Helicity Distributions and Orbital Angular Momentum at Small x

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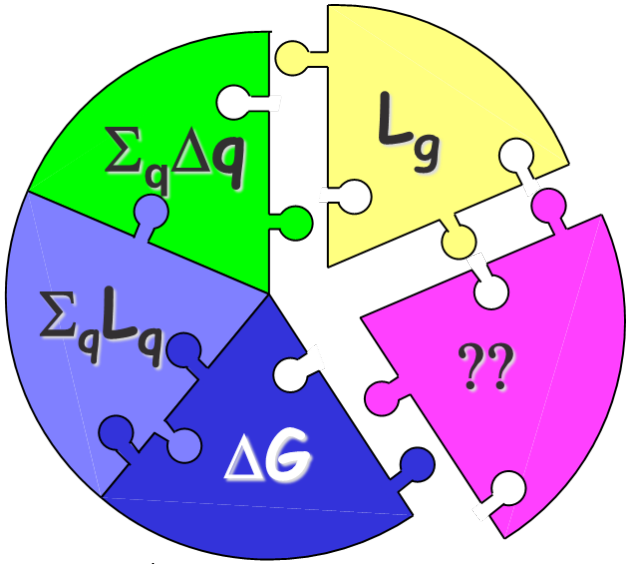
Based on work done with Dan Pitonyak and Matt Sievert (2015-2018, 2020-2021), Florian Cougoulic (2019-2020), Yossathorn Tawabutr (2020), Daniel Adamiak, Wally Melnitchouk, Nobuo Sato (JAM-small*x*, 2020-2021).

Proton Spin Puzzle



- Helicity sum rule (Jaffe-Manohar form):

$$\frac{1}{2} = S_q + L_q + S_g + L_g$$

with the net quark and gluon spin

$$S_q(Q^2) = \frac{1}{2} \int_0^1 dx\, \Delta\Sigma(x, Q^2) \qquad S_g(Q^2) = \int_0^1 dx\, \Delta G(x, Q^2)$$

- The helicity parton distributions are

$$\Delta f(x, Q^2) \equiv f^+(x, Q^2) - f^-(x, Q^2)$$

with the net quark helicity distribution

$$\Delta\Sigma \equiv \Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}$$

- L_q and L_g are the quark and gluon orbital angular momenta

Our goal

- The goal is to constrain theoretically the amount of proton spin and OAM coming from small x.

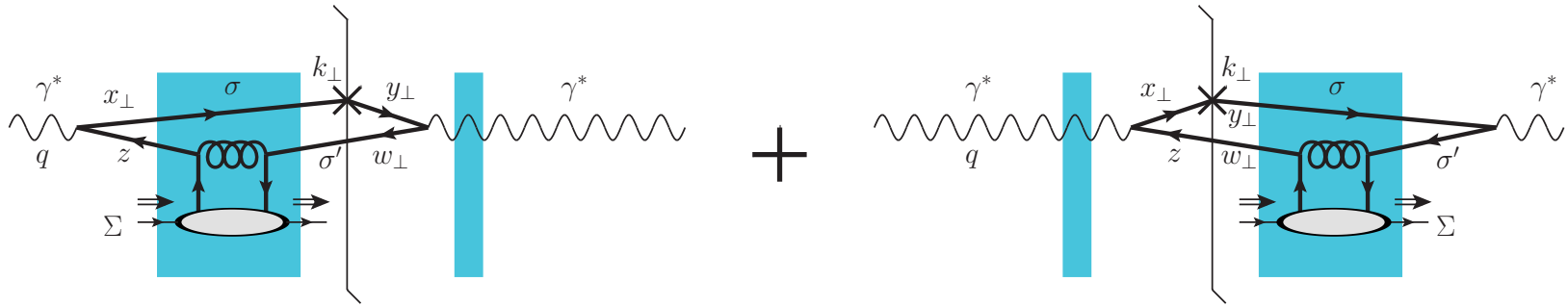
- Any existing and future experiment probes the helicity distributions and OAM down to some x_{min} .

$$S_q(Q^2) = \frac{1}{2} \int_0^1 dx\, \Delta\Sigma(x, Q^2) \qquad S_g(Q^2) = \int_0^1 dx\, \Delta G(x, Q^2)$$

$$L_{q+\bar{q}}(Q^2) = \int_0^1 dx\, L_{q+\bar{q}}(x, Q^2) \qquad L_G(Q^2) = \int_0^1 dx\, L_G(x, Q^2)$$

- At very small x (for the proton), saturation sets in: that region likely carries a negligible amount of proton spin. But what happens at larger (but still small) x?

Quark Helicity Observables at Small x



- One can show that the g_1 structure function and quark helicity PDF (Δq) and TMD at small-x can be expressed in terms of the polarized dipole amplitude (flavor singlet case):

$$g_1^S(x,Q^2) = \frac{N_c\,N_f}{2\,\pi^2\alpha_{EM}} \int_{z_i}^{1} \frac{dz}{z^2(1-z)} \int dx_{01}^2 \left[\frac{1}{2}\sum_{\lambda\sigma\sigma'} |\psi^T_{\lambda\sigma\sigma'}|^2_{(x_{01}^2,z)} + \sum_{\sigma\sigma'} |\psi^L_{\sigma\sigma'}|^2_{(x_{01}^2,z)}\right] G(x_{01}^2,z),$$

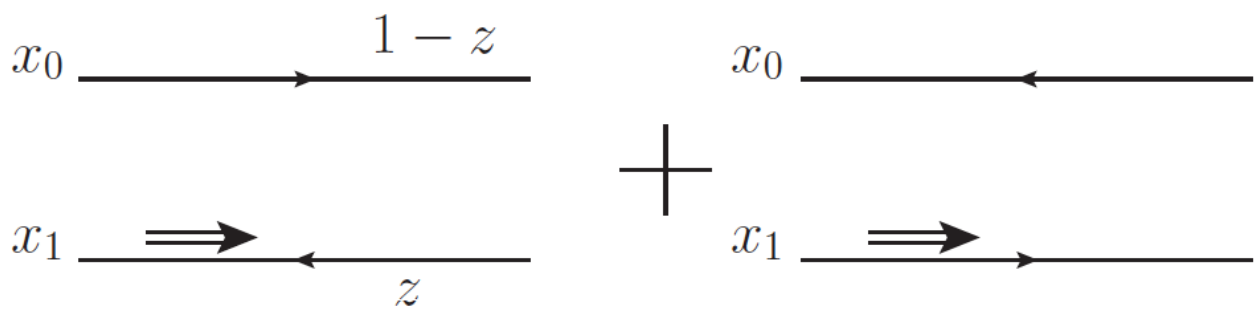
$$\Delta q^S(x,Q^2) = \frac{N_c\,N_f}{2\pi^3} \int_{z_i}^{1} \frac{dz}{z} \int_{\frac{1}{zs}}^{\frac{1}{zQ^2}} \frac{dx_{01}^2}{x_{01}^2}\, G(x_{01}^2,z), \quad = \Delta\Sigma(x,Q^2)$$

$$g_{1L}^S(x,k_T^2) = \frac{8\,N_c\,N_f}{(2\pi)^6} \int_{z_i}^{1} \frac{dz}{z} \int d^2x_{01}\, d^2x_{0'1}\, e^{-i\underline{k}\cdot(\underline{x}_{01}-\underline{x}_{0'1})}\, \frac{\underline{x}_{01}\cdot\underline{x}_{0'1}}{x_{01}^2 x_{0'1}^2}\, G(x_{01}^2,z)$$

- Here s is cms energy squared, $z_i=\Lambda^2/s$, $\quad G(x_{01}^2,z) \equiv \int d^2b\, G_{10}(z)$

Polarized Dipole

- All flavor-singlet small-x helicity observables depend on one object, "polarized dipole amplitude":



$$G_{10}(z) \equiv \frac{1}{2N_c}\,\mathrm{Re}\left\langle\!\!\left\langle \mathrm{T}\,\mathrm{tr}\left[V_{\underline{0}} V_{\underline{1}}^{pol\,\dagger}\right] + \mathrm{T}\,\mathrm{tr}\left[V_{\underline{1}}^{pol} V_{\underline{0}}^{\dagger}\right] \right\rangle\!\!\right\rangle(z)$$

unpolarized quark

polarized quark: eikonal propagation, non-eikonal spin-dependent interaction

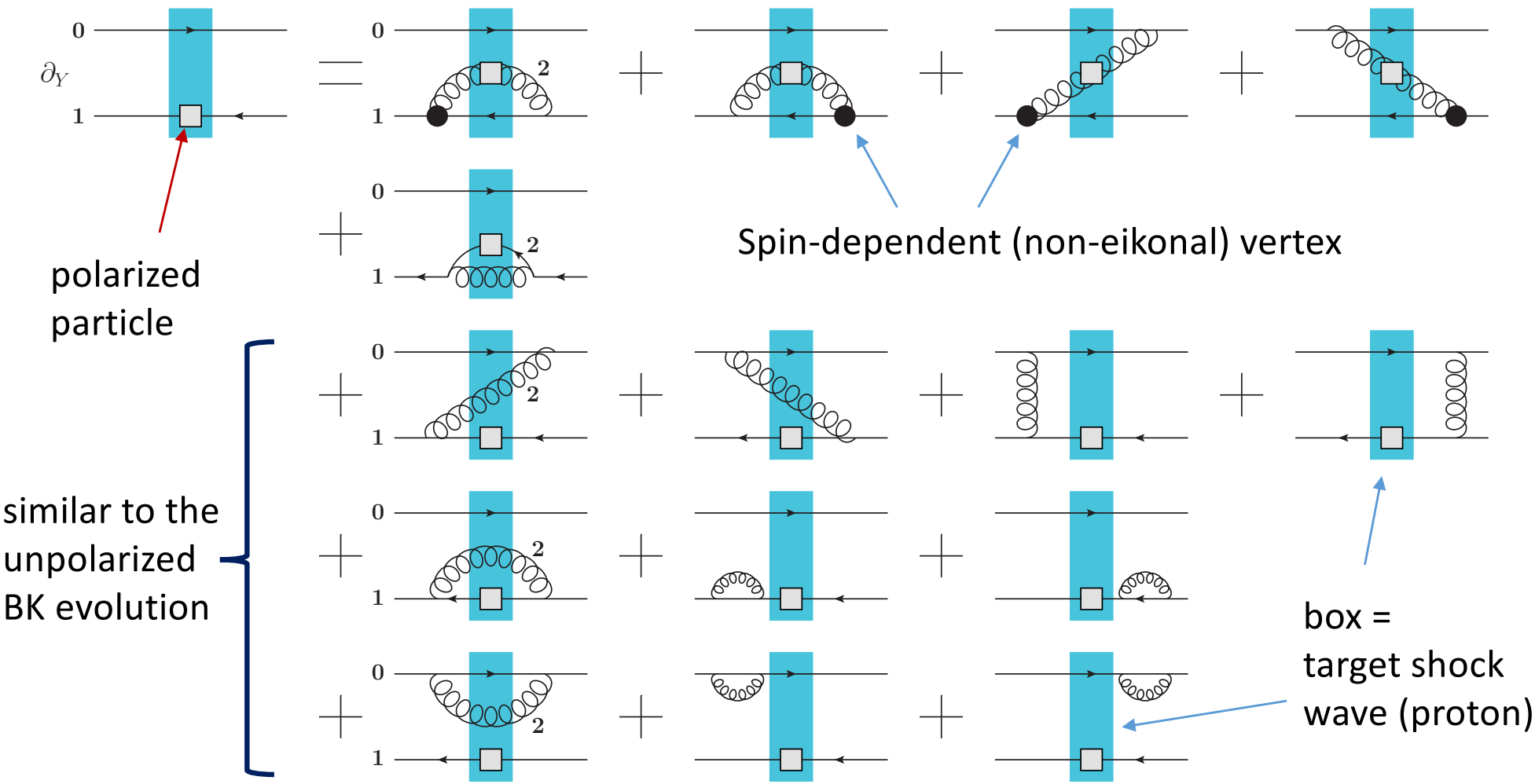
$$V_{\underline{x}} \equiv \mathcal{P}\exp\left[ig\int_{-\infty}^{\infty} dx^+\, A^-(x^+, 0^-, \underline{x})\right]$$

- Double brackets denote an object with energy suppression scaled out:

$$\left\langle\!\!\left\langle \mathcal{O} \right\rangle\!\!\right\rangle(z) \equiv zs\left\langle \mathcal{O} \right\rangle(z)$$

Evolution for Polarized Quark Dipole

One can construct an evolution equation for the polarized dipole:



Large-N_c Evolution

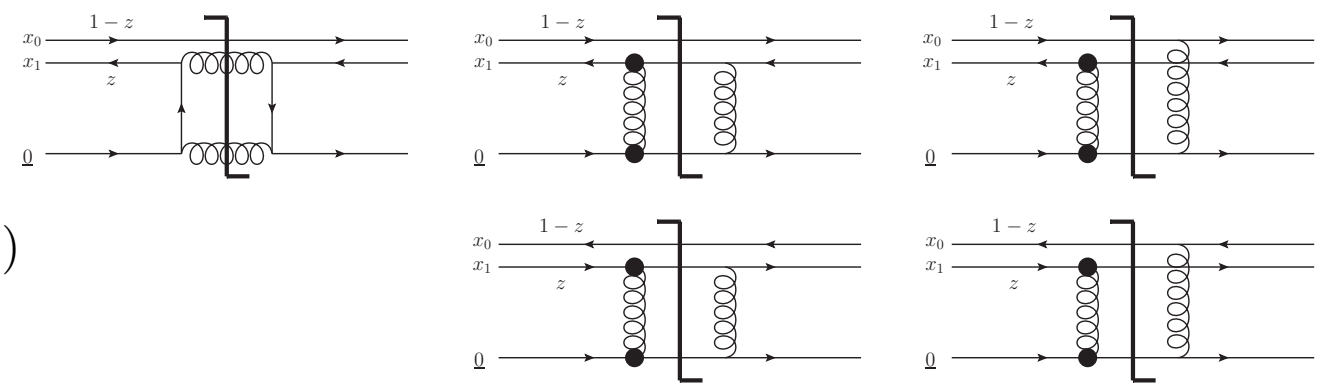
$$\alpha_s \, \ln^2 \frac{1}{x}$$

- In the strict DLA limit and at large N_c we get (here Γ is an auxiliary function we call the 'neighbor dipole amplitude') (KPS '15)

$$G(x_{10}^2, z) = G^{(0)}(x_{10}^2, z) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z' s}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[\Gamma(x_{10}^2, x_{21}^2, z') + 3\, G(x_{21}^2, z')\right]$$

$$\Gamma(x_{10}^2, x_{21}^2, z') = \Gamma^{(0)}(x_{10}^2, x_{21}^2, z') + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z'} \frac{dz''}{z''} \int\limits_{\frac{1}{z'' s}}^{\min\left\{x_{10}^2, x_{21}^2 \frac{z'}{z''}\right\}} \frac{dx_{32}^2}{x_{32}^2} \left[\Gamma(x_{10}^2, x_{32}^2, z'') + 3\, G(x_{32}^2, z'')\right]$$

- The initial conditions are given by the Born-level graphs

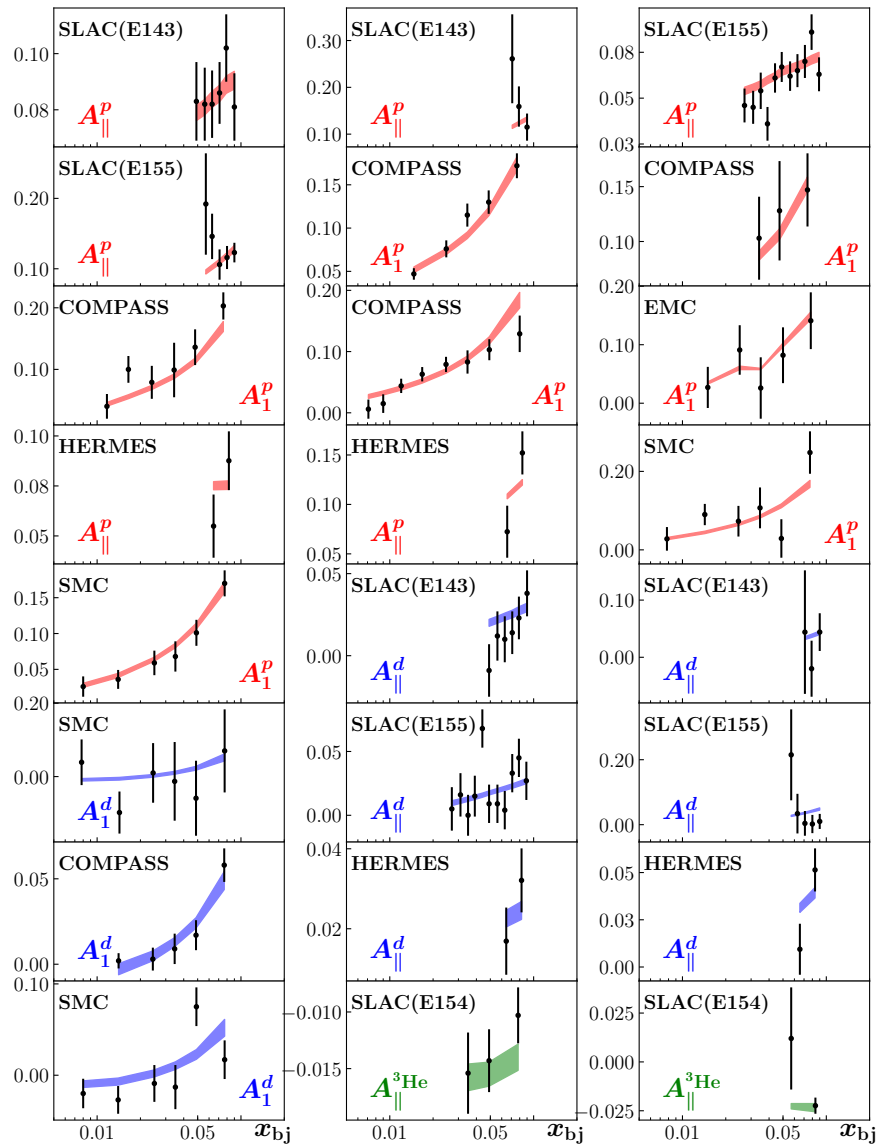


$$\Gamma^{(0)}(x_{10}^2, x_{21}^2, z) = G^{(0)}(x_{10}^2, z)$$

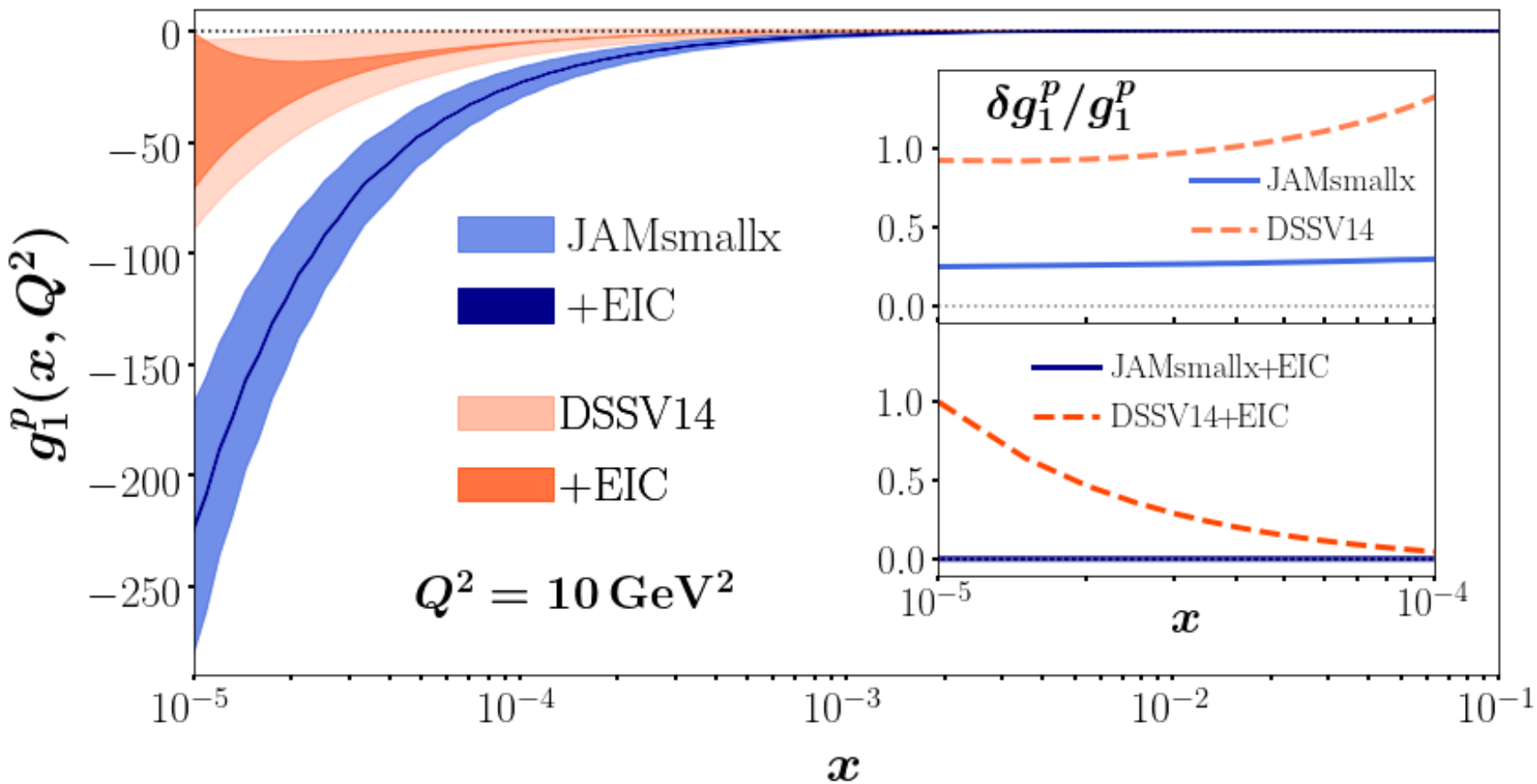
$$G^{(0)}(x_{10}^2, z) = \frac{\alpha_s^2 C_F}{N_c} \pi \left[C_F \ln \frac{zs}{\Lambda^2} - 2 \ln(zs\, x_{10}^2)\right]$$

Small-x Polarized DIS Data

- We have analyzed all existing world polarized DIS data with x<0.1 (122 data points) using the large-N_C KPS evolution with the Born-inspired initial conditions (8 parameters for 2 flavors, 11 parameters for 3 flavors).
- It worked well with chi^2/dof of about 1 and even smaller.
- Work in preparation with D. Adamiak, W. Melnitchouk, D. Pitonyak, N. Sato, M. Sievert in the JAM Collaboration framework.

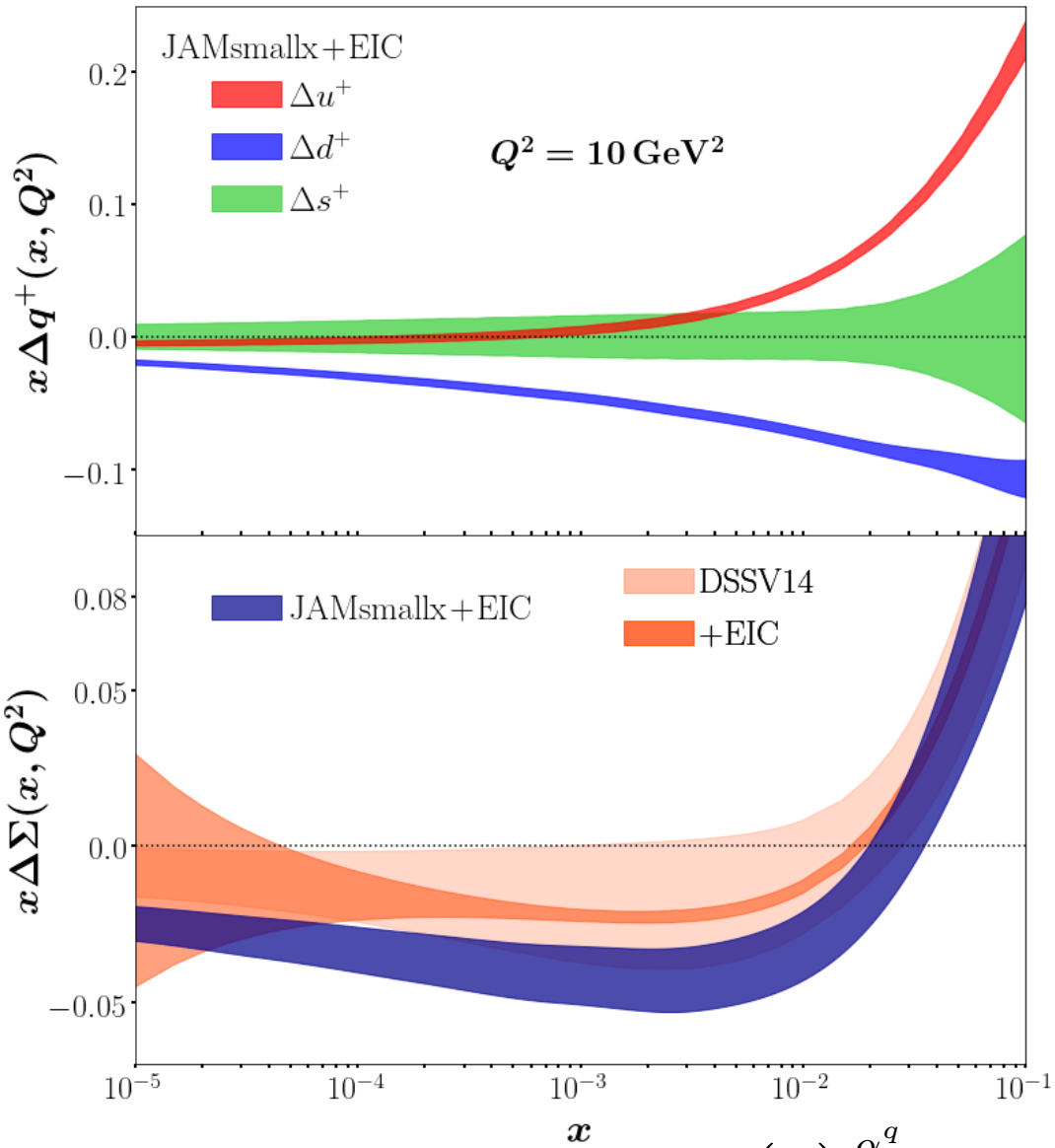


Prediction for g_1 structure function



Work in preparation with D. Adamiak, W. Melnitchouk, D. Pitonyak, N. Sato, M. Sievert in the JAM Collaboration framework. With the EIC pseudo-data we have 1096 data points.

Predictions for helicity PDFs



Work in preparation with D. Adamiak, W. Melnitchouk, D. Pitonyak, N. Sato, M. Sievert in the JAM Collaboration framework.

If we plug in $\alpha_S = 0.25$
we get $\alpha_h^q = 0.80$

$$\Delta q(x, Q^2) \sim \left(\frac{1}{x}\right)^{\alpha_h^q} \quad \text{with} \quad \alpha_h^q = \frac{4}{\sqrt{3}}\sqrt{\frac{\alpha_s\, N_c}{2\pi}} \approx 2.31\sqrt{\frac{\alpha_s\, N_c}{2\pi}}$$

Small-x Evolution at large N_c&N_f

- The resulting equations are

$$Q_{10}(zs) = Q_{10}^{(0)}(zs) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int\limits_{1/(z's)}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left\{ \frac{1}{2}\, \Gamma_{02,21}^{adj}(z') + \frac{1}{2}\, G_{21}^{adj}(z') + Q_{12}(z') - \bar{\Gamma}_{01,21}(z') \right\}$$

$$+ \frac{\alpha_s\, N_c}{4\,\pi} \int\limits_{\Lambda^2/s}^{z} \frac{dz'}{z'} \int\limits_{1/(z's)}^{x_{10}^2 z/z'} \frac{dx_{21}^2}{x_{21}^2}\, Q_{21}(z'),$$

$$G_{10}^{adj}(z) = G_{10}^{adj\,(0)}(z) + \frac{\alpha_s\, N_c}{2\pi} \int\limits_{\max\{\Lambda^2, 1/x_{10}^2\}/s}^{z} \frac{dz'}{z'} \int\limits_{1/(z'\,s)}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[\Gamma_{10,21}^{adj}(z') + 3\, G_{21}^{adj}(z') \right]$$

$$- \frac{\alpha_s\, N_f}{2\,\pi} \int\limits_{\Lambda^2/s}^{z} \frac{dz'}{z'} \int\limits_{1/(z'\,s)}^{x_{10}^2 z/z'} \frac{dx_{21}^2}{x_{21}^2}\, \bar{\Gamma}_{02;21}(z'),$$

$$\Gamma_{10,21}^{adj}(z') = \Gamma_{10,21}^{adj\,(0)}(z') + \frac{\alpha_s\, N_c}{2\pi} \int\limits_{\max\{\Lambda^2, 1/x_{10}^2\}/s}^{z'} \frac{dz''}{z''} \int\limits_{1/(z''\,s)}^{\min\{x_{10}^2, x_{21}^2 z'/z''\}} \frac{dx_{32}^2}{x_{32}^2} \left[\Gamma_{10,32}^{adj}(z'') + 3\, G_{32}^{adj}(z'') \right]$$

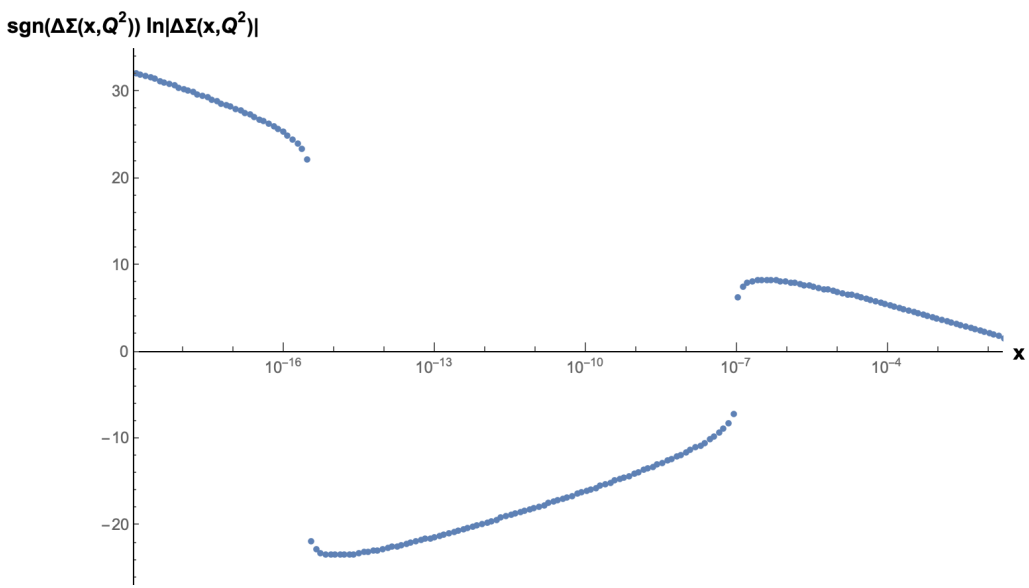
$$- \frac{\alpha_s\, N_f}{2\,\pi} \int\limits_{\Lambda^2/s}^{z'} \frac{dz''}{z''} \int\limits_{1/(z''\,s)}^{x_{21}^2 z'/z''} \frac{dx_{32}^2}{x_{32}^2}\, \bar{\Gamma}_{03;32}(z''),$$

$$\bar{\Gamma}_{10,21}(z') = \bar{\Gamma}_{10,21}^{(0)}(z') + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z'} \frac{dz''}{z''} \int\limits_{1/(z''s)}^{\min\{x_{10}^2, x_{21}^2 z'/z''\}} \frac{dx_{32}^2}{x_{32}^2} \left\{ \frac{1}{2}\, \Gamma_{03,32}^{adj}(z'') + \frac{1}{2}\, G_{32}^{adj}(z'') + Q_{32}(z'') - \bar{\Gamma}_{01,32}(z \right.$$

$$+ \frac{\alpha_s\, N_c}{4\,\pi} \int\limits_{\Lambda^2/s}^{z'} \frac{dz''}{z''} \int\limits_{1/(z''s)}^{x_{21}^2 z/z'} \frac{dx_{32}^2}{x_{32}^2}\, Q_{32}(z'').$$

Small-x Asymptotics at large N_c&N_f

- Large N_c&N_f can be solved only numerically, due to their complexity.
- This was done by Y. Tawabutr and YK in arXiv:2005.07285 [hep-ph].
- The solution exhibits an interesting qualitative change compared to large-N_c: it oscillates with ln(1/x)!



$$\alpha_h^q \approx 2.3\sqrt{\frac{\alpha_s\, N_c}{2\pi}}$$

$$\omega_q \approx \frac{0.22 N_f}{1+0.1265 N_f}\sqrt{\frac{\alpha_s\, N_c}{2\pi}}$$

$$\Delta\Sigma(x,Q^2)\Big|_{\text{large-}N_c\&N_f} \sim \left(\frac{1}{x}\right)^{\alpha_h^q} \cos\left[\omega_q \ln\frac{1}{x} + \varphi_q\right]$$

Quark and Gluon OAM

- Similar calculations for quark and gluon OAM lead to

$$L_{q+\bar{q}}(x,Q^2) = -\Delta\Sigma(x,Q^2) \sim \left(\frac{1}{x}\right)^{\frac{4}{\sqrt{3}}\sqrt{\frac{\alpha_s\, N_c}{2\pi}}}$$

and

$$L_G(x,Q^2) = \left(\frac{\alpha_h^q}{4}\,\ln\frac{Q^2}{\Lambda^2}\right)\,\Delta G(x,Q^2)$$

- The small-x asymptotics is (at large N_C)

$$L_{q+\bar{q}}(x,Q^2) = -\Delta\Sigma(x,Q^2) \sim \left(\frac{1}{x}\right)^{\frac{4}{\sqrt{3}}\sqrt{\frac{\alpha_s\, N_c}{2\pi}}},$$

$$L_G(x,Q^2) \sim \Delta G(x,Q^2) \sim \left(\frac{1}{x}\right)^{\frac{13}{4\sqrt{3}}\sqrt{\frac{\alpha_s\, N_c}{2\pi}}}$$

Conclusions

- Small-x physics can contribute to the resolution of the proton spin puzzle.
- Evolution equations are written at DLA with LLA corrections on the way. $\alpha_s \ln^2 \frac{1}{x}$ $\alpha_s \ln(1/x)$
- Initial effort at describing the existing polarized DIS data and making the predictions for EIC is almost complete (JAM-small*x*).
- More effort is needed in terms of improving the evolution kernels (e.g., *a la* BK) and solving the resulting evolution equations to describe the data and to generate more precise predictions for EIC. Helicity evolution mixes with the unpolarized BK/JIMWLK one, so a proper interface of the two is important.