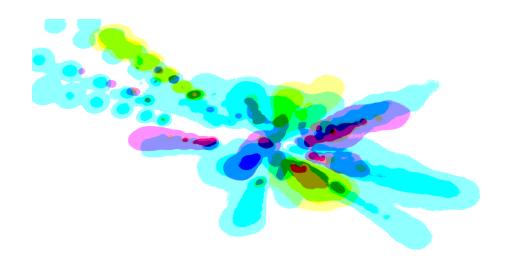
Separating fact from fantasy*: The chiral anomaly and the proton spin puzzle



Raju Venugopalan (BNL)

CFNS seminar, August 20, 2020

Separating fact from fantasy*: The chiral anomaly and the proton spin puzzle



R. L. Jaffe



A. Manohar

30th anniversary of their seminal paper which strongly inspired the work reported here:

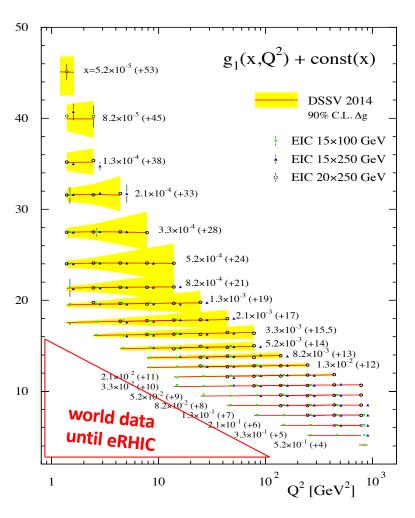
The G(1) Problem: Fact and Fantasy on the Spin of the Proton. Nucl. Phys., B337:509–546, 1990



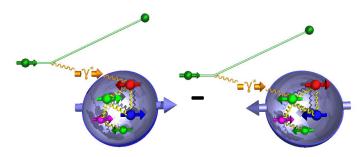
Work* in collaboration with Andrey Tarasov (The OSU and CFNS)

* https://arxiv.org/abs/2008.08104 and two follow-up papers in preparation

Resolving the proton's spin puzzle: the g₁ structure function

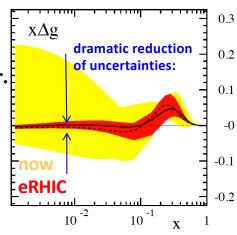


g₁ extracted from longitudinal spin asymmetry

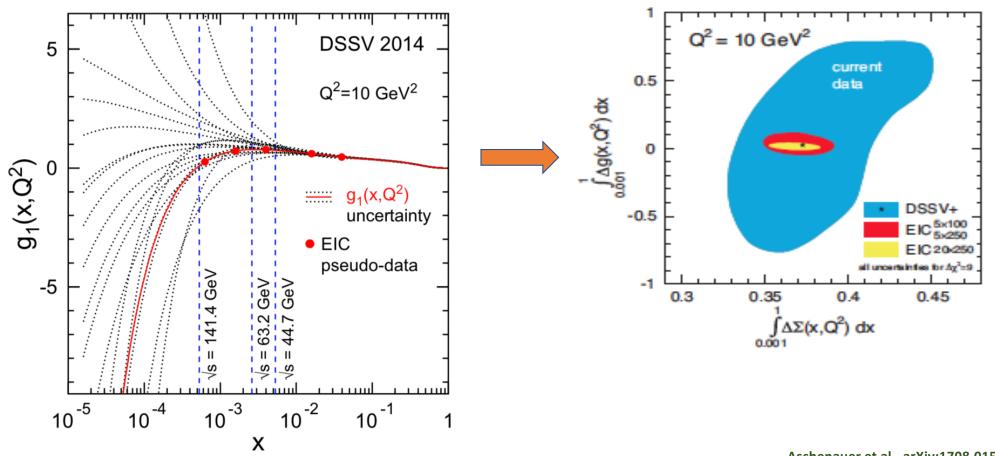


$$\Delta\Sigma(Q^2) \propto \int_0^1 dx \, g_1(x, Q^2) \to \text{quark contribution}$$

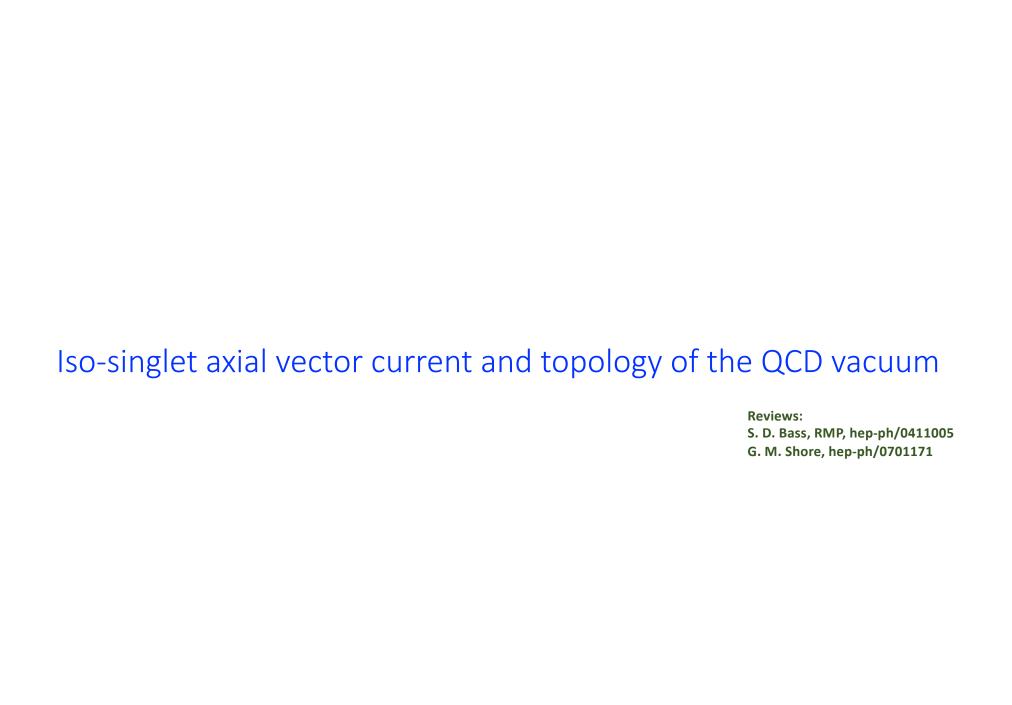
$$\frac{dg_1}{d\log(Q^2)} \stackrel{?}{\sim} -\Delta g(x, Q^2) \rightarrow \text{gluon contr.}$$



Resolving the proton's spin puzzle: the g₁ structure function

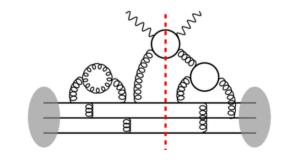


Aschenauer et al., arXiv:1708.01527 Rep. Prog. Phys. 82, 024301 (2019)



g₁ structure function: formal definitions

Hadron tensor
$$W^{\mu\nu}= {
m Im}\, rac{i}{\pi} \int d^4{f x}\, e^{i{f q}\cdot{f x}} \langle P,S| {\mathbb T}\, \hat j^\mu({f x}) \hat j^\nu(0)|P,S\rangle$$
 with $j^\mu=\sum_f e_f \bar\psi_f \gamma^\mu\psi_f$



Most generally,
$$g_1(x,Q^2)=rac{1}{8\lambda}\epsilon_T^{\mu\nu}\tilde{W}_{\mu\nu}(q,P,S)$$
 where $\widetilde{W}^{\mu\nu}$ is the antisymmetric part of $W^{\mu\nu}$ $S^\mu=rac{2\lambda}{m_P}P^\mu$ and $\lambda=\pm 1/2$

Generalized parton model ("leading twist"):
$$g_1(x_B,Q^2)=rac{1}{2}\sum_f e_f^2\left(\Delta q_f(x_B,Q^2)+\Delta \bar{q}_f(x_B,Q^2)\right)$$

Where the quark helicity pdf is defined to be

$$\Delta q(x) = \frac{1}{4\pi} \int dy^{-} e^{-iy^{-}xP^{+}} \langle P, S | \bar{\psi} (0, y^{-}, \mathbf{0}_{\perp}) \gamma^{+} \gamma_{5} \psi(0) | P, S \rangle$$

$$\Delta q(x) = -\mathbf{0} \longrightarrow -\mathbf{0}$$

Iso-singlet axial vector current

In general, the first moment of g₁:
$$\int_0^1 g_1(x,Q^2) = \frac{1}{18} \left(3F + D + 2 \, \Sigma(Q^2) \right)$$

Combination of triplet axial vector current (gives q_{Δ}) measured in B decay and octet axial vector current measured in hyperon decays

 $\Delta\Sigma(Q^2) = \sum_{n=1}^{N_f} \int_0^1 dx \left(\Delta q(x, Q^2) + \Delta \bar{q}(x, Q^2) \right)$ with the iso-singlet quark helicity given by

In the parton model picture, this mixes, under evolution, with other isospin blind moment $\Delta G = \int_0^1 dx \, \Delta g(x, Q^2)$

$$\frac{d}{d \ln Q^2} \begin{pmatrix} \Delta \Sigma \\ \Delta G \end{pmatrix} = \begin{pmatrix} \Delta P_{\Sigma \Sigma} & 2N_f \Delta P_{qG} \\ \Delta P_{Gq} & \Delta P_{GG} \end{pmatrix} \begin{pmatrix} \Delta \Sigma \\ \Delta G \end{pmatrix}$$

Splitting functions known to high loop order

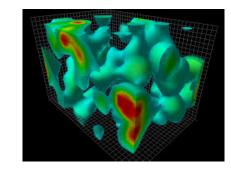
Moch, Rogal, Vermaseren, Vogt Talk by Vogelsang at POETIC IX, LBNL; DeFlorian, Vogelsang (2019)

Iso-singlet axial vector current and the chiral anomaly

$$S^{\mu} \Delta \Sigma = \langle P, S | \bar{\psi} \gamma^{\mu} \gamma_5 \psi | P, S \rangle \equiv \langle P, S | j_5^{\mu} | P, S \rangle$$

 $U_A(1)$ violation from the chiral anomaly:

$$\partial_{\mu}J_{5}^{\mu} = 2n_{f}\partial_{\mu}K^{\mu} + \sum_{i=1}^{n_{f}} 2im_{i}\bar{q}_{i}\gamma_{5}q_{i}$$



where the Chern-Simons current

$$K_{\mu} = \frac{g^2}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \left[A_a^{\nu} \left(\partial^{\rho} A_a^{\sigma} - \frac{1}{3} g f_{abc} A_b^{\rho} A_c^{\sigma} \right) \right]$$

For massless quarks, conserve
$$J_5^\mu - 2\,n_f K^\mu \longrightarrow \propto \Delta \Gamma(Q^2) = -\frac{\alpha_{\rm s}(Q^2)}{2\pi}N_{\rm f}\Delta g(Q^2)$$

So then the "real" $\Delta\Sigma$ is

$$\Sigma(Q^2) = \tilde{\Sigma}(Q^2) - \frac{\alpha_{\rm s}(Q^2)}{2\pi} N_{\rm f} \Delta g(Q^2)$$

offering a possible explanation of empirical small $\Delta\Sigma$ (in addition to flavor SU(3) violation)...

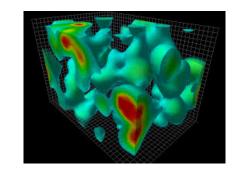
ca., 1988 Efremov, Teryaev Altarelli, Ross Carlitz, Collins, Mueller

Iso-singlet axial vector current and the chiral anomaly

$$S^{\mu} \Delta \Sigma = \langle P, S | \bar{\psi} \gamma^{\mu} \gamma_5 \psi | P, S \rangle \equiv \langle P, S | j_5^{\mu} | P, S \rangle$$

 $U_A(1)$ violation from the anomaly:

$$\partial_{\mu}J_{5}^{\mu} = 2n_{f}\partial_{\mu}K^{\mu} + \sum_{i=1}^{n_{f}} 2im_{i}\bar{q}_{i}\gamma_{5}q_{i}$$



where the Chern-Simons current

$$K_{\mu} = \frac{g^2}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \left[A_a^{\nu} \left(\partial^{\rho} A_a^{\sigma} - \frac{1}{3} g f_{abc} A_b^{\rho} A_c^{\sigma} \right) \right]$$

But, identification of CS charge with ΔG is intrinsically ambiguous

... the latter is gauge invariant, the former is not

$$K_{\mu} \to K_{\mu} + i \frac{g}{8\pi^{2}} \epsilon_{\mu\nu\alpha\beta} \partial^{\nu} \left(U^{\dagger} \partial^{\alpha} U A^{\beta} \right)$$
$$+ \frac{1}{24\pi^{2}} \epsilon_{\mu\nu\alpha\beta} \left[(U^{\dagger} \partial^{\nu} U)(U^{\dagger} \partial^{\alpha} U)(U^{\dagger} \partial^{\beta} U) \right]$$

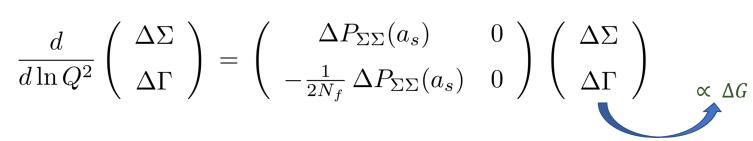
"Large gauge transformation"

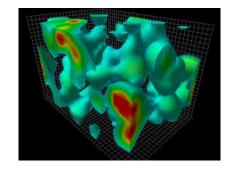
- deep consequence of topology

R. Jaffe: "Identification of K^{μ} with ΔG a source of much confusion in the literature (Varenna lectures, 2007)"

Iso-singlet axial vector current and the chiral anomaly

However, identifying CS — charge with ΔG "works" remarkably well ... (Vogelsang, POETIC IX)





NNLO splitting function computations confirm this

At LO, Altarelli, Lampe (1990) NNLO: Vogt, Moch, Rogal, Vermaseren, arXiv:0807.1238

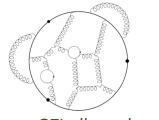
This suggests that $\Delta\Sigma$ only mixes with itself; likewise ΔG as defined, only depends on $\Delta\Sigma$ with the same splitting function

We will discuss later the deeper physics behind these expressions and their relation to the topology of the QCD vacuum

Alternative picture: topological charge screening of spin

Shore, Veneziano, PLB (1990); NPB (1992) Narison, Shore, Veneziano, hep-ph/9812333

Anomalous chiral Ward identities and extended PCAC from $U_A(1)$ breaking – physics of the η' from mixing of a "primordial massless η' and topological charge





OZI-suppressed

Example: Witten-Veneziano formula
$$m_{\eta'}^2=rac{2\,n_f}{f_\pi^2}\chi_{
m YM}(0)+O((rac{n_f}{N_c})^2)$$

where the topological susceptibility
$$\chi_{\rm YM}(p^2)=i\int dx\,e^{iP\cdot x}\langle 0|T(Q(x)Q(0)|0\rangle$$
 with $Q(x)=\frac{\alpha_S}{8\pi}{\rm Tr}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)$ In this picture, $\Sigma(Q^2)=\frac{1}{3m_N}\Delta C_1^S(\alpha_S)\left(g_{QNN}\chi(0)+g_{\eta'NN}\sqrt{\chi'(0)}\right)$

In chiral limit $\chi(0) \to 0$, $\Delta\Sigma$ "controlled" by the slope χ' at p²=0 – estimated to be small by Veneziano et al.

What about dynamics – can we reconcile this picture with the perturbative one?

Perturbative & nonperturbative interplay: The triangle graph

The key role of the $U_A(1)$ anomaly is seen from the structure of the triangle graph In the off-forward ($l^{\mu} \to 0$, $t=l^2 \to 0$) matrix element of < P',S $\mid J^5_{\mu}\mid P$,S>

$$\langle P', S|J_5^{\mu}|P, S\rangle = G_A(t)S_{\mu} + l \cdot S l_{\mu}G_P(t)$$

 p^{α}

The computation of this has an infrared pole $l^{\mu}E\tilde{E}$

proportional to $\frac{l^{\mu}}{l^2}F\tilde{F}$

Jaffe. Manohar (1990)

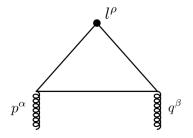
The perturbative/nonperturbative interplay gives a highly non-trivial result:

$$\langle P', S|J_5^{\mu}|P, S\rangle \mapsto \frac{l \cdot S l^{\mu}}{l^2} \kappa(t) + \left(S^{\mu} - \frac{l \cdot S l^{\mu}}{l^2}\right) \lambda(t)$$

The infrared pole in $G_A(t)$ must be canceled by a pole in $G_P(t)$ – the corresponding Wess-Zumino-Witten term for the η'

For that to hold, one must have $~\kappa(0)=\lambda(0)\propto F ilde{F}~$ the topological charge density

Perturbative & nonperturbative interplay: The triangle graph



with the (manifestly) gauge invariant result for the forward matrix element

$$\Sigma(Q^2) = \frac{n_f \,\alpha_s}{2\pi \,M_N} \lim_{l_\mu \to 0} \langle P', S | \frac{1}{il \cdot s} \text{Tr}\left(F\tilde{F}\right)(0) | P, S \rangle$$

Suprisingly (since known to Jaffe+Manohar & Veneziano et al) not addressed in a lot of the "pQCD" literature It is however deeply profound since it is intimately tied to the mechanism whereby the η' gets its mass

This result, generalization to $g_1(x,Q^2)$, and the interplay with non-perturbative physics can be explored efficiently in a worldline framework

TI . I .	1 1 1	i
I hinking hraner	IV about abomal	lies with worldlines
THIRING PROPER	ry about anomai	ICS WILL WOLIGITICS

Review: Schubert, Phys. Repts. (2001)

N. Mueller, RV: 1701.03331.1702.01233,1901.10492

Tarasov, RV: 1903.11624 and in preparation

The worldline formulation of QFT is equivalent to the string amplitude formalism of Bern and Kosower, as shown by Strassler

Bern, Kosower, NPB 379 (1992) 145; Bern, TASI lectures, hep-ph/9304249 Strassler, NPB 385 (1992) 145

World-line formalism: vector and axial vector fields

$$S[A, A_5] = \int d^4x \, \bar{\psi} \left(i \partial \!\!\!/ + A \!\!\!/ + \gamma_5 \!\!\!/ A_5 \right) \psi$$

One loop effective action $\Gamma[A,A_5]=\log\det\left(\chi\right)$ with $\chi=i\not\!\!\partial+\not\!\!A+\gamma_5\not\!\!A_5$

$$\Gamma[A, A_5] = \Gamma_R + i\Gamma_I \longrightarrow$$

Phase of the determinant..

An elegant way to represent the chiral anomaly which Fundamentally arises from non-invariance of path integral measure under a chiral rotation

Remarkably, this phase can be re-expressed in a form that is nearly identical to Γ_R !

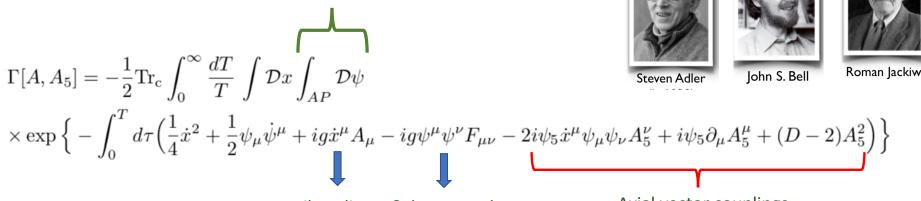
Useful mnemonic: *Odd* powers of γ_5 contribute to Γ_I and *Even* powers to Γ_R

D'Hoker, Gagne, hep-ph/9512080 See also Mondragon, Nellen, Schmidt, Schubert, hep-th/9502125

The triangle anomaly in the worldline formalism

Tarasov, RV

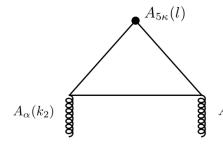
Point particle Bose and Grassmann path integrals



Wilson line Spin precession

Axial vector couplings

$$\langle P', S|J_5^{\kappa}|P, S\rangle = \int d^4y \frac{\partial}{\partial A_{5\kappa}(y)} \Gamma[A, A_5] \Big|_{A_5=0} e^{ily} \equiv \Gamma_5^{\kappa}[l]$$



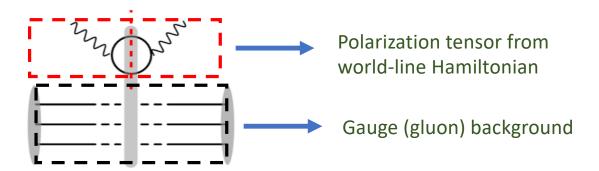
$$= \frac{1}{4\pi^2} \left(\frac{l^{\kappa}}{l^2} \right) \int \frac{d^4k_2}{(2\pi)^4} \int \frac{d^4k_4}{(2\pi)^4} \operatorname{Tr}_{c} F_{\alpha\beta}(k_2) \tilde{F}^{\alpha\beta}(k_4) (2\pi)^4 \delta^4(l+k_2+k_4)$$

Famous infrared pole of anomaly



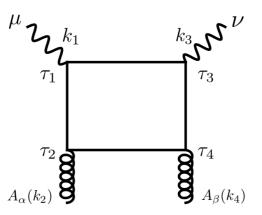
William A. Bardeen

Polarized DIS in the worldline formalism



The box diagram for polarized DIS $(g_1(x,Q^2))$

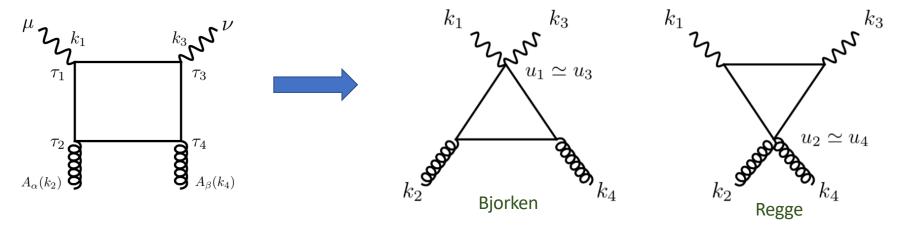
DIS with worldlines, Tarasov, RV (2019)



$$\Gamma_A^{\mu\nu}[k_1,k_3] = \int \frac{d^4k_2}{(2\pi)^4} \int \frac{d^4k_4}{(2\pi)^4} \; \Gamma_A^{\mu\nu\alpha\beta}[k_1,k_3,k_2,k_4] \; \mathrm{Tr_c}(\tilde{A}_\alpha(k_2)\tilde{A}_\beta(k_4))$$
 Polarization tensor Box diagram (antisymmetric piece)

We can compute the box explicitly in both the Bjorken limit of QCD ($Q^2 \to \infty$, $s \to \infty$, x = fixed) and the Regge limit ($x \to 0$, $s \to \infty$, $Q^2 = fixed$). The latter result is new

Finding triangles in boxes in Bjorken and Regge asymptotics



Remarkably, $g_1(x_B, Q^2)$ has the same structure in both limits, dominated by the triangle anomaly!

$$S^{\mu}g_1(x_B,Q^2)\Big|_{Q^2\rightarrow\infty} = \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N} \int_{x_B}^1 \frac{dx}{x} \, \left(1-\frac{x_B}{x}\right) \int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_{\mu}\rightarrow0} \frac{l^{\mu}}{l^2} \langle P',S| {\rm Tr}_c F_{\alpha\beta}(\xi n) \tilde{F}^{\alpha\beta}(0) |P,S\rangle \qquad + \mbox{non-pole} \ \frac{\Lambda^2 QCD}{Q^2} + \frac{1}{2} \left(1-\frac{x_B}{x}\right) \int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_{\mu}\rightarrow0} \frac{l^{\mu}}{l^2} \langle P',S| {\rm Tr}_c F_{\alpha\beta}(\xi n) \tilde{F}^{\alpha\beta}(0) |P,S\rangle \\ = \frac{1}{2} \left(1-\frac{x_B}{x}\right) \int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_{\mu}\rightarrow0} \frac{l^{\mu}}{l^2} \langle P',S| {\rm Tr}_c F_{\alpha\beta}(\xi n) \tilde{F}^{\alpha\beta}(0) |P,S\rangle \\ = \frac{1}{2} \left(1-\frac{x_B}{x}\right) \int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_{\mu}\rightarrow0} \frac{l^{\mu}}{l^2} \langle P',S| {\rm Tr}_c F_{\alpha\beta}(\xi n) \tilde{F}^{\alpha\beta}(0) |P,S\rangle \\ = \frac{1}{2} \left(1-\frac{x_B}{x}\right) \int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_{\mu}\rightarrow0} \frac{l^{\mu}}{l^2} \langle P',S| {\rm Tr}_c F_{\alpha\beta}(\xi n) \tilde{F}^{\alpha\beta}(0) |P,S\rangle \\ = \frac{1}{2} \left(1-\frac{x_B}{x}\right) \int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_{\mu}\rightarrow0} \frac{l^{\mu}}{l^2} \langle P',S| {\rm Tr}_c F_{\alpha\beta}(\xi n) \tilde{F}^{\alpha\beta}(0) |P,S\rangle \\ = \frac{1}{2} \left(1-\frac{x_B}{x}\right) \int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_{\mu}\rightarrow0} \frac{l^{\mu}}{l^2} \langle P',S| {\rm Tr}_c F_{\alpha\beta}(\xi n) \tilde{F}^{\alpha\beta}(0) |P,S\rangle \\ = \frac{1}{2} \left(1-\frac{x_B}{x}\right) \int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_{\mu}\rightarrow0} \frac{l^{\mu}}{l^2} \langle P',S| {\rm Tr}_c F_{\alpha\beta}(\xi n) \tilde{F}^{\alpha\beta}(0) |P,S\rangle \\ = \frac{1}{2} \left(1-\frac{x_B}{x}\right) \int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_{\mu}\rightarrow0} \frac{l^{\mu}}{l^2} \langle P',S| {\rm Tr}_c F_{\alpha\beta}(\xi n) \tilde{F}^{\alpha\beta}(0) |P,S\rangle \\ = \frac{1}{2} \left(1-\frac{x_B}{x}\right) \int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_{\mu}\rightarrow0} \frac{l^{\mu}}{l^2} \langle P',S| {\rm Tr}_c F_{\alpha\beta}(\xi n) \tilde{F}^{\alpha\beta}(0) |P,S\rangle \\ = \frac{1}{2} \left(1-\frac{x_B}{x}\right) \int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_{\mu}\rightarrow0} \frac{l^{\mu}}{l^2} \langle P',S| {\rm Tr}_c F_{\alpha\beta}(\xi n) \tilde{F}^{\alpha\beta}(0) |P,S\rangle \\ = \frac{1}{2} \left(1-\frac{x_B}{x}\right) \int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_{\mu}\rightarrow0} \frac{l^{\mu}}{l^2} \langle P',S| {\rm Tr}_c F_{\alpha\beta}(\xi n) \tilde{F}^{\alpha\beta}(0) |P,S\rangle \\ = \frac{1}{2} \left(1-\frac{x_B}{x}\right) \int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_{\mu}\rightarrow0} \frac{l^{\mu}}{l^{\mu}} \langle P',S| {\rm Tr}_c F_{\alpha\beta}(\xi n) |P,S\rangle \\ = \frac{1}{2} \left(1-\frac{x_B}{x}\right) \int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_{\mu}\rightarrow0} \frac{l^{\mu}}{l^{\mu}} \langle P',S| {\rm Tr}_c F_{\alpha\beta}(\xi n) |P,S\rangle \\ = \frac{1}{2} \left(1-\frac{x_B}{x}\right) \int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_{\mu}\rightarrow0} \frac{l^{\mu}}{l^{\mu}} \langle P',S| {\rm Tr}_c F_{\alpha\beta}(\xi n) |P,S\rangle \\ = \frac{1}{2} \left(1-\frac{x_B}{x}\right) \left(1-\frac{x_B}{x}\right) \left(1-\frac{x_B}{x}\right) \left(1-\frac{x_B}{x}\right) \left(1-\frac{x_B}{x}\right) \left(1-\frac{x_B}{x}\right) \left(1-\frac{x_B$$

$$S^{\mu} g_1(x_B,Q^2)|_{x_B \to 0} = \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N} \int_{x_B}^1 \frac{dx}{x} \int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_{\mu} \to 0} \frac{l^{\mu}}{l^2} \langle P', S | \mathrm{Tr}_{\mathbf{c}} F_{\alpha\beta}(\xi n) \tilde{F}^{\alpha\beta}(0) | P, S \rangle \quad + \text{non-pole } \frac{Q_S^{-2}}{M^2} << 1$$

Hence g_1 is topological in both asymptotic limits of QCD...its relation to $\Delta g(x,Q^2)$ is unclear

The r.h.s in the previous expressions is infrared divergent – how is this cured to give a finite result?

The role of pseudoscalar fields in resolving the $U_A(1)$ problem

We **did not** previously write down the most general form of the **imaginary part** of the worldline effective action :

D'Hoker, Gagne, hep-th/9508131

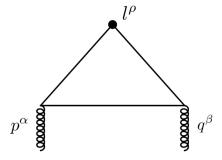
$$\begin{split} W_{\Im}[\Phi,\Pi,A] &= \frac{1}{8} \int_{-1}^{1} d\alpha \int_{0}^{\infty} dT \, \mathcal{N} \int \, \mathcal{D}x \mathcal{D}\psi \, \mathrm{Tr_c} \, \mathcal{J}(0) \, \mathcal{P}e^{-\int_{0}^{T} d\tau \mathcal{L}_{\alpha}} \\ &\text{with} \quad \mathcal{L}_{\alpha} = \frac{\dot{x}^2}{2\mathcal{E}} + \frac{1}{2} \psi_A \dot{\psi}_A - i \dot{x} \cdot A + \frac{i}{2} \mathcal{E}\psi_{\mu} F_{\mu\nu} \psi_{\nu} + \frac{1}{2} \mathcal{E}\alpha^2 \Phi^2 + \frac{1}{2} \mathcal{E}\Pi^2 \\ &\quad + \frac{i}{\epsilon} \mathcal{E}\psi_{\mu} \psi_5 D_{\mu} \Pi + \alpha \, \epsilon \, \psi_5 \psi_6 \big[\Pi, \Phi \big] \qquad \text{where Φ is the chiral condensate} \\ &\text{and} \quad \mathcal{J}(0) \propto \psi_5 \psi_6 \big\{ \Pi, \Phi \big\} \end{split}$$

Expanding out the worldline Lagrangian, the first nontrivial contribution to W_I is the Wess-Zumino-Witten term!

$$W_{\Im}[\Pi^{5}] = -\frac{i}{5} \int_{p^{1},\dots,p^{5}} (2\pi)^{4} \delta^{(4)}(p^{1} + \dots + p^{5}) (-4im) \operatorname{Tr}_{c}(\tilde{\Pi}_{1} \dots \tilde{\Pi}_{5}) \varepsilon_{\mu_{1} \dots \mu_{4}} p_{\mu_{1}}^{1} \dots p_{\mu_{4}}^{4} I'(p^{i})$$

The role of pseudoscalar fields in resolving the $U_A(1)$ problem

Tarasov, RV, in preparation

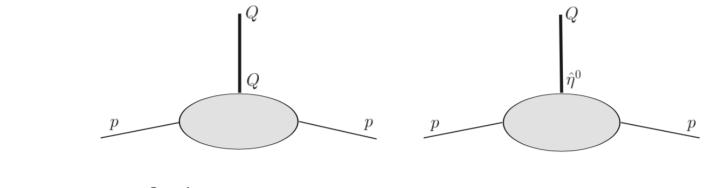


The WZW term has a contribution $\propto \eta' F \tilde{F}$ (Leutwyler (1996); Herrara-Sikody et al (1997); Leutwyler-Kaiser (2000))

The zero mode part of it exactly cancels the $\frac{l^{\mu}}{l^2}$ "perturbative" contribution to the anomaly

$$g_1(x_B, Q^2) \rightarrow \frac{\alpha_S}{8\pi} \lim_{l_\mu \to 0} \langle P', S | \frac{1}{il \cdot s} \operatorname{Tr} \left(F \tilde{F} \right) (0) | P, S \rangle$$

Conjecture: "Axion-like" effective action for polarized DIS



$$\Delta\Sigma(Q^2) = \frac{2}{3} \frac{1}{2m_N} \Delta C_1^S(\alpha_s) \left(\langle 0|T \ Q \ Q|0 \rangle \ g_{QNN} + \langle 0|T \ Q \ \hat{\eta}^0|0 \rangle \ g_{\hat{\eta}^0NN} \right)$$

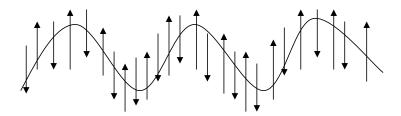
$$\approx 0 \qquad \qquad \chi'(p^2 = 0)$$

Narison, Shore and Veneziano (see hep-ph/0701171) argue that the result is dominated by the derivative of the topological susceptibility (χ'), which they compute (using QCD sum rules) to be in agreement with the HERMES and COMPASS data

 g_1 can also in principle be computed on the lattice (Kei-Fei Liu et al) though computing Off-forward matrix elements of non-local operators is



Conjecture: "Axion-like" effective action for Regge limit



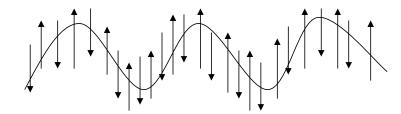
If we now assume (at small x), the background field couples to a large # of quark and gluon world-line trajectories, one can construct the *effective action for an ensemble of spinning, colored partons at a given x*<< 1:

$$g_1(x,Q^2) \propto \int [D\rho] W_Y^P[\rho] \int [D\psi] \tilde{W}_P^{P,S}[\psi] \qquad \text{Density matrix: coupling of charge and topological charge to nucleon} \\ \times \int [dA] \int d^4 X \, \omega(X) \exp\left(i S_{\rm YM}[A] + \frac{i}{N_c} {\rm Tr}_c \left(\rho \, U_{-\infty,\infty}\right)\right) \qquad \text{Color charge current} \\ \times \exp\left(\int d^4 X \left(-\frac{\omega^2}{2 \, \chi_{\rm YM}} - \sqrt{\frac{N_f}{2}} \omega \, \psi + \frac{1}{2} F^2 \psi \partial^2 \psi\right)\right) \qquad \text{Topological current}$$

Topological susceptibility:
$$\chi_{\mathrm{YM}} = \int d^4 X \langle \omega(X) \omega(0) \rangle_A$$

 $\omega(X)$ is the topological charge density, $F \psi$ is the η' field and F is the η' decay constant

Conjecture: "Axion-like" effective action for Regge limit



$$\text{The term} \quad \exp\left(\int d^4 X \left(-\frac{\omega^2}{2\,\chi_{\text{YM}}} - \sqrt{\frac{N_f}{2}}\omega\,\psi + \frac{1}{2}F^2\psi\partial^2\psi\right)\right)$$

can be rewritten as the "Veneziano effective action" of a glueball and η' field

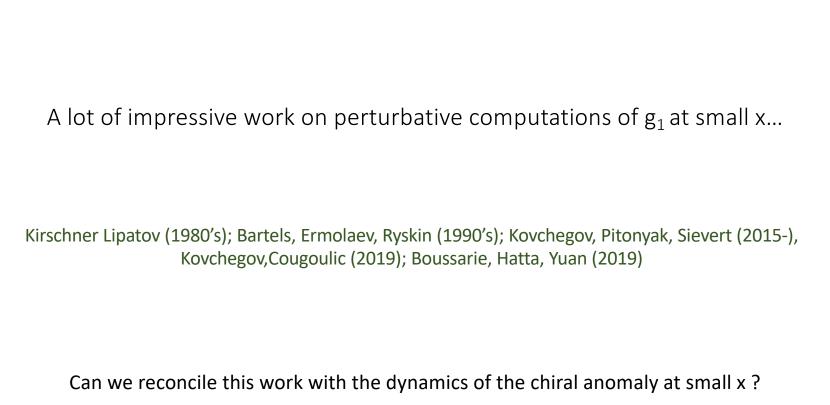
$$\exp\left(-\int d^4X\left(-\frac{G^2}{2\chi_{\rm YM}} + \frac{1}{2}\eta'\left(\partial^2 + m_{\eta'}^2\right)\eta'\right)\right)$$

ChPT formulation for the nonet, Herrera-Siklody et al, hep-ph/9610549

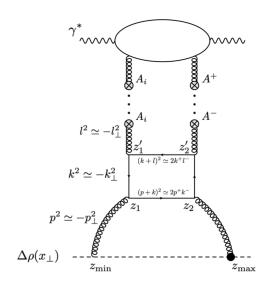
with $m_{\eta'}$ given by the Witten-Veneziano formula $~m_{\eta'}^2=rac{2N_f~\chi_{
m YM}}{F^2}$

or equivalently as
$$\ \exp\left(\int d^4X \left(F^2\psi\partial^2\psi+\left(\psi+\theta\right)\omega+\chi_{\mathrm{YM}}\left(\psi+\theta\right)^2\right)\right)$$





QCD evolution to small x

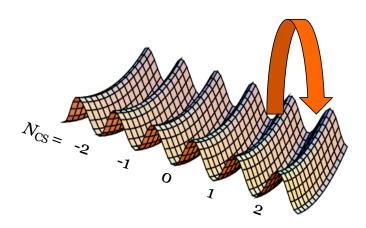


After smearing of the shockwave target in x^- , the effective action reproduces itself (in a double log approximation)

 renormalizing the "axion effective action" at small x, one should recover the triangle structure in the "box diagram"

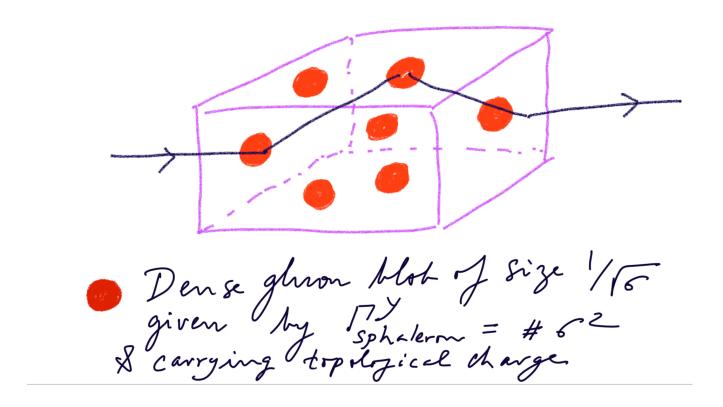
Fascinating interplay between gluon saturation (controlled by the saturation scale Q_S) and spin diffusion (controlled by the topological charge density ω)

Spin diffusion via sphaleron transitions in topologically disordered media



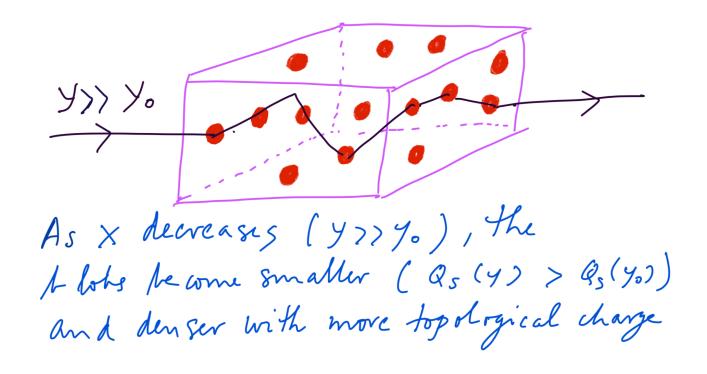
Over the barrier (sphaleron) transitions between different topological sectors of QCD vacuum...characterized by integer valued Chern-Simons # --analogous to proposed mechanism for Electroweak Baryogenesis

Spin diffusion via sphaleron transitions in topologically disordered media



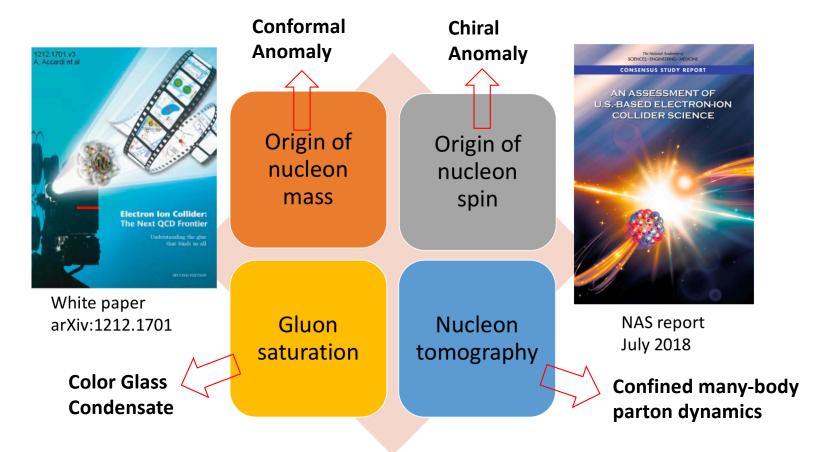
Helicity flip for massless quarks given by nL - nR = 2 $N_f \, \nu$ where $\Gamma^Y_{sphaleron} \propto \langle \nu^2 \rangle$

Spin diffusion via sphaleron transitions in topologically disordered media



Helicity flip for massless quarks given by $nL - nR = 2 N_f v$ where $\Gamma^Y_{sphaleron} \propto \langle v^2 \rangle$

Outlook: these ideas can be tested at the EIC!



Precision probes of the strong interplay between perturbative many-body parton dynamics and non-perturbative structure ("the ether") of the QCD vacuum