Investigating the virtual pion's gluon structure at small *x* with leading neutron events

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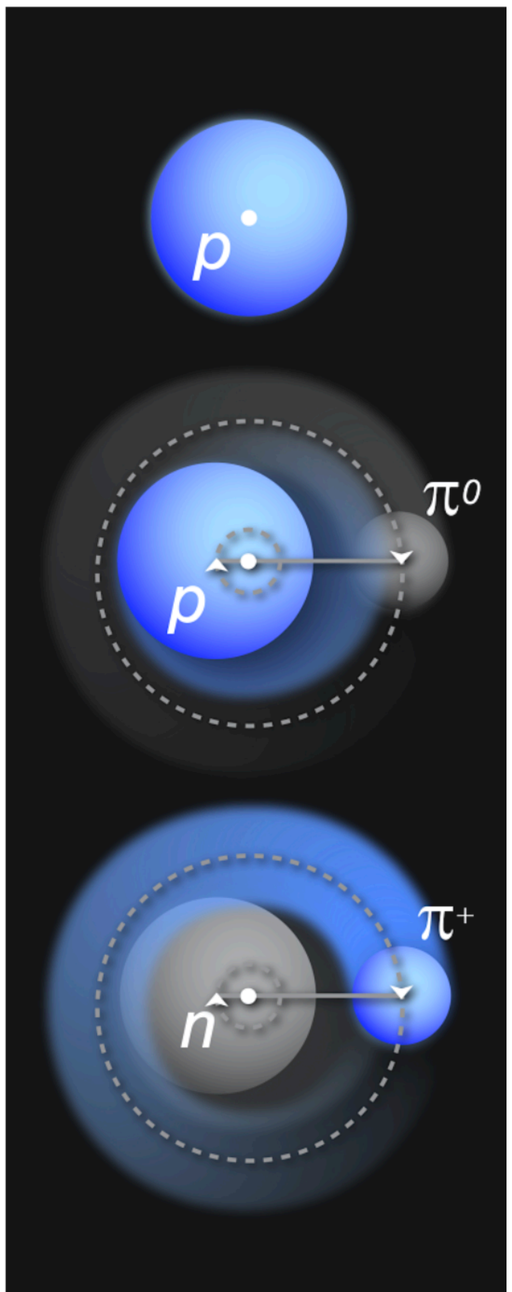
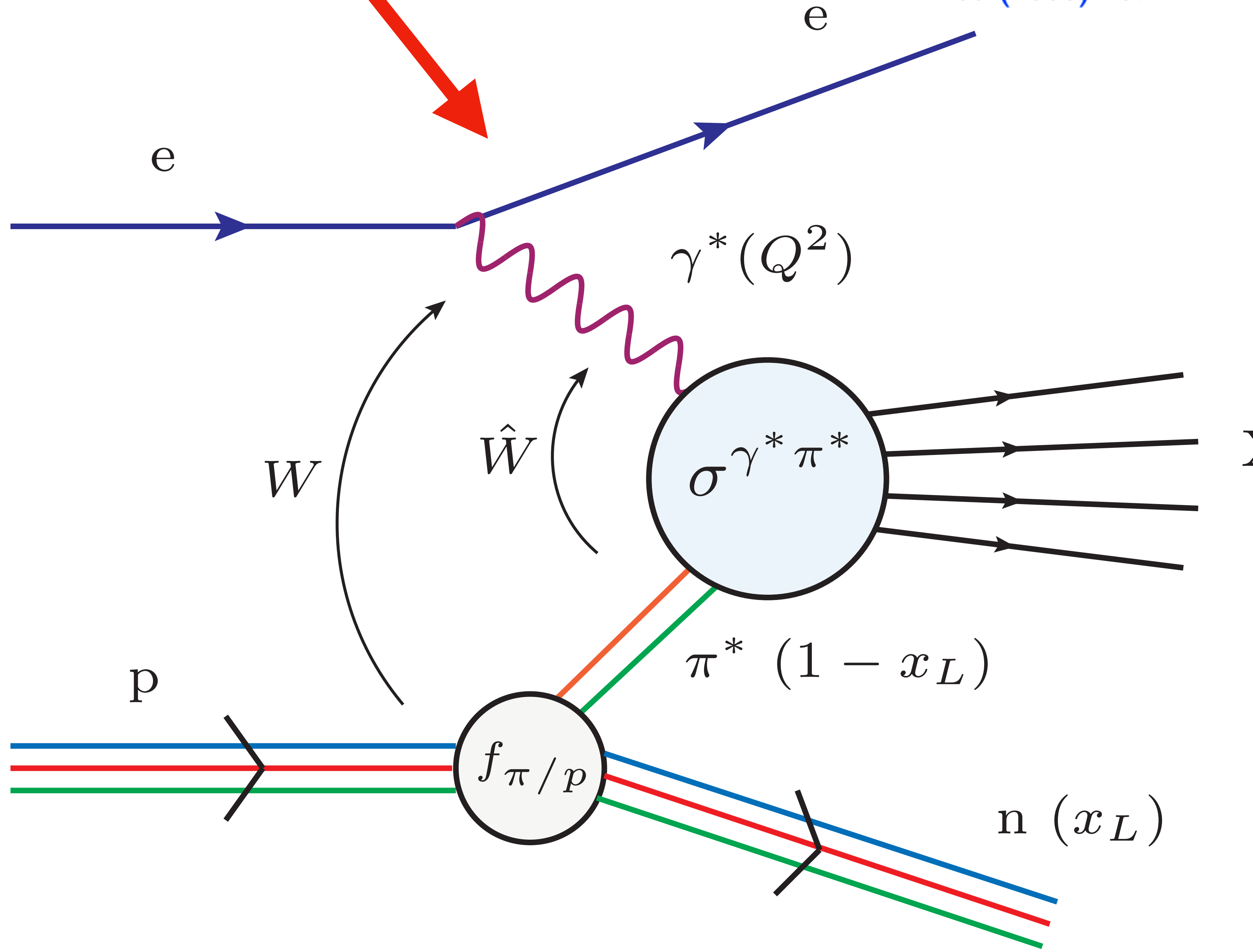
Arjun Kumar, TT: arXiv:2203.13314

The "Pion Cloud"

$$|p> \to \sqrt{1-a-b}\;|p_o> + \sqrt{a}\left(-\sqrt{\frac{1}{3}}\;|p_0\,\pi^0> + \sqrt{\frac{2}{3}}\;|n_0\,\pi^+>\right) + \sqrt{b}\left(-\sqrt{\frac{1}{2}}\;|\Delta_0^{++}\,\pi^-> - \sqrt{\frac{1}{3}}\;|\Delta_0^{+}\,\pi^0> + \sqrt{\frac{1}{6}}\;|\Delta_0^{0}\,\pi^+>\right)$$

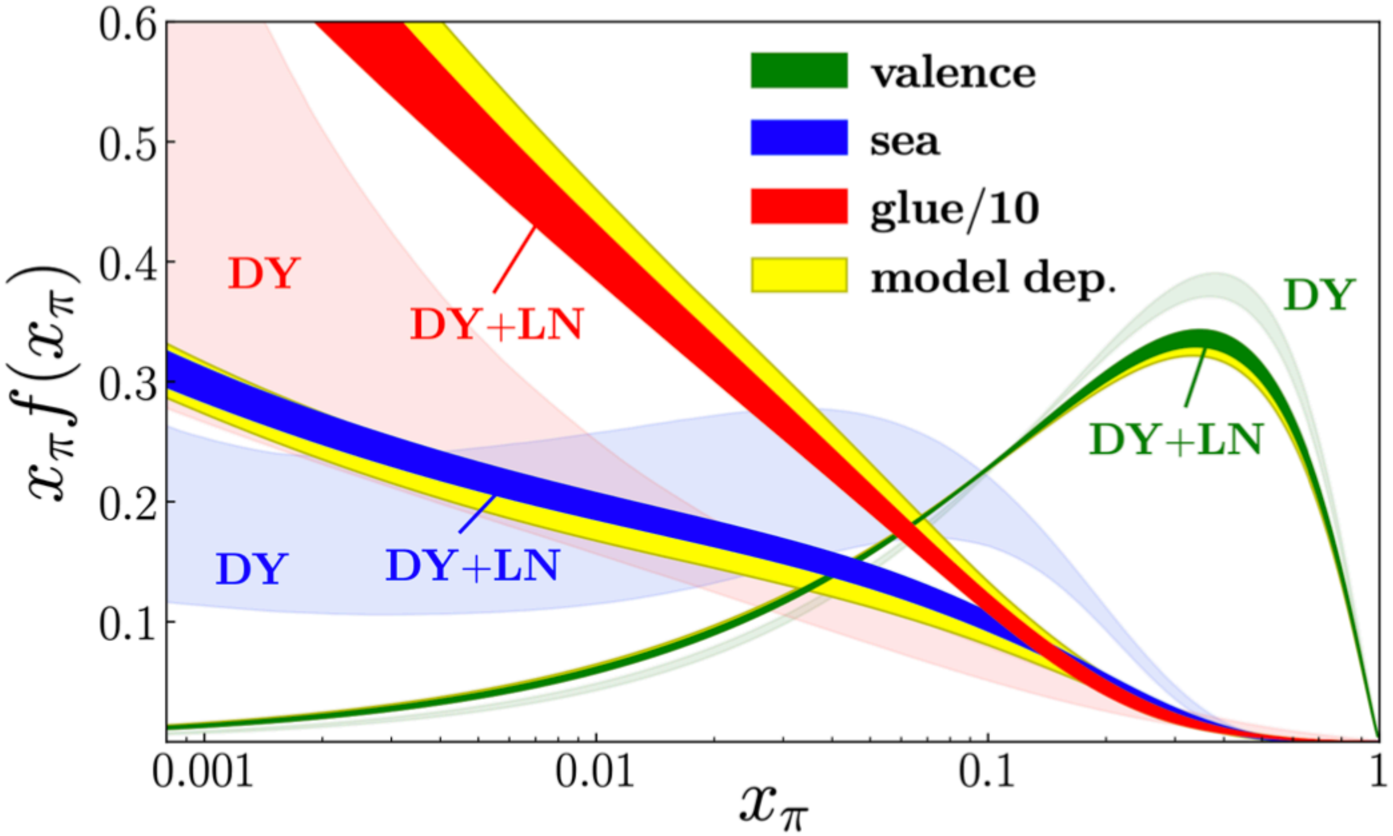
Chiral approach: a=0.24, b=0.12
Thomas, Melnitchouk & Steffens, PRL85 (2000) 2892



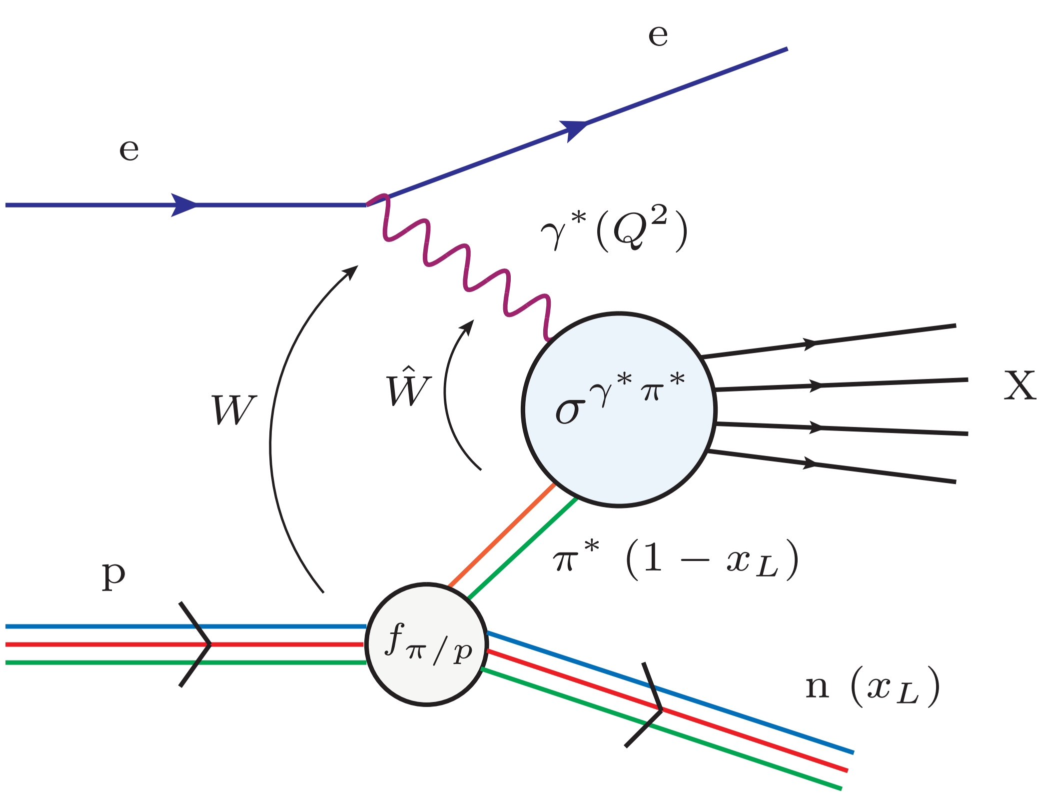


The "Pion Cloud"

$$|p> \to \sqrt{1-a-b}\ |p_o> + \sqrt{a}\left(-\sqrt{\frac{1}{3}}\ |p_0\ \pi^0> + \sqrt{\frac{2}{3}}\ |n_0\ \pi^+>\right) + \sqrt{b}\left(-\sqrt{\frac{1}{2}}\ |\Delta_0^{++}\ \pi^-> - \sqrt{\frac{1}{3}}\ |\Delta_0^{+}\ \pi^0> + \sqrt{\frac{1}{6}}\ |\Delta_0^{0}\ \pi^+>\right)$$



Barry et al PRL 121, 152001



Three Questions:

1. How different is the small-x gluon structure of the virtual pion compared to the proton?

2. Is Feynman scaling a signal for saturation?

Inclusive Questions

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3. How to calculate the t'-spectrum of the photon-pion iteraction? (Fourier transform of its shape)

Exclusive Question

Leading Neutron Structure Function

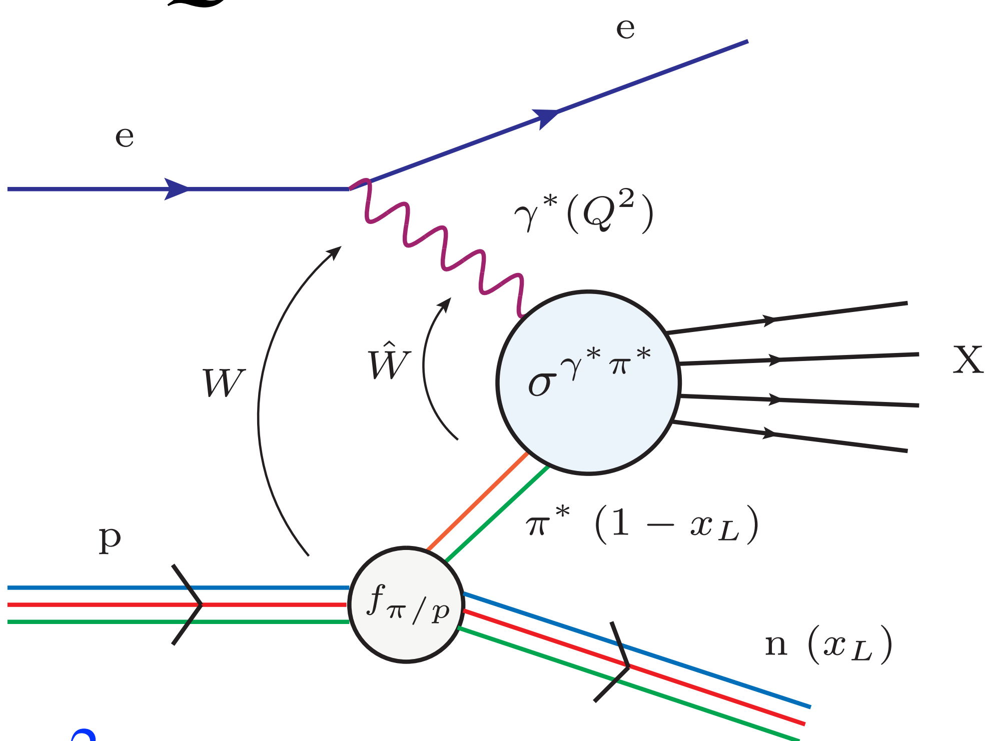
H1 EPJC 68 (2010), 381

J.D. Sullivan PRD 5 (1972), 1732

$$F_2^{LN}(x, Q^2, x_L) = \frac{Q^2}{4\pi^2\alpha_{\rm EM}} \frac{\mathrm{d}\sigma^{\gamma^* p \to Xn}}{\mathrm{d}x_L}$$

$$\hat{x} = \frac{Q^2 + m_f^2}{\hat{W}^2 + Q^2} \qquad \hat{W}^2 \simeq (1 - x_L)W^2$$

$$\frac{\mathrm{d}\sigma^{\gamma^* p \to Xn}}{\mathrm{d}x_L} = \int_{t_{min}}^{t_{max}} \frac{\mathrm{d}^2\sigma^{\gamma^* p \to Xn}}{\mathrm{d}x_L \mathrm{d}t} \mathrm{d}t$$



$$\frac{\mathrm{d}^2\sigma(W, Q^2, x_L, t)}{\mathrm{d}x_L \mathrm{d}t} = f_{\pi/p}(x_L, t)\ \sigma^{\gamma^*\pi^*}(\hat{W}^2, Q^2)$$

Carvalho, Gonçalves, Navarra, Spiering, Phys. Rev. D 103 (3) (2021) 034021

$$F_2^{LN}(W, Q^2, x_L) = \int_{t_{min}}^{t_{max}} f_{\pi/p}(x_L, t)\mathscr{K}(Q^2) F_2^{\pi}(W, Q^2, x_L)\mathrm{d}t$$

Pion Flux

Pion Structure Function

Pion Flux

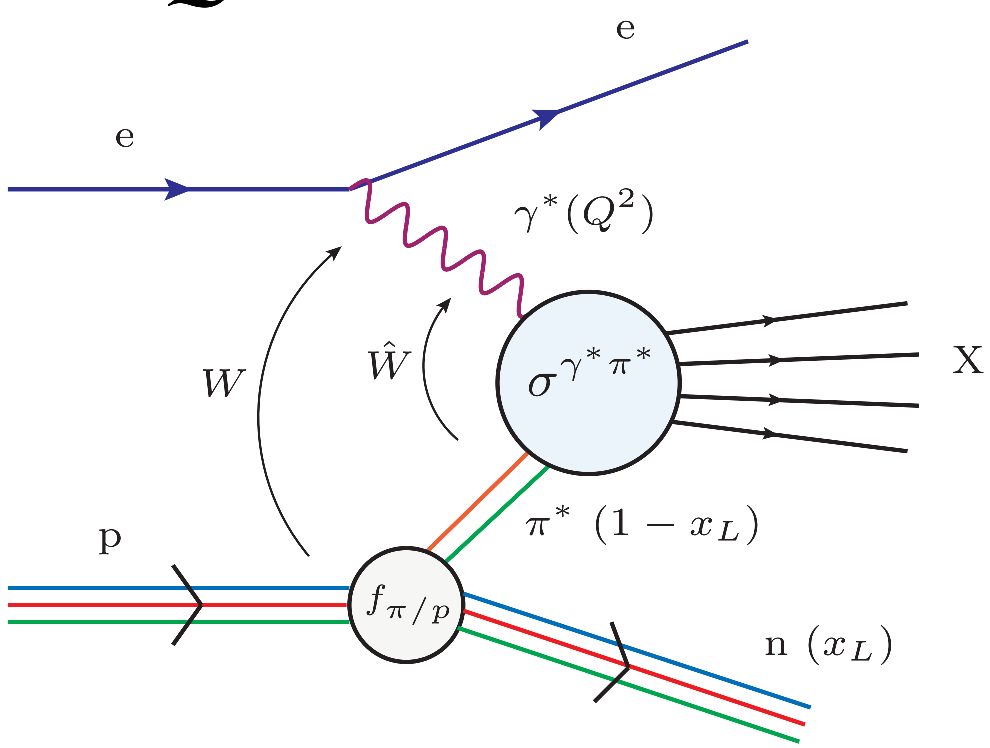
$$F_2^{LN}(W, Q^2, x_L) = \int_{t_{min}}^{t_{max}} f_{\pi/p}(x_L, t)\mathcal{K}(Q^2)F_2^{\pi}(W, Q^2, x_L)\mathrm{d}t$$

$$\hat{x} = \frac{Q^2 + m_f^2}{\hat{W}^2 + Q^2} \qquad \hat{W}^2 \simeq (1 - x_L)W^2$$

$$f_{\pi/p}(x_L, t) = \frac{1}{4\pi}\frac{2g_{p\pi p}^2}{4\pi}\frac{|t|}{(m_\pi^2 + |t|)^2}(1 - x_L)[F(x_L, t)]^2$$

$$F(x_L, t) = \exp\left[-0.036\frac{|t| + m_\pi^2}{(1 - x_L)}\right]$$

H. Holtmann and A. Szczurek *Phys.Lett.B* 338 (1994) 363-368



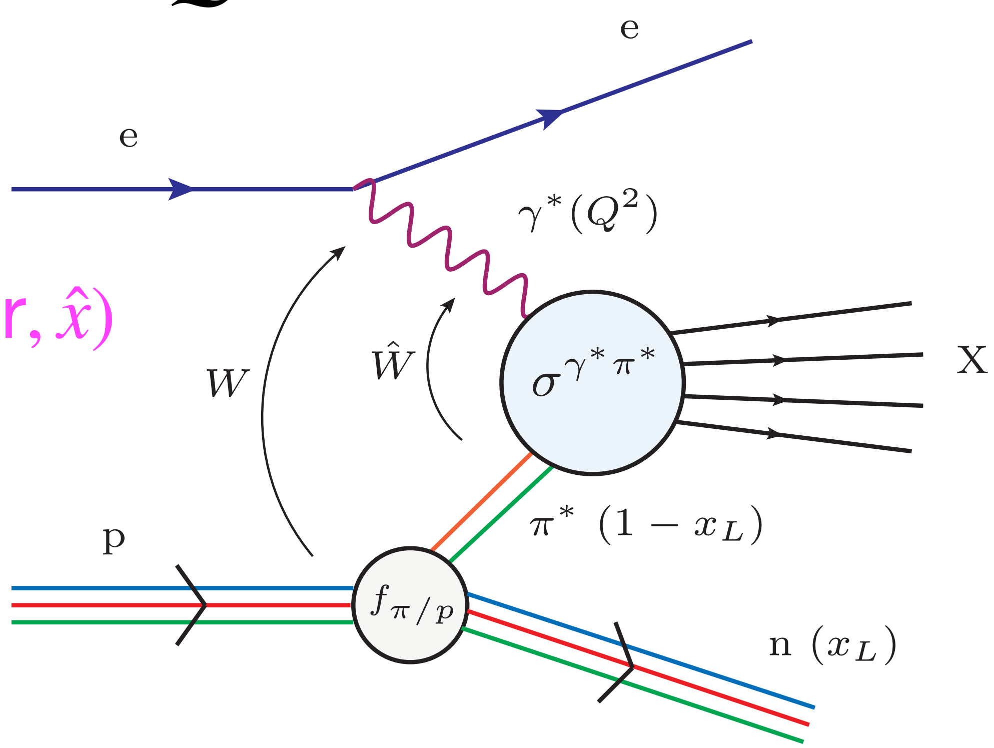
$$t \simeq -\frac{p_T^2}{x_L} - (1 - x_L)\left(\frac{m_n^2}{x_L} - m_p^2\right)$$

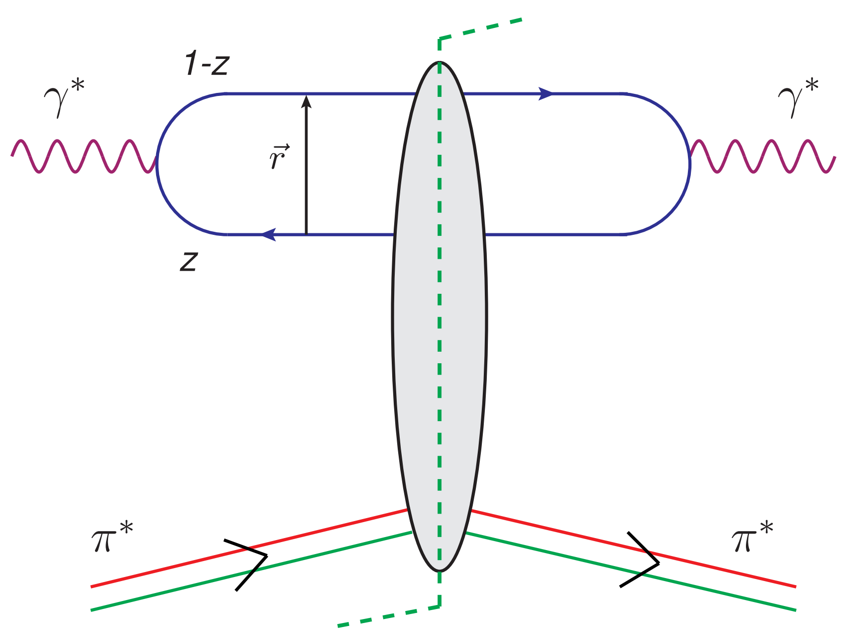
Pion Structure

$$F_2^{LN}(W,Q^2,x_L) = \int_{t_{min}}^{t_{max}} f_{\pi/p}(x_L,t)\mathscr{K}(Q^2)F_2^{\pi}(W,Q^2,x_L)\mathrm{d}t$$

$$F_2^{\pi} = \frac{Q^2}{4\pi^2\alpha_{\mathrm{EM}}}\left(\sigma_L^{\gamma^*\pi^*} + \sigma_T^{\gamma^*\pi^*}\right) \qquad \hat{x} = \frac{Q^2+m_f^2}{\hat{W}^2+Q^2} \qquad \hat{W}^2 \simeq (1-x_L)W^2$$

$$\sigma_{L,T}^{\gamma^*\pi^*}(\hat{x},Q^2) = \int \mathrm{d}^2\mathrm{b}\ \mathrm{d}^2\mathrm{r}\int_0^1 \frac{\mathrm{d}z}{4\pi}\,|\,\Psi_{L,T}^{f}(\mathrm{r},z,Q^2)\,|^2\,\frac{\mathrm{d}\sigma_{q\bar{q}}^{(\pi)}}{\mathrm{d}^2\mathrm{b}}(\mathrm{b},\mathrm{r},\hat{x})$$





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Pion Structure

$$F_2^{LN}(W, Q^2, x_L) = \int_{t_{min}}^{t_{max}} f_{\pi/p}(x_L, t)\mathcal{K}(Q^2)F_2^{\pi}(W, Q^2, x_L)\mathrm{d}t$$

$$F_2^{\pi} = \frac{Q^2}{4\pi^2\alpha_{\mathrm{EM}}}\left(\sigma_L^{\gamma^*\pi^*} + \sigma_T^{\gamma^*\pi^*}\right) \qquad \hat{x} = \frac{Q^2 + m_f^2}{\hat{W}^2 + Q^2} \qquad \hat{W}^2 \simeq (1 - x_L)W^2$$

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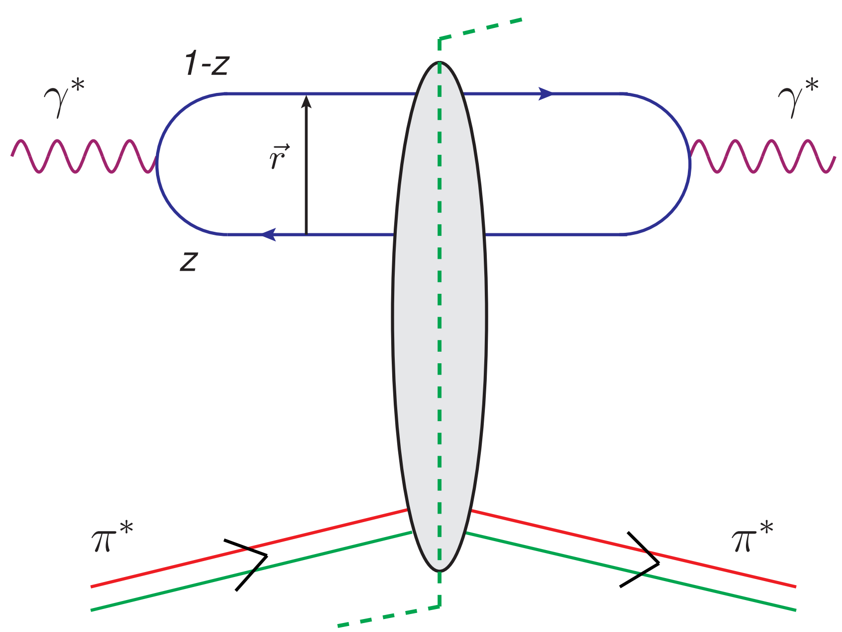


Universal gluon structure at small x:

$$\frac{\mathrm{d}\sigma_{q\bar{q}}^{(\pi)}}{\mathrm{d}^2\mathsf{b}}(\mathsf{b}, \mathsf{r}, \hat{x}) = R_g\ \frac{\mathrm{d}\sigma_{q\bar{q}}^{(p)}}{\mathrm{d}^2\mathsf{b}}(\mathsf{b}, \mathsf{r}, \hat{x})$$

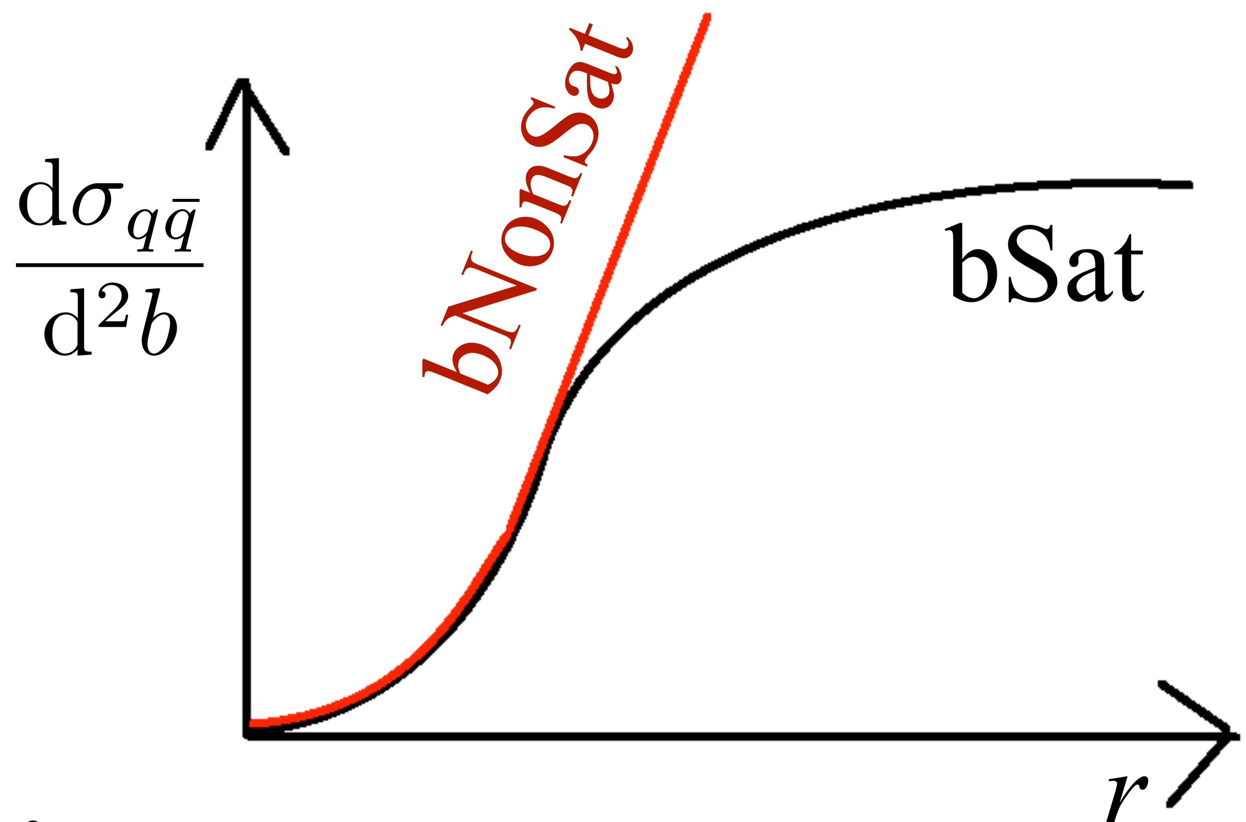
Two Dipole Models

bSat (IP-Sat)

$$\frac{d\sigma^{\rm sat}}{d^2\vec{b}} = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_C} r^2 \alpha_S(\mu^2) x g(x,\mu^2) T(b)\right)\right]$$

bNonSat

$$\frac{d\sigma^{\rm nosat}}{d^2\vec{b}} = \frac{\pi^2}{N_C} r^2 \alpha_S(\mu^2) x g(x,\mu^2) T(b)$$

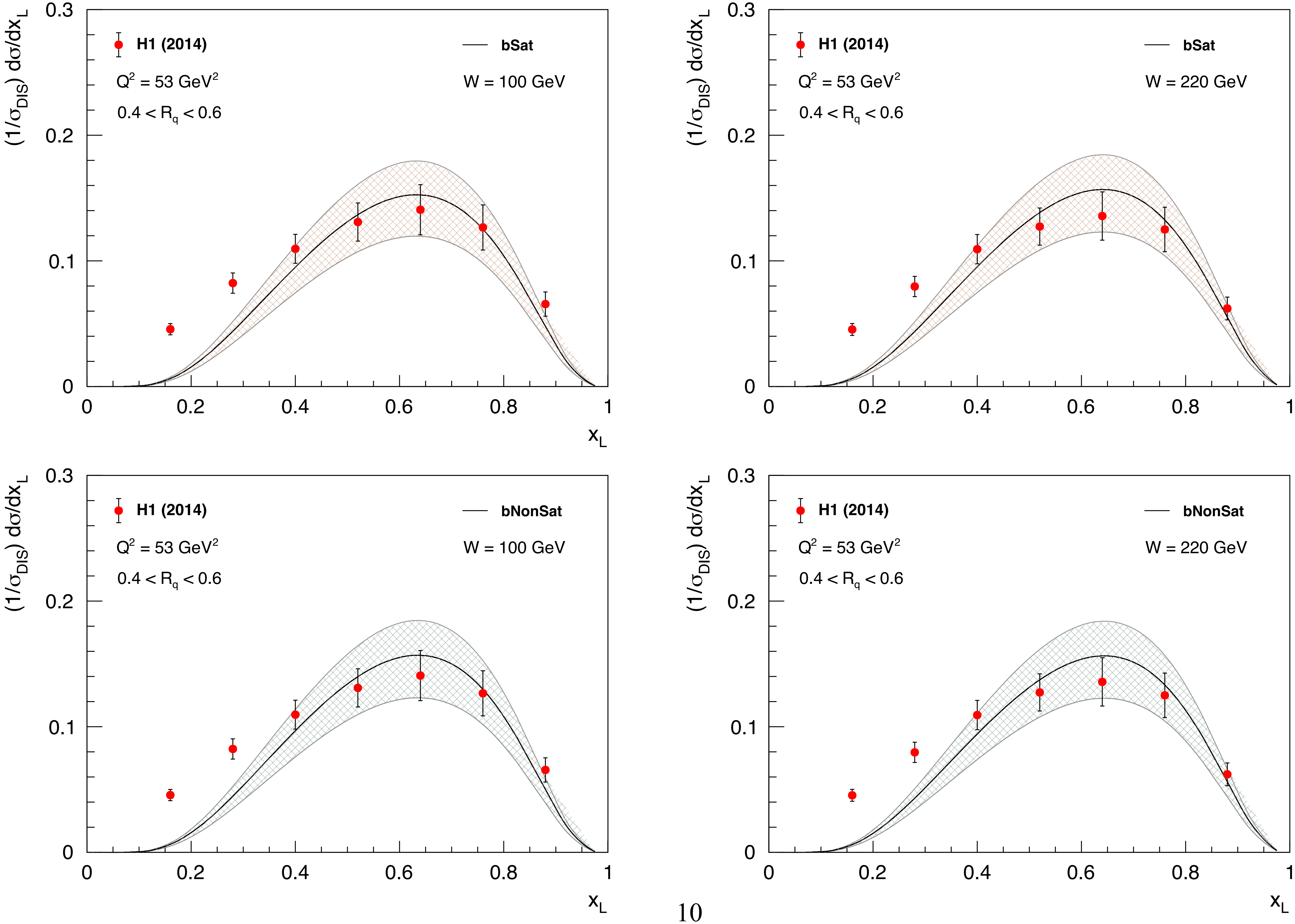


$$xg(x,\mu_0^2) = A_g x^{-\lambda_g}(1-x)^6$$

$$\mu^2 = \mu_0^2 + \frac{C}{r^2}$$

Feynman-x

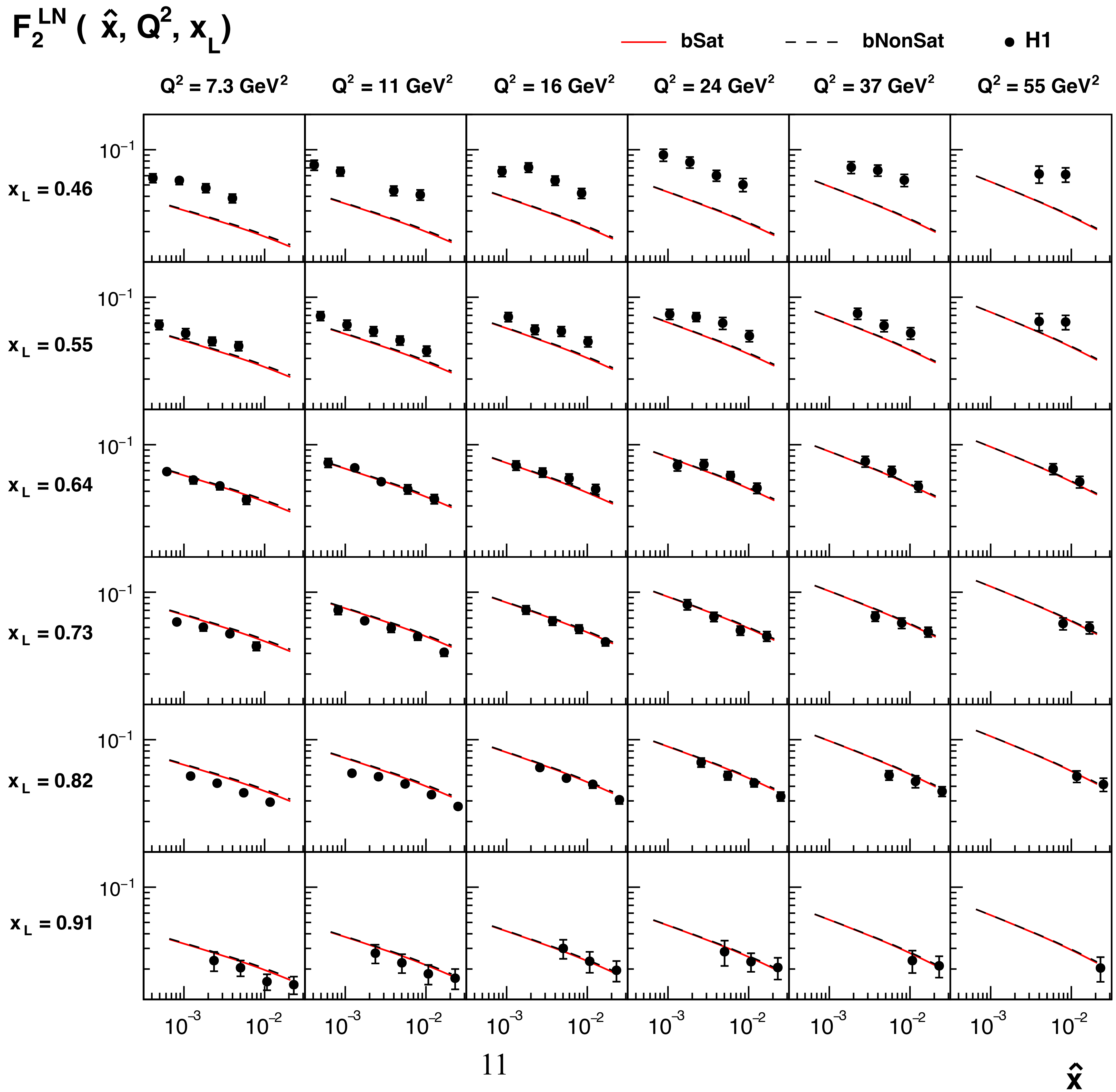
$$\frac{d\sigma^{(\pi)}_{q\bar{q}}}{d^2b}(b, r, \hat{x}) = R_g \, \frac{d\sigma^{(p)}_{q\bar{q}}}{d^2b}(b, r, \hat{x})$$



Pion Structure

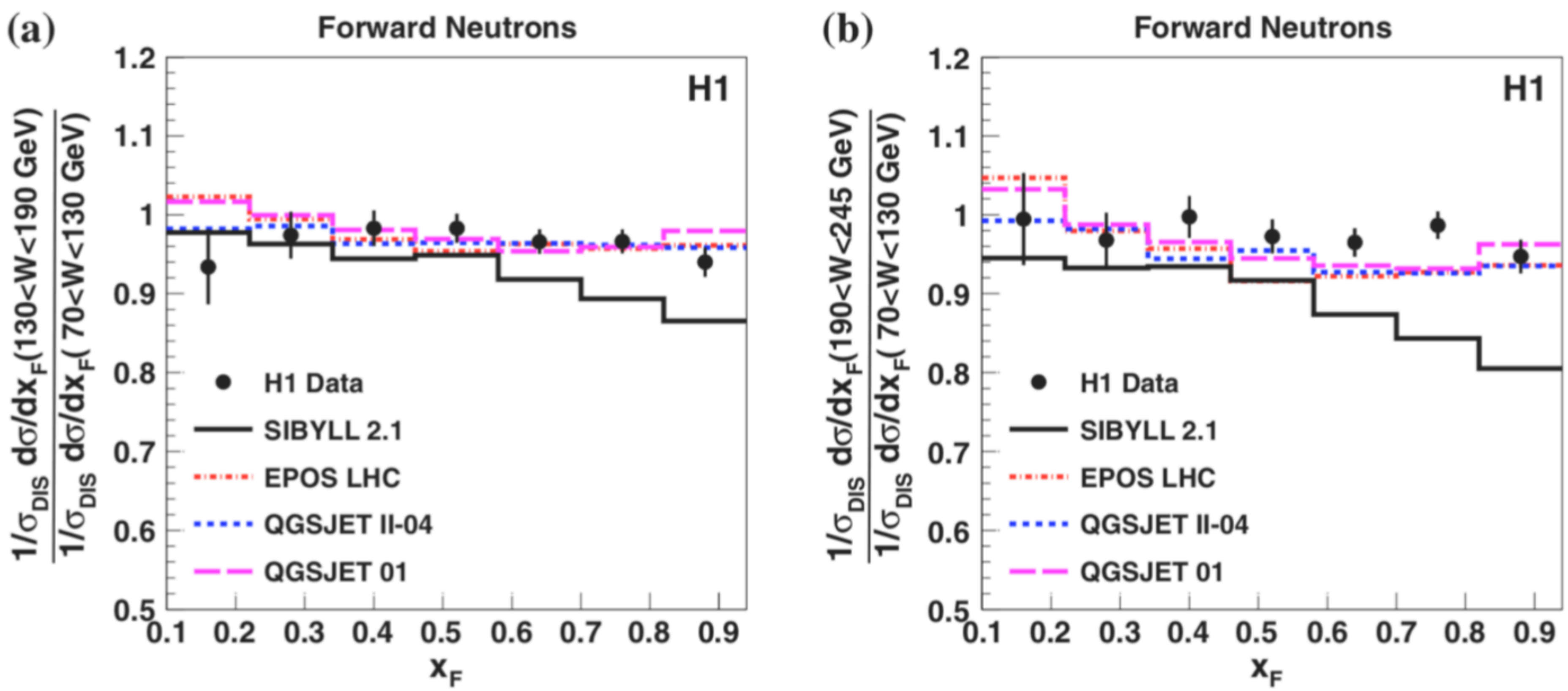
$$\frac{d\sigma_{q\bar{q}}^{(\pi)}}{d^2b}(b, r, \hat{x}) = R_g \frac{d\sigma_{q\bar{q}}^{(p)}}{d^2b}(b, r, \hat{x})$$

$R_g = 0.5$



Feynman scaling and Saturation

H1 EPJC 74 (2014), 2915



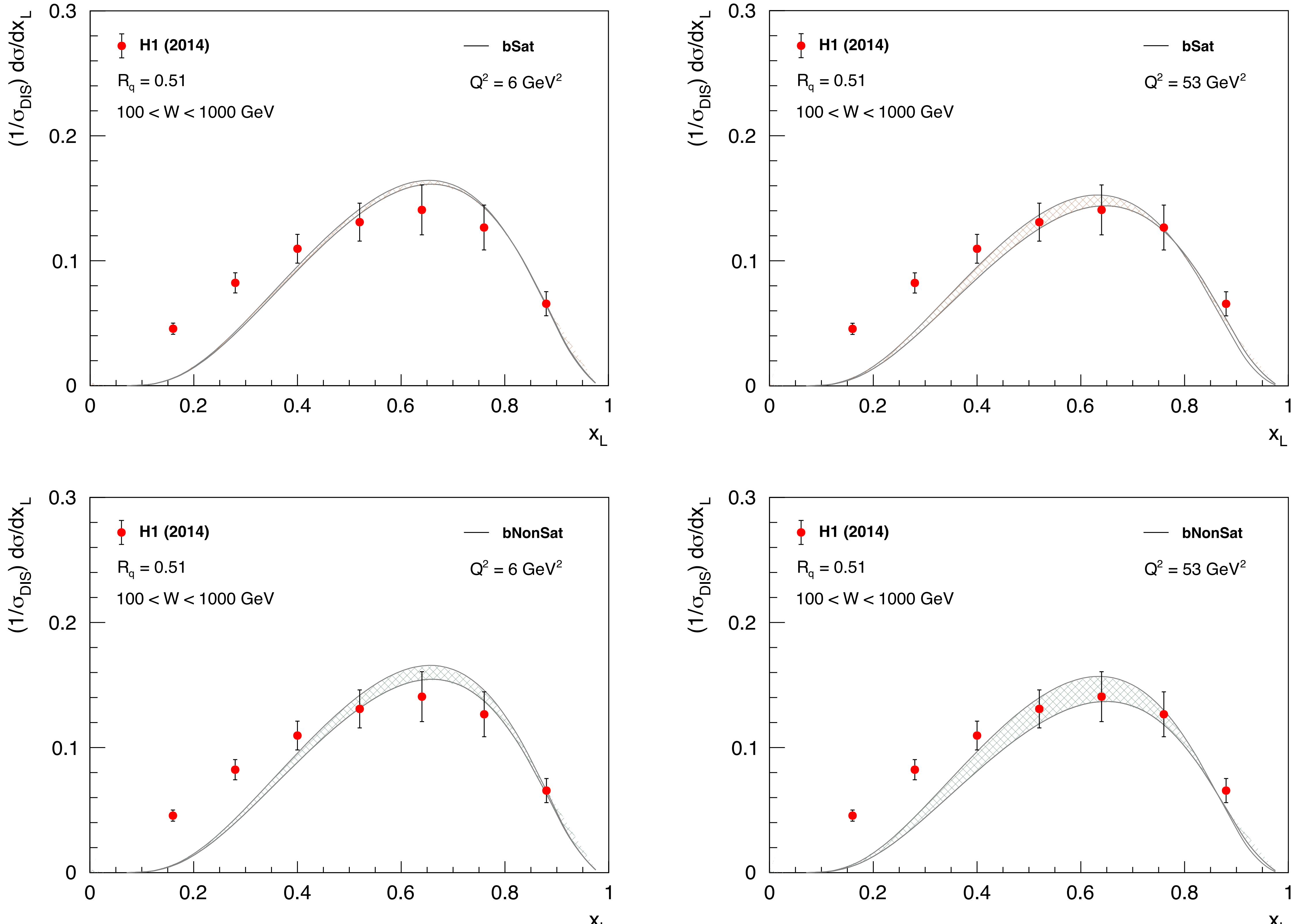
$x_F \equiv x_L$

Is Feynman scaling related to saturation?

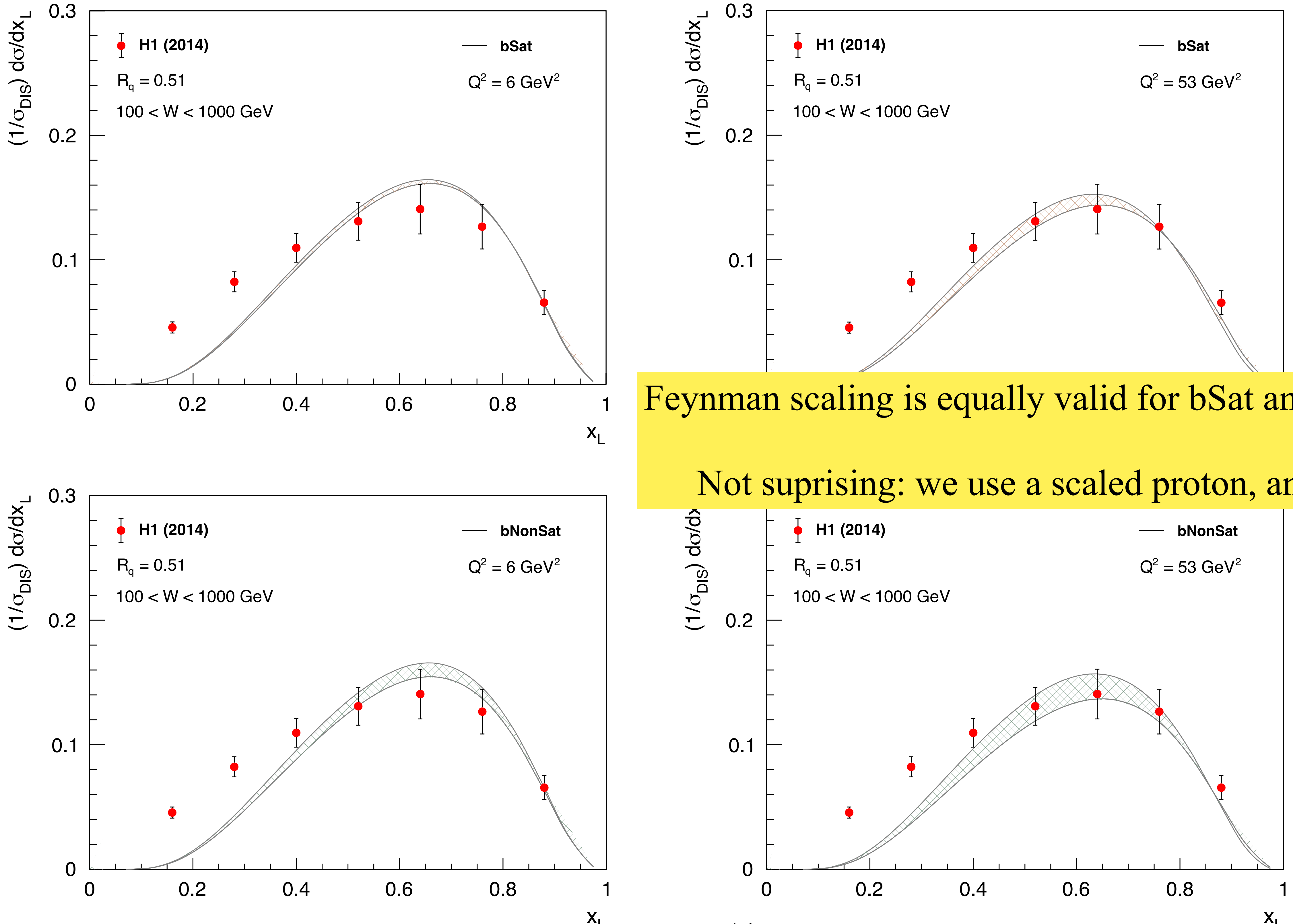
"We demonstrate the recently released H1 leading neutron spectra can be described using the color dipole formalism and that these spectra could help us to observe more clearly gluon saturation effects in future *ep* colliders"

Carvalho, Gonçalves, Spiering, Navarra PLB 752 (2016) 76 using bCGC model

Feynman scaling and Saturation



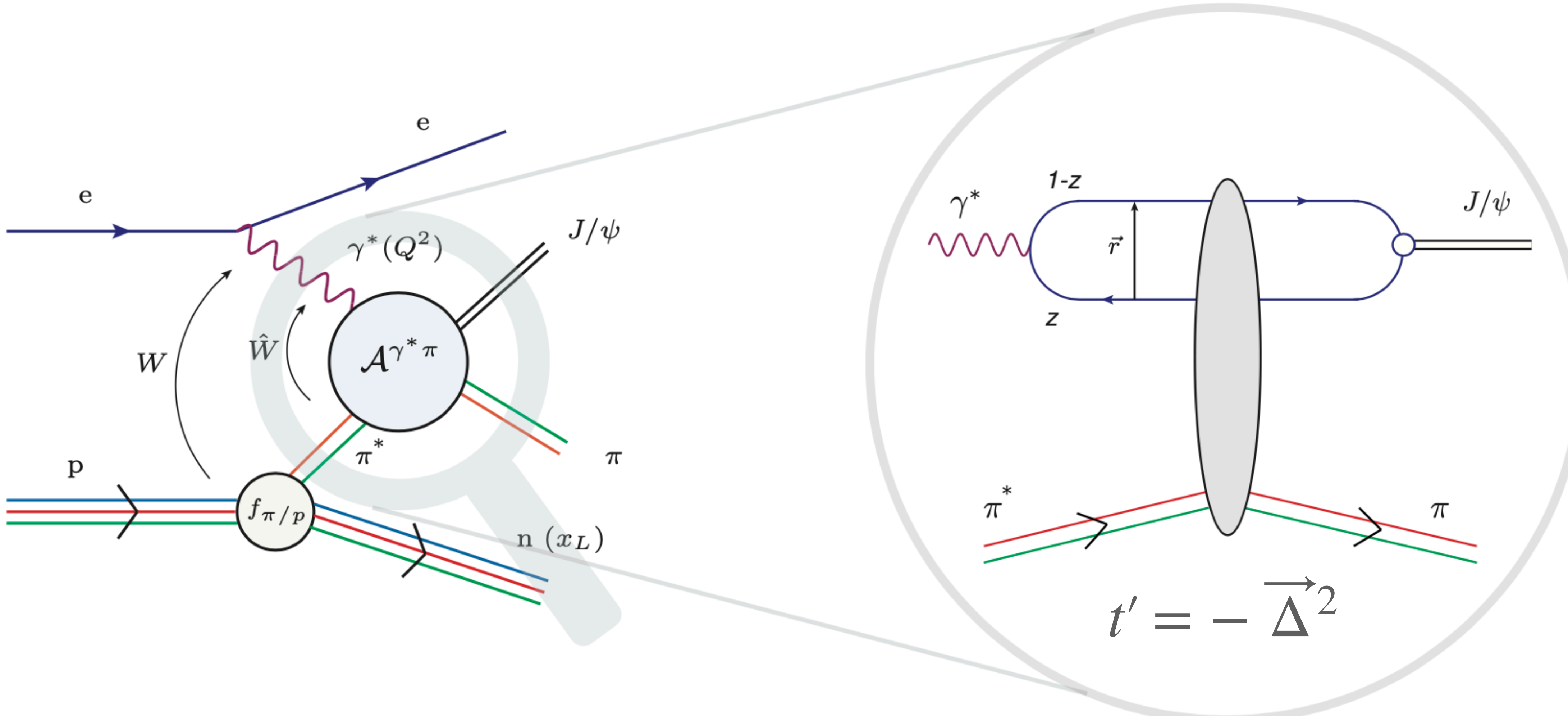
Feynman scaling and Saturation



Feynman scaling is equally valid for bSat and bNonSat.

Not suprising: we use a scaled proton, and $\hat{x} \geq x$

Exclusive Diffraction



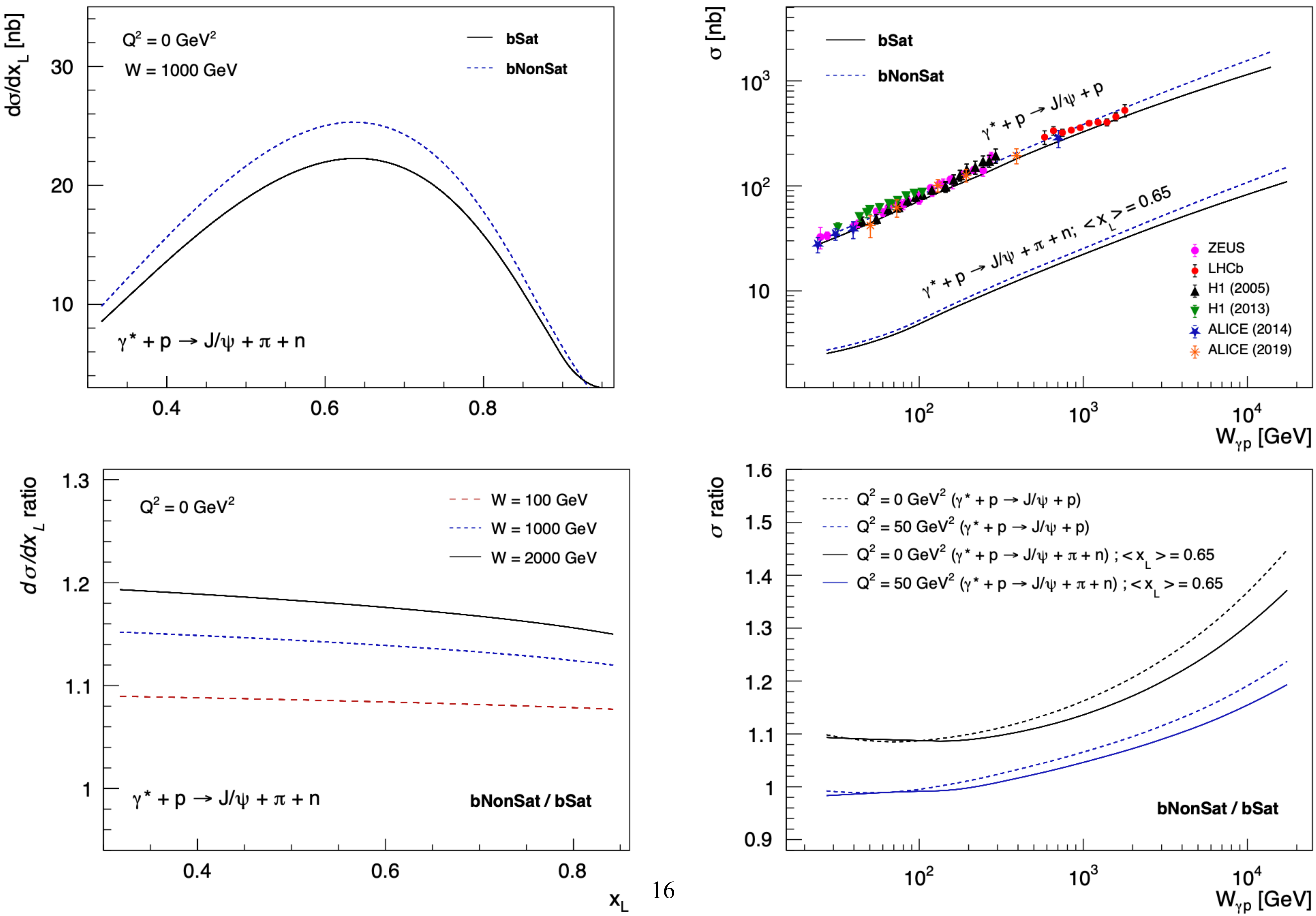
$$\mathscr{A}_{T,L}^{\gamma^*\pi^*\to J/\psi\pi}(\hat{x}, Q^2, \Delta) = i\int \mathrm{d}^2\vec{r}\,\mathrm{d}^2\overrightarrow{b}\,\frac{\mathrm{d}z}{4\pi}\left(\Psi^*\Psi_V\right)_{T,L}(Q^2, r, z)e^{-i[\overrightarrow{b}-(1-z)\vec{r}]\cdot\overrightarrow{\Delta}}\frac{\mathrm{d}\sigma_{q\bar{q}}^{(\pi)}}{\mathrm{d}^2\overrightarrow{b}}(\overrightarrow{b}, \vec{r}, \hat{x})$$

$$\frac{\mathrm{d}\sigma^{\mathrm{sat}}}{\mathrm{d}^2\overrightarrow{\mathrm{b}}} = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_C}r^2\alpha_S(\mu^2)xg(x,\mu^2)T(b)\right)\right]$$

$$\frac{\mathrm{d}\sigma_{q\bar{q}}^{(\pi)}}{\mathrm{d}^2\overrightarrow{b}}(\overrightarrow{b}, \vec{r}, \hat{x}) = R_g\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\overrightarrow{b}}$$

$$\frac{\mathrm{d}\sigma^{\mathrm{nosat}}}{\mathrm{d}^2\overrightarrow{\mathrm{b}}} = \frac{\pi^2}{N_C}r^2\alpha_S(\mu^2)xg(x,\mu^2)T(b)$$

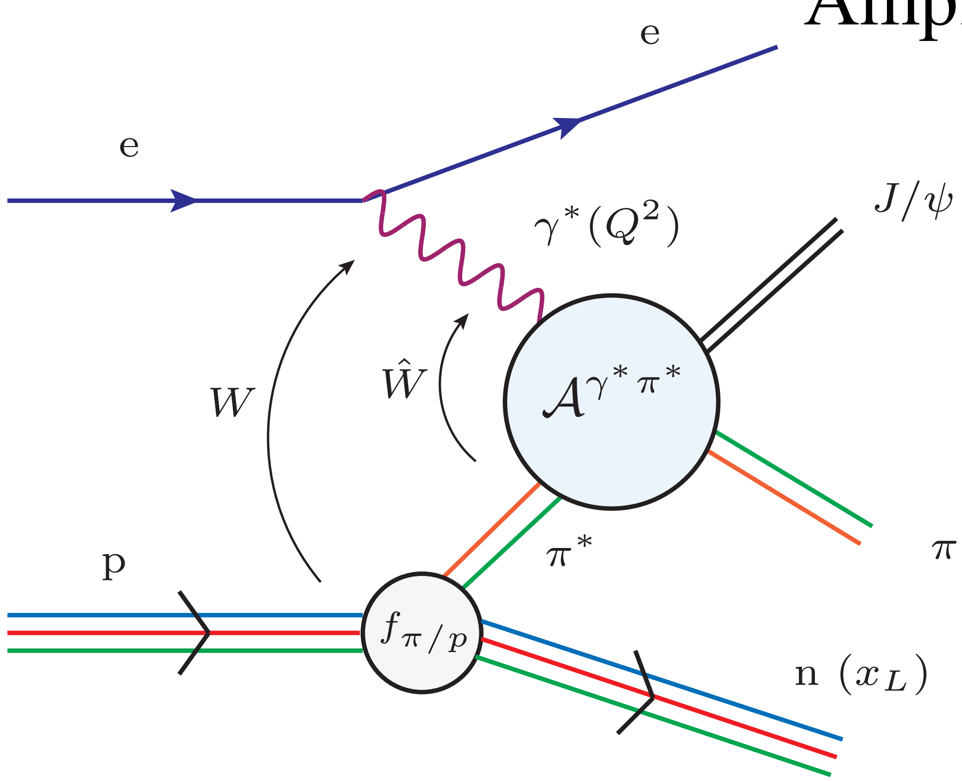
Exclusive Diffraction



Exclusive Diffraction: t'-spectrum

$$\mathscr{A}^{\gamma^*\pi^*\to J/\psi\pi}_{T,L}(\hat{x},Q^2,\Delta) = i\int \mathrm{d}^2\vec{r}\,\mathrm{d}^2\overrightarrow{b}\,\frac{\mathrm{d}z}{4\pi}\left(\Psi^*\Psi_V\right)_{T,L}(Q^2,r,z)e^{-i[\overrightarrow{b}-(1-z)\vec{r}]\cdot\Delta}\frac{\mathrm{d}\sigma^{(\pi)}_{q\bar{q}}}{\mathrm{d}^2\overrightarrow{b}}(\overrightarrow{b},\vec{r},\hat{x})$$

Amplitude is a Fourier transform from b to $\Delta = \sqrt{-t}$



$$\frac{\mathrm{d}\sigma^{\mathrm{sat}}}{\mathrm{d}^2\overrightarrow{\mathrm{b}}} = 2\left[1-\exp\left(-\frac{\pi^2}{2N_C}r^2\alpha_S(\mu^2)xg(x,\mu^2)T(b)\right)\right]$$

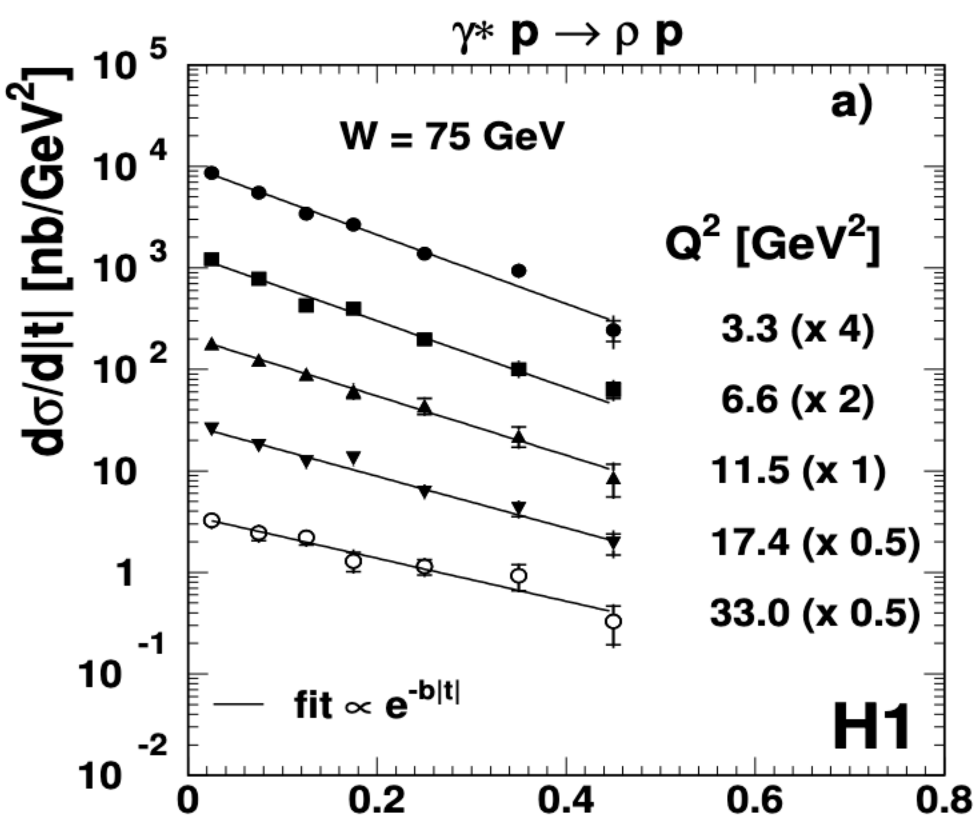
Proton: $T_p(b) = \dfrac{1}{2\pi B_p}e^{-\frac{b^2}{2B_p}}$, $B_p = 4\ \mathrm{GeV}^{-2}$

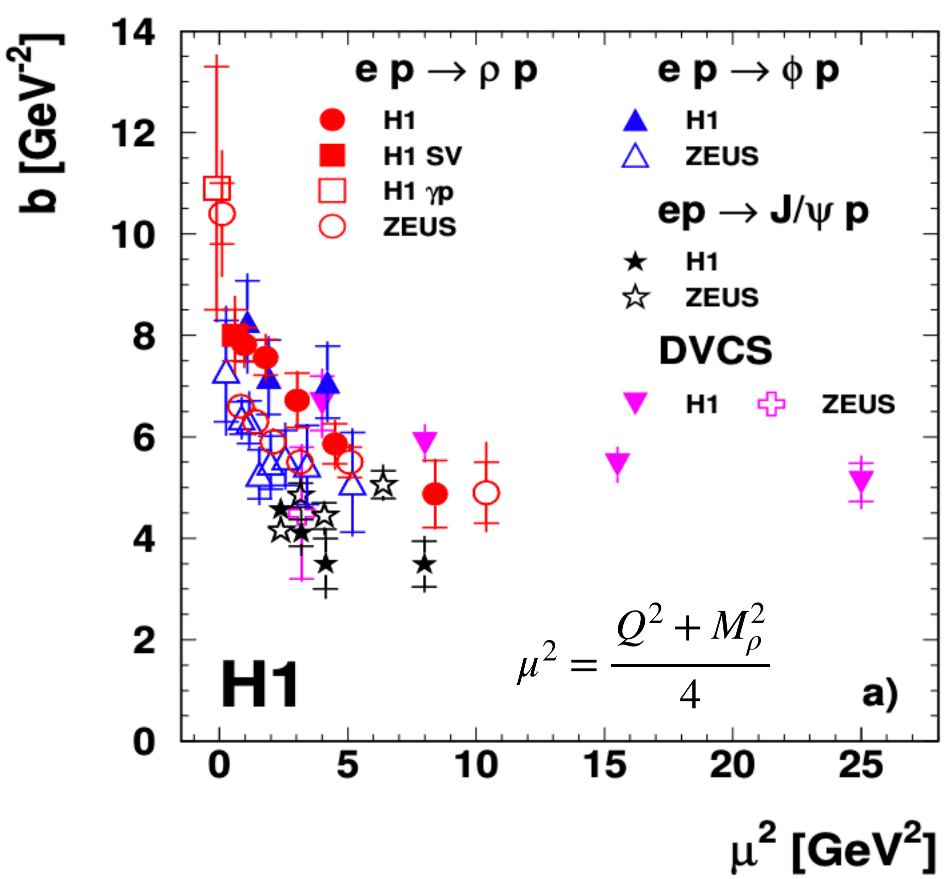
Virtual pion: $T_{\pi^*}(b) = \displaystyle\int_{-\infty}^{\infty}\mathrm{d}z\,\rho_{\pi^*}(b,z)$

$$\rho_{\pi^*}(b,z) = \frac{m_\pi^2}{4\pi}\frac{\exp\left(-m_\pi\sqrt{b^2+z^2}\right)}{\sqrt{b^2+z^2}}$$

Real pion: $T_\pi(b) = \dfrac{1}{2\pi B_\pi}e^{-\frac{b^2}{2B_\pi}}$

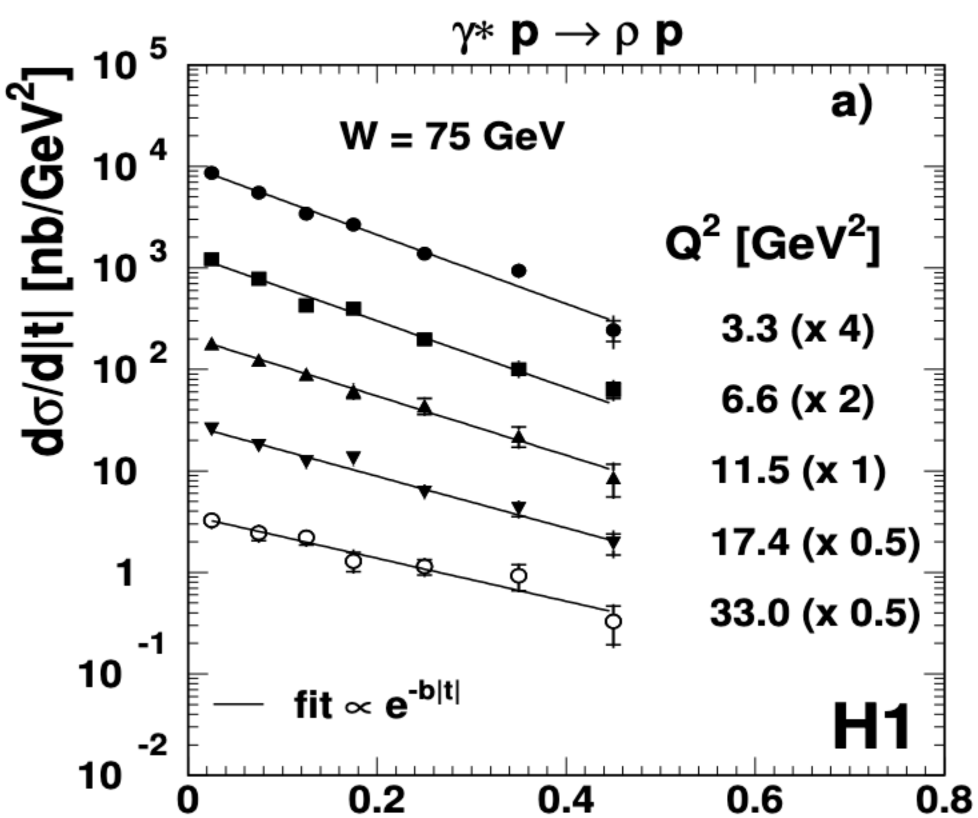
Exclusive Diffraction: *t'*-spectrum

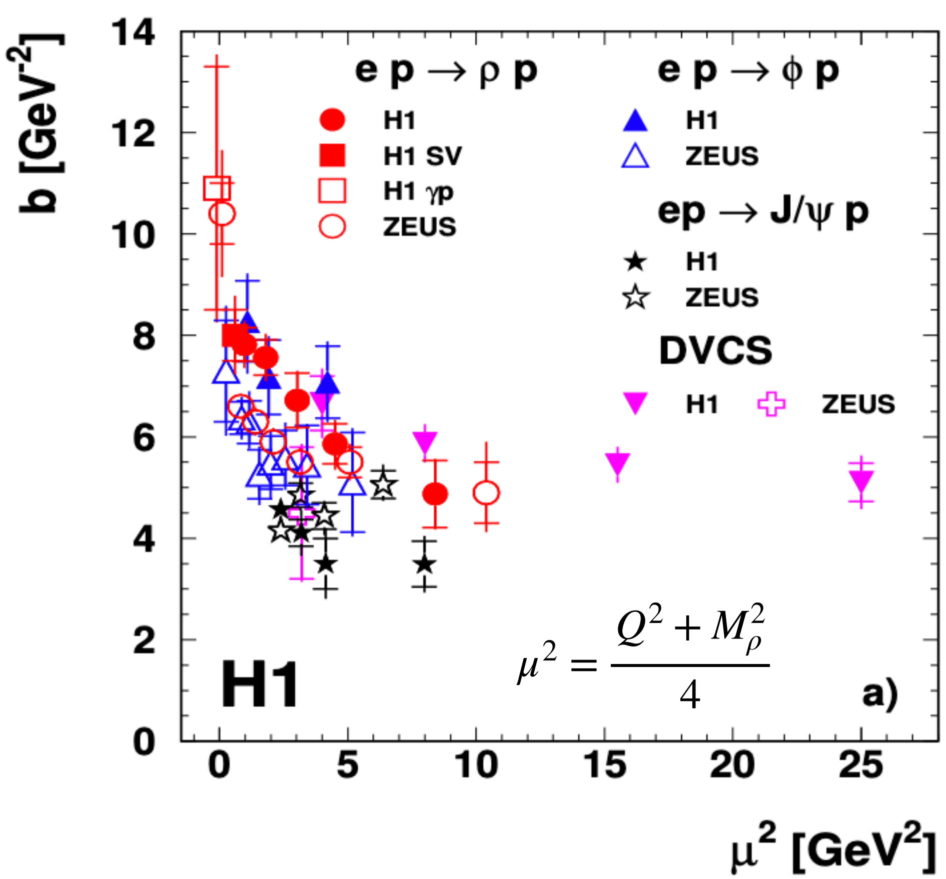




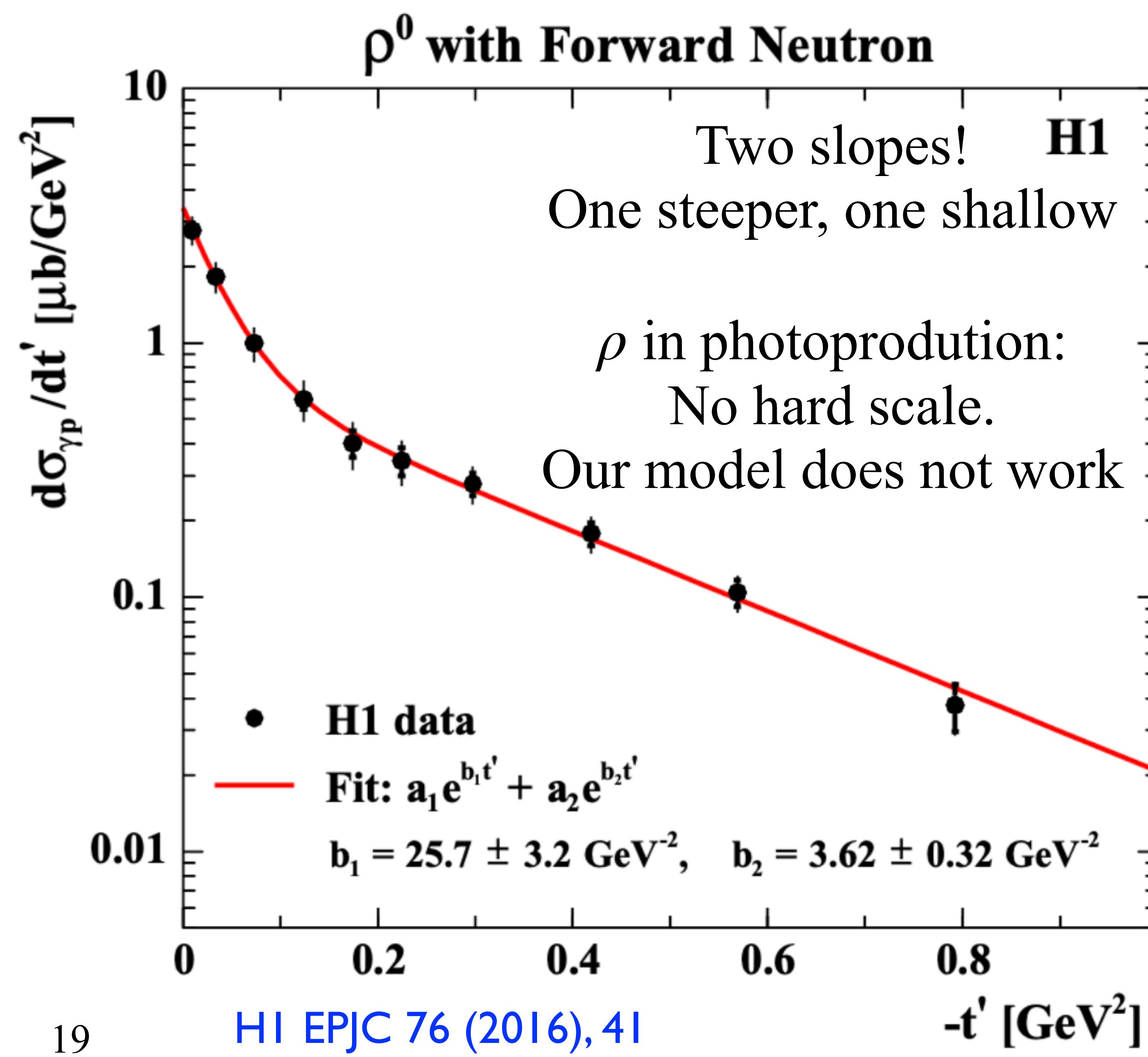
t-spectrum off the proton
slope ∝ (target size)²
Pion cloud is larger than the proton
Expect: Steeper *t*-spectrum.

Exclusive Diffraction: t'-spectrum



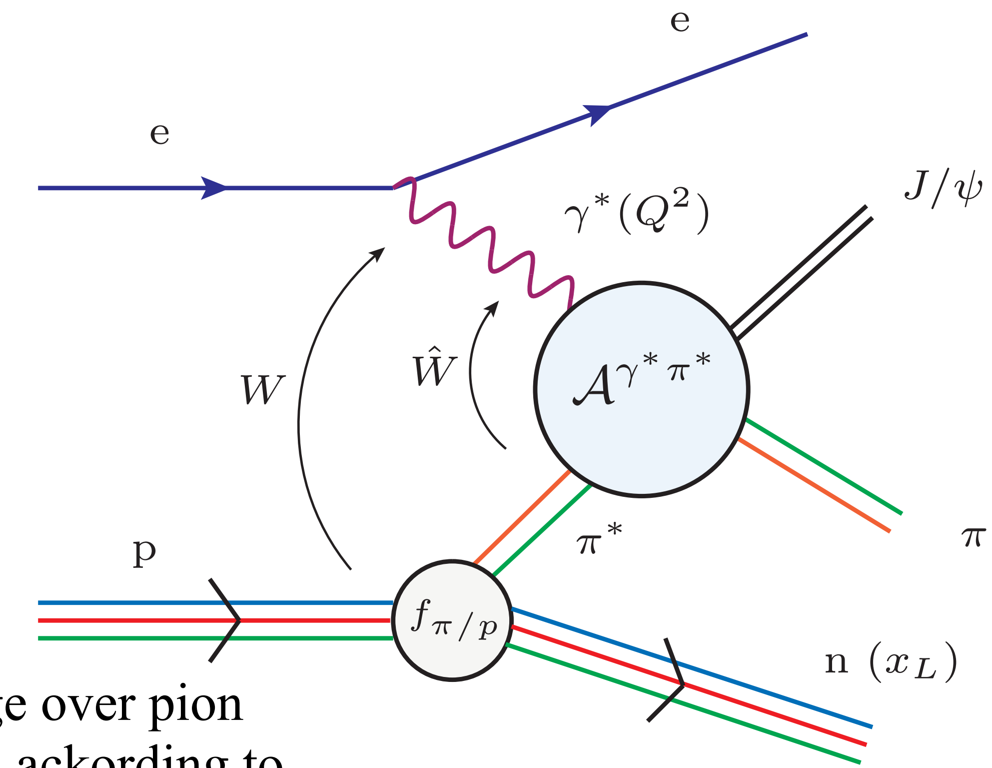


t-spectrum off the proton
slope ∝ (target size)²
Pion cloud is larger than the proton
Expect: Steeper t-spectrum.



H1 EPJC 76 (2016), 41

Exclusive Diffraction: t'-spectrum



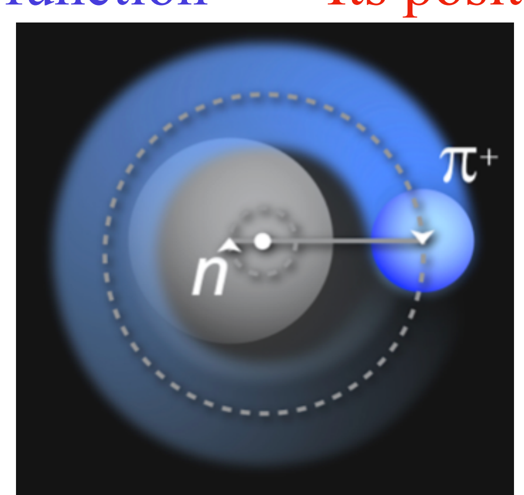
t-spectrum off the proton
slope ∝ (target size)²
Pion cloud is larger than the proton
Expect: Steeper t-spectrum.

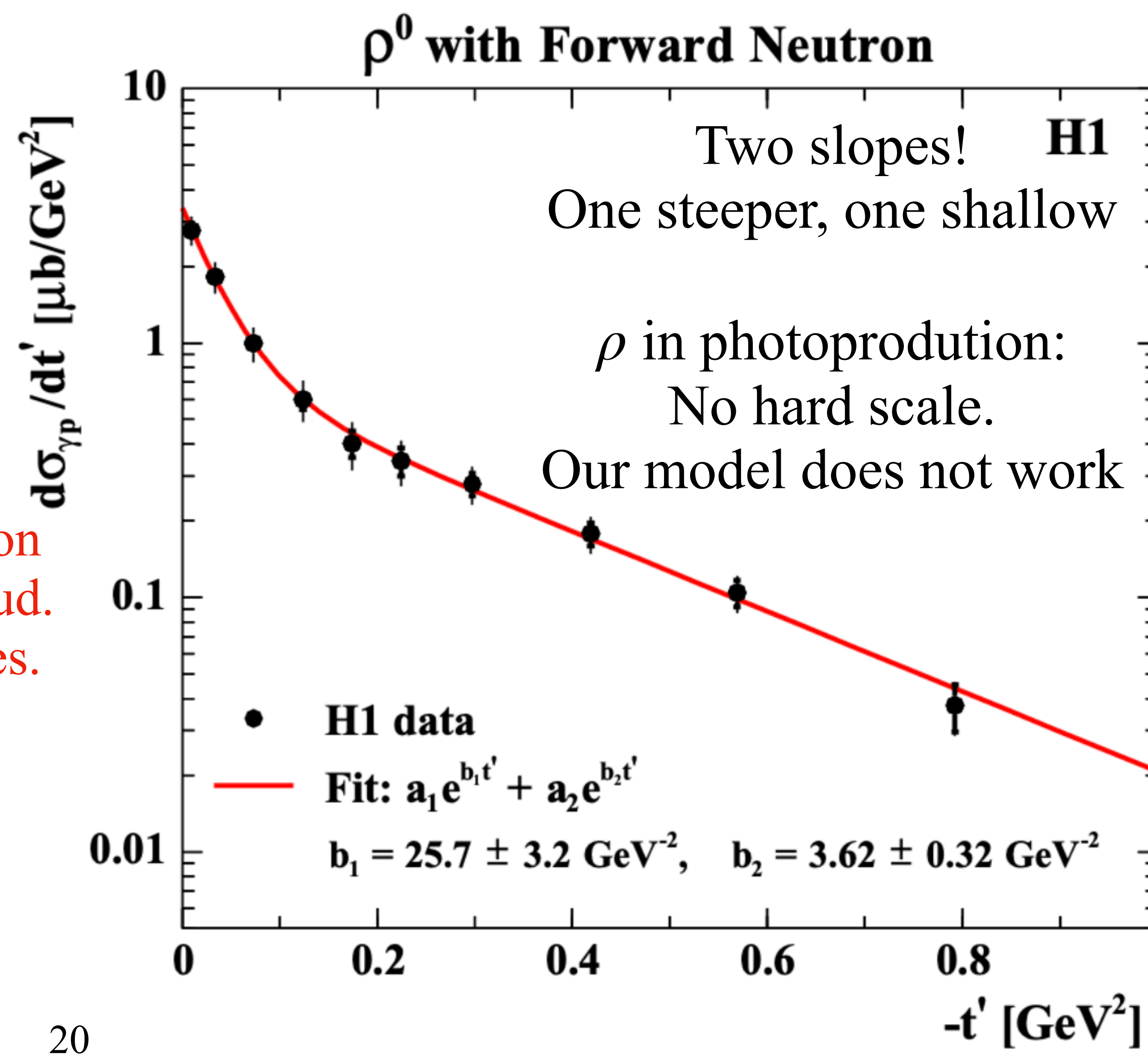
Average over pion positions ackording to Yukawa wave function

$$\langle |\mathscr{A}|^2 \rangle = \left|\langle \mathscr{A} \rangle\right|^2 + \left(\langle |\mathscr{A}|^2 \rangle - \left|\langle \mathscr{A} \rangle\right|^2 \right)$$

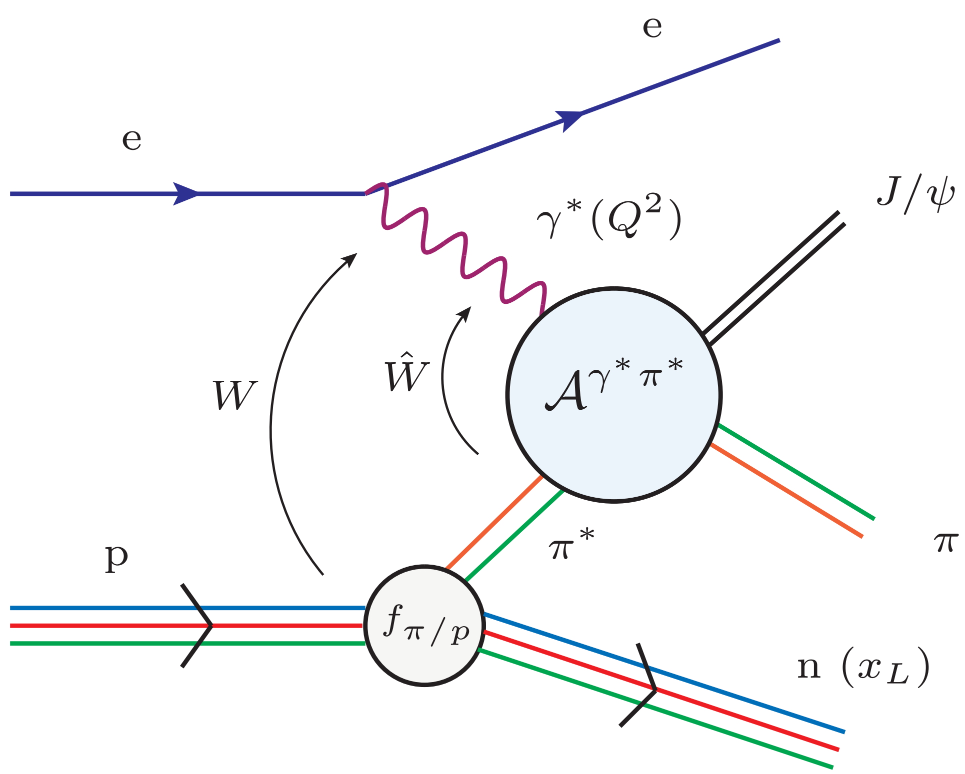
Coherently probes the entire pion cloud wave function

Resolves the real pion in the (one) pion cloud. Its position fluctuates.





Exclusive Diffraction: t'-spectrum



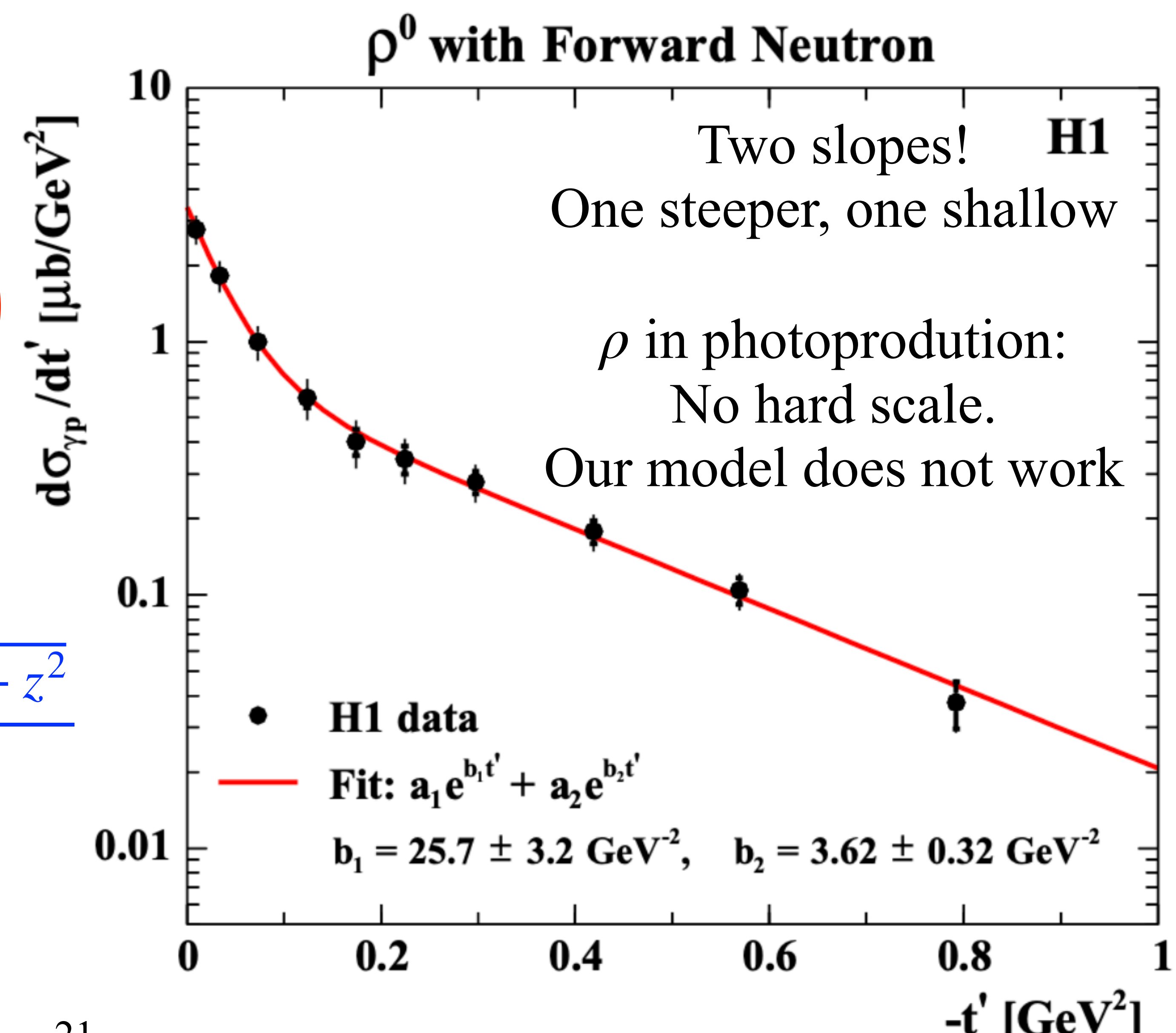
t-spectrum off the proton
slope ∝ (target size)²
Pion cloud is larger than the proton
Expect: Steeper t-spectrum.

$$\left\langle |\mathscr{A}|^2 \right\rangle = \left|\langle \mathscr{A} \rangle\right|^2 + \left(\langle |\mathscr{A}|^2 \rangle - \left|\langle \mathscr{A} \rangle\right|^2 \right)$$

Virtual pion: $$T_{\pi^*}(b) = \int_{-\infty}^{\infty} dz \rho_{\pi^*}(b,z)$$

$$\rho_{\pi^*}(b,z) = \frac{m_\pi^2}{4\pi} \frac{\exp(-m_\pi\sqrt{b^2+z^2})}{\sqrt{b^2+z^2}}$$

Real pion: $$T_\pi(b) = \frac{1}{2\pi B_\pi} e^{-\frac{b^2}{2B_\pi}}$$



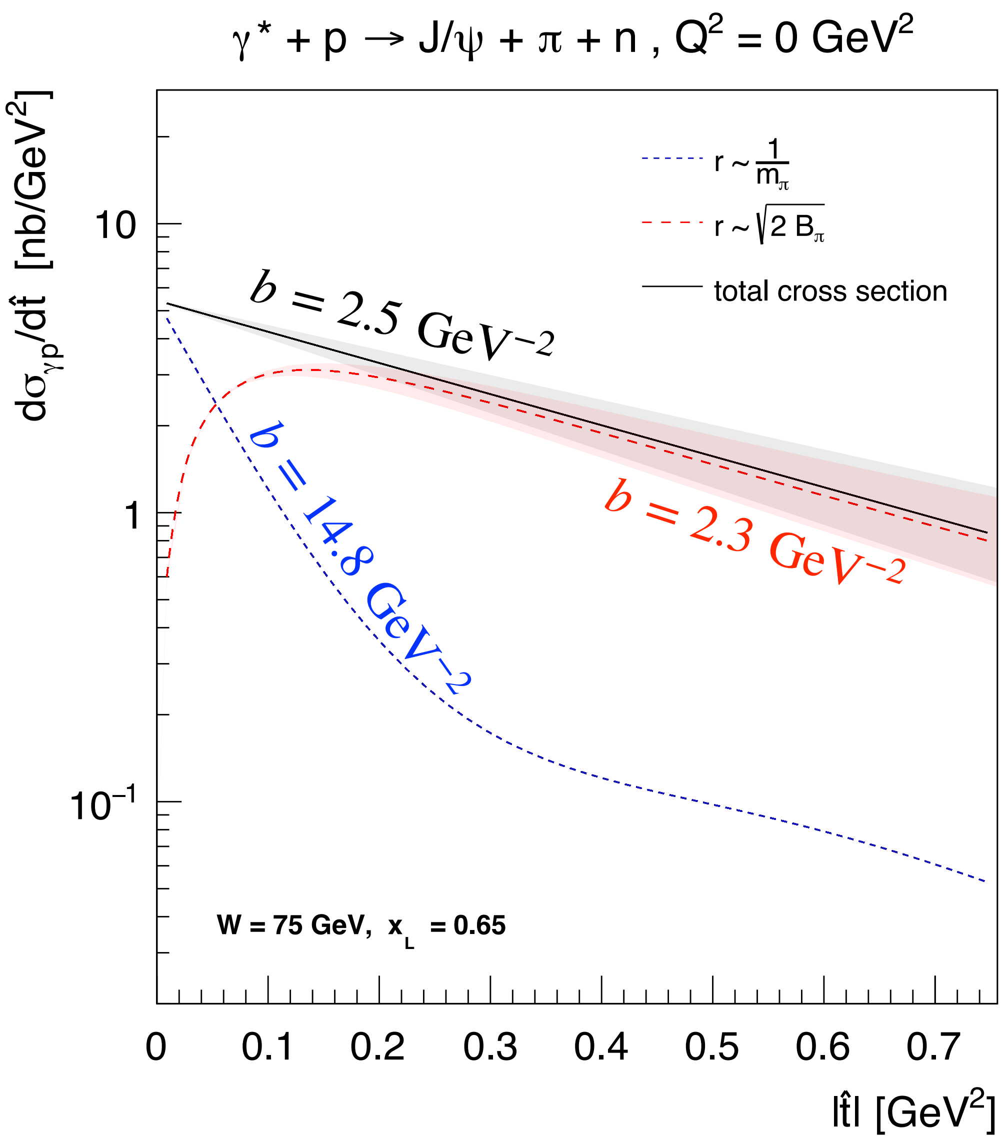
Exclusive Diffraction: t'-spectrum

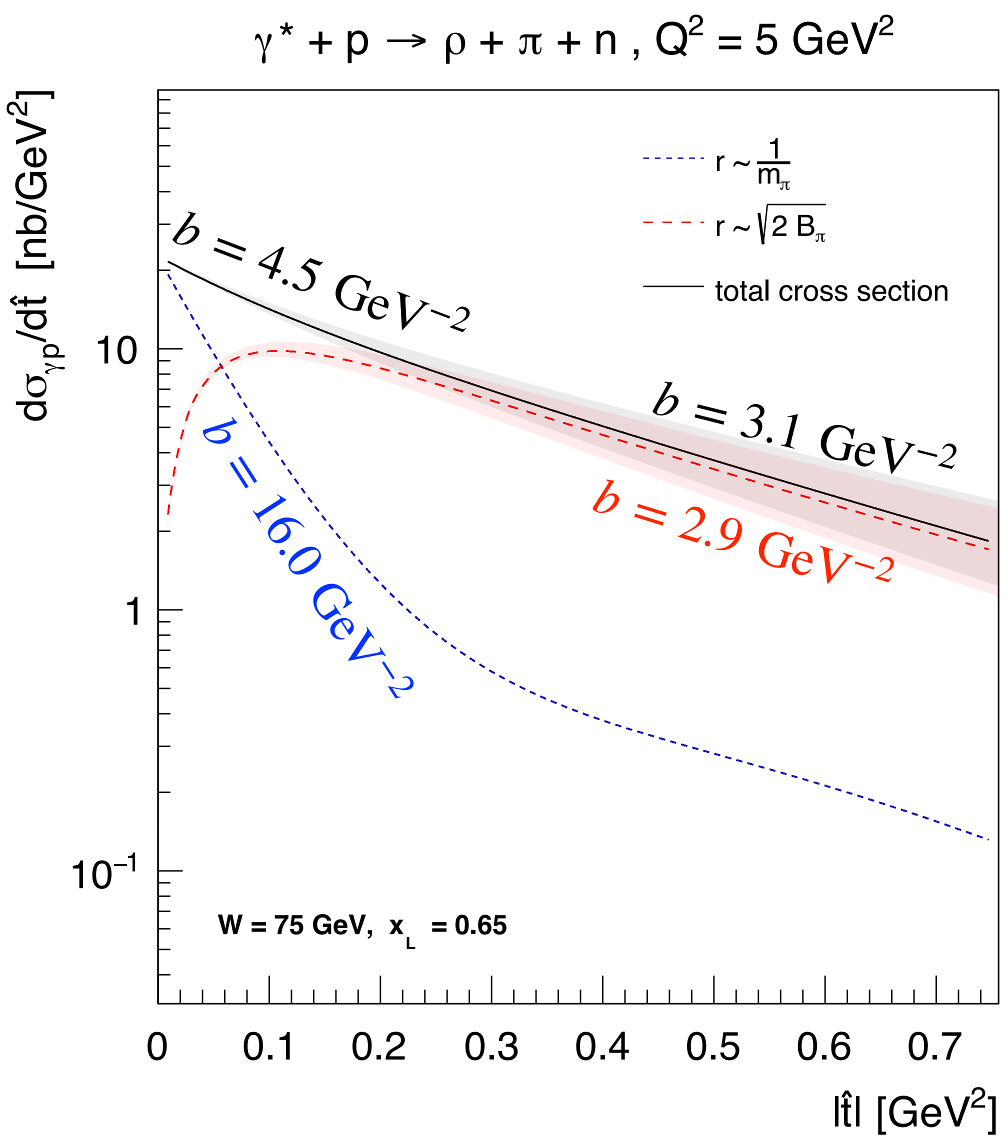
$$\left\langle |\mathscr{A}|^2 \right\rangle = \left| \langle \mathscr{A} \rangle \right|^2 + \left(\langle |\mathscr{A}|^2 \rangle - \left| \langle \mathscr{A} \rangle \right|^2 \right)$$

$$T_\pi(b) = \frac{1}{2\pi B_\pi} e^{-\frac{b^2}{2B_\pi}}$$

$$T_{\pi^*}(b) = \int_{-\infty}^{\infty} \mathrm{d}z \rho_{\pi^*}(b,z) \quad \rho_{\pi^*}(b,z) = \frac{m_\pi^2}{4\pi} \frac{\exp(-m_\pi \sqrt{b^2+z^2})}{\sqrt{b^2+z^2}}$$

$$B_\pi = 2 \pm 0.5 \text{ GeV}^{-2}$$





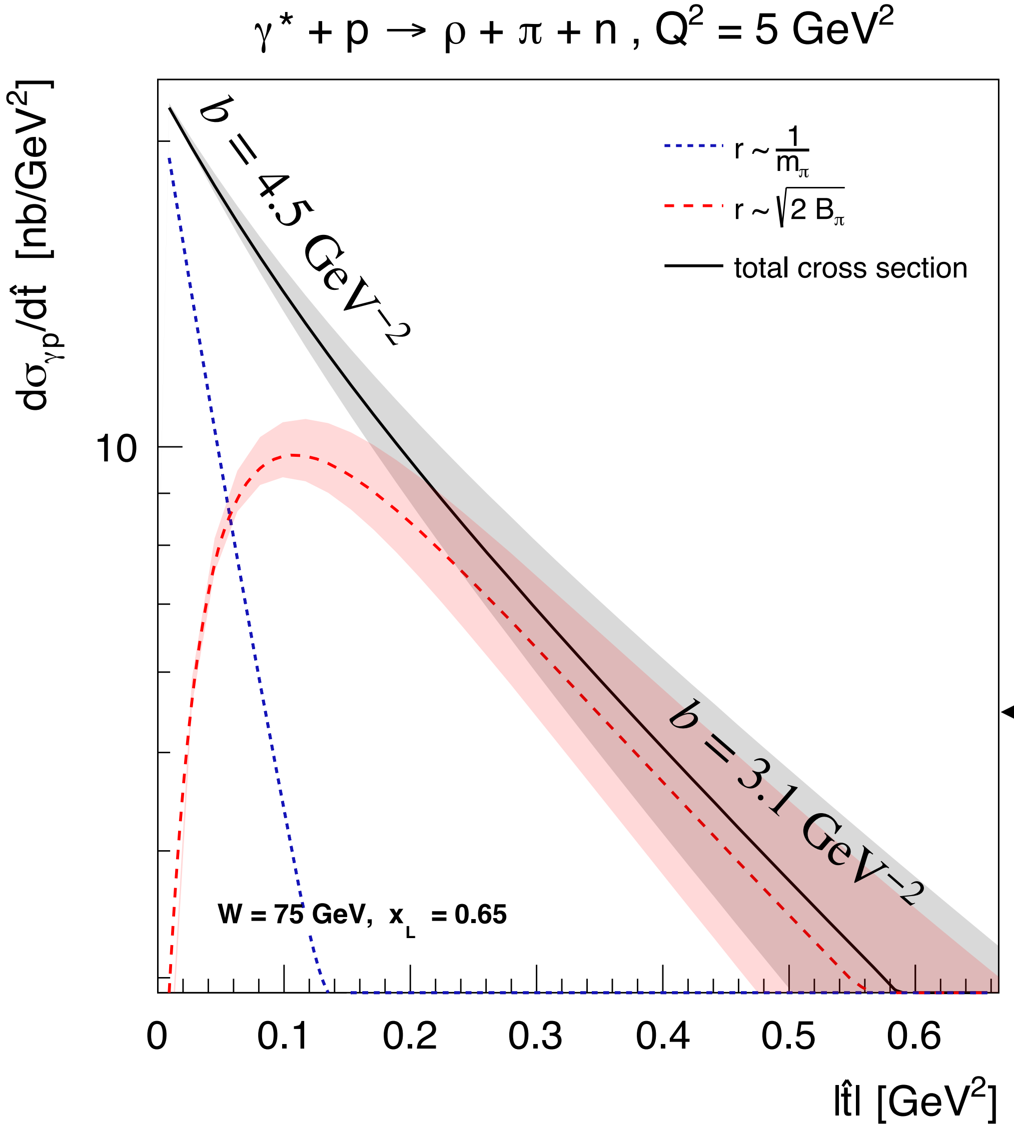
Exclusive Diffraction: $t\,'$-spectrum

$$\left\langle |\mathscr{A}|^2 \right\rangle = \left|\left\langle \mathscr{A} \right\rangle\right|^2 + \left(\left\langle |\mathscr{A}|^2 \right\rangle - \left|\left\langle \mathscr{A} \right\rangle\right|^2 \right)$$

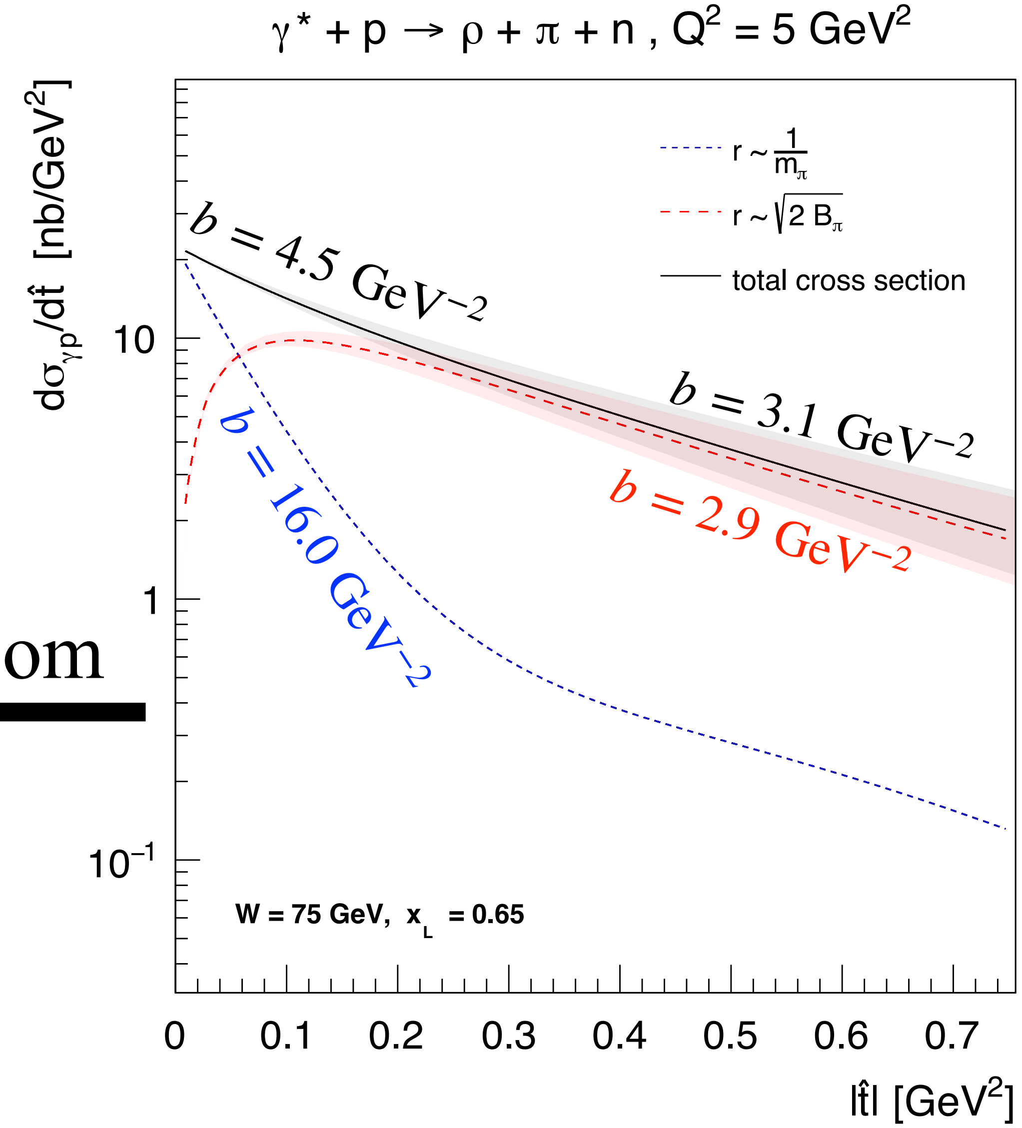
$$T_\pi(b) = \frac{1}{2\pi B_\pi} e^{-\frac{b^2}{2B_\pi}}$$

$$T_{\pi^*}(b) = \int_{-\infty}^{\infty} \mathrm{d}z \rho_{\pi^*}(b,z) \quad \rho_{\pi^*}(b,z) = \frac{m_\pi^2}{4\pi} \frac{\exp(-m_\pi\sqrt{b^2+z^2})}{\sqrt{b^2+z^2}}$$

$$B_\pi = 2 \pm 0.5\ \mathrm{GeV}^{-2}$$



zoom



Conclusions

Leading neutrons at small x: Yukawa theory meets pQCD

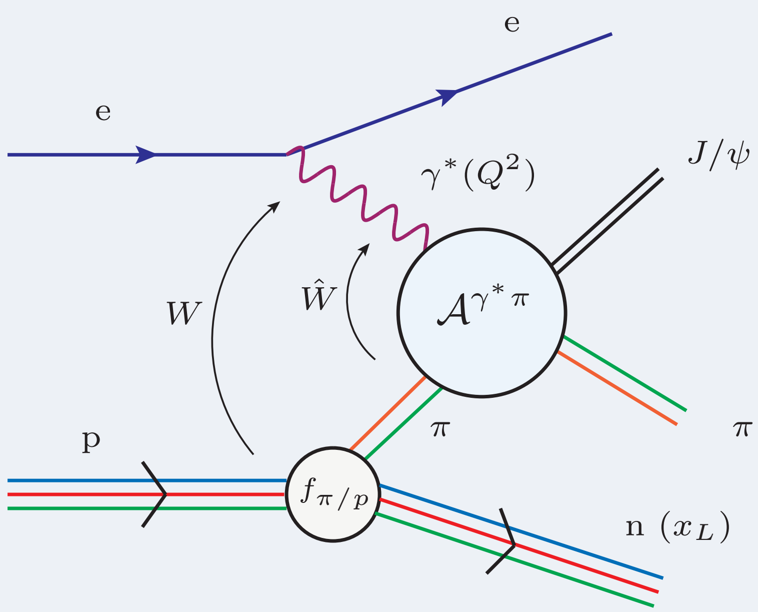
How different is the small-x gluon structure of the virtual pion compared to the proton?
We can describe F_2^{LN} well assuming universality between protons and pions up to a factor (for large x_L).

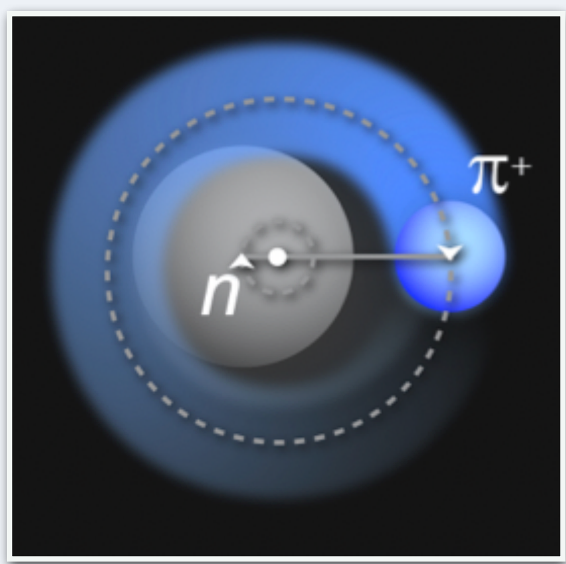
Is Feynman scaling a signal for saturation?
We saw no signal for saturation in the leading neutron data.
This is a consequence of the universality assumption.

The Leading Neutron t'-spectrum exhibits two distinct slopes.
We show how to calculate these slopes by resolving the pion wave function at higher $|t'|$
We have made predictions for the t'-spectrum for J/ψ and ρ in electroproduction at HERA
Part of *incoherent* p/A t-spectrum:
Not tagged as such at HERA
Too small cross section to be seen at RHIC

Looking forward to seeing the new analysis!

Backup





$$\sigma_{total} = \sigma_{yukawa} + \sigma_{fluctuations}$$

PROBING THE GLUON DISTRIBUTION

- The thickness function of pion:

$T_\pi(b) = \frac{1}{2\pi B_\pi} e^{-\frac{-b^2}{2B_\pi}}$, B_π is the transverse width of the pion

- No experimental data on |t| dependence which can restrict this parameter
  - Assume that the gluon to charge radius is same in pions and protons: $B_\pi = r_\pi^2 / r_p^2 B_p = (0.657/0.840)^2 . 4^{-2} \approx 2.44 \ GeV^{-2}$
  - Pion gluon radius from the Belle measurements at KEKB in hadron-pair production $\gamma^*\gamma \to \pi^0\pi^0$ which suggests $B_\pi \approx 1.33 - 1.96 \ GeV^{-2}$ Kumano et al PRD 97 (2018), 014020
  - H1 measured the |t| spectrum for exclusive ρ photo-production with leading neutrons in ep scattering, as this process lacks a hard scale we are not able to make a direct comparison, but this spectrum suggests $B_\pi \approx 2.3 \ GeV^{-2}$ H1 EPJC 76 (2016), 41
- We therefore present our results with bands for $B_\pi = 2 \pm 0.5 \ GeV^{-2}$