Progress towards an *ab initio*, Standard Model calculation of direct CP-violation in K decays

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Baryogenesis

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} = 6.14(25) \times 10^{-10}$$

- The Universe is matter dominated, but why?
- Most likely explanation is existence of baryogenesis mechanism.
- Sackarov conditions (1967):
  - B-number violation
  - Non-thermal interactions (e.g. during a phase transition)
  - C and CP violation.
- Amount of CP-violation in Standard Model far too small to account for observed value.
- Most BSM theories introduce additional direct CP-violation but a precise SM value does not yet exist which could be compared to experiment.

$\mathbf{K} \to \pi\pi$ Decays

- Direct CP-violation first observed in K→ππ decays.
- Two types of decay:

$$\begin{array}{llll} \Delta I = 3/2 & : K^+ \to (\pi^+\pi^0)_{I=2} & \text{with amplitude} & A_2 \\ \Delta I = 1/2 & : K^0 \to (\pi^+\pi^-)_{I=0} & \text{with amplitude} & A_0 \\ & \quad K^0 \to (\pi^0\pi^0)_{I=0} & & \end{array}$$

- Direct CP-violation:

$$\epsilon' = \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}} \left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \right)$$

  where

  $\omega = \mathrm{Re}A_2/\mathrm{Re}A_0$ and δ_I are ππ scattering phase shifts.

- ϵ' is highly sensitive to BSM sources of CPV.
- Strong interactions very important – origin of the ΔI=1/2 rule: preference to decay to I=0 final state.

[arXiv:1212.1474]

Lattice determination

MSbar renormalization
matrix using NPR and
perturbative matching
at high scale

$$\mathcal{A}_I = \sum_{i=1}^{7}\sum_{j=1}^{10} \mathcal{F}\frac{G_F}{\sqrt{2}} V_{us}^* V_{ud}[z_i(\mu) + \tau y_i(\mu)] Z_{ij}(\mu) \langle (\pi\pi)_I | Q_j(\mu) | K \rangle$$

Lellouch-Luscher
factor relates
finite to inf. vol.

MSbar perturbative
Wilson coeffs for
Weak effective
theory

10 Weak effective
four-quark operators
(7 independent)

$$\tau = -\frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} = 0.0014606 + 0.00060408i$$

This imaginary part is responsible for the CP-violation!
(everything else is pure-real)

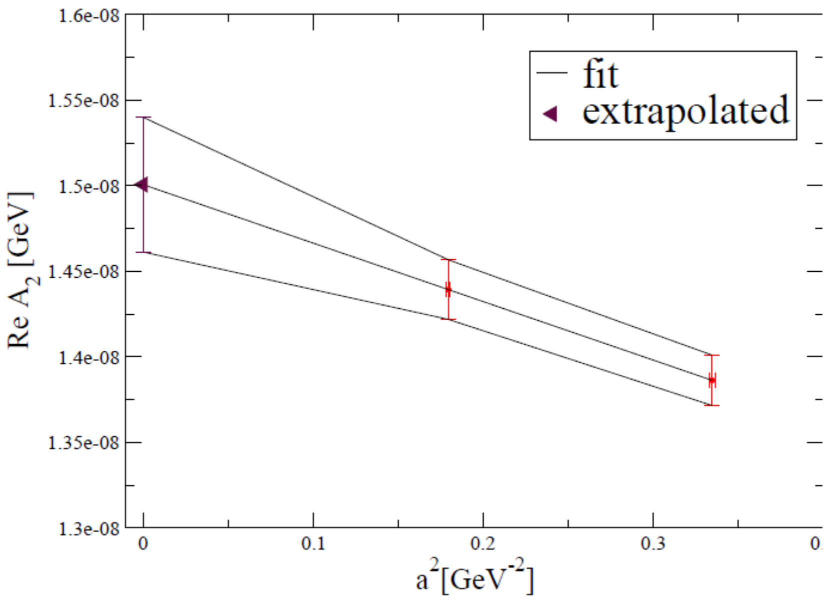
$\Delta I=3/2$ Calculation

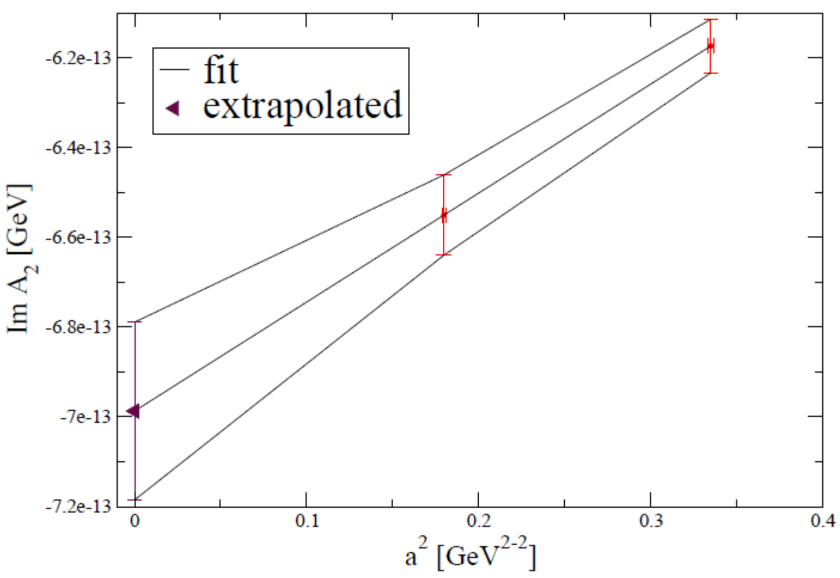
- Original physical measurement [Phys.Rev.Lett. 108 (2012) 141601]

$$\mathrm{Re}(A_2) = 1.38(5)_{\mathrm{stat}}(26)_{\mathrm{sys}} \times 10^{-8}\ \mathrm{GeV}$$
$$\mathrm{Im}(A_2) = -6.54(46)_{\mathrm{stat}}(120)_{\mathrm{sys}} \times 10^{-13}\ \mathrm{GeV}$$

20% sys error dominated by 15% discretization error

- Calculation has now been repeated on RBC & UKQCD 48^3x96 and 64^3x128 Mobius DWF ensembles with (5 fm)^3 volumes and a=0.114 fm, a=0.084 fm.
- Make full use of eigCG and AMA to translate over all timeslices. Obtain 0.7-0.9% stat errors on all bare matrix elements!





- New results published Monday: [arXiv:1502.00263]

$$\mathrm{Re}(A_2) = 1.50(4)_{\mathrm{stat}}(14)_{\mathrm{sys}} \times 10^{-8}\ \mathrm{GeV}$$
$$\mathrm{Im}(A_2) = -6.99(20)_{\mathrm{stat}}(84)_{\mathrm{sys}} \times 10^{-13}\ \mathrm{GeV}$$

10%, 12% total errors on Re, Im!

Systematic errors in $\mathrm{Im}A_2/\mathrm{Re}A_2$	48^3	64^3	cont
NPR (nonperturbative)	0.1%	0.1%	0.1%
NPR (perturbative)	7.6 %	6.7 %	7.6 %
Finite volume corrections	3.5 %	3.5 %	3.5 %
Unphysical kinematics	1.8 %	4.6%	4.6%
Wilson coefficients	12.0 %	10.5 %	12.0%
Derivative of the phase shift	0	0	0
Total	14.7%	13.7%	15.3%

TABLE XIII: Systematic error breakdown for $\mathrm{Im}A_2/\mathrm{Re}A_2$.

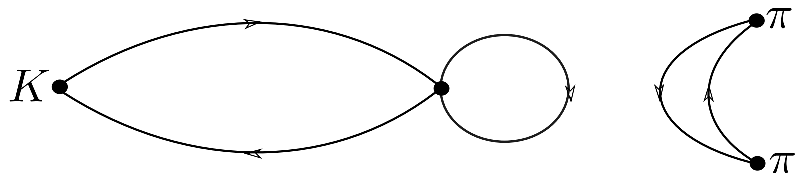
- Systematic error completely dominated by perturbative error on NPR and Wilson coefficients.
- Future considerations:
  - Higher order PT calculation of NPR and Wilson coeffs.
  - Step-scaling NPR to higher energy scale.

$\Delta I=1/2$ Calculation

- A_0 is significantly more difficult than A_2 for two reasons:

1) Disconnected diagrams

  - ππ has same quantum numbers as vacuum, hence there are disconnected diagrams of the form:



  - These are extremely noisy and dominate stat error.
  - We use A2A method with O(1000) exact low modes and stochastic high modes with spin, color and flavor dilution.
  - Disconnected diagram evaluated for all lattice sites for maximum statistical resolution.

2) Obtaining Physical Kinematics

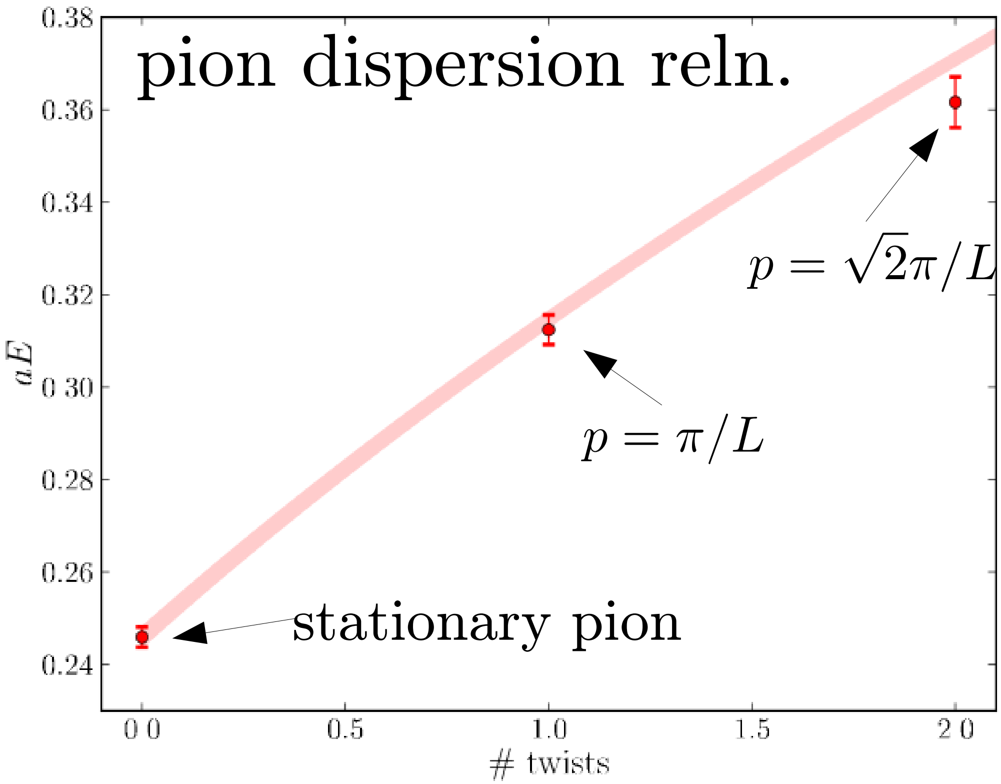
- Physical decay is energy conserving but lattice ground-state comprises two stationary pions (after explicit vacuum subtraction for I=0): $2\,E_\pi \ll m_K$
- Avoid multi-exponential fits by modifying spatial BCs to remove stationary pion state.
- Must measure $K^0 \to \pi^0\pi^0$ and $K^0 \to \pi^+\pi^-$ with I=0 final state. Boundary conditions must therefore:
  - ➔ Conserve isospin such that I=0 state can be isolated.
  - ➔ Give momentum to both charged and neutral pions.
- In I=2 calculation we used twisted BCs applied to the d-quark, but this satisfies neither of the above conditions.
- Instead, we use G-parity BCs:

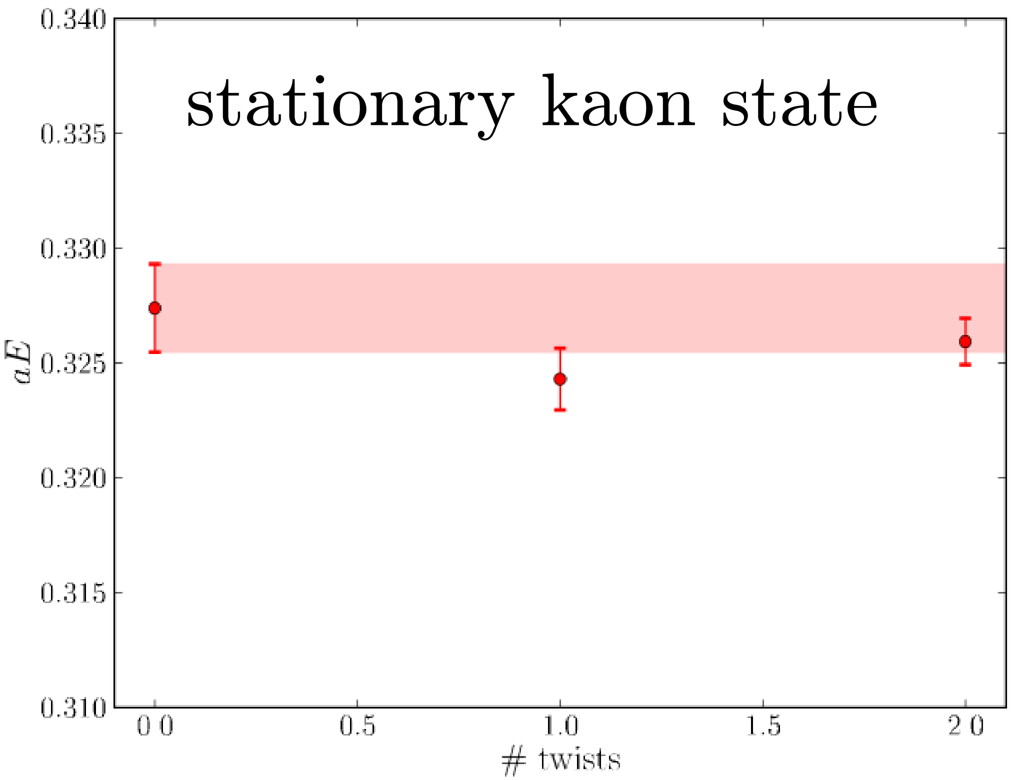
$$\hat{G} = \hat{C} e^{i\pi \hat{I}_y} \quad : \quad \begin{aligned} \hat{G}|\pi^\pm\rangle &= -|\pi^\pm\rangle \\ \hat{G}|\pi^0\rangle &= -|\pi^0\rangle \end{aligned}$$

- As a boundary condition: $\pi^i(x+L) = \hat{G}\pi^i(x) = -\pi^i(x)$ ➔ Discretized lattice momenta $|p| \in (\pi/L,\, 3\pi/L,\, 5\pi/L \ldots)$ Moving ground-state

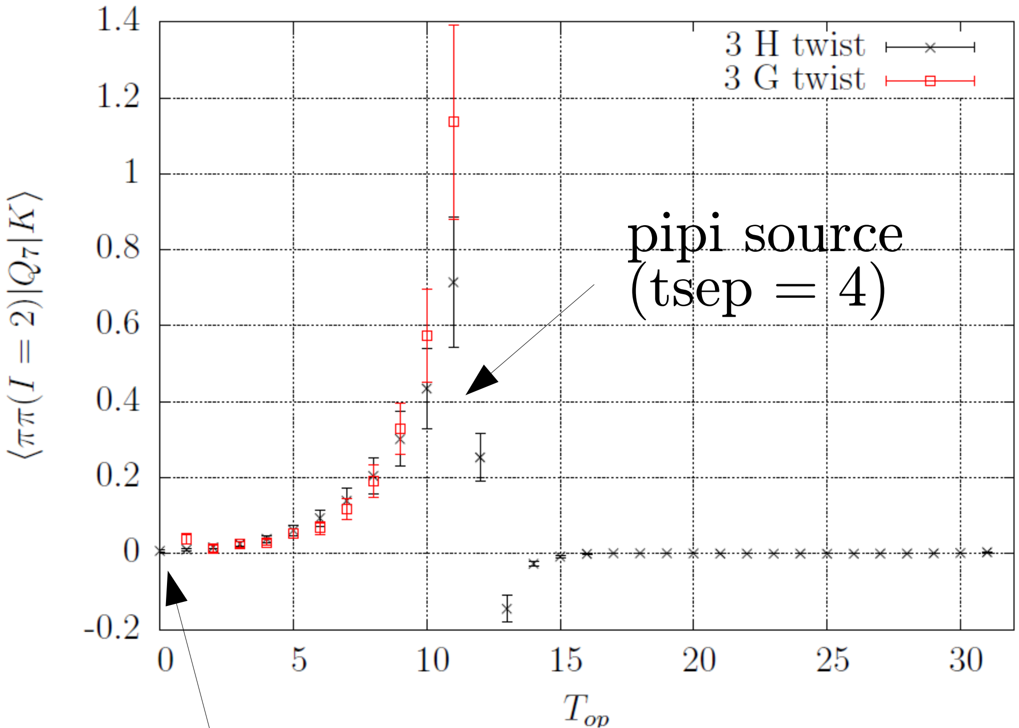
Demonstration

- 16^3x32 DWF+Iwasaki a^{-1} = 1.73(3) GeV with 420 MeV pion test configurations with GPBC in 0,1,2 directions.

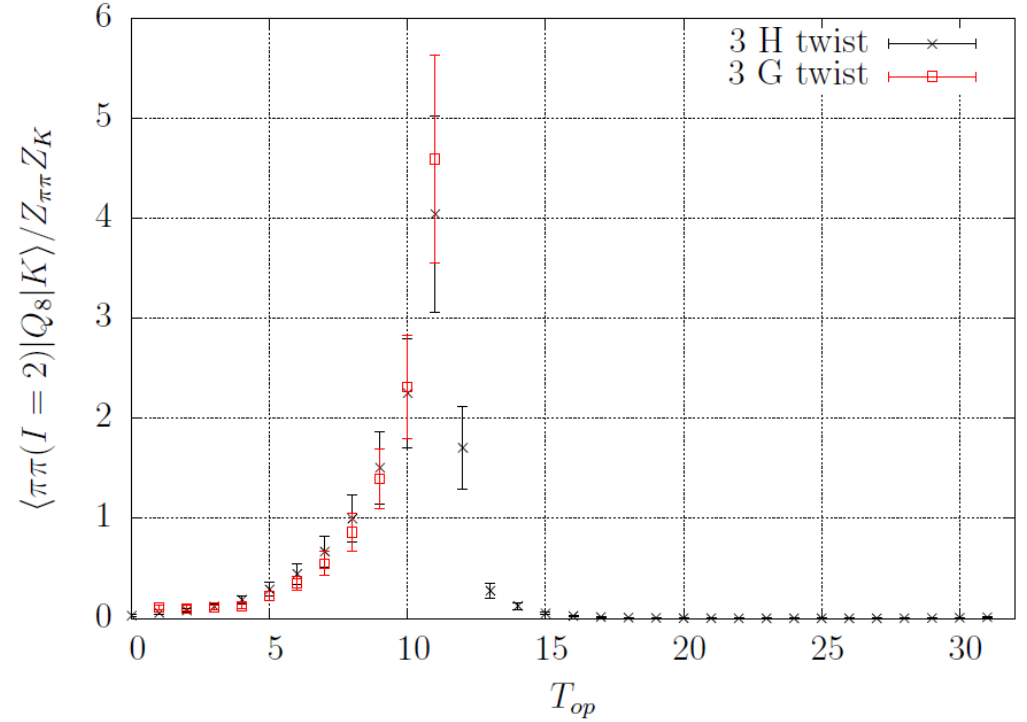




Direct comparison of ΔI=3/2 amplitudes between GPBC and twisted BCs, both in 3 directions, on 16^3 ensembles



kaon source



i	3 G twist	3 H twist
1	1.92(19)e-03	2.10(12)e-03
7	6.41(45)e-03	6.03(32)e-03
8	2.98(16)e-02	2.91(15)e-02

Table 7: Fitting results of $\frac{\langle \pi\pi|Q_i|K\rangle}{Z_K Z_{\pi\pi} e^{-E_{\pi\pi}\Delta T}}$. Fitting range [4:8].

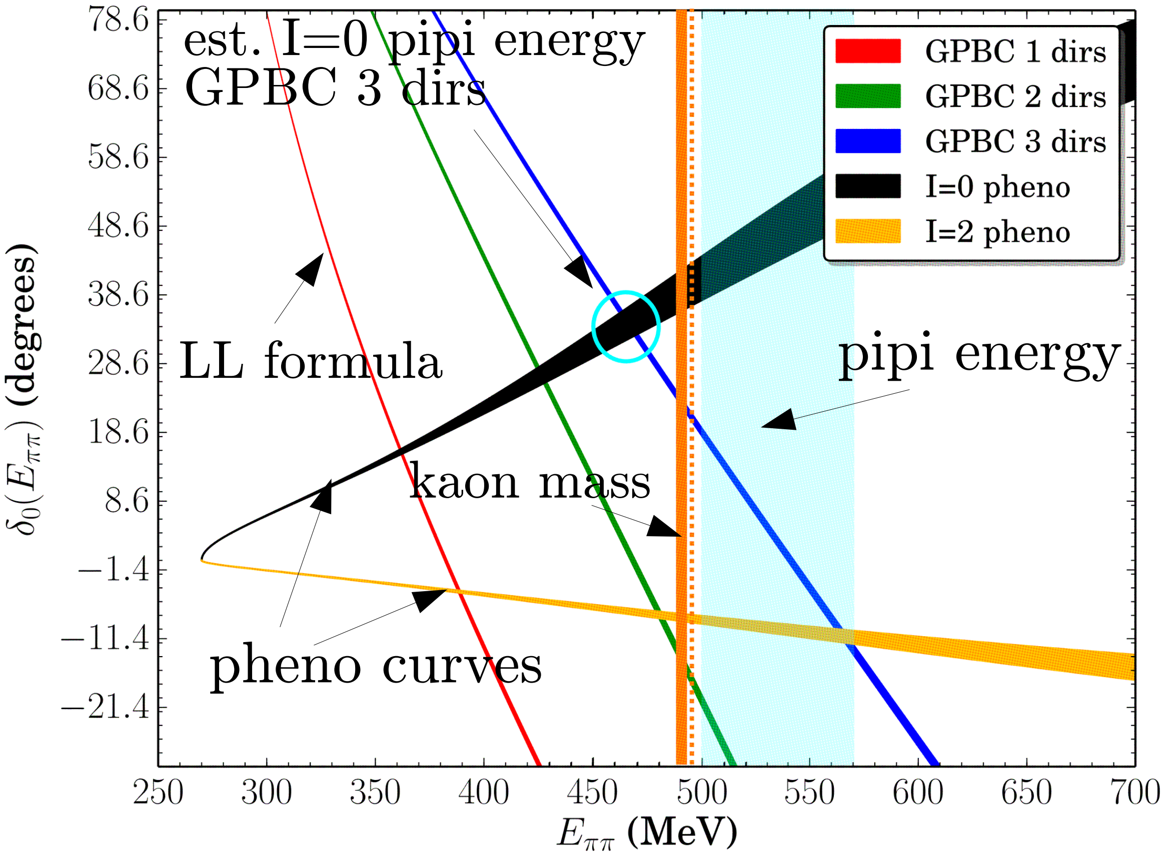
	3 G twist	3 H twist
$E_{\pi\pi(I=2)}$	0.922(25)	0.955(27)

Table 8: Fitting results of pipi(I=2)

Preliminary Results

Physical Ensemble

- To-date generated 988 configs (~688 thermalized).
- Utilizing USQCD 512-node BG/Q machine.



- For 32^3x64 a^{-1} ~1.38 GeV DWF+IDSDR β=1.75 close match with GPBC in 3 dirn.
- 4.6 fm^3 box.
- Measure (161 confs):

  E_π=274.7(14) MeV

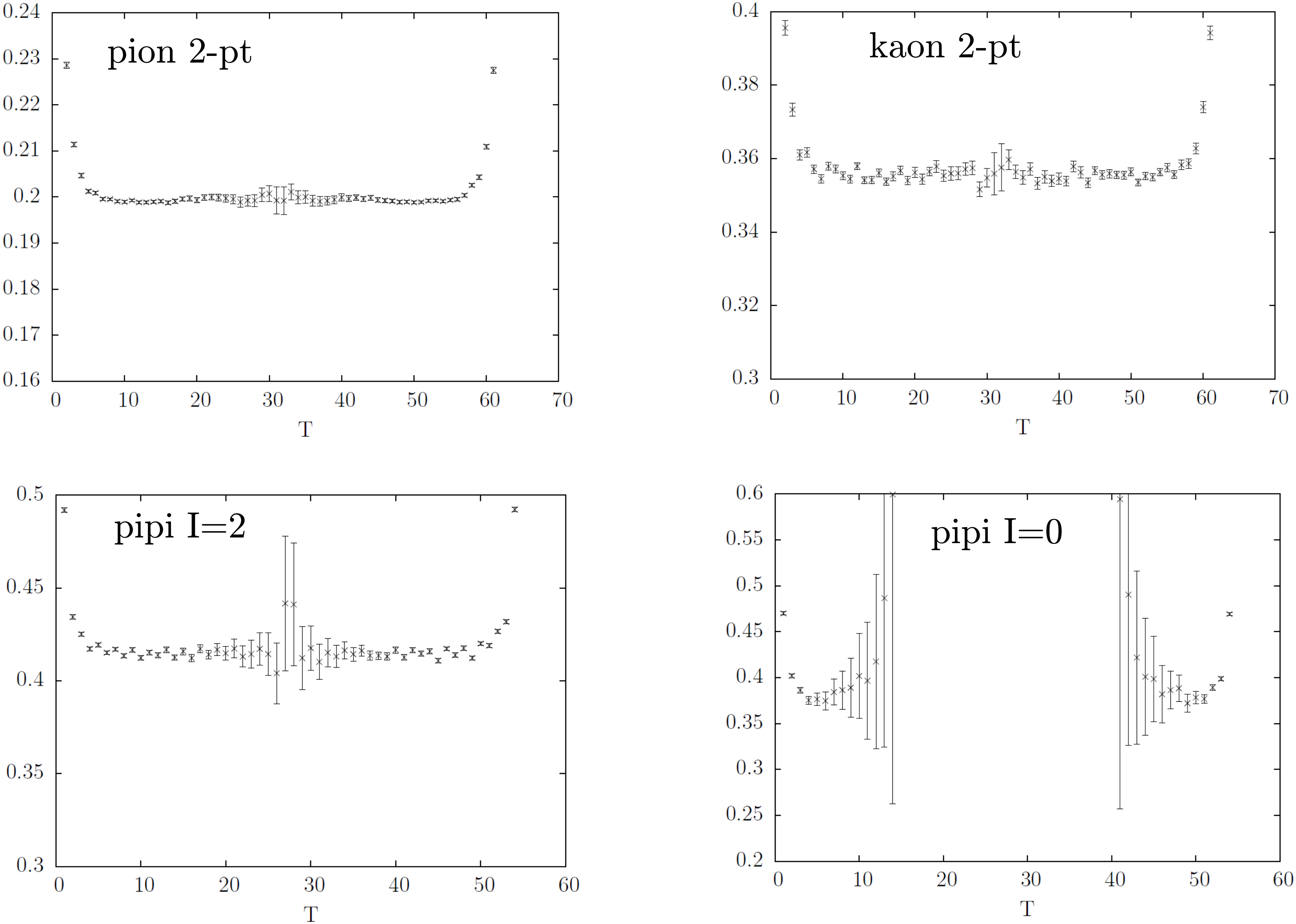
  m_π~143.2(11) MeV

  m_K=490.2(24) MeV

  $E_{\pi\pi}$(I=0) = 534(34) MeV

  $E_{\pi\pi}$(I=2) = 572(3) MeV

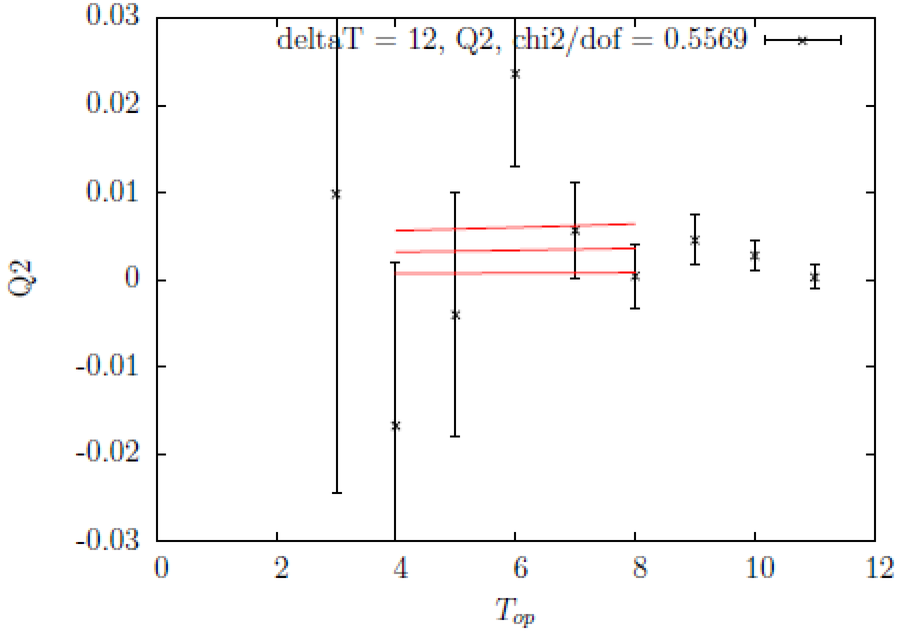
Effective Energies



- Measure with K-pipi separations of 10,12 and 14.
- Currently measured 161 configurations.
- Error is completely dominated by disconnected diagrams.

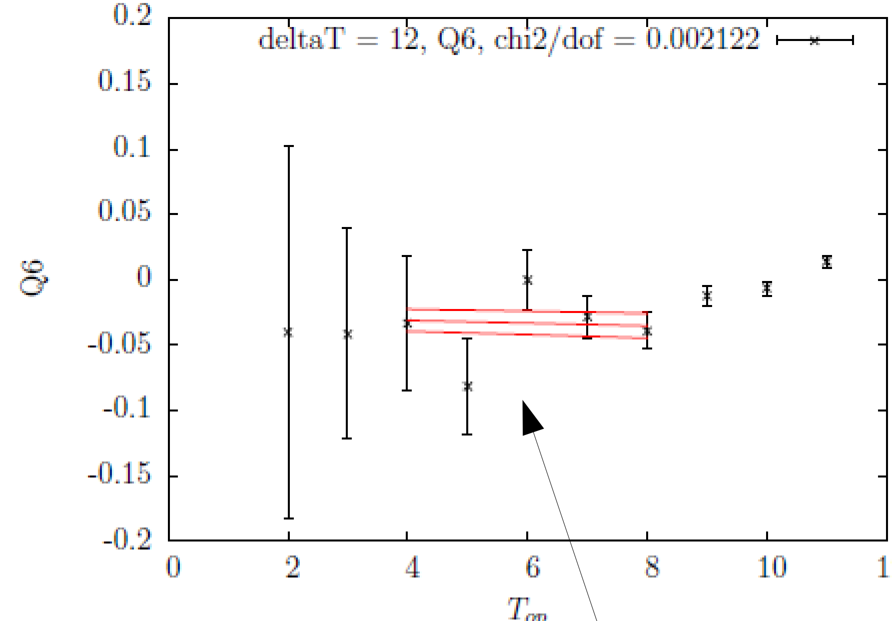
$$\frac{\langle \pi\pi(t{=}12)|Q_2(t{=}T_{op})|K(t{=}0)\rangle}{Z_K Z_{\pi\pi} e^{-m_{\pi\pi}*12}}$$

[Dominant contribution to Re(A_0)]



$$\frac{\langle \pi\pi(t{=}12)|Q_6(t{=}T_{op})|K(t{=}0)\rangle}{Z_K Z_{\pi\pi} e^{-m_{\pi\pi}*12}}$$

[Dominant contribution to Im(A_0)]



slight slope is due to small difference between kaon and pipi energies

- Currently obtain 35% errors on Im(A_0) and 30% on Re(A_0).
- Re(A_0) can be precisely determined in expt, so only Im(A_0) is important.
- Currently obtain stat error on ε' approx 2x experimental error if we use Re(A_0) from expt.

Systematic Errors

Two main sources of sys error:

- Discretization effects (~15%)
  - We currently measure using only one coarse lattice spacing.
  - Future calculations will need to be performed on multiple lattice spacings, like A_2 analysis.
- Wilson coefficients (unknown)
  - Unlike A_2, charm effects possibly play a significant role.
  - Our calculation is performed in the 3-flavor effective theory where the charm has been integrated out perturbatively.
  - Charm is light so it is not clear how reliable this is.
  - Ultimately we will need to perform a full 4-flavor dynamical calculation.
  - To estimate error we are looking into a direct comparison of a 3-flavor threshold calculation and a 4-flavor calculation on the same lattice (partially-quenched charm).

Conclusions

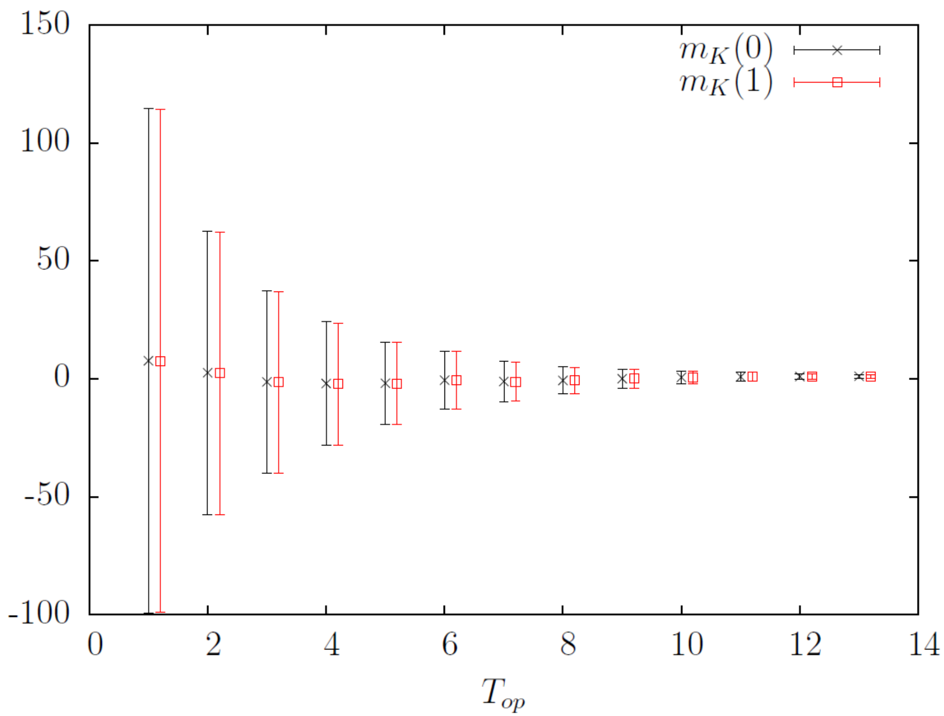
Conclusions and Outlook

- We have now measured A_2 with 2-3% stat error and 10% systematic.
- Sys. error is dominated by perturbative matching to MSbar and can be reduced by higher-order calculation or step-scaling to higher energies.

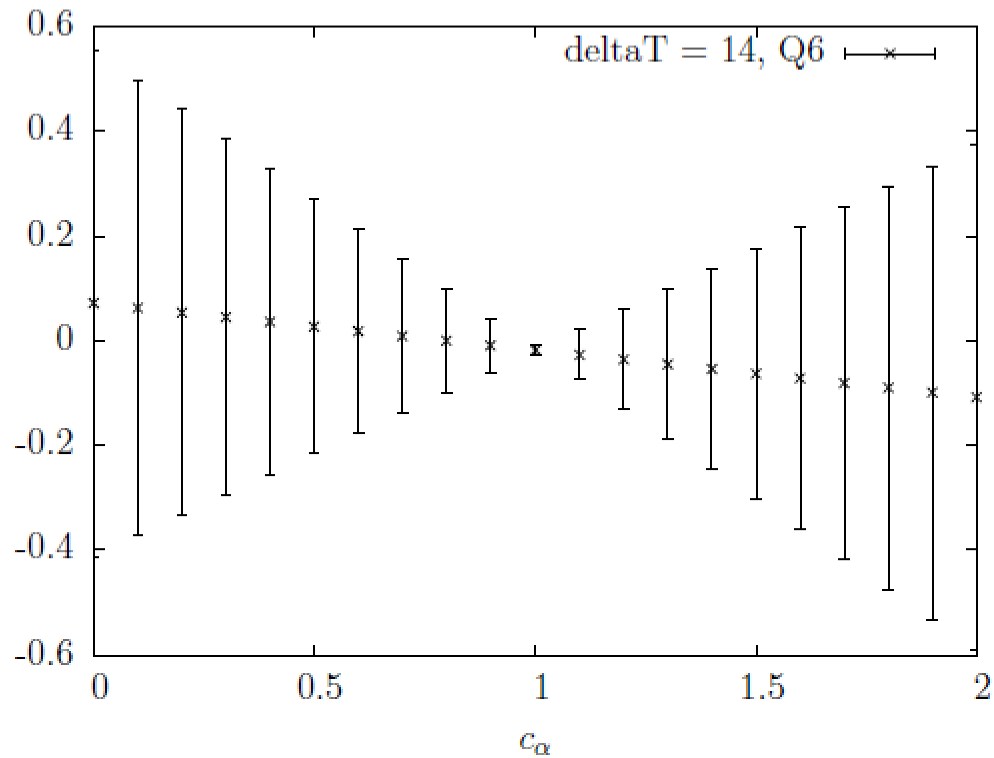
- A_0 calculation has begun using a single coarse lattice but with physical kinematics.
- Preliminary results from 161 meas give 35% stat errors on $\mathrm{Im}(A_0)$ and error on ε' about 2x expt. if we use $\mathrm{Re}(A_0)$ from expt.
- Sys. errors dominated by discretization effects and use of 3-flavor Wilson coeffs. Future calculations will need to be performed using multiple 4-flavor dynamical ensembles.

Subtraction term consistent with 0

$\frac{\langle \pi\pi(t{=}14)|\bar{s}\gamma_5 d(t{=}T_{op})|K(t{=}0)\rangle}{Z_K Z_{\pi\pi} e^{-m_K * t_{op} - m_{\pi\pi} * (14 - t_{op})}}$ term depending on Kaon mass. Using 69 configurations



Significant effect on stat error



$$\frac{\langle \pi\pi(t{=}14)|Q_6(t{=}T_{op}) - c_\alpha \alpha_i(t{=}T_{op})\bar{s}\gamma_5 d(t{=}T_{op})|K(t{=}0)\rangle}{Z_K Z_{\pi\pi} e^{-m_{\pi\pi} * 14}}.$$

Subtraction essential for signal to be obtained

G-parity BCs

- At quark level: $\hat{G}\begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} -C\bar{d}^T \\ C\bar{u}^T \end{pmatrix}$ where $C = \gamma^2\gamma^4$. in our conventions
- Gauge invariance requires gauge field to obey charge conjugation (complex conjugate) boundary conditions.
- New ensembles needed (true for all modifications of BCs due to disconnected diagrams).
- For stationary kaon eigenstates we must introduce a fictional partner to the strange quark; s'

  $|\tilde{K}^0\rangle = \left(|\bar{s}d\rangle + |\bar{u}s'\rangle\right)/\sqrt{2}$ is G-parity even
- Must take root of s/s' determinant to remove it from action; introduces non-locality that vanishes exponentially in L.

Implementation

- Dirac operator applied simultaneously to two fermion fields that mix at the global lattice boundary.
- Naively expect factor of 2 in cost due to two flavors.

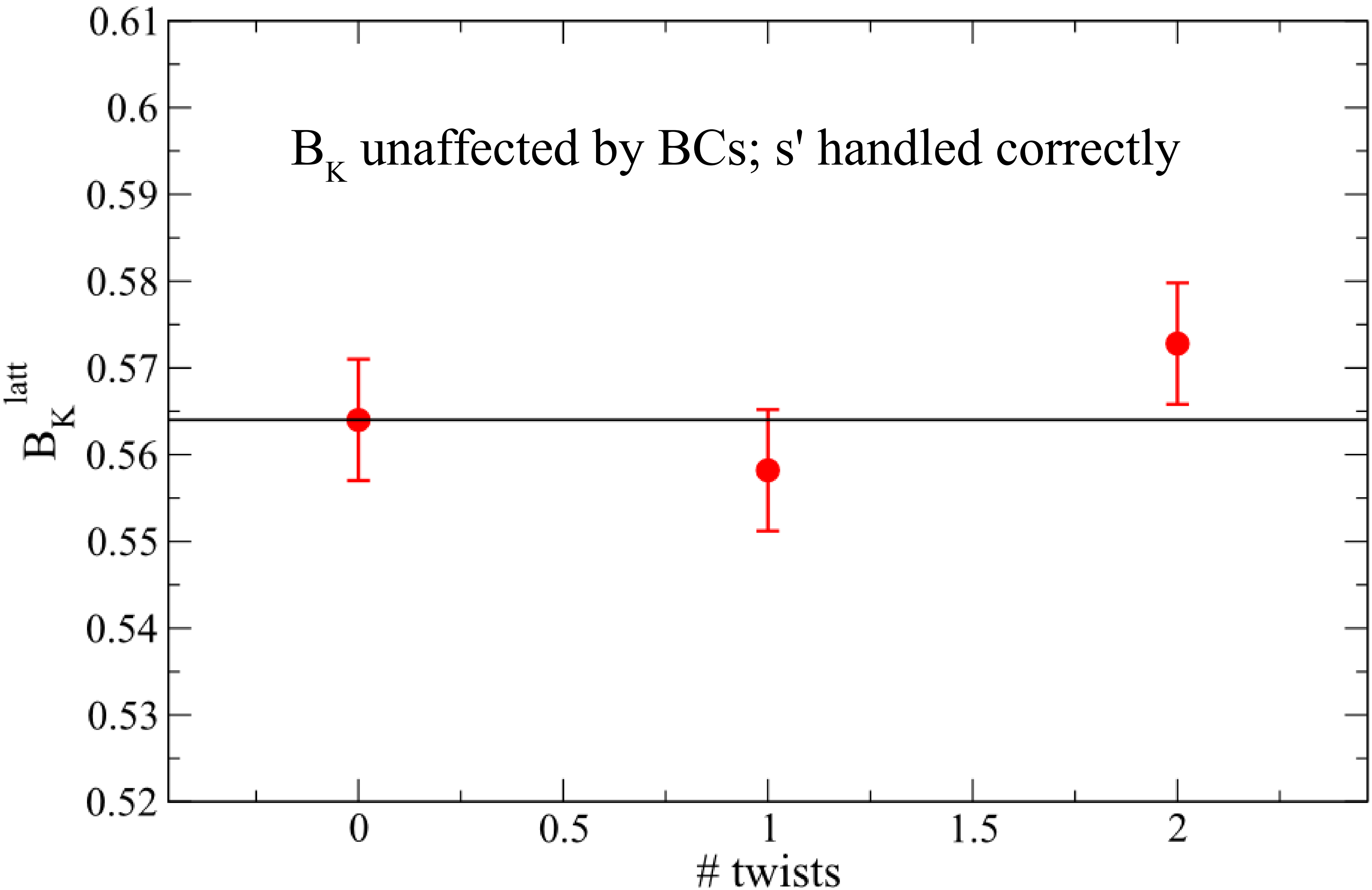
  However fields are intrinsically two-flavor; use of $M^{\dagger}M$ in HMC to ensure positive-definite matrix requires square-root of light determinant (fourth-root for s/s')

  RHMC needed throughout
- Standard double-precision multi-shift solver is quite slow due to linear algebra overheads coupled with finite BG/Q memory bandwidth.
- Developed optimized mixed-precision multi-shift inverter for RHMC in BFM/Bagel to optimize memory bandwidth usage on BG/Q.
- Alternative solution might be to use TWQCD's single-flavor action to avoid RHMC for light quarks.

Summary of code changes

- HMC and basic measurement code written in CPS, with modified BFM/Bagel solvers for BG/Q.
- Complex conjugate BCs on gauge fields required changes to virtually all aspects of the codebase:

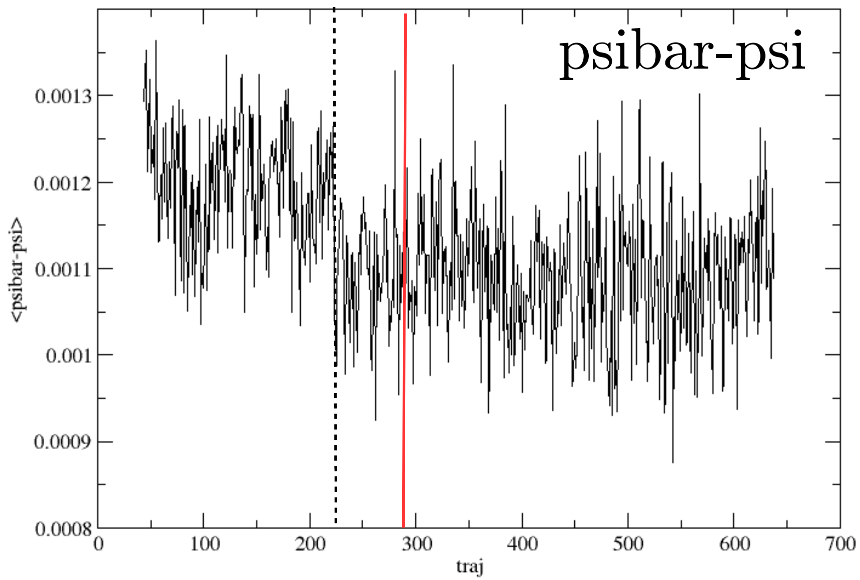
  - Gauge fixing algorithms
  - Plaquette and rectangle, plus staples.
  - Momentum field CC BCs, gauge force.
  - Memory layout reordering code.
  - Modified CPS+BFM/Bagel Dirac ops: Shamir DWF, Mobius DWF, twisted mass (for DSDR).
  - Fermion forces.
  - Eigenvalue algorithms: Ritz, Lanczos.
  - CPS propagator code.
  - Standard measurements: two-point correlators, B_K, Wilson flow, residual mass.
  - Multi-shift optimization

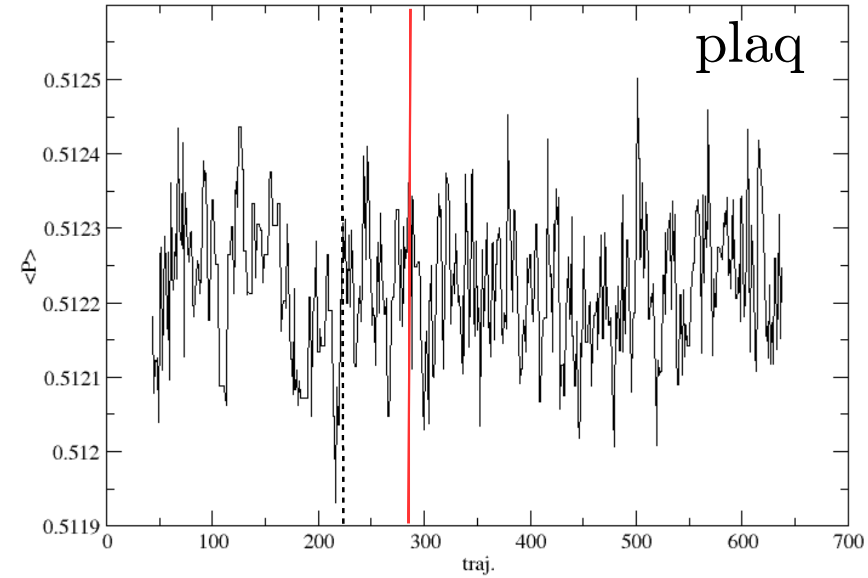


B_K unaffected by BCs; s' handled correctly
0.61
0.6
0.59
0.58
0.57
0.56
0.55
0.54
0.53
0.52
B_K^latt
0
0.5
1
1.5
2
twists

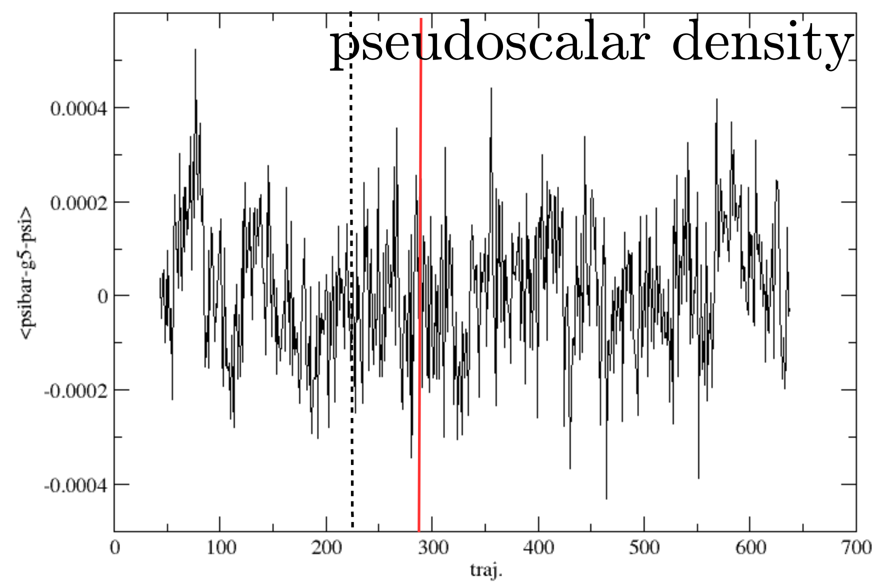
Ensemble Generation

- Ensemble generated on USQCD 512-node BGQ machine at BNL.
  - ~660 configurations to date.
  - Approx 400 thermalized.
  - 6.8 hours per configuration.
  - 89% Metropolis acceptance (88% theor.)





Dashed line: reduced quark mass
Red line: measurements begin (286)



ΔI=1/2 Decay

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  - ➔ Conserve isospin such that I=0 state can be isolated.
  - ➔ Give momentum to both charged and neutral pions.

- Conventional application of twisted BCs to the d-quark breaks both of these.
- Instead, we use G-parity:

$$\hat{G} = \hat{C} e^{i\pi \hat{I}_y} \quad : \qquad \hat{G}|\pi^{\pm}\rangle = -|\pi^{\pm}\rangle$$
$$\hat{G}|\pi^{0}\rangle = -|\pi^{0}\rangle$$

- As a boundary condition:

$$\pi^i(x+L) = \hat{G}\pi^i(x) = -\pi^i(x)$$

$\rightarrow$ Discretized lattice momenta

$$|p| \in (\pi/L,\, 3\pi/L,\, 5\pi/L \ldots)$$

Moving ground-state

Baryogenesis

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} = 6.14(25) \times 10^{-10}$$

- The Universe is matter dominated, but why?
- Most likely explanation is existence of baryogenesis mechanism.
- Sackarov conditions (1967):
  - B-number violation
  - Non-thermal interactions (e.g. during a phase transition)
  - C and CP violation.
- Why C *and* CP? Because C-breaking $\Gamma(X \to Y + B) \neq \Gamma(\bar{X} \to \bar{Y} + \bar{B})$ but allows

$$\Gamma(X \to q_L q_L) + \Gamma(X \to q_R q_R) = \Gamma(\bar{X} \to \bar{q}_L \bar{q}_L) + \Gamma(\bar{X} \to \bar{q}_R \bar{q}_R)$$

CP-violation prevents this.

- Amount of CP-violation in Standard Model far too small to account for observed value.
- Most BSM theories introduce additional direct CP-violation but a precise SM value does not yet exist which could be compared to experiment.