Computing the $K_L - K_S$ mass difference using lattice QCD

Multi-Hadron and Nonlocal Matrix Elements in Lattice QCD

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RBC and UKQCD Collaborations

Outline

- The $K_L - K_S$ mass difference
- Current lattice capability
- The $K_L - K_S$ mass difference from Euclidean space
- Finite volume correction
- Numerical experiments

RBC Collaboration

- BNL
  - Chulwoo Jung
  - Taku Izubuchi (RBRC)
  - Christoph Lehner
  - Amarjit Soni
- RBRC
  - Chris Kelly
  - Tomomi Ishikawa
  - Shigemi Ohta (KEK)
  - Sergey Syrityn
- Connecticut
  - Tom Blum
- Columbia
  - Ziyuan Bai
  - Xu Feng
  - Norman Christ
  - Luchang Jin
  - Robert Mawhinney
  - Greg McGlynn
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UKQCD Collaboration

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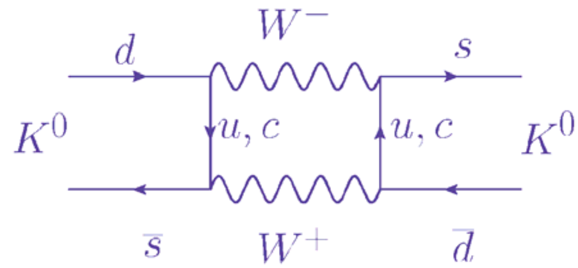
  - Andreas Juttner
  - Andrew Lawson
  - Edwin Lizarazo
  - Andrew Lytle (Mumbai)
  - Marina Marinkovic (CERN)
  - Antonin Portelli
  - Chris Sachrajda
  - Matthew Spraggs
  - Tobi Tsang

Physics Context

$K^0 - \overline{K^0}$ mixing

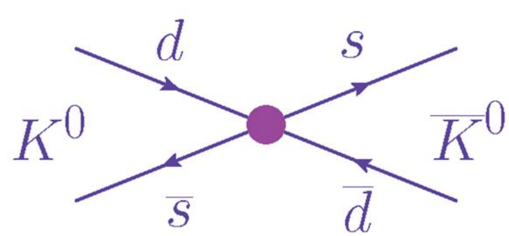
- ΔS=1 weak interactions allow $\overline{K}^0$ and K^0 to mix.



- New eigenstates are approximately:

$$K_+ = \frac{1}{\sqrt{2}}\left(K^0 + \overline{K}^0\right) \qquad K_- = \frac{1}{\sqrt{2}}\left(K^0 - \overline{K}^0\right)$$

- Small mass difference: ΔM_K = 3.483(6) x 10^{-12} MeV
- Effective $\frac{\alpha_{EM}}{\Lambda^2}(\bar{s}d)(\bar{s}d)$ operator would give ΔM_K = 3 x 10^{-12} MeV for Λ = 1000 TeV



$K^0 - \bar{K}^0$ mixing: Indirect CP Violation

- CP violation leads to K_L and K_S states which are not CP eigenstates:

$$K_S = \frac{K_+ + \bar{\epsilon} K_-}{\sqrt{1+|\bar{\epsilon}|^2}} \qquad K_L = \frac{K_- + \bar{\epsilon} K_+}{\sqrt{1+|\bar{\epsilon}|^2}}$$

- Here $\bar{\varepsilon}$ is closely related to $\epsilon_K = \bar{\epsilon} + i\,\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}$

- Where $|\varepsilon_K| = (2.228 \pm 0.011) \times 10^{-3}$

$K^0 - \overline{K^0}$ Mixing

- Time evolution of $K^0 - \overline{K^0}$ system given by familiar Wigner-Weisskopf formula:

$$i\frac{d}{dt}\begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix} = \left\{\begin{pmatrix} M_{00} & M_{0\bar{0}} \\ M_{\bar{0}0} & M_{\bar{0}\bar{0}} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{00} & \Gamma_{0\bar{0}} \\ \Gamma_{\bar{0}0} & \Gamma_{\bar{0}\bar{0}} \end{pmatrix}\right\}\begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix}$$

where:

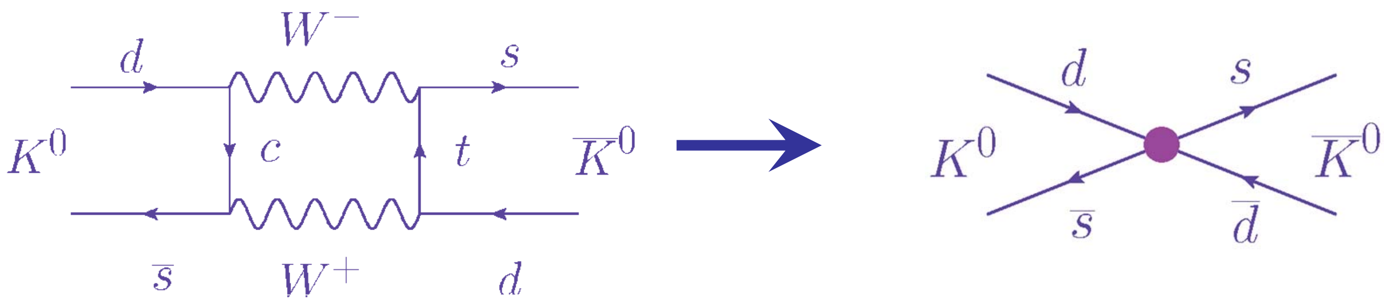
$$\Gamma_{ij} = 2\pi \sum_\alpha \int_{2m_\pi}^\infty dE \langle i|H_W|\alpha(E)\rangle\langle\alpha(E)|H_W|j\rangle\delta(E-m_K)$$

$$M_{ij} = \sum_\alpha \mathcal{P}\int_{m_\pi}^\infty dE \frac{\langle i|H_W|\alpha(E)\rangle\langle\alpha(E)|H_W|j\rangle}{m_K - E}$$

$K^0 - \overline{K^0}$ Mixing

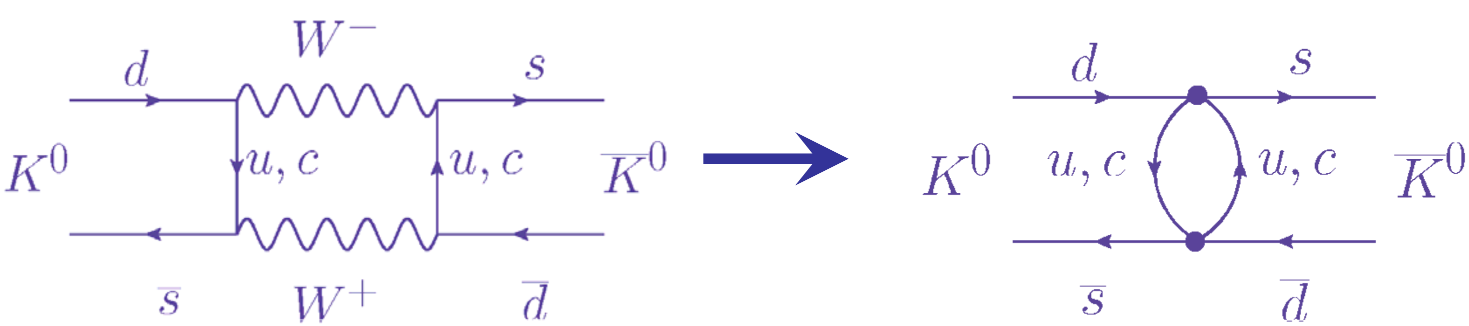
- CP violating: $p \sim m_t$

$$\bar{\epsilon} = \frac{i}{2}\left\{\frac{\mathrm{Im}\,M_{0\bar{0}} - \frac{i}{2}\mathrm{Im}\,\Gamma_{0\bar{0}}}{\mathrm{Re}\,M_{0\bar{0}} - \frac{i}{2}\mathrm{Re}\,\Gamma_{0\bar{0}}}\right\}$$



- CP conserving: $p \leq m_c$

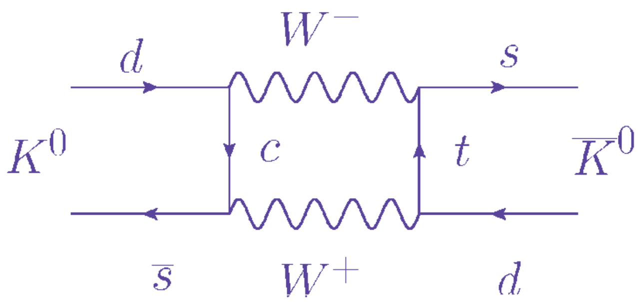
$$m_{K_S} - m_{K_L} = 2\mathrm{Re}\{M_{0\bar{0}}\}$$



$K^0 - \overline{K^0}$ Mixing

- CP violating: $p \sim m_t$

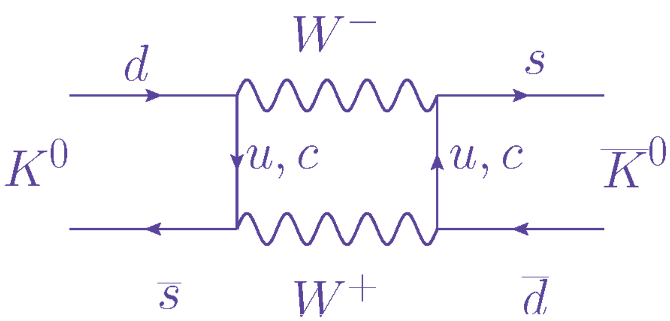
$$\overline{\epsilon} = \frac{i}{2}\left\{\frac{\mathrm{Im}M_{0\bar{0}} - \frac{i}{2}\mathrm{Im}\Gamma_{0\bar{0}}}{\mathrm{Re}M_{0\bar{0}} - \frac{i}{2}\mathrm{Re}\Gamma_{0\bar{0}}}\right\}$$



Long distance part is a small but important contribution: following talk of Ziyuan Bai

- CP conserving: $p \leq m_c$

$$m_{K_S} - m_{K_L} = 2\mathrm{Re}\{M_{0\bar{0}}\}$$



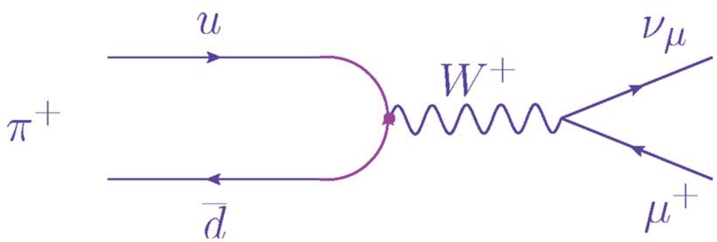
Long distance part is large. QCD perturbation theory fails at the 30% level.

Lattice QCD
in 2015

Current state-of-the-art

- Use chiral fermions: Mobius DWF
- Physical m_π=135 MeV and L = 4 – 6 fm.
- Generate 48^3 x 96 and 64^3 x 128.
- Many complex ingredients:
  - Highly optimized code (64 threads, SPI comms., wide-vector FP)
  - Sophisticated algorithms (deflation, FG $(\Delta t)^3$ integrator, multigrid)
  - Complex measurement strategies (NPR, G-parity BC, 5-pt functions, all-mode-averaging)
- Complete set of measurements took 5.3 hours on a 32-rack BG/Q machine (sustains 1 Pflops)

Simple state-of-the-art example: f_π



$$\langle 0|\bar{d}\gamma^5\gamma^\mu u|\pi^+(\vec{p})\rangle = f_\pi \frac{p^\mu}{\sqrt{4E_\pi(\vec{p})}}$$

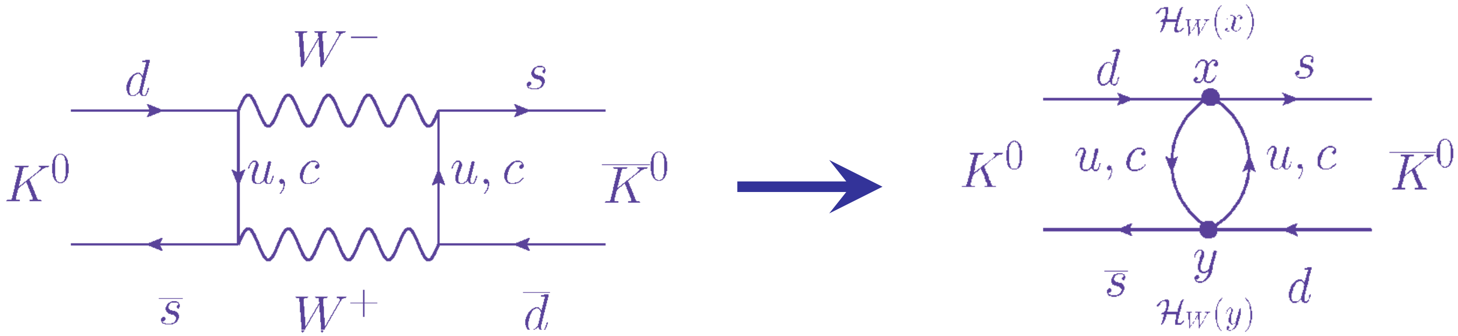
$$f_\pi = N\sum_{\vec{r}} \frac{\langle A^0(\vec{r},t)O_\pi(t=0)\rangle}{\langle O_\pi^\dagger(t)O_\pi(t=0)\rangle^{\frac{1}{2}}} e^{m_\pi t/2}$$

- 2012 (elaborate chiral fit): $f_\pi = 127(3)_{stat}(3)_{sys}$ MeV
- 2013 (m_π=135 MeV): $f_\pi = 130.0(0.3)_{stat}$ MeV (40 configs.)
- Experiment: $f_\pi = 130.4(0.04)(0.2)$ MeV

ΔM_K from Euclidean space

ΔM_K from Euclidean space

- Replace $W^{\pm}$ exchange by four-Fermi operator:



- Use 4-flavor theory incorporating GIM cancellation
- Amplitude is regular when the two weak operators collide: $x \to y$
- This long distance calculation is self-contained with no subtraction or perturbative correction needed!

ΔM_K from Euclidean space

- Start with $H = H_{\rm QCD} + H_{\rm W}$
- Calculate:

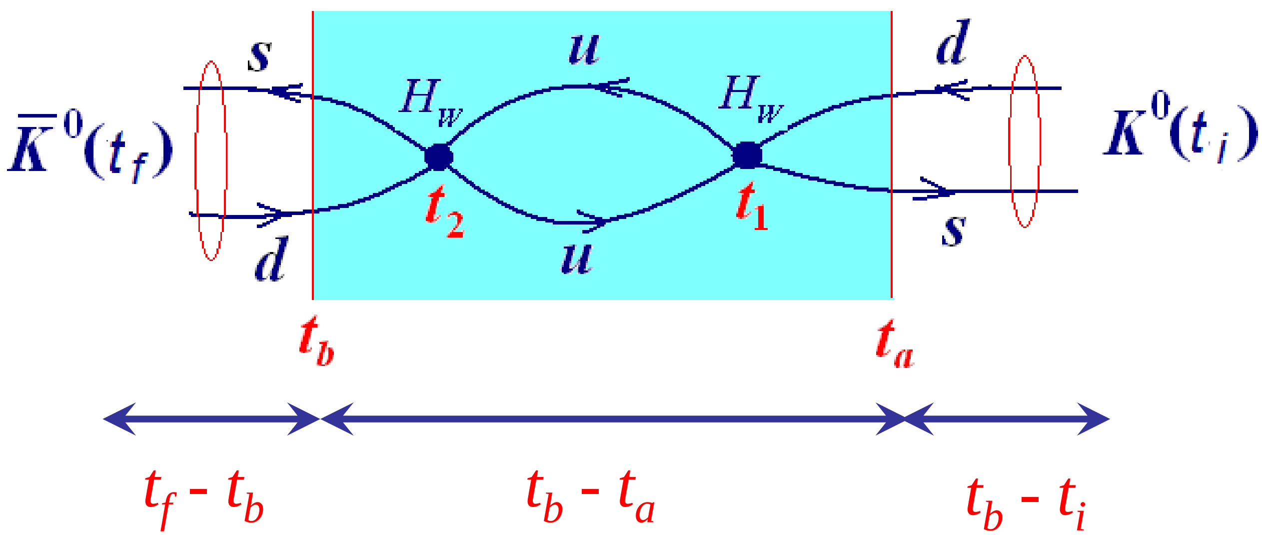
$$\langle K_\pm | e^{-\left(H_{\rm QCD}+H_{\rm W}\right)t} | K_\pm \rangle = e^{-\left(M_K + \Delta M_{K_\pm}\right)t} + \ldots$$
$$= e^{-M_K t}\left(1 + \Delta M_{K_\pm} t\right) + \ldots$$

- The … terms include $|0>$, $|\pi>$ and $|\pi\,\pi>$ states with energy below M_K
- These fall with increasing t exponentially less rapidly that the term of interest!

ΔM_K from Euclidean space

- Evaluate standard, Euclidean, 2nd order $\overline{K}^0 - K^0$ amplitude:

$$\mathcal{A} = \langle 0|T\left(K^0(t_f)\frac{1}{2}\int_{t_a}^{t_b} dt_2 \int_{t_a}^{t_b} dt_1 H_W(t_2) H_W(t_1) K^{0\dagger}(t_i)\right)|0\rangle$$



Interpret Lattice Result

$$\mathcal{A} = N_K^2 e^{-M_K(t_f-t_i)} \sum_n \frac{\langle \overline{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{M_K - E_n} \Big(\underbrace{-(t_b - t_a)}_{1.} \underbrace{- \frac{1}{M_K - E_n}}_{2.} \underbrace{+ \frac{e^{(M_K-E_n)(t_b-t_a)}}{M_K - E_n}}_{3.} \Big)$$

1. Δm_K^{FV}
2. Uninteresting constant
3. Growing or decreasing exponential:
   $E_n < m_K$ must be removed!

- Finite volume correction:

$$M_{K_L} - M_{K_S} = 2 \sum_n \frac{\langle \overline{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{M_K - E_n} + 2 \frac{d(\phi + \delta_0)}{dk}\Big|_{m_K} |\langle n_0 | H_W | K^0 \rangle|^2 \cot(\phi + \delta_0)\Big|_{M_K}$$

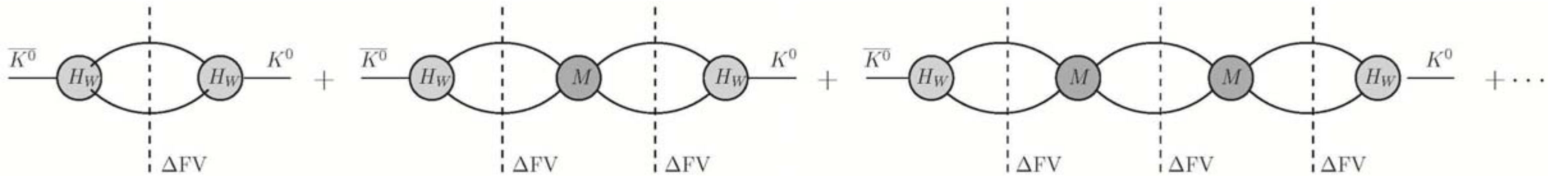
N.H. Christ, X. Feng, G. Martinelli, C.T. Sachrajda

Finite volume correction

- Exploit Kim, Sachrajda and Sharpe arXiv:hep-lat/0507006

$$\Delta_{\rm FV}\left\{\left\langle T\left(K^0(0)\int_{-\infty}^{\infty}H_W(t_1)dt_1\int_{-\infty}^{\infty}H_W(t_2)dt_2\int d^4x\overline{K}^0(x)e^{iqx}\right)\right\rangle\right\}\left(\frac{(q^2+M_K^2)}{i}\right)^2$$

$=$



$$= \quad \langle\overline{K}^0|H_W|(\pi\pi)^{in}\rangle F\frac{1}{1+\frac{i}{2}MF}\langle(\pi\pi)^{out}|H_W|K^0\rangle$$

$$= \quad \frac{qM_K\pi}{2}\left|\langle\overline{K}^0|H_W|(\pi\pi)^{in}\rangle\right|\left(\cot(\phi+\delta)+i\right)\left|\langle(\pi\pi)^{out}|H_W|K^0\rangle\right|$$

$$M_{K_L}-M_{K_S}=2\sum_n\frac{\langle\overline{K}^0|H_W|n\rangle\langle n|H_W|K^0\rangle}{M_K-E_n}+2\frac{d(\phi+\delta_0)}{dk}\bigg|_{m_K}|\langle n_0|H_W|K^0\rangle|^2\cot(\phi+\delta_0)\big|_{M_K}$$

N.H. Christ, X. Feng, G. Martinelli, C.T. Sachrajda

Numerical Experiments

Choice of weak operators

- Use four-Fermi operators in the four-flavor theory:

$$Q_1^{qq'} = (\overline{q}_i d_i)_{V-A} (\overline{q}'_j s_j)_{V-A} \qquad Q_2^{qq'} = (\overline{q}_i d_j)_{V-A} (\overline{q}'_j s_i)_{V-A}$$

$$\mathcal{H}_W = \frac{G_F}{2} \sum_{q,q'=u,c} V_{qd} V_{q's}^* \left(C_1 Q_1^{qq'} + C_2 Q_2^{qq'} \right)$$

- Use Rome-Southampton NPR and 4-flavor RI/SMOM / $\overline{\text{MS}}$-NDR matching from Lehner and Sturm
- Assume Cabibbo unitarity:

$0 = \lambda_u + \lambda_c + \lambda_t \approx \lambda_u + \lambda_c$ where $\lambda_q = V_{qd} V_{qs}^*$

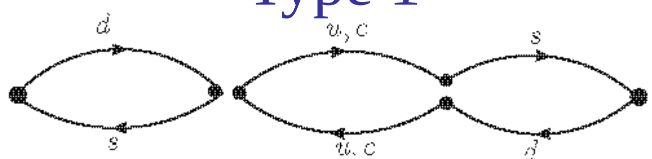
Lattice setup

- Must include charm quark (GIM *u*–*c* cancellation)
- Three calculations performed:

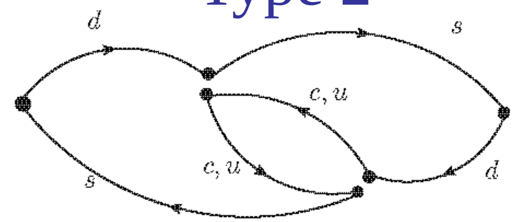
Jianglei Yu
- – 16^3 x 32, m_p = 420 MeV, types 1 & 2 (arXiv:1212.5931)
- – 24^3 x 64, m_p = 330 MeV, all graphs (arXiv:1406.0916)

Ziyuan Bai
- – 32^3 x 64, m_p = 170 MeV, all graphs

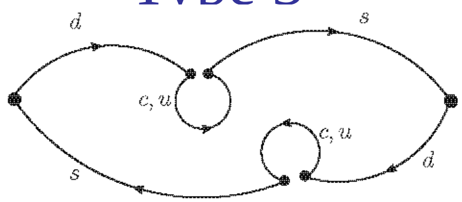
Type 1



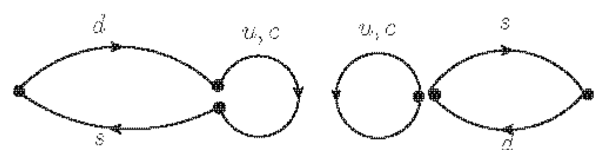
Type 2



Type 3

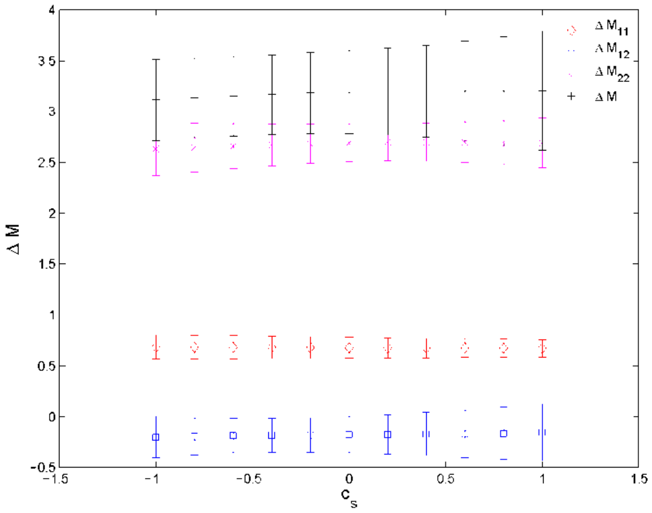


Type 4

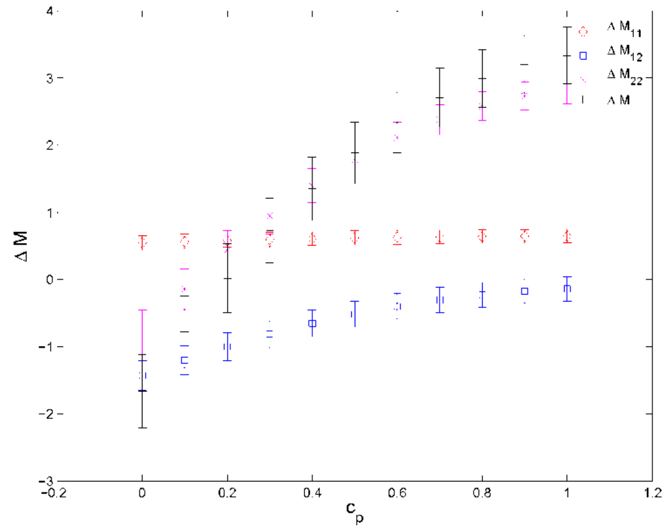


Exponentially growing terms

- The vacuum, π^0 and η require special treatment:
  - Calculate $\langle$ X | H_W | K^0 $\rangle$ directly and subtract, X = $|0\rangle$, π^0, η
  - Fit the exponential time dependence in the 4-point function
  - Adjust $c_s\, \overline{s}\, d$ or $c_p\, \overline{s}\, \gamma^5 d$ terms to completely remove two unwanted states.



vary c_s



vary c_p

Remove extra η contribution

- Calculate $\langle\, \eta \,|\, H_W \,|\, K^0 \,\rangle$ directly and remove
- Has an ~10% effect on the result

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η and η' Mesons from Lattice QCD

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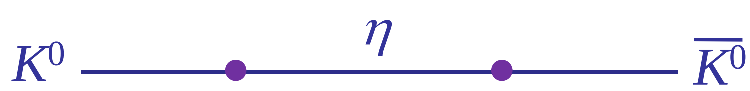
The large mass of the ninth pseudoscalar meson, the η', is believed to arise from the combined effects of the axial anomaly and the gauge field topology present in QCD. We report a realistic, 2 + 1-flavor, lattice QCD calculation of the η and η' masses and mixing which confirms this picture. The physical eigenstates show small octet-singlet mixing with a mixing angle of $\theta = -14.1(2.8)^\circ$. Extrapolation to the physical light quark mass gives, with statistical errors only, $m_\eta = 573(6)$ MeV and $m_{\eta'} = 947(142)$ MeV, consistent with the experimental values of 548 and 958 MeV.

DOI: 10.1103/PhysRevLett.105.241601 PACS numbers: 12.38.Gc, 11.15.Ha, 11.30.Rd, 14.40.Be

The relatively large mass of the ninth pseudoscalar meson, the η', provides a significant challenge for quantum chromodynamics (QCD), the component of the standard model which describes the interactions of quarks and gluons. On a naive classical level, there are nine conserved axial-vector currents. Given the vacuum breaking of the symmetries which these currents generate, this should diagrams to decrease exponentially with increasing time separation. For mesons this falloff roughly matches the exponential time dependence of the massive, Euclidean-space meson propagator, and good numerical signals can be seen over a large range of times. For terms in which the source and sink of the meson propagator are not joined by quark propagators, the needed exponential decrease comes

$$\eta' = \frac{1}{\sqrt{3}}\left(\bar{u}u + \bar{d}d + \bar{s}s\right)$$

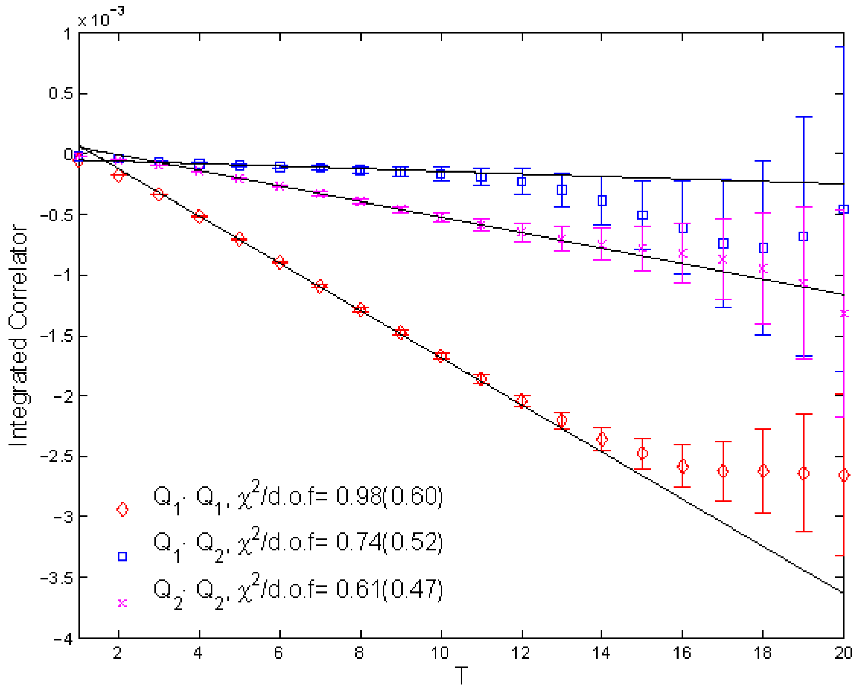
$$\eta = \frac{1}{\sqrt{6}}\left(\bar{u}u + \bar{d}d - 2\bar{s}s\right)$$



$$\frac{\left|\langle\eta|H_W|K^0\rangle\right|^2}{M_K - M_\eta}\left(-(t_b - t_a) - \frac{1}{M_K - M_\eta} + + \frac{e^{(M_K - M_\eta)(t_b - t_a)}}{M_K - M_\eta}\right)$$

Lattice results
(Jianglei Yu)

- N_f=2+1, 24^3 x 64, m_π = 330 MeV, $m_c^{\overline{MS}}$(2 GeV) = 949 MeV
- Incorporate GIM cancellation



- Large statistics (800 configurations, 64 measurements each).

Results

Δ_K	T_{min}	$Q_1 \cdot Q_1$	$Q_1 \cdot Q_2$	$Q_2 \cdot Q_2$	ΔM_K
	6	0.754(42)	-0.16(15)	2.70(18)	3.30(34)
7	7	0.755(42)	-0.18(15)	2.66(18)	3.23(34)
	8	0.751(42)	-0.18(15)	2.62(19)	3.18(35)

x 10^{-12} MeV

Diagrams	$Q_1 \cdot Q_1$	$Q_1 \cdot Q_2$	$Q_2 \cdot Q_2$	ΔM_K
Type 1,2	1.485(80)	1.567(38)	3.678(56)	6.730(96)
All	0.754(42)	-0.16(15)	2.70(18)	3.30(34)

x 10^{-12} MeV

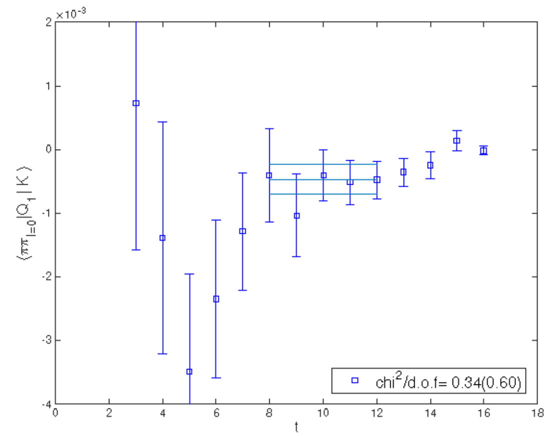
- Unphysical, m_π = 330 MeV
- Active charm but $m_c\, a$ = 0.55
- 80^2 x 96 x 192, 1/a=3.0 GeV calculations planned!

- Result:
  ΔM_K = 3.30(34) 10^{-12} MeV
- ΔM_K^{expt} = 3.483(6) 10^{-12} MeV
- Agreement fortuitous!

New m_π = 170 MeV calculation
(Ziyuan Bai)

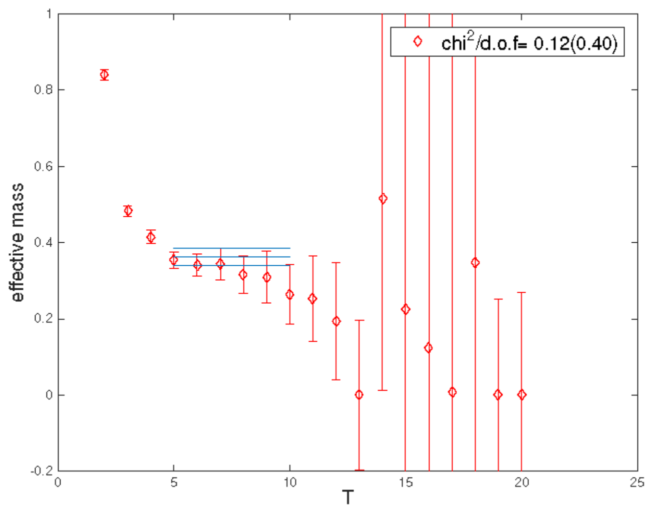
- N_f=2+1, 32^3 x 64, 1/a=1.37 GeV
- Charm: m_c = 592 and 750 MeV

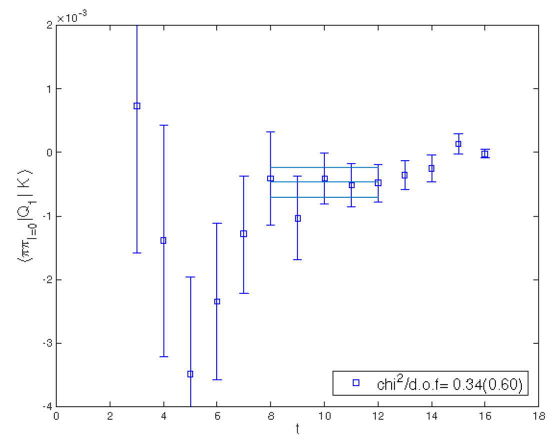
M_K	M_π	M_η	$E_{\pi\pi,I=0}$	$E_{\pi\pi,I=2}$
492.8(2)	171.9(2)	496(32)	335.8(13)	346.8(7)



Q1

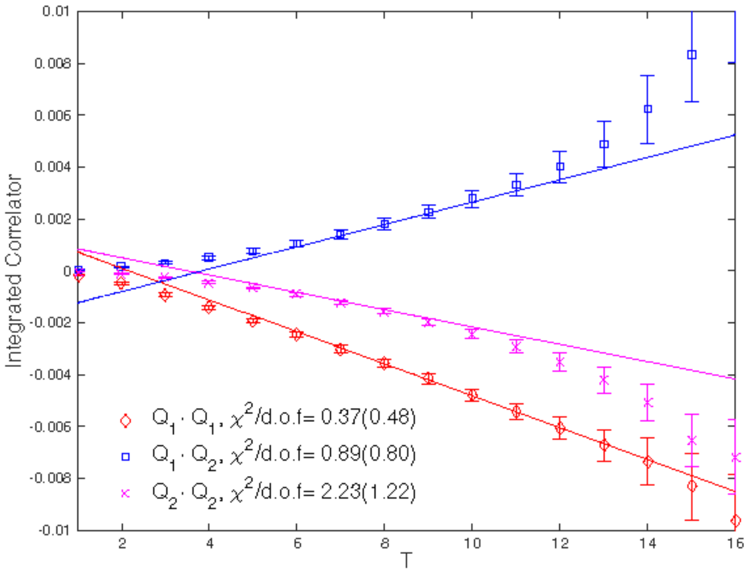
m_η





Q2

m_π = 170 MeV results
(Ziyuan Bai)



	$\Delta M_K \times 10^{+12}$
Types 1-4	5.76(73)
Types 1-2	4.19(15)
η	0
π	0.27(14)
$\pi\pi$, I=0	-0.097(49)
$\pi\pi$, I=2	-6.56(6) x 10^{-4}
Δ_{FV}	0.029(19)

- Use m_c = 750 MeV, fit for t ≥ 8
- Disconnected contribution small
- $\pi\pi$ contribution ~2% and FV correction ~0.5%

Outlook

- The $K_L - K_S$ mass difference in the standard model (and beyond) is a practical target for lattice QCD.
- A result with controlled 15-20% errors should be possible in ~2 years, as soon as $1/a \geq 3$ GeV ensembles become available.
- Sub-percent accuracy possible in 5-10 years!
- An exciting time for lattice QCD:
  - $K \rightarrow \pi\pi$, ΔI=3/2 and 1/2, ε'
  - $m_{K_L} - m_{K_S}$ and ε_K
  - $K^0 \rightarrow \pi^0 \bar{l} l$, $K^\pm \rightarrow \pi^\pm \bar{\nu}\nu$
  - Quark effects on g_μ - 2 at $O(\alpha^3)$