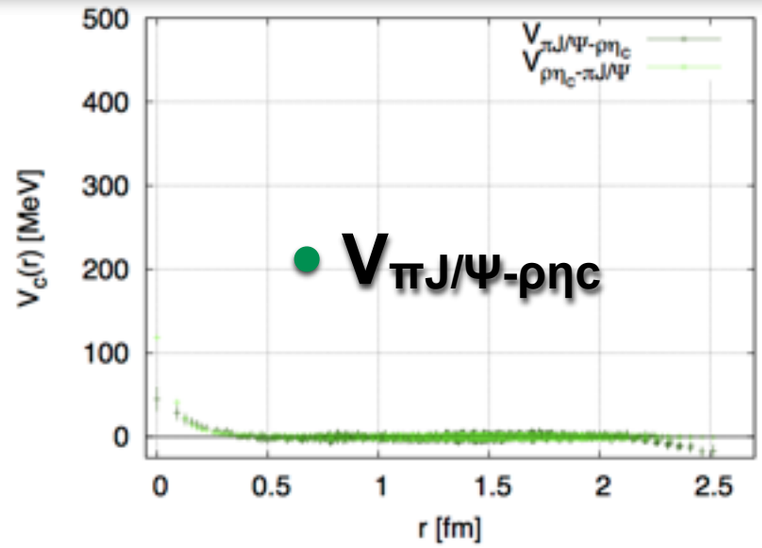
➡ **heavy quark spin symmetry**

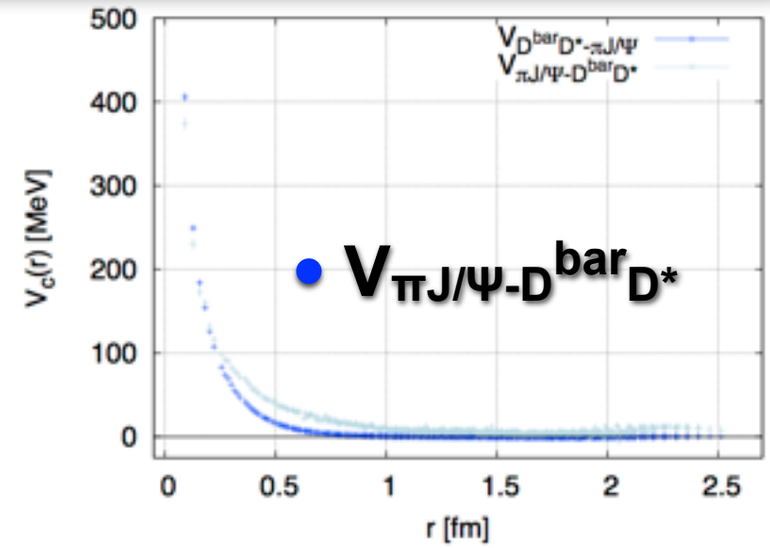
(charm quark spin-flip amplitude is suppressed)

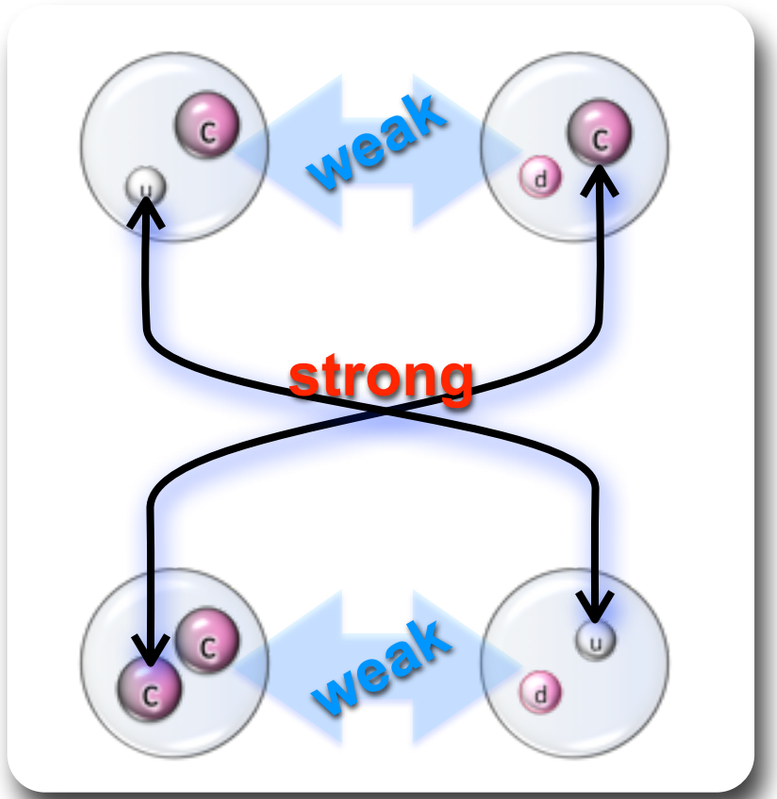


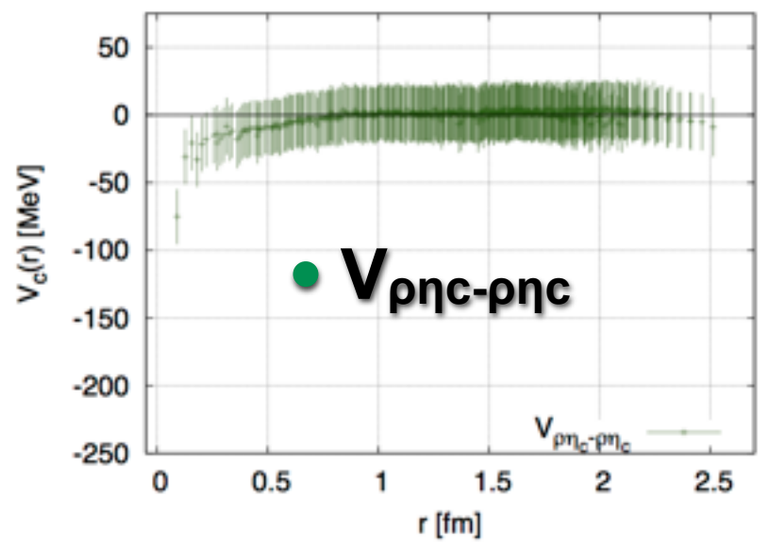
Potential matrix ($\pi J/\Psi$ - $\rho\eta_c$ - $D^{bar}D^*$)

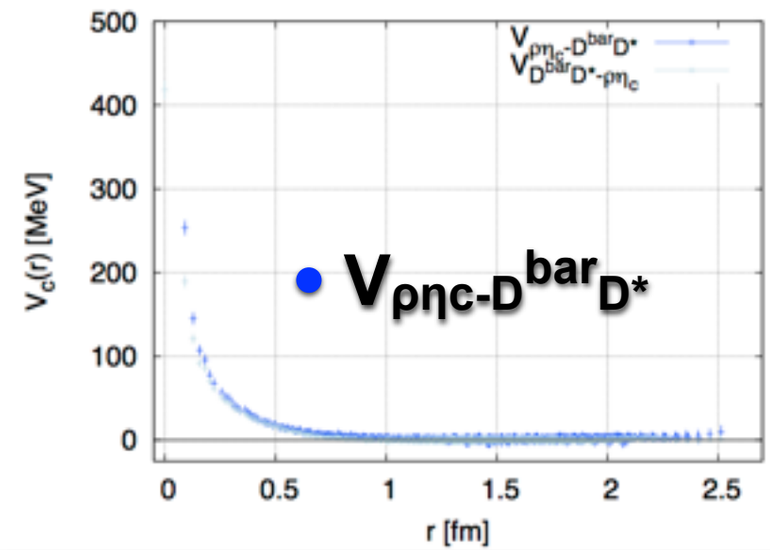


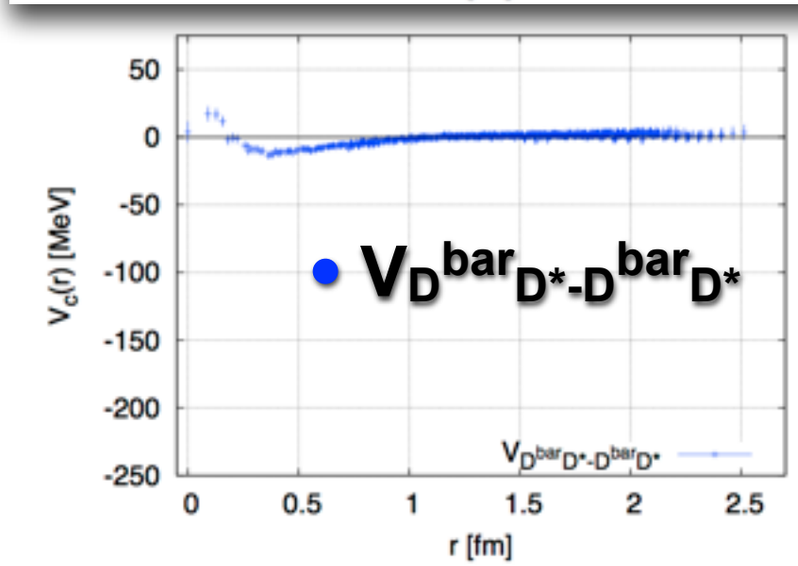






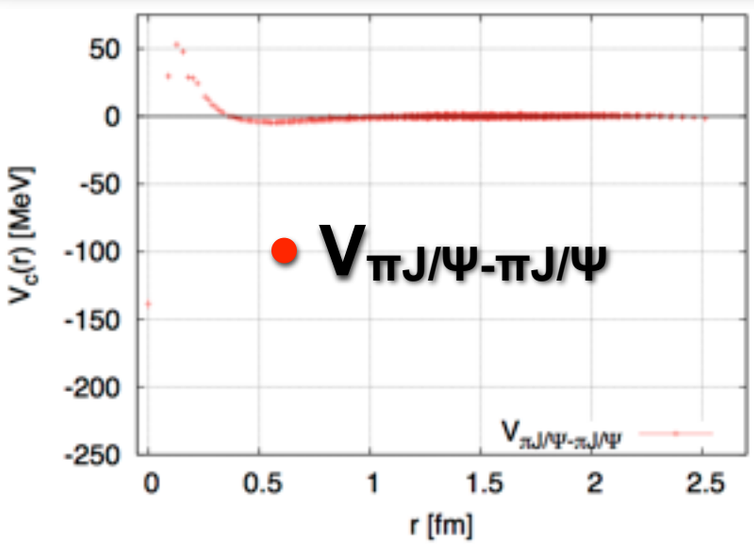


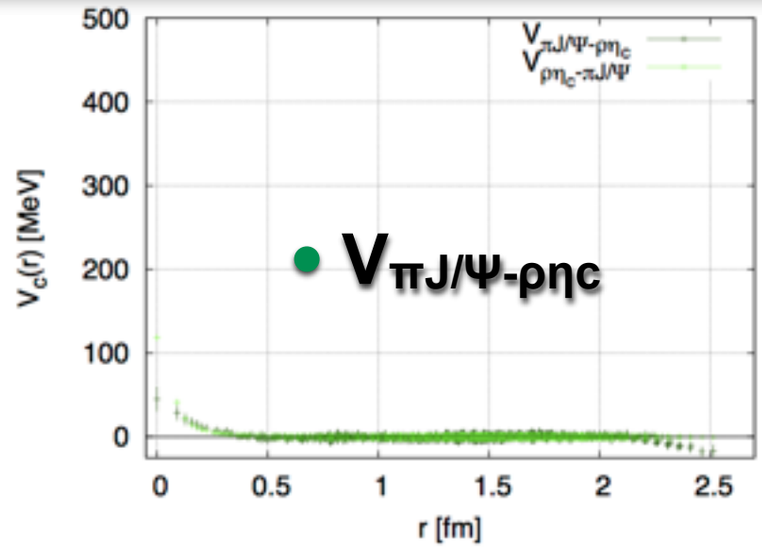


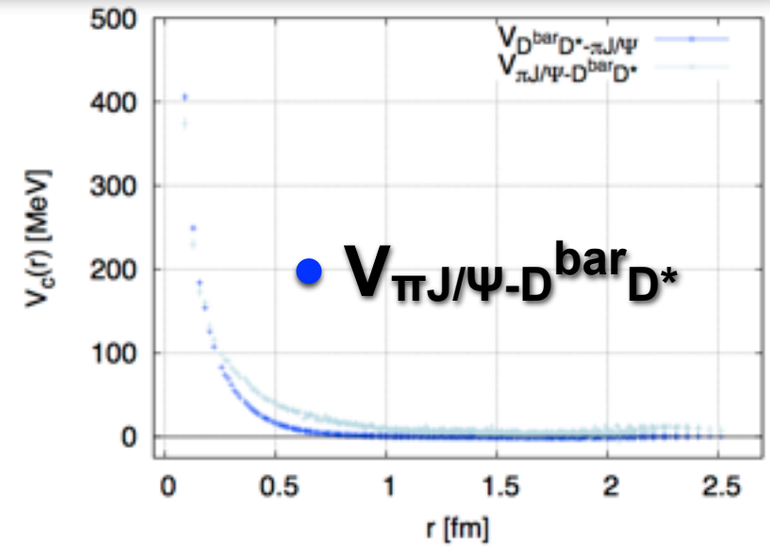


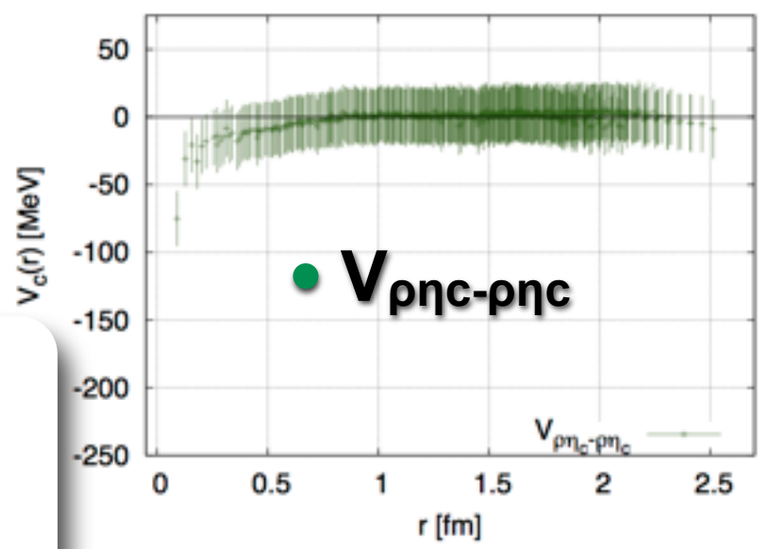
- **Strong off-diagonal $D^{bar}D^*$ potentials**
  - ✓ strong charm-quark-exchange interactions

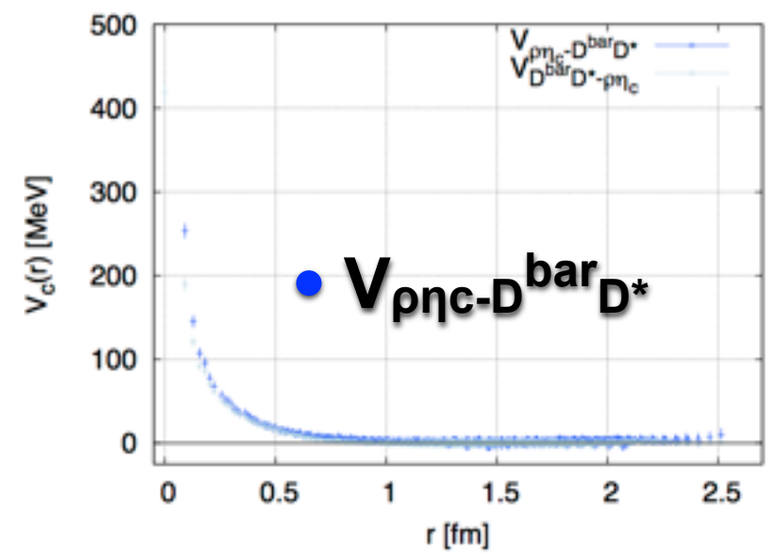
Potential matrix ($\pi J/\Psi$ - $\rho\eta_c$ - $D^{bar}D^*$)

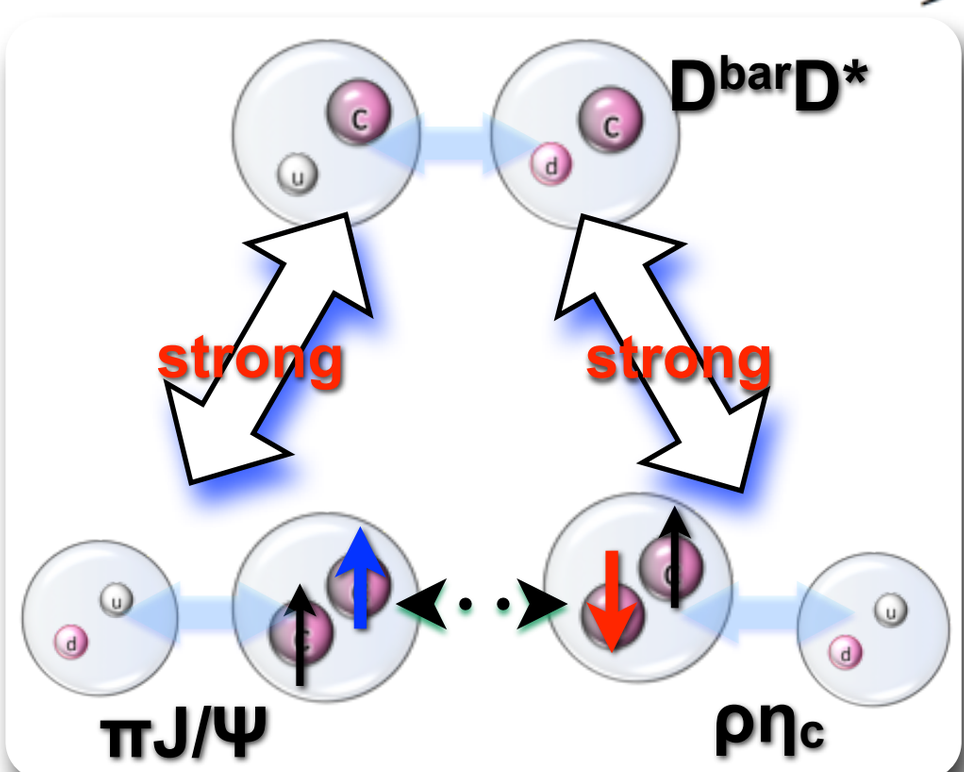


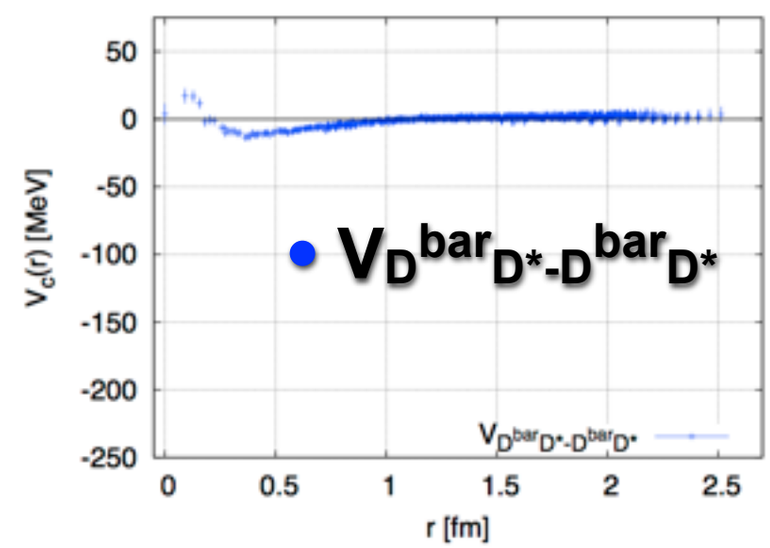






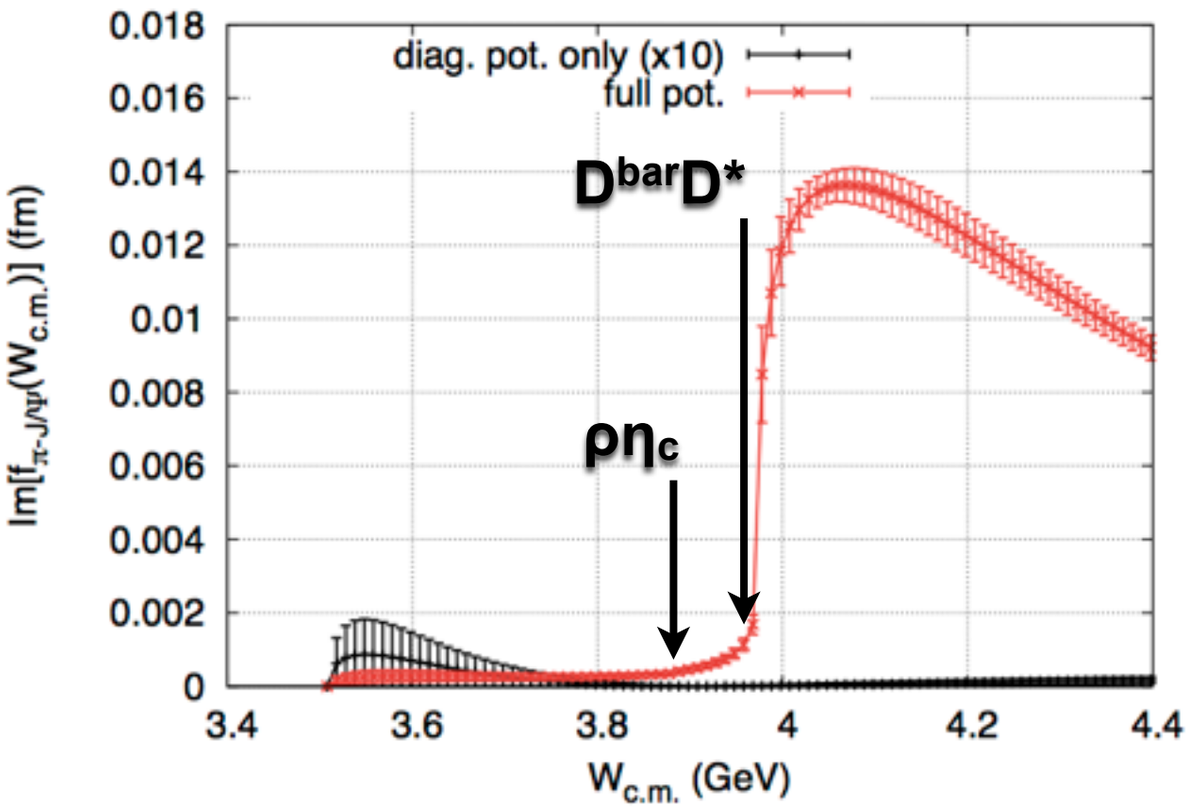




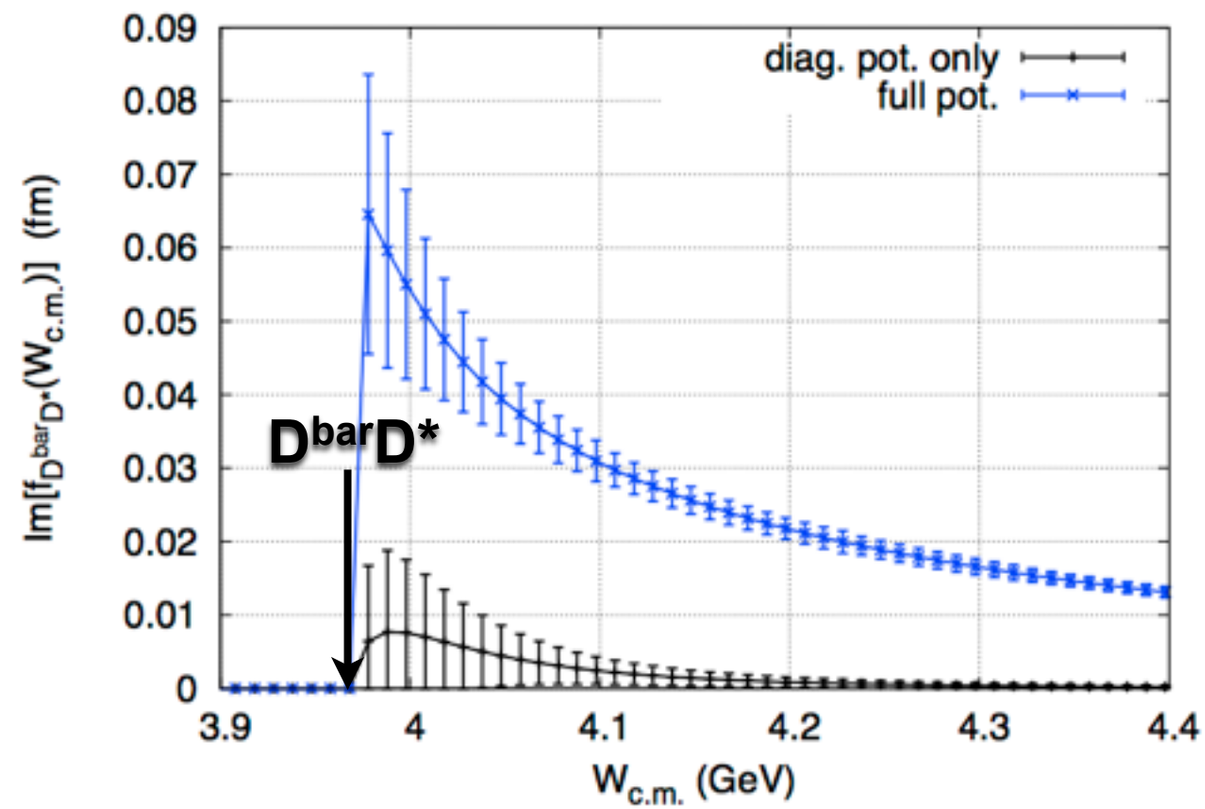


Invariant mass spectra of πJ/Ψ & $D^{bar}D^*$

- πJ/Ψ invariant mass (m_π=410MeV)



- $D^{bar}D^*$ invariant mass (m_π=410MeV)

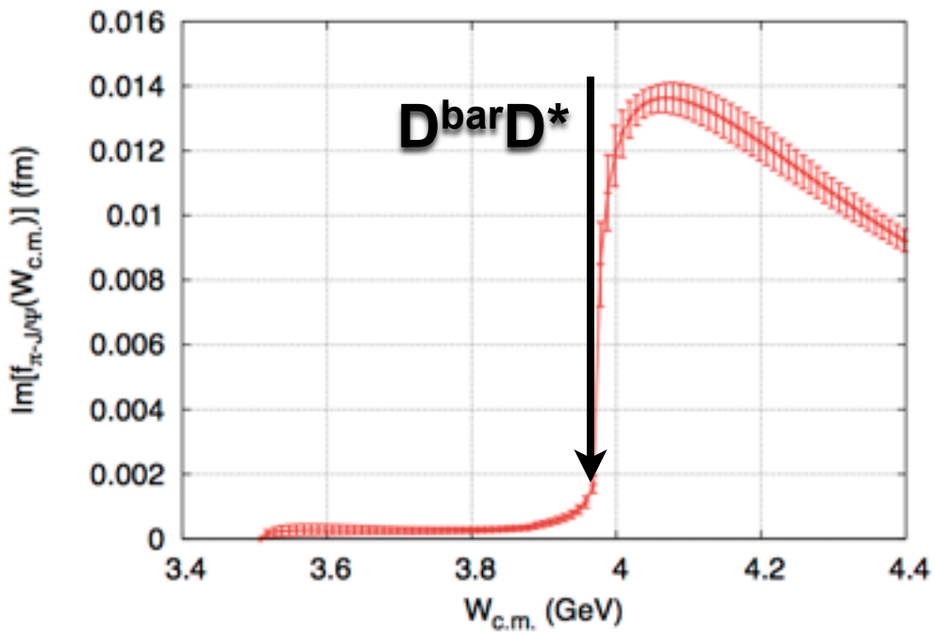


✓ enhancement near $D^{bar}D^*$ threshold due to large πJ/Ψ-$D^{bar}D^*$ coupling

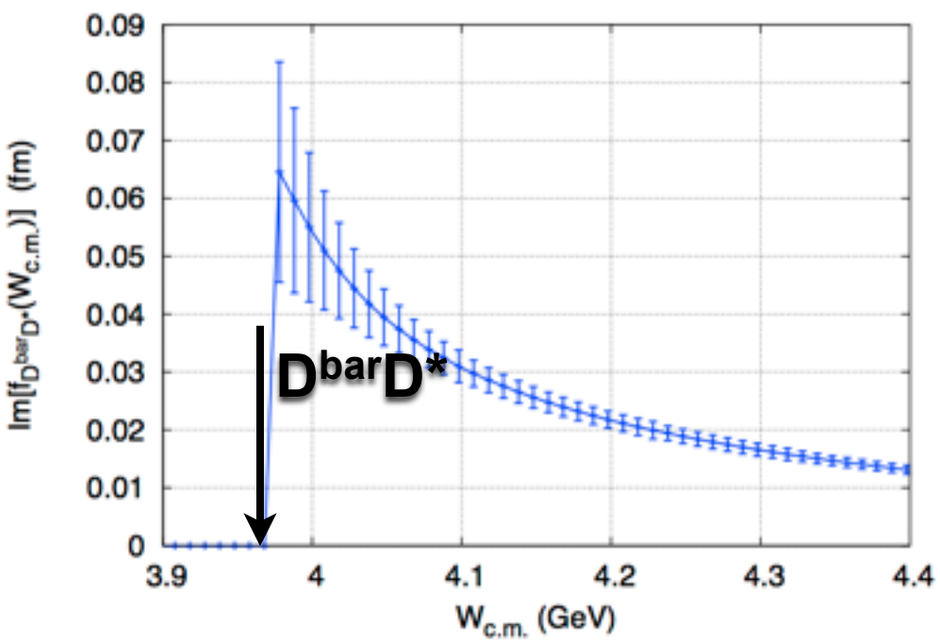
- peak in πJ/Ψ invariant mass
- sharp enhancement in $D^{bar}D^*$ invariant mass

LQCD results & EXP. results

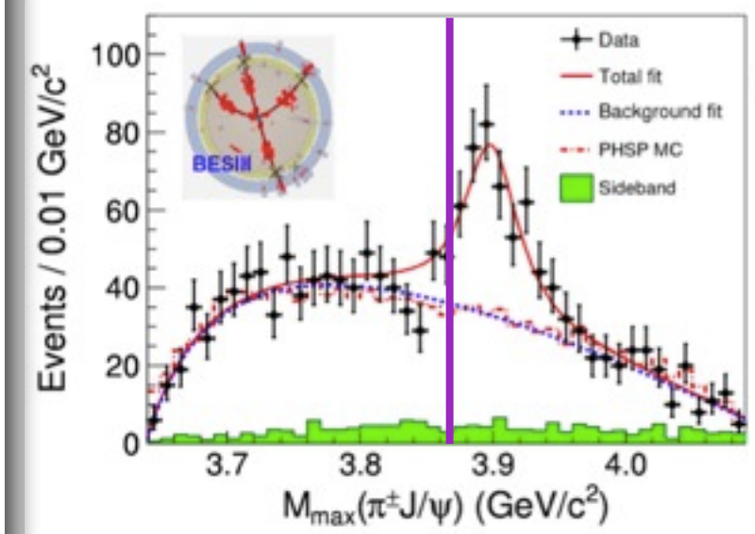
- **πJ/Ψ invariant mass (m_π=410MeV)**



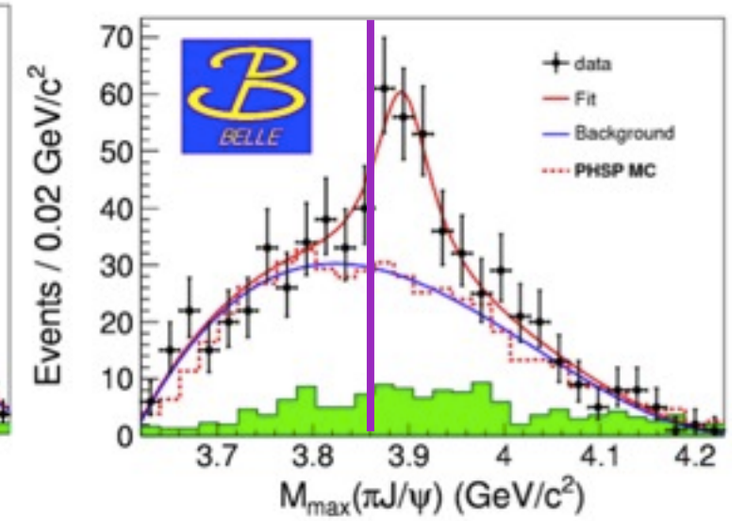
- **$D^{bar}D^*$ invariant mass (m_π=410MeV)**



- **e^+e^- --> π(πJ/Ψ) @ 4.26GeV**

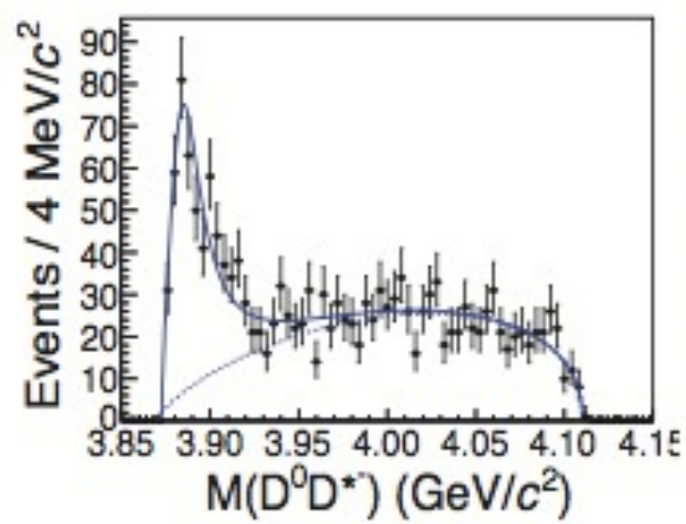


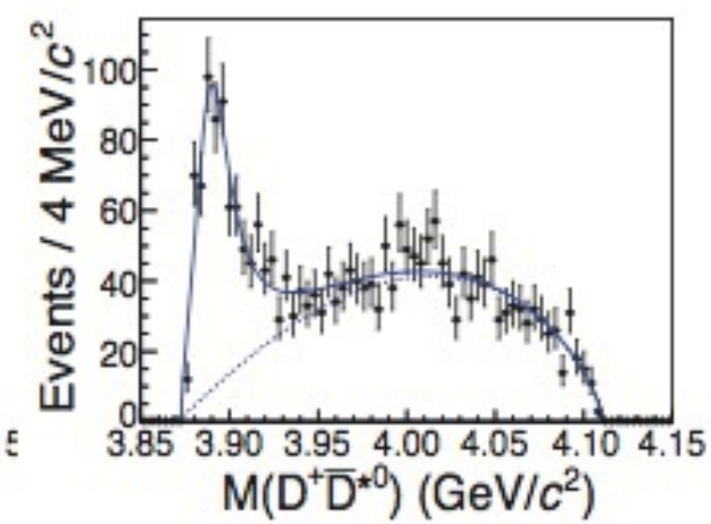
BESIII Coll., PRL110 (2013).



Belle Coll., PRL110 (2013).

- **e^+e^- --> $\pi^{+/-}$ $(D^{bar}D^*)^{-/+}$**

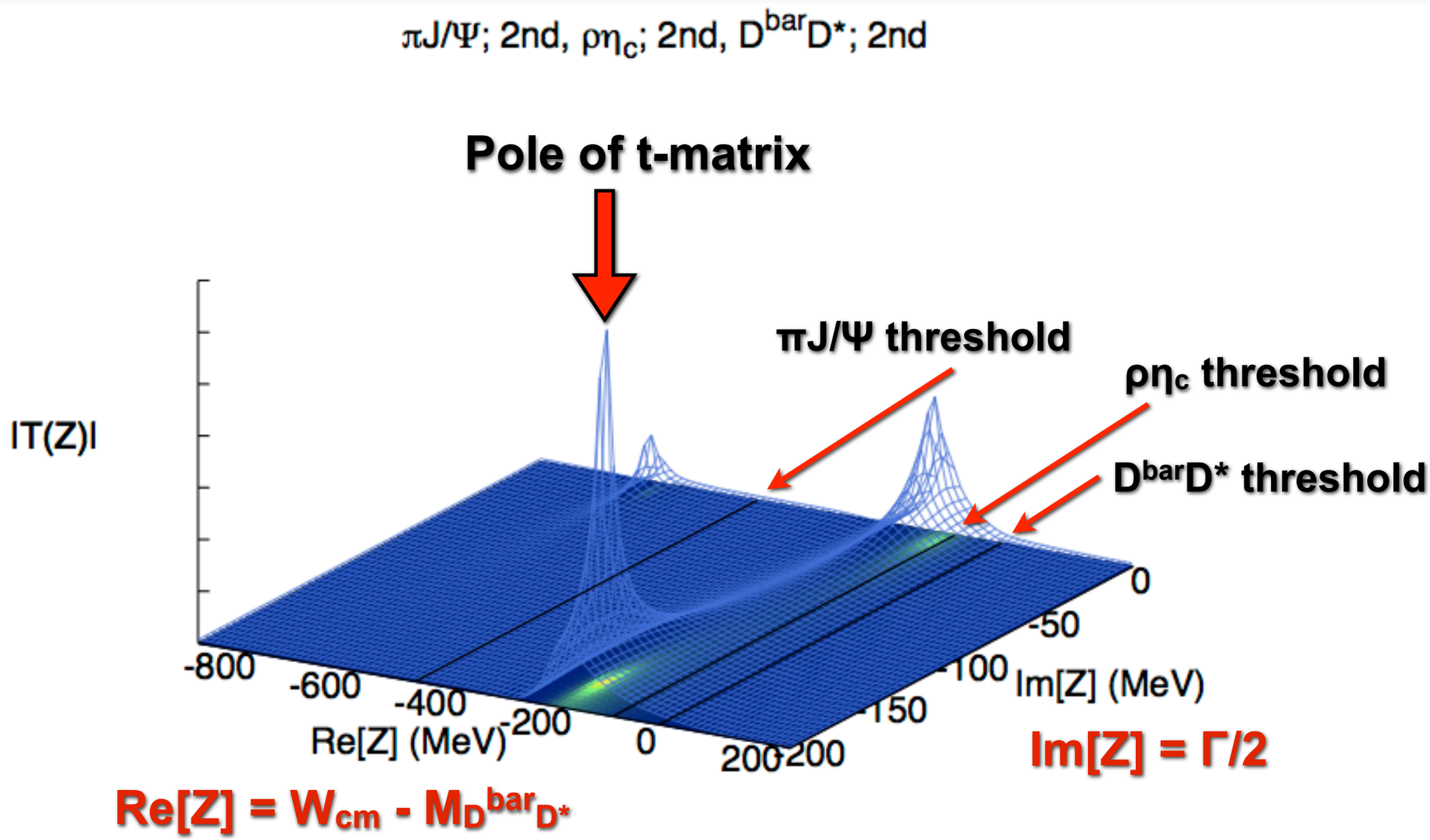




BESIII Coll., PRL112 (2014).

✔ **We observe similar line shapes of πJ/Ψ & $D^{bar}D^*$ inv. mass**

Pole search ($\pi J/\Psi$:2nd, $\rho\eta_c$:2nd, $D^{bar}D^*$:2nd)



- ✓ Pole for Zc(3900) on the most adjacent complex energy plane is found
- ✓ Origin of enhancement in 2-body invariant mass near $D^{bar}D^*$ threshold

Summary

✤ Applications of HAL QCD method to tetra-quarks, Tcc & Zc(3900)

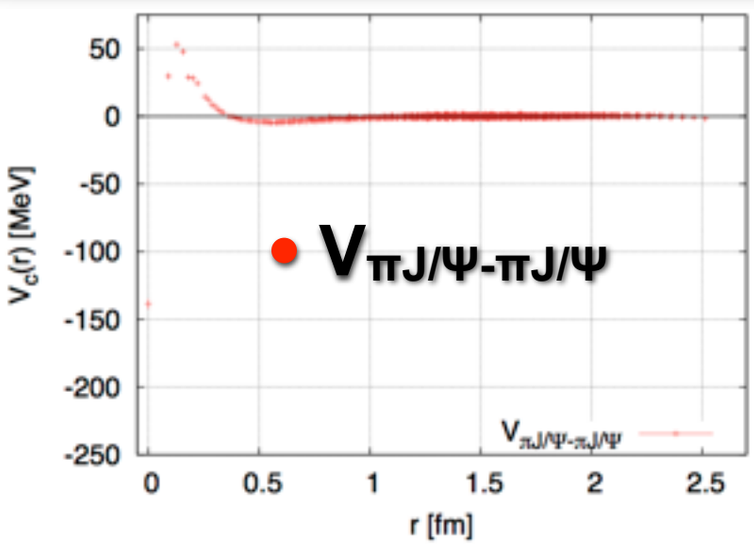
- T_{cc} search on the lattice@m_π=410--700MeV
  - ➡ T_{cc} is not bound for m_π>400MeV (T_{bb} is already bound)
  - ➡ sizable correlation of diquarks is found
    - I=0 good diquark channel : attractive
    - I=1 bad diquark channel : repulsive
- Zc(3900) in $I^G(J^P)=1^+(1^{+)}$ channel on the lattice@m_π=410MeV
  - ➡ Large channel coupling between $\pi J/\Psi$-$D^{bar}D^*$ is a key
  - ➡ Heavy quark spin symmetry is seen in c.c. potentials
    - Zc(3900) is neither simple $D^{bar}D^*$ molecule nor J/Ψ + π-cloud
    - pole on complex energy plane is found (w/ relatively large width)

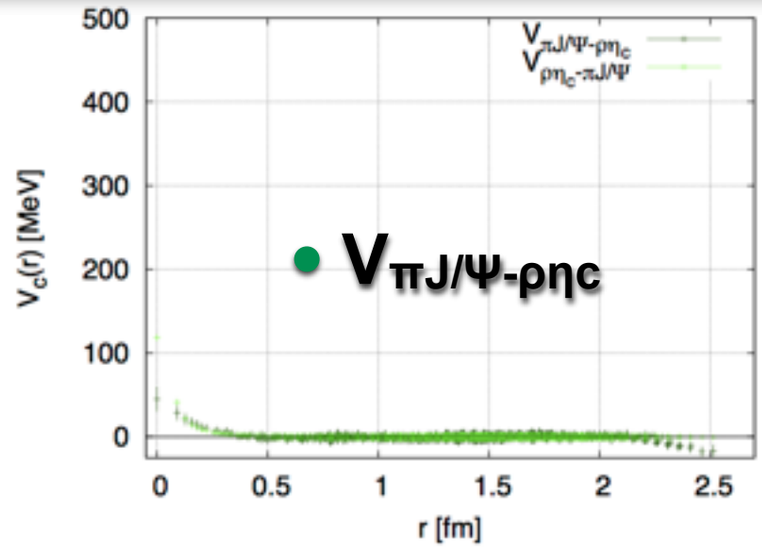
see also, S. Prelovsek et al., PRD91, 014504 (2015).

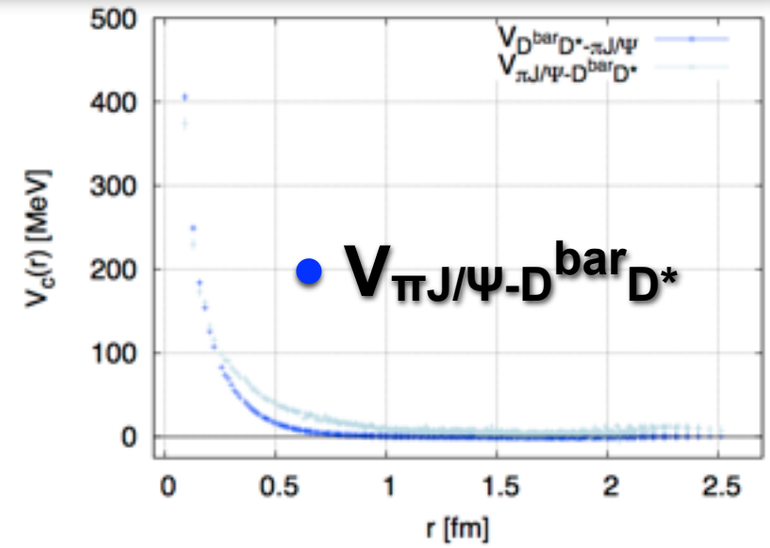
✤ Physical point simulation is the next step

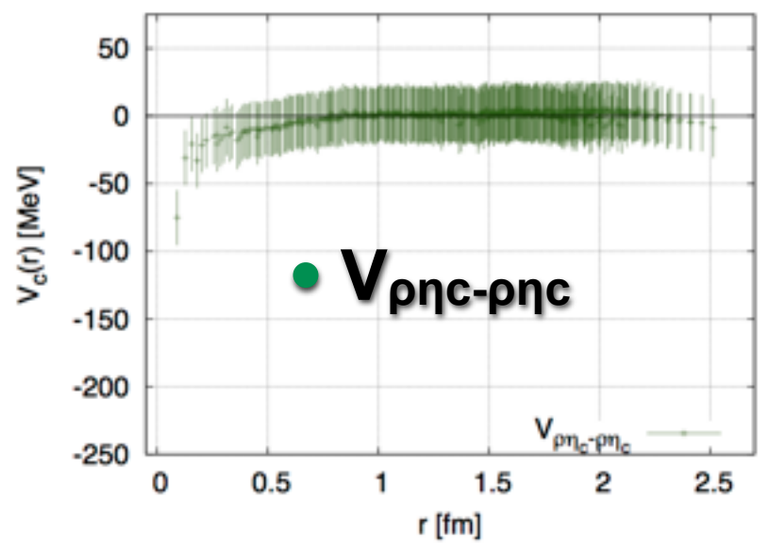
Backup

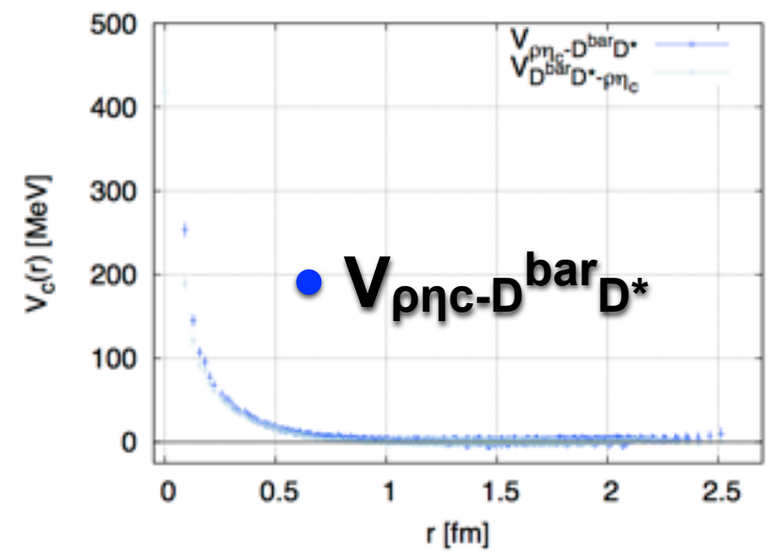
Potential matrix ($\pi J/\Psi$ - $\rho\eta_c$ - $D^{bar}D^*$)

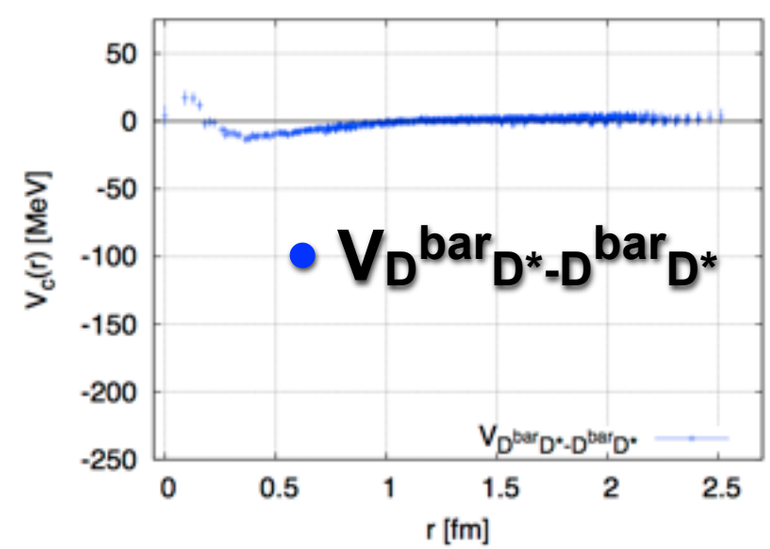










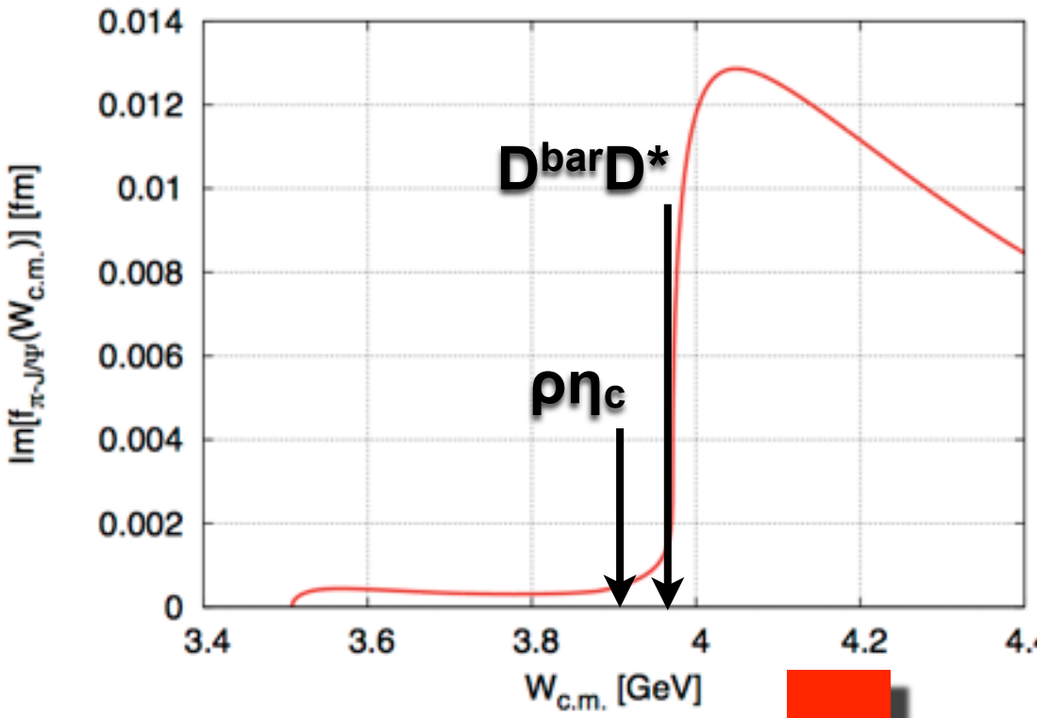


- **<u>Strong off-diagonal $D^{bar}D^*$ potentials</u>**
  - ✓ **heavy quark spin symmetry**
  - ✓ **strong charm-quark-exchange interactions**

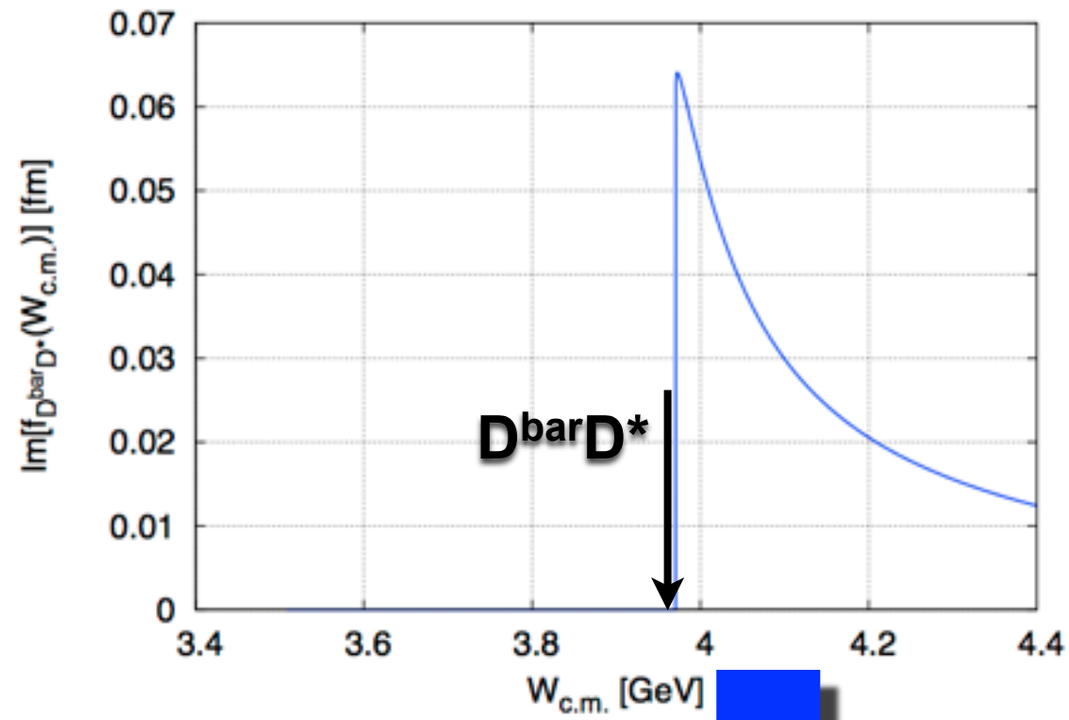
➡ $V_{DbarD^*-\pi J/\Psi} \sim V_{DbarD^*-\rho\eta c} >> V_{\pi J/\Psi-\rho\eta c}$

2-body invariant mass spectra

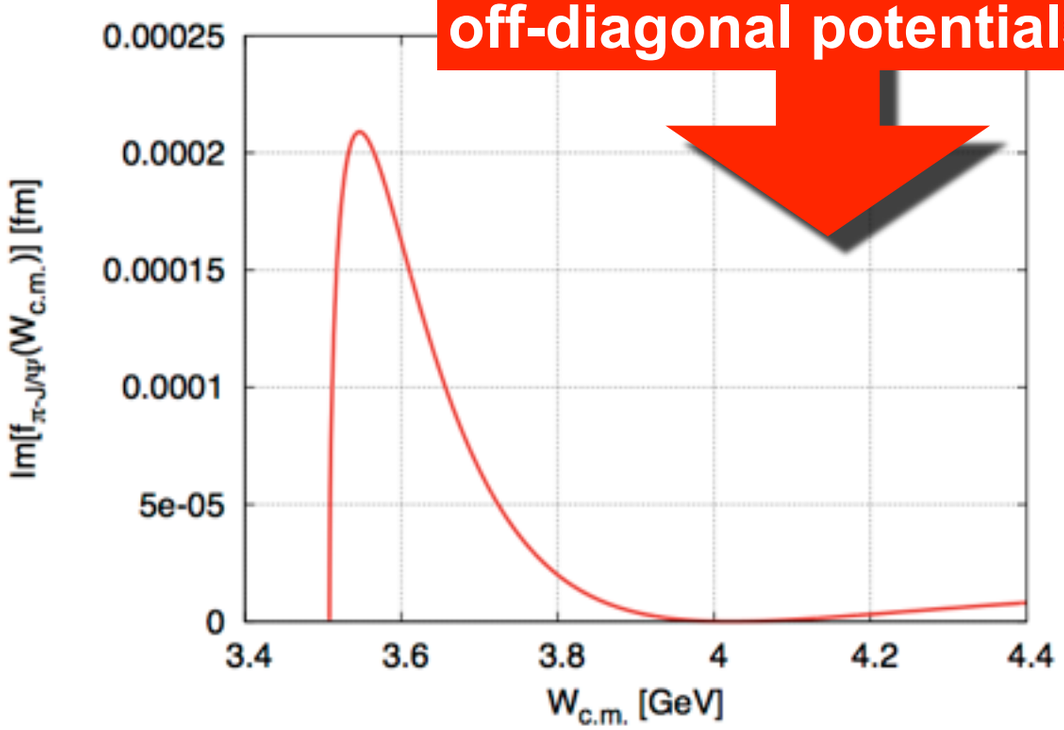
- πJ/Ψ invariant mass (m_π=410MeV)



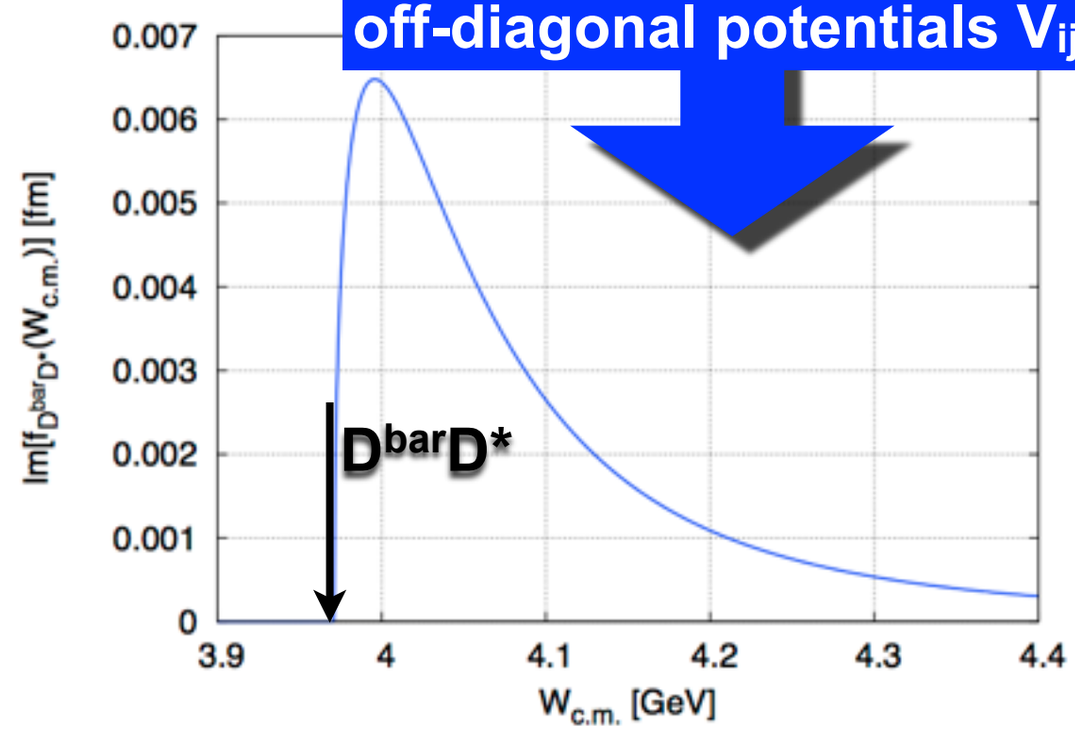
- $D^{bar}D^*$ invariant mass (m_π=410MeV)



off-diagonal potentials V_{ij}=0

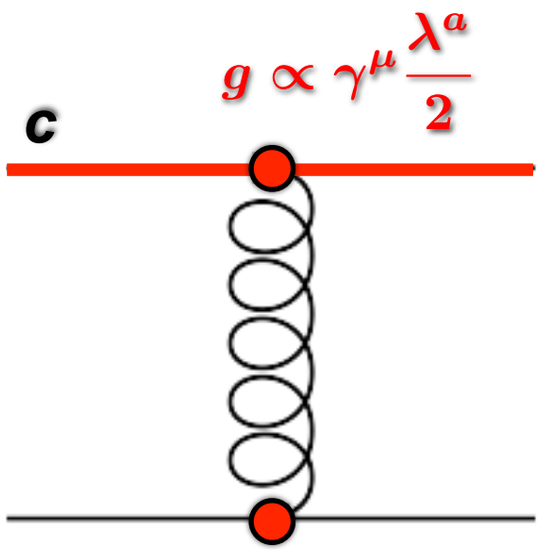


off-diagonal potentials V_{ij}=0



Backup

Tcc bound state



$$\sim \bar{c}\gamma^\mu \frac{\lambda^a}{2} c A^a$$ = color magnetic + color electric forces

- **Color magnetic interaction (CMI)** : hadron mass splitting

$$V_{ij}^{\mathrm{CMI}} \propto -\frac{\left(\vec{\lambda}(i)\cdot\vec{\lambda}(j)\right)\left(\vec{\sigma}(i)\cdot\vec{\sigma}(j)\right)}{M_i M_j}$$

- Color-spin matrix elements : $\langle v_{ij}\rangle = -\langle\left(\vec{\lambda}(i)\cdot\vec{\lambda}(j)\right)\left(\vec{\sigma}(i)\cdot\vec{\sigma}(j)\right)\rangle$

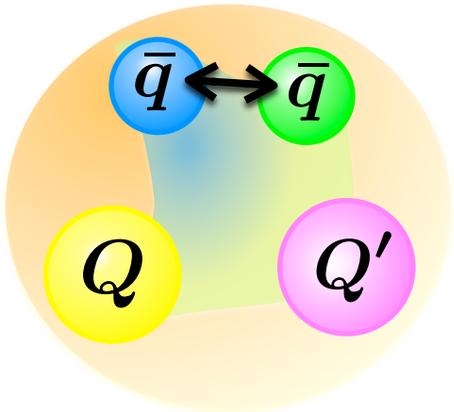
$\langle v_{ij}\rangle$	C=1	C=8	C=3	C=6^{bar}
S=0	-16	2	-8	4
S=1	16/3	-2/3	8/3	-4/3

- C=3, S=0 (I=0) : -8
- C=6^{bar}, S=1 (I=0) : -4/3
- C=3, S=1 (I=1) : 8/3
- C=6^{bar}, S=0 (I=1) : 4

attractive

repulsive

- CMI proportional to $1/M_i$: strongly attractive u^{bar}d^{bar}-diquark pair

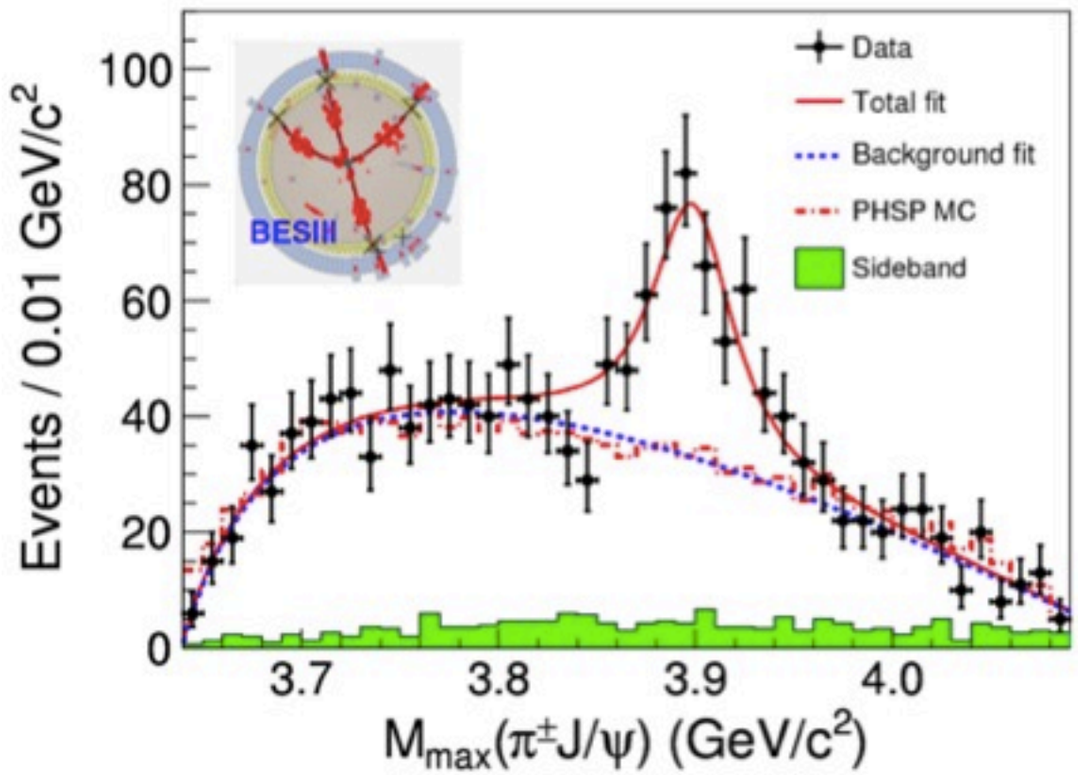


➡ Possibility of bound $T_{QQ'}$

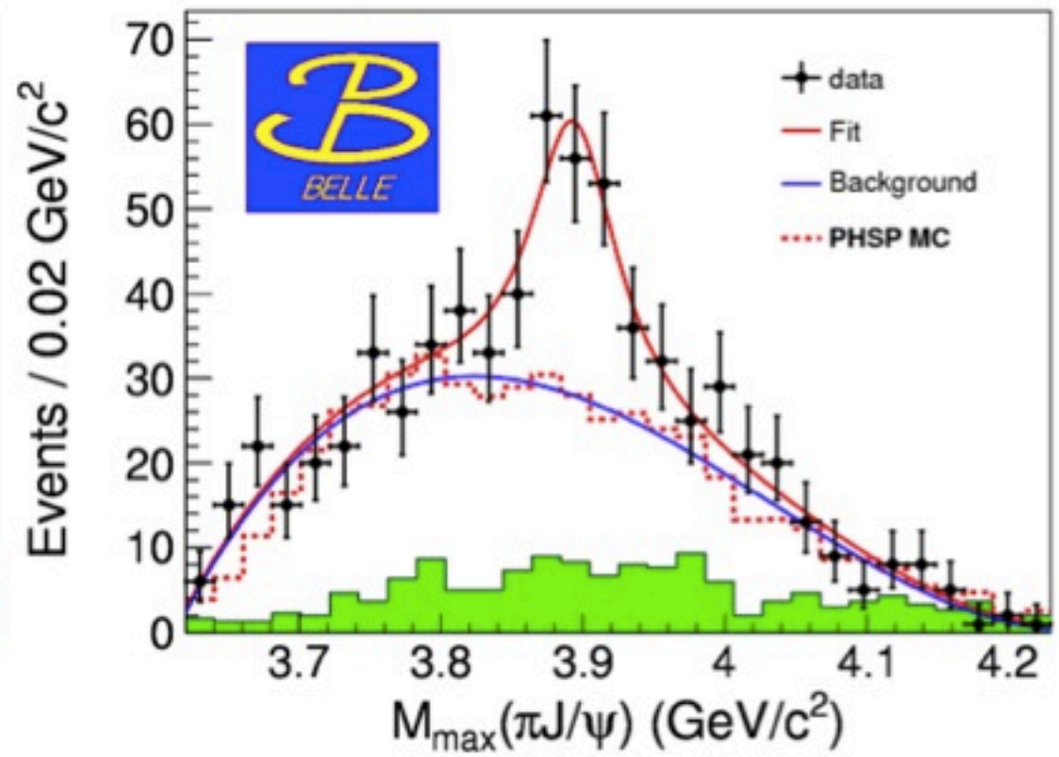
H. J. Lipkin, PLB172, 242 (1986).

Zc(3900) : experimental observations (I)

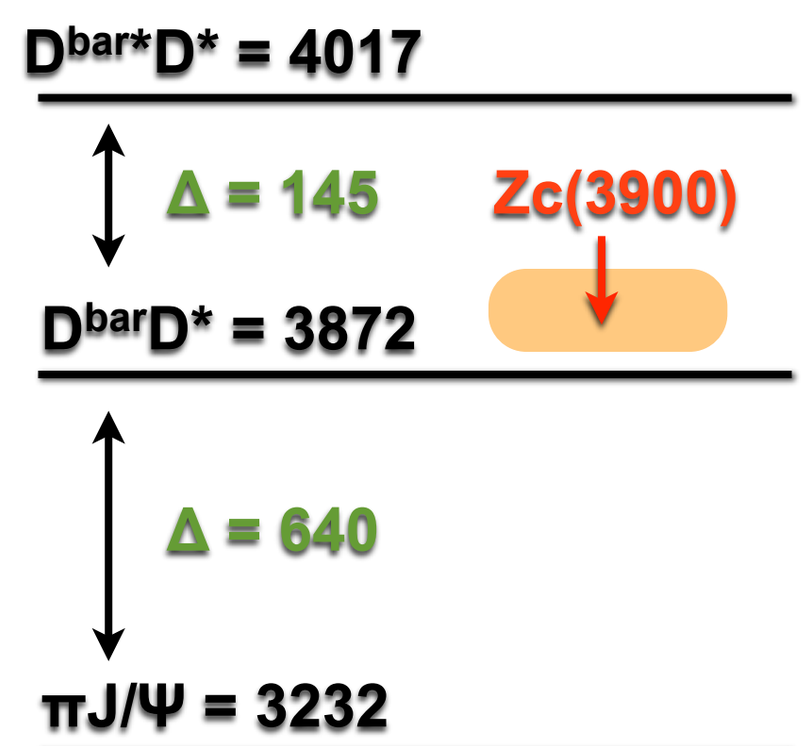
BESIII Coll., PRL110, 252001, (2013).



Belle Coll., PRL110, 252002, (2013).



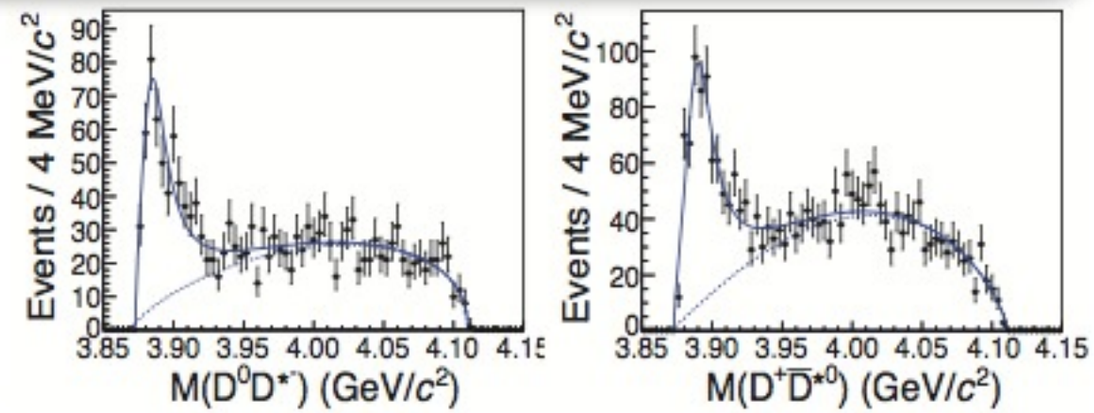
- Zc(3900) is observed in $\pi^{\pm}J/\Psi$ invariant mass
- Zc(3900) is charged state --> at least 4-quark?
- Isospin of Zc(3900) must be $I^G=1^+$
- M ~ 3900, Γ ~ 60 MeV from BW line shape
- Peak confirmed by CLEO Coll.

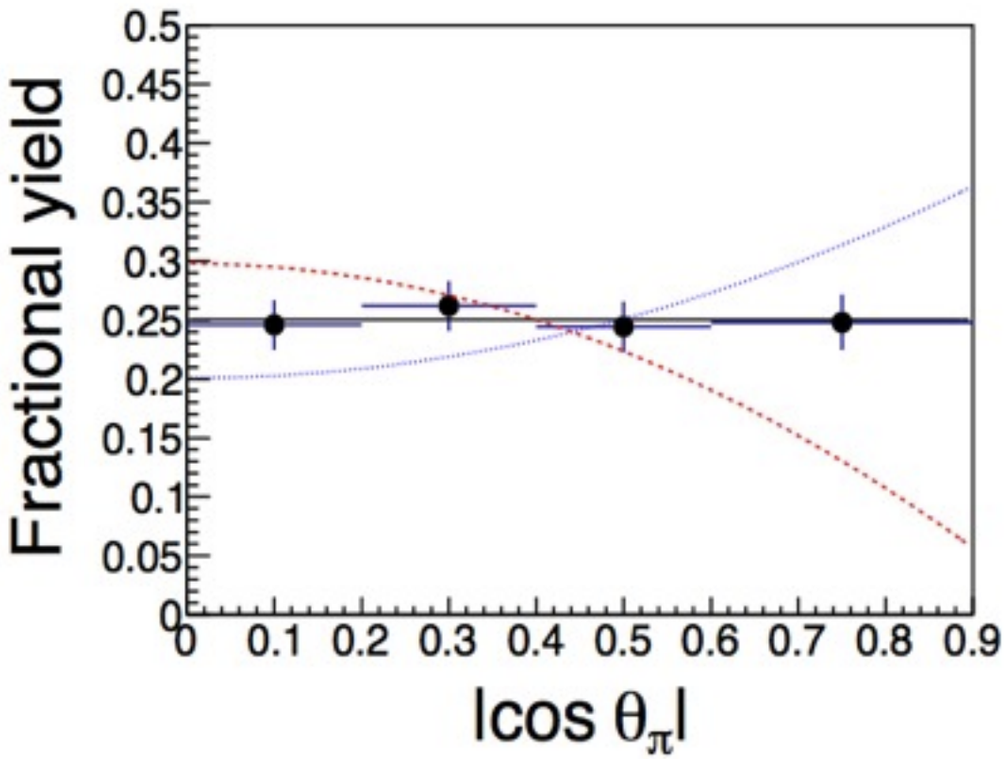


Zc(3900) : experimental observations (II)

- **What about J^P?**
  : $e^+e^- \to \pi^{+/-} (D^{bar}D^*)^{-/+}$ [$\pi^{+/-}$ Zc(3900)$^{-/+}$]

BESIII Coll., PRL112, 022001, (2014).





J^P of Zc(3900)$^{-/+}$

- **0^- : P-wave (J_z=1)--> $\sin^2\theta_\pi$**
- **0^+ : forbidden due to parity cons.**
- **1^- : P-wave --> $1+\cos^2\theta_\pi$**
- **1^+ : S/D-wave --> flat dist.**

- **What about coupling?**

Partial width of Zc(3900)$^{-/+}$

$$\frac{\Gamma(Z_c(3900) \to \bar{D}D^*)}{\Gamma(Z_c(3900) \to \pi J/\Psi)} \simeq 6.2$$

Zc(3900)$^{-/+}$ by BESIII analysis

- **$I^G J^P$=1^+1^+**
- **large coupling to $D^{bar}D^*$**
  --> consistent with small width
- **just above $D^{bar}D^*$ threshold**

S-wave $D^{bar}D^*$ molecule???

Zc(3900) : models

- **Tetraquark picture : diquark-antidiquark model** Maiani et al., PRD71 (2005).

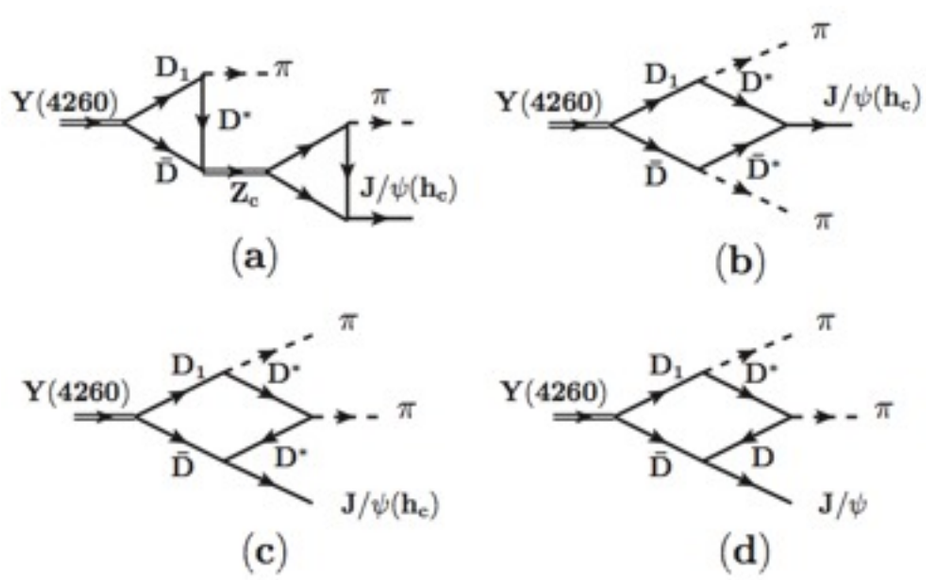
  ➡ diquark mass is unknown (fixed to reproduce X(3872))

  ✓ M=3882 MeV, $J^P=1^+$ (S-wave $D^{bar}D^*$ channel)

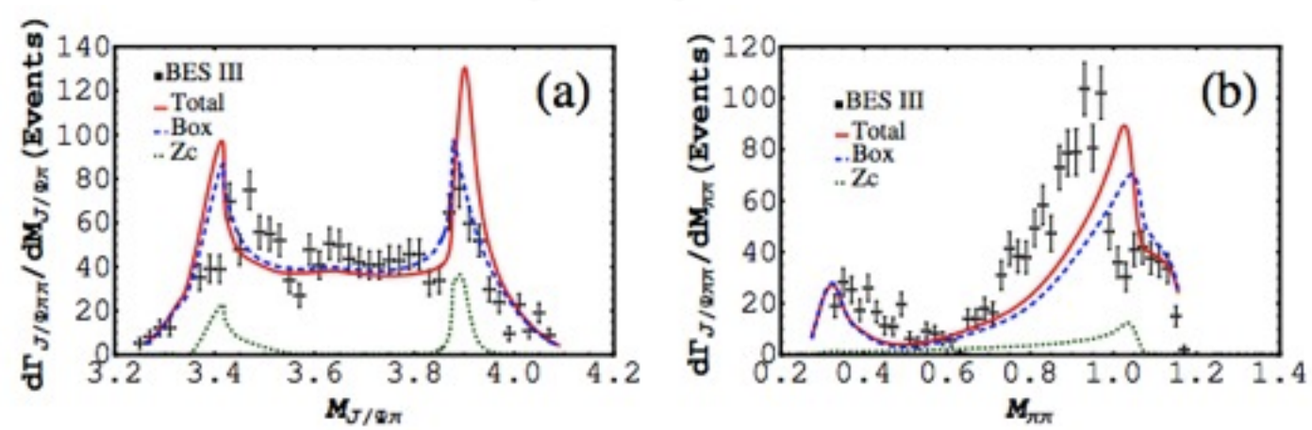
$$\frac{\Gamma^{\mathrm{Model}}(Z_c(3882) \to \bar{D}D^*)}{\Gamma^{\mathrm{Model}}(Z_c(3882) \to \pi J/\Psi)} = \frac{4(\mathrm{MeV})}{29(\mathrm{MeV})}$$

Maiani et al., PRD87 (2013).

- **Molecule picture :**
  - πJ/Ψ + π-cloud M.B. Voloshin, PRD87 (2013) 9, 091501.
  - **Zc(3900) pole + $D^{bar}D^*$ cloud :** Wang et al., PRL111 (2013).



➡ Y(4260) is assumed to be bound state of $DD_1(2420)$

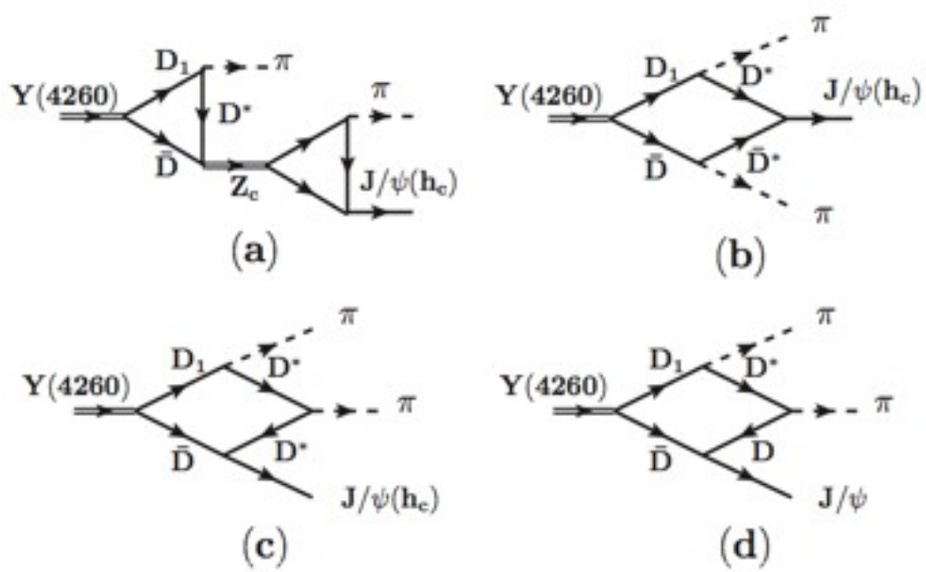


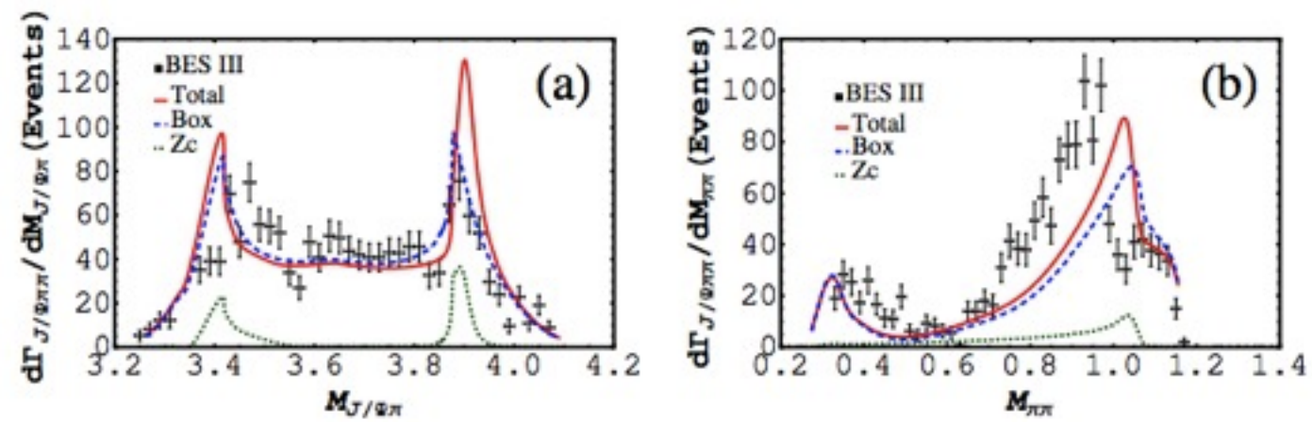
Zc(3900) : models for decay process

- **Zc(3900) pole + $D^{bar}D^*$ cloud :**

Wang et al., PRL111 (2013).

➡ **Y(4260) is assumed to be bound state of $DD_1(2420)$**





- **No Zc(3900) pole : initial-state pion exchange mechanism**

Chen-Liu-Matsuki, PRD88 (2013).