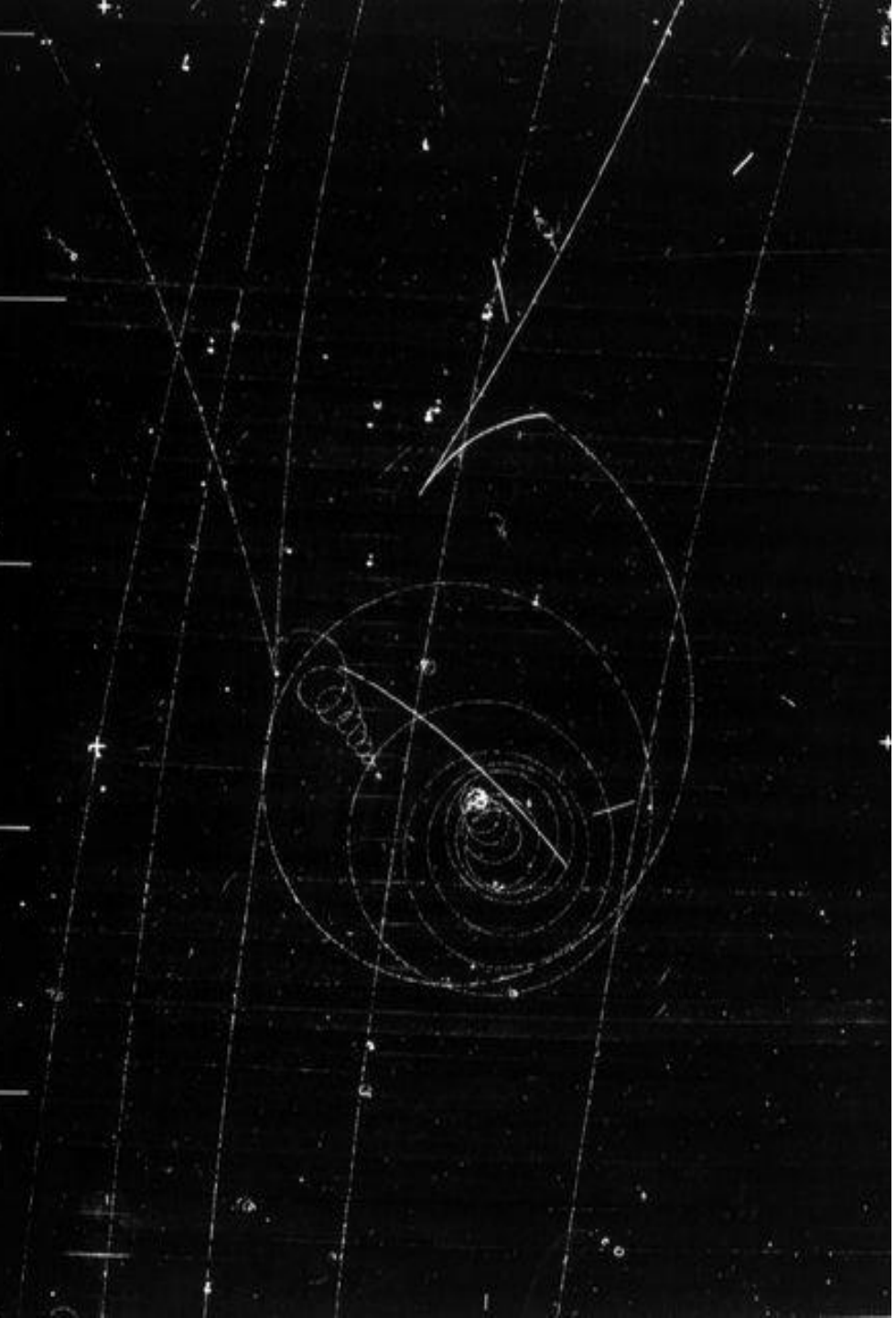

Extraction of the Λ Polarizing Fragmentation Function from Belle e^+e^- data

Marco Zaccheddu - Università degli Studi di Cagliari & INFN

In collaboration with: Umberto D'Alesio, Francesco Murgia

Based on [Phys. Rev. D 102, 054001 (2020)]

EIC opportunities for SnowMass 2021

Istituto Nazionale di Fisica Nucleare

Observation of Transverse $\Lambda/\overline{\Lambda}$ Hyperon Polarization in e^+e^- Annihilation at Belle

[Y. Guan et al., Phys. Rev. Lett. 122. 042001 (2019)]

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- $e^+e^- \rightarrow \Lambda(jet) + X$

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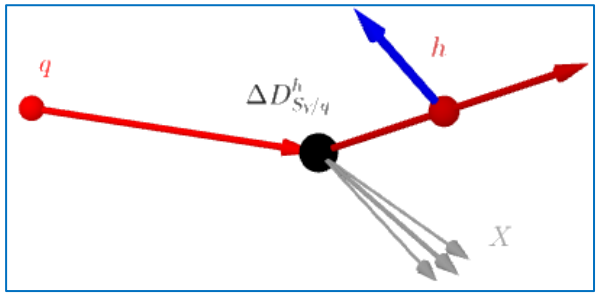
Λ TMD Polarizing FF

$$\Delta D^h_{S_Y/q} = \frac{k_\perp}{zM} D^\perp_{1T}$$



First extraction of the Λ pFF
[D'Alesio, Murgia, Zaccheddu, Phys. Rev. D 102, 054001 (2020)]

Similar analysis:

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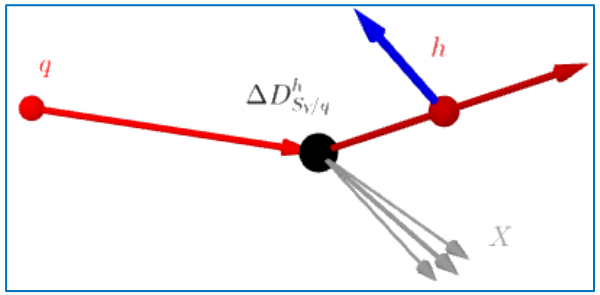

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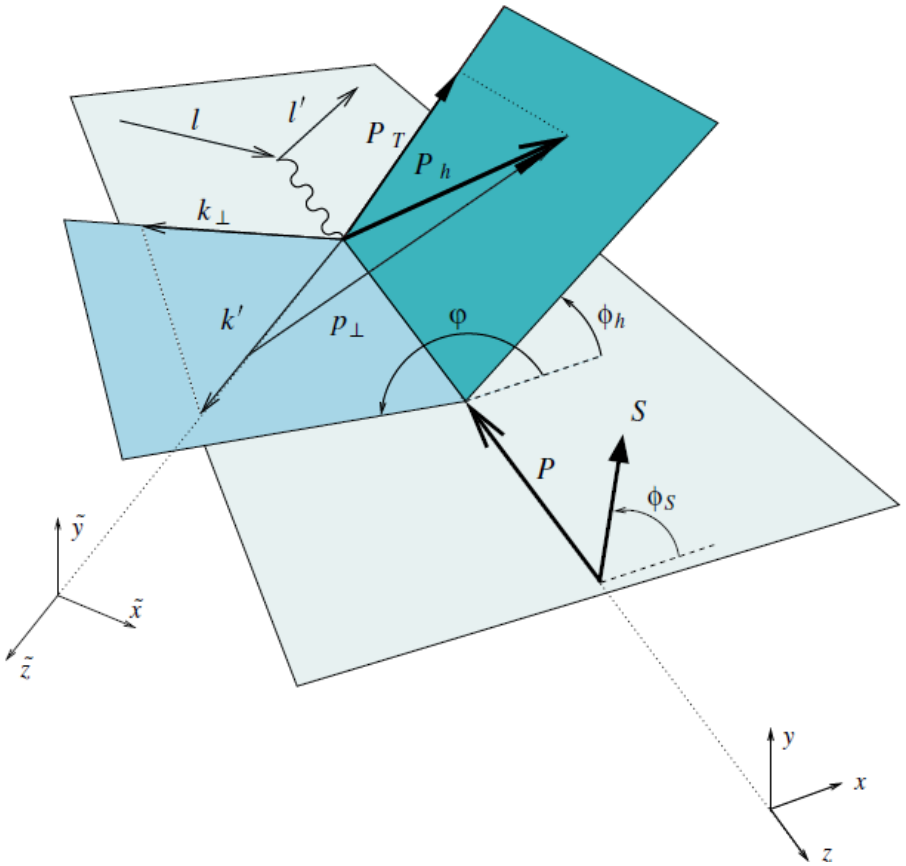
Prediction for SiDIS: $e^-P \to e^-\Lambda + X$



$e^+e^- \to h^{\uparrow}{}_1 h_2 X$: Hadron Frame

The polarization is measured along:

$$\hat{n} = -\hat{P}_{h_2} \times \hat{P}_{h_1}$$

$$\begin{aligned}\mathcal{P}_n(z_1, z_2) &= \sqrt{\frac{e\pi}{2}} \frac{1}{M_{\rm pol}} \frac{\langle p_\perp^2 \rangle_{\rm pol}^2}{\langle p_{\perp 1}^2 \rangle} \frac{z_2}{\left\{ [z_1(1 - m_{h_1}^2/(z_1^2 s))]^2 \langle p_{\perp 2}^2 \rangle + z_2^2 \langle p_\perp^2 \rangle_{\rm pol} \right\}^{1/2}} \\ &\times \frac{\sum_q e_q^2 \, \Delta D_{h_1^\uparrow/q}(z_1) D_{h_2/\bar{q}}(z_2)}{\sum_q e_q^2 \, D_{h_1/q}(z_1) D_{h_2/\bar{q}}(z_2)}\end{aligned}$$

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$e^+e^- \to \mathrm{h}_1 + (\mathrm{jet})\, X$: Thrust Frame

The polarization is measured along:

$$\hat{n} = \hat{T} \times \hat{P}_{h_1}$$

$$\mathcal{P}_T(z_1, p_{\perp 1}) = \frac{\sum_q e_q^2 \, \Delta D_{h_1^\uparrow/q}(z_1, p_{\perp 1})}{\sum_q e_q^2 \, D_{h_1/q}(z_1, p_{\perp 1})}$$

Within a phenomenological approach

$e^+e^- \to h^{\uparrow}{}_1 h_2 X$: Hadron Frame

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Introduce $p_\perp$ - parameterization for FFs:

$$\Delta D^h_{S_Y/q}(z, p_\perp) = \Delta D^h_{S_Y/q}(z) \sqrt{2e} \frac{p_\perp}{M_{pol}} \frac{e^{-p_\perp^2/\langle p_\perp^2 \rangle_{pol}}}{\pi \langle p_\perp^2 \rangle_h}$$

$$D_{h/q}(z, p_\perp) = D_{h/q}(z) \frac{e^{-p_\perp^2/\langle p_\perp^2 \rangle_h}}{\pi \langle p_\perp^2 \rangle_h}$$

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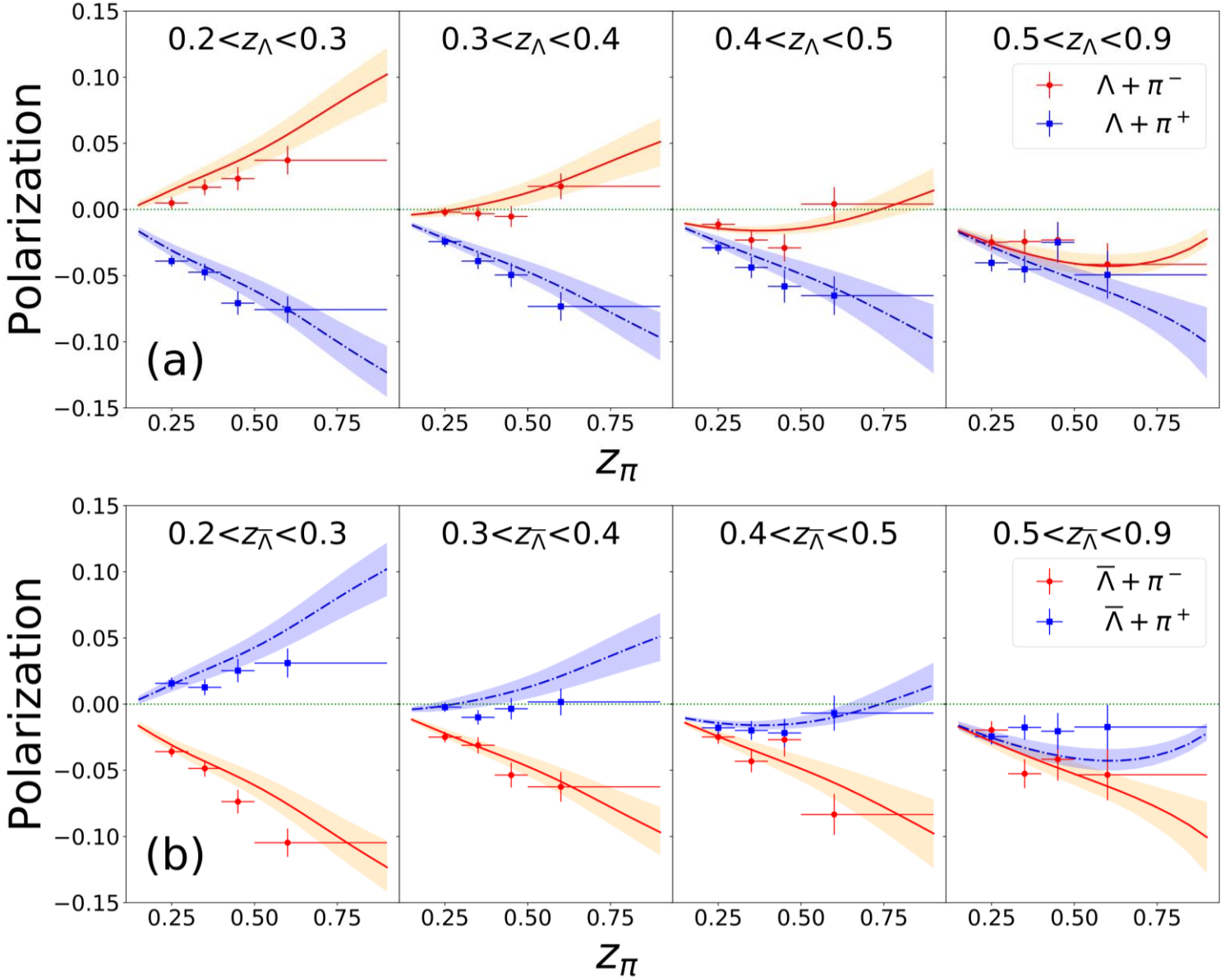
Within a phenomenological approach

The two extractions:

- Associated production: $e^+e^- \to h^{\uparrow}{}_1 h_2 X$
- Global: $e^+e^- \to h^{\uparrow}{}_1 h_2 X$ + $e^+e^- \to \mathrm{h}_1 + (\mathrm{jet})\, X$

Lead to consistent results

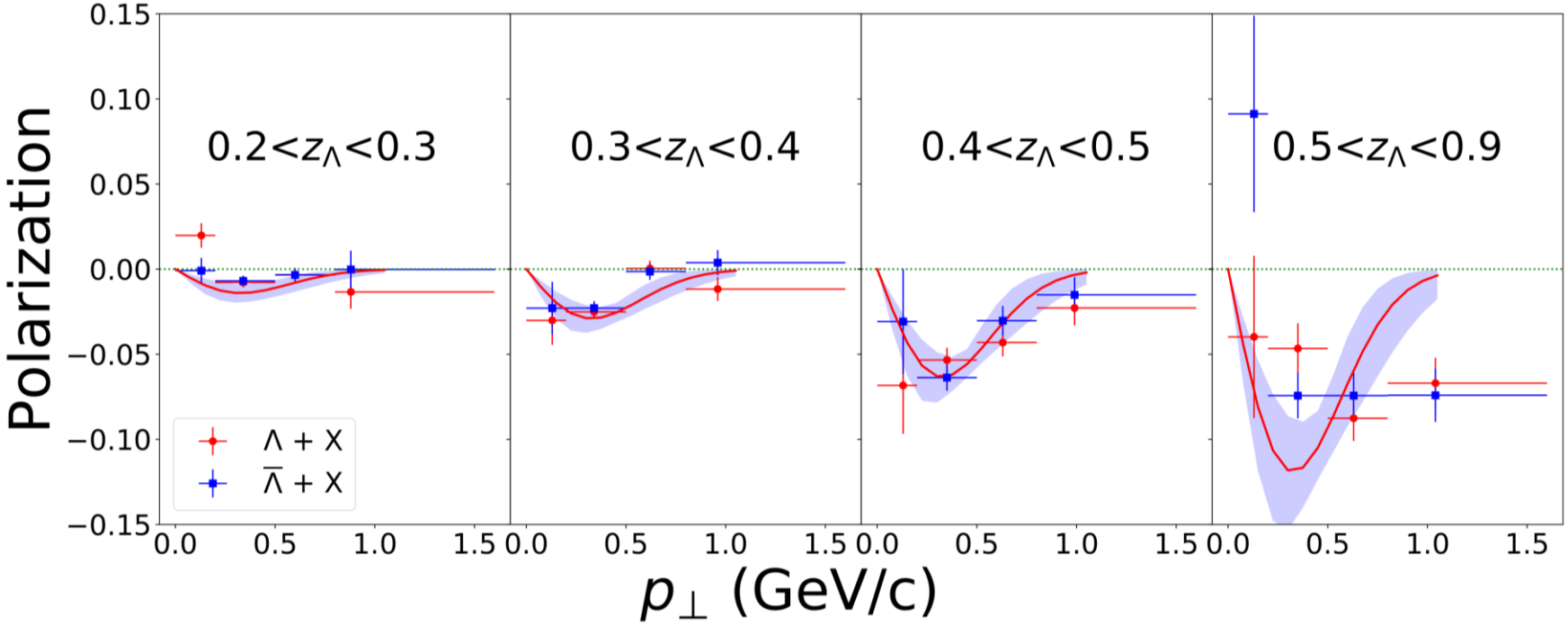
Lambda-pion



Bin excluded
$z_\pi = [0.5 - 0.9]$

$\chi^2_{dof} = 1.94$

Lambda-jet



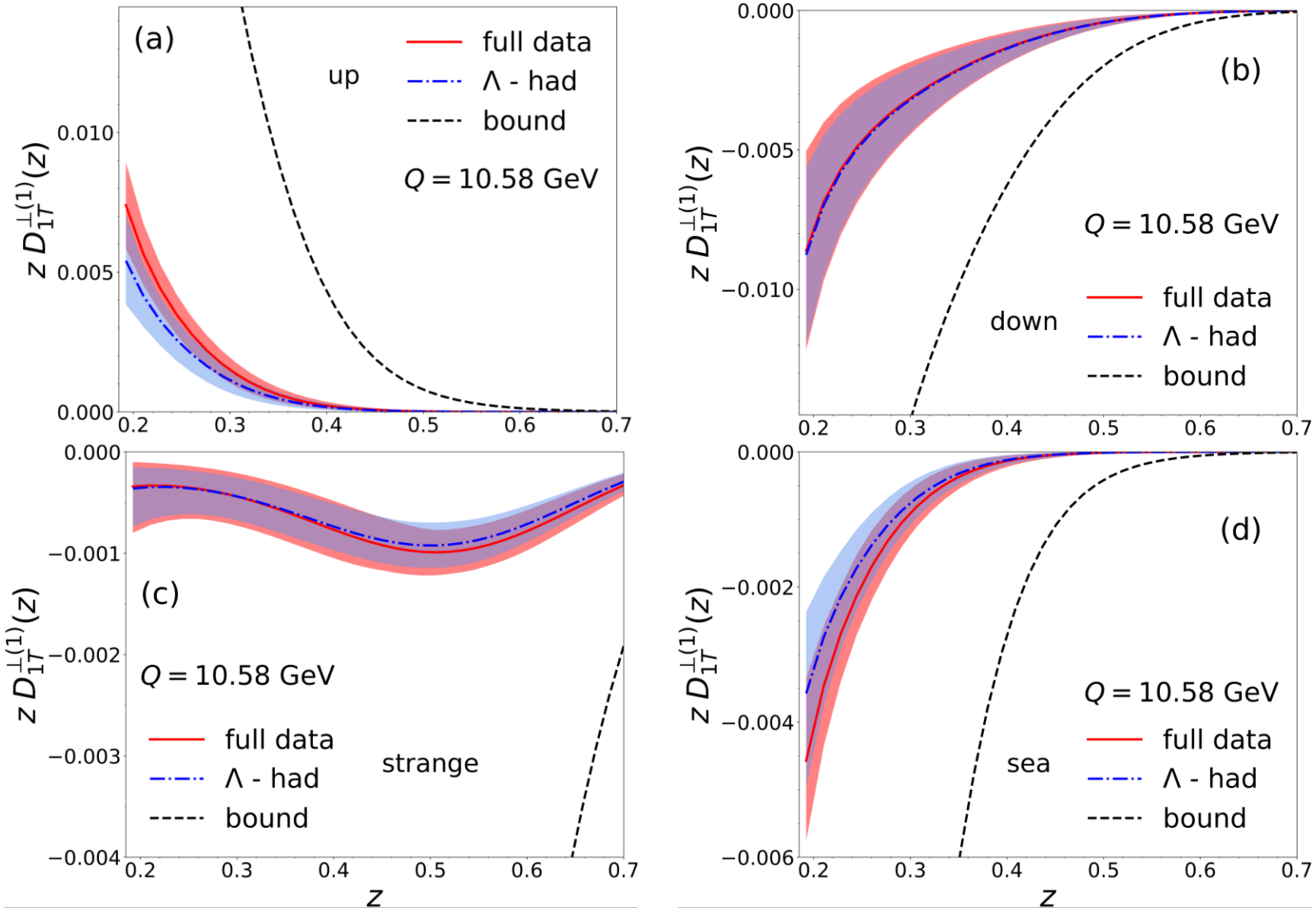
Bin excluded
$z_\Lambda = [0.5 - 0.9]$

$\chi^2_{dof} = 1.94$

Expected features:

- $P_T = 0$ when $p_\perp = 0$;
- $P_T(\Lambda) = P_T(\bar{\Lambda})$.

First moments



$$D_{1T}^{\perp(1)}(z) = \int d^2\mathbf{p}_\perp \frac{p_\perp}{2zm_h} \Delta D_{h^\uparrow/q}(z, p_\perp)$$

- Three different valence pFF and Sea

SiDIS – Polarized Lambda Production

$$P_T(x_B, z_h) = \frac{\sqrt{2e\pi}}{2M_p} \frac{< p_\perp^2 >_p^2}{< p_\perp^2 >} \frac{1}{\sqrt{< p_\perp^2 >_p + \xi_p^2 < k_\perp^2 >}} \times \frac{\sum_q f_{q/P}(x_B) \Delta D_{h^\uparrow/q}(z_h)}{\sum_q f_{q/P}(x_B) D_{h/q}(z_h)}$$

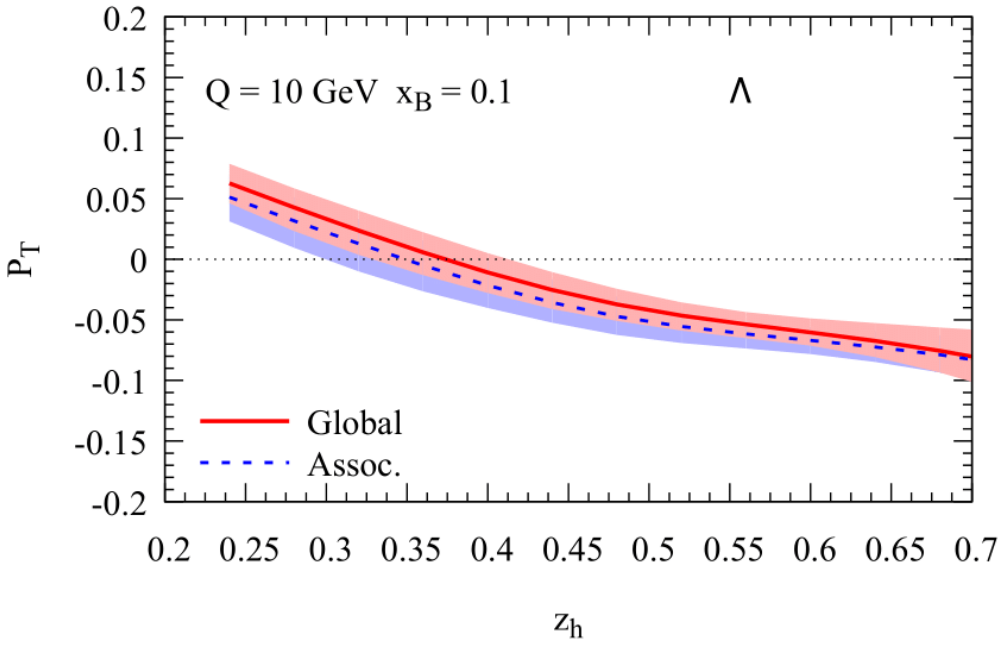
Proton PDF

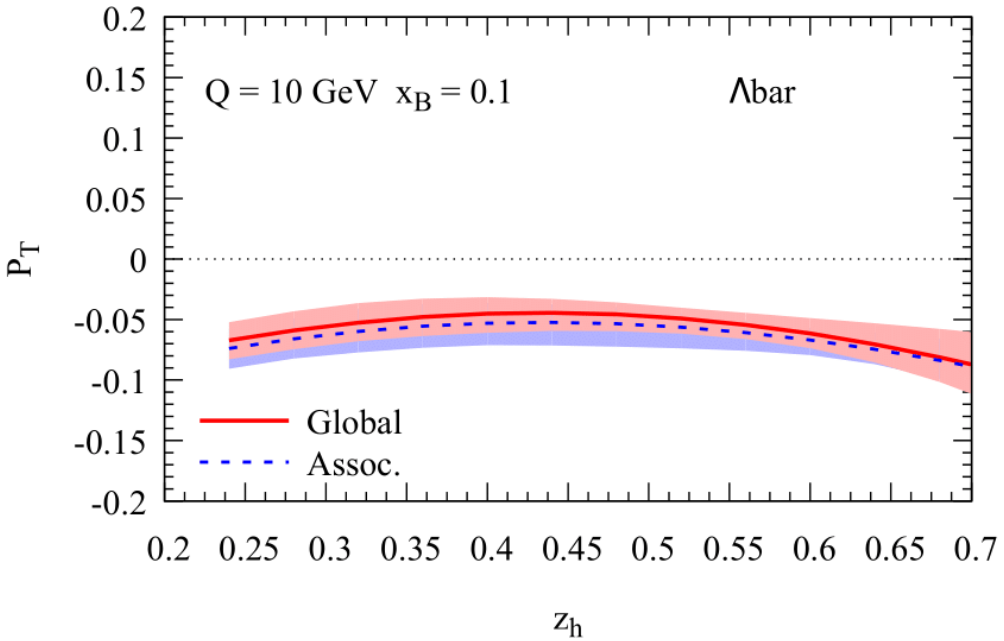
Lambda pFF

Prediction for the Λ polarization:

- x_B = 0.1

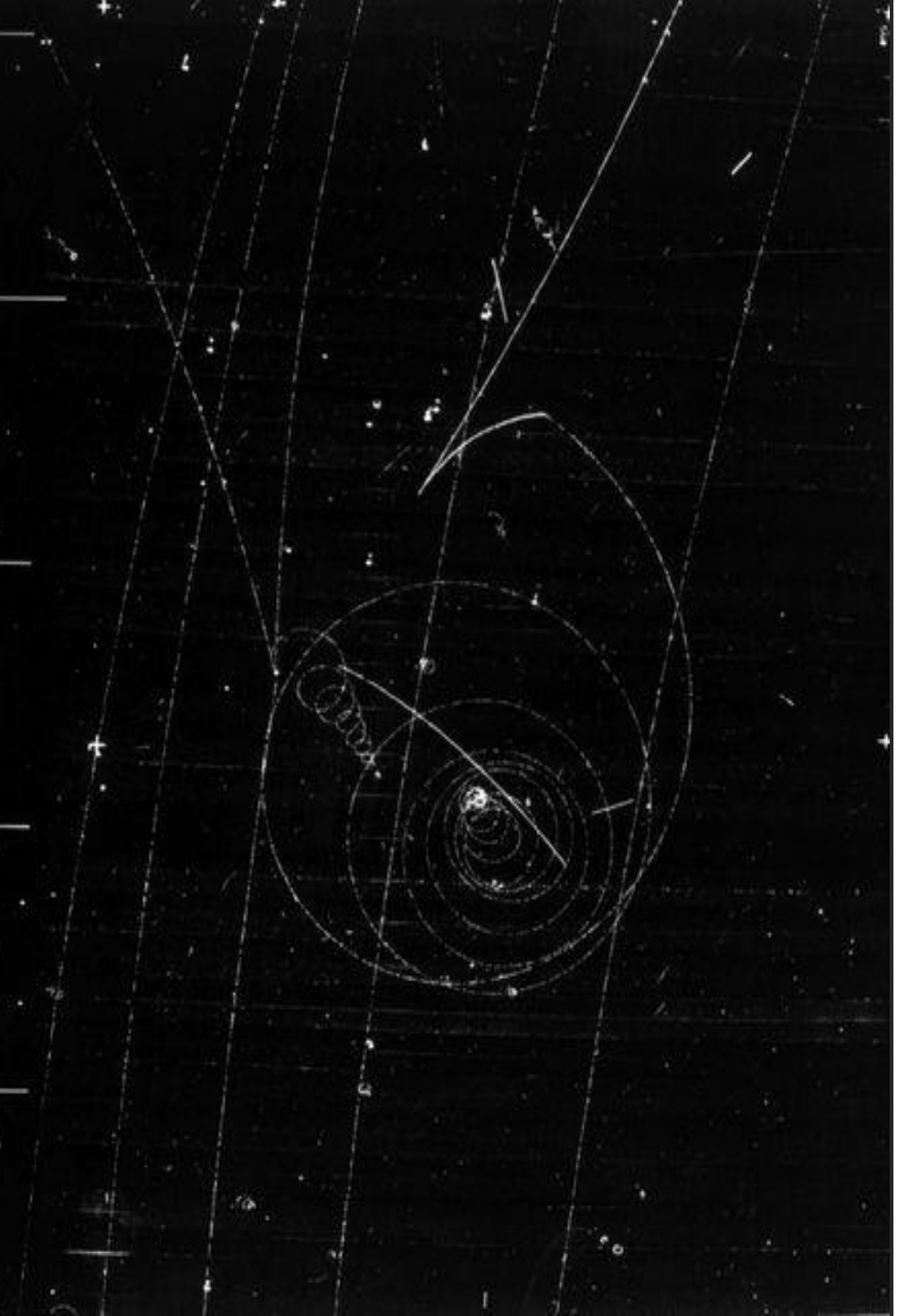
Similar pattern for x_B = 0.3





Outlook:

- EIC will allow to test the Lambda pFF universality;
- Improve flavor separation;
- Test TMD evolution



Backup Slides

TMD Fragmentation Functions for quarks : Spin - ½ hadrons

Helicity density matrix

$$\rho^{i,s_i}_{\lambda_i,\lambda_i'} = \frac{1}{2}\begin{pmatrix} 1+P_z^i & P_x^i - iP_y^i \\ P_x^i + iP_y^i & 1-P_z^i \end{pmatrix}$$

$$\rho^{h,S_h}_{\lambda_h,\lambda_h'}\,\hat{D}_{h/q,s_q}(z,\mathbf{k}_{\perp h}) = \sum_{\lambda_q,\lambda_q'} \rho^{q,s_q}_{\lambda_q,\lambda_q'}\,\hat{D}^{\lambda_h,\lambda_h'}_{\lambda_q,\lambda_q'}(z,\mathbf{k}_{\perp h})$$

8 independent TMD Fragmentation Functions

		Hadron		
	Pol. States	U	L	T
Quark	U	$\hat{D}_{h/q}$		$\Delta\hat{D}^{h}_{S_Y/q}$
	L		$\Delta\hat{D}^{h/q}_{S_Z/s_L}$	$\Delta\hat{D}^{h/q}_{S_X/s_L}$
	T	$\Delta^N D_{h/q^\uparrow}$	$\Delta\hat{D}^{h/q}_{S_Z/s_T}$	$\Delta\hat{D}^{h/q}_{S_X/s_T}$ / $\Delta^-\hat{D}^{h/q}_{S_Y/s_T}$

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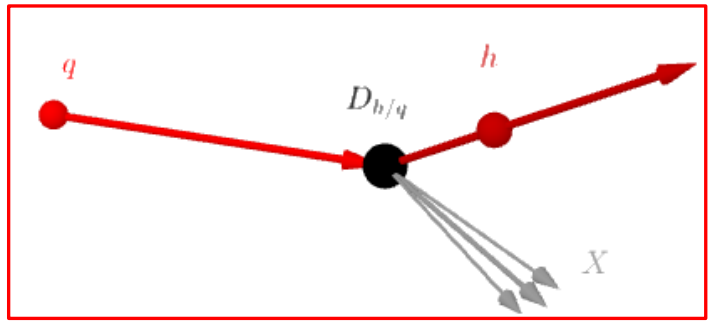
Unpolarized FF



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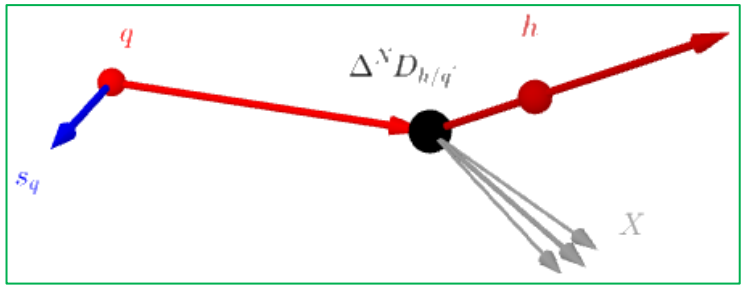
Collins FF



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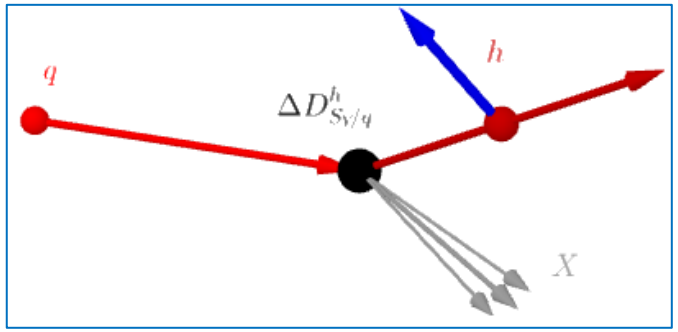
Polarizing FF



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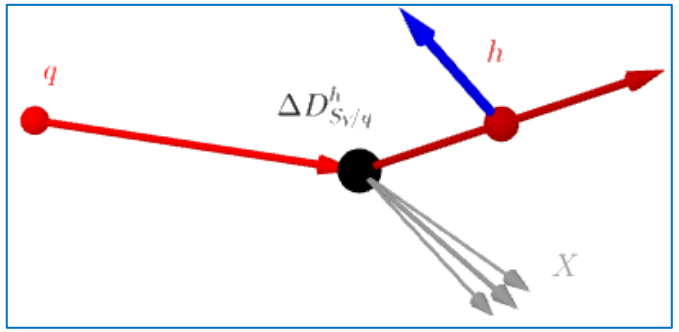

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Amsterdam notation:

Collins $\Delta^N D_{h/q^\uparrow} \longleftrightarrow H_1^\perp$

Polarizing $\Delta D^h_{S_Y/q} \longleftrightarrow D_{1T}^\perp$

$e^+e^- \to h_1 h_2 X$: Helicity Formalism

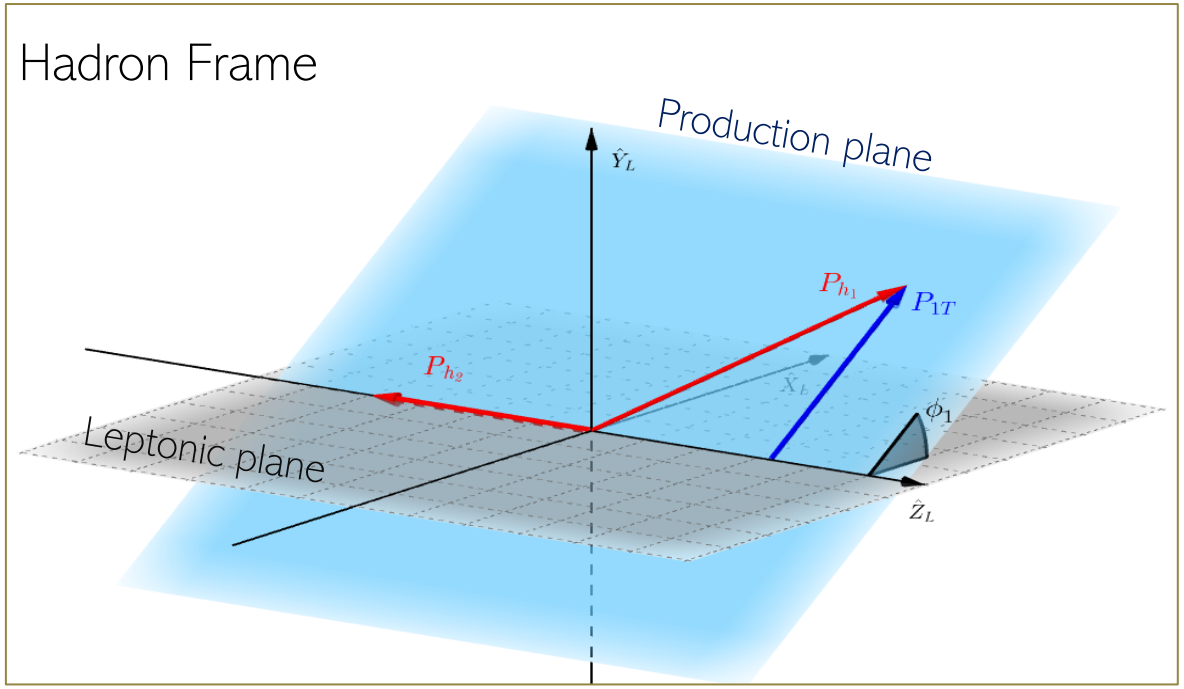
$$\rho^{h_1}_{\lambda_{h_1},\lambda'_{h_1}} \rho^{h_2}_{\lambda_{h_2},\lambda'_{h_2}} \frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta dz_1 d^2\mathbf{p}_{\perp 1} dz_2 d^2\mathbf{p}_{\perp 2}}$$
$$= \sum_{q_c} \sum_{\{\lambda\}} \frac{1}{32\pi s} \frac{1}{4} \hat{M}_{\lambda_c\lambda_d,\lambda_a\lambda_b} \hat{M}^*_{\lambda'_c\lambda'_d,\lambda_a\lambda_b} \hat{D}^{\lambda_{h_1},\lambda'_{h_1}}_{\lambda_c,\lambda'_c}(z_1,\mathbf{p}_{\perp 1}) \hat{D}^{\lambda_{h_2},\lambda'_{h_2}}_{\lambda_d,\lambda'_d}(z_2,\mathbf{p}_{\perp 2})$$

Scaling variables

- Light cone z
- Momentum fraction $z_p = 2|\mathbf{P}_h|/\sqrt{s}$
- Energy fraction $z_h = 2E_h/\sqrt{s}$

$$z_{h,p} \simeq z\left[1 \pm m_h^2/(z^2 s)\right]$$

Results consistent with [D. Boer, R. Jakob, and P.J. Mulders. Nucl. Phys. B 504 (1997)]



$e^+e^- \to h_1 h_2 X$: Helicity Formalism

$$\overbrace{\rho^{h_1}_{\lambda_{h_1},\lambda'_{h_1}}\,\rho^{h_2}_{\lambda_{h_2},\lambda'_{h_2}}}^{\text{Helicity matrices}}\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta dz_1 d^2\mathbf{p}_{\perp 1} dz_2 d^2\mathbf{p}_{\perp 2}}$$

$$=\sum_{q_c}\sum_{\{\lambda\}}\frac{1}{32\pi s}\frac{1}{4}\underbrace{\hat{M}_{\lambda_c\lambda_d,\lambda_a\lambda_b}\hat{M}^*_{\lambda'_c\lambda'_d,\lambda_a\lambda_b}}_{\text{Scattering Amplitudes}}\underbrace{\hat{D}^{\lambda_{h_1},\lambda'_{h_1}}_{\lambda_c,\lambda'_c}(z_1,\mathbf{p}_{\perp 1})\hat{D}^{\lambda_{h_2},\lambda'_{h_2}}_{\lambda_d,\lambda'_d}(z_2,\mathbf{p}_{\perp 2})}_{\text{TMD Fragmentation Functions}}$$

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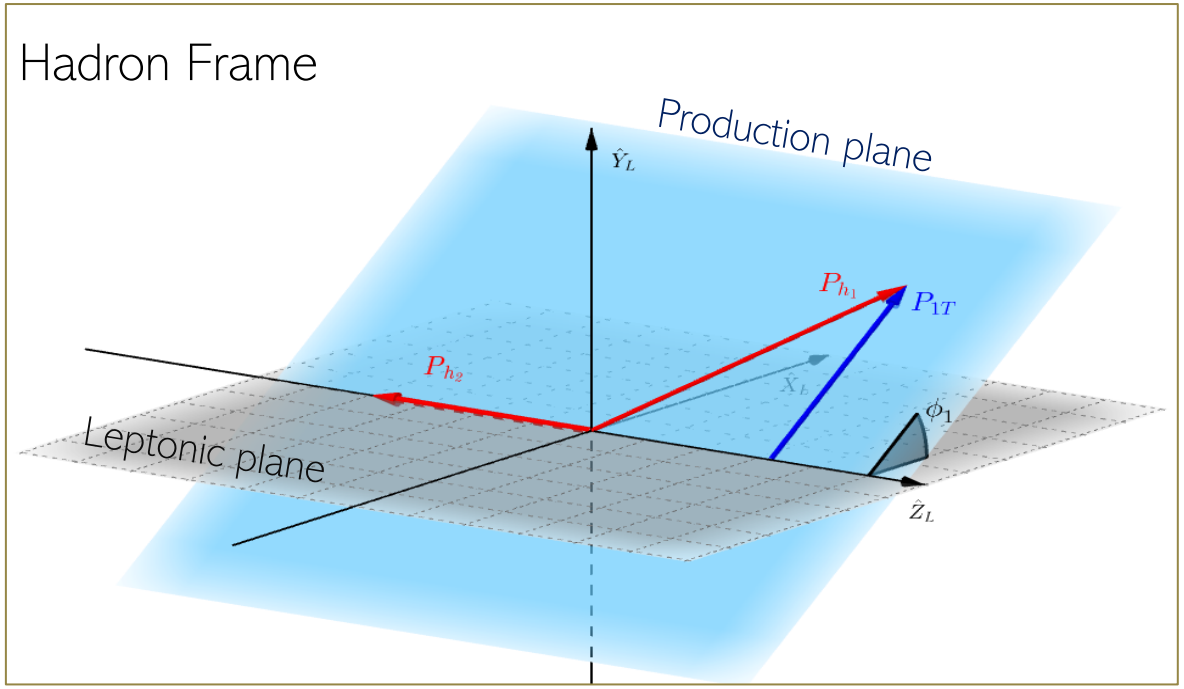

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$$=\sum_{q_c}\sum_{\{\lambda\}}\frac{1}{32\pi s}\frac{1}{4}\underbrace{\hat{M}_{\lambda_c\lambda_d,\lambda_a\lambda_b}\hat{M}^*_{\lambda'_c\lambda'_d,\lambda_a\lambda_b}}_{\text{Scattering Amplitudes}}\underbrace{\hat{D}^{\lambda_{h_1},\lambda'_{h_1}}_{\lambda_c,\lambda'_c}(z_1,\mathbf{p}_{\perp 1})\hat{D}^{\lambda_{h_2},\lambda'_{h_2}}_{\lambda_d,\lambda'_d}(z_2,\mathbf{p}_{\perp 2})}_{\text{TMD Fragmentation Functions}}$$

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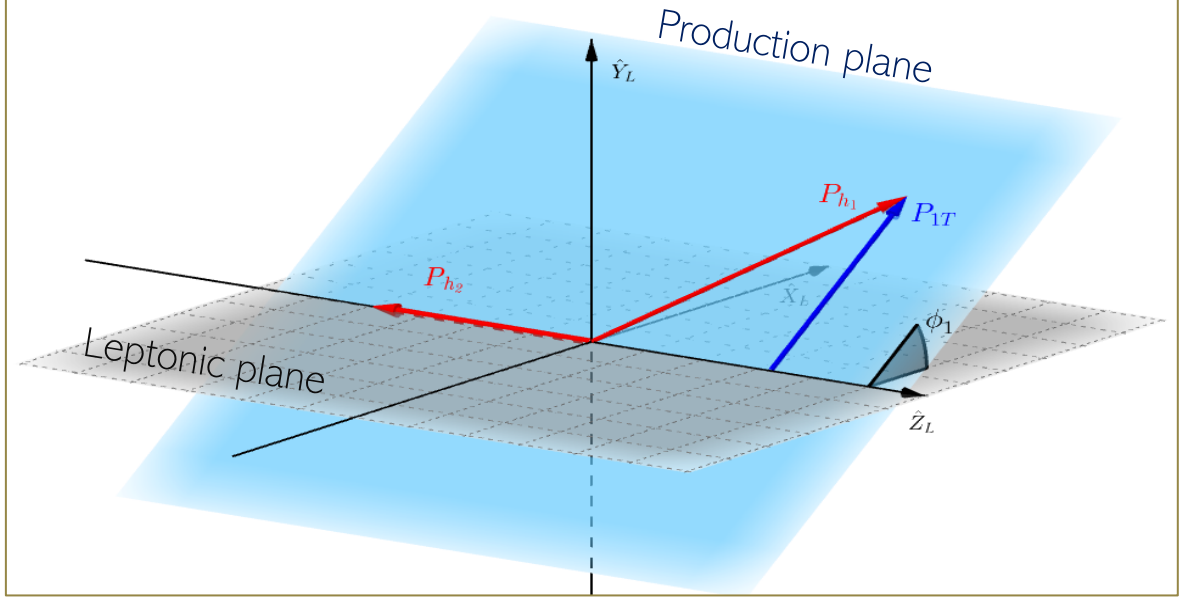
$$z_{h,p} \simeq z\left[1 \pm m_h^2/(z^2 s)\right]$$

Polarization vector

$$\mathcal{P}^{h_1} = P_x^{h_1}\hat{X}_1 + P_y^{h_1}\hat{Y}_1 + P_z^{h_1}\hat{Z}_1$$

Hadron Frame



$e^+e^- \to h_1 h_2 X$: Helicity Formalism

$$\overbrace{\rho^{h_1}_{\lambda_{h_1},\lambda'_{h_1}}\,\rho^{h_2}_{\lambda_{h_2},\lambda'_{h_2}}}^{\text{Helicity matrices}}\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta dz_1 d^2\mathbf{p}_{\perp 1} dz_2 d^2\mathbf{p}_{\perp 2}}$$

$$=\sum_{q_c}\sum_{\{\lambda\}}\frac{1}{32\pi s}\frac{1}{4}\underbrace{\hat{M}_{\lambda_c\lambda_d,\lambda_a\lambda_b}\hat{M}^*_{\lambda'_c\lambda'_d,\lambda_a\lambda_b}}_{\text{Scattering Amplitudes}}\underbrace{\hat{D}^{\lambda_{h_1},\lambda'_{h_1}}_{\lambda_c,\lambda'_c}(z_1,\mathbf{p}_{\perp 1})\hat{D}^{\lambda_{h_2},\lambda'_{h_2}}_{\lambda_d,\lambda'_d}(z_2,\mathbf{p}_{\perp 2})}_{\text{TMD Fragmentation Functions}}$$

Results consistent with [D. Boer, R. Jakob, and P.J. Mulders. Nucl. Phys. B 504 (1997)]

Scaling variables

- Light cone z
- Momentum fraction $z_p = 2|\mathbf{P}_h|/\sqrt{s}$
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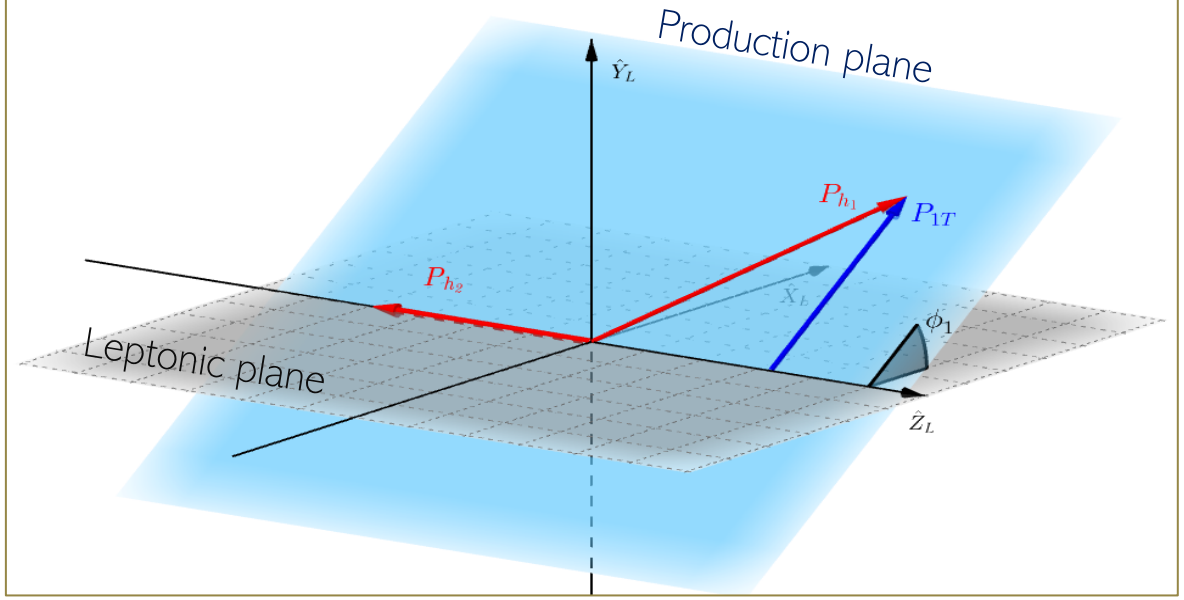
Polarization vector

$$\mathcal{P}^{h_1} = P_x^{h_1}\hat{X}_1 + P_y^{h_1}\hat{Y}_1 + P_z^{h_1}\hat{Z}_1$$

The polarization is measured along:

$$\hat{n} = -\hat{P}_{h_2} \times \hat{P}_{h_1}$$

Hadron Frame



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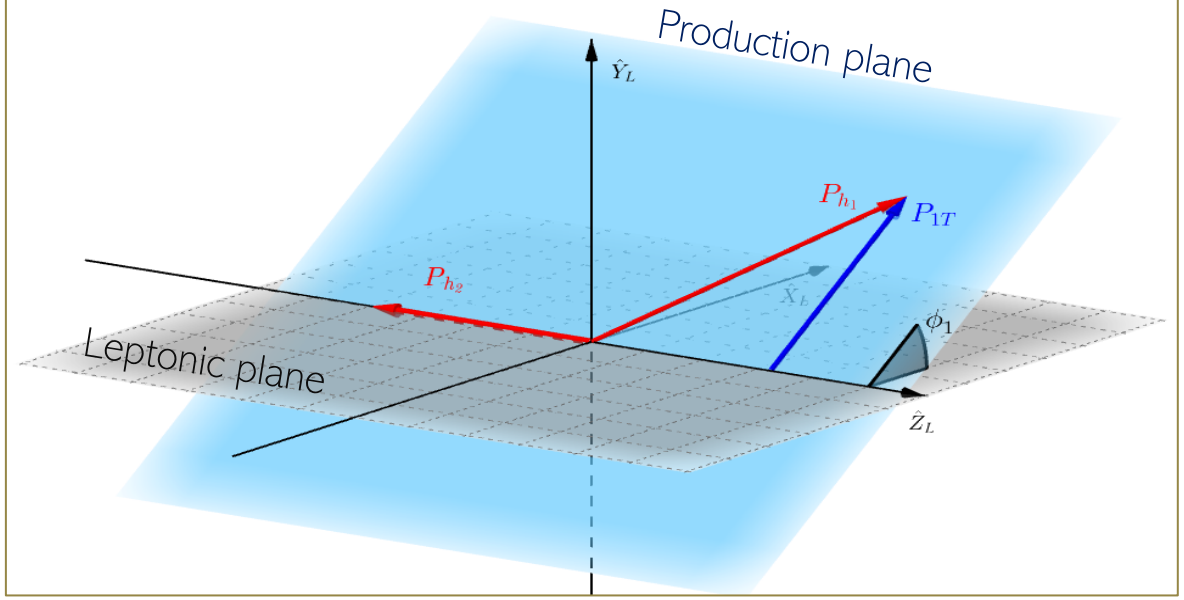
Hadron Frame



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The polarization is measured along:

$$\hat{n} = -\hat{P}_{h_2} \times \hat{P}_{h_1}$$

The polarization projection along $\hat{n}$:

$$\mathcal{P}^{h_1}\cdot\hat{n} = P_x^{h_1}\cos\tilde{\phi} + P_y^{h_1}\sin\tilde{\phi}$$

$e^+e^- \to h^{\uparrow}{}_1 h_2 X$: Hadron Frame

Introduce $p_\perp$ - parameterization for FFs:

$$\Delta D^h_{S_Y/q}(z, p_\perp) = \Delta D^h_{S_Y/q}(z)\sqrt{2e}\frac{p_\perp}{M_{pol}}\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle_{pol}}}{\pi\langle p_\perp^2\rangle_h}$$

$$D_{h/q}(z, p_\perp) = D_{h/q}(z)\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle_h}}{\pi\langle p_\perp^2\rangle_h}$$

- Fixed energy scale $\sqrt{s} = 10.58$ GeV
- NO Evolution
- Data depend only on energy fraction z_Λ - $z_{\pi,K}$
- $\langle p_\perp^2\rangle_h = 0.2\ GeV^2$ width of unp. FF

$e^+e^- \to h^{\uparrow}{}_1 h_2 X$: Hadron Frame

Introduce $p_\perp$ - parameterization for FFs:

$$\Delta D^h_{S_Y/q}(z, p_\perp) = \underbrace{\Delta D^h_{S_Y/q}(z)}_{\text{z dependence}} \sqrt{2e} \frac{p_\perp}{M_{pol}} \frac{e^{-p_\perp^2/\langle p_\perp^2 \rangle_{pol}}}{\pi \langle p_\perp^2 \rangle_h}$$

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$e^+e^- \to h^{\uparrow}{}_1 h_2 X$: Hadron Frame

Introduce $p_\perp$ - parameterization for FFs:

Linear term

$$\Delta D^h_{S_Y/q}(z, p_\perp) = \underbrace{\Delta D^h_{S_Y/q}(z)}_{\text{z dependence}} \sqrt{2e} \frac{p_\perp}{M_{pol}} \underbrace{\frac{e^{-p_\perp^2/\langle p_\perp^2 \rangle_{pol}}}{\pi \langle p_\perp^2 \rangle_h}}_{\text{Gaussian dependence on } p_\perp}$$

$$D_{h/q}(z, p_\perp) = D_{h/q}(z) \frac{e^{-p_\perp^2/\langle p_\perp^2 \rangle_h}}{\pi \langle p_\perp^2 \rangle_h}$$

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$$d^2\mathbf{p}_{\perp 1} \to d^2\mathbf{P}_{T1}$$

$$\int d^2\mathbf{P}_{1T} d^2\mathbf{p}_{\perp 2}$$

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$$d^2\mathbf{p}_{\perp 1} \to d^2\mathbf{P}_{T1}$$

$$\int d^2\mathbf{P}_{1T} d^2\mathbf{p}_{\perp 2} \longrightarrow$$

$$\mathcal{P}_n(z_1, z_2) = \sqrt{\frac{e\pi}{2}} \frac{1}{M_{\rm pol}} \frac{\langle p_{\perp 1}^2 \rangle_{\rm pol}^2}{\langle p_{\perp 1}^2 \rangle} \frac{z_2}{\left\{ [z_1(1 - m_{h_1}^2/(z_1^2 s))]^2 \langle p_{\perp 2}^2 \rangle + z_2^2 \langle p_\perp^2 \rangle_{\rm pol} \right\}^{1/2}} \times \frac{\sum_q e_q^2\, \Delta D_{h_1^\uparrow/q}(z_1) D_{h_2/\bar{q}}(z_2)}{\sum_q e_q^2\, D_{h_1/q}(z_1) D_{h_2/\bar{q}}(z_2)}$$

$e^+e^- \to h^{\uparrow}{}_1 h_2 X$: Hadron Frame

Introduce $p_\perp$ - parameterization for FFs:

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$$\Delta D^h_{S_Y/q}(z, p_\perp) = \underbrace{\Delta D^h_{S_Y/q}(z)}_{\text{z dependence}} \sqrt{2e}\, \frac{p_\perp}{M_{pol}} \underbrace{\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle_{pol}}}{\pi \langle p_\perp^2 \rangle_h}}_{\text{Gaussian dependence on } p_\perp}$$

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Consistent with [D. Callos, Z.B. Kang, and J. Terry, Phys. Rev. D 102, 096007 (2020)]

Fit and Results (1)

Belle data: $\sqrt{s} = 10.58$ GeV

- 128 points $\Lambda + h$, in bins of the energy fractions z_Λ - $z_{\pi,K}$
- 32 points $\Lambda(jet)$, in bins of z_Λ - $p_\perp$

Polarizing FF parametrization:

$$\Delta D^h_{S_Y/q}(z) = \mathcal{N}^p_q(z) \overbrace{D_{h/q}(z)}^{\text{Unpolarized FF}}$$

$$\mathcal{N}^p_q(z) = \mathcal{N}^p_q z^{\alpha_q}(1-z)^{\beta_q} \frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\beta_q^{\beta_q}}$$

- Normalization factor: $\mathcal{N}^p_q$, $|\mathcal{N}^p_q| \leq 1$
- Shape for high and low z: α_q β_q

Unpolarized FF set adopted:

- DSS07 for π, K
- AKK08 for $\Lambda + \bar{\Lambda}$

$q\bar{q}$ FF separation for AKK08:

$$D_{\Lambda/\bar{q}}(z_p) = (1-z_p) D_{\Lambda/q}(z_p)$$

Fit and Results (2)

Data selection:

- $\Lambda + \pi/K$: $z_{\pi.K} = [0.5 \ - 0.9]$ bin excluded → 96 data points
- $\Lambda(jet)$: $z_{\Lambda} = [0.5 \ - 0.9]$ bin excluded → 24 data points

Fit and Results (2)

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Fitted 8 parameters

Flav.	$\mathcal{N}_q^p$	α_q	β_q	$\langle p_\perp^2 \rangle_{pol}$
u	$\mathcal{N}_u^p$		β_u	
d	$\mathcal{N}_d^p$			$\langle p_\perp^2 \rangle_{pol}$
s	$\mathcal{N}_s^p$	α_s		
sea	$\mathcal{N}_{sea}^p$		β_{sea}	

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Fitted 8 parameters

Fitted Parameters	Value
Nu	$0.47^{+0.32}_{-0.20}$
Nd	$-0.32^{+0.13}_{-0.13}$
Ns	$-0.57^{+0.29}_{-0.43}$
Nsea	$-0.27^{+0.12}_{-0.20}$
α_s	$2.30^{+1.08}_{-0.91}$
β_{sea}	$2.60^{+2.60}_{-1.74}$
β_u	$3.50^{+2.33}_{-1.82}$
$< p_{\perp}^2 >_{pol}$	$0.10^{+0.02}_{-0.02}$

Full data Fit : $\chi^2_{dof} = 1.94$

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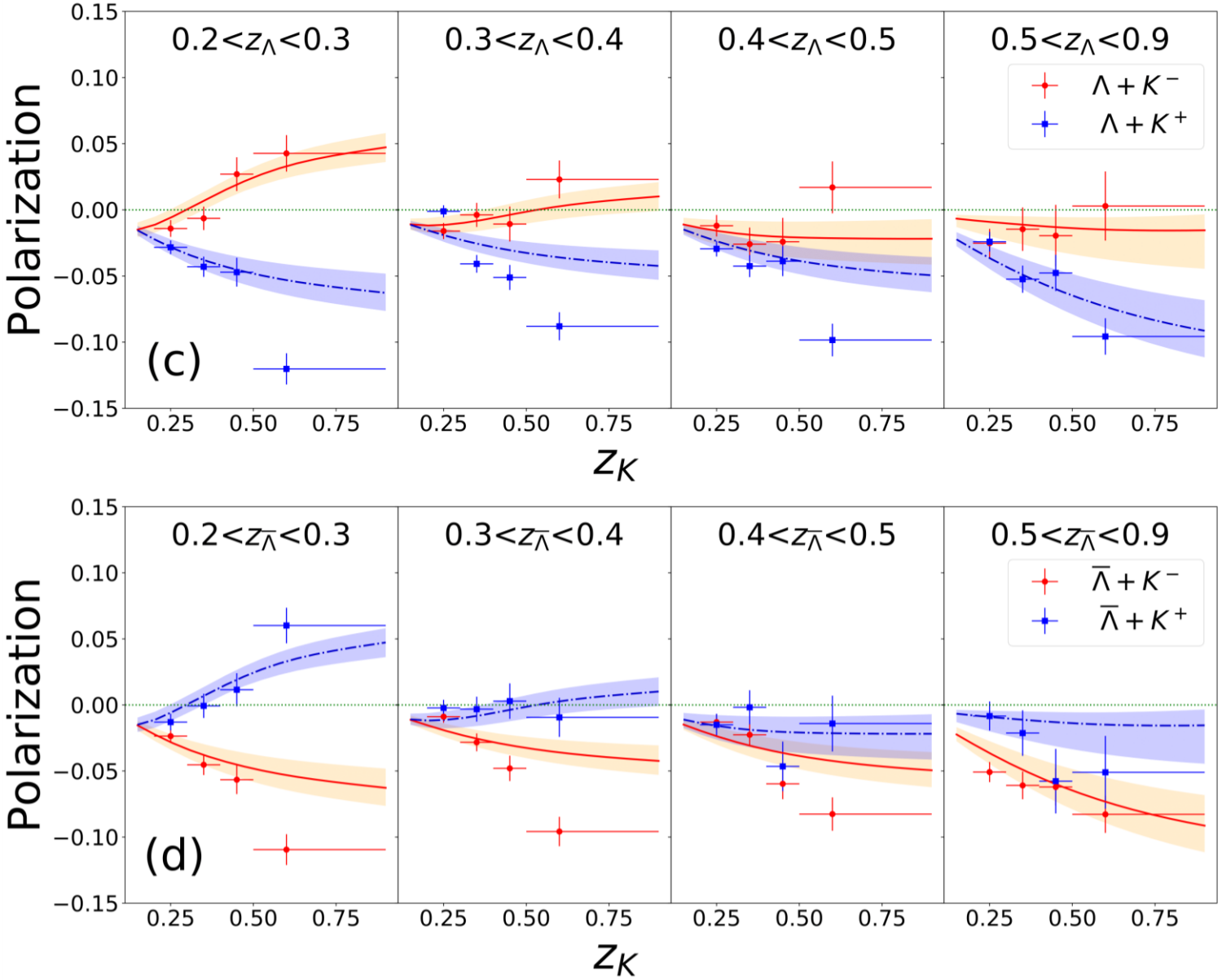
β_u	3.50 $^{+2.33}_{-1.82}$
$< p_{\perp}^2 >_{pol}$	0.10 $^{+0.02}_{-0.02}$

Full data Fit : $\chi^2_{dof} = 1.94$

$\Lambda + \pi/K$ data Fit : $\chi^2_{dof} = 1.26$

The two extractions are consistent

Lambda-kaon

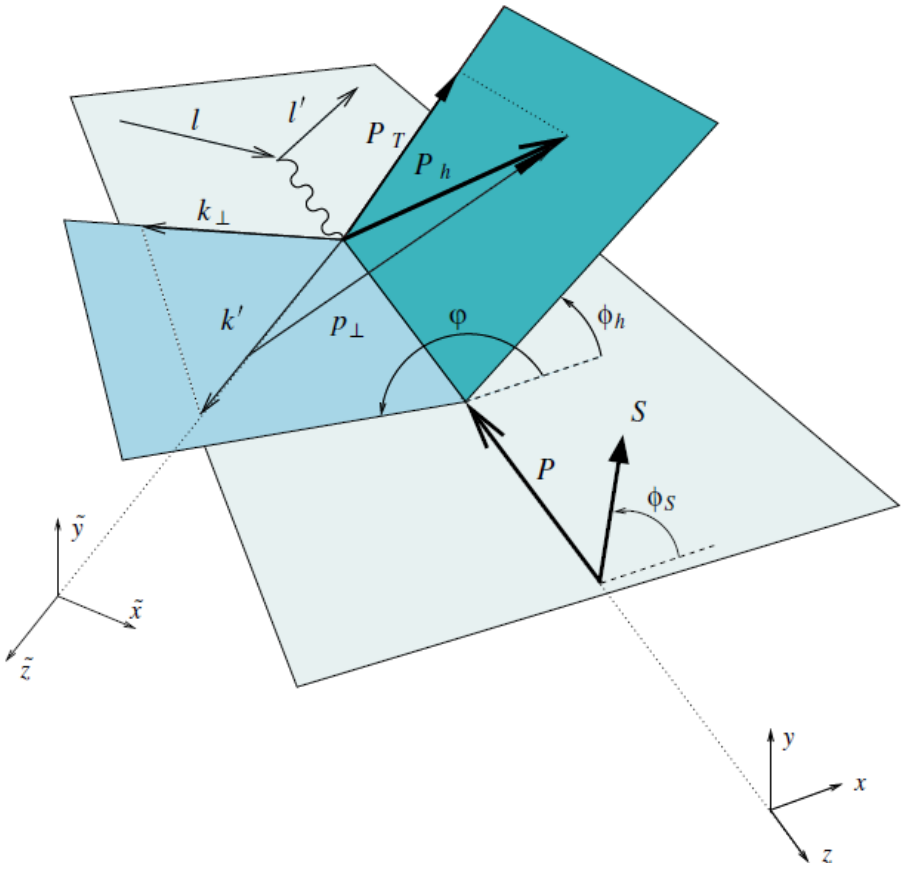


Bin excluded
$z_K = [0.5 - 0.9]$

$\chi^2_{dof} = 1.94$

SiDIS – Polarized Lambda Production

$e^- P \rightarrow e^- \Lambda$



$$P_T(x_B, z_h) = \frac{\sqrt{2e\pi}}{2M_p} \frac{< p_\perp^2 >_p^2}{< p_\perp^2 >} \frac{1}{\sqrt{< p_\perp^2 >_p + \xi_p^2 < k_\perp^2 >}} \times \frac{\sum_q f_{q/P}(x_B) \Delta D_{h^\uparrow/q}(z_h)}{\sum_q f_{q/P}(x_B) D_{h/q}(z_h)}$$

Proton PDF

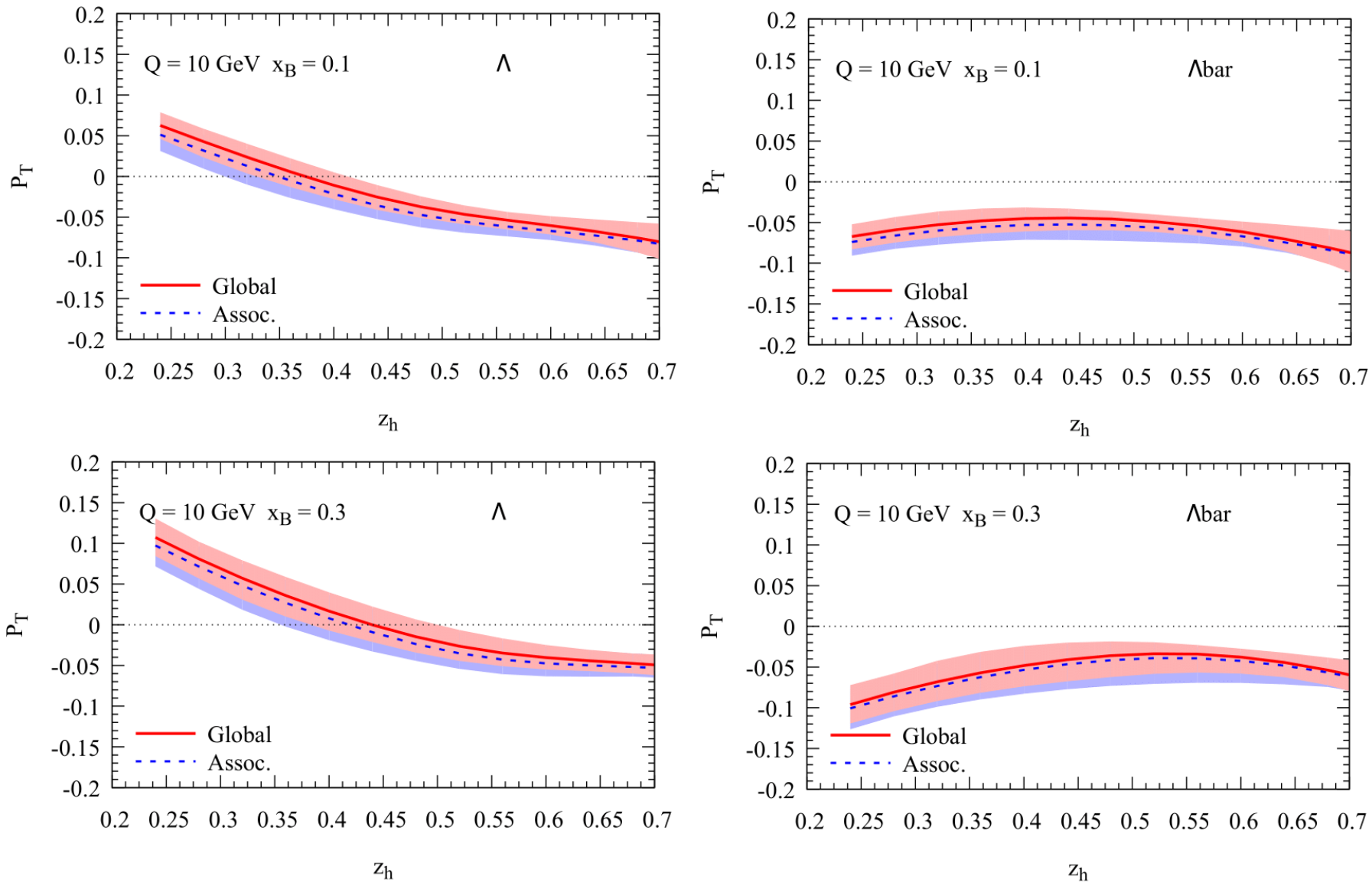
CTEQ6L1

Lambda Polarizing FF

x_B Bjorken-x
z_h energy fraction

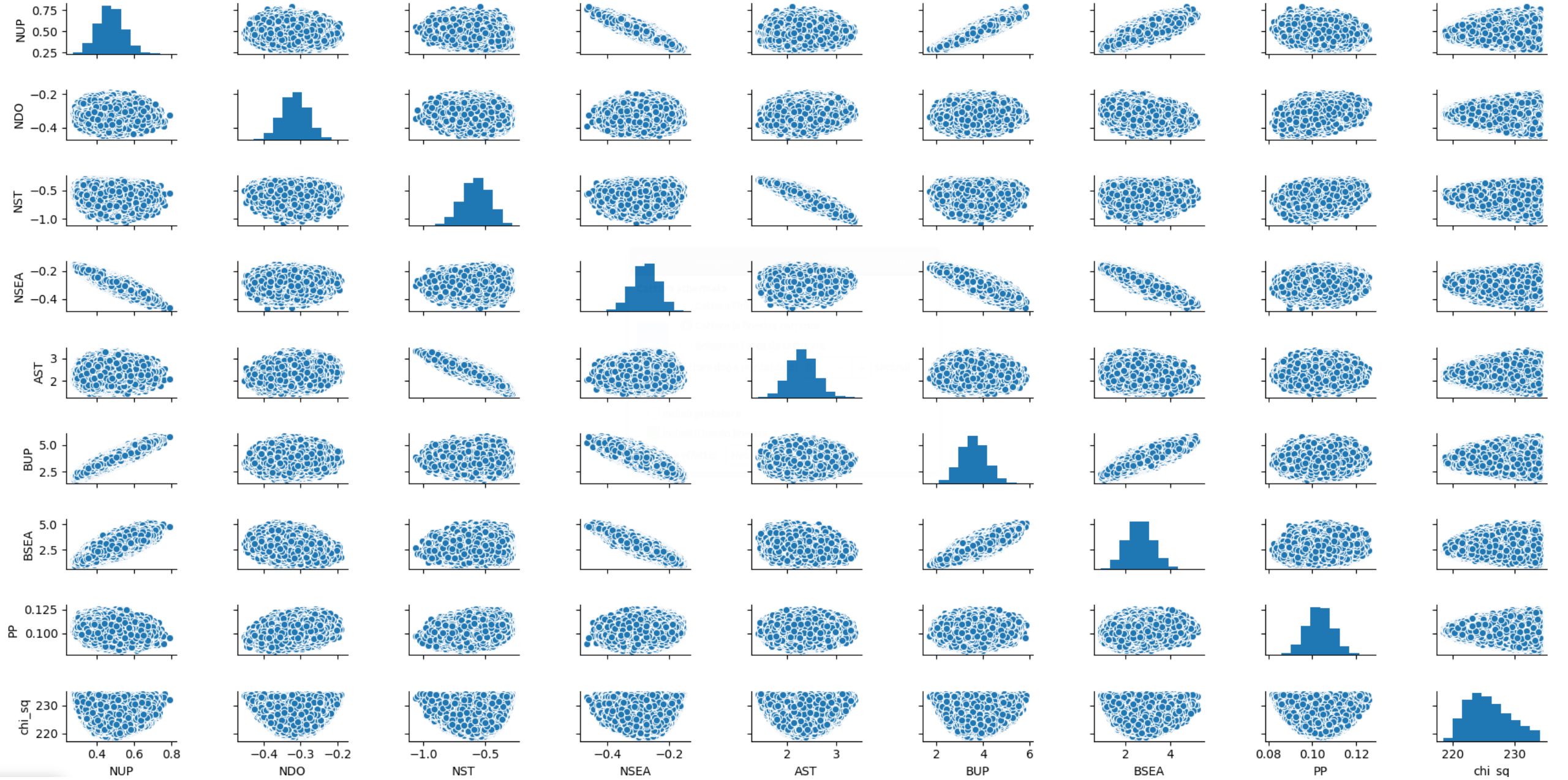
$$\xi_p = z_h \left(1 - \frac{m_h^2}{z_h^2 Q^2} \frac{x_B}{1 - x_B} \right)$$

SiDIS – Polarized Lambda Production



Prediction for the Λ polarization:

- x_B = 0.1
- x_B = 0.3


NUP
NDO
NST
NSEA
AST
BUP
BSEA
PP
chi_sq
0.75
0.50
0.25
−0.2
−0.4
−0.5
−1.0
−0.2
−0.4
3
2
5.0
2.5
5.0
2.5
0.125
0.100
230
220
0.4
0.6
0.8
−0.4
−0.3
−0.2
−1.0
−0.5
−0.4
−0.2
2
3
2
4
6
2
4
0.08
0.10
0.12
220
230

Statistical Uncertainty Band

Multivariate Normal Distribution

MINUIT:

- Best fit parameters μ : $\mathcal{N}_q^p \, \alpha_q \, \beta_q \, \langle p_\perp^2 \rangle_p$
- Covariance matrix Σ
- Minimum Chi-square χ^2

$\xrightarrow{\mu \ , \ \Sigma}$

$$p(x;\mu,\Sigma) = \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right)$$

Generate a random set of parameter

x: ($\mathcal{N}_q^p \ \ \alpha_q \ \ \beta_q \ \ \langle p_\perp^2 \rangle_p$)

$\longrightarrow$

Calculate their own Chi-square

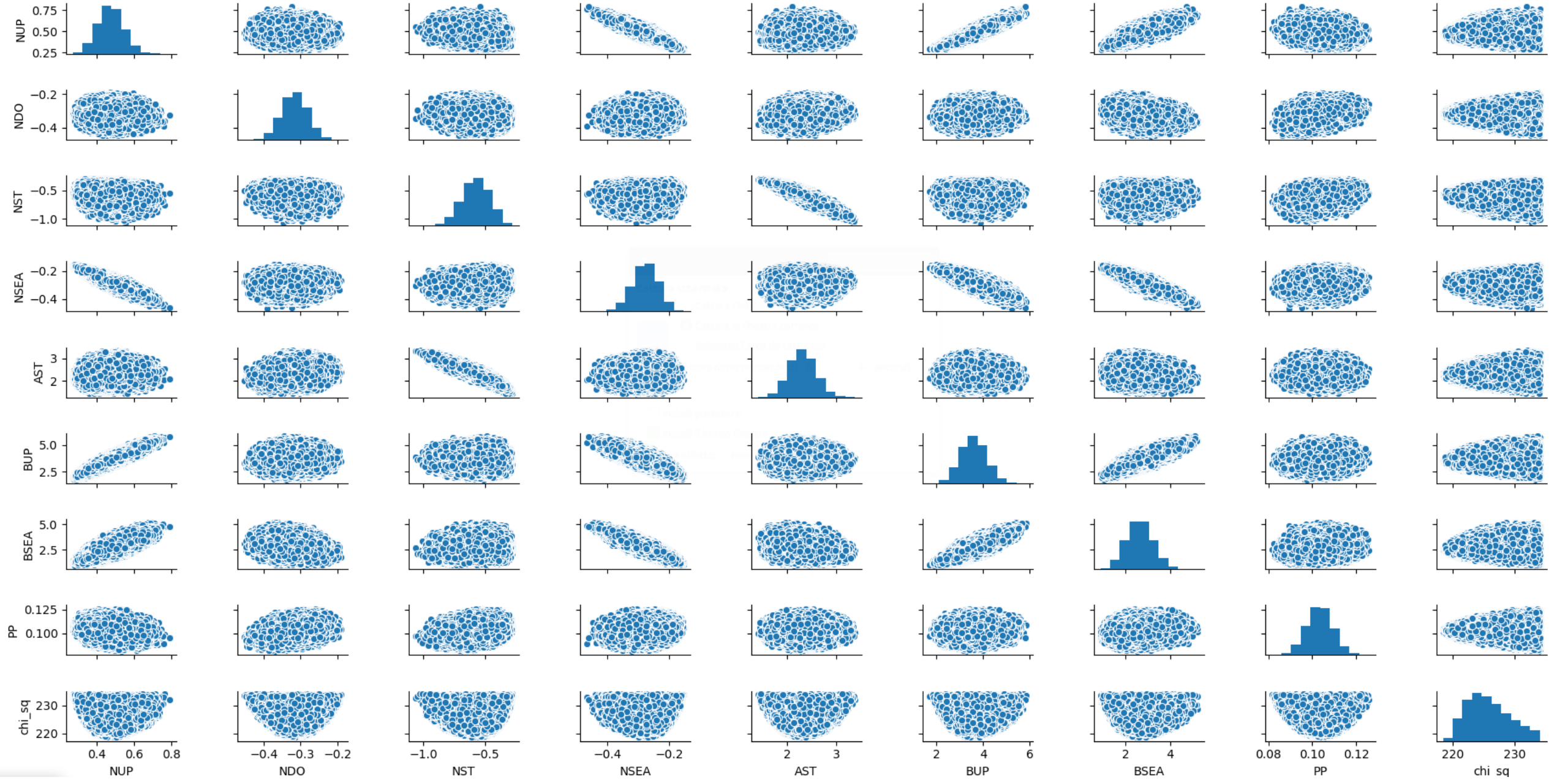
χ'^2

$\longrightarrow$

Keep set if

$$\chi^2 \leq \chi'^2 \leq \chi^2 + \Delta\chi^2$$

- Minimum Chi-square: χ^2 = 232,8
- Confidence interval $2\sigma \rightarrow$ 95,5% : $\Delta\chi^2$ = 15,79 for 8 parameters (χ^2 -distribution)


NUP
NDO
NST
NSEA
AST
BUP
BSEA
PP
chi_sq
0.75
0.50
0.25
−0.2
−0.4
−0.5
−1.0
−0.2
−0.4
3
2
5.0
2.5
5.0
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0.6
0.8
−0.4
−0.3
−0.2
−1.0
−0.5
−0.4
−0.2
2
3
2
4
6
2
4
0.08
0.10
0.12
220
230

$e^+e^- \to h^{\uparrow}{}_1 h_2 X$

h_1, h_2 unpolarized

$$\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta dz_1 d^2p_{\perp 1} dz_2 d^2p_{\perp 2}}$$

$$= \frac{6e^4 e_q^2}{64\pi\hat{s}} \Big\{ D_{h_1/q}(z_1, p_{\perp 1}) D_{h_2/\bar{q}}(z_2, p_{\perp 2}) \Big(1 + \cos^2\theta\Big)$$

Unpolarized FFs

Collins FFs

$$+ \frac{1}{4}\sin^2\theta \Delta^N D_{h_1/q^\uparrow}(z_1, p_{\perp 1}) \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2, p_{\perp 2}) \cos(2\varphi_2 + \phi_1^{h_1}) \Big\}$$

h_1 polarized: X , h_2 unpolarized

$$P_X^{h_1} \frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta dz_1 d^2p_{\perp 1} dz_2 d^2p_{\perp 2}}$$

$$= \frac{3e^4 e_q^2}{64\pi\hat{s}} \Delta D^{h_1}_{S_X/s_T}(z_1, p_{\perp 1}) \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2, p_{\perp 2}) \sin^2\theta \sin(2\varphi_2 + \phi_1^{h_1})$$

FF h_1 transv. Pol.

Collins FF

h_1 polarized: Y , h_2 unpolarized

$$P_Y^{h_1} \frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta dz_1 d^2p_{\perp 1} dz_2 d^2p_{\perp 2}}$$

Polarizing FF

Unpolarized FF

$$= \frac{6e^4 e_q^2}{64\pi\hat{s}} \Big\{ \Delta D^{h_1}_{S_Y/q}(z_1, p_{\perp 1}) D_{h_2/\bar{q}}(z_2, p_{\perp 2}) \Big(1 + \cos^2\theta\Big)$$

$$+ \frac{1}{2}\sin^2\theta \Delta^- D^{h_1}_{S_Y/s_T}(z_1, p_{\perp 1}) \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2, p_{\perp 2}) \cos(2\varphi_2 + \phi_1^{h_1}) \Big\}$$

Collins FF

FF h_1 transv. Pol.

h_1 polarized: Z , h_2 unpolarized

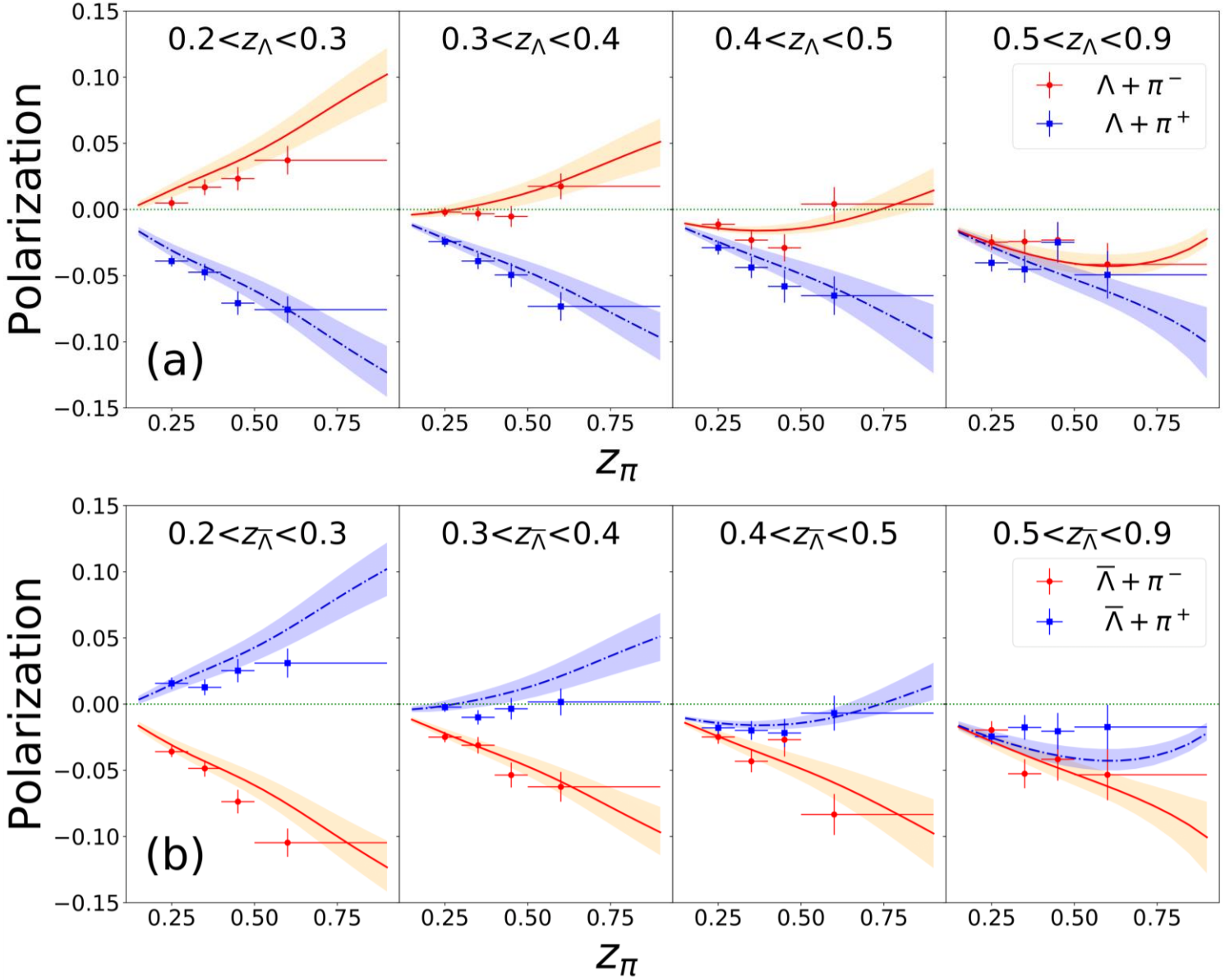
$$P_Z^{h_1} \frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{d\cos\theta dz_1 d^2p_{\perp 1} dz_2 d^2p_{\perp 2}}$$

$$= \frac{3e^4 e_q^2}{64\pi\hat{s}} \Delta D^{h_1}_{S_Z/s_T}(z_1, p_{\perp 1}) \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2, p_{\perp 2}) \sin^2\theta \sin(2\varphi_2 + \phi_1^{h_1})$$

FF h_1 long. Pol.

Collins FF

Lambda-pion



Data fitted:

- $z_\pi < 0.50$

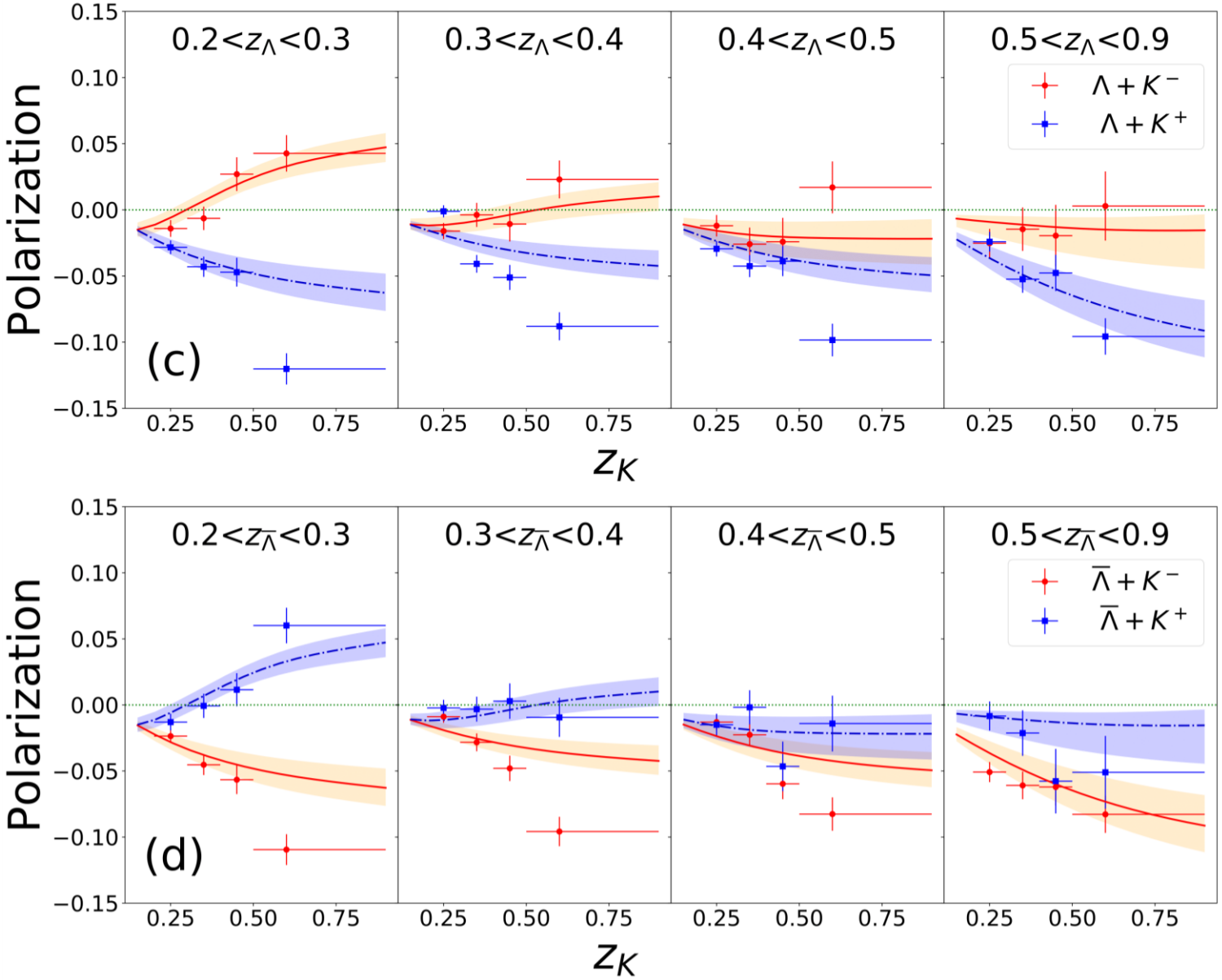
$\chi^2_{dof} = 1.94$

Lambda pion

- Charge-conjugation symmetry $P_n(\Lambda\pi^+) = P_n(\bar{\Lambda}\pi^-)$
- Data give information on the pFFs for u and d
- $P_n(\Lambda\pi^+)$ negative, dominated by down pFF
- $P_n(\Lambda\pi^-)$ positive, dominated by up pFF
- $P_n(\Lambda\pi^-)$ strong reduction due to the large suppression of the up pFF

- Small z_a sea pFFs become important and negative,
- $P_n(\Lambda\pi^+)$ up and down cancel each other, sea pFF leads to large, and negative, values of the transverse polarization
- $P_n(\Lambda\pi^-)$ sea pFF partial reduction of the up pFF

Lambda-kaon



Data fitted:

- $z_K < 0.50$

$\chi^2_{dof} = 1.94$

Lambda-kaon

- Charge-conjugation symmetry $P_n(\Lambda K^+) = P_n(\bar{\Lambda} K^-)$
- Similar pattern pion
- Data give information on the pFFs for u and s
- $P_n(\Lambda K^+)$ negative, dominated by strange pFF
- $P_n(\Lambda K^-)$ positive, dominated by up pFF
- $P_n(\Lambda K^-)$ strong reduction due to the large suppression of the up pFF

- Small z_a sea pFFs become important and negative,