Dijet production in DIS in the CGC

HERA 4 EIC Workshop

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R. Boussarie, H. Mäntysaari, FS, and B. Schenke. 2106.11301 [JHEP 09 (2021) 178]

P. Caucal, FS, and R. Venugopalan. 2108.06347 [JHEP 11 (2021) 222]

+ work in preparation (2207.XXXX)
Caucal, FS, Schenke, and Venugopalan

Outline

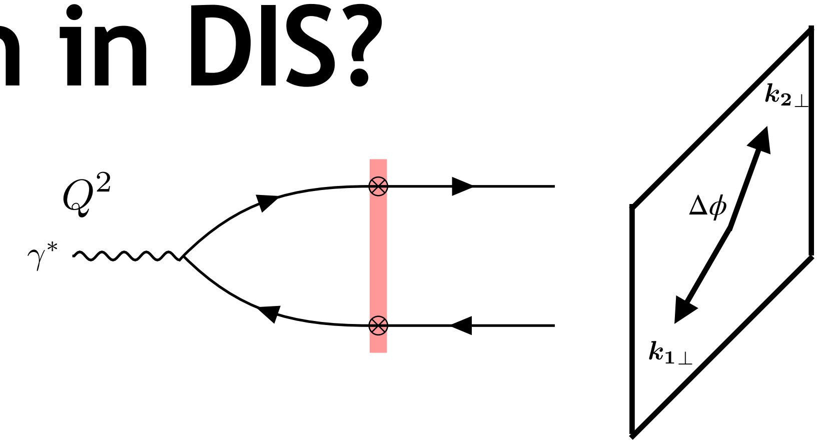
- Why dijet production?
- CGC beyond TMDs
- NLO corrections
- Back-to-back limit

Why inclusive dijet production in DIS?

Complementary process to fully inclusive

Study dependence on kinematic variables, correlations, etc

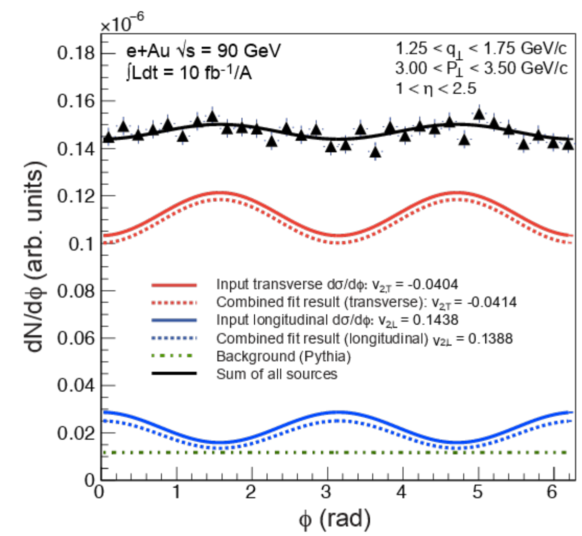
Kharzeev, Levin, McLerran (2005) Marquet (2007)



$$Q_Y(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{y}'_\perp, \boldsymbol{x}'_\perp) = \frac{1}{N_c}\langle \mathrm{Tr}\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)V(\boldsymbol{y}'_\perp)V^\dagger(\boldsymbol{x}'_\perp)\right]\rangle_Y$$

Simplest observable featuring the quadrupole.

Validity of JIMWLK factorization beyond the dipole

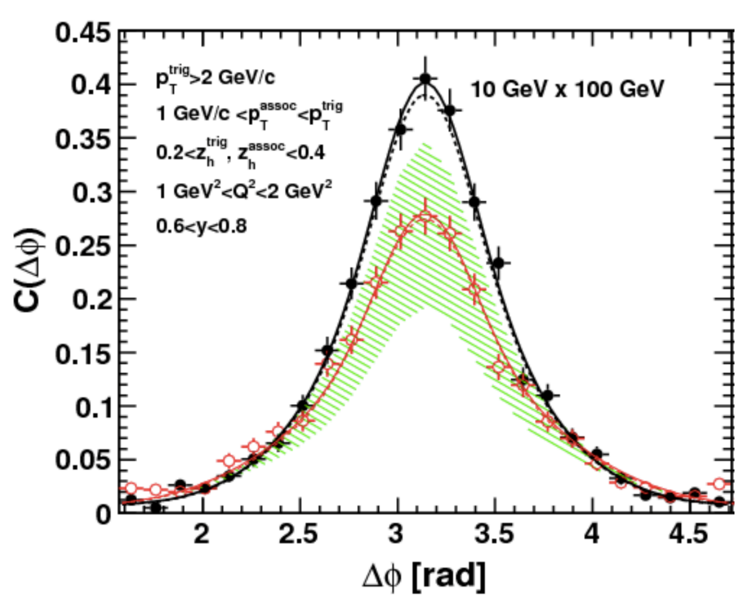


Dumitru, Skokov, Ullrich (2019)

In the back-to-back limit contact with the (transverse momentum dependent) TMD formalism

Weizsäcker-Williams TMD

Dominguez, Marquet, Xiao, Yuan (PRD 2011)

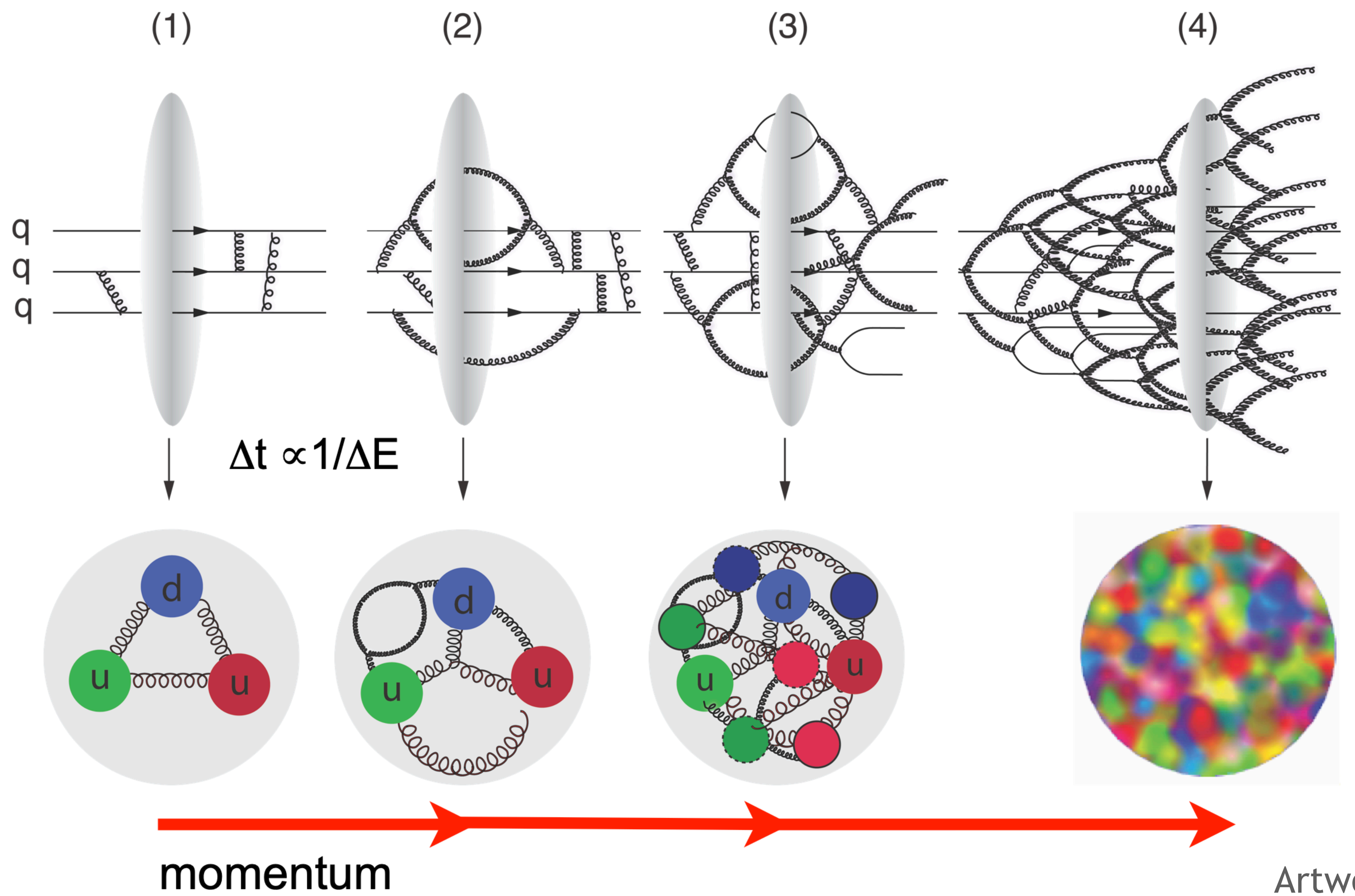


Suppression of back-to-back peak potential signature of gluon saturation (dihadrons)

Zheng, Aschenauer, Lee, Xiao (2014)

Jets are better proxies of hard partons (than hadrons)

Anatomy of nuclear matter in the high-energy limit



Artwork: T. Ullrich

Emergence of an energy and nuclear specie dependent momentum scale

Multiple scattering (higher twist effects)

Non-linear evolution equations (BK/JIMWLK)

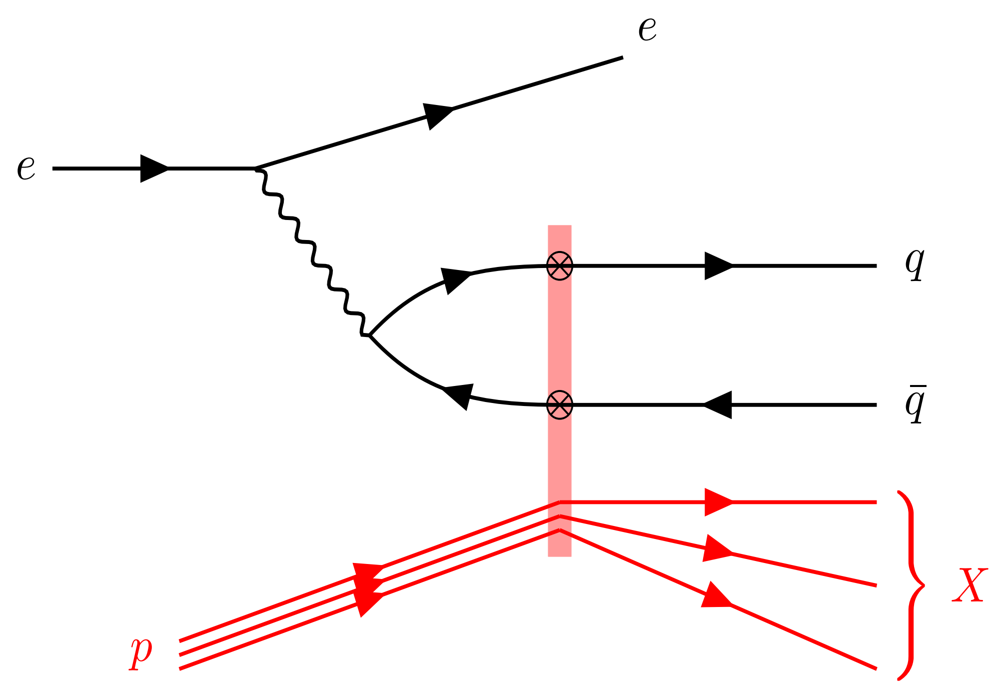
$$Q_s^2 \propto A^{1/3} x^{-\lambda}$$

Review of gluon saturation. *Morreale, FS (2021)*

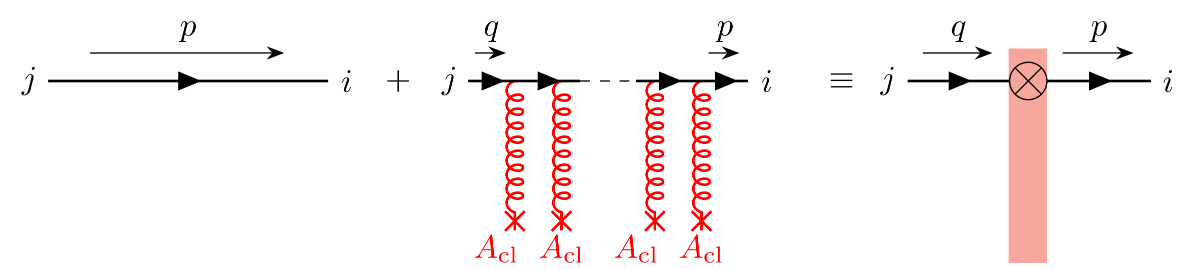
Dijet production beyond TMDs

Computation in the CGC: **resummation** of multiple scatterings

Dominguez, Marquet, Xiao, Yuan (2011)



Dense gluon field $A_{\rm cl} \sim 1/g$ **needs resummation of multiple gluon interactions**



$$V_{ij}(\boldsymbol{x}) = P \exp\left\{ ig \int dx^- A^{+,a}_{cl}(\boldsymbol{x}, x^-) t^a \right\}$$

Unpolarized differential cross-section:

$$\frac{\mathrm{d}\sigma^{\gamma^*_\lambda + A \to q\bar{q} + X}}{\mathrm{d}^2\boldsymbol{k}_{1\perp}\mathrm{d}^2\boldsymbol{k}_{2\perp}\mathrm{d}\eta_1\mathrm{d}\eta_2} \propto \int \mathrm{d}^8\boldsymbol{X}_\perp \; e^{-i\boldsymbol{k}_{1\perp}\cdot(\boldsymbol{x}_\perp - \boldsymbol{x}'_\perp)} e^{-i\boldsymbol{k}_{2\perp}\cdot(\boldsymbol{y}_\perp - \boldsymbol{y}'_\perp)}$$

$$\times \boxed{\langle \Xi_{\rm LO}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{y}'_\perp \boldsymbol{x}'_\perp)\rangle_Y} \boxed{\mathcal{R}^\lambda(\boldsymbol{x}_\perp - \boldsymbol{y}_\perp, \boldsymbol{x}'_\perp - \boldsymbol{y}'_\perp)}$$

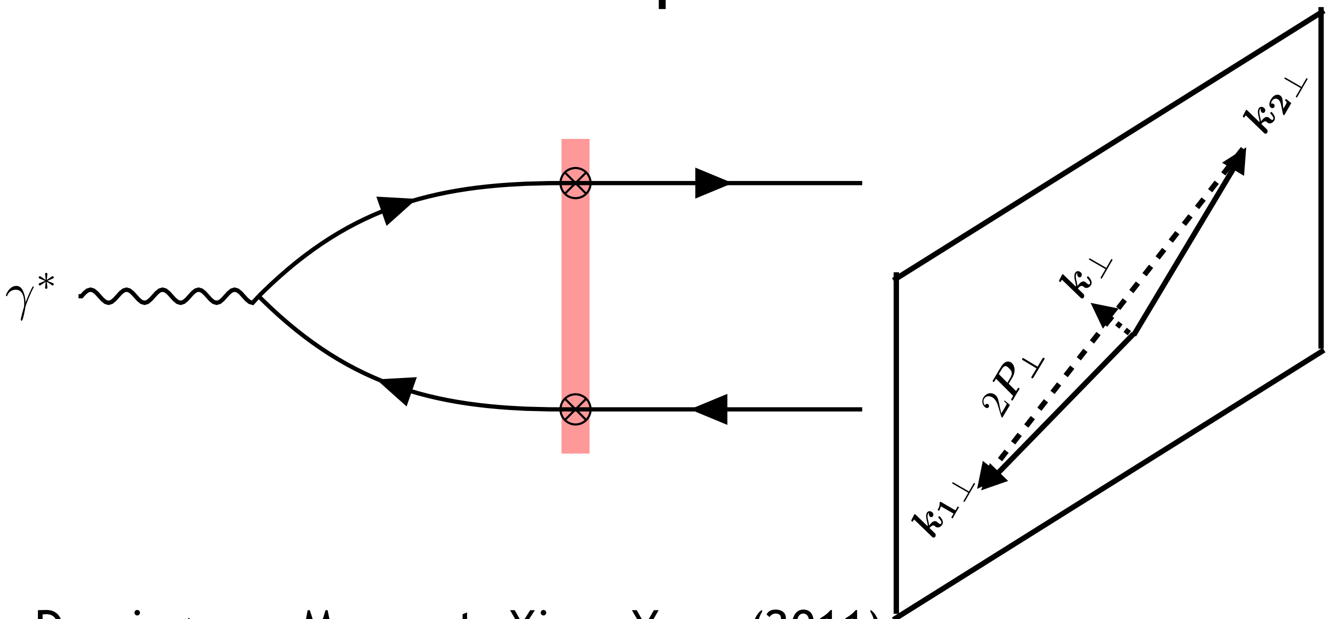
$$\Xi_{\rm LO}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{y}'_\perp \boldsymbol{x}'_\perp) = 1 - S^{(2)}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) - S^{(2)}(\boldsymbol{y}'_\perp, \boldsymbol{x}'_\perp) + S^{(4)}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{y}'_\perp, \boldsymbol{x}'_\perp)$$

dipoles

quadrupole

Dijet production beyond TMDs

Resummation of power corrections and genuine saturation corrections



R. Boussarie, H. Mäntysaari, FS, B. Schenke (2021)

$$\underbrace{\mathrm{d}\sigma_{\mathrm{CGC}} = \mathrm{d}\sigma_{\mathrm{TMD}} + \overbrace{\mathcal{O}\left(\frac{k_\perp}{Q_\perp}\right)}^{\text{kinematic}}}_{\mathrm{d}\sigma_{\mathrm{ITMD}}} + \overbrace{\mathcal{O}\left(\frac{Q_s}{Q_\perp}\right)}^{\text{genuine}}$$

Dominguez, Marquet, Xiao, Yuan (2011)

TMD valid $k_\perp, Q_s \ll P_\perp$

back-to-back hadrons/jets and transverse momenta larger than sat scale

Hard factor

Weizsäcker-Williams gluon TMD

$$d\sigma^{\gamma^* A \to q\bar{q}X} \sim \mathcal{H}^{ij}_{\mathrm{TMD}}(\boldsymbol{P}_\perp)\, \alpha_s x G^{ij}_{\mathrm{WW}}(x, \boldsymbol{k}_\perp)$$

$$+\mathcal{O}(k_\perp/P_\perp) + \mathcal{O}(Q_s/P_\perp)$$

Altinoluk, Boussarie, Kotko (2019) Boussarie, Mehtar-Tani (2020)

Improved TMD valid $Q_s \ll P_\perp$

transverse momenta larger than sat scale

Hard factor resums kinematic powers $k_\perp/P_\perp$

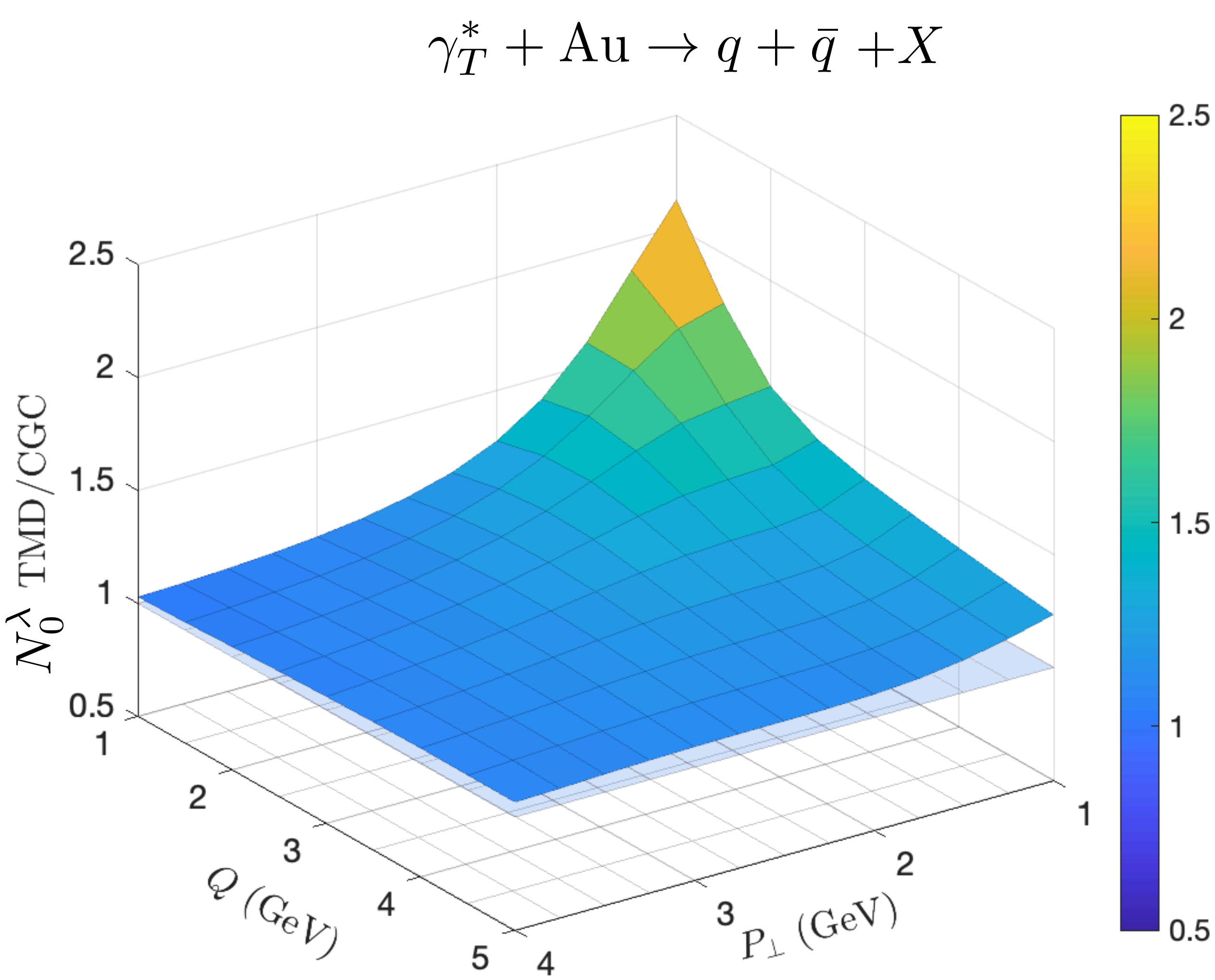
$$d\sigma^{\gamma^*_\lambda A \to q\bar{q}X} \sim \mathcal{H}^{\lambda,ij}_{\mathrm{ITMD}}(\boldsymbol{P}_\perp, \boldsymbol{k}_\perp)\, \alpha_s x G^{ij}_{\mathrm{WW}}(x, \boldsymbol{k}_\perp)$$

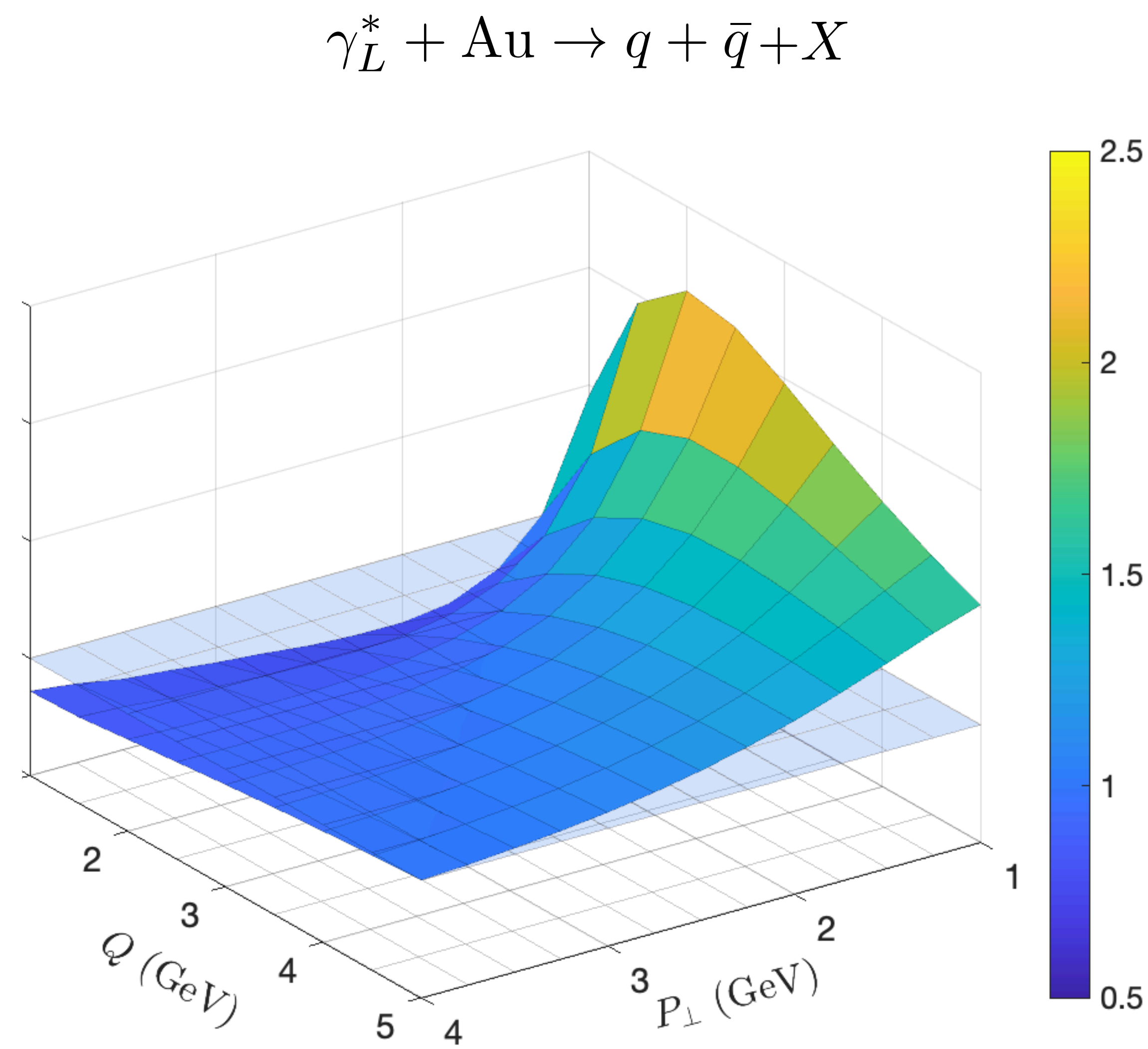
$$+\mathcal{O}(Q_s/P_\perp)$$

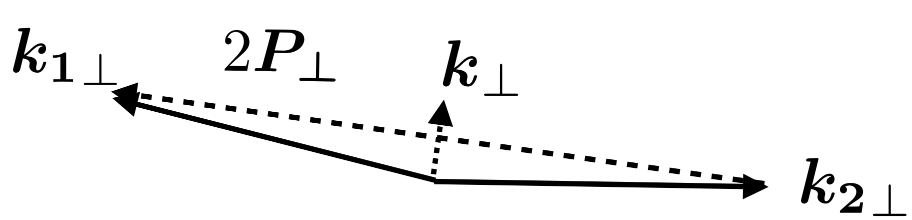
Dijet production beyond TMDs

Q^2 and $P_\perp$ dependence of genuine saturation

Near back-to-back $k_\perp \approx 0$ the ratio of CGC/TMD is sensitive to genuine twists







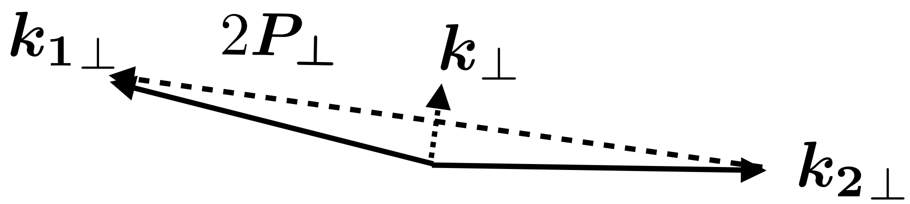
$$\frac{\mathrm{d}N^{\gamma_\lambda^*+A\to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{P}_\perp \mathrm{d}^2\boldsymbol{k}_\perp \mathrm{d}\eta_1 \mathrm{d}\eta_2} = N_0^\lambda(P_\perp, k_\perp)\left[1 + 2\sum_{k=1}^{\infty} v_{k,\lambda}(P_\perp, k_\perp)\cos(k\phi)\right]$$

$$\phi \equiv \phi_{\boldsymbol{k}_\perp} - \phi_{\boldsymbol{P}_\perp}$$

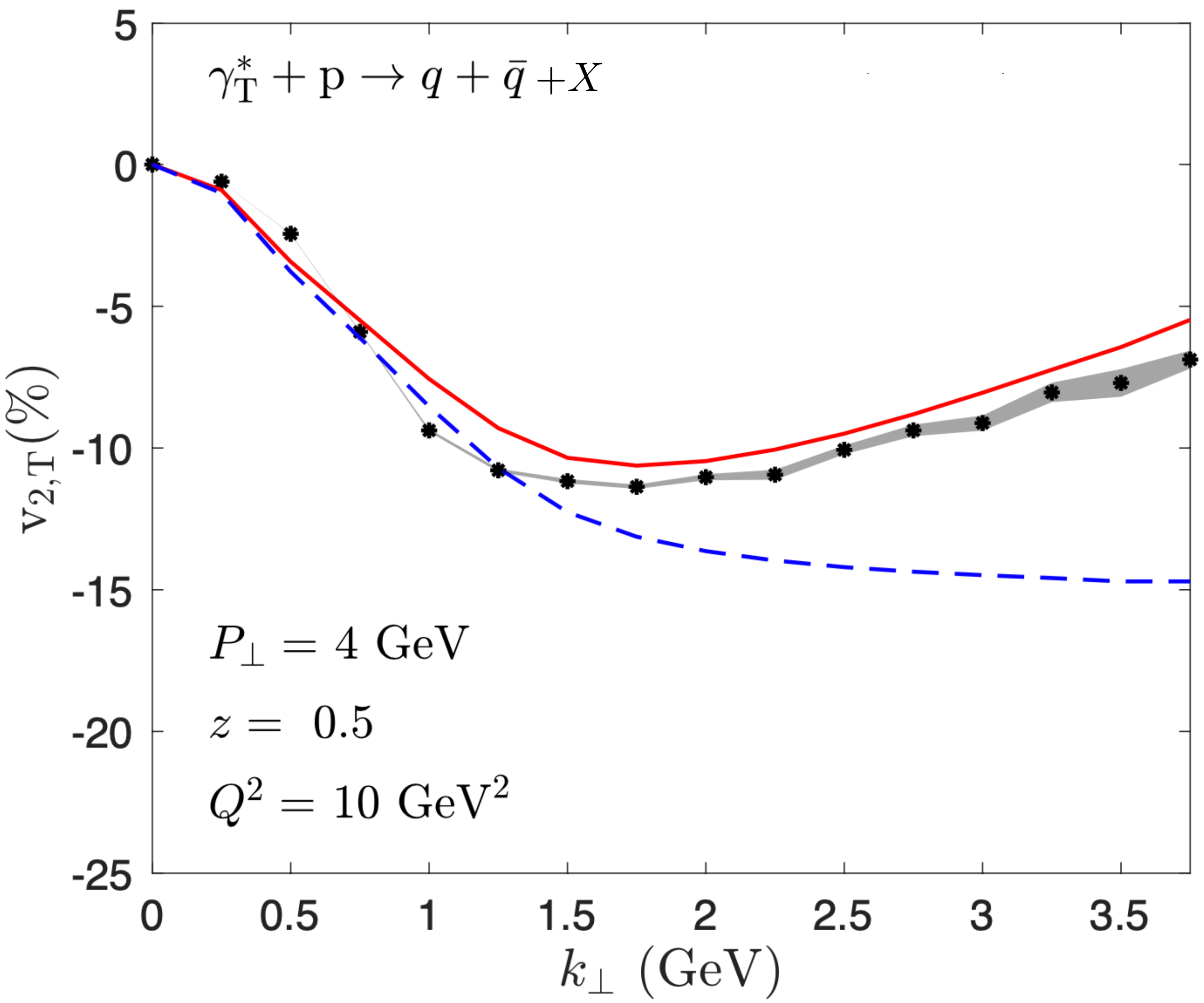
R. Boussarie, H. Mäntysaari, FS, B. Schenke (2021)

Dijet production beyond TMDs

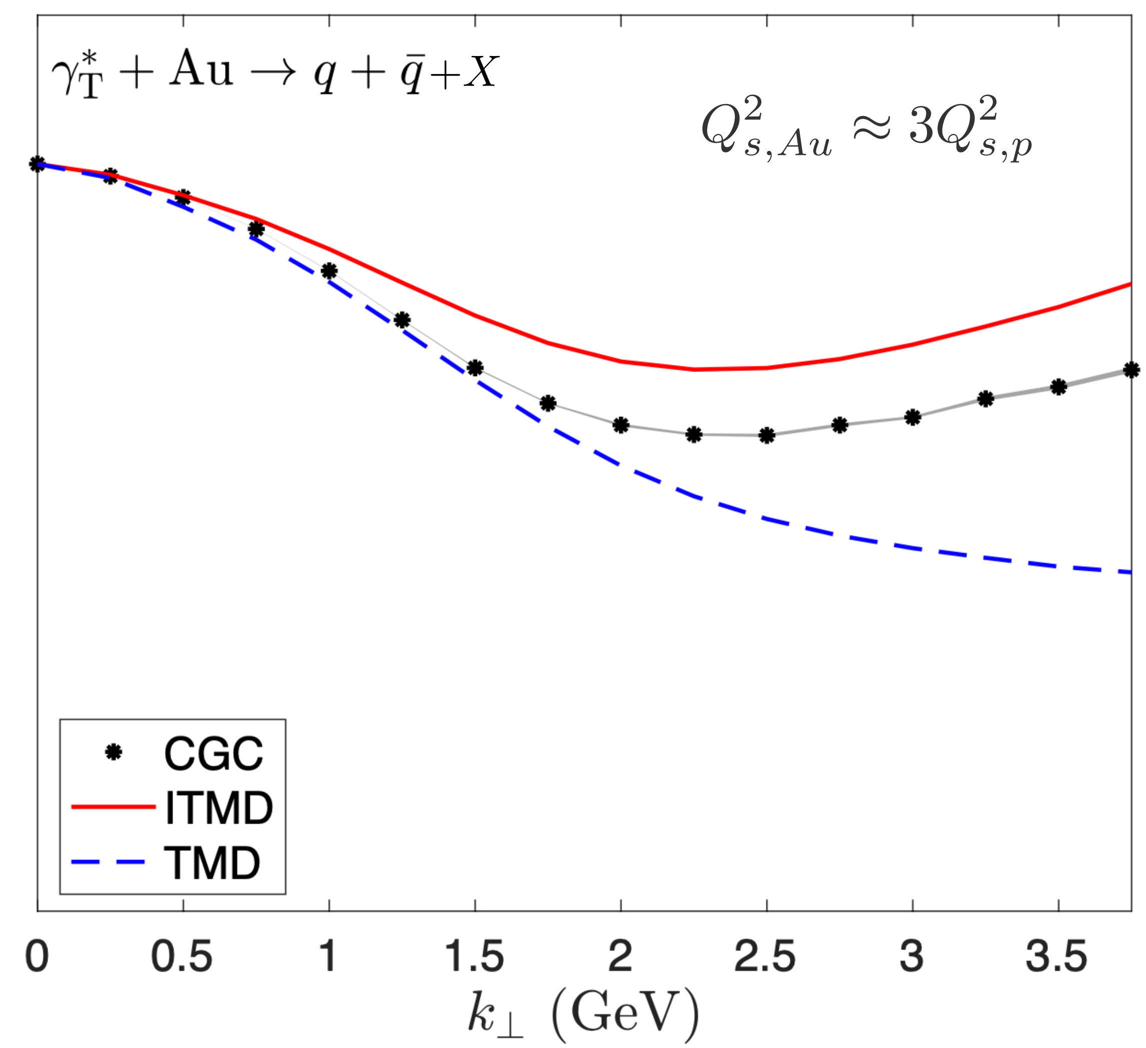
Momentum imbalance **elliptic** anisotropies: TMD vs ITMD vs CGC



$\mathrm{v}_2(k_\perp, P_\perp)$



proton ~ smaller Q_s^2



Gold nucleus ~ larger Q_s^2

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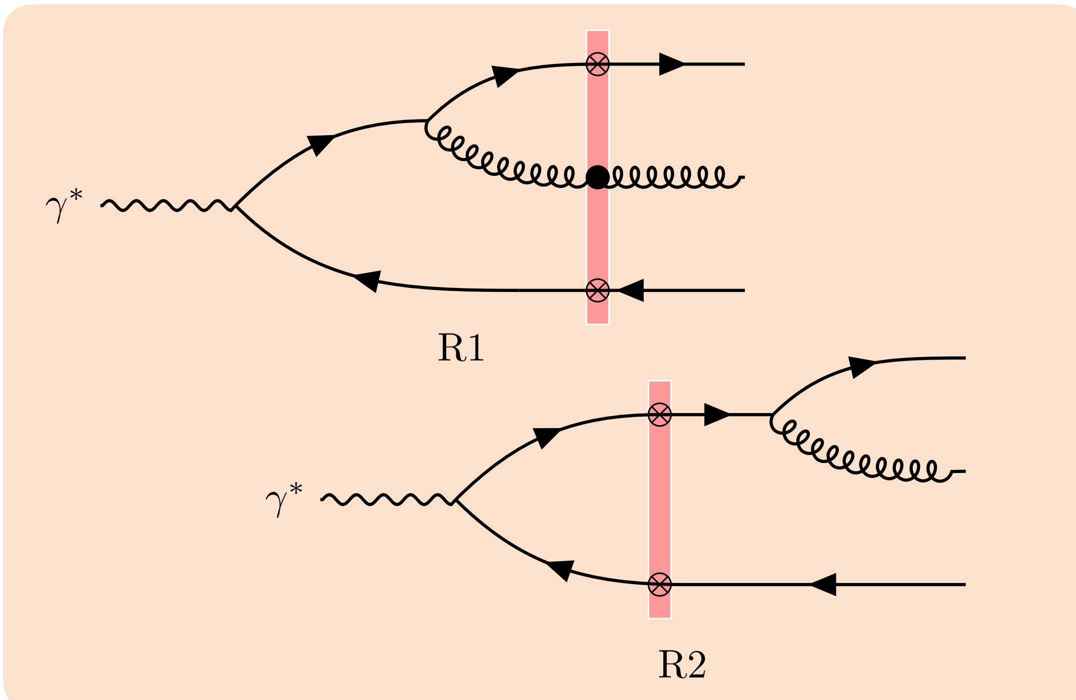
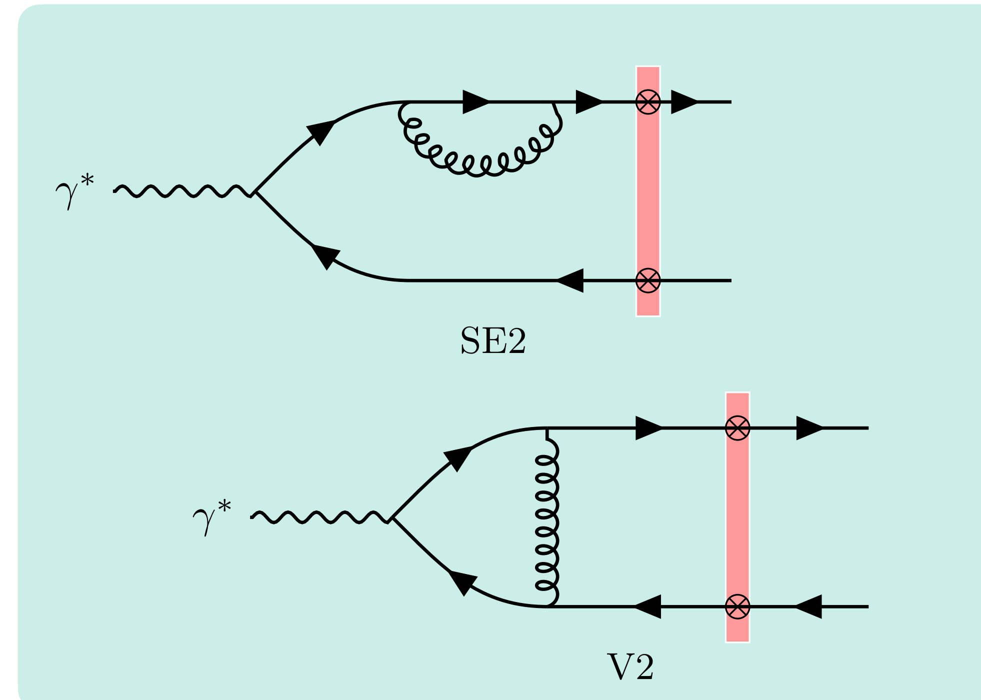
One-loop contributions

Amplitudes



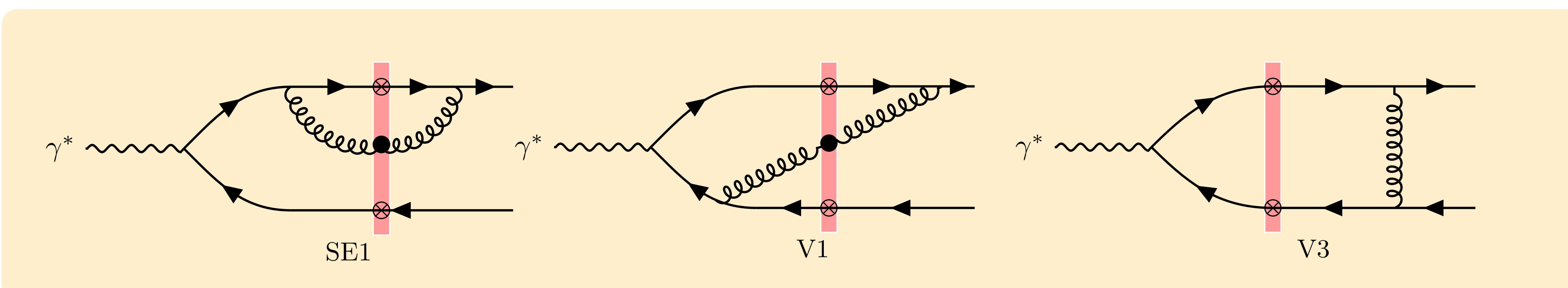


Ayala, Hentschinski, Jalilian-Marian, Tejeda-Yeomans (2017)

Altinoluk, Boussarie, Marquet, Taels (2020)
with LCPT and $Q^2 = 0$

Beuf (2016) with LCPT

Hänninen, Lappi, and Paatelainen (2017) with LCPT



Boussarie, Grabovsky, Szymanowski, Wallon (2016) for exclusive dijets

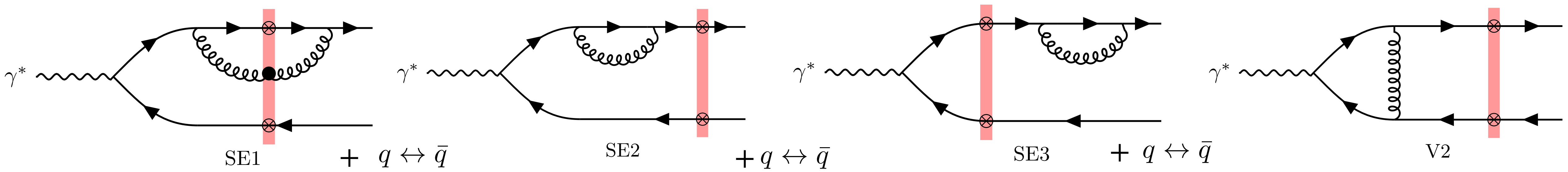
Taels, Altinoluk, Marquet, Beuf (2022) with LCPT and $Q^2 = 0$

Caucal, FS, Venugopalan (2021)
Computed all diagrams within covariant PT for inclusive dijets

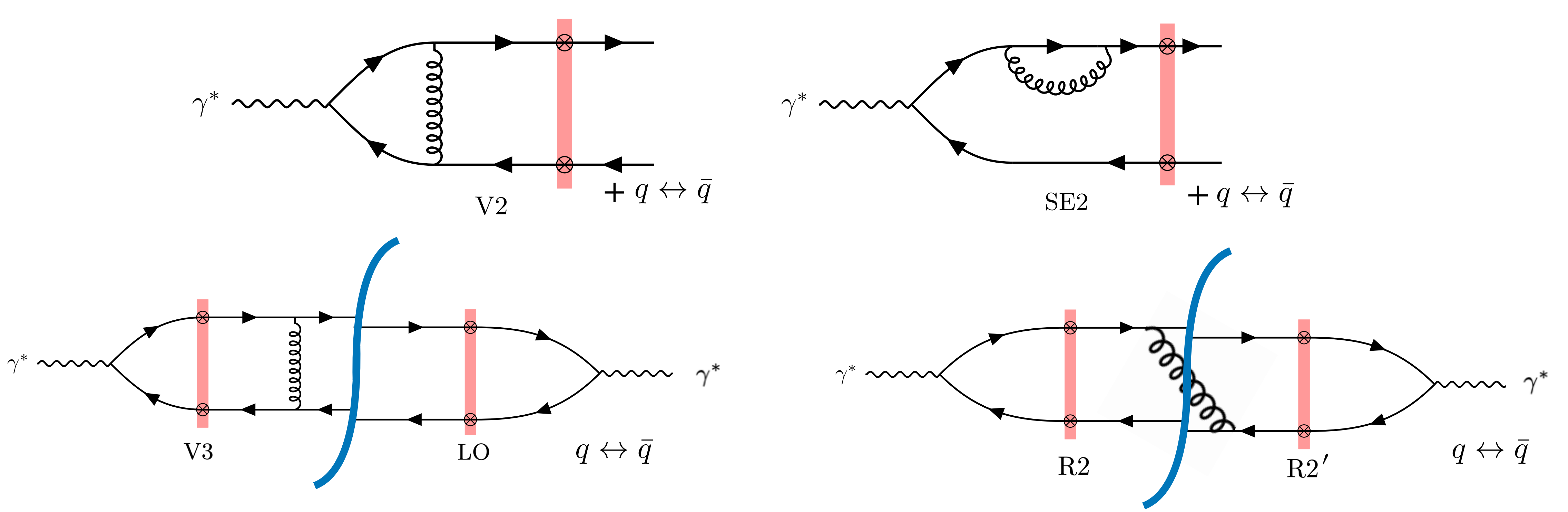
One-loop corrections

Cancellation of divergences: UV and soft

- UV divergences cancel among virtual diagrams: cancellations of poles $1/\epsilon$



- Soft divergences in real and virtual diagrams: cancellation of double logs/poles

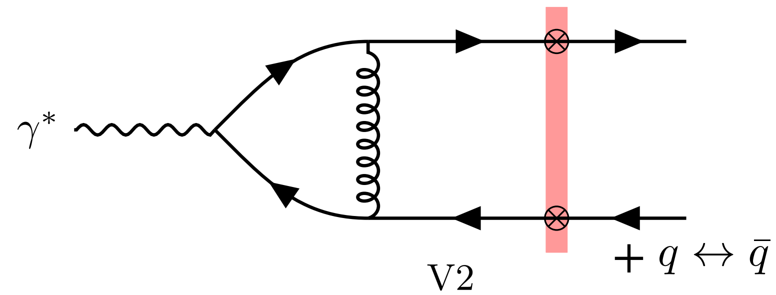


Caucal, FS, Venugopalan (2021)

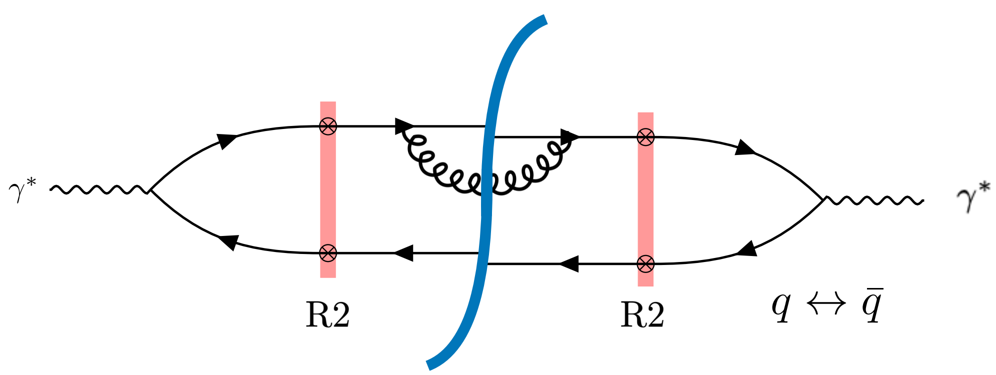
One-loop corrections

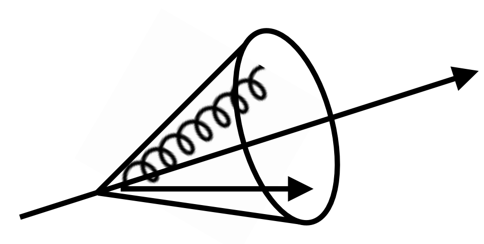
Cancellation of divergences: IR and collinear

- Remaining IR divergence in V2 manifesting as $1/\epsilon$ pole



- Need to define IRC safe observable → jets





Collinearity variable:

$$\mathcal{C}_{qg,\perp} = \frac{z_q}{z_j}\left(\boldsymbol{k_g}_\perp - \frac{z_g}{z_q}\boldsymbol{k}_\perp\right)$$

Small-cone condition:

$$\mathcal{C}^2_{qg,\perp} \leq \mathcal{C}^2_{qg,\perp}\big|_{\max} = R^2 \boldsymbol{p}_j^2 \min\left(\frac{z_g^2}{z_j^2}, \frac{(z_j - z_g)^2}{z_j^2}\right)$$

- Collinear divergence contains $1/\epsilon$ pole cancels IR pole

- Exclude slow gluon divergence phase space (must go into evolution)

$$\int_{z_f}^{z_j} \frac{\mathrm{d}z_g}{z_g}\mu^{\varepsilon} \int \frac{\mathrm{d}^{2-\varepsilon}\mathcal{C}_{qg,\perp}}{(2\pi)^{2-\varepsilon}} \frac{1}{\mathcal{C}^2_{qg,\perp}}$$

- Other jet algorithms can be implemented → modify impact factor (finite piece)

Caucal, FS, Venugopalan (2021)

One-loop corrections

Slow gluon limit and JIMWLK factorization

$$\mathrm{d}\sigma_{\mathrm{NLO}} = \boxed{\mathrm{d}\widetilde{\sigma}_0 \ln\left(\frac{z_f}{z_0}\right)} + \boxed{\int_0^z \frac{\mathrm{d}z_g}{z_g}\left[\mathrm{d}\widetilde{\sigma}_{\mathrm{NLO}} - \mathrm{d}\widetilde{\sigma}_0 \Theta(z_f - z_g)\right]} + \mathcal{O}(z_0)$$

Slow gluon piece   impact factor

$$\left.\frac{\mathrm{d}\sigma^{\lambda}_{\mathrm{NLO}}}{\mathrm{d}^2\boldsymbol{k_{1\perp}}\mathrm{d}\eta_1\mathrm{d}^2\boldsymbol{k_{2\perp}}\mathrm{d}\eta_2}\right|_{\mathrm{slow}} = \frac{\alpha_{\mathrm{em}}e_f^2 N_c}{(2\pi)^6}\delta(1-z_q-z_{\bar{q}})\int \mathrm{d}\boldsymbol{X}_\perp \mathcal{R}^{\lambda}_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\ln\left(\frac{z_f}{z_0}\right)$$

$$\begin{aligned}
\times \frac{\alpha_s N_c}{4\pi^2}\Bigg\langle \int \mathrm{d}^2\boldsymbol{z}_\perp \Bigg\{ & \frac{\boldsymbol{r}^2_{xy}}{\boldsymbol{r}^2_{zx}\boldsymbol{r}^2_{zy}}(2D_{xy} - 2D_{xz}D_{zy} + D_{zy}Q_{y'x',xz} + D_{xz}Q_{y'x',zy} - Q_{xy,y'x'} - D_{xy}D_{y'x'}) \\
& + \frac{\boldsymbol{r}^2_{x'y'}}{\boldsymbol{r}^2_{zx'}\boldsymbol{r}^2_{zy'}}(2D_{y'x'} - 2D_{y'z}D_{zx'} + D_{zx'}Q_{xy,y'z} + D_{y'z}Q_{xy,zx'} - Q_{xy,y'x'} - D_{xy}D_{y'x'}) \\
& + \frac{\boldsymbol{r}^2_{xx'}}{\boldsymbol{r}^2_{zx}\boldsymbol{r}^2_{zx'}}(D_{zx'}Q_{xy,y'z} + D_{xz}Q_{y'x',zy} - Q_{xy,y'x'} - D_{xx'}D_{y'y}) \\
& + \frac{\boldsymbol{r}^2_{yy'}}{\boldsymbol{r}^2_{zy}\boldsymbol{r}^2_{zy'}}(D_{y'z}Q_{xy,zx'} + D_{zy}Q_{y'x',xz} - Q_{xy,y'x'} - D_{xx'}D_{y'y}) \\
& + \frac{\boldsymbol{r}^2_{xy'}}{\boldsymbol{r}^2_{zx}\boldsymbol{r}^2_{zy'}}(D_{xx'}D_{y'y} + D_{xy}D_{y'x'} - D_{zx'}Q_{xy,y'z} - D_{zy}Q_{y'x',xz}) \\
& + \frac{\boldsymbol{r}^2_{x'y}}{\boldsymbol{r}^2_{zx'}\boldsymbol{r}^2_{zy}}(D_{xx'}D_{y'y} + D_{xy}D_{y'x'} - D_{y'z}Q_{xy,zx'} - D_{xz}Q_{y'x',zy}) \Bigg\}\Bigg\rangle_Y
\end{aligned}$$

JIMWLK LL Hamiltonian acting on LO color structure $\mathcal{H}_{\mathrm{JIMWLK}}\langle \Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{y}'_\perp \boldsymbol{x}'_\perp)\rangle_Y$

Evolution of quadrupole can be found in
Dominguez, Mueller, Munier, and Xiao (2011)

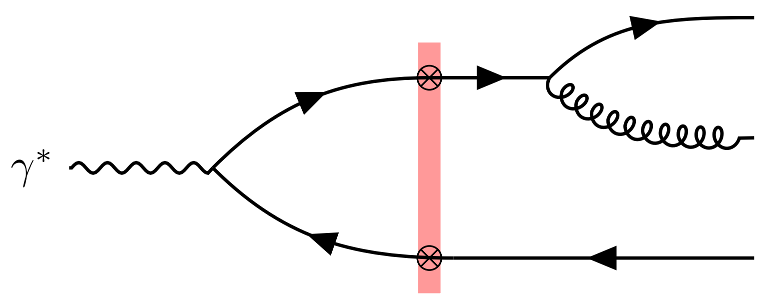
Small-x evolution of dipole and quadrupole!

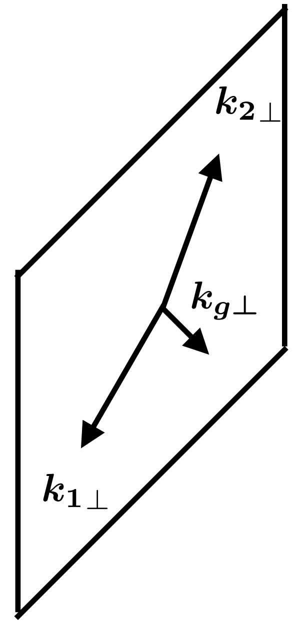
Caucal, FS, Venugopalan (2021)

Back-to-back limit

Origin of Sudakov double logs

In the back-to-back limit we expect the appearance of double and single Sudakov logarithms





$$\boldsymbol{k}_\perp = \boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp}$$

$$\boldsymbol{P}_\perp = z_2 \boldsymbol{k}_{1\perp} - z_1 \boldsymbol{k}_{2\perp}$$

resummation

$$\mathrm{d}\sigma \sim \mathcal{H}(\boldsymbol{P}_\perp, Q, z_1) \int \mathrm{d}^2\boldsymbol{b}_\perp \mathrm{d}^2\boldsymbol{b}'_\perp e^{-i\boldsymbol{k}_\perp\cdot(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)} \boxed{e^{-S_{\mathrm{Sud}}(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp, \boldsymbol{P}_\perp)}}\, xG(\boldsymbol{b}_\perp, \boldsymbol{b}'_\perp; x)$$

$$\boxed{= 1 - S_{\mathrm{Sud}}(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp, \boldsymbol{P}_\perp) + \ldots}$$

one-loop contribution

Sudakov factor $$S_{\mathrm{Sud}}(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp, \boldsymbol{P}_\perp) = \frac{\alpha_s N_c}{4\pi} \ln^2\left(\frac{\boldsymbol{P}_\perp^2(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)^2}{c_0^2}\right) + \ldots$$ *Mueller, Xiao, Yuan (2013)*

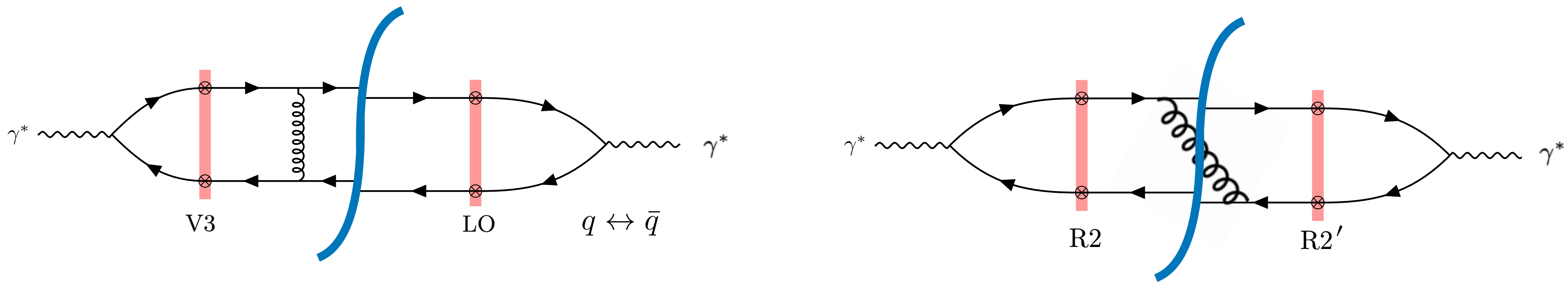
Single logs depend on jet algorithm… Computed in collinear factorization see *Sun, Yuan, Yuan (2015)*

Effect of soft gluon radiation on azimuthal anisotropies see Hatta, Xiao, Yuan, Zhou (2011)

Back-to-back limit

Where is the Sudakov in our computation?

- Take back-to-back limit of impact factor (real diagrams with soft divergence)



$$\mathrm{d}\sigma \sim \mathcal{H}(\boldsymbol{P}_\perp, Q, z_1) \int \mathrm{d}^2\boldsymbol{b}_\perp \mathrm{d}^2\boldsymbol{b}'_\perp e^{-i\boldsymbol{k}_\perp\cdot(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)} \left[1 \boxed{+ \frac{\alpha_s N_c}{4\pi} \ln^2\left(\frac{\boldsymbol{P}_\perp^2(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)^2}{c_0^2}\right)} + \cdots + \alpha_s \ln\left(\frac{z_f}{z_0}\right) \mathcal{K}_{LL}\otimes\right] xG(\boldsymbol{b}_\perp, \boldsymbol{b}'_\perp; x)$$

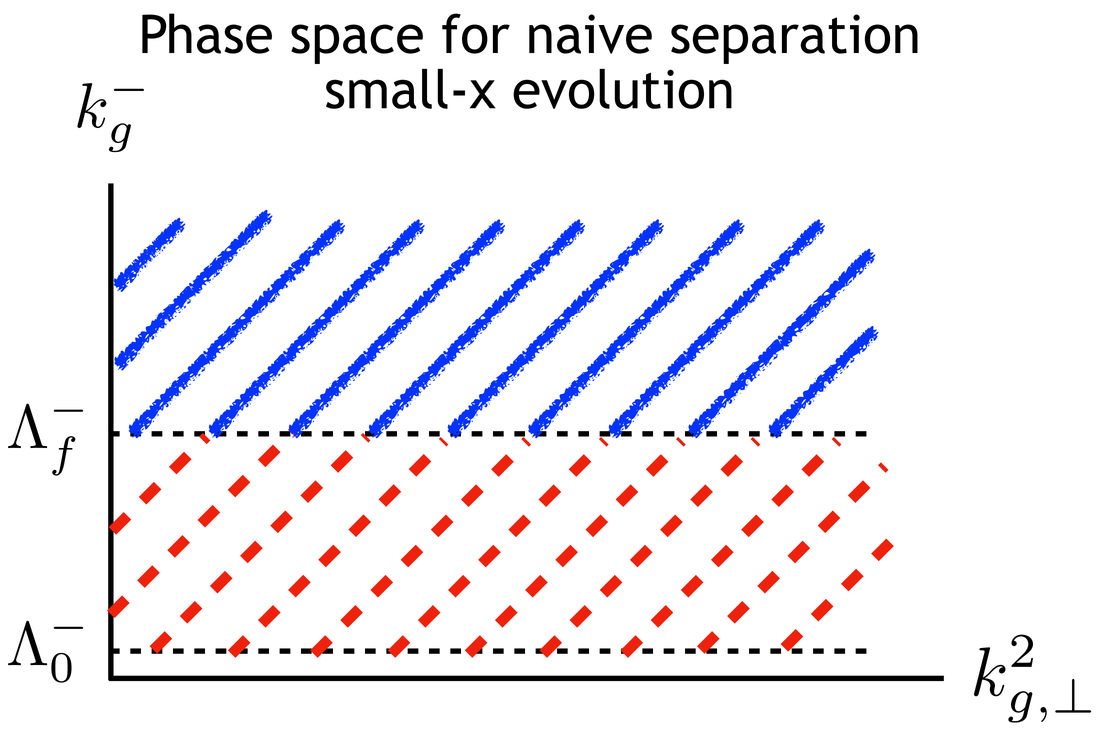
Sudakov double log but with positive sign!

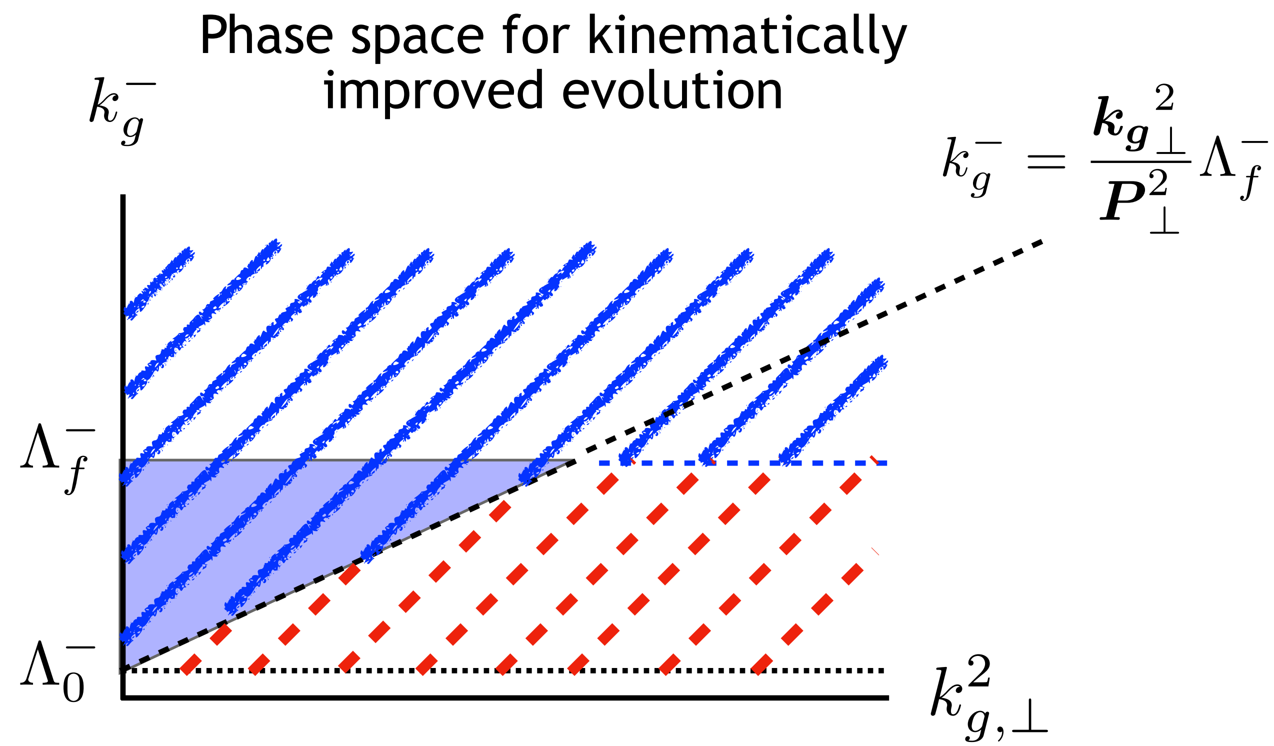
- Need kinematic constraint: disentangle phase space for soft and slow (small-x) gluons

Caucal, Schenke, FS, Venugopalan (in preparation)

Back-to-back limit

Sudakov resummation and kinematically improved small-x evolution





- Positive (wrong) Sudakov double log
- NLL small-x evolution will suffer from collinear divergences

- **Correct Sudakov double log**
- **Collinearly improved small-x evolution**

$$d\sigma \sim \mathcal{H}(\boldsymbol{P}_\perp, Q, z_1) \int d^2\boldsymbol{b}_\perp d^2\boldsymbol{b}'_\perp e^{-i\boldsymbol{k}_\perp\cdot(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)} \left[1 - \frac{\alpha_s N_c}{4\pi}\ln^2\left(\frac{\boldsymbol{P}_\perp^2(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)^2}{c_0^2}\right) + \cdots + \alpha_s \mathcal{K}_{LL,\mathrm{coll}} \otimes\right] xG(\boldsymbol{b}_\perp, \boldsymbol{b}'_\perp; x)$$

Caucal, Schenke, FS, Venugopalan (in preparation)

Taels, Altinoluk, Marquet, Beuf (2022) for photo-production.

Back-to-back limit

TMD factorization breaking at NLO

- LL small-x evolution of the Weizsäcker Williams TMD is not a closed equation (even at large Nc)!

See *Dominguez, Mueller, Munier, and Xiao (2011)*

- At NLO finite pieces containing correlators beyond the Weizsäcker Williams TMD

See also *Taels, Altinoluk, Marquet, Beuf (2022)*

- In the dilute limit, LL evolution follows BFKL (closed equation) and finite pieces simplify to dipole gluon TMD $\rightarrow$ TMD factorization: Sudakov resummation + small-x resummation

As conjectured in Mueller, Xiao, Yuan (2013)

- Saturation introduces factorization breaking terms at NLO. How large are these contributions? Expected to be enhanced for nuclei!

Caucal, Schenke, FS, Venugopalan (in preparation)

Summary

- Why dijet production?

  Saturation, linearly polarized gluons, azimuthal correlations, etc

- CGC beyond TMDs

  From Wilson lines to TMDs and beyond
  (kinematic and genuine twists)

- NLO corrections

  Cancellation of divergences, rapidity evolution, and impact factor

- Back-to-back limit

  Interplay between Sudakov resummation and kinematically improved evolution
  TMD factorization breaking

Future directions

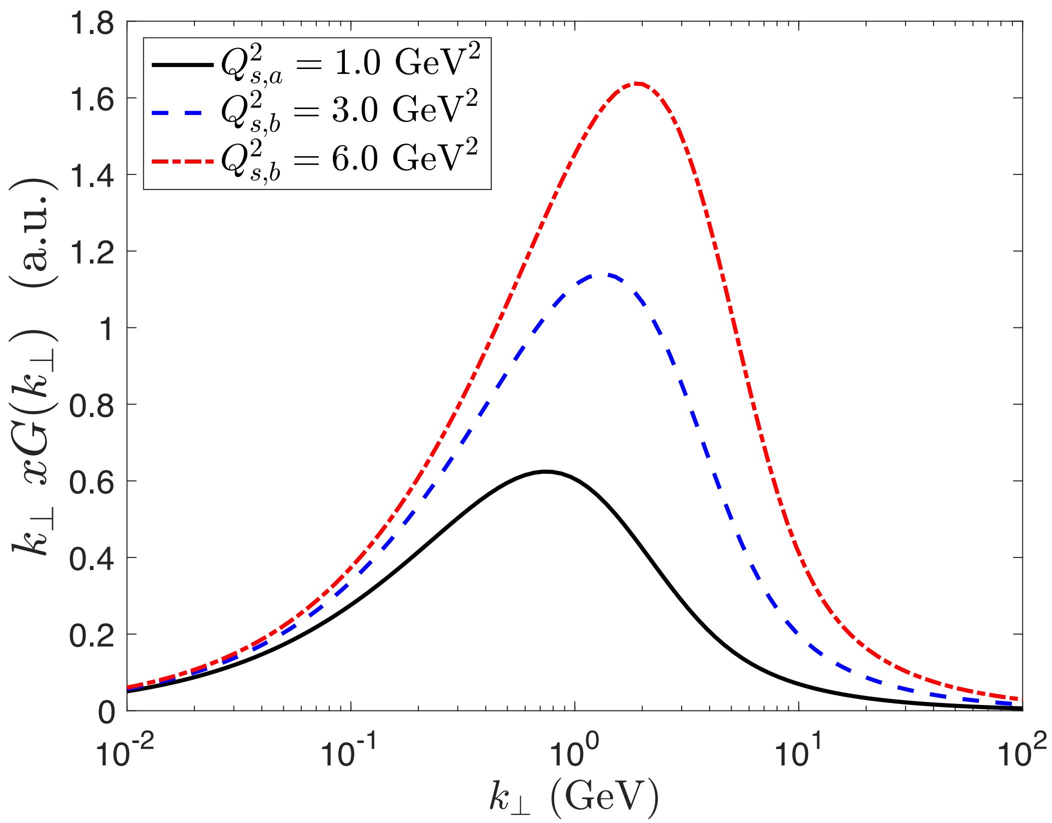
- Numerical predictions for dijets at NLO
- Alternative way to distinguish soft/rapidity divergences using SCET
- Connection to RHIC and LHC physics via UPCs (photo-production limit)
- Massive quarks: heavy quark and quarkonia production

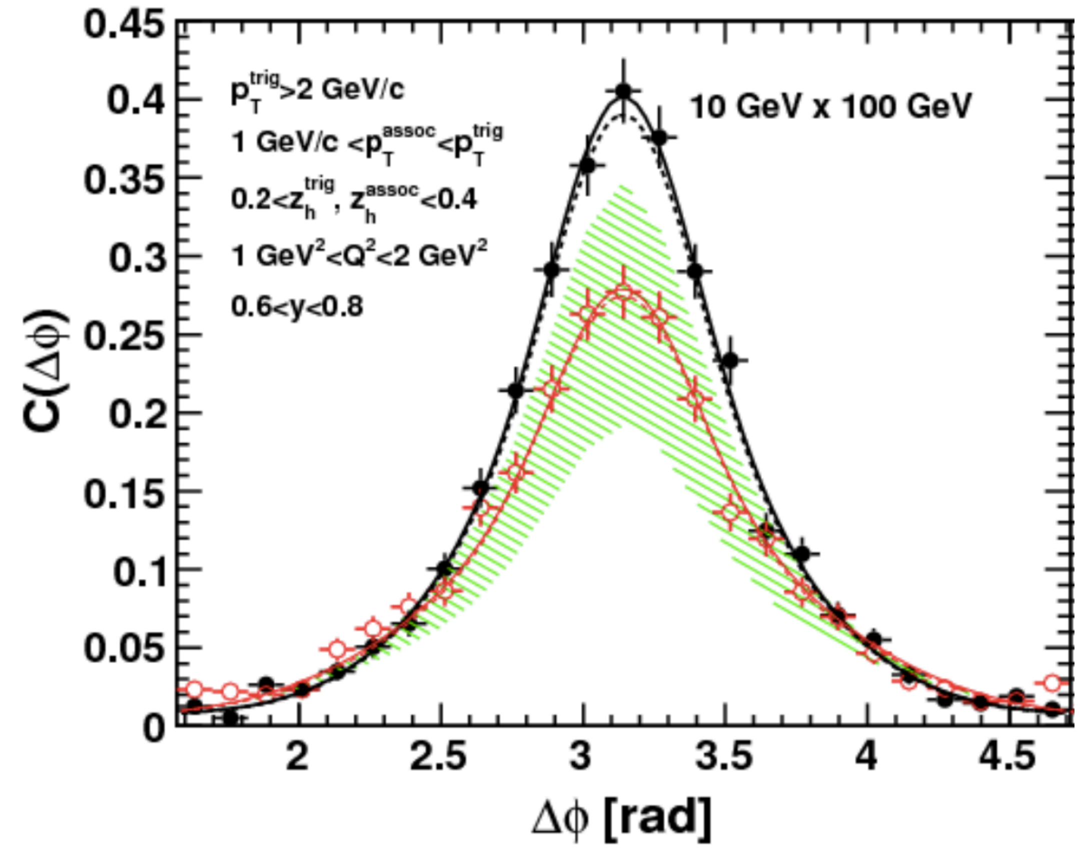
Back-up Slides

Dijet and dihadron production: TMD formalism

Forward dihadron azimuthal correlations and gluon saturation

Weizsäcker-Williams TMD at small-x → Dihadron suppression back-to-back peak

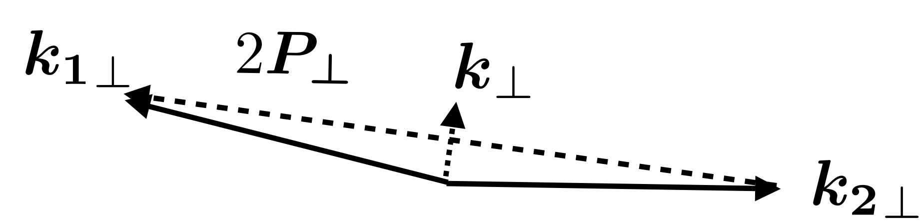




Typical momentum transfer from hadron/nucleus to

Momentum imbalance → $k_\perp \sim Q_s$ ← Saturation scale

Zheng, Aschenauer, Lee, Xiao (2014)

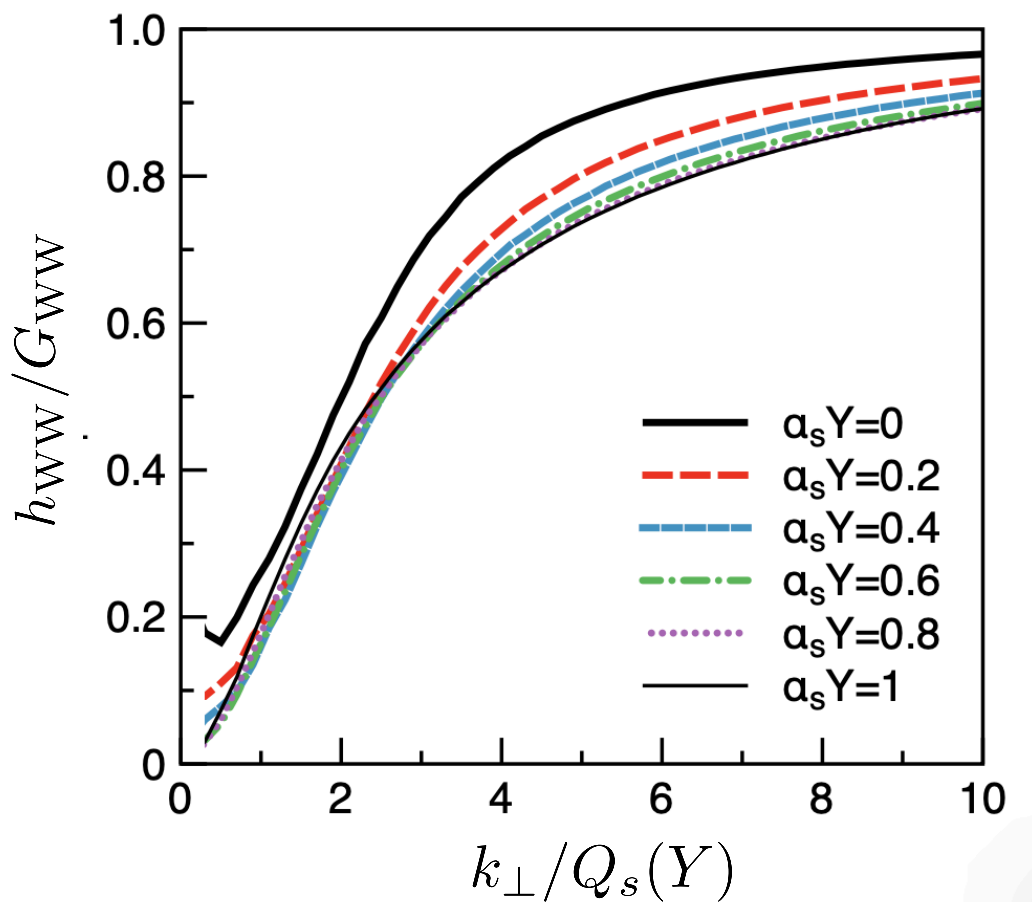


$\Delta\phi$ angle between $\boldsymbol{k_{1\perp}}$ and $\boldsymbol{k_{2\perp}}$

Dijet and dihadron production: TMD formalism

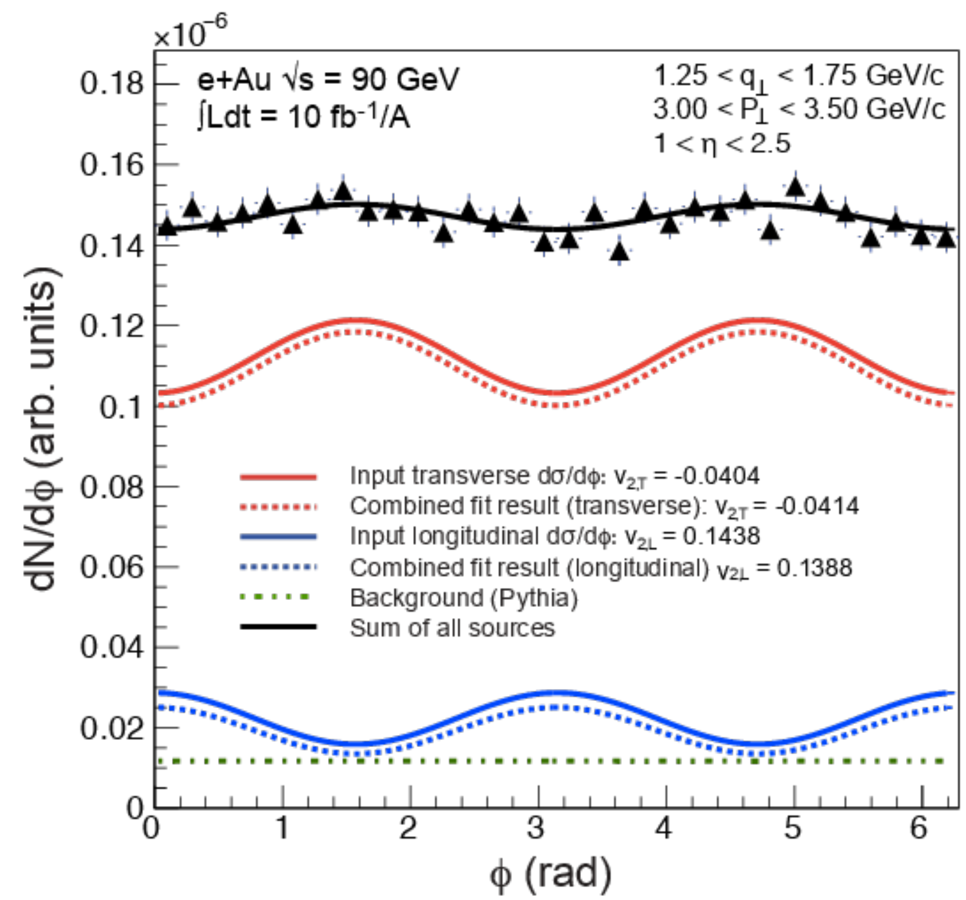
Forward dijet azimuthal asymmetries and linearly pol WW gluon TMD

Ratio of linearly polarized to unpolarized gluon WW



Dumitru, Lappi, Skokov (2015)

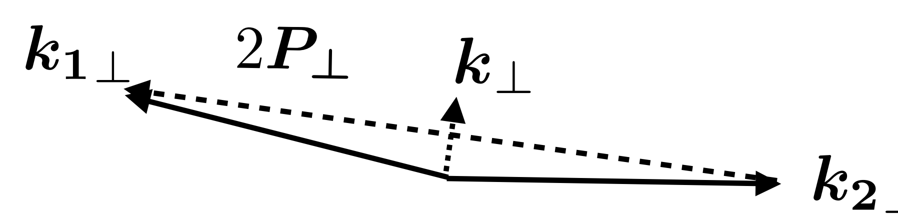
Dijet azimuthal asymmetries in momentum imbalance



Dumitru, Skokov, Ullrich (2018)

Accessed in azimuthal asymmetry of momentum imbalance in dijet pairs

Dominguez, Qiu, Xiao, Yuan (2011)



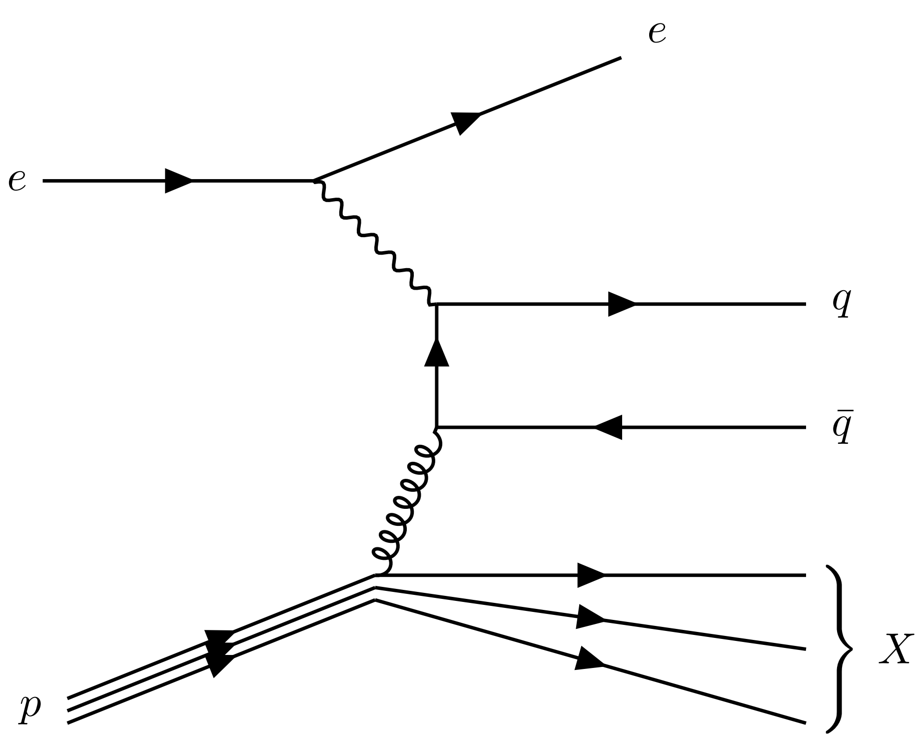
ϕ angle between $\boldsymbol{P}_\perp$ and $\boldsymbol{k}_\perp$

$$|v_2| \sim \frac{xh_{WW}}{xG_{WW}}$$

See Feng's talk tomorrow effects of soft gluon radiation!

Dijet and dihadron production: TMD formalism

The Weizsäcker-Williams gluon TMD



LO diagram for $q\bar{q}$ production

Bomhof, Mulders, Pijlman (2006)
Dominguez, Marquet, Xiao, Yuan (2011)
Dominguez, Qiu, Xiao, Yuan (2011)

$$d\sigma^{\gamma^* A \to q\bar{q}} \sim \mathcal{H}^{ij}_{\mathrm{TMD}}(\boldsymbol{P}_\perp)\, \alpha_s x G^{ij}_{\mathrm{WW}}(\boldsymbol{k}_\perp, x)$$

Perturbatively calculable

WW gluon TMD

Unpolarized part

$$xG^0_{WW}(k_\perp, x) = \delta^{ij} x G^{ij}_{\mathrm{WW}}(\boldsymbol{k}_\perp, x)$$

Linearly polarized part

$$xh^0_{WW}(k_\perp, x) = \left(2\frac{\boldsymbol{k}^i_\perp \boldsymbol{k}^j_\perp}{\boldsymbol{k}^2_\perp} - \delta^{ij}\right) x G^{ij}_{\mathrm{WW}}(\boldsymbol{k}_\perp, x)$$

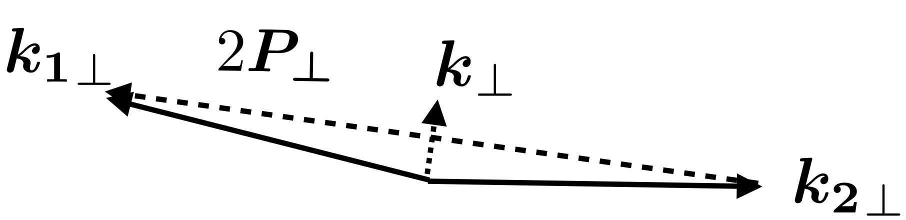
$$|\mathrm{v}_2| \sim \frac{xh_{\mathrm{WW}}}{xG_{\mathrm{WW}}}$$

in the angle

$$\phi \equiv \phi_{\boldsymbol{k}_\perp} - \phi_{\boldsymbol{P}_\perp}$$

Validity of TMD approach:

$k_\perp \ll P_\perp$ (i.e. back-to-back configuration)



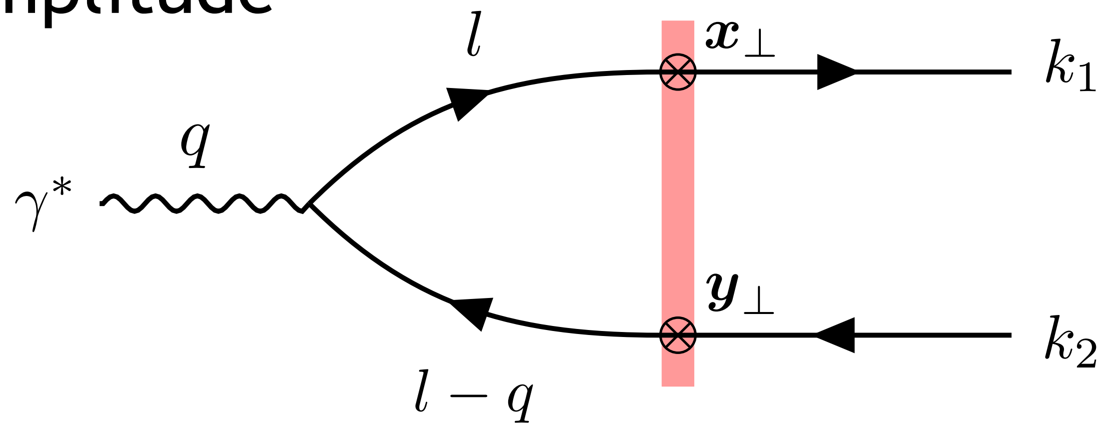
$$\boldsymbol{k}_\perp = \boldsymbol{k_1}_\perp + \boldsymbol{k_2}_\perp$$

$$\boldsymbol{P}_\perp = z_2 \boldsymbol{k_1}_\perp - z_1 \boldsymbol{k_2}_\perp$$

Review of Leading order

Anatomy of the amplitude

$$q^\mu = \left(-\frac{Q}{q^-}, q^-, \mathbf{0}_\perp\right)$$



LO

$$k_1^\mu = \left(\frac{\boldsymbol{k}_{1\perp}^2}{2z_1 q^-}, z_1 q^-, \boldsymbol{k}_{1\perp}\right)$$

$$k_2^\mu = \left(\frac{\boldsymbol{k}_{2\perp}^2}{2z_2 q^-}, z_2 q^-, \boldsymbol{k}_{2\perp}\right)$$

$$\int \frac{d^4 l}{(2\pi)^4} \bar{u}(k_1,\sigma_1)\mathcal{T}^q(k_1,l)S^0(l)\left(-iee_f \not\epsilon(q,\lambda)\right)S^0(l-q)\mathcal{T}^q(l-q,-k_2)v(k_2,\sigma_2)$$

*Loop integration variable l since both quark and anti-quark receive momentum from shock-wave.

<u>Dissecting amplitude</u>

Color correlator

Perturbative factor!

$$\mathcal{M}^\lambda_{\mathrm{LO},ij,\sigma_1\sigma_2} = \frac{ee_f q^-}{\pi}\int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp} e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{x}_\perp} e^{-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{y}_\perp} \boxed{\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) - \mathbb{1}\right]_{ij}} \boxed{\mathcal{N}^\lambda_{\sigma_1\sigma_2}(\boldsymbol{x}_\perp - \boldsymbol{y}_\perp)}$$

$$\mathcal{N}^\lambda_{\sigma_1\sigma_2}(\boldsymbol{r}_\perp) = -i(2q^-)\int \frac{\mathrm{d}^4 l}{(2\pi)^2} \frac{e^{i\boldsymbol{l}_\perp\cdot\boldsymbol{r}_\perp} N^\lambda_{\sigma_1\sigma_2}(l)\delta(k_1^- - l^-)}{(l^2+i\epsilon)((l-q)^2+i\epsilon)}$$

with Dirac/Lorentz structure:

$$N^\lambda_{\sigma_1\sigma_2}(l) = \frac{1}{(2q^-)^2}\left[\bar{u}(k_1,\sigma_1)\gamma^- \not{l} \not\epsilon(q,\lambda)(\not{q} - \not{l})\gamma^- v(k_2,\sigma_2)\right]$$

Leading order

Anatomy of the amplitude

- Evaluate Dirac/Lorentz structure with basic gamma matrix manipulations

$$N^{\lambda=0}_{\sigma_1\sigma_2}(l) = -2Q(z_1z_2)^{3/2}\delta_{\sigma_1,-\sigma_2} \qquad N^{\lambda=\pm1}_{\sigma_1\sigma_2}(l) = \boldsymbol{l}_\perp \cdot \boldsymbol{\epsilon}^\lambda_\perp \left[z_2\delta^\lambda_{\sigma_1} - z_1\delta^\lambda_{\sigma_2}\right]\delta_{\sigma_1,-\sigma_2}$$

- Evaluate loop integral $\mathrm{d}^4 l$

$$\mathcal{N}^\lambda_{\sigma_1\sigma_2}(\boldsymbol{r}_\perp) = -i(2q^-)\int \frac{\mathrm{d}^2\boldsymbol{l}_\perp}{(2\pi)^2}\int \mathrm{d}l^+ \boxed{\int \mathrm{d}l^-} \frac{e^{i\boldsymbol{l}_\perp\cdot\boldsymbol{r}_\perp}N^\lambda_{\sigma_1\sigma_2}(l)\boxed{\delta(k_1^- - l^-)}}{(l^2+i\epsilon)((l-q)^2+i\epsilon)}$$

- Compute l^- integral using eikonal delta function $\delta(k_1^- - l^-)$

$$\mathcal{N}^\lambda_{\sigma_1\sigma_2}(\boldsymbol{r}_\perp) = -i(2q^-)\int \frac{\mathrm{d}^2\boldsymbol{l}_\perp}{(2\pi)^2} e^{i\boldsymbol{l}_\perp\cdot\boldsymbol{r}_\perp}N^\lambda_{\sigma_1\sigma_2}(l)\boxed{\int dl^+ \frac{1}{(l^2+i\epsilon)((l-q)^2+i\epsilon)}}$$

- Compute l^+ via contour integration using residues

$$\mathcal{N}^\lambda_{\sigma_1\sigma_2}(\boldsymbol{r}_\perp) = \boxed{\int \frac{\mathrm{d}^2\boldsymbol{l}_\perp}{2\pi}\frac{N^\lambda_{\sigma_1\sigma_2}(l)e^{i\boldsymbol{l}_\perp\cdot\boldsymbol{r}_\perp}}{z_1z_2Q^2+\boldsymbol{l}_\perp^2}}$$

- Compute $\boldsymbol{l}_\perp$ with 2D Fourier transform

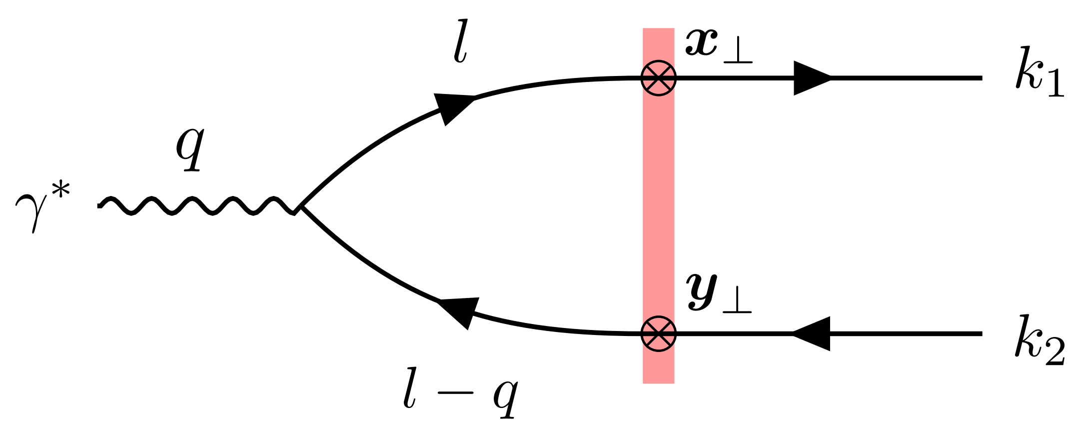
$$\mathcal{N}^{\lambda=0}_{\sigma_1\sigma_2}(\boldsymbol{r}_\perp) = -2(z_1z_2)^{3/2}QK_0(Q\sqrt{z_1z_2}r_\perp)\delta_{\sigma_1,-\sigma_2}$$

$$\mathcal{N}^{\lambda=\pm1}_{\sigma_1\sigma_2}(\boldsymbol{r}_\perp) = 2(z_1z_2)^{3/2}\left[z_2\delta^\lambda_{\sigma_1} - z_1\delta^\lambda_{\sigma_2}\right]\frac{iQ\boldsymbol{r}_\perp\cdot\boldsymbol{\epsilon}^\lambda_\perp}{\sqrt{z_1z_2}r_\perp}K_1(Q\sqrt{z_1z_2}r_\perp)\delta_{\sigma_1,-\sigma_2}$$

where K_0 and K_1 are Bessel functions (exponential decaying like)

Leading order

Anatomy of the cross-section



LO

Amplitude:

$$\mathcal{M}^{\lambda}_{\mathrm{LO},ij,\sigma_1\sigma_2} \propto \boxed{\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) - \mathbb{1}\right]_{ij}} \boxed{\mathcal{N}^{\lambda}_{\sigma_1,\sigma_2}(Q, z_1, \boldsymbol{x}_\perp - \boldsymbol{y}_\perp)}$$

$q\bar{q}$ interaction with nucleus $\qquad$ γ^* splitting to $q\bar{q}$

Unpolarized differential cross-section:

$$\frac{\mathrm{d}\sigma^{\gamma^*_\lambda + A \to q\bar{q} + X}}{\mathrm{d}^2\boldsymbol{k}_{1\perp}\mathrm{d}^2\boldsymbol{k}_{2\perp}\mathrm{d}\eta_1\mathrm{d}\eta_2} \propto \int \mathrm{d}^8\boldsymbol{X}_\perp \; e^{-i\boldsymbol{k}_{1\perp}\cdot(\boldsymbol{x}_\perp - \boldsymbol{x}'_\perp)} e^{-i\boldsymbol{k}_{2\perp}\cdot(\boldsymbol{y}_\perp - \boldsymbol{y}'_\perp)}$$
$$\times \boxed{\langle \Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{y}'_\perp \boldsymbol{x}'_\perp)\rangle_Y} \boxed{\mathcal{R}^{\lambda}(\boldsymbol{x}_\perp - \boldsymbol{y}_\perp, \boldsymbol{x}'_\perp - \boldsymbol{y}'_\perp)}$$

$$\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{y}'_\perp \boldsymbol{x}'_\perp) = 1 - S^{(2)}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) - S^{(2)}(\boldsymbol{y}'_\perp, \boldsymbol{x}'_\perp) + S^{(4)}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{y}'_\perp, \boldsymbol{x}'_\perp)$$

dipoles

quadrupole

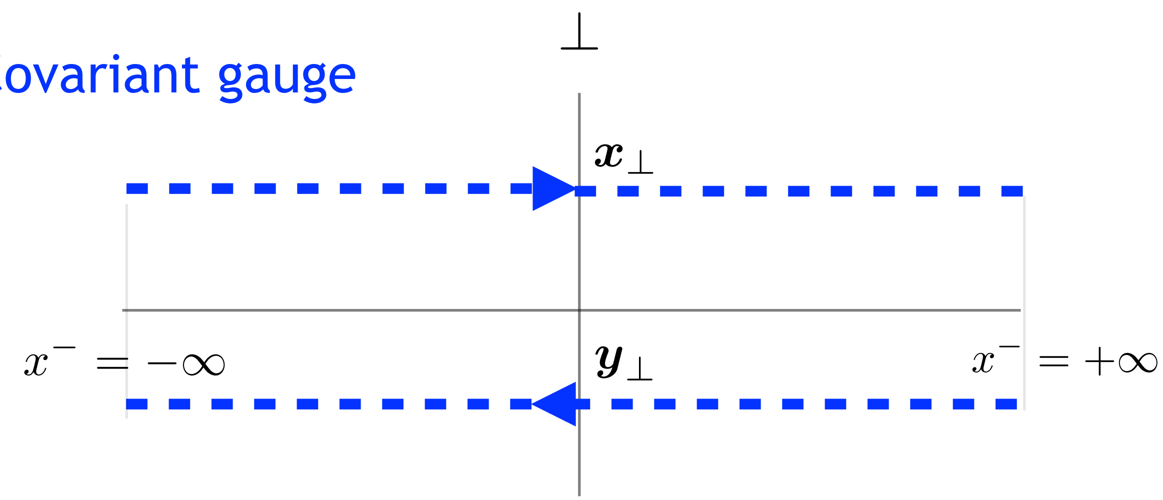
Dijet production beyond TMDs

Choosing the gauge: light-like Wilson lines vs transverse gauge link

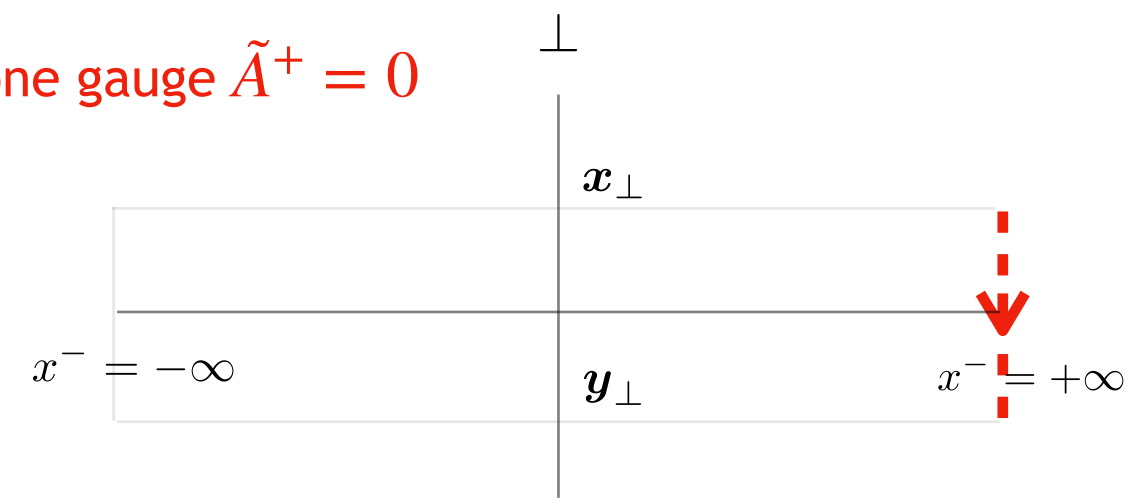
Boussarie, Mehtar-Tani (2020)

Pair of Wilson lines as transverse gauge link

Covariant gauge



Light-cone gauge $\tilde{A}^+ = 0$



$$\boxed{V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)} = \boxed{\mathcal{P}\,\exp\left[-ig\int_{\boldsymbol{y}_\perp}^{\boldsymbol{x}_\perp} \mathrm{d}\boldsymbol{z}_\perp \cdot \tilde{\boldsymbol{A}}_\perp(\boldsymbol{z}_\perp)\right]}$$

gA expansion:

$$= 1 - ig\int_{\boldsymbol{y}_\perp}^{\boldsymbol{x}_\perp} \mathrm{d}\boldsymbol{z}_\perp \cdot \tilde{\boldsymbol{A}}_\perp(\boldsymbol{z}_\perp) + \ldots$$

Small dipole expansion:

Altinoluk, Boussarie, Kotko (2019)

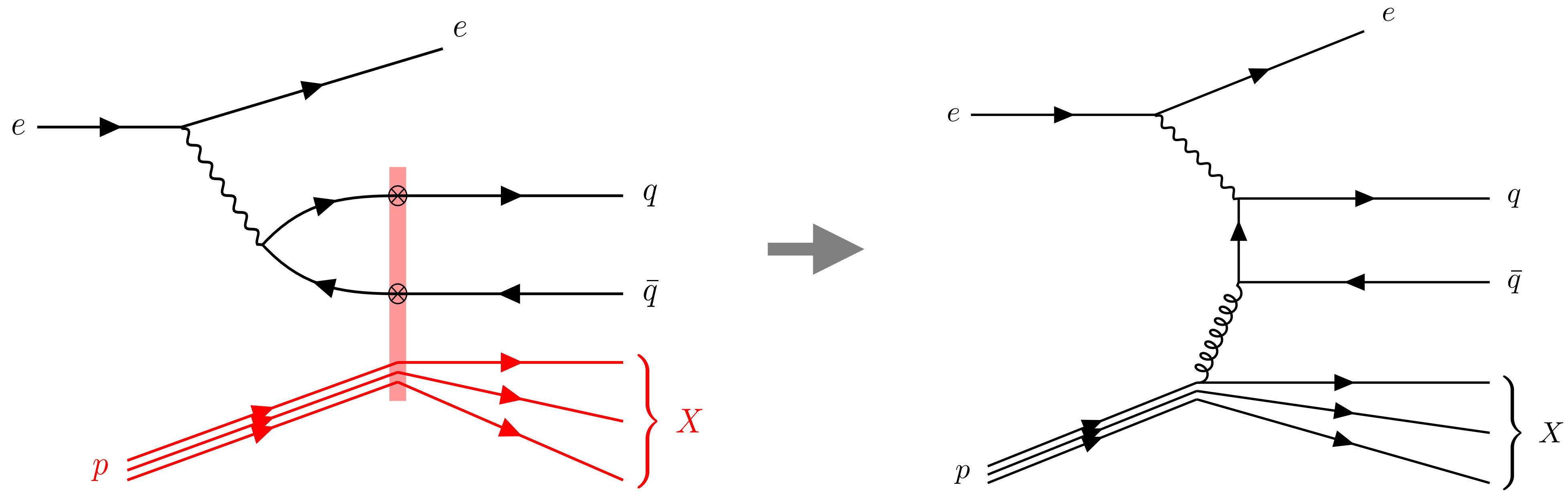
$$= 1 + ig\,\boldsymbol{r}_\perp \cdot \tilde{\boldsymbol{A}}_\perp(\boldsymbol{z}_\perp) + \ldots$$

$\boldsymbol{r}_\perp = \boldsymbol{x}_\perp - \boldsymbol{y}_\perp$

Dominguez, Marquet, Xiao, Yuan (2011)

Dijet production beyond TMDs

From CGC to Improved TMD



CGC

$$V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) = \mathcal{P}\,\exp\left[-ig\int_{\boldsymbol{y}_\perp}^{\boldsymbol{x}_\perp} \mathrm{d}\boldsymbol{z}_\perp \cdot \tilde{\boldsymbol{A}}_\perp(\boldsymbol{z}_\perp)\right]$$

Boussarie, Mehtar-Tani (2020)

Improved TMD

$$= 1 - ig\int_{\boldsymbol{y}_\perp}^{\boldsymbol{x}_\perp} \mathrm{d}\boldsymbol{z}_\perp \cdot \tilde{\boldsymbol{A}}_\perp(\boldsymbol{z}_\perp) + \ldots$$

TMD

Altinoluk, Boussarie, Kotko (2019)

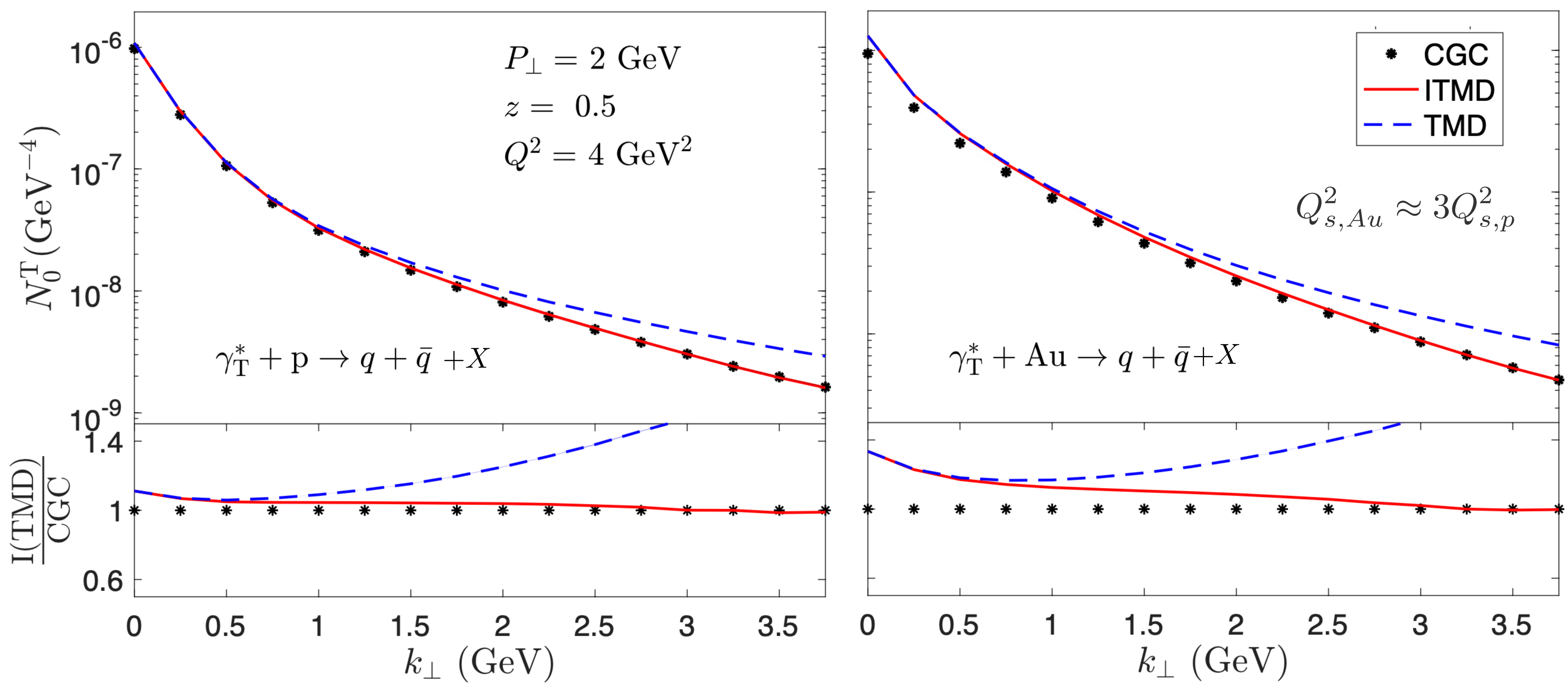
$$= 1 + ig\;\boldsymbol{r}_\perp \cdot \tilde{\boldsymbol{A}}_\perp(\boldsymbol{z}_\perp) + \ldots$$

$\boldsymbol{r}_\perp = \boldsymbol{x}_\perp - \boldsymbol{y}_\perp$

Dominguez, Marquet, Xiao, Yuan (2011)

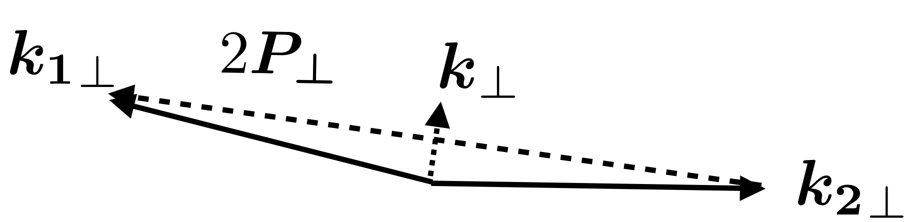
Dijet production beyond TMDs

Differential yield: TMD, ITMD and CGC



proton ~ smaller Q_s^2

Gold nucleus ~ larger Q_s^2



$$\frac{\mathrm{d}N^{\gamma^*_\lambda + A \to q\bar{q} + X}}{\mathrm{d}^2\boldsymbol{P}_\perp \mathrm{d}^2\boldsymbol{k}_\perp \mathrm{d}\eta_1 \mathrm{d}\eta_2} = N_0^\lambda(P_\perp, k_\perp)\left[1 + 2\sum_{k=1}^{\infty} v_{k,\lambda}(P_\perp, k_\perp)\cos(k\phi)\right]$$

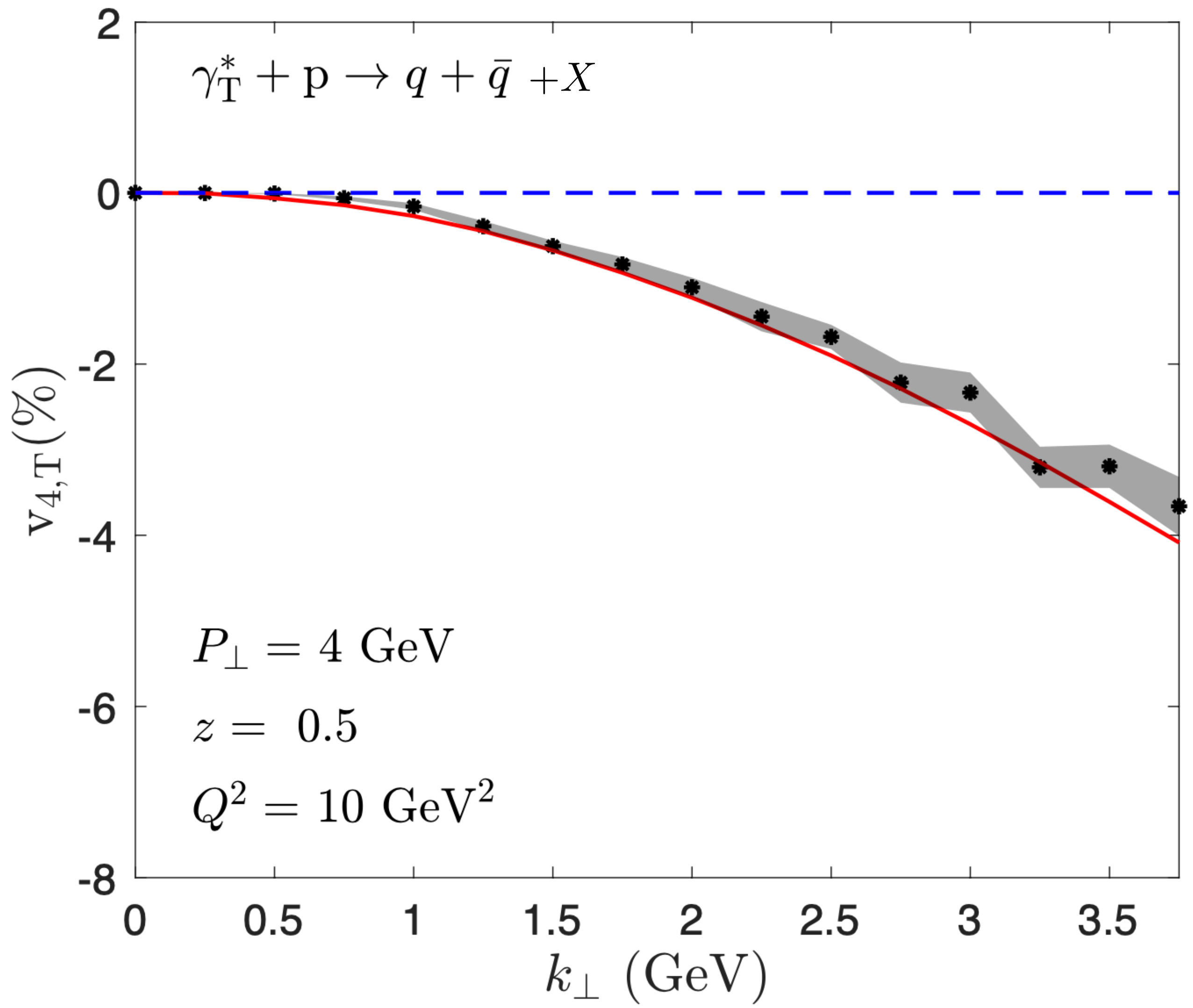
$$\phi \equiv \phi_{\boldsymbol{k}_\perp} - \phi_{\boldsymbol{P}_\perp}$$

R. Boussarie, H. Mäntysaari, FS, B. Schenke (2021)

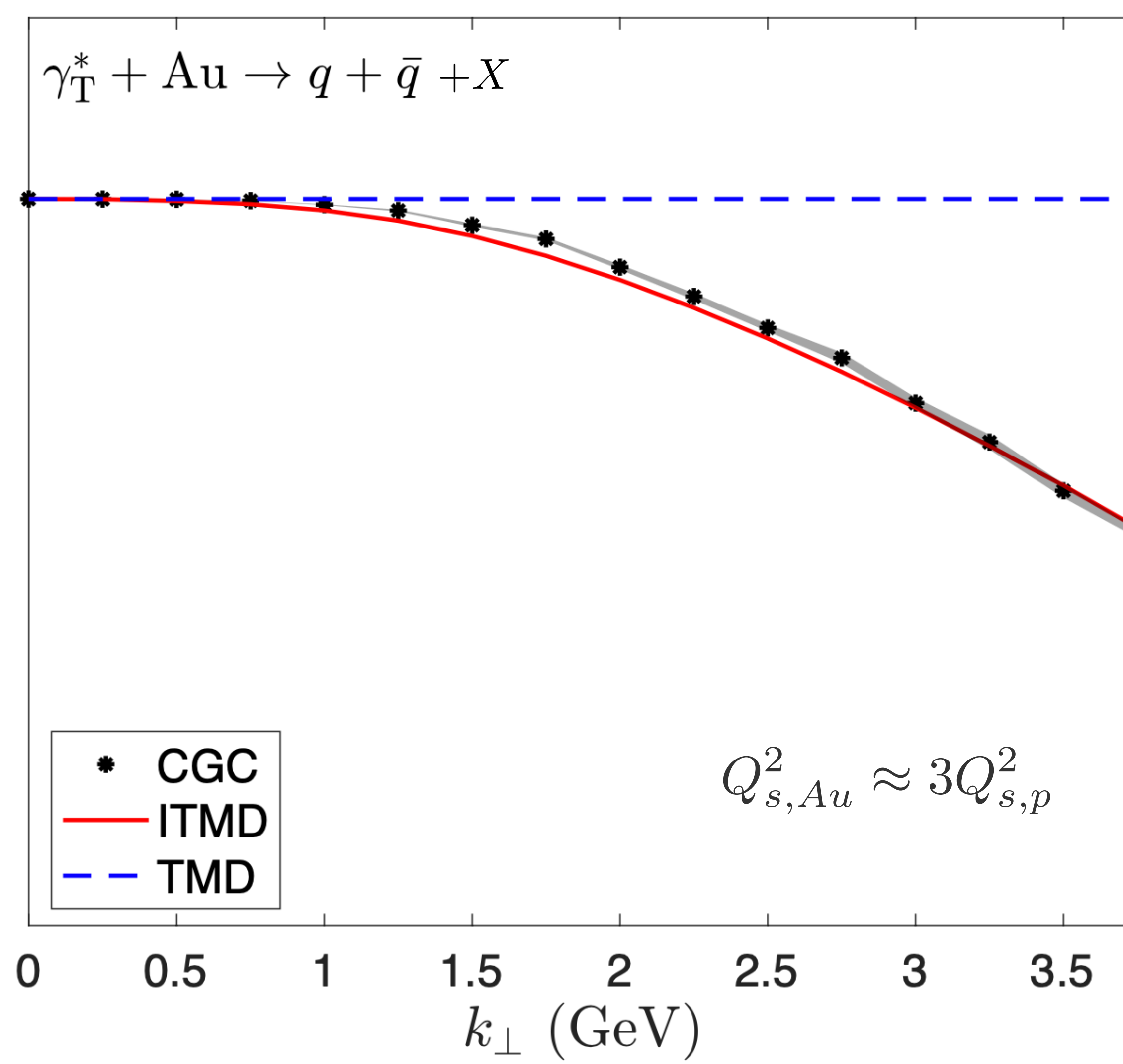
For a comparison of TMD, ITMD and CGC in pA see Fuji, Watanabe, Marquet (2020)

Dijet production beyond TMDs

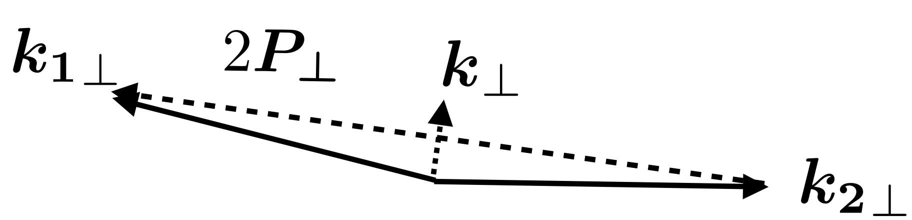
Momentum imbalance **quadrangular** anisotropies:
TMD vs ITMD vs CGC



proton ~ smaller Q_s^2



Gold nucleus ~ larger Q_s^2



$$\frac{\mathrm{d}N^{\gamma^*_\lambda + A \to q\bar{q} + X}}{\mathrm{d}^2\boldsymbol{P}_\perp \mathrm{d}^2\boldsymbol{k}_\perp \mathrm{d}\eta_1 \mathrm{d}\eta_2} = N_0^\lambda(P_\perp, k_\perp)\left[1 + 2\sum_{k=1}^{\infty} v_{k,\lambda}(P_\perp, k_\perp)\cos(k\phi)\right]$$

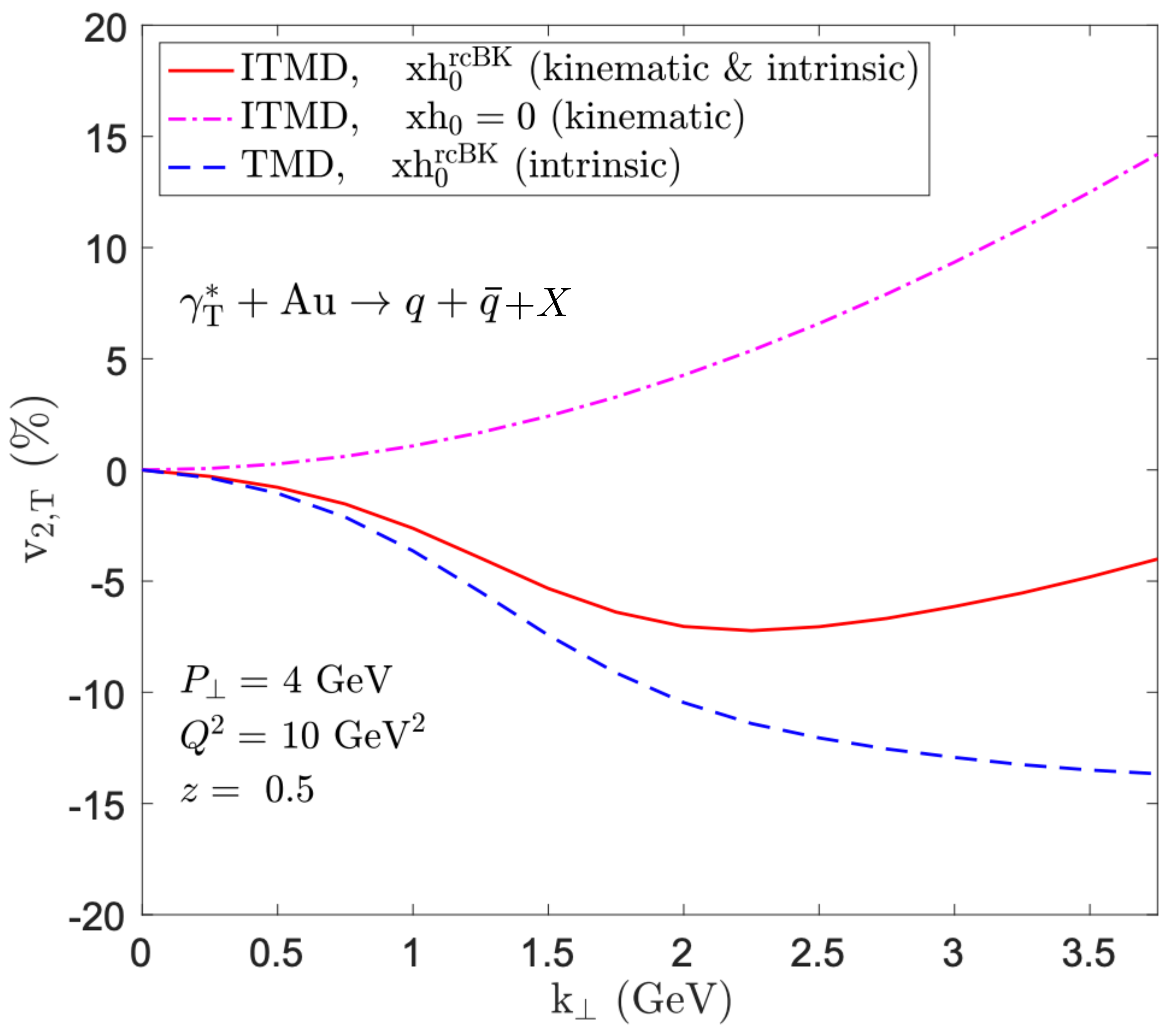
$$\phi \equiv \phi_{\boldsymbol{k}_\perp} - \phi_{\boldsymbol{P}_\perp}$$

R. Boussarie, H. Mäntysaari, FS, B. Schenke (2021)

Dijet production beyond TMDs

Momentum imbalance **elliptic** anisotropies: TMD vs ITMD

R. Boussarie, H. Mäntysaari, FS, B. Schenke (2021)



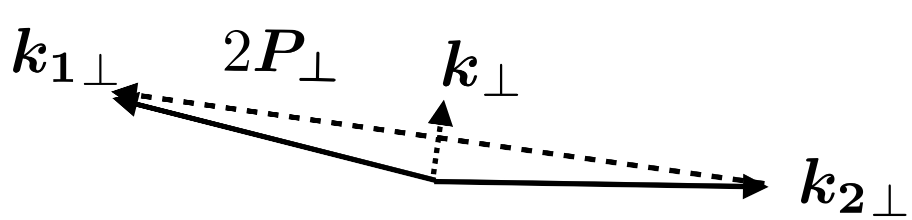
Dijet production in ITMD

Kinematic power corrections induce correlations!

$$d\sigma^{\gamma^* A \to q\bar{q}+X} \sim \boxed{\mathcal{H}^G_{\mathrm{ITMD}}(\boldsymbol{P}_\perp, \boldsymbol{k}_\perp)}\alpha_s x G^0_{\mathrm{WW}}(k_\perp, x)$$

$$+\boxed{\mathcal{H}^h_{\mathrm{ITMD}}(\boldsymbol{P}_\perp, \boldsymbol{k}_\perp)}\boxed{\alpha_s x h_{\mathrm{WW}}(k_\perp, x)\cos(2\phi)}$$

Elliptic anisotropy from linearly pol WW



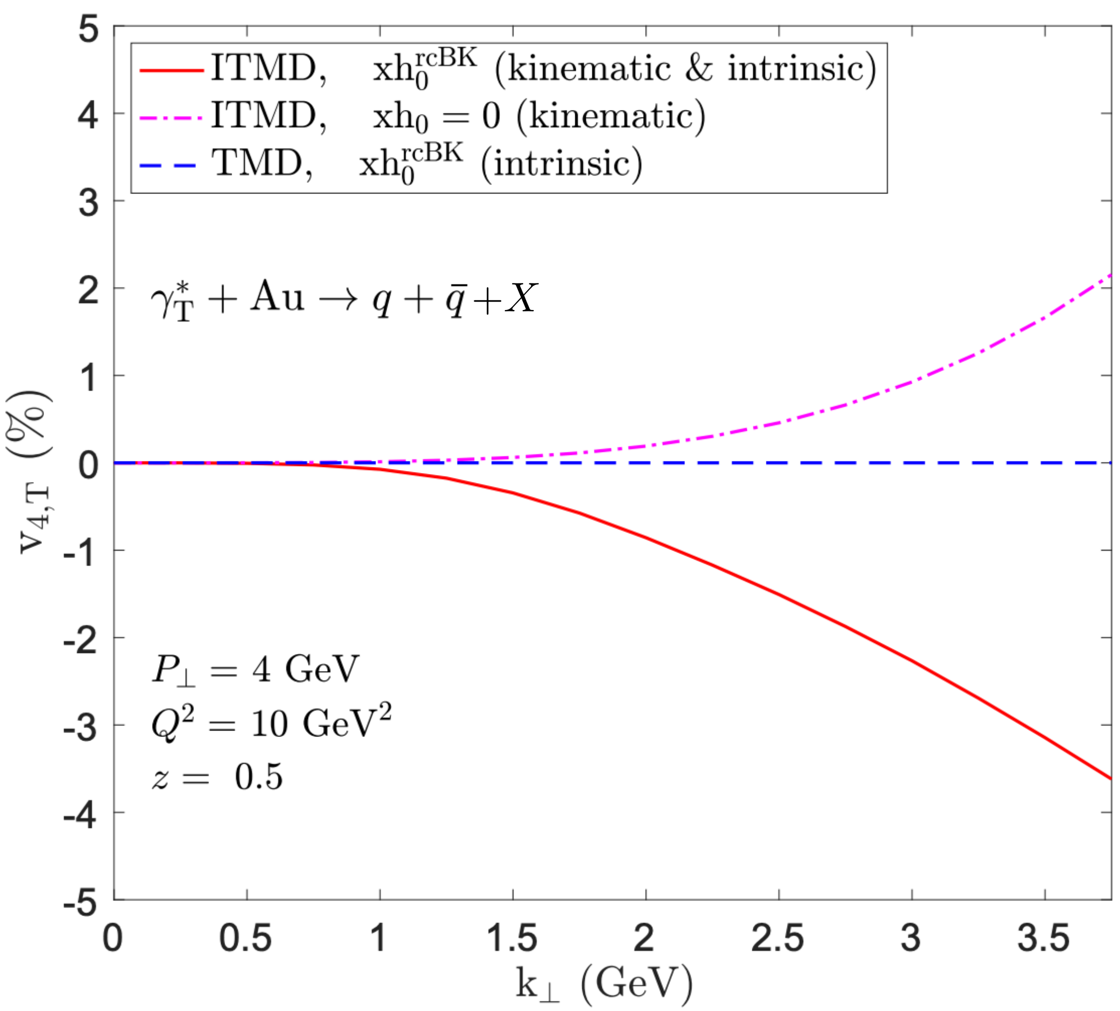
$$\frac{\mathrm{d}N^{\gamma^*_\lambda + A \to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{P}_\perp \mathrm{d}^2\boldsymbol{k}_\perp \mathrm{d}\eta_1 \mathrm{d}\eta_2} = N_0^\lambda(P_\perp, k_\perp)\left[1 + 2\sum_{k=1}^{\infty} v_{k,\lambda}(P_\perp, k_\perp)\cos(k\phi)\right]$$

$$\phi \equiv \phi_{\boldsymbol{k}_\perp} - \phi_{\boldsymbol{P}_\perp}$$

Dijet production beyond TMDs

Momentum imbalance **quadrangular** anisotropies:
TMD vs ITMD

R. Boussarie, H. Mäntysaari, FS, B. Schenke (2021)

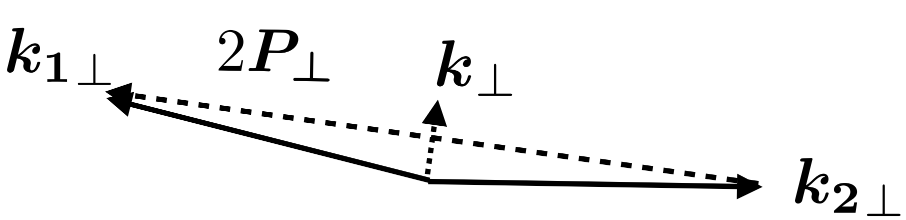


Dijet production in ITMD

Kinematic power corrections induce correlations!

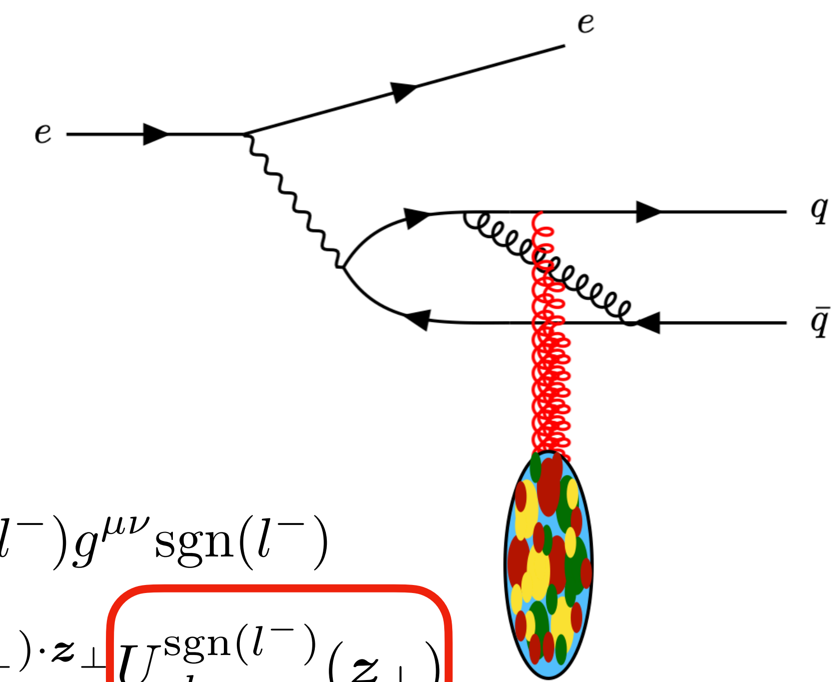
$$d\sigma^{\gamma^* A \to q\bar{q}+X} \sim \boxed{\mathcal{H}^G_{\rm ITMD}(\boldsymbol{P}_\perp, \boldsymbol{k}_\perp)} \alpha_s x G^0_{\rm WW}(k_\perp, x)$$
$$+ \boxed{\mathcal{H}^h_{\rm ITMD}(\boldsymbol{P}_\perp, \boldsymbol{k}_\perp)} \alpha_s x h_{\rm WW}(k_\perp, x) \cos(2\phi)$$

No quadrangular anisotropy in TMD



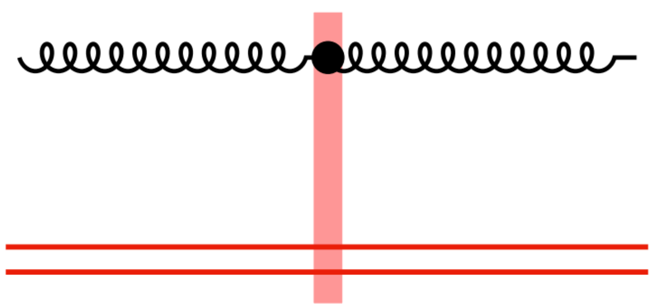
$$\frac{\mathrm{d}N^{\gamma^*_\lambda + A \to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{P}_\perp \mathrm{d}^2\boldsymbol{k}_\perp \mathrm{d}\eta_1 \mathrm{d}\eta_2} = N_0^\lambda(P_\perp, k_\perp)\left[1 + 2\sum_{k=1}^{\infty} v_{k,\lambda}(P_\perp, k_\perp)\cos(k\phi)\right]$$
$$\phi \equiv \phi_{\boldsymbol{k}_\perp} - \phi_{\boldsymbol{P}_\perp}$$

One-loop corrections



General remarks

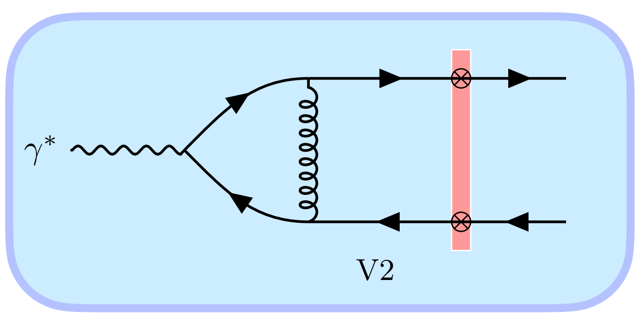
- Covariant PT (in light-cone gauge) with effective Feynman rules for eikonal multiple scattering



$$\mathcal{T}_g^{\mu\nu,ab}(l,l') = -(2\pi)\delta(l^- - l'^-)(2l^-)g^{\mu\nu}\mathrm{sgn}(l^-) \times \int \mathrm{d}^2\boldsymbol{z}_\perp e^{-i(\boldsymbol{l}'_\perp - \boldsymbol{l}_\perp)\cdot\boldsymbol{z}_\perp} \boxed{U_{ab}^{\mathrm{sgn}(l^-)}(\boldsymbol{z}_\perp)}$$

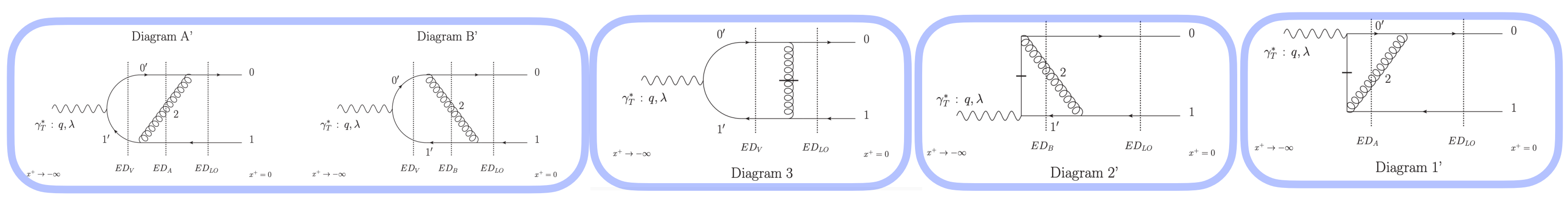
Adjoint Wilson line

- Separate regular and instantaneous pieces in the Dirac-Lorentz structures



Inspired from spinor-helicity techniques and LCPT

$$\boxed{\mathcal{N}_{\mathrm{V2}}} = \boxed{\mathcal{N}_{\mathrm{V2,reg}}} + \boxed{\mathcal{N}_{\mathrm{V2},g\mathrm{inst}}} + \boxed{\mathcal{N}_{\mathrm{V2},q\mathrm{inst}}} + \boxed{\mathcal{N}_{\mathrm{V2},\bar{q}\mathrm{inst}}}$$



Diagrams from Guillaume Beuf

- Regularization schemes

Dimensional regularization for transverse integrals
+ sharp cut-off Λ_0^- in longitudinal momentum

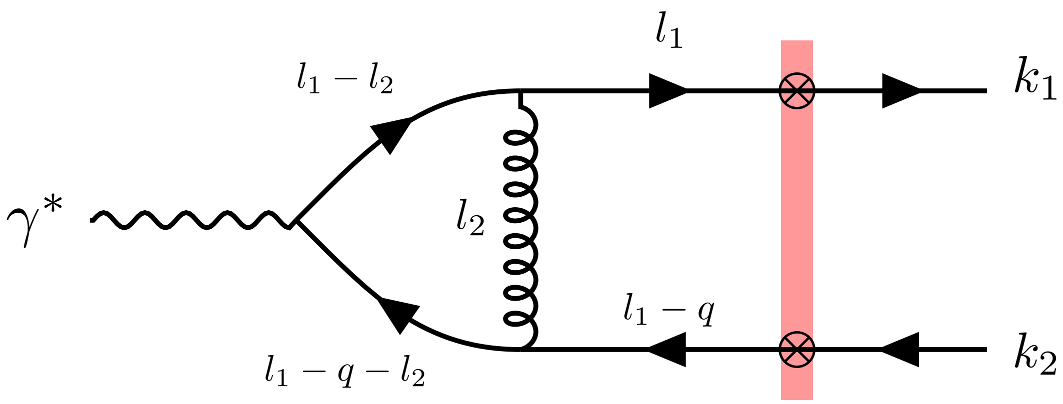
$$\int \mathrm{d}^{2-\varepsilon}\boldsymbol{l}_\perp \int_{\Lambda_0^-} \frac{\mathrm{d}l^-}{l^-}$$

+Small R cone algorithm

Similar regularization procedure:
Boussarie, Grabovsky, Szymanowski, Wallon (JHEP 2016)
Roy, Venugopalan (PRD 2019)

One-loop corrections

Connection to LCPT: an example



V2

$$\mathcal{C}_{\mathrm{V2},ij}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) = C_F \left[V(\boldsymbol{x}_\perp) V^\dagger(\boldsymbol{y}_\perp) - \mathbb{1}\right]_{ij}$$

$$\mathcal{M}^{\lambda}_{\mathrm{V2},ij,\sigma_1\sigma_2} = \frac{e e_f q^-}{\pi} \int_{\boldsymbol{x}_\perp, \boldsymbol{y}_\perp} e^{-i(\boldsymbol{k_{1\perp}} \cdot \boldsymbol{x}_\perp + \boldsymbol{k_{2\perp}} \cdot \boldsymbol{y}_\perp)} \mathcal{C}_{\mathrm{V2},ij}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) \mathcal{N}^{\lambda}_{\mathrm{V2},\sigma_1\sigma_2}(\boldsymbol{r}_{xy})$$

Perturbative factor

$$\mathcal{N}^{\lambda}_{\mathrm{V2},\sigma_1\sigma_2}(\boldsymbol{r}_{xy}) = g^2 \int_{l_1} \int_{l_2} \frac{(2q^-)\delta(k^- - l_1^-) N^{\lambda}_{\mathrm{V2},\sigma_1\sigma_2}(l_1, l_2) e^{i \boldsymbol{l_{1\perp}} \cdot \boldsymbol{r}_{xy}}}{[l_1^2 + i\epsilon]\,[(l_1 - l_2)^2 + i\epsilon]\,[(l_1 - l_2 - q)^2 + i\epsilon]\,[(l_1 - q)^2 + i\epsilon]\,[l_2^2 + i\epsilon]}$$

Dirac-Lorentz structure

$$N^{\lambda}_{\mathrm{V2},\sigma_1\sigma_2}(l_1, l_2) = \frac{1}{(2q^-)^2} \left[\bar{u}(k_1, \sigma_1) \gamma^- \not{l}_1 \gamma^\mu (\not{l}_1 - \not{l}_2) \not{\epsilon}(q, \lambda) (\not{l}_1 - \not{l}_2 - \not{q}) \gamma^\nu (\not{l}_1 - \not{q}) \gamma^- v(k_2, \sigma_2)\right] \Pi_{\mu\nu}(l_2)$$

One-loop corrections

Connection to LCPT: an example

Perturbative factor

$$\mathcal{N}^{\lambda}_{\mathrm{V2},\sigma_1\sigma_2}(\boldsymbol{r}_{xy}) = g^2 \int_{l_1}\int_{l_2} \frac{(2q^-)\delta(k^- - l_1^-)\, N^{\lambda}_{\mathrm{V2},\sigma_1\sigma_2}(l_1,l_2)\, e^{i\boldsymbol{l}_{1\perp}\cdot \boldsymbol{r}_{xy}}}{[l_1^2+i\epsilon]\,[(l_1-l_2)^2+i\epsilon]\,[(l_1-l_2-q)^2+i\epsilon]\,[(l_1-q)^2+i\epsilon]\,[l_2^2+i\epsilon]}$$

Dirac-Lorentz structure (DLS)

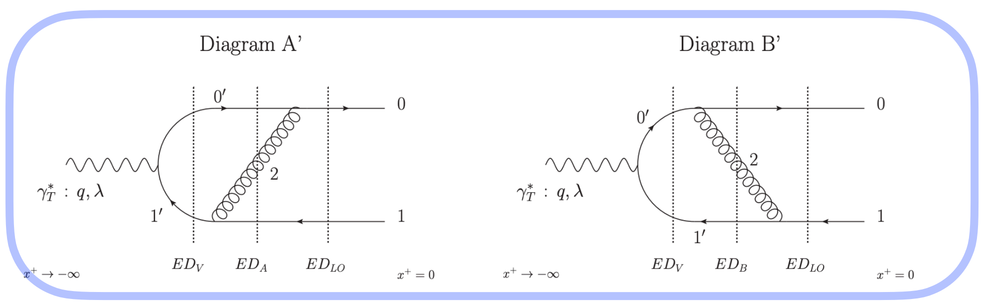
$$N^{\lambda}_{\mathrm{V2},\sigma_1\sigma_2}(l_1,l_2) = \frac{1}{(2q^-)^2}\left[\bar{u}(k_1,\sigma_1)\gamma^- \not{l}_1 \gamma^\mu (\not{l}_1 - \not{l}_2)\not{\epsilon}(q,\lambda)(\not{l}_1-\not{l}_2-\not{q})\gamma^\nu(\not{l}_1-\not{q})\gamma^- v(k_2,\sigma_2)\right]\Pi_{\mu\nu}(l_2)$$

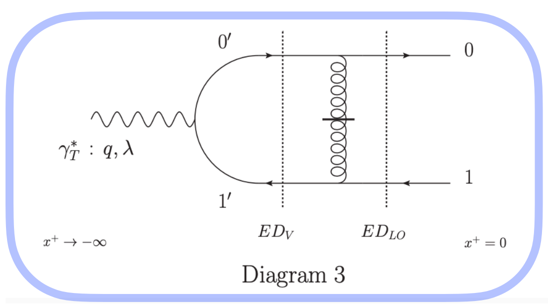
Useful decomposition (dissecting Dirac-Lorentz structure)

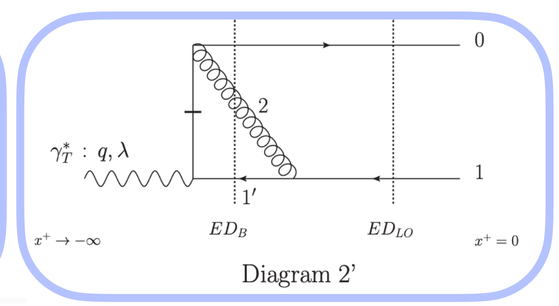
$$N_{\mathrm{V2}} = N_{\mathrm{V2,reg}} + l_2^2 N_{\mathrm{V2},g\mathrm{inst}} + (l_1-l_2)^2 N_{\mathrm{V2},q\mathrm{inst}} + (l_1-l_2-q)^2 N_{\mathrm{V2},\bar{q}\mathrm{inst}}$$

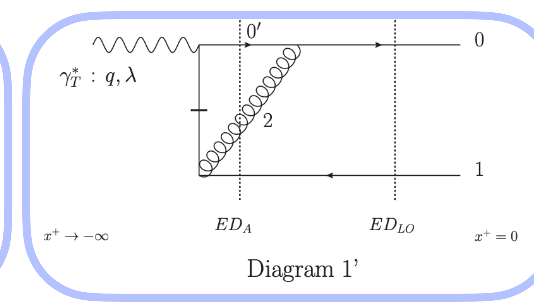
$$\mathcal{N}_{\mathrm{V2}} = \mathcal{N}_{\mathrm{V2,reg}} + \mathcal{N}_{\mathrm{V2},g\mathrm{inst}} + \mathcal{N}_{\mathrm{V2},q\mathrm{inst}} + \mathcal{N}_{\mathrm{V2},\bar{q}\mathrm{inst}}$$

After contour integration l_1^+ and l_2^+ one obtains light-cone energy denominators in LCPT







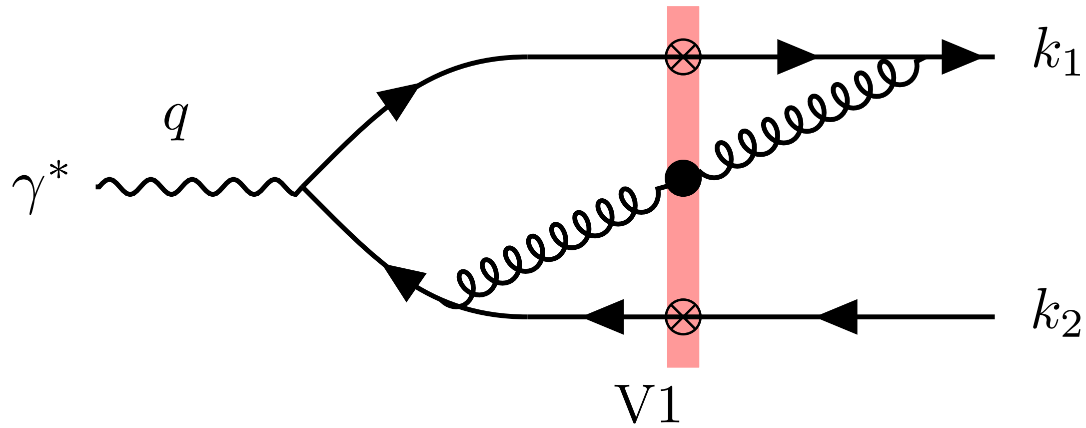


Diagrams from Beuf (2016)

These perturbative factors have been computed in Beuf (2016,2017), Hänninen, Lappi, and Paatelainen (2017)

One-loop corrections

Vertex with gluon crossing SW



$$\mathcal{C}_{\mathrm{V1},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp\boldsymbol{z}_\perp) = \left[t^a V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{z}_\perp)t_a V(\boldsymbol{z}_\perp)V^\dagger(\boldsymbol{y}_\perp) - C_F\right]_{ij}$$

$$\mathcal{M}^{\lambda}_{\mathrm{V1},ij,\sigma_1\sigma_2} = \frac{ee_f q^-}{\pi}\int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp} e^{-i(\boldsymbol{k_{1\perp}}\cdot\boldsymbol{x}_\perp+\boldsymbol{k_{2\perp}}\cdot\boldsymbol{y}_\perp)}\boxed{\mathcal{C}_{\mathrm{V1},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp)}\boxed{\mathcal{N}^{\lambda}_{\mathrm{V1},\sigma_1\sigma_2}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zy})}$$

Perturbative factor:

$$\boxed{\mathcal{N}^{\lambda=0}_{\mathrm{V1},\sigma_1\sigma_2}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zy})} = \frac{\alpha_s}{\pi^2}\int_{z_0}^{z_1}\frac{\mathrm{d}z_g}{z_g}e^{-i\frac{z_g}{z_1}\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{zx}}2(z_1z_2)^{3/2}QK_0\left(QX_V\right)\delta_{\sigma_1,-\sigma_2}$$
$$\times\left\{\left(1-\frac{z_g}{z_1}\right)\left(1+\frac{z_g}{z_2}\right)\left[1-\frac{z_g}{2z_1}-\frac{z_g}{2(z_2+z_g)}\right]\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{r}_{zy}}{\boldsymbol{r}^2_{zx}\boldsymbol{r}^2_{zy}}+\ldots\right\}$$

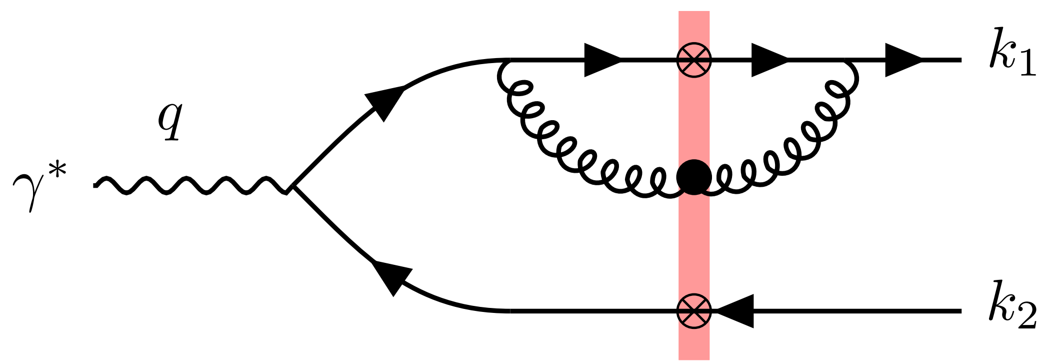
This contribution is UV finite!

$$X_{\mathrm{V}}^2 = z_2(z_1-z_g)\boldsymbol{r}^2_{xy} + z_g(z_1-z_g)\boldsymbol{r}^2_{zx} + z_g z_2\boldsymbol{r}^2_{zy}$$

Caucal, FS, Venugopalan (JHEP 2021)

One-loop corrections

Self energy with gluon crossing SW



$$\mathcal{C}_{\mathrm{SE1},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp) = \left[t^a V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{z}_\perp)t_a V(\boldsymbol{z}_\perp)V^\dagger(\boldsymbol{y}_\perp) - C_F\right]_{ij}$$

$$\mathcal{C}_{\mathrm{UV},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp) = C_F\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) - \mathbb{1}\right]_{ij}$$

UV finite piece

$$\mathcal{M}^\lambda_{\mathrm{SE1,UVfinite},ij,\sigma_1\sigma_2} = \frac{ee_f q^-}{\pi}\int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp} e^{-i(\boldsymbol{k_{1\perp}}\cdot\boldsymbol{x}_\perp+\boldsymbol{k_{2\perp}}\cdot\boldsymbol{y}_\perp)}\left[\mathcal{C}_{\mathrm{SE1},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp)\mathcal{N}^\lambda_{\mathrm{SE1},\sigma_1\sigma_2}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zx}) - \mathcal{C}_{\mathrm{UV},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp)\mathcal{N}^\lambda_{\mathrm{SE1,UV},\sigma_1\sigma_2}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zx})\right]$$

$$\mathcal{N}^{\lambda=0,\sigma\sigma'}_{\mathrm{SE1}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zx}) = -\frac{\alpha_s}{\pi^2}\int_{z_0}^{z_1}\frac{\mathrm{d}z_g}{z_g}\frac{1}{2}\left[1+\left(1-\frac{z_g}{z_1}\right)^2\right]\frac{e^{-i\frac{z_g}{z_1}\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{zx}}}{\boldsymbol{r}^2_{zx}}2(z_1z_2)^{3/2}QK_0\left(QX_{\mathrm{V}}\right)\delta_{\sigma_1,-\sigma_2}$$

$$\mathcal{N}^{\lambda=0}_{\mathrm{SE1,UV},\sigma_1\sigma_2}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zx}) = -\frac{\alpha_s}{\pi^2}\int_{z_0}^{z_q}\frac{\mathrm{d}z_g}{z_g}\frac{e^{-\frac{\boldsymbol{r}^2_{zx}}{2\xi}}}{\boldsymbol{r}^2_{zx}}2(z_1z_2)^{3/2}QK_0\left(Q\sqrt{z_1z_2}r_{xy}\right)\delta_{\sigma_1,-\sigma_2}$$

Hänninen, Lappi, Paatelainen (Annals Phys. 2017)

$$X_{\mathrm{V}}^2 = z_2(z_1-z_g)\boldsymbol{r}^2_{xy} + z_g(z_1-z_g)\boldsymbol{r}^2_{zx} + z_2z_g\boldsymbol{r}^2_{zy}$$

UV divergent piece

$$\mathcal{M}^\lambda_{\mathrm{SE1,UV},ij,\sigma_1\sigma_2} = \frac{ee_f q^-}{\pi}\int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp} e^{-i(\boldsymbol{k_{1\perp}}\cdot\boldsymbol{x}_\perp+\boldsymbol{k_{2\perp}}\cdot\boldsymbol{y}_\perp)}\,\mathcal{C}_{\mathrm{UV},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp)\mathcal{N}^\lambda_{\mathrm{SE1,UV},\sigma_1\sigma_2}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zx})$$

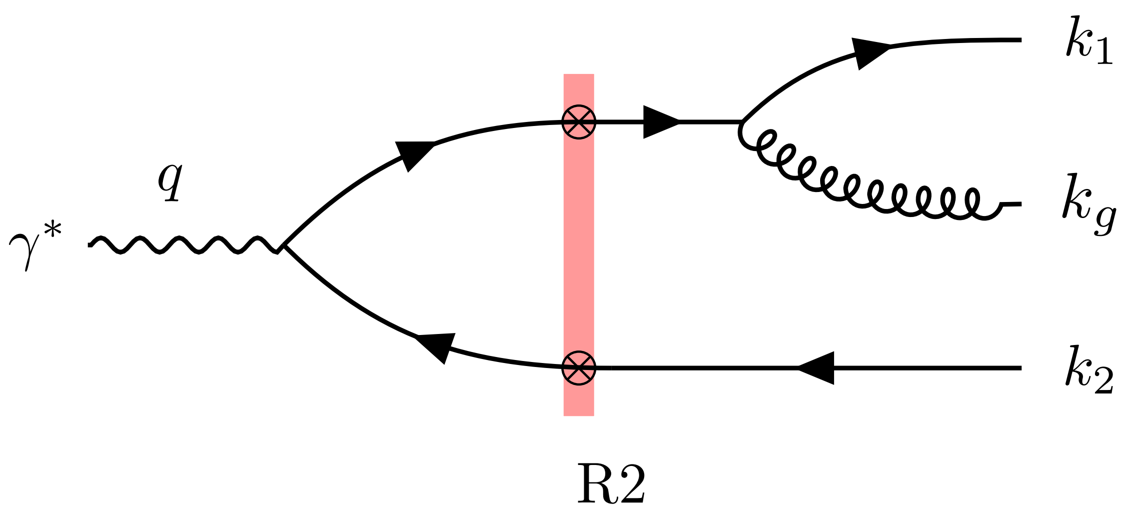
$$\mathcal{N}^\lambda_{\mathrm{SE1,UV},\sigma_1\sigma_2}(\boldsymbol{r}_{xy}) = \frac{\alpha_s}{2\pi}\left\{\left(2\ln\left(\frac{z_1}{z_0}\right)-\frac{3}{2}\right)\left(\frac{2}{\varepsilon}+\ln(2\pi\mu^2\xi)\right)-\frac{1}{2}+\mathcal{O}(\varepsilon)\right\}\mathcal{N}^\lambda_{\mathrm{LO},\varepsilon,\sigma_1\sigma_2}(\boldsymbol{r}_{xy})$$

UV pole

Caucal, FS, Venugopalan (JHEP 2021)

One-loop corrections

Real gluon emission after SW



$$\mathcal{C}_{\mathrm{R2},ija}(\boldsymbol{w}_\perp, \boldsymbol{y}_\perp) = \left[t_a V(\boldsymbol{w}_\perp) V^\dagger(\boldsymbol{y}_\perp) - t_a\right]_{ij}$$

$$\mathcal{M}^{\lambda\bar\lambda}_{\mathrm{R2},ija,\sigma_1\sigma_2} = \frac{e e_f q^-}{\pi} \int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp} e^{-i(\boldsymbol{k_{1\perp}}\cdot\boldsymbol{x}_\perp + \boldsymbol{k_{2\perp}}\cdot\boldsymbol{y}_\perp + \boldsymbol{k_{g\perp}}\cdot\boldsymbol{z}_\perp)} \mathcal{C}_{\mathrm{R2},ija}(\boldsymbol{w}_\perp, \boldsymbol{y}_\perp) \mathcal{N}^{\lambda\bar\lambda}_{\mathrm{R2},\sigma_1\sigma_2}(\boldsymbol{r}_{wy}, \boldsymbol{r}_{zx})$$

Perturbative factor:

$$\mathcal{N}^{\lambda=0,\bar\lambda}_{\mathrm{R2},\sigma_1\sigma_2}(\boldsymbol{r}_{wy}, \boldsymbol{r}_{zx}) = 2(z_1 z_2)^{3/2} Q K_0(Q X_{wy}) \delta_{\sigma_1,-\sigma_2} \frac{ig}{\pi} \frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{\epsilon}^{\bar\lambda *}_\perp}{\boldsymbol{r}^2_{zx}} \frac{[z_1 \delta^{\bar\lambda}_{\sigma_1} + (z_1+z_g)\delta^{\bar\lambda}_{\sigma_2}]}{z_1}$$

$$\mathcal{N}^{\lambda=\pm1,\bar\lambda}_{\mathrm{R2},\sigma_1\sigma_2}(\boldsymbol{r}_{wy}, \boldsymbol{r}_{zx}) = -2(z_1 z_2)^{3/2} \left[z_2 \delta^{\lambda}_{\sigma_1} - z_1 \delta^{\lambda}_{\sigma_2}\right] \frac{iQ\boldsymbol{r}_{wx}\cdot\boldsymbol{\epsilon}^{\lambda}_\perp}{X_{wx}} K_1(Q X_{wx}) \delta_{\sigma_1,-\sigma_2} \frac{ig}{\pi} \frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{\epsilon}^{\bar\lambda *}_\perp}{\boldsymbol{r}^2_{zx}} \frac{[z_1 \delta^{\bar\lambda}_{\sigma_1} + (z_1+z_g)\delta^{\bar\lambda}_{\sigma_2}]}{z_1}$$

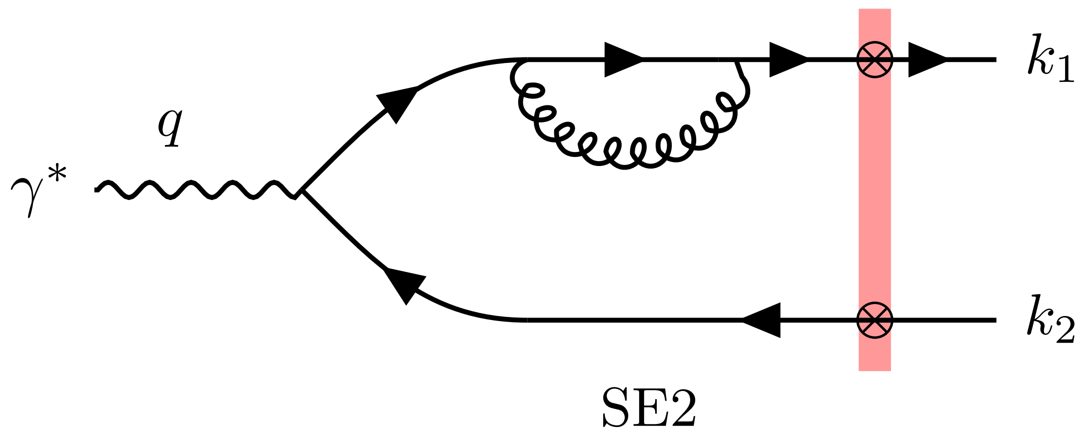
$$\boldsymbol{w}_\perp = \frac{z_1 \boldsymbol{x}_\perp + z_g \boldsymbol{z}_\perp}{z_1 + z_g}$$

$$X^2_{wy} = z_2(z_1+z_g) r^2_{wy}$$

Ayala, Hentschinski, Jalilian-Marian, Tejeda-Yeomans (2017)
with spinor-helicity techniques

One-loop corrections

Self energy with gluon before SW



$$\mathcal{C}_{\mathrm{SE2},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp) = C_F\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) - \mathbb{1}\right]_{ij}$$

$$\mathcal{M}^{\lambda}_{\mathrm{SE2},ij,\sigma_1\sigma_2} = \frac{ee_f q^-}{\pi}\int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp} e^{-i(\boldsymbol{k}_{1\perp}\cdot\boldsymbol{x}_\perp+\boldsymbol{k}_{2\perp}\cdot\boldsymbol{y}_\perp)}\,\mathcal{C}_{\mathrm{SE2},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp)\,\mathcal{N}^{\lambda}_{\mathrm{SE2},\sigma_1\sigma_2}(\boldsymbol{r}_{xy})$$

Perturbative factor:

$$\mathcal{N}^{\lambda=0}_{\mathrm{SE2},\sigma_1\sigma_2}(\boldsymbol{r}_{xy}) = \frac{\alpha_s}{2\pi}\left\{\left(-2\ln\left(\frac{z_1}{z_0}\right)+\frac{3}{2}\right)\left(\frac{2}{\varepsilon}+\frac{1}{2}\ln\left(\frac{z_1 z_2 Q^2 \boldsymbol{r}_{xy}^2}{4}\right)+\gamma_E-\ln\left(\frac{z_1 Q^2}{\tilde{\mu}^2}\right)\right)\right.$$
$$\left.+\left(\frac{1}{2}+3-\frac{\pi^2}{3}-\boxed{\ln^2\left(\frac{z_1}{z_0}\right)}\right)+\mathcal{O}(\varepsilon)\right\}\mathcal{N}^{\lambda=0}_{\mathrm{LO},\varepsilon,\sigma_1\sigma_2}$$

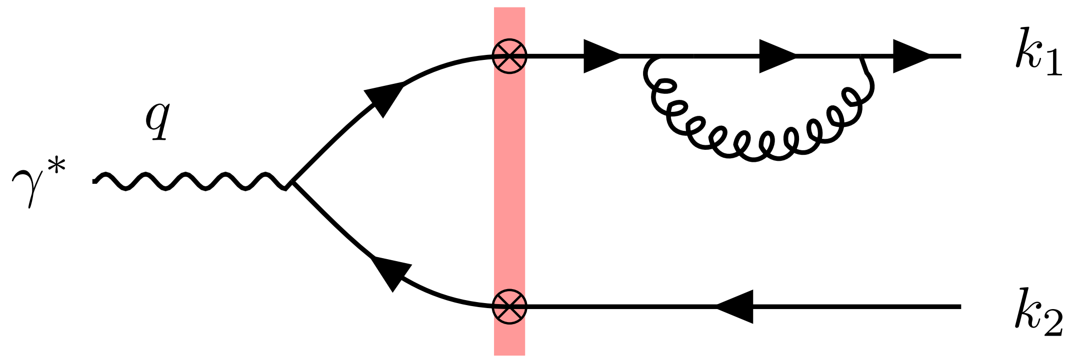
UV pole

double log

Beuf (2016,2017), Hänninen, T. Lappi, and R. Paatelainen (2017)
Loop corrections light-cone photon wave-function

One-loop corrections

Self energy with gluon after SW



SE3

$$\boxed{\mathcal{C}_{\mathrm{SE3},ij}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) = C_F \left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) - \mathbb{1}\right]_{ij}}$$

$$\mathcal{M}^{\lambda}_{\mathrm{SE2},ij,\sigma_1\sigma_2} = \frac{e e_f q^-}{\pi} \int_{\boldsymbol{x}_\perp, \boldsymbol{y}_\perp} e^{-i(\boldsymbol{k_{1\perp}}\cdot\boldsymbol{x}_\perp + \boldsymbol{k_{2\perp}}\cdot\boldsymbol{y}_\perp)} \boxed{\mathcal{C}_{\mathrm{SE3},ij}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp)} \boxed{\mathcal{N}^{\lambda}_{\mathrm{SE3},\sigma_1\sigma_2}(\boldsymbol{r}_{xy})}$$

Perturbative factor:

UV pole

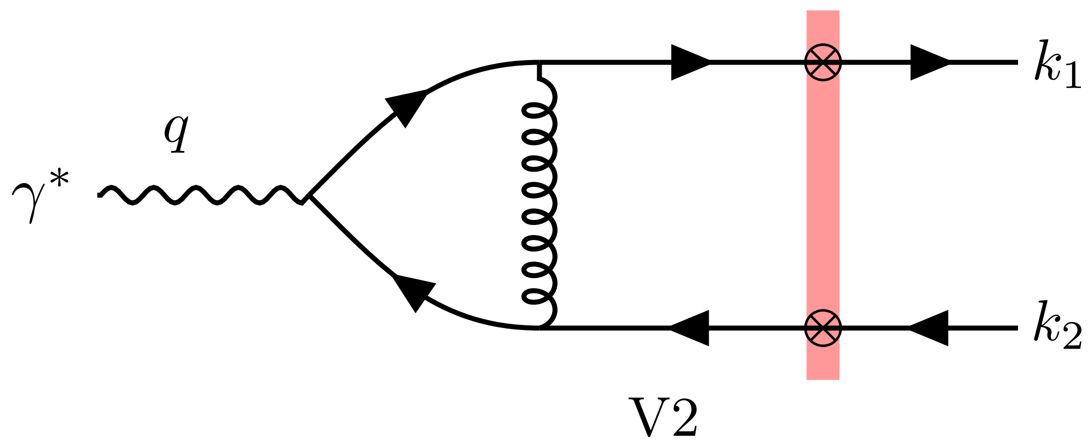
$$\boxed{\mathcal{N}^{\lambda}_{\mathrm{SE3},\sigma_1\sigma_2}(\boldsymbol{r}_{xy})} = -\frac{\alpha_s}{2\pi} \mathcal{N}^{\lambda}_{\mathrm{LO},\varepsilon,\sigma_1\sigma_2}(\boldsymbol{r}_{xy}) \left(\frac{2}{\varepsilon_{\mathrm{UV}}} - \frac{2}{\varepsilon_{\mathrm{IR}}}\right) \left\{2\ln\left(\frac{z_q}{z_0}\right) - \frac{3}{2}\right\}$$

IR pole

Self-energy contribution vanishes exactly in dim reg (IR and UV pole cancel each other out)
turns UV divergences into IR (massless quarks)

One-loop corrections

Vertex with gluon before SW



$$\mathcal{C}_{\mathrm{V2},ij}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) = C_F \left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) - \mathbb{1}\right]_{ij}$$

$$\mathcal{M}^{\lambda}_{\mathrm{V2},ij,\sigma_1\sigma_2} = \frac{e e_f q^-}{\pi} \int_{\boldsymbol{x}_\perp, \boldsymbol{y}_\perp} e^{-i(\boldsymbol{k_{1\perp}} \cdot \boldsymbol{x}_\perp + \boldsymbol{k_{2\perp}} \cdot \boldsymbol{y}_\perp)} \mathcal{C}_{\mathrm{V2},ij}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) \mathcal{N}^{\lambda}_{\mathrm{V2},\sigma_1\sigma_2}(\boldsymbol{r}_{xy})$$

Perturbative factor*:

UV pole

double logs

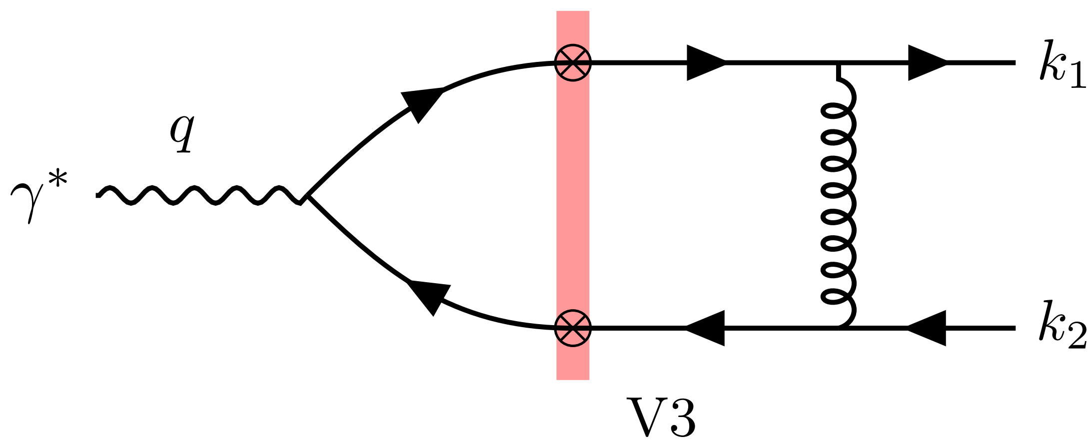
$$\mathcal{N}^{\lambda=0}_{\mathrm{V2},\sigma_1\sigma_2}(\boldsymbol{r}_{xy}) = \frac{\alpha_s}{2\pi} \left\{ \left(\frac{2}{\varepsilon} + \ln\left(\frac{\tilde{\mu}^2}{z_1 z_2 Q^2}\right)\right) \left[\ln\left(\frac{z_1}{z_0}\right) + \ln\left(\frac{z_2}{z_0}\right) - \frac{3}{2}\right] + \boxed{\ln^2\left(\frac{z_1}{z_0}\right) + \ln^2\left(\frac{z_2}{z_0}\right)} + \frac{1}{2}\ln^2\left(\frac{z_1}{z_2}\right) + \frac{\pi^2}{2} \right.$$
$$\left. + \left(2\ln\left(\frac{z_2}{z_0}\right) - \frac{3}{2}\right)\ln(z_1) + \left(2\ln\left(\frac{z_1}{z_0}\right) - \frac{3}{2}\right)\ln(z_2) - \frac{7}{2} - \frac{1}{2} + \mathcal{O}(\varepsilon) \right\} \mathcal{N}^{\lambda=0}_{\mathrm{LO},\varepsilon,\sigma_1\sigma_2}(\boldsymbol{r}_{xy})$$

*includes both regular and gluon instantaneous contribution

Beuf (2016,2017), Hänninen, T. Lappi, and R. Paatelainen (2017)
Loop corrections light-cone photon wave-function

One-loop corrections

Vertex with gluon after SW



$$\mathcal{C}_{\mathrm{V3},ij}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) = C_F \left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) - \mathbb{1}\right]_{ij}$$

$$\mathcal{M}^{\lambda}_{\mathrm{V3},ij,\sigma_1\sigma_2} = \frac{ee_f q^-}{\pi} \int_{\boldsymbol{x}_\perp, \boldsymbol{y}_\perp} e^{-i(\boldsymbol{k_{1\perp}}\cdot\boldsymbol{x}_\perp + \boldsymbol{k_{2\perp}}\cdot\boldsymbol{y}_\perp)} \boxed{\mathcal{C}_{\mathrm{V3},ij}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp)} \boxed{\mathcal{N}^{\lambda}_{\mathrm{V3},\sigma_1\sigma_2}(\boldsymbol{r}_{xy})}$$

Perturbative factor*:

$$\begin{aligned}
\mathcal{N}^{\lambda=0}_{\mathrm{V3},\sigma_1\sigma_2}(\boldsymbol{r}_{xy}) = -\frac{\alpha_s}{\pi} \int_0^{z_q} \frac{\mathrm{d}z_g}{z_g} 2(z_1 z_2)^{1/2}(z_1 - z_g)(z_2 + z_g) Q K_0\left(Q\sqrt{(z_1 - z_g)(z_2 + z_g)} r_{xy}\right) \delta_{\sigma_1,-\sigma_2} \\
\times \left\{ \left[(1 + z_g)\left(1 - \frac{z_g}{z_1}\right)\right] e^{i(\boldsymbol{P}_\perp + z_g(\boldsymbol{k_{1\perp}} + \boldsymbol{k_{2\perp}}))\cdot \boldsymbol{r}_{xy}} K_0(-i\Delta_{\mathrm{V3}} r_{xy}) \right. \\
- \left[1 - \frac{z_g}{2z_1} + \frac{z_g}{2z_2} - \frac{z_g^2}{2z_1 z_2}\right] e^{i\frac{z_g}{z_1}\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xy}} \boxed{\mathcal{J}_\odot\left(\boldsymbol{r}_{xy}, \left(1 - \frac{z_g}{z_1}\right)\boldsymbol{P}_\perp, \Delta_{\mathrm{V3}}\right)} \\
\left. + \sigma \left[\frac{z_g}{z_1} - \frac{z_g}{z_2} + \frac{z_g^2}{z_1 z_2}\right] e^{i\frac{z_g}{z_1}\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xy}} \mathcal{J}_\otimes\left(\boldsymbol{r}_{xy}, \left(1 - \frac{z_g}{z_1}\right)\boldsymbol{P}_\perp, \Delta_{\mathrm{V3}}\right) \right\} + (q \leftrightarrow \bar{q})
\end{aligned}$$

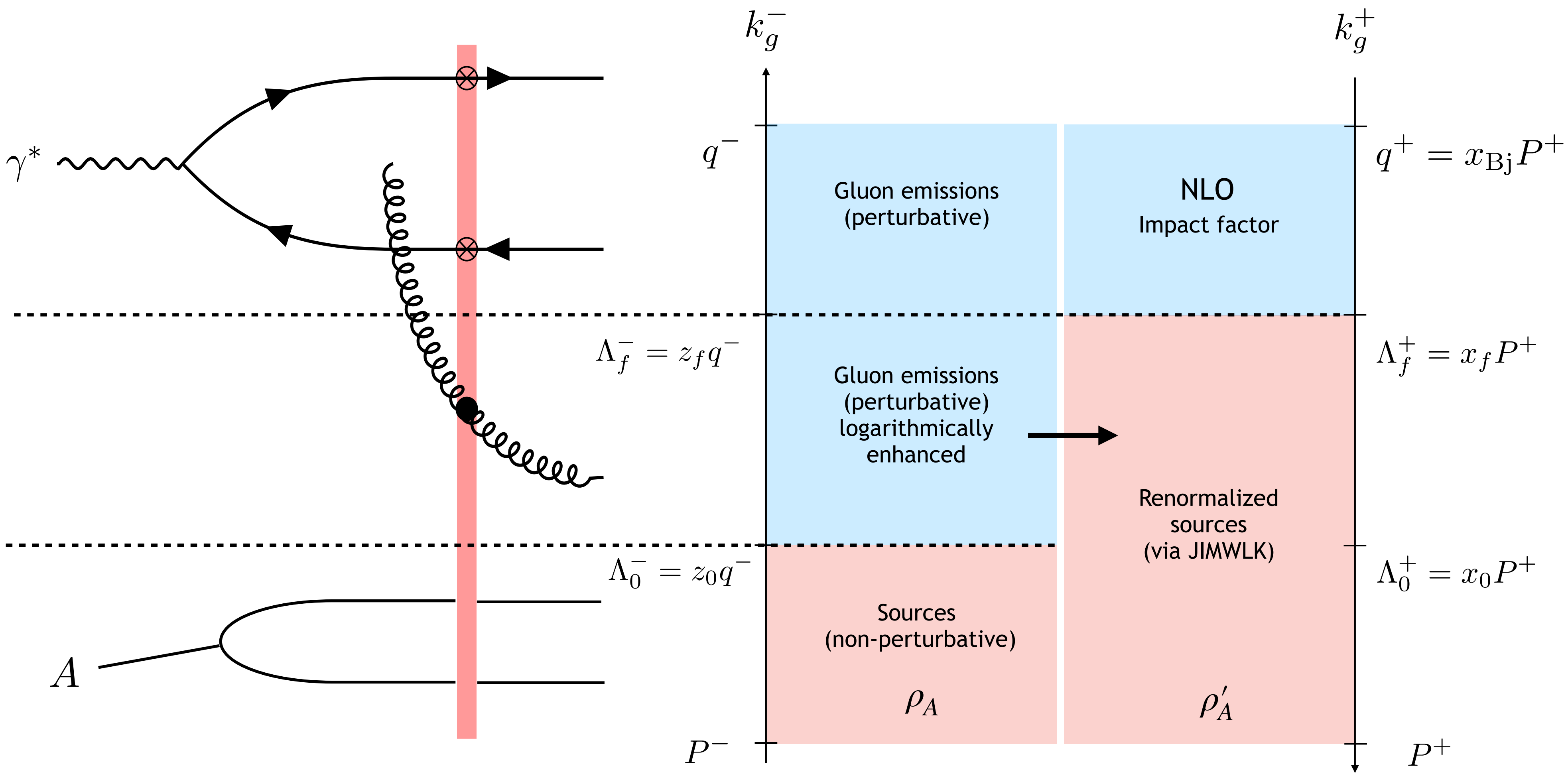
Contains a double log $\ln^2(z_0)$

*includes both regular and gluon instantaneous contribution

One-loop corrections

Rapidity (slow gluon) divergences and JIMWLK factorization

$$\mathrm{d}\sigma_{\mathrm{NLO,slow}} = \ln\left(\frac{z_f}{z_0}\right) \mathcal{H}_{\mathrm{JIMWLK}}\ \mathrm{d}\sigma_{\mathrm{LO}}$$



Slow gluon radiation $\mathrm{d}\sigma_{\mathrm{NLO,slow}} \propto \alpha_s \ln\left(\frac{z_f}{z_0}\right)$ can be large in resummed by redefining distribution of sources:

$$W_{x_0}[\rho_A] \to W_{x_f}[\rho_A']$$