

EIC opportunities at Snowmass

January 26<sup>th</sup>, 2021

# 3D proton tomography at the EIC TMD gluon densities

**Francesco Giovanni Celiberto**

ECT\*/FBK Trento & INFN-TIFPA

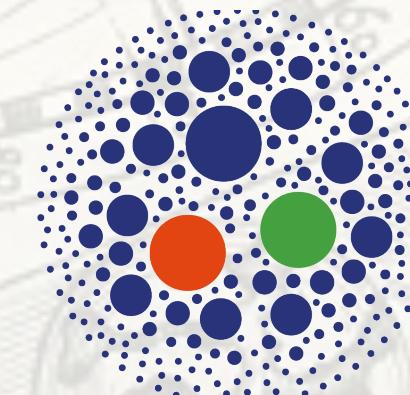
**ECT\***

EUROPEAN CENTRE FOR THEORETICAL STUDIES  
IN NUCLEAR PHYSICS AND RELATED AREAS

**FBK**  
FONDAZIONE  
BRUNO KESSLER  
FUTURE BUILT  
ON KNOWLEDGE



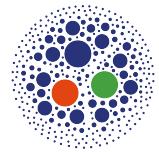
Trento Institute for  
Fundamental Physics  
and Applications



**HAS QCD**

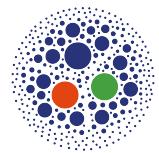
HADRONIC STRUCTURE AND  
QUANTUM CHROMODYNAMICS

# Gluon TMDs: a largely unexplored territory



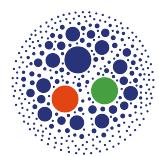
**Theory:** different **gauge-link** structures...

...more diversified kind of **modified universality!**



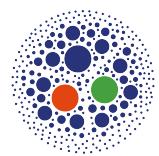
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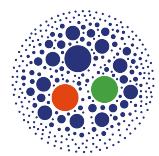
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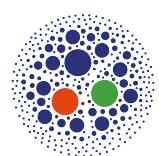


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## Motivation



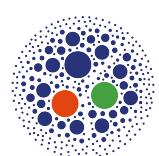
Gluon-TMD PDFs: *core* sector of **EIC** studies



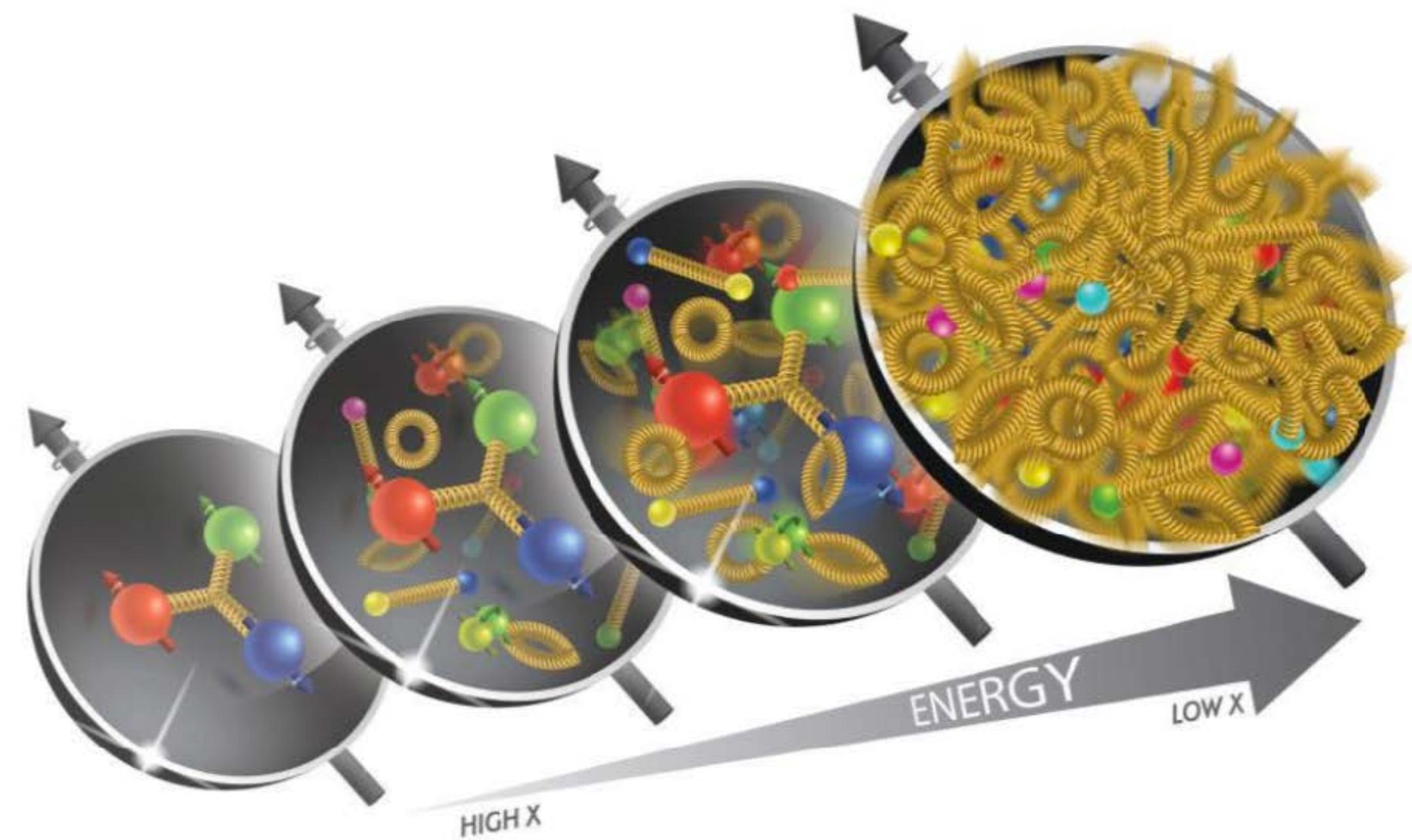
Need for a *flexible* model, suited to *pheno*



**Unpolarized** and **polarized gluon TMDs**



*Consistent* framework for quark TMDs

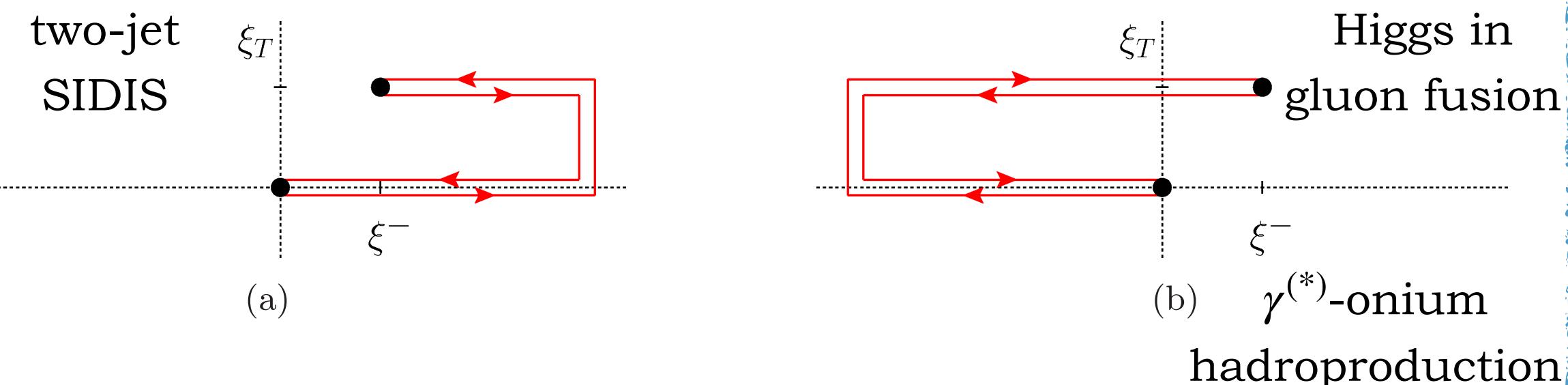


Significance of gluon-TMD studies  
in a wide range of  $x$

# Accessing WW and DP gluon TMDs

## Weiszäcker-Williams (WW)

(a) [ + , + ] or (b) [ - , - ]

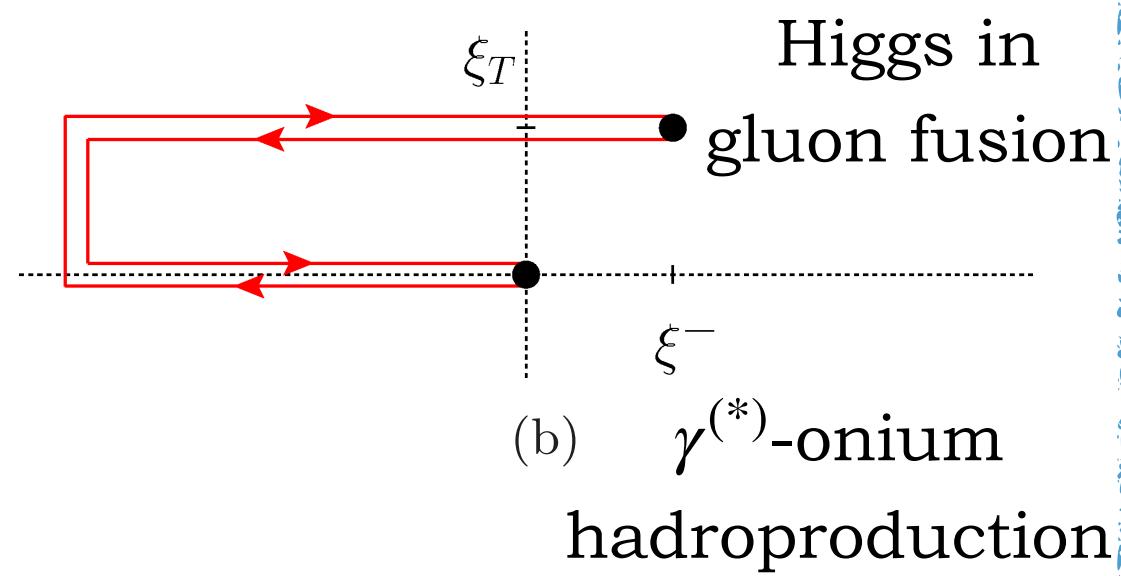
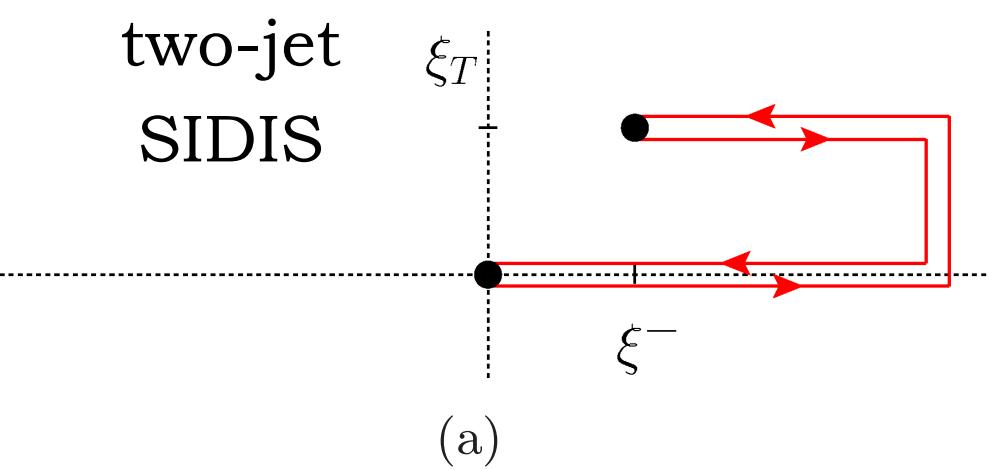


- \* Color flow annihilated within final/initial state
- \*  $f$ -type gluon TMDs  $\rightarrow f^{abc}$  color structure
- \* Modified universality:
$$f_1^{[+,+]} = f_1^{[-,-]},$$
$$f_{1T}^{\perp[+,+]} = -f_{1T}^{\perp[-,-]}$$
- \* Phenomenology: Higgs, quarkonia or  $\gamma\gamma$  in  $pp$ , two-jet SIDIS, heavy-quark pair SIDIS

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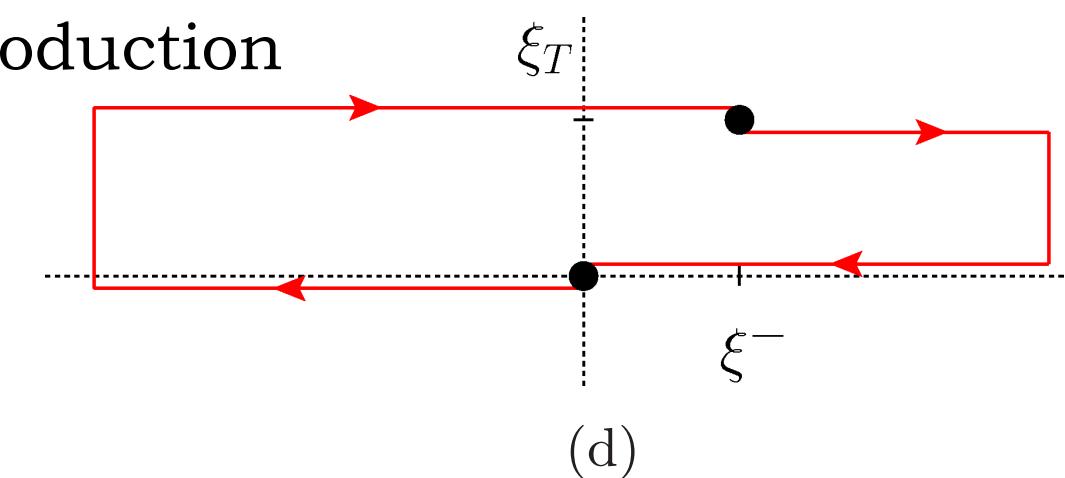
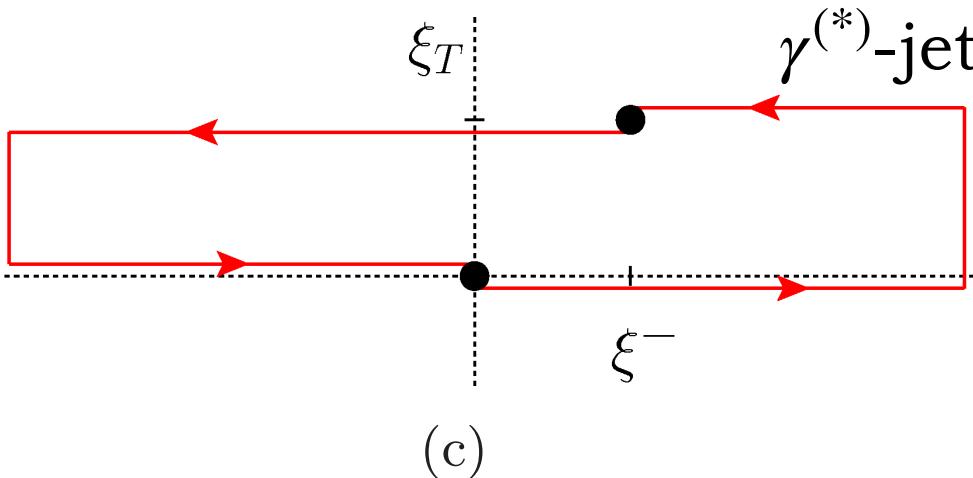
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## Dipole (DP)

(c) [ + , - ] or (d) [ - , + ]



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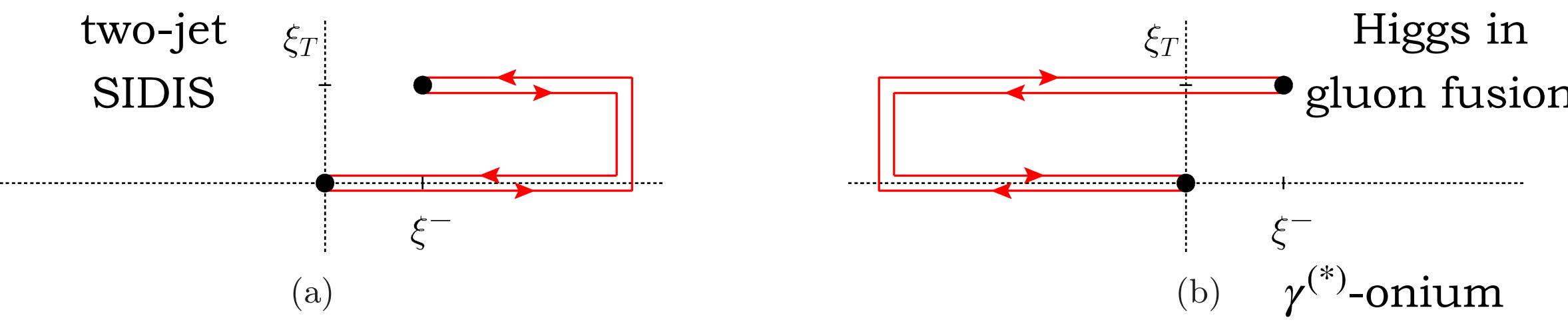
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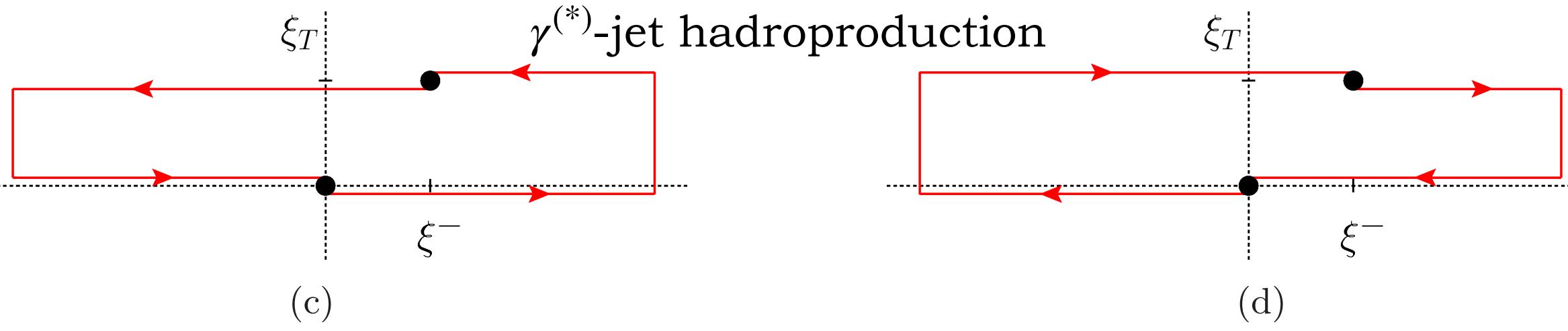
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Gauge link  $\rightarrow$  two main independent sets of TMDs, **not related** to each other

# **T-even and T-odd gluon TMD PDFs at twist-2**

gluon pol.

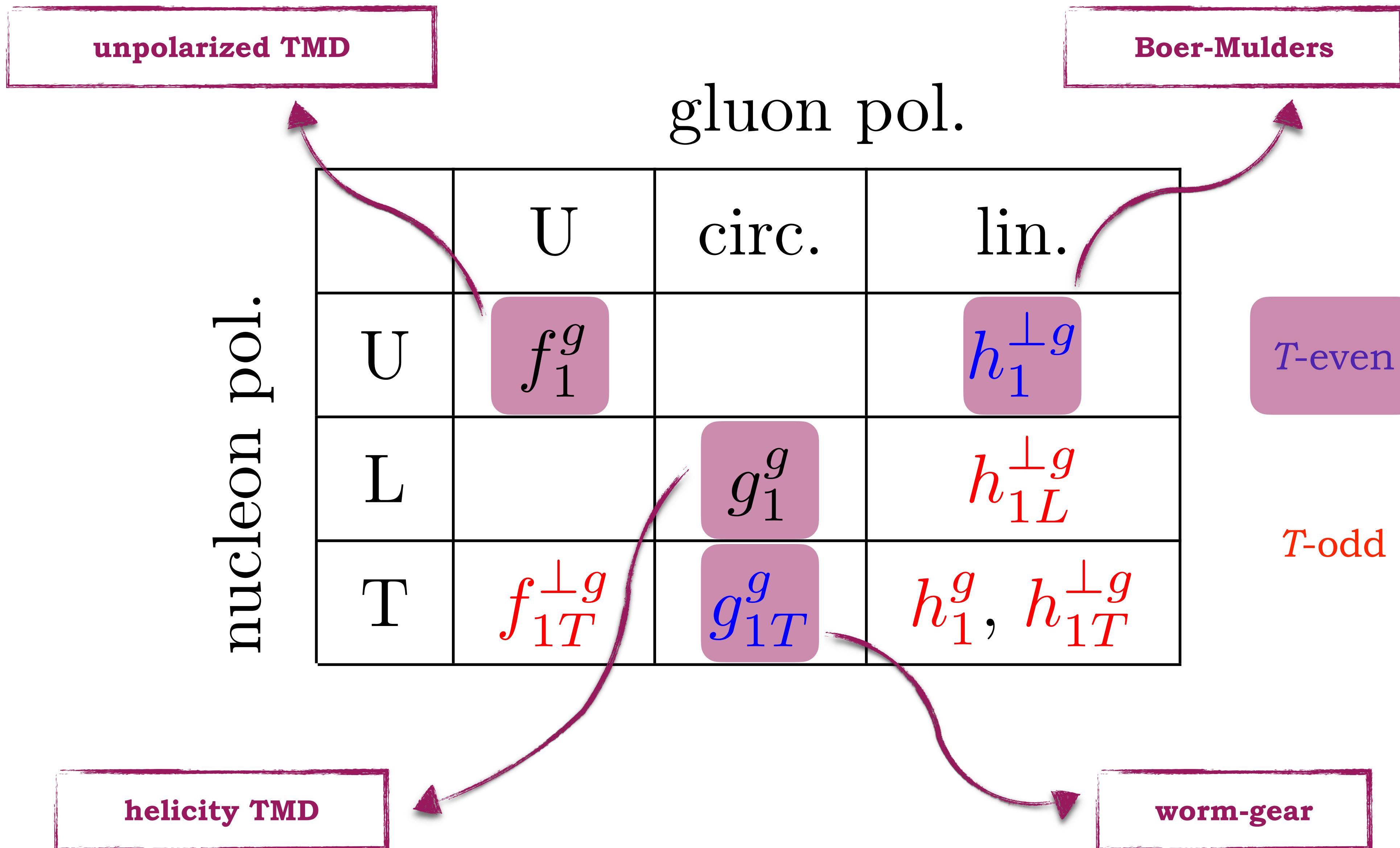
	U	circ.	lin.
U	$f_1^g$		$h_1^{\perp g}$
L		$g_1^g$	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	$g_{1T}^g$	$h_1^g, h_{1T}^{\perp g}$

*T-even*

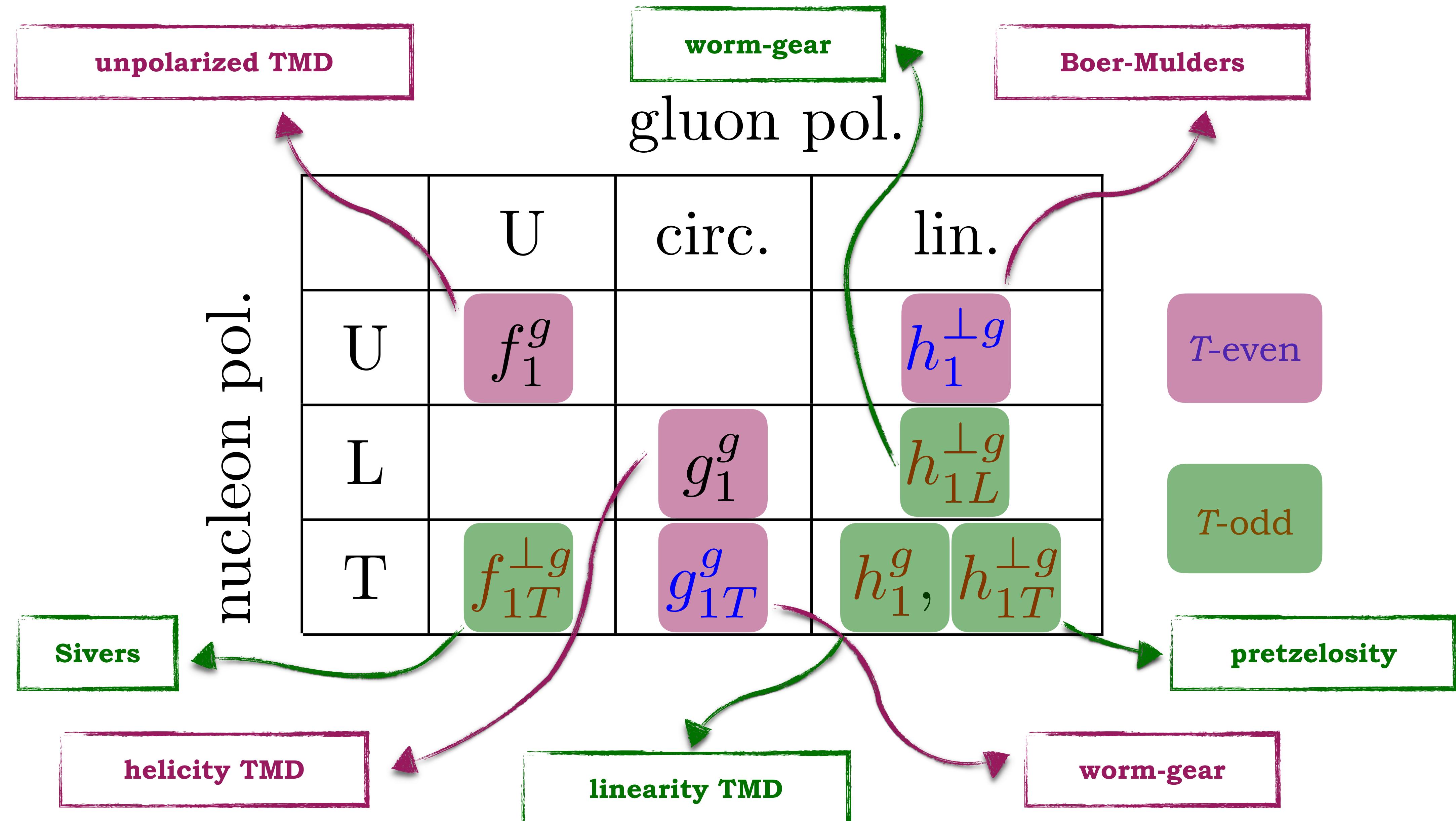
*T-odd*

nucleon pol.

# T-even and T-odd gluon TMD PDFs at twist-2

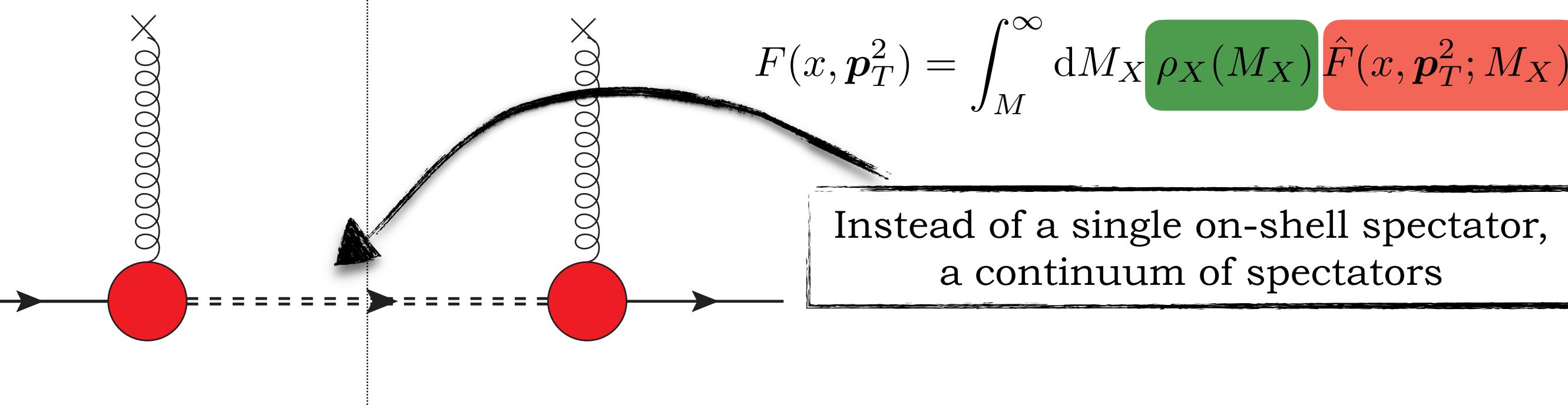


# T-even and T-odd gluon TMD PDFs at twist-2



# Our model at a glance

## Spectator-system spectral-mass function



Spectral function **learns** small- and moderate- $x$  info  
encoded in **NNPDF** collinear parametrizations

(NNPDF3.1sx + NNPDFpol1.1)

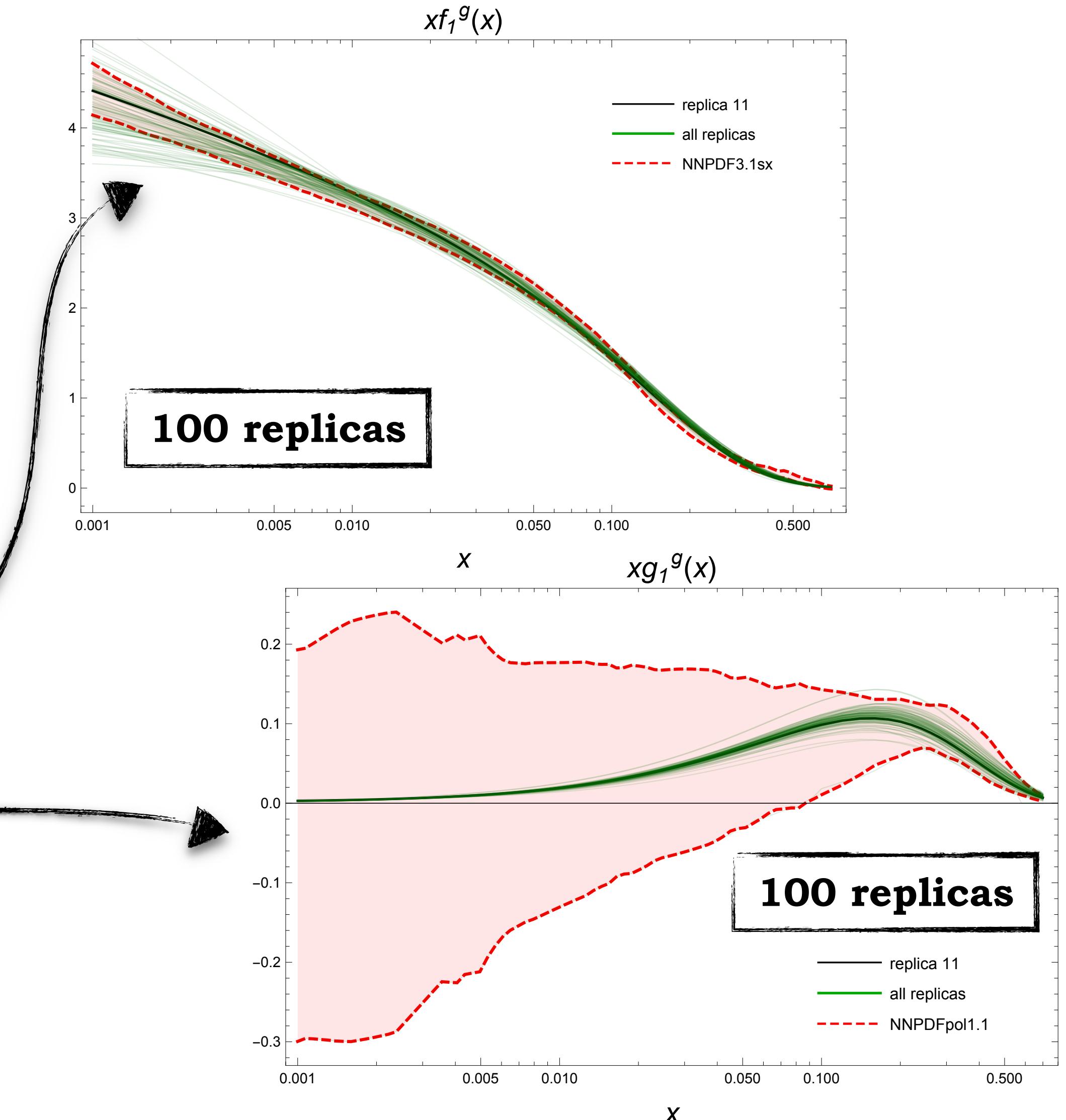
**Simultaneous fit** of  $f_1$  and  $g_1$  PDFs

Inclusion of small- $x$  resummation effects (**BFKL**)

Calculation of all twist-2  $T$ -even gluon TMDs

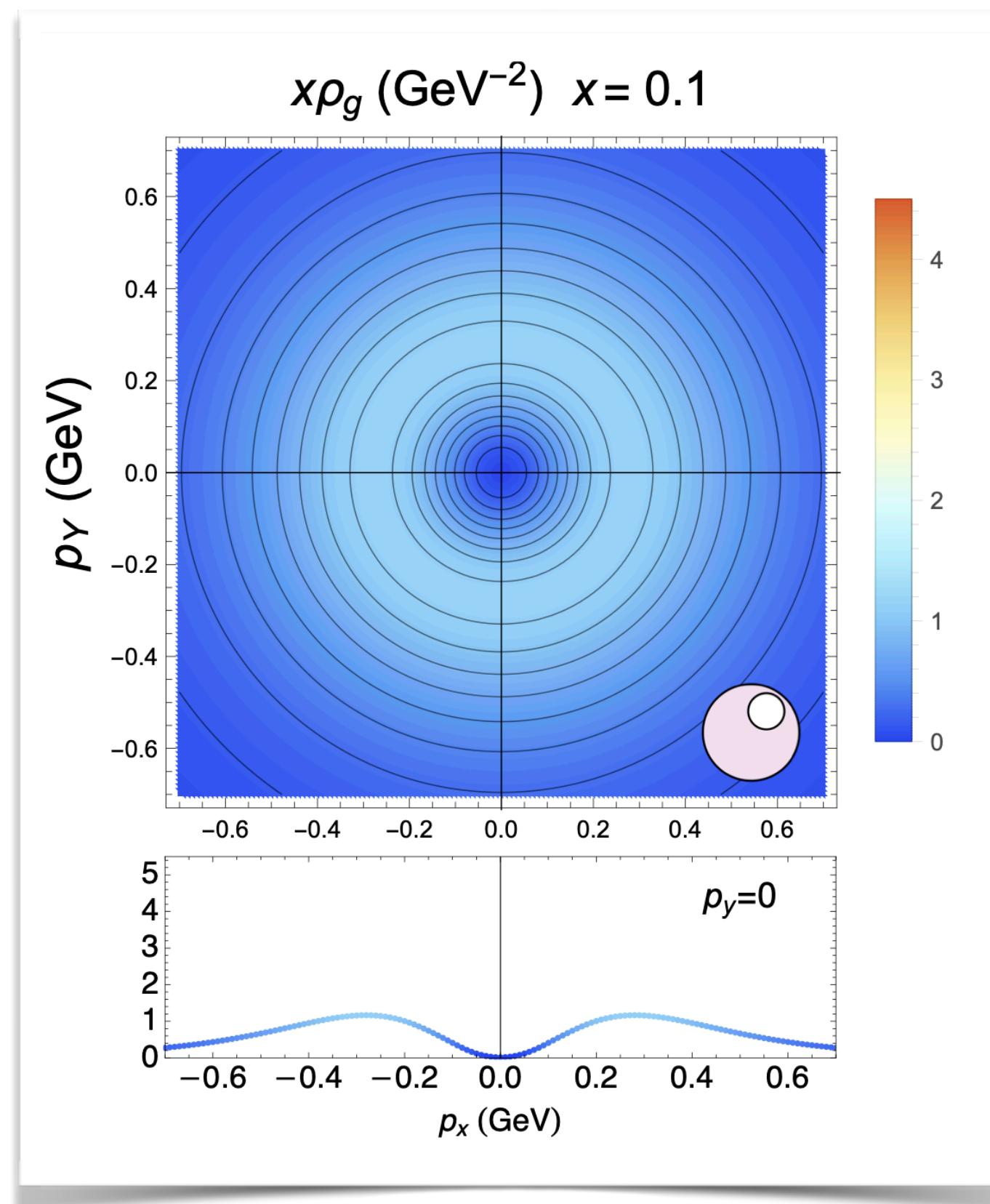
## Link with collinear factorization

$p_T$ -integrated TMDs **have to** reproduce PDFs  
at the lowest scale ( $Q_0$ ) *before* evolution



# 3D tomography: the gluon content in the proton

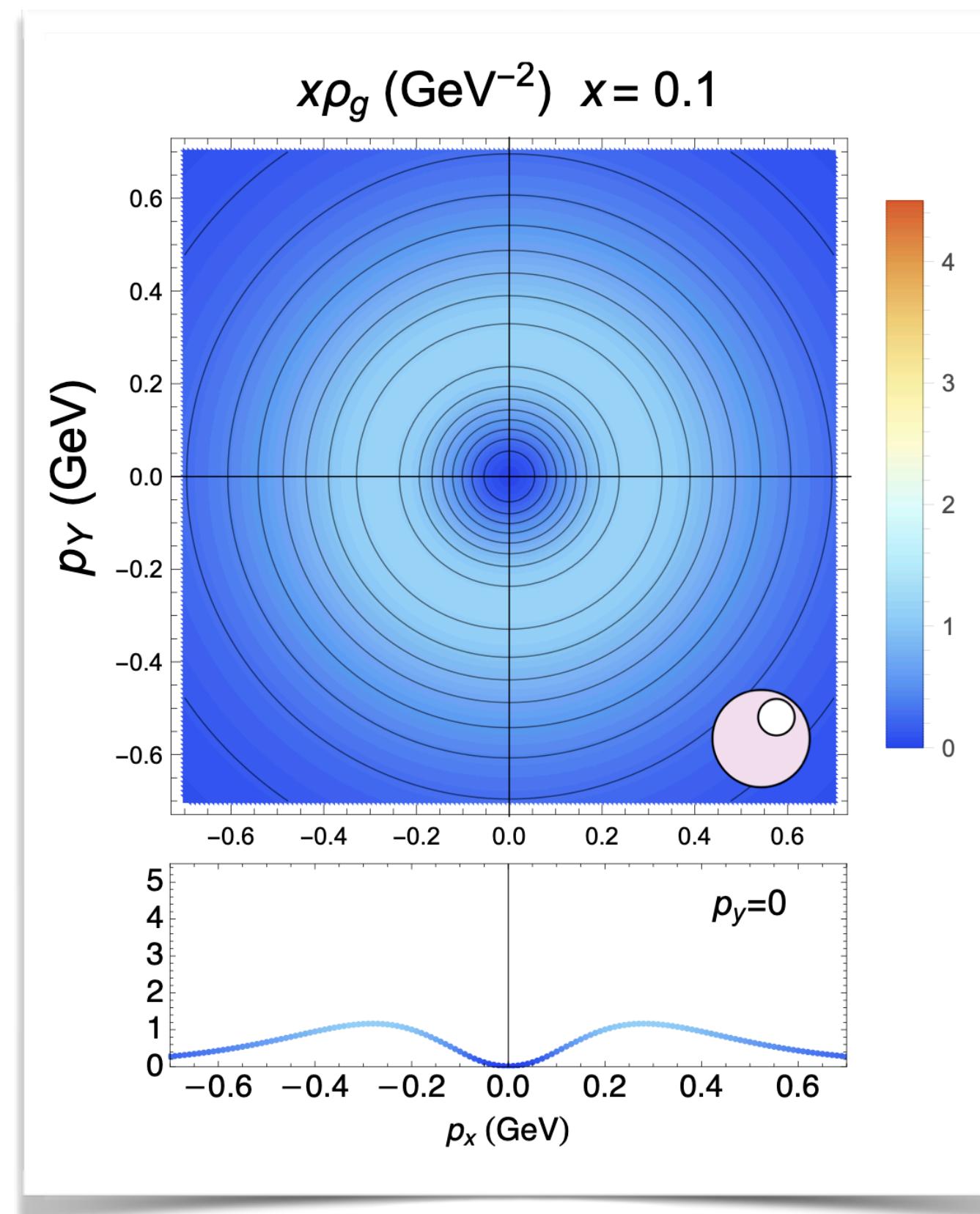
unpolarized TMD



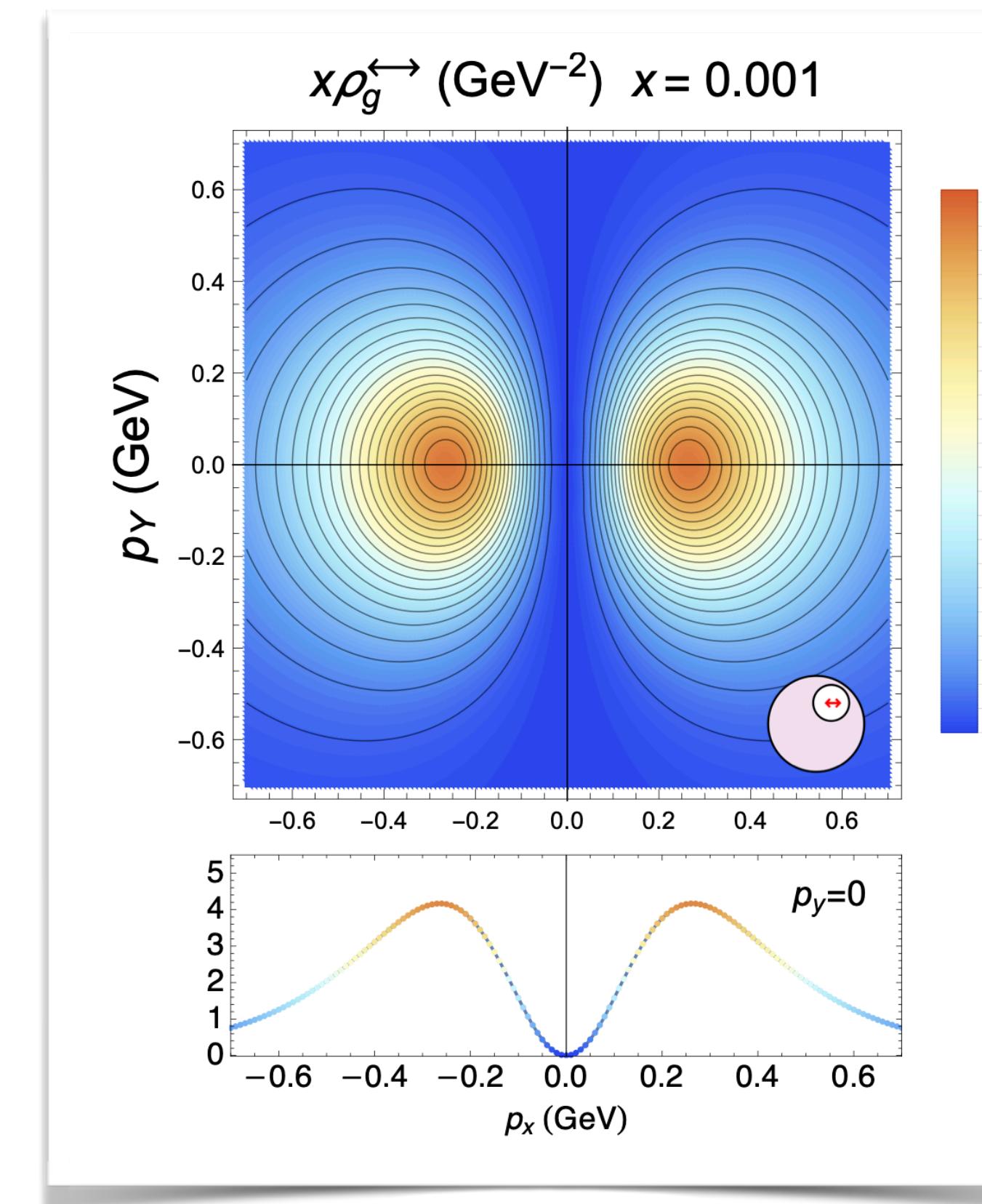
[A. Bacchetta, F.G.C., M. Radici, P. Taels, *Eur. Phys. J. C* **80** (2020) no.8 [[arXiv:2005.02288](https://arxiv.org/abs/2005.02288)]]

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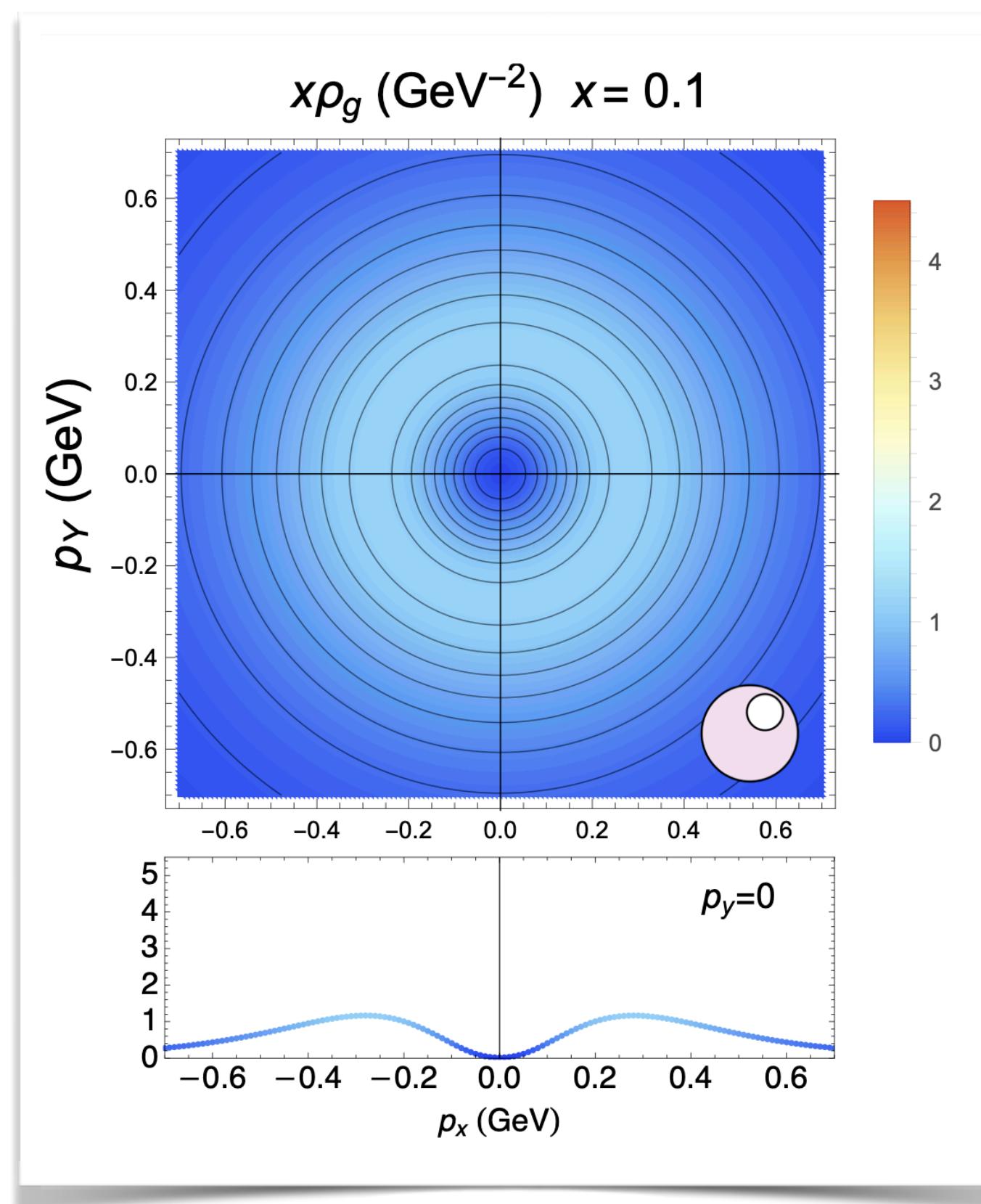
Boer-Mulders



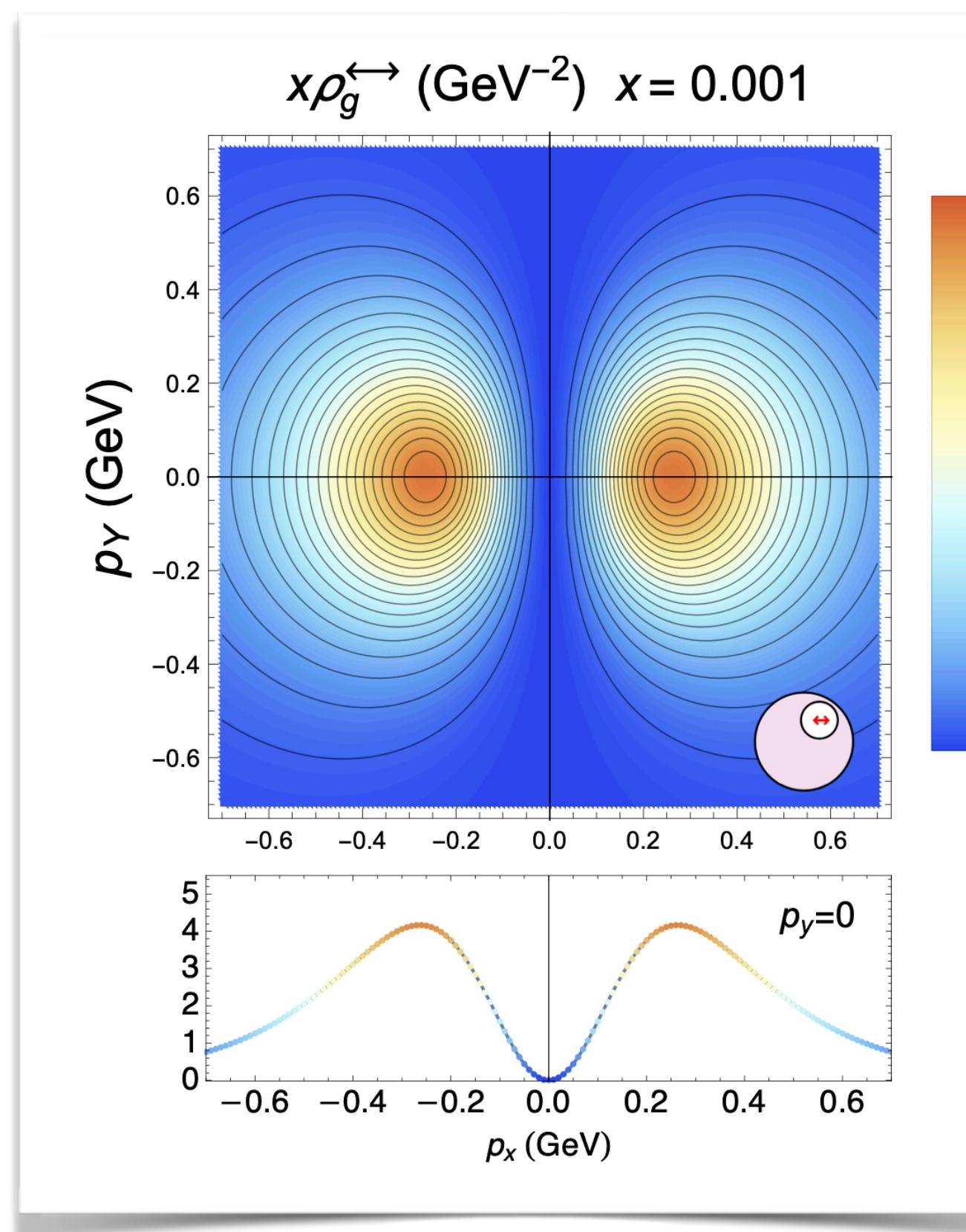
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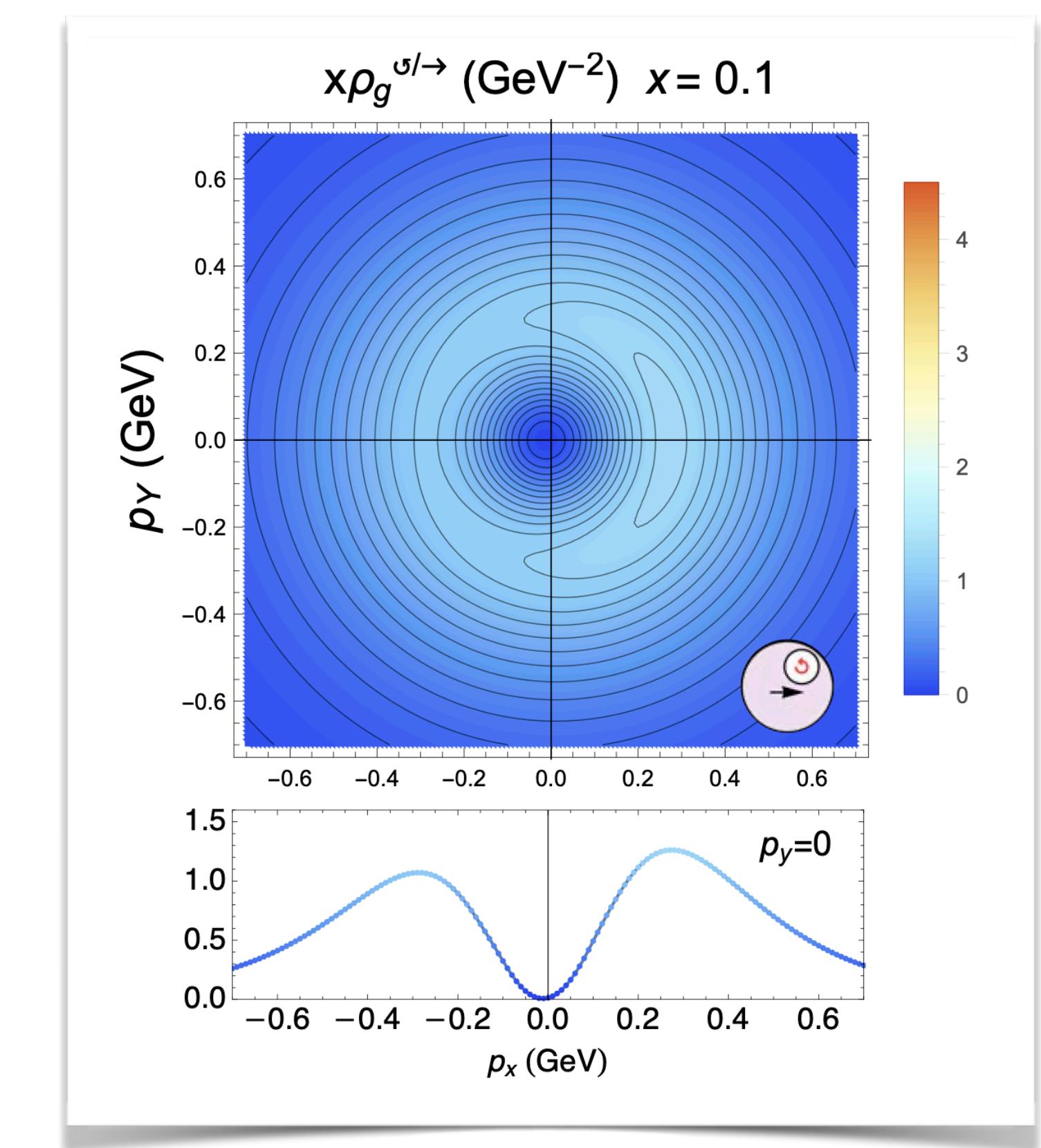
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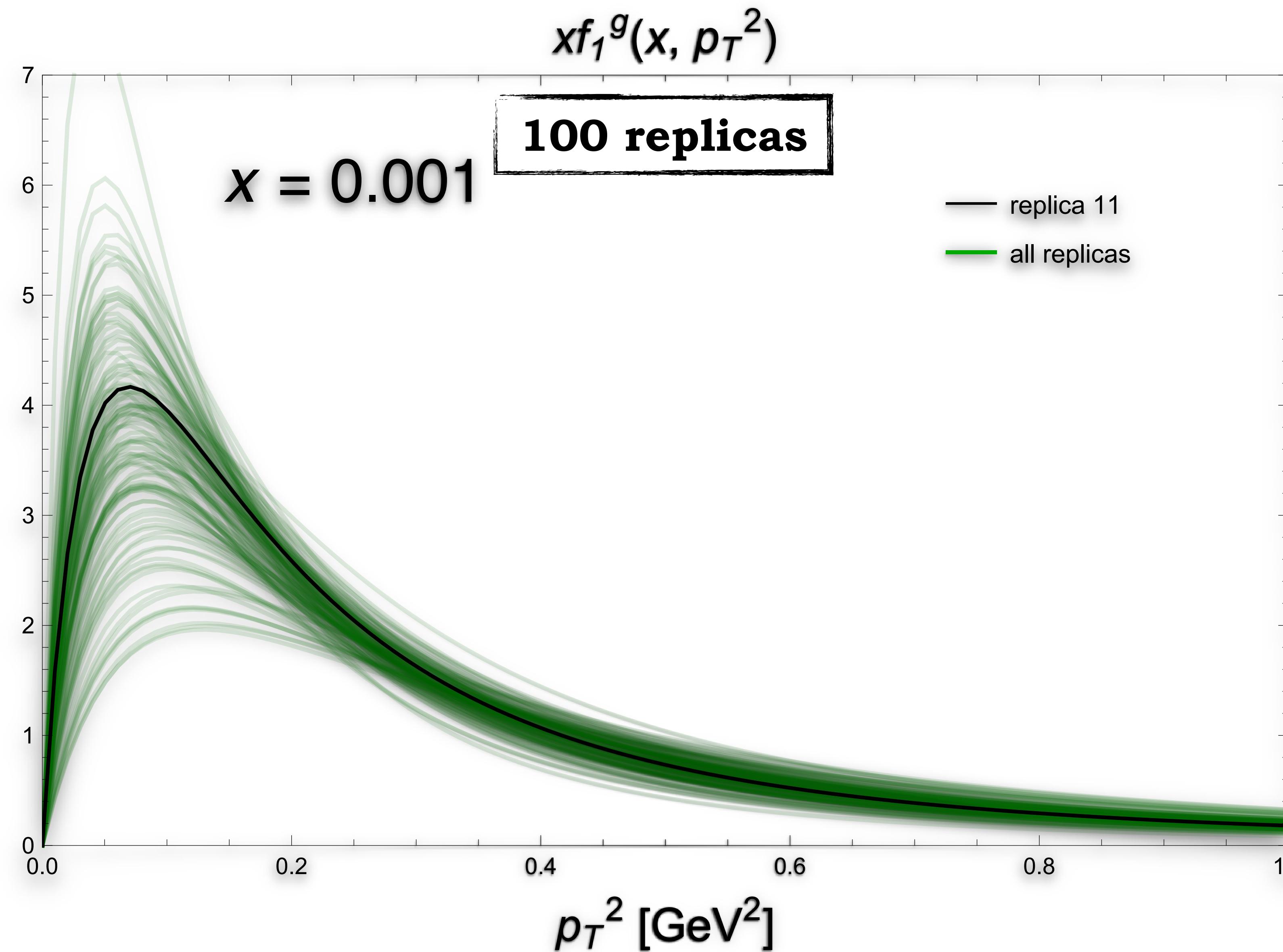


worm-gear

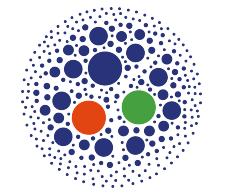


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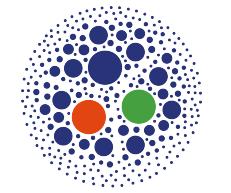
# Unpolarized gluon TMD



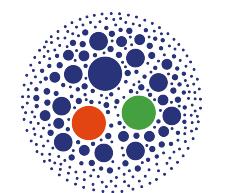
# Bottom line



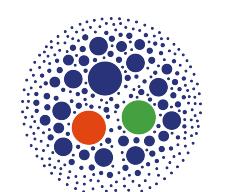
Each TMD shows a distinctive  $x$ - and  $p_T$ -behavior



Data on gluon TMDs will exclude many replicas  
and constrain parameters not yet so well constrained by collinear  
PDFs



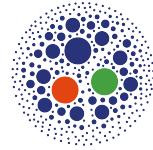
Simultaneous fit on two distinct PDFs provides with *corroborating evidence* of reliability of our model



Standard CSS  $\otimes \otimes$  evolution to be turned on

# Hadronic structure at small- $x$

Incomplete list of small- $x$  formalisms → *linear* (BFKL) or *saturation* (BK/JIMWLK) effects embodied



## Unintegrated parton densities

*A (hybrid) high-energy factorization established*

- \* **BFKL UGD**: pure small- $x$  evolution, Reggeons
- \* HEF, CCFM, PRA **uPDFs**: BFKL + collinear matching

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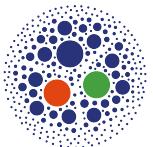


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## Small- $x$ improved collinear PDFs

*DGLAP description improved via BFKL*

- \* **ABF**: PDFs + small- $x$  resummed splitting

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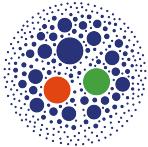
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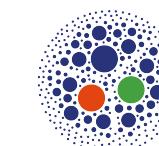
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*Nonperturbative content via an enhanced spectator model*

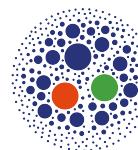
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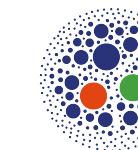
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Need for sub-eikonal corrections, neglected by BFKL

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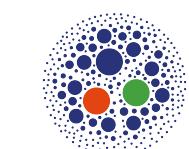
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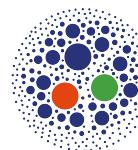
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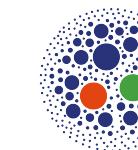
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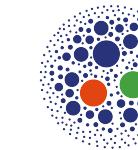


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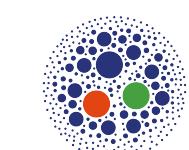


## CGC/JIMWLK gluon TMDs

*Gluon-recombination effects encoded*

\* **WW vs DP** gluon TMDs, **GTMDs**

\* **iTMD**: interpolating between TMD and BFKL regimes



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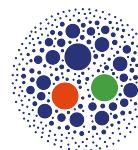
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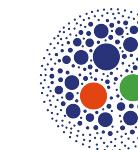
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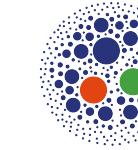


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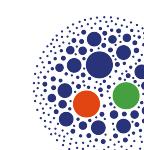


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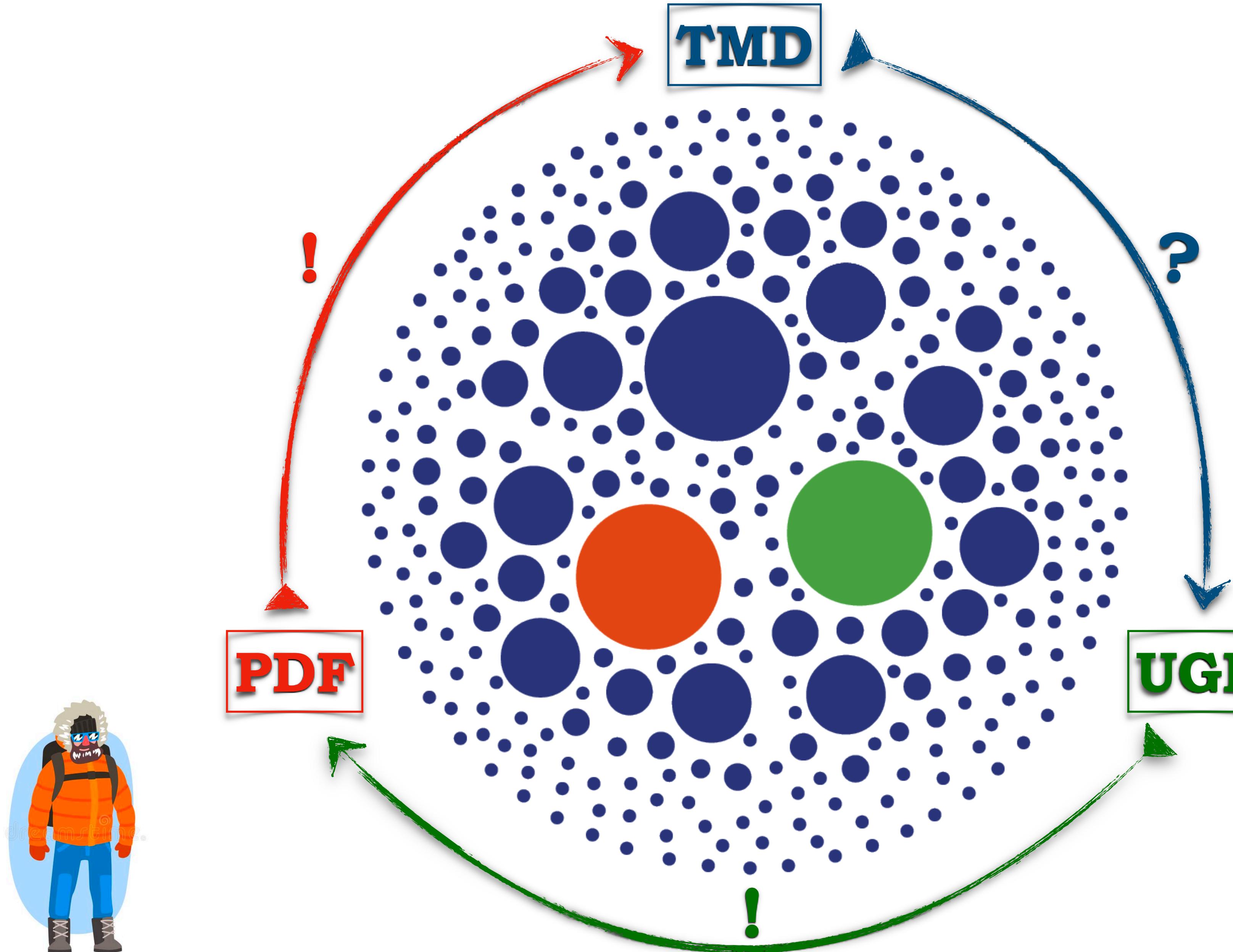
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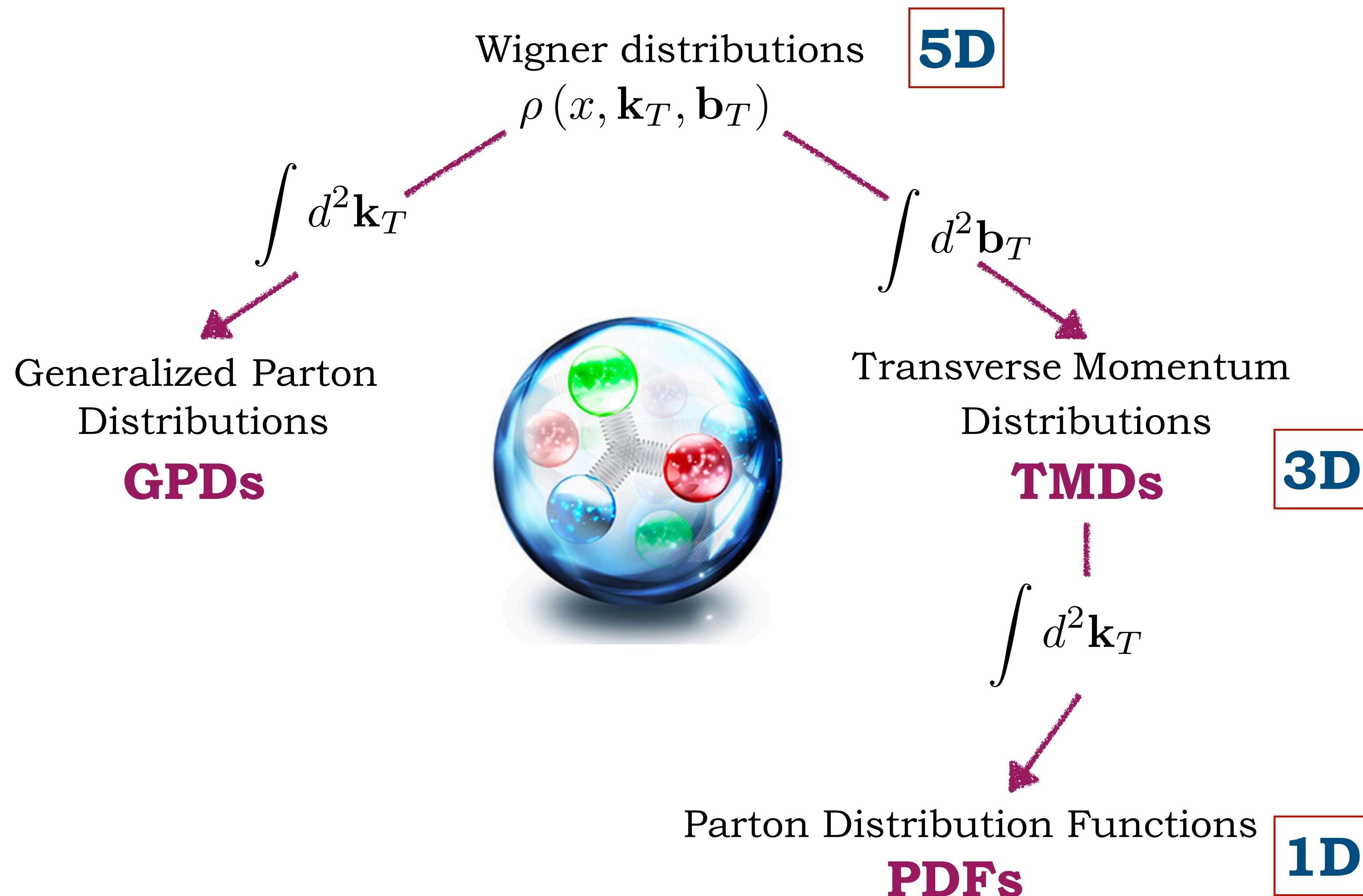
[A. Bacchetta, F.G.C., M. Radici, P. Taels (2020)]

# Mapping the proton content at small- $x$

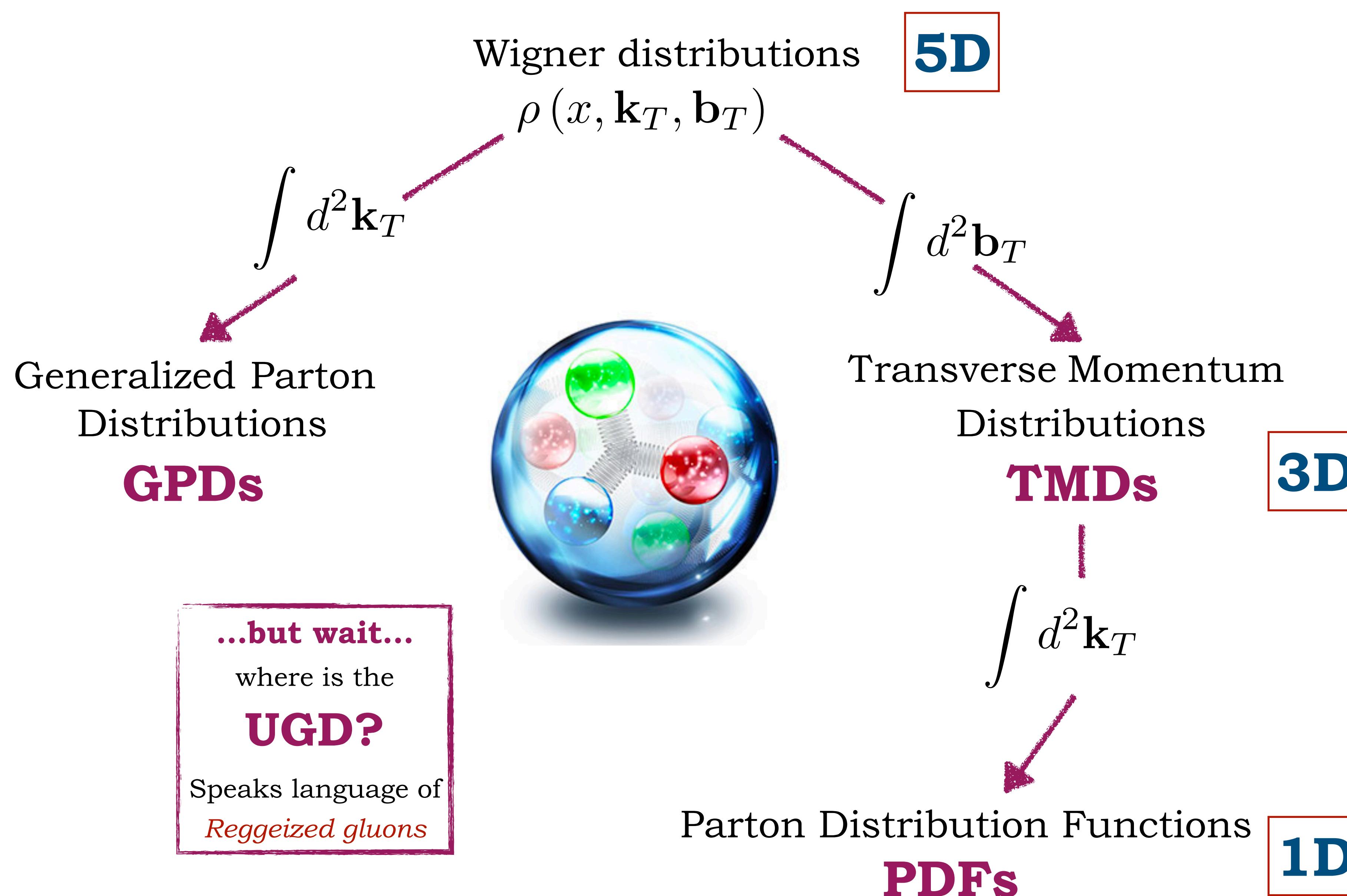


**Backup  
slides**

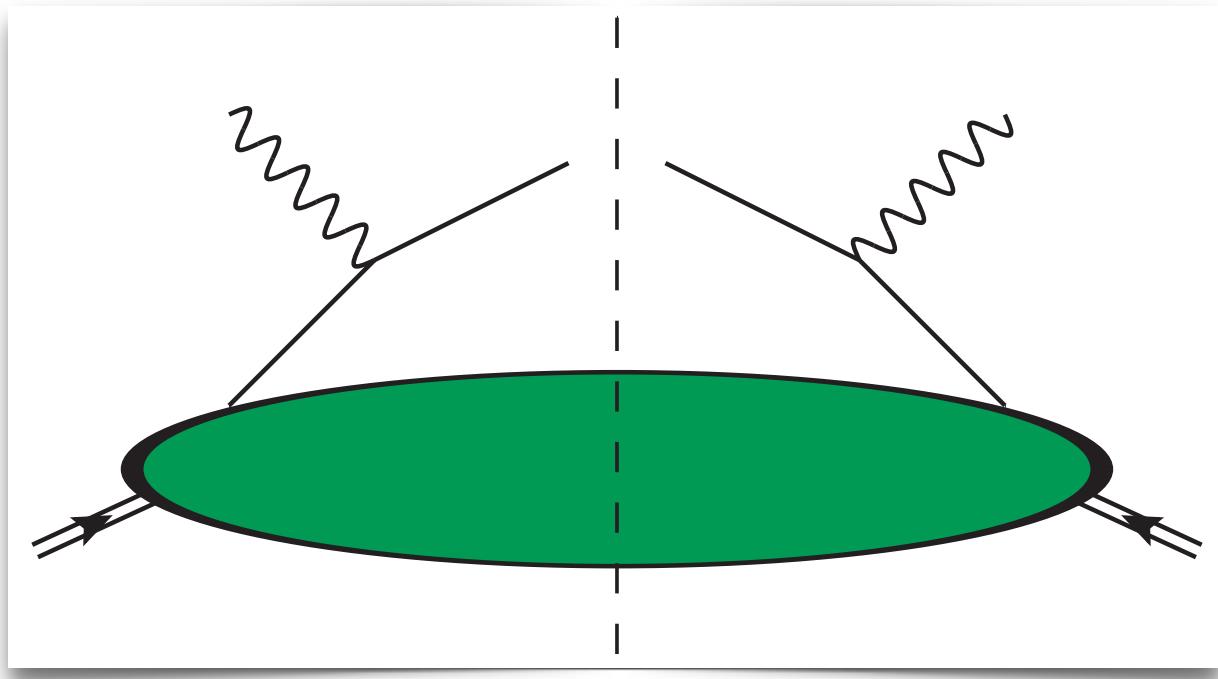
# Parton densities: an incomplete family tree



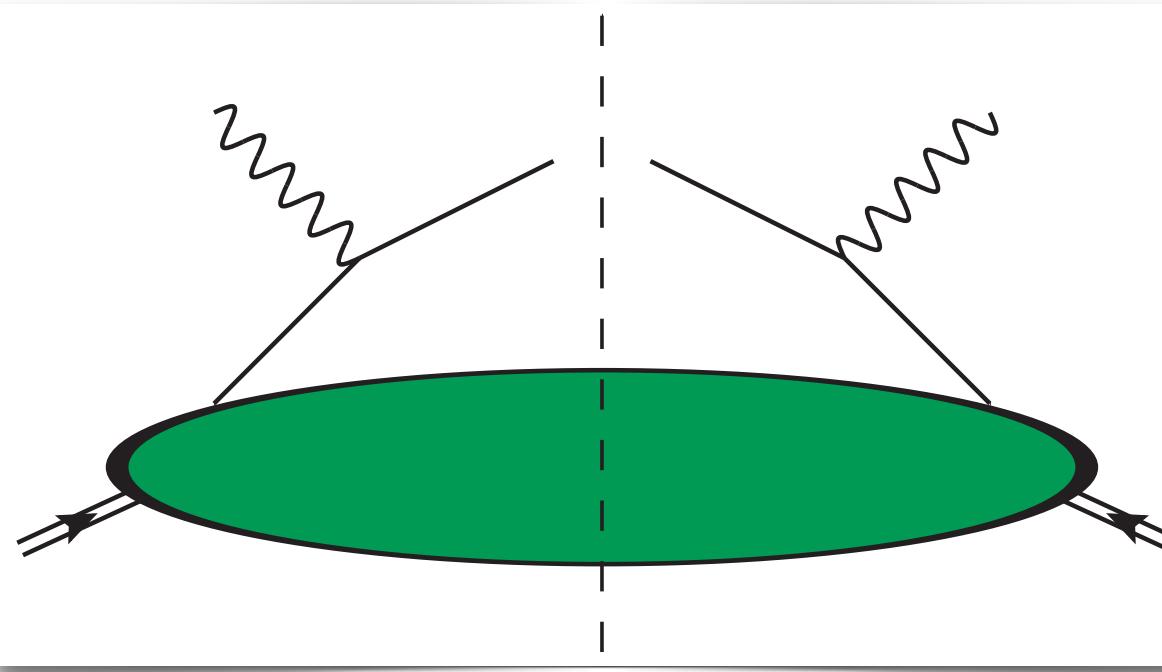
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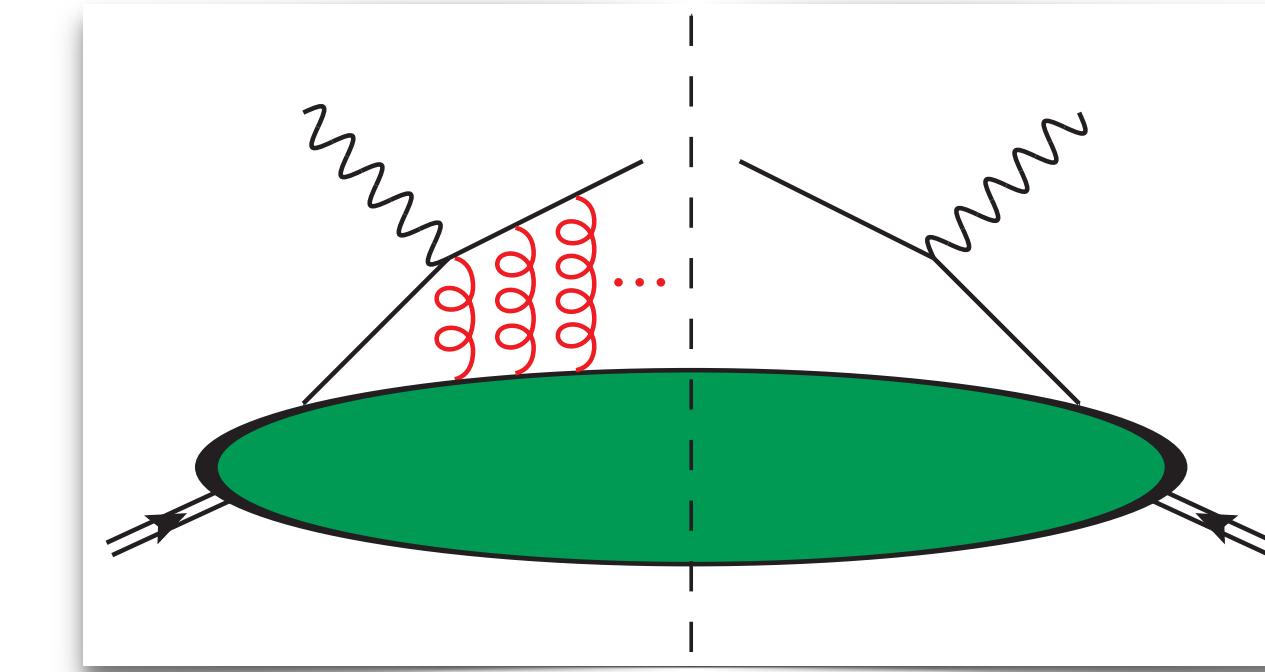
# Gauge links and process dependence



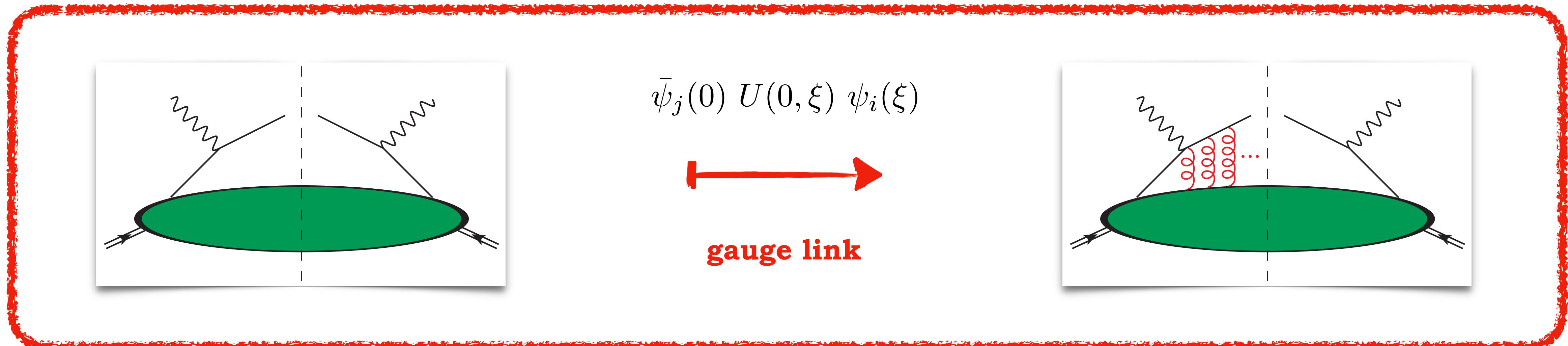
# Gauge links and process dependence


$$\bar{\psi}_j(0) \ U(0, \xi) \ \psi_i(\xi)$$


**gauge link**



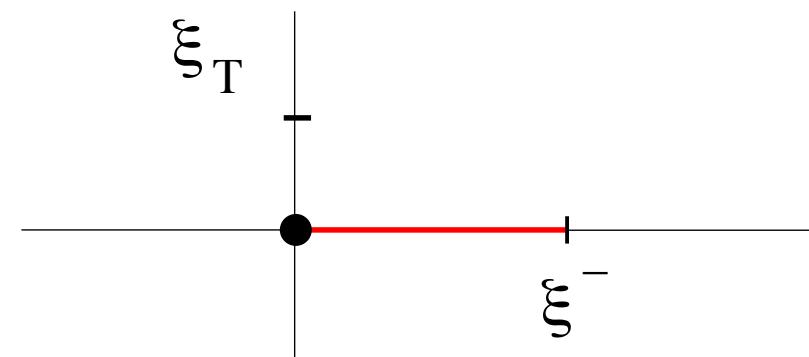
# Gauge links and process dependence



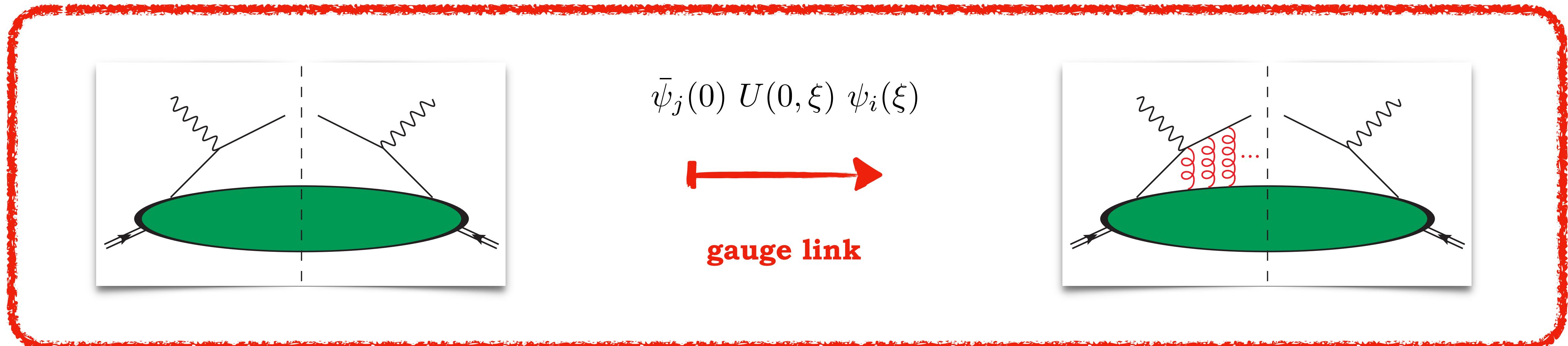
## Collinear PDFs

$$\Phi_{ij}(x) \doteq \int d^2 p_T \Phi_{ij}(x, p_T) = \int \frac{d\xi^-}{2\pi} e^{ip \cdot \xi} \langle P | \bar{\psi}_j(0) \psi_i(\xi) | P \rangle|_{\xi^+ = 0, \xi_T = 0}$$

- Light-cone:  $\xi^+ = 0, \xi = 0$
- **Straight** gauge link (unique!)
- $(A^+ = 0)$  light-cone: WL =  $\hat{1}$
- ✓ **Universality warranted**



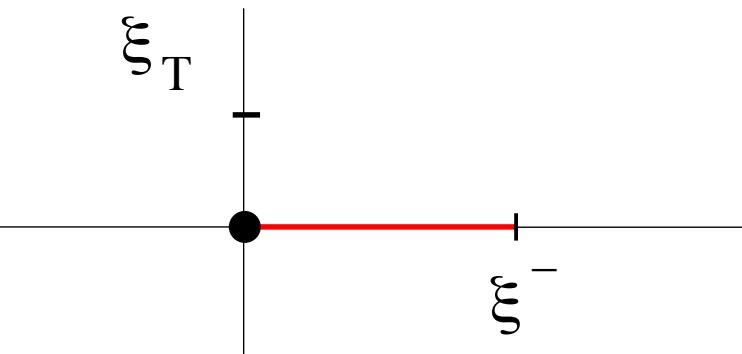
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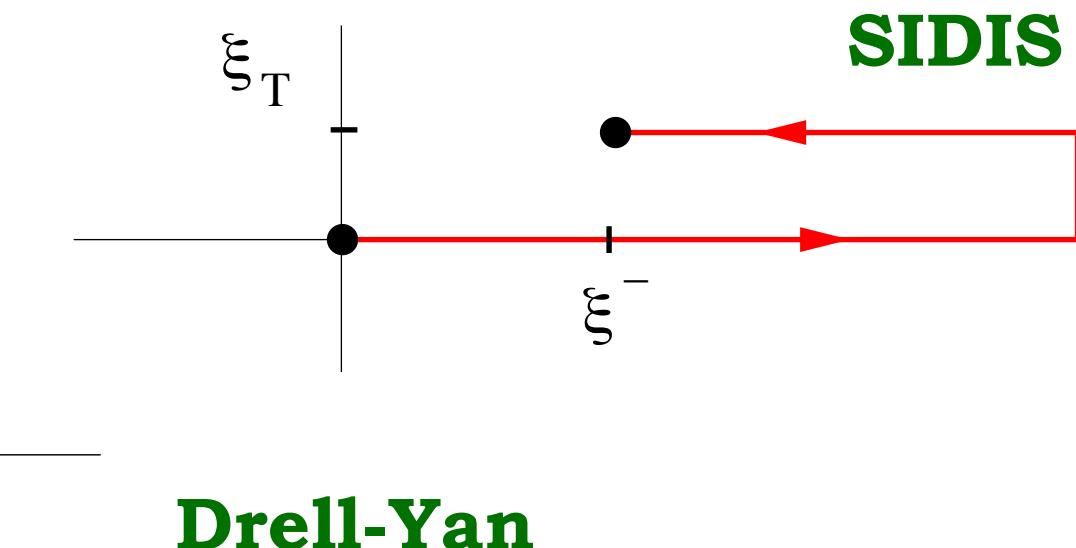
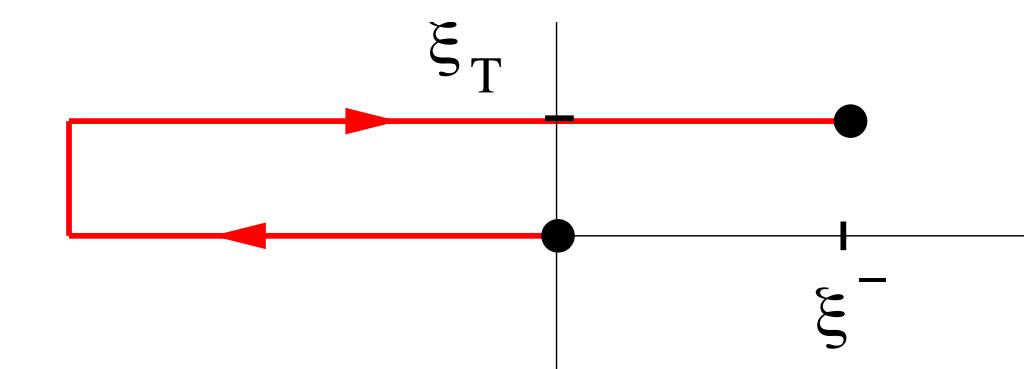
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- $(A^+ = 0)$  light-cone: WL =  $\hat{1}$
- Universality warranted**



## TMD PDFs

- Transverse gauge link not eliminated by gauge choice
- Staple-like gauge link (not unique!)

**!! Process dependence**

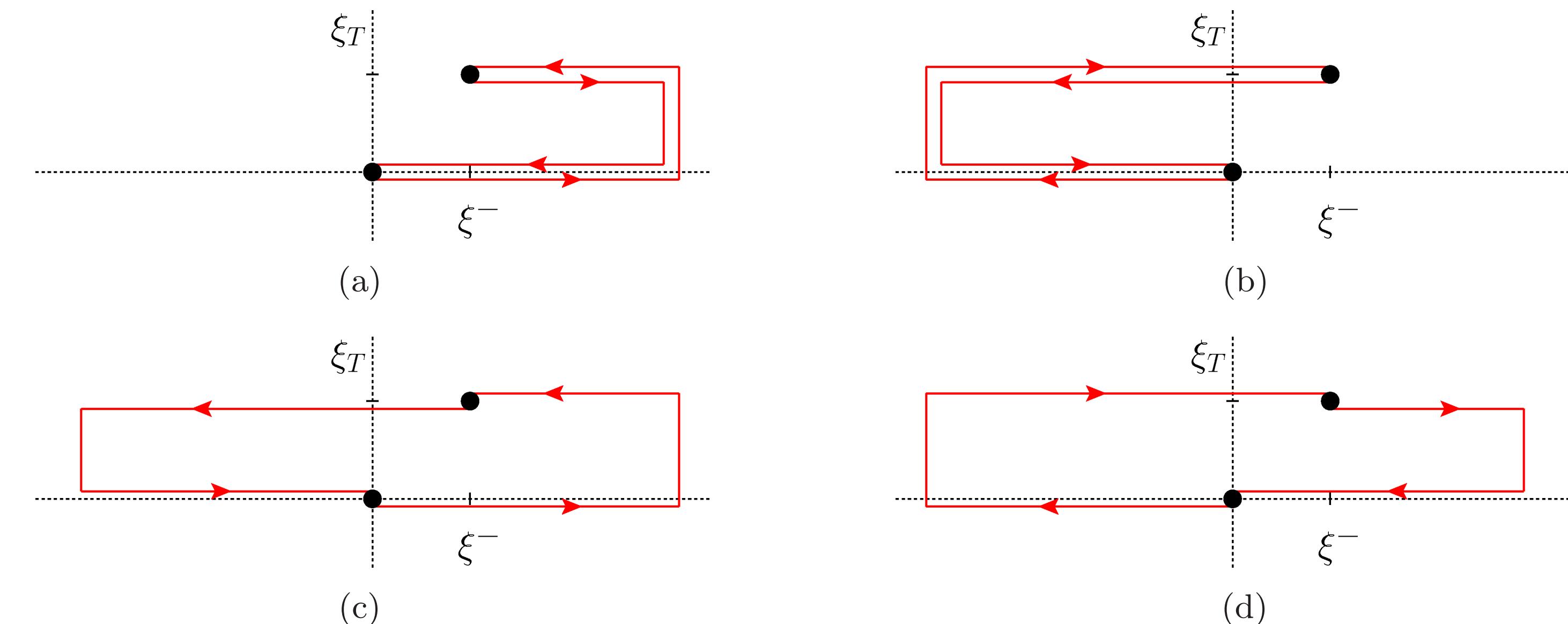


# Gluon TMDs: gauge links and modified universality

- \* **Single-spin asymmetries** → process dependence of TMDs via **gauge links**
- \* **Color flow** → integration paths of gauge links calculable
- \* Gluon TMDs → more complicated structure with respect to quark **staple links**
- \* **Factorization-preserving** processes → two main kinds of **modified universality**
- \* Different classes of processes → distinct gluon TMDs, **not related** to each other

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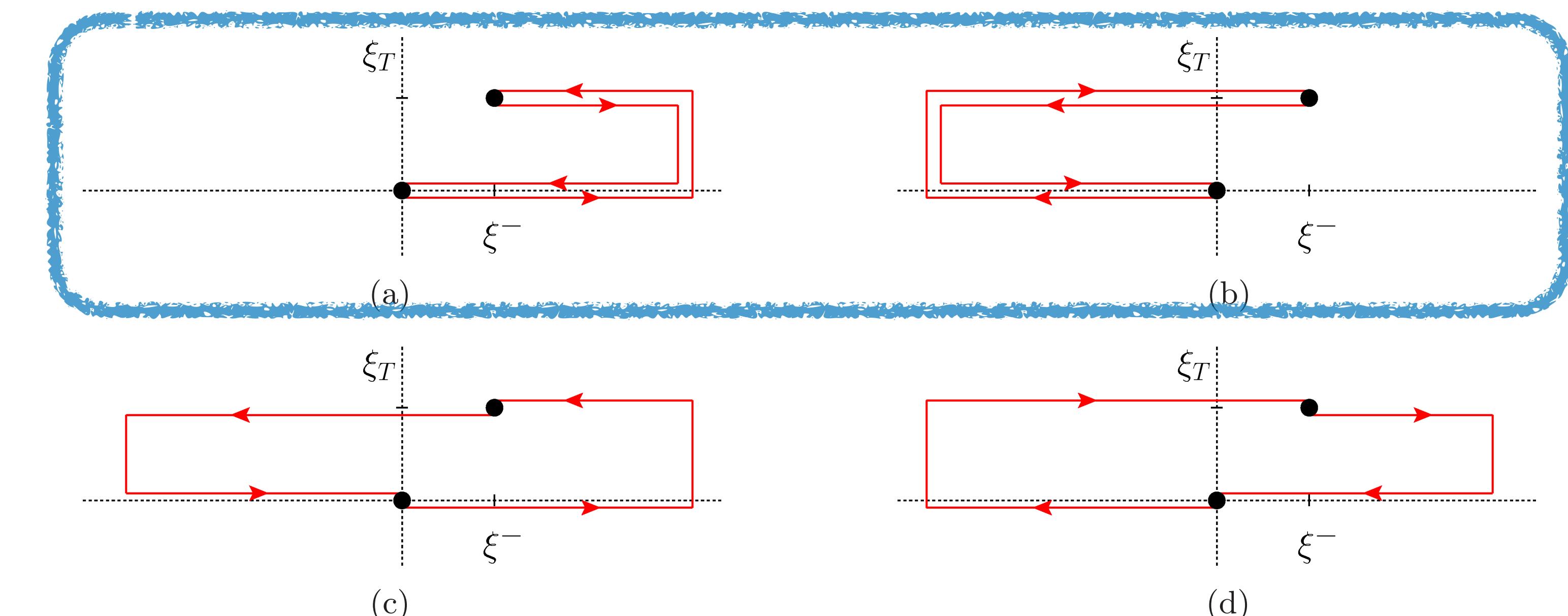


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## Weiszäcker-Williams (WW)

(a) [ + , + ] or (b) [ - , - ]

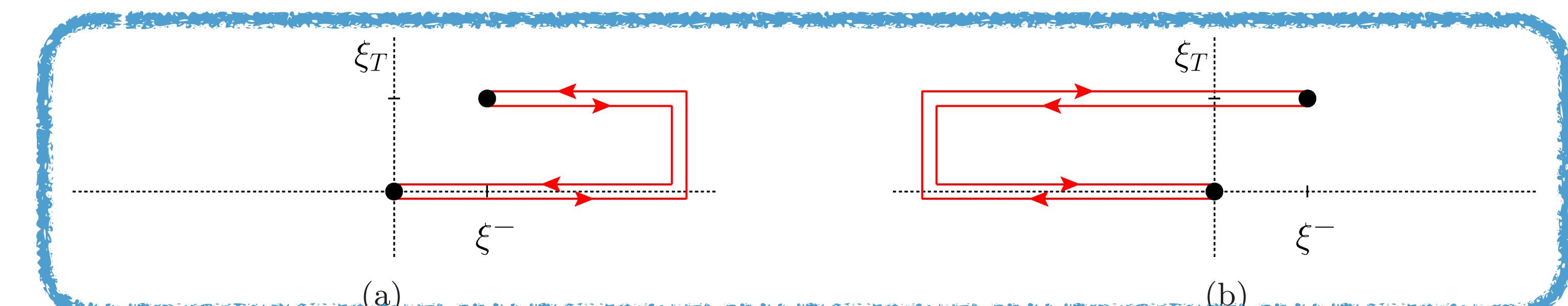


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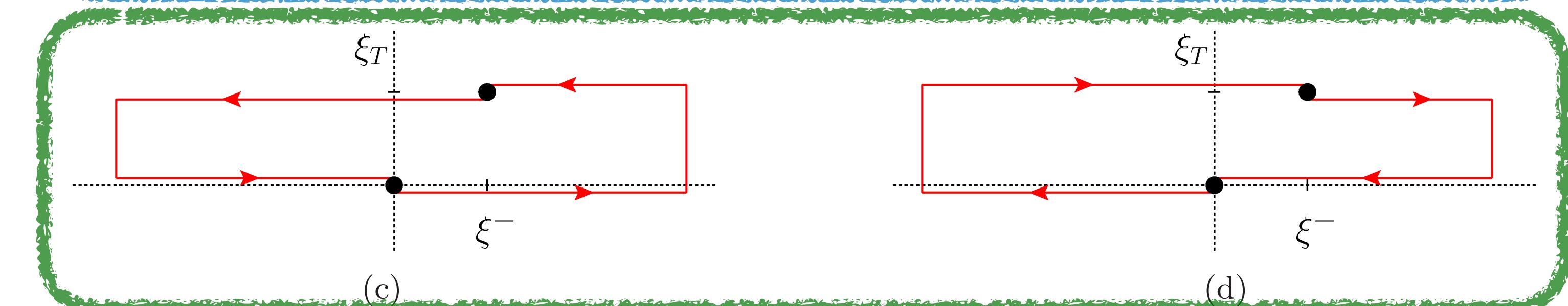
## Weiszäcker-Williams (WW)

(a) [ + , + ] or (b) [ - , - ]



## Dipole (DP)

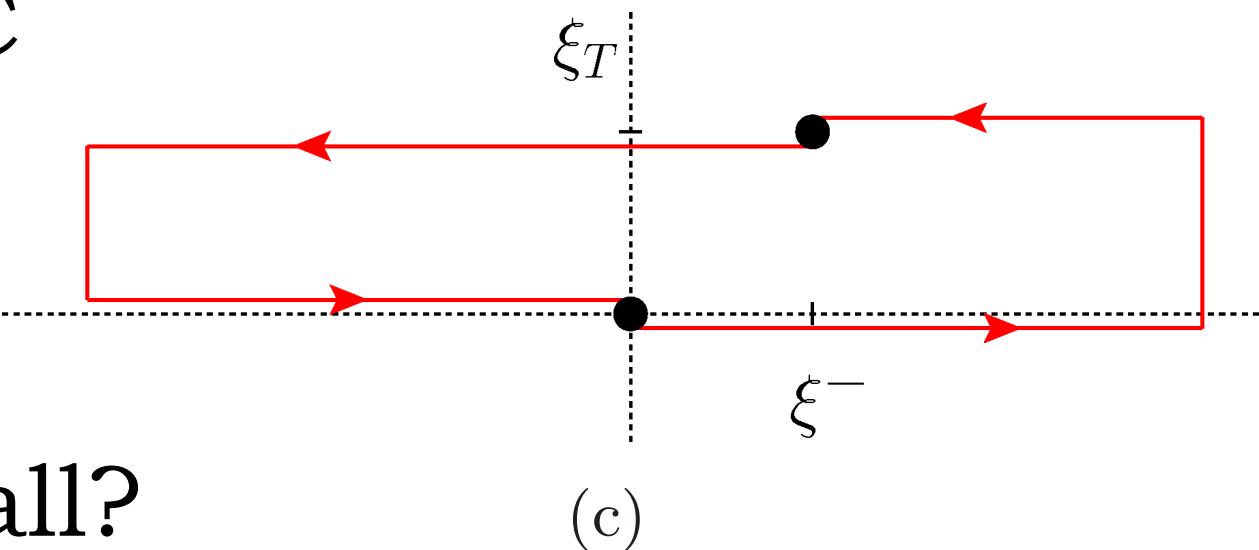
(c) [ + , - ] or (d) [ - , + ]



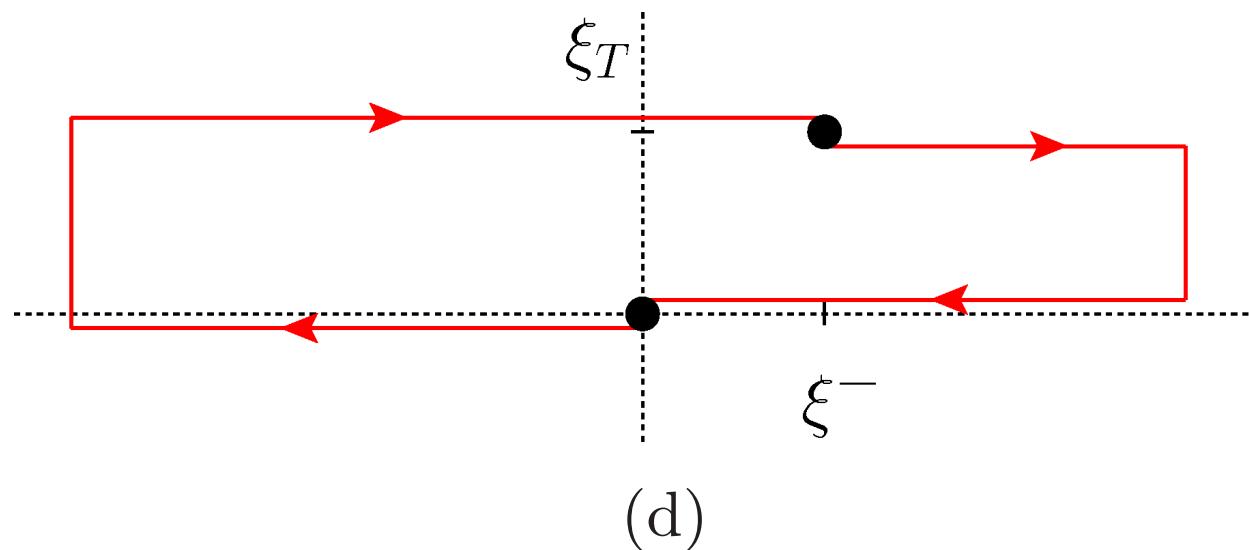
# Dihadron hadroproduction and factorization breaking

- \* Proof of factorization violation  [T.J. Rogers, P.J. Mulders (2010)]

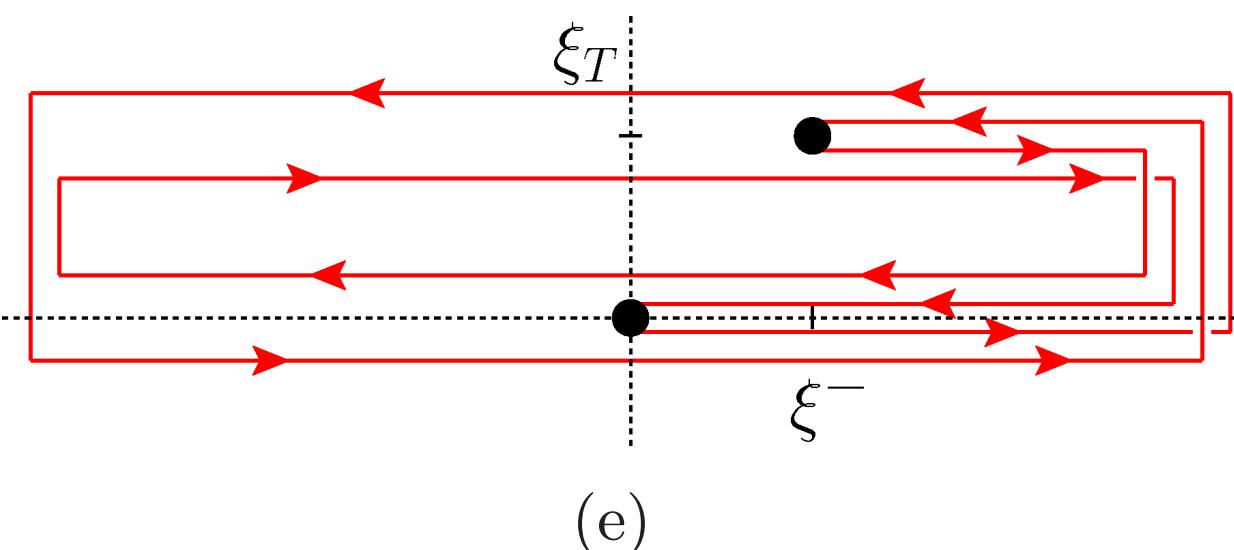
- \* Assumed factorization in SCET and CGC



- \* Significance of low- $x$  studies

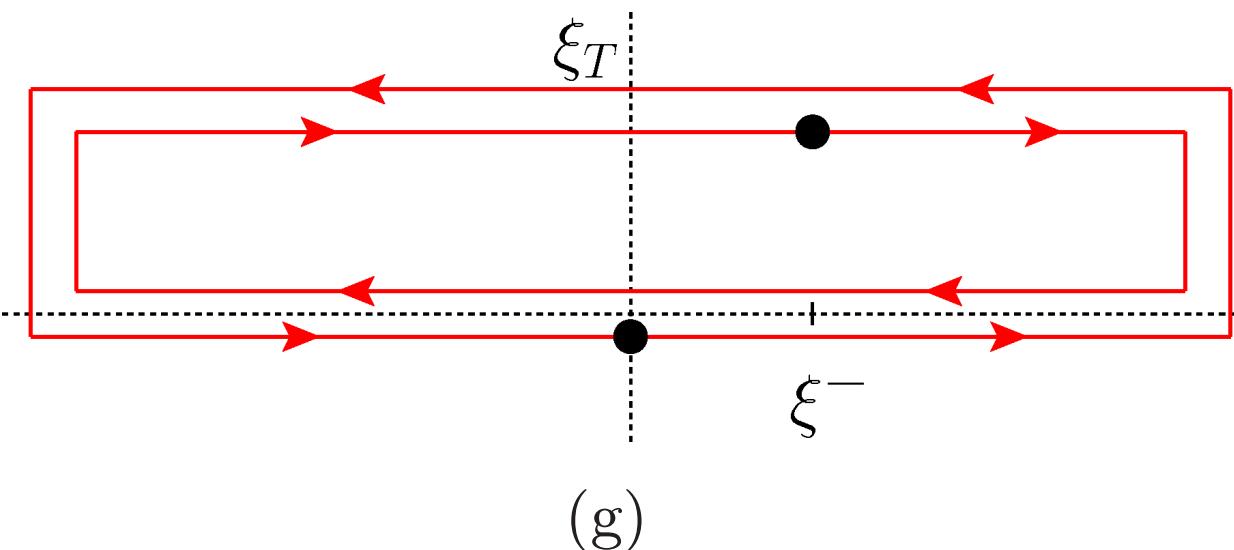


- \* Size of factorization-breaking effects small?



- \* DP TMDs:

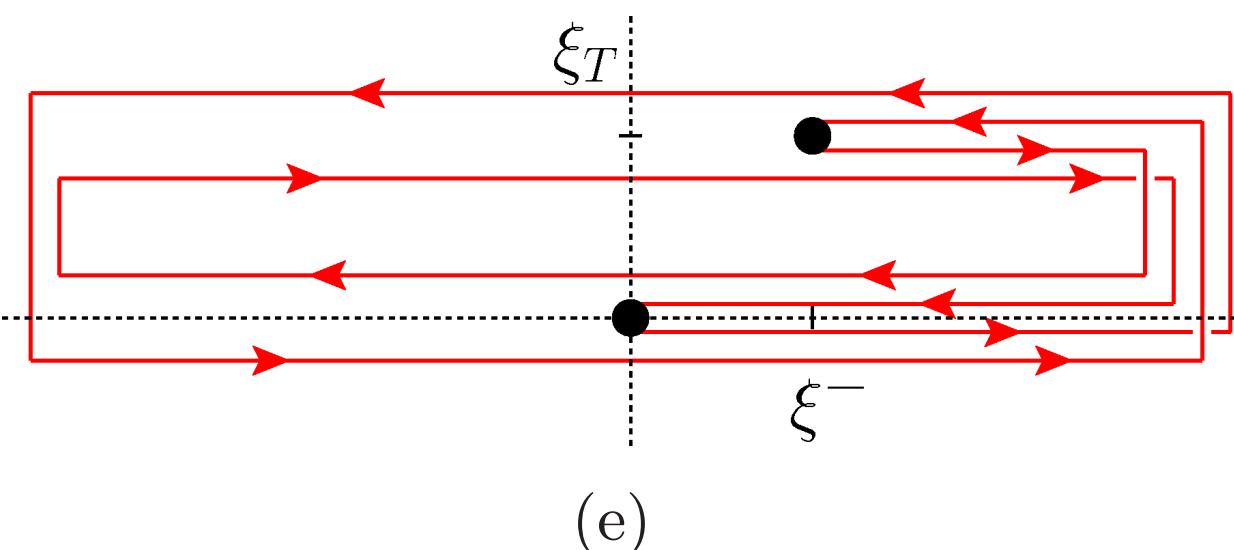
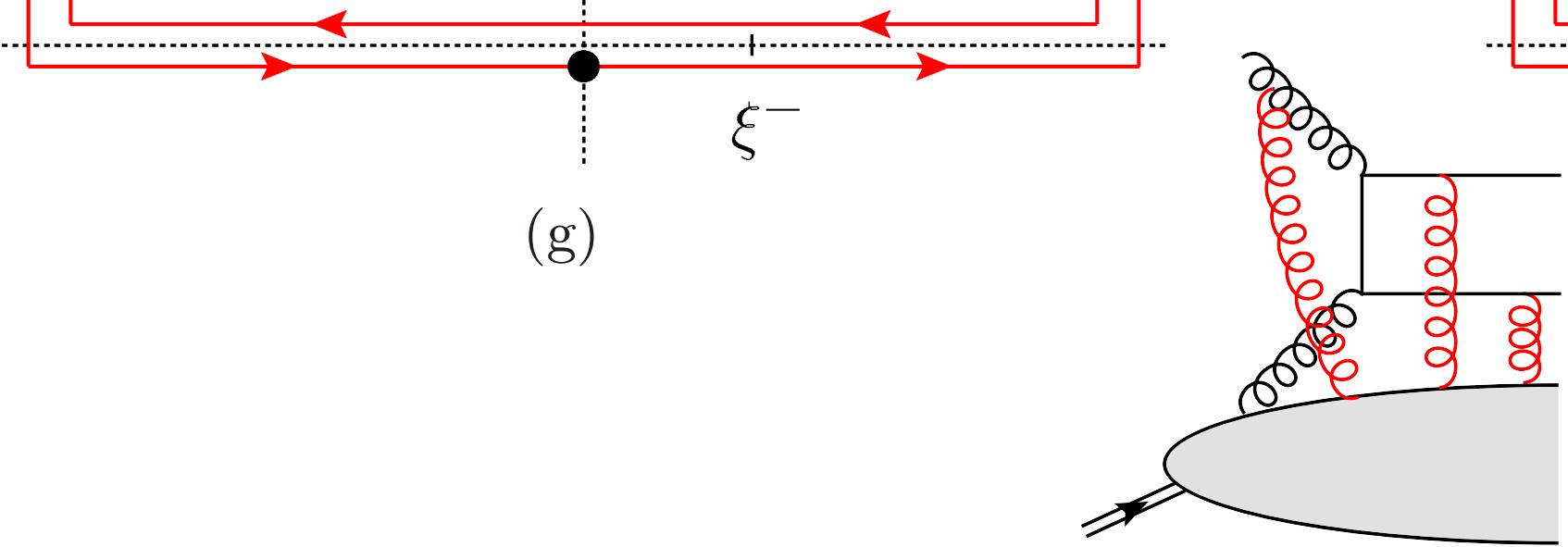
(c)  $[+, -]$  and (d)  $[-, +]$



- \* Appearance of new gauge **loop links**:

(e)  $[+ \square, + \square]$ , (f)  $[+, + \square]$ ,

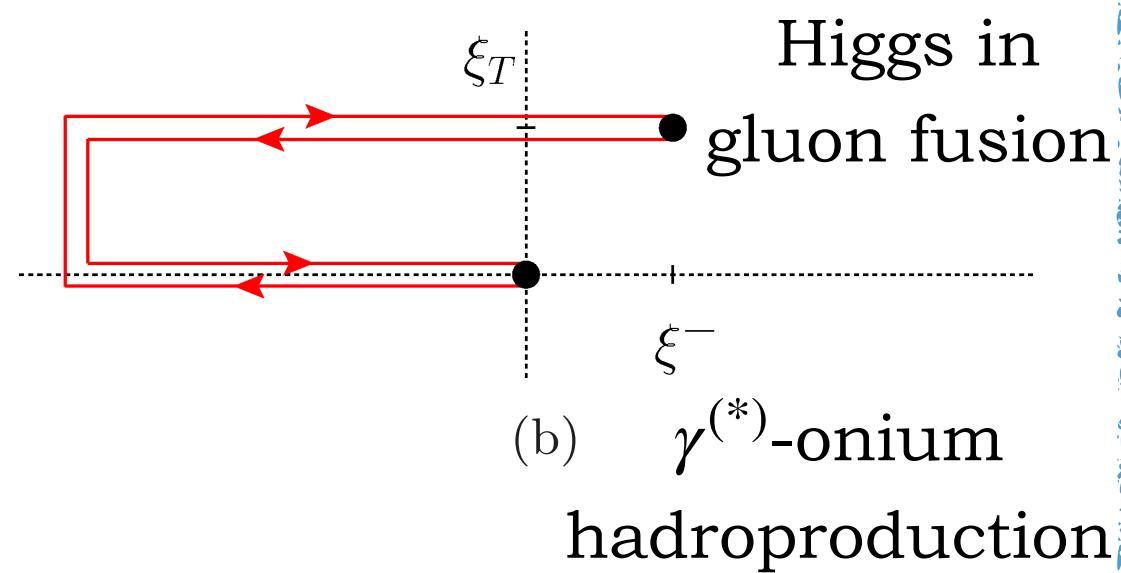
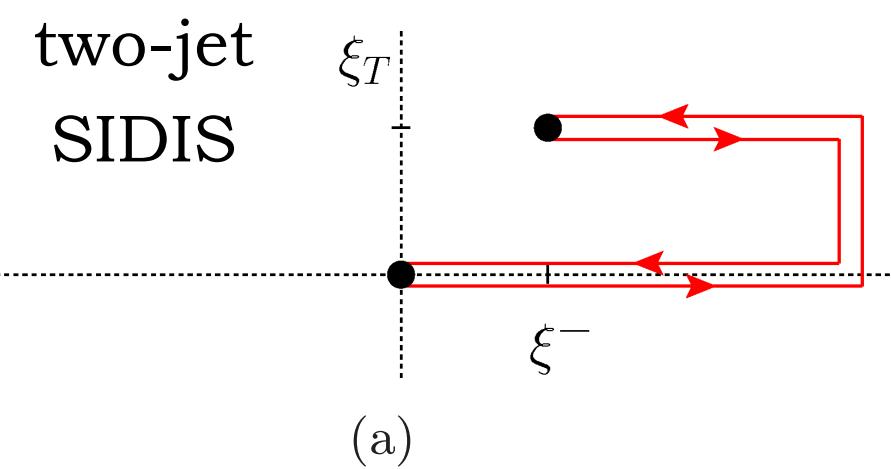
(g)  $[\square, \square]$ , and (h)  $[\square, \square]$



# Accessing WW and DP gluon TMDs

## Weiszäcker-Williams (WW)

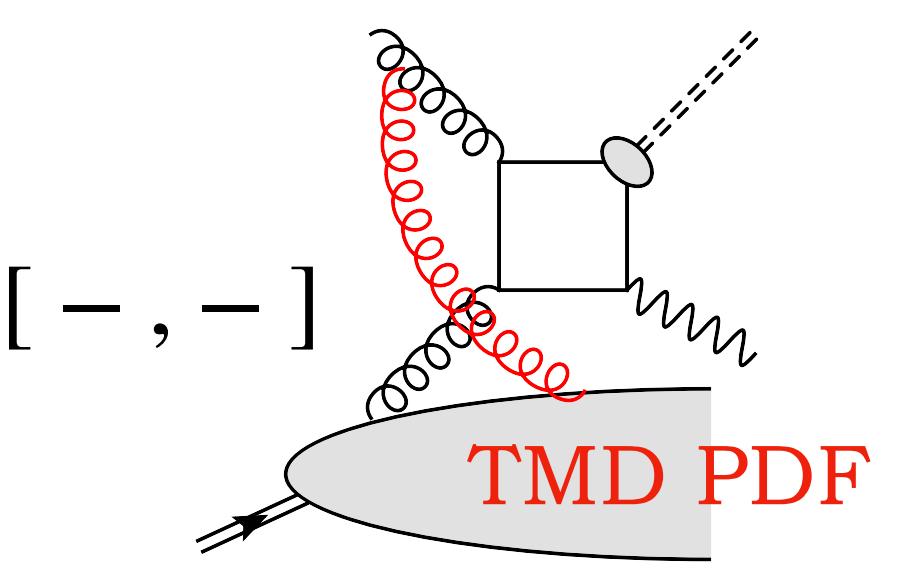
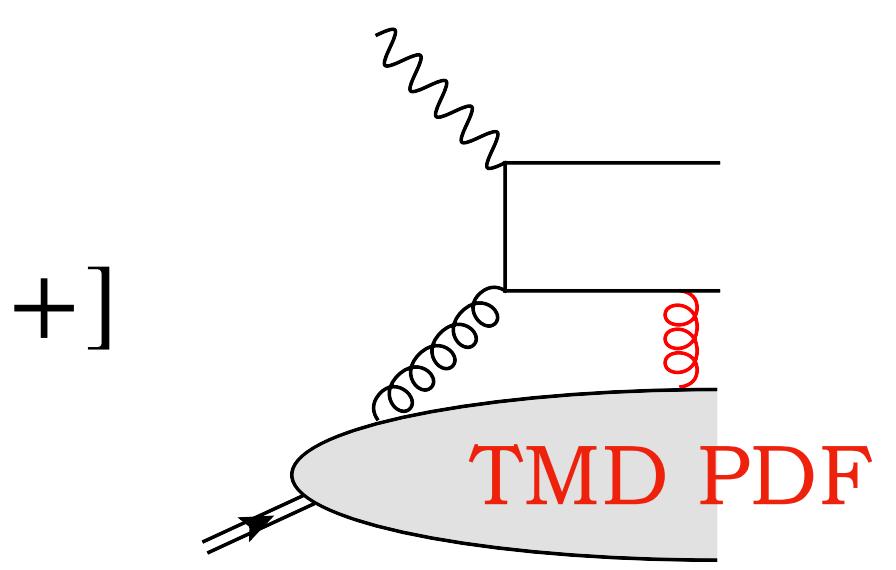
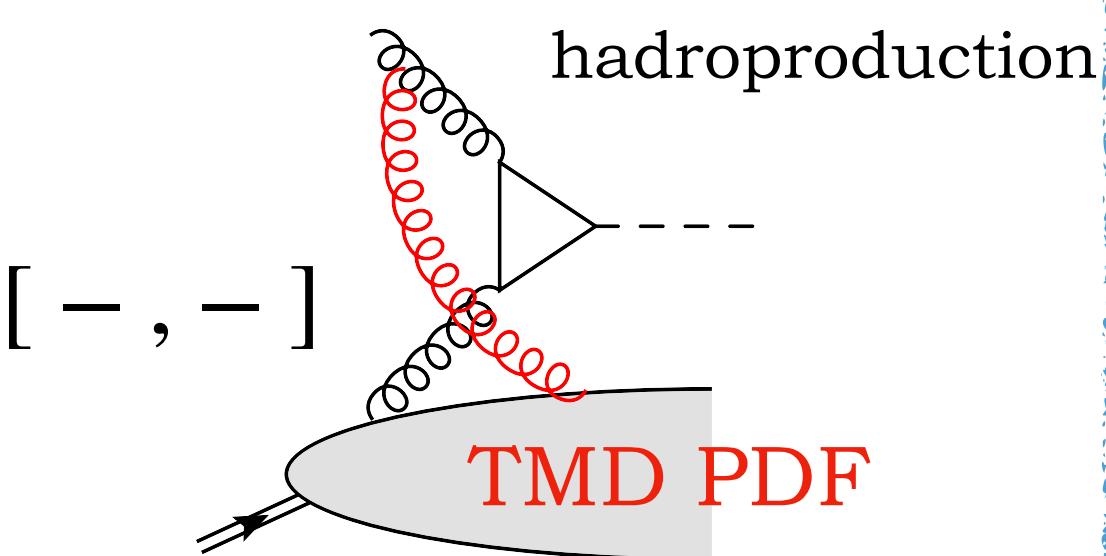
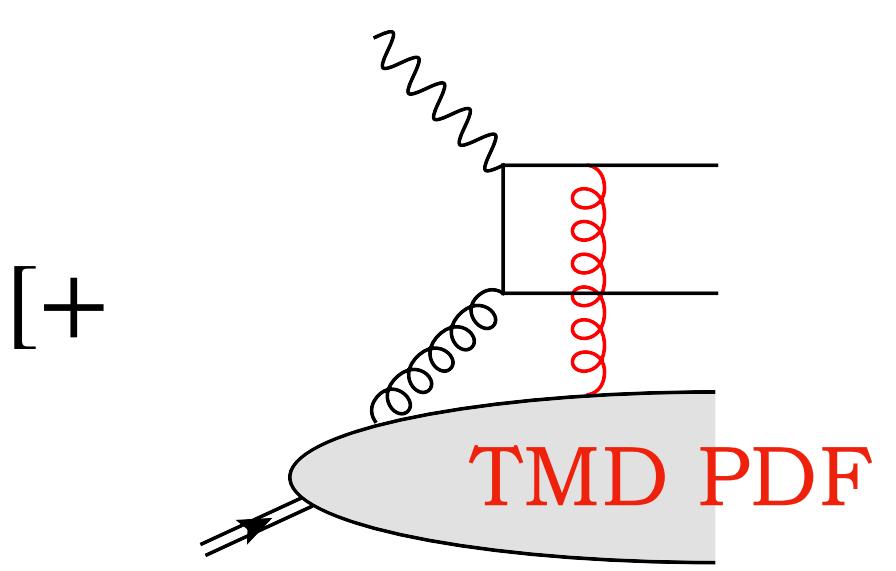
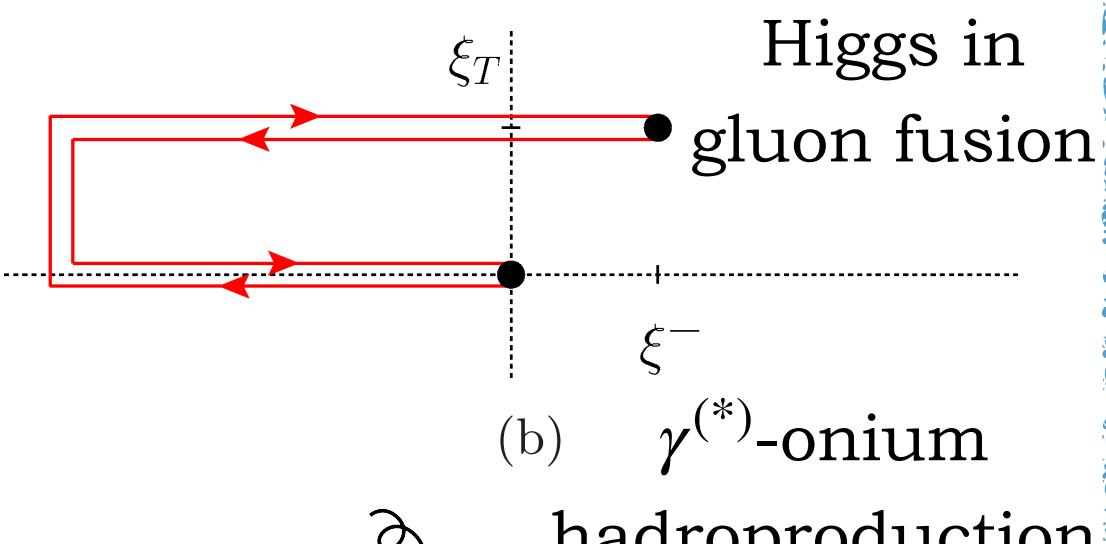
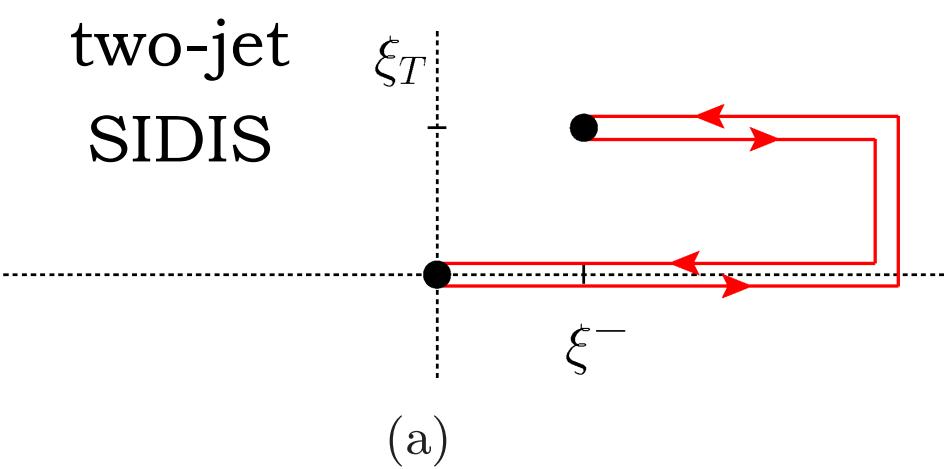
(a) [ + , + ] or (b) [ - , - ]



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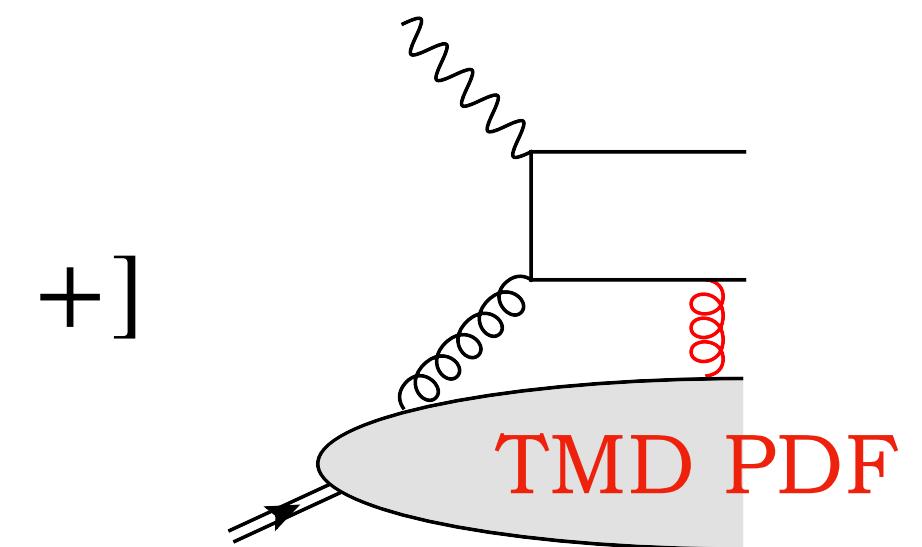
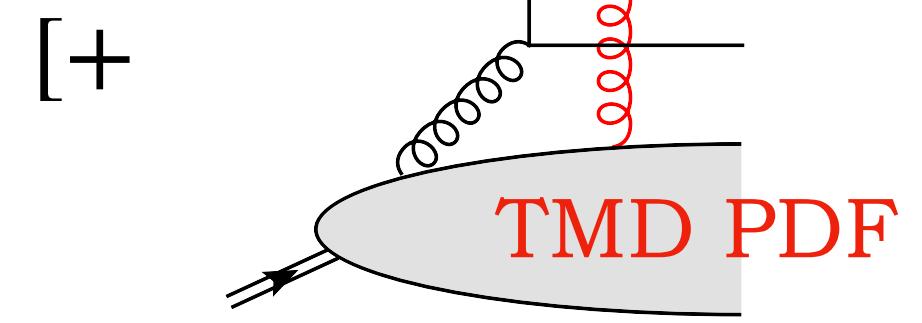
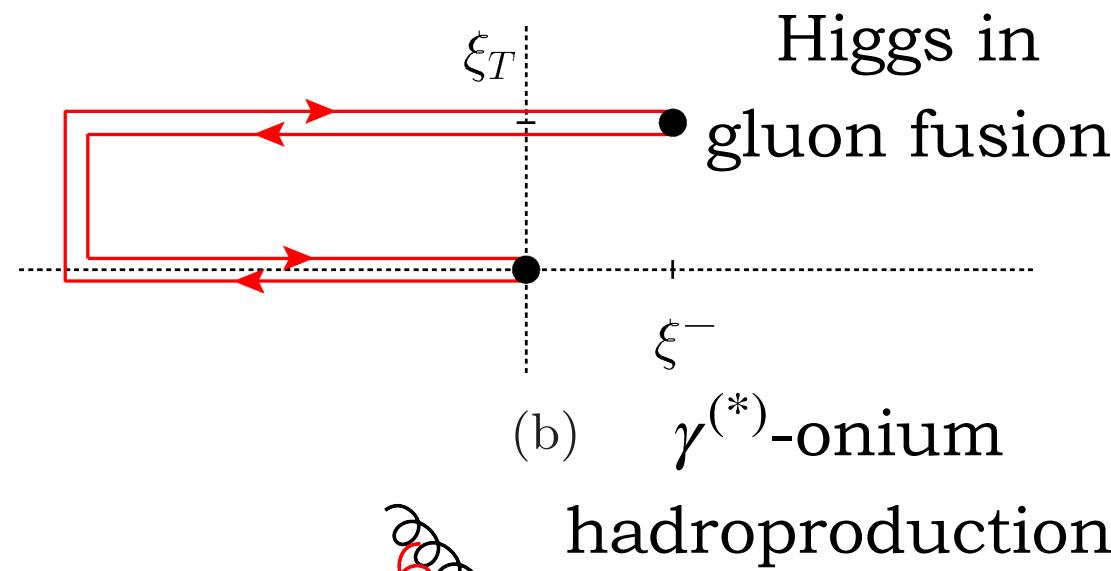
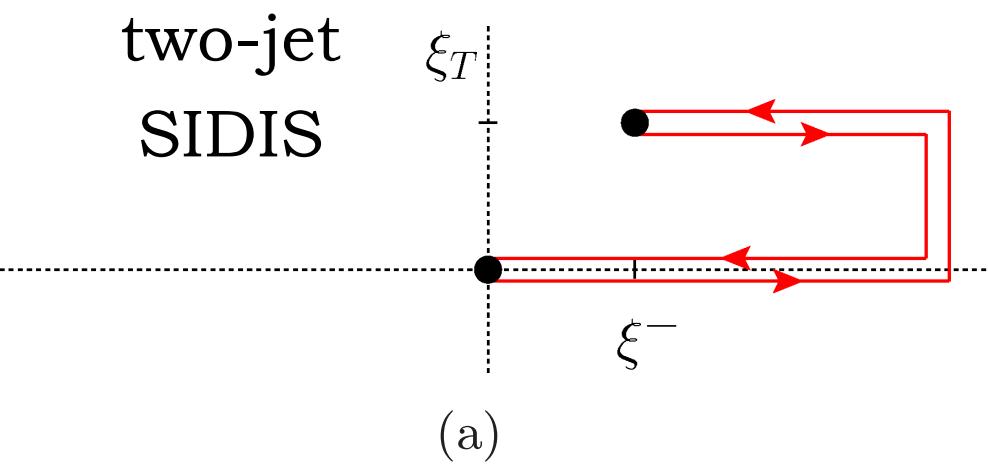
(a) [ + , + ] or (b) [ - , - ]



# Accessing WW and DP gluon TMDs

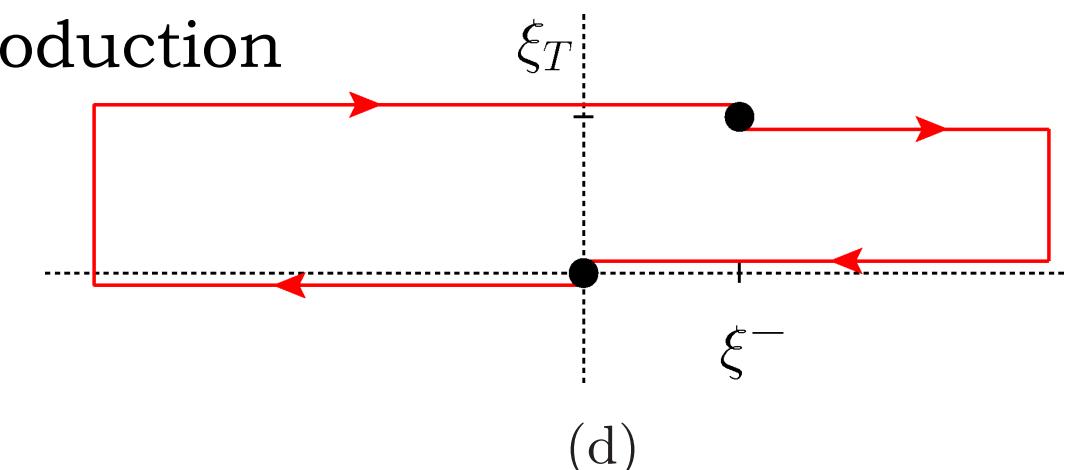
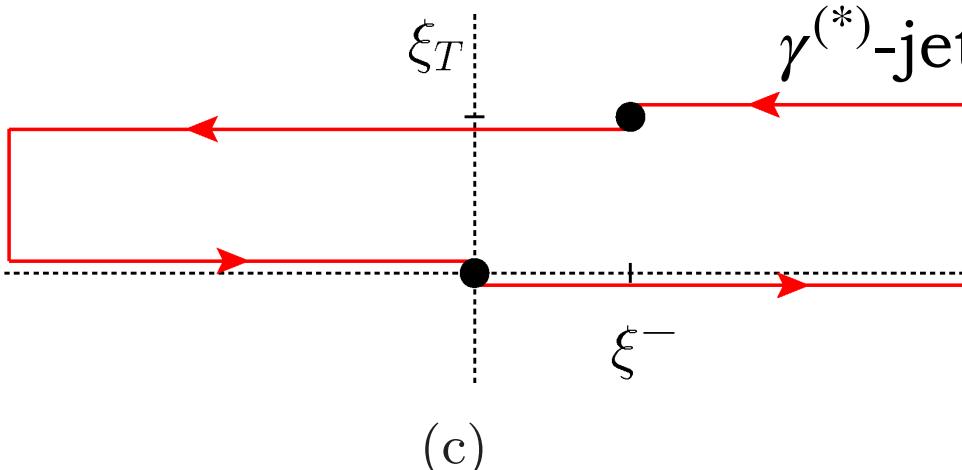
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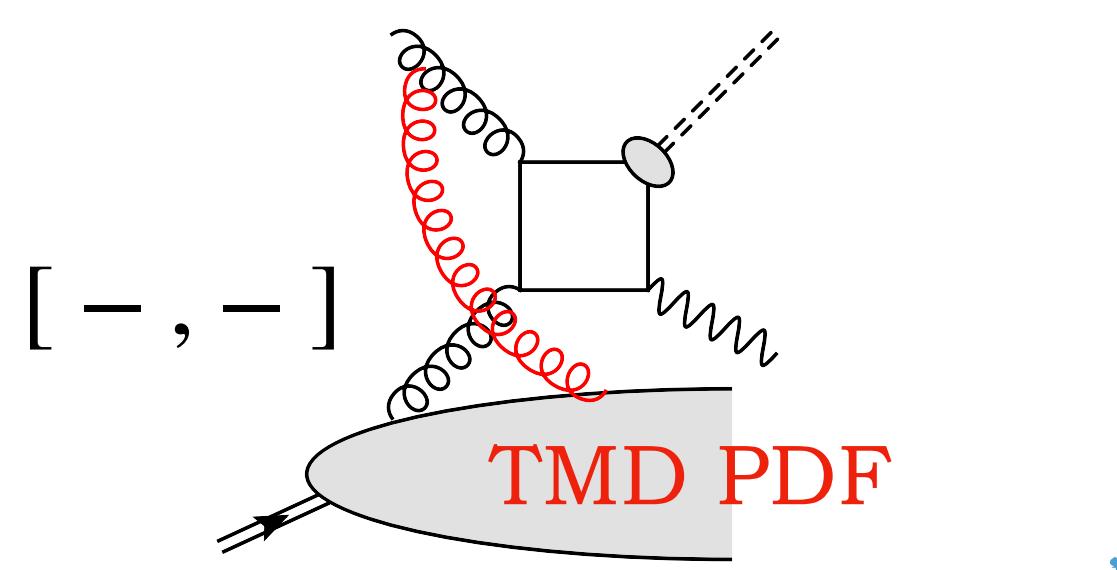
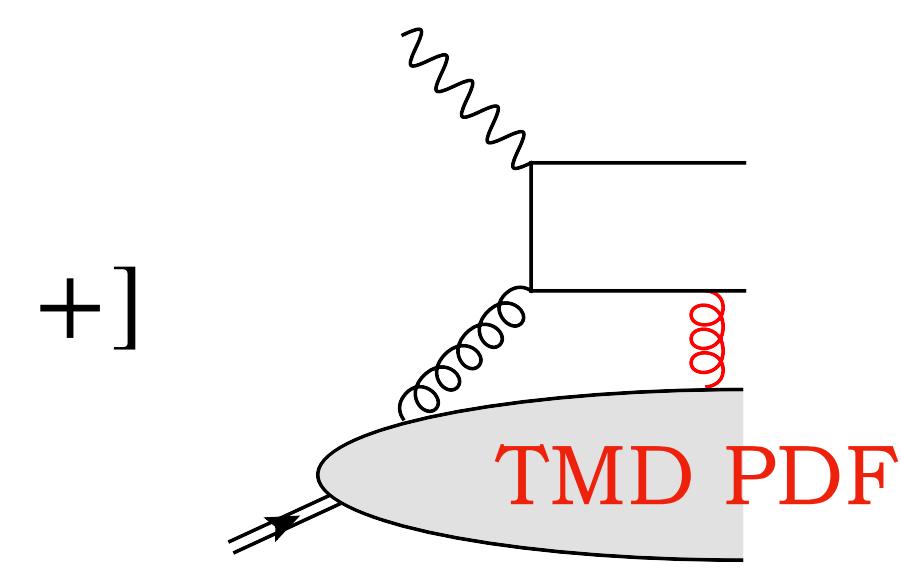
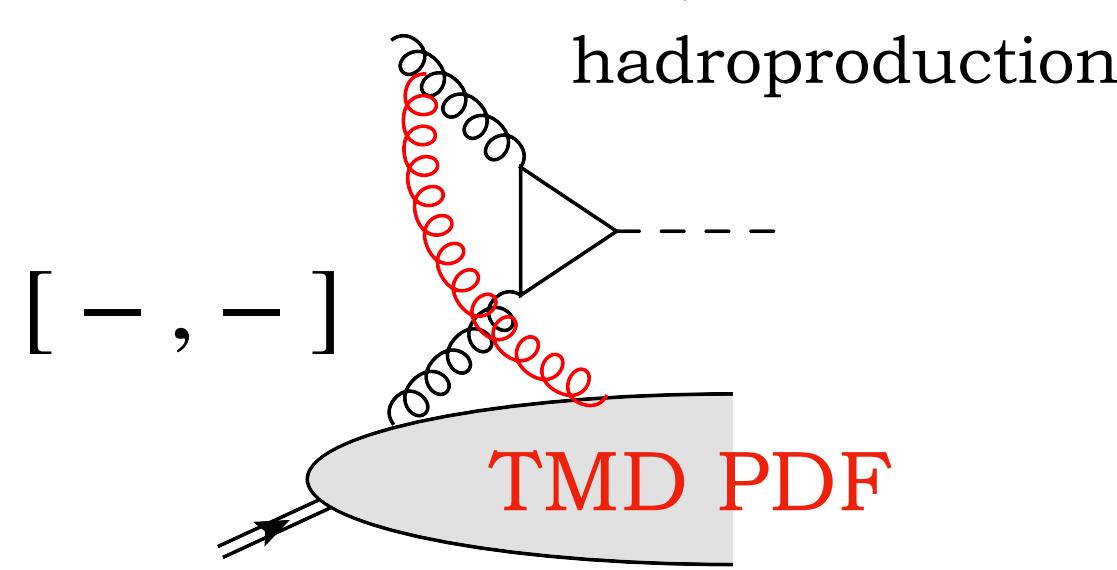
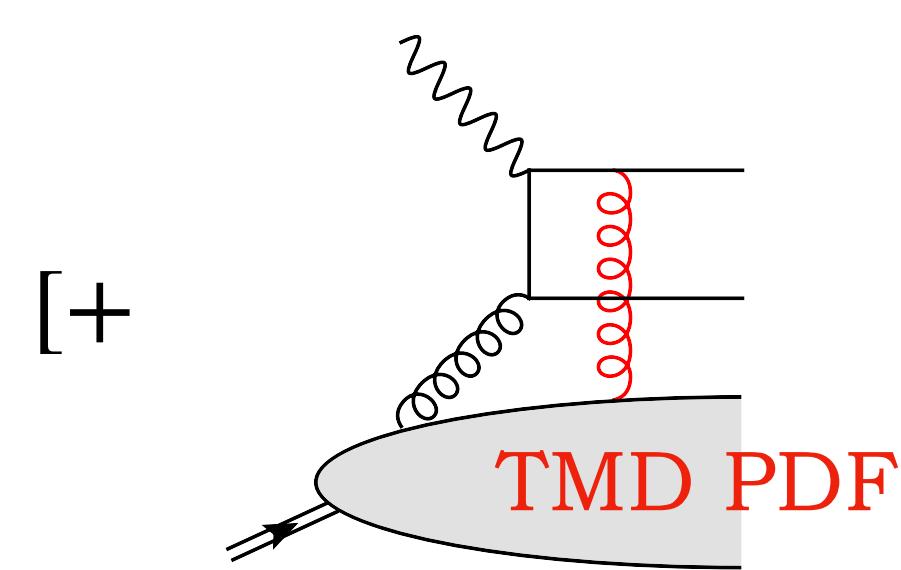
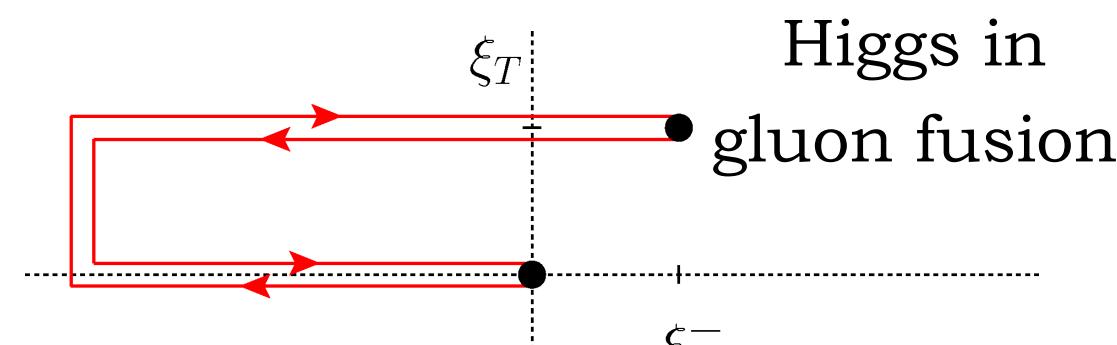
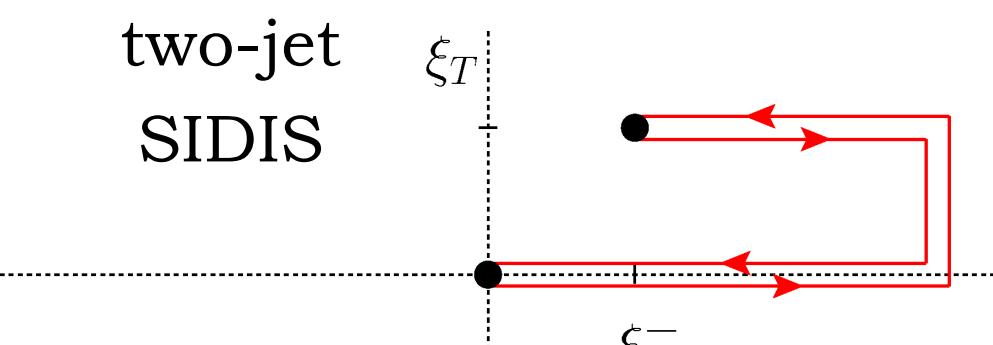
(c) [ + , - ] or (d) [ - , + ]



# Accessing WW and DP gluon TMDs

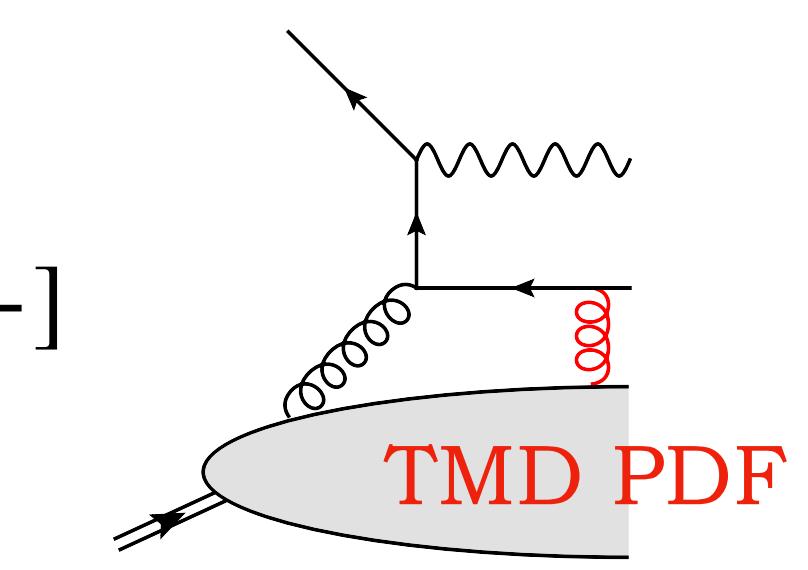
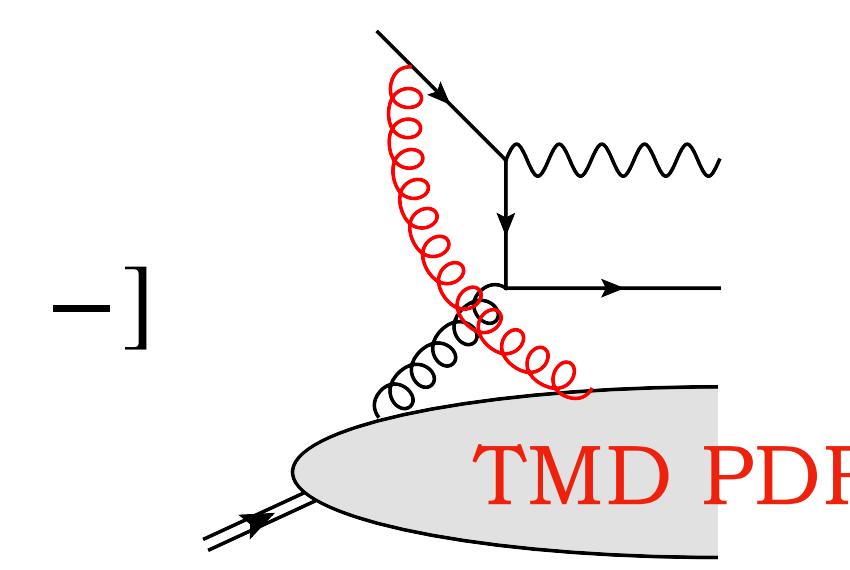
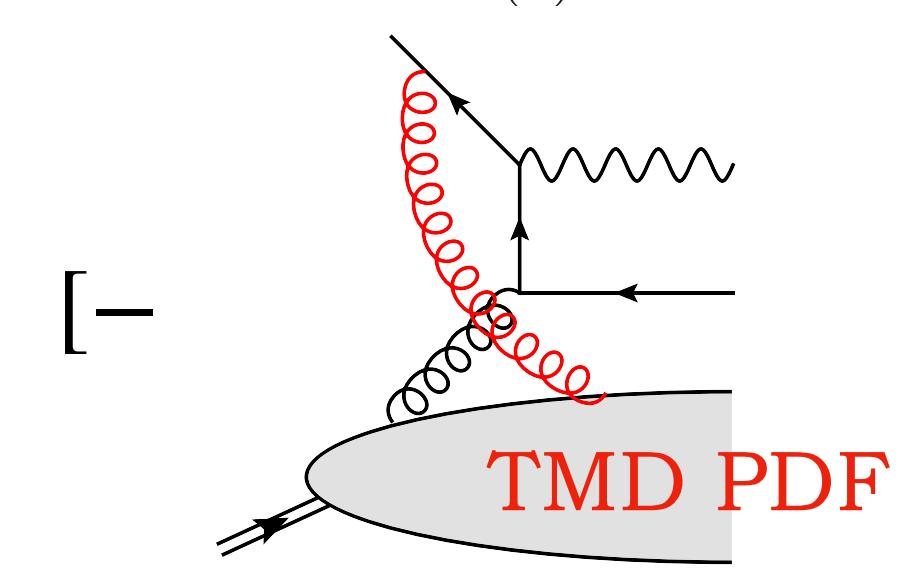
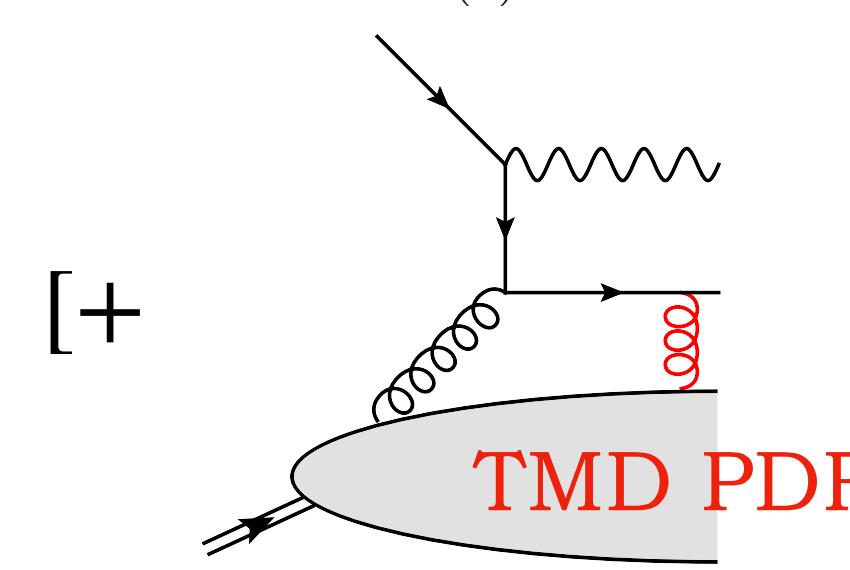
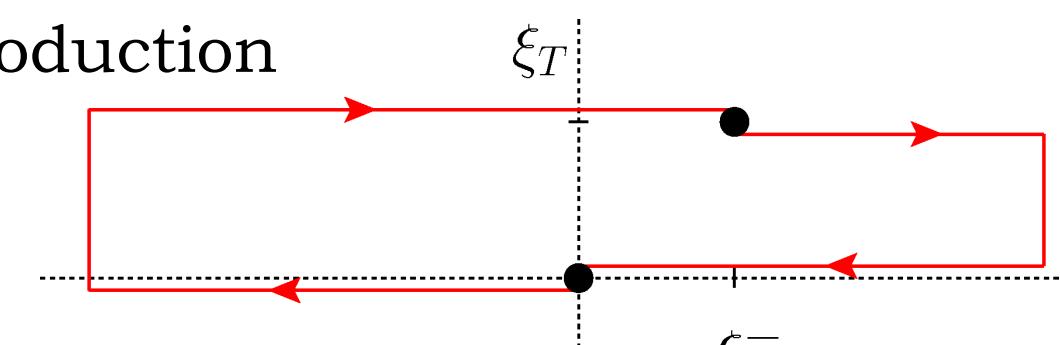
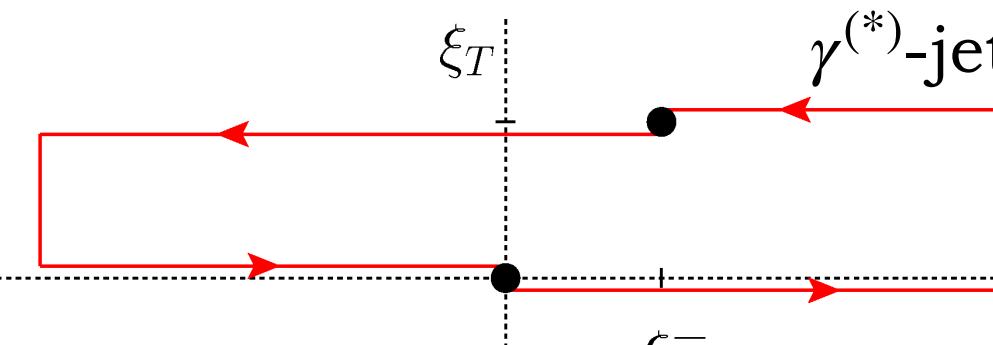
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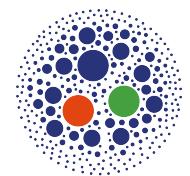


## Dipole (DP)

(c) [ + , - ] or (d) [ - , + ]

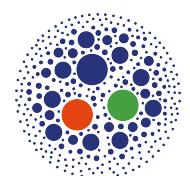
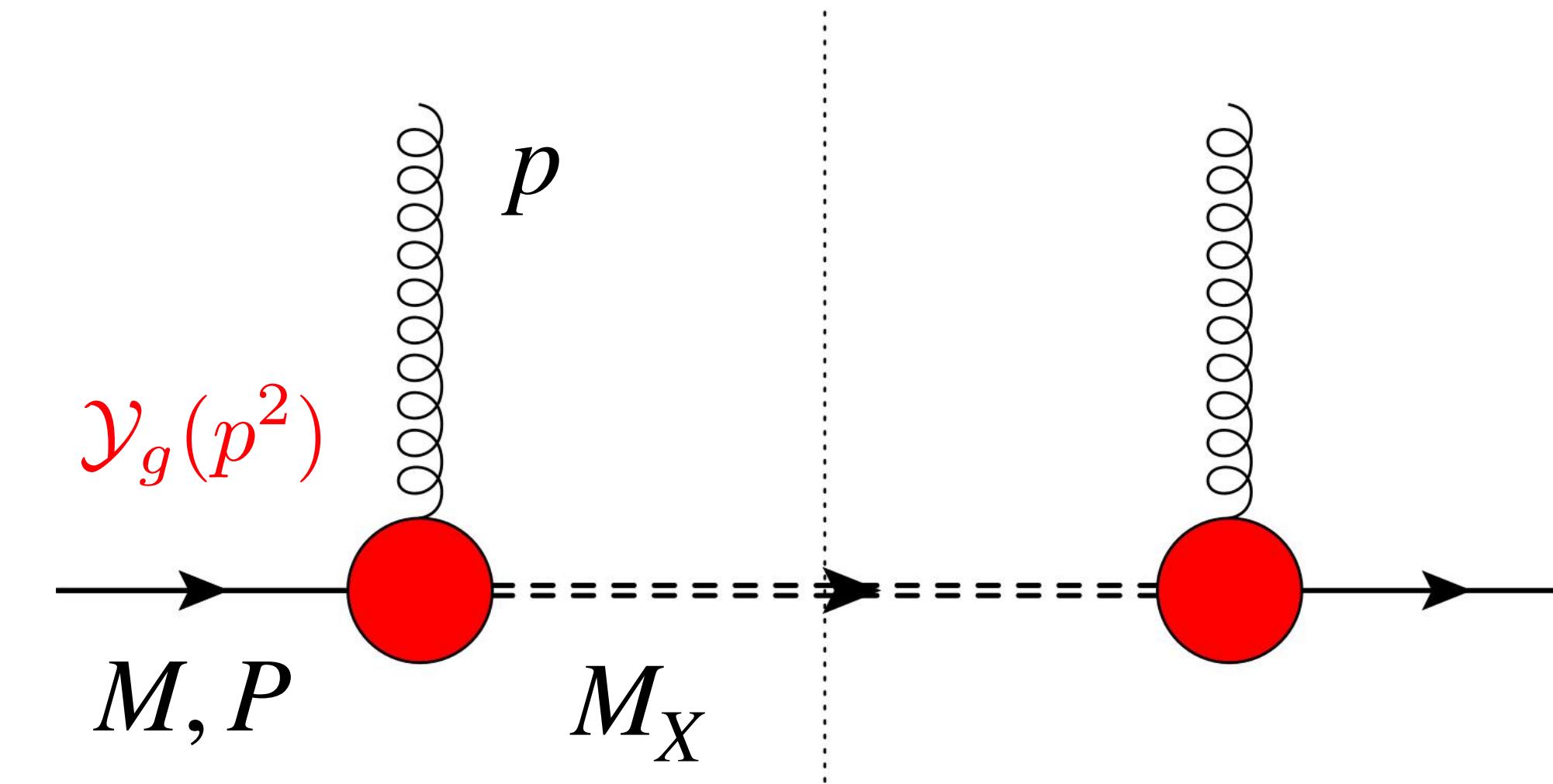


# Assumptions of the model



## Effective vertex

Lowest Fock state:  
**tri-quark spectator**  
on-shell and  
with mass  $M_X$



## Spin-1/2 spectator (gluon)

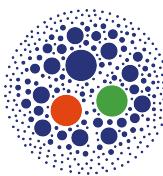
$$\Phi_g = \frac{1}{2(2\pi)^3(1-x)P^+} Tr \left[ (\not{P} + M) \frac{1 + \gamma^5 \$}{2} G_{\mu\rho}^*(p) G^{\nu\sigma}(p) \gamma_g^{\rho*} \gamma_{g\sigma}(\not{P} - \not{p} + M) \right]$$

$$\gamma_g^\mu = g_1(p^2) \gamma^\mu + i \frac{g_2(p^2)}{2M} \sigma^{\mu\nu} p_\nu$$



mimics proton form factors  
(conserved EM current  
of a free nucleon)

# Assumptions of the model



## Link with collinear factorization

$p_T$ -integrated TMDs **have to** reproduce PDFs at the lowest scale ( $Q_0$ ) *before* evolution

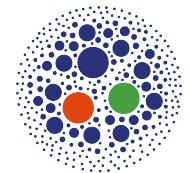


## Dipolar form factor(s)

$$g_{1,2}(p^2) = \kappa_{1,2} \frac{p^2}{|p^2 - \Lambda_X^2|^2}$$

1. Cancels singularity of gluon propagator
2. Suppresses effects of high  $p_T$
3. Compensates log divergences arising from  $p_T$ -integration
4. Adds three more parameters:  $\kappa_{1,2}$  and  $\Lambda_X$

# Assumptions of the model



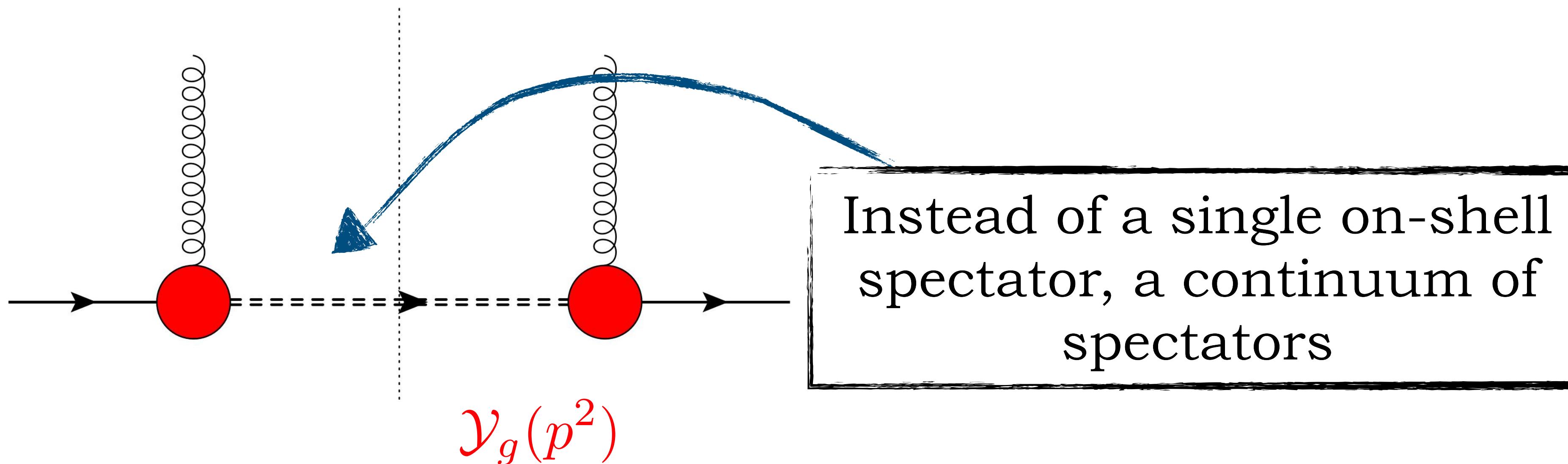
## Spectator-system spectral-mass function

$$F(x, \mathbf{p}_T^2) = \int_M^\infty dM_X \rho_X(M_X) \hat{F}(x, \mathbf{p}_T^2; M_X)$$

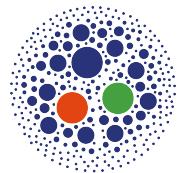
spectral-mass function

spectator-model TMD

[Inspired by G.R. Goldstein, J.O.G. Hernandez, S. Liuti (2011)]



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spectral-mass function

spectator-model TMD

∅ [Inspired by G.R. Goldstein, J.O.G. Hernandez, S. Liuti (2011)]

$$\rho_X(M_X; \{X^{(\text{pars})}\} \equiv \{A, B, a, b, C, D, \sigma\}) = \mu^{2a} \left[ \frac{A}{B + \mu^{2b}} + \frac{C}{\pi\sigma} e^{-\frac{(M_X - D)^2}{\sigma^2}} \right]$$

low- $x$  (high- $\mu^2$ ) tail  $\propto (a - b)$

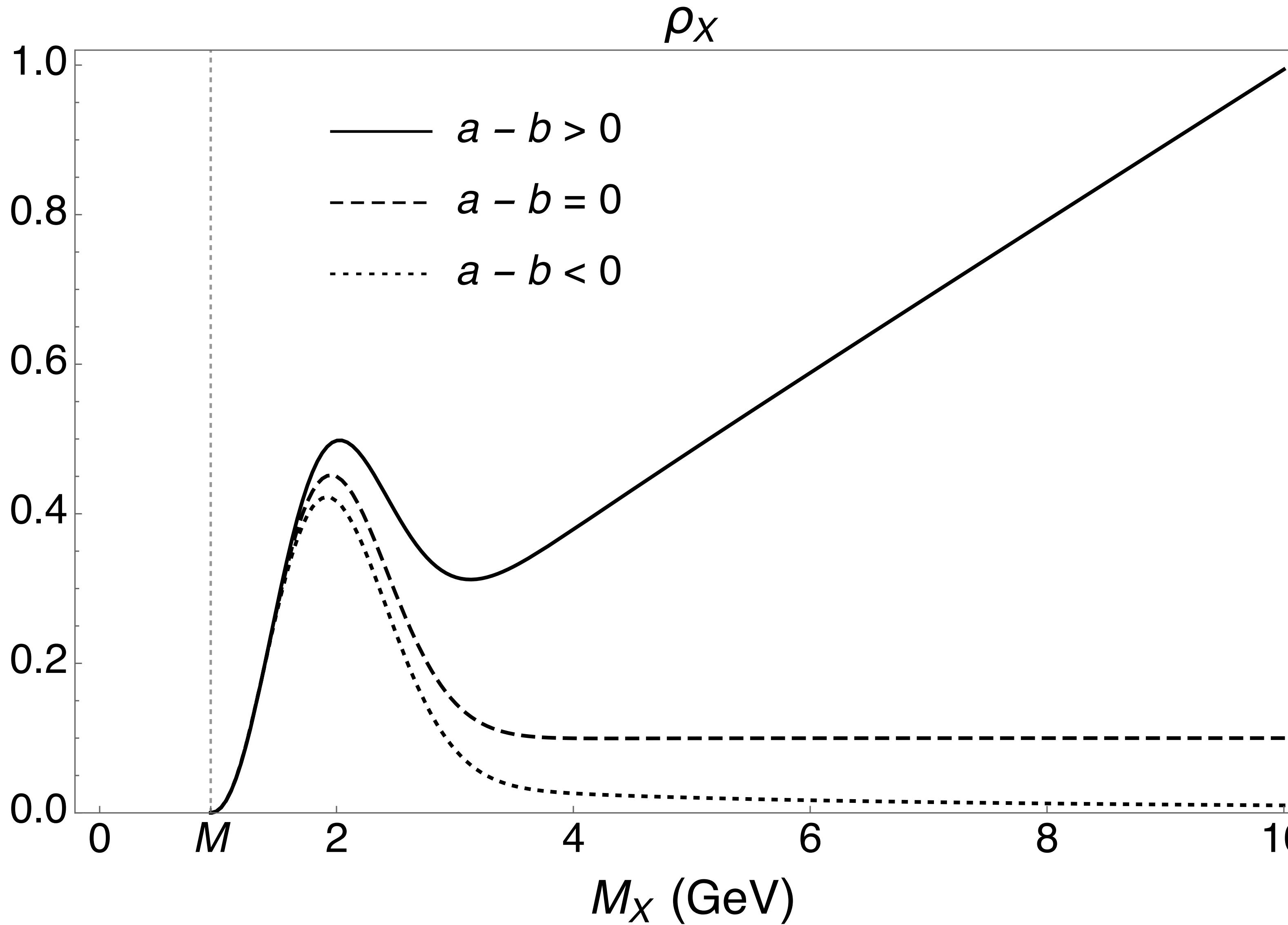
$q\bar{q}$  contributions energetically available at large  $M_X$

$$\mu^2 = M_X^2 - M^2$$

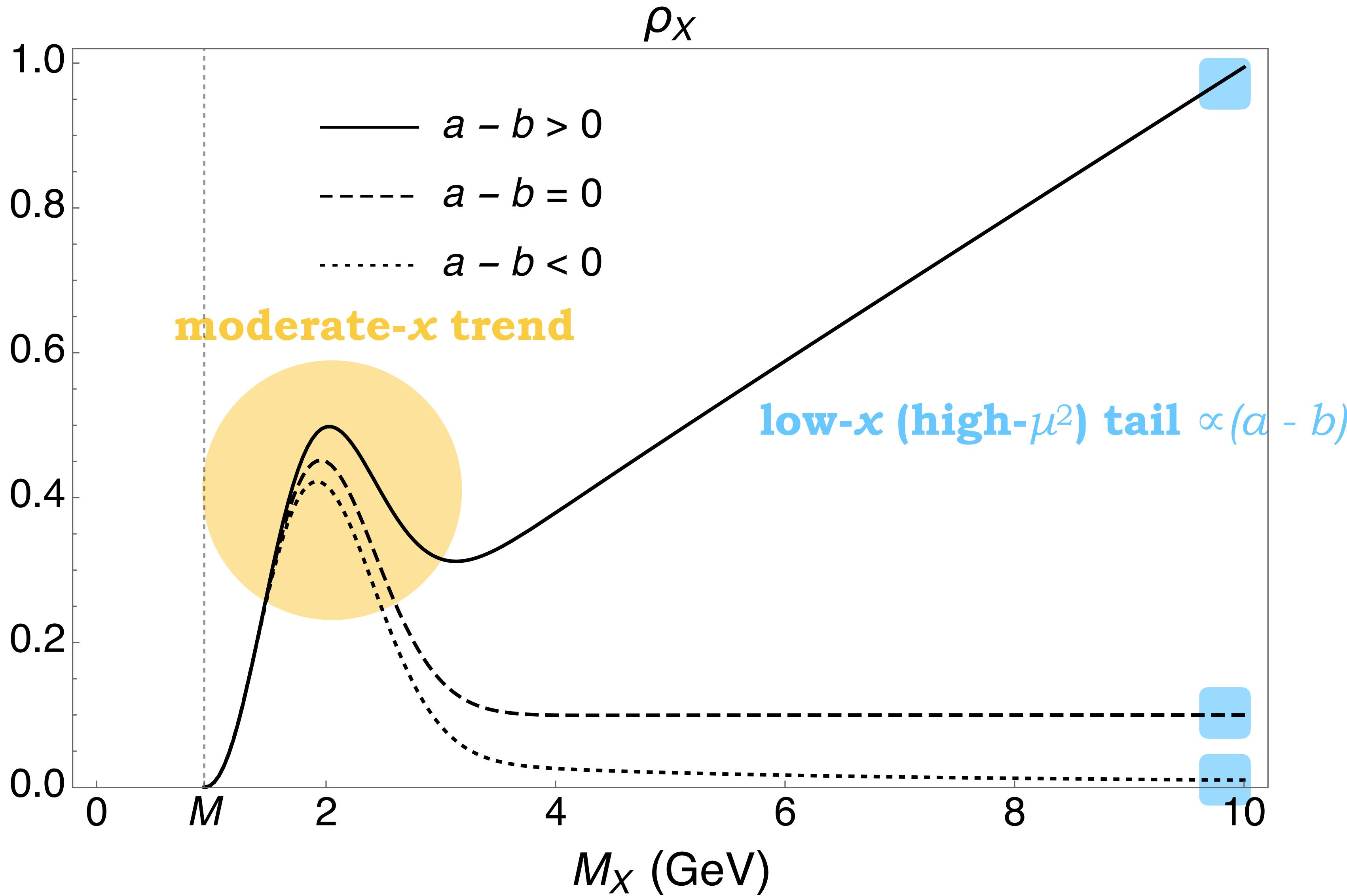
moderate- $x$  trend

pure tri-quark contribution at low  $M_X$

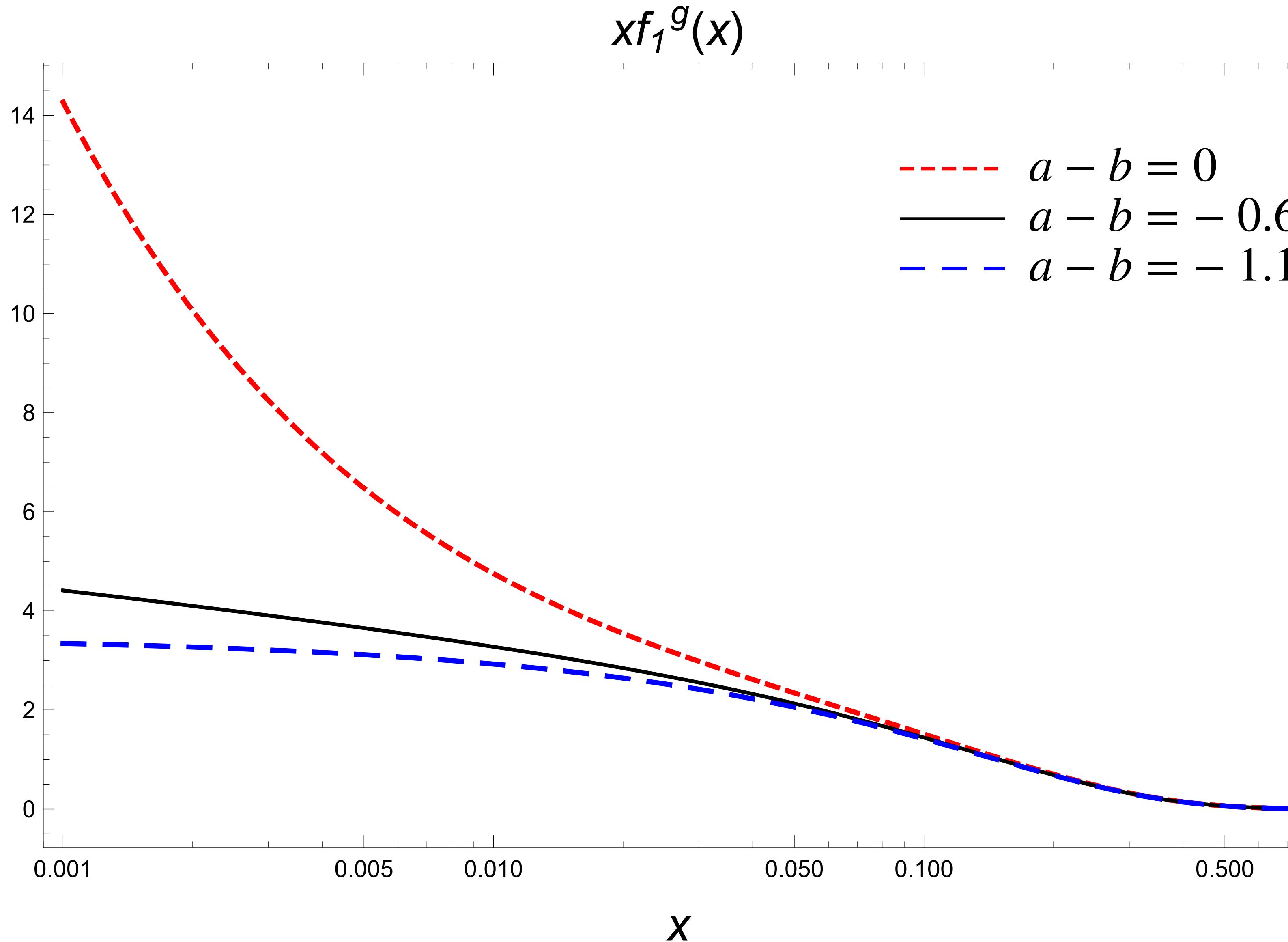
# Spectral function vs $(a - b)$



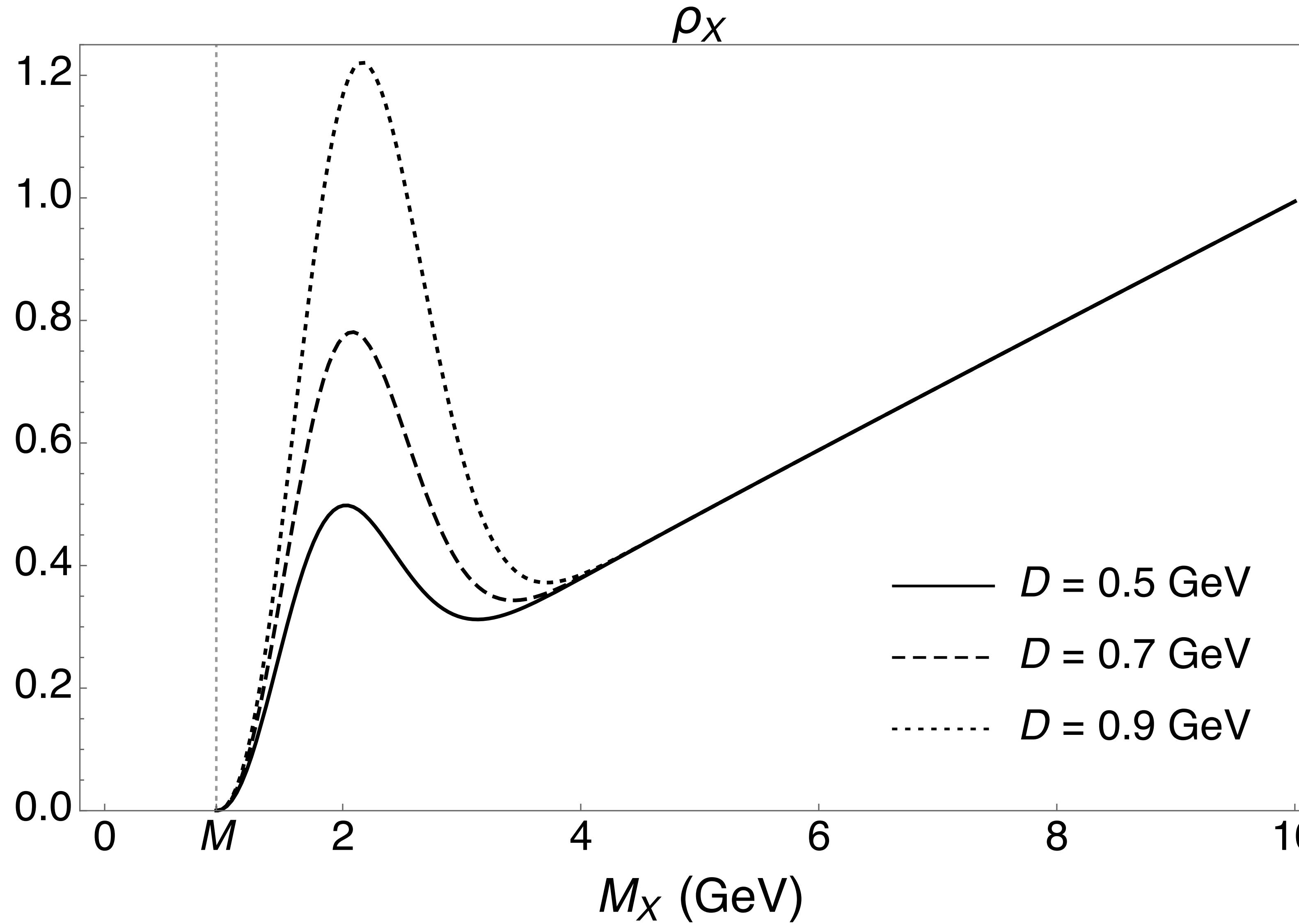
# Spectral function vs $(a - b)$



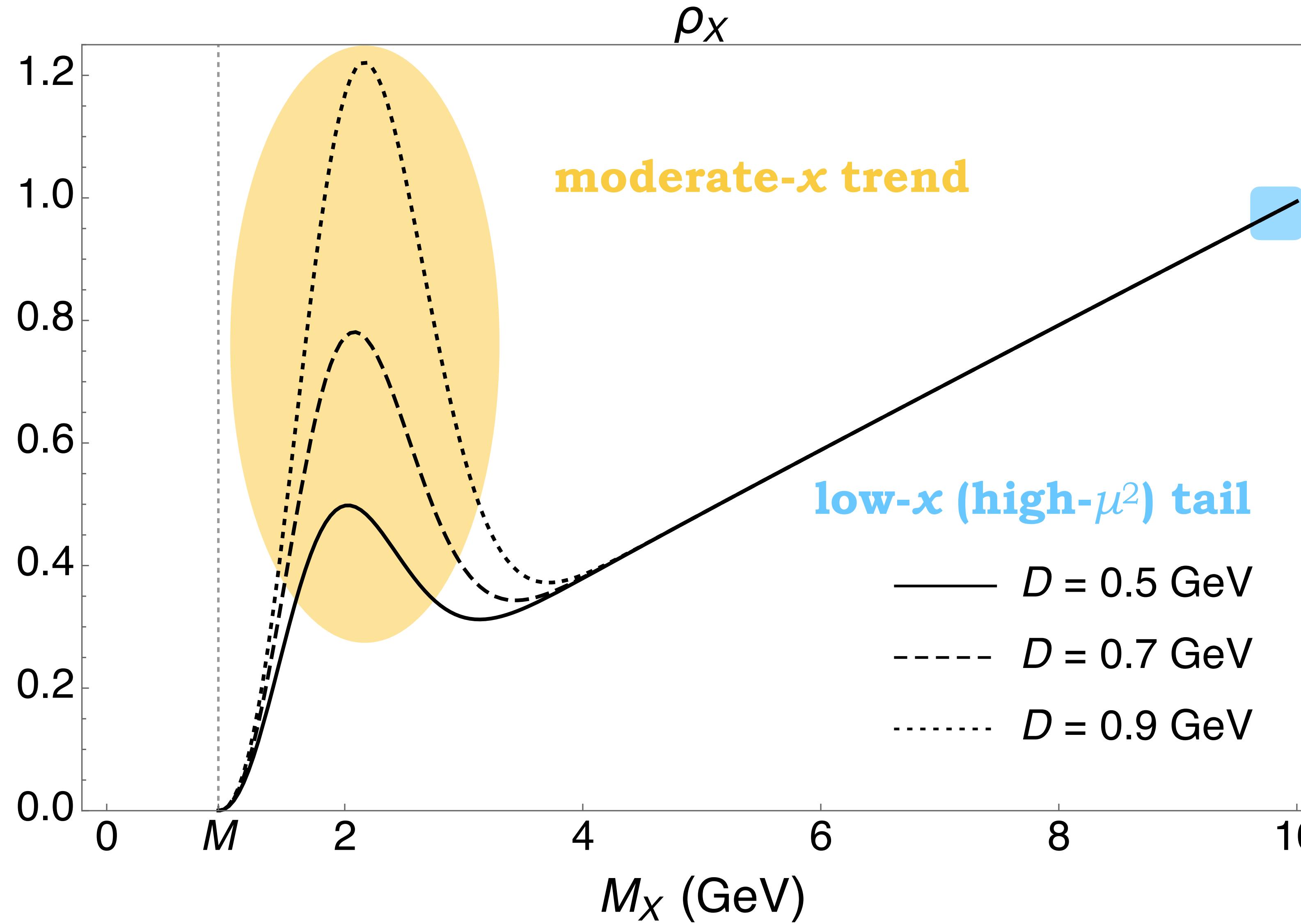
# $xf_1$ collinear PDF vs $(a - b)$



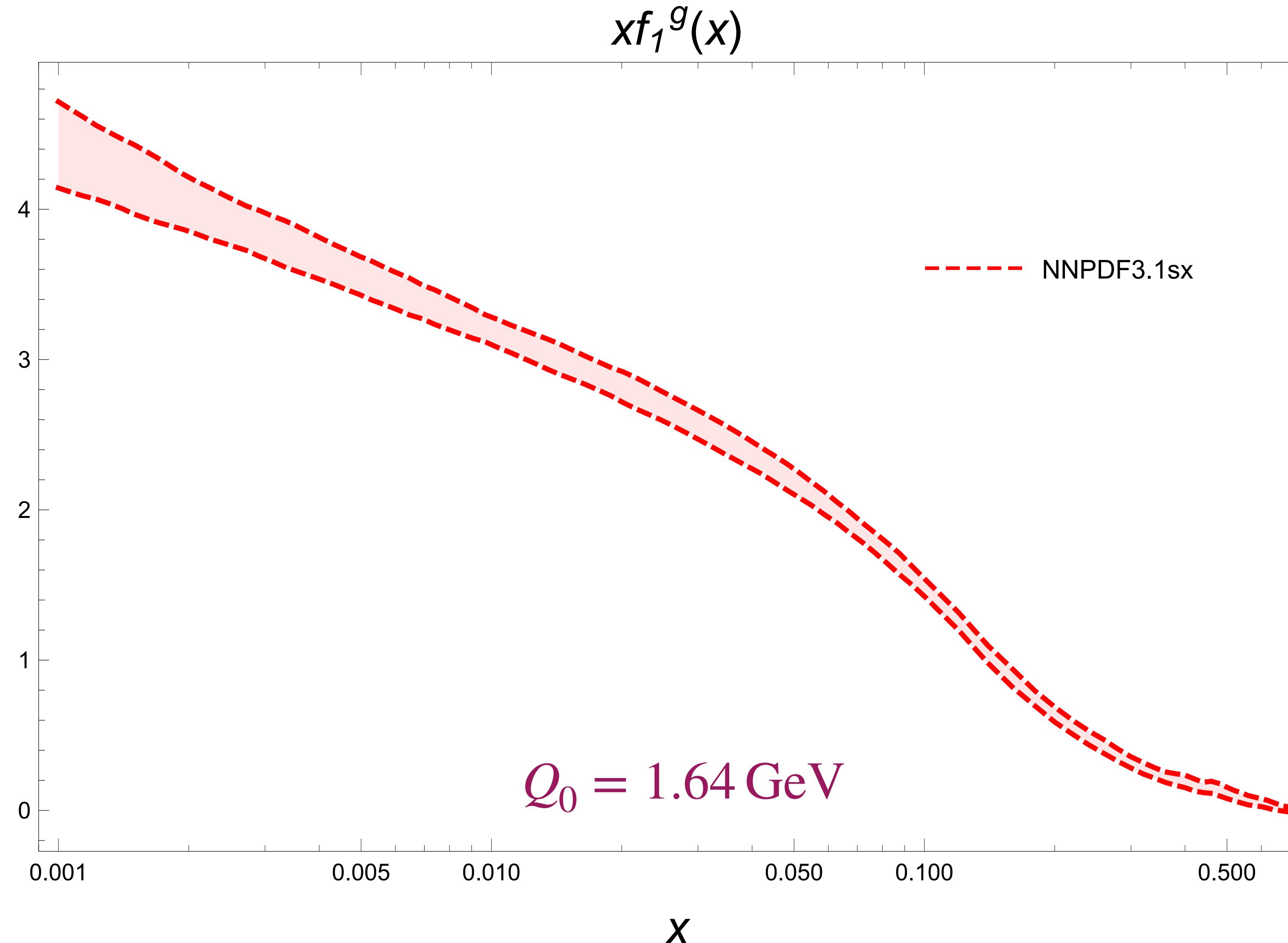
# Spectral function vs $D$



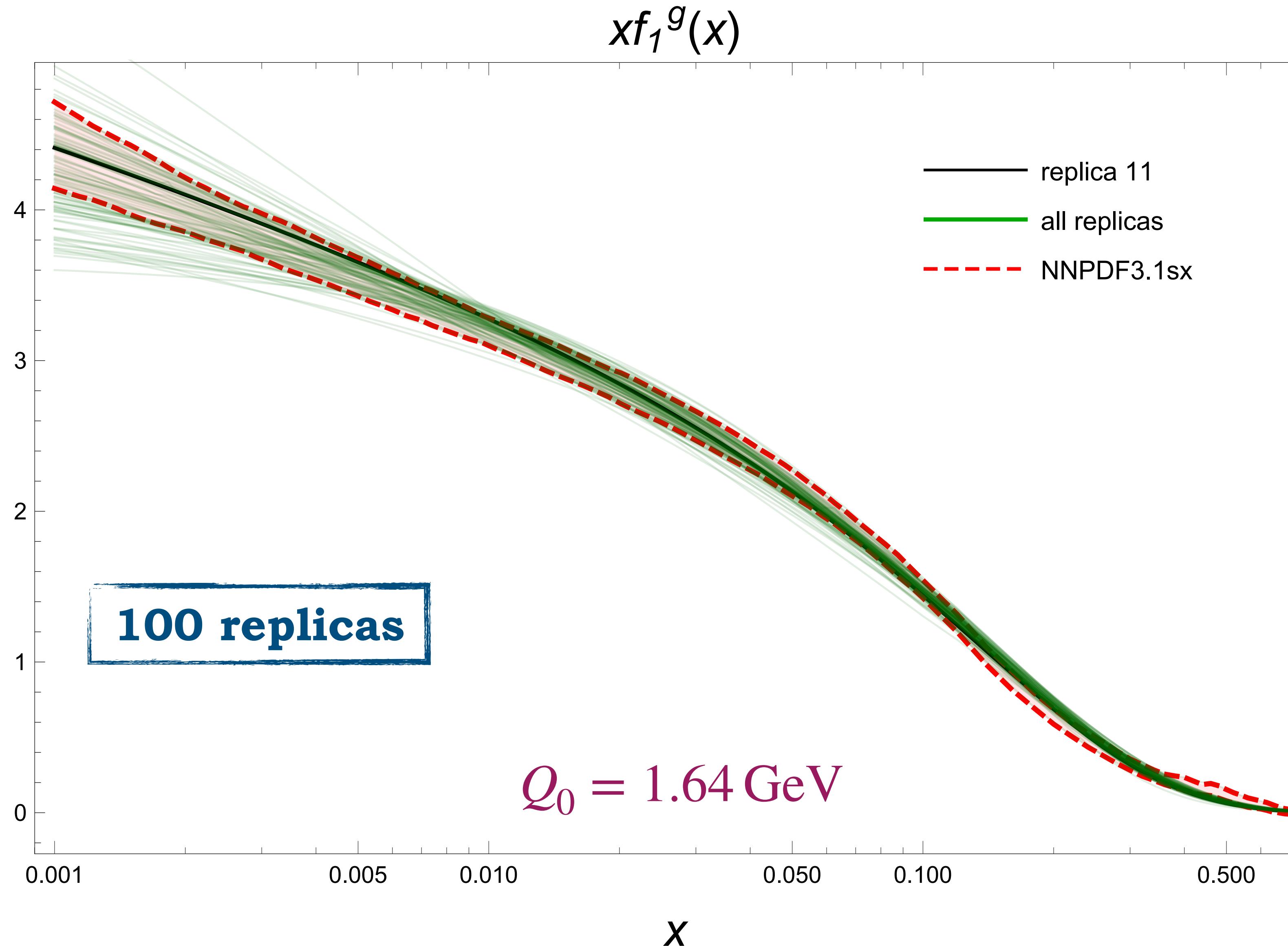
# Spectral function vs $D$



# Unpolarized gluon PDF

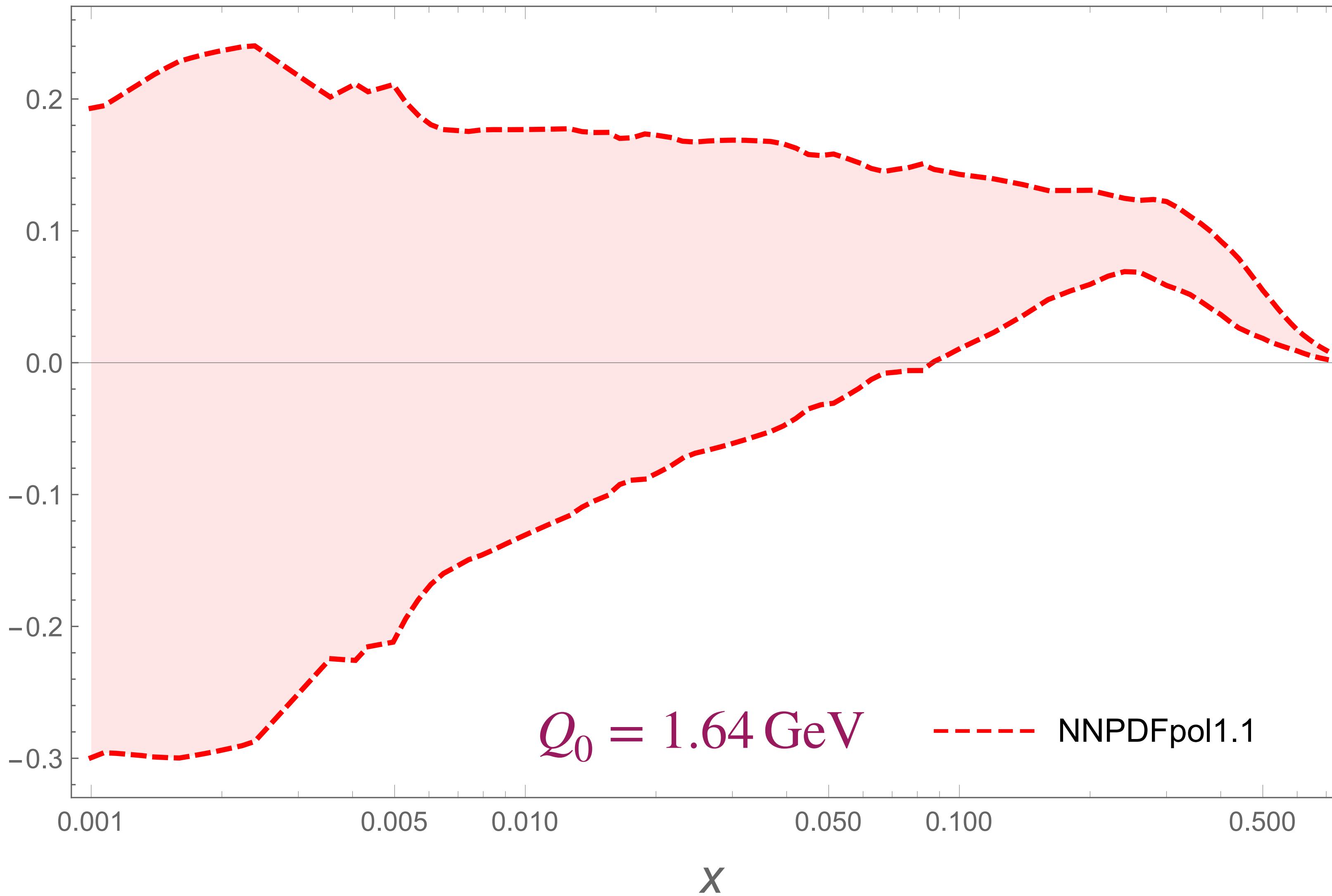


# Unpolarized gluon PDF



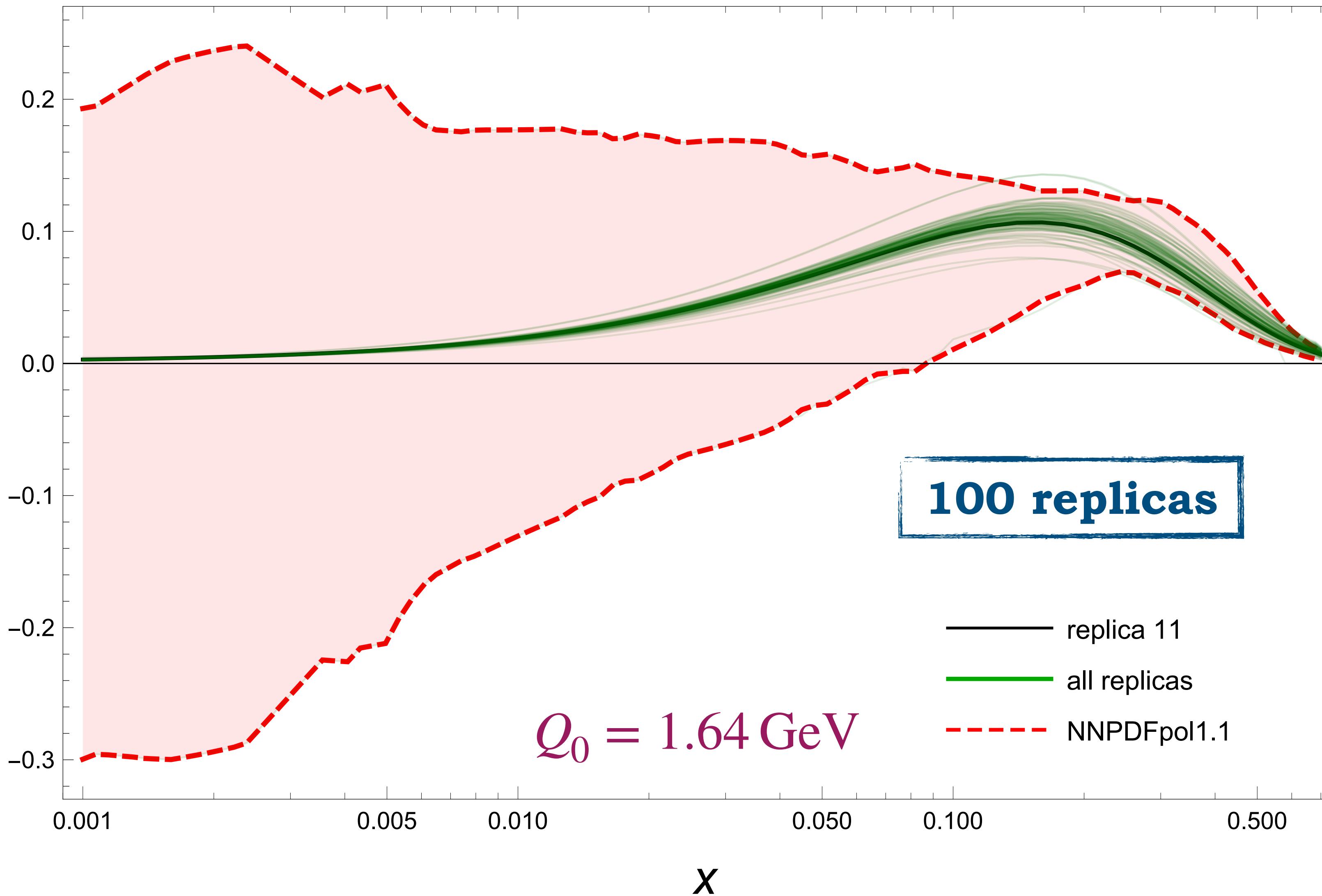
# Helicity gluon PDF

$xg_1^g(x)$



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$xg_1^g(x)$



# Fit specifics

$$\chi^2/\text{d.o.f.} = 0.54 \pm 0.38$$

no **overlearning**, just large errors for  $g_1$

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$$\langle x \rangle_g = \int_0^1 dx x f_1^g(x, Q_0)$$

$$S_g = \frac{1}{2} \langle 1 \rangle_{\Delta g} = \int_0^1 dx g_1^g(x, Q_0)$$

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$$\chi^2/\text{d.o.f.} = 0.54 \pm 0.38$$

no **overlearning**, just large errors for  $g_1$

$$\langle x \rangle_g = \int_0^1 dx x f_1^g(x, Q_0)$$

$$S_g = \frac{1}{2} \langle 1 \rangle_{\Delta g} = \int_0^1 dx g_1^g(x, Q_0)$$

Our model @  $Q_0 = 1.64$  GeV

$$\langle x \rangle_g = 0.424(9)$$

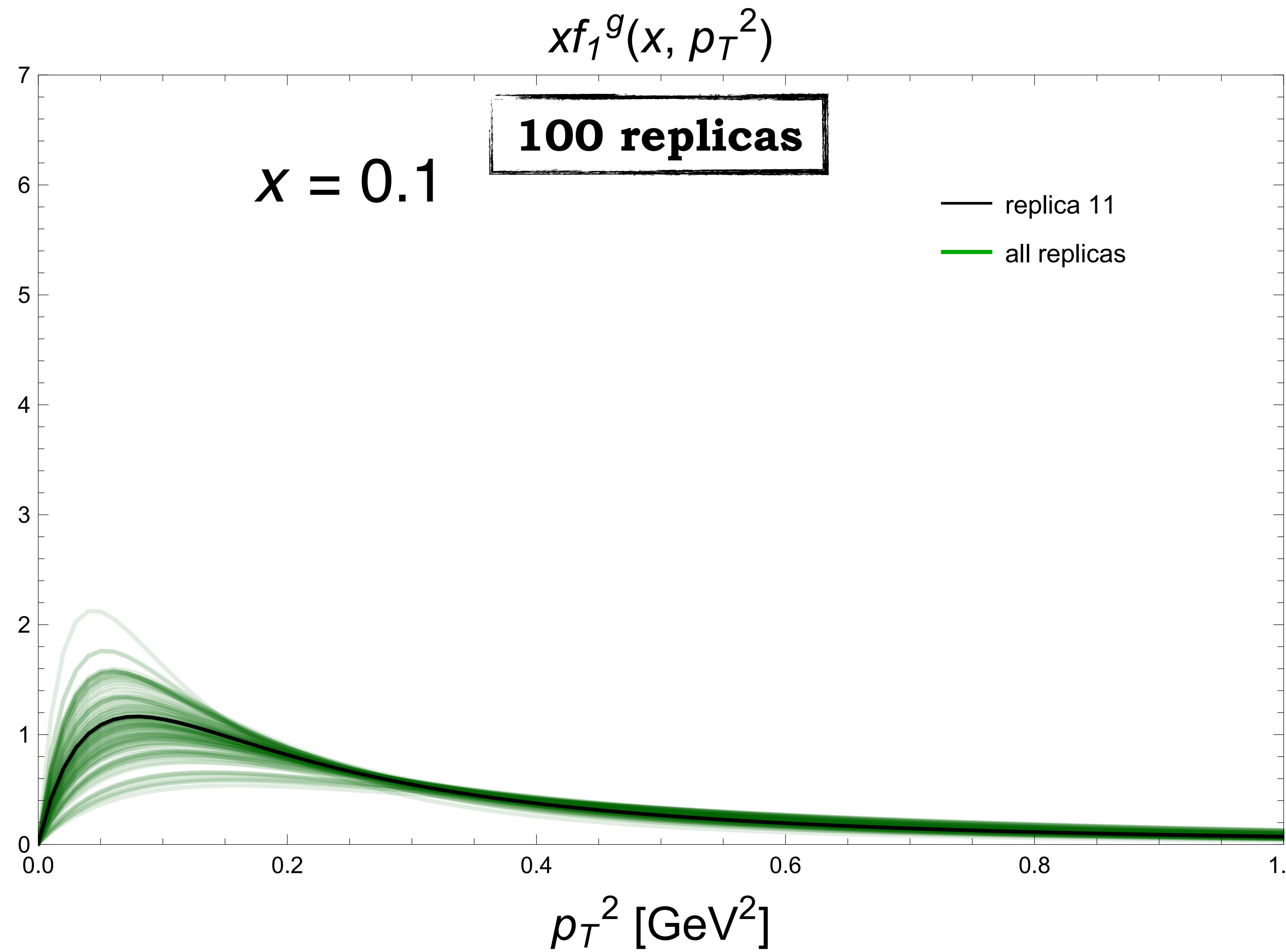
$$\langle S \rangle_g = 0.159(11)$$

Lattice @  $Q_0 = 2$  GeV

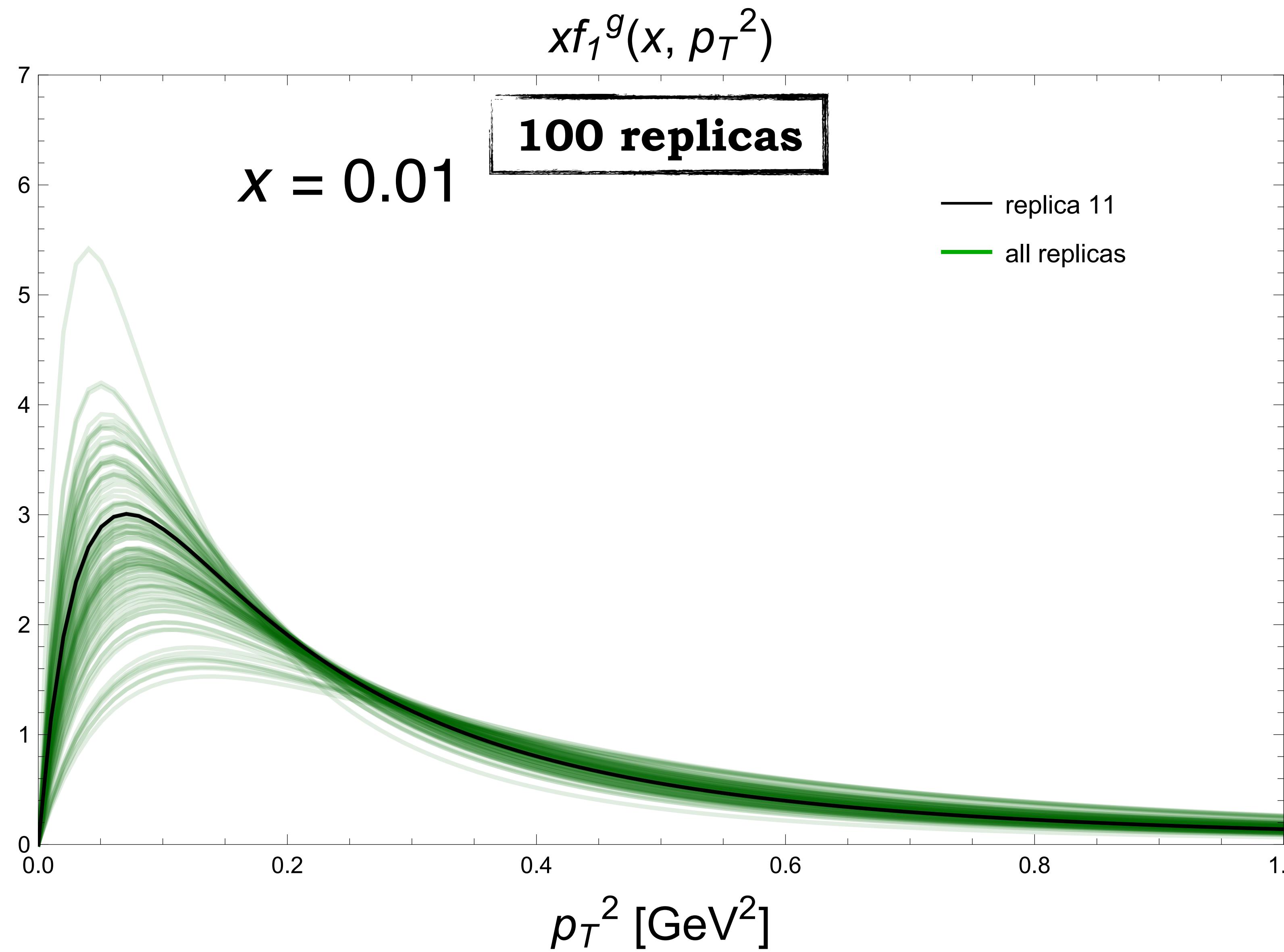
$$\langle x \rangle_g = 0.427(92)$$

$$\langle J \rangle_g = 0.187(46)$$

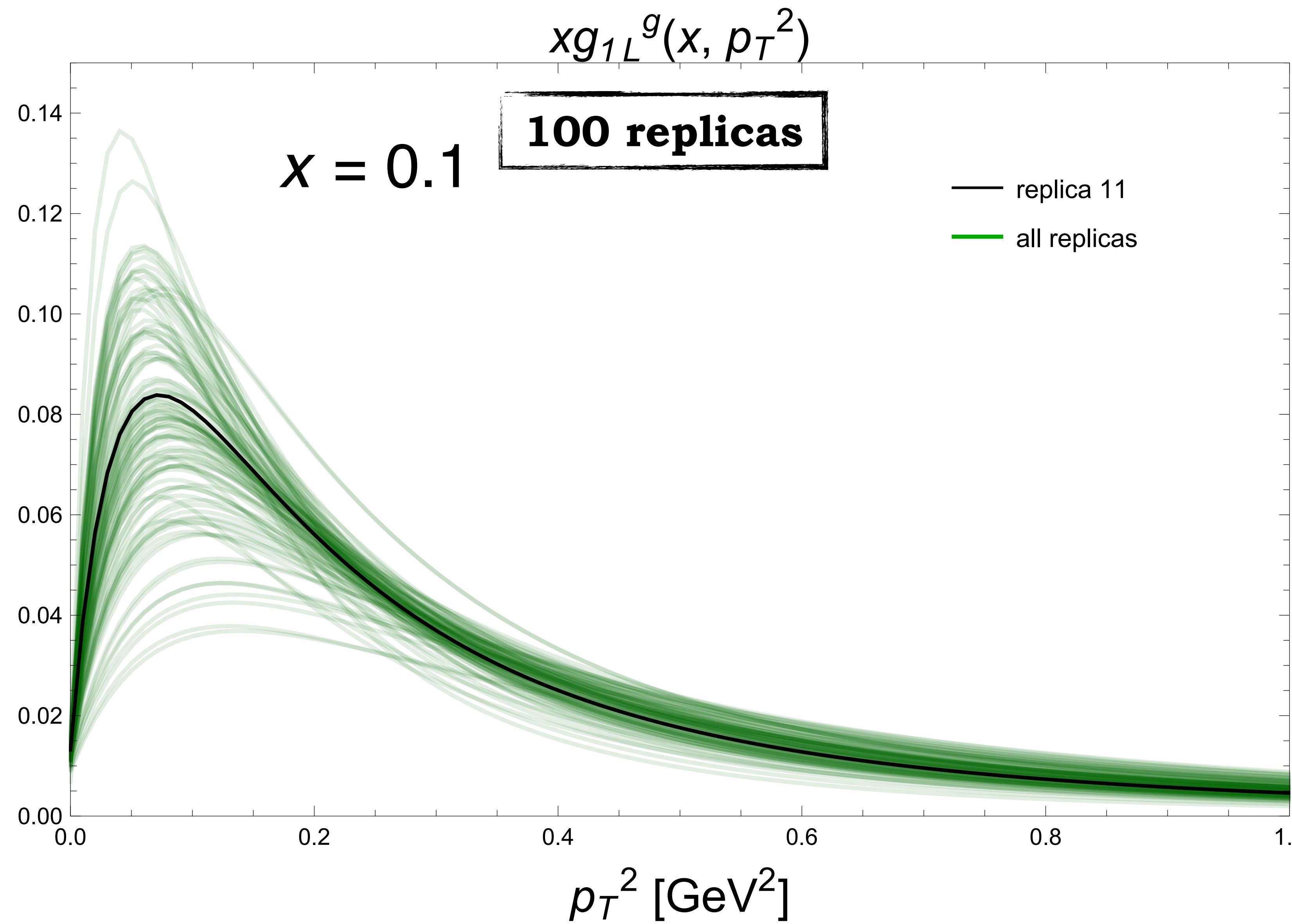
# Unpolarized gluon TMD



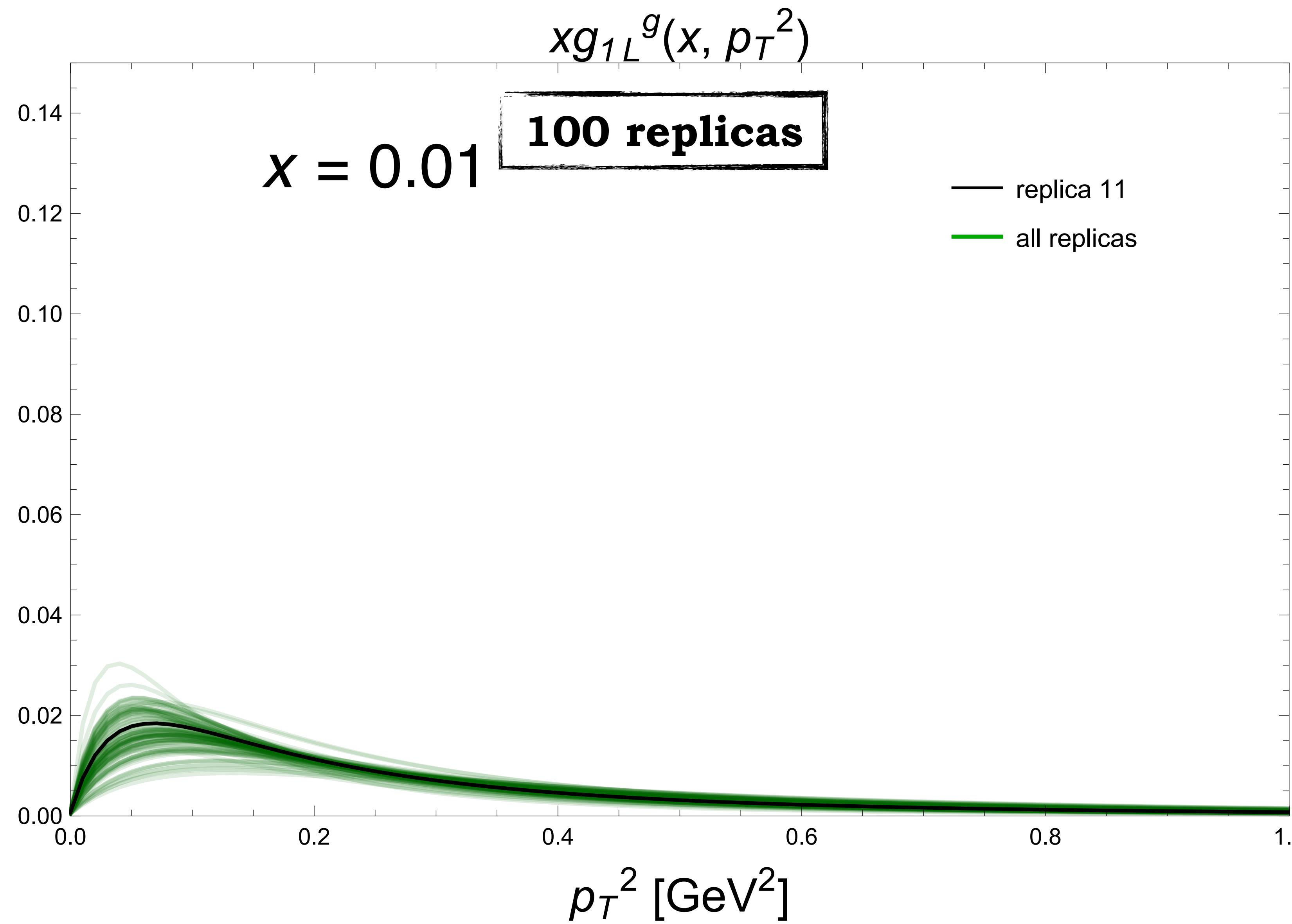
# Unpolarized gluon TMD



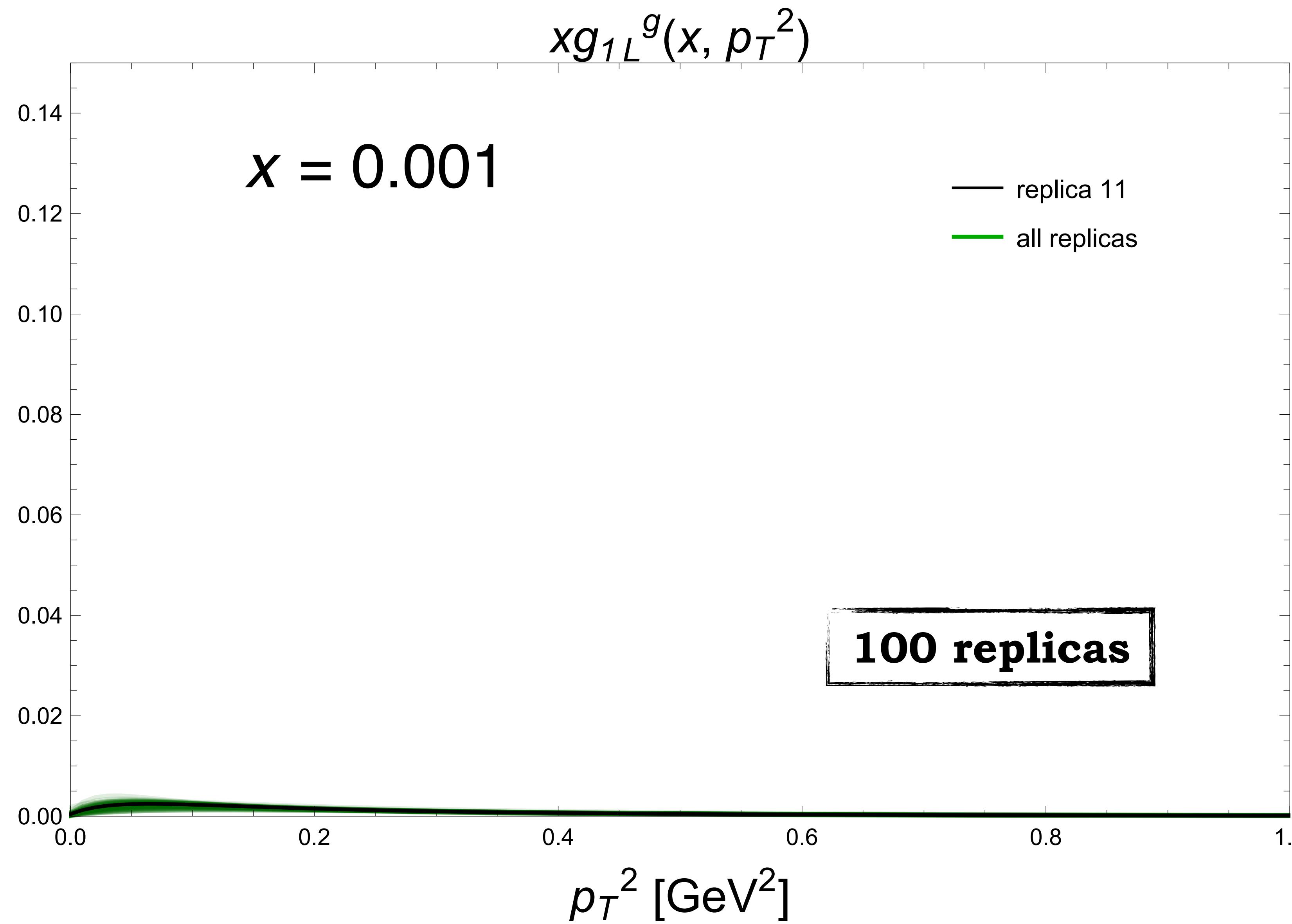
# Helicity gluon TMD



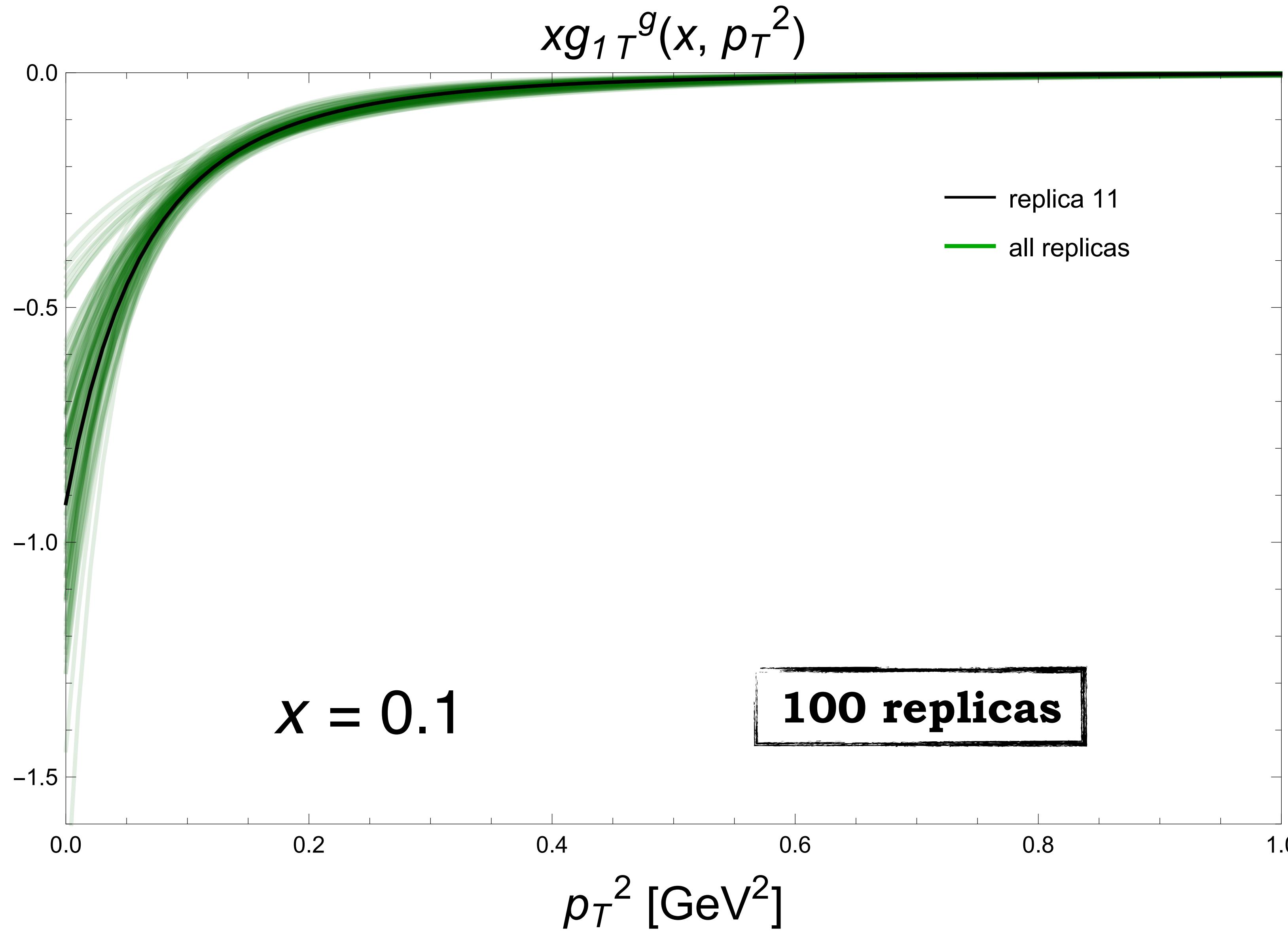
# Helicity gluon TMD



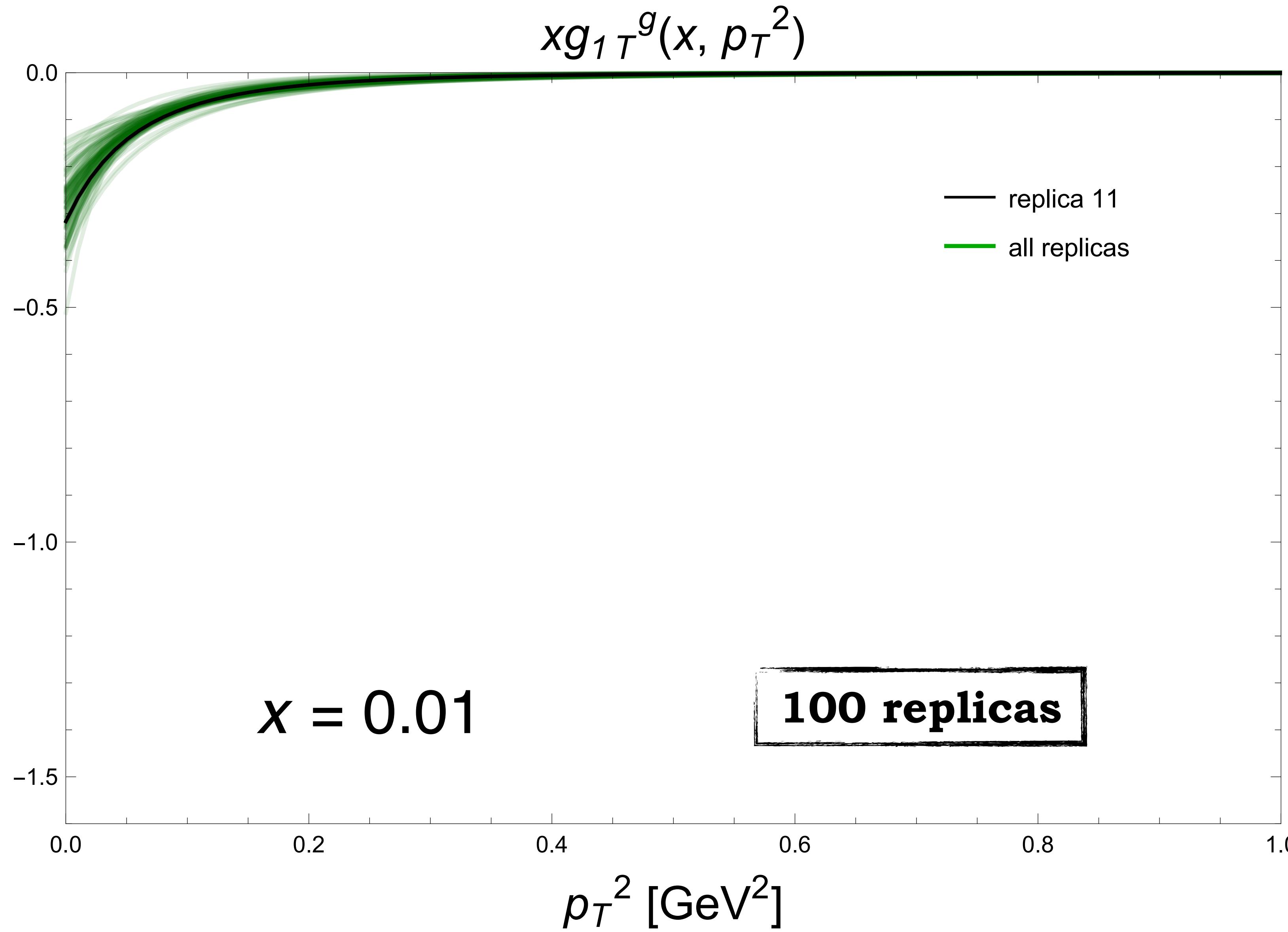
# Helicity gluon TMD



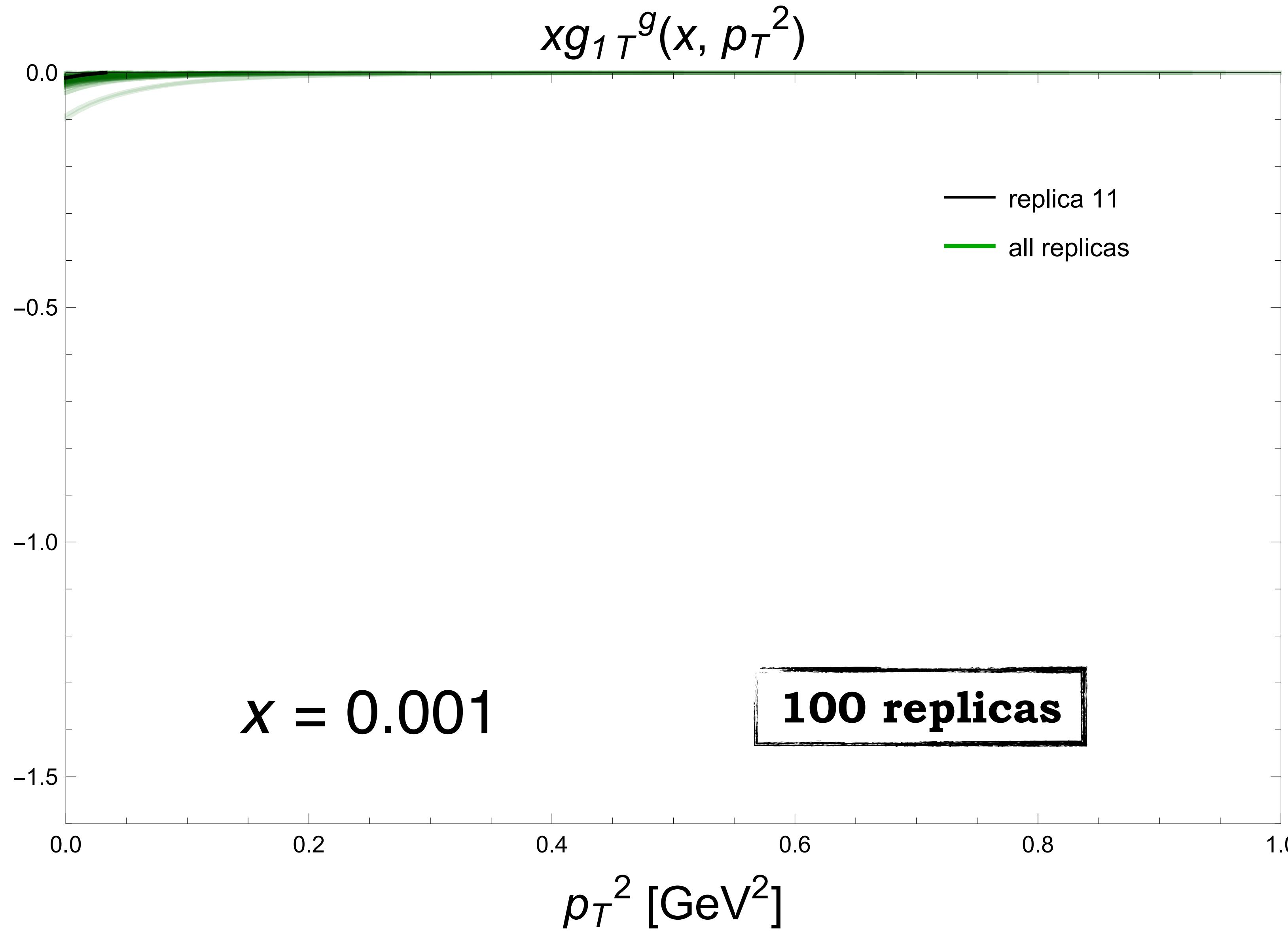
# Worm-gear gluon TMD



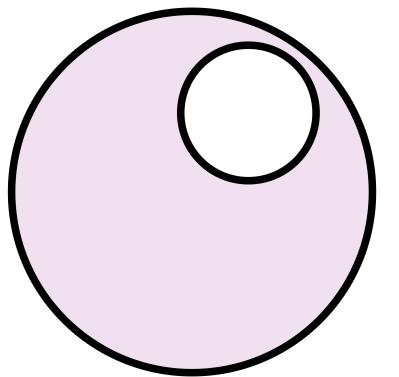
# Worm-gear gluon TMD



# Worm-gear gluon TMD



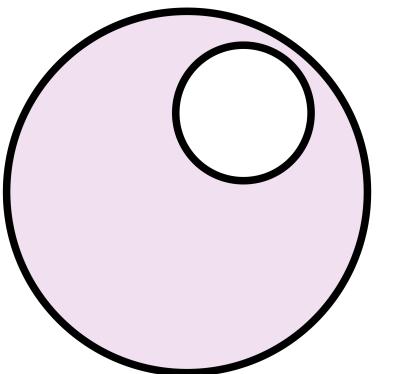
# $\rho$ -densities



**Unpolarized [u/u]**

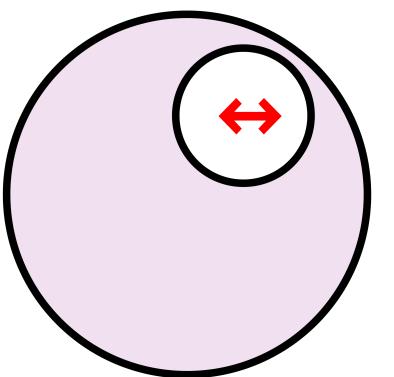
$$f_1(x, p_x, p_y)$$

# $\rho$ -densities



**Unpolarized [u/u]**

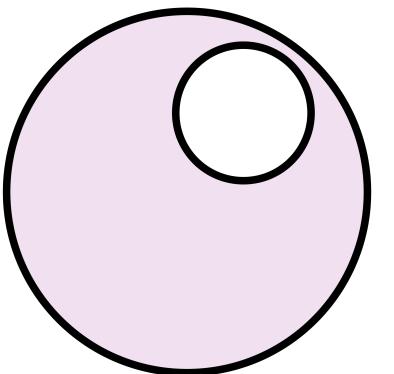
$$f_1(x, p_x, p_y)$$



**Boer-Mulders [ $\leftrightarrow/u$ ]**

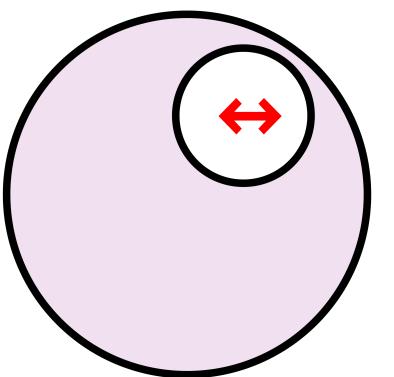
$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$

# $\rho$ -densities



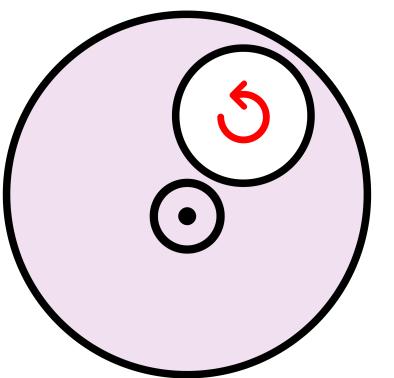
**Unpolarized** [u/u]

$$f_1(x, p_x, p_y)$$



**Boer-Mulders** [ $\leftrightarrow/u$ ]

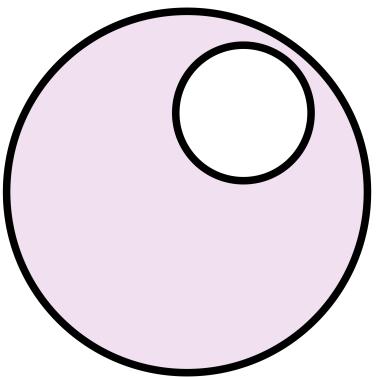
$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$



**Helicity** [ $\cup/+$ ]

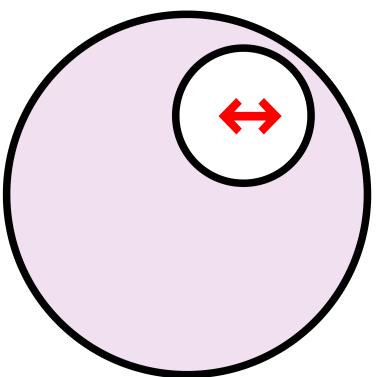
$$\frac{1}{2} \left[ f_1(x, p_x, p_y) + g_{1L}(x, p_x, p_y) \right]$$

# $\rho$ -densities



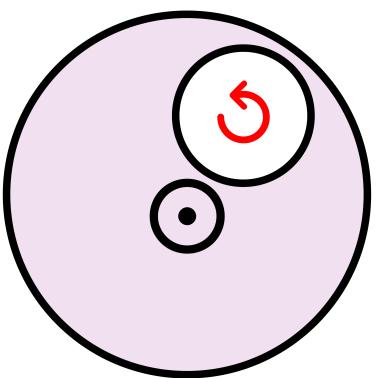
**Unpolarized** [u/u]

$$f_1(x, p_x, p_y)$$



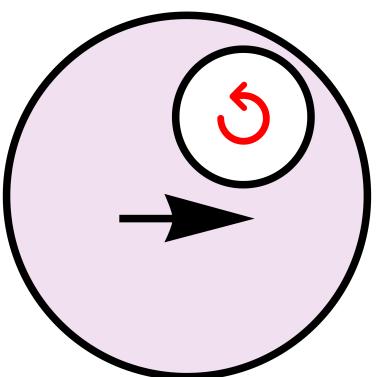
**Boer-Mulders** [ $\leftrightarrow/u$ ]

$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$



**Helicity** [ $\cup/+$ ]

$$\frac{1}{2} \left[ f_1(x, p_x, p_y) + g_{1L}(x, p_x, p_y) \right]$$

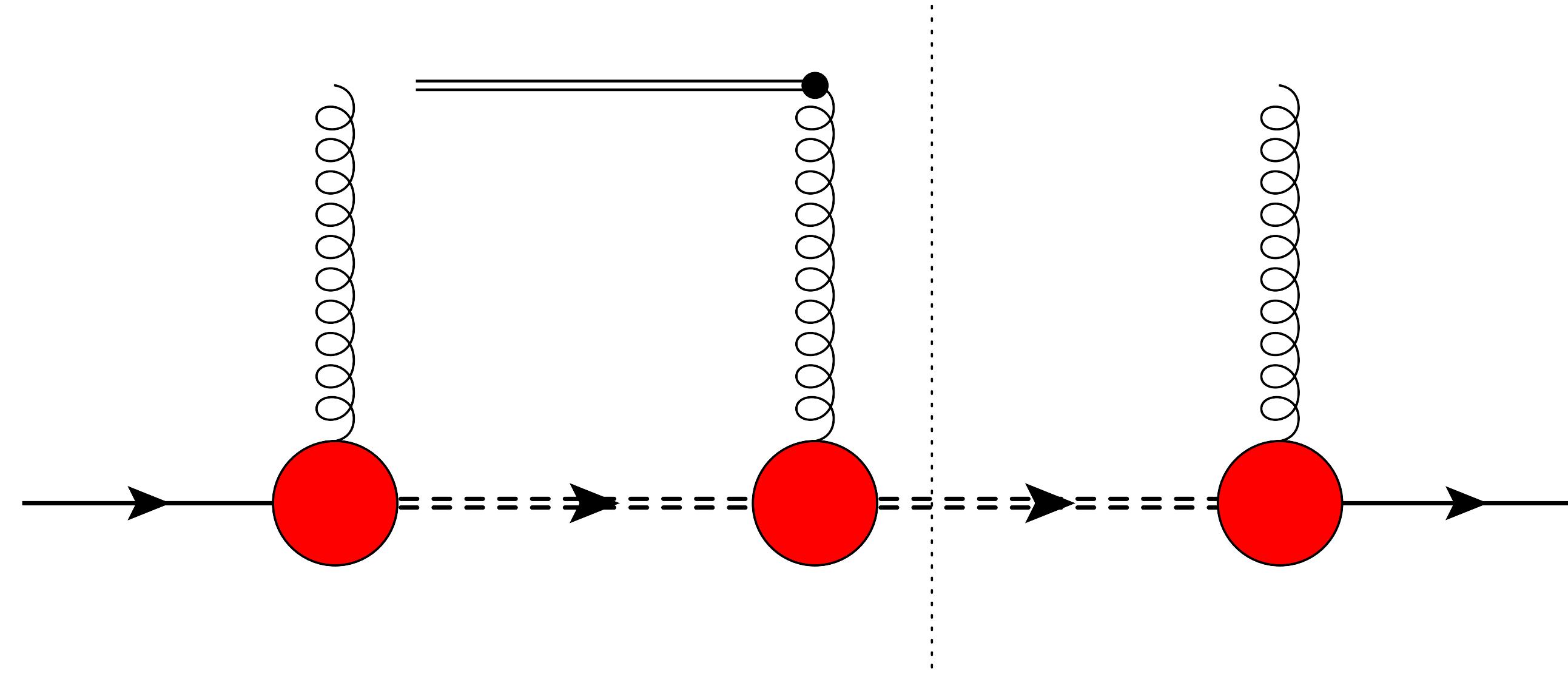


**Worm-gear** [ $\cup/\rightarrow$ ]

$$f_1(x, p_x, p_y) - \frac{p_x}{M} g_{1T}(x, p_x, p_y)$$

# T-odd gluon TMDs in a spectator model

- No residual gluon-spectator interaction at tree level
- Interference with one-gluon exchange (eikonal)*



- Leading-twist one-gluon-exchange of the gauge-link operator
- Sensitivity to WW/DP structures
- Calculation of **Sivers** function *underway!*

# Gluon TMDs: phenomenology

	$e p^\uparrow \rightarrow e' Q \bar{Q} X$ $e p^\uparrow \rightarrow e' j_1 j_2 X$	$p^\uparrow A \rightarrow h X$ ( $x_F < 0$ )	$p^\uparrow A \rightarrow \gamma^{(*)} \text{jet } X$	$p^\uparrow p \rightarrow \gamma \gamma X$ $p^\uparrow p \rightarrow J/\psi \gamma X$ $p^\uparrow p \rightarrow J/\psi J/\psi X$
$f_{1T}^{\perp g [+,+]}$	✓	✗	✗	✓
$f_{1T}^{\perp g [+,-]}$	✗	✓	✓	✗

**Boer-Mulders**

credits: D. Boer

	$pp \rightarrow \gamma \gamma X$	$pA \rightarrow \gamma^* \text{jet } X$	$ep \rightarrow e' Q \bar{Q} X$ $ep \rightarrow e' j_1 j_2 X$	$pp \rightarrow \eta_{c,b} X$ $pp \rightarrow H X$	$pp \rightarrow J/\psi \gamma X$ $pp \rightarrow \Upsilon \gamma X$
$h_1^{\perp g [+,+]} (\text{WW})$	✓	✗	✓	✓	✓
$h_1^{\perp g [+,-]} (\text{DP})$	✗	✓	✗	✗	✗

# Gluon TMDs: phenomenology

	$e p^\uparrow \rightarrow e' Q \bar{Q} X$ $e p^\uparrow \rightarrow e' j_1 j_2 X$	$p^\uparrow A \rightarrow h X$ ( $x_F < 0$ )	$p^\uparrow A \rightarrow \gamma^{(*)} \text{jet } X$	$p^\uparrow p \rightarrow \gamma \gamma X$ $p^\uparrow p \rightarrow J/\psi \gamma X$ $p^\uparrow p \rightarrow J/\psi J/\psi X$
$f_{1T}^{\perp g} [+,-]$	✓	✗	✗	✓
$f_{1T}^{\perp g} [+,-]$	✗	✓	✓	✗

**Boer-Mulders**

credits: D. Boer

	$pp \rightarrow \gamma \gamma X$	$pA \rightarrow \gamma^* \text{jet } X$	$ep \rightarrow e' Q \bar{Q} X$ $ep \rightarrow e' j_1 j_2 X$	$pp \rightarrow \eta_{c,b} X$ $pp \rightarrow H X$	$pp \rightarrow J/\psi \gamma X$ $pp \rightarrow \Upsilon \gamma X$
$h_1^{\perp g} [+,-]$ (WW)	✓	✗	✓	✓	✓
$h_1^{\perp g} [+,-]$ (DP)	✗	✓	✗	✗	✗

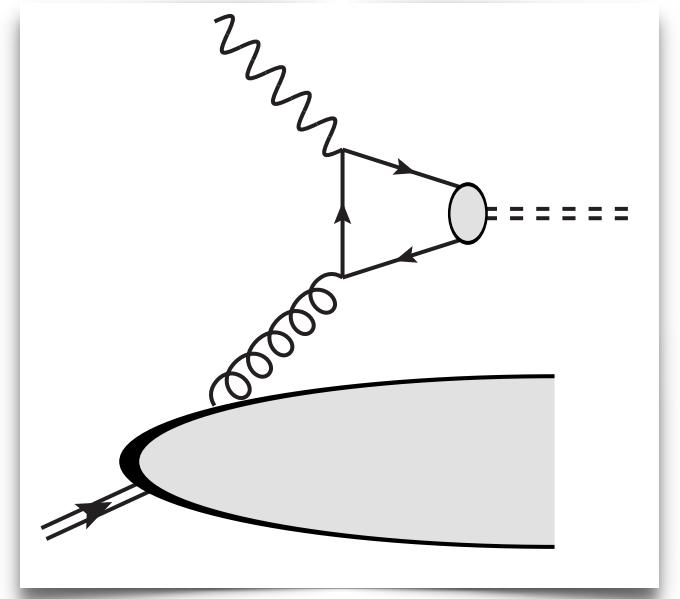
(pheno overview)  [D. Boer (2017)]

(azimuthal asym. in  $J/\Psi + \text{jet}$  SIDIS at the EIC)  [U. D'Alesio, F. Murgia, C. Pisano, P. Taels (2019)]

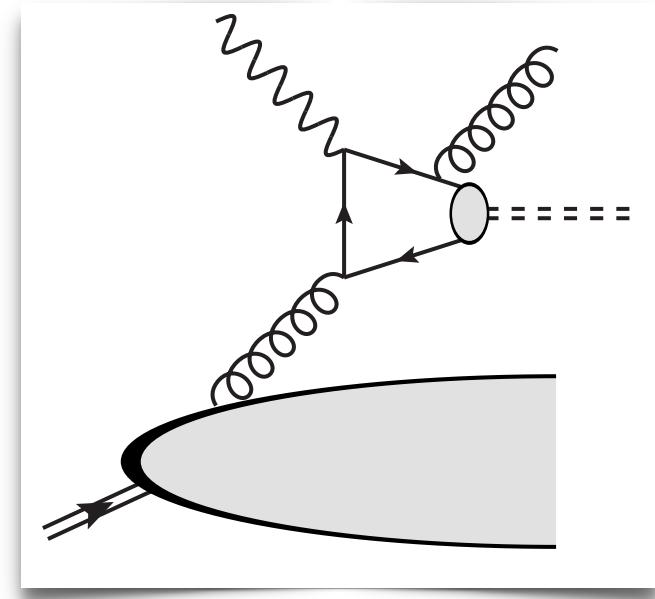
(matching high and low  $p_T$  in  $J/\Psi$  SIDIS)  [D. Boer, U. D'Alesio, F. Murgia, C. Pisano, P. Taels (2020)]

(gluon TMDs and NRQCD in  $J/\Psi$  at the EIC)  [A. Bacchetta, D. Boer, C. Pisano, P. Taels (2020)]

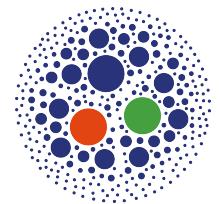
# Gluon TMDs and quarkonia



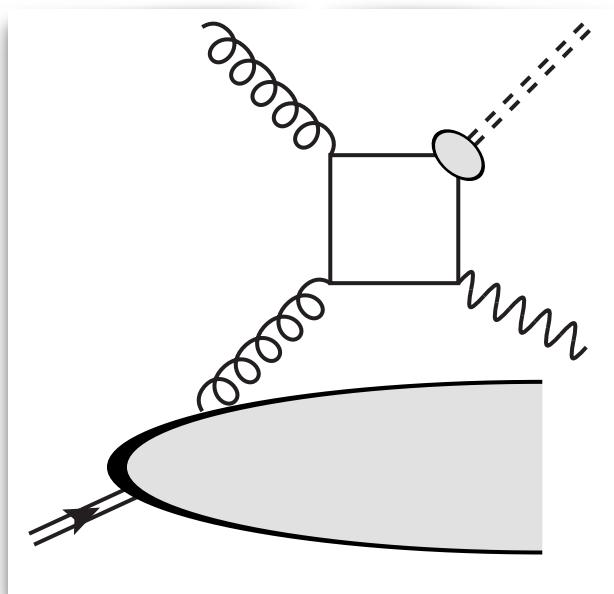
CO



CO + CS

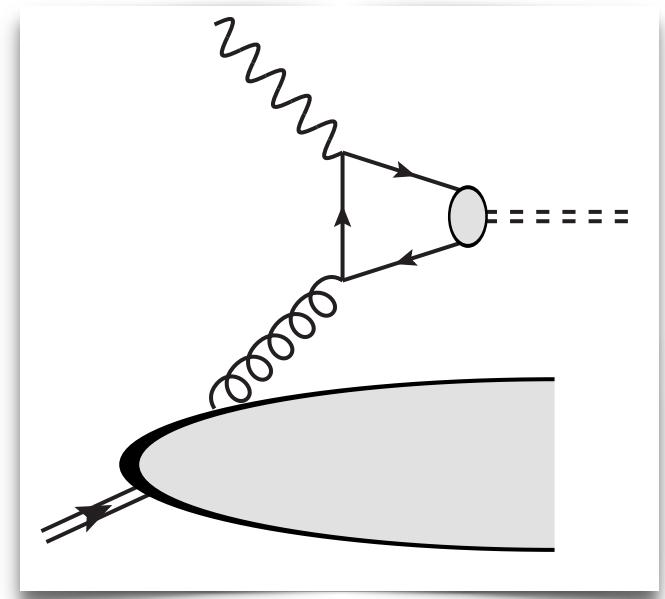


## Hadroproduction

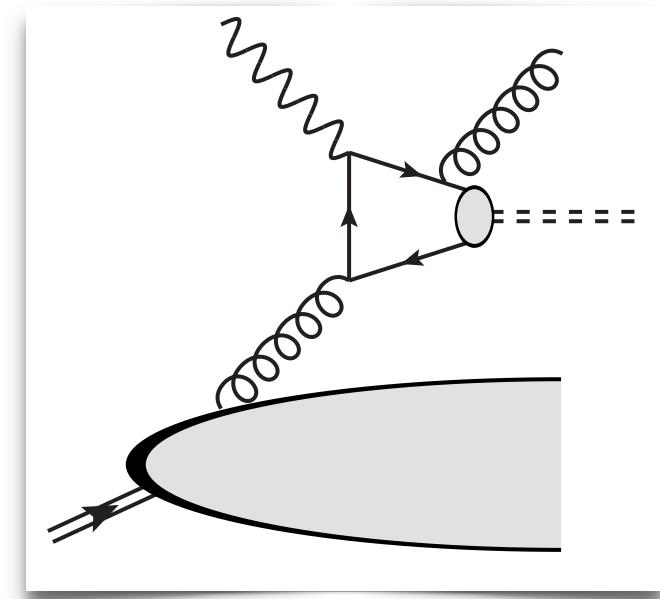


CO + CS

# Gluon TMDs and quarkonia



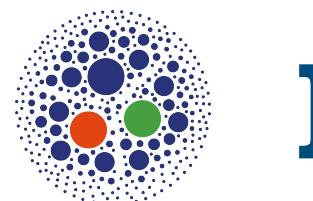
CO



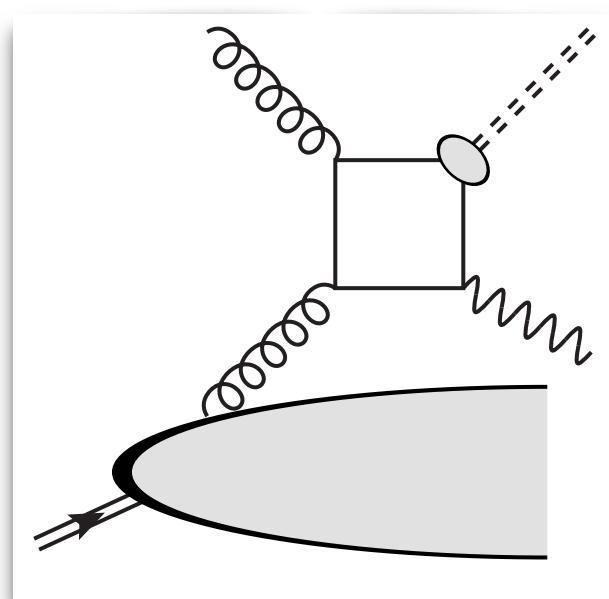
CO + CS

EIC Yellow Report Document  
(link to EICUG website)

🔗 [EICUG [*in preparation*]]



## Hadroproduction

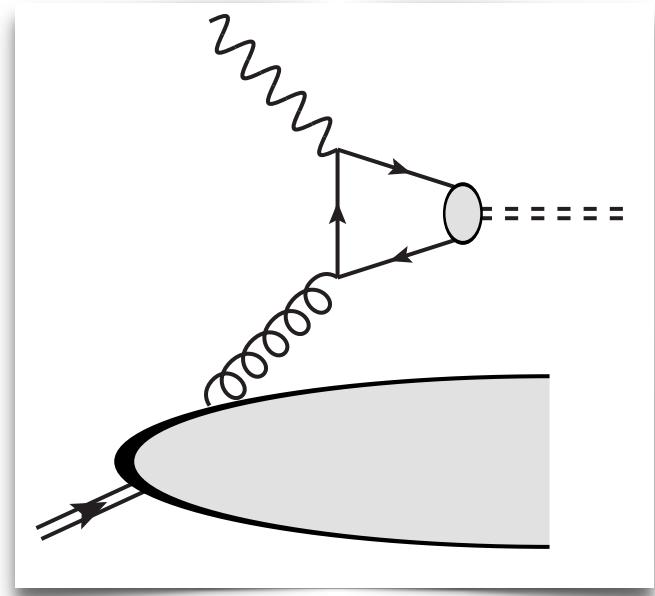


CO + CS

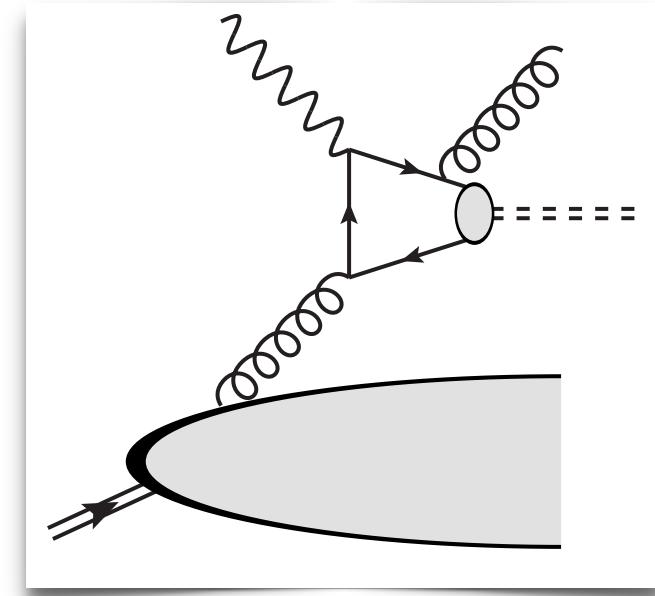
# Gluon TMDs and quarkonia



**SIDIS**



CO

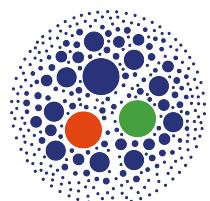


CO + CS

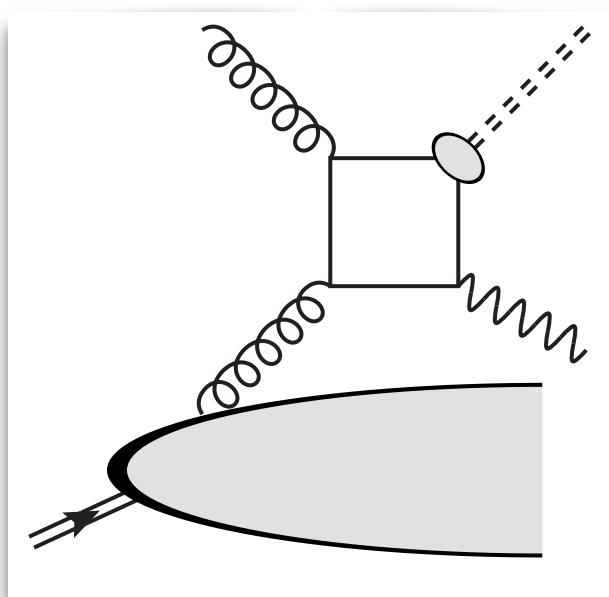
EIC Yellow Report Document  
(link to EICUG website)

🔗 [EICUG [*in preparation*]]

On the physics potential to study the gluon content of proton and deuteron at NICA SPD



**Hadroproduction**

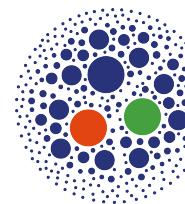


CO + CS

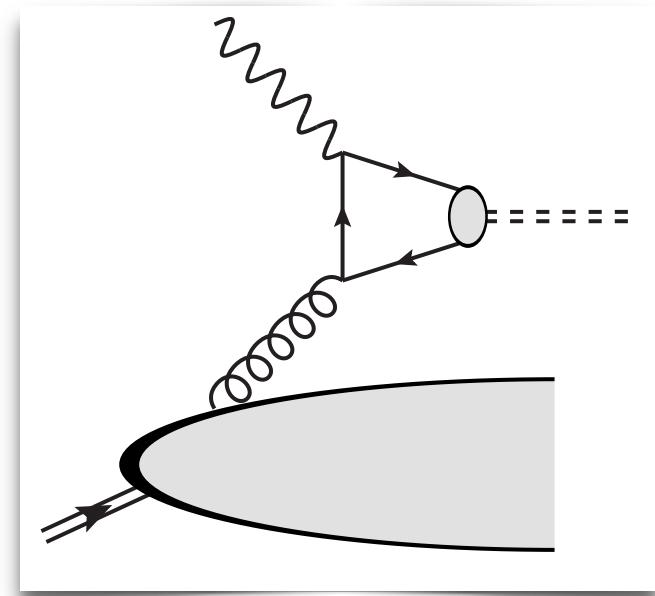
🔗 [NICA Collaboration [arXiv:2011.15005]]

**Backup**

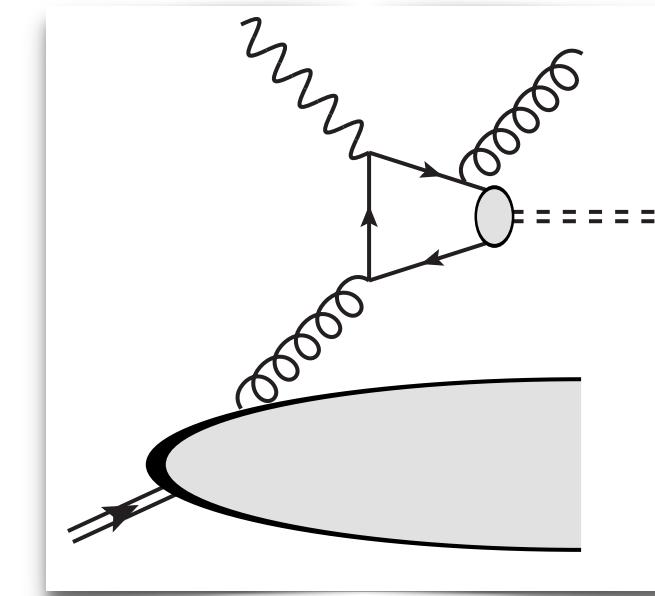
# Gluon TMDs and quarkonia



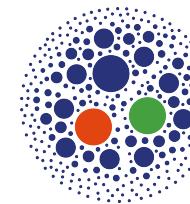
**SIDIS**



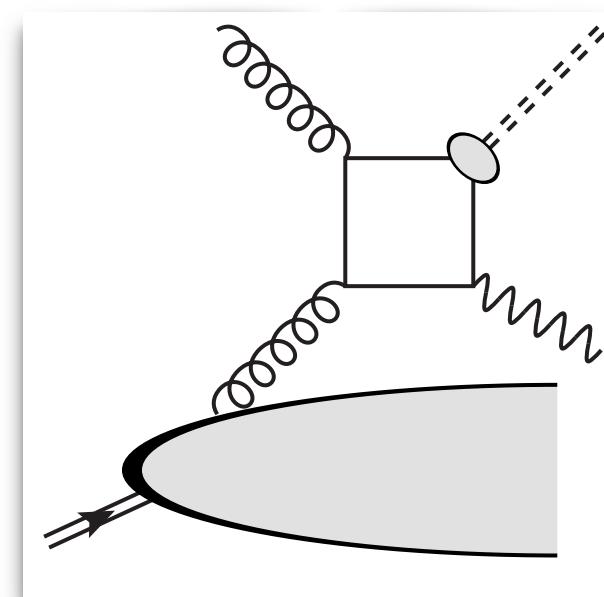
CO



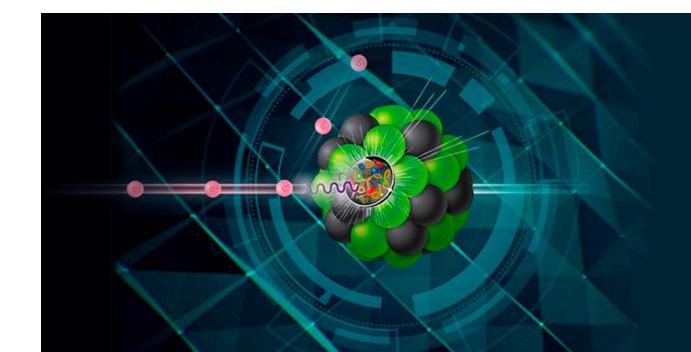
CO + CS



**Hadroproduction**



CO + CS



On the physics potential to study the gluon content of proton and deuteron at NICA SPD



🔗 [NICA Collaboration [[arXiv:2011.15005](#)]]

Perspectives for quarkonium studies at the high-luminosity LHC  
(link to QAT 2020 Workshop)

🔗 [Quarkonia As Tools Collaboration [[arXiv:2012.14161](#)]]

**Backup**