Lattice QCD calculation of long distance contribution to ϵ_K

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Outline

- Introduction and theoretic background.
- Evaluation of Im $M_{0\bar{0}}$ and ϵ_K.
- Preliminary results and short distance correction.
- Outlook and summary.

Introduction

- One of the most important test of the Standard Model: CP violating observables ϵ_K, with the experimental value.

$$|\epsilon_K| = 2.228(11) \times 10^{-3}$$

- All previous attempt to calculate ϵ_K involve only the short distance part, (evaluating the kaon bag parameter B_K). The estimate of long distance contribution is a few percent, but not preceisely calculated.

- Previous method for the calculation of ΔM_K, with the technique of evaluating the space time integrated four point correlator, can also be used to calculate the long distance part of ϵ_K.

Introduction

- Our major challenge includes:

  1. different from the ΔM_K calculation, we focus on the CP violating part of kaon mixing. More operators and more diagrams to evaluate on lattice.

  2. In the ΔM_K calculation, all the diagrams are convergent. However, in our ϵ_K calculation, most of our four point diagrams have an $\log(a)$ ultraviolet divergence and therefore treatment of the divergence is required.

Theoretical background: $K^0 - \overline{K^0}$ mixing

- Let i, j stand for K^0 and $\overline{K^0}$, from the kaon mixing theory we obtain:
- dispersive part:

$$M_{ij} = \mathcal{P} \sum_{\alpha} \frac{\langle i|H_W|\alpha\rangle\langle\alpha|H_W|j\rangle}{m_K - E_\alpha}$$

- absorptive part:

$$\Gamma_{ij} = \sum_{\alpha} 2\pi \langle i|H_W|\alpha\rangle\langle\alpha|H_W|j\rangle \delta(m_K - E_\alpha)$$

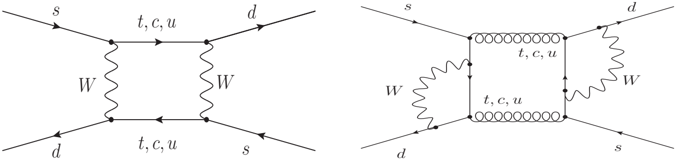
- We have the ϵ_K:

$$\epsilon_K = \frac{exp(i\pi/4)}{\sqrt{2}\Delta M_K}(\mathrm{Im}\, M_{0\bar{0}} + 2\xi\, \mathrm{Re}\, M_{0\bar{0}})$$

$$\xi = \frac{\mathrm{Im}\, A_0}{\mathrm{Re}\, A_0}$$

Evaluation of $\mathrm{Im}\, M_{0\bar{0}}$

- Two types of diagram contribute to the $\Delta S = 2$, $K^0 - \overline{K^0}$ mixing process:



box topology　　disconnected topology

FIGURE : two types of $\Delta S = 2$ diagram

- Write the sum over three types of quark (u,c,t) into two terms ($\lambda_i = V_{id} V_{is}^*$, and $\lambda_u + \lambda_c + \lambda_t = 0$ has been used).
  The usual way is to eliminate up:

$$\sum_{i=u,c,t} \frac{\lambda_i \not{p}}{p^2 + m_i^2} = \lambda_c \left\{ \frac{\not{p}}{p^2 + m_c^2} - \frac{\not{p}}{p^2 + m_u^2} \right\} + \lambda_t \left\{ \frac{\not{p}}{p^2 + m_t^2} - \frac{\not{p}}{p^2 - m_u^2} \right\} \quad (1)$$

Evaluation of Im $M_{0\bar{0}}$

- We choose to eliminate the **charm**.

$$\sum_{i=u,c,t} \frac{\lambda_i \not{p}}{p^2 + m_i^2} = \lambda_u \left\{ \frac{\not{p}}{p^2 + m_u^2} - \frac{\not{p}}{p^2 + m_c^2} \right\} + \lambda_t \left\{ \frac{\not{p}}{p^2 + m_t^2} - \frac{\not{p}}{p^2 - m_c^2} \right\} \tag{2}$$

- This choice has the following advantages:

  1. Take the product of the two internal quark lines:
  $\lambda_u \lambda_u$ term: has no imaginary parts (λ_u is real).
  $\lambda_t \lambda_t$ term: pure perturbative.
  Therefore, only focus on the $\lambda_u \lambda_t$ term.

  2. Connected diagrams for the current-current operators don't have a pion intermediate state (all internal quark involve charm).

EVALUATION OF $\mathrm{Im}\, M_{0\bar{0}}$: $\Delta S = 1$ WEAK HAMILTONIAN

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- We calculate $\langle \overline{K^0} | T H_W^{\Delta S=1}(x) H_W^{\Delta S=1}(y) | K^0 \rangle$ in four flavor theory. The $H_W^{\Delta S=1}$ is given by:

$$H_{eff}^{\Delta S=1} = \frac{G_F}{2}\left(\sum_{q,q'=u,c} V_{q's}^* V_{qd} \sum_{i=1,2} C_i Q_i^{q'q} - \lambda_t \sum_{i=3}^{6} C_i Q_i \right) \quad (3)$$

$$Q_1^{q',q} = \sum_{q,q'=u,c} V_{q',s}^* V_{q,d} (\bar{s}_i q'_j)_{V-A} (\bar{q}_j d_i)_{V-A} \quad (4)$$

$$Q_2^{q',q} = \sum_{q,q'=u,c} V_{q',s}^* V_{q,d} (\bar{s}_i q'_i)_{V-A} (\bar{q}_j d_j)_{V-A} \quad (5)$$

$$Q_3 = (\bar{s}_i d_i)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_j q_j)_{V-A} \quad (6)$$

$$Q_4 = (\bar{s}_i d_j)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_j q_i)_{V-A} \quad (7)$$

$$Q_5 = (\bar{s}_i d_i)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_j q_j)_{V+A} \quad (8)$$

$$Q_6 = (\bar{s}_i d_j)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_j q_i)_{V+A} \quad (9)$$

Evaluation of $\mathrm{Im}\, M_{0\bar{0}}$: 2nd order weak process.

- By multiplying two $H_W^{\Delta S=1}$, we get:

$$TH_W(x)H_W(y) = \frac{G_F^2}{2}\lambda_u\lambda_t \sum_{i=1}^{2}\sum_{j=1}^{6} C_i C_j Q_{i,j} \tag{10}$$

$$\begin{aligned} Q_{i,j} &= T\Big[2Q_i^{cc}(x)Q_j^{cc}(y) - Q_i^{uu}(x)Q_j^{cc}(y) - Q_i^{cc}(x)Q_j^{uu}(y) \\ &\quad -Q_i^{uc}(x)Q_j^{cu}(y) - Q_i^{cu}(x)Q_j^{uc}(y)\Big], (j = 1, 2) \\ Q_{i,j} &= T\left[(Q_i^{cc}(x) - Q_i^{uu}(y))Q_j(y) + Q_j(x)(Q_i^{cc}(y) - Q_i^{uu}(y))\right], (j = 3, ..., 6) \end{aligned}$$

- The C_i are the Wilson Coefficient.
- As in the ΔM_K calculation, evaluate the integrated correlator:

$$\mathcal{A} = \frac{1}{2}\sum_{t_2=1}^{t_a}\sum_{t_1=t_a}^{t_b}\langle 0|T\left\{\overline{K}^0(t_f)H_W(t_2)H_W(t_1)\overline{K}^0(t_i)\right\}|0\rangle \tag{11}$$

Evaluation of $\mathrm{Im}\, M_{0\bar{0}}$

- After insertion of a complete set of intermediate states, we have:

$$\mathcal{A} = N_K^2 e^{-M_K(t_f - t_i)} \left\{ \sum_n \frac{\langle \bar{K}^0 | H_w | n \rangle \langle n | H_w | K^0 \rangle}{M_K - M_n} \left(-T + \frac{e^{(M_K - M_n)T} - 1}{M_K - M_n} \right) \right\} \tag{12}$$

- By doing a linear fit with T, we can find the $M_{0\bar{0}}$.
- Two different parts for the intermediate states $|n\rangle$:

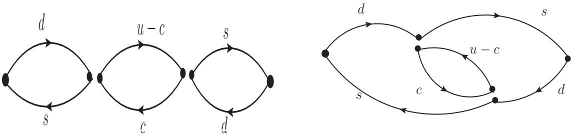
  1. $E_n > m_K$: contribution to the exponential terms is highly suppressed, leaving only terms proportional to T, plus constant terms.

  2. $E_n < m_K$: their exponentially growing term should be identified and subtracted.

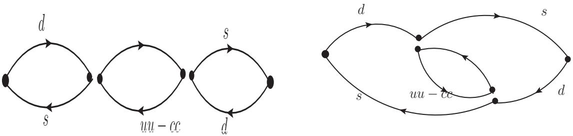
Evaluation of $\mathrm{Im}\, M_{0\bar{0}}$

- Four points diagrams:

type 1 type 2



$i = 1, 2,\ j = 1, 2$



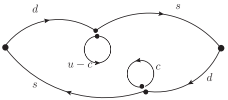
$i = 1, 2,\ j = 3, 4, 5, 6$

- In all these diagrams, the momentum in the internal quark lines are cutoff by a unphysical scale $\propto 1/a$ (inverse lattice spacing). The divergence piece should therefore be identified and corrected.

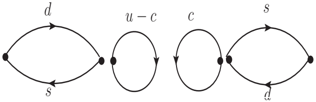
Evaluation of Im $M_{0\bar{0}}$

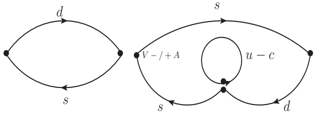
- Four points diagrams.

type 3



type 4





type 5

- Leave these to future work!

Simulation details

- $24^3 \times 64$ Iwasaki lattice, $m_\pi = 329$ MeV, and $m_K = 575$ MeV. quenched charm, $m_c = 0.363$ (949 MeV).
- Use wall source propagator for the kaon source and sink, and random volume source for the self loop.
- Use Lanczos algorithm with 300 eigenvectors.
- The main goal of this calculation:

  1.To understand the size of long distance contribution.

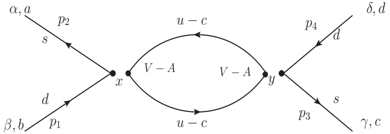
  2. Develop $\log(a)$ correction method.
- For the present analysis, we focus on type 1 and 2 diagrams :

  1. Smaller statistical noise.

  2. Short distance part easier to evaluate in continuum.

Short distance divergence

- For the case of ΔM_K calculation, our diagrams are convergent.



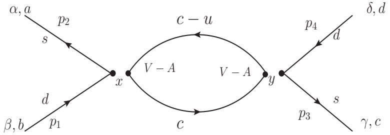
$$\int d^4 p \gamma^\mu (1-\gamma^5)\left(\frac{\not{p}-m_u}{\not{p}^2+m_u^2} - \frac{\not{p}-m_c}{\not{p}^2+m_c^2}\right)\gamma^\nu (1-\gamma^5)\left(\frac{\not{p}-m_u}{\not{p}^2+m_u^2} - \frac{\not{p}-m_c}{\not{p}^2+m_c^2}\right) \tag{13}$$

$$= \int d^4 p \gamma^\mu (1-\gamma^5)\frac{\not{p}(m_c^2-m_u^2)}{(\not{p}^2+m_u^2)(\not{p}^2+m_c^2)}\gamma^\nu (1-\gamma^5)\frac{\not{p}(m_c^2-m_u^2)}{(\not{p}^2+m_u^2)(\not{p}^2+m_c^2)} \tag{14}$$

- We get this equation because the $V - A$ structure of the vertex made the m on the numerator disappear. And the final expression is not divergent when p is high.

Short distance divergence

- In the ϵ_K calculation, with both the two weak operator Q_1, Q_2, one of the internal quark lines is a single charm.

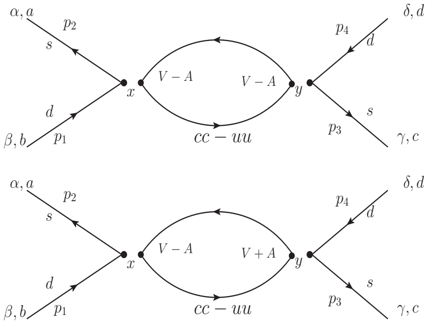


$$\int d^4p\gamma^\mu(1-\gamma^5)(\frac{\not{p}-m_c}{\not{p}^2+m_c^2}-\frac{\not{p}-m_u}{\not{p}^2+m_u^2})\gamma^\nu(1-\gamma^5)(\frac{\not{p}-m_c}{\not{p}^2+m_c^2}) \qquad (15)$$

$$=\int d^4p\gamma^\mu(1-\gamma^5)\frac{\not{p}(m_c^2-m_u^2)}{(\not{p}^2+m_u^2)(\not{p}^2+m_c^2)}\gamma^\nu(1-\gamma^5)(\frac{\not{p}}{\not{p}^2+m_c^2}) \qquad (16)$$

- Ultraviolet **logarithm** divergence, when the two weak vertex are close to each other. Cutoff by the unphysical scale $\propto 1/a$ on lattice. therefore the divergent part must be corrected.

- Gluonic penguin diagrams are also log divergent!



- Use a local operator $O_{LL} = (\bar{s}d)(\bar{s}d)$, to represent the short distance divergent part of $\langle \overline{K^0} | T H_W(x) H_W(y) | K^0 \rangle$
- Use Rome-Southamption method, work under momentum scale μ and find the coefficient $C_{i,j}^{lat}(\mu^2)$ that

$$\sum_{x,y} \langle \overline{K^0} | T Q_i(x) Q_j(y) | K^0 \rangle_{SD} = C_{i,j}^{lat}(\mu^2) \sum_x \langle \overline{K^0} | O_{LL}(x) | K^0 \rangle \tag{17}$$

- Then work in continuum under same μ, find the **physical** short distance part, which can be represented by a coefficient $C_{i,j}^{cont}(\mu^2)$. Finally, the correct correlator :

$$\mathcal{A} \to \mathcal{A} - C^{lat}(\mu^2) \sum_x \langle \overline{K^0} | O_{LL}(x) | K^0 \rangle + C^{cont}(\mu^2) \sum_x \langle \overline{K^0} | O_{LL}(x) | K^0 \rangle \tag{18}$$

Short distance divergence: Evaluation of C^{lat}

- To work out the coefficient C^{lat} on lattice, use the off-shell amputated Green function for our weak Hamiltonian and the O_{LL} operator, projected on the projection operator $P_{\alpha,\beta,\gamma,\delta}$:

$$\left(\Gamma^{amp}_{\alpha,\beta,\gamma,\delta}(p) - C^{lat}(\mu^2)\Gamma^{amp,SD}_{\alpha,\beta,\gamma,\delta}(p)\right) P_{\alpha,\beta,\gamma,\delta} = 0 \tag{19}$$

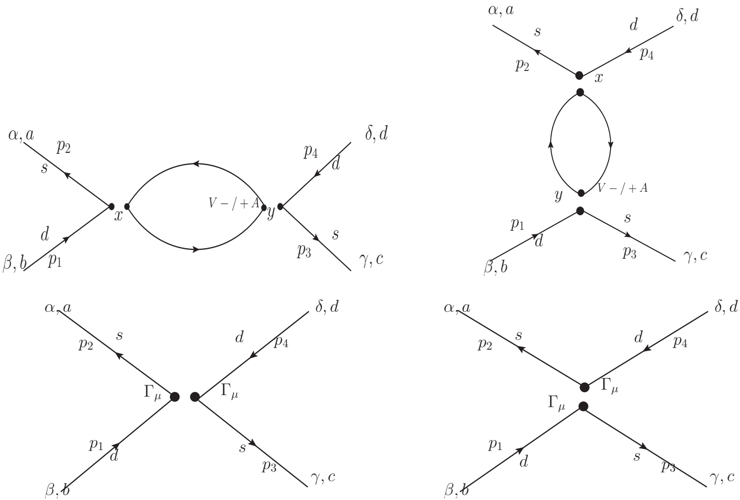
$$\Gamma_{\alpha\beta\gamma\delta}(p) = \langle s_\alpha(p_1)\bar{d}_\beta(p_2) \int d^4x_1 \int d^4x_2 H_W(x_1)H_W(x_2)s_\gamma(p_3)\bar{d}_\delta(p_4)\rangle. \tag{20}$$

$$\Gamma^{\mathrm{SD}}_{\alpha\beta\gamma\delta}(p) = \langle s_\alpha(p_1)\bar{d}_\beta(p_2) \int d^4x O_{LL}(x)s_\gamma(p_3)\bar{d}_\delta(p_4)\rangle. \tag{21}$$

- If the lattice momentum has relatively high scale, then the major contribution to $\Gamma_{\alpha\beta\gamma\delta}(p)$ is from short distance. Therefore C^{lat} can correctly represent the divergent part of our correlator.

Short distance divergence: Evaluation of C^{lat}

- choose momentum:
  $|p_1| = |p_2| = |p_3| = |p_4| = \mu, \quad |p_1 - p_2| = |p_3 - p_4| = \mu,$
  $p_1 + p_4 = p_2 + p_3$ (net momentum flow is 0).



Short distance divergence: Evaluation of C^{lat}

- Use:

$$p_1 = \frac{2\pi}{La}(M, M, 0, 0), \quad p_2 = \frac{2\pi}{La}(M, 0, M, 0)$$
$$p_3 = \frac{2\pi}{La}(0, M, 0, M), \quad p_4 = \frac{2\pi}{La}(0, 0, M, M) \tag{22}$$

- To study the short distance divergence of our correlator, we impose a space-time cutoff R:

$$\Gamma_{\alpha\beta\gamma\delta}(p) = \langle s_\alpha(p_1)\bar{d}_\beta(p_2) \int d^4x_1 \int_{|x_1-x_2|^2<R^2} d^4x_2 H_W(x_1)H_W(x_2)s_\gamma(p_3)\bar{d}_\delta(p_4)\rangle. \tag{23}$$

- Small R dependence when $R \geq 5$: C^{lat} really represent short distance part!

cutoff	3	4	5	7	none
C_{11}^{lat}	0.1726	0.1881	0.1903	0.1905	0.1904
C_{22}^{lat}	0.0489	0.0520	0.0522	0.0522	0.0522

Table : $C_{i,j}^{lat}$: coefficient for Q_iQ_j.

Short distance divergence: Evaluation of C^{lat}

- Energy scale μ dependence of $C^{lat}(\mu^2)$:

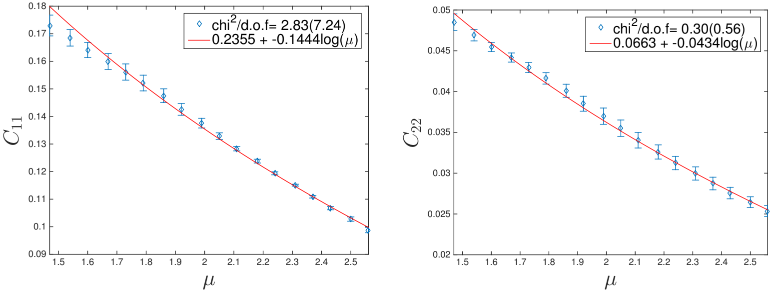


Figure : Short distance coefficients for energy scale $\mu = 1.41GeV - 2.6GeV$. Red line is a logarithm fit.

Preliminary results: Wilson coefficients

- We use the basis for operators $(Q_1^{uu}, Q_2^{uu}, Q_3, Q_4, Q_5, Q_6)$. The Wilson coefficient for Q_i^{cc}, Q_i^{cu}, and Q_i^{uc} will be the same to Q_i^{uu} $(i = 1, 2)$.
- We can find the Wilson coefficient in $\overline{MS}$ at $\mu = 2.15$ GeV:

$$C^{\overline{MS}} = \begin{pmatrix} -0.2967 & 1.1385 & 0.0217 & -0.0518 & 0.0102 & -0.0671 \end{pmatrix} \tag{24}$$

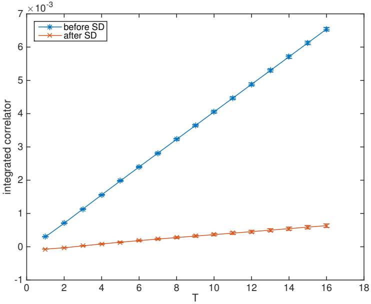
- Use NPR to find the lattice Wilson coefficient, renormalized at $\mu = 2.15$ GeV.

$$\sum_{i=1}^{6} C_i^{\overline{MS}} Q_i^{\overline{MS}} = \sum_{i=1}^{6} C_i^{lat} Q_i^{lat}, \tag{25}$$

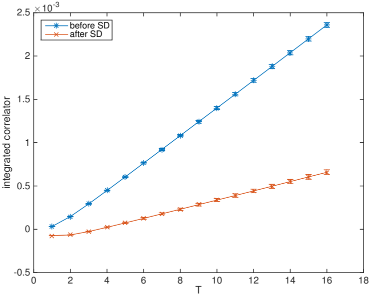
$$C^{\overline{lat}} = \begin{pmatrix} -0.2373(1) & 0.6885(1) & 0.0113(8) & -0.0213(10) & 0.0085(8) & -0.0256(8) \end{pmatrix} \tag{26}$$

Preliminary results

- Integrated correlator before and after short distance correcton:



$Q_1 Q_1$



$Q_2 Q_2$

Figure : Integrated correlator, only $Q_1 Q_1$ and $Q_2 Q_2$ are plotted.

Preliminary results

- Different contribution to Im $M_{0\bar{0}}$, before and after subtraction of short distance contribution. Error comes from both the fitting of correlator and the Wilson coefficients.

	before	after		before	after
$Q_1 Q_1$	-0.7241(61)	-0.0786(59)			
$Q_1 Q_2$	0.9112(101)	0.0637(099)	$Q_2 Q_2$	-2.3472(281)	-0.7816(282)
$Q_1 Q_3$	0.1181(60)	0.0506(27)	$Q_2 Q_3$	-0.0190(21)	0.0242(22)
$Q_1 Q_4$	-0.0116(11)	0.0161(12)	$Q_2 Q_4$	0.2216(81)	0.1182(51)
$Q_1 Q_5$	-0.0681(048)	-0.0035(009)	$Q_2 Q_5$	0.3461(245)	0.2917(208)
$Q_1 Q_6$	0.1577(059)	0.0828(051)	$Q_2 Q_6$	0.0863(121)	0.2914(137)

TABLE : Imaginary part of $M_{0\bar{0}}$, before/ after subtraction of short distance contribution.

Preliminary results

- We therefore obtain the $\lambda_u \lambda_t$ contribution to $M_{0\bar{0}}$. Before the subtraction of short distance part:

$$M_{0\bar{0}} = (3.16(10) - 1.32(4)) \times 10^{-15} MeV$$

- After the subtraction of short distance part:

$$M_{0\bar{0}} = (-1.79(99) + 0.75(41)) \times 10^{-16} MeV$$

- To obtain correct results, we still need to calculate the **physical** short distance contribution in continuum and add them back.
- We can find the ϵ_K from:

$$\epsilon_k = \frac{exp(i\pi/4)}{\sqrt{2}\Delta M_K}(\mathrm{Im}\, M_{0\bar{0}} + 2\xi\, \mathrm{Re}\, M_{0\bar{0}}) \tag{27}$$

- The value for ξ can be obtained from lattice calculation of kaon to two pion A_0. Currently, we can set this term to 0, and we use the experimental value $\Delta M_K = 3.483(6) \times 10^{-12} MeV$.

Preliminary results

- The long distance contribution to $\lambda_u \lambda_t$ part of ϵ_K is:
- before short distance correction.

$$|\epsilon_K| = 2.69(8) \times 10^{-4},$$

- after short distance correction.

$$|\epsilon_K| = 1.52(84) \times 10^{-5},$$

- The experimental value(with all parts):

$$|\epsilon_K| = 2.228(11) \times 10^{-3}$$

- We have relatively large error on the ϵ_K after short distance subtraction, because there is huge cancellation between different contribution to ϵ_K, but the error is not cancelled.

Outlook and summary

- To obtain complete calculation of $\lambda_u \lambda_t$ part of ϵ_K, we have to:

  1. Find the correct short distance contribution from continuum for the box diagram topology, and add it back to our results. Then our type 1 and 2 diagrams for the box topology is complete.

  2. Include all the diagrams (type 3, 4, 5), do the corresponding NPR to remove the short distance artifact, and find the short distance contribution in continuum for the disconnected topology and add to our results.

  3. Work on a lattice with more physical kinematics, including lighter pion, unquenched charm, and larger physical volume.