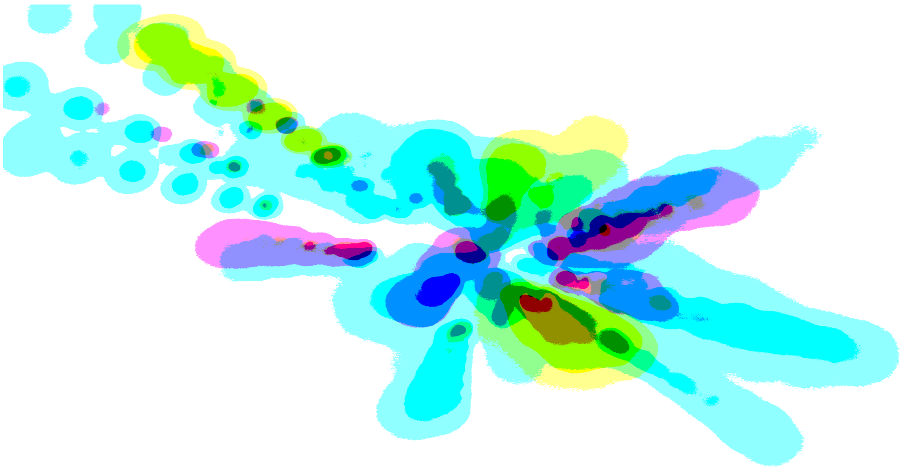
Gluon saturation and spin physics: a window into axion—like dynamics



Raju Venugopalan
(BNL)

Gluon saturation at the EIC: Snowmass meeting, January 28, 2021

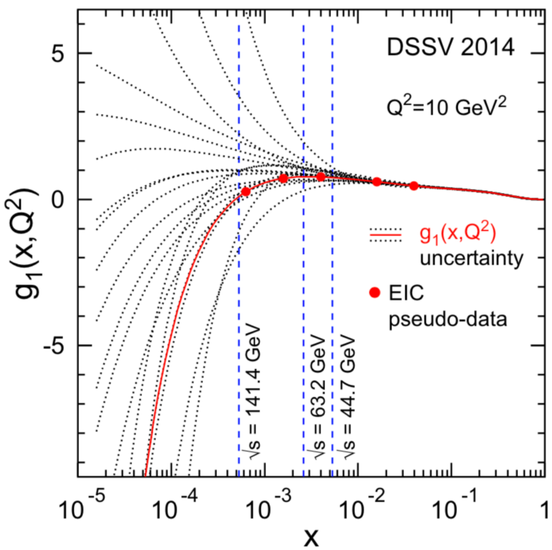
Gluon saturation and spin physics: a window into axion—like dynamics

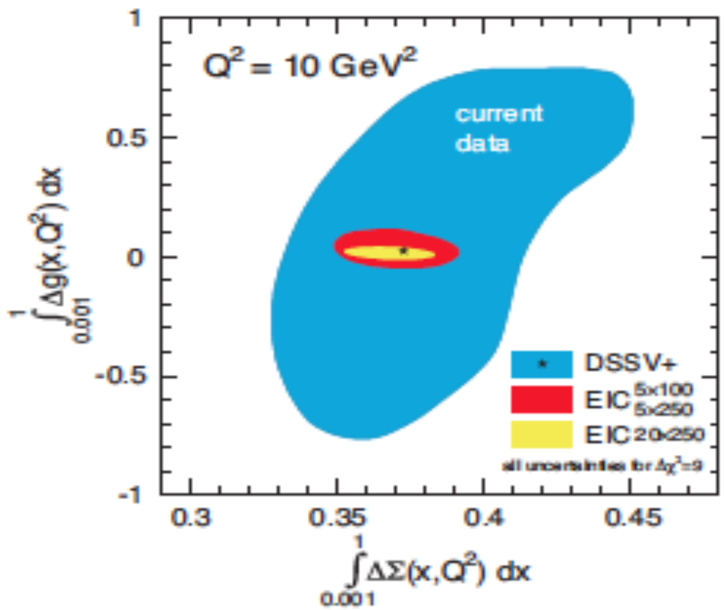
Work* in collaboration with Andrey Tarasov (The OSU and CFNS)

- The first paper is published in PRD: https://arxiv.org/abs/2008.08104
  and others are in preparation

Resolving the proton's spin puzzle: the g_1 structure function

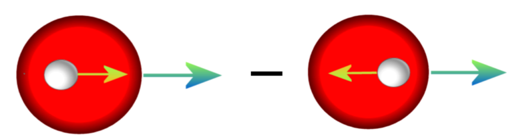
Aschenauer et al., arXiv:1708.01527
Rep. Prog. Phys. 82, 024301 (2019)





In the parton model, at leading twist

$$g_1(x_B,Q^2) = \frac{1}{2}\sum_f e_f^2\left(\Delta q_f(x_B,Q^2) + \Delta\bar{q}_f(x_B,Q^2)\right)$$

$\Delta q(x) =$ 

Most generally, $g_1(x,Q^2) = \frac{1}{8\lambda}\epsilon_T^{\mu\nu}\tilde{W}_{\mu\nu}(q,P,S)$

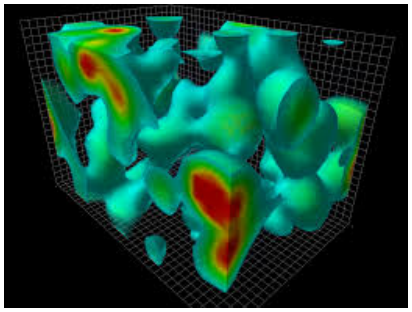
where $\widetilde{W}^{\mu\nu}$ is the antisymmetric part of $W^{\mu\nu}$
$S^\mu = \frac{2\lambda}{m_P}P^\mu$ and $\lambda = \pm 1/2$

Iso-singlet axial vector current and the chiral anomaly

$$\int_0^1 g_1(x,Q^2) = \frac{1}{18}\left(3F + D + 2\,\Sigma(Q^2)\right) \Longrightarrow S^\mu\,\Delta\Sigma = \langle P,S\,|\,\bar\psi\,\gamma^\mu\gamma_5\,\psi\,|\,P,S\rangle \equiv \langle P,S\,|\,j_5^\mu\,|\,P,S\rangle$$

$U_A(1)$ violation from the chiral anomaly
-famous anomaly equation:

$$\boxed{\partial_\mu J_5^\mu = 2n_f\partial_\mu K^\mu + \sum_{i=1}^{n_f} 2im_i\bar q_i\gamma_5 q_i}$$



Divergence of C-S current $\propto$ topological charge density

where the Chern-Simons current $K_\mu = \frac{g^2}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\left[A_a^\nu\left(\partial^\rho A_a^\sigma - \frac{1}{3}g f_{abc}A_b^\rho A_c^\sigma\right)\right]$

Identification of CS charge with ΔG is intrinsically ambiguous
… *the latter is gauge invariant, the former is not*

Jaffe, Manohar (1990)

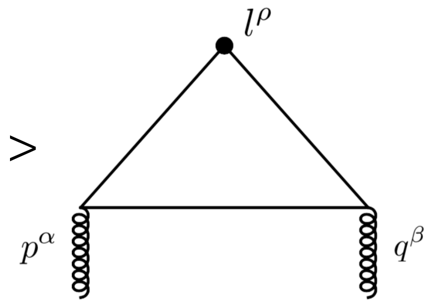
$$K_\mu \to K_\mu + i\frac{g}{8\pi^2}\epsilon_{\mu\nu\alpha\beta}\partial^\nu\left(U^\dagger\partial^\alpha U A^\beta\right) + \frac{1}{24\pi^2}\epsilon_{\mu\nu\alpha\beta}\left[\underbrace{(U^\dagger\partial^\nu U)(U^\dagger\partial^\alpha U)(U^\dagger\partial^\beta U)}\right]$$

"Large gauge transformation"
- deep consequence of topology

R. Jaffe: identification of K^μ with ΔG
a source of much confusion
in the literature (Varenna lectures, 2007)

Perturbative & nonperturbative interplay: The triangle graph

The key role of the $U_A(1)$ anomaly is seen from the structure of the triangle graph in the off-forward limit ($l^\mu \to 0$, $t = l^2 \to 0$) of the matrix element of $< P'$,S$| J_\mu^5|$P, S $>$



$$\langle P', S | J_5^\mu | P, S \rangle = G_A(t) S_\mu + l \cdot S \, l_\mu G_P(t)$$

The computation of this has an infrared pole proportional to $\frac{l^\mu}{l^2} F\tilde{F}$

Jaffe. Manohar (1990)

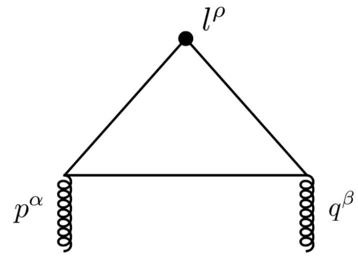
This perturbative/nonperturbative interplay gives a highly non-trivial result:

$$\langle P', S | J_5^\mu | P, S \rangle \to \frac{l \cdot S \, l^\mu}{l^2} \kappa(t) + \left(S^\mu - \frac{l \cdot S \, l^\mu}{l^2} \right) \lambda(t)$$

The infrared pole in $G_A(t)$ must be canceled by a pole in $G_P(t)$ – the corresponding Wess-Zumino-Witten term for the η'

For that to hold, one must have $\kappa(0) = \lambda(0) \propto F\tilde{F}$ the topological charge density

Perturbative & nonperturbative interplay: The triangle graph



Tarasov, RV, arXiv:2008.08104

with the (manifestly) gauge invariant result for the forward matrix element

$$\boxed{\Sigma(Q^2) = \frac{n_f\,\alpha_s}{2\pi\,M_N} \lim_{l_\mu \to 0} \langle P', S| \frac{1}{il \cdot s} \mathrm{Tr}\left(F\tilde{F}\right)(0)|P,S\rangle}$$

This result, generalization to $g_1(x,Q^2)$, and interplay with non-perturbative physics can be explored efficiently in a worldline framework

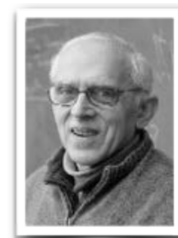
Not addressed in lot of "pQCD" literature
Deeply profound since intimately tied to mechanism (the "UA(1) problem") whereby the η' gets its mass

> The authors of refs. [12, 13] suggest that the triangle diagram provides a *local* probe of the gluon distribution in the target. If this were true, $\Delta\Gamma$ would be protected from infrared problems and the calculation would be reliable in the usual sense. However, we believe there are strong arguments that the triangle is not local in the sense required. It is therefore not necessarily protected from infrared effects, in particular from the non-perturbative effects which give the η' a mass★.

Jaffe, Manohar (1990)

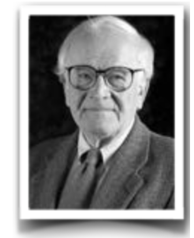
The triangle anomaly in the worldline formalism

Tarasov, RV



Steven Adler

John S. Bell



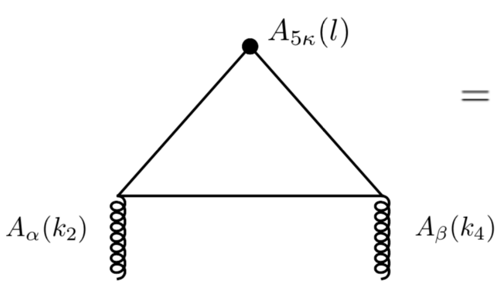
Roman Jackiw

$$\Gamma[A, A_5] = -\frac{1}{2}\mathrm{Tr}_c \int_0^\infty \frac{dT}{T} \int \mathcal{D}x \overbrace{\int_{AP} \mathcal{D}\psi}^{\text{Point particle Bose and Grassmann path integrals}}$$

$$\times \exp\Big\{ -\int_0^T d\tau \Big(\frac{1}{4}\dot{x}^2 + \frac{1}{2}\psi_\mu \dot{\psi}^\mu + \underbrace{ig\dot{x}^\mu A_\mu}_{\text{Wilson line}} - \underbrace{ig\psi^\mu\psi^\nu F_{\mu\nu}}_{\text{Spin precession}} \underbrace{- 2i\psi_5 \dot{x}^\mu \psi_\mu \psi_\nu A_5^\nu + i\psi_5 \partial_\mu A_5^\mu + (D-2)A_5^2}_{\text{Axial vector couplings}} \Big) \Big\}$$

$$\langle P', S | J_5^\kappa | P, S \rangle = \int d^4y \frac{\partial}{\partial A_{5\kappa}(y)} \Gamma[A, A_5]\Big|_{A_5=0} e^{ily} \equiv \Gamma_5^\kappa[l]$$

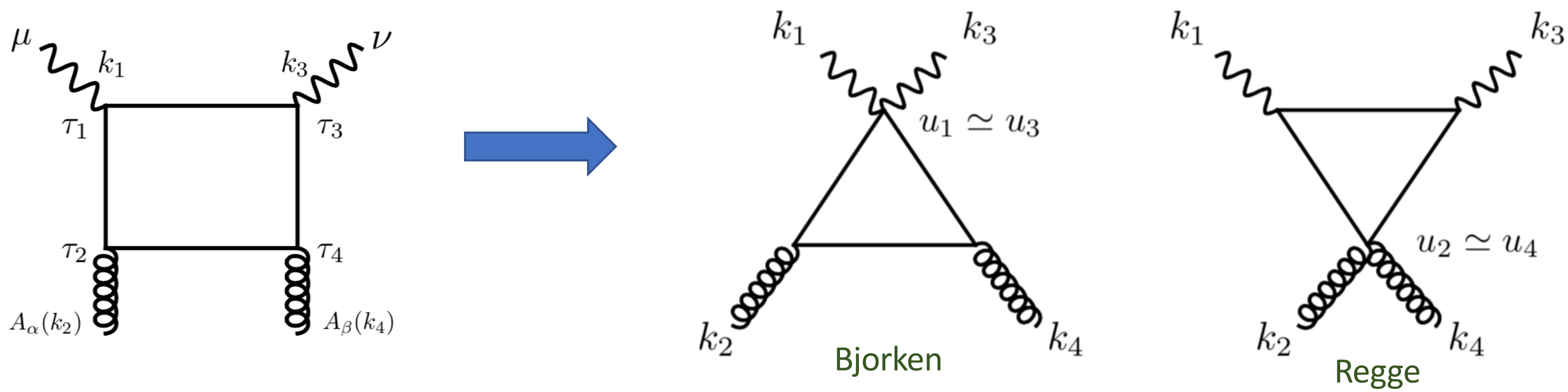
$$= \frac{1}{4\pi^2} \frac{l^\kappa}{l^2} \int \frac{d^4k_2}{(2\pi)^4} \int \frac{d^4k_4}{(2\pi)^4} \mathrm{Tr}_c F_{\alpha\beta}(k_2) \tilde{F}^{\alpha\beta}(k_4)\ (2\pi)^4 \delta^4(l + k_2 + k_4)$$



Famous infrared pole of anomaly

William A. Bardeen

Finding triangles in boxes in Bjorken and Regge asymptotics



Tarasov, RV, arXiv:2008.08104

Remarkably, box diagram for $g_1(x_B,Q^2)$ has the same structure in both limits, dominated by the triangle anomaly !

$$S^\mu g_1(x_B,Q^2)\Big|_{Q^2\to\infty} = \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N}\int_{x_B}^1 \frac{dx}{x}\left(1-\frac{x_B}{x}\right)\int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_\mu\to 0}\frac{l^\mu}{l^2}\langle P',S|\mathrm{Tr}_c F_{\alpha\beta}(\xi n)\tilde{F}^{\alpha\beta}(0)|P,S\rangle + \text{non-pole} \quad \frac{\Lambda^2{}_{QCD}}{Q^2} \quad << 1$$

$$S^\mu g_1(x_B,Q^2)|_{x_B\to 0} = \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N}\int_{x_B}^1 \frac{dx}{x}\int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_\mu\to 0}\frac{l^\mu}{l^2}\langle P',S|\mathrm{Tr}_c F_{\alpha\beta}(\xi n)\tilde{F}^{\alpha\beta}(0)|P,S\rangle + \text{non-pole} \quad \frac{x_B}{x} \quad << 1$$

Hence g_1 is topological in both asymptotic limits of QCD…its relation to $\Delta g(x, Q^2)$ is unclear

The role of pseudoscalar fields in resolving the $U_A(1)$ problem

We **did not** previously write down the most general form of the **imaginary part** of the worldline effective action :

D'Hoker, Gagne,hep-th/9508131

$$W_{\Im}[\Phi, \Pi, A] = \frac{1}{8}\int_{-1}^{1} d\alpha \int_0^\infty dT\, \mathcal{N} \int \mathcal{D}x\mathcal{D}\psi\, \mathrm{Tr_c}\, \mathcal{J}(0)\, \mathcal{P}e^{-\int_0^T d\tau \mathcal{L}_\alpha}$$

with $$\mathcal{L}_\alpha = \frac{\dot{x}^2}{2\mathcal{E}} + \frac{1}{2}\psi_A\dot{\psi}_A - i\dot{x}\cdot A + \frac{i}{2}\mathcal{E}\psi_\mu F_{\mu\nu}\psi_\nu + \frac{1}{2}\mathcal{E}\alpha^2\Phi^2 + \frac{1}{2}\mathcal{E}\Pi^2$$

$$+\boxed{i\,\varepsilon\,\psi_\mu\psi_5 D_\mu\Pi} + \alpha\,\varepsilon\,\psi_5\psi_6[\Pi, \Phi]$$

and $\mathcal{J}(0) \propto \psi_5\psi_6\{\Pi, \Phi\}$

where Φ is the chiral condensate

Expanding out the worldline Lagrangian, the first nontrivial contribution to W_I is the Wess-Zumino-Witten term !

This WZW term can be re-written as $\propto \eta_0 F\tilde{F}$

Leutwyler (1996); Herrara-Sikody et al (1997); Leutwyler-Kaiser (2000)

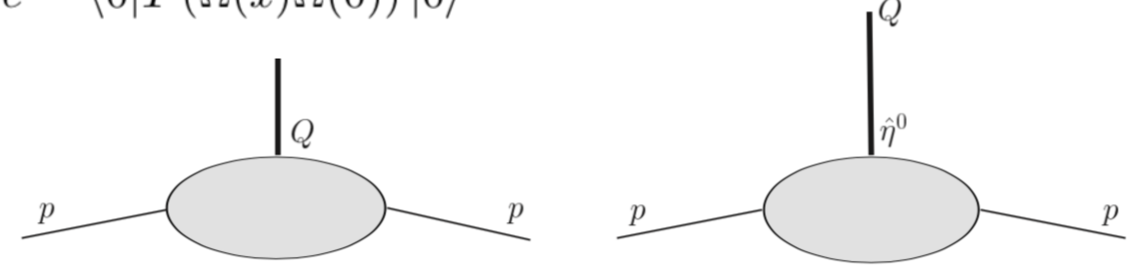
The zero mode part of it exactly cancels the $\frac{l^\mu}{l^2}$ "perturbative" contribution to the anomaly

Topological charge screening of spin

Topological charge $\Omega = \frac{\alpha_S}{8\pi}\mathrm{Tr}\left(F\tilde{F}\right)$

Topological susceptibility $\chi(t) = i\int d^4x\, e^{il\cdot x}\,\langle 0|T\left(\Omega(x)\Omega(0)\right)|0\rangle$

Shore, Veneziano, PLB (1990); NPB (1992)
Narison, Shore, Veneziano, hep-ph/9812333



$$\Sigma(Q^2) = \frac{1}{3m_N}\Delta C_1^S(\alpha_S)\left(g_{QNN}\chi(0) + g_{\eta' NN}\sqrt{\chi'(0)}\right)$$

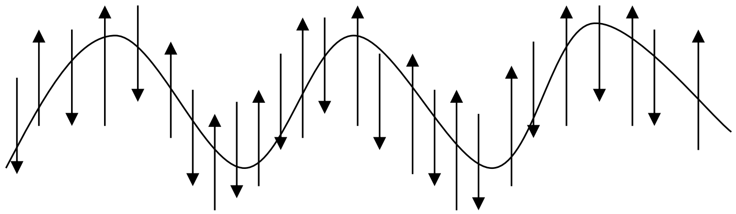
≈ 0

Narison, Shore and Veneziano (see hep-ph/0701171) argue that the result is dominated by the derivative of the topological susceptibility (χ') in the QCD vacuum

- which they compute (using QCD sum rules) to be in agreement with the HERMES and COMPASS data

Lattice computations: χQCD collaboration, arXiv: 1806.08366v2

Conjecture: "Axion-like" effective action for Regge limit



If (at small x), the background field couples to a large # of quark and gluon world-line trajectories, one can construct from their density matrix the *effective action for an ensemble of spinning, colored partons*

$$g_1(x_B,Q^2) \propto \int [D\rho]\, W_Y^P[\rho] \int [D\eta_0]\, \tilde{W}_Y^{P,S}[\eta_0]$$
$$\times \int [dA]\, \Omega(X)\, \exp\left(iS_{\rm YM}[A] + \frac{i}{N_c}\mathrm{Tr}_c\,(\rho\, U_{-\infty,\infty})\right)$$
$$\times \exp\left(\int d^4X\left(-\frac{\Omega^2}{2\chi_{\rm YM}} - \sqrt{\frac{N_c}{2}}\Omega\,\eta_0 + \frac{1}{2}F^2\eta_0\partial^2\eta_0\right)\right)$$

Axion-like effective action

Veneziano, Mod. Phys. Lett. (1989)

Hatsuda, PLB (1990)

Can equivalently be written as: $\exp\left(-\int d^4X\,\left(F^2\eta_0\partial^2\eta_0 + (\eta_0+\theta)\Omega + \chi_{\rm YM}(\eta_0+\theta)^2\right)\right)$

With this form of the effective action, we see explicitly how the pole of the anomaly is canceled by η_0 exchange

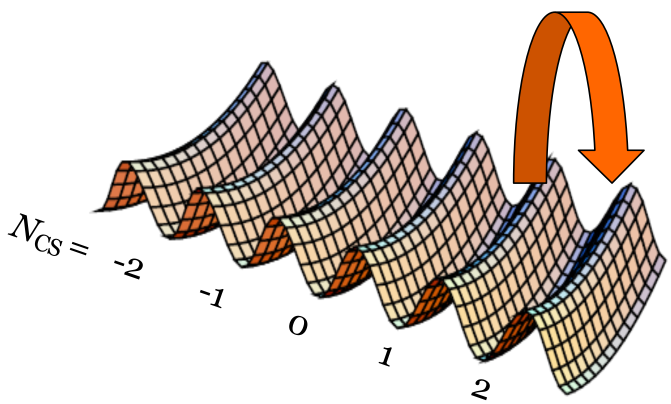
Tarasov, RV, in preparation

Spin diffusion via sphaleron transitions in topologically disordered media

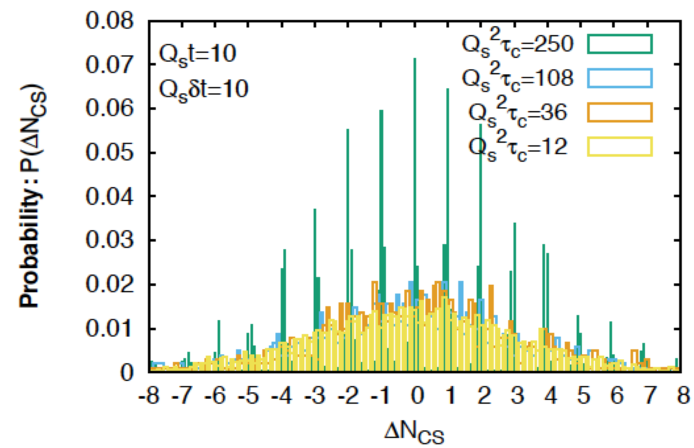
Two scales – the height of the barrier given by $m_{\eta'}^2 = 2n_f \frac{\chi_{\rm YM}}{F^2}$ (Witten-Veneziano formula)

- the saturation scale Q_S

When $Q_S^2 >> m_{\eta'}^2$ over the barrier gauge configurations dominate over instanton configurations



Topological transitions in overoccupied gauge fields

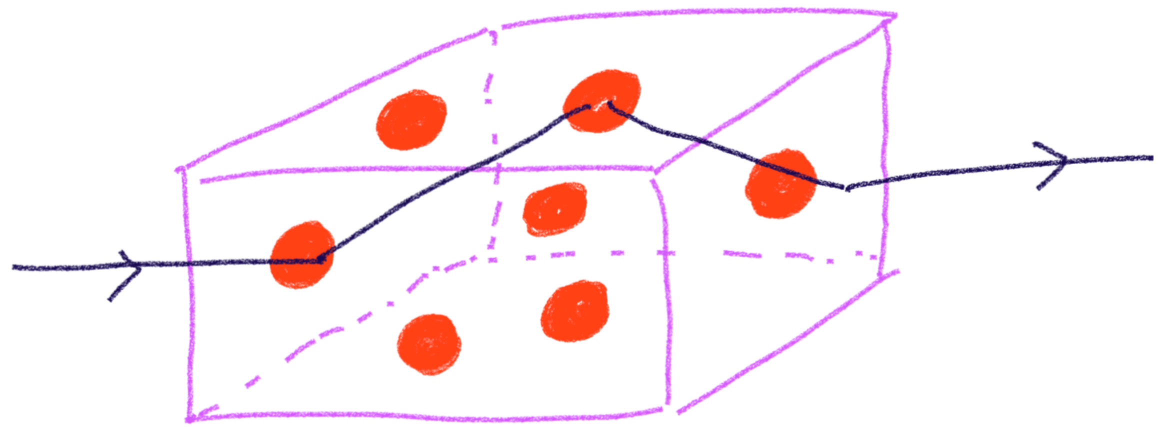


Mace, Schlichting, RV: PRD (2016) 1601.07342

Over the barrier (sphaleron) transitions between different topological sectors of QCD vacuum…
characterized by integer valued Chern-Simons #
--analogous to proposed mechanism for Electroweak Baryogenesis

Axion-like dynamics in a hot QCD plasma - McLerran,Mottola,Shaposhnikov (1990)

Spin diffusion via sphaleron transitions in topologically disordered media

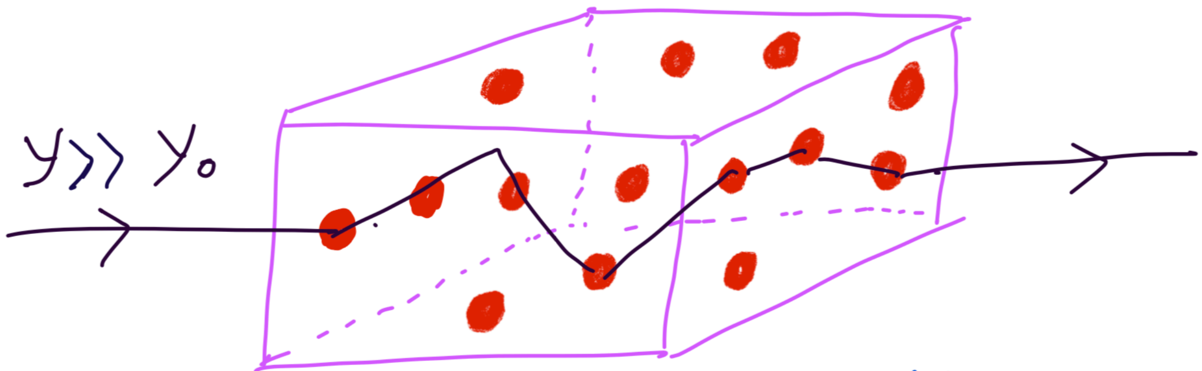


Dense gluon blob of size $1/\sqrt{\sigma}$ given by $\Gamma^{Y}_{\text{sphaleron}} = \# \, \sigma^2$ & carrying topological charge

Helicity flip for massless quarks given by $n_L - n_R = n_f \, \nu$, (Atiyah – Singer index theorem)

where ν is the topological charge and $\Gamma^{Y}_{\text{sphaleron}} \propto \langle \nu^2 \rangle$

Spin diffusion via sphaleron transitions in topologically disordered media

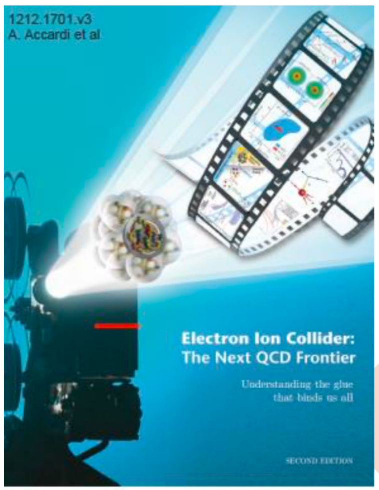


As x decreases ($y \gg y_0$), the blobs become smaller ($Q_s(y) > Q_s(y_0)$) and denser with more topological charge

Expect very rapid quenching of g1 at small x_B: interplay between QCD evolution of the topological charge and the saturation scale

Stay tuned for quantitative studies fleshing out this picture.

Outlook: these ideas can be tested at the EIC !



White paper
arXiv:1212.1701

Conformal Anomaly

Chiral Anomaly

Origin of nucleon mass

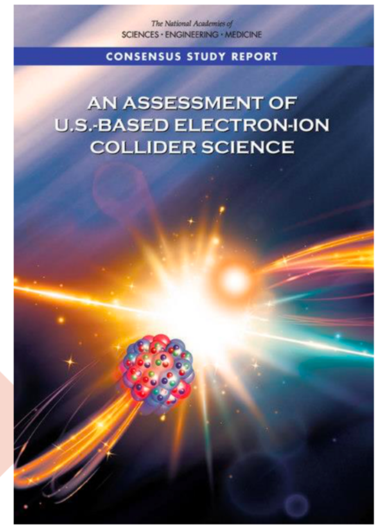
Origin of nucleon spin

Gluon saturation

Nucleon tomography

Color Glass Condensate

Confined many-body parton dynamics



NAS report
July 2018

Precision probes of the strong interplay between perturbative many-body parton dynamics and non-perturbative structure ("the ether") of the QCD vacuum