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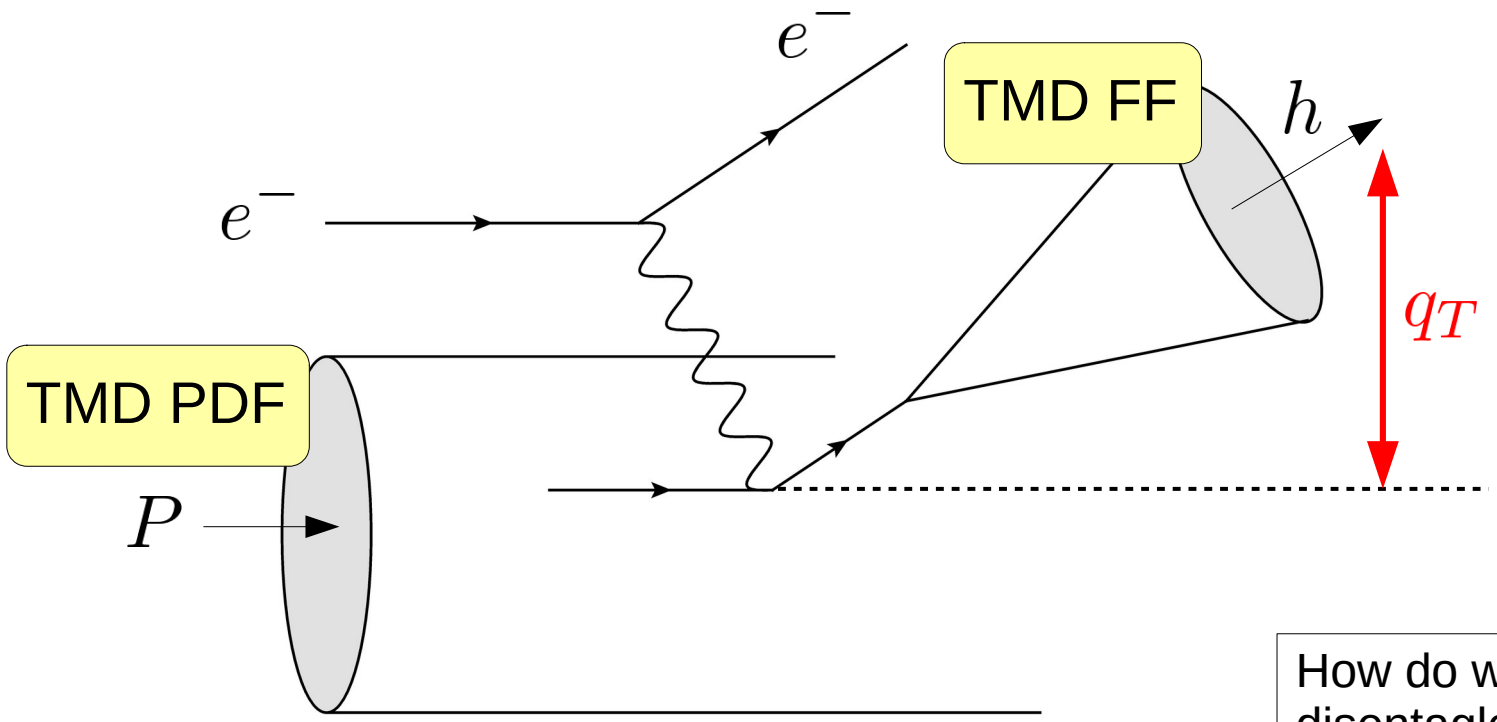
In collaboration with M. Boglione

# Toward a universal definition of TMD Fragmentation Functions

Center for Frontiers in Nuclear Science
Workshop series
EIC opportunities for Snowmass

# SIDIS: $e\,p \to h\,X$



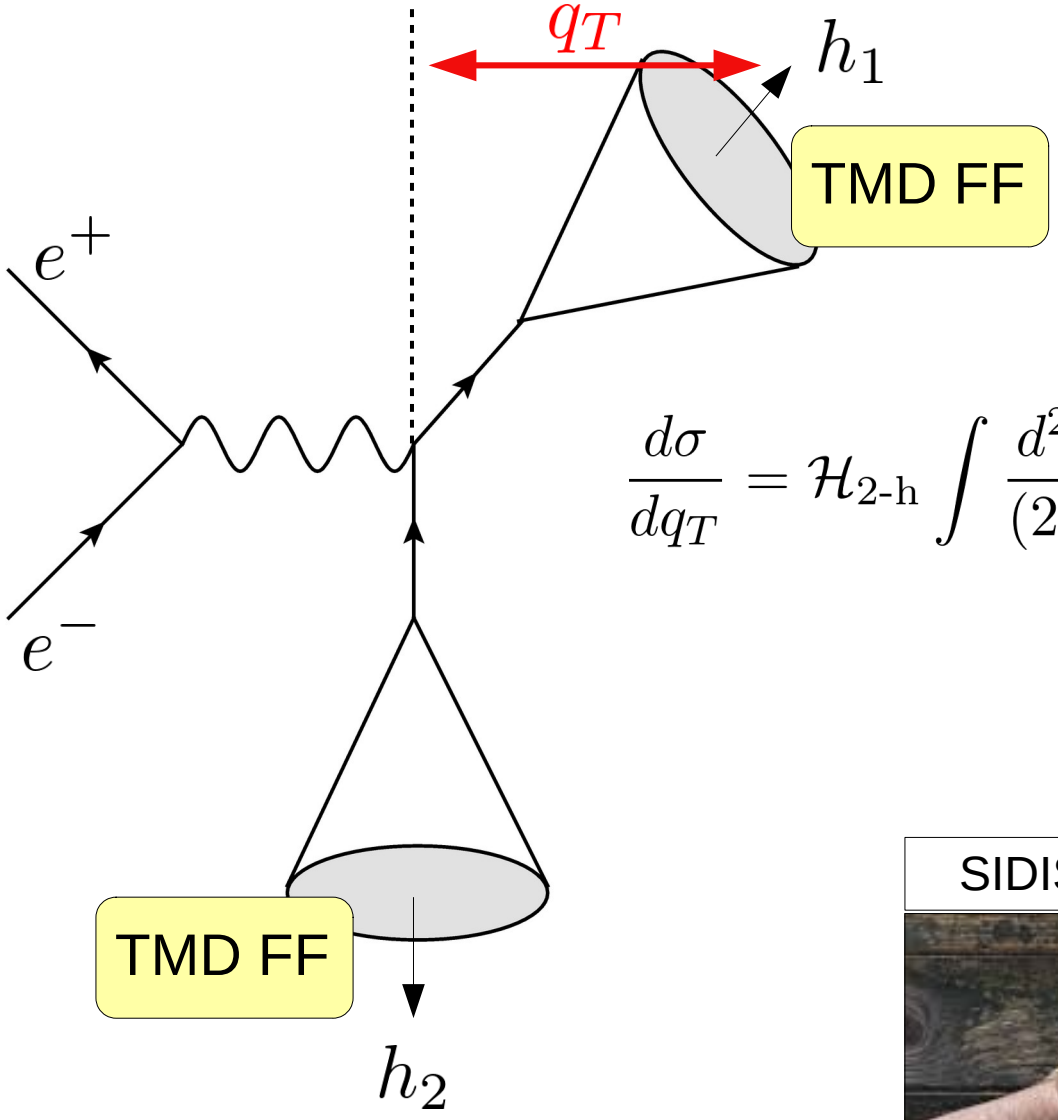

$$\frac{d\sigma}{dq_T} = \mathcal{H}_{\rm sidis} \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{b}_T}\, F(b_T)\, D(b_T)$$

How do we disentagle the convolution?

3D-picture of partons inside the target

3D-picture of partons hadronizing into the detected hadron

# Double hadron production: $e^+ e^- \to h_1 h_2 X$

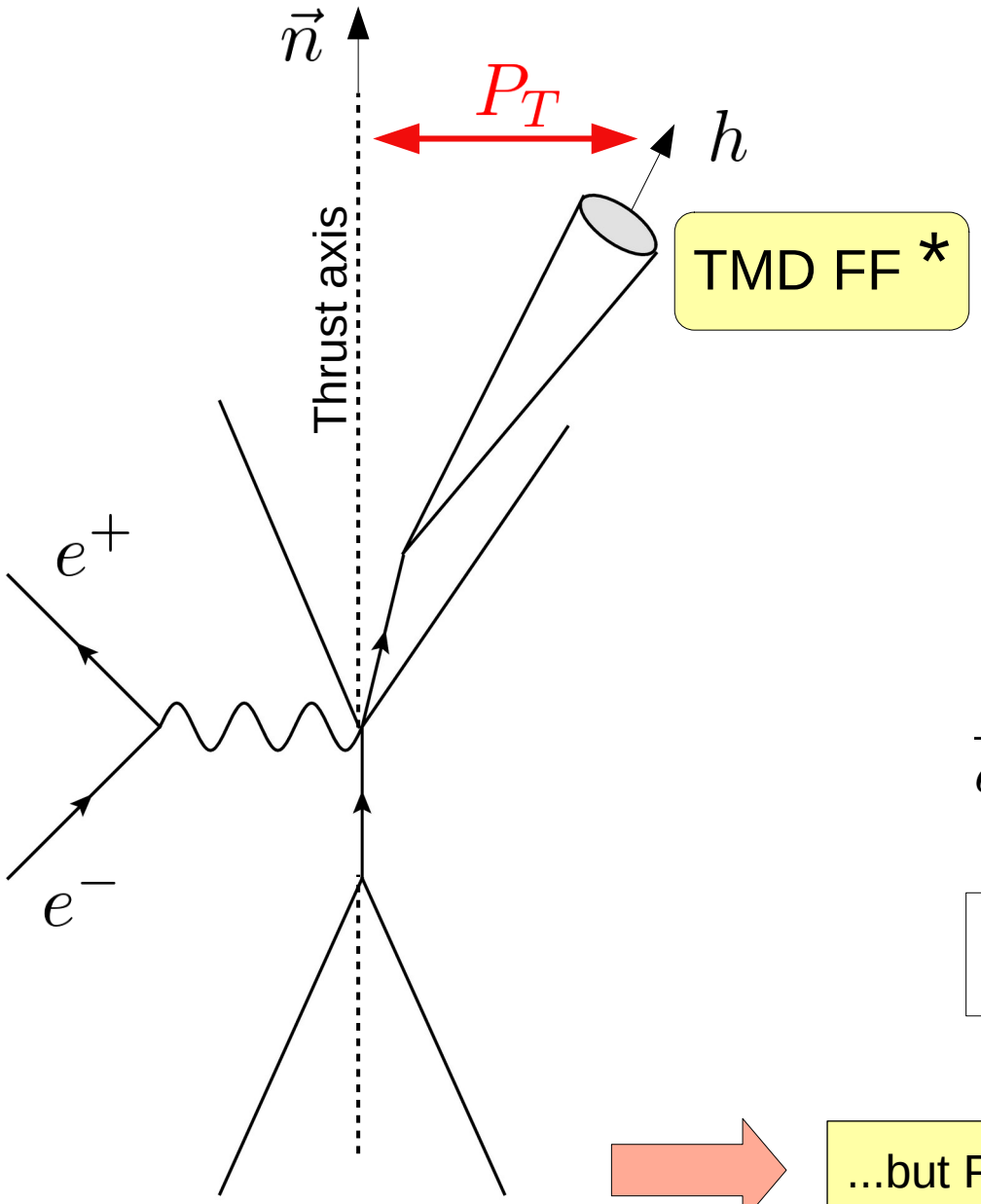



3D-picture of partons hadronizing into the detected hadrons

$$\frac{d\sigma}{dq_T} = \mathcal{H}_{2\text{-h}} \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T \cdot \vec{b}_T} D_1(b_T)\, D_2(b_T)$$

SIDIS + double hadron production

# Single hadron production: $e^+ e^- \to h X$ (thrust)




TMD FF *

M. Boglione, A. Simonelli, *Universality-breaking effects in $e^+e^-$ hadronic production processes*, Eur. Phys. J. C (accepted)

M. Boglione, A. Simonelli, *Factorization of $e^+e^- \to H X$ cross section, differential in $z_h$, $P_T$ and thrust, in the 2-jet limit*, JHEP (accepted)

3D-picture of partons hadronizing into the detected hadron

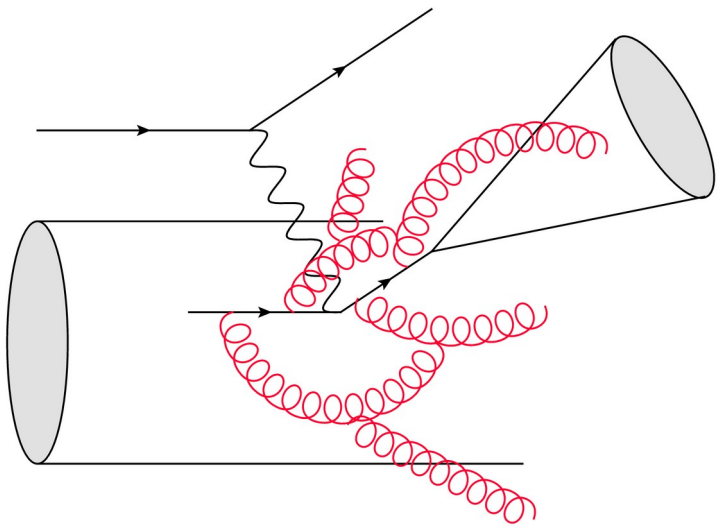
$$\frac{d\sigma}{dP_T} = d\hat{\sigma} \otimes D^\star(P_T)$$

One of the cleanest way to access TMD Fragmentation Function*...

...but PAY ATTENTION TO THE STAR ( * )

# Role of the Soft Gluons

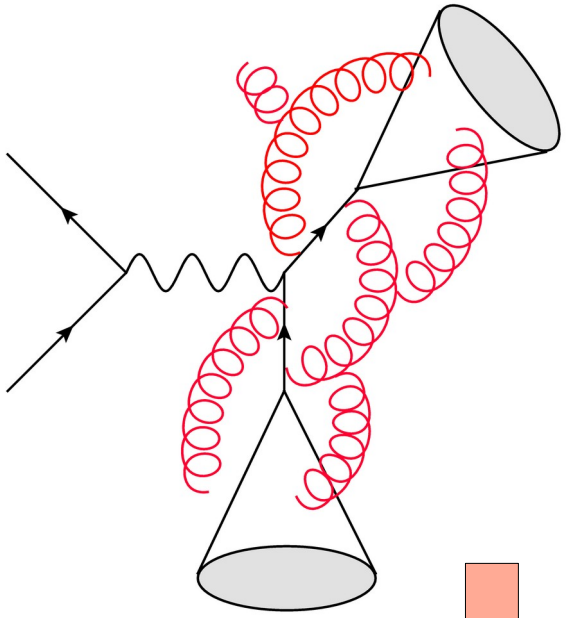
## SIDIS



$$\frac{d\sigma}{dq_T} = \mathcal{H}_{\text{sidis}}$$

$$\int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{b}_T}\, F(b_T)\, D(b_T)$$

## Double hadron production



$$\frac{d\sigma}{dq_T} = \mathcal{H}_{\text{2-h}}$$

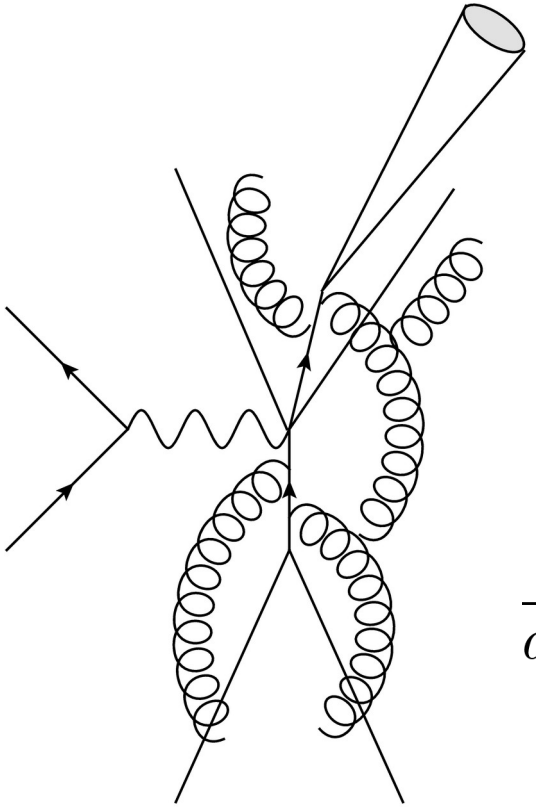
$$\int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{b}_T}\, D_1(b_T)\, D_2(b_T)$$

Soft Gluons Factor:

- Non-Perturbative contribution
- Evenly shared by the TMDs

# Role of the Soft Gluons

Single hadron production



$$\frac{d\sigma}{dP_T} = d\hat{\sigma} \otimes D^{\star}(P_T)$$

Soft Gluon Factor:

- Perturbative (computable) contribution (soft thrust function in the partonic cross section).
- The TMD FF***** is **free** from any soft gluon contributions

Meaning of the star ( * )

Once the relation between FF and FF* is known we can combine all the processes and disentagle the non-perturbative terms.

# Two TMD FFs definitions

$$D = D^{\star}\sqrt{M_S}$$

**SQUARE ROOT DEF.**
Usual definition of TMDs. It includes (half of) the Soft Gluon Factor that contributes to the cross section of SIDIS and 2-hadron production.

**FACTORIZATION DEF.**
**Pure collinear** TMD, totally free from any soft gluon contribution.

**SOFT MODEL**:
(Half of) the Soft Gluon Factor appearing in the cross section of SIDIS and 2-hadron production.

The two definitions are equivalent, but we have to make a choice:

CRUCIAL STEP

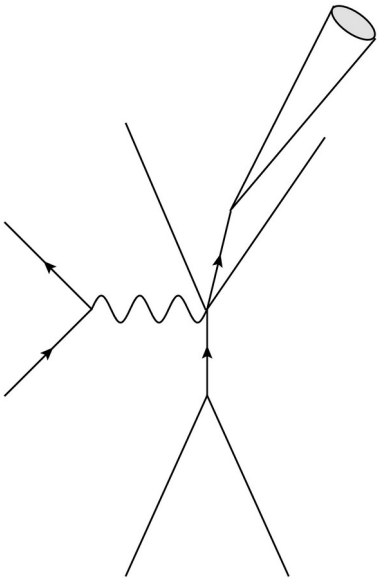
This choice defines our **work plan**.

- Same for Drell-Yan, SIDIS and 2-hadron production. (2-h class universality).
- Non-perturbative function (phenomenology).

# WORK PLAN (one of the possible recipes)

0. DEFINITION CHOICE:

We adopt the **factorization definition** for the TMDs ( * ).



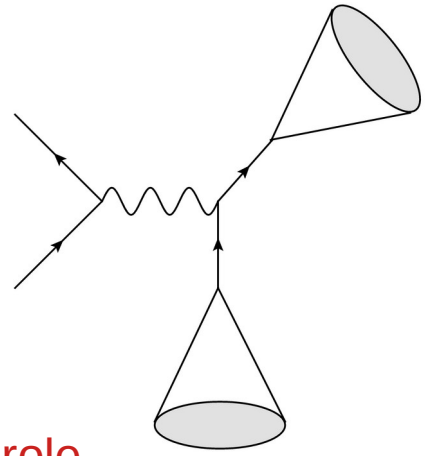
1. Extraction of the TMD FF D* from single hadron production (with thrust), where it is the only non-perturbative function.

Soon we will present the extraction of the unpolarized TMD FF for charged pions $D_{1,\,\pi^{\pm}}(z, P_T)$ extracted from BELLE collaboration data.

Boglione, Gonzalez, Simonelli,
*in preparation*

BelleCollaboration, R. Seidl et al.,
*Phys. Rev. D99 (2019)*, no. 11 112006

2. Extraction of the TMD FF D from the double hadron production.
In terms of the factorization definition, there are two non-perturbative functions:

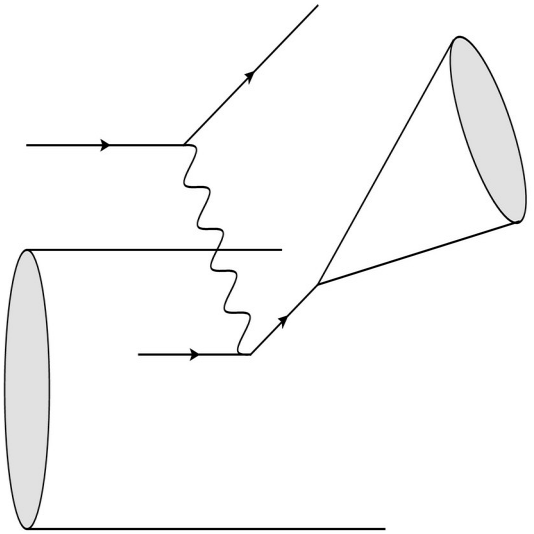


- D*, known from step 1.
- Soft Model $M_S$, obtained as ratio: $M_S = D/D^{\star}$

Notice that in this scheme the Soft Factor acquires a central and active role in phenomenological analyses.

# WORK PLAN (one of the possible recipes)

3. Go back to SIDIS.
In the cross section there are three non-perturbative functions:



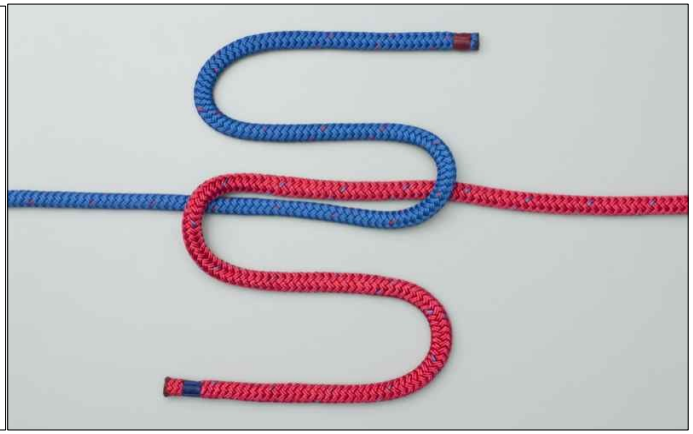
- D*, known from step 1.
- Soft Model $M_S$, known from step 2.
- F*, i.e. the TMD PDF in the factorization definition.

Extraction of the TMD PDF F* (which is now the last remaining unknown)

SIDIS +

double hadron prod. +

single hadron prod.
(with thrust)



The **EIC** will be instrumental for the success of this work plan, since it will give access to SIDIS over a wide range of energy scales (large enough for factorization to work)

# WORK PLAN (one of the possible recipes)

4. The non-perturbative functions D* and F* (TMDs), together with the soft model $M_S$, are building blocks of any cross sections that involve up to two hadrons.

- 1-had: single hadron prod
- 2-had: DRELL-YAN | **SIDIS** | double hadron prod


5. In principle, this scheme can be extended even **beyond**, to processes that involve more than two hadrons (e.g. $p\,p \to h\,X$ …).

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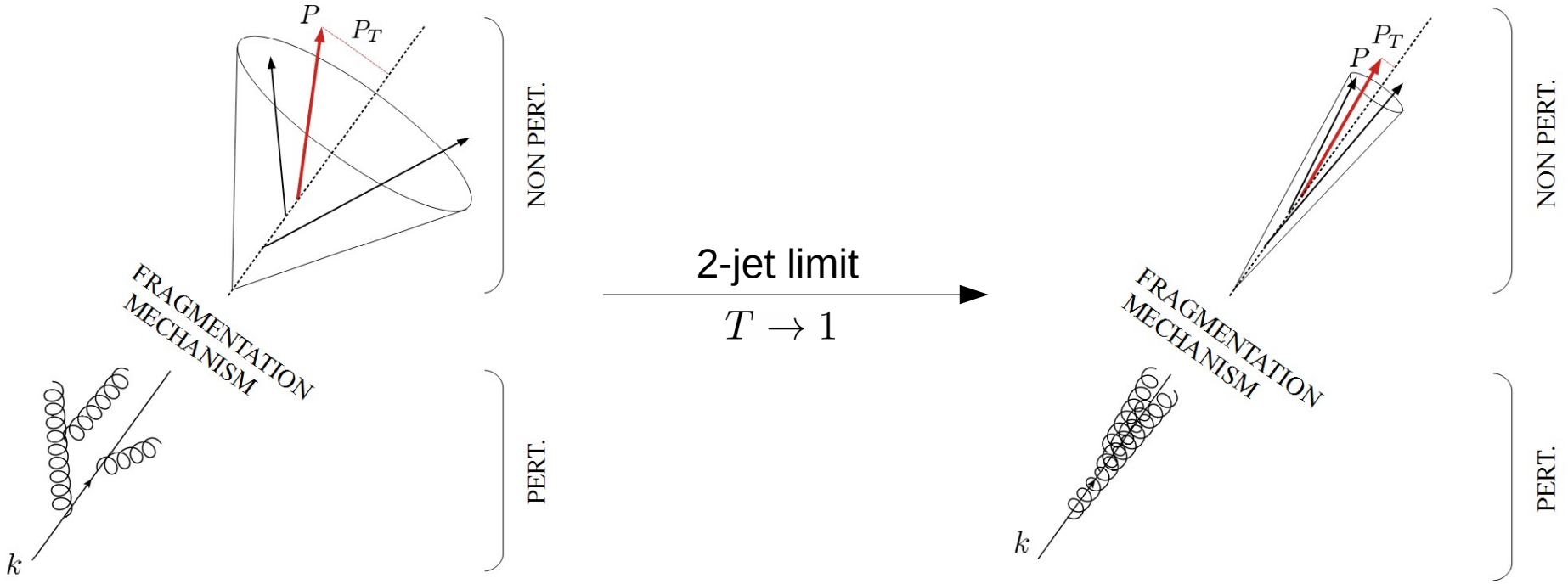
# BACKUP SLIDES

# Cross section of single hadron production (thrust)

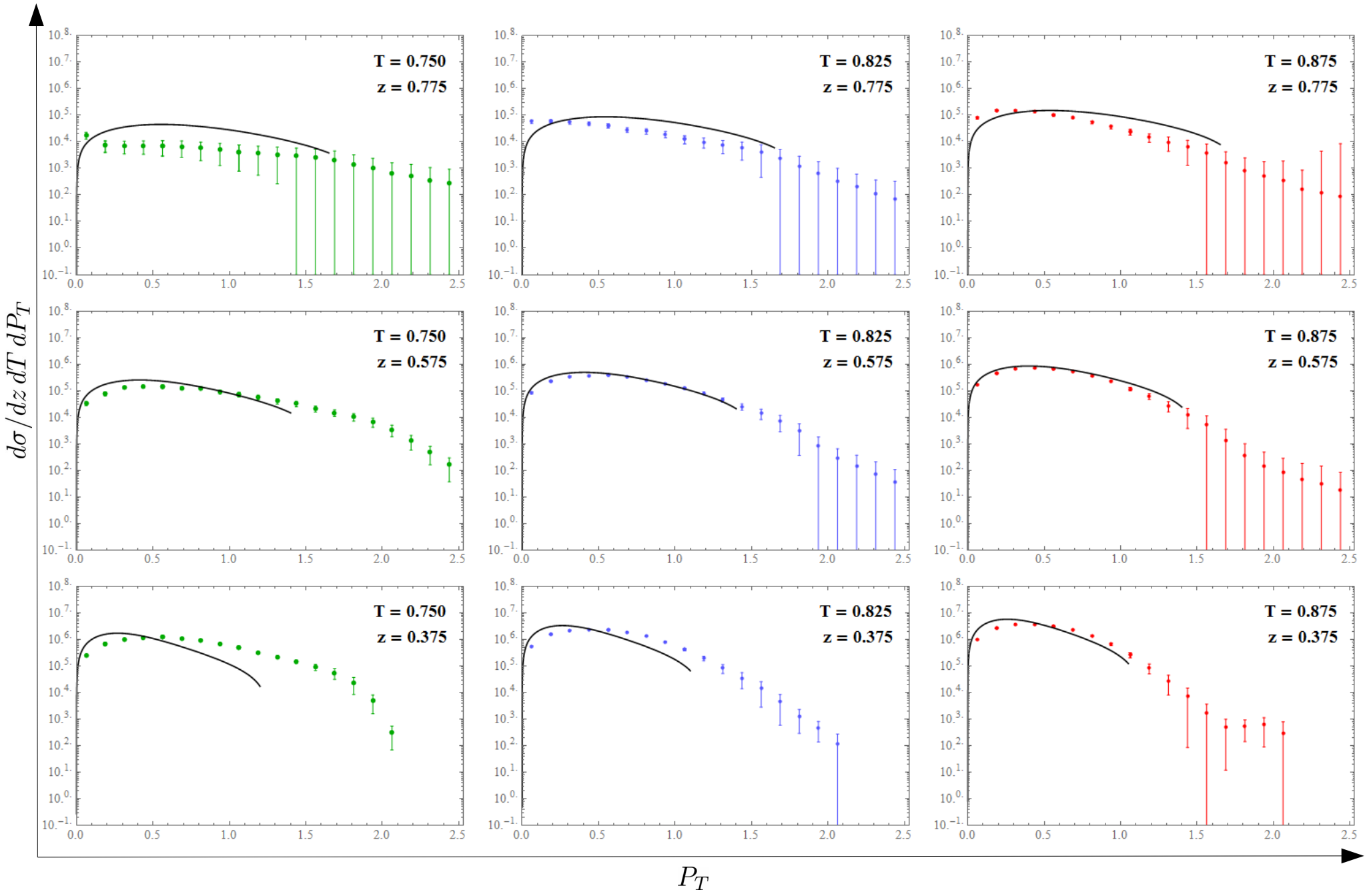
$$\frac{d\sigma}{dz_h\, dT\, dP_T^2} = \pi \sum_f \int_{z_h}^1 \frac{dz}{z}\, \frac{d\hat{\sigma}_f}{dz_h/z\, dT}\, D_{1,\pi^\pm/f}(z,\, P_T,\, Q,\, (1-T)\, Q^2)$$

Only fermions,
the fragmenting gluon
is suppressed by $\mathcal{O}(1-T)$

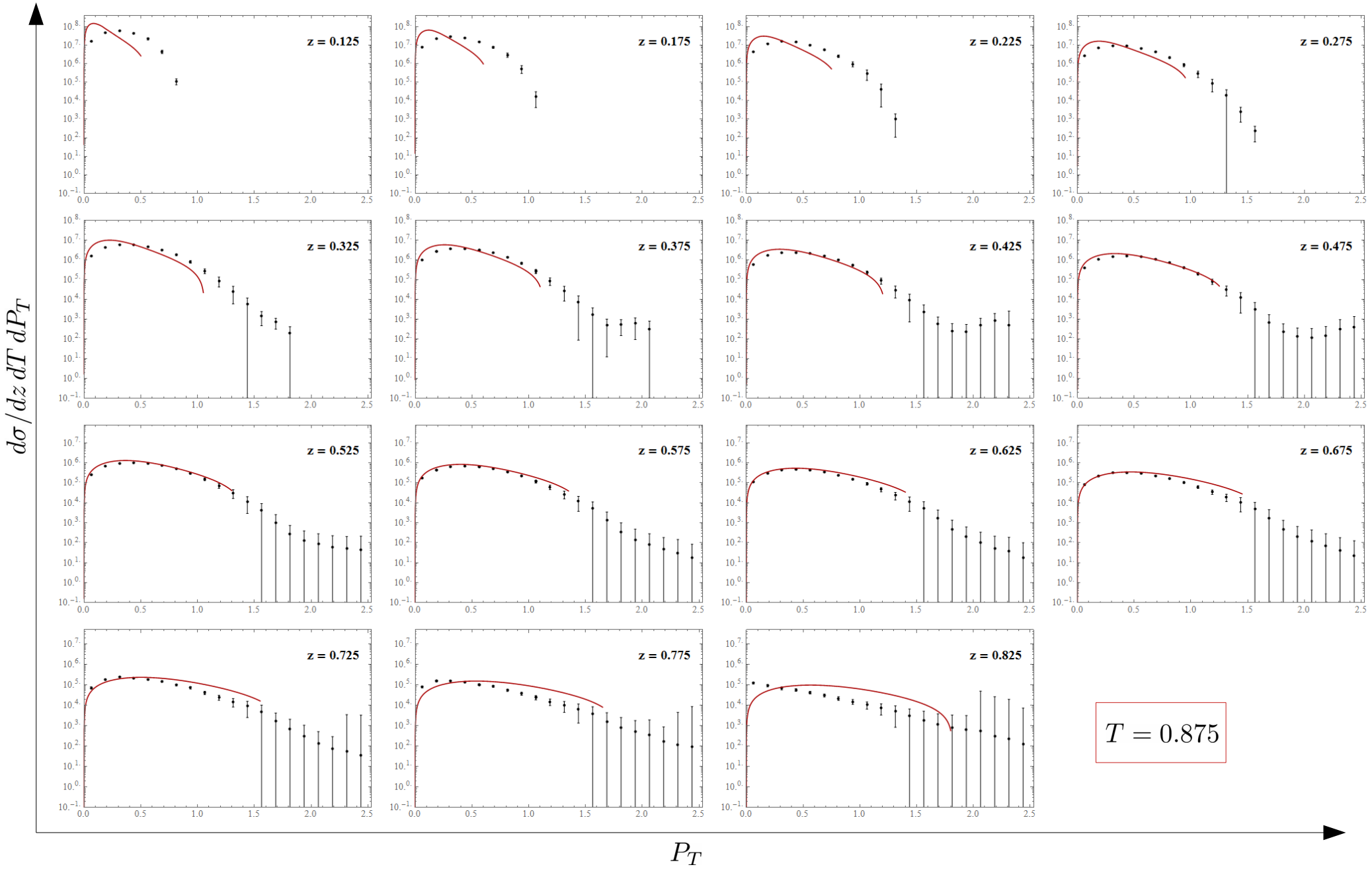
The TMD FF acquires a
dependence on thrust
through its rapidity cut-off.



# Preliminar results

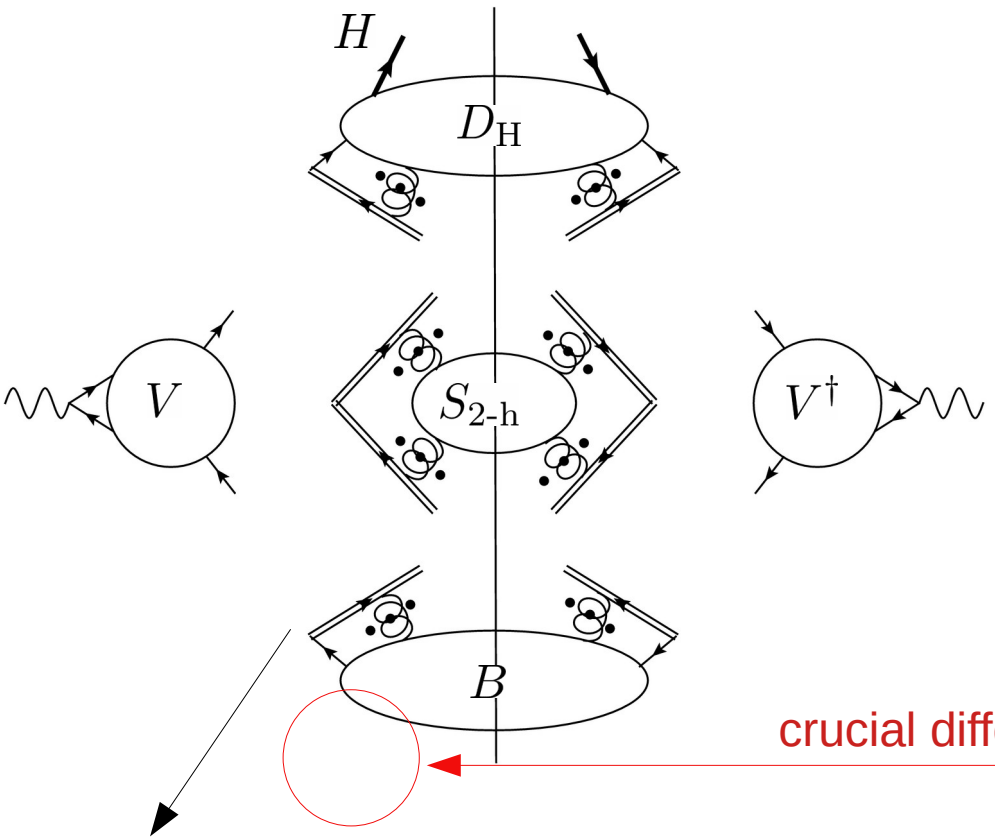


# Preliminar results

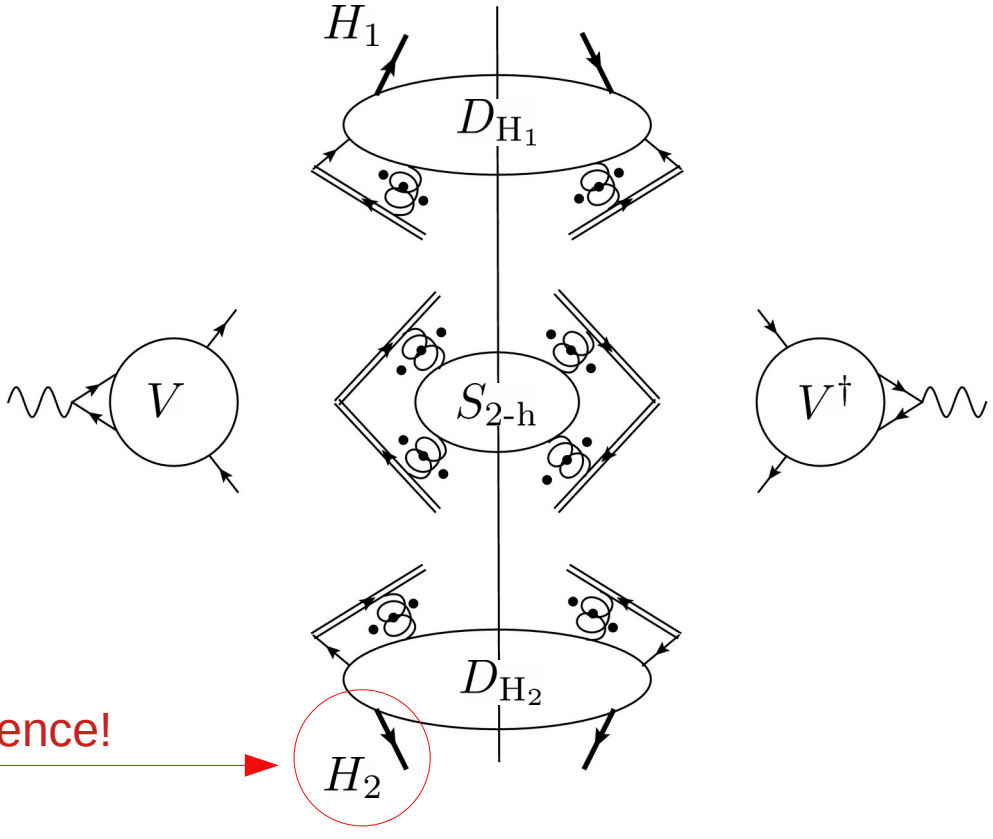


# Single vs Double hadron

$e^+e^- \to H\,X \quad (T \sim 1)$




$e^+e^- \to H_1\,H_2\,X$




crucial difference!

$\bar{q}$ plays the role of a hard real emission.

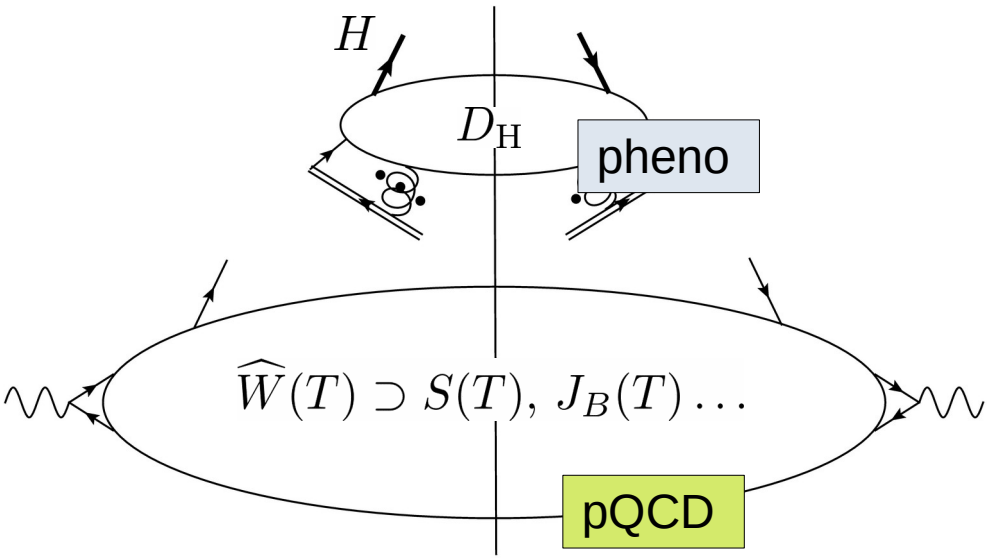
J. Collins, *Foundations of perturbative QCD*.

- $B$ can be considered a “hard part”.
- $S_{2\text{-h}}$ turns out to be unity ($\mathbb{1}$).

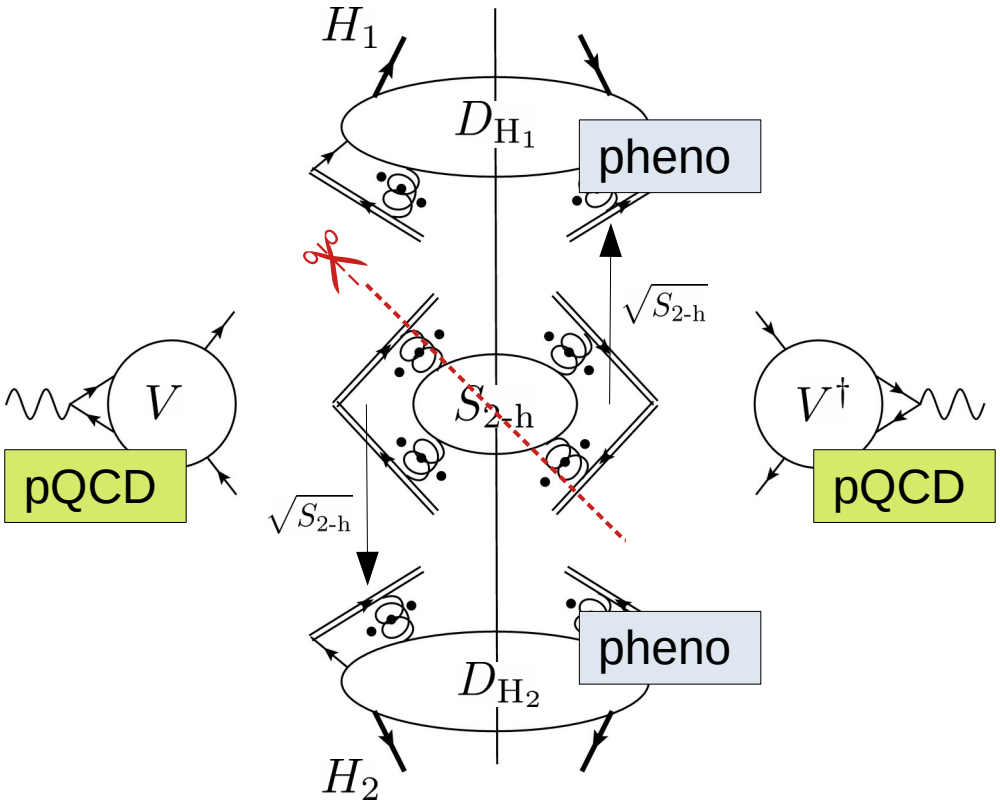
REARRANGEMENT OF BLOBS in the SINGLE-HADRON case

# Single vs Double hadron

$e^+e^- \to H\,X \quad (T \sim 1)$




$e^+e^- \to H_1\,H_2\,X$




Different TMDs !

$$\widetilde{D}^{\mathrm{sqrt}}_{H_1/f} = \widetilde{D}_{1,\,H/f}\,\sqrt{M_S}$$

## Soft Model

- Long-distance behavior of the 2-h Soft Factor $S_{2\text{-h}}$ .
- Pivotal role of Soft Factor